AMERICAN HANDBO

FOR

LECTRICAL ENGIN

A REFERENCE BOOK F[illegible]R [illegible]

ENGINEERS AND [illegible]

OF ENGINEERING,

COMPILED BY A STAFF OF [illegible]

HAROLD PENDER, E[illegible]

[illegible] OF ELECTRICAL ENGINEERING [illegible]

DIRECTOR [illegible] ELECTRICAL ENGINEERING [illegible]

MASSACHUSETTS INSTITUTE OF TECHNOLOGY [illegible]

FIRST EDITION

FIVE THOUSAND

NEW YORK

JOHN WILEY & SONS, I

LONDON: CHAPMAN & HALL, L

AMERICAN HANDBOOK
FOR
LECTRICAL ENGINEERS

A REFERENCE BOOK FOR PRACTICING
ENGINEERS AND STUDENTS
OF ENGINEERING

COMPILED BY A STAFF OF SPECIALISTS

HAROLD PENDER, EDITOR-IN-CHIEF

OFESSOR OF ELECTRICAL ENGINEERING, UNIVERSITY OF PENNSYLVANIA ; FORMERLY
DIRECTOR OF ELECTRICAL ENGINEERING RESEARCH LABORATORY,
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

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JOHN WILEY & SONS, INC.
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1914

'he Publishers and the Editor-in-Chief will be grateful to readers
; volume who will kindly call attention to any errors of omission
imission therein. It is intended to make our publications stand
:ks of study and reference, and, to that end, the greatest accuracy
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ors; but it is the endeavor of the Publishers to have them remove
l it is therefore desired that the Editor-in-Chief may be aided in hi
k of revision, from time to time, by the kindly criticism of readers.

JOHN WILEY & SONS, INC.
432 Fourth Avenue, New York.

First Edition printed in
September, 1914

Composition and Electrotyping by the STANHOPE PRESS, Boston, Mass.
Printing and Binding by BRAUNWORTH & COMPANY, Brooklyn, N.Y.

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PREFACE

PREFACE

In the summer of 1910 the writer was invited by the publishers of Kent's *Mechanical Engineers' Pocket-Book* and of the *American Civil Engineers' Pocket-Book* (Mansfield Merriman, Editor-in-Chief), to act as Editor-in-Chief of a Handbook of Electrical Engineering. Although there were at that time two electrical engineering handbooks in general use in this country, it seemed that a new handbook of electrical engineering, treating in a somewhat different manner the various subjects which naturally come within the scope of such a reference book, would fill a want felt by both practicing engineers and students of engineering.

The Handbook here offered to the engineering profession embodies the following features:

1. The Handbook has been prepared primarily for the practicing engineer. With this end in view the matter has been so arranged as to be most readily found, and all *theoretical discussions have been segregated into separate articles.* It is fully realized that the practicing engineer sometimes is desirous of finding a clear and concise statement of fundamental principles and theory, but when he wants specific data or practical information he does not care to search through a maze of theoretical discussions. Consequently, in this book fundamental or "theoretical" principles are fully but concisely treated in articles dealing with such matters and nothing else, e.g., such articles as *Electricity and Magnetism, Principles of; Alternating Currents; Electrochemistry, Principles of; Mechanics, Principles of; Hydraulics; etc.* Therefore, in articles dealing with practical matters only enough is said of theory to indicate the general principle of which the matter in hand may be a specific application.

2. The Handbook is primarily for Electrical Engineers, but the general arrangement of the subject matter and the method of treatment adopted will make the book a useful reference book for mechanical, civil, mining and other engineers who have occasion to utilize any of the numerous applications of electricity in their special fields. Considerable space has been devoted to those matters pertaining to the applications of motors in all branches of modern industry; see the article on *Motors, Industrial Applications of,* and the numerous cross references there given.

3. Although the Handbook deals primarily with electrotechnical matters, a large amount of space has been devoted to those mechanical and civil engineering subjects which are closely related to electrical engineering practice. To an electrical engineer who is fortunate enough to possess a good mechanical engineer's handbook and a good civil engineer's handbook, this may seem a duplication of effort. However, young engineers at the beginning of their professional careers usually seem satisfied with a handbook covering only their particular "brand" of engineering. To such electrical engineers a concise treatment of the elements of mechanical and civil engineering should prove particularly helpful. Moreover, the convenience of having under one cover the essential mechanical and civil engineering data will prove a convenience even to those electrical engineers who may possess other reference books dealing with these matters in greater detail.

4. Numerous mathematical tables and relations are given in the Handbook. Among these may be mentioned the articles on *Logarithms, Trigonometric Functions, Hyperbolic Functions, Exponential Functions, Derivatives, Integrals, Indeterminate Forms, Equations,* etc. The table of trigonometric functions is

so arranged that the three functions, sine, cosine and tangent, of each tenth of a degree are given at one place on the page; this arrangement will be found particularly useful in alternating-current calculations, since usually at least two of these functions are needed simultaneously. The table of hyperbolic functions, together with a statement of the meaning of such functions and the various formulas used in their application, should prove valuable to those having to make calculations of long-distance power-transmission lines. The table of exponential functions will be found very useful in making calculations of transient electrical phenomena and also for other calculations in which ϵ^x or ϵ^{-x} occur.

5. In spite of the fact that this book contains upwards of 2000 pages, many of the articles have been greatly condensed. However, it has been the consistent endeavor of the editorial staff to make each article sufficiently complete so that the information given therein should be of the greatest practical value. A handbook is not a treatise, and one should therefore not expect to find such detail as is given in an ordinary textbook; yet, on the other hand, a reference book of this kind should contain enough explanation and examples to make clear the application of the data given. This idea has been kept in mind throughout.

6. Although this book has been prepared primarily for the practicing engineer, it is believed that the method of treatment adopted will render many of the articles suitable as the bases for courses of lectures in technical schools. A teacher usually has his own method of presenting a subject, and sometimes a textbook proves more of a handicap than a help; yet every teacher recognizes the desirability of putting in the hands of his students some sort of synopsis or syllabus which will serve as a general guide.

7. The encyclopedic arrangement of this book is a departure in handbook construction, but now that the book is completed the writer is more firmly convinced than ever that it is the rational arrangement. A handbook is a reference book. Therefore the object of any arrangement is to render the data given in the book readily accessible. Experience has shown that the alphabetical arrangement is eminently fitted for an encyclopedia of general information; a like arrangement is equally applicable to a reference book dealing only with engineering topics. The subject matter of this book has therefore been disposed of in articles. At the beginning of each article is given a list of references to related articles. Besides, there is a detailed index covering some 54 pages at the end of the book. The user of the book will undoubtedly find it more convenient at first to refer directly to this detailed index, but as soon as he becomes acquainted with the main article headings, he will be able to turn directly to the article containing the information he may be seeking without referring to the index.

8. One feature of this Handbook which should prove particularly helpful is that the same plan of treatment has been consistently followed in all the articles, at least wherever possible. This applies particularly to articles dealing with apparatus or machinery, the plan of treatment being:

a. General Description and Definitions.
b. Brief Statement of Application.
c. Principle of Operation.
d. Design.
e. Testing.
f. Performance.
g. Specifications.
h. Installation.
i. Operation.

some instances it was found advisable to vary this scheme more or less, the general sequence of topics was adhered to as closely as possible. At beginning of each article occupying more than fifteen pages is given a brief le of contents.

. It is believed that the cost data given in the various articles will prove ticularly valuable when properly used. These data in most instances are en as unit costs, and usually the ordinary *range* of cost is given. It has been experience of the writer that students and recent technical graduates are lly lacking in even the roughest idea of the cost of apparatus and structures. is primarily to supply a rough idea of such costs that the cost curves and ures are given in the various articles. The cost data given will also be found table for preliminary estimates; but for close estimating, current prices uld of course be obtained from the makers of the apparatus, or, in the case construction work, some one having personal experience in similar work uld be consulted.

10. The bibliography given at the end of each article is intended to direct e reader to more extended information in treatises and current periodicals. e space available for these bibliographies has made it necessary to omit any important works and technical articles The references are usually ose with which the writer of the article in question is most familiar and which has found most useful. They are therefore in no sense complete, but will be und a very useful guide in the search for additional information. In many nstances a blank space of a half page or more is left after the bibliography; this space may be advantageously utilized by the insertion of references to new books and articles as they appear.

11. The articles in this Handbook have been prepared by an editorial staff of experts; this staff is given on pp. III and IV. The name of the writer of each article is given at the end of the article.

12. The preparation of the various articles has extended over a period of approximately three years. However, none of the plates were cast until the summer of 1914. Before casting, all the galleys were carefully revised in order to bring the articles up-to-date, and in some instances large sections of the articles were rewritten. In particular, all the articles dealing with electrical apparatus and machinery were revised in July, 1914, to bring them into agreement with the new Standardization Rules of the American Institute of Electrical Engineers (see p. 1295), presented to the Board of Directors of the Institute on July 10, 1914.

The Editor-in-Chief wishes to take this opportunity of expressing his sincere appreciation of the hearty coöperation of the entire editorial staff. Naturally, to make the treatment of the various articles in a work of this kind uniform throughout, much rearrangement of material and modification in its original presentation has been necessary. Each associate editor has been quick to realize this point, and has been unsparing in his efforts to bring about a concise and uniform treatment of the various subjects covered in the book. To Mr. H. F. Thomson, who has acted as Assistant Editor during the past two years, the Editor-in-Chief is particularly indebted for valuable assistance throughout the most arduous part of the editorial work.

The writer also wishes to express the appreciation of the associate editors for the valuable assistance given by their personal friends, and by various engineering firms and manufacturing companies, in the preparation of the various articles.

The plate proof of the entire book has been carefully read by Mr. N. S. Marston of the Massachusetts Institute of Technology. He has done this work with extreme thoroughness, checking every formula and table either by direct calculation, or by reference to its original source. It is hoped that all serious errors have thus been eliminated.

In conclusion, the writer wishes to express his appreciation of the spirit of coöperation and generosity shown by the publishers throughout the preparation of this book.

HAROLD PENDER.

EAST BLUEHILL, ME.,
September 5, 1914.

TOPICAL LIST OF ARTICLES

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ABBREVIATIONS AND SYMBOLS. — (*See also Units and Conversion Factors.*) At the meeting of the International Electrotechnical Commission in Berlin, in 1913, a list of 36 symbols for electrotechnical and related quantities was adopted. The American Institute of Electrical Engineers has also adopted (1914) a list of such symbols, which for the most part is in agreement with the list of the I.E.C., but contains certain additional symbols not included in the I.E.C. list. In the following table both lists are given. See paragraph 495 of the *Standardization Rules of the A.I.E.E.* for symbols for photometric quantities.

STANDARD SYMBOLS AND ABBREVIATIONS

Name of quantity	Symbol for the quantity		Name of Unit, adopted by A.I.E.E.	Abbreviation for the unit to be used only after numerical values	
	Adopted by I.E.C. (*See Note 1*)	Adopted by A.I.E.E.		Adopted by I.E.C.	Adopted by A.I.E.E.
Acceleration due to gravity	g	g	centimeter per second per second	—	cm. per sec. per sec.
Acceleration due to gravity, Standard (=980.665 cm. per sec. per sec.) (*Note 5*)	—	g_0	centimeter per second per second	—	cm. per sec. per sec.
Admittance	—	Y, y	mho	—	—
Angles	α, β, γ, etc.	—	—	—	—
Angular velocity ($2\pi f$)	ω	ω	radian per second	—	—
Capacity (Capacitance)	C	C	farad	F	—
Conductance	G	g	mho	—	—
Conductivity (*Note 6*)	—	γ	mho per centimeter	—	mho per cm.
Current	I	I, i	ampere	A	—
Dielectric constant	ϵ	ϵ or k	—	—	—
Dielectric field intensity or electric force	—	F	—	—	—
Dielectric flux	—	ψ	—	—	—
Dielectric flux density	D	D	—	—	—
Efficiency	η	η	per cent	—	(*Note 7*)
Electromotive force (e.m.f.)	E	E, e	volt	V	—
Energy	W (U)	U or W	joule or watt-hour	—	—
Frequency	f (ν)	f	cycle per second	—	~
Impedance	Z ($\mathfrak{Z}$)	Z, z	ohm	(*Note 2*)	—
Inductance (or coefficient of self-induction)	L ($\mathfrak{L}$)	L	henry	H	—

STANDARD SYMBOLS AND ABBREVIATIONS (*Continued*)

Name of quantity	Symbol for the quantity: Adopted by I.E.C. (*See Note 1*)	Symbol for the quantity: Adopted by A.I.E.E.	Name of unit adopted by A.I.E.E.	Abbreviations for unit to be used after numerical values: Adopted by I.E.C.	Abbreviations for unit to be used after numerical values: Adopted by A.I.E.E.
Inductance, mutual (or coefficient of mutual induction)	M ($\mathfrak{M}$)	M	henry	H	—
Length	l (*Note 3*)	l	centimeter	cm.	cm.
Magnetic flux (*Note 4*)	Φ ($\mathcal{F}$)	Φ, ϕ	maxwell	—	—
Magnetic flux density	B ($\mathcal{B}$)	$B, \mathcal{B}$	gauss (*Note 4*)	—	—
Magnetization, intensity of	J ($\mathcal{J}$)	J	—	—	—
Magnetizing force or magnetic field intensity	H ($\mathcal{H}$)	$H, \mathcal{H}$	gilbert per centimeter or gauss	—	gilbert per cm.
Magnetomotive force (m.m.f.)	—	$\mathcal{F}$	gilbert (*Note 4*)	—	—
Mass	m (*Note 3*)	m	gram	—	g.
Period	T	—	—	—	—
Permeability	μ	μ	—	—	—
Phase displacement (or phase angle)	ϕ	θ, ϕ	degree or radian	—	(*Note 8*)
Potential difference (p.d.)	—	V, v or E, e	volt	V	—
Power	P	P, p	watt	W	—
Quantity of electricity	Q	Q, q	coulomb or ampere-hour	C	—
Reactance	X ($\mathfrak{X}$)	X, x	ohm	(*Note 2*)	—
Reluctance	S ($\mathcal{R}$)	$\mathcal{R}$	—	—	—
Resistance	R	R, r	ohm	(*Note 2*)	—
Resistivity (*Note 6*)	ρ	ρ	ohm-centimeter	—	ohm-cm.
Revolutions per unit time	n	n	revolution per second	—	rev. per sec.
Susceptance	—	b	mho	—	—
Susceptibility	κ	κ	—	—	—
Temperature, absolute	T, Θ	—	degree centigrade	—	—
Temperature, centigrade	$t (\theta, \vartheta)$	T, t, θ	degree centigrade	—	deg.cent. (*Note 8*)
Time	t (*Note 3*)	t	second	—	sec.
Turns or number of conductors	—	N	convolution or turn	—	—
Voltage	—	E, e or V, v	volt	V	—
Work, mechanical	A (W)	W or A	joule or watt-hour	—	—

NOTES. — (1) The symbols in brackets are recommended in case the principal symbol is unsuitable; instead of the script letters heavy-faced or other special type may be used. (2) One or other of the symbols O and Ω is recommended provisionally to represent the ohm. The symbol Ω should no longer be employed for the megohm. (3) In dimensional equations the capital letters L, M and T for length, mass and time respectively are to be employed. (4) An additional unit for m.m.f. is the "ampere-turn", for flux the "line", and for magnetic flux density "maxwell per sq. in." (5) This has been the accepted standard value for many years and was formerly considered to correspond accurately to 45° latitude and sea level. Later researches, however, have shown that the most reliable value for 45° and sea-level is slightly different; but this does not affect the standard value given above. (6) The numerical values of these quantities are *ohms resistance* and *mhos conductance* between two opposite faces of a cm. cube of the material in question, but the correct names are as given, not ohms and mhos per cm. cube as commonly stated. (7) The symbol % is commonly used for per cent, but is not recommended by the A.I.E.E. (8) The symbol ° is commonly used for degree, ° C. for degree centigrade and ° F. for degree Fahrenheit; these symbols are not recommended by the A.I.E.E.

In addition to the symbols for the units given in the above table, the I.E.C. adopted (1913) the following signs to be used only after numerical values.

Volt-coulomb	**VC**	Kilovolt-ampere	**kVA**
Watt-hour	**Wh**	Kilowatt-hour	**kWh**
Volt-ampere	**VA**	Sign for milli-	**m**
Ampere-hour	**Ah**	Sign for kilo-	**k**
Milliampere	**mA**	Sign for micro- or micr-	**μ**
Kilowatt	**kW**	Sign for mega- or meg-	**M**

SPECIAL RULES IN REGARD TO SYMBOLS. — The following rules were also adopted at the Berlin meeting (1913) of I.E.C., and are concurred in by the Standards Committee of the A.I.E.E. The latter committee also recommends that vector quantities be printed in bold-face capitals.

Instantaneous values of electrical quantities which vary with the time to be represented by small letters. In case of ambiguity they may be followed by the subscript "t."

Virtual (*i.e., effective or r.m.s.*) or constant values of electrical quantities to be represented by capital letters.

Maximum values of periodic electrical and magnetic quantities to be represented by capital letters followed by the subscript "m."

In cases where it is desirable to distinguish magnetic quantities from electric quantities, magnetic quantities should be represented by capital letters of either script, heavy-faced or other special type. Script letters should not be used except for magnetic quantities.

Angles should be represented by small Greek letters.

Dimensionless and specific quantities should be represented wherever possible by small Greek letters.

Ordinary numerals as exponentials shall exclusively be employed to represent powers. (In consequence, it is desirable that the expression $\sin^{-1}x$, $\tan^{-1}x$, employed in certain countries, be expressed by arc sin x, arc tan x.)

The comma and the full-stop shall be employed for separating decimals according to the custom of the country, but the separation between any three digits constituting a whole number shall be indicated by a space and not by a full-stop or a comma (1 000 000).

For the multiplication of numbers and geometric quantities indicated by two letters, it is recommended to use the sign × and the full-stop only when there is no possible ambiguity.

To indicate division in a formula it is recommended that the horizontal bar and the colon be employed. Nevertheless the oblique line may be used when

there is no possibility of ambiguity; when necessary, ordinary brackets ( ), square brackets [ ], and braces { } may be employed to obtain clearness.

INSTITUTE (A.I.E.E.) STYLE. — The Editing Committee of the American Institute of Electrical Engineers issues a little pamphlet called "Suggestions to Authors," in which are given the rules of the Institute in regard to manuscript submitted for publication. The following list of abbreviations is taken from the last edition (1913) of these rules.

Name	A.I.E.E. style	Name	A.I.E.E. style
Alternating current	spell out, or a-c. as adjective.	Kilowatts	kw.
		Kilowatt-hours	kw-hr.
		Magnetomotive force	m.m.f.
Amperes	spell out	Mean effective pressure	spell out
Brake horse power	b.h.p.	Miles	mi.
Boiler horse power	boiler h.p.	Miles per hour per second	mi. per hr. per sec.
British thermal units	B.t.u.		
Candle power	c.p.	Millimeters	mm.
Centigrade	cent.	Milligrams	mg.
Centimeters	cm.	Minutes	min.
Circular mils	cir. mils	Meters	m.
Counter electromotive force	counter e.m.f.	Meter-kilograms	m-kg.
		Microfarad	spell out
Cubic	cu.	Ohms	spell out
Diameter	spell out	Per	spell out
Direct current	spell out, or d-c. as adjective.	Percentage	per cent, or % in tabular matter.
Electric horse power	e.h.p.	Pounds	lb.
Electromotive force	e.m.f.	Power-factor	spell out
Fahrenheit	fahr.	Revolutions per minute	rev. per min., or r. p. m. in tabular matter.
Feet	ft.		
Foot-pounds	ft-lb.		
Gallons	gal.		
Grains	gr.	Seconds	sec.
Grams	g.	Square	sq.
Gram-calories	g-cal.	Square-root-of-mean-square	effective, or r.m.s.
High-pressure cylinder	spell out		
Hours	hr.	Ton-mile	spell out
Inches	in.	Tons	spell out
Indicated horse power	i.h.p.	Volts	spell out
Kilogram	kg.	Volt-amperes	spell out
Kilogram-meters	kg-m.	Watts	spell out
Kilogram-calories	kg-cal.	Watt-hours	watt-hr.
Kilometers	km.	Watts per candle power	watts per c.p.
Kilovolts	kv.	Yards	yd.
Kilovolt-amperes	kv-a.		

1. Use "Fig.," not "Figure." Example: "Fig. 3" and not "Figure 3."

2. In all decimal numbers having no units, a cipher should be placed before the decimal point. Example: "0.32 lb." not ".32 lb."

3. Use the word "by" instead of "x" in giving dimensions. Example: "8 by 12 in." not "8x12 in."

4. Never use the characters (′) and (″) to indicate either feet and inches, or minutes and seconds as period of time.

5. Do not use the expression "rotary" or "rotary converter"; use "converter" or "synchronous converter."

6. Do not use a descriptive adjective as a synonym for the noun described. Example: a "spare transformer," not a "spare"; a "portable instrument," not a "portable"; "automatic apparatus," not "automatics"; a "short circuit," not a "short."

7. Do not use the words "primary" and "secondary" in connection with transformer windings. Use instead "high-tension" and "low-tension."

[H. PENDER.]

ACIDS.—The properties of some of the more important acids used in the arts are given in the following paragraphs.

Aqua Regia is a mixture of one part of nitric acid and three parts of hydrochloric acid. It obtains its name from the fact that it will dissolve gold. It is used as an oxidizing agent and for dissolving metals, such as platinum, gold, etc., which are insoluble in other acids. It should be used as soon as possible after being prepared since it loses its characteristic properties after standing a short time. It is distinguished by its reddish yellow color and its chlorine-like odor.

Hydrochloric Acid or Muriatic Acid (HCl) is an aqueous solution of hydrogen chloride, a colorless, pungent gas. While the pure acid is colorless, the commercial acid is a yellowish liquid, the color being due to impurities. The concentrated acid has a specific gravity of 1.16 and contains 32 per cent of HCl. The dilute acid has a specific gravity of 1.09 and contains 18.4 per cent of HCl. Hydrochloric acid is used extensively in the manufacture of chlorine, hydrogen and bleaching powder and is prepared as a by-product of the soda manufacture. The acid is recognized by its odor and by the dense fumes it makes with ammonia.

Hydrofluoric Acid (HF) is an aqueous solution of anhydrous hydrogen fluoride, a colorless, fuming liquid. The acid, when saturated, has a specific gravity of 1.25. Its gas is poisonous if inhaled and causes swellings and pain if applied to the skin. It readily dissolves glass and must be kept in platinum, lead, rubber or wax vessels. The acid is most commonly used for etching glass.

Nitric Acid (HNO_3) is a colorless fuming liquid with a specific gravity of 1.55 at 0° C. Commercial nitric acid has a specific gravity of 1.414 at 15° C. and contains 68 per cent of the pure acid. The commercial acid is yellowish in color and the pure acid becomes yellow if exposed to the light. Nitric acid is a powerful oxidizing agent and decomposes organic substances. The acid is used in the manufacture of many chemical substances. Concentrated nitric acid is best recognized by the red fumes, which are given off when the acid is acting upon a metal.

Sulphuric Acid or Oil of Vitriol (H_2SO_4) is a colorless oily liquid. The concentrated acid has a specific gravity of 1.854 at 0° C. and contains 1.5 per cent of water. Commercial sulphuric acid, sometimes known as "brown acid," has a specific gravity of 1.720 and usually contains arsenic as an impurity. Sulphuric acid boils at 290° C., the temperature increasing to 338° C. as the boiling continues. The acid has a strong affinity for water and is frequently used as a drying agent. When diluted with water, the mixing should be performed gradually as great heat is evolved. The acid should be added to the water as the addition of water to the acid may produce violent explosions due to the ebullition of the water.

Sulphuric acid is used in the manufacture of many chemical substances, in dyeing and in refining petroleum. It is a fairly good conductor of electricity and is used in the lead type of storage batteries (q.v.). The acid is recognized by its weight, by its carbonizing action upon organic bodies and by the white precipitate formed by the addition of barium chloride.

[R. G. Hudson.]

ALLOYS.—In this article is given a brief description of some of the more common alloys used in engineering work, together with specific data on some of their more important properties. It should be noted that there may be a considerable departure from the quantitative values given. Traces are not included in the chemical compositions given. The various alloys are listed alphabetically. Additional data on their various physical properties will be found in the articles on *Heat and Thermal Properties; Resistance and Conductance; Strength and Elasticity; Weight of Materials; Wires, Resistor; etc.*

Aluminum Brass (70.5 Cu, 26.4 Zn, 3.1 Al).—The tensile strength is about 21 tons per square inch, the elastic limit 8.5 tons per square inch and the elongation 50 per cent. Aluminum brass is used when very accurate castings are desired, e.g., for pumps, valves, pinions and propellers. It can be rolled and forged while hot but is not easily worked when the aluminum content exceeds 4 per cent.

Aluminum Bronze (95 Cu, 5 Al).—The tensile strength is about 28 tons per square inch, the elastic limit 12 tons per square inch and the elongation 75 per cent. Aluminum bronze containing less than 7.5 per cent Al is very ductile. With more than 7.5 per cent Al the alloy becomes brittle but increases in tensile strength. Tubes, propellers and propeller shafts are sometimes made of aluminum bronze. It has not been found to withstand intense heat for any length of time without fracturing. It is sometimes drawn into wire for use as an electrical resistance.

Amalgams are alloys of mercury and other metals. When newly made amalgams are plastic but harden in a short time without appreciable expansion or contraction. The common metals combined with mercury to form amalgams are tin, copper, cadmium, bismuth, silver and gold. Amalgams are used for silvering glass and as a cement for metals and porcelain.

Anti-friction Metals are alloys of copper, tin, zinc, lead and antimony in various combinations. These alloys are commonly used in bearings for revolving shafts and for valve packings. See *Babbitt and Bearing Metals* below.

Babbitt Metal (4 Cu, 69 Zn, 19 Sn, 5 Pb, 3 Sb) is used extensively for bearings, the composition stated being that used for car bearings of the Pennsylvania Railroad.

Bearing Metals for use in specific cases are illustrated by the following examples: Locomotive (82 Cu, 8 Zn, 10 Sn), railway car (90 Cu, 10 Sn), low-speed bearing (16 Sn, 84 Pb). There are a large number of anti-friction metals in use, the object in the composition of each of them being to procure a metal which is as hard as possible but plastic enough to be moulded by the shaft into a shape offering a minimum friction.

Brass is an alloy of copper and zinc and often contains small percentages of lead, tin, arsenic, antimony, bismuth and iron. *Cast brass* usually consists of 66 per cent of copper and 34 per cent of zinc. *Low brasses*, suitable for hot rolling, contain from 55 per cent to 63 per cent of copper. *High brasses*, suitable for cold rolling and drawing, contain from 60 per cent to 70 per cent of copper. In drawing brass it must be annealed and cleaned in acid at frequent intervals to prevent fracture. The ductility of brass is impaired if the lead content exceeds 0.1 per cent, but in the case of brasses intended for turning about 2 per cent of lead is often added so that the brass may be turned at higher speed and possess a better finish. Brass is made harder by the addition of about 1 per cent of tin and is found to better withstand corrosion due to salt water. The presence of small quantities of arsenic, antimony or bismuth in brass is liable to cause it to crack when rolled. The addition of 1 per cent to 3 per cent of iron to brass produces a harder and stronger alloy. The tensile strength of commercial

brass ranges from 15 to 40 tons per square inch and the elongation varies from 10 per cent to 40 per cent depending upon the composition.

Fusible (Low Melting Point) Metals consist of various alloys of bismuth, lead, tin and cadmium. The chemical composition of the common fusible metals and their melting points are given in the following table:

Alloy	Composition				Melting point °C.
	Bi	Pb	Sn	Cd	
Newton's alloy	50.0	31.25	18.75	.....	95
Rose's alloy	50.0	26.00	24.00	.....	100
Darcet's alloy	50.0	25.00	25.00	.....	93
Wood's alloy	50.0	24.00	14.00	12.00	66–71
Lipowitz' alloy	50.0	27.00	13.00	10.00	60

Lower melting points are obtained by the addition of mercury.

German Silver is an alloy of copper, nickel and zinc in various combinations. It is sometimes called nickel silver, argentan, packfong, silveroid, silverite or electrum. German silver is extensively used because of its ductility; it can be readily rolled, hammered and drawn. It is hard, tough and not easily corroded. Its composition varies from (50 Cu, 30 Ni, 20 Zn) to (57 Cu, 7 Ni, 36 Zn), the nickel content decreasing as the zinc increases. The usual impurities found in German silver are iron, lead and tin. The presence of iron in the alloy makes it stronger, harder and more elastic. Tin makes the alloy brittle. Lead is sometimes added to render the alloy more workable but should not be added when the alloy is to be rolled. German silver is drawn into wires for use as an electrical resistance and has been drawn into tubes for use in locomotive boilers. See *Wires, Resistor*.

Gun Metal is a bronze consisting of about 90 per cent of copper and 10 per cent of tin. The tensile strength varies from 12 to 16 tons per square inch, depending upon the method of working. It is mechanically strong and elastic and withstands severe shocks without fracture. If a small amount of lead is added to gun metal, the alloy is more easily turned or filed. Better castings are made by the addition of a small amount of zinc and the alloy is made harder by adding a small percentage of iron.

Manganese Bronze is an alloy containing copper, tin, zinc, lead, iron and manganese in various combinations. Alloys containing from 4 per cent to 6 per cent of manganese possess high tensile strength at high temperatures and are therefore used for fire-box stays. High tensile strength at ordinary temperatures is obtained, however, by the addition of very small quantities of manganese. The composition of manganese bronze for certain specific uses is as follows: For hydraulic machinery (82 Cu, 8 Sn, 5 Zn, 3 Pb, 2 Mn); for forging (58.6 Cu, 38.4 Zn, 1.6 Fe, 0.02 Mn). The tensile strength ranges from 27 to 38 tons per square inch but if the alloy is cold-rolled, a tensile strength of 50 tons per square inch may be obtained. Manganese bronze is often used in place of brass or copper, when a higher tensile strength is required. It is not easily corroded and may be bent when hot or cold.

Non-expansive Alloys are composed of iron, nickel and carbon. *Platinite* contains 46 per cent of nickel and 0.15 per cent of carbon. *Invar* contains 36 per

from 8×10^{-7} to 25×10^{-7} per degree centigrade. These alloys are used extensively in scientific instruments for standard measures of length and in incandescent lamps where the wire connections fused into the glass must not expand enough to fracture the glass.

Phosphor Bronze consists of copper, tin and phosphorus in various proportions, the phosphorus content rarely exceeding 2 per cent. The most useful property of phosphor bronze is its hardness and resistance to wear. Phosphor bronzes containing (A) 8 to 10 per cent of tin and 0.5 to 0.7 per cent of phosphorus are used for valves, pumps, propellers and boiler fittings, (B) 10 to 12 per cent of tin and 0.7 to 1 per cent of phosphorus are used for worms and gears and (C) 10 to 12 per cent of tin and 1 to 1.5 per cent of phosphorus are used for worms, gears and bearings where the wear is excessive. The tensile strength ranges from 10 to 15 tons per square inch, the elastic limit from 5 to 7 tons per square inch and the percentage elongation from 2 to 6 per cent.

Resistance Alloys. — See article on *Wires, Resistor*.

Solder is an alloy of tin and lead. *Hard* or tin solder contains 50 per cent of tin and 50 per cent of lead and melts at 370° F. *Soft* or plumber's solder contains 33.3 per cent of tin and 66.6 per cent of lead and melts at 441° F.

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[R. G. HUDSON.]

ALTERNATING CURRENTS. — (*See also Electricity and Magnetism, Principles of; Generators, Alternating-current; Motors, Alternating-current; Resistance and Conductance, Electric; Skin Effect; Transformers; Wave Analysis, etc.*). The following is a brief table of contents giving the main divisions of this article:

GENERAL DEFINITIONS. — (*See also Standardization Rules of the A.I.E.E.*). To avoid repetition the following definitions are given in terms of electric current; they also apply to electromotive forces, potential differences or to any other functions of time.

An alternating current is defined as a current which varies continuously with time from a constant maximum value in one direction to an equal maximum value in the opposite direction and back again to the same maximum in the first direction, repeating this cycle of values over and over again in *equal* intervals of time.

Period, Frequency and Alternations. — The period of an alternating current is the time taken for the current to pass through a complete cycle of positive and negative values.

The frequency or number of cycles per second is the number of periods per second.

The number of alternations per minute is the total number of times per minute that the current changes in direction, from positive to negative and from negative to positive. In engineering practice the number of cycles is usually referred to the second as the unit of time and the number of alternations is referred to the minute as the unit of time.

Let T be the period, f the frequency or number of cycles per second and a the number of alternations per minute, then

$$f = \frac{1}{T} \qquad \text{and} \qquad a = 120f = \frac{120}{T}. \tag{1}$$

The constant

$$\omega = 2\pi f = \frac{2\pi}{T} \tag{1a}$$

is sometimes called the angular velocity or angular frequency of the current. The name periodicity for this quantity is not recommended.

Instantaneous, Maximum and Average Values. — The instantaneous value of an alternating current is the value of the current at any instant. Instantaneous values of current, potential difference and electromotive force will be designated by small letters throughout this article, viz., i, v and e.

The maximum value of an alternating current is the numerical value of its maximum instantaneous value. Maximum values will be designated by capital letters with the subscript "m."

The average value of an alternating current for which the positive and negative half cycles are equal, which is usually the case, is defined as the numerical value of the average of its instantaneous values for a half cycle; the average over a complete cycle is of course zero. The general expression for the average value of a symmetrical current wave over a half cycle is

$$I_{\text{aver.}} = \frac{2}{T}\int_{t_0}^{t_0+\frac{1}{2}T} i\,dt, \tag{2}$$

Effective or R.M.S. Values. — The square root of the mean of the squares of the instantaneous values of an alternating current over a complete period is called the effective or r.m.s. value of the alternating current. In specifying the value of an alternating current as so many amperes this effective value is always meant unless specifically stated otherwise. In the same manner the square root of the mean of the squares of the instantaneous values of an alternating potential difference over a complete period is called the effective value of the alternating potential difference. When the value of an alternating potential difference is specified as so many volts, this effective value is always meant unless specifically stated otherwise.

The reason for selecting this particular function of the instantaneous values of an alternating current or potential difference as the measure of the current or potential difference is that the deflection of all instruments used in alternating-current measurements is a function of this effective value. See *Ammeters*, *Electrodynamometers*, *Voltmeters*, *etc.* Moreover, the average power dissipated as heat in a resistance r, when an alternating current of effective value I flows through it, is rI^2.

Effective values will be designated throughout this article by capital letters without subscripts.

The general expression for the effective value of an alternating current is

$$I = \sqrt{\frac{1}{T}\int_0^T i^2\,dt}, \tag{3}$$

and similarly for an alternating potential difference.

Form Factor. — The form factor of an alternating current is defined as the ratio of its effective to its average value, viz.,

$$\text{Form factor} = \frac{I}{I_{\text{aver.}}}, \tag{4}$$

and similarly for an alternating potential difference.

Crest or Peak Factor. — The crest factor, also called the peak factor or amplitude factor, of an alternating current is defined as the ratio of its maximum to its effective value, viz.,

$$\text{Crest factor} = \frac{I_{m.}}{I}. \tag{5}$$

Instantaneous and Average Power. — Let v be the value at any instant of the potential drop from any point 1 to any other point 2, and let i be the instantaneous value of the current from 1 to 2 at this same instant; then the power input at this instant is

$$p = vi. \tag{6}$$

When v and i are *both* positive (i.e., in the direction from 1 to 2, say) or when they are *both* negative, the power input is positive, but when v is positive and i negative or vice versa, the power input is negative, i.e., there is an actual power output.

The average value of the product vi over a complete period for both v and i (or over any whole number of periods) is the average power input or output, usually called simply the power input or output (input when the average of vi is positive, output when the average of vi is negative), the word average being understood. That is, the average power input is

$$P = \frac{1}{T}\int_0^T p\,dt = \frac{1}{T}\int_0^T vi\,dt, \tag{7}$$

T being a complete period. For the actual measurement of alternating-current power see *Wattmeters*.

Power Factor.— Only in certain special cases (*see below*) is the average power input P equal to the product of the effective value V of the potential difference by the effective value I of the current; it can never be greater and as a rule is less. The ratio of the average power P to the product of the effective value V of the potential difference by the effective value I of the current is called the power factor of the circuit between the terminals considered, i.e.,

$$\text{Power factor} = \frac{P}{VI}. \tag{8}$$

When V is expressed in volts and I in amperes then P must be in watts; when V is expressed in kilovolts and I in amperes P must be in kilowatts.

Volt-Amperes, Kilovolt-Amperes (kv-a.) — The product of the effective volts across the terminals of a circuit by the effective amperes through it is called the volt-amperes taken by the circuit; this product divided by 1000 is called the kilovolt-ampere input. Or, when V is in volts and I in amperes

$$\text{volt-amperes} = VI, \tag{9}$$

$$\text{kilovolt-amperes} = \frac{VI}{1000}. \tag{9a}$$

Kilovolt-amperes are usually abbreviated kv-a. or K.V.A., the former abbreviation being that recommended by the American Institute of Electrical Engineers and used in this book.

Effective Resistance (r) **and Conductance** (g). — The effective resistance of any portion of a circuit to an alternating current is the quotient of the average rate P_h at which heat is developed by this current, either directly in the substance through which it passes or indirectly as a consequence of the hysteresis and eddy-current losses produced by its magnetic field (*see Magnetic Properties of Iron*), divided by the square of the effective value I of the total current (conduction plus displacement or charging current) through this portion of the circuit, viz.,

$$r = \frac{P_h}{I^2}. \tag{10}$$

Similarly, calling V the effective value of the potential difference across the given portion of the circuit the effective conductance g of this portion of the circuit is defined by the relation

$$g = \frac{P_h}{V^2}. \tag{11}$$

In general, both r and g depend upon both the frequency and the wave-shape (*see below*) of the current and voltage but in many instances they may be considered as practically constant irrespective of the frequency or wave-shape. See the articles on *Resistance and Conductance* and *Skin Effect* for further discussion.

Impedance (z) **and Admittance** (y). — Let V be the effective value of the potential drop through any portion of a circuit due to its effective resistance, self-inductance and capacity (*see Electricity and Magnetism, Principles of*), i.e., if there is any other source of e.m.f. in the given portion of circuit (e.g. a generator or motor) V is the effective potential drop which the same current would produce through this portion of circuit were this external source of e.m.f. removed. Then the quotient of the potential drop V by the current I in this portion of the circuit, viz.,

$$z = \frac{V}{I}, \tag{12}$$

is defined as the impedance of this portion of the circuit.

The reciprocal of the impedance, viz.,

$$y = \frac{1}{z} = \frac{I}{V}, \tag{13}$$

is defined as the admittance of the given portion of the circuit.

Impedance and admittance, as thus defined, both depend upon the frequency and wave-shape. Impedance is expressed in the same units as resistance (e.g. ohms), and admittance in the same units as conductance (e.g. mhos); see *Units and Conversion Factors*.

Reactance (x) and Susceptance (b). — The square root of the difference between the square of the impedance and the square of the effective resistance of a given portion of an electric circuit is defined as the reactance x of this portion of the circuit, viz.,

$$x = \sqrt{z^2 - r^2}. \tag{14}$$

The reactance of a coil of inductance L to a sine-wave current of frequency f is

$$x = 2\pi f L. \tag{14a}$$

Similarly, the susceptance b of the given portion of the circuit is defined by the relation

$$b = \sqrt{y^2 - g^2}. \tag{15}$$

The susceptance of a condenser of capacity C to a sine-wave voltage of frequency f is

$$b = 2\pi f C. \tag{15a}$$

The simple relations expressed by (14a) and (15a) hold only for *sine-wave* currents and voltages; see Pender, H., *Principles of Electrical Engineering*, N. Y., 1912. See also the articles in this book on *Capacity and Charging Current* and *Inductance and Inductive Reactance*.

Inductive and Condensive (or Capacity) Reactance and Susceptance. — The reactance of a circuit may be due either to the back e.m.f. set up as a consequence of the varying magnetic field of the current or to a back e.m.f. set up by a condenser or its equivalent, or to both. In the first case the reactance and susceptance are said to be "inductive" and in the second case "condensive." A condensive reactance or susceptance is equivalent to a *negative* inductive reactance or susceptance; e.g., the inductive susceptance of a condenser to a sine-wave voltage is $-2\pi f C$.

Equivalent* Resistance, Impedance and Reactance. — Sometimes in calculating alternating-current circuits it is convenient to consider a motor or other load developing a back e.m.f. as equivalent to a single resistance and reactance. Let P be the total power taken by the load, V the voltage between its terminals, I the current; then the equivalent resistance is defined as

$$R = \frac{P}{I^2}, \tag{16}$$

the equivalent impedance as

$$Z = \frac{V}{I} \tag{16a}$$

and the equivalent reactance as

$$X = \sqrt{Z^2 - R^2}. \tag{16b}$$

The difference between the *effective* resistance and the *equivalent** resistance is

* The distinction here made between equivalent and effective is not always observed; the term equivalent resistance is frequently used in the same sense as effective resistance.

that the first takes into account only the power dissipated as heat, whereas the latter takes into account the total power, of which only a part is heat, the rest being converted into some other form, e.g. mechanical power.

SIMPLE HARMONIC OR SINE-WAVE CURRENTS AND VOLTAGES. — A simple harmonic or sine-wave current is one which varies with time according to the sine formula

$$i = I_m \sin(\omega t + \theta),$$

where t represents time in seconds, measured from any arbitrarily chosen instant, I_m the maximum value of the current, $\omega = 2\pi f = \frac{2\pi}{T}$, where f is the frequency in cycles per second and T the period as a fraction of a second, and θ a constant, called the "phase angle," which depends upon the instant chosen as the zero of time. See the section on *Harmonic Motion* in the article on *Mechanics, Principles of,* for a full discussion of this equation and its physical significance.

Difference in Phase Between a Sine-wave Current and a Sine-wave Voltage of the Same Frequency. — In general, when a sine-wave electromotive force is impressed on a circuit the resulting current is likewise a sine function of time (after a very brief interval, see *Transient Electric Phenomena and Oscillations*) having the same frequency, but the e.m.f. and current do not reach their maximum values simultaneously. Let the current and the potential drop in the direction of the current be represented respectively by the two equations

$$i = I_m \sin \omega t,$$
$$v = V_m \sin(\omega t + \theta),$$

where t is the time measured from the instant when $i = 0$ and is increasing in the *positive* direction. The current reaches its maximum value when $t = \frac{\pi}{2\omega}$, while the potential drop reaches its maximum value when $t = \frac{\pi}{2\omega} - \frac{\theta}{\omega}$. Hence when θ is positive the potential drop reaches its maximum value $\frac{\theta}{\omega}$ seconds *before* the current reaches its maximum, or the current reaches its maximum value $\frac{\theta}{\omega}$ seconds *after* the potential drop reaches its maximum; when θ is negative the current reaches its maximum value $\frac{\theta}{\omega}$ seconds *before* the potential drop reaches its maximum. In the first case the current is said to "lag behind" the potential drop, and in the second case the current is said to "lead" the potential drop. The angle θ is called the "difference in phase," or simply the phase angle, between the current and potential drop.

In general, when i and v are expressed by the formulas

$$i = I_m \sin(\omega t + \theta_i),$$
$$v = V_m \sin(\omega t + \theta_v),$$

i reaches its first maximum at an interval of time $\frac{1}{\omega}(\theta_i - \theta_v)$ ahead of v, and therefore i leads v by the angle $\theta_i - \theta_v$. Note the order of the subscripts: *i leads v* by the angle $\theta_i - \theta_v$; or *v leads i* by $\theta_v - \theta_i$. A negative lead is of course equivalent to an actual lag, and a negative lag is equivalent to an actual lead.

Currents and Voltages in Phase, in Quadrature and in Opposition. — When the phase difference is zero the current and potential drop are said to be "in phase"; when the phase difference is $\frac{\pi}{2}$ radians or 90° the current and potential drop are said to be "in quadrature"; when the phase difference is π radians or 180° the current and potential drop are said to be "in opposition."

Effective and Average Values, Form Factor and Crest Factor of Sine-wave Currents and Voltages. — When the current and voltage vary according to the sine law, as explained above, the following relations hold:

$$\text{Effective value} = \frac{\text{Maximum value}}{\sqrt{2}}; \qquad (17)$$

$$\text{Average value} = \frac{2}{\pi} \times (\text{maximum value}); \qquad (17a)$$

$$\text{Form factor} = \frac{\pi}{2\sqrt{2}} = 1.11; \qquad (17b)$$

$$\text{Crest factor} = \sqrt{2} = 1.414. \qquad (17c)$$

For any other relation between the instantaneous values of the current or voltage and time, i.e., for any other shape of current or voltage wave, *these relations do not hold;* see definitions above and article on *Wave Analysis.*

Power and Power Factor for Sine-wave Current and Voltage. — Let the voltage drop from terminal No. 1 to terminal No. 2 through any piece of apparatus be $v = \sqrt{2}\,V \sin(\omega t + \theta_v)$ and the current from terminal No. 1 to terminal No. 2 be $i = \sqrt{2}\,I \sin(\omega t + \theta_i)$, where V and I are the effective values and therefore $\sqrt{2}\,V$ and $\sqrt{2}\,I$ are the maximum values. Then the *instantaneous* power input is

$$p = vi = VI\,[\cos(\theta_v - \theta_i) - \cos(2\,\omega t + \theta_v + \theta_i)]. \qquad (18)$$

A study of Fig. 1 will show the physical meaning of this expression. The *average* power input is

$$P = VI \cos(\theta_v - \theta_i), \qquad (18a)$$

where $(\theta_v - \theta_i)$ is the difference in phase between the current and voltage. Putting θ for this difference in phase, viz., $\theta = \theta_v - \theta_i$, equation (18a) may be written

$$P = VI \cos\theta. \qquad (18b)$$

Whence the power factor of the load supplied to the apparatus is, from equation (8),

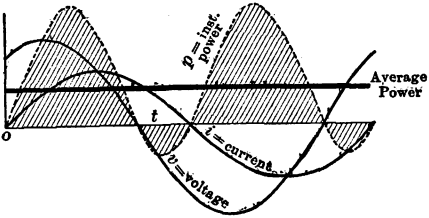

Fig. 1.

$$\cos\theta = \frac{P}{VI}. \qquad (19)$$

Power-factor Angle. — Since in the case of sine-wave currents and voltages the power factor is equal to the cosine of the angle which expreses the difference in phase between them, this difference in phase is frequently called the "power-factor angle." When the wave shape is not a pure sine

curve, the power factor cannot be interpreted as the cosine of the phase difference, for phase difference has no definite meaning except in reference to sine waves; see definitions above. A non-sinusoidal voltage and current may both reach their zero values at the same instant, as in the case of an arc (*see Arc, Electric*), and in a sense may be said to be "in phase," but the power factor as defined by equation (8) may be far from unity.

Leading and Lagging Power Factor. — The power factor is always a positive quantity, but the power-factor angle may be either positive or negative, i.e., the current may lag behind or lead the voltage drop by any angle between 0° and 90°, or lag behind or lead the potential drop by any angle between 0° and 90°.* When the current lags behind the reference voltage by an angle between 0° and 90° the power factor is stated as such a fraction or percentage, *lagging*, e.g., a power factor of 80% lagging, and when the current leads the power factor is stated as such a fraction or per cent, *leading*. In the first case the power-factor angle is taken as positive, and in the second case negative.

Equivalent Sine-wave Currents and Voltages. — In very few instances are the actual currents and voltages in a circuit pure sine-waves, but many of the ordinary calculations of alternating-current circuits may be made with sufficient accuracy by assuming them as sine-waves of the same *effective* values as the actual waves, and differing in phase by the angle whose cosine is equal to the actual power factor, i.e., by the angle

$$\theta = \cos^{-1}\frac{P}{VI}, \tag{19a}$$

where P is the average power, V the effective value of the actual voltage and I the effective value of the actual current.

Vector Representation of Sine-wave Currents and Voltages. — Consider any sine function

$$i = I_m \sin \omega t.$$

The value of i at any instant may be represented graphically, see Fig. 2, by the vertical projection (i.e., the vertical distance from P_1 to OX) of a point P_1 at the end of a radius $OP_1 = I_m$ which revolves† at an angular speed ω about a fixed point O, the angle ωt being measured from the horizontal fixed line OX. Similarly, any other sine function

$$v = V_m \sin(\omega t + \theta)$$

may be represented by the vertical projection of the point P_2 at the end of a radius $OP_2 = V_m$ also revolving about O with an angular speed ω, the angle between OP_1 and OP_2, *when the frequency of both i and*

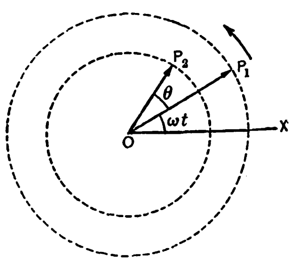

Fig. 2.

* When there is an actual power input into a portion of a circuit (e.g., a motor) it is most convenient to refer the current to the voltage *drop* through this portion of circuit in the direction of the current; when there is an actual power output (e.g., a generator) it is most convenient to refer the current to the voltage *rise* through this portion of the circuit, i.e., to the electromotive force in this portion of the circuit, in the direction of the current. When the potential *rise* is used as the reference, then for a power-factor angle between − 90° and 0° and between 0° and + 90°, the apparatus gives out power, and for a power-factor angle between − 90° and − 180° and between + 90° and + 180° the apparatus absorbs power.

† Counter-clockwise rotation has been adopted (1911) as standard by the International Electrotechnical Commission.

v is the same, remaining fixed in value and equal to the difference in phase θ between v and i. That is, v and i may be represented by rotating vectors (*see article on Vectors*) and when of the same frequency the relative position of the two vectors remains fixed. Similarly any number of currents and voltages of the same frequency may be represented by rotating vectors which remain fixed with respect to one another.

Instead of referring the various rotating vectors to a *fixcd* line OX, this line of reference may also be considered as rotating with the same speed ω as the various vectors, or any one of the vectors may be chosen as the line of reference, for example, the vector OP_1 in Fig. 2. The rotating vectors referred to this rotating line of reference are then fixed with respect to this line of reference, and the entire diagram may be considered as fixed, as in Fig. 3, the originally chosen fixed line of reference OX rotating in the opposite direction with an angular speed ω.

Instead of making the vectors equal in length to the maximum values of the sine functions they may be chosen equal in length to their *effective* values. This merely introduces a factor $\sqrt{2}$, so that when any vector is considered as rotating the instantaneous value of the quantity which it represents is equal to $\sqrt{2}$ times the perpendicular distance from its end to the fixed line of reference.

Addition of Sine-wave Currents or of Sine-wave Voltages. — Since the effective values and phase relations of sine-wave currents and voltages may be represented by vectors, sine-wave currents are added in exactly the same manner as vectors, or forces, are added, and similarly for sine-wave voltages. The addition of vectors is fully treated in the article on *Vectors*, q.v. To add any two sine-wave currents or voltages not only must their effective values be known but also their phase relation; *the resultant of two alternating voltages of effective values V_1 and V_2 is never the arithmetical sum of V_1 and V_2,* except when the two voltages are exactly in phase, and similarly for alternating currents.

In-phase and Quadrature Components. — In Fig. 3, considering OP_1 as equal to the effective value I of the current and OP_2 as representing the effective value V of the voltage, the voltage V may be considered as made up of two components, viz.:

$V_1 = V \cos\theta$ in phase with I,

$V_2 = V \sin\theta$ in quadrature with I.

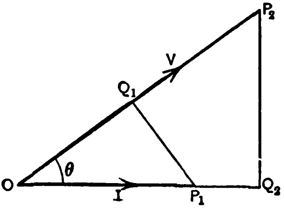

Fig. 3.

The average power corresponding to the component $V_1 = V \cos\theta$ is, from equation (18b), $IV_1 = IV \cos\theta$, and is equal to the total power corresponding to V and I. The average power corresponding to the component $V_2 = V \sin\theta$, since the angle between the current and this component of the voltage is 90°, is equal to zero. The voltage component $V_1 = V \cos\theta$ is therefore frequently called the "power" component of the voltage, and the component $V_2 = V \sin\theta$ is frequently called the "wattless" component of the voltage. These terms, however, are not recommended. It is preferable to refer to these two components as the in-phase and quadrature components respectively. The terms active and reactive components are also used.

Similarly, the current I may be considered as made up of two components, viz.:

$I_1 = I \cos\theta$ in phase with V,

$I_2 = I \sin\theta$ in quadrature with V.

The first component is called the in-phase component of the current and the second the quadrature component of the current.

Resistance Drop and Reactance Drop. — When a sine-wave current of effective value I is established in a coil which has an effective* resistance r and inductance L, the drop of voltage V through the coil is represented by the vector diagram shown in Fig. 4. The reactance of such a coil, from the definition given by equation (14a), is then $x = 2\pi fL$. The voltage drop rI, due to the resistance of the coil, is in phase with the current I, whereas the voltage drop $2\pi fLI = xI$, due to the reactance of the coil, is 90° ahead of the current. A "resistance drop" i.e., the drop of potential due to a current through a resistance, is always in phase with the current which causes it, and an *inductive* reactance drop is always 90° ahead of the current which produces it. (A *condensive* reactive drop is always 90° behind the current which causes it, and is therefore directly opposite in phase to an inductive reactance drop produced by the same current.)

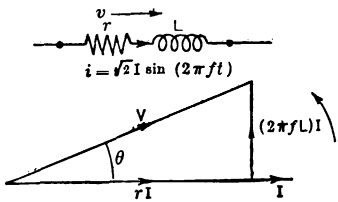

Fig. 4.

Impedance of a Coil to a Sine-wave Current. — From Fig. 4 and the definition of impedance, equation (12), it is evident that the impedance of a coil of resistance r and inductance L is

$$z = \sqrt{r^2 + (2\pi fL)^2}. \tag{20}$$

Leakage Current and Charging Current of a Condenser. — When a sine-wave voltage of effective value V is established across a condenser which has an effective† conductance g and capacity C, the total current I through it is represented by the vector diagram shown in Fig. 5. The *condensive* susceptance of a condenser from the definition given in equation (15a) is then $b = 2\pi fC$, or the *inductive* susceptance is $b' = -b = -2\pi fC$. The conduction or leakage current gV is in phase with the voltage drop through the condenser, whereas the charging or capacity current (or displacement current) $2\pi fCV = bV$ is 90° ahead of the voltage.

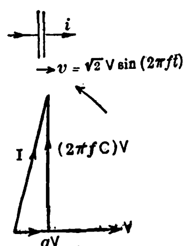

Fig. 5.

Admittance, Impedance, Effective Resistance and Reactance of a Condenser. — From Fig. 5 and the definitions given by equations (12) to (15) it follows that the admittance of a condenser to a sine-wave voltage is

$$y = \sqrt{g^2 + (2\pi fC)^2}; \tag{21}$$

the impedance is

$$z = \frac{1}{y} = \frac{1}{\sqrt{g^2 + (2\pi fC)^2}}; \tag{21a}$$

* When the coil has a non-magnetic core and the frequency is low, say 60 cycles or less, the effective resistance is practically equal to its ohmic or d-c. resistance; see article on *Skin Effect.*

† Even for moderate frequencies, 25 cycles or more, the effective conductance of a condenser is in general many times its ohmic conductance; see article on *Condensers, Electric.*

the effective resistance is

$$r = \frac{g}{g^2 + (2\pi fC)^2}; \tag{21b}$$

and the effective *condensive* reactance is

$$x_C = \frac{2\pi fC}{g^2 + (2\pi fC)^2}, \tag{21c}$$

which is equivalent to an *inductive* reactance of

$$x = -x_C = \frac{-2\pi fC}{g^2 + (2\pi fC)^2}. \tag{21d}$$

Only when the effective conductance g is zero is the reactance of a condenser equal to $-\frac{1}{2\pi fC}$, but in many instances the value of g is so small compared with $2\pi fC$ that the inductive reactance may be taken as $-\frac{1}{2\pi fC}$; the error in this approximate expression is less than 1 per cent for g less than 10 per cent of $2\pi fC$.

Relations between Effective Resistance, Reactance and Impedance and Effective Conductance, Susceptance and Admittance for Sine-wave Currents and Voltages. — From the definitions given above, equations (12) to (15), it may be shown that for sine-wave currents and voltages of a given frequency the following relations hold for any portion of a circuit:

$$\left.\begin{array}{ll} z = \sqrt{r^2 + x^2}, & y = \sqrt{g^2 + b^2}, \\ z = \dfrac{1}{y}, & y = \dfrac{1}{z}, \\ r = \dfrac{g}{y^2}, & g = \dfrac{r}{z^2}, \\ x = \dfrac{b}{y^2}, & b = \dfrac{x}{z^2}, \end{array}\right\} \tag{22}$$

where r = effective resistance, x = reactance (taken positive when inductive), z = impedance, g = effective conductance, b = susceptance (taken positive when inductive) and y = admittance, all for the given portion of circuit. Equations (20) to (21d) are special cases of equation (22).

Impedances in Series. — Let z_1, z_2, z_3, etc., be the impedances of the several portions of a circuit all connected in series (same current through each). Then the resultant impedance is

$$\left.\begin{array}{ll} & Z = \sqrt{R^2 + X^2}, \\ \text{where} & R = r_1 + r_2 + r_3 + \text{etc.}, \\ \text{and} & X = x_1 + x_2 + x_3 + \text{etc.}, \end{array}\right\} \tag{23}$$

where r_1, r_2, r_3, etc., are the effective resistances and x_1, x_2, x_3, etc., the reactances (condensive reactances to be considered as negative) of the several impedances. When there is no external source of e.m.f. in any portion of the circuit, then the resultant power factor is $\cos\theta$ where

$$\tan\theta = \frac{x_1 + x_2 + x_3 + \text{etc.}}{r_1 + r_2 + r_3 + \text{etc.}}. \tag{23a}$$

Example. — An alternating current of 100 amperes is to be supplied to a receiver which has an equivalent resistance r_1 of 2 ohms and an equivalent

reactance x_1 of 0.5 ohm. The line has a resistance r_2 of 0.1 ohm and an inductive reactance x_2 of 1.5 ohms. The equivalent resistance of the line and receiver is then $R = 2 + 0.1 = 2.1$ ohms and the equivalent reactance of the line and receiver is $X = 0.5 + 1.5 = 2.0$ ohms. Hence the equivalent impedance of the line and receiver is $Z = \sqrt{(2.1)^2 + (2.0)^2} = 2.90$ ohms. The impedance of the receiver alone is $z_1 = \sqrt{(2)^2 + (0.5)^2} = 2.06$ ohms and the impedance of the line alone is $z_2 = \sqrt{(0.1)^2 + (1.5)^2} = 1.50$ ohms. The arithmetical sum of z_1 and z_2 is 3.56, which is 23 per cent greater than the true impedance of the line and receiver.

When the current supplied to the receiver is 100 amperes, the voltage at the receiver is $V = 100 \times z_1 = 100 \times 2.06 = 206$ volts and the voltage at the generator is $V_0 = 100 \times Z = 100 \times 2.90 = 290$ volts, see Fig. 6, that is, the voltage at the receiver is $290 - 206 = 84$ volts less than at the generator. The total potential drop in the two wires forming the line, however, is $100 \times z_2 = 100 \times 1.50 = 150$ volts, which is 79 per cent greater than the true difference between the potential drops across the generator and across the receiver terminals.

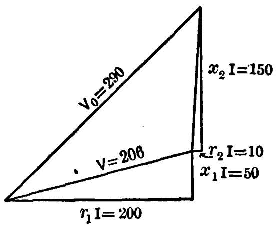

Fig. 6.

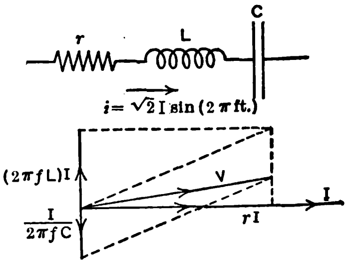

Fig. 7.

Resonance of a Coil and Condenser in Series.—Consider the circuit shown in Fig. 7. When a sine-wave current of effective value I is established in such a circuit the voltage drop across the resistance is $V_r = rI$ and is in phase with I. The voltage drop across the inductance is $V_L = (2\pi fL)\,I$ and *leads* I by 90.° The voltage drop across the capacity (a condenser with *negligible* conductance) is $V_c = \frac{I}{2\pi fC}$ and *lags behind* I by 90°. Hence the resultant voltage across the entire circuit is

$$V = \overline{V_r + V_L + V_C} = I\sqrt{r^2 + \left(2\pi fL - \frac{1}{2\pi fC}\right)^2}. \tag{24}$$

The reactance is therefore

$$x = \sqrt{z^2 - r^2} = \left(2\pi fL - \frac{1}{2\pi fC}\right).$$

When the inductance L, capacity C and frequency f are so related that

$$f = \frac{1}{2\pi\sqrt{LC}} \tag{25}$$

the reactance of the circuit is zero, the impedance is equal to the resistance, the power factor is unity and the current corresponding to a given voltage V is $I = \frac{V}{r}$; that is, the current is a maximum and depends only upon the resistance

of the circuit. The frequency corresponding to this condition is the same as the frequency with which the current and p.d. would oscillate were the condenser short-circuited by the inductance; i.e., this frequency corresponds* to the free period of such a circuit (*see Transient Electric Phenomena and Oscillations*). In general, a condenser and coil in series are said to be in resonance with the impressed frequency when the current for a given impressed voltage is a maximum, and the power factor therefore unity. When the conductance of the condenser is negligible the resonant frequency is given by equation (25).

When resonance obtains in a series circuit the voltage across the coil and that across the condenser may be many times the impressed voltage. For example, when the inductance L is 1 henry, the capacity C is 7.04 microfarads, and the frequency is 60 cycles, the inductive reactance is $x_L = 2\pi \times 60 \times 1 = 377$ ohms, the condensive reactance is $x_C = \dfrac{1}{2\pi \times 60 \times 7.04 \times 10^{-6}} = 377$ ohms, and the circuit is in resonance. For a resistance of 1 ohm in the coil and an impressed e.m.f. of 100 volts, the voltage across the coil is $\sqrt{(100)^2 + (377 \times 100)^2} = 37{,}700$ volts and the voltage across the condenser is $377 \times 100 = 37{,}700$ volts.

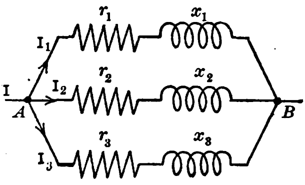

Fig. 8.

Impedances in Parallel.— Let z_1, z_2, z_3, etc., be the impedances of several branch circuits as shown in Fig. 8, and let the resistances and reactances constituting these impedances be r_1, r_2, r_3, etc., and x_1, x_2, x_3, etc., respectively.† First calculate the corresponding conductances g_1, g_2, g_3, etc., and susceptances b_1, b_2, b_3, etc., using equations (22). Then the resultant admittance, *provided there are no external sources of e.m.f. in any of the branches*, is

$$Y = \sqrt{G^2 + B^2},$$

where

$$\left.\begin{aligned} G &= g_1 + g_2 + g_3 + \text{etc.}, \\ B &= b_1 + b_2 + b_3 + \text{etc.}, \end{aligned}\right\} \qquad (26)$$

and the resultant power factor is $\cos\theta$ where

$$\tan\theta = \frac{b_1 + b_2 + b_3 + \text{etc.}}{g_1 + g_2 + g_3 + \text{etc.}}. \qquad (26a)$$

Resonance of a Coil and Condenser in Parallel. — When a coil of negligible resistance and a condenser of negligible conductance are connected in parallel, the resultant current established through the circuit by a given impressed voltage will be *zero* (infinite impedance) when the inductance L, capacity C and frequency f bear the following relation:

$$f = \frac{1}{2\pi\sqrt{LC}}. \qquad (27)$$

Compare with equation (25). Under these conditions the coil and condenser are said to be in resonance with the impressed frequency. When the coil has an appreciable resistance and the condenser an appreciable conductance they are said to be in resonance when the resultant current is a minimum for a given impressed voltage, and the power factor therefore unity.

* Approximately only when r is large; see *Transient Electric Phenomena and Oscillations*.

† Condensive reactances to be considered negative.

NON-SINUSOIDAL CURRENTS AND VOLTAGES. — The general expression for any alternating current, or in fact of any continuous periodic function, is

$$i = \sqrt{2}\, I_1 \sin(\omega t + \theta_1) + \sqrt{2}\, I_2 \sin(2\,\omega t + \theta_2) + \sqrt{2}\, I_3 \sin(3\,\omega t + \theta_3) + \ldots,$$

where I_1, I_2, I_3, etc., represent effective values of each of the terms. That is any current or voltage wave may be considered as made up of a "fundamental" sine-wave, having the same frequency as that of the actual wave, and "harmonic" sine-waves having frequencies which are integral multiples of the frequency of the fundamental. Alternating currents and voltages in practice usually contain one or more of the *odd* harmonics; *even* harmonics practically never occur in ordinary electric circuits supplied from commercial forms of generators. However, as noted above, it is permissible in most instances to assume sine-wave currents and voltages, since the harmonics present are usually relatively weak compared with the fundamental. In certain instances, however, it is necessary to analyze a wave into its fundamental and harmonics. For methods of experimentally determining the shape of current and voltage waves see the articles on *Oscillographs* and *Braun Tube;* for the analysis of the curves themselves see the article on *Wave Analysis*.

Effective Value of a Non-sinusoidal Wave. — The effective value of such a wave can be obtained directly from equation (3), or if the effective values I_1, I_2, I_3, etc., of the harmonic and fundamentals are known the effective value of the resultant wave is

$$I = \sqrt{I_1^2 + I_2^2 + I_3^2 + \ldots} \qquad (28)$$

A like relation holds for the effective value of a voltage wave. Similarly, if for example a 25-cycle e.m.f., say E_{25}, and a direct e.m.f., say E_d, are acting in series in the same circuit, the resultant e.m.f. of the combination is

$$E = \sqrt{E_{25}^2 + E_d^2}.$$

Power Corresponding to Non-sinusoidal Currents and Voltages. — Let I_1 be the effective value of the fundamental (first harmonic) of the current, V_1 the effective value of the fundamental of the voltage and θ_1 the difference in phase between these two fundamentals, both being of the same frequency; let I_2, V_2 and θ_2 be the corresponding quantities for the second harmonic; let I_3, V_3 and θ_3 be the corresponding values for the third harmonic, etc. Then the average power is

$$P = V_1 I_1 \cos\theta_1 + V_2 I_2 \cos\theta_2 + V_3 I_3 \cos\theta_3 + \text{etc.}$$

That is, each harmonic contributes an amount to the total power equal to the power it would develop were the other harmonics not present. If, for example, the third harmonic is not present in the current wave, then this harmonic contributes nothing to the average power even though there may be a large third harmonic in the voltage wave. Again, when a 25-cycle alternating electromotive force E_{25} and a direct electromotive force E_d are acting in series on the same circuit the power developed is the sum of the powers which each would develop if they acted separately, but the resultant e.m.f. of the combination is not $E_{25} + E_d$, but, as noted above, is $\sqrt{E_{25}^2 + E_d^2}$.

Calculation of Networks when the Currents and Voltages are Non-sinusoidal. — See below under *Symbolic Notation*, equations (31) and (32), and the accompanying text.

SYMBOLIC NOTATION FOR EXPRESSING SINE-WAVE CURRENTS AND VOLTAGES. — Since sine-wave currents and voltages may be represented by vectors equal in length to the effective values of these quan-

tities and making definite constant angles with one another, two mutually perpendicular axes of references may be chosen, and each current and voltage resolved into two components, one along the X-axis or horizontal axis and one along the Y-axis or vertical axis. The component along the X-axis may be expressed as an ordinary algebraic quantity, and the component along the Y-axis may also be expressed as an algebraic quantity with the symbol "j" written in front of it to indicate that it is perpendicular to the X-component, That is, any current may be written

$$\underset{\cdot}{I} = I_1 + jI_2,$$

where I_1 is the horizontal or X-component of the current and I_2 the vertical or Y-component. Until one gets familiar with this notation it is best to indicate the symbolic nature of the currents, voltages, impedances, etc., by writing dots under them; but these dots may be dispensed with in the actual solution of problems by this method when one keeps clearly in mind that in all operations the various quantities are to be treated as *complex* quantities throughout.

A voltage drop in symbolic notation is expressed in a similar manner, viz.,

$$\underset{\cdot}{V} = V_1 + jV_2,$$

and an electromotive force as

$$\underset{\cdot}{E} = E_1 + jE_2,$$

the *same* axis of reference being used for the currents, voltage drops and electromotive forces.

Impedance and Admittance in Symbolic Notation. — In this notation an impedance of resistance r and reactance* x is represented by the complex quantity

$$\underset{\cdot}{z} = r + jx, \tag{29}$$

and an admittance of conductance g and susceptance* b by the complex quantity

$$\underset{\cdot}{y} = g - jb. \tag{29a}$$

These expressions are independent of the axes of reference chosen.

Currents, Voltages, Impedances and Admittances as Complex Numbers. — In all operations involving the addition or subtraction of currents or voltages and in all operations involving products of currents and impedances or products of voltages and admittances, these quantities when written in symbolic notation may be treated as complex quantities (*see article on Complex Quantities*), *provided all the currents and voltages are referred to the same axis of references*. In all such operations the symbol "j" may be considered as mathematically equivalent to $\sqrt{-1}$. Hence when a term of the form $A_1 + jA_2$ occurs in the denominator of any fraction, the fraction may be "rationalized" by multiplying numerator and denominator by $A_1 - jA_2$, which gives

$$\frac{1}{A_1 + jA_2} = \frac{A_1 - jA_2}{A_1^2 + A_2^2}. \tag{30}$$

In any resulting expression for a current or voltage when thus rationalized the "real" part represents the actual component of the current or voltage in the direction of the X-axis and the "j" part, i.e., the sum of the terms which are multiplied by j, represents the actual component of the current or voltage along the Y-axis.

* Taken positive when inductive, negative when condensive, e.g., the admittance of a condenser is $g + j\,(2\pi fC)$.

Solution of Alternating-current Networks. — When all the currents, voltages, impedances and admittances are expressed in symbolic notation, the following relations hold:*

1. The sum of all the currents flowing to any point in any network of conductors is zero. That is, at any point

$$\dot{I} + \dot{I}' + \dot{I}'' + \ldots = 0, \tag{31}$$

where the currents are all expressed in symbolic notation and are all referred to the *same* line of reference.

2. The sum of all the impedance drops in a given direction around any closed loop in any network of conductors is equal to the sum of all the *externally induced e.m.f.'s* acting in this loop in this direction. That is, around any closed loop

$$\dot{z}\dot{I} + \dot{z}'\dot{I}' + \dot{z}''\dot{I}'' + \ldots = \dot{E} + \dot{E}' + \dot{E}'' + \ldots, \tag{32}$$

where the currents, impedances and electromotive forces are all expressed in symbolic notation, and the *currents and e.m.f.'s are all referred to the same axis of reference.* That is, the currents are to be expressed as $\dot{I} = I_1 + jI_2$, $\dot{I}' = I_1' + jI_2'$, etc., and the e.m.f.'s as $\dot{E} = E_1 + jE_2$, $\dot{E}' = E_1' + jE_2'$, etc., where all the components of the currents and e.m.f.'s with the subscript 1 are parallel to one another and all those with the subscript 2 are parallel to one another and lead the first set of components by 90 degrees.

The electromotive forces due to inductance and capacity are taken account of by the impedance; the electromotive forces represented by the E's in equation (32) are the externally induced electromotive forces such as those due to generators or motors or to the mutual inductance of the two windings of a transformer.

In applying equations (31) and (32) to the calculation of the currents and potential drops in any network of circuits, care must be taken to designate clearly the sense of the vectors representing the currents and e.m.f.'s. This is most conveniently done by numbering all the junction points in the network, and designating each current and e.m.f. by a double subscript written in the order corresponding to the assumed direction of the current or e.m.f. vector. For example, in Fig. 9, the e.m.f. from 0 to 1 is represented by $\dot{E}_{01}$ while the e.m.f. from 1 to 0, which is equal to $-\dot{E}_{01}$, is represented by $\dot{E}_{10}$. In the figure, then, the net electromotive force from 1 to 2 is

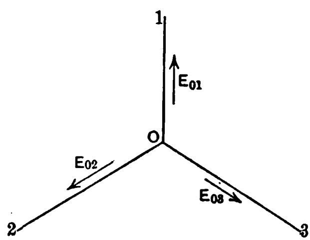

Fig. 9.

$$\dot{E}_{12} = \dot{E}_{10} + \dot{E}_{02},$$

or

$$\dot{E}_{12} = -\dot{E}_{01} + \dot{E}_{02}.$$

Each equation of the form (31) or (32) is in reality equivalent to two equations, since the sum of all the "real" terms on one side must be equal to the sum of all the real terms on the other side, and similarly, the sum of all the "j" terms on one side must be equal to the sum of all the "j" terms on the other side, the denominators of all fractions having been cleared of "j" terms by the transformation given by equation (30). This is merely another way of stating the fact that the component in any direction of the *resultant* of any number of vectors

* These relations are simply Kirchhoff's Laws (*see Electricity and Magnetism, Principles of*) expressed in a convenient form for alternating currents.

must be equal to the algebraic sum of the components in this direction of all the individual vectors. Applying these two laws to any network enables one, therefore, to calculate both components of every p.d. and every current, when the impedances and the electromotive forces are known.

These equations hold only when *the currents and e.m.f.'s are all simple harmonic functions* of the *same* frequency *and the resistances and reactances are constant.* When, however, the resistances, inductances and capacities are constant, a similar set of equations holds for each frequency that may be present. Since the equations are all linear in the I's and E's, the currents and e.m.f.'s of any given frequency will be uninfluenced by the presence of currents or e.m.f.'s of any other frequency. Hence, when the harmonics present in each e.m.f. are known, the harmonics present in each current may be calculated by solving the equations corresponding to the frequency of this particular harmonic, these equations being exactly the same as would hold were all the other harmonics absent.

Note particularly that the above equations do not hold for transient currents; they apply only after the transient terms have become zero. See *Transient Electric Phenomena and Oscillations.*

Difference in Phase between a Current and Voltage in Symbolic Notation. — Let the current be represented by

$$\underset{\cdot}{I} = I_1 + jI_2$$

and the voltage drop in the same sense as the current be represented by

$$\underset{\cdot}{V} = V_1 + jV_2.$$

Then the current lags behind the voltage drop by the angle θ, where

$$\tan\theta = \frac{V_2I_1 - V_1I_2}{V_1I_1 + V_2I_2}. \tag{33}$$

Power Corresponding to a Current and Voltage in Symbolic Notation. — Let the current and voltage drop be represented by the same expressions as in the preceding paragraphs. Then the average power is

$$P = V_1I_1 + V_2I_2. \tag{34}$$

Note that this expression is equal to the real part of the product of the complex numbers $V_1 + jV_2$ and $I_1 + jI_2$ *with the sign between the two terms reversed.*

Example of the Use of the Symbolic Method. — An impedance z_1 has a resistance of 3 ohms and an inductive reactance of 4 ohms; a second impedance z_2 has a resistance of 8 ohms and a condensive reactance of 6 ohms. z_1 is then represented symbolically as $\underset{\cdot}{z_1} = 3 + j4$ and z_2 as $\underset{\cdot}{z_2} = 8 - j6$.

Let these two impedances be connected in series, and let an e.m.f. of 100 volts be impressed across them. Choosing the vector representing the over-all potential drop as the axis of reference, and calling the current I, then $(3 + j4 + 8 - j6)\,\underset{\cdot}{I} = 100 + j0$, whence

$$\underset{\cdot}{I} = \frac{100}{11 - 2j} = \frac{1100 + j200}{121 + 4} = 8.8 + j1.6.$$

Hence the effective value of the current is

$$I = \sqrt{(8.8)^2 + (1.6)^2} = 8.94 \text{ amperes}$$

and the current leads the over-all potential drop by the angle

$$\tan^{-1}\frac{1.6}{8.8} = 10.3°.$$

The potential drop across the first impedance is

$$\underset{\cdot}{V'} = (3 + j4)\,(8.8 + j1.6) = 26.4 - 6.4 + j\,(35.2 + 4.8) = 20 + j40,$$

which has the effective value

$$V' = \sqrt{(20)^2 + (40)^2} = 44.7 \text{ volts},$$

and leads the over-all potential drop V by the angle

$$\tan^{-1}\frac{40}{20} = 63.5°.$$

The potential drop across the second impedance is

$$V'' = (8 - j6)(8.8 + j1.6) = 70.4 + 9.6 - j(52.8 - 12.8) = 80 - j40,$$

which has the effective value

$$V'' = \sqrt{(80)^2 + (40)^2} = 89.4 \text{ volts},$$

and lags behind the over-all potential drop V by the angle

$$\tan^{-1}\frac{40}{80} = 26.5°.$$

The power input into the first impedance is

$$W' = 8.8 \times 20 + 1.6 \times 40 = 240 \text{ watts}.$$

The power input into the second impedance is

$$W'' = 8.8 \times 80 - 1.6 \times 40 = 640 \text{ watts}.$$

The total power input is

$$W = 8.8 \times 100 + 1.6 \times 0 = 880 \text{ watts},$$

which of course is the sum of W' and W''.

POLYPHASE SYSTEMS. -- A polyphase alternating-current system is a network (i.e., combination of circuits) supplied from a generator or generators which develop two or more electromotive forces differing in phase from one another by a constant angle; see *Distribution of Electric Energy; Generators, Alternating-current; Transformers.* The two kinds of polyphase circuits commonly employed are the two-phase and three-phase circuits.

Star and Mesh Connections. — Consider n separate coils or windings, which may be mounted on a common armature or be entirely distinct, as for example n groups of lamps. When these n windings are connected end to end so that they are all in series, forming a closed chain, as in Fig. 10, and terminals

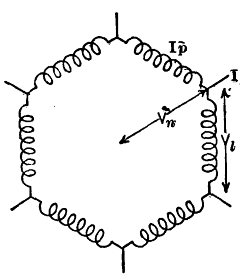

Fig. 10.

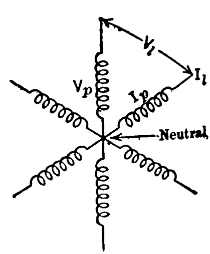

Fig. 11.

are brought out from the n junctions, they are said to be connected in "mesh." When one terminal of each of these windings is connected to a common junction point, as in Fig. 11, and terminals are brought out from the free ends, the windings are said to be connected in "star," and the common point is called the "neutral point."

n-Phase System. — When such a group of n windings, as shown in Fig. 10 or 11, is connected to a generator or other source of e.m.f., having n separate windings and therefore developing n different e.m.f.'s which differ in phase from one another, the system is called an n-phase system, each winding being called a "phase." For example, when there are three separate windings on the generator, three line wires and three windings constituting the load, the system is a three-phase system.

Balanced Systems. — When the e.m.f.'s, if any, in the n-phases of any system of connection are all equal in effective value and differ successively in phase by $360/n$ degrees and if the currents in these windings are also equal in effective value and differ successively in phase by $360/n$ degrees, the system is said to be a "balanced" n-phase system. When the e.m.f.'s in the various parts of a system are equal in effective value and differ successively in phase by $360/n$ degrees, and the impedances in the various windings or phases are all equal, both as regards resistance and reactance, then the currents are necessarily equal in effective value and differ successively in phase by $360/n$ degrees, and the system is balanced.

Phase and Line Currents and Voltages. — The current in any winding or phase of an n-phase system, see Figs. 10 and 11, is called the "phase current," and the drop (or rise) of potential *through* this winding is called the "phase voltage." The current in the line leaving any terminal of an n-phase system is called the "line current" and the voltage between *adjacent* line wires or terminals is called the "line voltage," except in the special case of a two-phase connection, see below, when the voltage between diametrically opposite terminals is called the line voltage. In the case of a star-connection the voltage between any terminal and the neutral point is called the "voltage to neutral"; in the case of a balanced mesh connection by voltage to neutral is meant the voltage which would exist between any terminal and the neutral of a star-connection connected to the terminals of the actual device, the impedance of all the legs of the star-connection forming this "artificial neutral" being equal and sufficiently large not to take an appreciable current.

The relations between these various currents and voltages for a balanced n-phase system are as follows:

	Mesh	Star
Number of phases	n	n
Line current	I_l	I_l
Line voltage	V_l	V_l
Phase current	$I_p = \dfrac{1}{2\sin\frac{\pi}{n}} I_l$	$I_p = I_l$
Phase voltage	$V_p = V_l$	$V_p = \dfrac{1}{2\sin\frac{\pi}{n}} V_l$
Voltage to neutral	$V_n = \dfrac{1}{2\sin\frac{\pi}{n}} V_p$	$V_n = V_p$
Total volt-amperes	$nV_pI_p = \dfrac{n}{2\sin\frac{\pi}{n}} V_lI_l$	$nV_pI_p = \dfrac{n}{2\sin\frac{\pi}{n}} V_lI_l$

Two-phase or Quarter-phase System. — Strictly, the so-called single-phase system is a star-connected two-phase system, since the currents *from* the two terminals are in opposite directions at any instant, the current leaving by one and entering by the other. However, in practice the name two-phase system is used for a system supplied from a generator or other source of e.m.f. having two windings in which are developed two e.m.f.'s differing in phase by 90°; i.e., a two-phase system is in reality two distinct single-phase systems each with two terminals. Two of the four terminals may be connected to each other, in which case but three line wires are required. Or, the two single-phase systems may be connected at their middle points; in this case the two-phase system may be considered as a four-phase, or, as it is usually called, a "quarter-phase" system. See the articles on *Distribution of Electric Energy* and *Transformers.*

Three-phase System. — Delta and Y-Connections. — For a three-phase system the generators and motors are designed with three windings or phases which may be connected either in mesh, usually called a "delta-connection" in this case, since the diagram of the three windings forms a Greek delta, or the three windings may be connected in star, usually called a "Y-connection" in this case, since the diagram of the three windings forms a Y. The relations between line and coil currents and voltages for a *balanced* three-phase system are as follows:

	Delta	Y
Line current	I_l	I_l
Line voltage	V_l	V_l
Phase current	$I_p = \frac{I_l}{\sqrt{3}}$	$I_p = I_l$
Phase voltage	$V_p = V_l$	$V_p = \frac{V_l}{\sqrt{3}}$
Voltage to neutral	$V_n = \frac{V_l}{\sqrt{3}}$	$V_n = V_p$
Total volt-amperes	$3V_pI_p = \sqrt{3}V_lI_l$	$3V_pI_p = \sqrt{3}V_lI_l$

Calculation of Balanced Three-phase Circuits. — Any problem in regard to a *balanced* three-phase circuit may therefore be solved by reducing all parts of the circuit to an equivalent Y-connection, provided the currents and e.m.f.'s are sine-waves. The transformations are made as follows:

Any Δ-connected motor or generator is considered as equivalent to a Y-connected generator or motor in which

$$E_y = \frac{E_\Delta}{\sqrt{3}}, \qquad r_y = \frac{r_\Delta}{3}, \qquad x_y = \frac{x_\Delta}{3},$$

where the quantities E_y, r_y and x_y are the e.m.f., resistance and reactance per phase of the Y-connected machine equivalent to the e.m.f., resistance and reactance per phase of the actual Δ-connected machine.

Each of the line wires is in series with a corresponding phase of the equivalent Y-connected machine.

When all parts of the circuit have thus been reduced to equivalent Y's, each of the three-phases may be treated as a single-phase circuit, each circuit considered completed by a wire having *zero* impedance connecting all the neutrals together, since all the neutrals are at the same potential.

The voltages thus calculated are the voltages to neutral and the currents are line currents. To find the line voltage multiply the calculated voltage by $\sqrt{3}$; similarly, to find the actual phase current in the Δ-connected generator or load divide the calculated current by $\sqrt{3}$.

Example of Three-phase Calculation. — Energy is supplied from a generating station to a substation 50 miles away at a rate of 20,000 kilowatts. The system is a balanced three-phase system and operates at a frequency of 25 cycles. The transmission line consists of three No. 0000 B. & S. copper wires spaced six feet between centers. It is desired to find (1) the voltage between wires at the generating station when the voltage between wires at the substation is 60,000 volts, and the power factor at the substation is 80 per cent, with the current *lagging*, (2) how much power is lost in the transmission line and (3) what is the power factor at the generating station. The electrostatic capacity of the line may be neglected.

The current per wire is

$$I = \frac{20{,}000{,}000}{\sqrt{3} \times 60{,}000 \times 0.8} = 241 \text{ amperes.}$$

The voltage to neutral at the substation is

$$V_n = \frac{60{,}000}{\sqrt{3}} = 34{,}600.$$

The component of this voltage in phase with the line current is $0.8 \times 34{,}600 = 27{,}700$ volts and the component 90° ahead of the line current is $0.6 \times 34{,}600 = 20{,}800$ volts, since $\cos\theta = 0.8$ and $\sin\theta = 0.6$.

The resistance per mile of a No. 0000 wire is 0.258 ohm; its reactance per mile at 25 cycles is 0.303 ohm. Hence the total resistance of each wire is 12.9 ohms and the total reactance of each wire 15.2 ohms. The resistance drop in each wire is then $12.9 \times 241 = 3110$ volts and is in phase with the line current and the inductive drop in each wire is $15.2 \times 241 = 3660$ volts and is 90° ahead of the line current.

At the generator end the voltage to neutral in phase with the line current is then $27{,}700 + 3110 = 30{,}810$ volts and the voltage to neutral 90° ahead of the current is $20{,}800 + 3660 = 24{,}460$ volts. The resultant voltage to neutral at the generator end is then $\sqrt{(30{,}810)^2 + (24{,}460)^2} = 39{,}300$ volts, and therefore the line voltage at the generating station is

$$V_l' = \sqrt{3} \times 39{,}300 = 68{,}000.$$

The power lost in the line is equal to $3RI_l^2$, where R is the total resistance of each wire and I_l the line current. Hence the power lost in the line is

$$3 \times 12.9 \times (241)^2 \text{ watts} = 2250 \text{ kilowatts.}$$

The total power delivered to the line and substation is then 22,250 kilowatts. Hence the power factor at the generating station is

$$\frac{22{,}250{,}000}{\sqrt{3} \times 68{,}000 \times 241} = 0.784 = 78.4 \text{ per cent.}$$

Measurement of Power in Three-phase Circuits. — See *Wattmeters*.

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[H. PENDER.]

ALUMINUM. — (*See also Electrochemical Processes, Industrial; Lightning Protectors; Wires and Cables, Bare.*) The method of manufacturing aluminum wire is similar to that employed in making copper wire (*see Copper*).

MECHANICAL PROPERTIES. — The more important mechanical properties of aluminum are discussed below.

Necessity of Stranding Aluminum Wire. — Aluminum wire (i.e., solid wire) is little used for aerial spans owing to the danger of failure where accidental abrasions have occurred. Cables not only reduce the above danger, but have greater breaking load, owing to the superior tensile strength of small wires.

Tensile Strength. — The tensile strength of aluminum has not been studied so thoroughly as that of copper, but it is well known that like copper, aluminum is much stronger in small, than in large, wires. The following table by H. M. Hobart, which is for solid wires, illustrates this point.

Diameter, inches	0.05	0.10	0.15	0.20	0.25	0.30	0.35
Tensile strength, lb. per sq. in.	33,300	28,800	26,200	24,800	23,800	23,200	23,000

The Standard Electric Co., of California, uses a solid aluminum wire, 0.294 inch diameter, having a tensile strength of 22,800 pounds per square inch, and a conductivity of 59.9 per cent.

Elongation. — The ultimate elongation of hard-drawn aluminum wire varies from 2.0 per cent to 3.7 per cent (in 1 meter length) as the diameter is increased from 1 to 4½ mm. (*H. M. Hobart.*)

Modulus of Elasticity. — Different authorities give values from 9×10^6 to 10×10^6 in pounds per square inch, the former being given by the Aluminum Company of America.

Elastic Limit. — The Aluminum Company of America in their booklet on "Properties of Aluminum," Pittsburg, 1909, give the following figures for elastic limit in tension, the metal being 99 per cent pure.

	Lb. per sq. in.
Castings	8,500
Sheet	12,500 to 25,000
Wire	16,000 to 33,000
Bars	14,000 to 23,000

Density. — A density of 2.70 grams per cubic centimeter is given by the U. S. Bureau of Standards (*Circ. No. 31*) as a good average value for commercial hard-drawn aluminum. This is equivalent to 0.0975 pounds per cubic inch. Hobart gives 2.71 for the density. F. J. Brislee (*Lond. Elec. Review, Jan. 5, 1912*) gives 2.708 as the density of cast aluminum and 2.705 for hard-drawn rod.

THERMAL PROPERTIES. — See article on *Heat and Thermal Properties*.

CONDUCTIVITY AND RESISTIVITY. — The Aluminum Company of America give as the average of many thousands of separate determinations, the figure of 2.828 microhms per centimeter cube at 20° C. as the conductivity of commercial aluminum wire.

Effect of Hardness on Conductivity. — The resistance of aluminum depends upon its hardness; for example, the resistance of aluminum wire of hard-

mately 2 per cent greater than that of the same wire when thoroughly annealed and consequently soft. (*H. M. Hobart.*)

Temperature Coefficient of Resistivity. — This is given by M. Dusaugey as 0.00402 per degree centigrade.

COMPARISON OF ALUMINUM AND COPPER. — Aluminum is usually cheaper than copper of the same length and resistance and is said to be cheaper to install (*C. B. Smith, El. W., 1912, Vol. 59, p. 96*). It is, however, subject to the disadvantage of lower tensile strength which often makes it more expensive than copper, except for very high voltages.

The table below compares the various items for wires having the same length and same resistance and is based on the following assumptions:

	Copper	Aluminum
Per cent conductivity	98	61
Tensile strength, lb. per sq. in.	55,000	25,000
Density	8.89	2.70
Price per pound	P	p

COMPARISON OF COPPER AND ALUMINUM WIRES FOR EQUAL RESISTANCES PER UNIT LENGTH

Item	Copper	Aluminum
Cost	1	$0.488 \times \frac{p}{P}$
Cross-section	1	1.63
Diameter	1	1.28
Weight	1	0.488
Breaking strength	1	0.731
Carrying capacity	1	1.13

Disadvantage of Low Tensile Strength. — The lower tensile strength of aluminum for equal length and conductance as compared with copper affects the cost of an aerial line in two ways; 1st, by making it necessary to erect the spans with a greater sag or less length in order to reduce the stresses, thereby either increasing the height or the number of poles, and 2nd, by making it necessary to increase the distance between wires on account of the increased sag. The increase in the height of poles for the same spacing amounts to about 10 per cent, (*C. L. Johnson*).

Effect of Large Elongation of Aluminum. — The extraordinarily great elongation of aluminum enables it to withstand severe mechanical overloads by stretching and thus increasing the sag. However, in dealing with a single solid wire this cannot be relied on, as a scratch or a single imperfection will often cause the wire to break without any appreciable elongation. This is one reason why cables are to be preferred to single-wire conductors.

Effect of Low Melting Point of Aluminum. — The distance between aerial wires is also influenced by the fact that aluminum has a lower melting point than copper, making it necessary to keep wires at different potentials well separated in order to avoid the danger of short-circuits through foreign bodies. This danger has, perhaps, been exaggerated in the past as the author has the

record of a case where a copper wire was thrown across two aluminum cables with 60,000 volts difference of potential between them and an arc was formed between the cables forming an arch 12 to 15 feet high, but with no injury to the cables except the formation of a few minute beads on their surface.

Formation of Sleet. — While for equal length and resistance, aluminum wires are larger than copper wires, they do not necessarily collect a proportionally greater load of sleet, as it is found that a film of grease left over from the drawing process often entirely prevents the adhesion of sleet. Conclusive information on this subject is, however, lacking, especially with regard to wire that has been in use for several years.

Corona Formation. — At very high potentials, such as 100,000 volts, aluminum conductors possess a marked advantage over copper in the lower corona loss due to their greater diameter for the same conductance.

Corrosion. — E. Huber-Stockar gives the results of a number of recent tests on the corrosion of aluminum in moist air, salt air, salt water, various gases, acids, etc., which indicate that, in respect to its chemical stability, aluminum is in general the equal of copper. There has been much said about the tendency of electrostatic repulsion to reduce impure condensation on aluminum conductors, but this author rejects such statements as unworthy of the slightest credence.

Bare aluminum conductors laid in the earth have been tried for railway return feeders, with the result that they were rapidly corroded by galvanic action.

Mr. Dusaugey says that aluminum containing over 4 per cent of impurities is very liable to corrode rapidly. This is especially the case if sodium is present.

At ordinary temperatures aluminum is covered with a layer of oxide which protects it against the influence of the weather and of many chemicals.

Example of Relative Cost. — According to the official publications of the Ontario Hydro-Electric Commission on a line consisting of two three-phase circuits, each comprising three 0000 A.W.G. cables, the six cables cost $1450 per mile as compared with $2050 per mile for copper cables (copper being at 16 cents per pound and aluminum at 23.5 cents per pound), showing a saving of nearly 30 per cent on the cables alone. This saving was reduced to 5.6 per cent only on the total cost of the line, partly because the actual towers weighed 1.72 tons against 1.57 tons for towers for an equivalent copper line, and partly because the cost of cables was only 30 per cent of the total cost of the line, including erection but excluding rights-of-way. (*C. L. Johnson.*) Owing to a tariff of 3½ cents per pound the price of aluminum is higher in the United States than in Canada and Europe, so that the saving would have been considerably less at United States prices.

SPREE-ALUMINUM. — E. Huber-Stockar gives considerable data on an aluminum alloy, known under the name of spree-aluminum. It has nearly double the strength of pure aluminum. Its breaking length is 10.7 kilometers, as compared with 10.3 kilometers for steel, 6.7 kilometers for aluminum and 5.1 kilometers for hard copper. The use of spree-aluminum cable shows a saving in cost of about 7.3 per cent as compared with copper cable of equal conductance.

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[W. A. DEL MAR.]

AMMETERS. — (*See also Balances, Current; Electricity and Magnetism, Principles of; Electrodynamometers; Galvanometers; Voltmeters.*) An ammeter is essentially a galvanometer (q.v.) or electrodynamometer (q.v.) of rugged construction, provided with a pointer moving over a scale forming an integral part of the instrument and calibrated to read directly in amperes. The moving element is usually supported in jeweled bearings.

CLASSIFICATION AND USE OF AMMETERS. — Ammeters which have found general commercial application may be classified as follows:

Moving-magnet Type, in which a permanent magnet or polarized vane is caused to move under the influence of a winding which carries the current to be measured. The principle of operation is similar to that of a Thomson galvanometer (*see Galvanometers*). The restoring force is usually a fixed permanent magnet. This type of meter measures average values and is therefore suitable for direct currents only.

Soft-iron Vane (or Plunger) Type, in which a soft-iron vane or plunger is caused to move under the influence of a winding carrying the current to be measured; a fixed piece of iron may be included near the moving vane or plunger, so magnetized with relation to the latter that either attraction or repulsion between it and the vane or plunger takes place under the influence of the current. The restoring force may be either gravity or a spring. This type of meter measures effective (r.m.s.) values* and may be used for either alternating or direct currents, but such meters are subject to slight errors in the latter case, due to the tendency of the vane or plunger to hold its magnetism.

Thomson Inclined-coil Ammeter (Fig 1). — This is a moving-vane instrument, the essential features of which are shown in Fig. 1. The stationary coil makes an angle of approximately 45 degrees with the shaft upon which the soft-iron vane is mounted. The vane, which is mounted on the shaft, also makes an angle of approximately 45 degrees with the shaft. When the pointer is in the zero position the plane of the vane coincides very nearly with the plane of the coil, being therefore nearly perpendicular to the magnetic field which would be produced by a current in the coil. When a current is sent through the coil the vane tends to set itself parallel to the magnetic field, since by so doing the reluctance of the path of the flux produced by the coil is diminished (*see Electricity and Magnetism, Principles of*). The torque exerted on the vane by the magnetic field is balanced by the opposing torque of the spring, so that for a definite value of the current through the coil the needle takes up a definite position.

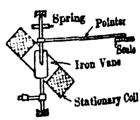

Fig. 1.

Moving-coil Type, in which a permanent magnet maintains a strong field in a fixed location, and the current to be measured is sent through a moving coil, entering or leaving by springs that furnish the restoring force or through auxiliary spirals. The principle of operation is the same as that of a D'Arsonval galvanometer (*see Galvanometers*). This type of meter measures average values and is suitable for direct currents only. Precision d-c. ammeters are usually of this type.

* Moving-vane or plunger instruments do not measure r.m.s. values exactly, on account of the lack of exact proportionality between the instantaneous torque and the square of the instantaneous current, but by careful attention to details in the design such instruments can be made nearly perfect in this respect, so that all things considered, they are sometimes to be preferred to the dynamometer-type instruments. For precise work calibration of such instruments made with direct current should not be used on

Electrodynamometer Type, in which the effect of one conductor carrying a current moving under the influence of a fixed coil carrying the same current, and joined in series to the moving coil, is utilized (*see Electrodynamometers*). In these instruments the restoring force is sometimes gravity, but is usually due to springs, and the current is led into and out of the moving coil through these springs or through flexible conductors constructed to carry relatively large currents and to have a minimum restoring force. This type of instrument measures effective (r.m.s.) values and may be used to measure either alternating or direct currents. Precision a-c. instruments are usually of the electrodynamometer type.

Hot-wire Type, in which the expansion of a wire or strip due to the heat developed within it, caused by the current passing through it, is made to move an indicating pointer. The "thermocouple" ammeter, described below, may also be classified as a hot-wire ammeter, although the principle of operation is quite different. Hot-wire meters measure effective (r.m.s.) values and are useful for both alternating- or direct-current measurements.

Induction Type, in which a metal cylinder or disk with pointer attached is caused to rotate under the action of a rotating magnetic field, usually produced from a single-phase source by means of a phase-splitting device. The principle of operation is similar to that of an induction watt-hour meter (*see Watt-hour Meters*), the motion of the moving element being opposed by a spring. This type of instrument measures effective values and is suitable for alternating currents only and generally for only a limited range of frequency.

Thermocouple Ammeters. — For measuring small alternating currents, particularly when the frequency is high (500 cycles per second or more), a thermocouple (*see Pyrometers*) may be used; the current to be measured is passed through a wire resistor (*see Wires, Resistor*) to which is soldered one junction of the thermocouple, the free ends of which are connected to an ammeter (or galvanometer) of any of the types noted above. The heating of the resistor wire sets up a direct current in the thermocouple circuit which, in turn, depends upon the effective value of the current through the heater wire. The limitations and errors in the use of such meters are discussed at length in a paper on *High Frequency Ammeters* in Scientific Paper, No. 206, Bull. Bur. Stds., 1913, Vol. 10.

Other Types of Ammeters. — Successful instruments for special needs have been made that would not ordinarily be classed under the above headings, and other instruments might be referred to which combine in a single instrument more than one of the subdivisions.

Portable and Switchboard Instruments. — Switchboard instruments are usually of more rugged construction than portable instruments and are somewhat less precise in their indications. Portable instruments are provided with suitable binding posts on their bases and are usually used in a horizontal position. Switchboard meters are usually provided with back connections and are used in a vertical position. In the so-called "edgewise" instruments the meter mechanism is horizontal but the pointer is so designed that it registers on a scale in a vertical plane.

Curve-tracing Ammeters. — A continuous record of current may be obtained by using an ammeter provided with a clock mechanism which slowly rotates a chart (disk-shaped or mounted on a drum) or a continuous paper ribbon under the end of the pointer which carries a pen kept continuously inked. The moving element in such meters is comparatively heavy since the deflecting force must be great enough to overcome the friction of the pen and prevent "sticking."

METER SCALES. — A uniform scale is usually desirable in any kind of meter, and with permanent-magnet types such a scale is readily obtained, since

the deflecting torque is proportional to the current. In moving-vane, electrodynamometer types and other types in which the deflecting torque depends upon the square of the current, very small deflections are obtained for low values of the current and the scale divisions are crowded for low values.

DAMPING. — For ordinary purposes it is essential that the pointer of the meter come quickly to rest. The moving coil of the moving-coil instruments is usually wound on a light metallic frame (aluminum) which is mounted between the poles of the permanent magnet, and a soft-iron core fills the air gap between the poles, other than that necessary to allow free motion of the coil. The result is that the metal frame moves through a very strong magnetic field and eddy currents are set up in the frame which effectually damp its motion. Although it is not difficult to make the instruments perfectly aperiodic, it is seldom advisable to have as much damping as this, for a slight underdamping gives opportunity to observe at all times that the movement of the instrument is perfectly free.

Magnetic (eddy current) damping or air damping by means of light vanes moving in a more or less perfectly closed air chamber is used with the other types of instruments.

ELIMINATION OF EFFECT OF STRAY FIELDS. — Due to the low reluctance of iron compared with air, a soft-iron shield placed around the meter mechanism will prevent, at least to a large degree, any external magnetic field from producing any effect on the moving element of the meter. Many types of switchboard and portable instruments are thus shielded. Permanent-magnet instruments are affected by only comparatively strong fields.

Astatic Instruments. — Another means of avoiding errors due to stray fields is by making the moving element astatic. The principle of this construction is to divide the moving element into two parts, so arranged that the deflecting torque acting on the two parts due to the field of the instrument are additive, whereas any stray field will produce equal and opposite torques on these two parts.

Thomson Astatic Meters. — The Thomson astatic ammeters and voltmeters are well-known instruments of this class. These instruments are also unique in that the deflecting torque and the controlling torque are both caused by the same magnetic field. This field is produced by an electromagnet wound for any specified voltage and provided with binding posts separate from the current posts of the instrument, or by means of a permanent magnet. Two moving coils are mounted diametrically opposite each other upon an aluminum disk. The disk and coils cut across the field which is directed parallel to the shaft carrying the disk. Two small pieces of soft iron are rigidly mounted on the shaft, the magnetic field entering these pieces of iron radially, tending to hold the shaft and moving coils in their initial position.

When current passes through the coils of the moving element, the lines of force parallel to the shaft produce a torque which tends to turn the shaft and cause the needle to travel across the scale. This action is opposed by the magnetic field at right angles to the shaft acting on the two pieces of soft iron. The instrument has no controlling springs and since both the deflecting and controlling force are due to the same electromagnet, the accuracy is not affected by changes in magnetic strength. These instruments are particularly suited to service where very strong external fields are present.

USE OF SHUNTS AND CURRENT TRANSFORMERS. — (*See also articles on Shunts; Transformers, Instrument.*) The current which may be led into the moving element of an ammeter (moving-coil or electrodynamometer type) is limited by the current-carrying capacity of the springs or special lead-

10 amperes. To increase the range of such instruments, and also to avoid the use of heavy conductors in the instruments, shunts or current transformers (the latter for alternating currents only) are used. Ammeters for use with current transformers are usually designed to take a maximum current of 5 amperes, but their scales are calibrated to read directly the current in the primary of the transformer.

Millivoltmeter Used as Ammeter. — Direct-current ammeters designed for use with shunts are usually designed to carry but a very small current (usually 0.025 ampere). A non-inductive resistance, made of a wire resistor having practically zero temperature coefficient, is permanently connected, within the case of the instrument, in series with the moving coil. The combined resistance of the moving coil and resistance coil is thus made large compared with the resistance of the shunt, and the error is not large for ordinary changes in temperature. The ammeter proper is then essentially a millivoltmeter (*see Voltmeters*) and measures the drop of potential through the shunt to which it is connected; the scale of the ammeter, however, is usually calibrated to read directly in amperes for one or more standard shunts designed for use with it.

Voltage Drop Across Shunts. — Shunts for use with direct-current ammeters of the moving-coil type are usually designed for a drop of 60 millivolts at full load, although for some special requirements, where the indicating instrument must be located at a considerable distance from the shunt, double-drop shunts having 120 millivolts are used, and even higher drops are used to meet very special requirements. For precision use with portable instruments the resistance coil is so designed that a drop of 200 millivolts is required for full-scale reading, the high resistance with zero temperature coefficient practically eliminating all temperature errors.

Use of Portion of Bus Bar as Shunt. — Sometimes the shunt is dispensed with altogether and the millivoltmeter is attached directly to a portion of the bus bar or main conductor. This arrangement is usually not desirable, because the variation of temperature in the bus bar due to the current flowing through it causes larger errors than the variation due to changes in room temperature. Some arrangements have been made to compensate for these, but are not very generally used.

Voltmeter and Shunt for Measuring Alternating Currents. — An a-c. voltmeter with shunt is not in general suitable for measuring an alternating current, since such meters are not readily constructed to read low voltages and therefore a relatively large drop in the shunt would be required, also the division of current between the shunt and voltmeter depends (1) upon the inductances of the shunt and meter, (2) upon the "skin effect" (q. v.) in the shunt. However, hot-wire voltmeters with shunts are sometimes used for alternating-current measurements and are satisfactory at commercial frequencies.

PRECAUTIONS IN USE OF PORTABLE AMMETERS. — The following precautions, which in the main also apply to other portable electrical instruments, should be observed:

Ammeters are subject to most of the errors which affect the accuracy of voltmeters, but usually to a less degree; see article on *Voltmeters*.

An ammeter should always be connected in series with the load. Never connect it across the mains.

All contact surfaces of nuts, terminals, etc., must be clean.

Transformers and wires carrying heavy currents should be kept at a safe distance from all meters.

Portable instruments should be always used in a horizontal position, unless specifically designed for use otherwise.

The tapping of the dial of the instrument before making a reading in order to make sure that the moving element does not "stick," is liable to do more harm than good.

It is a good plan in testing to place a low-resistance shunt or short-circuit block * or switch around the terminals of the meter and to connect a rheostat in series with it and the load. The plug of the short-circuit block should be gradually withdrawn, or the switch gradually opened, and the needle of the ammeter watched to be sure that the current is in the right direction and does not exceed the rated current of the instrument.

A heavy short-circuit in nearby conductors, even though of extremely short duration, is liable to demagnetize the magnet of a permanent magnet instrument. After such a short-circuit the meter should be recalibrated, even though the short-circuit causes no mechanical injury.

Transformers and shunts are designed for use with any ammeter, but for very precise work the transformer and meter, or shunt and meter, should be calibrated together or proper corrections made by calculation; see below under *Calibration of Ammeters* and also the article on *Wattmeters*.

When it is desired to measure the average value of a pulsating current (e.g., from a rectifier), permanent-magnet instruments should be used.

Meters containing iron should not be used for a frequency differing largely from that for which the meter is designed. For high-frequency measurements (above 500 cycles per second), some type of hot-wire or thermocouple meter should be used. Ordinary hot-wire instruments used with shunts, or so designed that two or more sections of the hot wire are connected in parallel, are liable to give incorrect readings at frequencies above 500 cycles per second.

Instruments containing iron are also subject to errors due to distorted current waves; for precise work on badly distorted waves they should be compared with an electrodynamometer-type or hot-wire instrument, using the same current in the comparison as that to be measured.

CALIBRATION OF AMMETERS. — The readings of two or more ammeters may be compared by connecting them in series with a rheostat (q.v.) and suitable source of e.m.f. An ammeter may thus be compared with a standard meter, and a curve drawn showing the true amperes corresponding to the dial readings. It is usually more convenient to plot the correction to be made to the reading against the actual reading. Corrections to be added to the reading are usually designated as + and corrections to be subtracted from the reading as −.

Instead of using a standard ammeter, a standard millivoltmeter and a standard resistor (*see Resistors, Standard*) may be employed for calibration on direct current. The standard resistor is connected in series with the ammeter to be calibrated and the millivoltmeter is connected across the potential terminals of the resistor. Let V be the reading of the millivoltmeter in volts, R the resistance of the resistor and r the resistance of the millivoltmeter and then the current measured is

$$I = \frac{V}{R} + \frac{V}{r}.$$

The last term V/r is usually negligible.

An accurate and convenient way of calibrating an ammeter on direct current is to use a potentiometer (q.v.), when such an instrument is available.

Calibration of A-C. Ammeters. — Alternating-current indicating instruments whose deflections are independent of frequency and wave form when

* For light testing two blocks of metal mounted on an insulating base with a tapered hole between the two, into which fits a tapered metal plug, makes a convenient short-

calibrated on direct-current indicate correctly effective values when used on alternating current. Electrodynamometer-type instruments usually give readings of equal accuracy on alternating or direct current at commercial frequencies and wave form. Eddy currents in the metal supports and conductors, however, may cause slight errors in some makes, even at commercial frequencies, and for high frequencies errors are liable to occur in all types unless special precautions are taken to avoid eddy currents. Soft-iron and induction-type ammeters should be compared with an electrodynamometer-type meter which has been previously calibrated on direct current.

PRECISION OF AMMETERS AND OTHER ELECTRICAL METERS. — It is customary to express the accuracy of an electrical instrument provided with a pointer and scale in terms of the per cent of *full scale reading;* i.e., calling S the full scale reading, e the *maximum* error of a single reading at any point of the scale, as a fraction of a single scale division, then the percentage error is $p = \frac{100\,e}{S}$. It should be noted, however, that the scale reading is usually less than the full amount, and therefore, the error as a per cent of the actual reading will be greater the smaller the actual scale reading. On the other hand, the actual errors in graduating and reading scales are not as large at the lower part of the scale as they are higher up, so that although the error as a per cent of the actual reading is greater the nearer to zero the reading is, this error is not quite inversely proportional to the size of the deflection.

Again, the average error, expressed as a fraction of a single scale division, for several readings taken at different parts of the scale will probably be less than the error of a single observation expressed as a fraction of a single scale division, corresponding to the stated percentage error of the instrument.

From the above it is evident that any brief statement of the accuracy of such instruments, although carefully guarded, are apt to be misunderstood, and may give an erroneous idea of the actual precision which is obtainable with the instrument. However, merely as an indication of the relative degree of precision of the various types of meters, the following figures are given. Portable meters with a stated full scale percentage accuracy of 0.5 per cent are readily obtainable from reliable makers, and when properly calibrated the readings may be relied on to an accuracy of about 0.2 per cent of full scale deflection. The stated full scale percentage accuracy of switchboard instruments is usually not so good, being about ½ to 1 per cent for a high-grade instrument and less for a cheap low-grade instrument, or one having a small scale with heavy divisions.

INSTALLATION OF SWITCHBOARD INSTRUMENTS. — The same general considerations apply to all switchboard instruments, i.e., ammeters, voltmeters, wattmeters, etc. Instruments previous to installation should be kept in a dry, cool place. Switchboard instruments should not be put in place until all the work on the boards or in the vicinity of their location has been done so that they may not be damaged by violent shocks.

During shipment moving parts, etc., are sometimes blocked and instructions furnished with instruments by makers should be carefully followed in unpacking and assembling.

It is not customary to ground the cases of d-c. instruments or of a-c. instruments above 650 volts; these are usually protected by insulated covers. Instruments on secondary circuits below 650 volts are usually grounded.

Determination of Capacity of Instruments. — In providing instrument equipment for switchboard service the relation between ammeters, voltmeters, wattmeters and the circuit conditions should be carefully considered so that proper sizes may be provided. The size should be so chosen that a good indication can be obtained under all load conditions, and at the same time the

highest sustained loads can be accurately indicated. Ammeters or current coils of wattmeters should not ordinarily be arranged with shunts or current transformers having a smaller ampere capacity than the instrument scales. Where standard transformer sizes do not exactly meet requirements, it is sometimes advisable to use instruments having 4-ampere current coils on transformers with 5-ampere secondaries, in order to limit the maximum scale readings. This expedient or other means of reducing the maximum wattmeter scale reading may sometimes be employed with advantage in switchboard wattmeters when the power factor is low.

REPAIRS OF INSTRUMENTS. — Most instruments in use at the present time, both for portable and switchboard use, are constructed with the idea of having their parts readily accessible for repair. Instruments, however, are usually, in comparison with other apparatus, very delicate, particularly with reference to the pivots and jewel bearings. Any extended repairs should, therefore, not be attempted unless equipment and experience suitable for the work in hand are available.

COST OF AMMETERS AND VOLTMETERS. — The following figures are approximate only, and should be used only as a rough guide. Shunts and instrument transformers are not included; see articles on *Shunts* and *Transformers, Instrument*.

COST OF AMMETERS AND VOLTMETERS

Type of instrument	Switchboard instruments	Portable instruments
Moving-coil permanent-magnet type:		
Small instruments of cheap construction	\$2–\$3	\$6–\$10
Small medium-grade instruments	\$6–\$8	....
High-grade, large size	\$15–\$50	\$25–\$75*
Moving-magnet type	\$3–\$5	\$2–\$10
Iron-vane type:		
Low-grade	\$5–\$8	\$15–\$25
High-grade	\$20–\$35	\$40–\$60
Hot-wire type, frequency work:		
Low-grade	\$5–\$10	......
High-grade	\$20–\$40	\$20–\$40
Electrodynamometer type:		
Low-grade	......	\$20–\$35
High-grade	\$25–\$50	\$50–\$75
Induction type, high-grade	\$18–\$40	\$30–\$50

* External shunts with low temperature coefficient for use with portable moving-coil millivoltmeters, reading 200 millivolts, cost as follows:

10-ampere shunt	\$12–15	200-ampere shunt	\$30–35
50-ampere shunt	20–25	500-ampere shunt	40–45
100-ampere shunt	25–30	1000-ampere shunt	60–75

Multipliers for 300-volt voltmeters, giving full scale reading for both 600 and 1500 volts cost from \$15 to \$25.

BIBLIOGRAPHY. — Jansky, C. M., *Electrical Meters*, N. Y., 1913; *Electrical Meterman's Handbook*, Published by Nat. Elec. Lt. Assoc., 1912; *Testing Electrical Measuring Instruments*, Cir. No. 20 Bur. Stds., 1909; Dellinger, J. H., *High Frequency Ammeters*, Bull. Bur. Stds., 1913, Vol. 10.

[L. T. Robinson.]

AMPERE-HOUR METERS. — (*See also Ammeters; Wattmeters.*) Ampere-hour or quantity (coulomb) meters may be divided into practically two classes, i.e., electrolytic meters and electromagnetic or motor meters. They are used in this country chiefly for storage-battery work; small capacity ampere-hour meters are also used, particularly abroad, as watt-hour meters, being calibrated in this case to read directly in kilowatt-hours at some definite voltage.

ELECTROLYTIC METERS. — This type of meter is used extensively in Europe, but is rarely used in this country. The meter generally consists of some form of glass container or "U" tube containing an electrolyte, such as water, mercurous nitrate or caustic soda. The action of the current passing through the solution decomposes it and since the rate of decomposition is proportional to the strength of the current, the deposit of metal or the number of cubic centimeters of gas evolved can be calibrated to measure the quantity of current. (*See Electrochemistry, Principles of.*) Such meters can also be used as watt-hour meters (q.v.) in which case they are usually calibrated to read directly in kilowatt-hours at some assumed voltage. In some types the entire current to be measured is passed through the electrolyte and in others the main current passes through a shunt and only a very small part of the current passes through the electrolyte. In a satisfactory design of electrolytic meter the drop at full load does not exceed 1½ to 2 per cent on a 200-volt circuit, and is accurate to within 2 per cent or better (depending on the design) at all loads between 10 per cent and 150 per cent load.

MOTOR METERS. — The essential parts of the motor ampere-hour meter are the motor element, the brake- or speed-regulating device and the register or gear train which records the integrated current passed through the meter. The motor consists of an armature rotating in a magnetic field produced either by a permanent magnet or by an electromagnet. The meters are generally arranged so that only a shunted portion of the current flows through the armature and when electromagnet fields are used they are connected *in series* with the armature. The construction of these motors is otherwise similar to that of a watt-hour meter (q.v.). The commutator and other electromagnetic types of motor meters are all designed with the idea of producing a device which will be cheaper and if possible more rugged than the ordinary watt-hour meter. The absence of a constantly excited "potential circuit" in the meter also avoids the constant loss of energy which takes place in such a circuit irrespective of the load. In the motor meter the drop across the shunt varies in meters of different manufacture from about 50 to 100 millivolts.

Commutator Ampere-hour Meter. — The commutator type of meter in its simplest form consists of a permanent magnet for the fields and an armature with three or more coils and a three- or four-part commutator. In some designs the armature coils are mounted flat on an aluminum disk which in addition to supporting the armature coils produces the retarding or braking action in connection with the field magnet. In another design there is a stationary iron core between the poles of the field magnet and a drum armature rotating in the gap between the stationary iron core and the pole faces of the field magnet. No braking device is used and the motor runs at a speed proportional to the impressed e.m.f., taking only sufficient power to overcome the friction in the bearings and in the gear train.

To compensate for friction at light loads a slight auxiliary potential is sometimes maintained across the brushes by placing the brushes in series with a high resistance across the line or source of power. This arrangement is adjusted so that the meter will not run with no current through the main shunt, but at light load the extra potential difference at the brushes will be sufficient

to speed up the motor enough to make up for loss due to friction of the different moving parts.

Oscillating Motor Ampere-hour Meter. — A special form of the electromagnetic motor meter differing from all other designs is the oscillating meter. In principle, this meter is a clock which is controlled by the current. The hair spring of the clock is replaced by a disk carrying a few small iron wires. This special balance wheel is supported between two coils which carry the current to be measured or a shunted portion of it. The magnetic field from the coils magnetizes the iron wires and in conjunction with the main spring of the clock causes the special balance wheel to oscillate at a rate proportional to the strength of the magnetic field and therefore proportional to the current passing through the meter. The movement of the clock is transferred to a registering mechanism geared to register ampere-hours, or kilowatt-hours at a constant voltage.

Mercury Ampere-hour Meter. — The motor meter of the mercury type utilizes mercury to carry current to the armature, the mercury really taking the place of the brushes and commutator of the ordinary direct-current motor. One form of the meter consists in principle of a copper disk or copper thimble armature mounted in jeweled bearings and immersed in the mercury. The current is led into the mercury chamber through suitable terminals and since mercury has about forty times the resistance of copper, the greater part of this current goes through the copper (approximately diametrically across the disk) and out the other terminal by way of the mercury. This arrangement applies to meters with permanent-magnet fields. In case electromagnet fields are used, the winding is generally divided in two parts on each side of the mercury chamber; also, since the driving torque is proportional to the square of the current, a braking system must be used to produce a retarding force varying with the square of the speed, such as air or fluid friction. When this is done the rate of revolution of the armature when running at a constant speed is proportional to the current passed through the meter. Another method is to use a compensating device to compensate for the increase of fluid friction with increase of speed; this device takes some such form as an auxiliary coil, in series with the motor element, which produces a counter-flux reducing the flux cutting and retarding the disk.

The mercury motor meters utilize iron or in some cases moulded compound for the mercury well, and in the design care must be taken to provide non-spillable joints at the point where the spindle for the armature enters the mercury chamber and to design the magnet so as to insure permanence. All metal in the meters should be of such composition as not to be readily affected by mercury even though not in direct contact with it.

The calibration curve of a mercury motor meter falls off rapidly below 20 per cent load and also falls off rapidly for loads over 100 per cent. Between these limits it is accurate to within 2 per cent or better, depending on the design.

CALIBRATION OF AMPERE-HOUR METERS. — Ampere-hour meters are adjusted and calibrated as follows:

Electrolytic Meters. — As noted above, in the modern types of electrolytic meters mercury is deposited in a glass tube or water is decomposed, the gases passing off in the atmosphere. In the first type a marked scale is placed beside the registering tube into which the electrolyzed mercury falls. This mercury is run back into the mercury reservoir after the reading is taken, by tipping up the cell which is hinged at the top and held vertical by a removable clip at the bottom. To calibrate the instrument an ordinary ammeter may be placed in series with it and the time that the current passes noted on a watch. In testing the types in which gases are evolved and pass off into the atmos-

per division of the scale, which can be done with a graduated burrette. Only the water is decomposed by the passage of the current. To renew the electrolyte after the reading is taken at the end of a period, it is only necessary to add water.

Motor Meters. — In testing motor meters several adjustments are necessary since a shunt, a series-adjusting resistance and the gear train are involved and in addition care must be taken to see that there is no excessive friction in any of the moving parts. The general method is to use suitable standards and with a constant current through the meter to time the revolutions of the armature spindle over at least one-minute intervals, or preferably to determine the time with a stop watch for a certain number of revolutions. Knowing the shunt constant (ratio of total to shunt current) and the register or gear-train ratio, correct speed at a certain current can be calculated and the shunt or adjusting resistance in series with the motor element adjusted to give the correct speed. To determine the accuracy for any set of readings the following formula can be used.

$$\frac{(\text{No. of rev.}) \times (\text{meter constant})}{\text{No. of seconds}} = \text{current.}$$

The "meter constant" is marked on each meter; it is the number of ampere-seconds per revolution of the disk. Meters of this type are not usually as accurate as modern induction and commutator types of watt-hour meters.

WEIGHTS AND COSTS. — Electrolytic meters weigh from 3 to 4 lb. to 15 to 16 lb. Mercury meters of American manufacture weigh about 8 lb. exclusive of shunts. Electrolytic meters cost abroad from \$4 to \$5 each. Motor meters, of both the commutator and mercury types, are made in this country and sell for \$25 to \$30 for small capacities with shunt self-contained. Above 200 and 300 amperes external shunts are furnished. These shunts range in price from \$3 to \$4 for 300 amperes to \$25 or \$30 for 3000 or 4000 amperes.

BIBLIOGRAPHY. — See Bibliography in article on *Ammeters*.

[L. T. ROBINSON.]

ANGLES. — (*See also Trigonometric Functions; Trigonometry.*) Plane angles may be expressed in degrees or in radians. The degree is arbitrarily taken as 1/360th of the plane angle about a point; the radian is defined as the angle subtended by an arc of unit length in the circumference of a circle having unit radius. The angle in radians subtended by any arc of any circle is

$$\text{Angle in radians} = \frac{\text{arc}}{\text{radius}}.$$

The relation between radians and degrees is

$$1 \text{ radian} = 57.30 \text{ degrees.}$$
$$1 \text{ degree} = 0.01745 \text{ radians.}$$

Angle of Curvature. — In railroad practice the angle of curvature is the angle subtended by a chord 100 feet in length, the curve being the arc of a true circle. The relation between degree of curvature D and radius of curvature R is

$$R = \frac{50}{\sin \frac{D}{2}} \text{ feet.}$$

For D small, the chord is very approximately equal to the arc, consequently under these conditions

$$R = \frac{5730}{D} \text{ feet,}$$

where D is in degrees. The error involved in this approximate formula is less than 2 parts in 1000 for D less than 10°, and to less than 1 per cent for D less than 30°. (*See also Railways, Location and Permanent Way for.*)

Solid Angles. — The solid angle at any point P subtended by any surface S is equal to the portion of the surface of a sphere of unit radius which is cut out by a cone having its apex at the point P and its base coinciding with the perimeter of S. The total solid angle about a point is then equal to 4π. The unit of solid angle as thus defined is called a steradian. A solid angle of 2π steradians is called a hemisphere and a solid angle of $\frac{\pi}{2}$ steradians is called a spherical right angle. See table in *Units and Conversion Factors.*

[W. A. Del Mar.]

ARC, ELECTRIC. — (*See also Distribution Systems; Furnaces, Electric; Illumination, Interior and Street; Lamps, Arc; Rectifiers.*) An electric arc is an incandescent vapor bridge consisting of material electrically impelled from a negative to a positive electrode. A spark is also an incandescent vapor bridge, but it differs from an arc, in not depending upon the electrodes for its material medium. The establishment of an arc requires the expenditure of energy for the latent heat of evaporation of the electrode and for the motion of the vapor stream. As no energy can be expended for these purposes until the current flows, an arc cannot start spontaneously. The following expedients are, therefore, adopted to start arcs.

(1) Bringing the conductors into contact with each other and separating them after the current has commenced to flow.

(2) Stressing the dielectric between the conductors until it breaks down electrically and becomes conducting.

(3) Using a subsidiary arc to furnish the initial vapor bridge. The first of these methods is used in carbon-arc lamps and the last in the mercury-arc rectifier.

CARBON ARC. — Until recently the only arc used for illumination was the arc between carbon electrodes, though at the present time there are several materials used, such as magnetite, mercury and the mixtures of various substances with carbon.

Crater. — The vapor column of the carbon arc, impinging on the positive electrode, raises the end of the latter to a very high temperature and volatilizes the carbon. The effect of this is to cause a hollow to be burned in the tip of the positive electrode. This hollow is called the crater.

Temperature. — The maximum temperature of the arc cannot be measured by direct means and the various estimates, which have been made, range from 3200° C. to 6000° C. Using Wien's law connecting temperature and wave length corresponding to maximum radiated energy, Lummer and Pringsheim found that this wave length was 0.7 μ and the corresponding temperature somewhere between 3750° C. and 4200° C., the latter figure being based upon the assumption of black body radiation.

Sources of Luminosity. — The light of the arc is derived either from the vapor column or from a body heated to incandescence by the vapor column.

In the old carbon electrode arc no special effort was made to utilize the luminosity of the vapor column, the incandescent spot or crater on the positive carbon being relied upon for practically the entire light. For this reason the positive carbon was always placed above the negative in order that the crater might better shed its light downward.

The flame arc, on the other hand, owes the greater part of its luminosity to the vapor column and very little or none to the heat of the positive electrode.

Hissing. — When the crater is surrounded by the hot gases constituting the arc, the arc is silent, but if the crater extends over the tip of the carbon and comes into contact with cool air, a peculiar hissing sound is evolved.

Characteristic of Arcs between Carbon Electrodes. — A peculiarity of the solid carbon arc is that, with any particular length of arc, if the current be increased, the difference of potential across the carbons will decrease. This occurs continuously until a certain point, when in the open arc the voltage drops quite suddenly. If the current is still increased the voltage will again become steady at a much lower value. Between the values before and after the drop, the arc is unstable

Fig. 1

and hisses. The beginning of the hissing period is indicated by the dotted line in Fig. 1, and the hissing continues throughout the lower part of the curve.

The hissing point being absent in inclosed arcs, the curve for such arcs is quite continuous.

Steadying Resistance. — In order to maintain a steady current in an electric circuit it is necessary that any decrease in the voltage across the terminals of the circuit shall produce a decrease in the current. Hence, as the arc characteristic shows an increase of current with decreasing voltage there must be placed in circuit with the arc a resistance great enough to compensate for this tendency, if the arc is to be kept in a stable condition. The method of calculating the resistance required to maintain a stable arc for a given current is as follows:

Let e = generator terminal volts = OE,
v = arc terminal volts,
x = resistance in circuit, exclusive of arc and generator resistance,
i = current in amperes,
then, since $e = v + ix$,

the resistance x must be such that

$$x = \frac{e - v}{i}.$$

On the arc characteristic (Fig. 2) find the point P corresponding to the current i and set off $OE = e$ along the vertical axis. Then join EP and continue EP until it cuts the horizontal axis at X. The resistance x is then equal to $\tan \angle EXO$. If EX cuts the curve not only at P, but at S above P, it would appear that the e.m.f. e could support either of two arcs. The point S, although affording a mathematical solution, is not physically possible, as it would involve an increase of voltage, producing a decrease of current, as may be seen by moving EX upward, parallel to its original position. Hence the arc corresponding to P is the only one possible. If EX cuts the curve not only at P, but at some point below P, say T, the same reasoning shows the arc at P to be unstable, while that at T is stable.

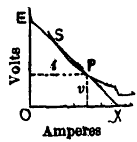

Fig. 2

For any point P, therefore, the arc is stable only if the line EX does not cut the curve below P. Hence, to find the proper value of the resistance x, for a given current i, draw a line tangent to the arc characteristic curve at the point having the abscissa i, then x must be equal to or greater than the tangent of the angle made by this line with the axis of abscissas.

Generator Voltage and Hissing Current. —

Let E = generator voltage,
V = potential difference between carbons just before current is increased to hissing point,
I = maximum amperes which will not produce hissing
D = drop in volts from silent to hissing,
i = rise of current from silent to hissing,

then $$E = V + \frac{I}{i} D \quad \text{or} \quad i = \frac{D}{E - V} I.$$

Thus, if the generator voltage is great compared to the arc voltage, and therefore the steadying resistance great, the rise of current at hissing will be less than when the voltage is small.

Resistance of the Arc.—The relation between the drop of potential across the arc and the current flowing through it depends upon whether the arc is in the steady state corresponding to the current flowing or whether the current has been changed without giving time to the vapor column and electrodes to accommodate themselves to the new strength of current. Let V be the drop of potential across a direct-current arc and A the current flowing. If the current be increased by a small amount dA for so short a time that it produces no effect upon the arc itself, it will be accompanied by a rise in potential difference dV, and the ratio $\frac{dV}{dA}$ will be the resistance of the arc. This may be accomplished experimentally by superimposing upon the direct-current arc a small rapidly-alternating current and measuring the value of the alternating p.d. and current. Unless a very high frequency (about 100,000 cycles) is used, the ratio $\frac{dV}{dA}$ will be negative, due to the fact that the power factor of the arc is not unity for low frequencies and this ratio, therefore, does not represent the resistance (*see Duddell's papers in Bibliography*).

Back Electromotive Force of the Arc.—The product of the direct current of an arc and its resistance, measured as described above, is the ohmic drop in the arc. This is usually less than the actual p.d. by 7 to 15 volts, which represent a back electromotive force. This back electromotive force really consists of two, the larger near the positive electrode, opposing the flow of current, and the smaller near the negative, helping the flow. By varying the direct current carried by the arc, the back e.m.f. is not noticeably affected, but the resistance tends to become infinite for very small currents, and small for large currents. A variation in the length of the arc between solid carbons produces no effect upon the back e.m.f., whereas with cored carbons the back e.m.f. decreases with increase of length. The value of the back e.m.f. depends upon the make of carbons employed. In his experiments which established the existence of a true back e.m.f. in the arc, Duddell used an alternating current of about 1 per cent of the strength of the direct current and having a frequency of 100,000 ~ per second.

ARC AS TELEPHONE RECEIVER. — A direct-current arc may be used as a telephone receiver which can be clearly heard in a quiet room at a distance of 10 or 12 feet. The arrangement of apparatus, which gives the best results, is shown in Fig. 3, which is Duddell's improvement of the H. Simon circuit. In Duddell's experiments AB was a solenoid about 30 cm. long, wound with about 1000 turns of No. 18 D.S.C. wire and having an iron core of about 15 mm. diameter. Six hundred turns of the solenoid were connected to the microphone M and battery B and four hundred to the arc and condenser C.

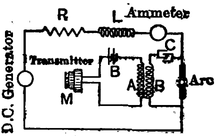

Fig. 3

The arc was between cored carbons of 11 to 13 mm. diameter separated 20 to 30 mm. and took 10 to 12 amperes. The condenser capacity was 2 or 3 microfarads. The resistance R was the usual ballast resistance and the choke coil L served the purpose of confining the telephonic currents to the arc circuit.

ARC AS A TELEPHONE TRANSMITTER. — The arc may be used as a telephone transmitter, but it is unsatisfactory on account of hisses and splits due to the irregularities of the carbons and to the access of air to the crater. The arrangement of circuits is shown in Fig. 4, the apparatus R, L and C being the same as in the receiver circuit.

MUSICAL ARC. — A direct-current arc of suitable length and current between solid carbons will give a musical note, if it be shunted with a con-

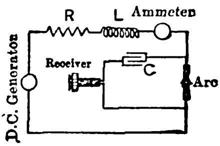

Fig. 4

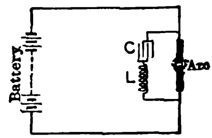

Fig. 5

denser in series with a *choke* coil, as shown in Fig. 5. The pitch corresponds to the periodic time

$$T = 2\pi\sqrt{LC},$$

where

C = capacity of condenser, farads,
L = inductance of choke coil, henrys.

The choke coil must not have an iron core, or the hysteresis and eddy currents will destroy the effect. A closed circuit, such as a ring of wire placed near the inductance coil, has the same effect. Further details of the musical arc are given below.

THE ARC AS A SOURCE OF HIGH-FREQUENCY CURRENT. — The musical arc owes its pitch to the action of the arc in transforming a part of the direct current into alternating current, the frequency of which can be varied between very wide limits by altering the self-inductance and capacity with which it is shunted. The inclosed arc works just as well as the open arc. In some of Duddell's experiments, the *alternating current* through the condenser circuit was as large as 5 amperes. Only condensers suitable for high voltages should be used, as, although the arc p.d. may be quite low, the condenser p.d. rises to several hundred volts. Alternating currents of frequencies of from 500 cycles to 500,000 cycles per second are easily obtainable.

Arcs between solid carbons always work well as converters, while those between cored carbons will not work under any conditions. In fact the general conditions under which the arc works as a converter are that $\frac{dV}{dI}$ must be negative and numerically greater than the resistance of the condenser circuit, exclusive of the condenser itself. The expression $\frac{dV}{dI}$ should be taken as referring to small instantaneous changes of the arc voltage and current.

Fig. 6 shows a means of operating several arcs together so as to obtain a greater output than would be possible with a single arc. Direct current is supplied to the arcs from the mains, through the choking coils L_1 and L_2, the ballast resistance R_1 and R_2 and ammeters A_1 and A_2. Alternating current is delivered by the arcs to the circuit containing the capacity C and inductance L.

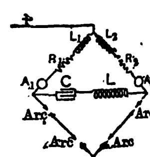

Fig. 6

STEEL BURNING BY MEANS OF ARC. — In addition to its application for welding and electrochemical manufactures (q.v.), the thermal effect of the arc is utilized for burning steel. A recent application of this was made by the contractors of the Pennsylvania Railroad (East River tunnel), who used a carbon

with an asbestos shield, one foot in diameter. The operators worked with asbestos masks and aprons and dark-colored eye glasses. While this sufficed to shield them, other workers some distance away became temporarily blind a few hours after exposure to the glare.

Direct current was used, the carbon electrode being connected to the positive and the steel to be cut to the negative feeders. The current was varied by means of water rheostats, the voltage at the tool varying between 45 and 60 volts. The current varied from 250 to 400 amperes per tool for burning off rivet heads and light section plates and from 600 to 800 amperes for burning plates 4 inches thick. The best results were obtained with 40 volts, 600 amperes and a ½-inch to ¾-inch arc. A fair day's work (8 hours) removed 300 rivet heads, although a record of 350 was reached. In the same time 4 feet 6 inches of 4-inch plate could be burned off. (*H. Japp, Proc. A.S.C.E. 1909, Vol. 35, No. 9, p. 1230.*)

Portable plants, consisting of a gasoline engine driven generator of about 25 kw. capacity, mounted on a truck, are now in use for wrecking buildings. Such an outfit is capable of burning off a 15-inch I-beam in 20 minutes, using the full-rated output of the generator.

The same apparatus is used for filling blowholes in castings. The casting must be heated to a dull red heat before the arc is applied, in order to avoid strains due to local expansion. The heat of the arc may be applied not only as above described, but also by deflecting an arc between two carbons against the casting by means of a magnet. In either case, the filling metal is introduced into the arc and allowed to flow into the blowhole.

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[W. A. DEL MAR.]

AUTOMOBILES, ELECTRIC. — (*See also Batteries, Storage, Alkaline Type and Lead Type; Batteries, Storage, Applications of.*) There are two types of electric automobiles, namely the battery car and the "gasoline-electric" car. The former type carries a storage battery from which energy is delivered to the driving motor; the latter type carries no battery but electric energy is supplied to the motor by a small generator which is itself driven by a gasoline engine. The battery car only is discussed in this article. A battery car is commonly referred to as "an electric," regardless of whether it belongs to the passenger or the commercial class.

HISTORICAL DEVELOPMENT. — The first battery-driven automobile in this country was built by Fred M. Kimball in Boston in 1888. This machine had 6 cells of lead storage battery and could travel 10 miles on good roads at an average speed of 5 m.p.h. In 1891 there was exhibited at the Mechanics' Fair in Boston an electric surrey built by the Holzer-Cabot Electric Co. Little more was done in the development of electrics until about 1900, when various makes of electric pleasure cars and trucks began to appear. By 1903 a brisk business in electric vehicles had sprung up. Some of these early machines were failures, due either to poor mechanical construction or to crude operating methods which failed to give reliable service. The only batteries then available were of comparatively heavy types, such as had been used previously in stationary service. The battery situation was soon met by the battery manufacturers in the grid type of pasted lead plate. The Edison battery was first placed on the market in January, 1905. But the electric vehicle business did not assume large proportions until about 1910, when the electric light companies in the large cities began to look toward vehicle charging at night as a possible source of load for increasing the load factors of their systems. With the stimulus of the lighting companies the business began to develop rapidly, there being 37 manufacturers of electric vehicles in this country in 1913. It is estimated that at the close of 1913, there were 25,000 passenger and 10,000 commercial electrics in use in this country. (*A. Williams.*)

APPLICATIONS OF ELECTRICS. — The three types of vehicles which are at present available for urban street haulage are the horse-drawn wagon, the gasoline motor car and the electric battery car. In comparing these three types for use in any given service, the chief factors to be considered are relative speed, distance between stops, length of stops required by the service, expense per vehicle and reliability (i.e., number of days per year upon which the wagon can be depended for operation). The first three of these factors may be determined by trial or by observation.

In considering the matter of speed it should be remembered that with motor cars the wear and tear on the machine increases approximately as the square of the speed, so that the speed should be such as to give minimum cost of service. The last two of the above factors, namely expense and reliability, can be absolutely determined for any set of operating conditions only by several years of use of each type of vehicle. It is possible, however, to closely estimate the "expected" values of expense from the experience of others in similar kinds of work. See section below on *Costs.*

The electric truck has found its widest commercial application in city services where the hauls are of moderate length, say from two to ten miles. For very short hauls where the standing time of the wagon is a large proportion of the working time, so that the work per day of each delivery unit is not largely influenced by the speed of the unit, the horse-drawn wagon can perform the work cheaper than an electric truck. For the so-called "long hauls" the gasoline truck would be expected to work to better advantage than an electric

truck because of higher running speed or greater distance capacity in a day, or both. The rated values of speed and distance on a single battery charge for a representative line of electric trucks are given in Table V below. The effect of hills or poor pavements would be equivalent to a decrease in the rated values of both speed and mileage. Systems of operating in which discharged batteries are exchanged for charged batteries during the day in order to increase the mileage capacity beyond the rated value have been used with signal success in a few instances, but in general such a system is not sufficiently flexible to meet required conditions.

DESIGN. — The necessity of producing cars which would travel at moderate speeds with a low rate of energy consumption has forced the designers of electric automobiles to use equipment which would meet this requirement, even though in some instances less efficient equipment would cost less or might wear longer than that used. Thus tires of low-energy consumption, batteries with large capacity per unit of weight and "non-friction" bearings in transmission and axles are used almost universally. Consequently an electric automobile is essentially a high-grade machine, for any sacrifice in equipment directly influences the operating qualities.

Weight and Speed. — In the design of an electric battery car the two primary features are the total weight and the speed under normal conditions. In special cases a car may be built to meet unusual requirements of distance traveled per charge, in which case either the speed or weight must depart from the limitations of common practice. Any decrease in the total weight of the car will result in an increase in either speed or distance capacity or both; or if the weight is brought back to the original value by the use of additional battery capacity, the speed or distance may be still further increased. This has been one of the chief considerations leading to the universal adoption of lighter batteries, such as the Edison and "thin" pasted lead plate, in place of the "standard" pasted lead plate batteries which were extensively used a few years ago. General practice as regards weight and speed for various types of electric automobiles is indicated in Table I.

TABLE I.—SPEEDS AND WEIGHTS OF ELECTRIC AUTOMOBILES

Class of car or rated capacity of truck	Approximate weight in lb., including empty body	Approximate speed in miles per hour	Weight allowance for body, lb.*
2-passenger runabout	2,200	15 to 25	....
2-passenger Victoria	2,400	15 to 21	....
2-passenger coupé	2,600	15 to 20	....
4-passenger brougham	3,000	14 to 18	....
1,000-lb. truck	3,500	12 to 14	500
2,000-lb. truck	4,500	10 to 12	600
4,000-lb. truck	6,000	8 to 10	800
7,000-lb. truck	8,500	6.5 to 8	1100
10,000-lb. truck	9,500	6 to 7	1400
12,000-lb. truck	11,500	6 to 6.5	1600

* Recommended by Commercial Vehicle Committee of Auto. Chamber of Com. (formerly National Assoc. of Auto. Mfgr., Inc.), March 4, 1912.

Similar data for a representative line of electric trucks are given in Table V below.

Load Efficiency. — By load efficiency of a motor truck is meant the ratio of the load on the machine to the gross weight of the machine, including chassis, body and load. Thus, from the above table, a representative value for the rated load efficiency of a 5-ton capacity electric truck is $\frac{10,000}{10,000 + 9500} = 0.51$. The rated load efficiencies for gasoline trucks of large capacity, 2 tons or more, are practically the same as the values for electric trucks, but small capacity gasoline cars of good design may have a rated load efficiency as high as 30 per cent as compared to about 24 per cent for electric cars with the customary lead battery equipments.

Motors. — Four-pole series motors are now used almost exclusively for electric automobiles. In the majority of cases but a single motor is used on each vehicle. The field coils are usually arranged in two groups which may be connected first in series and then in parallel. All manufacturers do not rate their motors on the same basis, but good practice is indicated by that of the General Electric Co., whose motors will carry their normal rated load continuously with a maximum temperature rise of 65° C. above the surrounding air at 25° C. or will carry 2½ times their rated load for 1 hour with a maximum temperature rise of 75° C. Ratings are based on bench tests, so that when the motor is installed in a car the overload capacity is usually increased due to the improved ventilation. Large overload capacity is necessary to meet severe street and grade conditions, and unusually good commutating characteristics are required.

An automobile motor must also operate satisfactorily over a considerable range in the voltage furnished by the battery in the charged and discharged conditions. The motor speeds which are used with single reduction drives range from 750 to 1000 r.p.m.; the speeds which are used with double reduction drives range from 1000 to 2000 r.p.m.

Number of Motors. — All electric passenger cars and most electric trucks are now being equipped with a single motor. It was formerly common practice to use two motors on the larger trucks on account of flexibility in control, but a single motor is now being used on account of space limitations, saving in weight and better electrical efficiency. The peculiar construction of the Couple-gear (*see below*) equipment requires a departure from this practice. Otherwise, either two or four motor equipment is used now only when unusually high tractive effort is required.

Motor Characteristics. — Along with the tendency during recent years to standardize battery equipments at 40 to 44 lead cells and 60 Edison cells, the tendency among motor manufacturers has been to develop a line of motor frames, for each of which two windings could be provided. Thus an 80- or 85-volt motor is usually installed with a 40- to 44-cell lead battery and a 60-volt motor with a 60-cell Edison battery. The windings are so proportioned that a motor has approximately the same speed and torque at both voltage ratings. This development has made it unnecessary to alter either the motor suspension or the mechanical transmission system in changing a car's battery equipment from lead to Edison or vice versa. 24-, 36- and 48-volt motors can also be obtained.

The characteristic curves for a typical Westinghouse motor with the 60-volt and 80-volt windings are given in Figs. 1A and 1B. The relatively small change in efficiency for overload with the field coils in parallel is noteworthy. The "torque ratio" of this motor, i.e., the ratio of torque at 2½ times rated current to torque at rated current, is approximately 5 with both windings.

The following table gives a partial list of General Electric automobile motors with their normal ratings.

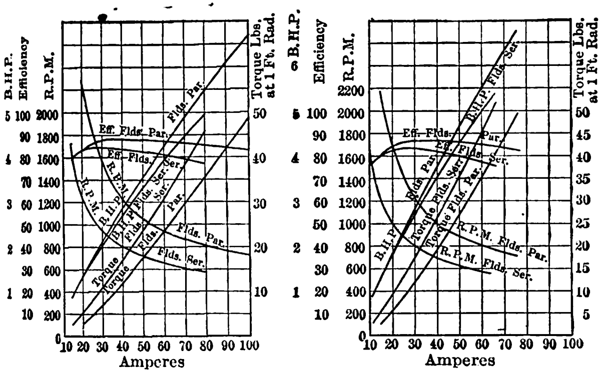

Fig. 1. Characteristic Curves of Typical Automobile Motors

TABLE II.—RATINGS OF G.E. AUTOMOBILE MOTORS

Designation	Rated volts	Rated amperes	Speed, r.p.m.	Approximate weight, lb.
G.E. 1022	60	60	1100	380
G.E. 1022	85	40	1200	380
G.E. 1026	60	40	1200	310
G.E. 1026	85	28	1200	310
G.E. 1027	60	85	900	660
G.E. 1027	85	60	900	660
G.E. 1028	36	35	1800	150
G.E. 1028	48	26	1800	150
G.E. 1030	60	70	850	500
G.E. 1030	85	50	850	500
G.E. 1031	24	40	1600	120
G.E. 1036	60	32	1200	265
G.E. 1036	85	22	1200	265
G.E. 1037	60	28	1800	185
G.E. 1037	85	20	2000	185

Before recommending motor equipment for automobile service the motor manufacturers usually require that the purchaser submit data upon the weight, speed, battery, wheel dimension, etc., for the car on which the motor is to be used.

Size of Motor Required.—In selecting the size of motor the voltage should be chosen on the basis of 1.9 volts per cell for lead batteries and 1.0 volt per cell for Edison batteries. The average discharge voltages of both types of batteries at their normal rates are higher than these values, but it has been found desirable to install motors on the basis of these values so that at high rates of discharge the terminal voltage may not fall too far below the rated motor voltage.

The current rating of the motor (and also of the battery) should be approximately the value of the current required to drive the car at the rated speed on hard level asphalt. The current input into a vehicle motor when driving a car at constant speed is given by the expression

$$I = \frac{2\,vWr}{E\epsilon},$$

where v = speed of car in miles per hour,
W = total weight of car and load in tons,
r = car resistance in pounds per ton of total car weight,
E = motor terminal voltage,
ϵ = over-all efficiency of motor, controller and transmission.

The value of ϵ at the rated speed is the product of the efficiencies of motor and transmission, usual values falling between 60 and 75 per cent. For value of r see next paragraph.

Car Resistance. — The car resistance depends upon the following conditions: speed, grade, diameter of wheels, load per square inch of tire contact or per inch of tire width, construction and composition of tire, method of attaching tire to wheel rim, and road surface. The effect of speed is to change r in a manner dependent upon the type of tire, the value of r having a minimum usually at a speed between 6 and 10 m.p.h.; air resistance varies approximately as the square of the speed and need not be considered for speeds of less than 20 m.p.h. The effect of up-grades is to increase r by 20 lb. per ton for each per cent grade. The car resistance with pneumatic tires on hard, level asphalt is given by Churchward (*Soc. of Auto. Eng. Handbook, 1913*) as ranging from 15 lb. per ton for special tires designed for electric runabouts up to 35 lb. per ton for the standard type of tire used on gasoline touring cars; the resistance is greatly affected by the air pressure, increasing rapidly as the air pressure is reduced. Churchward also gives the car resistance with solid tires on hard level asphalt as ranging from 18 to 26 lb. per ton. Other tests have shown a car resistance of 30 lb. per ton for a small capacity electric truck equipped with solid tires (*El. W., May 17, 1913, Vol. 61, p. 1040*).

Effect of Road Surface. — Two sets of values of relative car resistance on various level road surfaces as compared to the value on hard, level asphalt are given in the following table. One set of values is given by Churchward in the *Soc. of Auto. Eng. Handbook* (1913), and the other set is based on tests conducted at the Massachusetts Institute of Technology in 1913 (*El. W., May 17, 1913, Vol. 61, p. 1040*).

TABLE III. — RELATIVE VALUES OF CAR RESISTANCE

Road surface	Churchward	M.I.T.
Asphalt, hard	1.0	1.0
Wood block	1.15	1.1
Macadam	1.15 to 3	1.15
Granite block	1.75	2.0
Dirt road	1.1 to 2.0	...
Brick	....	1.4
Snow, packed	....	1.3
Snow, fairly hard, without grips	....	1.7
Snow, fairly hard, with grips	....	1.9
Snow, soft, about 3 in. deep	....	2.1
Sand	20	

Transmission Systems. — The transmission systems between motor and wheel which are in common use may be grouped in two classes, known as the "chain" and the "shaft" or gear drives. A single-speed reduction is used in a few makes of cars, but a double reduction is used on a majority of machines. If the first reduction is made by chain and the second by gear, or vice versa, the transmission system is usually designated according to the means employed for the final reduction; e.g., if the final reduction is accomplished by a chain, the system is referred to as a chain drive although the first reduction may be by gears.

Chain Drive. — With chain drive the motor is mounted transversely and a double-speed reduction is used almost without exception. The connection between motor and countershaft is usually by silent chain (Morse or Reynold type, see article on *Chains and Chain Drive*), although spur and herringbone gears are also used; see article on *Gears and Gearing*. The final drive is usually by roller chain, either (*a*) between countershaft and differential in the rear axle, or (*b*) between countershaft and a sprocket on each of the rear wheels. Chain-driven passenger cars are usually equipped with (*a*) and chain-driven commercial cars with (*b*). The chain method of drive possesses the advantages of low first cost and ease of adjustment; its disadvantages are noise, rapid wear due to collection of dirt unless properly housed, poor efficiency when not properly adjusted, and unequal wear on the two sides of the car. The chain drive has been losing favor during recent years among both designers and operators.

Shaft or Gear Drive. — With shaft drive the motor is usually mounted lengthwise of the car, and either a single- or double-speed reduction is used. For single reduction both bevel and worm gears are used as the connection between propeller shaft and differential in the rear axle. For double reduction the motor is usually connected to the propeller shaft by spur gear, herringbone gear or silent chain, and the propeller shaft is connected to the differential by a bevel gear. The propeller shaft is usually fitted with universal joints. The shaft drive has become very popular in passenger cars because it runs quieter than chain drive, and because it can be easily protected from dirt by housing. The shaft drive has the disadvantage of greater weight below the springs than in the case of the chain drive, which tends to increase the wear on tires.

Worm Drive. — There has recently been a decided tendency among designers of passenger cars and light trucks to use the single-reduction worm gear instead of the double-reduction bevel or chain arrangements, the advantages claimed for the worm being better transmission efficiency and less trouble from adjustment.

Walker Balanced Gear. — In the trucks built by the Walker Vehicle Co. the motor is built into the rear axle, which is made hollow and of a sufficient diameter to contain the motor. The armature shaft is also hollow, so that the drive shafts may extend through from the sockets of the differential to the center of each rear wheel. The wheels are driven by a spur-gear reduction mounted inside the wheels. The wheels have steel-plate sides instead of spokes, so that the gears may run in oil and are thoroughly protected from dirt.

Couple-Gear Wheel. — A device built by the Couple-Gear Freight Wheel Co. has the electric motor mounted inside the wheel. The motor armature carries a pinion on either end which engages with a gear rack on either side of the wheel. The sides of the wheels are dished steel plates. These wheels are being used on both front-wheel-drive and four-wheel-drive machines Being located in the wheels, they do not interfere with the use of the ordinary steering knuckle arrangement.

"Differential" or Differential Gear. — All cars equipped with a single motor have a differential gear to permit the transmission of power from motor to both wheels and yet permit the wheels to revolve at different speeds. The working parts of the bevel-gear type of differential are indicated in Fig. 2. The large gear *D* is driven by the pinion *S* on the propeller shaft. Mounted within *D* and carried on spindles in the plane of *D* are four small bevel pinions *P* (only two are shown in the figure). Bevel gears *G* and *G′*, attached to the ends of the axle shafts, mesh with the pinions *P*. Thus if *G* is stationary or presents greater resistance than *G′*, the pinions *P* will roll over the surface of *G* and also upon their own spindles, turning *G′*. If the resistance of both wheels is the same, the pinions *P* will not revolve on their spindles and the gears *G* and *G′* will be revolved at the same speed. (*Description abstracted from the Electric Vehicle Handbook, by Cushing and Smith.*)

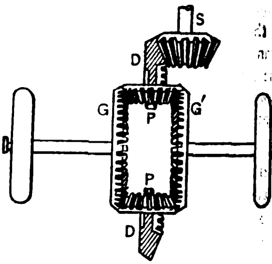

Fig. 2.

Efficiency of Transmission. — (*See also article on Gears and Gearing.*) Beaumont gives the following figures on the transmission efficiency of different types of automobile mechanisms.

Source of loss of power	Efficiency, per cent
One chain, and one and one-half pairs of bearings	89.5
One set of gears, two pairs of bearings	82.0
One set of gears, equivalent of two chains, three pairs of bearings	74.0
Two sets of gears, four pairs of bearings	70.0

The following figures, given by F. Burgess (*Trans. Soc. Auto. Eng., 1912, Vol. 7, Part 2, p. 196*), are the results of an efficiency test of a straight type worm and worm gear for rear-axle drive of electric automobiles. The worm gear was made of phosphor bronze and had 39 teeth; the worm was made of case-hardened steel and had 4 threads.

R.p.m. of worm	Temperature of worm gear, °F.	Input, trans. dyn., h.p.	Output, brake h.p.	Efficiency, per cent
1393	74	1.64	1.03	61.2
1416	86	3.41	3.17	93.2
1370	90	5.48	5.13	93.7
1389	94	6.72	6.24	93.0
1400	108	9.43	8.5	90.2

Batteries. — There are three types of batteries in general use in electric vehicles, namely, the pasted lead, the "Iron-clad" lead, and the Edison. The performance characteristic (except life) of the first two are identical and are described in the article on *Batteries, Storage, Lead Type;* the characteristics of the Edison battery are described in the article on

Type. The desirable characteristics of a battery for vehicle service are given in the article on *Batteries, Storage, Applications of.*

Number of Cells; Number of Plates. — Practice among electric automobile manufacturers is rapidly becoming standardized upon the installation of 40, 42 or 44 cells of lead battery and 60 cells of Edison battery in both passenger and commercial classes of cars, as these numbers permit of charging from a 115-volt circuit with minimum loss in the rheostat. The exceptions are chiefly in the lower-priced pleasure cars in which from 24 to 38 lead cells are sometimes used, and the small capacity delivery wagons in which 48 Edison cells are commonly used. A 24-volt battery is usually used on industrial trucks. For a given watt-hour capacity a battery of a small number of cells is cheaper to buy and maintain than one of a large number of cells, but is more expensive to charge from constant-potential d-c. mains on account of rheostat losses.

As pointed out above the rated current output of a vehicle battery for 4 hours should not be exceeded by the current required to drive the car at rated speed on hard, level asphalt (*see section above on Size of Motor*). Thus when the current under these normal conditions is known, the number of plates per cell can be determined from a knowledge of the discharge rates of the various types of plates which are under consideration. For A-type Edison batteries the normal (5-hr. rate) is 7.5 amp. per positive plate; conservative designers use this rate in applying batteries rather than the value of 9.5 amp. which could be obtained for 4 hours. The discharge rates per positive plate of 5¾ by 8⅝ in. for 4 hours and for the so-called "normal" times of discharge for the types of vehicle batteries made by the Electric Storage Battery Co. are given in the accompanying table.

Type of plate	"Normal" discharge		Amperes per positive plate discharged during 4 hr.
	Time, hours	Amperes per positive plate	
Exide	4	7	7
Hycap	5	5.5	6.5
Thin	6	4.25	5.8
Iron-clad	4.5	7	7.6

It will be noted that the above rule of installing a battery whose 4-hour discharge rate is approximately equal to the current required for running at rated speed under good street conditions will furnish a minimum practicable battery capacity, and that this capacity may have to be increased materially in order that the car may meet the necessary mileage requirements when operating on grades or poor roads. There is a tendency among electric car manufacturers to recommend battery equipment for given conditions which is much more liberal than they recommended a few years ago.

Construction of Battery Box. — In passenger cars a tight wood-lined compartment is usually supplied for carrying the battery. In commercial cars the battery compartment is also wood lined, and provision is commonly made for ventilation to facilitate charging in warm weather. The details of a typical underslung battery box are shown in Fig. 3. With this construction strips may be laid in the spaces between the floor boards in order to make the compartment entirely inclosed, as is frequently desirable in cold weather. A

tight covering should be provided with an underslung battery box to prevent dirt and water falling upon the battery from the floor of the car.

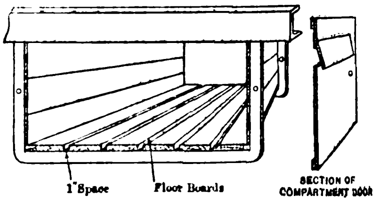

Fig. 3. Typical Underslung Battery Box

Controllers.—Controllers are usually of the drum type with two running positions, known as the "field-series" and the "field-parallel" positions. The motor field coils are arranged in two groups for connection either in series or in parallel. The cells of the battery are connected permanently in series. In starting, either 2 or 3 resistance notches are used before the "field-series" notch is reached. A shunt on the fields is frequently used between the "field-series" and the "field-parallel" notches. For normal full-speed running the two groups of field coils are in parallel. An emergency speed is obtained by shunting a resistance around the fields when connected in parallel. It was formerly customary to split the battery into two groups which could be operated in series-parallel in order to obtain 4 economical running speeds, but trouble was experienced from the unequal discharge of the two halves of the battery. In the magnetic type of control which is used to some extent in passenger cars the changes in connection are affected by relays operated by an auxiliary electric circuit. Some forms of regenerative control have been tried with only moderate success, although in hilly localities regenerative control may be desirable on account of protecting the mechanical brakes.

Tires.—Tires for use on electric automobiles are generally more resilient than the standard types of tires commonly used on gasoline automobiles. "High efficiency" or special electric tires, both solid and pneumatic, have been developed by most of the tire manufacturers in response to a demand by the operators of electric cars for tires with low consumption of energy. Standard types of tires may consume as much as 100 per cent more energy than the special electric tires. Experience has shown that the smooth starting characteristic of an electric motor produces much less abrasion than the uneven acceleration with a gasoline motor, so that the rubber compounds in tires for electric cars are usually softer than the compounds in tires for gasoline cars and yet the lives of both types in point of distances traveled are approximately the same. Electric pneumatic tires are usually of the cord type; i.e., the "fabric" consists of parallel strands of cotton impregnated with rubber gum. Some types of solid electric tires, such as the Motz, have the sides undercut at intervals to allow for the "flow" of the rubber when under compression.

Carrying Capacities of Tires.—In general a tire of small transverse tread will consume less energy than a tire of the same compound with large tread when operating on a smooth, hard surface; also, a large wheel diameter gives less energy consumption than a small one as well as improving the riding qualities on uneven surfaces. The Society of Automobile Engineers have recommended the use of 32-, 36- and 40-in. tires as standard practice. (*See Trans. Soc. Auto. Eng., 1914.*) But in selecting tire equipment it should be remembered that there is a limit to the load which any size of tire will stand without permanent injury by disintegration. The maximum carrying capacities per wheel at present (1914) recommended by the B. F. Goodrich Co., are given in the following table, as are also the recommended inflation pressures for pneu-

matic tires of the fabric type; the recommended inflation pressures for cord tires are approximately 80 per cent of the values in the table.

TABLE IV. — MAXIMUM CARRYING CAPACITIES OF TIRES

Size of tire, inches	Maximum load, pounds per wheel				Recommended inflation pressure, lb. per sq. in.
	Solid		Pneumatic*		
	Single	Dual	Rear wheel	Front wheel	
2	500	......	....	....	...
2½	750	1,950	....	....	...
3	950	2,475	375	450	55
3½	1375	3,675	500	600	60
4	1750	5,000	750	900	70
5	2000	6,000	1100	1300	90
6	3000	8,000	....	....	110
7	4000	10,000	....	....	...

* The allowable load per wheel for a given width of tire increases slightly with the wheel diameter. The sizes quoted in the table are 32 by 3, 32 by 3½, 36 by 4, and 36 by 5 in.

Examples of Commercial Car Design. — The following table gives data from the standard specifications of a number of the electric trucks made by the General Vehicle Co.

TABLE V. — DATA ON "G.V." ELECTRICS

Rated load capacity, lb.	750	1000	2000	4000	7000	10,000
Speed, miles per hour	12	12	10	9	8	7
Miles per battery charge, approx.	45	45	45	45	40	35
Lead battery equipment:						
Number of cells	44	44	44	44	44	44
Number of pasted plates	9	11	13	17	21	25
Number of iron-clad plates	7	9	11	15	17	19
Edison battery equipment:						
Number of cells	48	60	60	60	60	60
Designation	A-4	A-5	A-6	A-8	A-10	A12
Tires, front, in.	32×2½	36×3	36×3½	36×4	36×6	36×7
Tires, rear, in.	32×2½	36×3	36×3½	36×3D*	36×3½D*	36×5D*
Weight, chassis and lead battery, lb.	2700	3325	4370	6000	8225	9300
Per cent weight on rear wheels†	50	50	50	50	50	50
Ampere rating of motor on 85 volts	16	20	28	28	40	50

* Dual. † Chassis and battery.

PERFORMANCE; ENERGY CONSUMPTION.—The energy consumed by an electric automobile will depend very largely upon the care of the driver in coasting up to stops instead of braking directly from full speed; other im-

portant factors in determining the energy consumption are efficiencies of battery and driving mechanism, tire equipment, nature of roads and grades, number of stops and miles per day, etc. However, for a car which is operated under similar conditions day after day, the energy consumption per mile should be reasonably uniform, so that any considerable increase in energy consumption may indicate either careless driving or poorly adjusted mechanism, as for instance a dragging brake. Table VI gives a series of approximate values for energy consumption per mile which are based upon a large number of reports upon the operation of electric cars in large cities; these reports were collected in connection with the Vehicle Research study conducted during 1912 to 1914 by the Electric Research Laboratory of the Massachusetts Institute of Technology. These figures are representative of experience with modern types of cars under average city conditions; the consumption for a given set of conditions may vary from the figures by as much as 20 per cent, either above or below.

TABLE VI.—ENERGY CONSUMPTION IN KILOWATT-HOURS PER MILE

Type of car	Lead battery	Edison battery
2-passenger	300	400
1,000-pounds	550	750
2,000-pounds	650	900
4,000-pounds	830	1150
7,000-pounds	1100	....
10,000-pounds	1400	....

Garage Load Curves.—Fig. 4 shows a typical daily load curve for a public garage handling 75 electric passenger cars; Fig. 5 shows a similar typical daily curve for the private garage of a large department store (*see paper by E. E. Witherby, Proc. N. E. L. A., 1913*). The "off-peak" character of the electric vehicle load is indicated by these curves. In order to attract such off-peak load most light and power companies are now offering special rates for vehicle charging, frequently with the provision that no charging shall be done during the time of the power company's daily peak load.

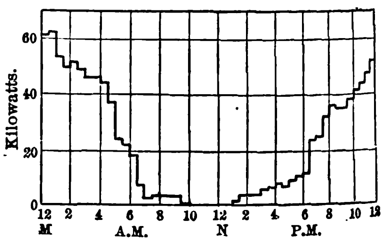

Fig. 4. Daily Load Curves of a Public Garage Handling Passenger Cars

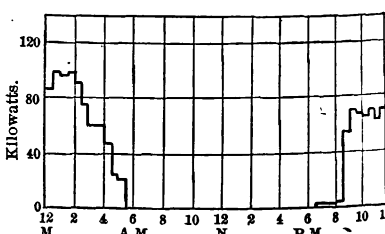

Fig. 5. Daily Load Curves of a Private Garage Handling Trucks

OPERATION.—In the operation of electric automobiles particular attention should be paid to the proper care of the storage batteries. The battery manufacturing companies issue complete instructions for the proper handling of their cells. The

manufacturers of the battery, as well as the manufacturers of the car, are usually ready to confer with the operator of a car to the end that he may obtain satisfactory service from his machine. They should be consulted on the first indication of trouble in their respective portions of the equipment, should the cause of the trouble not be understood.

The following points upon the care and operation of electric cars should be observed:

1. A battery must always be charged with direct current and in the right direction.
2. Never bring an exposed flame near a battery while charging or immediately afterwards.
3. Do not allow the battery temperature to exceed 110° F.
4. Keep the cells filled to the proper level by adding distilled water only. Never put acid in an Edison battery under any circumstances.
5. Keep the outside of the cells free from foreign substances, both solid and liquid.
6. For boosting a lead battery during a specified short period, the maximum current rate I which may be used without reaching the gassing point is the quotient of the ampere-hours Q previously discharged (read from ampere-hour meter), divided by 1 plus the hours H available for boosting, viz., $I = \frac{Q}{1 + H}$.
7. The mechanism of a car should be inspected carefully at least once in two weeks.
8. A car should be entirely overhauled at least once each year, in order that worn parts may be located and replaced.

COSTS. — The initial cost of passenger electrics depends so largely upon the character of body and fittings that it is impossible to quote any but the most general figures; prices range from $1500 to $5000, the most popular cars selling at from $2500 to $3000. It is likewise impossible to quote definite figures on the cost of operating pleasure cars; prices for electricity for private charging vary from about 3 cents per kw-hr. upward; the cost of storage in public garages ranges from $15 to $30 per month, plus a charge for electricity at from 4 cents per kw-hr. upward.

The initial cost of commercial electrics ranges from about 40 cents per pound for large trucks to 55 cents per pound for light cars, based upon usual equipment of solid tires, lead batteries and express or stake bodies. (*For typical weights see Tables I and V.*) Special equipment such as winches or dumping bodies is additional. The operating expense will vary widely for different conditions, so that it is impossible to predetermine the total cost of operating a truck of a given rating without knowing the requirements of the service, the nature of the roads, and the general method of handling and caring for the car. The data in the following table are deduced from estimates by Pender and Thomson given in *Vehicle Research Bulletin No. 3*, issued in 1913 by the Massachusetts Institute of Technology. The figures are based upon reports of the operation of a large number of cars during from 1 to 4 years in city trucking and delivery services.

TABLE VII. — COST OF OPERATING ELECTRIC TRUCKS

Rated load capacity, lb.	**1000**	**4000**	**7000**	**10,000**
Miles per year considered	10,500	9100	8850	8000
Cents per mile for:				
Tires, repairs and battery*	6.8	8.3	11.2	14.3
Electricity at 3 c per kw-hr	1.5	2.2	3.0	3.6
Dollars per year for:				
Garage and lubricants	215	235	255	285
Driver and helper	1,000	1140	1210	1210
Depreciation, interest and insurance	380	464	532	685
Total annual expense, dollars	2,455	2794	3252	3610

* Pasted plate lead battery.

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[H. F. Thomson.]

AUTO-TRANSFORMERS AND COMPENSATORS. — (*See also Starters, Motor; Transformers.*) An auto-transformer or single-circuit transformer, also called a "compensator," consists of a transformer having the usual iron core but only one electrical circuit instead of two. This circuit is tapped at various points as shown in Fig. 1, and the primary and secondary circuits, while independent outside the transformer, unite in the same winding in the transformer. If an alternating voltage is impressed across the points *ab*, a magnetizing current will flow in the winding setting up an alternating flux which will link every turn and induce therein an alternating voltage. The voltage between any two taps, as *ac*, is proportional to the number of turns between the taps; thus any ratio of voltages may be obtained. If the secondary *ac* is loaded a current will flow in the primary and the primary and secondary currents will flow in the two parts of the winding as indicated in the figure.

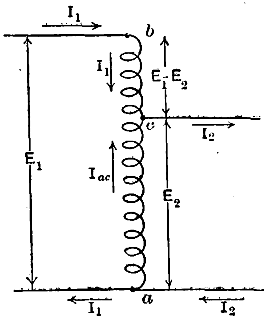

Fig. 1. Auto-transformer

VOLTAGE AND CURRENT RELATIONS. — Let N_1 be the total number of turns between a and b, N_2 the turns between a and c, E_1 the voltage across the terminals a and b and E_2 the voltage across the terminals a and c. Then, neglecting the resistance and reactance of the winding,

$$E_2 = \frac{N_2}{N_1} E_1.$$

Let I_1 be the current entering the terminal b, I_2 the current in the *external* circuit connecting a and c, and I_{ac} the current *in the transformer winding* between a and c. Then, neglecting the resistance and reactance of the winding and the magnetizing current,

$$I_2 = \frac{N_1}{N_2} I_1,$$

$$I_{ac} = I_2 - I_1 = \frac{N_1 - N_2}{N_2} I_1.$$

The current in the turns between b and c is

$$I_{bc} = I_1.$$

Auto-transformer Versus the Two-circuit Transformer.—For $N_2 = \frac{1}{2} N_1$ the current in the turns between a and c would be just equal, neglecting the exciting current, to the current in the turns from b to c (but opposite in direction). Consequently for a 2 to 1 transformation but one winding of an ordinary two-circuit transformer could be used, provided a tap was available at the middle point of this winding, and the rated output of the transformer could be obtained without the current in this winding exceeding its rated value. Since under these conditions there would be no current in the second winding, the heating of the transformer would be less than it would be were the transformer used as an ordinary two-circuit transformer, and therefore a greater output could be obtained without exceeding the nominal temperature rise.

In general, for the same power input into the connected load an auto-transformer requires but $\frac{E_1 - E_2}{E_1}$ of the copper required for a two-circuit transformer,

where E_1 is the high-tension and E_2 the low-tension voltage. The higher the ratio of transformation the less the saving in copper. There is also a serious objection to an auto-transformer of high ratio of transformation, in that an accidental ground on the high-tension lead, *b* in Fig. 1, would establish a high voltage between the low-tension leads and the ground, which may cause a dangerous shock to a person touching either low-tension lead or may cause other damage. This may be partially prevented by grounding the common point of the high- and low-tension circuits, the point *a* in Fig. 1, in which case an accidental ground on the high-tension side would produce a short-circuit, which would open the circuit breaker in the primary circuit, *provided the resistance between the two grounds is low and the circuit breaker operates properly.* These two provisions, however, may not always be realized even though reasonable care is taken, and it is therefore not considered good practice to use an auto-transformer of high ratio of transformation, except in special cases where economy of space is an important factor and danger from shock can be guarded against, e.g., on a-c. locomotives.

RATING OF AN AUTO-TRANSFORMER.—In commercial practice the rating of an auto-transformer in volt-amperes is taken equal to the difference between the high- and low-tension voltages multiplied by the rated current in that part of the winding, *bc* in Fig. 1, across which this voltage exists. The capacity of an auto-transformer for a given work bears to the capacity of a two-circuit transformer for the same purpose the ratio $\frac{E_1 - E_2}{E_1}$. For example, to step down the voltage of the supply mains from 500 to 400 volts and supply a load of 100 kv-a. would require an auto-transformer having a rating of $\frac{500 - 400}{500} \times 100 = 20$ kv-a. as against a two-circuit transformer having a rating of 100 kv-a. The weight, dimensions and cost of an auto-transformer are very nearly the same as for an ordinary two-circuit transformer having the same voltage and kv-a. rating.

APPLICATIONS.—Auto-transformers are used chiefly where the required change in voltage is small, e.g., for motor starters (*see Starters, Motor*), and for balancing the voltage between two or more circuits. The smaller the ratio of transformation the greater is the gain in cost and efficiency resulting from the use of an auto-transformer instead of a two-circuit transformer. Auto-transformers are also used to provide a neutral for the Edison three-wire system; see *Distribution Circuits*. In single-phase railway work single-phase auto-transformers are used on the locomotives for transforming the trolley voltage (from 3000 to 6000 volts) to the motor voltage (about 500). This is a rather high ratio of transformation for an auto-transformer, but as the saving in weight is very important and as one terminal is grounded, it has proven satisfactory for this service; see *Locomotives, Electric.*

DESIGN.—The design of an auto-transformer is quite similar to that of a two-circuit transformer (q.v.) and the leakage reactance is calculated in the same way. For low-leakage reactance the primary and secondary coils must not be entirely separated, but each must be divided into sections and intermixed to as great an extent as possible. Having found the magnetizing current, core-loss resistance and reactance in the usual manner, the calculations follow the same methods as for power transformers. In general, the efficiency at unity power factor is given by the equation

$$\text{Efficiency} = \frac{E_2 I_2}{E_2 I_2 + (r_1 - r_2) I_1^2 + r_2 (I_2 - I_1)^2 + \text{core-loss}},$$

where r_1 is the total resistance of the winding measured between the high-tension terminals and r_2 the resistance measured between the low-tension terminals. The efficiency is better than that of a two-circuit transformer of the same kv-a. rating.

TESTS, SPECIFICATIONS, ETC. — See *Transformers*.

DIMENSIONS, WEIGHT AND COSTS. — For the same kv-a. rating the dimensions, weight and cost are very nearly the same for auto-transformers as for ordinary two-circuit transformers; see *Transformers*.

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[W. I. Slichter.]

BALANCES, CURRENT. — (*See also Ammeters; Electrodynamometers.*) The Kelvin balance consists of a system of two coils mounted one at each end of a rigid beam and free to move between two pairs of field coils. The balance may be used as an ammeter, voltmeter or wattmeter. It may be used for either a-c. or d-c. measurements, but for d-c. work it has been practically superseded by laboratory standard voltmeters. For a-c. work it is used only as a laboratory standard for calibrating other instruments.

A current balance when carefully constructed may be used for the absolute measurement of electric current, since the force of attraction between the coils can be calculated in terms of their dimensions and the current flowing. It has been successfully used to determine in absolute measure the electrochemical constant or Faraday (*see Electrochemistry, Principles of*).

Principle of Operation. — The principle of the current balance is similar to that of the electrodynamometer. The connections for current measurements are shown in Fig. 1. The current passes through all the coils in series and as a result of the action of the magnetic fields, the left-hand end of the beam is depressed and the right-hand end is elevated. The beam on which the middle coils are mounted has an index pointer and carries a weight that may be slid along a scale on the beam. With no current on and the sliding weight at zero on the scale, a counter weight on the swinging system is so adjusted that the index pointer on the system is opposite a fixed mark and the system is balanced.

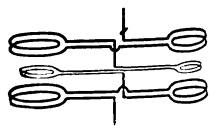

Fig. 1. Connections for Current Balances

Passing current through this system destroys the balance which is restored by sliding the weight along the beam. Since the torque exerted on the moving system is proportional to the square of the current, the current flowing will be proportional to the square root of the distance through which the weight has to be moved to balance the beam.

In the case of the wattmeter the current passes through the fixed coils, while the coils on the moving system serve as the potential coils. In this case the power is directly proportional to the distance through which the weight has to be moved to balance the system.

Range and Cost. — Kelvin balances are built in 5 standard sizes. The useful range and approximate cost of each instrument is given below.

Type	Range, amperes	Approx. cost
Centi-ampere	0.025–1	$150
Deci-ampere	0.25–10	150
Deca-ampere	5–100	150
Hekto-ampere	30–600	150
Kilo-ampere	100–2500	250

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[H. Pender and H. R. Ranken.]

BATTERIES, PRIMARY. — (*See also Cells, Standard; Electrochemistry, Principles of.*) A primary battery or cell is a device for the *direct* transformation of chemical energy into electrical energy. For convenience of treatment primary batteries may be classified as wet batteries, dry batteries and standard cells. The latter are treated in the article on *Cells, Standard.* The wet battery at one time was largely used in laboratory testing, for telephones, bells and other devices requiring small amounts of energy. In recent years it has been largely supplanted by the dry battery and small storage battery cells (*see Batteries, Storage*). Dry batteries are now (1913) used, in this country alone, to the extent of some 50,000,000 a year, chiefly for gas-engine ignition and telephone work (*see Gas Engines and Telephone Instruments*).

Poles and Electrodes. — The pole or terminal of a battery which is at the *higher* potential is called the *positive* pole or terminal, the other pole being called the negative pole or terminal. The negative pole is the anode or *positive electrode* or plate and the positive pole is the cathode or *negative electrode* or plate. For example, in a copper-acid-zinc battery the copper is the positive pole but the negative electrode or plate.

THEORY OF PRIMARY BATTERIES. — The modern theory of primary batteries is fully discussed in the article on *Electrochemistry, Principles of.* Only one or two points of practical moment will be mentioned here.

Electromotive Force. — A battery consists essentially of two metallic conductors or poles dipping into an electrolyte. Copper or carbon is commonly employed for the positive pole and zinc for the negative pole. The electrolyte may be sulphuric or nitric acid or sal-ammoniac, caustic soda or other salt. For a battery made of given materials the open-circuit e.m.f. is always the same provided the temperature, degree of concentration of the electrolyte and the purity of the materials are the same. The terminal e.m.f. or potential difference (p.d.) on closed circuit is always less than the open-circuit e.m.f. due (1) to the internal resistance of the battery, (2) to the polarization of the battery and (3) to the exhaustion of the battery.

Internal Resistance. — Let E be the initial open-circuit e.m.f. of the cell and V be the p.d. across its terminals, when it is supplying a given current I, then the "apparent" internal resistance of the cell is

$$r = \frac{E - V}{I}.$$

This is not the true resistance of the cell, for its net e.m.f. when a current I is being drawn from it is less than the initial open-circuit e.m.f. due to polarization (*see below*) and partial exhaustion. If, however, after measuring the p.d. on closed circuit the current is interrupted and the open-circuit e.m.f., say E', be measured *immediately* (i.e., before the polarized condition of the cell has had time to change) then the true internal resistance of the cell is

$$r' = \frac{E' - V}{I}.$$

Polarization. Depolarizers. — Polarization is the name applied to the changes produced in the relative concentrations of the electrolyte at the two poles of a cell or to the production at the poles of new chemical substances (such as hydrogen) as a result of the flow of current through it. A depolarizer is any substance which when placed in the electrolyte or on the poles of the cell will partially or wholly prevent these changes. Polarization always tends to reduce the effective e.m.f. of the cell. When a cell which has been polarized is open-circuited, the relative changes in concentration gradually disappear, due

to diffusion, and any new substances formed also tend to diffuse uniformly through the electrolyte, with the result that the open-circuit e.m.f. returns in time to nearly its original value, provided the active materials are not exhausted.

The chief cause of polarization in a cell is the formation of hydrogen gas at the positive pole or to the transfer of the metal from the negative pole to the positive pole. The depolarizer is usually an oxidizing agent which reduces the hydrogen or metal liberated at the positive pole to a form readily soluble in the electrolyte and thereby prevents its accumulation at the positive plate. This is the principle on which depends the depolarizing action of the various metallic oxides, such as manganese peroxide or cupric oxide. The same result may be obtained by surrounding the positive pole by a solution of a salt of itself which has the same acid radical as the electrolyte surrounding the negative pole, but which is less soluble, the two solutions being kept practically separated by a porous cup which renders the diffusion of one into the other very slow.

TYPICAL WET BATTERIES. — Numerous forms of wet batteries have been used; only some of the more common forms can be described here. The materials used and the e.m.f. developed and approximate range of internal resistance are listed in the following table.

TYPICAL WET BATTERIES

	Daniell	**Gravity**	**Bunsen**	**Chromic acid**	**Edison-Lalande**	**Leclanché type**
Positive pole....	Cu	Cu (b)	C	C	C	C
Negative pole....	Zn (a)	Zn (a)	Zn (a)	Zn (a)	Zn (a)	Zn
Electrolyte....	H_2SO_4 or $ZnSO_4$	$ZnSO_4$ or $MgSO_4$	H_2SO_4	H_2SO_4 or NaCl	KOH or NaOH	MnO_2
Depolarizer......	$CuSO_4$	$CuSO_4$	HNO_3	CrO_3	CuO	NH_4Cl
Separator........	Porous pot	None	Porous pot	With or without pot	None	With or without pot
E.m.f., volts.....	1.07 to 1.14	1	1.9–1.95	2	0.75	1.5
Resistance, ohms	0.3 to 30	0.1 to 6	Low	0.5 to 4	0.02 to 0.1	1 to 5

a. Amalgamated. b. In the Krüger cell copper-plated lead is used.

Daniell Cell (Fig. 1). — A typical form of this cell is shown in Fig. 1. Other forms, differing in certain details, are Muirheads cell, the Siemens & Halske Daniell cell, Minotto's cell (which may also be classed as a gravity cell, *see below*), etc. J is a glass or glazed earthenware jar containing a concentrated solution of copper sulphate, P is a porous pot containing dilute sulphuric acid (about 10 per cent by volume) or zinc sulphate solution or both, C is the positive copper pole and Z the negative zinc pole which is usually amalgamated. The chemical reaction which takes place may be represented by the formula

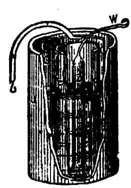


Fig. 1. Porous Pot Daniell Cell

$$Zn + CuSO_4 = Cu + ZnSO_4$$

A current of 1 ampere for 1 hour deposits 1.186 grams of copper and liberates 1.219 grams of zinc. Hence

0.042 oz. copper is deposited per ampere-hour,
0.043 oz. zinc is used up per ampere-hour,
0.164 oz. copper sulphate crystals are used up per ampere-hour,
0.106 oz. zinc sulphate is formed per ampere-hour.

The latter when crystallized out will form 0.189 oz. zinc sulphate crystals. These figures do not include any loss of zinc due to local action, which may amount to 10 per cent or more.

E.M.F. of Daniell Cell.—The e.m.f. of a Daniell cell varies from about 1.07 volts to 1.14 volts, depending on the density of the copper sulphate solution and on the amount of zinc sulphate present in the dilute sulphuric acid. The cell has its highest e.m.f. at the start when the sulphate of copper solution is saturated and no sulphate of zinc has formed. Hence, in order that the e.m.f. shall remain more nearly constant, it is better to start with *both solutions saturated*. The resistance of the cell will be higher and its e.m.f. lower than when dilute sulphuric acid is used, but this lower value of about 1.10 volts will be maintained nearly constant while the cell is sending a current.

Internal Resistance of Daniell Cell.—This depends not only upon the dimensions of the cell but also upon the porosity of the pot or other separating medium. The type of cell shown in Fig. 1, having a pot 7 inches high which is quite porous, has an internal resistance of about 0.3 ohm. The resistance of the Siemens type of Daniell cell is from 10 to 15 ohms, and the resistance of Minotto's modification of the Daniell cell, in which a layer of sawdust or sand is used in place of the porous cup, may be as high as 30 ohms.

The resistance of a Daniell cell, like that of liquids generally, *diminishes* with *increase* of temperature.

Local Action.—Impurities in the zinc form with the zinc small short-circuited voltaic cells, resulting in a wasting away of the zinc without producing a current in the external circuit. This can be largely prevented by amalgamating the zinc, i.e., coating it with mercury, which is readily done by thoroughly cleaning the zinc by dipping it into dilute sulphuric acid and then rubbing mercury over its surface. In the Daniell cell metallic copper also forms in the pores of the porous cup where the zinc touches it; this can be prevented by covering with paraffin those portions of the cup which may come into contact with the zinc.

Care of Daniell Cells.—The type of cell shown in Fig. 1 must be taken to pieces when not in use. If it has to be put to one side for only an hour or two, it will be sufficient to lift the porous pot with the contained zinc rod bodily out of the cell, and to place it in another empty jar, or stand it in a dish while out of use.

Gravity Battery (Fig. 2).—The principle of this cell is the same as that of the Daniell cell, except that no porous pot is used, the copper sulphate and zinc sulphate being maintained separate by gravity. There are various modifications in the form of construction, known as the Meidinger cell, Calland cell, Kelvin tray battery, Krüger cell, etc. The gravity cell is still used in telegraph and telephone work.

The copper electrode is placed at the bottom of the cell, and is then covered with copper sulphate crystals. The zinc electrode is then put in place and the jar either filled with dilute zinc sulphate or with dilute sulphuric acid. When first set up the internal resistance is high, but if the cell is short-circuited for a considerable time the resistance is reduced due to the formation of zinc sulphate. To prevent evaporation the solution is covered with a layer of mineral oil.

Care of Gravity Battery. — The resistance of the cell increases rapidly with decrease of temperature; it should therefore be kept at a reasonably high temperature, say 70° F. A gravity cell must, of course, not be moved about, or if moved great care must be taken to avoid the two liquids being mixed together. To prevent the copper sulphate wandering to the zinc plate, it is well to allow the cell to send a weak current through an external circuit of considerable resistance even when the cell is not in ordinary use. The electrolyte should be renewed when the blue sulphate solution turns brown. The line of separation between the copper and zinc sulphates, or the "blue line," should be about halfway between the two electrodes.

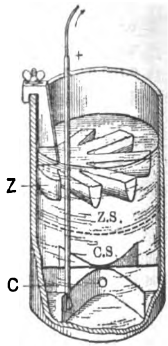


Fig. 2. Gravity Daniell Cell

Bunsen Cell. — In the Bunsen cell a zinc plate is placed in dilute sulphuric acid, as in the Daniell, but the copper plate is replaced by one of carbon and the copper sulphate solution by strong nitric acid, which is generally said to act as the depolarizer. The Grove cell differs from the Bunsen only in the use of platinum in place of carbon. Both cells give a high e.m.f., from 1.9 to 1.95 volts, and have low internal resistances, so they may be used for producing fairly large currents. When working the cells give off dark brown fumes of nitric peroxide, NO_2, and should be placed in the open air or under a chimney.

The chemical reaction which takes place may be represented by the formula $Zn + H_2SO_4 + 2\,HNO_3 = ZnSO_4 + 2\,H_2O + 2\,NO_2$.

Care of Bunsen Battery. — A Bunsen or Grove battery must be taken to pieces at the end of each day's use, since the mixing of the liquids through the walls of the very porous cup used to separate them would render the battery practically useless the next day. The porous pots should be placed in water after use, so that all the zinc sulphate solution may be dissolved out of the pores of the earthenware, for, otherwise, when the pots are dried the zinc sulphate solution will crystallize in the pores and cause the pots to fall to pieces.

Chromic Acid Cell (Fig. 3). — The chromic acid or potassium bichromate cell was devised by Poggendorff. In the original type of cell no porous pot was employed. In the Fuller cell, shown in Fig. 3, a porous pot is used. The depolarizer used is chromium peroxide, popularly called chromic acid, which may be purchased ready prepared, or may be formed by heating potassium or sodium bichromate with sulphuric acid (1 part by weight of the bichromate, 3 parts of acid and 9 parts of water). In the type of cell shown in the figure the wire connected to the zinc rod is well amalgamated or coated with gutta-percha to insulate it. In the porous pot containing the zinc, there is put a quantity of mercury to maintain the amalgamation, and either dilute sulphuric acid or a solution of common salt NaCl. The chromic acid solution is placed in the jar containing the carbon plate.

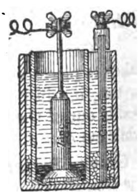


Fig. 3. Fuller's Mercury Bichromate Cell

The chemical reaction which takes place may be represented by the formula $3\,Zn + 2\,CrO_3 + 6\,H_2SO_4 = Cr_2(SO_4)_3 + 3\,ZnSO_4 + 6\,H_2O$.

This cell has an e.m.f. of about 2 volts, and is suitable for producing a fairly strong current for a short time. When much used the cell becomes saturated with the potassium and chromium sulphates, and a double salt, chrome alum, $K_2Cr_2(SO_4)_4$, crystallizes out and sticks so firmly to the bottom of the cell that it is somewhat difficult to remove.

Edison-Lalande Cell. — In this type of cell the positive pole is a plate of compressed cuprous oxide (CuO), the surface of which is reduced to metallic copper. The cuprous oxide acts as the depolarizer. The negative pole is amalgamated zinc and the electrolyte a strong solution (1 to 3 by weight) of caustic potash or of caustic soda.

A layer of heavy oil is poured over the solution to prevent evaporation and "creeping." No local action or polarization takes place in this cell; under normal conditions it is an easy matter to set it up to give any required number of ampere-hours, and to so proportion the constituents that they are all exhausted at the same time. This is a matter of considerable importance where closed-circuit working is employed, as in some systems of telegraphy and in alarm circuits. Although the e.m.f. of the Edison-Lalande cell is low, its resistance is also low, and the cell is capable of producing large currents. The cell may be left set up for months without deterioration.

The chemical reaction which takes place may be represented by the following formula

$$Zn + CuO + 2\,KOH = Cu + H_2O + K_2ZnO_2.$$

Leclanché Cells. — In the original form of this type of cell the positive pole was a plate of carbon embedded in a mixture of solid manganese peroxide and broken carbon contained in a porous pot. The electrolyte is sal ammoniac, NH_4Cl, and the negative pole zinc. The manganese peroxide acts as the depolarizer. The only object of the porous pot was to hold the carbon and manganese peroxide. In the later forms of this cell, such as the "agglomerate," Corsak, etc., the porous pot is dispensed with and a mixture of carbon and manganese peroxide are moulded together with a suitable binder, or the mixture of carbon and manganese peroxide is held together by a wrapping of canvas or sacking.

The chemical reaction which takes place may be represented by the formula

$$Zn + 2\,MnO_2 + 2\,NH_4Cl = Mn_2O_3 + H_2O + 2\,NH_3 + ZnCl_2.$$

If, however, too little sal ammoniac be present, zinc oxide or zinc oxychloride is formed instead of zinc chloride, and the solution becomes milky; hence when this happens, more sal ammoniac should be added.

The e.m.f. of a Leclanché cell is about 1.5 volts, but falls rapidly when the cell is used to send a strong current. It will, however, regain its value if the cell be left for some time unused, and it does not sensibly diminish when the cell is put to one side, even for some months. Hence, while the Leclanché cell is much inferior to the Daniell cell for the purpose of sending a steady current for an hour or two, it is much superior to the Daniell cell for producing intermittent currents at any time during the course of a year or more — for example, such currents as are employed for the ringing of electric bells, for house telephones, and for railway signaling.

Care of Leclanché Cells. — When the sal ammoniac becomes exhausted it should be thrown out and a new solution made. Three or 4 ounzes of sal ammoniac for a jar of ordinary size is required, the jar to be filled about one-third with water before putting in the electrodes.

DRY BATTERIES. — The modern dry battery may be looked upon as a modification of the Leclanché cell, the chief difference being that only enough water is added to the electrolyte to moisten an absorbent layer of pulp-board, blotting paper, cheese cloth or starch paste, this lining separating the positive and negative poles. The negative pole, which also serves as a container, is a hollow zinc cylinder, 6 inches high and 2.5 inches in diameter; the bottom of

this cylinder is also usually made of zinc. About 80 per cent of the dry batteries made in this country have these dimensions. The positive pole is a carbon rod, which may be either cylindrical or fluted. The absorbent layer above mentioned is placed next the zinc and is saturated with a solution of sal ammoniac and zinc chloride. The zinc chloride is necessary to reduce the rapid deterioration which would otherwise take place on open circuit. The space between this lining and the carbon electrode is filled with a mixture of granulated carbon and manganese peroxide, the latter being the depolarizer. The top of the cell is usually sealed with a pitch composition.

E.M.F. of Dry Batteries. — In new cells of practically all types the open-circuit e.m.f. is between 1.5 and 1.6 volts. The decrease in e.m.f. when the cell stands on open circuit is very slight, being only about 0.1 after the cell has stood many months. An open-circuit e.m.f. materially less than 1.5 volts is generally an indication of serious deterioration or of some other defect. The effect of temperature on the open-circuit e.m.f. is slight, amounting to only a few hundredths of a volt for all ordinary temperature ranges. Due to the relatively rapid polarization and increase of internal resistance with use, the average terminal voltage during the useful life of the cell is only about 1 volt.

Internal Resistance. — The internal resistance of a high-grade dry cell when new is usually less than 0.1 ohm, which may increase to 0.5 ohm within 9 or 12 months, even though the cell is not used during this time. The polarization of the cell in actual service causes a much greater decrease in the terminal e.m.f. than does the internal resistance drop, and therefore the internal resistance test is of little practical value.

Short-circuit Current. — Nine out of every ten users of dry cells consider the short-circuit current, i.e., the current produced through an ammeter having a relatively small resistance connected directly between the poles of the battery, as a direct measure of the value of the cell. There are other factors, however, which must be considered, such as the temperature of the cell, the service for which it is to be used, etc. According to D. L. Ordway (*Trans. Am. Electroch. Soc., 1910, Vol. 17, p. 346*) a standard 2.5 by 6-inch dry cell should give when new a short-circuit current (external resistance not over 0.01 ohm) of from 18 to 25 amperes; a cell giving a short-circuit current much above 25 or below 16 amperes should be looked upon with suspicion. A cell giving a short-circuit current much in excess of 25 amperes is liable to polarize rapidly, whereas if the short-circuit current is much under 16 amperes, it is probable that the cell has been made a long time or that cheap materials have been used.

The effect of temperature on the short-circuit current is pronounced. Between 10 and 80° C. the current increases about 1 ampere for each 10° increase in temperature. This effect is even more pronounced at very low temperatures. The short-circuit current returns to its normal value when the cell is restored to normal temperature.

Shelf-life. — The shelf-life of a dry cell of the ordinary sizes is usually considered as the time in months that the cell may stand on open circuit without its short-circuit current falling below 10 amperes. This current is about half the short-circuit current when the cell is new, and represents a value which is probably lower than the minimum point at which a dealer could dispose of a cell to the average consumer. The average shelf-life of high-grade cells is from 10 to 12 months, though the very best cells have a shelf-life of from 12 to 15 months. Many makes of cells have a shelf-life of only 8 to 10 months; and cells are on the market having a shelf-life of only 1 or 2 months.

The shelf-life is increased by storing the cells at a low temperature. Ordway gives the following results on standard dry cells kept in storage at the stated

temperatures. These cells give initially a short-circuit current of about 20 amperes.

Temperature at which cells were stored, ° C.	0	25	50	75
Short-circuit current at 25° C. after 5 months	18.1	17.4	0.5	0.4

Ampere-hour and Watt-hour Capacity. — The short-circuit current, however, is not a measure of the ampere-hours obtainable from the cell, as is indicated in the following table, taken from Ordway's paper. The ampere-hours given are those obtained from the various cells when discharged continuously through a resistance of 16 ohms until the closed-circuit voltage fell to 0.5 volt.

Brand of cell	A	D	G	J	M	P	V	X
Short-circuit current	33.0	24.5	22.5	20.9	20.1	19.2	11.6	6.6
Ampere-hours	24.0	33.5	40.0	18.2	11.7	30.3	13.6	4.5

The letters are arbitrary designations.

Although the short-circuit current falls off with the age of a cell, even though the cell is not used, the ampere-hour capacity does not decrease in the same ratio. Ordway gives the following tests on samples from the same lot.

	Freshly made	9 months after manufacture
Short-circuit current	22.4	3.6
Ampere-hour capacity (through 2 ohms to 0.25 volt)	24.9	20.2

Effect of Rate of Discharge on Capacity. — Ordway gives the following results of tests on standard 2.5 by 6-inch cells, when the cells are discharged continuously through 2, 4, 8, 16, 24, 32 and 40 ohms respectively. By "end point in volts" is meant the terminal voltage per cell at the end of the stated number of hours.

HOURS OF CONTINUOUS SERVICE OF 2.5 BY 6-INCH DRY CELLS

End point in volts	Resistances used in ohms						
	2	4	8	16	24	32	40
1.2	4.3	10	39	142	260	414	549
1.0	9.3	35	94	296	548	889	1148
0.8	16.5	51	143	414	751	1078	1550
0.6	28.2	76	225	954	1240	1600	1763
0.4	55	207	648	1197	1711	2280	2040
0.2	160	450	882	1318	1914	2626	3140

WATT-HOURS FROM 2.5 BY 6-INCH DRY CELLS DISCHARGED CONTINUOUSLY

End point in volts	Resistances used in ohms						
	2	4	8	16	24	32	40
1.2	3.7	4.3	8.1	15.2	18.8	21.7	23.8
1.0	6.7	13.0	16.5	26.9	33.4	39.8	42.0
0.8	9.7	16.3	21.5	32.8	40.3	44.6	50.6
0.6	12.5	19.4	26.6	48.9	49.5	52.7	53.2
0.4	15.4	27.3	39.1	52.6	54.3	58.2	54.8
0.2	19.8	32.6	41.5	53.3	55.2	59.3	57.1

Capacity on Intermittent Service. — Neither the ampere-hour nor watt-hour capacity of a battery on *continuous* service is a measure of its capacity on intermittent service. What the user wishes to know is the actual number of hours of service that he can obtain from a battery when used to operate a definite piece of apparatus, which as a rule takes current only intermittently, e.g., for telephone or ignition service. From curves given in Ordway's paper for 3 cells connected in series and discharged through the stated resistances the following table has been made up.

HOURS OF INTERMITTENT SERVICE OF 2.5 BY 6-INCH DRY CELLS

(The hours given are the hours the cell is actually supplying current)

End point in volts, 3 cells	Continuous discharge			5 minutes each hour night and day			5 minutes each hour, 8 hours per day, 6 days per week		
	5 ohms	10 ohms	20 ohms	5 ohms	10 ohms	20 ohms	5 ohms	10 ohms	20 ohms
3.6	4	12	30	4	45	80	3	15	45
3.0	9	25	65	29	65	145	12	30	60
2.4	15	40	100	48	90	190	23	45	75
1.8	25	60	150	65	130	225	35	70	100
1.2	42	140	475	80	160	250	52	100	150
0.6	95	...	...	93	200	290	70	115	165

From the above table it is apparent (1) that the terminal voltage falls off more rapidly on continuous discharge than on intermittent discharge, counting hours of *actual discharge* only, but (2) that during the latter stages the terminal voltage falls off more rapidly for the intermittent discharge, this latter effect occurring earlier as the external resistance is increased. The second effect is probably due to the deterioration of the battery while standing on open circuit. It should be kept in mind that the length of time the batteries were in service ... the intermittent tests were respectively 12 and 40 times the period of

Proper Arrangement of Cells for Best Results. — The data given in the above tables may be used as a basis for the determination of the number and proper arrangement of cells for a given service. For example, if a certain piece of apparatus has a resistance of 8 ohms and cannot be operated at a voltage under 0.8, then 143 hours of service can be obtained from a single battery. If however 4 cells in parallel are used, the drain on each cell will be one-fourth as great as before, which is equivalent to discharging each cell through 32 ohms. Therefore the 4 cells will give 1078 hours of service, or 270 hours per cell. If the 4 cells were connected in series, then drain on each would be 4 times as great as for a single cell, which is equivalent to discharging each cell through 2 ohms, but each cell could discharge to 0.2 volt instead of 0.8. The 4 cells in series would then give 160 hours, or only 40 hours per cell. The parallel arrangement is therefore the best in this case.

For ignition service 6 cells in series are usually required in order to obtain a sufficiently high voltage. On heavy service of this kind two or more such series groups connected in parallel are usually more economical than a single group. For telephone service 3 cells in series are usually required in order to obtain a sufficiently high voltage. For such light service a single group of cells is more advantageous than two or more groups in parallel; any gain which might at first sight be effected is more than offset by the deterioration caused by local action on standing. This may be seen by a consideration of the data on intermittent service given above.

Tests of Dry Batteries. — The proper testing of dry batteries has been the subject of considerable discussion during the last few years. See *Trans. Am. Electrochem. Soc.*, 1909 to date. A preliminary report was submitted by a committee of this society in April, 1912. The following is a brief summary.

In measuring the terminal voltage of a dry battery a voltmeter having a fairly high resistance, 300 ohms or more, should be used. The ammeter and leads for measuring the short-circuit current should have a combined resistance of 0.01 ohm. The internal resistance test is not recommended.

Service Tests. — Dry batteries to be used in telephone work should be tested by discharging 3 cells, connected in series, through 20 ohms rèsistance for a period of 2 minutes, each hour, during 24 hours per day and 7 days per week, until the closed-circuit voltage of the battery at the end of a period of contact falls to 2.8 volts (0.93 volt per cell). Report the results as the number of days during which the closed-circuit voltage remains above the limiting value of 2.8 volts.

For ignition service discharge 6 cells connected in series through 16 ohms resistance for 2 periods of 1 hour each per day, seven days per week. Determine at the end of every 12th period of closure the current which the 6 cells are capable of sending through a 0.5-ohm coil in series with an ammeter, the two being in parallel with the 16-ohm coil. The test is considered completed when the current through the 0.5-ohm coil at the end of a period of closure falls below 4 amperes. Report the results as the number of hours of actual discharge to this limiting value of the current.

For flash-light service discharge the battery through a resistance of 4 ohms for every cell in series, for a period of 5 minutes, once each day, until the closed circuit voltage at the end of a discharge period falls to 0.75 volt per cell in series. Report the result as the number of minutes of actual discharge until the voltage reaches this limiting value.

Full details for making these tests, together with a description of proper automatic arrangements for opening and closing the circuit at the designated times, will be found in the *Trans. Am. Electrochem. Soc.*, 1912, Vol. 21, p. 282. The timing device is an ordinary cheap clock provided with suitable contacts by means of which the hand closes the circuit.

Shelf-life Test. — Keep the cells open-circuited in a dry room at normal temperature. Determine the short-circuit current at the end of every 8 weeks. Report results as the number of months before the short-circuit current falls below 10 amperes.

COST OF DRY BATTERIES. — Dry batteries of standard size (2.5 by 6 inches) cost at retail from 10 to 25 cents, depending upon the quality of the battery. When bought by the barrel, good dry batteries may be had for 10 cents apiece.

BIBLIOGRAPHY. — Cooper, W. R., *Primary Batteries*, London, 1901; Ayrton & Mather, *Practical Electricity*, New York, 1911. For recent data on dry batteries see the *Trans. Am. Electrochem. Soc.*, particularly Ordway, D. L., *Some Characteristics of the Modern Dry Cell*, 1910, Vol. 17, p. 341, and *Report of Committee on Dry Cell Tests*, 1912, Vol. 21, p. 275 (which contains references to numerous other articles).

[H. PENDER AND H. R. RANKEN.]

BATTERIES, STORAGE, ALKALINE TYPE. — (*See also Automobiles, Electric; Batteries, Storage, Applications of; Batteries, Storage, Lead Type.*) The alkaline type of storage battery consists in its best-known form, i.e., the Edison battery, of an iron-nickel element immersed in dilute caustic-potash solution. The alkaline type of storage battery was first exploited commercially by Edison in 1904. At the present time (1914) it is being used extensively for the propulsion of electric vehicles, the operation of railroad block signals, the electric lighting of trains and the ignition and starting of gasoline engines.

THEORY OF ALKALINE STORAGE BATTERY.* — Numerous active elements have been used in the alkaline type of storage battery. The elements in the Edison type of alkaline battery consist of nickel hydroxide for the active material of the positive plate, iron for the active material of the negative plate and dilute potassium hydrate solution for the electrolyte. On discharge the iron is oxidized and the high nickel oxide is reduced to a lower oxide, while the electrolyte is not appreciably changed.

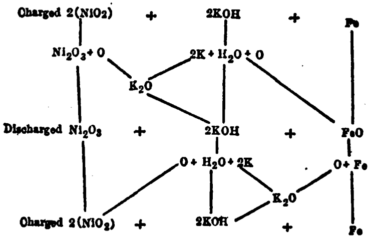
As far as the writer knows, an exhaustive study of all of the secondary reactions which take place during charge and discharge of this type of battery has not been made. It is generally conceded that the ultimate reaction of charge and discharge is simply a transference of oxygen backwards and forwards between the two electrodes. For this reason, only sufficient electrolyte need be provided to form a conductor between the positive and negative plates. A test of the electrolyte outside the pores of the plate shows that the specific gravity during charge and discharge does not vary to an appreciable extent.

The following simple reaction may be taken to indicate the probable final reaction on charge and discharge:

$$\begin{array}{cccccc} 2\,NiO_2 + 2\,KOH + & Fe & = & Ni_2O_3 + 2\,KOH + & FeO \\ +\text{ Plate} & -\text{ Plate} & & +\text{ Plate} & -\text{ Plate} \end{array}$$

This equation when read from left to right is the equation of discharge; when read from right to left it is the equation of charge. For simplicity, the water of the electrolyte has been eliminated from the equation. This, however, makes no difference, since it will be noted that the electrolyte indicated by 2 KOH remains unchanged whether the cell be charged or discharged.

Density of Electrolyte in Pores of Plates. — Although the density of the body of the electrolyte remains practically unchanged, it has been noted that, when the battery is on charge, the electrolyte becomes more dense in the pores of the iron plate and less dense in the pores of the nickel plate. A possible explanation of this action might be indicated by the accompanying diagram.



As neither potassium nor potassium oxide can exist in the presence of water, the reactions as shown in this diagram are necessarily hypothetical. However, although these elements cannot exist under these conditions, it is probable that the potassium oxide (K_2O) is formed in the pores of the cathode and immediately reacts with the water present in the electrolyte at this point, the result being a concentration of the solution at the cathode (iron plate) and a weakening of the solution at the anode (nickel plate).

* By T. Milton.

"Gassing." — In the alkaline type of cell, as in the lead type of cell, a certain amount of the water (H_2O) is broken up on charge by the electrolytic action of the charging current. The rate at which gas is evolved during a charge at the normal rate of current remains approximately constant during the first half of the charge, and increases rapidly during the latter portion of the charge. Of the total amount of gas evolved during a 7-hr. charge, about one-half is evolved during the first 5 hr. of the charge. As the oxygen and hydrogen thus liberated form an explosive mixture, provision should be made with a battery of any considerable size to carry the gases away to prevent their becoming a source of danger.

DESIGN.* — The only alkaline type of battery in use at present (1914) in the United States is that made by the Edison Storage Battery Co. This battery is made up of a positive plate having as the active material a high nickel oxide, a negative plate having as the active material powdered iron, and an electrolyte consisting of dilute potassium hydrate solution.

Positive Plate. — The positive or nickel plate consists of perforated steel tubes, nickel plated, filled with alternate layers of nickel hydroxide and pure metallic nickel in thin flakes. The nickel is added to give the necessary conductivity to the active material.

The tube is formed from a perforated ribbon of steel, nickel plated, and has a spiral lapped seam. This tube, after being filled with active material, is reinforced with steel bands, which prevent the tube expanding away from and breaking contact with its contents. The tubes are flanged at both ends and held in contact with a steel supporting frame or grid made of cold-rolled steel, nickel plated.

Negative Plate. — The negative or iron plate consists of a grid of cold-rolled steel, nickel plated, holding a number of rectangular pockets filled with powdered iron oxide. These pockets are made up of very finely perforated steel, nickel plated. After the pockets are filled they are inserted in the grid and subjected to pressure between dies which corrugate the surface of the pockets and force them into contact with the grid.

After the plates have been prepared, as outlined above, the positive or nickel plate is further oxidized electrolytically and the nickel hydroxide converted to a high oxide of nickel, probably NiO_2. The negative or iron plate is reduced electrolytically, the powdered iron oxide being converted into metallic iron. See section above on *Theory* for an explanation of the chemical action.

Separation and Insulation. — After the plates are assembled into a complete element, narrow strips of hard rubber are inserted between the plates to insulate them from each other. The side insulator is provided with grooves to insulate the complete element from the steel container. At the ends of the element, that is, between the outside negative plates and the container, are inserted smooth sheets of hard rubber. At the bottom, the element rests upon a hard-rubber rack or bridge, insulating the plates from the bottom of the container.

Assembly. — The jar is made from cold-rolled sheet steel, nickel plated. The walls of the jar are corrugated to give the greatest amount of strength with minimum weight.

The cover is of sheet steel, nickel plated, provided with three mountings, two being pockets for containing stuffing boxes about the terminal posts. The third mounting is an opening for filling the cell with electrolyte, and for the occasional (every four or five discharges) addition of distilled water to take the place of that which is lost during charge. This opening is provided with a

* By T. Milton.

hinged cap, on the under side of which is loosely hung a spherical segment of hard rubber. The latter, when the cap is closed, seats upon the circular opening beneath and is lifted by escaping gas, permitting its free egress. It closes the cell however to the admission of foreign gases or other material from without. The cap is opened only for filling and is held open or closed by a flat steel spring.

The plates are grouped on steel connector rods with steel spacing washers, all being attached to a steel terminal post. The terminal posts are insulated from the cover by means of hard and soft rubber washers and bushings.

The cells are assembled in wood trays, the positive and negative terminals of adjacent cells being connected together by copper connecting links. Each individual cell is insulated from the adjacent cell and from the containing wood tray.

RATING AND PERFORMANCE.—It is standard practice to rate an Edison battery in terms of the amperes it will give continuously for 5 hours. The rated capacity in ampere-hours is then 5 times this current rate. The normal rate of charge is the same as the normal rate of discharge and the time of normal charge is given as 7 hours. In the trade designations employed, e.g., A-6, the letter designates the size of the plate and the numeral the number of positive plates per cell, i.e., in the A-6 cell there are 6 positives; see below under *Dimensions, Weight and Cost*. The values of the rated current and output for each size of cell are given in the following table; the average voltage per cell during discharge is 1.2 volts.

RATED PERFORMANCE OF EDISON CELLS

Size of cell	Normal rate of charge and discharge, amperes	Rated capacity		Watt-hours per pound of cell for rated capacity	Average watts during discharge at normal rate
		Ampere-hours	Watt-hours		
B-2	8	40	48	10.4	9.6
B-4	16	80	96	13.0	19.2
B-6	22.5	112.5	135	13.7	27
A-4	30	150	180	13.3	36
A-5	37.5	187.5	225	13.4	45
A-6	45	225	270	14.1	54
A-8	60	300	360	13.1	72
A-10	75	375	450	13.2	90
A-12	90	450	540	13.2	108

The capacity of new cells increases for at least twenty cycles of charge and discharge. This betterment may be as much as 30 per cent above rating and comes from an improvement of conditions in the nickel electrode which is brought about by regular charging and discharging. It is expedited by over-charging. The capacity then decreases slowly with use until the electrolyte is renewed, which should be done before the capacity has fallen to the rated value. The capacity again increases for a few charges and subsequently falls.

Voltage Characteristics.—The charge and discharge voltage characteristics of an Edison battery vary with the conditions, among which are temperature, condition of electrolyte, time since last discharge or charge, etc. Typical

charge and discharge voltage curves at the normal rate are given in Fig. 1. It will be noted that the voltage rises rapidly at the beginning of charge, decreases somewhat during the second hour, and then rises gradually during the remainder of the charge. This hump in the voltage curve is characteristic of the Edison battery, and may lead to confusion as to battery's condition of charge. The value and duration of this hump may vary considerably even on successive charges of the same battery. The final charging voltage, with current on, is approximately 1.8 volts per cell, but ranges from 1.7 to 1.95 volts per cell. The rise in voltage near the end of charge is not sufficient to serve as a very satisfactory criterion of complete charge, but the Edison Storage Battery Co. recommends that if the extent of the previous discharge is unknown, the charge at the normal rate should be continued until the voltmeter reading has remained constant for 30 minutes at about 1.8 volts per cell.

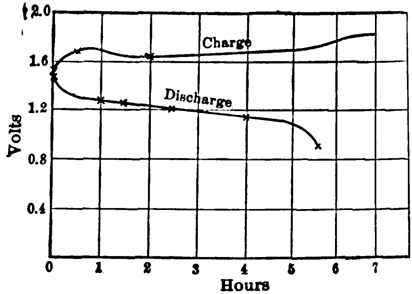


Fig. 1. Voltage Curves at Normal Rate

Effect of Temperature on Charging Voltage. — The normal average temperature of a battery on charge is between 90 and 100° F. The values of average and maximum charging voltage are higher for low temperatures of the electrolyte than for high temperatures. The range is indicated by the results of a series of tests given in the accompanying table.

Temperature, deg. Fahr.	Average voltage	Maximum voltage
35	1.88	1.94
55	1.81	1.92
75	1.76	1.88
95	1.70	1.84
115	1.67	1.77

Discharge Voltage. — On discharge the voltage performance of an Edison battery is more uniform than on charge, although during the early portion of the discharge the voltage is liable to vary considerably from the values indicated in Fig. 1. This depends chiefly on the length of time the cell has stood on open-circuit since the completion of the charge. The average discharge voltage is 1.2 volts per cell when discharged to 0.9 volt per cell (with current on) at the normal (5-hour) rate.

As with lead cells the open-circuit voltage is valueless as an indication of the state of charge. The open-circuit voltage in general will be about 20 per cent higher than the corresponding terminal voltage at normal rate of discharge.

Specific-gravity Characteristics. — The normal specific gravity of the electrolyte is about 1.200, and decreases slowly as the cell is used. (*See section below on Tests of Electrolyte.*) The specific gravity does not change materially on charge or discharge. A slight concentration has been noted on

discharge, but the variation is too small to have commercial significance. The specific gravity decreases 0.002 with each 10° F. increase in temperature.

Variation of Capacity With Rate of Discharge. — The ampere-hour capacity of an Edison battery is practically independent of the rate of discharge, provided the discharge is carried to a sufficiently low voltage. In this connection

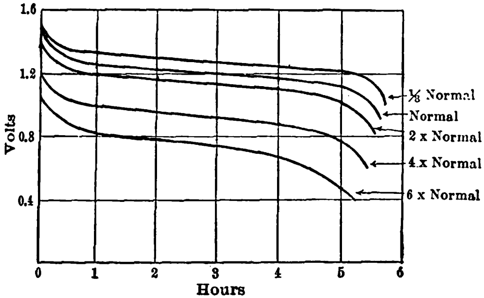


Fig. 2. Discharge at Various Rates

it may be pointed out that no harm is done an Edison battery by discharging it to a complete short-circuit. The total ampere-hour outputs of a battery when discharged to zero voltage at various current rates from one-third normal to four times normal, after a normal charge in each case, vary by only 2 per cent. As the low voltage part of a discharge cannot be considered useful, it is customary to consider the discharge completed for any rate of discharge when the voltage has reached a value differing from 0.9 volt per cell by an amount equal to the difference between the resistance drop in the cell corresponding to this rate and that corresponding to the resistance drop at the normal rate. Thus

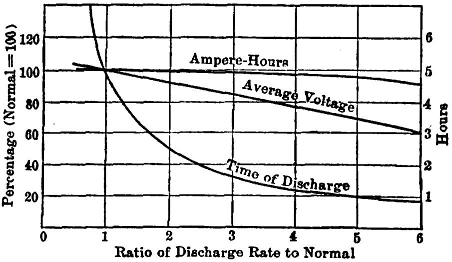


Fig. 3. Relation of Output Characteristics to Rate of Discharge

for a cell with an internal resistance (*for values see paragraph below on Internal Resistance*) of 0.003 ohm and a normal rate of 30 amperes, the terminating voltage for a discharge at 60 amperes would be $0.9 - 0.003\ (60 - 30) = 0.81$ volt. Fig. 2 shows a series of voltage curves for discharges at various rates following normal charges. The discharge has been discontinued in each case in accordance with the above rule. The variations of average voltage, ampere-hour output and time of discharge, with discharge rate are shown in Fig. 3, where values of these factors for the normal (5-hour) rate are each taken as 100 per cent.

Variation of Capacity with Rate of Charge. — The energy contained in a newly charged Edison cell depends upon both the rate and length of charge. Fig. 4 gives the variation in the time of discharge at normal rate for various charge rates and periods for a particular battery. In this particular test when the cell was discharged at the normal 5-hour rate to 0.9 volt, after a 7-hour

charge at this same rate it gave this current for 5.9 instead of 5 hours. The curve for the normal rate of charge shows that the capacity for a 10-hour charge is 13.5 per cent and for a 14-hour charge 22.0 per cent greater than for a normal (7-hour) charge. The results of a series of charges of various lengths and subsequent discharges, both at the normal rate, are given in the accompanying table.

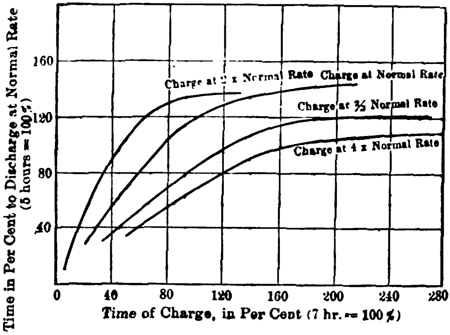


Fig. 4. Capacity at Normal Rate Discharge after Charging at Various Rates and Periods

Length of charge, hours	5.5	7.0	10.0
Length of discharge, hours	5.0	5.8	6.4
Average discharge voltage	1.19	1.202	1.203
Ampere-hour efficiency, per cent	91	83	64
Watt-hour efficiency, per cent	66	59	45

Variation of Efficiency with Length of Charge. — As shown by Fig. 4 the capacity of a cell depends upon the rate as well as upon the length of charge. For charge and discharge at the normal rate, the watt-hour efficiency is approximately 72 per cent for a short charge (i.e., one hour or less), it drops to about 58 per cent for a normal or 7-hour charge, and has smaller values for larger charges, being approximately 30 per cent for a 15-hour charge. The normal rate of charge has been chosen by the manufacturers such that for an output of approximately the rated number of ampere-hours at the normal rate the efficiency is higher for the normal rate of charge than for a rate which is considerably higher or lower than the normal.

Variation of Capacity with Temperature. — With the Edison battery, as with all alkaline batteries, there is a critical electrolyte temperature for each rate of discharge below which the capacity falls to a low value. The higher the rate of discharge the higher is this critical temperature. If the electrolyte throughout the cell has been chilled below the critical point for a given rate of discharge, the full capacity can be obtained as soon as the cell is warmed above the critical range. A series of tests, reported by W. E. Holland (*Central Station, Nov., 1911*), showed that the temperature should be kept above 55° F. if a large current is to be required. In the operation of Edison batteries in electric automobiles in cold weather it is recommended that the batteries be given a warming charge shortly before the car is taken from the garage. See also next paragraph.

Rate of Cooling of Electrolyte. — The rate of cooling of the electrolyte

contact with the battery, the temperature of this air, and the rate at which current flows from the battery. Another series of tests reported by Holland (*Central Station, Nov., 1911*) showed that in an open box a battery which had been charged in a room temperature of approximately 65° F. could stand idle in still air at 8° F. for a period of about 4 hours before the electrolyte dropped to 55° F. When placed in a closed box it took about 13 hours for the battery to cool from an initial temperature of 90° F. (which a battery would have at the end of a normal charge) to the critical temperature for high discharge rates. These tests pointed out the efficacy of a closed battery box for keeping cells warm. (*See also Automobiles, Electric.*)

Internal Resistance.—The effective internal resistance of a cell determines the immediate change in voltage at the cell terminals with a sudden change in the discharge rate. For method of measuring see section on *Testing*, below. The value in ohms of the mean effective internal resistance for an A-type cell discharging at normal rate is approximately equal to 0.012 ohm divided by the number of positive plates as given by the designation of the cell size, and for a B-type cell is approximately equal to 0.024 ohm divided by the number of positive plates. For two or more cells in series the total effective internal resistance is the product of this value by the number of cells in series. The internal resistance of Edison cells is such as to cause an immediate drop in terminal voltage of between 7 and 8 per cent with a sudden increase in current equal to the normal rate. The virtual internal resistance increases slightly during the progress of a discharge.

Retention of Charge.—The rate of loss of charge of a battery during idleness varies with the temperature at which it stands. The loss is slight in the cold and becomes greater as the temperature increases. The variation for cells charged under normal conditions and discharged after resting for various periods in one case at normal temperature of 75° F., and in another case at 35° F. is shown in Fig. 5. Accordingly a cell which has not been used for 10 days following a normal charge may be expected to retain at least 85 per cent of its initial capacity. It will be noted that the *rate* of loss of capacity after the second day is practically the same at both temperatures.

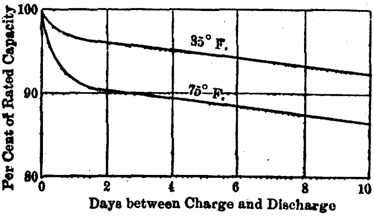


Fig. 5. Retention of Charge when Standing Idle

TESTING.—There are described below only those tests which are of interest in the commercial operation of the batteries. The methods of conducting several of the tests are the same as for the lead type of battery and are described more fully in the article on *Batteries, Storage, Lead Type*.

Tests of Electrolyte.—The normal specific gravity of the electrolyte at 80° F. is 1.200 as measured by hydrometer but may range from 1.160 to 1.230. Specific gravity is not necessarily a true indication of the suitability of the electrolyte, since harmful impurities may get into the electrolyte when watering or in other ways. The manufacturers recommend that the electrolyte be removed when the specific gravity has fallen to 1.160; this will usually be required after from 8 to 10 months of daily service. It is advisable to discharge the battery before renewing the electrolyte, and to give it a 12-hour charge, at the normal rate after renewing.

Test for Internal Resistance. — The internal resistance of a battery may be determined by momentarily opening the switch during discharge and noting the rise in voltage. This rise represents the *IR* drop in the battery due to its internal resistance and cell connections. Hence the value of the internal resistance at the given point of discharge will be the quotient of the volts rise divided by the current rate. This value divided by the number of cells in series will give the internal resistance per cell. Since on interrupting the current the voltage may rise for some time, due to change in the e.m.f., a better voltage reading may be obtained by momentarily reducing the current flow instead of opening the circuit. In this case the resistance is found by dividing the difference in voltage at the two rates by the change in current.

Capacity; Efficiency. — In testing for the capacity of an Edison battery at any given rate, the discharged battery should first be charged for 7 hours at the normal (5-hour) discharge rate. The battery is then discharged at the desired constant-current rate until the terminal voltage has reached the value as determined by the rule given in the paragraph above on *Variation of Capacity with Rate of Discharge*. The capacity in ampere-hours is the product of time of discharge in hours by the rate of discharge in amperes. The watt-hour efficiency of the battery is the ratio of the energy given out on the discharge to the energy put in on the charge. The term "volt efficiency" is used to express the relation of the average voltage of discharge to the average voltage of charge. For charge and discharge at the normal rate the value of the volt efficiency will be close to 72 per cent for any length of charge not extremely short or long. Thus 72 per cent is the limit of possible efficiency at the normal rate. The watt-hour efficiency on any normal rate test may be calculated accurately enough for all practical purposes by taking 72 per cent of the ampere-hour efficiency, i.e., by taking 72 per cent of the ratio of ampere-hours output to ampere-hours input.

Test Electrodes. — The best substance for a third electrode in analyzing voltage curves of alkaline batteries is a partially reduced oxide which will undergo reduction in the electrolyte without polarization, and which is insoluble in the electrolyte. Either cupric oxide or the high nickel oxide of a charged Edison positive fills these conditions for alkaline cells. For a discharge at normal rate the readings of the cell electrodes to a test electrode of cupric oxide are: iron, approximately 0.44 volt, nickel, 0.9 to 0.5 volt; for a test electrode of nickel oxide the readings are: iron, approximately 1.28 volts, nickel − 0.1 to − 0.4 volt.

INSTALLATION. — The A- and B-types of cells are usually assembled by the manufacturer in wooden trays holding from 2 to 12 cells each, according to requirements. There is an air space between cells for ventilation and insulation. The standard form of tray is made "bottomless," in order to prevent short-circuits between cells from the collection of dirt and moisture. If A-type cells are to be used where they will be pulled horizontally for filling, etc., trays with bottoms are used on account of the greater strength. Data on typical assemblies in bottomless trays are given in the following table:

Size of cell	B-2	A-4	A-8	A-12
Width of standard tray, in.	6⅛	6⅛	6⅛	9
Over-all height (filler cap closed), in.	9⅝	14 5/16	14⅞	15½
Length of trays, in.:				
2-cell tray	5	7¾	12⅞	13¾
5-cell tray	10¾	17⅛	29¾	32½
8-cell tray	16⅝	27¼	47⅜	....
12-cell tray	24⅜	40½	....	....

For the extra high container which is sometimes furnished (designated by *H*) the over-all height for B-type is 2 in. greater and for A-type 3 in. greater than the figures given in the above table.

The over-all height with filler cap open is about 1½ in. greater than the height with filler cap closed.

If a battery is to be used for stationary service a clearance of 6 in. above cells should be allowed so as to permit the proper filling of the cells in place. In vehicles from which the battery is removed for filling the clearance need be only ¾ in.

OPERATION.—An Edison battery should never be operated in any manner except in accordance with the instructions received from the Edison Storage Battery Co. Disobeying these instructions may result in forfeiture of the guarantee with regard to the life of the cell. As the detailed instructions differ somewhat for various services, only general directions which apply to all services are noted below.

Charging.—The batteries are usually shipped in a discharged condition. The initial charge should continue for 12 hours at the normal rate. A similar overcharge should be given after 30 days of service, one after 60 days of service, and another after each renewal of electrolyte. The normal charging rates are given above in the section on *Rating and Performance;* the time of a normal charge is 7 hours. The boosting rate may be as high as desired provided the temperature does not exceed 115° F. in any of the cells. The best results are obtained when the temperature is kept between 75 and 95° F. Under average operating conditions the charge in ampere-hours necessary to replace a discharge is from 15 to 25 per cent greater than the discharge. The Edison Storage Battery Co. recommends that if an ampere-hour meter is used, it should be set to operate 20 per cent slow on charge.

Precautions as to Gases.—The battery should be well ventilated while charging, and no open flame or arcing contact should be allowed near the cells while charging or immediately afterward, as the evolved gases may be exploded.

Standing Idle.—An Edison battery may be allowed to stand idle in any state of discharge provided the level of the electrolyte is kept above the plates. After long idleness an overcharge may be required to bring the battery up to rated capacity.

Watering Cells.—The level of the electrolyte must be kept above the plates by adding distilled water from time to time. The frequency of adding water will depend upon the amount and rate of charging. The manufacturers of the battery supply an indicating filler which is of assistance in preventing slopping and over-filling.

Cleaning.—The trays and containers must be kept dry, and dirt or other foreign material must not be allowed to collect between or under cells. Dirt and dampness may cause leakage which may result in corrosion of the containers. The outside of the cans may be cleaned with a steam or air blast. If a container becomes leaky it should be returned to the manufacturer for repair.

DIMENSIONS, WEIGHT AND COST.—The Edison battery is made in two standard types, the plates of the A-type being approximately 4¾ by 9¼ in. and those of the B-type 4¾ by 4⅝ in. Plates of other dimensions can be obtained for special purposes. The sizes of cells are commonly designated by the type letter and a number which indicates the number of positive plates, as for instance, an A-6 cell has 6 positive and 7 negative plates. The several sizes of cells together with data on the dimensions and weight of each are given in the following table:

DIMENSIONS AND WEIGHTS OF EDISON CELLS

Size of cell	Over-all dimensions of cell, in inches			Weight in pounds	
	Length	Width	Height	Complete cell	Average per cell with trays and connectors
B-2	1.5	5.1	8.8	4.6	5.5
B-4	2.6	5.1	8.8	7.4	8.7
B-6	3.8	5.1	8.8	11.0	12.0
A-4	2.7	5.1	13.4	13.3	14.5
A-5	3.2	5.1	13.4	16.8	18.5
A-6	3.8	5.1	13.4	19.0	21.0
A-8	5.0	5.3	14.0	27.0	30.0
A-10	6.2	5.5	14.0	34.0	37.5
A-12	7.4	5.5	14.6	41.0	45.0

For services, such as the lighting of railroad cars, which require that the batteries work for long periods without the addition of water, the A-type plates are assembled in containers (designated by H) about 3 in. higher in each case than those indicated in the table, in order to obtain additional space above the plates for the electrolyte. The weight per cell is then about 15 per cent greater than the above.

The cost of Edison cells assembled in trays is approximately $1.00 per pound, including weight of trays and connectors. This is equivalent to a cost of approximately $420 per kilowatt at the normal (5-hour) rate of discharge or $125 per kilowatt at the one-hour rate of discharge. The manufacturers guarantee that these cells will show at least their rated capacity after being used a specified time, the usual guarantee period for A-type cells being 4 years and for B-type cells, 5 years.

BIBLIOGRAPHY. — Holland, W. E., *Effect of Low Temperature on the Alkaline Storage Battery*, Central Station, Nov., 1911; Kammerhoff, Meno, *Der Edisonakkumulator*, Berlin, 1910; Lyon, H., *The Manufacture and Performance of the Edison Storage Battery*, Jour. of Indus. and Eng. Chem., Dec., 1911; Foljambe, E. S., *The Edison Storage Battery, Its Conception and Method of Manufacture*, Com'l Car Jour., Jan., 1914; Smith, H. H., *The Edison Storage Battery in Service*, Cent. Sta., Nov., 1912; *Manual of Storage Battery Practice*, printed by Assoc. of Edison Ill. Co., 1912; Publications of the Edison Storage Battery Co., especially *The 1910 Edison Storage Battery*, by Holland, W. E.

[H. F. THOMSON.]

BATTERIES, STORAGE, APPLICATIONS OF. — (*See also Batteries, Storage, Alkaline Type; Batteries, Storage, Lead Type.*) The following list shows some of the more important applications of storage batteries.

Central Stations:
- Emergency reserve — "Stand-by service," p. 88.
- Load or voltage regulation, p. 90.
- Taking peaks, p. 95.
- Day load on small systems, p. 96.
- Exciter reserve, p. 92.
- Remote-control switch operation, p. 93.

Isolated Plants:
- Mine hoists, steel mills and other heavy motor regulation (*see Index*).
- Carrying entire load during certain hours of light load.
- Load and voltage regulation in office buildings or hotels, where electric elevators are in service, p. 98.
- Giving 24-hour service in residences.
- Operation of drawbridges, p. 101.

Other uses are:
- Regulation of long feeders, see *Trolley Systems, Overhead*.
- Propulsion of pleasure cars, trucks, street cars, submarine boats, launches, etc., p. 100.
- Gas-engine ignition, p. 102.
- Railway passenger-car lighting, see *Lighting of Trains by Electricity*.
- Railway signaling, see *Signaling, Railway*.
- Telephone and telegraph (q.v.)
- Portable and small stationary lamps.
- Fire and burglar alarm (q.v.)
- Electroplating, p. 102.
- Dental and other surgical work.

Probably the largest application of storage batteries at the present time is for "stand-by" service in central stations, for peak and regulating work in railway power stations, in electric vehicles and in the steel-mill power stations where the load fluctuations are very rapid and of great magnitude.

Extent of Application in the United States. — The first application in America of storage batteries to central-station service was in 1886. In 1912, there were in service in the United States, for central-station "stand-by" work, 185 storage batteries of the lead-lead acid type, having a combined capacity of approximately 147,000 kw.-hr. at the one-hour rate of discharge. Twenty-one of these are of the Faure or Pasted type, the rest being of the Planté type, or a combination of Planté and Faure types.

In railway power stations there were in service in the United States in 1912 batteries of the lead-lead acid type aggregating approximately 120,000 kw-hr. at the one-hour rate of discharge. All of these batteries are of the Planté or combined Planté and Faure type.

There were in service in steel-mill power plants in the United States in 1912 twelve batteries of the lead-lead acid type aggregating approximately 7000 kw-hr. at the one hour rate. All of these batteries are of the Planté or Planté Faure type.

There were in service in the United States in 1912 approximately 500,000 cells of the lead-lead acid type, operating telephones, lighting railway cars and operating signals. There are probably over 400,000 lead storage-battery cells in operation in electric automobiles.

STAND-BY BATTERIES IN CENTRAL LIGHTING STATIONS.— Formerly the storage battery in a central lighting station served a dual purpose: first, as a source of current in an emergency, and second, to take short peaks of load, thus cutting down the generating capacity in service and increasing the economy of plant operation. In certain cases the batteries are used in this manner to-day, but owing to the desire always to have the full battery reserve in case of emergency, the peak battery feature is being abandoned and the stand-by feature only is being retained.

This method is considered by some authorities to be even more conducive to economy of station operation than that in which the battery was also used on the peak, inasmuch as the presence of a fully charged battery behind the generating apparatus permits station operation at a much higher load factor than were the battery fully or partly discharged. Furthermore, as one of the important factors in the life of the battery is the amount of work it does, this life is greatly increased if the battery be used only for emergency service.

Connections for Battery and Its Auxiliaries, Stand-by Service.— Fig. 1 shows a general standard scheme for the operation of a battery on a 3-wire lighting system, either for peak work or stand-by emergency work. As will be noted in this scheme, the neutral of the battery is connected to the neutral of the system.

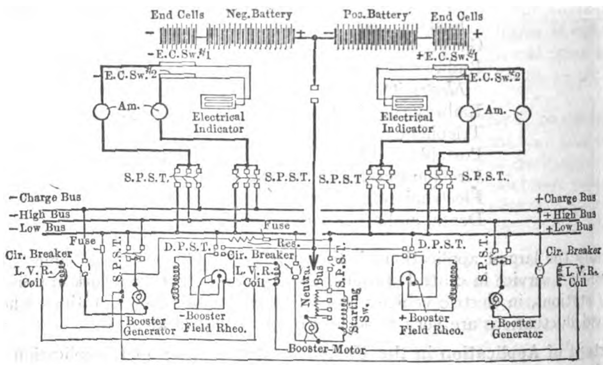


Fig. 1. Connections for Stand-by Service

Charging. — For stand-by emergency work it is standard practice to take the battery off the bus for charge and in most cases the battery is floated on only one bus. In this service the battery is kept fully charged and floating on the bus-bars continuously, except during charge. Except in case of an emergency discharge having been required from the battery, it will only be necessary to give the battery an occasional overcharge about once a week or once in two weeks in order to keep the active material in proper condition. Since this overcharge is given at the normal rate for only a few hours, once every week or every two weeks, a time for this overcharge can be selected when it will be safe to take the battery off the bus. It will also be noted that on charge, sufficient voltage is obtained for charging the complete battery by adding to the bus voltage the voltage of a simple charging booster. It is necessary with the 3-wire system to have two generators, one on each side of the system, in order to be able to charge the two halves of the battery at different rates in case the battery has become unbalanced on discharge. The diagram shows a simple shunt motor driving these two booster generators.

End-Cells and Switches for Stand-by Service. — The connections between the two sides of the battery and the outside bus-bars of the system are made through "end-cell switches," by means of which the number of cells on the system can be varied at will within certain limits. This scheme shows two end-cell switches in parallel on each side of the system, this number of switches being installed in this case in order to provide sufficient capacity for the discharge.

The number of end-cell switches necessary on each side of the system depends upon the current to be carried, upon the number of bus-bar voltages to be maintained from the battery simultaneously, and upon whether or not it is desired to keep the battery on the bus during charge. It is frequently required to float the battery on more than one bus. In such cases there must be one end-cell switch on each side of the system for each bus. If it is not desired to keep the battery on the bus during charge, and if the battery is to float on only one bus, one end-cell switch on each side of the system will be sufficient, provided the one end-cell switch has sufficient current-carrying capacity.

If, however, it is deemed advisable to keep the battery on the bus at all times, it will usually be necessary to have at least two end-cell switches on each side of the system and the number of end cells required will be greater than the number of end cells required where the battery can be taken off the bus for charge. This point is referred to again in the next section.

It will be noted from the diagram in Fig. 1 that on discharge the cells are connected in series and that the voltage on discharge is controlled by means of the end cells.

Importance of Well-designed End-cell Switches. — It should be noted that the end-cell switches used with this standard scheme are by far the most important of the auxiliary apparatus and the efficacy of the battery is absolutely dependent on the successful operation of these end-cell switches. The factors necessary for successful operation of the end-cell switches are, first, there must be no chance of interruption of the battery discharge; second, the switch must be able to carry the required current without dangerous heating or sparking. End-cell switches having a continuous current-carrying capacity of 10,000 amperes are now on the market. These switches can carry 40,000 amperes for six minutes and the contact brushes can be moved from point to point without injurious sparking or heating.

Number and Capacity of Cells for Stand-by Service. — The number and the capacity of the cells depends upon the lowest bus voltage maintained under normal operating conditions, the minimum allowable voltage at the end of discharge in case of emergency, and the load required to be carried during emergency.

If the battery can be taken off the bus for charge, the number of cells in the main battery will be equal to the lowest bus voltage (between outside bus bars) divided by 2.1. The number of cells in the entire battery will be equal to the lowest allowable voltage in emergency divided by the final voltage per cell at the rate of discharge required by the emergency. The difference between the number of cells in the main battery and the total number of cells will, of course, be the number of end cells. When the total number of cells in the battery is determined it will become necessary to put half the number of cells, including half the end cells, on each side of the system.

If it is deemed necessary to keep the battery on the bus at all times during charge and discharge, the total number of cells will be determined as indicated above, but the number of cells in the main battery will be equal to the bus voltage divided by 2.7, since this is the average cell voltage at the end of charge. In this case, at least two end-cell switches on each side of the system will be

required. On charge one end-cell switch on each side of the system is connected to the main bus and is set at such a position that the total voltage of the cells in circuit during charge equals the bus voltage. The other end-cell switch is set so as to include the cells being charged. That is, at the start of charge, the circuit from the charging booster to one end-cell switch will include all of the cells, and the circuit from the lighting bus to the other end-cell switch will only include sufficient cells to give the lighting-bus voltage. As the charge proceeds, the end-cell switch in the charging circuit will be shifted to cut out end cells as their charge is completed, and the end-cell switch connecting the battery with the lighting bus will be shifted to cut out cells as the voltage of the battery comes up, so that the voltage of the cells in circuit on the lighting bus can be held at the lighting-bus voltage.

It is seen that with this operation the end-cells included between the two end-cell switches will have to carry the charging current plus or minus any current transferred between the battery and the lighting bus. It is thus apparent that with this method of operation, unless especial care is taken to float the battery at the true lighting-bus voltage, the end cells will be worked unequally. It is also seen that, if it is desired to keep the battery on the bus at all times, the amount of end-cell copper is considerably increased and the cost of the battery installation is proportionately increased, for the number of end-cells required is greater than the number of end cells required when the battery can be taken off the bus for charge. On account of this increase in cost and the possibilities of unequal work on the end cells, it is desirable to install the simpler scheme shown in Fig. 1.

Determination of Size of Stand-by Battery.—As an example of the method used in determining the number and capacity of cells required for this class of work, suppose it were desired to install a battery to take care of an emergency of 20,000 amperes for 20 minutes at a normal bus voltage of 230 volts with a minimum allowable voltage during emergency of 225 volts. The 20-minute rate of discharge is 8 times the normal rate, as shown in Figs. 3 and 6 in the article on *Batteries, Storage, Lead Type*, and it will be noted from Fig. 6 that the final voltage per cell at the end of this rate is 1.5 volts. The total number of cells will be 225 divided by 1.5, or 150 cells. If it is desired to use standard practice, taking the battery off the bus for charge, the number of cells in the main battery will be equal to the normal bus voltage, 230, divided by 2.1, or 110 cells.

It will, therefore, be necessary, in this case, to install a battery having 150 cells, including 40 end cells, each cell having a capacity of 20,000 amperes for 20 minutes. This battery will be installed with 75 cells, including 20 end cells, on each side of the neutral. In standard stand-by practice the 20 end cells on each side of the neutral will be connected to the end-cell switches, with one cell per switchpoint for a few of the cells and with 2 or 3 cells per switchpoint for the remainder of the end cells. A certain number of single-cell connections will be required in order to take care of the variation in the bus voltage, as it may be necessary to cut in and out cells while the battery is floating. The rest of the equipment necessary should be in accordance with the diagram shown in Fig. 1.

LARGE REGULATING BATTERIES FOR RAILWAY WORK, ETC.—Often the load (particularly a railway load) on the power house is subject to frequent and violent fluctuations, causing severe strains on the generating apparatus in service and in many cases requiring the operation of a greater number of units than would be necessary to furnish a steady average load. This not only results in a lower load factor and consequent loss of economy, but it reduces the surplus capacity of the generating station by the extra

amount of apparatus required in operation. Another loss in economy is due to the rapid changes in the cut-off point of reciprocating engines, due to the violent fluctuations of load. Fig. 2 shows a simplified wiring diagram of the scheme generally used.

Carbon Regulator. — The carbon regulator shown in Fig. 2 consists of piles of carbon disks arranged in two groups on opposite sides of the fulcrum of a lever, the lever being subjected at one end to the tension of a spring and at the other end to the pull of a coil connected in series with the main bus-bar. Variations in the force applied to opposite ends of this lever will increase the pressure on one group of carbon disks and decrease the pressure on the other, thus varying their relative resistances. These piles are connected in series across the battery terminals, and the field winding of the booster exciter is connected between the middle point of the battery and a point in the circuit between the two groups of carbon disks. Variation in the resistances of these two piles, produced by variation of the pressure of the lever, will cause current to flow in one direction or the other through the field of the exciter, thus controlling the operation of the booster and compelling the battery to charge and discharge with small variations of current in the series coil.

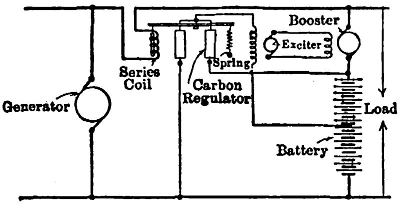


Fig. 2. Connections for Regulating Service

In this manner the load on the generator can be maintained constant within a few per cent on either side of the average value for which the apparatus has been adjusted. The load on the generator is determined by the tension of the spring, which may be adjusted to balance any average current in the series coil. If the average load on the line should exceed for a considerable period of time the generator load for which the spring is adjusted, the battery will be subjected to a sustained discharge which may in time exhaust its available capacity. If, on the other hand, the average load on the line falls off for a considerable period of time, the battery will be subjected to a sustained charge which may produce excessive and unnecessary overcharge. To follow these variations in average load, it is necessary to adjust the spring from time to time.

Automatic "Average" Adjustor. — In order to eliminate the necessity for such frequent adjustment by hand in cases where the average load is continually changing and where the generator is capable of handling these comparatively slow changes, the automatic average adjustor has been designed. This piece of apparatus consists of a small motor connected by a reducing gear to the drum which controls the tension of the carbon regulator spring. The armature of this motor is connected across the main bus-bars through a fixed resistance which maintains a practically constant current through the armature. The field of the motor is connected across the terminals of the booster.

Whenever the booster voltage is in the direction to discharge the battery, the motor field is energized and the motor revolves in the direction to increase the tension of the spring, thus gradually increasing the load on the generator and relieving the battery of its discharge until the booster voltage comes back to zero. If, on the other hand, the booster voltage is in the direction to charge the battery, the motor of the average adjustor will operate in the opposite direction, gradually relieving the tension on the spring and reducing the average load on the generator until the booster voltage is again brought back to zero

and the battery is again restored to its floating condition. This transfer of load between the battery and generator is brought about slowly so that the battery is still free to relieve the generator of all quick fluctuations of load. The battery, however, is relieved from a considerable amount of sustained peak charge and discharge.

Load-limiting Device. — To prevent the average adjustor from throwing load on the generator beyond its capacity, a load-limiting device has been designed. This consists of an electrolytic valve in series with the field of the motor. Ordinarily this valve is short circuited by a switch. When the drum which regulates the tension of the spring has traveled to a certain point corresponding to the maximum permissible load on the generator, a projection mounted on the shaft of this drum engages with the switch and opens it, whereupon the valve prevents current from passing through the motor field in a direction to cause a further increase in the spring tension. The valve will, however, permit the current to flow in the opposite direction so that as soon as the load has decreased sufficiently to cause the battery to charge, the motor will operate so as to relieve the tension on the spring and at the same time permit the switch to close, thus restoring the original adjustment.

Automatic Current Stop. — While the capacity of a storage battery to withstand an excessive momentary overload without injury is practically unlimited, the booster must be protected by an automatic circuit breaker. Whenever the setting of this circuit breaker is exceeded on heavy overload and the breaker opens, the entire maximum load is thrown on the generator. This usually results in opening the generator circuit breaker also and interrupting the supply of current to the line. To avoid this and permit the battery to furnish the maximum current which the booster can handle without danger of overloading the latter or opening the circuit, there has been designed an automatic current stop. This stop is arranged to automatically prevent the battery from discharging above a certain predetermined value, for which this stop has been adjusted. All load demand above this predetermined value is thrown upon the generator. The generator is thus subjected only to the excess of load above the maximum capacity of the battery booster, instead of having the entire load of the line suddenly thrown on it when the battery circuit breaker opens. The adjustment of this automatic current stop can be changed within certain limits to meet the requirements.

Batteries for Controlling Alternating-current Loads. — For controlling fluctuations of an alternating-current load a solenoid which is responsive to the energy component of the alternating current is substituted for the series coil shown in Fig. 2. The general operation of this scheme is the same as described above, except that it is, of course, necessary to discharge the battery on to the alternating-current bus through suitable alternating-current-direct-current transforming apparatus. There are in service at present several battery installations which regulate fluctuating alternating-current loads. Most of these installations are based on the carbon regulator principle, as outlined above.

Number of Cells and Capacity of Cells and Booster. — The number and capacity of cells required for regulating work depends upon the bus voltage and upon the nature of the fluctuating load. It is also frequently desirable to utilize a regulating battery for emergency and peak work, and the extent of the emergency and peak requirements will also be a factor in fixing the capacity of the battery. The number of cells installed will be equal to the bus voltage divided by 2.1, since this is the floating voltage per cell. A reference to Fig. 2 shows that during the regulation of load fluctuations all of the battery discharge passes through the armature of the booster generator. It is thus seen that it ... for the booster to generate sufficient voltage to make up for the drop

in battery voltage on discharge. This drop in battery voltage depends upon the capacity of the battery, and the amount and duration of the discharge. Before the booster can be properly designed it will be necessary to have complete data on these points.

Emergency Work. — For emergency work, where the battery is carrying the entire load without the assistance of any generating apparatus, it may or may not be necessary to keep the booster in circuit, depending upon whether or not it is desired to maintain a constant bus voltage. If it is not desired to maintain a constant bus voltage, a saving can be effected by cutting the booster out of circuit and allowing the battery to discharge directly on to the system.

Peak Work.— For peak work, where the battery must work in parallel with the generators, it will either be necessary to keep the booster in circuit in order to maintain the same voltage as the generator or else to give the generators a drooping characteristic so that the battery can discharge in parallel with the generators without the aid of the booster.

Current Capacity. — If it is desired to float the battery on the system at all times in order to regulate load fluctuations, it will be possible to float the battery about 75 per cent fully charged. If this battery must have a definite capacity always available for peak or emergency work, it is seen that the battery must be of sufficient capacity, when 75 per cent fully charged, to deliver the load called for by the emergency or peak conditions.

Determination of Size of Regulating Battery. — In all cases where a battery is to be considered for any one of the above conditions or any combination of them, it is recommended that the complete data summarized below be sent to the battery manufacturer with a request for a complete report covering the battery and accessories recommended to suit the conditions. The data necessary are as follows:

Number, type and capacity of boilers, if the plant is a steam plant.
Schedule of boiler operation, if the plant is a steam plant.
Number, type and rating of generators.
Schedule of generator operation.
Bus voltage maintained.
If the generators are of the alternating-current type, give frequency, number of phases, and the number and rating of any static transformers used.
If the station in question is a substation, describe the complete substation equipment along these same lines.
Give complete 24-hour load curve showing half-hour readings of the total load on the station bus bars. A graphic recording meter chart will be sufficient, if such an instrument is available.
If possible, obtain five-second readings on the total indicating ammeter or wattmeter, over a period of one-half hour during the time of maximum fluctuations.
If the battery is required to assist the generators on the peak, give the amount of load in kilowatts required from the battery and the length of time this load is required. It would be well to have an exact load curve showing the shape of this peak, if it is possible to obtain such a record.
If the battery is to be used for emergency, give the amount of the load and the time over which this emergency extends. In all cases state the exact nature of the load required, that is, whether it is a motor load, a lighting load, etc.

BATTERIES FOR REMOTE-CONTROL SWITCH OPERATION. — It is standard practice in central stations of any magnitude to install a storage battery of sufficient capacity to give ample insurance against interruption of

the switch-control circuit. It is the usual practice to utilize this battery, not only for the operation of the remote-control switches, but for the operation of the signal lamps and a sufficient number of emergency lamps throughout the station. In some stations the remote-control switches are operated from the exciter bus bars, but it is better practice to operate these switches together with their signal lamps and the emergency lamps from a separate source of power.

Control Battery Connections. — Fig. 3 shows a scheme which is more or less standard with the larger central-station companies. This figure shows a motor-generator set consisting of an induction motor driving a direct-current generator whose voltage can be varied from 110 volts up to 168 volts, the latter voltage being required to completely charge the 62 cells in series.

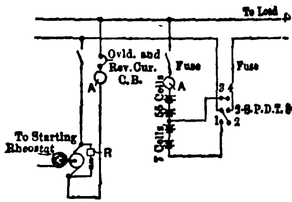


Fig. 3. Connections for Control Battery

Under normal operation the battery is floated on the 130-volt bus with the motor-generator set, the battery switches being thrown into positions 1 and 2. It will be noted that with this arrangement all 62 of the cells are floating on the bus. When it becomes necessary to charge the battery, the battery switches are thrown into positions 1 and 4, and the generator voltage is raised sufficiently to start the charge of the entire 62 cells. With this arrangement the voltage at the load bus bars will be the voltage of 55 cells. At the finish of charge the battery switches are thrown into positions 3 and 4. It will be noted that in these positions the group of 7 end cells is out of circuit, as they have received their complete charge, and the motor-generator set will be furnishing power to charge the 55 cells and to carry the load.

Size of Battery. — With this arrangement the voltage at the bus bars may vary from 145 volts to the *minimum* voltage of the battery on discharge. The number of cells for this scheme can be varied to suit the voltage conditions. The capacity of the battery is fixed by the maximum current demand, the total ampere-hour current demand over the period of estimated possible shutdown, and the minimum allowable voltage on the load bus bars. With these conditions known, it will be possible with the aid of trade catalogues showing the charge and discharge curves of the battery and the battery capacity at various rates, to select a battery to suit the conditions.

Control Battery on Exciter Bus. — If the oil-switch load is furnished from the exciter bus it will be necessary to float the emergency battery on the exciter bus, or to supply automatic means for throwing the battery on the exciter bus when the voltage of the exciter bus drops below a certain predetermined value. If this method is used, unless the battery is of sufficient capacity to carry the entire exciter load, it will be necessary to place a reverse-current circuit breaker between the battery and the exciter load, so that when the exciter bus "goes dead" the battery will be automatically cut off from the exciter bus and left on the remote-control switch bus.

EXCITER BATTERIES. — It is standard practice with the larger central-station companies to install a battery of sufficient capacity to carry the total maximum exciter load for from one hour to two hours continuously. The number of cells will depend upon the voltage of the exciter bus, which is usually 110 but is sometimes as high as 250 volts.

If it is desired to maintain constant the voltage on discharge, it will be necessary to utilize end cells by means of an end-cell switch. In this case the number of cells in the main battery will be equal to the bus voltage divided by 2.1 and

the total number of cells in the battery will be equal to the bus voltage divided by the voltage per cell at the end of discharge for the discharge rate used. The difference between the number of cells in the main battery and the number of cells in the whole battery will be the number of end cells. These end cells can be connected to the points on the end-cell switch singly or in pairs, depending upon the voltage regulation desired. If it is not desired to maintain a constant voltage on an exciter bus, the end cells will not be required. The total number of cells will be kept on the bus bars and as the voltage of the battery falls the field excitation of the generators can be maintained constant by changing the generator field rheostats.

The ampere capacity of the cells will be determined by the amount and duration of the load to be carried. Knowing the ampere capacity, the length of time and the minimum allowable voltage during the period, the number of cells and the type of battery can be determined by the use of the charge and discharge curve and battery rating given in trade catalogues.

LINE BATTERIES FOR RAILWAY WORK. — (*See also Trolley Systems.*) The functions of a storage battery floating on an electric railway line at a distance from the power house are as follows:

1. To improve the line voltage and displace a certain amount of feed wire.
2. To relieve the power house of fluctuations of load.
3. To keep the cars moving when the power supply is temporarily interrupted.
4. To supply power for the operation of a few cars or lights at night when the power house is shut down.

Location. — Before any calculations can be made, the location of the battery must be decided. Where there are no other determining factors, the battery would ordinarily be located about three-quarters of the distance from the power house to the end of the line, or, if the line is fed from both ends, midway between the feeding points. In some cases the location of the maximum momentary load, due, for example, to a severe grade or to the passing of two cars at a siding, will influence the location of the battery. In other cases the location of a suitable piece of property owned by the company, or the location of a car barn where attendance would be available, may be the determining factor.

Line Battery without Boosted Feeder. — In this class of line-battery installation the number of cells is determined by the average line voltage at the proposed battery site on the basis of approximately 2.1 volts per cell. Thus, if the average voltage at the battery site, as determined by voltage readings taken at frequent intervals, such as five or ten seconds, over a considerable length of time, should be found to be 500 volts, the number of cells suitable for this average voltage would be 238. If it is found that, owing to changes in load conditions at different hours of the day or on different days of the week, the average voltage varies considerably for prolonged periods of time, it may be necessary to provide means for changing the number of cells connected to the line. This may be accomplished by separating a group of cells at one end of the battery, and arranging suitable switches so that these cells may be connected either in series with the main battery when the average voltage is high or in parallel with an equal number of cells of the main battery when the average voltage is low. It is found preferable to connect these end cells in parallel with a portion of the main battery, and thus keep them active rather than disconnect them from the system entirely.

Calculation of Size of Battery Required. — See article on *Trolley Systems, Overhead*, and *Bulletin No. 134* of The Electric Storage Battery Co.; also Chap. 44 of Lyndon's *Storage Battery Engineering*.

BATTERIES FOR SMALL CENTRAL STATIONS AND ISOLATED PLANTS. — The objects of a battery installation in small electric lighting and power plants may be any one or all of the following:

1. Twenty-four-hour service with but a few hours daily operation of engine.
2. Reduction in size of engine and dynamo otherwise required.
3. Improved voltage regulation.
4. A source of current during breakdown of machinery, or while repairs are being made.

Twenty-four-hour Service. — In this service the battery supplies the current during the hours when the engine is not running. As the capacity of a battery of normal size in many instances in this class of work, especially in residential service, is not generally exhausted in daily operation, it may usually be charged by running the engine and dynamo only two or three hours each day at the most convenient time. In summer, when the lighting hours are shorter, it may be sufficient to charge only once in two or three days.

Reduction in Size of Generator. — If the plant is so designed that the combined capacities of the dynamo and the battery are equal to the maximum load requirements, a smaller engine and dynamo may be installed than would be required if the dynamo alone had to carry the total maximum load. The battery will then be discharged in parallel with the dynamo, to assist it by taking a part of the total load on special occasions, as when the maximum number of lights is in use.

Improved Regulation. — Objectionable fluctuations of voltage and consequent flickering of lights, due, for example, to the operation of an electric elevator or other motor work, may be eliminated by floating the battery across the lighting bus.

Emergency. — In case of temporary derangement of the engine or dynamo, which may occur when the lighting service is most important, or in case repairs to the machinery are required, a battery will serve to tide over the break and avoid any interruption in the current supply.

Small Regulating Battery with Counter Cells. — Fig. 4 shows an ideal arrangement for small battery lighting plants where the capacity of the battery does not greatly exceed 300 ampere-hours at the normal rate. This figure shows 62 cells in the main battery with 8 counter cells, the whole being designed to operate on a 110-volt bus-bar. It will be noted that the battery is charged with two halves in parallel. During charge each half is placed across the bus-bars in series with a fixed resistance. At the beginning of charge the counter cells are all in circuit and assist the fixed resistances in reducing the bus voltage to the proper charging voltage. The counter cells are cut out of circuit as the battery voltage rises. On discharge, all of the cells in the main battery and the full number of counter cells are connected in series. As the voltage falls on discharge the counter cells are cut out of circuit, thus maintaining a steady voltage at the bus-bars. The number of cells and counter

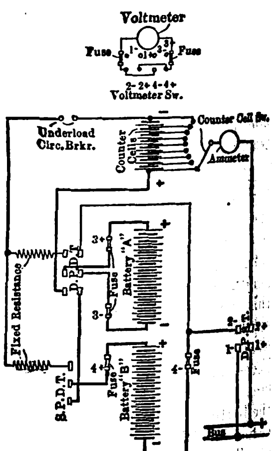


Fig. 4. Connections of Small Regulating Battery

cells shown in the figure are selected to maintain a steady voltage of 110 volts at the normal rate of discharge. The counter cells are composed of unformed battery plates or grids, and since they have very little active material they have practically no capacity. Each counter cell will, however, furnish an opposing voltage of approximately 2.3 volts; this counter voltage is practically constant over the range in current demand for an ordinary small station.

Small Regulating Battery with Charging Booster. — For larger-sized plants this method of operation is not economical owing to the loss in the charging rheostats and the loss in the counter cells. For a straight lighting plant where a battery of more than 400 ampere-hours capacity is required, it is usual to use the scheme shown in Fig. 5. It will be noted in this scheme that the cells in series are connected to the bus for discharge, the voltage being controlled by cutting in and out the end cells at one end of the series. On charge the required voltage to complete the charge is obtained by adding to the bus-bar voltage the voltage of a small booster generator.

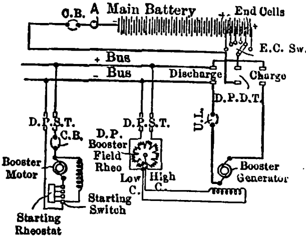


Fig. 5. Connections of Small Battery with Booster

Size of Battery. — For ordinary small plants which operate at a bus voltage of 110 the standard equipment consists of 64 cells, including 12 end cells. The capacity and type of the cells are fixed by the ampere-hour requirements and the minimum-voltage requirements on discharge. For any bus voltage other than 110 volts the complete number of cells, including the end cells, can be determined by dividing the minimum allowable bus voltage by the minimum voltage per cell at the end of discharge at the rate which will be used. The number of cells in the main battery will be equal to the bus voltage divided by 2.1. The difference between the number of cells in the main battery and the total number of cells will give the number of end cells. An end-cell switch having one point more than the number of end-cells required will be necessary.

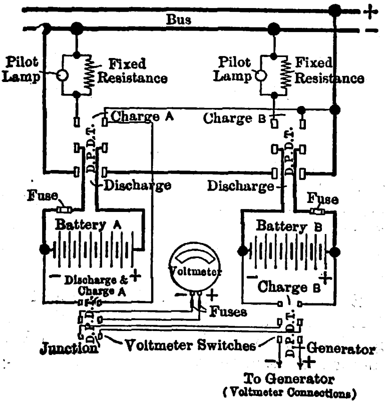


Fig. 6. Connections of Small Battery without Booster Cells

Small Battery without Voltage Regulation. — If voltage regulation on discharge is not required, the scheme shown in Fig. 6 can be used. It will be noted that this scheme is practically the same as shown in Fig. 4 except that the counter cells have been omitted and only 56 cells are used.

Small Battery on 3-Wire System. — In small central stations or isolated plants which utilize the 3-wire 110 to 220-volt system, the battery is arranged in two halves, one on each side of the neutral. A group of end cells on each side of the system and two booster generators direct con-

nected to one motor are used. This scheme of operation is shown in general in Fig. 1 which is described in detail above in the section on *Stand-by Batteries*.

Small Battery for Low-voltage Lighting. — Since the advent of the tungsten lamp, 32-volt plants for private residences and other small power plants are growing very popular. Fig. 7 shows a standard scheme utilizing a 32 to 42-volt generator, driven by a gasoline engine, and 16 cells of storage battery. It will be noted from the diagram shown that the lights can be carried entirely by either the battery or the generator, or by the battery and generator in parallel. It will also be noted that on charge the voltage of the generator is raised and three counter cells are placed in series between the generator and the lights, thus preventing excessive voltage at the lights.

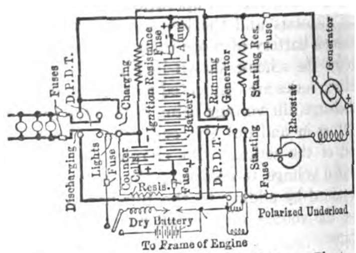


Fig. 7. Connections for Small Power Plant

Referring to Fig. 7 it will be noted that there is shown a polarized underload circuit breaker which is used for opening the circuit in case of trouble with the engine during charge. This is a reverse-current circuit breaker, which permits the current to drop to zero and even to reverse slightly before the circuit breaker will open. This provision avoids the difficulty of frequent opening of the breaker when the gas engine misses an explosion in normal operation. The polarizing coil is connected across the counter cells with a resistance in series for adjustment.

Current for ignition is taken from the terminals of the counter cells. The dry-battery cells shown are utilized for sparking the engine when the plant is first installed until the storage-battery cells have been charged.

The ignition resistance shown is for charging the counter cells whenever the dynamo is running. Ordinarily the engine ignition is obtained from the counter-cell circuit, the generator supplying the average current required and the counter cells acting as a reservoir.

The starting resistance shown in the diagram is used for starting up the engine from the battery. The generator is then used as a motor.

Size of Battery. — The proper type and capacity of cell will depend upon the requirements. It is customary to install a battery having a capacity at the normal rate sufficient to carry the ordinary lighting load for from 2 to 5 days. With a battery of this capacity it will be necessary to run the engine and generator only a few hours each day in order to keep the battery fully charged. Trade bulletins describing in detail these small plants can be obtained from the manufacturers.

SMALL REGULATING BATTERIES FOR ELEVATORS, ETC. — Fig. 8 shows a scheme of load regulation which has been adopted as standard for situations where it is desirable to operate a lighting load and a fluctuating motor load from the same generator. This scheme is known as the "constant-current" scheme.

Elevator Load Regulation. —This scheme finds its greatest use in office buildings, hotels, etc., where electric elevators are in service. The load on such a plant is a very fluctuating one on account of the heavy current required for starting the elevators. This starting current is generally about twice the elevator running current. Moreover, as the elevators are not in service con- ... less than the running

current per elevator. There is generally, therefore, a considerable difference between the average load on the station and the maximum momentary load. This difference varies with the number and type of elevators, the character of elevator service and the magnitude of the lighting load. If the fluctuations

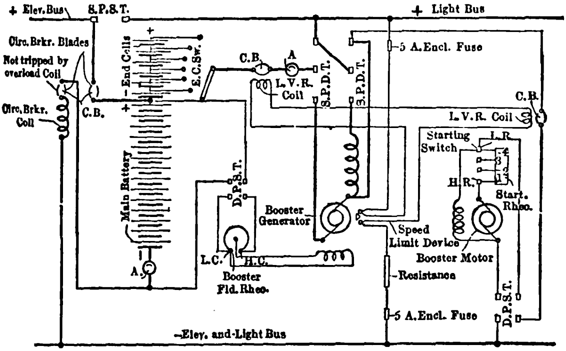


Fig. 8. Connections for "Constant-current" Regulation

above and below the true average are a comparatively large percentage of the true average load, they are liable to cause fluctuations in the generator voltage with consequent flickering of lights. In addition to this objection, the generating capacity necessary without a battery must be increased and it is necessary to operate the whole plant at a poor load factor.

Constant-current Scheme. — Reference to Fig. 8 shows that the battery and the elevator circuit are in multiple, and that the booster is in series between the lighting bus and the elevator bus. The booster in the constant-current scheme, as the name would imply, is wound in such a manner that the current going through it is rendered constant. Therefore, the parallel system of battery and elevator circuit can only draw a constant current from the generator, and it follows that all of the fluctuations due to the elevator load must be taken by the battery. When the elevator current is greater or less than the average, the difference is given or absorbed by the battery. It is important to note that the battery voltage applied to the elevator motors is that of the battery alone and is therefore variable. Ordinarily this is not a disadvantage as the heavy load is caused by the starting of motors, in which case high voltage is not an object.

It will be noted that the scheme given in Fig. 8 shows end cells. These are used for regulating the voltage on the lights when the generator is shut down and the booster is out of circuit, the battery carrying the entire lighting and elevator load.

Other Uses of Small Regulating Batteries. — Electric-elevator service has been taken as an example, as this is the class of work usually encountered. This scheme may be used, however, for other fluctuating loads, such as are due to electric cranes, shop tramways, mine hoists, etc., and in general for any fluctuating load where the maximum load is greatly in excess of the average load and constant potential on the motor bus is not required.

For fluctuating loads where constant potential on the power bus is required, the scheme described above under *Large Regulating Batteries* and shown in Fig. 2 should be used.

Number and Capacity of Cells. — The number of cells in the main battery is usually determined by dividing the bus voltage by 2.1. The number of end cells necessary will be determined by the minimum allowable voltage and the final cell voltage at the end of discharge when the battery is carrying the entire load. The capacity of the battery will be determined by the amount of the fluctuations to be taken care of by the battery and the amount of load to be taken out of the battery when the generating apparatus is shut down.

Booster. — It will be noted in Fig. 8 that the diagram shows a differentially-wound booster. The desired regulation is obtained by means of the series field, which carries the constant current between the lighting bus and the elevator bus, and which is designed to produce a booster voltage in the direction to oppose this current. The relation between the series field and the shunt field is adjusted for any given average load.

A straight shunt booster with the field controlled by the carbon regulator as shown in Fig. 2 can be utilized with this constant-current scheme, if the solenoid is connected between the lighting bus and the elevator bus and if the carbon regulator actuated by this solenoid is utilized to control the field of the exciter as shown in Fig. 2. In all other respects this carbon regulator constant-current scheme operates according to the same general principle as the scheme for the differentially-wound booster.

It will be noted also that with the scheme shown in Fig. 2 the booster is in series with the battery and carries only the battery current, whereas in the constant-current scheme shown in Fig. 8 the booster carries the average current of the fluctuating load. The relative magnitude of the fluctuations as compared with the average will determine which booster will be least expensive and may, therefore, be a factor in deciding between the two schemes.

Two- and Three-wire Systems. — The scheme shown in Fig. 8 is for a two-wire system. This same general scheme is utilized for a three-wire system where the motor load is connected across the outside wires and the lights are balanced on each side of the neutral. Where the three-wire system is used, the general principles of the scheme are exactly the same as shown in Fig. 8, except that it will be necessary to have one booster generator and one set of end cells on each side of the system.

ELECTRIC-VEHICLE BATTERIES. — The use of storage batteries for the propulsion of electric vehicles is increasing rapidly. The voltage and capacity of the battery necessary to operate any given vehicle is usually specified by the manufacturer of the vehicle, as the design of the vehicle determines the number of watt hours per car mile under certain definite conditions. Unless the vehicle user is a battery expert and a vehicle expert, it is not advisable that he specify either the number or capacity of cells to be used.

Desirable Characteristics of Vehicle Batteries. — The following are the chief characteristics of a good vehicle battery:

1. Low internal resistance, because of high current demand required for acceleration or for ascending grades.

2. Small capacity temperature coefficient, in order that the capacity of the battery shall not be unduly affected by chilling of the cells.

3. Voltage characteristic as nearly flat as possible under average conditions of discharge.

4. Construction of sufficient ruggedness to prevent breakage of parts in ordinary service.

5. Assembly such as to minimize possible labor and attention for flushing, cleaning, removing short-circuits, etc.

6. Ability to stand some abuse, as the average vehicle operator is not a battery

It is impossible for a battery to possess all these qualities to the same extent. With regard to the first three requirements the lead battery possesses peculiar advantages, whereas the Edison battery is superior in the last three features.

The use of batteries for vehicle work is treated in greater detail in the article on *Automobiles, Electric*.

DRAWBRIDGE BATTERIES. — It is becoming standard practice to install storage batteries in connection with electrically-operated drawbridges, chiefly as an insurance against the interruption of power from the generating station. The voltage of drawbridge motors is usually 220 or 500. The number of cells and the capacity of the cells to meet the requirements will depend upon the maximum and minimum allowable voltages, and the total energy required from the battery before recharging.

Size of Drawbridge Battery. — Assume a 2000-ton bascule lift bridge with a 185-foot span. This bridge is to be operated with 220-volt motors. The time required to open or close is 1½ minutes, or a total of 3 minutes for a complete opening and closing. Assume the following conditions: The battery will be required to open and close the bridge 40 times on one charge, the maximum allowable voltage during bridge operation will be 250 and the minimum allowable voltage 200; the average current demand over a complete cycle will be 100 amperes with a maximum momentary current demand of 320 amperes lasting a few seconds.

It is seen that the total energy in ampere hours required for the 40 complete openings and closings of the bridge would be equal to $\frac{3 \times 100 \times 40}{60} = 200$ ampere hours during a total period of $3 \times 40 = 120$ minutes or 2 hours. The battery, therefore, must have a capacity not less than 200 ampere hours at the 2-hour rate of discharge. Bearing in mind the voltage limits specified above, an inspection of Figs. 3 and 6 in the article on *Batteries, Storage, Lead Type*, will show that the battery capacity will be fixed by the voltage limits in this case and not by the ampere-hour requirements. In any case that may arise, if there is any doubt as to which requirement will fix the capacity of the battery, select a battery having the ampere-hour capacity required and then examine Fig. 6 to see if a battery having this ampere-hour capacity will meet the voltage requirements.

For a maximum allowable voltage of 250 and a floating voltage of 2.1 volts per cell, there would be required 120 cells. Referring to Fig. 6 and using the upper curves, for plates 10¾ inches square, we will assume that the battery must not discharge at a higher rate than 4 times the normal rate. At 4 times the normal rate the voltage drops almost immediately to 1.9 volts per cell. With 120 cells the voltage would drop to 228 volts. It is noted that the normal rate of this battery will be 80 amperes, and the average discharge of 100 amperes will be 1¼ times the normal rate. By interpolating a curve between the normal curve and four times the normal curve, it is seen that with an average discharge at this rate for 2 hours, the voltage per cell at the end of the two hours, with 100 amperes flowing, will be approximately 1.9 volts, or the voltage of the 120-cell battery will be 228 volts. If, now, the current were increased to 320 amperes (4 times the normal rate), and this current flow continued for an appreciable period of time, the voltage would drop further, by about 20 volts, leaving a voltage of 208 at the battery terminals. It is, therefore, evident that the voltage requirements will be met by a 120-cell battery having a capacity of 80 amperes at the normal rate.

A battery having a normal rate of 80 amperes could be discharged at 1¼ times the normal rate for something over 4 hours. It is, therefore, seen that the battery which we selected has more than ample ampere-hour capacity to

meet the requirements, its capacity being fixed by the momentary current demand.

It usually will be found that the capacity of cells required for drawbridge operation is fixed by the maximum discharge rate and not by the ampere-hour capacity at the average rate.

Size of Charging Generator. — If a separate power plant is installed for the operation of the drawbridge, it will be well to install an engine-generator set having an available voltage range of from 220 to 325 volts. There may be required 2.7 volts per cell for charge, which with 120 cells would call for 324 volts. If the battery installed is of sufficient capacity to carry the bridge a complete day or more on one discharge, it is good practice to install a generator of only sufficient capacity to charge the battery at the normal rate and to operate the bridge at all times from the battery. In the case cited above the capacity of the generator would be approximately 80 amperes. If it is required ordinarily to supply the bridge from the generator and hold the battery for emergencies, the generator must have sufficient capacity to carry the maximum current required by the bridge motors.

IGNITION BATTERIES FOR PORTABLE AND STATIONARY GAS ENGINES. — It is the usual practice with portable gas engines, automobiles, etc., to use for ignition a 6-volt portable battery put up in a wooden case. The capacity most frequently used is 60 ampere hours, though capacities ranging from 40 to 100 ampere hours are used, depending upon the requirements.

For stationary engines of large capacity it is customary to use a voltage ranging from 90 to 110 volts. Glass-jar batteries of sufficient ampere-hour capacity to meet the requirements are installed in a cabinet in the main engine room. The number of cells in the main battery necessary to float on the system will be equal to the bus voltage divided by 2.1. The capacity of the battery will depend upon the number of ampere-hours required on one charge and the minimum allowable voltage during a discharge. These requirements being known, the correct number and type of cells can be determined from the characteristics of the batteries (*see Batteries, Storage, Lead Type and Alkaline Type*), in connection with trade catalogues.

Occasionally, there is installed in addition to the cells in the main battery a group of end cells which can be cut in on discharge by means of a single switch and thus hold up the voltage of the battery. The number of cells in this end-cell group usually varies from 3 to 8, depending upon the voltage requirements.

Usually a large ignition battery is charged by placing the two halves in parallel, each half being connected to the bus through a fixed resistance somewhat after the fashion explained above and indicated in Fig. 6.

BATTERIES FOR STEADY-VOLTAGE REQUIREMENTS. — It is frequently required to obtain a source of direct current with a steady voltage, as, for instance, in electroplating, instrument calibration, telephone work, laboratory work, etc. In such cases there should be chosen a battery that has more capacity at the normal rate than the ampere hours actually required before recharging, in order that the battery may be worked on the flat portion of the discharge curve.

For instance, to furnish 10 amperes continuously for 8 hours at about 20 volts, with a variation of 1 per cent between the beginning and the end of the discharge, the time of discharge must be limited to about 50 per cent of the total time. A cell having a capacity of 10 amperes for 16 hours should, therefore, be chosen.

BIBLIOGRAPHY. — See Bibliography in article on *Batteries, Storage, Lead Type.*

[TALIAFERRO MILTON.]

BATTERIES, STORAGE, LEAD TYPE. — (*See also Batteries, Storage, Applications of; Batteries, Storage, Alkaline Type; Electrochemistry, Principles of.*) The following is a brief outline of the contents of this article:

TYPES OF LEAD CELLS. — A secondary or storage cell is any voltaic couple which can be regenerated, after exhaustion, by passing a current through it in a direction opposite to the direction of current flow when the couple delivers energy to the external circuit. Three types of storage batteries are now in use in this country, the Planté type, the Faure type, and the alkaline or Edison type. The special features of the alkaline type are treated in a separate article, viz., *Batteries, Storage, Alkaline Type.*

Planté Type. — The principle of the storage battery was discovered in 1801 by Gautherot, a Frenchman. However, practically nothing further was done until 1860 when Gaston Planté constructed a storage cell consisting of two lead strips immersed in dilute sulphuric acid. Planté found that, by giving this cell a long charge by passing the current through in one direction, and then giving the cell a long charge in the opposite direction, and repeating this cycle many times, the capacity of the cell was considerably increased. Little improvement was made in the Planté cell until a number of years after Planté's discovery.

At present the Planté type of storage battery is made by preparing a pure lead plate with a large superficial area exposed and then oxidizing the surface electrolytically so that it is covered with lead peroxide. A plate formed thus is a positive plate. To form a negative plate the peroxide, after electrolytic formation, is reduced to metallic sponge lead by reversal of current. In this manner a thin layer of active material which is porous and which adheres firmly to the supporting lead plate is produced.

Use of Planté Type. — The Planté type is used chiefly where weight and space are of no great importance.

Faure or Pasted Plate. — In 1880 Camille A. Faure, in France, and Charles F. Brush, in America, simultaneously developed the "pasted" type of storage battery. In this type of storage battery the active material, on both the positive and negative plates, instead of being formed electrolytically, as was done by Planté, was applied, in the form of a paste, to a stiff lead-antimony alloy supporting grid. This type of plate is now commonly called the Faure type of plate, and it was the discovery of this type that gave the storage battery its first commercial impetus.

Use of the Faure or Pasted Plate Type. — The Faure type is used chiefly where it is desired to obtain the greatest possible capacity with a minimum of weight and space occupied. This type is used chiefly for vehicle propulsion and for central-station "stand-by service."

THEORY. — The electrochemical theories of the storage battery are quite complicated and the various authorities on these subjects do not agree on all details. In this outline no attempt will be made to cover the various theories in detail; only the simplest fundamental chemical actions will be described.

Positive and Negative Plates. — The terms positive and negative are employed throughout this article in accordance with engineering usage; that is, the positive plate is the one from which the current flows on discharge and the negative plate is the one into which current flows on discharge. In a lead battery the positive plate, on which the lead peroxide is formed, has a comparatively hard surface of a reddish-brown or chocolate color, while the negative plate, which carries the sponge lead, has a much softer surface of a grayish color.

Chemical Reactions. — The active elements of the lead-lead acid type of battery consist of lead peroxide (PbO_2) on the positive plate, sponge lead (Pb) on the negative plate and dilute sulphuric acid (H_2SO_4) for the electrolyte.

Whatever the secondary reactions may be, it is agreed that the final result on discharge is the formation of lead sulphate ($PbSO_4$) on both the positive and negative plates, the SO_4 radical of the sulphuric acid combining with the lead of both plates to form this compound, resulting in the formation of some water (H_2O), with a consequent decrease in the specific gravity of the electrolyte. On charge the electric current splits up the lead sulphate ($PbSO_4$), returning the SO_4 radical to the electrolyte, oxidizes the positive plate to its original condition of lead peroxide (PbO_2) and reduces the negative plate to its original condition of sponge lead (Pb). This action may be represented as follows:

$$\underset{+\text{ Plate}}{PbO_2} + 2\,H_2SO_4 + \underset{-\text{ Plate}}{Pb} = \underset{+\text{ Plate}}{PbSO_4} + 2\,H_2O + \underset{-\text{ Plate}}{PbSO_4}$$

This equation read from left to right is the equation of discharge; if read from right to left, it is the equation of charge. In practice, on charge, towards the end of charge some of the water (H_2O) is split up by the current into its component parts, hydrogen (H) and oxygen (O), the hydrogen being liberated at the negative plate and the oxygen at the positive plate. This occurs whenever the density of charging current is greater than can be utilized in decomposing the lead sulphate remaining in the plates.

DESIGN. — The capacity of a storage-battery plate depends not only upon the amount of active material but also upon the active surface exposed to the electrolyte. Plates should therefore be designed to expose a maximum amount of surface consistent with the strength of the supporting grid or lead base. In order that the active material of the plates can be acted upon by the electrolyte, it should be as porous as is consistent with its proper support by, and proper contact with, the grid or supporting lead base.

Sponge lead and lead peroxide possess little mechanical strength. Lead peroxide is a poor conductor. Since mechanical conditions require a certain amount of rigidity in the plates and since the current generated by the active materials must be carried away, a battery plate must necessarily consist of two parts, viz., the grid or supporting base and the active material which in the lead battery is finely-divided porous sponge lead or porous lead peroxide, and in the alkaline battery is finely-divided nickel and nickel hydroxide in the positive plate and iron oxide in the negative plate.

Lead Plates. — There are certain ideal requirements to be met by a lead storage-battery plate, among which may be mentioned:

1. The grid and active material should be so proportioned as to obtain, as far as possible, uniform current distribution over the entire surface.

2. The active material should be applied to, or formed on, the grid in such a manner that good electrical contact exists when the plate is new and this contact should be maintained during the life of the plate.

3. In pasted plates and pellet plates the grid should be of material that is not injuriously affected by the electrolyte.

4. The material of the grid should be such as to insure a minimum of local action between itself and the active material.

5. The surface of the active material exposed to the action of the electrolyte should be as great as is compatible with proper mechanical strength and a suitable provision for the natural expansion of the active material in use.

6. The plates should be so made and assembled as to allow the maximum possible diffusion of the electrolyte through and around the plates and through the containing vessel.

7. The grid or lead base should be of ample cross section to carry the current generated under working conditions without undue loss.

8. The lugs which collect the current from the grids should be of ample cross section, and when designed for heavy currents they should be so arranged as to distribute the current uniformly through the grid.

9. The negative, sponge-lead plate should be constructed so that the sponge lead retains its porous character and does not grow hard and dense in service and thus lose its capacity.

10. The lead and the electrolyte should be pure; otherwise secondary reactions will be set up, with a consequent decrease in efficiency.

Of the many lead plates on the market to-day, the most commonly used are the "pasted," Planté and composite or "pellet" plates. Cuts showing the construction of the various types of plates may be found in manufacturers' catalogues; the essential features of design are briefly discussed below.

Pasted Plates. — The pasted plate is usually made by applying to a hard lead-antimony grid a paste made of some oxide of lead, usually litharge (PbO) or red lead (Pb_3O_4), and some liquid and other substances. In the so-called "Iron-clad" battery made by the Electric Storage Battery Co., the active material is held in perforated hard rubber tubes. Various substances are used to mix with the lead oxide, the idea being to increase the hardness, porosity, toughness and conductivity. Some of the substances used by different manufacturers include anthrocine, glycerine, graphite, potassium silicate, asbestos, ammonium-sulphate, etc.

"Forming" of Pasted Plates. — After the grid is filled with the paste, the plate is dried. After being completely dried a number of plates are assembled in a forming bath of dilute sulphuric acid with dummy lead plates for the opposite electrode and the forming charge is given by passing the proper current through the voltaic couple thus formed. Positive plates are formed by connecting the plates to be formed as the anode; the current oxidizes the lead oxide further to lead peroxide (PbO_2). Negative plates are formed by passing the current in the opposite direction, reducing the lead oxide to sponge lead. After the forming charge the plates are dried and are ready for the market.

Planté Plates. — Of the various types of Planté plates, among those most commonly in use may be mentioned the central-web type; the cast-lead having no central web; and the composite or "pellet" type. The main idea in any of these methods of manufacture is to produce a large surface on which to form the active material.

Central-web Type. — In the central-web type there is a solid sheet or "web" of pure lead on which the ribs are formed. This web prevents the circulation of the electrolyte through the plate. The ribs are formed from the original lead plate, either by rolling, spinning or cutting. In the rolling and spinning processes the plate is formed from a lead blank by means of a number of steel disks placed side by side and separated by small spacers on a shaft. The lead blank is passed between two sets of disks by forward and backward movements. The disks gradually work deeper into the plate and squeeze up

lead into the spaces between the adjacent steel disks. In the cut type of plate the ribs are formed from the lead sheet by a tool, which at each stroke turns up one complete rib. The cutting edge works at an angle so that the finished ribs stand out from the surface. The ribs may incline upward from the central web and thus form pockets to hold the active material and prevent its falling away.

The disadvantage of the *web* type is the web itself. There is invariably a tendency towards unequal work on the two sides of the positive plate, this tendency being caused by difference in the plate spacing, unequal capacity of the negative plate on either side, inequality in the shape of the ribs, etc. Where the active material and the active surface of a plate are disposed in planes perpendicular to the face of the plate and extend *through* the plate, excessive action on one side simply works the plate a little further through from that side, the effect on the active material, however, being uniform throughout. With a plate provided with a central web, preventing such action, any inequality of work will charge or discharge one side more than the other, producing a tendency to buckle.

Cast-lead Type. — The type of plate which has no central web is made by casting pure soft lead in a mold, casting having the advantage of allowing for distributing metal in the plate without limitations in manufacturing process. The plate as it comes from the mold consists of a great number of short vertical ribs running entirely through the plate and bound together by transverse ribs to give strength to the plate. In this manner a large surface can be obtained and in a plate having no central web the electrolyte can circulate through the plate, and the active material will be uniformly worked throughout even though the amount of work on the two sides of the plate be unequal. The best-known form of this plate is the "Tudor" positive.

Composite or "Pellet" Type.— The composite or pellet-type plate is made by rolling up into pellets pure soft-lead corrugated ribbons. These pellets or buttons are then forced by pressure into circular openings in a grid composed of a hard-lead-antimony alloy. The openings in the grid are beveled towards the center so that when the pellets are formed the swelling action causes them to rivet tightly into place. The corrugations on the lead ribbon allow the electrolyte to circulate through the plate and thus expose the full surface of the closed lead spiral to the action of the electrolyte. It will be noted that in this type of plate additional mechanical strength is obtained from the supporting grid. The best-known form of this plate is the "Manchester" positive.

"Forming" of Planté Plates. — In all Planté positives, after the ribs or corrugations have been formed on the lead blank or the pellets have been placed in the hard grid, the plates are assembled in a sulphuric-acid bath containing some corrosive chemical, called a "forming agent," together with dummy lead plates. The forming agents used by various manufacturers are usually kept as trade secrets; the nature and method of using such agents determines largely the quality of the battery. To form the positive plates the dummies are connected as the cathodes, and a current is passed through the couple thus formed. The electrolytic action of this current causes lead peroxide (PbO_2) to be formed from the lead of the ribs. The strength and duration of the current produce the desired thickness of lead peroxide on the ribs or pellets.

Negative Planté plates are made from positive plates by electrolytically reducing the lead peroxide to sponge lead. In most types of Planté negatives, however, it is necessary to form them with an initial capacity considerably in excess of the capacity of the positive plate. This is due to the fact that in actual service the sponge lead shrinks and loses its spongy nature, thereby reducing its capacity, since insufficient surface is exposed to the action of the

"Permanizing" Process. — One of the large manufacturers of storage batteries has recently developed a process for making the Planté negative plate permanent. This method consists of injecting into the sponge lead a material which prevents the shrinking of the sponge lead and thus enables it to maintain its spongy character and its initial capacity. This permanizing process can be applied to the pasted negative plate as well as to the Planté negative plate. The most widely used form of the permanized negative plate is one in which the sponge lead, formed by reducing lead oxide electrolytically, is retained in boxes made by casting a soft-lead perforated sheet on a hard lead-antimony grid. This negative is known as the "Box negative."

Applications of the Various Types of Plates. — In stationary work requiring frequent discharges, where expert attention is available at all times, and where the conditions of operation are properly suited to its use, the all-lead Planté type of plate with through-and-through circulation is the plate that should be used. Where the battery is to receive little attention and where the conditions of service are variable and uncertain, the composite or "pellet" type of plate is the safest plate to use, on account of its rugged structure and its ability to stand great abuse. As stated above, for stand-by service and for vehicle service and other service where high capacity with a minimum weight and space is desired, the pasted type of plate is more desirable.

Electrolyte. — The electrolyte used with the lead type of battery is always a dilute solution of sulphuric acid. The specific gravity of the electrolyte, when the battery is fully charged, varies from about 1.210 for stationary batteries to 1.300 for automobile ignition batteries. These values have been adopted as standard by all the leading manufacturers.

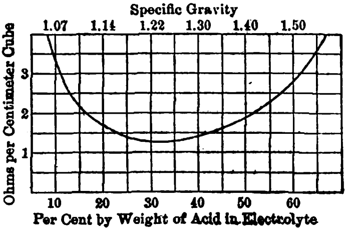


Fig. 1.

The proper specific gravity to use varies with the conditions. Fig. 1, showing the variation with specific gravity of the resistance of one cubic centimeter of electrolyte, shows that the resistance of dilute sulphuric acid is least at a specific gravity of from 1.224 to 1.240, this resistance increasing if the specific gravity be either increased or decreased. There are numerous other conditions which influence the selection of the proper specific gravity.

The curves in Fig. 2 show the specific gravity of various mixtures, both by weight and by volume, of one part of 1.840 specific gravity acid with from 1/10 to 7 parts of water. There is also a curve showing the percentage, by weight, of 1.840 specific gravity acid in mixtures of various specific gravities. These curves are approximately correct at 60° F. Unless a compensating hydrometer is used in determining the specific gravity, allowance must be made for temperature variation, on the basis of an increase of one point (i.e., one one-thousandth) in gravity for each 3 degrees Fahrenheit *decrease* in temperature, and vice versa; for instance, electrolyte that has a specific gravity of 1.210 at 70° F. will have a specific gravity of 1.213 at 61° F., and 1.207 at 79° F.

Impurities in Electrolyte. — The electrolyte should be free from organic substances, iron, chlorine, copper, arsenic, mercury, nitrates, acetates and the slightest possible trace of platinum. The various battery manufacturers issue exact specifications to the acid manufacturers, specifying the maximum amount of these injurious ingredients which the acid may contain. Electrolyte that is not approved by the company furnishing the battery should never be used.

Preparation of Electrolyte. — In preparing the electrolyte, sulphuric acid, approved by the battery manufacturer, should be diluted with sufficient pure distilled water to bring the mixture to the required specific gravity. The acid should be poured into the water; *never pour the water into the acid.* If the water is poured into the acid, the heat formed by the mixture is sufficient to cause sputtering and damage may ensue.

The sulphuric-acid manufacturing companies furnish electrolyte for battery work in such large quantities that they carry a stock of various standard mixtures. It will usually be found cheaper and more convenient to purchase the electrolyte ready-mixed than to purchase the concentrated sulphuric acid and prepare the mixture on the ground. The latter course, however, is sometimes adopted where the amount of acid used is considerable and where the item of freight saving is appreciable.

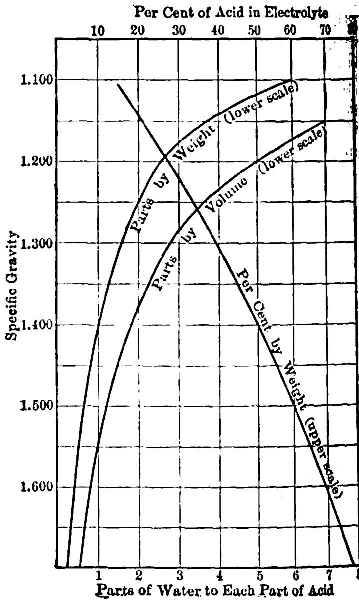


Fig. 2.

Containers for Lead Batteries. — The containing receptacles for holding the battery plates and electrolyte are usually rubber jars, glass jars or lead-lined wooden tanks.

Rubber Jars. — Hard-rubber jars are used exclusively in vehicle and portable batteries. In these batteries the plates are supported on ribs at the bottom of the jars. To reduce cleaning to a minimum, these ribs should be of sufficient height to leave ample space in the bottom of the jar for the reception of the active material which falls away from the plate. If the sediment in the bottom of the jars is allowed to reach the plates and short-circuit them, serious damage will result. The jars themselves should be made of the very best hard rubber which is not affected by sulphuric acid.

Glass Jars are ideal containers for small batteries when they are properly installed. The glass jar has the advantage that the plates and electrolyte can always be seen. The use of the glass jars is limited to the smaller sizes on account of their liability to break, due to strains left in the glass after annealing, temporary strains set up by unequal temperatures between the inside and outside of the jars, and to strains due to weight of the plates and electrolyte. Glass-jar manufacturers have, up to the present time, been unable to produce a thoroughly reliable jar larger than 21 by 13 by 18 inches.

Lead-lined Tanks. — In the larger sizes of batteries lead-lined tanks are standard. Tanks are generally made from specially selected yellow pine, dovetailed together without the use of nails or other metallic fastenings. The upper edges should be slightly beveled inward so that the moisture will drain

lated; these bottoms are usually constructed of slats across the tanks, separated by a small spacing to provide drainage. To further facilitate drainage under the lining, the upper surface of these slats should be grooved crosswise. Before the tanks are lined they should be coated inside and outside with two coats of acid-resisting paint, and a third coat should be added outside during installation. The tanks should be lined with lead of 3 to 4 pounds per square foot, depending on their size. The lining should extend over the upper edge of the tank and a short distance down the sides. The outer edge of the lining should be provided with drip points projecting clear of the tank, so that any drip from the lining will clear the wood of the tank and the tank supports. Especial attention should be paid to the seaming of the linings. All seams should be burned with the hydrogen flame with pure lead, without the use of any flux. The lower corners of the lining should be reënforced by puddling with lead. The upper corner should be reënforced by burning on an additional thickness of sheet lead. Each tank should be so built that it is self-supporting without the use of any braces or reënforcements. If this is done, any tank in the battery can be removed and replaced without affecting the remaining tanks in any way. A poorly constructed wood tank is bound to cause trouble. Special attention should be paid to this detail in preparing specifications.

Covers. — While covers are not absolutely necessary for stationary batteries, it is considered good practice to cover all cells. Covers should preferably be made of glass of sufficient weight to prevent excessive breakage in handling. Covers will more than pay for themselves in reducing evaporation and keeping out dirt. They are absolutely necessary in situations, such as steel mills, where the air is liable to contain particles of foreign matter injurious to the battery.

Support of Plates in Container. — As stated above, in the portable and vehicle types of batteries, the plates are supported on ribs at the bottom of the jar.

In stationary types, using glass jars or lead-lined tanks, it is preferable to support the elements from the top. In glass jars, plates are supported on the top edges of the jars by means of lugs cast or burned on to the plates. In lead-lined tanks it is, of course, necessary to insulate the plates from the lead lining. The plates are, therefore, supported in the tanks by vertical sheets of glass resting on the bottoms of the tanks. The bottom of the tank lining, under the glass sheets, should be heavily reënforced. These glass sheets should be ground top and bottom and the lower corners should be cut off at an angle of about 45°, to avoid injury to the lead lining during installation by sharp corners.

Separation and Separators. — Since the positive and negative groups, forming the complete element in a cell, are assembled with positive and negative plates alternating, it is necessary that some means be provided to prevent contact between adjacent positive and negative plates. Such contact would mean a short-circuit and might result in injury to the element. Various forms of separators have been used for keeping the plates apart. The most common practice to-day, and the one considered the best for stationary work, is to use wooden diaphragms. Each wooden diaphragm should consist of a thin sheet of porous wood mounted in slotted dowels which will allow free passage of the electrolyte between the plates. The wood should be specially treated to insure the absence of elements which in conjunction with the electrolyte might form acids injurious to the plates.

In vehicle batteries rubber separators are generally used in addition to the wood separator. That is, a rubber separator is placed on each side of each positive plate and one wood separator between each positive plate and the adjacent negative plate. In the so-called "iron-clad" battery, made by the

Electric Storage Battery Co., the active material of the positive plate is contained in rubber tubes, and therefore additional rubber separators are not necessary. Ignition batteries use wood separators only. Portable batteries use rubber only. Car-lighting batteries use wood only or rubber only. Yacht-lighting batteries use rubber only.

Sediment Space. — In all types of batteries, no matter what container is used, there should be a certain amount of free space between the bottoms of plates and the bottoms of jars. This space is designed to receive the sediment or active material which falls away from the plates. The amount of sediment space required will depend upon the service, and no fixed rule can be given to cover this point. If not prohibitive from space or expense limitations it would be well to have this sediment space great enough to take all of the active material which will be shed from the plates during their useful life.

Assembly of Parts. — All portable, automobile, car-lighting and other batteries that will be subject to jar or vibration should be assembled in the containing vessel in such a manner that the plates cannot move. They should be packed in as tightly as is possible with due consideration to the amount of electrolyte necessary and the avoidance of injury to the plates when being put into or taken out of the jars. Hard-rubber covers should be used and these covers should be so sealed as to avoid any splashing of the electrolyte. The crates containing the jars should be constructed, as far as possible, without the use of metallic fasteners and they should be thoroughly painted with acid-resisting paint. In portable batteries that will receive rough handling, if several jars are assembled in a single crate, they should be imbedded thoroughly in an acid-proof elastic compound.

Cells assembled in crates should be connected by burning together the adjacent positive and negative terminals. Cell connections should never be soldered. The main terminals of the series in one crate should be brought out and fastened in such a manner that these terminals will not be attacked by the acid.

The methods of assembling and connecting stationary types of cells are described below in the section on *Installation*.

METHODS OF TESTING. — The tests described below are intended to cover only those tests that are of interest in the commercial operation of batteries.

Test of Specific Gravity. — The specific gravity of the electrolyte is the most accurate guide as to the state of charge of a lead-type storage battery. Specific instructions furnished by the manufacturer of the battery are given, showing the range in specific gravity over a given amount of charge and discharge and the operator should be guided by the instructions of the manufacturer. The test of the specific gravity is made by means of a hydrometer having a suitable scale for the type of cell to be tested. In stationary types of batteries the hydrometer has a scale reading of 1150 to 1250 and is left floating in one cell. In all portable types of batteries, and ordinarily in vehicle- and car-lighting batteries, it is usually necessary to draw some of the electrolyte from the cell in order to test its specific gravity with the hydrometer, which should have a scale reading of 1150 to 1300. Hydrometer syringes, with hydrometer contained in a glass barrel, can be obtained on the market for this purpose. There is also on the market a regular acid-testing set consisting of a syringe for withdrawing the acid and a test tube into which the acid is poured from the syringe, in order that the hydrometer may be floated for reading.

Test for Impurities in Electrolyte. — The proper testing of electrolyte for impurities requires not only some knowledge of chemistry, but also experience

for the ordinary user to attempt it. Furthermore, it is usually entirely unnecessary that he should, since the leading battery manufacturers make analyses for their customers free of charge. The harmful impurities most likely to be present are iron, hydrochloric acid, oxides of nitrogen, sulphurous acid, arsenic, organic matter and platinum. In order to insure freedom from the latter, it is advisable to specify acid which has not been concentrated in platinum.

Test of Internal Resistance. — The most accurate method of determining the internal resistance of a cell is to subject it to a discharge current of a certain amount, and after the voltage has become sufficiently constant to permit accurate reading with the discharge current flowing, the current is instantly interrupted and the rise of voltage noted. This rise of voltage divided by the current will give the internal resistance. If this test is made on a battery of more than one cell in series the result shows the internal resistance of the entire battery, including the cell connections.

Test of Capacity. — The capacity of a storage cell depends on various conditions, such as the temperature, the rate of discharge, the strength of the electrolyte, the character of service to which it has previously been subjected and the attention it has received. The "normal capacity" of a storage cell is usually expressed in ampere hours at the 8-hour rate at 70° F., down to a certain definite voltage per cell. For instance, when it is said that a cell has a capacity of 100 ampere hours, it is *usually* meant that this cell can be discharged at a rate of 12½ amperes continuously for 8 hours at 70° F., down to the limiting voltage specified by the battery manufacturer. In lead cells this limiting voltage is usually taken at 1.75 volts per cell. The "watt-hour" capacity of a battery is equal to the ampere-hour capacity multiplied by the average voltage during discharge.

The "available" ampere-hour and watt-hour capacity of a battery varies with the rate of discharge, the available capacity decreasing with increase of rate. The higher the rate of discharge the lower the limiting voltage at end of discharge. Therefore, to test the capacity at any given rate of discharge, the limiting voltage at that rate as specified by the manufacturer should be known.

Bearing in mind the above facts, to test the capacity of a storage battery at any given rate, the battery should be first charged at the normal rate, as specified by the manufacturer, until the voltage and gravity in all of the cells will rise no further and until all cells gas freely at both the positive and negative plates. The capacity of the battery, as a whole, will be limited by the capacity of the lowest cell and all cells should be as nearly as possible in a condition of full charge before the discharge is started to test the capacity of the battery. Individual cell readings should be taken of the voltage and gravity while the charging current is flowing.

When it is certain that the battery is fully charged, discharge the battery at a constant current at the desired rate, down to the limiting voltage at this rate. The ampere-hour capacity will be equal to the constant current in amperes, multiplied by the time of the discharge in hours; and the watt-hour capacity of the battery will be the ampere-hour capacity multiplied by the average voltage during discharge.

In order to be sure that the battery is in good condition it is well to take several complete discharges before making the final charge, preceding the final test discharge.

On each charge and discharge observations should be taken of the voltage, the current, the temperature and the specific gravity. The temperature of the room should be held as nearly constant as possible throughout the test. In order to facilitate the plotting of curves, the readings should be taken at intervals of time which are even factors of an hour.

Cadmium Test. — When a strip of cadmium is placed across the top of the plates in a cell, and properly insulated from them, there will be a difference in potential, as measured by a voltmeter, between the cadmium and the positive plates and between the cadmium and the negative plates; the sum or difference (depending on the relative directions of the two potential differences) between these two readings being equal to the internal voltage of the cell.

Such a test, when properly interpreted, is a valuable guide to the capacity and condition of the positive and negative plates separately. The results are, however, very apt to be misleading to one who has not had a wide experience in storage-battery work.

Test of Efficiency. — The ampere-hour efficiency of a battery is the ratio of the output on discharge in ampere-hours to the input on charge in ampere-hours; the watt-hour efficiency of a battery is the ratio of the output on discharge in watt-hours to the input on charge in watt-hours.

The efficiency of a battery depends upon numerous conditions, chief among which are the charge and discharge rates and the temperature.

It has been noted above that the *available* capacity of a battery depends upon the discharge rate. For instance, if a battery is discharged at the one-hour rate to the allowable final voltage limit, the energy taken out at this rate is approximately only fifty per cent of the energy that could be taken out at the normal rate. However, if, at the end of one hour's discharge at the one-hour rate, the rate of discharge be reduced to the normal rate, much of the available *normal* capacity of the battery can be obtained.

It is thus seen that any statement of the true efficiency of a battery should include the charge and discharge rates. It is also seen that in order to make a true efficiency test an average of several charges and discharges should be taken, and the battery should be given several discharges and charges before the actual test is begun. Failure to observe these conditions may result in figures that are misleading. The writer has seen test figures showing an efficiency of more than one hundred per cent; he has also seen figures showing efficiencies far below the actual efficiency.

To determine the efficiency of the battery, get all the cells in satisfactory condition by preliminary charge and discharge, as outlined above under the capacity test, and then take a preliminary discharge down to the final voltage specified by the manufacturer for this rate of discharge. The first test charge should then be made at the normal rate until the cells are fully charged. The first test discharge should then be taken at the specified rate down to the same final voltage used in the preliminary discharge. The watt-hour efficiency of the battery will then be the ratio of the total watt-hours discharged to the total watt-hours charged. If greater accuracy is desired, this cycle should be repeated several times (say, from four to six times) and the watt-hour efficiency will then be the ratio of the total watt-hours of all the discharges to the total watt-hours of all the charges.

RATING AND PERFORMANCE. — The capacity of a storage-battery plate of a given size varies somewhat with the conditions, chief among which are the specific gravity of electrolyte, the temperature of electrolyte, the amount of electrolyte, the original formation of the plate, the design of the grid, the age of the plate, the length of time between charge and discharge and the final voltage limit specified.

Rating of Stationary Batteries. — The rate of discharge of a storage battery is the number of amperes that it will supply continuously for 8 hours, for 3 hours or for 1 hour. The 8-hour rate is the so-called "normal" rating, the 3-hour rate is approximately twice the normal rating and the 1-hour rate is approximately 4 times the normal rating. The *capacity* of a storage battery is usually

stated in ampere hours. Since the capacity varies with the rate of discharge, it is necessary to specify the rate of discharge in stating the capacity in ampere-hours. If the rate is not specified, the normal rate is assumed.

Certain plates especially designed for high rates of discharge are rated in terms of the amperes they will supply for 6 hours, this rating being designated as "normal." The corresponding 1-hour rate is 4 times this 6-hour rate.

While the above is standard for ordinary batteries, plates for certain classes of service are designed to give the normal rate for from 7 to 7½ hours.

The capacity of a storage battery is sometimes expressed in kilowatts, but this is ambiguous unless the rate of discharge is specified.

Rating of Vehicle and Ignition Batteries. — It is standard practice to rate a vehicle battery in terms of the amperes it will give continuously for 4, 4½ or 5 hours, specifying the battery to have a capacity of so many ampere-hours at one of these rates. In most portable types of ignition batteries the batteries are rated at so many ampere-hours at the "service rate," i.e., the rate of discharge (amperes) corresponding to the service for which they are designed.

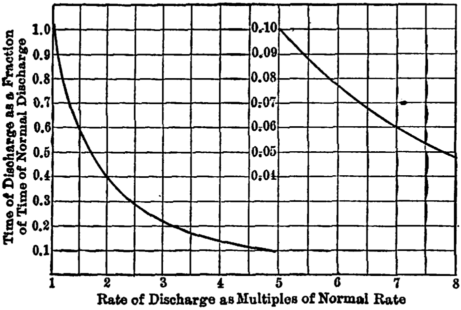


Fig. 3.

Variation of Rate of Discharge with Time of Discharge. — Fig. 3 shows an average capacity curve for a Planté plate, the time for a complete discharge at any rate being shown as a fractional part of the normal time and the rate of discharge for any given time being shown as a multiple of the normal rate.

From these curves it will be noted that a cell which will give its normal rate for 8 hours will give 4 times that rate for about 1⅛ hours; and a cell which will give 4 times the normal rate for 1 hour will give the normal rate for only 7 hours. These are approximately correct relations for the average Planté cell. Trade catalogues usually specify the normal rate for 8 hours and 4 times the normal rate for 1 hour. This is due to the fact that a cell which is regularly worked at the normal rate will tend to hold its 8-hour capacity, while a cell which is regularly worked at 4 times the normal rate will tend to lose some of its 8-hour capacity and finally give only 1 hour at 4 times the normal rate.

Variation of Capacity with Rate of Discharge. — It will be noted from the capacity curves that the available capacity in ampere-hours decreases with increase of rate of discharge. This is largely due to the time required for fresh electrolyte to penetrate into the pores of the active material of the plate. At the higher rates of discharge the diffusion of electrolyte is not sufficiently rapid to reach the more remote portions of the active material, and these portions are not fully available at these high discharge rates. As should be expected, if, after all of the available capacity at any given rate has been delivered

by the battery, the battery be allowed to rest or the rate of discharge be decreased, additional capacity can be obtained from the battery.

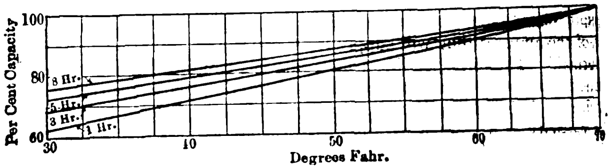


Fig. 4.

Variation of Capacity with Temperature. — Fig. 4 shows the variation of capacity with temperature at various rates of discharge. This change of capacity is temporary and the capacity will be restored to its original value when the temperature is restored to its original value.

Voltage and Specific-gravity Characteristics. — The charge and discharge voltage characteristics of a battery vary with the conditions. Fig. 5 shows typical average voltage and specific-gravity charging characteristics of a Planté type of cell designed for stationary service. It will be noted that the specific gravity increases gradually until all of the lead sulphate in the plates has been reduced, the (SO_4) radical being returned to the electrolyte to form sulphuric acid, which accounts for the steady increase in specific gravity and for the fact that the specific gravity will rise no further after the battery is completely charged.

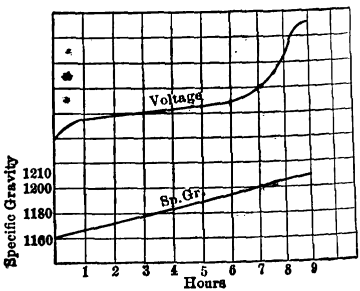


Fig. 5.

Final Charging Voltage (Fig. 5). — The scale of volts on the upper curve has been purposely omitted, but this curve shows the general shape of charging curve for any type of lead battery. The final charging voltage of any lead type of battery varies over a considerable range, according to the type of cell, age of plates, temperature of electrolyte, strength of electrolyte, etc. The final charging voltage may vary anywhere from 2.4 to 2.8 volts per cell. It will be noted that the voltage rises rapidly at the beginning and at the end of charge.

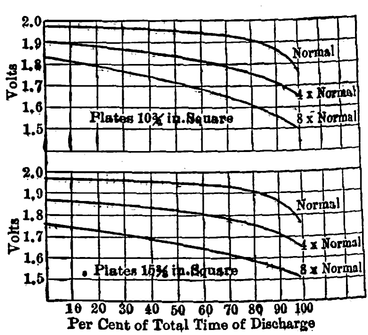


Fig. 6.

Discharge Voltage. — Fig. 6 shows discharge curves at the normal, four times the normal and eight times the normal rates, respectively, for two different sizes of Planté plates The upper set of curves are for plates 10¾ inches square and the lower set of curves are for plates 15¾ inches square.

These curves, used in conjunction with the capacity curves shown in Fig. 3, will enable anyone to determine the approximate performance of any lead battery when the capacity at any given rate is known.

The voltage readings at the very beginning of the discharge are liable to vary more or less from those shown on the curves depending upon the length of time the cell has been allowed to stand on open circuit after the completion of the charge.

Open-circuit Voltage. — The open-circuit voltage of a lead cell is of no value as an indication of its state of charge. A healthy cell will regain its full voltage on open circuit, even though fully discharged, if sufficient time is allowed for depolarization.

Internal Resistance. — The true internal resistance of a cell may be determined by the method explained above in the paragraph on *Test of Internal Resistance*. This internal resistance determines the *immediate* change of voltage at the cell terminals with any sudden change of discharge rate. In the ordinary commercial types of lead cells at 70° F. the internal resistance is such as to cause an immediate drop in terminal voltage of between 5 per cent and 7 per cent, with a sudden increase of current equal to four times the normal rate of discharge, the drop being proportional to the change in current. If this change in current is not strictly momentary, the change in terminal voltage will be greater, due to the effect of polarization. The internal resistance of a cell increases with reduction of temperature; at 0 °F. the internal resistance is twice as great as at 70° F.

DIMENSIONS AND WEIGHTS. — In determining the dimensions and weights of a storage battery to meet the requirements, it is suggested that the proper type of battery be determined in accordance with the curves and examples given above and a battery be selected from trade catalogues to meet the specifications. When the battery has been thus chosen, the dimensions and weights of individual cells can be obtained from the trade catalogues. In order to determine the total installed dimensions and weights, it will be necessary to make a layout according to standard practice. The distances between adjacent cells and the dimensions of aisles are given below in the section on *Installation*.

SPECIFICATIONS, CONTRACTS AND PROPOSALS. — (*See also article on Specifications.*) In preparing specifications for a storage-battery plant and accessories, special care should be taken to avoid drawing up specifications in such a manner as needlessly to embarrass the manufacturers who are requested to bid.

Preliminary Specifications. — It is recommended that preliminary specifications of a very general nature be prepared. These specifications should contain complete detailed data of the conditions to be met, so that the manufacturers who are requested to bid can readily determine the number and capacity of cells, the proper scheme of operation, the correct design of booster, switchboard and other accessories recommended by them to meet the requirements in hand. That is, the preliminary specifications should state plainly just what the battery is intended to accomplish. These preliminary specifications should be sent to the manufacturer with a request for a complete proposal.

Manufacturer's Proposal. — This proposal should give detail specifications covering the apparatus recommended by the manufacturer. In order to obtain the best results, it is recommended that after these proposals are received, the prospective purchaser confer with the bidders that are to be considered, and that complete final specifications be prepared, covering all details of the necessary equipment. These final specifications can then be sent to the manufacturers for their final bids.

Final Specifications. — As a guide in preparing the final specifications the points enumerated below should be thoroughly covered:

Number of cells;
Number of plates in elements initially installed;
Size of plates;
Capacity of the elements initially installed;
Charging rate of the elements initially installed;
Size of the containing vessels expressed in the number of plates which the vessel can contain;
Separators and supports;
Electrolyte;
Insulation;
Assembling and lead burning;
Bus bars, both plain and reënforced;
End-cell switches;
All copper work and cables;
Switchboard in detail;
Detailed specifications for booster and exciter apparatus according to the A.I.E.E. Standardization Rules.
Foundation for booster and switchboard;
Testing instruments;
Battery room;
Erection;
Skilled and unskilled labor;
First charge;
Operation;
Test;
Freight, cartage;
Delivery;
General scheme of operation;
Temporary work;
Cutting of walls, etc.;
Access, storage and hoisting;
Acceptance.

If specifications are drawn, covering all of these points clearly and if the specifications are attached to, and form part of, the contract, no trouble should arise in the future from misunderstandings.

Contract. —The contract should cover the price, time of payments, guarantee, protection from patent litigation and insurance of the material during construction.

INSTALLATION AND ERECTION. — *Never install any battery without following explicitly the detailed instructions furnished by the manufacturer who made the battery.*

If the battery is of considerable capacity, detail drawings showing the method of installation should be obtained from the battery manufacturer. The leading manufacturers issue printed instructions and detail drawings describing the method of installation. Much time and money can be saved by obtaining these data from the battery manufacturers.

Spacing of Glass Jars. —The spacing between cells along rows varies from 1½ inches with the smaller types of multiple plate-glass cell batteries to 3½ inches for the largest size multiple plate-glass cell batteries. This space is the space from jar to jar. It is standard practice to place the jars on wood or glass sand trays filled with sand; the distance between the sand trays along the rows varies from ½ inch to 2⅛ inches, depending upon the size of the glass jars. As the outside dimensions of the sand trays are not usually given in trade catalogues, it would be well in figuring the length of the rows to use the spacing between glass jars as given above, for the outside dimensions of the glass jars are usually stated.

With glass-jar batteries the aisle space between the rows should never be less than 24 inches, and aisles of 36 inches are recommended. In the case of double rows or where a row is installed near a wall or partition, the distance between rows, or the distance between a row and the wall, should not be less than 6 inches.

All batteries should be installed in one tier, if space is available, though glass-

in two or three tiers, if more space is not available. In installing batteries in more than one tier, sufficient headroom should be allowed in each tier to permit the removal of the elements from the jars without displacing the jars. The minimum headroom for a single-tier glass-jar battery varies from 4½ feet to 5 feet; for two tiers from 5 feet to 7½ feet and for three tiers from 7 feet to 12 feet, depending upon the size of the battery. In all cases the ceiling of the battery room should be of sufficient height to allow for the passage of a man without stooping.

Spacing of Lead-lined Tanks. — Lead-lined tank batteries should always be installed in one tier, if space is available. More than one tier should not be installed without consulting with the battery manufacturer. With lead-lined tanks the distance between tanks along rows is from 2 to 2¼ inches. The aisle space between rows should never be less than 30 inches and should always be sufficient to allow the tank to be pulled out into the aisle for repairs. Where space will not allow the tank to be pulled out its full length the aisle should have at least sufficient width to allow the tank to be stood on end in the aisle. In double rows or in rows next to the wall, at least 18 inches should be allowed between rows or between the tanks and the walls so that a man can get in for examination. The battery room should have sufficient headroom for a man to walk upright. With large-sized batteries where the amount of gases liberated on overcharge is considerable, sufficient headroom should be allowed for proper ventilation.

Dimensions of Containers. — For any given type of plate, either in glass jars or lead-lined wood tanks, there is one dimension for all jars or tanks which is practically constant, independent of the number of plates in the cell. This is the dimension parallel to the horizontal edge of the plates. This dimension is usually specified in trade catalogues as the "width." The other dimension is the dimension across the row and is usually specified in trade catalogues as the "length."

Dimensions of Battery Room. — Keeping in mind these definitions one can determine from trade catalogues the approximate inside dimensions of a battery room necessary to house the battery in question. Note, however, that any manufacturer will gladly furnish a sketch showing an ideal layout for a given battery and such a layout should always be obtained from the manufacturer, if there is sufficient time. If there is available space in a building already built, the battery manufacturer will gladly prepare a sketch showing the layout of any given battery in the available space.

Insulation of Cells. — Cells in stationary batteries, whether in glass jars or in wood tanks, must be insulated from the ground and from each other.

Glass jars are usually mounted in glass or wooden sand trays which rest on glass insulators. The glass insulators, in turn, rest on wooden racks or stringers, these racks or stringers being mounted on vitrified brick set on the battery room floor.

For lead-lined wood-tank cells the standard practice at present is to mount each tank on a sufficient number of oil insulators. These insulators consist of a glass insulator provided with a circular trough partly filled with oil, this oil surrounding a central section which supports the tank. Over the insulator is placed a lead-alloy cap, designed to exclude spray or other foreign matter from the oil. This cap rests on the central section of the glass insulator, there being no connection between the tank and the glass except at this section. The oil around this central section of glass is for the purpose of preventing a film of acid from collecting thereon; if, by any chance, acid or water should get into the trough, it would sink to the bottom, the oil would float on top, and perfect insulation would be maintained.

The glass insulator rests upon a heavy earthenware truncated cone, having three feet. Between this cone and the glass is a Y-shaped lead washer. Thus three-point support is obtained between the floor and the earthenware, and between the earthenware and the glass. This form of support is not necessary between the glass and the tank, because the wood and lead cap take up any surface inequalities.

Connections between Cells. — In glass-jar batteries of small size, elements in adjacent cells are usually bolted together. In the larger-sized glass-jar batteries and in all lead-lined wood-tank batteries the positive plates in one cell are burned to the negative plates in the adjacent cell through the medium of a lead bus bar. The bus bars at the ends of rows, and wherever current taps are made, are reënforced by a bar of copper embedded in the lead.

Battery-room Design. — Though no battery should be installed without specific instructions from the manufacturer, the following points should receive special attention.

Floor. — The floor of the battery room should be made acidproof and should be graded to drain. A wood floor should never be used, as it is bound to become acid soaked and eventually be destroyed. For small batteries a cement floor can be used, but, unless it is kept thoroughly washed, acid will eat holes in it. A glazed tile floor is preferable.

Ventilation. — All battery rooms should be well ventilated. With small batteries natural ventilation is usually sufficient; with large-capacity batteries, where the gassing during overcharge is considerable, artificial ventilation should be provided. Exhaust ventilation is preferable to compression ventilation.

Temperature. — The temperature of the battery room should be kept as near 70° F. as possible.

Exposed Metal. — The amount of exposed iron or metal work in the battery room should be reduced to a minimum on account of the action of acid fumes upon it. Any iron or metal work that must be in the battery room should be protected with an acid-resisting paint.

OPERATION. — *Never operate any battery in any manner except in accordance with the instructions of the battery manufacturer furnishing the battery.*

With all large battery plants there should be supplied by the manufacturer the proper testing instruments and detail instructions for the operation of the battery under the conditions obtaining. The blank forms furnished by the manufacturer should be filled in at regular intervals and mailed to the manufacturer for analysis and recommendations.

No general instructions for the proper operation of a battery are given in this article, since the different manufacturers differ in their opinions on certain points of operation.

Printed instructions for operating any type of battery can be obtained free of charge from the manufacturer.

Attention from Operator. — The successful operation of a battery plant does not require much time from the operator, but attention given the battery should be systematic and absolutely in accordance with the instructions furnished by the manufacturer. If the manufacturer's instructions are followed implicitly and systematically, it will be found that the battery can be kept in good condition with a small amount of labor and time. If the operator neglects to follow the regular instructions and does not appeal to the manufacturer until he gets into trouble, the usual result will be considerable loss in time and money.

Addition of Water. — Pure distilled water, or natural water that has been analyzed and approved by the battery manufacturer, should be added to the

of pure, natural water, and if distilled water cannot be purchased cheaply, a distilling outfit will usually be found necessary with large sizes of batteries. Standard water stills, for this purpose, can be purchased on the market.

Addition of Acid. — *Acid should never be added to a battery except upon the recommendation of the battery manufacturer.* The manufacturer should specify the specific gravity and the proper amount of acid.

REPAIRS. — Very few repairs, other than battery plates, should be required. For replacing the plates in the larger sizes of batteries, a lead-burning outfit and the necessary lead-burning material will usually be required. No one but a skilled lead burner should attempt to burn in plates.

Smaller sizes of batteries, with bolted connections between adjacent cells, can be purchased from the manufacturer with each element assembled.

Leaky Tanks. — A leaky tank should be repaired immediately. No one but an expert sheet-lead burner should be allowed to repair a leaky lead-lined tank. To repair the leaky lead-lined tank it will be necessary to cut loose from the bus bars all of the plates in the leaky tank and remove the tank into the aisle. If it is necessary to keep the rest of the battery in commission while the repairs are being made, the cell or cells cut out should be jumped by a jumper of sufficient current-carrying capacity.

FIRST COST. — It is impossible to give exact figures for the total cost of installation of a complete battery plant of any size. An analysis of the first costs of a great many plants now in operation shows that the first cost per kilowatt for the entire battery installation, exclusive of building, varies from $75.00 per kilowatt to $250.00 per kilowatt. The kilowatt rating as used here and elsewhere in this article is an arbitrary rating determined by multiplying the one-hour discharge rate in amperes (four times the normal rate) by double the number of cells.

ANNUAL COST. — The total annual cost includes operation, maintenance, depreciation and interest.

Operating Cost. — The operating cost will consist largely of labor. All material except necessary water should be included under maintenance.

Maintenance and Depreciation. — The cost of maintenance of a battery varies with the conditions. Other things being equal, the life of a battery is dependent upon the work done. The maintenance cost of a battery varies from 4 per cent to 10 per cent per annum. When the battery is used only for strictly emergency service the maintenance cost may be less than 4 per cent.

These figures cover not only maintenance, but also what is commonly included under the term depreciation. The plate being the essential component of the battery, plate renewals, which are made from time to time, involve all improvements that have been made in design and construction and serve to bring the battery up to date. An increase of voltage or capacity to meet changed conditions may be effected by the addition of cells or by increasing the number of plates in each cell, without sacrifice of the original investment. In the opinion of the writer there is, therefore, little, if any, true depreciation in a battery installation which is properly maintained.

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[TALIAFERRO MILTON.]

BEARINGS. — (*See also Friction; Lubricants and Lubrication; Shafting.*) The common types of shaft bearings may be classified as follows:

Plain Cylindrical Bearings. — The usual form of bearing is a metal cylinder of cast iron, brass, bronze or gun-metal, mounted in a pedestal, bracket or frame, which serves as a support for the shaft. The portion of the shaft within the bearing is called the journal. Plain bearings are usually lined with some soft metal, such as babbitt metal or other white metal. In a self-aligning bearing the seating of the bearing in the pedestal is made a portion of a sphere, so that the bush may automatically align itself with the shaft. Cylindrical bearings are sometimes provided with an oil bath within the pedestal just below the bearing proper; rings or chains running loosely over the journal through slots in the bearing and dipping into this bath supply a steady stream of oil to the rubbing surfaces.

Roller Bearings. — In this type of bearing the journal is supported on rollers which are in turn supported by smaller shafts in smaller bearings. The rollers support the journal of the main shaft.

Step Bearing. — A step bearing is essentially a large pivot bearing used at the end of vertical shafts carrying a heavy load. The end of the shaft is provided with a steel plate or "step," which rests on a second plate or bearing. The bearing proper may be either of the ball or roller type, or may be a plane surface with one or more grooves into which oil can be forced under pressure. For heavy loads on a plain step bearing the pressure of the oil must be sufficient to actually raise the step by a small amount so that a thin film of oil is formed between the step and the bearing.

Thrust Bearings. — Thrust bearings are used on horizontal shafts when the shaft is subjected to a horizontal thrust. They are similar to step bearings for vertical shafts, except that a collar fitting around the shaft is used instead of a plate at the end of the shaft. Steamship shaft bearings have numerous collars on the shaft, with thrust blocks between them. Roller or ball bearings may be used instead of collars.

Ball Bearings. — A ball bearing consists essentially of a track of curved cross section which is filled with a set of balls.

Pivot Bearings. — Various types of pivot bearings are used. The pivot may be either a flat surface, a pointed cone, a truncated cone, a hemisphere or a surface of special form, such as Shiele's "tractrix."

ALLOWABLE LOADS AND SPEEDS. — If the pressure on a bearing is too great the oil film between journal and bearing surface will be destroyed and the bearing will overheat and "seize." Various formulas have been developed for the allowable pressure in terms of the speed and type of bearing.

Safe Loads, Line Shaft and Mill Bearings. — F. W. Taylor (*Trans. A.S.M.E., 1905*), as the result of an investigation of line shaft and mill bearings that were running near the limit of durability and heating, yet not dangerously heating, gives the formula

$$p = \frac{400}{v},$$

where p = pressure in pounds per square inch of projected area (i.e., product of diameter of the journal by its length) and v = peripheral velocity of journal in feet per second.

The formula is applicable to bearings in ordinary shop or mill use on shafting which is intended to run with the care and attention which such bearings usually receive, and gives the maximum or most severe duty to which it is safe to subject

ordinary *chain* or *oiled* ball- and socket-bearings which are *babbitted*. It is not safe for ordinary shafting to use *cast-iron boxes*, with either sight feed, wick feed or grease-cup oiling, under as severe conditions as $p \times v = 200$.

Safe Loads, Miscellaneous Plain Bearings. — Archbutt and Deeley's *Lubrication and Lubricants* gives the following table of allowable pressures in pounds per square inch of projected area of different bearings:

Crank pin of shearing and punching machine, hard steel, intermittent load bearing	3000
Bronze crosshead neck journals	1200
Crank pins, large slow engine	800–900
Crank pins, marine engines	400–500
Main crankshaft bearing, fast marine	400
Same, slow marine	600
Railway coach journals	300–400
Flywheel shaft journals	150–200
Small engine crank pin	150–200
Small slide block, marine engine	100
Stationary engine slide blocks	25–125
Same, usual case	30– 60
Propeller thrust bearings	50– 70
Shafts in cast-iron steps, high speed	15

Safe Loads, Roller Bearings. — The following table gives the safe load in pounds for Mossberg roller bearings (*Trans. A.S.M.E., 1905*). D = diameter of journal, in inches; d = diameter of roll, in inches; N = number of rolls; P = safe load on journals, in pounds. The rolls are enclosed in a bronze supporting cage.

D	d	N	P	D	d	N	P	D	d	N	P
2	1/4	20	3,500	6	11/16	24	50,000	15	1 3/8	28	255,000
2 1/2	5/16	22	7,000	7	13/16	22	70,000	18	1 3/8	32	325,000
3	3/8	22	13,000	8	7/8	22	90,000	20	1 1/2	34	400,000
4	7/16	24	24,000	9	1	24	115,000	24	1 1/2	38	576,000
5	9/16	24	37,000	12	1 1/4	26	175,000	..	...	..	......

Surface speed of journal from 0 to 50 feet per minute. Length of journal 1 1/2 diameters. The rolls are made of tool steel not too high in carbon, and of spring temper. The journal or shaft should be made not above a medium spring temper. The box should be made of high-carbon steel and tempered as hard as possible.

Safe Loads, Ball Bearings. — The following formula is given by Mr. Henry Hess, 1910. See also paper by Mr. Hess in *Trans. A.S.M.E., 1907*.

Let

W = total safe load on bearing in pounds,
n = number of balls,
d = diameter of balls in *eighths of an inch*.

Then for *radial* (cylindrical) bearings

$$W = Knd^2,$$

where K ranges from 0.9 to 9 depending upon the condition and type of bearing and the hardness of the balls, but for ordinary speeds is practically independent of the speed. (*See Kent's Mechanical Engineers' Pocket-Book.*)

For *thrust* bearings

$$W = \frac{K_1 n d^2}{\sqrt[3]{N}},$$

where N is the number of revolutions per minute, which may range from 3000 down to 1 revolution per minute, as for crane hooks and similar elements. For high quality ball K_1 ranges from 25 to 40 for a race having a cross section of radius $1.66 \times$ (radius of balls). For unhardened steel, occasionally used for very large races, $K_1 = 0.5$.

In both types of bearings the balls must be carefully selected to make sure that all that are used in the same bearing do not vary among one another by more than 0.0001 inch. A ball that is more than that larger than its fellows will sustain more than its proportion of the load, and may therefore be overloaded and will in turn overload the races.

FRICTION OF BEARINGS. — The coefficient of friction of a well-lubricated plain cylindrical bearing is practically independent of the projected area of the journal, the pressure per unit area remaining constant, and in the case of plain bearings is also practically independent of the nature of the rubbing surfaces, provided these are smooth.

Friction of Plain Bearings. — The coefficient of friction of plain bearings however, does depend to a very great extent upon the following factors:

1. The method of lubrication.
2. The nature of the lubricant.
3. The temperature of the lubricant.
4. The peripheral velocity of the journal.
5. The pressure of the journal on the bearing surface.

1. The more perfectly the bearing is bathed in oil the less will be the coefficient of friction. The friction coefficient of a scantily lubricated bearing may be from 6 to 10 times the coefficient when an oil bath is used.

2. As a rule the lower the viscosity of an oil the less will be the coefficient of friction. There are other factors, however, which must be taken into account in selecting the proper lubricant for any service (*see article on Lubricants and Lubrication*).

3, 4 and 5. The variation of the friction coefficient with temperature, speed and pressure is shown by the curves in Fig. 1. The pressures are nominal

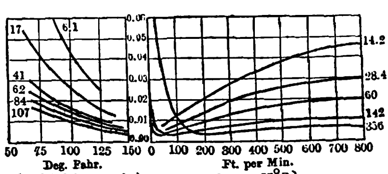

Fig. 1. Numbers at end of curves are pressures in lb. per sq. in.

pressures, i.e., pounds per square inch of projected area of journal. These curves are taken from a paper by Stribeck (*Zeit. Ver. Deutsch. Ing., 1902, Vol. 46, p. 1341*), and are test results on a Sellers bearing with oil rings; journal 13 inches long, 2.75 inches in diameter, lubricated with "gas motor oil." These curves are typical of ordinary bearings.

POWER LOST IN BEARINGS. — Let

f = coefficient of friction,
W = weight on journal or pivot in pounds,
r = radius in inches,
d = diameter in inches,
S = space in feet through which sliding takes place per minute.
r_1 = inner radius in inches,
r_2 = outer radius in inches,
n = number of revolutions per minute,
a = the half-angle of the cone, i.e., the angle of the slope with the axis.

Type of bearing	Friction torque in in.-lb.	Power loss ft.-lb. per min.
Flat surfaces	....	fWS
Shafts and journals	$\frac{1}{2}\ fWd$	$0.2618 fW\ dn$
Flat pivots	$\frac{2}{3} fWr$	$0.349 fWrn$
Collar-bearing	$\frac{2}{3} fW \frac{r_2^3 - r_1^3}{r_2^2 - r_1^2}$	$0.349 fWn \frac{r_2^3 - r_1^3}{r_2^2 - r_1^2}$
Conical pivot	$\frac{2}{3} fWr \operatorname{cosec} a$	$0.349 fWrn \operatorname{cosec} a$
Conical journal	$\frac{2}{3} fWr \sec a$	$0.349 fWrn \sec a$
Truncated-cone pivot	$\frac{2}{3} fW \frac{r_2^3 - r_1^3}{r_2 \sin a}$	$0.349 fWn \frac{r_2^3 - r_1^3}{r_2 \sin a}$
Hemispherical pivot	fWr	$0.5236 fWrn$
Tractrix, or Schiele's "anti-friction" pivot	fWr	$0.5236 fWrn$

It should be noted that for peripheral speeds of 100 feet and over the coefficient of friction is approximately proportional to the square root of the peripheral velocity of the journal. These particular curves also indicate that the coefficient of friction is approximately inversely proportional to the square root of the pressure. Other experimenters, however, have found that this variation is inversely as the first power of the pressure.

Friction of Roller and Ball Bearings. — The friction coefficient of a well-made annular ball bearing is 0.001 and 0.002 of the total load on the journal and is independent of the speed and load. The friction coefficient of a good roller bearing under normal loads and speed is from 0.0035 to 0.014; it rises very much if the load is light. It increases also when the speeds are very low, though not so much as with plain bearings. (*Henry Hess.*)

Lubrication is absolutely necessary with ball-and-roller bearings, although the contrary claim is often advanced. Under favorable conditions an almost imperceptible film is sufficient; a sufficient quantity to immerse half the lowest ball should always be provided as a rust preventive. Rust and grit must be kept out of ball-and-roller bearings. Acid or rancid lubricants are as destructive as rust. (*Henry Hess.*)

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[Wm. Kent.]

BELTS AND BELTING. — (*See also Ropes and Rope Drive.*) Belts are usually made of tanned leather, though rubber belts are frequently employed, as well as various woven fabrics. Rubber belts are made of two or more layers of canvas connected together with a rubber composition, and then heated until the rubber vulcanizes. The "ply" of a rubber belt is the number of layers of canvas. Leather belts are referred to as single, light-double, medium-double, standard-double and 3-ply.

Thickness of Belts. — The following figures are from a paper by Samuel Webber (*Am. Mach., May 11, 1909*). The thickness of leather belts, however, is variable, depending on the hide and process of manufacture.

BELT THICKNESS, INCHES

Leather		Rubber, 30-oz. duck, new rubber vulcanized	
Single thickness	1/6	3-ply	0.18
Light-double	1/4	4-ply	0.24
Medium-double	5/16	5-ply	0.30
Standard-double	1/8	6-ply	0.35
3-ply	9/16	7-ply	0.40
		8-ply	0.45

Weight. — The average weight of leather per cubic inch is 1/30 pound (*Barth*). The average weight of rubber belting made of 30-ounce duck is about 0.045 pound per cubic inch.

Strength. — The strength of the solid leather in belts is from 2000 to 5000 pounds per square inch; at the lacings, even if well put together, only from 1000 to 1500. If riveted, the joint should have half the strength of the solid belt. Rubber belts have approximately the same tensile strength as leather belts. The working tension on the driving side is generally taken at not over one-third of the strength of the lacing, or from one-eighth to one-sixteenth of the strength of the solid belt.

Belt Tension. — Let

T_1 = actual, or "total," tension in pounds per square inch on driving side of belt,
T_2 = actual tension in pounds per square inch on slack side of belt,
T_0 = "initial" tension in belt, i.e., the tension in each side when the belt is at rest,
$T = T_1 - T_2$ = effective tension in pounds per square inch, i.e., T corresponds to the force actually driving the driven pulley,
T_c = the tension produced in the belt due to centrifugal action as it goes around the pulleys,
m = weight of belt per cubic inch in pounds,
f = coefficient of friction between belt and pulley,

n = ratio of arc of contact between belt and pulley to an arc of 180°, i.e., if the arc of contact is 180°, $n = 1$,

$C = 1 - \epsilon^{-nf\pi}$. The value of $\epsilon^{-nf\pi}$ may be found directly from the table in the article on *Exponential Functions*, putting $nf\pi = x$,

V = velocity of belt in feet per minute.

Then

$$T_c = \frac{mV^2}{9660}, \qquad (1)$$

$$T = C(T_1 - T_c), \qquad (2)$$

$$T_2 = T_1 - C(T_1 - T_c), \qquad (3)$$

$$T_0 = \left(\frac{\sqrt{T_1} + \sqrt{T_2}}{2}\right)^2 \text{ approximately.*} \qquad (4)$$

Barth recommends that for belts running at various speeds the initial tension T_0 should not be the same in all belts, but that the belts should be tightened initially to such a tension that under load the sum

$$A = T_1 + \tfrac{1}{2} T_2 \qquad (5)$$

is a constant. Barth also states, basing his conclusion in part on the studies of the economics of belting made by F. W. Taylor (*Trans. A.S.M.E., Vol. 15, p. 204*), that the best values of A for greatest economy are the following:

	Machine belts	Counter-shaft belts
Maximum value of A, belt first put on	320	240
Minimum value of A, belt to be retightened	240	160

These values of A are for full-load conditions. The corresponding values of the various tensions are given below.

Belt Friction and Belt Slip. — The value of the factor C depends upon the coefficient of belt friction, which is subject to considerable variation, depending upon the condition of the belt and pulleys. Barth gives the following formula for f for leather belts on cast-iron pulleys

$$f = 0.54 - \frac{140}{500 + V}.$$

Barth also gives a formula for the belt slip which may be written

$$s = \frac{V_1 - V_2}{V} = \frac{320}{V}\left(\frac{1 + 0.0055\,V}{85 + 0.03\,V}\right),$$

where V_1 and V_2 are the peripheral velocities of the driving and driven pulleys respectively.

* See paper by C. G. Barth, *Trans. A.S.M.E.*, 1909, Vol. 31, p. 29. In this paper Barth also gives a more exact formula, and takes into consideration the difference in the relation between T_1, T_2 and T_0 for vertical and horizontal belts.

Power Loss Due to Belt Slip is proportional to the slip, i.e., if the slip is 0.03 the power lost is 3 per cent of the power transmitted by the belt. Under normal conditions the power loss due to belt slip is from 2 to 4 per cent. The power loss in mill shafting due to the friction of the shaft bearings ranges from 10 to 60 per cent, depending upon the type of bearing, belt tension, etc., See article on *Bearings*.

Power Transmitted by Belt. — Let

V = velocity of belt in feet per minute,
w = width of belt, in inches,
t = thickness of belt, in inches,
m = weight, in pounds, of 1 cubic inch of belt,
T_1 = actual tension in driving side of belt in pounds per square inch of belt cross-section,
s = slip of belt as a fraction of belt speed.

Then the horse-power transmitted by the belt is

$$P = \frac{wtCVT_1}{33{,}000}\left(1 - \frac{mV^2}{9660\,T_1}\right),$$

where C, defined above, depends on the belt friction and arc of contact between belt and pulley.

As a rough approximation C may be taken equal to 0.6, and the second term in the bracket zero, at least for low speeds, and the formula then becomes

$$P = \frac{wtVT_1}{55{,}000},$$

or the horse-power transmitted per square inch of belt cross-section is

$$p = \frac{VT_1}{55{,}000}.$$

The power lost in the transmission from one pulley to the other, excluding the loss due to shaft friction, is sP, the driving pulley absorbing approximately $(P + 0.5\,sP)$ horse-power and the driven pulley receiving $(P - 0.5\,sP)$ horse-power.

The following tables are derived from curves in the paper by Barth above referred to.

HORSE-POWER TRANSMITTED BY BELTS DRIVING MACHINES
(For $A = T_1 + 0.5\,T_2 = 240$)

Velocity, ft. per min.	500	1000	2000	3000	4000	5000	6000
Initial tension, T_0	124	120	121	128	136	144	152
Centrifugal tension T_c	0+	3	13	31	56	86	124
Difference, $T_0 - T_c$.........	123	117	108	97	80	58	28
Tension on tight side, T_1...	210	212	211	207	198	187	173
Tension on slack side, T_2...	60	54	57	68	84	107	134
Effective pull, $T_1 - T_2$	150	158	154	139	114	80	39
Sum of tensions $T_1 + T_2$	270	268	269	274	282	294	307
H.p. per sq. in. of section..	2.27	4.79	9.33	12.64	13.82	12.12	7.09

HORSE-POWER TRANSMITTED BY BELTS DRIVING COUNTER-SHAFTS

(For $A=T_1+0.5\ T_2=160$.)

Velocity of belt, ft. per min.	500	1000	2000	3000	4000	5000
Initial tension, T_0	82	81	83	89	96	102
Tension on tight side, T_1	140	141	140	134	125	114
Tension on slack side, T_2	40	38	41	53	69	92
Effective pull, T_1-T_2	100	103	99	81	56	22
Sum of tensions	180	179	181	187	194	206
H.p. per sq. in. of section	1.51	3.12	6.04	7.36	6.79	3.33

Proper Size of Belt to Transmit a Given Amount of Power. — Using the same notation as in the preceding paragraph, the proper cross-section of the belt in square inches is approximately

$$wt = \frac{55{,}000\ P}{VT_1},$$

or exactly

$$wt = \frac{33{,}000\ P}{CVT_1\left(1 - \frac{mV^2}{9660\ T_1}\right)}.$$

The various "handy formulas" for determining the proper cross-section of belt are all of the form of the approximate formula, and differ only in regard to the proper value to assign to the tension T_1. A working tension, that is the difference between the tensions on the tight and slack sides of the belt, of 45 pounds for a single (thickness) belt 1 inch wide is commonly used, giving a nominal tension of $6 \times 45 = 270$ pounds per square inch. Taking the coefficient C at 0.6, the actual tension in a belt designed in accordance with this rule is 450 pounds per square inch. The value of C varies through a wide range, say from 0.25 to 0.90, chiefly due to the variability of the coefficient of friction.

The following table, deduced from Barth's curves, may be considered as representative of modern practice.

CROSS-SECTION OF LEATHER BELT PER HORSE-POWER TRANSMITTED, IN SQUARE INCHES

Speed, ft. per min.	500	1000	2000	3000	4000	5000	6000
Machine belts	0.44	0.209	0.107	0.079	0.072	0.082	0.141
Countershaft belts	0.662	0.320	0.166	0.136	0.147	0.300	.....

All belts should have a contact area with the smaller pulley of at least 165 degrees. If this is not possible to obtain under normal conditions an idler should be provided to increase the belt contact.

Pulley Face and Belt Speed.* — The following table may be used for determining the pulley face and best belt speed for moderate-speed machines. For slow-speed machines the pulley sizes are the same as those used with moderate machines of the same frame dimensions.

PULLEY FACE AND BELT SPEED

Horse-power	Kilowatts	Belt width in inches	Pulley face in inches	Belt speed in feet per minute for moderate-speed machines
1 to under 5	1 to under 4	2	2½	2000
5 " 10	4 " 8	3	3½	2500
10 " 15	8 " 11	4	4½	2500
15 " 20	11 " 15	5	6	2500
20 " 25	15 " 19	6	7	3000
25 " 30	19 " 23	7	8	3000
30 " 40	23 " 30	8	9	3000
40 " 50	30 " 37	10	11	3000
50 " 60	37 " 45	12	13	3000
60 " 75	45 " 56	14	15	3000
75 " 100	56 " 75	16	17	3500
100 " 125	75 " 93	16	17	4000
125 " 150	93 " 112	18	19	4000
150 " 200	112 " 159	20	21	4500
200 " 250	159 " 186	22	23	5000
250 " 300	186 " 224	26	27	5000
300 " 350	224 " 261	30	31	5000
350 " 400	261 " 298	32	33	5000
400 " 450	298 " 336	36	37	5000
450 " 550	336 " 410	40	42	5000
550 " 650	410 " 485	44	46	5000
650 " 750	485 " 560	48	50	5000
750 " 850	560 " 634	52	54	5000
850 " 950	634 " 709	56	58	5000
950 " 1050	709 " 783	60	62	5000

Relations of Driving and Driven Pulleys.* — The maximum ratio between the diameters of the driving and driven pulleys should not exceed 6 : 1. The ratio will determine the distance between the pulley centers, and good proportions are given in the accompanying table.

CARE AND OPERATION OF BELTS. — The following is taken from an article by F. W. Taylor (*Trans. A.S.M.E., Vol. 15, p. 204*).

The best distance from center to center of shafts is from 20 to 25 feet.

Idler pulleys work most satisfactorily when located on the slack side of the belt about one-quarter way from the driving pulley.

Belts are more durable and work more satisfactorily made narrow and thick, rather than wide and thin.

It is safe and advisable to use a double belt on a pulley 12 inches diameter or larger; a triple belt on a pulley 20 inches diameter or larger; a quadruple belt on a pulley 30 inches diameter or larger.

As belts increase in width they should also be made thicker.

Approximate Ratio	Minimum Distance Between Centers in Feet
2 : 1	8
3 : 1	10
4 : 1	12
5 : 1	15
6 : 1	20

The ends of the belt should be fastened together by splicing and cementing instead of lacing, wiring or using hooks or clamps of any kind.

Belts should be cleaned and greased every five to six months.

Belt-clamps having spring-balances between the two pairs of clamps should be used for weighing the tension of the belt accurately each time it is tightened.

Double leather belts, when treated with great care and run night and day at moderate speed, should last for 18 years (the tension being adjusted in accordance with Barth's rules).

In figuring the total expense of belting, and the manufacturing cost chargeable to this account, by far the largest item is the time lost on the machines while belts are being relaced and repaired.

The total stretch of leather belting exceeds 6 per cent of the original length.

The stretch during the first six months of the life of belts is 15 per cent of their entire stretch (the tension being adjusted in accordance with Barth's rules).

A double belt will stretch 0.81 per cent of its length before requiring to be tightened (the tension being adjusted in accordance with Barth's rules).

The most important consideration in making up tables and rules for the use and care of belting is how to secure the minimum of interruptions to manufacture from this source.

The average double belt when running night and day in a machine shop will cause an interruption to manufacture not oftener than once in sixteen months (the tension being adjusted in accordance with Barth's rules).

The oak-tanned and fulled belts showed themselves to be superior in all respects except the coefficient of friction to either the oak-tanned not fulled, the semi-rawhide, or rawhide with tanned face.

Belts of any width can be successfully shifted backward and forward on tight and loose pulleys. Belts running between 5000 and 6000 feet per minute and driving 300 h.p. are now being daily shifted on tight and loose pulleys to throw lines of shafting in and out of use.

The best form of belt-shifter for wide belts is a pair of rollers twice the width of belt, either of which can be pressed onto the flat surface of the belt on its slack side close to the driven pulley, the axis of the roller making an angle of 75° with the center line of the belt.

Dressings for Leather Belts. — We advise that no belt dressing should be used except when the belt becomes dry and husky, and in such instances we recommend the use of a dressing. Where this is not used beef tallow at blood-warm temperature should be applied and then dried either by artificial heat

or the sun. The addition of beeswax to the tallow will be of some service if the belts are used in wet or damp places. Our experience convinces us that resin should never be used on leather belting. (*Fayerweather and Ladew.*)

Some forms of belt dressing, the compositions of which have not been published, appear to have the property of increasing the coefficient of friction between the belt and the pulley, enabling a given power to be transmitted with a lower belt tension than with undressed belts. C. W. Evans (*Power, Dec., 1905*) gives a diagram, plotted from tests, which shows that three of these compositions gave increased transmission for a given tension, ranging from about 10 per cent for 90 pounds tension per inch of width to 100 per cent increase with 20 pounds tension.

Dressings for Rubber Belts. — Rubber belts will be improved, and their durability increased, by putting on with a painter's brush, and letting it dry, a composition made of equal parts of red lead, black lead, French yellow and litharge, mixed with boiled linseed oil and japan enough to make it dry quickly. The effect of this will be to produce a finely polished surface. If, from dust or other cause, the belt should slip, it should be lightly moistened on the side next the pulley with boiled linseed oil. (*From circulars of manufacturers.*)

Lacing of Belts. — In punching a belt for lacing, use an oval punch, the longer diameter of the punch being parallel with the sides of the belt. Punch two rows of holes in each end, placed zigzag. In a 3-inch belt there should be four holes in each end — two in each row. In a 6-inch belt, seven holes — four in the row nearest the end. A 10-inch belt should have nine holes. The edge of the holes should not come nearer than ¾ inch from the sides, nor ⅞ inch from the ends of the belt. The second row should be at least 1¾ inches from the end. On wide belts these distances should be even a little greater.

Begin to lace in the center of the belt and take care to keep the ends exactly in line, and to lace both sides with equal tightness. The lacing should not be crossed on the side of the belt that runs next the pulley. In taking up belts observe the same rules as in putting on new ones.

Setting a Belt on Quarter-twist. — A belt must run squarely on to the pulley. To connect with a belt two horizontal shafts at right angles with each other, say an engine-shaft near the floor with a line attached to the ceiling, will require a quarter-turn. First, ascertain the central point on the face of each pulley at the extremity of the horizontal diameter where the belt will leave the pulley, and then set that point on the driven pulley plumb over the corresponding point on the driver. This will cause the belt to run squarely on to each pulley, and it will leave at an angle greater or less, according to the size of the pulleys and their distance from each other.

In quarter-twist belts, in order that the belt may remain on the pulleys, the central plane on each pulley must pass through the point of delivery of the other pulley. This arrangement does not admit of reversed motion.

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[Wm. Kent.]

BLOCKS AND TACKLE. — (*See also Ropes and Rope Drive; Chains and Chain Drive.*) A simple block used with rope tackle consists of one or more pulleys mounted in a casing or shell. They may be single, double, triple, etc., depending upon the number of sheaves or pulleys. In chain blocks, instead of simple pulleys, spur or worm and wheel gearing (*see Gears and Gearing*) or differential pulleys are used. In the triplex block, made by the Yale and Towne Co., the power is transmitted to the hoisting-chain wheel by means of a train of spur gearing operated by the hand chain. In the duplex block the power is transmitted through a worm wheel and screw.

Fig. 1 shows schematically a differential pulley. The two upper pulleys are rigidly fastened together and rotate as one piece. The rims of these pulleys are shaped to mesh with the links of the chain and prevent the latter from slipping. Let D_1 and D_2 be the respective diameters of the larger and smaller upper pulleys. Then if the diameter of the lower pulley is $(D_1 + D_2)/2$, the pull P' required to raise a load W, neglecting friction, is

$$P' = \frac{D_1 - D_2}{2D_1} \cdot W.$$

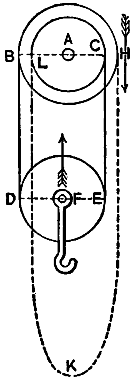

Fig. 1.

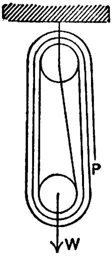

Fig. 2.

ACTUAL PULL REQUIRED; EFFICIENCY OF BLOCK AND TACKLE. — In the case of a simple block and tackle the pull P' required to raise a load W is, neglecting friction,

$$P' = \frac{W}{N},$$

where N is the number of lengths of rope shortened when the lower block rises (in Fig. 2 $N = 6$). The actual pull required is this theoretical pull P' divided by the efficiency expressed as a fraction, or

$$P = \frac{100\,W}{Ne},$$

where e = the per cent efficiency. The distance in feet through which the pulling rope must be pulled to raise the load 1 foot is equal to the number (N) of lengths of rope shortened.

In the case of a differential pulley the actual pull is

$$P = \frac{100\,(D_1 - D_2)\,W}{2D_1e},$$

where e is the per cent efficiency and the other symbols as above. The distance through which the hand chain must be pulled to raise the load 1 foot is

$$\frac{2D_1}{D_1 - D_2}.$$

In any case, if the rope or chain pulls on the lower block at an angle, the block will be pulled out of the line drawn between the load and the upper block, and the effective pull will be less than the actual pull on the rope in the ratio of the cosine of the angle the pulling rope makes with the vertical, or line of action of the resistance, to unity.

Efficiency of Rope Blocks. — S. L. Wonson, *Eng. News*, June 11, 1903, gives the following:

Number of rope lengths shortened, *N*	Manila rope, 1¼ to 2 in.		¾-inch wire rope	
	Ratio of load to pull	Efficiency, per cent, *e*	Ratio of load to pull	Efficiency, per cent, *e*
2	1.91	96	....	..
3	2.64	88	2.73	91
4	3.30	83	3.47	87
5	3.84	77	4.11	82
6	4.33	72	4.70	78
7	4.72	67	5.20	74
8	5.08	64	5.68	71
9	5.37	60	6.08	68
10	....	..	6.46	65
12	....	..	7.08	59

Pull Required with Chain Blocks. — The following table is taken from a catalogue of the Yale and Towne Co., 1908.

Capacity in tons, (2000 lb.)	Pull in pounds required on hand-chain to lift full loads			Feet of hand-chain to be pulled by operator to lift load one foot high		
	Triplex	Duplex	Differential	Triplex	Duplex	Differential
¼	...	...	72	...	...	18
½	62	68	122	21	40	24
1	82	87	216	31	59	30
1½	110	94	246	35	80	36
2	120	115	308	42	93	42
3	114	132	557	69	126	38
4	124	142	...	84	155	..
5	110	145	...	126	195	..
6	130	145	...	126	252	..
8	135	160	...	168	310	..
10	140	160	...	210	390	..

For loads from 10 to 20 tons two hand-chains are provided.

SAFE LOAD. — Blocks are usually designed to carry a load as great as that of the rope or chain which fits the grooves in the sheaves (*see Ropes and Rope Drive; Chains and Chain Drive*).

BIBLIOGRAPHY. —See the works on *Machine Design* given in the Bibliography in article on *Bearings;* also circulars of Yale & Towne Mfg. Co., N. Y., and Boston & Lockport Block Co.

[Wm. Kent.]

BLOWERS AND COMPRESSORS. — (*See also Draft, Mechanical; Fans.*) A blower is any kind of apparatus that is used for blowing or forcing air or other gas into or out of a room or other receptacle. When it is used to draw air from a room or vessel and discharge it into another vessel or into the external atmosphere it is commonly called an "exhauster." A fan is a blower in which motion is given to the air by means of thin blades or vanes (*see Fans*). A blowing engine is a large machine in which air is compressed in and discharged from a cylinder which is fitted with a reciprocating piston. When a blowing engine delivers air at high pressures, say 30 pounds per square inch and upwards, it is called an air compressor. Following is a brief description of the several kinds of blowers.

Displacement Blowers. — Into a closed tank containing air a stream of water is caused to flow, displacing the air and blowing it out through an opening provided for it. By using two tanks with suitable valves, the action may be made continuous.

Hydraulic Blowers. — A large tank is surmounted by a vertical pipe into the top of which water enters in such a way as to "entrain" or drag air along with it, which is separated from the water when it reaches the tank and is blown out through an air pipe while the water escapes through another opening. Tests of a compressor of this kind at Magog, P. Q., Canada, showed a capacity of from 967 to 1165 cubic feet of air per minute at a pressure of 53 pounds per square inch, and an efficiency of from 60 to 70 per cent. See paper by W. O. Webber, *Trans. A.S.M.E.*, Vol. 22, p. 599.

Jet Blowers. — A jet of water, compressed air, or steam flowing into a short pipe of a diameter greater than that of the jet will induce a current of air to flow through the pipe, which may thus be used as a blower or exhauster. The steam jet is in common use for blowing air into furnaces and gas producers, and is used as an exhauster to increase the draft of chimneys. It is usually very wasteful of steam, and should be used only in emergencies, when blowing machines are not available, or when the steam used to drive the jet is also used for other purposes, as for combining with the carbon of the fuel in gas producers.

Positive Rotary Blowers. — These are built like rotary pumps, with two rotating shafts, geared together so as to turn in opposite directions, each carrying an "impeller," a casting approximating a figure 8 in section; the curve of the two impellers is so shaped that they touch or nearly touch on lines parallel to the shafts during the whole period of rotation, and also nearly touch the casing which surrounds them. A blower thus built delivers a constant quantity of air at each revolution, differing in this respect from centrifugal fans which deliver a varying quantity according to the resistance the air meets at the outlet. The economical range of these blowers is between 8 ounces and 8 pounds pressure per square inch. For higher pressures then 8 pounds the blowing engine is more economical.

Blowing Engines. — A cylinder with a reciprocating piston when used for blowing air is called a blowing engine, whether it is driven by a steam engine, a water wheel or an electric motor. Blowing engines are commonly used to furnish air to blast furnaces and Bessemer converters, at pressures of from 4 to 30 pounds per square inch. They are simple in construction, but are usually large, heavy and costly relative to the quantity and pressure of air furnished by them, on account of their moderate speeds, and there is a tendency to supplant them by

High-speed Centrifugal Compressors. — These are built on the principle of steam turbines or high-pressure centrifugal pumps and are driven at very high

speeds, 1800 to 3500 r.p.m. The General Electric Co. makes a line of these compressors for pressures ranging from 1 to 3.25 pounds per square inch, and for delivering 800 to 28,000 cubic feet of free air per minute. By using several such compressors in series, one delivering into another, pressures of 100 pounds per square inch may be obtained.

POWER AND EFFICIENCY.—The useful work done by a blower is that equivalent to the isothermal compression of the air to the pressure in the receiver (or to the static plus the velocity pressure in the delivery main if the air were cooled to the atmospheric temperature) plus the work of delivering it against that pressure. If P_1 = absolute pressure at inlet in pounds per square foot; P_2 = pressure in the receiver; V_1 = volume of entering air at the pressure P_1, in cubic feet per minute, then useful work is foot-pounds per minute = $P_1V_1 \log_e \frac{P_2}{P_1}$. The useful or "air horse-power" is this quantity divided by 33,000.

When P_2 is but slightly in excess of P_1 the work per minute is P_2V_1 nearly, the error being less than 5 per cent when $P_2 = 1.1\ P_1$, and less than 1 per cent when $P_2 = 1.01\ P_1$.

When the air is compressed adiabatically, that is, without cooling during compression, the *useful* work is the same as in isothermal compression, but the actual work done in the cylinder (not including work lost by friction and leakage) is greater, or

$$3.463\ P_1V_1 \left\{ \left(\frac{P_2}{P_1}\right)^{0.29} - 1 \right\}.$$

The efficiency of blowing machines (i.e., the ratio of the air horse-power to the horse-power required to drive the machine) ranges from 95 per cent in the case of a slow-moving engine with large inlet and outlet valves, down to almost zero in poorly designed rotating machines. Centrifugal compressors have an efficiency under normal load of from 40 to 60 per cent (*see Fans*). A 39 by 84-inch positive blower, made by the Connersville Blower Co., is reported to have given an efficiency ranging from 68.5 per cent at an air pressure of 0.5 pound per square inch to 86 per cent at a pressure of 3.5 pounds per square inch, the displacement of air in each case being approximately 18,000 cubic feet per minute. The efficiency of steam-jet blowers is very low.

SPECIFICATION FOR BLOWER.*—The following memoranda are intended to assist in writing specifications. See also under *Fans* and article on *Specifications*.

Principal Characteristics and Conditions of Service.—Purpose of blowers.

Style and Description; Details of Construction.—Shall have horizontal, upward or downward discharge. Location of air outlet with reference to rotating parts. Hand or automatic closing of dampers when blower stops. How coupled to motor or engine, i.e., direct or belted. If direct driven, give details of coupler to disconnect blower from motor. Lubrication. Tools. Enclosure of moving parts to reduce danger.

Performance and Tests.—Cubic feet of air per minute. Air pressure, ounces or inches of water column at stated temperature.

ELECTRIC DRIVE OF COMPRESSORS.†—(*See also Motors, Industrial Applications of.*) As compressors of the reciprocating type must be driven at a low speed, the synchronous motor is well adapted for driving them, chiefly

* By W. A. Del Mar.
† By D. B. Rushmore.

because this motor operates normally at unity power factor and can be efficiently operated at leading power factors to counteract the lagging power factor due to induction motors driving other apparatus on the system. Slow-speed induction motors must be built with a large number of poles and will then operate at very low power factors with a comparatively low efficiency. Other reasons favoring the synchronous motor drive for this service are the steady load and the low torque required in starting, as a by-pass is generally provided so that the compressor works only against atmospheric pressure when starting. The motor should be able to be started from the alternating-current side and should for this reason be provided with the usual "squirrel-cage" winding. There is usually sufficient flywheel effect in the machine itself to keep within a reasonable angle of displacement, so that no trouble is experienced from "hunting."

For driving the centrifugal type of compressor, which requires a comparatively high speed, the synchronous motor is not as well adapted, and the best operation can generally be secured by the use of high-speed induction motors. For large units or where a low starting current is desirable the phase-wound type of motor should preferably be selected, although for smaller units the squirrel-cage type will as a rule prove satisfactory.

For direct-current installations compound-wound motors are generally used for driving reciprocating compressors and shunt-wound motors for the centrifugal type.

DIMENSIONS, WEIGHTS AND COSTS. — There is so great a variety of blowers and compressors that it is impossible in the space available to give a representative table of dimensions, weights and costs. The reader is referred to the manufacturers' catalogues and price lists. Representative manufacturers are the Ingersoll-Rand Co., the American Blower Co., the Sturtevant Co., and the Connersville Blower Co. (*See also Kent's Mechanical Engineers' Pocket-Book.*)

BIBLIOGRAPHY. — Ennis, W. D., *Applied Thermodynamics*, N. Y., 1910; Innes, C. H., *Air Compressors and Blowing Engines*, London, 1906; Miller, W. E., *Motor Driven Air Compressors*, G. E. Rev., 1909, Vol. 12, p. 459; and catalogues of manufacturers.

[Wm. Kent.]

BOILERS, STEAM. — (*See also Chimneys; Condensers, Steam; Conveyors; Draft, Mechanical; Feed-water Heaters; Fuels; Pipes and Piping; Power Stations; Pumps; Separators, Steam; Steam.*) The simplest form of boiler is a closed metal cylinder partly filled with water, with a pipe for the introduction of feed water, another pipe for the escape of steam and means for maintaining a fire under the boiler. Modifications of this elementary form are made: (1) for the purpose of insuring that as large a fraction as possible of the heat energy generated by the burning fuel shall be absorbed by the water and as small a fraction as possible shall be lost in the escaping hot gases and by radiation; and (2) for the purpose of increasing the heating surface and therefore the evaporative capacity of the boiler, with as great an economy of space occupied and of cost of the metallic structure as possible, without at the same time endangering the safety of the boiler, or decreasing its durability, facility for cleaning or other desirable qualities.

DEFINITIONS. — Certain special terms used in reference to boilers are defined below.

Boiler Horse-power. — This term was originally introduced to indicate the size of boiler required for an average reciprocating engine, the boiler being said to have the same horse-power as that of the engine. Due to improvements in the design of steam engines a boiler of a given horse-power rating may be ample to supply an engine (e.g., a compound condensing engine) of 3 times the rating of the boiler. The American Society of Mechanical Engineers has adopted the following definition:

A boiler horse-power is equivalent to the evaporation of 34.5 pounds of water per hour from feed water at 212° F. to saturated steam at the same temperature.

Adopting Marks and Davis's figures for the properties of steam (*see Steam*) 34.5 pounds of steam from and at 212° F. is equivalent to 33,479 B.t.u. per hour, or to an evaporation of 30.018 pounds from 100° F. feed-water temperature into steam at 70 pounds gage pressure.

It is customary in the trade to consider 10 square feet of heating surface as equivalent to a boiler horse-power, for stationary boilers. The term boiler horse-power is not used in connection with locomotive or marine boilers, and there is a tendency to discontinue its use for stationary boilers, expressing their size in square feet of heating surface, and their evaporative capacity in pounds of water evaporated from and at 212° F. per hour.

Equivalent Evaporation. — Factor of Evaporation. — For the purpose of reducing the results of a boiler test made under certain conditions of feed-water temperature and steam pressure and quality to a common standard, it is customary to use the equivalent evaporation from and at 212° F. as that standard. The pounds evaporated under actual conditions are multiplied by a "factor of evaporation," F, which is determined by the formula

$$F = \frac{H - h}{970.4},$$

in which H = total B.t.u. above 32° F. in 1 pound of steam at the actual pressure and degree of superheat, and h = total B.t.u. above 32° F. of 1 pound of the feed water. The values of H and h are given in steam tables (*see Steam*). If the steam contains x per cent of moisture, use for H in the above formula

$$H = H'\left(1 - \frac{x}{100}\right) + \frac{h'x}{100},$$

where H' is the total heat of saturated steam at the given pressure and h' is the heat of the liquid at the given pressure, as given in steam tables.

Boiler Efficiency. — The efficiency of the boiler alone is usually defined as the ratio of the number of B.t.u. absorbed by the water per pound of combustible *actually* burnt, to the number of B.t.u. in one pound of the combustible. The over-all efficiency of the boiler and grate is the ratio of the B.t.u. absorbed by the water per pound of *coal as fired* to the number of B.t.u. in one pound of this coal. This over-all efficiency differs from the efficiency of the boiler alone according to the amount of coal lost through the grates.

"Economy" or Water Rate. — Boiler efficiency is also frequently expressed in terms of the number of pounds of water evaporated from and at 212° F. per pound of combustible in the coal actually burnt (this corresponds to the efficiency of the boiler alone), or per pound of coal as fired (this corresponds to the efficiency of boiler and grate). The number of pounds of water evaporated from and at 212° F. per pound of coal is frequently referred to as the "economy" of the boiler, or its "water rate."

The relation between efficiency and economy may be expressed as follows.

Let ϵ = boiler efficiency, in per cent,
B = B.t.u. per pound of coal,
w = economy, i.e., the equivalent number of pounds water evaporated per pound of coal.

Then

$$w = \frac{B\epsilon}{97,040}.$$

For example, if the coal contains 10,000 B.t.u. per pound and the boiler efficiency is 70 per cent, then the economy is $10,000 \times 70/97,040 = 7.2$ pounds water per pound coal.

CLASSIFICATION OF BOILERS. — Boilers may be classified as *externally* and *internally fired*, *water tube* and *fire tube*, *through tube* and *return tube*, *horizontal* and *vertical*. An *internally fired* boiler is one in which the furnace is built inside of the boiler, and its roof and sides form a part of the heating surface of the boiler; the locomotive and Scotch marine, the Lancashire and the vertical-tubular boiler are of this type. An *externally fired* boiler is one in which the furnace is separate from the boiler itself and is usually placed underneath it, but sometimes at the side of it in a structure lined with fire-brick. A *fire tube* boiler is one in which the hot gases pass through the tubes, whereas the water is contained in an external shell through which the tubes pass. A *water tube* boiler is one in which the water is contained in the tubes and the hot gases pass around them. A *through tube* boiler is one in which the hot gases pass directly through the tubes from the fire to the smoke flue, whereas in a *return tube* boiler the gases are caused to pass under the shell containing the tubes to the farther end, thence upward through the "back connection," or rear of the setting and return through the tubes to the "breeching" or flue connection to the chimney. The term "return tubular" is applied only to fire tube boilers.

APPLICATIONS OF VARIOUS TYPES. — Externally fired *return tube* boilers in sizes up to 200 horse-power and for pressures up to 150 pounds per square inch are very commonly used. Boilers of this type are comparatively cheap and do not require expensive setting. In large power stations externally-fired *water tube* boilers are almost invariably used in this country. These boilers are built in sizes up to 2300 horse-power and for all practicable pressures. Babcock and Wilcox, Heine, Sterling and Wickes boilers are of the water tube type. In the first two makes the tubes are slightly inclined, in the third they are quite steeply inclined, and in the last they are vertical.

Selection of Type of Boiler. — In selecting the type of boiler to be used in any particular case the following points should be taken into consideration: (1) steam pressure desired; (2) space available; (3) danger of explosion; (4) liability to minor troubles which cause temporary stoppage; (5) durability; (6) ease of making repairs; (7) facilities for cleaning and removing scale and for inspection; (8) water-storage capacity; (9) rapidity at which steam can be raised; (10) steadiness of water level; (11) dryness of steam; (12) cost, including setting.

The question of economy of fuel is not generally dependent on the type of boiler, as the same economy may be obtained with all types provided. In order to secure maximum economy the following requirements are essential: (1) sufficient heating surface; (2) that the fuel can be thoroughly burned in the furnace, so that no unburned gases are allowed to reach the heating surface, and (3) that the path of the gases through the boiler is properly baffled so that they are not "short-circuited," leaving part of the heating surface out of the current.

DESIGN AND CONSTRUCTION. — The essential parts of the boiler in the case of a water tube boiler are the tubes, steam drums and mud drums, grates and setting. The tubes are usually 4 inches in diameter in large stationary boilers, and are usually more or less inclined. The steam drums are large cylindrical vessels which serve as a reservoir for the water and steam. The mud drums are arranged to collect a considerable part of the sediment formed by evaporation of the water. Suitable blowpipes are provided for draining off the water and for discharging the sediment and scale-forming material. The water level in the boiler is usually indicated by a gauge glass or try cocks, or both, connected either directly to the boiler shell or to a water column. Suitable manholes and handholes are provided to give access to the various parts of the boiler.

Heating Surface. — For maximum fuel economy with any kind of boiler it should be proportioned so that at least one square foot of heating surface should be given for every 3 pounds of water to be evaporated from and at 212° F. per hour. Still more liberal proportions are required if a portion of the heating surface has its efficiency reduced by: (1) tendency of the heated gases to short-circuit, that is, to select passages of least resistance and flow through them with high velocity, to the neglect of other passages; (2) deposition of soot from smoky fuel; (3) incrustation. If the heating surface is clean, and the heated gases pass over it uniformly, little if any increase in economy can be obtained by increasing the heating surface beyond the proportion of 1 square foot to every 3 pounds of water to be evaporated, and with all conditions favorable but little decrease of economy will take place if the proportion is 1 square foot to every 4 pounds evaporated; but in order to provide for driving of the boiler beyond its rated capacity, and for possible decrease of efficiency due to the causes above named, it is better to adopt 1 square foot to 3 pounds evaporation per hour as the minimum standard proportion.

For maximum commercial economy, taking into consideration not only the cost of fuel but also the interest, depreciation and taxes on the cost of the plant, a higher rate of driving may be required, even as high as 6 or 8 pounds per square foot of heating surface per hour at the time of maximum or peak load, if the high loads last only a few hours each day.

Measurement of Heating Surface. — The usual rule is to consider as heating surface all the surfaces that are surrounded by water on one side and by flame or heated gases on the other, using the external diameter for

Grates. — Three kinds of grates are employed, the stationary, the rocking or shaking, and the traveling grate. The grate bars are made of cast iron. Rocking grates have the advantage of permitting clearing the fire-bed of ash and clinker without opening the fire door and require less manual labor than the stationary grate. A traveling grate is one form of a mechanical stoker (*see Stokers, Mechanical*).

Grate Surface. — The amount of grate surface required per horsepower and the proper ratio of heating surface to grate surface are extremely variable, depending chiefly upon the character of the coal and upon the rate of draft. With good coal, low in ash, approximately equal results may be obtained with large grate surface and light draft, and with small grate surface and strong draft, the total amount of coal burned per hour being the same in both cases. With good bituminous coal, like Pittsburgh, low in ash, the best results apparently are obtained with strong draft and high rates of combustion, provided the grate surfaces are cut down so that the total coal burned per hour is not too great for the capacity of the heating surface to absorb the heat produced.

With coals high in ash, especially if the ash is easily fusible, tending to choke the grates, large grate surface and a slow rate of combustion are required, unless means, such as shaking grates, are provided to get rid of the ash as fast as it is made.

The following table is adapted from a more extensive one given in Gebhardt's *Steam Power Plant Engineering*.

Nature of plant	Number of plants	Character of fuel	Average ratio of heating surface to grate surface
Central stations	10	Illinois screenings, 15 to 20% ash	65
Central stations	14	Bituminous	60
Central stations	1	Bituminous	31
Central stations	9	Anthracite	40
Mfg. plants	20	Anthracite	35
Office building	6	Bituminous	48

Calculation of Required Heating Surface and Grate Area. — From the data given above the proper grate area and heating surface may be calculated as follows:

Example 1. — Steam required per hour at the "peak of the load," 6000 pounds; 3000 pounds ordinarily. Steam per pound of the poorest coal under the given conditions of feed-water temperature and steam pressure at maximum rate of driving, 6 pounds. This corresponds to $6000/6 = 1000$ pounds of coal per hour. Maximum rate of combustion under worst conditions, 25 pounds of coal per square foot of grate per hour. The required grate surface is then $1000/25 = 40$ square feet. Heating surface for maximum economy at ordinary driving will be $3000/3 = 1000$ square feet, requiring $6000 \times 3/3000 = 6$ pounds of steam per square foot of heating surface per hour at highest rate. Ratio of heating to grate surface, $1000/25 = 40$ to 1.

Example 2. Let 3000 pounds of steam be required per hour uniformly. The coal is of good quality so that 9 pounds of water will be evaporated per pound of coal when the rate of driving does not exceed 3 pounds of evaporation per square foot of heating surface per hour. The rate of combustion of this coal may be as high as 30 pounds per square foot of grate per hour, but to have more easy firing, 20 pounds is chosen. Then,

$$\begin{aligned}\text{Heating surface} &= 3000/3 = 1000 \text{ square feet;}\\ \text{Coal per hour} &= 3000/9 = 333 \text{ pounds;}\\ \text{Grate surface} &= 333/20 = 16.7 \text{ square feet;}\\ \text{Ratio of heating to grate surface} &= 1000/16.7 = 60 \text{ to } 1.\end{aligned}$$

Air Passages through Grates. — The usual practice is to make the air opening 30 to 50 per cent of the total grate area. With coal free from clinker much smaller openings may be used.

Flues and Other Gas Passages. — Rules are usually given making the area of gas passages bear a certain ratio to the area of the grate surface; thus a common rule for horizontal tubular boilers is to make the area over the bridge wall 1/7 of the grate surface, the flue area 1/8, and the chimney area 1/9.

For average conditions with anthracite coal and moderate draft, say a rate of combustion of 12 pounds of coal per square foot of grate per hour, and a ratio of heating to grate surface of 30 to 1, this rule is as good as any, but it is evident that if the draft were increased so as to cause a rate of combustion of 24 pounds, requiring the grate surface to be cut down to a ratio of 60 to 1, the areas of gas passages should not be reduced in proportion. The amount of coal burned per hour being the same under the changed conditions, and there being no reason why the gases should travel at a higher velocity, the actual areas of the passages should remain as before, but the ratio of the area to the grate surface would in that case be doubled.

Mr. Barrus states that the highest efficiency with anthracite coal is obtained when the tube area is 1/9 to 1/10 of the grate surface, and with bituminous coal when it is 1/6 to 1/7, for the conditions of medium rates of combustion, such as 10 to 12 pounds per square foot of grate per hour, and 12 square feet of heating surface allowed to the horse-power.

Boiler Settings. — The boiler proper is usually mounted in a brick setting with suitable iron buckstays. The setting also forms the walls of the furnace. Common red brick is usually employed for the setting except for those parts which are exposed to a sufficiently high temperature to require the use of fire brick. Sometimes a separate furnace, or "Dutch oven," directly in front of the boiler, is used.

OIL BURNERS AND FURNACES. — In burning liquid fuel in steam-boiler or other furnaces it is of the utmost importance that the fuel be completely atomized and that each minute particle be surrounded by sufficient air for its complete combustion. Failure in this results in the formation of smoke and soot. The following methods of injecting the oil and air into the furnace are used in connection with steam boilers:

(1) *Steam injection.* A fine jet of steam at high pressure is employed to inject both oil and air into the furnace. Furnaces fired by this method are very successful in burning oil without smoke, if the oil is not fed to the furnace at a rate faster than that for which the burner is designed. (2) *Air injection.* Air at about 40 pounds pressure per square inch is used to inject and atomize the oil. Burners of this type are very successful, and they give higher temperatures than steam injectors. The objection to them is the cost of the air-compressing machinery. (3) *Mechanical injectors.* The oil under pressure

delivered in a very fine spray. Some burners of this type have given excellent results. With these burners it is necessary to supply the oil under a high pressure, say from 50 to 80 pounds per square inch and to heat it before reaching the burner to a temperature of from 120° to 180° F. in order to reduce its viscosity.

More important than the burner is the furnace. It must be of sufficient size to give ample room for complete combustion of the gases formed by decomposition of the oil before these gases are allowed to be chilled by contact with the heating surfaces of the boiler. For economy of oil the air supply should be regulated so that the excess of air is not more than 50 per cent above that theoretically required for complete combustion.

SUPERHEATERS. — To obtain the advantages of superheated steam (*see Steam*), various forms of superheaters are employed. A superheater consists essentially of a number of small tubes, through which the steam is passed, arranged to be heated by the hot gases in the boiler or from some external source. In this country the superheater is nearly always placed within the boiler setting. Frequent cleaning of the exterior walls of the superheater tubes with steam is necessary to prevent the accumulation of soot. It is customary to provide for the flooding of the superheaters when the boiler is banked and when steam is being raised; this procedure may be undesirable when the feed water contains much scale-forming impurity, as superheater tubes are not readily cleaned internally.

PERFORMANCE OF BOILERS. — The number of boiler horse-power (i.e., the number of pounds of steam from and at 212° F. divided by 34.5) which can be obtained from a boiler depends upon the heating surface of the boiler, the grate area of the furnace, the quality of coal used, the method of firing, etc. Consequently the horse-power rating of a boiler is at best but an exceedingly rough indication of its capacity, and therefore the modern tendency is to rate a boiler in terms of the number of square feet of heating surface. If the horse-power rating is also given, this is usually taken arbitrarily for water tube boilers as the number of square feet of heating surface divided by 10; that is, 1 boiler horse-power is taken as equivalent to 10 square feet of heating surface. Some large modern boilers, however, are operated normally under conditions such that the boiler horse-power actually developed is twice the rating on this basis. Builders of fire tube boilers usually rate them on the basis of 11 to 12 square feet per boiler horse-power.

Boiler Capacity. — The capacity of a boiler may be expressed in terms of the boiler horse-power which may be obtained from it, or, more definitely, in terms of the number of pounds of water which can be evaporated in it. The capacity of a boiler, by which is meant the quantity of steam it will make in a given time, depends (1) upon the quantity of heat that can be generated in the furnace, and (2) upon the percentage of this heat that is absorbed by the boiler and not wasted in the chimney gases or by radiation.

The first essential of boiler capacity for a given heating surface is furnace capacity, and this depends chiefly upon three things, area of grate surface, quality of fuel and force of draft. It may also depend upon the kind of grate, as plain or shaking grate, or mechanical stoker; upon the roof of the furnace, whether formed of the heating surface of the boiler, or of fire brick; and upon the introduction of air above the fire in addition to that which passes through the grate. The second essential of boiler capacity is boiler efficiency. Other things being equal, anything that increases the efficiency of a boiler increases its capacity in the same ratio.

Factors Affecting the Efficiency of a Boiler. — Boiler efficiency depends: (1) on the quality of the coal (or other fuel) as regards its content of

moisture and volatile matter (*see article on Fuels*); (2) on the completeness with which the coal is burned in the furnace, before the gases of combustion touch the heating surface; (3) on the combustion being effected with the least possible excess of air, above 20 per cent excess being necessary to avoid imperfect combustion; (4) on the cleanness of the heating surface, inside and out; (5) on the proper baffling of the gases, so as to avoid short-circuiting; (6) on the heat lost by radiation; and (7) on the extent of the heating surface relative to the quantity of heat generated, or to the rate of driving of the boiler, as measured by the heat absorbed in a given time per square foot of heating surface (*see section on Heating Surface, below*). In general the efficiency of a boiler may be represented approximately by the equation

$$\epsilon = \frac{T_1 - T_2}{T_1},$$

in which T_1 is the temperature of the fire and T_2 the temperature of the chimney gases. The temperature T_1 is proportional to the heat generated in the furnace, assuming the specific heat of the gases to be constant, and T_2 to the heat loss in the chimney. The loss by radiation is not considered in the formula, but this is generally not over 2 to 3 per cent when a boiler is driven at a normal rate.

Values of Boiler Efficiency. — The *highest* efficiency that is obtained with ordinary boilers and furnaces, fired by hand, is about 79 per cent with anthracite and 76 per cent with bituminous coal. As high as 81 per cent may possibly be obtained with bituminous coal with very large boilers, provided with mechanical stokers, and 82 per cent with oil fuel. With economizers, heating the feed water by the waste heat of the gases from the boiler, the combined efficiency of boiler and economizer may be as high as 90 per cent.

The necessary conditions for obtaining these high figures of efficiency are: (1) that the coal is of good quality, low in moisture; (2) that the coal is burned uniformly and with uniform conditions of air supply, so that the grate is not obstructed by clinker or imperfectly covered by coal; (3) that the rate of driving does not exceed 3 pounds evaporation per square foot of heating surface per hour; (4) that the air supply does not exceed 20 pounds per pound of carbon burned; (5) that the combustion of the gases is completed in the furnace, so that no unburned gases touch the heating surface of the boiler and become chilled below the point of ignition; (6) that the path of the gases through the boiler is so baffled as to cause them to traverse uniformly all parts of the heating surface; (7) that the heating surface is free from soot and dust on the outside and from scale or grease on the inside.

The conditions of high efficiency above described are obtained only in the best-managed plants, where the chimney gases are constantly analyzed as a check on the air supply (*see below under Tests of Boilers*). In the average plant in everyday practice the efficiency may be anywhere from 10 to 30 per cent below the best possible.

Effect of Rate of Evaporation on Efficiency. — On the basis of 10 square feet of heating surface per boiler horse-power (34.5 pounds of steam from and at 212° F.) approximately 3 pounds of steam are evaporated per square foot of heating surface when the boiler is developing its rating at normal steam pressure and temperature. If there is sufficient grate surface and draft the evaporation may be increased to 6 or in emergency to 15 pounds of steam per square foot of heating surface. The efficiency, however, falls off with increased rate of driving. The following figures are taken from a test by D. S. Jacobus (*Trans. A.S.M.E., 1911*) on a large Stirling boiler (23,650 square feet of heating surface).

Equivalent evaporation per square foot =	3.24	3.40	3.72	4.18	5.22	5.62	6.40	6.67	6.75	7.29
Over-all efficiency =	81.2	81.0	80.3	79.2	77.1	77.9	76.4	76.7	75.6	75.8

Still higher rates of driving and the corresponding efficiencies are given below, these figures being taken from a test of Babcock & Wilcox boilers designed for the United States warships "Cincinnati" and "Wyoming" (*Industrial Engineering, March, 1911*). The results are as follows:

Equivalent evaporation per square foot =	8.42	8.75	9.03	9.58	10.1	10.5	13.7	14.8
Over-all efficiency =	70.4	70.6	72.1	68.2	70.1	71.0	64.5	63.3

These efficiencies are all exceptionally high for the given rates of driving, but the relative efficiencies for the different rates are representative of good average practice.

Thickness of Fire and Efficiency. — Too thin a fire results in excess of air and too thick a fire in a deficiency, unless sufficient draft is provided by means of fans or tall chimneys (*see Draft, Mechanical*). There are so many factors to be considered that the proper thickness for maximum economy in any particular case can be determined only from actual test. Under ordinary conditions of hand-firing the thickness of fire for best efficiency ranges from 6 to 12 inches.

Air Supply and Efficiency. — The air supply for maximum efficiency, all other conditions remaining constant, is about 17 or 18 pounds per pound of carbon.

With an air supply lower than 17 pounds the efficiency is likely to fall off, on account of imperfect combustion. If the air supply is greater than 20 pounds the efficiency falls off very rapidly, the excess air supply causing a reduction of temperature in the furnace and an increase of heat carried into the chimney.

Heat Balance and Distribution of Losses. — By the "heat balance" is meant a statement accounting for ultimate distribution of the B.t.u. in the fuel supplied to the boiler. Such a statement is usually made up as follows. The figures given illustrate the ordinary range for the various items.

HEAT BALANCE PER POUND OF COMBUSTIBLE

Item	B.t.u.	Per cent
1. Heat absorbed by the boiler	......	80–50
2. Loss due to heating moisture in coal	......	0.2–2.0
3. Loss due to heating moisture formed by the burning of hydrogen	......	0.5–4.0
4. Loss due to heat carried away in the dry chimney gases	......	12–30
5. Loss due to incomplete combustion of carbon (CO in flue gas)	......	0.0–3.0
6. Loss due to		
a. Unconsumed hydrogen and hydrocarbons (smoke and soot)	......	0.0–1.5
b. To heating the moisture in the air	......	0.2–1.5
c. To carbon lost through grate	......	0.5–3.0
d. To radiation	......	1.0–3.0
e. Unaccounted for	......	1.0–10.0
Total	11,000 to 15,500	100

The five items under 6 are frequently given as a single item.

TESTING OF BOILERS. — The principal observations made during an "evaporation" test of a boiler are pounds of water evaporated, pounds of coal fired, steam pressure and feed-water temperature. Records of the water evaporated and the coal fired should be made for each hour of the test if possible. The steam pressures and feed-water temperatures are recorded every half-hour or oftener, and averaged for the whole test. All conditions should as nearly as possible be the same at the end as at the beginning of the test. When the furnaces are hand fired, tests of 8 to 10 hours duration are usually sufficient, but when mechanical stokers are used it is difficult to obtain an equal quantity and condition of the coal in the furnace at the beginning and end of the test, and a large error in the results may be made if the test is much shorter than 24 hours. The average moisture in the coal should be determined and the ash and refuse should be weighed.

In all important tests the code prepared by a committee of the American Society of Mechanical Engineers (*Code of 1913*) should be followed. The principal data and results given in this code are as follows:

PRINCIPAL DATA AND RESULTS OF BOILER TEST.

1.	Grate surface (width ...., length ....)	sq. ft.
2.	Total heating surface	sq. ft.
3.	Date	
4.	Duration	hr.
5.	Kind and size of coal	
6.	Steam pressure by gauge	lb.
7.	Temperature of feed water entering boiler	° F.
8.	Percentage of moisture in steam or number of degrees of superheating	per cent or ° F.
9.	Percentage of moisture in coal	per cent
10.	Dry coal consumed per hour	lb.
11.	Dry coal consumed per sq. ft of grate surface per hour	lb.
12.	Equivalent evaporation per hour from and at 212°	lb.
13.	Equivalent evaporation per hour from and at 212° per sq. ft. of heating surface	lb.
14.	Rated capacity per hour, from and at 212°	lb.
15.	Percentage of rated capacity developed	per cent
16.	Equivalent evaporation from and at 212° per lb. of dry coal	lb.
17.	Equivalent evaporation from and at 212° per lb. of combustible	lb.
18.	Calorific value of 1 lb. of dry coal by calorimeter.	B.t.u.
19.	Calorific value of 1 lb. of combustible by calorimeter	B.t.u.

20. Efficiency of boiler, furnace and grate:

$$100 \times \left(\frac{\text{Item } 16 \times 970.4}{\text{Item } 18}\right) \ldots\ldots\ldots\ldots \text{ per cent}$$

21. Efficiency of boiler and furnace:

$$100 \times \left(\frac{\text{Item } 17 \times 970.4}{\text{Item } 19}\right) \ldots\ldots\ldots\ldots \text{ per cent}$$

Flue Gas Analysis and Heat Balance. — See Kent's *Mechanical Engineers' Pocket-Book* and Gebhardt's *Steam Power Plant Engineering*.

SPECIFICATIONS FOR STEAM BOILERS. — As nearly all steam boilers are now made by large manufacturing concerns who have their own specifications as to details of construction, quality of material, etc., it is rarely the case that an engineer of a power plant or a consulting engineer is called on to make an original design of a boiler or to furnish detailed specifications for one. What is usually done is to call for bids on general specifications, naming the quality of coal to be used, the kind of furnace or stoker, the horse-power to be developed (34.5 pounds of water evaporated from and at 212° F. = 1 h.p.), when the boiler is driven at a rate not to exceed 3.5 pounds per square foot of heating surface per hour, evaporated from and at 212° F., and the overload capacity desired for higher rates of driving, or "the peak of the load." The bids received are then tabulated as to the details, such as heating surface, grate surface, size of combustion chamber, size of tubes, diameter of shell, quality of steel, style of riveting, factor of safety, space occupied, guarantees of capacity and economy, etc., and a selection made of the boiler that appears to be the most suitable for the location. Of two boilers that have the same heating surface the one that has the largest grate surface is the one that has the greatest overload capacity, if the draft is the same in both cases. Guarantees of capacity and economy may be disregarded when bids are for ordinary forms of boilers and furnaces, for the capacity and economy of two boilers of the same size of grate and heating surface will be practically the same whatever the type or form of the boiler, but they will vary greatly from causes independent of the boilers themselves, such as quality of coal, method of firing, kind of furnace, size of combustion chamber, etc.

OPERATION OF BOILERS. — To obtain the best results from boilers, considerable care must be paid to the firing and regulation of air supply.

Firing. — There are three common methods of hand firing: (1) the alternate in which fresh coal is fired on one side of the grate at a time, the combustion of volatile matter being assisted by the heated air passing through the other side; (2) the spread-firing, in which small amounts are spread thinly over the entire fire bed; and (3) the coking, in which a thick bed of fresh coal is fired directly in front of the door and allowed to coke, after which it is pushed back into the furnace. The best method for any particular kind of fuel, style of furnace and rate of driving should be determined by experiment.

Mechanical stokers (*see Stokers, Mechanical*) afford the following advantages: viz., the uniform feeding of coal, the close regulation of the air supply at all times, and consequently smokeless and efficient combustion with a wide range of loads, and a great reduction of labor cost in handling and firing fuel. In small plants with poor load factors the expense of installation is generally considered prohibitive, but in plants of 1500 kilowatts or more a considerable net annual saving can usually be obtained.

Air Supply. — (*See also Chimneys; Draft, Mechanical.*) The extent of the flue losses is determined by the amount of gas discharged, the temperature at which it is discharged, the amount of incompletely burned gas discharged and the amount of latent heat carried away in water vapor. The minimum economic amount of flue gas would obtain with the amount of air supplied to the fire exactly equal to the theoretical requirements of complete combustion. The admission of less air tends to the production of CO. More air dilutes the flue gas, increases the total amount of gas rejected and, in the same ratio, increases the amount of heat rejected, the stack temperature remaining the same. The theoretical air supply per pound of combustible (*see Fuels*) is approximately 12 pounds, but since the air and fuel cannot be mixed with perfect intimacy it is necessary to considerably increase this amount to prevent the formation of CO. See above under ***Air Supply and Efficiency***.

CO_2 Recorders. — The per cent of CO_2 in the flue gas is an index of the air excess, provided combustion is complete. With perfect combustion and no excess air 20.9 per cent CO_2 should be obtained with pure carbon as fuel, or 18.6 per cent with a fuel composed of 95 parts C and 5 parts H. In practice more air must be supplied and 14 per cent CO_2 represents about the upper limit which should be attempted. 12 per cent CO_2 is good, 10 per cent fair and 8 per cent poor. Several types of CO_2 recorders have been devised. If properly maintained and interpreted they form a valuable check on the regulation of the air supply. Their indications, however, give no clue to the presence of CO, and are directly significant only when the formation of CO is prevented.

Flue-gas Temperature. — Because of the great heating surface which would be required it is not practicable to reduce flue-gas temperature to less than 75 degrees above the feed water. Economizers or flue-gas feed-water heaters (*see Feed-water Heaters*) assist in lowering the temperature of discharge but add considerably to the space required, increase the cost of installation and maintenance and increase the total draft required. Without economizers it is seldom economical to attempt to reduce the gas temperature below 450° F.

Smoke represents a minor element of loss *per se*, seldom exceeding 1 per cent, but it is generally an index of inefficient combustion. Smoke is usually due to a poorly regulated air supply, an imperfect mixture of air and fuel, or the too-rapid volatilization of hydrocarbons and their premature chilling in a half-burned state. The absence of smoke, however, is not a final index of efficiency, for the air supply may be excessive. See also article on *Smoke Prevention*.

Feed Water. — (*See Feed-water Heaters; Pumps and Pumping Engines.*)

Cleaning Tubes. — The tubes of a boiler should be kept clean, both internally and externally. For blowing the soot off the external surface the tubes of water tube boilers or off the internal surface of the tubes of fire tube boilers a steam jet blower is ordinarily used. For removing scale from the interior of water tubes different forms of scrapers or chippers are used, also rotary cutting devices driven at a high speed by steam, water or other power. For removing scale from the water side of fire tubes of horizontal tubular boilers the usual method is the operation of chipping and hammering by a man working inside of the boiler. Sometimes the scale is softened by the use of soda or other chemicals, before chipping, and a strong jet of water from a hose assists in detaching the scale. In locomotive practice it is often necessary to cut the tubes out of the boiler and run them through a cleaning machine, and then to weld a piece of each tube on one end to restore its original length, and to reset the tubes in the boiler.

DIMENSIONS OF BOILERS. — An ordinary fire tube boiler occupies a floor space of from 1.5 to 2.0 square feet per boiler horse-power (on the basis of 12 square feet of heating surface per horse-power): A water tube boiler occupies from 0.5 to 1.0 square foot of floor space per boiler horse-power (on the basis of 10 square feet of heating surface per horse-power). With either type the height ranges from as low as 7 to as high as 30 feet depending not only on the horse-power but also on the style, whether horizontal or vertical. One boiler is sometimes placed directly on top of another, making a "double-deck" arrangement. For complete dimensions see the manufacturer's catalogues.

COST OF BOILERS. — The selling price of boilers per rated horse-power (10 square feet of heating surface = 1 horse-power) ranges from about $8 to $25, depending upon the size, the pressure they are to sustain, the style and design, etc. The lowest prices named are for large-sized ordinary horizontal tubular boilers (fire tube) for low pressures. For power plants of 1000 horse-power and over the price will range usually between $10 and $15 for pressures

not over 150 pounds per square inch. For higher pressures and for boilers provided with superheaters the prices will be higher. For boilers of less than 100 horse-power the price per horse-power increases as the size decreases. The prices named are for the boilers with the usual fittings of grates, steam and water gauges, blow-off and stop valves, on board cars at the boiler works, and do not include the cost of erection nor of brickwork, flues or chimneys. (*See also Power Station.*)

C. H. Benjamin (*Eng., N. Y., Nov. 15, 1902*) gives the following rules for estimating the cost of boilers; P = boiler horse-power; the cost is in dollars.

(1) Horizontal water-tube boilers, 125 pounds pressure, 10 square feet of heating surface per boiler horse-power,

Cost of boiler $= 500 + 9.20\,P$.
Cost of setting $= 400 + 0.80\,P$.

(2) Vertical water tube boiler as in (1),

Cost of boiler $= 500 + 8.50\,P$.

(3) Horizontal return tubular boilers, 12 square feet of heating surface per horse-power,

Cost of boiler $= 100 + 6.50\,P$.
Cost of setting $= 300 + 0.70\,P$.

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[WM. KENT.]

BONDS, RAILWAY TRACK. — (*See also Third Rail Systems; Trolley Systems; Wires and Cables.*) Rail bonds are electrical conductors for bridging the joints of rails. They consist either of a series of thin strips of annealed copper, or of one or more cables of copper wire, the ends of which are usually pressed or cast into solid copper terminals. Ribbon is more compact, but stranded wire is more flexible; the latter should always be used if space permits. Sometimes the terminals are made by upsetting the ends of a stranded conductor, so as to form a crude bond head. The terminals of the headed bond are then heated to a welding heat in furnaces, and pressed to the proper size and shape, the separate wires in the terminals becoming a solid mass of homogeneous copper. After being cleansed of all foreign substances the terminals are shaved to the proper size.

TYPES OF BONDS. — Bonds may be classified, according to the method of fastening them to the rail, as soldered bonds, brazed bonds and bonds applied by mechanical pressure.

Soldered Bonds (Figs. 1 and 2) usually consist of a series of thin strips of annealed copper with tinned terminals as shown in Figs. 1 and 2. They are

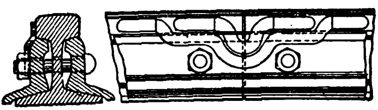

Fig. 1.

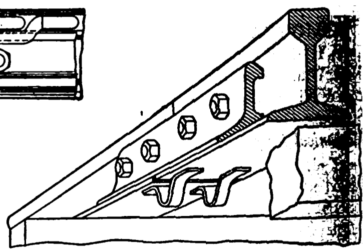

Fig. 2.

soldered direct to the head, foot or web of the rail. One or more bonds per joint may be used.

Brazed Bonds resemble soldered bonds except that the terminals are enveloped in brass. They are brazed or welded to the rail by heat generated electrically in a carbon electrode which constitutes one jaw of a clamp holding the bond against the rail.

Expanded and Compressed Terminal Bonds. — Bonds fastened to the rail by mechanical pressure may be divided into two general classes, expanded terminal and compressed terminal bonds.

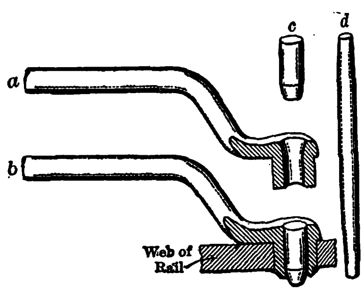

Fig. 3.

Pin-expanded Terminal Bonds (Fig. 3) have their heads drilled with an axial hole, through which a tapered steel pin *d* is driven, forcing the copper outward and against the steel. This type of bond is fastened to the web of the rail.

Compressed Terminal Bonds (Figs. 4 and 5). — There are two kinds of compressed terminal bonds, in one of which direct pressure is applied at both

ends of the head, and in the other, at one end only. The first type of bond is usually applied to the web of the rail by means of a heavy screw or hydraulic press (Fig. 8) which engages the bond head and causes it to compress longitudinally and expand laterally as the pressure is applied, bringing the copper into

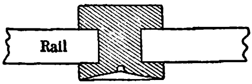

Fig. 4.

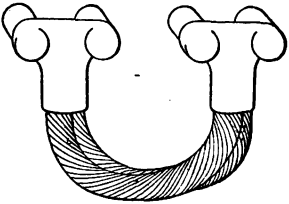

Fig. 5.

firm contact with the steel and spreading the projecting end of the terminal into a button-shaped rivet-head, as shown in Fig. 4. The second type of bond (Fig. 5) is applied only to the head of the rail, the terminal lugs being set in holes therein and expanded into contact by means of hammer blows.

Compressed Terminal Soldered Bond. — A type of bond which has been found very successful at the Detroit River Tunnel for third-rail work is a compressed terminal head bond soldered to the rail. The combination of mechanical adhesion and soldering has resulted in a bond of unusual durability. (*H. B. P. Wrenn.*)

Plastic Bonds. — In addition to the types of bonds described above, there is the plastic bond which in one form consists of a copper plate with amalgamated terminals pressed against amalgamated surfaces of the rail by the fish-plate. In another form it consists of an amalgamated copper plug set in a hole drilled vertically through the rail head and into the fish-plate.

Exposed Versus Concealed Bonds. — Whether soldered, brazed, expanded or compressed, bonds may be either exposed or concealed (Fig. 6) under the fish-plates. The former condition is preferable, if there is no likelihood of theft, as it permits inspection to be easily made. Where the bonds are exposed to theft, as, for example, on track rails unprotected by paving, concealed bonds are almost a necessity.

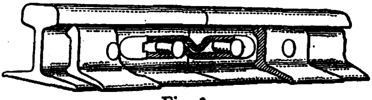

Fig. 6.

While concealed bonds are necessarily applied to the web of the rail, exposed bonds may be applied to the foot or head. Head bonds have the advantage of greater contact surface at the terminal studs, while foot bonds are less exposed to mechanical violence. Web bonds, unless concealed, have to be excessively long in order to span the fish-plates.

USES OF VARIOUS TYPES OF BONDS. — Bonds are used for track rails, third rails and girders of elevated and subway lines. Soldered bonds and compressed terminal head bonds find their best application in third-rail work, where good electrical contact is of greater importance than mechanical strength. Expanded terminal web bonds, especially of the concealed type with two stranded conductors, are regarded as the best for heavy track work, where mechanical strength and ease of installation are of the utmost importance.

SUBSTITUTES FOR BONDING.— Several efficient substitutes for bonding are now in use, such as electrical welding, cast welding, Thermit welding and the "Romapac" continuous rail system.

Electrical Welding is performed by clamping an iron bar to the web of the rails, and bringing the bar and the neighboring part of the rail to a white heat by means of an electric current.

Cast Welding is accomplished by setting a mold around the rail joint and pouring molten iron around it. The Thermit process is a modification of this, the iron being liberated at a white heat from a mixture of iron oxide and aluminum, which is ignited in a crucible.

Romapac Continuous Rail System (Fig 7). — This system differs from all forms of welding in not requiring heat at the joints, adhesion being obtained by a process of cold rolling. The rail consists of two pieces, a head and base, the head being provided with depending flanges which are rolled or crimped on to the base after the latter has been laid. The head and base pieces are staggered as shown in Fig. 7, thereby affording a continuous electrical path through the steel.

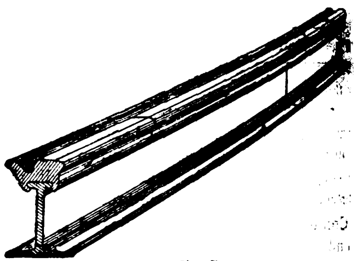

Fig. 7.

SELECTION OF TYPE AND SIZE OF BOND. — Considerations determining the choice between concealed and exposed bonds are liability of theft, electrolytic corrosion, facility of inspection and injury to fish-plate bolts of old rails.

The choice between mechanical adhesion and soldering depends largely upon the importance of rapid installation, the mechanical stresses to be withstood in service, and the facilities for the use of drills, presses, etc.

Single vs. Double Bonding. — Joints are sometimes bonded with one and sometimes with two bonds. Double bonding has the advantages of less chance of complete failure and greater carrying capacity for a given cross-section of copper. It, however, has the disadvantages of being more expensive and giving uncertain results in testing.

Selection of Cross-sectional Area of Bond. — The cross-sectional area of a rail bond should, as a rule, be not greater than is necessary to keep its temperature at a safe working amount, unless greater area is required for mechanical strength. The resistance of the bonded joint is of secondary importance unless very high, because the resistance of the joints is usually a mere fraction of the total track-rail resistance (*see below*).

Carrying Capacity of Bonds. — The carrying capacity of rail bonds is not very well known as the manufacturers do not supply any data on this subject for their various products. The excellent heat conductivity of copper and the large heat storage capacity of steel rails tend to make the carrying capacity of bonds in cold weather considerably greater than that of free wire of the same size, especially if the bonds are short. In hot weather, however, the rails and consequently the bonds are likely to become hot from exposure to the sun's rays, thereby reducing the effective carrying capacity of the bonds.

Tests to determine the ultimate carrying capacity of soldered bonds indicated that a 500,000 circular mil bond of the type shown in Fig. 1 will carry 3500 amperes continuously without injury, but will melt off in 5 or 10 minutes at 10,000 amperes. On the basis of this test assuming the cooling surface per unit

length of a bond to be proportional to the square root of its cross-section, the safe carrying capacity of a bond of A circular mils cross-section is

$$I = \frac{A}{5.6\sqrt[4]{A}}.$$

Or, the cross-section required for a given current is

$$A = 10\, I \sqrt[3]{I}.$$

Resistance of Bonds.—The total resistance of a bond is the sum of the resistances of the copper in the bond and the contact resistance between the body of the bond and the terminals and the contact resistance between the terminals and the rail. The following table gives the resistance of a well-bonded joint at 75° F., including the body resistance and the two contact resistances at each end.

RESISTANCE OF BONDED JOINT IN MICROHMS (10^{-6} OHMS)

Size A. W. G. or B. & S. and C. M.	Diameter of terminals, inches	Length of bond in inches, between centers of terminals							
		9	12	15	18	24	30	36	42
0	5/8	28	33	39	44	55	65	76	87
00	5/8	32	39	46	52	66	80	93	106
000	3/4	38	46	55	63	80	97	114	131
0000	7/8	44	54	65	75	95	116	136	157
250,000	7/8	50	62	75	87	111	136	160	185
300,000	1	60	75	91	107	137	169	200	230
400,000	1	72	91	112	131	171	210	250	290
500,000	1	87	112	137	162	211	260	310	360

Bonding Efficiency and "Equivalent" Length of Joint.—The bonding efficiency* of a bonded rail system (having one bonded joint to each rail length) is the ratio of its conductance to that of an equal length of perfectly continuous rail without any breaks or joints. By "equivalent" length of a joint is meant the length of continuous rail having a resistance equal to that of the bonded joint. Let

B = bonding efficiency,
l = distance in feet measured along the rail between centers of bond terminals,
l' = equivalent length of bonded joint in feet,
L = length in feet of a full rail section,
r = resistance of rail per foot,
R_j = resistance of bonded joint.

Then

$$l' = \frac{R_j}{r},$$

$$B = \frac{1}{1 + \frac{l' - l}{L}}.$$

* The efficiency of individual bonds is sometimes stated as the ratio of the distance along the rail between centers of bond terminals to the equivalent length $= l/l'$.

For values of *r* see article on *Rails*. The equivalent length of a properly bonded joint ranges from 1 foot to 6 feet, depending upon the size of rail, number and cross-section of bonds, length of bond, etc. (*See also section on Rebonding, below.*)

It is common practice to bond a joint with a bond having one-half the conductance of the rail. Hence for a 12-inch bond, and 30-foot rail section, assuming 9 inches between terminals measured along the rail, $l = 0.75$ and $l' = 2$, giving a bonding efficiency of 96 per cent.

In the Romapac system the resistance of the completed rail is usually less than that of an ordinary rail of the same cross-sectional area, because the peculiar shape of the rail necessitates a rolling mill process which lowers the resistivity of the rail metal without changing its chemical composition.

Selection of Length of Bond. — The length of concealed bonds is necessarily determined by the dimensions of the rail and fish-plate, and by the bolt-hole drilling. The length should never be less than ten inches for single bonding, and should generally be greater, it being usual practice to place the terminals between the first and second bolt holes. In double bonding it is customary to place one terminal of each bond between the first and second bolt holes of each rail, and the other terminals beyond the second bolt holes. It is not unusual to use concealed bonds 2 feet long for double-bonded rails on electrified steam roads. Exposed bonds are usually made as short as is consistent with the flexibility requisite to withstand vibration and other rail movements.

Effect of Vibration and Expansion of Rails. — The continual bending of a copper wire or ribbon will cause local hardening and crystallization of the metal, and eventually lead to fracture. It is, therefore, important to save track bonds from vibration as much as possible, and they should be initially designed to withstand the vibration they are likely to experience in use. The best way to accomplish this is to use long thin wires of soft annealed copper entirely free from surface imperfections.

According to the A. S. & W. Co., Cat. No. 3, wires of from 0.040 to 0.045 inch diameter give the best general service for short bonds.

Concealed ribbon bonds, and to a less extent wire bonds, frequently break at the loop between rail bolts, first, because the operation of crimping slightly hardens the copper in the loop, and second, because the travel of the rails, due to their expansion and contraction, is wholly taken up in the loops, causing the copper to crystallize and break at this point. Of course this action is aggravated by any severe jarring or vibration of the joint (*C. R. Sturdevant, Proc. Am. El. Ry. Assn., 1911, p. 756*).

SPECIFICATIONS. — (*See also article on Specifications and Contracts.*) Specifications for rail bonds should state the exact service conditions under which the bond is to be used, the style of adhesion desired, the part of the rail they are to be applied to, the style of conductor (ribbon, solid wire or strand), the cross-sectional area, the contact area of the stud, the formed length between centers of terminals, and the fish-plate and bolt layout, if the bonds are to be concealed.

INSTALLATION. — The foremost consideration in the installation of bonds is the cleanliness of the bonds and bond holes, or other adhesion surface. Unless this is secured the bonds will be electrically defective whatever their mechanical strength may be.

Soldered Bonds. — The rail surface is brightened by means of a carborundum or emery wheel, then tinned using an acid flux. The bond is then clamped in place and the rail and bond heated by means of a blow-torch, to a temperature at which the solder will melt and cause the bond to adhere firmly

Brazed Bonds. — The preliminary processes are the same as for soldered bonds except that a special clamp is used, the terminals of which are the electrodes of an electrical circuit, one being of copper and the other of carbon.

The surface of the rail being previously ground bright at the point where the weld is to be made, the brass-enveloped bond terminal is placed in position against the rail at this point, held there and pressed against the rail by the carbon electrode, the copper electrode being in contact with the opposite side of the rail. The current on passing from one electrode to the other, traverses the bond terminal and rail, the carbon becoming incandescent. The incandescent carbon pressing the copper against the rail quickly transmits sufficient heat at exactly the point where it is required, to produce the weld. To make a weld, the current is applied for a period of from 45 seconds to two minutes, depending on conditions.

Welding Outfit. — It is claimed by the manufacturers (The Electric Railway Improvement Co., of Cleveland, O.) that an average of over 100 bonds per day are readily installed by their car operating with four men, a bonder and three helpers. This car is about 6 feet, 10 inches long by 5 feet, 10 inches wide and carries an 18-kilowatt rotary converter and transformer, with the necessary apparatus for their safe operation. To weld an average-sized rail bond to the rail, an alternating current of about 2000 amperes at 5 volts is employed. On d-c. railways this is obtained by converting and transforming about 20 amperes at 500 volts taken from the trolley wire.

Pin-expanded Bonds. — The rail is drilled, usually through the web, with or without lubricant, using some form of drill especially adapted to this service. Drilling without lubricant has the advantage of giving a perfectly clean hole, but is believed by some to cause excessive wear of the drills. It is doubtful, however, whether the small amount of oil which would be used could be kept constantly at the cutting edges, the only places where it would be useful. Dry drilling has been found successful on many railroads. Advocates of lubrication are divided over the choice of lubricants, some preferring soapy water or caustic alkali solution, and some a clean lard oil. If oil is used, it should be wiped out with a clean cloth saturated with gasoline. According to the A. S. & W. Co., Cat. No. 3, oil lubrication will increase the joint resistance less than 3 per cent. Lubricants containing water are likely to cause rust, especially if the drilling gang precedes the bond installers by any considerable time. The hole having been drilled, the bond head is inserted into it and a long taper punch lubricated with grease is driven entirely through the terminal. Then a short drift pin is driven home, as shown in Fig. 3. The diameter of the hole is usually increased about 1/8 inch by the expansion process. From 100 to 200 bonds may be installed with one taper punch before it is worn down to an appreciable extent.

This type of bond requires a smaller equipment in tools and materials than most other types and does not necessitate the use of any apparatus which obstructs the track and thereby endangers traffic.

Compressed-terminal Web and Foot Bonds. — The drilling having been performed as for a pin-expanded bond, and the bond heads inserted into their respective holes, a screw or hydraulic compressor, as shown in Fig. 8, is applied at both ends of the bond head, the conical point of the press fitting into the conical depression of the bond. Pressure is applied, either until a collar on the ram touches the rail, or until the head of the bond acquires the proper shape. Where no collar is used the point of the press (if of the screw type) sometimes cuts into the bond head; this may be avoided by placing a small amount of flake graphite mixed with oil in the depression of the bond head.

Compressed-terminal Head Bonds. — A four-spindle drill is used to drill four holes simultaneously in the rail heads. It is important to avoid drilling

the holes too deep lest the copper should not touch bottom and therefore be unable to expand laterally. If, on the other hand, the hole is too shallow, expansion will occur too soon.

Pressure and Contact Resistance. — It has been found by P. M. Hall, P. C. Smith and C. B. Starbird (*St. Ry. Jour., Sept. 14, 1907*) that the pressure

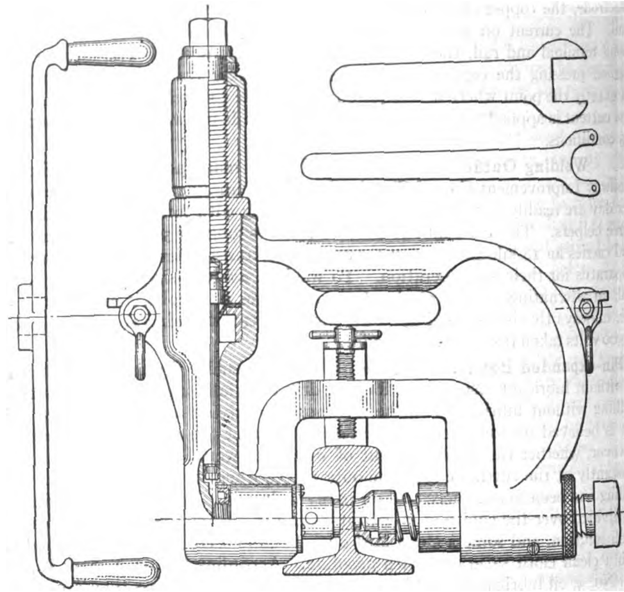

Fig. 8.

of a compressed-terminal rail bond against the steel rail, which gives a reasonably low-resistance value, is from 25,000 to 30,000 pounds per square inch of contact surface. To obtain this pressure a compressor giving a direct pressure of 25 tons per square inch of terminal steel section is required. This pressure being within the elastic limit of steel, the metal of the rail does not take a permanent set. Furthermore, if the contact surface of the bond terminal be increased, no appreciable decrease of resistance will occur unless the pressure is correspondingly increased. The contact resistance between annealed cast copper and steel is from 30 to 60 per cent higher than the resistance between annealed rolled copper and steel.

The copper of a bond head is hardened by the pressure it is subjected to, and, like the steel, is distorted within its elastic limit, causing the surfaces to adhere even if the pressure is reduced to one-third its original value, say 10,000 pounds per square inch. Between these two pressures, the electrical resistance does not vary. Expansion due to heat, therefore, has no effect upon the resistance of bonds.

TESTS AFTER INSTALLATION. — Every rail bond in service should be periodically tested and a complete record of the tests kept. The frequency of the tests will depend upon local conditions, once in 9 months being an average

of 14 urban railways, and once in 12 months an average of 11 interurban railways (*see Proc. Am. El. Ry. Assn., 1911, p. 751*).

Resistance Test. — The usual method of testing is to measure the drop of potential across the bonded joint and find simultaneously the length of continuous rail in which the same drop occurs, i.e., the "equivalent" length of the bonded joint. Several ingenious instruments have been devised for making this comparison with ease and accuracy.

Differential Voltmeter Method. — The most accurate type consists of a differential voltmeter and three contact pieces for attachment to the rail. One winding of the differential voltmeter is connected to a pair of contact pieces A and B, which are placed in contact with the rail just over the centers of the bond terminals, see Fig. 9, and the other winding to a pair of contact pieces B and C, which span a variable length of continuous rail. The distance BC is varied by moving the contact C until the voltmeter shows no deflection. This indicates that the potential drop between A and B is equal to that between B and C, or in other words, that a length of continuous rail has been found which has the same resistance as the bonded joint. The tester should make sure that there is current in the rail while he is making the test.

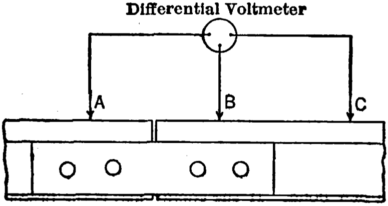

Fig. 9.

Conant Bond Tester. — R. W. Conant (Cambridge, Mass.) makes a bond tester in which the balancing is done by sound. A clockwork interrupter, an induction coil, and a telephone receiver is connected across a Wheatstone bridge (*see Bridges, for Electric Measurements*). The interrupted current causes sounds in the telephone receiver, which sounds diminish and disappear when the drop in the rail is balanced, by the bridge adjustment, to equal the drop across the joint. This type of apparatus has the advantage of being independent of the traction current in the rails, and is, therefore, constant in its sensitiveness.

Herrick Test Car. — A. B. Herrick has devised a recording instrument, which being placed on a car, is able to test the bonding of a railroad while running the car over it. The Herrick test car carries autographic recording apparatus, which measures by means of millivoltmeters, the potential drop across each joint, recording on sensitized paper by an electric spark, the scale of the record being usually about 1 inch to 60 feet of track. Wire-brush contacts made of steel of composition similar to that of the rail measure the drop across joints through which low-voltage current is circulated from a small motor generator set on the car, consisting of a 7½ horse-power 600-volt compound-wound motor direct connected to a 5-kilowatt 5-volt compound-wound generator and a small exciter. Some of the Herrick cars are equipped with sets, consisting of a 5-horse-power 600-volt compound-wound, 1200 r.p.m. motor, direct connected by flexible insulated coupling to a 2½-kilowatt compound-wound 2- to 6-volt generator, the motor and generator being of open type. The car trucks are insulated from each other, current passing through the rail from one truck to the other, while the bond is under measurement. The apparatus can also automatically mark bad bonds by squirting paint on the rail.

The advantages of this method of testing are as follows: 1. Uniform impressed low-voltage current through joints while testing; 2. Heavy car passing

over joint breaks any transient connection made by fish-plate; 3. Autographic record produced by the bonds themselves of their condition.

Mechanical Strength. — The mechanical adhesion of soldered bonds may be tested by means of a lever as shown in Fig. 10. It may be used as soon as the terminals are cool. The operation of testing consists simply in submitting each bond terminal to a predetermined pull. A properly soldered bond should stand a shearing force of 1200 pounds per square inch of contact. Calling S this shearing force per square inch, A the square inch of contact, and P the pull, as registered on the balance, then

$$P = \frac{ASl}{L}.$$

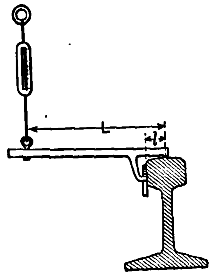

Fig. 10.

REBONDING. — The resistance at which a joint should be rebonded depends upon how much potential drop is permissible in the tracks, and upon the relative cost of the energy loss and the cost of rebonding. The latter, in turn, depends upon the probable life of a new bond. If the energy loss is the primary consideration, the resistance at which rebonding becomes economical is given by the formula below.

Let

R = amount by which the resistance of the existing bond has increased over that of a new bond, ohms,
B = cost of rebonding once, dollars, allowing credit for scrap material,
I = root-mean-square current in the bond, amperes,
C = cost of an increment of energy, dollars per kw. hour,
T = expected life of the bond in days, i.e., the number of days in which its resistance is expected to increase R ohms, or in which it is to be removed for any reason whatsoever. This may be estimated from previous experience on the railway under consideration, the usual limitations being the life of the rails to which they are attached, corrosion by electrolysis, fracture due to crystallization and loosening of the terminals.

Then it is economical to rebond if R is equal to or greater than

$$\frac{B}{0.024\,I^2CT}.$$

It is usual to state the resistance of bonds in terms of the number of feet of rail having the same resistance. Let r be the resistance of the rail in ohms per foot; then the increased resistance R will be represented by an increased length, expressed in feet, equal to $\frac{R}{r}$.

It should also be noted that bonds in different parts of a railway system carry different amounts of current, those nearer the station bus carrying more than those at comparatively remote points. Hence, the economical resistance at which to rebond is less the nearer the bond is to the bus, a consideration that complicates the use of the above formula. Furthermore, as the economical resistance is inversely proportional to the square of the current, a slight error in the estimation of the root-mean-square current will lead to a considerable error in the resistance. Such errors are unavoidable, a circumstance that, taken in conjunction with the probable error in the estimated life, renders the above formula a mere approximation. It is, therefore, usual for railway companies

to select some arbitrary total resistance at which to replace their bonds. In the Proc. Am. Elec. Ry. Assn., 1911, this resistance is given by 22 railways in terms of the equivalent length of continuous rail. This length ranged from 2 to 12 feet, with an average of 6.6 feet.

REPAIRS. — If a soldered bond becomes loose, it may often be resoldered, but defective bonds of other types are usually scrapped, unless the defect is of a very trivial nature. Pin-expanded bonds may, however, be reëxpanded into larger holes, but with some loss in efficiency. Soldered bonds entail a comparatively large and increasing expense for repairs when applied to track rails, in spite of the fact that no failures may occur for several months after installation.

COSTS. — The cost of bonding is extremely variable, depending upon the type and size of bond, cost of labor, etc. The following cost data should therefore be used with caution.

COST OF BONDING

	Cost per joint		
	Labor	**Material**	**Total**
a. 2-500,000 C.M. bonds soldered to head of third rail................	$0.66	$1.23	$1.89
b. 2-500,000 C.M. pin-expanded concealed bonds applied to track rail.	0.69	2.05	2.74
c. 2-400,000 C.M. compressed terminal concealed bonds applied to track rail...............................	....	....	1.90
d. 2-0000 bonds of same type as *c*......	0.50	1.00	1.50

Note. — In *a* and *b* the item for material includes inspection and the labor item includes foreman's salary; solder, acids and tools, not included.

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[W. A. Del Mar.]

BRAKES AND BRAKING SYSTEMS. — (*See also Cars, Electric; Railways, Energy Requirements for.*) In order to stop a car or train a torque must be applied to the wheels in a direction opposite to the direction of motion of the car. This may be accomplished by applying a frictional retarding force to the wheel rims, by applying a retarding force to the axle by means of a magnetically operated friction clutch, by applying a reverse torque to the axles by operating the motors as generators, or by applying a frictional force to the rails directly by a "track brake." The method of applying a retarding frictional force to the wheel rims is the most general one and lends itself most readily to the system of manipulating the brakes by compressed air. This system has done much to improve the safety of travel on railroads. The other methods have all been tried and some have been put into practical operation, but only to meet special local conditions.

FRICTIONAL RESISTANCES IN BRAKE-SHOE SYSTEM. — The application of the usual brake-shoe system makes use of the frictional adhesion between the wheels and the track and between the brake shoes and the wheels. Both these quantities vary throughout a considerable range and it is therefore necessary to adjust the pressure between the various members so that it is possible to rely on a definite minimum value.

Adhesion between Wheel Rim and Rails. — The coefficient of adhesion between the wheel rims and rails varies from less than 15 per cent to over 30 per cent depending upon the condition of the track and the relative motion between the track and wheel rim. (*See Railways, Energy Requirements for.*) An adhesion of 15 per cent can usually be depended upon with normal track, and this can be increased to 25 per cent by the use of sand. But these values only obtain while the wheels are rolling on the track. If they begin to slide the coefficient decreases considerably. For this reason the braking effort must always be controlled so that the wheels do not slip.

Adhesion between Brake Shoe and Wheel Rim. — When the brakes are applied, the retarding force is applied below the center of gravity of the car body. The latter is therefore subjected to a couple and tends to press downward at the forward end and upward at the rear end, thus changing the distribution of weight on the axles and decreasing the adhesion on some axles or trucks. It is therefore not possible to figure on using for braking purposes the same weight per axle as exists at standstill. For this reason the brake-shoe adhesion must be less than the track adhesion. The coefficient of adhesion between the customary cast-iron brake shoe and the steel tire of the wheel varies with the speed, and decreases as the time of application increases. As the speed increases the coefficient drops off, being a maximum of from 30 to 25 per cent at speeds from 0 to 5 mi. per hr., 20 per cent at 20 mi. per hr., 14 per cent at 40 mi. per hr., and 7.5 per cent at 60 mi. per hr. Thus at high speeds a heavy pressure may be applied without stopping the wheels, while as the speed of the car diminishes the pressure on the brake shoes must be decreased in order to prevent gripping the wheels and causing them to slide on the track.

Effect of Angular Momentum. — In addition to overcoming the linear momentum of the cars the brakes must overcome the angular momentum of the gears and motor armatures. The effect of the latter is to introduce a tendency of the whole motor to rotate around the car axle and introduce additional strains on the gears and on the trucks. For this reason brake shoes hung between the wheels of a truck are better than those hung on the outside of the wheels.

HAND BRAKES are always provided on cars and locomotives whether power brakes are employed or not, as they are necessary to hold a car left out of service on a grade, because the air brakes will not hold a car standing idle for any

length of time. When a car is descending a very steep grade it is customary to set the hand brakes to hold the speed of the car and reserve the power brakes for emergency or for stopping the car. In hand braking equipment the "foundation" brakes (*see next paragraph*) are actuated through a drum or lever system which is hand operated.

POWER BRAKES. — The "foundation" brakes are that part of the brake equipment usually furnished separately from the power-braking equipment, and consist of the brake shoes, hangers, equalizers, levers, etc., back to the brake cylinder. To this is attached the desired form or make of power brake. Of the various forms of power brakes in use, viz., air, electric, regenerative and electro-pneumatic, the air-brake is the most generally used.

Air-Brakes. — In electric railway practice there are three systems of air-brakes in use, each of which is best suited to a definite type of service and has its particular field, as follows: (1) straight air-brake system for cars always operated singly; (2) emergency straight air-brake system for cars operated in trains of two or three but never more than three cars; (3) automatic air-brake system for cars operated in trains of any number of cars.

Straight Air-brake System. — The equipment for the straight air-brake system consists essentially of a motor-driven compressor, a reservoir, a brake cylinder, a motorman's valve, a train pipe, and the foundation brakes. The brakes are applied by direct pressure, that is by admitting air from the reservoir directly to the cylinder. The advantages of the system are that it is quick-acting and the braking effort is easily controlled to any value desired. Its disadvantage is that in trains a leak in the train pipe renders the brakes ineffective so that there is no means of applying the brakes on the trail-cars if the train should break apart. Sometimes instead of a compressor the motor-car carries a large reservoir which is charged at stations.

Emergency Straight Air-brake System. — This system involves a special valve and an extra pipe on each car so arranged that if the air pressure in the train pipe drops, due to cars breaking apart, this valve automatically turns the pressure of the reservoir into the brake cylinders and applies the brakes. The equipment of the motor car includes an extra pipe and each trail car has a reservoir, two pipes, automatic valve, brake cylinder and foundation brakes. The usual operation in service is like that of the straight air-brake.

Automatic Air-brake System. — This system involves the use of an auxiliary reservoir and a "triple valve" on each car. Whenever the air pressure in the train pipe is reduced, either intentionally or accidentally, this triple valve turns the air pressure of the auxiliary reservoir on each car into the brake cylinders. The brakes are applied by the motorman by opening the service pipe to the air by means of the engineer's valve. The engineer's valve has three positions, "off," "lap," and "on." By turning the handle to the "on" position the train pipe is opened to the atmosphere and the air continuously escapes. This applies the brakes with a continuously increasing pressure. When the desired pressure has been reached the handle is turned to the "lap" position, when the pressure of air in the train pipe and the pressure of the brakes on the wheels remains constant. By turning the handle to the "off" position pressure is restored in the train pipe, the brakes are released, and the reservoirs recharged.

Quick-action Triple Valve. — In any air-brake system an appreciable time elapses between the action of turning the engineer's valve and the actual application of the brakes. If there are many cars in a train the brakes may be applied on the leading cars sometime before they are applied on the rear cars and the result is that the rear cars bump into the forward cars, and may cause a derailment, particularly if this happens on a curve. To guard against this, the

"quick-action triple valve" is used on long trains. This consists of a special form of valve which causes a local intensification of the change in pressure in the train pipe on each car. The action is somewhat similar to a relay which is affected by slight changes in pressure and causes greater changes, and thus applies or releases the brakes more quickly.

Electromagnetic Brakes. — Any braking system making use of the electric current is not as reliable as the air system since if either the trolley comes off or the motors fail to pick up as generators the brakes are inoperative. Electromagnetic braking may be applied either to the axles or directly to the track.

Disc Brakes. — This system comprises two cast-iron discs mounted concentrically on the car axle. One is keyed to the axle and the other anchored to the truck frames. A coil is placed in one of these discs and when a current flows in the coil the two discs are attracted to each other by the magnetism and the friction between them retards the car. The current for the coils may be taken from the trolley or from the motors operating as generators. In the latter case the motors supply an additional retarding force. The braking effort is controlled by resistance in series with the magnet coils.

Track Brakes. — The principle of electromagnetic braking may also be applied to a track brake in which a magnet coil when energized draws a shoe against the rail, the pressure being regulated by a resistance in series with the magnet coil.

Regenerative Braking. — By supplying the field circuits of the motors with a current of proper value and direction by means of separate excitation they may be operated as generators and made to exert a strong retarding effort on the car axles. This separate excitation may be obtained by connecting the field windings of all the motors on a car in series with themselves and with a regulating resistance to the trolley, or by means of a motor generator set (on a locomotive) supplying a low potential of the proper value. The energy developed by the motors when acting as generators may be dissipated in rheostats or may be returned to the distributing system. In the latter case the voltage generated by the motors must be accurately controlled. Ordinarily, if the energy is returned to the line, the braking effort can only be obtained at speeds in the proximity of full speed. Very special means must be provided to make it possible to return energy to the line at low speeds.

Electro-pneumatic Brakes. — This system has been proposed as an improvement over the quick-acting air-brake system. It involves controlling the application of the brakes on each car by means of an electromagnet receiving current from the locomotive cab. By energizing this circuit the motorman can set the air brakes on all cars practically simultaneously. Each car would be equipped with the usual brake cylinders and reservoir and merely the application of the brakes would be controlled electrically.

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[W. I. SLICHTER.]

BRAUN TUBE. — (*See also Oscillographs.*) A Braun tube is a special form of vacuum tube with accessory coils or condensers which may be used either for obtaining the shape of an alternating current or voltage wave or for obtaining a diagram the area of which is proportional to the power developed per cycle in an alternating-current circuit. A modification of the tube by Ryan (*see Bibliography*) is particularly well adapted to the measurement of small losses such as occur in the dielectrics of condensers, and small lengths of cables, and for measuring the corona loss (*see article on Corona, Electric*) on short lengths of wires.

CONSTRUCTION OF SIMPLE BRAUN TUBE. — The construction of the tube is illustrated in Fig. 1. It consists of a glass vacuum tube having

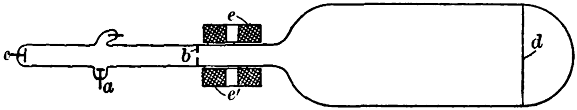

Fig. 1. Simple Braun Tube

electrodes *a* and *c*, a diaphragm *b* with a small hole in it, a thin mica disk *d*, coated with chemicals that fluoresce when struck with cathode rays, and coils *e* and *e'* through which passes the current whose wave shape is to be obtained. With the type of electrode shown in the figure about 20,000 volts between cathode and anode are required to excite the tube. A modified form of electrode, which is maintained at a red heat by an auxiliary current, requires a lower voltage for excitation.

THEORY. — When anode *a* and cathode *c* are connected to a suitable source of *constant* high electromotive force, a stream of cathode particles (electrons) are shot off perpendicularly from the cathode and travel normally in straight lines toward the screen *b*. Only a small pencil of rays passes through the hole in the center of the diaphragm, and this pencil makes a bright spot where it falls on the screen *d*.

The alternating current to be studied is passed through the coils *e* and *e'*, and sets up a magnetic field perpendicular to the stream of cathode particles. This alternating field deflects the ray at each instant by an amount proportional to the instantaneous value of the current, and the spot of light travels back and forth across the screen. By looking at a reflection of the screen in a mirror revolving synchronously with the current through the magnets, the spot of light will appear to trace a wave having the same form as the alternating current which is passed through the coils.

By substituting a revolving film for the revolving mirror and driving the film synchronously with the current through the electromagnets, oscillograms of the alternating currents can be made. The synchronous driving and satisfactory adjustment of mirror or film involve expensive and cumbersome apparatus.

Ryan's Wave Indicator. — To avoid the use of a moving mirror or film Prof. Harris F. Ryan has developed a method whereby the alternating-current wave can be studied directly from the mica screen and a photograph of the screen taken with an ordinary camera. In place of the two coils *e* and *e'* in Fig. 1, four coils at right angles to one another, as shown in sectional view in Fig. 2, are used. The current to be observed is passed through the coils *e* and *e'* and a known sine-wave current is passed through the coils *f* and *f'* at the same time. The spot of light on the screen is then given a motion that is the

resultant of two motions at right angles to each other and proportional to the instantaneous values of their corresponding currents. Owing to the persistence of vision, this results in the production of a closed card upon the screen traced by the spot of light. The wave-form of the current through e and e' may be deduced from an analysis of the photograph or "card."

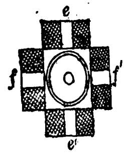

Fig. 2. Sectional View of Coils for Ryan's Wave Indicator

Ryan's Electrostatic Power Indicator. — Professor Ryan has also used the Braun tube as an "electrostatic power diagram indicator," in which the cathode ray is made to trace a diagram that incloses an area proportional to the energy per cycle. For high alternating voltages a condenser is used in place of one set of coils (say ff'); the condenser is so arranged that the electrostatic field, which is proportional to the voltage, deflects the ray in one direction and the magnetic field, which is proportional to the current, deflects it in a direction at right angles to the first deflection.

COSTS. — A Braun-Ryan tube with the accessory coils but without condensers costs about $35.

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[H. Pender and H. R. Ranken.]

BRICKS AND BRICK MASONRY. — (*See also Buildings, Allowable Unit Stresses in; Cement; Concrete.*) Brick construction may be made more ornamental than concrete, but is generally more expensive to place on account of the necessity of handling each brick as an individual unit. The proportion of building and engineering construction in which bricks are used, although still extensive, has been falling off in recent years with the increasing use of plain and reinforced concrete.

Kinds of Bricks. — Ordinary or clay bricks are made by moulding a rectangular block of nearly pure clay, or clay and clean sand, and subsequently burning it in a kiln for from one to two weeks. Sand lime bricks are also used to some extent. They are made from sand cemented by lime and are hardened by being subjected to steam pressure at from 100 to 150 pounds per square inch for about one-half a day. Fire bricks are a special kind of clay bricks, which are made to stand high temperatures. The refractory properties of fire bricks depend chiefly upon the amount of silicon contained. The amount of iron oxide in fire bricks should not exceed 6 per cent. Vitrified bricks are clay bricks which have been annealed by slow cooling.

Physical Properties. — A common, practical test for a good brick is that it will give a clear, ringing sound when struck with a hammer. The color of a brick is not a reliable indication of its strength or properties. The water absorbed in a specified time by a number of bricks is sometimes considered to indicate their relative strengths, as water acts as a lubricant on the material and causes crushing to occur more readily than when dry, but there is considerable difference of opinion as to the value of this test.

The compressive strength of dry bricks ranges from 500 pounds per square inch for soft bricks to over 10,000 pounds per square inch for pressed bricks. The strength of the individual bricks, however, is of little importance except for comparing various kinds, as the strength of brick masonry is usually limited by the strength of the mortar used.

The specific gravity of bricks ranges from 1.6 to 2.6. See also paragraph below on *Brick Masonry* and article on *Weights of Materials*.

Several foreign countries have adopted legal standard sizes for brick, but the only standard sizes in the United States are those specified by the National Brick Mfg. Assoc. as follows:

Common brick...........................	8¼ by 4 by 2¼ inches
Pressed brick...........................	8⅜ by 4 by 2⅜ inches
Paving brick...........................	8½ by 4 by 2½ inches

BRICK MASONRY. — Bricks are laid in lime or cement mortar, the mortar forming a cushion which fills the interstices and keeps out water. Joints, i.e., the spaces between adjacent bricks which are filled with mortar, are usually made from ¼ to ⅜ inch thick in outside walls, and from ⅜ to ½ inch thick in inner walls. In ordinary building work, where the weight of the wall itself is the greater part of the load, lime mortar is usually used because it is cheap. Where strength is required of brickwork, a rich Portland cement mortar should be used; see article on *Cement*. Tests upon the ultimate compressive strength of brick piers are quoted in detail in Johnson's *Materials of Construction*, N. Y., 1910, and Baker's *Masonry Construction*, N. Y., 1910. The values of loads allowed upon brick masonry in various localities range from 100 to 300 pounds per square inch; see *Buildings, Allowable Unit Stresses in*. A brick wall itself weighs from 100 to 145 pounds per cubic foot, depending somewhat upon the quality of the brick and the thickness of joints.

Measurement of Brickwork. — Bricks are usually sold by the thousand. Brickwork is commonly measured by the cubic yard in place, but the units of

per thousand brick and per square yard of surface area are also used. The relation between the number of bricks used and the volume of finished brickwork depends upon the size of brick, thickness of joints and shape of the structure. For solid brick walls the number of bricks per cubic yard ranges from about 400 with ⅝-inch joints to about 500 with ¼-inch joints. An allowance of 3 or 4 per cent excess should be made in estimating to provide for waste and breakage.

Cost of Brickwork.— Hudson River common bricks were quoted at $6.00 per thousand in the *Engineering News*, March 5, 1914. Lime for mortar was quoted at $0.97 to $1.10 per 200-pound barrel. The total cost of brick masonry including labor and materials ranges from $6.00 to $15.00 per cubic yard, depending upon the locality, type of construction and quality of brick and mortar. An average figure for ordinary brick walls is from $8.00 to $10.00 per cubic yard.

[H. F. Thomson.]

BRIDGES FOR ELECTRICAL MEASUREMENTS. — (*See also Inductance and Inductive Reactance; Resistance and Conductance, Electric; Resistors, Standard; Wires and Cables, Insulated.*) Numerous arrangements of electric circuits, known under the general term of "bridges," are used for the comparison of unknown with known resistances, capacities and inductances. The fundamental principle of an electric bridge is the adjustment of the component circuits or arms of the bridge in such a manner that the drop in potential V in the arm formed by the circuit to be tested is to the drop of potential V_r in an arm having known constants as the drop of potential V_a in one "ratio" arm is to the drop V_b in the second "ratio" arm.

A bridge may be made up of separate resistance boxes (*see Resistors, Standard*), but where frequent tests are to be made it is more desirable to have all the resistances, keys, etc., mounted together in a single box. Complete bridges of this kind can be obtained from instrument manufacturers.

SLIDE-WIRE BRIDGE (Fig. 1). — This is the simplest form of bridge but is seldom used in commercial testing, as its range is limited. In principle it is the same as a Wheatstone bridge (*see below*), the difference being a structural one only, in that a wire is used for the "ratio" arms.

The slide wire, AB in Fig. 1, is generally a wire of uniform cross-section stretched along a meter scale which is divided into millimeters. All connections in the X and R circuits are made of heavy low-resistance material. A slider is provided for sliding over and making contact on the wire. When the bridge is balanced (i.e., no current through the galvanometer),

$$X = \frac{AR}{1000 - A},$$

where A is the distance in millimeters between the slider and the left-hand end of the bridge. The slide wire can be made uniform to 1/10 per cent and the error in observation may be even smaller than this. When using a ratio greater than 1 to 1 and when a low resistance is being measured, the errors are greater. It is possible when using an even ratio to make very exact comparisons of the standard and unknown resistances by reversing them and taking the mean of the two measurements.

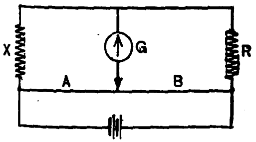

Fig. 1. Slide-wire Bridge

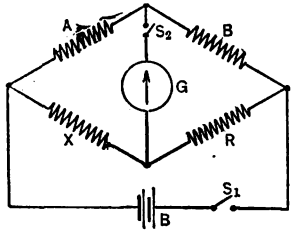

Fig. 2. Wheatstone Bridge

WHEATSTONE BRIDGES. — Fig. 2 shows the arrangement of resistances in the ordinary type of Wheatstone bridge used for comparing resistances. A and B represent two coils or sets of coils, usually referred to as the "ratio" coils, whose relative resistances must be known but whose actual resistance values are unnecessary; R is a standard variable resistance, or resistance box (*see Resistors, Standard*); X is the unknown resistance; G is a sensitive galvanometer connected across two points of the diamond and B is a battery connected across the other two points of the diamond.

For no deflection of the galvanometer when the switches S_1 and S_2 are closed, i.e., when the bridge is balanced,

$$X = \frac{A}{B} \cdot R,$$

where the letters represent the resistances of the four arms.

Construction. — Wheatstone bridges having the various parts mounted in one box may be divided into three classes: (1) portable bridges, in which not only the resistance coils and keys but also the battery and galvanometer are mounted in one box; (2) laboratory bridges, in which the coils and in some cases the keys are mounted in one box, but for which separate galvanometers and batteries must be provided; and (3) precision bridges, arranged in the same manner as laboratory bridges but made with greater care and capable of greater precision.

Since the construction of the resistance coils, blocks, plugs and other minor details of bridges is practically the same as for resistance boxes (*see Resistors, Standard*); only the arrangement of coils and connections will be described here.

Rheostat Arrangements. — The coils forming the rheostat (the variable resistance R in Fig. 2) may be arranged on the 1, 2, 3, 4 plan as shown in Fig. 3, on the 1, 2, 2, 5 plan, on the decade plan of Fig. 4, or on any other plan. (*See Resistors, Standard.*)

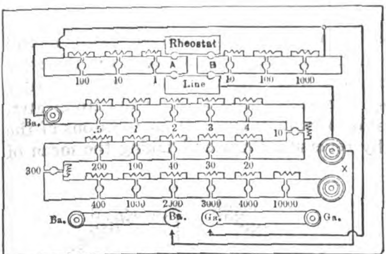

Fig. 3. Wheatstone Bridge (Post Office Type)

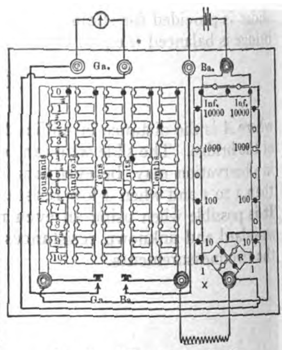

Fig. 4. Wheatstone Bridge (Anthony Type)

The advantage of the decade arrangement is that few plugs are required, and, therefore, there is less likelihood of error due to the contact resistance. A very good form of the decade type of rheostat is used in the Anthony type of Wheatstone bridge, shown in Fig. 4. In this bridge the coils may be joined in series or multiple or in any desired combination of series and multiple.

Plug and Dial Arrangements. — The resistances of the various arms of the bridge may be varied either by the insertion of plugs between the heavy metal terminals of the coils provided on the top of the box, or a dial arrangement similar to that employed on ordinary rheostats (but more carefully made to reduce the contact resistance) may be used.

Ratio Coils (Figs. 5 to 7). — The connections of the ratio arms in a common construction of dial bridge is shown in Fig. 5. The values of the coils are so chosen that the ratio of these coils is always that given by the stamping for various settings of the contact S. An advantage of this type of ratio coils is

that there is no error due to contact resistance in the ratio coils, as the contact is in series with the battery.

The arrangement of an improved form of ratio coils is shown diagrammatically in Fig. 6. This arrangement of ratio coils has the advantage over the old post-office arrangement shown in Fig. 7, in that there are but two plugs to operate and the coils can be checked by reversal.

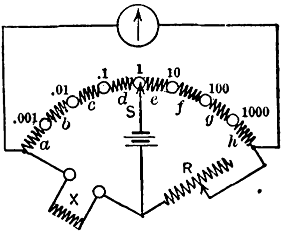

Fig. 5. Dial Type Ratio Coils

Precision of Measurements by Wheatstone Bridge. — The degree of accuracy with which a resistance can be measured by means of a Wheatstone bridge is determined by the following conditions:

1. The accuracy to which the resistances of the various coils is adjusted. The range of accuracy is from about 0.1 per cent for portable bridges to 0.02 per cent for precision bridges.

2. The relative value of the coil resistances and the contact resistances at plugs and terminals. A well-fitting, clean plug has a resistance of from 0.0001 to 0.00005 ohm; a poorly fitting or greasy plug a resistance of 0.01 ohm or higher.

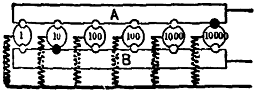

Fig. 6. Improved Arrangement of Ratio Coils

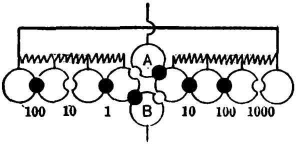

Fig. 7. Post-Office Arrangement of Ratio Coils

The contact resistance at a binding post between the binding post and wire, even when well clamped, ranges from 0.001 to 0.0001 ohm.

3. The relative value of the coil resistances and the insulation resistances between the coils. Dirt and grease on the top of the box may introduce a considerable error. Hard rubber, which is commonly used for the tops of resistance boxes, is also liable to have a thin film of acid formed on it when exposed for a long period to the action of the moisture and impurities in the air, which attack the sulphur in the rubber. Such a film will greatly reduce the insulation resistance between the lugs on top of the box.

4. The effect of changes of temperature in changing the resistances of the coils and in producing thermoelectric effects. The thermal e.m.f. in a copper-brass-copper circuit is about 2×10^{-6} volts per degree C. difference in temperature between the two junctions. Thermoelectric troubles can usually be avoided by reversing the connections of the battery to the bridge and taking the mean of the values of the resistances corresponding to the balance when the battery current flows through the bridge first in one direction and then in the other.

5. The maximum current that the coils will safely carry; this ranges from about 0.5 ampere for 1-ohm coils down to 0.005 ampere for a 5000-ohm coil.

6. The resistances of the galvanometer and battery, particularly the former. The battery resistance is usually small.

7. The relative resistances of the various arms of the bridge.

8. The sensitiveness of the galvanometer, i.e., the deflection per unit current.

Best Galvanometer Resistance. — Referring to Fig. 2, the best galvanometer resistance is

$$G = \frac{(A + X)(B + R)}{A + B + R + X},$$

where the letters designate the resistances of the various branches.

Best Location of Galvanometer. — Knowing the galvanometer and battery resistances (the galvanometer resistance is usually the larger), connect the one having the higher resistance so that it joins the junction of the two arms of the bridge having the highest resistances to the junction of the two arms having the lowest resistances.

Precautions in Making Measurements. — The following rules should be observed in using a Wheatstone bridge:

1. Do not employ a battery having an e.m.f. of over 5 volts; a lower value is desirable if sufficient sensitiveness can be obtained.
2. Always shunt the galvanometer during preliminary adjustments. The shunt circuit should be opened when the final balance is made.
3. See that all binding posts are screwed up tightly and all plugs firmly inserted. After withdrawing a plug those adjacent to it should be retightened.
4. For a preliminary balance use a 1 to 1 ratio.
5. In manipulating the keys be careful not to touch the metal work, as the heat and moisture of the hand is likely to set up appreciable electromotive forces.
6. Always close the battery switch first and then the galvanometer switch, to avoid momentary deflections of the galvanometer due to the transient e.m.f.'s set up while the currents are establishing themselves.
7. If the contact or lead resistances are appreciable relative to the resistances of the coils with which they are in series, these resistances should be separately determined.

Care of Bridge. — In order that a bridge or resistance box may remain in first-class condition, it must be carefully protected from dust and moisture by being covered when it is not in use. To insure high insulation the top must be kept clean, especially between the blocks. The plugs must be kept free from grit, grease or from contact with mercury. Grease may be removed by the use of a little benzol. Never use sand or emery paper for cleaning the plugs or holes, for the surfaces of the taper will be spoiled by this treatment. These materials work into the metal and cannot be removed, thus causing the plugs to "cut." If it becomes absolutely necessary to clean the plugs and sockets, a little of the very finest whiting may be employed. Extra holes of the proper taper bored in the brass blocks are convenient for holding the plugs when they are not in use, and also for attaching a movable terminal. Never apply undue force in inserting the plugs. (*From Laboratory Notes by Prof. F. A. Laws.*)

KELVIN BRIDGE. — The ordinary form of Wheatstone bridge is not suitable for measuring with accuracy a resistance of 0.1 ohm or less, due to the contact resistances introduced at the binding posts, this contact resistance being of the order of 0.0001 to 0.001 ohm. An arrangement of circuits to avoid these contact resistances devised by Lord Kelvin (*Wm. Thomson*) is shown in Fig. 8, and is known as the Kelvin or Thomson bridge. There will be no current in the galvanometer when

$$X = \frac{A}{B} \cdot R + \frac{bd}{a + b + d}\left(\frac{A}{B} - \frac{a}{b}\right).$$

If $\frac{a}{b}$ is made equal to $\frac{A}{B}$, then

$$X = \frac{A}{B} \cdot R.$$

It is also well to make the resistance of d extremely low in comparison with X or R since the lower the resistance of d the less an error in $\frac{a}{b} = \frac{A}{B}$ will affect the accuracy of measurement.

In the above formulas X and R are the resistances between the arrowheads, which represent the "potential terminals." Such terminals are always provided on low-resistance standards (*see Resistors, Standard*).

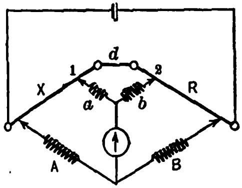

Fig. 8. Kelvin Bridge

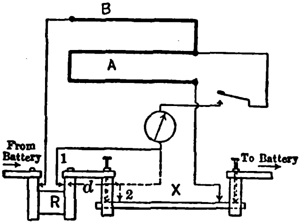

Fig. 9. Modification of the Kelvin Bridge

Simple Modification of the Kelvin Bridge. — When suitable resistances are not available for a and b, these resistances may be omitted and the galvanometer terminal be connected first at 1 and then at 2, and A adjusted for a balance in each case; let A_1 and A_2 be the corresponding values of A. Then

$$X = R\frac{A_1(B + A_2)}{B(B + A_1)}.$$

This method is very convenient for measuring the resistance X of a low-resistance shunt (*see Shunts*). The only standard resistances required are a low-resistance standard R and a resistance box with three terminals (or two separate resistance boxes). Fig. 9 shows such an arrangement.

HOOPES' CONDUCTIVITY BRIDGE. — This bridge is also a modification of the Kelvin bridge, designed for the rapid determination of the relative conductivity (*see Resistance and Conductance*) of samples of wire. It is extensively used in wire factories.

A diagram of the connections is shown in Fig. 10.

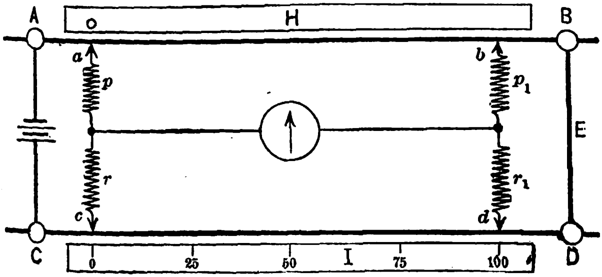

Fig. 10. Hoopes' Conductivity Bridge

The standard A–B and the unknown C–D are of the same metal; consequently if care be taken that they are at the same temperature, all corrections due to temperature are avoided. The arms p, r, p_1, r_1 are in the same case and are

made of material of low-temperature coefficient so that their relative values will not change. They are adjusted so that $p = r$ and $p_1 = r_1$; consequently at balance the resistance of $C-D$ equals the resistance of $A-B$.

The sample $C-D$ is placed alongside a scale I divided into 100 parts, so that the graduations represent percentages of the total length of the scale. Accompanying the standard wire $A-B$ is a scale H, on which are laid off a number of points corresponding to the weights of the standard length (38 inches) of a range of sizes of sample wires.

To make a conductivity reading, the weight of the standard length of the sample $C-D$ is found to within an accuracy of 1/20 per cent. The contact b is set at the point on scale H corresponding to this weight, the contacts a and c being at the zero points of their respective scales. After the case has been closed a sufficient length of time to allow both the standard and sample to assume the same temperature, the contact d is moved until the galvanometer shows no deflection; this will occur when the resistance between a and b is equal to that between c and d. The scale reading corresponding to the position of d for a balance is equal to the per cent conductivity.

The Hoopes' bridge is so designed that the standard wire with its scale is removable from the bridge and so that a single standard covers a range of sizes equal to 3 numbers of B. & S. gauge. Any number of standards can be supplied with a bridge, so that it can cover an extensive range of sizes and can also be used for wires of different materials. In order to keep the standard wire and the test wire at the same temperature the bridge is mounted in a metal-lined case and the scale read through a glass window in the case, the window being closed by a metal screen when readings are not being taken.

COSTS. — A good slide-wire bridge costs about \$20, a portable Wheatstone bridge from \$50 to \$125, depending upon the design, a laboratory Wheatstone bridge from \$25 to \$75 and a high-precision standard bridge from \$200 to \$400. A Hoopes' conductivity bridge with a single standard, covering 3 sizes of B. & S. gauge wire, costs about \$500; additional standards cost about \$50.

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[H. PENDER AND H. R. RANKEN.]

BUILDINGS, ALLOWABLE UNIT STRESSES IN.—(*See also Brick; Cement; Concrete; Iron, Pig and Cast; Iron, Wrought; Steel; Structures, Simple; Timber.*) The following tables give the allowable unit stresses and loads in accordance with the building laws of the respective cities, as corrected to recent dates (*see Bibliography*).

TABLE I.—STEEL AND IRON

Loads in pounds per square inch

	New York	Chicago	Philadelphia	St. Louis	Boston
Compression:					
Rolled steel	16,000	14,000	16,250	F.S. 4	16,000
Cast steel	16,000	14,000	......	F.S. 4	......
Wrought iron	12,000	10,000	12,500	F.S. 4	12,000
Cast iron (in short blocks)	16,000	10,000	11,700	......	16,000
Steel pins and rivets (bearing)	20,000	20,000	......	F.S. 4	18,000
Wrought-iron pins and rivets (bearing)	15,000	......	......	F.S. 4	15,000
Tension:					
Rolled steel	16,000	16,000	16,250	F.S. 4	16,000
Cast steel	16,000	16,000	16,250	F.S. 4	16,000
Wrought iron	12,000	12,000	12,500	F.S. 4	12,000
Cast iron	3,000	......	......	F.S. 8	3,000
Bending (extreme fiber stress):					
Rolled-steel beams	16,000	16,000	16,250*	F.S. 4	16,000
Rolled-steel pins, rivets and bolts	20,000	25,000	......	F.S. 4	22,500
Riveted steel beams (net flange section)	14,000	16,000	16,250*	F.S. 4	......
Riveted wrought-iron beams (net flange section)	12,000	20,000	12,500	F.S. 4	......
Rolled wrought-iron beams	12,000	12,000	12,500	F.S. 4	12,000
Riveted wrought-iron pins	15,000	......	......	......	18,000
Cast iron, compression side	16,000	10,000	17,500	F.S. 8	16,000
Cast iron, tension side	3,000	3,000	3,750	F.S. 8	3,000
Shear:					
Steel, web plates	9,000	10,000	10,000	......	10,000
Steel, shop rivets and pins	10,000	12,000	11,000	......	10,000
Steel, field rivets and pins	8,000	10,000	9,000	......	......
Wrought iron, web plates	6,000	......	7,500	......	9,000
Wrought iron, shop rivets and pins	7,500	......	9,000	......	9,000
Wrought iron, field rivets	6,000	......	7,000	......	......
Cast iron	3,000	......	......	......	......

F.S. = Factor of Safety.
* 14,500 for mild steel.

TABLE II. — TIMBER

Loads in pounds per square inch

	New York	Chicago	Philadelphia	St. Louis	Boston
Compression (short lengths):					
Oak, with grain	900	900	......	......	......
Oak, across grain	800	500	......	......	600
Yellow pine, with grain	1000	1100 *L* 800 *S*	750 *L*	......	......
Yellow pine, across grain	600	250	550 *L*	......	500 *L*
White pine, with grain	800	700	......	......	......
White pine, across grain	400	200	......	......	250
Spruce, with grain	800	......	500	......	......
Spruce, across grain	400	......	300	......	250
Hemlock, with grain	500	500	350	......	......
Hemlock, across grain	500	150	250	......	......
Tension:					
Yellow pine	1200	1300 *L* 1000 *S*	1800 *L*	F.S. 6 F.S. 6	....
White pine	800	800	......	F.S. 6	......
Spruce	800	......	1250	F.S. 6	......
Oak	1000	1200	......	F.S. 6	......
Hemlock	600	600	1000	F.S. 6	......
Bending (extreme fiber stress):					
Yellow pine	1200	1300 *L* 1000 *S*	1600 *L*	F.S. 6	1500 *L*
White pine	800	800	......	F.S. 6	1000
Spruce	800	......	1100	F.S. 6	1000
Oak	1000	1200	......	F.S. 6	1000
Hemlock	600	600	900	F.S. 6	......
Shear:					
Yellow pine, with grain	70	130 *L* 120 *S*	100 *L*	......	100 *L*
Yellow pine, across grain	500	......	1125	......	......
White pine, with grain	40	80	......	......	80
White pine, across grain	250	......	......	......	......
Spruce, with grain	50	......	75	......	80
Spruce, across grain	320	......	750	......	......
Oak, with grain	100	200	......	......	150
Oak, across grain	600	......	......	......	......
Hemlock, with grain	40	60	62	......	......
Hemlock, across grain	275	......	625	......	......

F.S. = Factor of Safety. *L* = Longleaf Yellow Pine. *S* = Shortleaf Yellow Pine.

TABLE III.—STEEL AND IRON COLUMNS

Loads in pounds per square inch*

City	Medium steel	Wrought iron	Cast iron
New York......	$15,200 - 58\frac{L}{R}$ (a)	$14,000 - 80\frac{L}{R}$ (a)	$11,300 - 30\frac{L}{R}$ (b)
Chicago.........	$16,000 - 70\frac{L}{R}$ (a)	$12,000 - 60\frac{L}{R}$	$10,000 - 60\frac{L}{R}$ (b)
Philadelphia....	$\dfrac{16,250}{1+\dfrac{L^2}{11,000R^2}}$	$\dfrac{12,500}{1+\dfrac{L^2}{15,000R^2}}$	$\dfrac{11,700}{1+\dfrac{L^2}{400D^2}}$
St. Louis.......	$S = 12,440$ for $\frac{L}{R} = 40$	....	$S = 8,900$ for $\frac{L}{R} = 10$
	$S = 8,925$ for $\frac{L}{R} = 120$	....	$S = 5,600$ for $\frac{L}{R} = 25$
Boston.........	$\dfrac{16,000}{1+\dfrac{L^2}{20,000R^2}}$	$\dfrac{12,000}{1+\dfrac{L^2}{20,000R^2}}$	$S = 11,000$ for $\frac{L}{R} = 10$
			$S = 9,200$ for $\frac{L}{R} = 70$

* All values obtained either by formulas or from tables shall be reduced for eccentric loading.

L = unsupported length in inches.
D = diam. or least side in inches.
R = least radius of gyration in inches.
S = stress in lb. per sq. in.

(a) $\frac{L}{R} > 120$ not allowed.
(b) $\frac{L}{R} > 70$ not allowed.

TABLE IV.—TIMBER COLUMNS

Loads in pounds per square inch

City	Yellow pine (b)	White pine	Oak
New York.......	$S = 820$ for $\frac{L}{D} = 10$	$S = 650$ for $\frac{L}{D} = 10$	$S = 730$ for $\frac{L}{D} = 10$
	$S = 460$ for $\frac{L}{D} = 30$	$S = 350$ for $\frac{L}{D} = 30$	$S = 390$ for $\frac{L}{D} = 30$
Chicago..........	$1100\left(1 - \frac{L}{80D}\right)$	$700\left(1 - \frac{L}{80D}\right)$	$900\left(1 - \frac{L}{80D}\right)$
Philadelphia......	$750\left(1 - \frac{L}{100D}\right)$	....	....
Boston (a).......	$S = 900$ for $\frac{L}{D} = 10$	$S = 630$ for $\frac{L}{D} = 10$	$S = 810$ for $\frac{L}{D} = 10$
	$S = 700$ for $\frac{L}{D} = 30$	$S = 490$ for $\frac{L}{D} = 30$	$S = 630$ for $\frac{L}{D} = 30$

L = unsupported length in inches.
D = diam. or least side in inches.
S = stress in lb. per sq. in.

(a) $\frac{L}{D} > 30$ not allowed.
(b) Longleaf.

TABLE V.—LIVE LOADS ON FLOORS AND ROOFS

Pounds per square foot

Kind of building	New York	Chicago	Philadelphia	St. Louis	Boston
Dwellings, hotels	60	40 to 50	70	60	50 to 100
Office buildings:					
First floor	150	50	100	150	100
Above first floor	75	50	100	70	100
Schools	75	75 to 100	....	100	60 to 125
Buildings for public assembly	90	100	120	100	200
Stores	120	100	120	150	125
Factories	150	100	150	150	250
Roofs:					
Pitch < 20 degrees	50	25 (*a*)	30	40 (*b*)	40 (*b*)
Pitch > 20 degrees	30 (*a*)	25 (*a*)	30	....	....

(*a*) Measured in horizontal plane. (*b*) Flat.

TABLE VI.—MASONRY AND BUILDING MATERIALS

Loads (compression) in pounds per square inch

Item	New York	Chicago	Philadelphia	St. Louis	Boston
Concrete (P), 1 : 2 : 4	230	400 M 350 H	208 (*b*)	250 (*d*)	417
Concrete (P), 1 : 2½ : 5 (*f*)	208	350 M 300 H	....	....	....
Rubble stonework:					
Portland cement mortar	140	100 NC	139 (*b*)	......	......
Lime mortar	70	60 NC	70	......	......
Brickwork:					
Portland cement mortar, 1 : 3	250	250 (*a*)	209 (*b*)	300 (*c*)	278
Lime mortar, 1 : 4	111	100	111 (*c*)	120 (*c*)	111 (*e*)
Granite (according to test)	1000–2400	......	......	......	834
Limestone (according to test)	700–2300	......	......	......	556
Marble (according to test)	600–1200	......	......	......	556
Sandstone (according to test)	400–1600	......	......	......	417

P = Portland cement.
M = Machine mixed.
H = Hand mixed.
NC = Not coursed.

a = Pressed.
b = Kind of concrete not specified.
c = Mixture not specified.
d = Mixture not leaner than 1 : 3 : 5.
e = Mixture 1 : 6.
f = Mixture 1 : 2 : 5 for Chicago.

TABLE VII.—BEARING CAPACITY OF SOILS

Tons per square foot

Item	New York	Chicago (a)	Philadelphia	St. Louis
Clay, soft	1	....	...	According to test, maximum not to exceed 3.
Clay, hard	4	....	3.5	
Clay, dry	3	1.75 (a)	3.5	
Sand, firm and coarse	4	2.5 (a)	3.5	
Sand, fine and dry, firm	3	2.5 (a)	...	
Clay and sand, wet	2	1.5	...	
Clay and gravel, well cemented	..	....	6	
Loam	3	....	...	

(a) In beds at least 15 feet thick.

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[C. M. SPOFFORD.]

BUS-BARS AND BUS-BAR STRUCTURES. — (*See also Circuit Breakers; Power Stations; Substations; Switches; Switchgear Equipment for Power Stations; Wires and Cables.*) Bus-bars, or "omnibus bars," as they were originally called, are the common circuits into which the various generators deliver their output and from which the different feeders draw their supply of power. In large-capacity, moderate-voltage a-c. plants the bus-bars are usually placed in structures and connected to the various generators, feeders, etc., by suitable wiring. The bus-bars and their connections, together with the bus-bar structures, form one of the most important parts of large plants, as the entire energy of the station is usually concentrated thereon.

BUS-BAR SYSTEMS. — Where there is only a single set of bus-bars either in d-c. or a-c. stations the connections are said to be arranged on the "single-throw" system; when the connections can be made to either of two sets of bus-bars the system is spoken of as "double-throw"; but if the connections can be made to both sets of bus-bars instead of only to either set, the system is spoken of as the "selector system." Occasionally three or more sets of bus-bars are used.

If there is only one set of bus-bars, but with switches provided for dividing it into one or more sections, it is spoken of as a "sectioned bus." Where there are two sets of these sectioned bus-bars connected together at the ends, the system forms a "ring bus." In many high-voltage plants having step-up transformers each generator normally connects to the low-tension side of its own transformer but switches are provided so that any transformer or generator can connect to a bus; such a bus is spoken of as a "relay bus." Where a number of feeders connect to a bus which in turn connects to the main bus through a switch or breaker, such a bus is spoken of as a "group bus."

Uses of Various Systems. — The various systems — single bus, double bus, relay bus, group bus, etc., — all have their advantages and disadvantages. The single bus is naturally the cheapest, simplest and least flexible, and trouble on the bus is apt to shut down the plant. The other systems are more flexible and also more expensive as they require more apparatus. In every installation a compromise must be effected between cost and flexibility, and each case must be considered on its own merits. In small low-voltage plants bus-bar trouble is almost unknown and a single-throw system is usually employed. In high-voltage, large-capacity plants, although bus-bar trouble is rare, a more flexible system than the single-throw is often advisable.

BUS-BAR MATERIAL. — Depending on the current and voltage, bus-bars may be made of wire, rod, tubing, cable or strap, either bare or insulated. Solid wire is seldom used for more than 200 amperes, rod is used for less than 1000 amperes, tubing for 300 to 600 amperes, cable up to 1000 amperes, while strap is used up to any capacity.

Strap Bars for Heavy Currents. — Strap for bus-bars, particularly for heavy currents, possesses several advantages over other shapes, the chief ones being the ease with which additional straps may be added and the excellent radiating surface secured. Straps are made in various sections, a typical one being 3 inches by 1/8 inch. Where more than one strap is required a space of 1/8 inch is kept between bars, making the so-called laminated bus. The connections from switches, circuit breakers, etc., to the bus are made of one or more similar straps suitably interleaved and clamped together. For very heavy currents straps of larger section are used.

Tubing for Small Currents and High Voltages. — For extremely high voltages with their correspondingly small current, copper tubing for bus-bars and

connections has many advantages over rods, wire or strap. These advantages are principally increased stiffness for the same amount of material, large and effective radiating surface, and the facility of making connections by flattening out the tubing at the point desired and bolting the tubing together at such points. On extremely high-voltage circuits tubing of approximately 1 inch outside diameter is not apt to be troubled by the brush discharge or corona effect that is sometimes noted with small wires or straps having sharp edges. In some plants iron piping is used for bus-bars and connections.

BUS-BAR STRUCTURES. — (*See also Power Stations; Substations; Switchgear.*) In large-capacity a-c. plants of 13,000 volts or less, with generators connected directly to the bus, the current that can be developed on a short-circuit is something enormous, and every precaution has to be taken to prevent trouble from spreading if it ever starts. For this reason it has become customary to employ masonry compartments and cellular construction for the oil circuit breakers and bus-bars. In higher-voltage plants open wiring possesses several decided advantages. The vertical walls and septums of the circuit-breaker and bus-bar structures are usually built of brick or concrete, and the horizontal shelves between the bus-bars are ordinarily made of concrete, sandstone, soapstone, slate or marble. These substances are named in the order of their increasing cost. In some instances the bus-bar structures have been made of asbestos lumber, transite or similar material.

Marble is undoubtedly the best material as far as insulation and absorption qualities go, but its high cost and its liability to crumble when exposed to a bad arc has caused the adoption of cheaper materials of slightly poorer insulating qualities. Slate, the next material tried, is a very uncertain insulator for high-voltage work and it has been generally superseded by soapstone, sandstone or concrete. Where space is at a premium, soapstone is used almost exclusively, as it can be drilled, machined, etc., and smaller clearance distances can be used than would be permissible with sandstone or concrete. Where there is a chance to secure a reasonable distance between bare metal parts and the shelves or barriers, concrete, either plain or reinforced, can be used to advantage.

Supports for Bus-Bars. — Low-tension bus-bars, when not too heavy, can be supported by the wall bushing for the lead. For heavier work, or where bushings are not used, the bus-bars are supported on porcelain pillars, petticoat insulators or similar devices resting on the bus-bar shelf or attached to the wall.

For supports for high-tension bus-bars and connections it is customary to employ line insulators either of the pillar type, pin type, or suspension type, depending on the voltage.

OPEN WIRING FOR BUS-BARS. — Some engineers are of the opinion that the cellular construction should be used for large-capacity circuits of any voltage, and bottom-connected breakers have been designed that work well with the inclosed bus-bar construction for high-voltage plants.

The writer is of the opinion that the open system of wiring is preferable for any voltage higher than that for which generators can be conveniently wound. This opinion is based on the following reasons:

1. For the same kilovolt-ampere capacity back of an arc the current established is approximately inversely proportional to the voltage, and consequently the violence of the arc and its destructive effects are less on a high-voltage than on a low-voltage system.

2. The distance from wire to ground has to be greatly reduced from what could be obtained with open wiring in the same space, as the conductivity of the

fireproof barriers is sufficiently good to permit large currents to flow with high voltages, in case an arc or a dead ground is established.

3. A more expensive building and more costly construction are usually needed for inclosed bus-bars and wiring than are required for open wiring.

4. Inspection and repairs are more difficult when bus-bars, wiring, disconnecting switches and similar appliances are boxed in masonry compartments, and the conductors are visible and accessible only by the removal of doors. Inspection will be more frequent and thorough and incipient trouble will be noticed far sooner with open wiring than with inclosed, as the station attendant can see everything in a walk of a few minutes, and will not have to remove many doors and visit two or three floors to examine the condition of the apparatus.

CONNECTIONS TO BUS-BARS. — Where the currents exceed 600 or 800 amperes, it is usual to employ laminated copper straps for connecting to bus-bars; for smaller currents, cable, wire, rod or tubing is used. Cable and, to a certain extent, bare wire are used for connections involving bends or long runs through conduits, but for straight runs or simple bends rod or tubing can be used. Tubing, though more costly than rod or wire, is stiffer for the same section, and can often be flattened out for making connections to studs, bars, etc., without the necessity of additional terminals.

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[S. Q. Hayes.]

CALORIMETERS, FUEL. — (*See also Fuel.*) A fuel calorimeter is an instrument for determining the heating value of a fuel. Its essential features are a closed chamber in which a weighed sample of the fuel can be quickly and completely burned, a vessel of water surrounding this chamber, into which all the heat generated by the combustion is transferred, a delicate thermometer for measuring the rise of temperature of the water, means for igniting the fuel, and provisions for preventing loss of heat from the apparatus by radiation or by the escape of the gases and vapors produced by the combustion. The most approved form of the instrument is Mahler's modification of Berthelot's calorimeter. The combustion chamber is a strong cylindrical steel vessel enameled on the inside, called a "bomb," into which about 1 gram of powdered coal, contained in a small platinum dish, is placed. Oxygen under pressure of 20 to 25 atmospheres is introduced, and the coal is ignited by an electric spark and burned explosively. The bomb is set in a water pail of thin brass, which is heavily felted and surrounded by a double-walled vessel filled with water of the temperature of the room. A stirring apparatus is used in the pail to circulate the water around the bomb. The thermometer is finely graduated. Readings of the temperature are made and recorded every minute until the maximum is reached, and a few minutes afterward to obtain a correction for radiation. The weight of the water, together with the water equivalent of the bomb and pail, multiplied by the rise in temperature, corrected for radiation and other minor errors, gives the number of heat units generated by the combustion of the coal.

The Junker calorimeter is commonly used to determine the heating value of fuel gas. A measured volume of gas is burned with air or oxygen in a vessel which is surrounded with water, and the calculations are made in the same way as those for the Mahler calorimeter.

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[WM. KENT.]

CALORIMETERS, STEAM. — (*See also Steam.*) For the purpose of determining the percentage of moisture in the steam, in a boiler or engine test, a steam calorimeter is used. Several forms of this instrument have been used but the most common is that of Professor Peabody, known as the throttling calorimeter. The action of this instrument depends upon the fact that the heat of saturated steam is greater the greater the pressure, and consequently if the pressure is reduced by throttling, the heat rendered available will convert the moisture into steam and in general produce more or less superheating.

A ½-inch pipe, closed at the end and perforated with several ⅛-inch holes in its walls, is inserted into the main steam pipe so that steam may enter these holes. The other end of the calorimeter pipe is throttled by an orifice 1/16 inch diameter through which the steam escapes into a chamber which has an outlet to the atmosphere. The temperature and pressure of the steam on each side of the orifice are observed. The steam in the chamber is superheated more or less, according to the amount of moisture contained in the sample drawn from the steam main.

The per cent of moisture in the steam is then

$$W = 100 \frac{H - h - K(T - t)}{L},$$

where H = total heat and L = the latent heat of saturated steam at the pressure of the steam in the main pipe; h = total heat of saturated steam at the pressure in the discharge chamber of the calorimeter (= 1150.4, corresponding to a pressure of 14.7, when this chamber opens directly to the atmosphere); K = specific heat of superheated steam (= 0.48 approximately); T = actual temperature in the discharge chamber; t = temperature of saturated steam at the pressure in the discharge chamber (= 212 when this chamber opens directly to the atmosphere). The above formula becomes

$$W = 100 \frac{H - 1150.4 - 0.48(T - 212)}{L},$$

when the discharge chamber opens directly to the atmosphere, and when the atmospheric pressure is 14.7 pounds per square inch.

When the steam is very moist, so as to reduce the superheating on the discharge side to 0°, the instrument fails, and a separating calorimeter, which is simply a small steam separator (q.v.), must be used between the throttling calorimeter and the steam pipe to collect the greater quantity of moisture. The moisture collected in the separator is then added to that determined by the calorimeter. The instrument must be thoroughly felted to reduce the error due to radiation. There is also usually a considerable error in obtaining steam of an average quality from the main pipe by the perforated tube.

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[Wm. Kent.]

CAMBRIC, VARNISHED. — (*See also Insulating Materials; Wires and Cables, Insulated.*) Varnished muslin, variously known as varnished cloth or varnished cambric, is an insulating material which is superseding both rubber (q.v.) and impregnated paper (q.v.) for many purposes. It consists of strips of cotton fabric coated with insulating varnish, wound helically around the conductor with a thin layer of plastic non-hardening compound between turns. This compound prevents the absorption of moisture, precludes air spaces and permits the layers of fabric to slide upon each other when the cable is bent.

The muslin is prepared by coating it with a mixture whose principal constituents are boiled linseed oil, resin and benzine. This mixture dries and the oil oxidizes in contact with the air leaving a hard smooth surface. Several coats are thus applied until the desired thickness is obtained. The plastic material between layers is a mixture of petrolatum or crude vaseline and resin. Different manufacturers use other constituents, with considerable improvement in the quality of the insulation. The exact nature of the impregnating compounds is kept secret.

SPECIFIC RESISTANCE. — The value of K in the formula $M = K \log \frac{D}{d}$ (*see article on Rubber*) ranges from $K = 500$ to $K = 4000$, the usual figure being between 700 and 1200, and anything over 2000 exceptional.

Temperature Coefficient of Resistance. — The resistance of varnished cloth is more affected by temperature changes than that of rubber or impregnated paper. The amount of variation differs in different makes, so that the accompanying table should be considered as an approximation only.

Temperature, ° F.	Per cent of resistance at 60° F.
60	100
65	61.4
70	36.8
75	23.8
80	14.9
85	10.5
90	7.90
95	6.14
100	5.26

DIELECTRIC STRENGTH. — The varnished fabric is usually 12 or 15 mils thick, and will stand a puncture test of about one kilovolt for five minutes. A 250,000 circular mil cable, insulated with 3/8 inch of varnished cambric, when subjected to a gradually increasing potential, punctured at 115 to 118 kilovolts. A well-designed varnished cloth cable can be subjected to a one-half hour test of double the working voltage and to an instantaneous voltage of three times the working voltage. Varnished-cloth cables seem to have the property of withstanding comparatively high instantaneous rises of voltage such as occur at the time of surges (*see Transmission Lines*), and hence there is no necessity of subjecting them to prolonged tests of two and one-half or three times the working voltages, which have a tendency to overheat the insulation. (*H. W. Fisher.*)

SPECIFIC INDUCTIVE CAPACITY. — Varnished cloth is made with specific inductive capacities from 4 to 6. The former figure is only occasionally attained, the average being about 5. A maximum of 6 may be relied upon, as it allows a slight margin over the ordinary variations of manufacture.

SPECIFICATIONS. — See article on *Wires and Cables, Insulated.*

BIBLIOGRAPHY. — Fisher, H. W., *Varnished Cloth Cables for Power Houses and Distributing Stations*, El. Journ., April, 1906; see also the bibliography in the article on *Rubber.*

[W. A. Del Mar.]

CAPACITY AND CHARGING CURRENTS. — (*See also Alternating Currents; Condensers, Electric; Insulating Materials, Testing of; Transient Electric Phenomena and Oscillations; Transmission Lines; Wires and Cables, Insulated.*) In the section on *Capacity and Condensers*, in the article on *Electricity and Magnetism, Principles of*, are given the formulas for capacities in series and in parallel and for the energy stored in a charged condenser. For units and their interrelations see the articles *Units, Practical Electrical*, and *Units and Conversion Factors*. Commercial forms of condensers are described in the article on *Condensers, Electric*. The following is a brief table of contents of this article.

GENERAL RELATIONS AND DEFINITIONS. — Consider any number of conductors 0, 1, 2, 3, etc., either (1) at a great distance from all other conductors or (2) completely surrounded by a hollow conducting shell, the inside surface of which shell is to be considered as one of the conductors, say No. 0, of the system. The electrostatic condition of such a system of conductors is uninfluenced by any electrostatic effects produced outside the system; it may therefore be called an "electrostatically independent system."

Potential Coefficients (A). — Any conductor of an electrostatically independent system may be chosen as a conductor of reference; let this reference conductor be designated as conductor No. 0. Let v_{10}, v_{20}, v_{30}, etc., represent the potential drop from No. 1 to No. 0, from No. 2 to No. 0, from No. 3 to No. 0, etc., and let q_0, q_1, q_2, q_3, etc., represent the charges on No. 0, No. 1, No. 2, No. 3, etc. Then, if the relative positions of the various conductors and insulators in the field remain unaltered and the specific inductive capacities (*see Electricity and Magnetism, Principles of*) of the various insulating materials between the conductors are *constant* (not necessarily the same for each insulating material, however), the following relations hold for all values of the charges on and potential drops between conductors irrespective of how the conductors may be connected: *

$$\left.\begin{aligned} v_{10} &= A_{11}q_1 + A_{12}q_2 + A_{13}q_3 + \text{etc.}, \\ v_{20} &= A_{12}q_1 + A_{22}q_2 + A_{23}q_3 + \text{etc.}, \\ v_{30} &= A_{13}q_1 + A_{23}q_2 + A_{33}q_3 + \text{etc.}, \\ q_0 &= -(q_1 + q_2 + q_3 + \text{etc.}), \end{aligned}\right\} \tag{1}$$

where the A's are all *constants* depending upon the distances apart of the con-

* By wires of small cross-section and length compared with the dimensions of the conductors.

ductors and the nature of the insulating medium between them. The coefficients A in these equations may be called the "potential coefficients" of the system of conductors. It should be noted particularly that *these coefficients are independent of how the conductors may be charged and of how they may be interconnected* (provided the connecting wires are small compared with surfaces of the conductors). In certain simple cases these coefficients A are readily calculated; see below.

Electrostatic Induction Coefficients (B). — The above equations may also be written

$$\left.\begin{aligned} q_1 &= B_{11}v_{10} + B_{12}v_{20} + B_{13}v_{30} + \text{etc.}, \\ q_2 &= B_{12}v_{10} + B_{22}v_{20} + B_{23}v_{30} + \text{etc.}, \\ q_3 &= B_{13}v_{10} + B_{23}v_{20} + B_{33}v_{30} + \text{etc.}, \end{aligned}\right\} \qquad (2)$$

etc.

$$q_0 = -(q_1 + q_2 + q_3 + \text{etc.}),$$

where the B's are also constants and may be expressed directly in terms of the potential coefficients A by solving equations (1) for q_1, q_2, q_3, etc. The constants B are called the "electrostatic induction coefficients," and like the constants A are independent of how the conductors may be charged and of how they may be interconnected. The B's may be expressed directly in terms of the normal and grounded capacities of the various conductors; see below.

Normal Capacity of Two Conductors (C). — By the normal capacity between any two conductors is meant the capacity of the condenser formed by these two conductors when all the other conductors are connected to one another and to the conductor of reference. The normal capacity between any two conductors of a system, say Nos. 1 and 2, is, then, from equation (2),

$$C_{12} = \frac{B_{11}B_{22} - B_{12}^2}{B_{11} + B_{22} + 2\,B_{12}}. \qquad (2a)$$

When the arrangement of the conductors is perfectly symmetrical (as in a three-conductor cable), $B_{11} = B_{22}$ and the normal capacity between 1 and 2 is

$$C_{12} = \frac{1}{2}(B_{11} - B_{12}). \qquad (2b)$$

Grounded Capacity (C_g). — By the grounded capacity of any conductor of a system is meant the capacity of the condenser formed by this conductor as one "plate" and all the other conductors, including the conductor of reference, connected together as the other plate.* The grounded capacity of conductor No. 1, say, is then, from equation (2),

$$C_{1g} = B_{11}. \qquad (2c)$$

That is, the "electrostatic coefficient of self-induction" of any given conductor is the same as the grounded capacity of this conductor.

Capacity to Neutral (C_0). — In calculating the charging current, voltage drops, etc., in a single-phase or balanced three-phase transmission line it is sometimes convenient to consider the actual capacity between wires as made up of two capacities in series, each of twice the actual capacity between wires. This double capacity, viz.,

$$C_0 = 2\,C_{12}, \qquad (2d)$$

is called the capacity to neutral, since this capacity multiplied by the voltage to neutral, in either a single-phase or balanced three-phase line, gives the charge per wire, which charge is also in phase with the voltage to neutral. This prod-

* The term "grounded" arises from the fact that the conductor of reference is usually the ground.

uct, however, does not give the charge per wire when the system is unbalanced; the general equations (2) must then be used.

Charging Current and Capacity Susceptance. — The charging currents taken by the various conductors of a system are found by differentiating equations (2) with respect to time; this gives the instantaneous values of the charging currents. In the case of sine-wave voltages all of frequency f, the effective values and phase relations of the charging currents in each conductor, in terms of the voltage drops to the conductor of reference, *all expressed in vector notation* (*see Alternating Currents*), are as follows:

$$\left.\begin{aligned} I_1 &= j\,2\pi f\,(B_{11}V_{10} + B_{12}V_{20} + B_{13}V_{30} + \ldots\ldots\ldots\ldots\ldots) \\ I_2 &= j\,2\pi f\,(B_{12}V_{10} + B_{22}V_{20} + B_{23}V_{30} + \ldots\ldots\ldots\ldots\ldots) \\ I_3 &= j\,2\pi f\,(B_{13}V_{10} + B_{23}V_{20} + B_{33}V_{30} + \ldots\ldots\ldots\ldots\ldots) \\ &\text{etc.,} \\ I_0 &= -\,(I_1 + I_2 + I_3 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots) \end{aligned}\right\} \quad (3)$$

Note that the quantities in the brackets are to be added *vectorially*.

In the case of a system of but two conductors, i.e., a simple condenser, these relations reduce to

$$I = j\,2\pi fCV, \quad (3a)$$

when V is the voltage drop through the condenser, I the current in the direction of this drop, and C the capacity of the condenser. That is, the charging current of a simple condenser leads the voltage drop by 90° and is equal numerically to the product of this voltage by $2\pi fC$. The factor

$$b = 2\pi fC \quad (3b)$$

is called the capacity susceptance * of the condenser. Capacity susceptance is expressed in mhos or micromhos, being of the same dimensions as conductance. Numerically, the charging current of a simple condenser may be then expressed as

$$I = bV. \quad (3c)$$

When V is in volts and b in mhos, the current I is in amperes: For either a single-phase or balanced three-phase line the charging current per wire may also be expressed as $I = b_0V_0$, where $b_0 = 2\pi fC_0 = 2\,b$, and may be called the "capacity susceptance to neutral." When b_0 is in micromhos (see tables below) and V_0 is in volts to neutral, the charging current per wire is

$$I = 10^{-6}b_0V_0. \quad (3d)$$

In a single-phase line $V_0 = V/2$ and in a balanced three-phase line $V_0 = V/\sqrt{3}$, when V in each case is the voltage between wires.

FORMULAS FOR THE CAPACITY OF SIMPLE CONDENSERS. — Let

K = specific inductive capacity of medium between conductors; medium assumed uniform; for air $K = 1$,
C = capacity of the condenser formed by the two conductors,
$C_0 = 2\,C$ = capacity to neutral.

Two Concentric Spheres. — Let r' = internal radius of outer sphere and r = external radius of inner sphere, both in centimeters, and R' and R the corresponding dimensions in inches.

$$C = \frac{Krr'}{r' - r} \quad \text{statfarads,}$$

$$= 2.822 \times 10^{-6}\frac{KRR'}{R' - R} \quad \text{microfarads.}$$

* The inductive susceptance of a condenser is $-2\pi fC$; see *Alternating Currents*.

Two Parallel Plates. — Unless the plates are large compared with their distance apart no simple formula can be deduced, since the electrostatic field at the edge of the plates is not uniform. Let $S =$ surface, in square inches, of contact between the metal plate and dielectric (the dielectric sheet is usually larger than the metal sheet, hence S is generally the surface, one side only, of the metal plate); $D =$ thickness of dielectric in inches, $K =$ specific inductive capacity, and let s and d be the dimensions in centimeters corresponding to S and D respectively. Then for D small compared with S, the capacity of two parallel plates is

$$C = \frac{Ks}{4\pi d} \quad \text{statfarads}$$
$$= 2.246 \times 10^{-7} \frac{KS}{D} \quad \text{microfarads.}$$

Stack of Plates. — Let $N =$ the number of *metal* plates in the stack; there will then be $N - 1$ effective dielectric sheets or $N - 1$ parallel plate condensers in series. Using the same notation as above, the capacity of a stack of N metal plates, *connected in series*, is

$$C = \frac{Ks}{4\pi d\,(N-1)} \quad \text{statfarads}$$
$$= 2.246 \times 10^{-7} \frac{KS}{D\,(N-1)} \quad \text{microfarads.}$$

Round Wire in Concentric* Sheath. — Dimensions as in Fig. 1; since the *ratio* only is involved, it is immaterial what units are used for D and d provided *both* are expressed in the *same* unit. For a length of cable long compared with its diameter,

$$C = \frac{K}{2 \log_\epsilon \frac{D}{d}} \quad \text{statfarads per centimeter}$$

$$= \frac{7.354 \times 10^{-3} K}{\log_{10} \frac{D}{d}} \quad \text{microfarads per 1000 ft.}$$

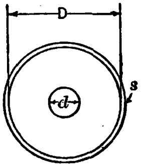

Fig. 1.

Two Parallel Round† Wires. — Dimensions as in Fig. 2; since the *ratio* only is involved, it is immaterial what units are used for D and d provided *both* are expressed in the *same* unit. For a length of line large compared with the distance apart of the wires, the exact formula ‡ for the capacity *between wires* is

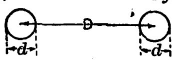

Fig. 2.

* When the wire is off center by a distance m (center of wire to center of sheath),

$$C = \frac{K}{2 \cosh^{-1}\alpha} \quad \text{statfarads per centimeter}$$

$$= \frac{16.93 \times 10^{-3} K}{\cosh^{-1}\alpha} \quad \text{microfarads per 1000 ft.,}$$

where $$\alpha = \frac{D^2 + d^2 - 4\,m^2}{2\,Dd}.$$

† When the wires are far apart compared with the linear dimensions of their cross-section, the second group of formulas also applies approximately to wires of any shape of cross-section provided d is taken equal to the perimeter of the cross-section divided by π, i.e., equal to the "equivalent" diameter of the cross-section.

‡ Taking into account the non-uniform distribution of the charge on each wire; see *Pender and Osborne, Elec. World, 1910, Vol. 56, p. 667.*

$$C = \frac{K}{4 \cosh^{-1} \dfrac{D}{d}} \quad \text{statfarads per centimeter}$$

$$= \frac{8.467 \times 10^{-3} K}{\cosh^{-1} \dfrac{D}{d}} \quad \text{microfarads per 1000 feet.}$$

When D is greater than 10 d the following formulas for the capacity *between wires* may be used instead of the above with an error of less than 0.1 per cent:

$$\left.\begin{aligned} C &= \frac{K}{4 \log_{\epsilon} \dfrac{2D}{d}} && \text{statfarads per centimeter} \\ &= \frac{3.677 \times 10^{-3} K}{\log_{10} \dfrac{2D}{d}} && \text{microfarads per 1000 feet} \\ &= \frac{19.41 \times 10^{-3} K}{\log_{10} \dfrac{2D}{d}} && \text{microfarads per mile.} \end{aligned}\right\} \quad (4)$$

The *capacity to neutral* in all cases is $C_0 = 2\,C$. Tables of capacity to neutral for various sizes of wires and various spacings, when separated by air ($K = 1$), are given in the tables below. Note that these tables and the above formulas are strictly applicable to ordinary overhead lines only when the distance from the wires to other conductors, particularly the earth, is large compared with their distance apart. However, the effect of the earth is usually small in most practical cases (see below), and the formulas and tables give a very fair approximation to the actual capacities.

The capacities of standard strands given in the following tables are calculated by the same formula as for smooth round wires using for the diameter d the diameter of the strand; see *Wires and Cables, Bare*. The values as thus calculated are therefore not exact, but the error is probably less than 3 per cent for all practical cases.

CAPACITY TO NEUTRAL* OF SMOOTH ROUND WIRES

Microfarads per 1000 FEET of each wire of a single-phase or of a symmetrical three-phase line

Size of wire, A.W.G.	Diam. of wire, inches	Inches between wires, center to center							
		1	3	6	9	12	18	24	30
0000	0.4600	0.01199	0.006608	0.005192	0.004618	0.004282	0.003884	0.003643	0.003477
000	0.4096	0.01099	0.006317	0.005013	0.004477	0.004161	0.003783	0.003555	0.003396
00	0.3648	0.01016	0.006055	0.004847	0.004344	0.004045	0.003688	0.003470	0.003319
0	0.3249	0.009458	0.005812	0.004692	0.004218	0.003936	0.003597	0.003390	0.003245
1	0.2893	0.008855	0.005587	0.004546	0.004100	0.003833	0.003511	0.003313	0.003174
2	0.2576	0.008332	0.005381	0.004408	0.003988	0.003735	0.003428	0.003239	0.003107
4	0.2043	0.007455	0.005010	0.004157	0.003781	0.003553	0.003274	0.003102	0.002980
6	0.1620	0.006753	0.004688	0.003933	0.003595	0.003388	0.003134	0.002975	0.002863
8	0.1285	0.006177	0.004406	0.003732	0.003426	0.003238	0.003005	0.002859	0.002755
10	0.1019	0.005693	0.004155	0.003551	0.003273	0.003100	0.002886	0.002751	0.002655
12	0.08081	0.005277	0.003931	0.003386	0.003132	0.002974	0.002776	0.002651	0.002562
14	0.06408	0.004921	0.003730	0.003235	0.003003	0.002858	0.002675	0.002558	0.002475
16	0.05082	0.004611	0.003549	0.003099	0.002885	0.002750	0.002580	0.002472	0.002394

Size of wire, A.W.G.	Feet between wires, center to center								
	3	4	5	6	8	10	15	20	25
0000	0.003351	0.003171	0.003043	0.002947	0.002806	0.002706	0.002542	0.002436	0.002361
000	0.003276	0.003103	0.002981	0.002889	0.002753	0.002657	0.002498	0.002396	0.002323
00	0.003204	0.003039	0.002922	0.002833	0.002702	0.002610	0.002456	0.002358	0.002287
0	0.003135	0.002977	0.002864	0.002779	0.002653	0.002564	0.002416	0.002320	0.002251
1	0.003069	0.002917	0.002809	0.002727	0.002606	0.002520	0.002376	0.002284	0.002217
2	0.003006	0.002860	0.002756	0.002677	0.002560	0.002477	0.002338	0.002249	0.002184
4	0.002887	0.002752	0.002656	0.002582	0.002474	0.002396	0.002266	0.002182	0.002121
6	0.002777	0.002652	0.002563	0.002494	0.002392	0.002319	0.002197	0.002118	0.002061
8	0.002676	0.002559	0.002476	0.002412	0.002317	0.002248	0.002133	0.002059	0.002004
10	0.002581	0.002473	0.002395	0.002335	0.002245	0.002181	0.002073	0.002002	0.001951
12	0.002493	0.002392	0.002319	0.002262	0.002178	0.002118	0.002016	0.001949	0.001900
14	0.002411	0.002316	0.002247	0.002194	0.002115	0.002058	0.001961	0.001898	0.001852
16	0.002334	0.002245	0.002180	0.002130	0.002056	0.002002	0.001910	0.001850	0.001806

* The capacity *between* wires equals one-half the values given in this table.

CAPACITY TO NEUTRAL* OF SMOOTH ROUND WIRES

Microfarads per MILE of each wire of a single-phase or of a symmetrical three-phase line

Size of wire, A.W.G.	Diam. of wire, inches	Inches between wires, center to center							
		1	3	6	9	12	18	24	30
0000	0.4600	0.06332	0.03490	0.02741	0.02438	0.02261	0.02051	0.01924	0.01836
000	0.4096	0.05802	0.03336	0.02647	0.02364	0.02197	0.01998	0.01877	0.01793
00	0.3648	0.05366	0.03198	0.02559	0.02293	0.02136	0.01947	0.01832	0.01752
0	0.3249	0.04995	0.03069	0.02477	0.02227	0.02078	0.01899	0.01790	0.01713
1	0.2893	0.04676	0.02951	0.02400	0.02165	0.02024	0.01854	0.01749	0.01676
2	0.2576	0.04400	0.02842	0.02328	0.02106	0.01972	0.01810	0.01710	0.01640
4	0.2043	0.03937	0.02645	0.02195	0.01997	0.01876	0.01729	0.01638	0.01573
6	0.1620	0.03566	0.02475	0.02077	0.01898	0.01789	0.01655	0.01571	0.01512
8	0.1285	0.03262	0.02326	0.01971	0.01809	0.01710	0.01587	0.01510	0.01455
10	0.1019	0.03006	0.02194	0.01875	0.01728	0.01637	0.01524	0.01453	0.01402
12	0.08081	0.02787	0.02076	0.01788	0.01654	0.01570	0.01466	0.01400	0.01353
14	0.06408	0.02599	0.01970	0.01709	0.01586	0.01509	0.01412	0.01351	0.01307
16	0.05082	0.02434	0.01874	0.01636	0.01523	0.01452	0.01362	0.01305	0.01264

Size of wire, A.W.G.	Feet between wires, center to center								
	3	4	5	6	8	10	15	20	25
0000	0.01769	0.01674	0.01607	0.01556	0.01482	0.01429	0.01342	0.01286	0.01246
000	0.01730	0.01639	0.01574	0.01525	0.01454	0.01403	0.01319	0.01265	0.01227
00	0.01692	0.01604	0.01543	0.01496	0.01427	0.01378	0.01297	0.01245	0.01207
0	0.01656	0.01572	0.01512	0.01467	0.01401	0.01354	0.01275	0.01225	0.01189
1	0.01621	0.01540	0.01483	0.01440	0.01376	0.01330	0.01255	0.01206	0.01171
2	0.01587	0.01510	0.01455	0.01413	0.01352	0.01308	0.01235	0.01187	0.01153
4	0.01525	0.01453	0.01402	0.01363	0.01306	0.01265	0.01196	0.01152	0.01120
6	0.01467	0.01400	0.01353	0.01317	0.01263	0.01225	0.01160	0.01118	0.01088
8	0.01413	0.01351	0.01307	0.01273	0.01223	0.01187	0.01126	0.01087	0.01058
10	0.01363	0.01306	0.01264	0.01233	0.01186	0.01152	0.01094	0.01057	0.01030
12	0.01316	0.01263	0.01224	0.01194	0.01150	0.01118	0.01064	0.01029	0.01003
14	0.01273	0.01223	0.01187	0.01159	0.01117	0.01087	0.01036	0.01002	0.009777
16	0.01232	0.01185	0.01151	0.01125	0.01085	0.01057	0.01008	0.009768	0.009536

* The capacity *between* wires equals one-half the values given in this table.

CAPACITY TO NEUTRAL* OF STANDARD STRANDS

Microfarads per 1000 FEET of each conductor of a single-phase or of a symmetrical three-phase line

Size of cable, C.M. or A.W.G.	Diam. of strand, inches	Inches between conductors, center to center							
		1	3	6	9	12	18	24	30
1,000,000	1.152	.....	0.0105	0.00725	0.00617	0.00558	0.00492	0.00454	0.00428
750,000	0.998	.....	0.00959	0.00683	0.00586	0.00533	0.00472	0.00437	0.00414
500,000	0.814	0.0254	0.00856	0.00630	0.00547	0.00501	0.00447	0.00415	0.00394
350,000	0.681	0.0181	0.00783	0.00591	0.00517	0.00476	0.00427	0.00398	0.00378
250,000	0.575	0.0147	0.00725	0.00558	0.00492	0.00454	0.00409	0.00383	0.00364
0 000	0.528	0.0135	0.00699	0.00542	0.00480	0.00444	0.00401	0.00376	0.00358
000	0.470	0.0122	0.00666	0.00523	0.00465	0.00431	0.00390	0.00366	0.00349
00	0.418	0.0112	0.00637	0.00504	0.00450	0.00418	0.00380	0.00357	0.00341
0	0.373	0.0103	0.00610	0.00488	0.00437	0.00407	0.00371	0.00349	0.00333
1	0.332	0.00958	0.00586	0.00472	0.00424	0.00396	0.00361	0.00341	0.00326
2	0.292	0.00891	0.00561	0.00456	0.00411	0.00384	0.00352	0.00332	0.00318
4	0.232	0.00790	0.00520	0.00429	0.00389	0.00365	0.00336	0.00318	0.00305
6	0.184	0.00712	0.00486	0.00405	0.00369	0.00348	0.00321	0.00304	0.00293

Size of cable, C.M. or A.W.G.	Feet between conductors, center to center								
	3	4	5	6	8	10	15	20	25
1,000,000	0.00410	0.00383	0.00365	0.00351	0.00331	0.00317	0.00295	0.00281	0.00271
750,000	0.00396	0.00371	0.00354	0.00341	0.00322	0.00309	0.00288	0.00274	0.00265
500,000	0.00378	0.00355	0.00339	0.00327	0.00310	0.00298	0.00278	0.00266	0.00257
350,000	0.00363	0.00342	0.00328	0.00316	0.00300	0.00289	0.00270	0.00258	0.00250
250,000	0.00351	0.00331	0.00317	0.00307	0.00292	0.00281	0.00263	0.00252	0.00244
0 000	0.00345	0.00326	0.00312	0.00302	0.00287	0.00277	0 00260	0.00249	0.00240
000	0.00337	0.00318	0.00306	0.00296	0.00282	0.00272	0.00255	0.00245	0.00237
00	0.00329	0.00312	0.00299	0.00290	0.00276	0.00267	0.00251	0.00240	0.00233
0	0.00322	0.00305	0.00293	0.00284	0.00271	0.00262	0.00247	0.00237	0.00229
1	0.00315	0.00299	0.00288	0.00279	0.00266	0.00257	0.00242	0.00233	0.00226
2	0.00308	0.00292	0.00281	0.00273	0.00261	0.00252	0.00238	0.00229	0.00222
4	0.00295	0.00281	0.00271	0.00263	0.00252	0.00244	0.00230	0.00222	0.00215
6	0.00284	0.00271	0.00261	0.00254	0.00244	0.00236	0.00223	0.00215	0.00209

* The capacity *between* conductors equals one-half the values given in this table.

CAPACITY TO NEUTRAL* OF STANDARD STRANDS

Microfarads per MILE of each conductor of a single-phase or of a symmetrical three-phase line

Size of cable, C.M. or A.W.G.	Diam. of strand, inches	Inches between conductors, center to center							
		1	3	6	9	12	18	24	30
1,000,000	1.152	.....	0.0554	0.0383	0.0325	0.0294	0.0260	0.0240	0.0226
750,000	0.998	.....	0.0506	0.0361	0.0309	0.0281	0.0249	0.0231	0.0218
500,000	0.814	0.134	0.0452	0.0333	0.0289	0.0264	0.0236	0.0219	0.0208
350,000	0.681	0.0955	0.0413	0.0312	0.0273	0.0251	0.0225	0.0210	0.0200
250,000	0.575	0.0776	0.0383	0.0295	0.0260	0.0240	0.0216	0.0202	0.0192
0 000	0.528	0.0713	0.0369	0.0286	0.0253	0.0234	0.0212	0.0198	0.0189
000	0.470	0.0644	0.0352	0.0276	0.0245	0.0227	0.0206	0.0193	0.0184
00	0.418	0.0590	0.0336	0.0266	0.0238	0.0221	0.0201	0.0189	0.0180
0	0.373	0.0544	0.0322	0.0258	0.0231	0.0214	0.0196	0.0184	0.0176
1	0.332	0.0506	0.0309	0.0249	0.0224	0.0209	0.0191	0.0180	0.0172
2	0.292	0.0470	0.0296	0.0241	0.0217	0.0203	0.0186	0.0175	0.0168
4	0.232	0.0417	0.0275	0.0227	0.0205	0.0193	0.0177	0.0168	0.0161
6	0.184	0.0376	0.0256	0.0214	0.0195	0.0184	0.0169	0.0161	0.0154

Size of cable, C.M. or A.W.G.	Feet between conductors, center to center								
	3	4	5	6	8	10	15	20	25
1,000,000	0.0216	0.0202	0.0193	0.0185	0.0175	0.0168	0.0156	0.0148	0.0143
750,000	0.0209	0.0196	0.0187	0.0180	0.0170	0.0163	0.0152	0.0145	0.0140
500,000	0.0200	0.0188	0.0179	0.0173	0.0164	0.0157	0.0147	0.0140	0.0135
350,000	0.0192	0.0181	0.0173	0.0167	0.0159	0.0153	0.0143	0.0136	0.0132
250,000	0.0185	0.0175	0.0168	0.0162	0.0154	0.0148	0.0139	0.0133	0.0129
0 000	0.0182	0.0172	0.0165	0.0160	0.0152	0.0146	0.0137	0.0131	0.0127
000	0.0178	0.0168	0.0161	0.0156	0.0149	0.0143	0.0135	0.0129	0.0125
00	0.0174	0.0165	0.0158	0.0153	0.0146	0.0141	0.0132	0.0127	0.0123
0	0.0170	0.0161	0.0155	0.0150	0.0143	0.0138	0.0130	0.0125	0.0121
1	0.0166	0.0158	0.0152	0.0147	0.0141	0.0136	0.0128	0.0123	0.0119
2	0.0162	0.0154	0.0149	0.0144	0.0138	0.0133	0.0126	0.0121	0.0117
4	0.0156	0.0148	0.0143	0.0139	0.0133	0.0129	0.0122	0.0117	0.0114
6	0.0150	0.0143	0.0138	0.0134	0.0129	0.0125	0.0118	0.0114	0.0111

* The capacity *between* conductors equals one-half the values given in this table.

25-CYCLE CAPACITY SUSCEPTANCE TO NEUTRAL*
SMOOTH ROUND WIRES

Charging current in amperes per mile = (*susceptance from table*) × (*volts to neutral*) × 10^{-6}

Micromhos per MILE of each wire of a single-phase or of a symmetrical three-phase line

Size of wire, A.W.G.	Diam. of wire, inches	Inches between wires, center to center							
		1	3	6	9	12	18	24	30
0000	0.4600	9.948	5.483	4.306	3.830	3.552	3.222	3.023	2.884
000	0.4096	9.115	5.241	4.158	3.714	3.451	3.139	2.949	2.817
00	0.3648	8.430	5.024	4.020	3.602	3.356	3.059	2.878	2.752
0	0.3249	7.847	4.821	3.891	3.499	3.265	2.983	2.812	2.691
1	0.2893	7.346	4.636	3.770	3.401	3.180	2.913	2.748	2.633
2	0.2576	6.912	4.465	3.657	3.309	3.098	2.844	2.686	2.576
4	0.2043	6.185	4.155	3.448	3.137	2.947	2.716	2.573	2.471
6	0.1620	5.602	3.888	3.263	2.982	2.811	2.600	2.468	2.375
8	0.1285	5.125	3.654	3.096	2.842	2.686	2.493	2.372	2.286
10	0.1019	4.722	3.447	2.946	2.715	2.572	2.394	2.283	2.203
12	0.08081	4.378	3.261	2.809	2.598	2.466	2.303	2.199	2.126
14	0.06408	4.083	3.095	2.685	2.492	2.371	2.218	2.122	2.053
16	0.05082	3.824	2.944	2.570	2.393	2.281	2.140	2.050	1.986

Size of wire, A.W.G.	Feet between wires, center to center								
	3	4	5	6	8	10	15	20	25
0000	2.779	2.630	2.525	2.444	2.328	2.245	2.108	2.020	1.957
000	2.718	2.575	2.473	2.396	2.284	2.204	2.072	1.987	1.928
00	2.657	2.520	2.424	2.350	2.242	2.165	2.038	1.956	1.896
0	2.602	2.470	2.375	2.305	2.201	2.127	2.003	1.924	1.868
1	2.547	2.419	2.330	2.262	2.162	2.089	1.972	1.895	1.840
2	2.493	2.372	2.286	2.220	2.124	2.055	1.940	1.865	1.811
4	2.396	2.283	2.203	2.141	2.052	1.987	1.879	1.810	1.760
6	2.305	2.199	2.126	2.069	1.984	1.924	1.822	1.756	1.709
8	2.220	2.122	2.053	2.000	1.921	1.865	1.769	1.708	1.662
10	2.141	2.052	1.986	1.937	1.863	1.810	1.719	1.661	1.618
12	2.067	1.984	1.923	1.876	1.807	1.756	1.672	1.617	1.576
14	2.000	1.921	1.865	1.821	1.755	1.708	1.628	1.574	1.536
16	1.935	1.862	1.808	1.767	1.705	1.661	1.584	1.535	1.498

* The susceptance *between* wires equals one-half the values given in this table.

25-CYCLE CAPACITY SUSCEPTANCE TO NEUTRAL* STANDARD STRANDS

Charging current in amperes per mile = (*susceptance from table*) × (*volts to neutral*) × 10^{-6}

Approximate micromhos per MILE of each conductor of a single-phase or of a symmetrical three-phase line

Size of cable, C.M. or A.W.G.	Diam. of strand, inches	Inches between conductors, center to center							
		1	3	6	9	12	18	24	30
1,000,000	1.152	.....	8.70	6.01	5.10	4.62	4.08	3.77	3.55
750,000	0.998	.....	7.94	5.67	4.85	4.41	3.91	3.63	3.42
500,000	0.814	21.0	7.10	5.23	4.54	4.14	3.71	3.44	3.27
350,000	0.681	15.0	6.48	4.90	4.29	3.94	3.53	3.30	3.14
250,000	0.575	12.2	6.01	4.63	4.08	3.77	3.39	3.17	3.02
0 000	0.528	11.2	5.79	4.49	3.97	3.67	3.33	3.11	2.97
000	0.470	10.1	5.53	4.33	3.85	3.57	3.24	3.03	2.89
00	0.418	9.26	5.28	4.18	3.74	3.47	3.16	2.97	2.83
0	0.373	8.54	5.06	4.05	3.63	3.36	3.08	2.89	2.76
1	0.332	7.94	4.85	3.91	3.52	3.28	3.00	2.83	2.70
2	0.292	7.38	4.65	3.79	3.41	3.19	2.92	2.75	2.64
4	0.232	6.55	4.32	3.57	3.22	3.03	2.78	2.64	2.53
6	0.184	5.90	4.02	3.36	3.06	2.89	2.65	2.53	2.42

Size of cable, C.M. or A.W.G.	Feet between conductors, center to center								
	3	4	5	6	8	10	15	20	25
1,000,000	3.39	3.17	3.03	2.90	2.75	2.64	2.45	2.32	2.25
750,000	3.28	3.08	2.94	2.83	2.67	2.56	2.39	2.28	2.20
500,000	3.14	2.95	2.81	2.72	2.57	2.47	2.31	2.20	2.12
350,000	3.02	2.84	2.72	2.62	2.50	2.40	2.25	2.14	2.07
250,000	2.90	2.75	2.64	2.54	2.42	2.32	2.18	2.09	2.03
0 000	2.86	2.70	2.59	2.51	2.39	2.29	2.15	2.06	1.99
000	2.79	2.64	2.53	2.45	2.34	2.25	2.12	2.03	1.96
00	2.73	2.59	2.48	2.40	2.29	2.22	2.07	1.99	1.93
0	2.67	2.53	2.43	2.36	2.25	2.17	2.04	1.96	1.90
1	2.61	2.48	2.39	2.31	2.22	2.14	2.01	1.93	1.87
2	2.54	2.42	2.34	2.26	2.17	2.09	1.98	1.90	1.84
4	2.45	2.32	2.25	2.18	2.09	2.03	1.92	1.84	1.79
6	2.36	2.25	2.17	2.10	2.03	1.96	1.85	1.79	1.74

* The susceptance *between* conductors equals one-half the values given in this table.

60-CYCLE CAPACITY SUSCEPTANCE TO NEUTRAL * SMOOTH ROUND WIRES

Charging current in amperes per mile = (*susceptance from table*) × (*volts to neutral*) × 10^{-6}

Micromhos per MILE of each wire of a single-phase or of a symmetrical three-phase line

Size of wire, A.W.G.	Diam. of wire, inches	Inches between wires, center to center							
		1	3	6	9	12	18	24	30
0000	0.4600	23.87	13.16	10.33	9.191	8.524	7.732	7.253	6.922
000	0.4096	21.87	12.58	9.979	8.912	8.283	7.532	7.076	6.760
00	0.3648	20.23	12.06	9.647	8.645	8.053	7.340	6.907	6.605
0	0.3249	18.83	11.57	9.338	8.396	7.834	7.159	6.748	6.458
1	0.2893	17.63	11.13	9.048	8.162	7.630	6.990	6.594	6.319
2	0.2576	16.59	10.71	8.777	7.940	7.434	6.824	6.447	6.183
4	0.2043	14.84	9.972	8.275	7.529	7.073	6.518	6.175	5.930
6	0.1620	13.44	9.331	7.830	7.155	6.745	6.239	5.923	5.700
8	0.1285	12.30	8.769	7.430	6.820	6.447	5.983	5.693	5.485
10	0.1019	11.33	8.271	7.069	6.515	6.171	5.745	5.478	5.286
12	0.08081	10.51	7.827	6.741	6.236	5.919	5.527	5.278	5.101
14	0.06408	9.798	7.427	6.443	5.979	5.689	5.323	5.093	4.927
16	0.05082	9.176	7.065	6.168	5.742	5.474	5.135	4.920	4.765

Size of wire, A.W.G.	Feet between wires, center to center								
	3	4	5	6	8	10	15	20	25
0000	6.669	6.311	6.058	5.866	5.587	5.387	5.059	4.848	4.697
000	6.522	6.179	5.934	5.749	5.482	5.289	4.973	4.769	4.626
00	6.379	6.047	5.817	5.640	5.380	5.195	4.890	4.694	4.550
0	6.243	5.926	5.700	5.531	5.282	5.105	4.807	4.618	4.483
1	6.111	5.806	5.591	5.429	5.188	5.014	4.731	4.547	5.415
2	5.983	5.693	5.485	5.327	5.097	4.931	4.656	4.475	4.347
4	5.749	5.478	5.286	5.139	4.924	4.769	4.509	4.343	4.222
6	5.531	5.278	5.101	4.965	4.762	4.618	4.373	4.215	4.102
8	5.327	5.093	4.927	4.799	4.611	4.475	4.245	4.098	3.989
10	5.139	4.924	4.765	4.648	4.471	4.343	4.124	3.985	3.883
12	4.961	4.762	4.614	4.501	4.336	4.215	4.011	3.879	3.781
14	4.799	4.611	4.475	4.369	4.211	4.098	3.906	3.778	3.686
16	4.645	4.467	4.339	4.241	4.090	3.985	3.800	3.683	3.595

* The susceptance *between* wires equals one-half the values given in this table.

60-CYCLE CAPACITY SUSCEPTANCE TO NEUTRAL*
STANDARD STRANDS

Charging current in amperes per mile = (susceptance from table) × (volts to neutral) × 10^{-6}

Approximate micromhos per MILE of each conductor of a single-phase or of a symmetrical three-phase line

Size of cable, C.M. or A.W.G.	Diam. of strand, inches	Inches between cables, center to center							
		1	3	6	9	12	18	24	30
1,000,000	1.152	.....	20.9	14.4	12.3	11.1	9.80	9.05	8.52
750,000	0.998	.....	19.1	13.6	11.6	10.6	9.39	8.71	8.22
500,000	0.814	50.5	17.0	12.6	10.9	9.95	8.90	8.26	7.84
350,000	0.681	36.0	15.6	11.8	10.3	9.46	8.48	7.92	7.54
250,000	0.575	29.3	14.4	11.1	9.80	9.05	8.14	7.62	7.24
0 000	0.528	26.9	13.9	10.8	9.54	8.82	7.99	7.46	7.13
000	0.470	24.3	13.3	10.4	9.24	8.56	7.77	7.28	6.94
00	0.418	22.2	12.7	10.0	8.97	8.33	7.58	7.13	6.79
0	0.373	20.5	12.1	9.73	8.71	8.07	7.39	6.94	6.63
1	0.332	19.1	11.6	9.39	8.44	7.88	7.20	6.79	6.48
2	0.292	17.7	11.2	9.09	8.18	7.65	7.01	6.60	6.33
4	0.232	15.7	10.4	8.56	7.73	7.28	6.67	6.33	6.07
6	0.184	14.2	9.65	8.07	7.35	6.94	6.37	6.07	5.81

Size of cable, C.M. or A.W.G.	Feet between cables, center to center								
	3	4	5	6	8	10	15	20	25
1,000,000	8.14	7.62	7.28	6.97	6.60	6.33	5.88	5.58	5.39
750,000	7.88	7.39	7.05	6.79	6.41	6.15	5.73	5.47	5.28
500,000	7.54	7.09	6.75	6.52	6.18	5.92	5.54	5.28	5.09
350,000	7.24	6.82	6.52	6.30	5.99	5.77	5.39	5.13	4.98
250,000	6.97	6.60	6.33	6.11	5.81	5.58	5.24	5.01	4.86
0 000	6.86	6.48	6.22	6.03	5.73	5.50	5.16	4.94	4.79
000	6.71	6.33	6.07	5.88	5.62	5.39	5.09	4.86	4.71
00	6.56	6.22	5.96	5.77	5.50	5.32	4.98	4.79	4.64
0	6.40	6.07	5.84	5.66	5.39	5.20	4.90	4.71	4.56
1	6.26	5.96	5.73	5.54	5.32	5.13	4.83	4.64	4.49
2	6.11	5.81	5.62	5.43	5.20	5.01	4.75	4.56	4.41
4	5.88	5.58	5.39	5.24	5.01	4.86	4.60	4.41	4.30
6	5.66	5.39	5.20	5.05	4.86	4.71	4.45	4.30	4.18

* The susceptance *between* conductors equals one-half the values given in this table.

Single Round* Wire Parallel to the Ground. — Dimensions as in Fig. 3. The dotted circle represents the "image" of the wire in the plane. The capacity of the actual condenser formed by the wire and the earth (assumed equivalent to an infinite conducting plane parallel to the wire) is the same as the capacity to neutral of the fictitious condenser formed by the wire and its image, the distance between the two wires of this fictitious condenser being $D = 2H$. Hence using the approximate expressions, since the wire is practically always more than 10 times its diameter above the other, the capacity *between the wire and the earth* is

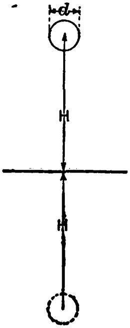

Fig. 3.

$$C = \frac{K}{2\log_e \frac{4H}{d}} \quad \text{statfarads per centimeter}$$

$$= \frac{7.354 \times 10^{-3} K}{\log_{10} \frac{4H}{d}} \quad \text{microfarads per 1000 feet}$$

$$= \frac{38.83 \times 10^{-3} K}{\log_{10} \frac{4H}{d}} \quad \text{microfarads per mile.}$$

SYSTEMS OF THREE CONDUCTORS. — Three practical examples of an electrostatically independent system consisting of three conductors only are (1) three parallel overhead wires at a distance above the earth large compared to their distances apart, (2) two wires in a lead sheath and (3) two overhead wires and the earth. In the first case any one of the three wires may be considered as the conductor of reference; in the second case it is convenient to choose the sheath of the cable as the conductor of reference, whether grounded or not, and in the third case to choose the earth as the conductor of reference. The general equations (1) and (2) are applicable to either case. Designating the reference conductor as No. 3 instead of No. 0, and the other two conductors as No. 1 and No. 2 respectively, the general equations may be written

$$\left.\begin{aligned} v_{13} &= A_{11}q_1 + A_{12}q_2, & q_1 &= B_{11}v_{13} + B_{12}v_{23}, \\ v_{23} &= A_{12}q_1 + A_{22}q_2, & q_2 &= B_{12}v_{13} + B_{22}v_{23}, \\ & & q_3 &= -(q_1 + q_2). \end{aligned}\right\} \tag{5}$$

v_{13} and v_{23} indicate the potential drop from 1 to 3 and from 2 to 3 respectively. Solving the first set of equations for q_1 and q_2 gives

$$B_{11} = \frac{A_{22}}{A_{11}A_{22} - A_{12}^2}, \quad B_{22} = \frac{A_{11}}{A_{11}A_{22} - A_{12}^2}, \quad B_{12} = \frac{-A_{12}}{A_{11}A_{22} - A_{12}^2}. \tag{5a}$$

Possible Combinations of Three Conductors which may be used as a Condenser — "Part Capacities." — The following combinations are possible:

No. 1 against† No. 2 with No. 3 insulated,
No. 2 against No. 3 with No. 1 insulated,
No. 3 against No. 1 with No. 2 insulated,
No. 1 against No. 2 and No. 3 connected together,
No. 2 against No. 3 and No. 1 connected together,
No. 3 against No. 1 and No. 2 connected together.

* When the wire is at a height large compared with the linear dimensions of its cross-section these formulas also apply approximately to wires of any shape of cross-section provided d is taken equal to the perimeter of the cross-section divided by π.

† By "against" is here meant that the source of e.m.f. is connected between the two conductors designated, e.g., in the fourth case No. 1 is charged positively say, and an equal and opposite charge divides between 2 and 3.

The capacities corresponding to these various combinations may be called "part capacities," and may be designated by subscripts thus: C_{12} = capacity of No. 1 against No. 2 with No. 3 insulated (= normal capacity between 1 and 2), $C_{1(23)}$ = capacity of No. 1 against No. 2 and No. 3 connected together (= grounded capacity of No. 1), and so on for the others. The values of these part capacities can be deduced directly from equations (5), viz.,

$$\left.\begin{aligned} C_{12} &= \frac{1}{A_{11}+A_{22}-2A_{12}}, & C_{1(23)} &= B_{11} \\ C_{23} &= \frac{1}{A_{22}}, & C_{2(31)} &= B_{22} \\ C_{31} &= \frac{1}{A_{11}}, & C_{3(12)} &= -(B_{11}+B_{22}+2B_{12}) \end{aligned}\right\} \quad (6)$$

Three Overhead Parallel Wires, Effect of Earth Neglected.—Arrangement of wires and dimensions as in Fig. 4; distance apart of wires and diameters both in the *same* unit of length, but this unit may be either centimeters or inches. From equation (20c) in the article on *Electricity and Magnetism, Principles of*, the following closely approximate values of the A's (accurate to within less than 0.1 per cent when the distance apart of the wires is greater than 10 diameters) may be deduced, taking K as unity, since the dielectric in the case under consideration is air:

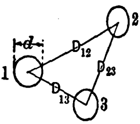

Fig. 4.

$$A_{11} = 2\log_\epsilon \frac{4D_{13}^2}{d_1 d_3}, \quad A_{22} = 2\log_\epsilon \frac{4D_{23}^2}{d_2 d_3}, \quad A_{12} = 2\log_\epsilon \frac{2D_{13}D_{23}}{d_3 D_{12}}, \quad (7)$$

all in c.g.s. electrostatic units for a length of one centimeter.*

From these relations the values of the coefficients B may be calculated for any arrangement of three parallel wires.

Equilateral Triangle Arrangement. — A common arrangement of the three wires of a three-phase transmission line is to place them so that their centers form the three vertices of an equilateral triangle. In this case $D_{12} = D_{23} = D_{13} = D$, say; and if all three wires are of the same diameter $d_1 = d_2 = d_3 = d$, say, then $A_{11} = A_{22} = 4\log_\epsilon \frac{2D}{d}$ and $A_{12} = 2\log_\epsilon \frac{2D}{d}$ in c.g.s. electrostatic units, and therefore, from equation (6) the normal capacity between any two of the wires with the third wire insulated is

$$C_{12} = \frac{1}{4\log_\epsilon \frac{2D}{d}} \quad \text{statfarads per centimeter,}$$

$$= \frac{3.677 \times 10^{-3}}{\log_{10} \frac{2D}{d}} \quad \text{microfarads per 1000 ft.,}$$

which is the same as the capacity between two parallel wires by themselves, see above.

Equilateral Triangle Arrangement with Balanced Three-phase Voltages. — For sine-wave voltages between the wires equal in effective value to V and differing in phase by 120 degrees, the above equations for the charges and

* To find the values of the B's in microfarads per 1000 feet directly from these values of the A's change 2 $\log_\epsilon$ to 136 $\log_{10}$.

charging currents give the following relations between the *effective* values of the voltages, charges and charging currents, for each of the three wires:

$$Q = C_0 V_0, \qquad I = 2\pi f C_0 V_0,$$

where

$$V_0 = \frac{V}{\sqrt{3}} = \text{voltage to neutral},$$

$$C_0 = 2\,C_{12} = \text{capacity to neutral.}$$

When the C's are in microfarads the charge Q is in microcoulombs and the charging current in micro-amperes.

The charge on any particular wire is in phase with the voltage between that wire and the neutral (*see Alternating Currents*) and the charging current for that wire is 90 degrees ahead of the voltage drop from that wire to the neutral. For the same voltage V between wires in a single-phase system as in a three-phase balanced system, the charging current per wire in the three-phase system with the equilateral triangle arrangement of wires is $\frac{2}{\sqrt{3}} = 1.155$ times the charging current per wire in the single-phase system.

For any other arrangement of wires and for an unbalanced three-phase system, the general equations (5) and the general formulas for A_{11}, A_{22} and A_{12} given above must be used.

Two-conductor Cable. — Arrangement of wires and dimensions as shown in Fig. 5. The sheath forms the conductor of reference and when designated as conductor No. 3, general equations (5), (5a) and (6) apply directly to this case also. For the symmetrical arrangement of the wires shown in the figure, $A_{11} = A_{22}$, whence, dropping the first subscript,

$$B_1 = \frac{A_1}{A_1^2 - A_2^2}, \qquad B_2 = \frac{-A_2}{A_1^2 - A_2^2}.$$

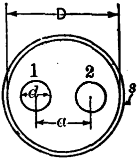

Fig. 5.

Russell (*Alternating Currents, Vol. 1*) gives the following values of A_1 and A_2, assuming a dielectric filling the space between the wires and the sheath to have the *same** specific inductive capacity throughout:

$$A_1 = \frac{2}{K}\log_e \frac{D^2 - a^2}{Dd}, \qquad A_2 = \frac{2}{K}\log_e \frac{D^2 + a^2}{2\,Da}, \tag{8}$$

both in c.g.s. electrostatic units for a length of one centimeter. To reduce to practical units for a length of 1000 ft. change $2\log_e$ to $136\log_{10}$.

Normal Capacity of a Two-conductor Cable. — Substituting Russell's values for A_1 and A_2 in equation (6) gives

$$C_{12} = \frac{K}{4\log_e\left(\frac{2a}{d}\cdot\frac{D^2 - a^2}{D^2 + a^2}\right)} \quad \text{statfarads per centimeter}$$

$$= \frac{3.677 \times 10^{-3} K}{\log_{10}\left(\frac{2a}{d}\cdot\frac{D^2 - a^2}{D^2 + a^2}\right)} \quad \text{microfarads per 1000 ft.}$$

Grounded Capacity of a Two-conductor Cable. — The capacity in this case, from equations (6), is

$$C_{1(23)} = B_{11} = \frac{A_1}{A_1^2 - A_2^2}.$$

*This is seldom realized in practice, since the dielectric is made up of insulation, fillers and braids having different specific inductive capacities; see *Wires and Cables, Insulated.*

By substituting Russell's values of the A's an approximate value for this capacity may be obtained; it will be greater than the normal capacity C_{12}.

Effect of the Earth on the Capacity of Two Overhead Wires.—As a fair approximation the earth may be considered as a conducting plane of infinite extent and the wires as parallel to this plane. Such a combination is electrostatically equivalent to two wires and their two "images" at a distance below the earth equal to the distance of the actual wires above the earth, see Fig. 6. Let q_1 and q_2 represent the charges on the actual wires and q_3 the actual charge on the earth, and V_{13} and V_{23} the potential differences between the two wires and the earth respectively (the p.d. between the two wires $v_{12} = v_{13} - v_{23}$); equations (5), (5a) and (6) then apply to this case also. The values of the A's in this case are (putting $K = 1$, since the dielectric is air)

$$A_{11} = 2 \log_\epsilon \frac{2\,D_{1a}}{d_1},\ A_{22} = 2 \log_\epsilon \frac{2\,D_{2b}}{d_2},\ A_{12} = 2 \log_\epsilon \frac{D_{1b}}{D_{12}}, \tag{9}$$

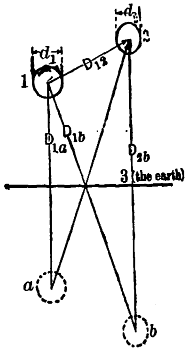

Fig. 6.

all in c.g.s. electrostatic units per centimeter length. To find the values of the B's directly in microfarads for a length of 1000 feet change 2 $\log_\epsilon$ to 136 $\log_{10}$.

Consider the special case where both wires are at the same height H, say, and both of the same diameter d; then from equation (9), putting D for the distance between their centers,

$$A_{11} = A_{22} = 2 \log_\epsilon \frac{4\,H}{d},\quad A_{12} = 2 \log_\epsilon \frac{2H}{D'}, \tag{9a}$$

where

$$D' = \frac{D}{\sqrt{1+\left(\frac{D}{2H}\right)^2}} = \text{"equivalent" distance apart.}$$

Then from equations (6)

$$C_{12} = \frac{1}{4 \log_\epsilon \frac{2\,D'}{d}} \quad \text{statfarads per centimeter.} \tag{10}$$

Comparing the formula for C_{12} with that for two wires by themselves, equation (4), it is evident that the effect of the earth in this case is to increase the capacity by an amount equal to the increase in capacity which would result from decreasing the distance between the wires from the actual distance D to the "equivalent" distance D'. For D small compared with the height H, which is usually the case, the equivalent distance D' is practically equal to D and the effect of the earth is therefore negligible.

Effect of the Earth When One Wire is Grounded.—Let No. 2 be grounded. Consider the same case as in the preceding paragraph, i.e., both wires at the same height H and of the same diameter d. From equation (5a)

$$B_{11} = B_{22} = 2C_{12}\left(\frac{A_{11}}{A_{11}+A_{12}}\right), \qquad B_{12} = -\ 2C_{12}\left(\frac{A_{12}}{A_{11}+A_{12}}\right),$$

where C_{12} has the value given by equation (10) and the A's the values given by (9a). The charge taken by conductor No. 1 is then $B_{11}v$, the charge taken

by conductor No. 2 is $B_{12}v$, and the charge taken by the earth $(B_{11} + B_{12})v$, where v is the drop of potential from No. 1 to No. 2.

Effect of the Earth When Middle or Neutral Point of the System is Grounded. — In this case $v_{23} = - v_{13}, = v_0$, where v_0 = voltage to neutral. From equation (5) it is then evident that grounding the middle point of the system has no effect upon the charging current.

SYSTEMS OF FOUR OR MORE CONDUCTORS. — The method of calculating the capacities for a three-conductor system, described in detail above, may be readily extended to a system of any number of circuits, independent or otherwise, made up of any number of conductors.

Relations Between the *A's* and *B's* for a Four-conductor System. — For a four-conductor system the relations are as follows: put

$$M = A_{11}A_{22}A_{33} + 2\,A_{12}A_{13}A_{23} - (A_{11}A_{23}^2 + A_{22}A_{13}^2 + A_{33}A_{12}^2),$$

then

$$\left.\begin{aligned} B_{11} &= \frac{A_{22}A_{33} - A_{23}^2}{M}, & B_{12} &= -\frac{A_{12}A_{33} - A_{13}A_{23}}{M}, \\ B_{22} &= \frac{A_{11}A_{33} - A_{13}^2}{M}, & B_{23} &= -\frac{A_{23}A_{11} - A_{12}A_{13}}{M}, \\ B_{33} &= \frac{A_{11}A_{22} - A_{12}^2}{M}, & B_{13} &= -\frac{A_{13}A_{22} - A_{12}A_{23}}{M}, \end{aligned}\right\} \quad (11)$$

There are 32 part capacities in the general case of a four-conductor system but in cases of symmetry they reduce to a lesser number.

Three-conductor Cable. — Arrangement and dimensions as shown in Fig. 7. The sheath forms the conductor of reference; let it be designated as conductor No. 0. From symmetry $A_{11} = A_{22} = A_{33} = A_1$, say, and $A_{12} = A_{23} = A_{13} = A_2$, say. Equations (11) then reduce to

$$B_{11} = B_{22} = B_{33} = \frac{A_1 + A_2}{(A_1 - A_2)\,(A_1 + 2\,A_2)} = B_1, \text{ say},$$

$$B_{12} = B_{23} = B_{13} = \frac{-A_2}{(A_1 - A_2)\,(A_1 + 2\,A_2)} = B_2, \text{ say},$$

and the capacity between any pair of wires is

$$C_{12} = \tfrac{1}{2}(B_1 - B_2).$$

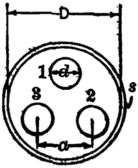

Fig. 7.

For sine-wave voltages equal in effective value and differing in phase by 120 degrees, the effective value of the charging current per wire is then, from equation (3),

$$I = 2\,\pi f\,(B_1 - B_2)\,V_0 = 2\,(2\,\pi f C_{12})\,V_0,$$

where $V_0 = \frac{V}{\sqrt{3}}$, the voltage V being the p.d. between wires. This current leads the voltage drop to neutral, V_0, by 90 degrees.

Calculated Value C_{12} for a Three-conductor Cable. — Russel (*Alternating Currents, Vol. 1*) gives the following approximate values for A_1 and A_2, assuming the insulation fillers and braids to have the same specific inductive capacity (which is not usually the case),

$$A_1 = \frac{0.1360 \times 10^9}{K} \log_{10}\left(\frac{D^2 - 1.33\,a^2}{Dd}\right) \text{ and } A_2 = \frac{0.0680 \times 10^9}{K} \log_{10}\left(\frac{0.75\,D^6 - 1.78\,a^6}{a^2D^2\,(3\,D^2 - 4\,a^2)}\right)$$

in practical units for a length of 1000 feet. Using these values the normal capacity between wires is

$$C_{12} = \frac{3.677 \times 10^{-3} K}{\log_{10} \frac{2\,a\rho}{d}} \quad \text{microfarads per 1000 feet,}$$

where

$$\rho = \sqrt{\frac{(3\,D^2 - 4\,a^2)^3}{(3\,D^2)^3 - (4\,a^2)^3}}.$$

That is, the capacity is the same as that of two parallel wires by themselves but at a distance ρa between centers instead of the actual distance a.

Effect of the Earth on the Capacity of Three Overhead Wires. — For any kind of arrangement, as shown in Fig. 8 (the dotted circles are the "images" of the actual wires), the general expressions for the A's in c.g.s. electrostatic units * for a length of one centimeter are as follows:

$$A_{11} = 2 \log_\epsilon \frac{2\,D_{1a}}{d_1}, \quad A_{22} = 2 \log_\epsilon \frac{2\,D_{2b}}{d_2}, \quad A_{33} = 2 \log_\epsilon \frac{2\,D_{3c}}{d_3},$$

$$A_{12} = 2 \log_\epsilon \frac{D_{1b}}{D_{12}}, \quad A_{13} = 2 \log_\epsilon \frac{D_{1c}}{D_{13}}, \quad A_{23} = 2 \log_\epsilon \frac{D_{2c}}{D_{23}}.$$

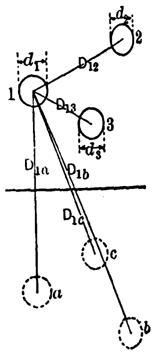

Fig. 8.

The values of the B's may then be calculated from equation (11) and the charging currents from equation (3). For any numerical case the calculations are tedious, but not difficult. Ordinarily the formulas given above neglecting the effect of the earth are sufficiently accurate for all practical purposes; compare with the effect of the earth on a two-wire line.

Electrostatic Induction From One Circuit to Another. — The general equations (1), (2) and (3) together with the method of calculation of the A's given in the last paragraph make it possible to calculate the induced voltages from one line to another, e.g., the voltages induced from a high-tension line to a telephone line. The method is straightforward, but tedious. The discussion given above indicates how such a problem may be attacked; space is not available for a detailed discussion here.

BIBLIOGRAPHY. — Russell, Alex., *Alternating Currents*, London, 1906; Ferguson, *Elements of Electrical Transmission*, N. Y., 1911; Perrine, *Conductors for Electric Distribution*, N. Y., 1907.

* To find from these A's the values of the B's in microfarads per 1000 feet change $2 \log_\epsilon$ to $136 \log_{10}$.

[H. PENDER.]

CAR BARNS AND INSPECTION SHEDS. — (*See also Cars, Electric; Locomotives, Electric; Railways.*) In every electric railway system it is necessary to provide car barns and inspection sheds where the cars may be taken regularly for a systematic inspection. In some cases these barns are used for the storage of cars and in special cases are equipped as regular repair shops. Provision for inspection is necessary, provision for storage is optional, although it is customary to provide storage at least for those cars which are idle during the daytime. The expense of providing for the storage under cover of all the cars that are idle between midnight and 6 a.m. is of doubtful value, for the deterioration of the cars left in the open is likely to be less than the fixed charges on the increased capacity of the sheds. The extent to which repairs are made by the railway company differs according to the policy and size of the road, and thus the size of the repair shop varies considerably.

Location. — The location of the car barn may be determined by several considerations. It may be located in the city on account of the convenience of the employees, the possibility of placing the general offices in the same building, and on account of the fact that the city service is an important part of the whole system. It may be located in the suburbs where real estate is cheap and ample space available. It may be located at a convenient point such as the center of gravity of the system, so that the dead mileage of the cars will be a minimum. Finally, it may be located adjacent to the power station, so that the repair shops of the two departments may be combined and thus become more efficient.

Danger of Fire. — Wherever the car barn is located considerable attention should be given to the possibility of fire and it should be detached from all neighboring buildings, particularly those of a hazardous character, as the danger from fire in a car barn is very great. There should be available a sufficient supply of water for fire protection and the building should be equipped with automatic fire sprinklers.

Lay-out Tracks, Inspection Pits, etc. — In arranging the scheme of tracks consideration is given to the convenience of getting cars in and out past cars undergoing repairs, and without interfering with the main-line traffic. It is also well to make provisions for getting all the cars out of the barn quickly in case of fire, and for this purpose the tracks are sometimes inclined towards the exits so that the cars will run out by gravity when the brakes are released. A transfer table on which a car may be run and transferred laterally to any track in the barn is sometimes provided, and to avoid wasting the space that the pit for such a table would occupy, the table is frequently operated on tracks at the same level as that of the main tracks. In this case the cars must mount a slight incline to get upon the table. A great many motors, particularly those used on city cars and single-truck cars, are of the split-frame type, which are inspected by swinging the lower half of the frame downward. To inspect these motors a pit is required of sufficient depth for a man to work comfortably underneath the cars, and with provision for illumination both general and by drop light. The illumination of a car barn should be both general and local. The general illumination should consist of incandescent lights suspended from overhead between the tracks and spaced about a car length apart. These may be run five in series on the trolley circuit, although an ungrounded system is better. The intensity of illumination on the floor should be from 1 to 1.5 foot-candles or lumens per square foot. Frequent sockets for drop lights must also be provided for many portions of the car equipment are in dark corners.

Repair Shops. — While some roads only make very minor repairs to their cars others maintain very complete repair shops so that general recommendations are difficult to make. Harding, in *Electrical Railway Engineering*, gives the following list of tools as desirable, named in order of importance:

1 screw-cutting lath, 14-inch swing.
1 vertical drill press, 24 inches.
1 tool-grinding wheel.
1 armature stand for rewinding armatures.
2 forges.
1 automatic-power hack saw.
1 oven for baking insulation.
1 commutator slotting machine.
1 wheel-turning lath.
1 hydraulic-wheel press.

BIBLIOGRAPHY. — Harding, C. F., *Electric Ry. Engineering*, N. Y., 1911; Numerous papers in the *Electric Railway Journal*, N. Y.

[W. I. SLICHTER.]

CARS, ELECTRIC. — (*See also Control Systems for Railway Motors; Collectors, Current; Locomotives, Electric.*) The cars used on electric railways have developed rapidly but gradually from the small two-axle car with one motor on the platform geared to the axle by a chain, to the large steel double-truck car now used in high-speed interurban service. Cars may be divided into two classes, viz., single-truck and double-truck. Each of these classes may be subdivided into types according as the cars are open or closed.

SINGLE-TRUCK CARS have a seating capacity as high as 32 passengers. Their characteristics are given in the accompanying table.

Item	Dimension
Over-all length	28 to 31 ft.
Rigid wheel base	5 to 7 ft.
Weight loaded and equipped	12 to 20 tons
Maximum speed	25 m.p.h
Maximum motor capacity, total	80 h.p.
Number of motors	2

They are only used in city service with frequent stops and usually have a cylinder-type controller at each end with hand brakes, although there are cases where these cars have been fitted with remote control and air brakes.

Considerable attention is being given at present to the subject of equipping these cars so that during the hours of heavy traffic they may be run in trains of two, controlled by one motorman. This causes less congestion in the streets for a given service and obviates the waste of operating throughout the day cars of large capacity designed to meet rush-hour conditions.

DOUBLE-TRUCK CARS.— The double-truck type of car has an over-all length of from 30 to 70 feet, a seating capacity of from 32 to 70 passengers and a maximum speed of from 40 to 70 m.p.h. One of these cars consists of a body mounted on two, four-wheel trucks. The car body under-frame is supported on a cross piece in the truck known as the bolster and is kept in position by a king-pin fitted into a king-pin center plate on the car body. The truck may swivel around this king-pin through quite an angle, guided by curved plates on each member.

Construction of Trucks. — The truck consists of axles, carried in journal boxes in side frames, transoms which connect the side frames, and the bolster which is carried by the transoms by means of springs (either spiral or elliptical) and a spring plate swung from the transoms. Coil springs are inserted between the journals and the side frames. The bolster may be of the "rigid," "floating" or "swinging" type depending upon the amount of play in the bolster. The rigid bolster is employed in locomotives only. The swinging bolster is most generally used for passenger cars, particularly in high-speed service, as it makes a much more easy riding construction. The three-axle truck is common in high-speed steam railroad practice, but the two-axle truck is generally used in electric railway practice as it allows more room for the motors. Four-wheel trucks are sometimes called "bogie" trucks or "swiveling" trucks to distinguish them from the "pony" or "radial" trucks having two wheels, which are used on steam locomotives.

Motor-car trucks have wheels of from 33 to 36 inches in diameter, and usually weigh from 10,000 to 15,000 pounds depending upon the size of the motors and body they are intended to carry. The rigid wheel base is from 6 to 8 feet.

The problem of making a motor-driven truck guide properly is considerably more difficult than guiding the trucks of a car that is hauled, as there is a tendency for the motor-driven truck to get out of line and become cramped between the rails. On this account and on account of the weight of the motors, trucks for a motor car must be stronger and heavier than for a trailer car.

Suspension of Motors. — The motors are usually hung between the axles on a single-truck car and between each axle and the transom on a double-truck car. In the "nose" suspension a lug is cast on the side of the motor frame away from the axle and this is attached to the transom with or without springs. The other side of the motor is carried by arms containing bearings which encircle the car axles. Thus about one-half of the weight of the motor is carried as a dead weight on the car axle, and the rest by the truck framing. In the "cradle" suspension both motors of a truck are carried by a cradle which is suspended from the car axles by springs. By this arrangement the entire weight of the motors is spring supported and carried by the axles, thus relieving the transom and side frames of the weight of the motors.

Maximum Traction Truck. — This is a special form of bogie truck with two axles having wheels of different diameters. The equalizing arrangements in this truck are such that about three-quarters of the total weight comes on the main axle which has the larger wheels and carries the motor, and the balance comes on the other axle which has small wheels. This arrangement gives a double-truck car with only two motors and yet provides that about three-quarters of the total weight shall be on driving axles, thus yielding "maximum traction." This type of truck is used on cars which are too long and heavy to use the two-axle rigid truck but which have such slow speed that only two motors are required. It is not adapted to high-speed work.

CAR BODIES may be classified as closed, convertible, semi-convertible and open.

Closed Cars.— The small closed type for city service is always of wood and has longitudinal seats. The general dimensions are given in the table below. Large closed cars with double trucks for interurban or rapid-transit service have both transverse and longitudinal seats, the proportion depending upon the proportion of long-haul traffic. The longitudinal seats give an arrangement permitting convenience of movement into and out of the car and allow more room for standing, but are not comfortable for long trips. Transverse seats are more comfortable for a long trip and for a service employing a high rate of acceleration.

Convertible Cars are constructed so that all the side panels of the body may be folded either up into the roof, or down to the floor leaving only the upright posts carrying the weight of the roof, thus converting a closed car into an open car at will.

Semi-convertible Cars. — A semi-convertible car, as its name implies, is arranged so that it may be partly opened in the summer time. It makes a very good car for high-speed interurban service because it is not advisable to operate open cars in high-speed service.

Open Cars usually have the transverse seats extending clear across the car. Access is obtained by steps running the length of the car on each side. They may be of the single- or double-truck type. They provide the maximum seating capacity for a given weight.

Cars for Rapid Handling of Traffic. — In large cities where the service is congested special means must be provided to aid and direct the rapid loading and unloading of cars. In a rapid transit service having stations this is accomplished by placing doors in the sides of the cars at the center and at or near the ends, and requiring all passengers to enter by the end doors and leave by the

center doors. Both center and end doors are operated by compressed air and controlled by a guard at one end of the car. For street service the pre-payment or "pay as you enter" car is employed. This is built with extra long platforms at each end. The passengers are required to deposit their fare in a box as they enter the car from the rear platform, where the conductor stands, and to leave by the front door and platform. The rear platform is made roomy to take care of congestion while the conductor is making change. Cars of this type recently installed in Chicago and Philadelphia are arranged for both entrance and exit of passengers at the front end, the conductor's station also being at this end.

Construction of Car Bodies. — Most of the car bodies used on electric railways are constructed of wood, but for underground roads or for roads operating a high-class suburban service of multiple-unit trains the cars are preferably constructed of steel to reduce the dangers from collision and fire. This construction is demanded by consideration of safety but it is directly contrary to the general tendency of the present day to reduce the weight of cars as much as possible. The proportion of weight of car to weight of load is very high and it entails a considerable waste of energy to propel this dead weight. The weight of car per passenger or seat capacity is a figure that should receive careful consideration and it should be kept at the minimum value compatible with strength and safety.

DIMENSIONS AND WEIGHTS OF TYPICAL EQUIPMENTS. — There are indicated in the following table the principle data for electric cars which are being operated in various typical services.

Service	City	City	City	City P. A. Y. E.	Suburban	Interurban	Subway	N. Y. C. R.R.	W. S. R.R.
Seating capacity	14	24	32	32	38	40	52	64	52
Number of trucks	1	1	1	2	2	2	2	2	2
Length over-all, ft.	22	30	32	41	38	40	50	60	49
Length of body, ft.	16	20	21	28	28	29	41	50	38
Number of motors	2	2	2	2	2	4	2	2	4
H.p. of each motor	40	40	40	70	70	50	200	200	75
Weight of body, lb.	6,000	9,530	7,500	15,800	15,670	19,200	34,300	55,000	35,110
Weight of trucks, lb.	4,600	4,800	4,500	10,400	10,600	16,000	22,500	14,940	23,000
Weight of electric equipment, lb.	5,990	6,962	7,062	9,500	8,350	16,542	21,510	25,460	21,680
Weight, total, lb.	16,590	21,292	19,062	35,700	34,620	51,742	78,310	95,400	79,790
Weight per passenger, lb.	1,185	887	594	1,118	911	1,293	1,506	1,490	1,534
Weight of load, lb.	2,240	3,640	4,760	4,760	5,600	5,889	7,560	9,240	7,560

BIBLIOGRAPHY.— Burch, E. P., *Electric Traction for Railway Trains*, N. Y., 1911; *Car Builders' Dictionary*, N. Y., 1913; Dawson, P., *Electric Traction on Railways*, N. Y., 1909; *Electric Railway Dictionary*, N. Y., 1911; Parshall and Hobart, *Electric Railway Engineering*.

[W. I. SLICHTER.]

CASTINGS, IRON AND STEEL. — These are made by pouring the molten material into sand or iron molds of the desired shape and allowing it to cool. Sand molds are generally used, and are made by packing molding sand about a wooden pattern having the shape of the desired casting and somewhat larger dimensions to allow for the contraction of the metal in cooling. The removal of the pattern leaves a hollow in the sand of the shape of the object to be cast. In order to have sufficient consistency the sand should either contain some clay and be somewhat moist, giving a "green sand" mold, or it should be mixed with a binding material giving a "dry sand mold."

Simple patterns may be made in one piece, but if the pattern be at all complicated it must be made in two or more pieces to permit withdrawal from the sand without disturbing the latter.

Steel castings are more difficult to make than iron castings, as they are more porous; thin steel sections should be avoided for such castings, as the steel cools too rapidly when thin.

In all castings sudden changes in the thickness of the metal should be avoided, as the unequal cooling of sections of different thicknesses causes temperature stresses and may crack the metal. This is particularly true in steel castings.

Care should be exercised in designing castings to make them such that the patterns can be readily removed from the sand molds.

Process of Making a Mold. — A wooden or iron box called a flask, consisting of sides only, is used in making the molds. This is divided into a bottom portion called the drag, and an upper portion called the cope; see Fig. 1. Intermediate portions, called checks, are used for complicated patterns.

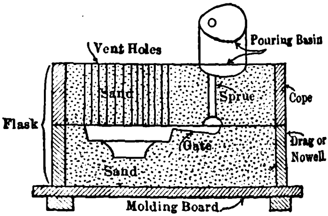

Fig. 1.

A hole is made through the cope to convey the metal to the mold. Molds made of "dry sand" should be baked in an oven before pouring. The surface of the mold is frequently coated with "blacking."

If holes are to be made in the casting they are formed by cores of hard-baked sand inserted after the pattern is removed and are held in place by studs of metal, called "chaplets," and by impressions in the mold made by projections on the pattern called "prints."

Cost of Iron and Steel Castings. — The cost of an iron casting, exclusive of the pattern, depends somewhat upon the intricacy of the pattern; representative costs in 1914 were for simple patterns of moderate size, 3 to 5 cents per pound; for large engine and dynamo castings 3 to 4.25 cents per pound; for small intricate castings 6 to 10 cents per pound. Malleable iron castings cost about 10 to 25 per cent more, and steel castings about 50 to 100 per cent more.

BIBLIOGRAPHY. — See bibliographies in the articles on *Iron, Pig and Cast; Steel.*

[C. M. SPOFFORD.]

CELLS, STANDARD. — (*See also Batteries, Primary; Electrochemistry, Principles of; Potentiometers.*) The need of a standard of electromotive force was recognized in the early days of the electrical industry, and the Daniell cell (*see Batteries, Primary*) was largely used for this purpose. This cell is readily prepared from ordinary chemicals, and is still used to some extent as a rough and ready standard. Such a cell, however, deteriorates quite rapidly, and has therefore been supplanted, for all accurate measurements, by other cells which possess to a greater degree the necessary characteristics of permanence and reproducibility. There are at present two types of standard cells in general use, the Clark and the Weston cells. The latter is the more generally used at the present time.

Pure Chemicals Required. — In order that the e.m.f. of any form of primary standard cell may accord with the stated value, great care must be exercised in the preparation of the materials, the processes for which have been carefully worked out by Kahle and later by Wolff and Waters (*Bull. Bureau of Standards, 1907, Vol. 4, p. 1*).

CLARK CELL. — In Fig. 1 is shown the Kahle's H-form of the Clark cell, which is one of the most satisfactory of the several forms in which the Clark cell is made. The container is of glass, which is mounted in a metal case with insulated binding posts connected to the two platinum terminals. The saturated solution of zinc sulphate forms the electrolyte, the paste of mercurous sulphate and zinc sulphate acts as a depolarizer, the mercury forms the positive pole and the zinc the negative pole. Detail directions for the preparation of the cell are given in the Bulletin of the Bureau of Standards, 1907, Vol. 4, p. 1.

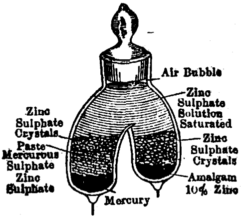

Fig. 1. Kahle's H-form of Clark Cell

Electromotive Force of Clark Cell. — The e.m.f. of the Clark cell, as determined at the time of the adoption of the international units (*see Units, Practical Electrical*), was given as 1.434 volts at 15° C. Subsequent investigations have shown that its true value is 1.4328 international volts at 15° C. At any other temperature $t°$ C. its e.m.f. is

$$E_t = 1.4328\,[1 - 0.00077\,(t - 15)].$$

The chief objections to the Clark cell are its relatively high temperature coefficient and the length of time required for the e.m.f. to attain a constant value after a change in the temperature of the cell.

Carhart-Clark Cell. — To reduce the temperature coefficient and time lag of the saturated or "normal" Clark cell, Carhart devised a form of Clark cell in which the zinc sulphate solution was just saturated at 0° C. with no excess of crystals. The e.m.f. of this type of cell is approximately

$$E = 1.440\,[1 - 0.00039\,(t - 15)].$$

For accurate work, however, such cells must be calibrated, as the e.m.f. of individual cells may differ appreciably.

WESTON OR CADMIUM CELL. — In Fig. 2 is shown the construction of the "normal" Weston or cadmium cell. The container is of glass suitably mounted in a case provided with binding posts to which the platinum wires are attached. This cell differs from the Clark cell chiefly in the use of cadmium amalgam and cadmium sulphate in place of the zinc amalgam and zinc sulphate

for the negative pole and electrolyte respectively. The particular advantage in the use of cadmium instead of zinc arises from the low temperature coefficient and small temperature lag thus obtainable. It also has a longer life and polarizes less rapidly than the Clark cell.

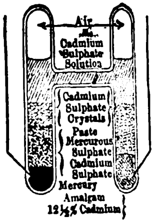

Fig. 2. Weston Cell

Electromotive Force of Weston Normal Cell. — The e.m.f. at 20° C. of the Weston normal cell (the word normal designating that the cadmium sulphate solution is saturated) is 1.0830 international volts. At any other temperature t° C. its e.m.f. is

$$E_t = 1.01830 - 10^{-6}[40.6\,(t-20) + 0.95\,(t-20)^2 + 0.001\,(t-20)^3].$$

(*See Bulletin Bureau of Standards, 1909, Vol. 5, p. 309.*) The e.m.f.'s of normal cadmium cells as set up by various observers following the same specifications differ among each other by less than 1 part in 10,000.

Weston Secondary Standard Cell. — The Weston Instrument Co. make a portable form of cadmium cell, which differs from the normal cadmium cell chiefly in that the cadmium sulphate solution is just saturated at 4° C., and therefore does not contain an excess of cadmium sulphate crystals at ordinary temperatures. This type of cell is to be used between the temperatures of 4° and 40° C. The temperature coefficient is much smaller than that of the normal cell and is negligible for any ordinary measurements; each cell is accompanied by a certificate stating its e.m.f. The extreme variation among 145 cells of this type was found to be 0.0009 volt. This is at present the best form of secondary standard cell, being remarkably permanent. The e.m.f. is approximately 1.0186 international volts; the resistance, roughly, 200 ohms.

PRECAUTION IN USING STANDARD CELLS. — No appreciable current can be taken from a standard cell without alteration of its e.m.f., due to polarization. It is found that the change is not permanent, for the cell gradually recovers its original e.m.f.; of course the cell is unreliable until the recovery is complete. Consequently standard cells should be used only where their e.m.f. is opposed to an equal p.d., as in connection with potentiometers. They should always be protected by a key, which is closed only momentarily, and for preliminary adjustments a high resistance, several thousand ohms at least, should be in series with the cell. When an approximate balance has been obtained this resistance should be cut out and the final balance then made.

COSTS. — A normal Clark cell costs about $12, a normal cadmium cell $15, a portable Weston cell $15.

BIBLIOGRAPHY. — Wolff and Waters, *Bull. Bur. of Stand.*, 1907, Vol. 4, p. 1; Wolff, *Bull. Bur. of Ltd.*, 1909, Vol. 5, p. 309.

[H. Pender and H. R. Ranken.]

CEMENT. — (*See also Concrete.*) The cements used in engineering construction are Portland, natural and Puzzolan (or slag) cements. Of these Portland cement is the most reliable and should always be used in reinforced concrete construction. The processes of manufacture and general characteristics of these cements are given in the following paragraphs, the descriptions being copied from the report of the Joint Committee as published in *Proc. Am. Soc. C. E.*, Feb., 1913.

PORTLAND CEMENT. — This is the finely pulverized product resulting from the calcination to incipient fusion of an intimate mixture of properly proportioned argillaceous and calcareous materials. Portland cement should be used in reinforced concrete construction and in any other construction that will be subject to shocks or vibrations or stresses other than direct compression.

Specifications. — The following specifications are taken by permission from the 1912 *Year Book of the Am. Soc. Test. Mat.*

Specific Gravity. — The specific gravity of cement shall be not less than 3.10. Should the test of cement as received fall below this requirement, a second test may be made on a sample ignited at a low red heat. The loss in weight of the ignited cement shall not exceed 4 per cent.

Fineness. — Portland cement shall leave by weight a residue of not more than 8 per cent on the No. 100, and not more than 25 per cent on the No. 200 sieve.

Time of Setting. — It shall not develop initial set in less than 30 minutes; and must develop hard set in not less than one (1) hour, nor more than ten (10) hours.

Tensile Strength. — The minimum requirements for tensile strength in pounds for briquettes 1 sq. in. in cross-section shall be as follows, and the cement shall show no retrogression in strength within the periods specified:

Age	Neat cement	One part cement, three parts standard Ottawa sand
24 hours in moist air	175	....
7 days (1 day in moist air, 6 days in water)	500	200
28 days (1 day in moist air, 27 days in water)	600	275

Constancy of Volume. — Pats of neat cement about 3 in. in diameter, ½ in. thick at the center, and tapering to a thin edge, shall be kept in moist air for a period of 24 hours.

(*a*) A pat is then kept in air at normal temperature and observed at intervals for at least 28 days.

(*b*) Another pat is kept in water maintained as near 70° Fahr. as practicable, and observed at intervals for at least 28 days.

(*c*) A third pat is exposed in any convenient way in an atmosphere of steam, above boiling water, in a loosely closed vessel for 5 hours.

These pats, to pass the requirements satisfactorily, shall remain firm and hard, and show no signs of distortion, checking, cracking or disintegrating.

Sulphuric Acid and Magnesia. — The cement shall not contain more than 1.75 per cent of anhydrous sulphuric acid (SO_3) nor more than 4 per cent of magnesia (MgO).

NATURAL CEMENT. — This is the finely pulverized product resulting from the calcination of an argillaceous limestone at a temperature only sufficient to drive off the carbonic acid gas. Natural cement does not develop its strength as quickly, nor is it as uniform in composition, as Portland cement. Natural cement may be used in massive masonry where weight rather than strength is the essential feature. Where economy is the governing factor, a comparison may be made between the use of natural cement and a leaner mixture of Portland cement that will develop the same strength.

Specifications. — The following specifications are taken from the 1912 *Year Book of the Am. Soc. Test. Mat.* by permission.

Fineness. — Natural cement shall leave by weight a residue of not more than 10 per cent on the No. 100, and 30 per cent on the No. 200 sieve.

Time of Setting. — It shall not develop initial set in less than 10 min., and shall not develop hard set in less than 30 min., or in more than 3 hours.

Tensile Strength. — The minimum requirements for tensile strength in pounds for briquettes 1 sq. in. in cross section shall be as follows, and the cement shall show no retrogression in strength within the periods specified:

Age	Neat cement	One part cement, three parts standard Ottawa sand
24 hours in moist air	75	....
7 days (1 day in moist air, 6 days in water)	150	50
28 days (1 day in moist air, 27 days in water)	250	125

Constancy of Volume. — Pats of neat cement about 3 in. in diameter, ½ in. thick at the center, tapering to a thin edge, shall be kept in moist air for a period of 24 hours.

(*a*) A pat is then kept in air at normal temperature.
(*b*) Another is kept in water maintained as near 70° Fahr. as practicable.

These pats are observed at intervals for at least 28 days, and, to pass the tests satisfactorily, should remain firm and hard and show no signs of distortion checking, cracking or disintegrating.

PUZZOLAN OR SLAG CEMENT. — This is the finely pulverized product resulting from grinding a mechanical mixture of granulated basic blast-furnace slag and hydrated lime. Puzzolan cement is not nearly as strong, uniform nor reliable as Portland or natural cement, is not used extensively, and never in important work; it should be used only for foundation work underground where it is not exposed to air or running water.

COST OF CEMENT. — As the cost of transportation is an important item in the cost of cement, the market price depends largely upon the place of sale. Quotations are given in the first number each month of *Engineering News*. The following prices are from the issue of this paper for June 5, 1913.

Portland cement at New York,	$1.58 per bbl. in wood with 40 cents rebate on return of barrel.
Portland cement at Los Angeles,	$1.25 per bbl. in bulk at mill.
Portland cement at Chicago,	$1.05 per bbl. in bulk at mill.
Portland cement at Boston,	$1.72 per bbl. in wood with 40 cents rebate on return of barrel.
Portland cement at Boston,	$1.42 per bbl. in paper (4 bags = 1 bbl.).
Portland cement at Pittsburg,	$1.58 per bbl. in bags, with 10 cents per bag rebate on return.
Natural cement at New York,	$0.95 per bbl.
Natural cement at Boston,	$1.15 per bbl.

BIBLIOGRAPHY. — See *Bibliography* in article on *Concrete*.

[C. M. SPOFFORD.]

CHAINS AND CHAIN DRIVE. — Chains for hoisting and similar purposes may be made with straight open links, twisted links or with links with transverse studs to prevent the links from collapsing under heavy loads. These types of chains are known as straight-link chains, twisted chains and stud chains, respectively. The twisted chain is usually employed when the chain has to be wrapped around a smooth drum. Crane chains are straight-link chains carefully made to fit the corrugations of the winding drum.

Chains for transmitting power are of various forms. The more common types are block chains, roller chains and various makes of "silent" chains. In all these chains the links are made of several pieces of metal, the longitudinal pieces being riveted, bolted or screwed to the end pieces or studs. In the block chain, an example of which is the ordinary bicycle chain, the end pieces are solid blocks of metal; in the roller chain the end pieces form studs which carry small rollers. With block or roller chains the power is transmitted from or to the sprocket directly by the studs or rollers, whereas in the various types of "silent" chains, e.g., the Morse and Renold chains, the longitudinal pieces of the links are provided with lugs or fingers which mesh with the teeth of the sprocket wheel and thus pull it around.

Link belts are essentially power transmitting chains made by mounting a number of single chains side by side, so that they all move as a unit. Link belts are used both for power transmission and for belt conveyors.

DATA ON ORDINARY CHAINS (*Penn. R.R. Specifications, 1903*)

Nominal diameter of wire, inches	Description	Maximum length of 100 links, inches	Weight per foot, lb.	Proof test, lb.	Breaking weight, lb.
5/32	Twisted	103 1/8	0.20	....	....
3/16	Twisted	96 1/4	0.35	....	....
3/16	Perfection twisted	151 1/4	0.27	....	....
1/4	Straight-link	102	0.70	1,600	3,200
5/16	Straight-link	114 3/4	1.10	2,500	5,000
3/8	Straight-link	114 3/4	1.60	3,600	7,200
3/8	Crane chain	113 5/8	1.60	4,140	8,280
7/16	Straight-link	127 1/2	2.07	4,900	9,800
7/16	Crane	126 1/4	2.07	5,635	11,270
1/2	Straight-link	153	2.50	6,400	12,800
1/2	Crane	151 1/2	2.60	7,360	14,720
5/8	Straight-link	178 1/2	4.08	10,000	20,000
5/8	Crane	176 3/4	4.18	11,500	23,000
3/4	Straight-link	204	5.65	14,400	28,800
3/4	Crane	202	5.75	16,560	33,120
7/8	Crane	252 1/2	7.70	22,540	45,080
1	Crane	277 3/4	9.80	29,440	58,880
1	Straight-link	280 1/2	9.80	25,600	51,200
1 1/8	Crane	303	12.65	38,260	76,520
1 1/4	Crane	353 1/2	15.50	46,000	92,000
1 1/2	Crane	416 5/8	22.50	66,240	132,480
1 3/4	Crane	479 3/4	30.00	90,160	180,320
2	Crane	555 1/2	39.00	117,760	235,520

SIZE, PITCH, STRENGTH AND WEIGHT OF CHAINS. — The size of an ordinary chain in which the links are made of wire or rods is specified by the diameter of the wire or rod. The pitch of a chain is the distance between the centers of successive links, usually expressed in inches. The proof test of a chain is the specified load it must carry without deformation, and is usually taken as one-half the breaking load. The ordinary safe load is usually taken as about two-thirds of the proof test, or one-third the breaking load.

The safe working load is dependent on the amount of rivet-bearing surface, speed, size of sprockets, etc., and ranges from 1/5 to 1/40 of the tensile strength.

STRENGTH OF ROLLER AND BLOCK "DIAMOND" CHAINS

Pitch, inches	Tensile strength, pounds	
	Roller	Block
1/2	1,200	....
5/8	1,200	....
3/4	4,000	....
1	6,000	1200–2500
1 1/4	9,000	....
1 1/2	12,000	5000
1 3/4	19,000	....
2	25,000	....

CHAIN DRIVE. — The advantages of chain drive are positive speed ratio, capability of transmitting large amounts of power at low speeds, not seriously affected by moisture, oil or grease, no stretch, and for short transmissions greater efficiency than leather, rubber or fiber belting. Roller chains should not as a rule be run at speeds of over 1000 feet per minute, and block chains not over 700 feet per minute.

Wherever possible, the distance between centers of shafts should permit of adjustment in order to regulate the sag of the chain. A chain should be adjusted, in proportion to its length, to show slack when running, care being taken to have it neither too tight nor too loose, as either condition is destructive.

The principal cause of trouble within the chain itself is elongation. It is the result of stretch of material or natural wear. Sudden jars or jolts beyond the limit of elasticity of the material will quickly render the chain useless. If for any reason a link elongates unduly it should be replaced at once, as one elongated link will eventually ruin the entire chain.

To minimize wear, chains should be kept well greased and protected from mud and grit, cleaned often, and when replaced put back so that they run in the same direction and same side up. A new chain should never be applied to a much-worn sprocket.

Sprockets. — Properly proportioned and machined sprockets are essential to successful chain gearing. For block chain these are obtained as follows:

Sprockets should be gauged to discover thick teeth and inaccurate diameters. A poor chain may operate on a good sprocket, but a bad sprocket will ruin a good chain. Sprockets of 12 to 60 teeth give best results. Fewer may be used, but cause undue elongation in the chain, wear the sprockets and consume too much power. Eight-tooth sprockets ruin almost every roller chain applied to them, and ten and eleven teeth are fitted only for medium and slow speeds with other conditions unusually favorable.

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[WM. KENT.]

CHIMNEYS. — (*See also Draft, Mechanical.*) A chimney serves as a means for establishing a sufficient draft through a furnace to produce the combustion of the fuel. The amount of air *required* per hour depends upon the quantity of fuel burned per hour and the quality of fuel. The amount actually used depends also on the manner of firing, or the judgment and skill of the fireman. The *force required* to produce this draft in turn depends upon the quality of the fuel, the method of firing, the thickness of the fuel bed, the design of the furnace, the length and cross section of the gas passages, the height and cross section of the chimney, the location of the chimney with respect to neighboring buildings and hills, and the direction and velocity of the wind which may be blowing. The *force available* for producing the draft depends upon the height and cross section of the chimney and the difference in temperature of the hot gases in the chimney and the cold air outside. On account of the number of these variables and their great range of variation from time to time, it is practically impossible to deduce a rational formula for the size of the chimney required for the combustion of fuel at a given rate. The following approximate method for determining the height and cross section of a chimney, however, has been found satisfactory in practice. The formula connecting the height, cross section and rate of fuel consumption is an empirical one, deduced by the author from numerous actual cases, and first published in 1884. It has been extensively employed with satisfactory results by numerous engineers since that time.

INTENSITY OF DRAFT AND HEIGHT. — The difference in weight between the hot gases inside the chimney and the weight of an equal column of the external air is the cause of the draft produced by the chimney. The pressure due to this difference is called the "intensity" of the draft and is usually measured in inches of water column required to balance it. The measurement is made by a draft gauge, usually a U-tube partly filled with water, one leg connected by a pipe to the interior of the flue between the boiler and chimney and the other open to the external air. For finer and more accurate readings some form of multiplying gauge is used.

Let t_1 be the average temperature of the chimney gases, t_2 the temperature of the external air, both in degrees Fahrenheit, and let H be the height of the chimney in feet. Then the total intensity of draft produced by the chimney, expressed in inches of water column for normal atmospheric pressure, is

$$F = H\left(\frac{7.64}{460 + t_2} - \frac{8.00}{460 + t_1}\right).$$

For an atmospheric pressure of P inches of mercury, multiply the left-hand side by $\frac{P}{30}$.

The intensity of the draft and consequently the velocity and the volume of the hot gases will of course be greater the greater the temperature of the hot gases, but as the temperature increases the volume of a given weight increases, that is, their density decreases. When the temperature of the gases exceeds a certain value the increase in the velocity due to a further increase of temperature will be more than offset by the decrease in density. According to Rankine the temperature corresponding to the maximum value of the product of volume and density, that is the maximum weight of gases discharged, is, under ordinary conditions, 600° F., when the external temperature is 60°. Hence the total intensity of draft F_0 corresponding to maximum weight of discharged gases may be found approximately by substituting 600 in the above formula for t_1, and 60 for t_2. The formula then reduces to $F_0 = 0.007\ H$.

The actual intensity of the draft is usually less than that calculated from the

temperature taken at the bottom of the chimney, on account of the cooling of the gases as they ascend in the chimney and other causes. Taking the reduction at 20 per cent the formula expressing the relation of intensity of draft to height of chimney becomes $F_n = 0.0057\, H$ on the assumption of a temperature of 600° at the bottom of the chimney.

MINIMUM HEIGHT. — Hence when the intensity of the draft necessary to produce the necessary current of air through the breeching, boiler and furnace is known, the minimum height of chimney which will produce this draft is

$$H = 175\, F_n \text{ feet,}$$

where F_n is the net intensity of draft in inches of water required.

The net intensity of draft required depends upon the resistance of the gas passages between the ash pit and base of stack. Gebhardt gives the following approximate rules, which, however, should be used only as a very rough guide, since the loss of draft in the various parts of the gas passages depends upon the size and shape of these passages, the velocity of the gases, etc.

Loss of draft in breeching: 0.1 inch of water column per 100 feet of flue.

Loss of draft in breeching: 0.05 inch for each right angle bend.

Loss of draft in boiler: 0.2 to 0.4 inch, depending upon type of boiler.

Loss of draft in furnace: depends upon the kind of coal, the kind of grate, the caking, the clinker, the thickness of the bed of coal and the area of the grate. The loss of draft is greatest with small anthracite coal (No. 3 Buckwheat) and least for high-grade bituminous coal. The following table, from curves published by the Stirling Co., gives roughly the loss of draft in the furnace which may be expected under average conditions.

APPROXIMATE LOSS OF DRAFT BETWEEN ASH PIT AND FURNACE, INCHES OF WATER

Lb. of coal per sq. ft. grate surface per hour	10	15	20	25	30	40	50
No. 3 buckwheat	0.4	0.8	1.3	....	....	....	....
No. 1 buckwheat	0.2	0.4	0.7	1.0	....	....	....
Pea coal	0.15	0.3	0.5	0.7	0.9	....	....
Semi-bituminous (run of mine)	0.1	0.15	0.2	0.3	0.4	0.6	1.0
Bituminous (slack)	0.07	0.1	0.15	0.2	0.3	0.4	0.6
Bituminous (run of mine)	0.05	0.08	0.10	0.15	0.2	0.3	0.4

Example: Find the minimum height of chimney for a boiler burning 30 pounds of bituminous run-of-mine per hour per square foot of grate surface, the flue being 100 feet long with two right angle bends

$$F_n = 0.1 + 2 \times 0.05 + 0.4 + 0.2 = 0.8,$$

assuming a loss of 0.4 inch in the boiler

$$H = 170 \times 0.8 = 136 \text{ feet.}$$

In any particular case the loss of draft in the various parts of the gas passages may differ considerably from the figures given above, and consequently the height of chimney calculated on the above assumptions should be looked upon

merely as a rough approximation. The final design of the chimney should be left to an engineer of experience in this kind of work.

According to C. L. Hubbard (*Am. Elec., Mar., 1904*), the following heights have been found to give good results in plants of moderate size (500 horse-power or less), with sufficient draft to force the boilers 20 to 30 per cent above their rating:

With free-burning bituminous	75 feet
With anthracite of medium and large size	100 feet
With slow-burning bituminous	120 feet
With anthracite pea	130 feet
With anthracite buckwheat	150 feet
With anthracite slack	175 feet

For plants of 700 or 800 horse-power or over, the chimney should not be less than 150 feet high regardless of the kind of coal used. It is evident that the figures in the above table are subject to wide variations with different qualities of coal as regards caking and non-caking, clinkering and non-clinkering, etc., with different proportions of grate to heating surface and with different kinds of grates and furnaces.

On account of the elimination of the grate in oil-burning boilers and the injection of oil under pressure the height of a chimney for an oil-burning boiler is less than for a coal-burning boiler of the same capacity. According to Gebhardt a height of from 80 to 90 feet is sufficient to force oil-burning boilers to 50 per cent above their rating. In large oil-burning plants, however, the chimney is usually designed on the coal-burning basis, in order to permit the use of coal should this subsequently prove desirable.

CROSS SECTION OF CHIMNEY.—When the height of the chimney has been decided upon, the proper cross section may be determined as follows: Let

C = pounds of coal to be burned per hour,
H = height of chimney in feet,
A = internal cross section of chimney in square feet,
D = internal diameter of chimney, if round, in inches,
S = side, internal, of chimney, if square, in inches,
E = "effective" internal cross section in square feet,
P = total rated boiler horse-power.

Then

$$E = \frac{3C}{50\sqrt{H}}, \qquad D = 13.54\sqrt{E} + 4, \qquad S = 12\sqrt{E} + 4,$$

$$A = 0.0546\,D^2 = \frac{S^2}{144}.$$

Allowing 5 pounds of coal per rated boiler horse-power per hour the formula for E may be written

$$E = \frac{3P}{10\sqrt{H}}.$$

Five pounds of coal per boiler horse-power per hour is a liberal allowance to cover the contingencies of poor coal being used, and of the boilers being driven beyond their rated capacity. In large plants with economical boilers, good fuel and other favorable conditions, the maximum rate of coal consumption will be considerably less than 5 pounds (*see Boilers*). Calling R

the maximum rate of coal consumption in pounds per hour per rated boiler horse-power, the formula for E may be written

$$E = \frac{3\,RP}{50\sqrt{H}}.$$

STABILITY — THICKNESS OF WALLS. — All chimneys of any considerable size should consist of an outer stack of sufficient strength to give stability to the structure, and an inner stack or core, independent of the outer one, to protect the latter from the high temperature of the hot gases. This core sometimes extends up to a height of but 50 or 60 feet above the base of the chimney, but better practice is to extend it the full height or to 100 to 120 feet in chimneys over 120 feet high. The core is usually constructed of fire brick; the outer stack may be built of common brick or radial brick (the Custodis chimney is an example of the latter) or of steel or of reinforced concrete. To determine whether a chimney is stable, treat it as a cantilever loaded throughout its length with a load equal to the total pressure (*see Elasticity and Strength; Wind Pressure*); at no section of the chimney should the stress in the walls exceed the safe working stress for the material of which it is constructed.

Brick Chimneys. — A general rule for diameter of base of brick chimneys, approved by many years of practice in England and the United States, is to make the diameter of the base one-tenth of the height. If the chimney is square or rectangular, make the diameter of the inscribed circle of the base one-tenth of the height. The "batter" or taper of a chimney should be from 1/16 to 1/4 inch to the foot on each side. The brickwork should be one brick (8 or 9 inches) thick for the first 25 feet from the top, increasing 1/2 brick (4 or 4 1/2 inches) for each 25 feet from the top downwards. If the inside diameter exceeds 5 feet, the top length should be 1 1/2 bricks and if under 3 feet, it may be 1/2 brick for 10 feet.

Reinforced Concrete Chimneys began to come into extensive use in this country in 1901 and several hundred of them are now in use. Reinforced concrete chimneys built by the Weber Co., of Chicago, consist of two parts, the lower double shell and the single shell above. The inside shell is usually 4 inches thick, while the thickness of the outside shell depends upon the height and varies from 6 to 12 inches. The single shell is from 4 to 10 inches thick. The height of the double shell depends upon the height of the chimney, nature and temperature of the gases, etc. The bending forces caused by wind pressure are taken up by vertical steel reinforcement; the resistance of the concrete itself against tension is not considered in calculation.

Steel Chimneys are largely used, especially for tall chimneys of iron-works, from 150 to 300 feet in height. The advantages claimed are: greater strength and safety; smaller space required; smaller cost, by 30 to 50 per cent, as compared with brick chimneys; avoidance of infiltration of air and consequent checking of the draft, common in brick chimneys. They are usually made cylindrical in shape, with a wide-curved flare for 10 to 25 feet at the bottom. A heavy cast-iron base-plate is provided, to which the chimney is riveted, and the plate is secured to a massive foundation by holding-down bolts. No guys are used.

Sheet-iron Smokestacks are relatively cheap in first cost, but depreciate rapidly. A heavy foundation is not necessary, but the stack may be supported directly on the boiler breeching. The stacks are not self-supporting but must be held upright by guy wires.

WEIGHT OF SHEET-IRON SMOKESTACKS PER FOOT (Porter Mfg. Co.)

Diam., in.	Thickness, W.G.	Lb. per ft.	Diam. in.	Thickness, W.G.	Lb. per ft.	Diam., in.	Thickness, W.G.	Lb. per ft.
10	No. 16	7.20	26	No. 16	17.50	20	No. 14	18.33
12	"	8.66	28	"	18.75	22	"	20.00
14	"	9.58	30	"	20.00	24	"	21.66
16	"	11.68	10	No. 14	9.40	26	"	23.33
20	"	13.75	12	"	11.11	28	"	25.00
22	"	15.00	14	"	13.69	30	"	26.66
24	"	16.25	16	"	15.00	...	.....	.....

FOUNDATIONS. — Chimney foundations are as a rule constructed of concrete except where the nature of the soil necessitates the use of piles or a grillage of timber or steel. (For the bearing power of various soils see section on *Foundations* in article on *Power Stations*.) For masonry chimneys the foundation is designed to give the necessary support for the shaft without particular reference to its weight and shape. In steel and reinforced concrete chimneys the weight and shape of the foundation are important factors in securing the proper stability of the chimney, since the shaft is usually securely fastened to the foundation and the two form practically one mass.

SIZES OF FOUNDATIONS FOR HALF-LINED STEEL CHIMNEYS
(Selected from circular of Phila. Engineering Works.)

Diameter, clear, feet	3	4	5	6	7	9	11
Height, feet	100	100	150	150	150	150	150
Least diam. foundation, ft. and in.	15–9	16–4	20–4	21–10	22–7	23–8	24–8
Least depth foundation, feet	6	6	9	8	9	10	10
Height, feet	.....	125	200	200	250	275	300
Least diam. foundation, ft. and in.	.....	18–5	23–8	25	29–8	33–6	36
Least depth foundation, feet	.....	7	10	10	12	12	14

BREECHING. — The area of the flue or breeching leading from the boilers to the stack is usually made 20 per cent greater than that of the stack. When several boilers discharge into the same flue its section may be tapered and proportioned to the number of boilers. When two flues enter the stack on opposite sides at the same level a diaphragm is inserted between the two openings.

COSTS. — The cost of chimneys in large power stations ranges from $1.85 to $2.50 per rated boiler horse-power. Christie (*Chimney Design and Theory*) gives the following costs of chimneys 150 feet high and 8 feet internal diameter.

Common red brick	$8500
Radial brick	6800
Steel, self-supporting, full lined	8300
Steel, self-supporting, half lined	7800
Steel, self-supporting, unlined	5800
Steel, guyed	4000

The following approximate cost of radial brick chimneys (adapted from Gebhardt, *Steam Power Plant Engineering*) will serve to indicate the variation in cost with height and diameter:

COST OF RADIAL BRICK CHIMNEYS

Diameter in feet	Height in feet					
	75	125	150	175	200	250
4	$1400	$3500	.....	.....	.....	.....
6	2000	4300	.....	.....	.....	.....
8	2700	4700	$6200	$7100	$9,300	.....
10	3700	5100	7100	7900	10,500	$16,500
12	.....	.....	7800	9000	11,100	18,300
14	.....	.....	8300	9700	12,500	21,500
16	.....	.....	.....	.....	.....	24,300

Sheet-iron smokestacks cost from 3.5 to 6.5 cents per pound, the higher unit price referring to the smaller sizes.

Brick and concrete stacks involve little expense for maintenance, 2 per cent being a liberal allowance for both maintenance and depreciation. Steel and sheet-iron stacks require occasional painting to prevent excessive corrosion.

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[Wm. Kent.]

CIRCUIT BREAKERS. — (*See also Bus-Bars; Switchboards; Switches; Switchgear Equipment for Power Stations.*) A circuit breaker is a device to open an electric circuit which is normally held closed by the action of a latch, toggle or similar mechanism. This tendency to open is the feature that usually distinguishes a circuit breaker from a switch, as the latter, although used to open electric circuits, is without any latch or similar device and hence will remain in the closed position. This distinction between switches and circuit breakers is not always followed, as circuit breakers which open in oil are frequently spoken of as "oil switches."

TYPES OF CIRCUIT BREAKERS. — When a circuit breaker automatically opens the circuit as the result of certain abnormal conditions, such as overload, the circuit breaker is said to be "automatic," but if the breaker does not open unless tripped by the attendant it is spoken of as "non-automatic." An automatic breaker usually can be released by hand also. A breaker designed to be closed directly by the operator is spoken of as "direct controlled," while a breaker mounted at a distance is "remote controlled," The latter can be "hand operated" if worked through a system of bell cranks, levers or similar devices; "electrically operated" if worked by motor or solenoid; or "pneumatically operated" if controlled by compressed air.

Small circuit breakers, particularly of low voltage, are usually direct controlled, and large-capacity high-voltage breakers are usually remote controlled, some auxiliary source of power being employed. For such distant-controlled breakers automatic operation is secured through relays (*see Relays*). Relays are also frequently used with direct-controlled breakers.

Underload, Over-voltage, Under-voltage and Reverse Current Breakers. — The principal demand for circuit breakers is to have them open the circuit when the current reaches a certain predetermined value, and breakers are designed with this end in view. They are also built for underload conditions to open on minimum current, for over-voltage to open when the voltage exceeds a certain amount, for under-voltage to open when the voltage falls below a certain minimum value, for reversal when the current flows in the opposite direction through the breaker from that which was intended. It is, of course, possible to combine these various features of overload, underload, reversal, etc., in one and the same breaker. (*See Relays.*)

PRINCIPLE OF OPERATION. — The opening of a circuit breaker is usually secured by the releasing of a latch or the upsetting of a toggle joint that keeps the breaker closed. Various schemes have been adopted to secure the adjustment or "calibration" of the overload tripping device, so that it may be made to operate within reasonably wide limits, usually 80 to 160 per cent of the normal rating.

Breakers Opening in Air. — Fig. 1 shows the general arrangement of the magnetic circuit of a typical carbon-break circuit breaker, arranged for hand or solenoid operation, as used on heavy-capacity d-c. and low-voltage a-c. circuits. The current passes in at one stud across the contact brush and back through the other stud, thus forming a loop or turn in the electric circuit. A U-shaped iron frame is placed around the lower stud and the movable armature completes the magnetic circuit.

The current passing through the breaker magnetizes the iron circuit and tends to lift the movable armature. At a certain predetermined current the attraction overcomes the force of gravity, and the raising of the moving arm opens the latch or upsets the toggle which holds the breaker closed.

Two methods of securing the necessary range or calibration can be employed, one by varying the air gap between the stationary and moving iron, and the

other by giving the electromagnet more work to do by increasing the weight to be lifted. This latter method can employ either additional weights at a fixed

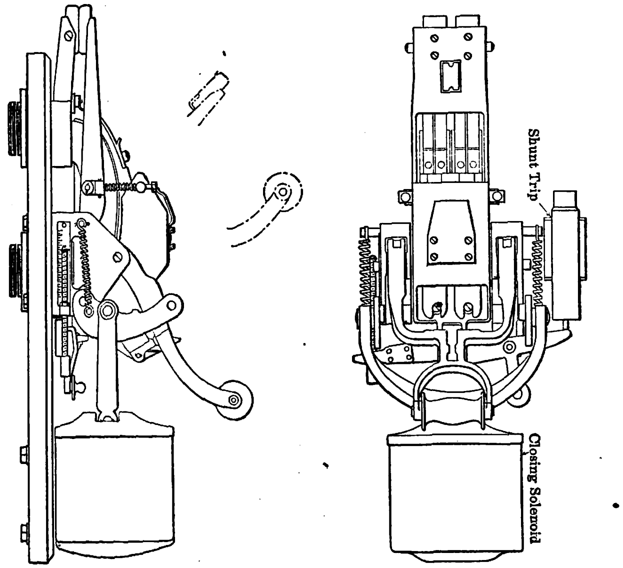

Fig. 1. Carbon-break Circuit Breaker

distance from the fulcrum, or a fixed weight at a variable distance from the fulcrum.

For small currents the electrical circuit is carried more than once around the iron circuit, which results in a solenoid design with a movable plunger. With this scheme the tripping range is secured by adding weights to the solenoid or varying the air gap.

Oil Circuit Breakers (Figs. 2 to 6). — With high-voltage oil circuit breakers the tripping coil is usually a solenoid of a fairly large number of turns, which is connected in series with the main circuit or serves as the secondary of a current transformer whose primary is connected in the high-tension circuit. For polyphase circuits two or more tripping coils are used.

DESIGN. — For different classes of service various types of circuit breakers are used, and the functions of these types overlap more or less. The following table gives the approximate values of the maximum normal current and voltage for which these various types are used at present.

In connection with the carbon-break circuit breaker 24,000 amperes is the largest size in actual service, but larger breakers could be built if desired. 2400 volts is the maximum d-c. voltage at present used for railway service in America, but carbon breakers can be built for higher d-c. voltages if desired; they have been used up to 22,000 volts for a-c. service.

The fuse type has not been built for more than 100 amperes or 66,000 volts, but could be made for higher voltages if desired.

Type	Maximum current, amperes	Maximum voltage
Carbon break	24,000	2,400
Fuse type	100	66,000
Magnetic blow-out	10,000	750
Oil break	4,000	150,000

The magnetic blow-out type has been built in capacities up to 10,000 amperes for d-c. railway service, but in large sizes has been practically superseded by the carbon break.

The oil-break type is in service for capacities up to 4000 amperes and for voltages up to 150,000, but designs have been made for larger currents and higher voltages and these can be supplied to meet whatever demand may exist.

Air-break Carbon-type Breakers are made in a variety of designs; Fig. 1 shows a typical single-pole carbon-break circuit breaker. As shown in the figure, the circuit-breaker mechanism consists of a swinging arm carrying the main and the arcing contacts actuated by a handle and toggle-joint mechanism. The main contacts are of copper and are laminated to insure a perfect contact with the copper blocks which are fastened to the base and connected to the line. Above the copper contacts are the arcing contacts of carbon. These contacts consist of one or more carbons carried by a swinging arm and pressed firmly against stationary contacts when the breaker is closed. These carbons are mounted on a pivot to insure proper adjustment.

When the breaker opens the current is gradually shifted through the copper shunts to the carbon contacts, and thus no arc is formed until the final break takes place between the carbon contacts at the top. This arrangement, combined with the natural tendency of an arc to rise, prevents any injury to the breaker by the arc.

Heavy-capacity carbon-break circuit breakers are also made motor-operated, in which case the breaker has a device that disconnects the worm gear and shuts down the motor when the breaker has closed and its toggle has locked. It is also so arranged that if an overload or short-circuit exists when the breaker is closed, the breaker will immediately trip out even though the control switch is held in the "close" position. This feature of being "nonclosable on overload" is one that is embodied in a large number of breakers of all kinds, and is a very valuable and useful one.

Switch and Breaker for Rotary Converters. — A combination motor-operated switch and circuit breaker has been supplied for various rotary converters and 600-volt feeder circuits supplying current for third-rail electrifications. One motor is provided for each combined circuit breaker and switch, and it has suitable clutches, shafts, operating rods and mechanism, so that in the act of closing the circuit breaker is first thrown in and the closing device is then disconnected before the swtich is thrown in, so that in case of trouble the breaker can immediately trip out and open the circuit.

Air-break Fuse-type Breaker. — This type is a modification of the carbon-break type and is intended for moderate capacity circuits for voltages of 6000 to 60,000. This breaker consists essentially of a stationary and a movable arm mounted on suitable insulators supporting the line connections. These arms are hinged together at one end. The free ends are held together ...

a strong spring by a piece of aluminum fuse wire that passes through a blow-out tube. When the current exceeds a certain amount, the fuse wire melts and the free end of the movable arm swings away from the stationary arm, the arc being ruptured in a blow-out tube.

Magnetic Blow-out Type Breaker.—This type of breaker is provided with auxiliary contacts that open in a strong magnetic field which blows out the arc formed at these auxiliary contacts. The main contacts are solid copper blocks bridged by a laminated copper brush. These open first when the breaker operates, leaving the final arc to be broken on the auxiliary contacts, which are protected by the magnetic blow-out device and which can be readily renewed at comparatively small expense. Although breakers of this type have been installed on the switchboards of power plants, they have been practically superseded by carbon breakers for this work. For such service as the protection of the circuits on a railway car, requiring a few hundred amperes at 600 volts, the magnetic blow-out principle is used to advantage, as the discharge vent from the blow-out compartment can be so set that the vapors from the arc will do no damage.

Oil Circuit Breaker (Figs. 2 to 6).—The essential feature of this type of breaker is the opening of the circuit under oil and the smothering of the arc in a restricted space. These breakers are used almost to the exclusion of all other designs on a-c. circuits of large capacity and high voltage. Oscillograph tests show that the circuit is interrupted at the time of zero point of the alternating-current wave, and, therefore, there is little tendency to set up surges in the circuit. The rapid interruption of a direct current, however, is apt to set up surges, and therefore the oil-break design is not commonly used with d-c. service.

Oil-break circuit breakers are designed to meet various conditions of current, voltage and amount of power that they might be called upon to handle in case of a short-circuit. For moderate amounts of power where the size and cost of the breaker is to be kept to a minimum it is often possible to locate all the poles in one oil tank. For slightly larger amounts of power each pole is in a separate oil tank but all the poles are mounted on the same frame. For still greater amounts of power at moderate voltages each pole is in a separate tank and each tank in a separate compartment. For very high-voltage work each pole is in a separate steel tank of such substantial construction as to be proof against any explosion due to the effects of a short-circuit.

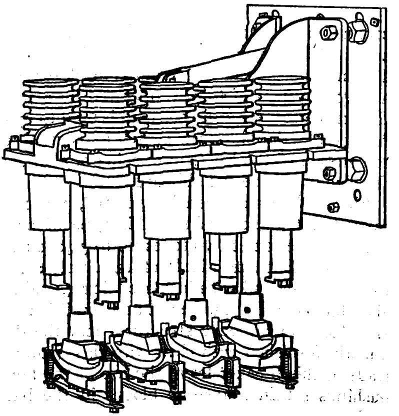

Fig. 2. 300-ampere, 15,000-volt, 4-pole Circuit Breaker

Fig. 2 shows a 300-ampere, 15,000-volt, 4-pole breaker. This type is built in capacities up to 2000 amperes at 600 volts, and up to 200 amperes at 22,000 volts. In this type of breaker each pair of contacts forming each pole is in a separate tank with insulating lining. This breaker is readily arranged for hand operation or electrical operation, can be

mounted on a wall, framework, switchboard panel, or on suitable supports, and may be placed in a masonry structure if desired.

Fig. 3 shows a solenoid-operated breaker of a design that is built in single-pole units in capacities up to 4000 amperes at 600 volts, and 100 amperes at 33,000 volts. Each pole of the breaker is intended for mounting in a masonry

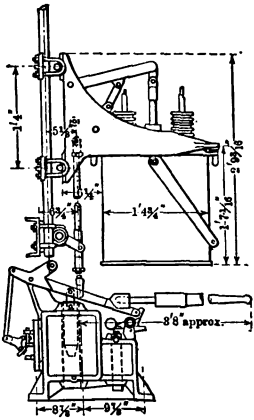

Fig. 3. 300-ampere, 15,000-volt, Solenoid Operated, Wall Mounting Oil Circuit Breaker

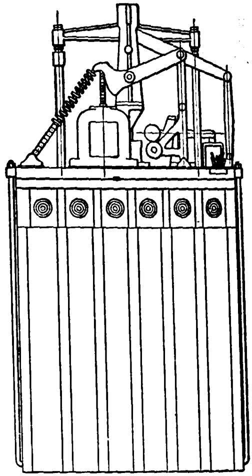

Fig. 4. Three-pole, Solenoid Operated, 13,200-volt, Oil Circuit Breaker

compartment, or on a framework, and 2-, 3-, or 4-pole units are made by connecting the mechanisms of 2, 3, or 4 poles to the same solenoid operating system. The mechanism and the terminal insulators are mounted on a suitable base and a simple system of toggles operated by a powerful solenoid is used for closing the breakers. A second solenoid is used to upset the toggle and trip the breaker. A two-pole, double-throw switch is mounted on the breaker and is operated by the motion of the levers in opening or closing the breaker; this switch controls the signals on the switchboard.

Fig. 4 shows a solenoid-operated, top-connected breaker of 600 amperes capacity at 13,200 volts. This type of breaker is built in capacities up to 3000 amperes and in voltages up to 33,000 and is used in stations of the largest capacity. Both terminals of each pole are in a single oval steel tank with welded seams and with insulated lining, and each tank is provided with a gauge glass for observing the height and condition of the oil. The breakers in service are placed in masonry structures and the doors of the compartment are frequently furnished with clear wire-glass panes to permit ready inspection. The leads of this breaker leave the top of the tank and pass out through porcelain bushings set in soapstone blocks in the back wall of the structure. These leads usually come out in separate compartments that keep them isolated from each other. The leads to the bus-bars, feeders, generators, etc., may all run upward or all downward, or some of them up and some down.

Fig. 5 shows one element and the controlling device for an 88,000-volt, 3-pole, 300-ampere breaker arranged for distant electrical control. Each pole of the

breaker is located in a welded steel tank with treated lining. All its mechanism is mounted complete on its cast-iron top. A 3-pole breaker is made up of three such elements, entirely independent of each other except that they are connected by a single operating rod. The spacing of the poles can thus be made to suit the station wiring. For mechanical operation the solenoid is replaced by a bell-crank device. The condenser bushing leads that form the stationary terminals can readily be unclamped and removed through the cover. The series transformers for the operation of ammeters and relays are clamped directly around the condenser bushing leads, which form the single-turn primary. This permits the use of a simple, compact, and cheap form of series transformer.

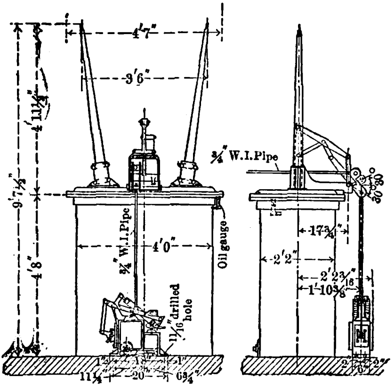

Fig. 5. Solenoid Operated, 88,000-volt, Oil Circuit Breaker, Indoor Type

Fig. 6 shows the corresponding 88,000-volt electrically-operated breaker intended for outdoor service. Its main features correspond closely with those in Fig. 5 previously described. For outdoor service the condenser-bushing leads of the breaker are covered with a series of porcelain insulators and the space between the bushing and the insulators is filled with a moistureproof compound. The operating mechanism is covered by a metallic hood and the pull rods connecting the poles of the breakers pass through pipes. The various joints are made thoroughly waterproof. Outdoor breakers of this design have been operating very satisfactorily on the lines of the Dominion Power and Transmission Co., near Hamilton, Ontario; the Niagara, Lockport and Ontario Power Co.'s lines near Niagara Falls, New York; and the Southern Power Company's circuits, and other plants.

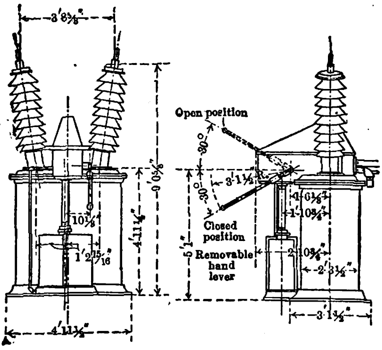

Fig. 6. Solenoid Operated, 88,000-volt, Oil Circuit Breaker, Outdoor Type

RATING — (*See below for Table of Sizes.*) The rating of a circuit breaker is usually given as the normal current which it will carry continuously with a temperature rise not exceeding 28° C. for carbon breakers and 50° C. for oil breakers, above the surrounding air. This rating has no direct relation to the maximum short-circuit current which the breaker will safely interrupt. (*See below under Ultimate Capacity.*) The tripping range of the breaker is usually

from 80 to 160 per cent of this normal rating. The cross-section of the conductors and the area of contact surfaces depend so much upon the character of design, the cooling effect of bodies of oil and masses of metal, that no definite current density can be given as an average value for the various types of breakers.

Ultimate Breaking Capacity. — (*See table below.*) When a short circuit or ground occurs on a line, the rush of current thereby produced depends upon: (1) the total resistance and reactance of all parts of the circuit through which flows this current or other currents induced thereby, (2) the demagnetizing action of these currents on the fields of the generators producing them, and (3) the lag of this demagnetizing action behind the current producing it. The ability of a breaker to interrupt, without damage to itself, a current of a given magnitude depends upon various features of design, and it is evidently impossible to state how much synchronous or transforming apparatus may be safely connected directly to a breaker of a given rating, unless the conditions of operation are also fully specified. In the table below the "ultimate capacity" of various breakers is given in terms of the total capacity of synchronous apparatus, in normal kilovolt-amperes, which may feed directly into a short circuit through the given breaker when provided with an instantaneous release without causing damage to the breaker upon opening. The breakers are considered to be connected with no other reactance in the circuit than the inherent reactance of the synchronous apparatus assumed as 8 per cent.

Effect of Circuit Characteristics on Ultimate Capacity. — The ultimate kilovolt-ampere capacity of the breakers given in the table below would be approximately as follows under the conditions stated.

Ultimate capacity for any other reactance in the circuit increases directly in proportion to the total reactance.

When a breaker is separated from all sources of energy by one or more transformers which have a total kilovolt-ampere rating very much less than the kilovolt-ampere rating of the generators, the kilovolt-ampere capacity of the breaker need not be greater than the kilovolt-ampere rating of these transformers, provided their reactance is not less than 8 per cent. When a breaker is provided with a time-limit overload release of 2 seconds or more, its capacity in kilovolt-amperes will be approximately twice that given in the table.

When a breaker is used on a circuit of lower voltage than that for which it is designed, the kilovolt-ampere capacity of the breaker will be increased in the ratio of the voltage decrease. Most careful consideration should be given the relation of breakers to other apparatus and bus bars with the view of minimizing the risk of danger from the expulsion of inflammable gases, realizing that the risk increases as circuit voltage and station capacity increase.

INSTALLATION. — Carbon-break circuit breakers are usually mounted at the top of a switchboard panel and ample head room is provided to prevent an arc from causing damage. Magnetic blow-out breakers, when placed on a switchboard, are usually located at the top for similar reasons. Fused circuit breakers are usually placed on the station wall or a framework with ample space around them. Oil circuit breakers of the smaller sizes can be mounted directly on the rear of a switchboard panel or on an auxiliary framework or in masonry compartments. The larger capacity breakers are almost invariably placed with each pole in a separate masonry compartment. The very high voltage breakers are mounted in the open.

DIMENSIONS, WEIGHTS AND COSTS. — Although the dimensions, weights, costs, etc., of breakers of various manufacturers naturally differ and the same manufacturer frequently builds various grades for the same capacity, the information given below will be an approximate guide for breakers made by

The dimensions of height, breadth and depth for carbon breakers refer to the switchboard mounting. The depth is given from the front of the panel, i.e., the length of the studs projecting through the panel is neglected. With the oil-break types for switchboard mounting the dimensions are from the back of the panel, i.e., the length of handle sticking through the board is neglected. For the wall-mounted circuit breakers the dimensions include the cells although the weights and prices do not. For the high-voltage breakers the dimensions are for the minimum spacing recommended for any particular voltage. The dimensions are in every case the maximum over-all dimensions except as mentioned.

The costs given below are the approximate selling prices at the time of the preparation of this handbook (1913) and are sufficiently accurate for the preparation of preliminary estimates, but for accurate work should be checked by actual quotations from a reliable builder.

RATING, ULTIMATE CAPACITY, DIMENSIONS, WEIGHTS AND COSTS

Mark	Amperes	Volts	Ultimate capacity, kv-a.	Height, inches	Width, inches	Depth, inches	Weight, pounds	Prices, dollars
A	200	750	......	9 13/16	2½	8¼	5	22.00
A	800	750	......	16	4⅝	12¾	30	47.00
A	2000	750	......	27⅝	9 9/16	13¾	200	157.00
A	4000	750	......	27⅝	9 9/16	13¾	300	255.00
B	300	7,500	2,600	24½	11 9/16	23⅜	155	71.00
C	300	15,000	5,000	25 7/16	13½	17	220	93.00
C	600	9,000	7,500	28 7/16	15⅞	18⅝	270	130.50
C	1200	6,600	7,500	29 11/16	20½	23⅝	420	194.50
C	2000	2,500	7,500	32 7/16	21⅝	23	600	507.00
D	300	15,000	12,500	45 15/16	46	32⅞	775	293.00
E	3000	15,000	40,000	99⅞	64¼	47 9/16	5500	2200.00
E	600	15,000	60,000	91¾	47½	41 13/16	3500	657.00
F	300	44,000	60,000	71⅝	97¼	38¾	900	564.00
F	300	66,000	60,000	90 7/16	121¾	46¼	1800	835.00
F	300	88,000	60,000	113½	139⅛	59½	3600	1236.00
F	300	110,000	60,000	127 11/16	137¾	68½	7200	1800.00
G	300	44,000	60,000	71⅝	97¼	38¾	1125	1019.00
G	300	110,000	60,000	127 11/16	137¾	68½	9000	2596.00

A = Single-pole, carbon-break circuit breaker (Fig. 1).
B = 3-pole, hand-operated oil circuit breaker — all contacts in same tank.
C = 3-pole, hand-operated, oil breaker, separate tanks — common frame (Fig. 2).
D = 3-pole, solenoid-operated breaker — separate tanks — cell mounting (Fig. 3).
E = 3-pole, solenoid-operated oil breaker — separate tanks — cell mounting (Fig. 4).
F = 3-pole, solenoid-operated oil breaker — steel tanks — indoor mounting (Fig. 5).
G = 3-pole, solenoid-operated oil breaker — steel tanks — outdoor mounting (Fig. 6).

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[S. Q. HAYES.]

COLLECTORS, CURRENT. — (*See also Cars, Electric; Locomotives, Electric; Third-Rail Systems; Trolley Systems, Overhead; Trolley Systems, Underground.*) Current collectors for electric cars or locomotives are divided into three classes in accordance with the form of the working conductor from which they collect current, as follows:

(*a*) Overhead Trolley, which may be of the wheel, scraping bow or roller type.

(*b*) Third-Rail Shoes, which may be of the over-running or under-running type.

(*c*) Underground Conduit Plow.

WHEEL TROLLEY. — The wheel trolley consists of a grooved brass or copper wheel held in bearings in a prong called a "harp" at the end of a steel pole which is pressed upward by a system of springs and levers. The trolley wire is from 18 to 22 feet above the rails. The pole presses the wheel upward against the wire with a pressure of from 20 to 40 pounds, the higher pressure being used for the higher speeds. For single cars requiring a current less than 200 amperes at full speed this type of current collector is most satisfactory. The current which it can collect is limited at the various speeds as indicated in the accompanying table. If greater currents are required, resort must be had to the third rail. The trolley wheel is not very satisfactory on cars in trains as each trolley pole requires the attention of a conductor to replace it at curves and switches, and moreover, it is difficult to manipulate the trolley pole from a platform between two cars.

Miles per hour	Amperes
5	1200
15	600
40	350
60	200

BOW OR SCRAPER TROLLEY. — This type of trolley is used to collect current from a high-voltage conductor because it is self-adjusting and needs no attention. At high voltages its current capacity, limited at high speeds to about 100 amperes, is sufficient to supply power for a train of considerable size but it would not have sufficient capacity to supply power for heavy trains at 600 volts. The bow may be held up either by simple springs in the same manner as the trolley pole or it may be mounted upon a pantagraph mechanism which is held up by springs and folded down by a compressed-air cylinder. A special form of overhead construction must be used for the bow trolley as the bow would strike the downwardly projecting ears and the guy wires of the ordinary trolley construction. The roller trolley may be used in place of the bow on the same form of mechanism. The roller has a greater current capacity and causes less wear on the working conductor than the bow.

THIRD-RAIL SHOES. — A third rail will collect currents as high as 2000 amperes at low speeds and 600 amperes at 60 miles per hour. As they are self-adjusting two or three may be placed on a locomotive or car just as well as one, so that there is practically no limit to the current that can be collected in this way. In fact it is always customary to put two on each side of each locomotive or car in order to prevent a cessation of current when passing over breaks in the third rail due to switches or crossings. The relative position of the third rail with respect to the running rails is a very important matter which has not been thoroughly standardized as yet, but has been the subject of much discussion. It must be placed so that all the rolling stock on the road will clear it.

Over-running vs. Under-running Types. — The third rail for top contact or for over-running shoe is cheaper to install, to protect and to maintain and is most generally used. The under-contact rail is less liable to trouble from sleet,

UNDERGROUND CONDUIT PLOW. — The underground plow consists of an insulated steel plate hung from a movable structure on the car. On the two sides of this plate and thoroughly insulated from it are two shoes pressed outward from the plate by springs. These shoes press against the two working conductors which are usually steel Tees separated from each other by about 6 inches and supported on some form of ceramic insulator. The current is led from the shoes to the car body by flexible insulated conductors.

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[W. I. SLICHTER.]

COMPLEX QUANTITIES. — (*See also Vectors.*) The square root of a negative quantity is called an "imaginary" quantity, or a pure imaginary. A quantity consisting of the sum or difference of a real quantity and an imaginary quantity is called a "complex" quantity. For example, $\sqrt{-3}$ is a pure imaginary, and $2+\sqrt{-3}$ is a complex quantity. All the rules of ordinary algebra apply to pure imaginaries and complex quantities. For example, $\sqrt{-3}$ may be written $\sqrt{-1}\sqrt{3}$, and in general $\sqrt{-a}$, where a is a positive quantity, may be written $\sqrt{-1}\sqrt{a}$. The square root of minus one is called the imaginary unit and is usually represented by the symbol j (writers on pure mathematics use the symbol i), that is,

$$j=\sqrt{-1}.$$

Any complex quantity may then be written

$$a+jb,$$

where a and b are both real quantities.

Geometrical Representation of a Complex Quantity. — A positive real quantity may be represented by a line drawn in a given direction; a negative real quantity may be represented by a line drawn in the opposite direction. Multiplying a quantity by -1 then reverses its direction. Also, since multiplying a real quantity by $\sqrt{-1}$ twice is equivalent to multiplying it by -1, the operation of multiplying once by $\sqrt{-1}$ may be represented by turning the line representing the quantity through 90° in the positive direction of rotation. The positive direction of rotation is taken as the opposite direction to that in which the hands of a clock move. Hence, a complex quantity $a+jb$ may be represented by the line OP in the figure, where $OA=a$ and $AP=b$. The complex quantity $a+jb$ is then completely specified by a line of length $\sqrt{a^2+b^2}$ making an angle θ, with the axis of reference OX where $\tan\theta=\frac{b}{a}$.

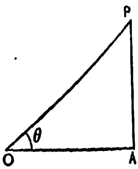

Fig. 1

The length $M=\sqrt{a^2+b^2}$ is called the magnitude of the complex quantity and the angle $\theta=\tan^{-1}\frac{b}{a}$ is called its angle. From the figure it is evident that the complex quantity $a+jb$ may also be written

$$a+jb=M(\cos\theta+j\sin\theta).$$

Expanding $\cos\theta$ and $\sin\theta$ into series (*see Series*) and adding, the resultant series obtained is the series for $\epsilon^{j\theta}$; hence

$$a+jb=M\epsilon^{j\theta}. \qquad (1)$$

From the above definitions and equation (1) it is evident that complex numbers possess the following properties:

Addition of Two Complex Quantities. —

$$(a+jb)+(a_1+jb_1)=(a+a_1)+j(b+b_1).$$

Subtraction of Two Complex Quantities. —

$$(a+jb)-(a_1+jb_1)=(a-a_1)+j(b-b_1).$$

Multiplication of a Complex Quantity by a Complex Number. —

$$(a+jb)\,(a_1+jb_1) = aa_1 - bb_1 + j\,(ab_1 + a_1b)$$

or, putting $a+jb = M\epsilon^{j\theta}$ and $a_1+jb_1 = M_1\epsilon^{j\theta_1}$

where $M = \sqrt{a^2+b^2}$, $M_1 = \sqrt{a_1{}^2+b_1{}^2}$,

$$\tan\theta = \frac{b}{a},$$

and

$$\tan\theta_1 = \frac{b_1}{a_1}$$

we have $(a+jb)\,(a_1+jb_1) = M\epsilon^{j\theta}M_1\epsilon^{j\theta_1} = MM_1\epsilon^{j(\theta+\theta_1)}.$

Hence the product of two complex quantities is in general a complex quantity which has a magnitude equal to the product of the magnitudes of the two quantities and an angle equal to the sum of the angles of the two quantities.

Division of a Complex Quantity by a Complex Number. —

$$\frac{a+jb}{a_1+jb_1} = \frac{(a+jb)(a_1-jb_1)}{(a_1+jb_1)(a_1-jb_1)} = \frac{aa_1+bb_1-j\,(ab_1-a_1b)}{a_1{}^2+b_1{}^2}$$

or

$$\frac{a+jb}{a_1+jb_1} = \frac{M\epsilon^{j\theta}}{M_1\epsilon^{j\theta_1}} = \frac{M}{M_1}\,\epsilon^{j(\theta-\theta_1)}.$$

Hence the quotient of two complex quantities is in general a complex quantity which has a magnitude equal to the quotient of the magnitudes of the two quantities and an angle equal to the difference of the angles of the two quantities.

Example. — Suppose

$$I = \frac{10+j\,15}{3+j\,1};$$

then by the rule for division

$$I = 4.5 + j\,3.5,$$

that is, I contains a real part 4.5 and an imaginary part j 3.5.

Equations Containing Complex Quantities. — Since a real quantity cannot be equal to an imaginary quantity it follows that any equation of the form

$$A+jB = A_1+jB_1,$$

where A, B, A_1 and B_1 are all real quantities (which may, however, consist of any number of terms), is equivalent to the two equations

$$A = A_1$$

and

$$B = B_1.$$

Also, if one member of an equation reduces to the form $A+jB$, then the other member of this equation must likewise contain an equal real and an equal imaginary part.

[W. A. Del Mar.]

CONCRETE. — (*See also Cement; Concrete, Reinforced.*) Concrete as used by engineers in construction is generally formed of an artificial mixture of Portland cement (*see Cement*) and an aggregate consisting of sand and gravel or broken stone. These ingredients are mixed with water either by hand or by machine mixers. When in a semi-liquid form the material can be shoveled or poured into molds, and will gradually harden in either air or water, forming an artificial stone of high strength. The compressive strength of concrete varies with the quality and proportion of the materials; the tensile strength is very low and is usually neglected.

SPECIFICATIONS. — The quality of the materials can be determined only by careful tests, but for construction of minor importance and magnitude the following simple specifications should give good results.

Cement. — The cement shall be first-class American Portland cement of standard brand, guaranteed to conform to the Standard Specifications of the American Society for Testing Materials. (*See Cement.*) Such cement may be delivered either in bags or barrels but must be kept dry during storage.

Sand. — The sand shall be clean and coarse and free from dust, vegetable, loam or other organic matter, and other impurities, and should pass, when dry, a screen with holes $\frac{1}{4}$ of an inch in diameter.

Gravel. — The gravel shall consist of clean pebbles, and contain no foreign matter. It shall be screened over a $\frac{1}{4}$-inch mesh; the sand passing through the screen may be remixed in definite proportions with the gravel. If dirty or clayey, the gravel should be washed by a hose before mixing.

Broken Stone. — The broken stone shall consist of hard and durable stone such as trap, granite and limestone. Unless otherwise specified, all stone shall pass through a $2\frac{1}{2}$-inch screen.

Water. — The water used for mixing shall be free from oil, acid and other injurious substances.

PROPORTION OF INGREDIENTS. — The amount of each ingredient is usually measured by volume. The concrete is designated by the proportion by volume of each of the ingredients in the following order: Cement, Sand, Stone or Gravel. For example, a 1 : 2 : 4 mixture is one consisting of one barrel of cement, two barrels of coarse sand, and four barrels of loose gravel or broken stone (standard cement barrels contain 3.8 cu. ft; four bags of packed cement may be considered as equal to one barrel).

The following proportions are somewhat generally adopted for different classes of work.

1:1:2 or $1:1\frac{1}{2}:3$ for water tanks and standpipes carrying considerable pressure and required to be water-tight.

1:2:4 for arches, reinforced floors, beams, columns, engine foundations subject to vibration, sewers, and in general for structures subjected to bending stresses of some magnitude. A mixture as rich as this is also desirable where concrete is to be deposited under water since in such a case some of the cement may be washed away from the mixture.

$1:2\frac{1}{2}:5$ for bridge abutments and piers when laid in air, retaining walls and ordinary machine foundations.

1:3:6 for heavy walls, ordinary foundations, backing for stone masonry, etc.

Quantity of Ingredients.— The following rule devised by Wm. B. Fuller may be used for approximate determination of the quantity of the various ingredients.

Let c = number of parts of cement,
s = number of parts of sand,
g = number of parts of gravel or broken stone,

then $\dfrac{11}{c+s+g} = N$ = number of barrels of Portland cement required per cu. yd. of concrete,

$N \times s \times \dfrac{3.8}{27}$ = number of cu. yd. of sand required for one cu. yd. of concrete,

$N \times g \times \dfrac{3.8}{27}$ = number of cu. yd. of stone or gravel required for one cu. yd. of concrete.

For tables giving more accurate values see **Taylor and Thompson**, *Concrete Costs*, N. Y., 1912.

MIXING. — The ingredients consisting of sand and aggregate should be thoroughly mixed while dry until the mass is uniform in color and homogeneous. Mixing should preferably be done by machine. When necessary to mix by hand, the mixing should be done on a water-tight platform and the ingredients should be turned not less than six times. Enough water should be used to produce a mixture which will flow readily into its place in forms or elsewhere.

Concrete Mixing in Freezing Weather. — Reinforced concrete should not be mixed or deposited in freezing weather unless special precautions are taken to keep the materials free from ice, and to protect the concrete from freezing until it has been thoroughly hardened. To accomplish these results the aggregate may be heated, salt water used in mixing, and the concrete carefully covered after it has been deposited. Portland cement concrete which is to be deposited in large masses may be laid in freezing weather provided the surface appearance is not important, and provided that hardened surfaces be thoroughly cleaned from frost as well as dirt before a new surface is laid. Natural cement concrete should never be laid in freezing weather.

Concrete Shrinkage and Temperature Changes. — Cracks in cement may be caused by contraction in setting, or by temperature changes, as well as by excessive stress. To preserve a good appearance it is common to establish artificial cracks in long walls, sidewalks, etc. For spacing of such cracks see reference in *Bibliography*. The insertion of steel reinforcing bars even when not needed to carry stresses due to applied loads is an expedient often adopted to prevent the occurrence of such cracks. The amount of reinforcement to be used for this purpose should generally be not less than one-third of one per cent.

Effect of Sea Water and of Acids on Concrete. — The data available relating to the effect of sea water upon concrete indicate that there may be some chemical action resulting in a softening and disintegration of the surface; to offset this action as far as possible, concrete laid in sea water should be carefully proportioned and well mixed. There is also great danger from disintegration due to frost in cold climates in concrete lying between high water and low water. To protect against this the concrete may be protected by a surface of stone in the zone exposed to such action.

Thoroughly hardened concrete of good quality may be considered as resisting the action of acids and mineral oils as well as other building materials. Oils containing fatty acids may produce injurious effects by combining with the lime in the concrete, resulting in a disintegration of the latter.

Waterproofing Concrete. — Concrete may be made reasonably impervious to water under moderate pressures by using a rich mixture and by careful pro-

portioning and mixing. In the case of building foundations and other structures where leakage is not permissible it is usually advisable to protect the concrete by a separate coating applied either on the outside or inside of the wall or floor. The usual method of waterproofing the exterior is to construct a so-called membranous coating consisting of alternate layers of pitch and tarred felt. The inner surface of concrete basement walls and floors of many of the important buildings in New York City and other large American cities have been waterproofed by the use of a patented compound called "Hydrolithic Cement." Many so-called waterproofing powders, pastes, etc., are manufactured which are claimed to make concrete entirely water-tight if incorporated with it during mixing, but their value is problematical.

COMPRESSIVE STRENGTH AND WORKING STRESSES.—In the Joint Committee Report (*Proc. Am. Soc. C. E.*) the following table of compressive strength of thoroughly set concrete is given:

COMPRESSIVE STRENGTHS OF DIFFERENT MIXTURES OF CONCRETE

Set 28 days under favorable conditions

In Pounds per Square Inch

Aggregate	1:1:2	1:1½:3	1:2:4	1:2½:5	1:3:6
Granite, trap rock....................	3300	2800	2200	1800	1400
Gravel, hard limestone and hard sandstone........................	3000	2500	2000	1600	1300
Soft limestone and sandstone.........	2200	1800	1500	1200	1000
Cinders.............................	800	700	600	500	400

The following recommendations are also made:

When compression is applied to a surface of concrete of at least twice the loaded area, a stress of 32.5 per cent of the compressive strength may be allowed for static loads. This value may also be used for extreme fiber stress in a reinforced concrete beam.

For concentric compression on a plain concrete column or pier, the length of which does not exceed 12 diameters, 22.5 per cent of the compressive strength may be allowed for static loads.

For reinforced structures see the full report of the Committee in the *Proc. A.S.C.E.*, Feb., 1913.

Modulus of Elasticity.—The value of the modulus of elasticity of concrete has a wide range, depending on the materials used, the age, the range of stresses between which it is considered, as well as other conditions. It is recommended that in computations for the position of the neutral axis and for the resisting moment of reinforced concrete beams and for the compression of concrete in columns it be assumed as:

(*a*) One-fifteenth of that of steel, when the strength of the concrete is taken as 2200 lb. per sq. in. or less.

(*b*) One-twelfth of that of steel, when the strength of the concrete is taken as greater than 2200 lb. per sq. in., or less than 2900 lb. per sq. in., and

(*c*) One-tenth of that of steel, when the strength of the concrete is taken as

COST OF CONCRETE. — The following table* gives approximate costs of concrete in place. The values include superintendence, overhead charges and general expense but not office expense nor profit.

Item	Cost per cubic yard	
	Range	Average
Mass concrete as in dams, piers, foundations, etc.:		
Labor only	\$0.75 to \$2.50	\$1.25
Material and labor	3.00 to 9.00	5.50
Concrete in tunnels and conduits:		
Labor only	1.00 to 3.00	2.00
Material and labor	4.50 to 8.00	6.25
Concrete reservoirs and standpipes:		
Labor only	0.75 to 3.00	1.50
Material and labor	3.50 to 13.00	7.00
Concrete buildings:		
Labor only	0.75 to 4.00	1.50
Material and labor	4.50 to 9.00	6.50
Concrete bridges:		
Labor only	0.50 to 2.50	1.50
Material and labor	4.00 to 8.00	6.00
Concrete sewers:		
Labor only	0.75 to 1.75	1.50
Material and labor	3.50 to 8.00	6.00
Granolithic sidewalks:		
Labor only	1.00 to 3.00	1.75
Material and labor	6.25 to 9.00	7.00
Granolithic sidewalks:		
Labor only	2¢ to 4¢ per sq. ft.	2¾¢ per sq. ft.
Material and labor	5¢ to 14¢ per sq. ft.	10½¢ per sq. ft.

BIBLIOGRAPHY. — Taylor & Thompson, *Concrete, Plain and Reinforced*, N. Y., 1910; Taylor & Thompson, *Concrete Costs*, N. Y., 1912; *Handbuch für Eisenbetonbau*, Berlin, 1912; Turneaure and Maurer, *Principles of Reinforced Concrete Construction*, N. Y., 1909; Brooks, *Reinforced Concrete*, N. Y., 1911; Hool, *Reinforced Concrete Construction*, N. Y., 1912; *Proc. Am. Soc. C.E.*, Feb., 1913; *Jour. of Ass'n of Eng. Soc.*, Vol. 44, p. 202; *Proc. Am. Ry. Eng. Assn.*, Vol. 13, p. 450.

[C. M. Spofford.]

* Copied by permission from *Concrete Costs* by Taylor and Thompson, which should be consulted for detailed information.

CONCRETE, REINFORCED. — (*See also Cement; Concrete; Structures, Simple.*) Reinforced concrete beams, girders and slabs are made of Portland cement concrete with the addition of steel bars or steel mesh to resist tensile stresses. They are usually fixed at the ends by being built into columns, walls, or other girders and are continuous over intermediate supports; they are, therefore, subject to negative bending moments at the ends and at intermediate supports and positive bending moments between supports. It is common to use bottom reinforcement throughout the length and top reinforcement across

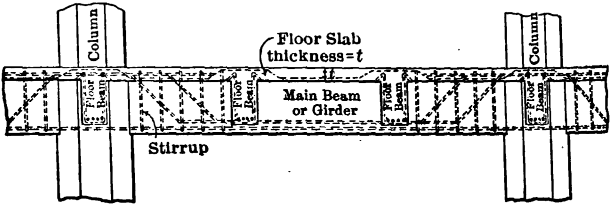

Fig. 1.

the supports. Vertical or inclined reinforcement is also usually necessary to carry the diagonal tensile stresses occurring in the web. Fig. 1 shows the general character of a reinforced concrete beam and shows methods of making connections to column and floor beams.

Character of Reinforcement. — The steel reinforcement may consist of round or square bars of which the diameter or side seldom exceeds 1½ inches. The square rods are sometimes twisted when cold, this type of bar being known as the Ransome bar. There are also various forms of corrugated or otherwise deformed bars on the market, some of which are shown in Fig. 2. Steel mesh is seldom used for ordinary beams, but is frequently employed in floor slabs. The steel reinforcement should preferably be of the same quality as that used for ordinary steel structures, although the use of a slightly less ductile material may be warranted in some cases where the steel is unlikely to be subjected to injurious shocks.

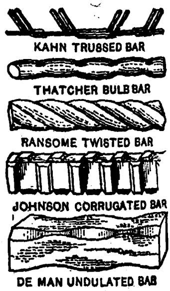

Fig. 2.

Proportions of Concrete; Forms. — The proportions of concrete for beams and other reinforced concrete structures subjected to flexure is commonly 1:2:4 (*see Concrete*). The forms or molds in which reinforced concrete beams are constructed are usually of wood bolted together, although steel forms are sometimes employed. In general, the size of the beam should be such that the forms can be manufactured from standard size boards. Forms should be watertight, and should be left in place until concrete has set sufficiently to carry the load to which it may be subjected at time of removal of forms.

Fireproofing. — In order to make reinforced concrete beams and columns fireproof the steel reinforcement must be protected by a reasonable amount of concrete. A common provision is that the metal in girder and columns be protected by a minimum of 2 inches of concrete; that the metal in beams be protected by a minimum of 1½ inches of concrete; and that the metal in floor slabs be protected by a minimum of 1 inch of concrete. It is also advisable that all

Spacing of Reinforcement. — Lateral spacing should not be less than three diameters center to center, with a clear spacing of not less than one inch. The distance from the side of the beam to the center of the nearest bar should be not less than two diameters. More than two layers should not be used unless securely tied together by metal connections particularly at or near points where bars are bent.

Electrolysis and Corrosion in Reinforced Concrete. — The best experimental data available indicate that no danger from electrolysis need be anticipated to the steel reinforcement in carefully mixed and proportioned Portland cement concrete free from salt or calcium chloride. Portland cement concrete may be considered to thoroughly protect embedded steel from corrosion provided the concrete is properly proportioned and mixed. If the concrete is porous, corrosion may be expected. See also article on *Electrolysis of Grounded Structures.*

Reinforced Concrete Beams. — The table on the following page gives the safe uniformly distributed loads per lineal foot for a series of reinforced concrete beams one (1) inch in width. The constants upon which the table is based are as follows: 1:2:4 concrete with ultimate compressive strength of 2000 pounds at 28 days. Allowable stress in concrete = 650 and in steel = 16,000 lb. per sq. in. Modulus of elasticity of steel 15 times that of concrete. Area of steel = 0.77 per cent area of beam above center of reinforcement.

FORMULAS FOR REINFORCED CONCRETE STRUCTURES. — Detail formulas for design cannot be given here; see references in *Bibliography.*

Reinforced Concrete Slabs. — These resemble beams, are much used for floors of buildings, and usually form an integral part of the floor beams and girders as indicated in Fig. 1. They may be computed in the same manner as reinforced concrete beams (*see above*) provided the shears and moments due to the outer forces are known. The distribution of the load may be determined by the application of the following formula recommended by the Joint Committee (*see Concrete*).

$$r = \frac{l^4}{l^4 + b^4},$$

in which r = proportion of load carried by the transverse reinforcement, l = length, and b = breadth of slab. For various ratios of l/b the values of r are as given in following table.

$\frac{l}{b}$	r
1	0.50
1.1	0.59
1.2	0.67
1.3	0.75
1.4	0.80
1.5	0.83

Using values above specified, each set of reinforcement is to be calculated in the same manner as slabs having supports on two sides only, but the total amount of reinforcement thus determined may be reduced 25 per cent, by gradually increasing the rod spacing from the third point to the edge of the slab.

If length of the slab exceeds one and five-tenths its width the entire load should be assumed as carried by transverse reinforcement.

Columns of Reinforced Concrete. — For such columns it is customary to limit to 15 the ratio of unsupported length to least width, and to consider the effective area as that within the protective coating (*see section on Fireproofing, above*), or for hooped columns or columns reinforced with structural shapes to the area within the hooping or structural shapes. The reinforcement may con-

sist either of longitudinal bars or of longitudinal bars connected by bands, hoops, or spirals, or of rigid structural forms. The following formulas may be used in design. Let A_c = cross-section of concrete, A_s = cross-section of steel, $A = A_c + A_s$ = total cross-section, n = ratio between modulus of elasticity of steel and concrete, usually 15, P = total safe load, f_c = allowable unit stress in concrete, f_s = allowable unit stress in steel, $p = A_s/A$. Then

$$P = f_c(A_c + nA_s) = f_cA[1 + (n - 1)p],$$

$$f_c = \frac{P}{A[1 + (n - 1)p]},$$

$$f_s = nf_c.$$

ALLOWABLE LOAD (POUNDS) ON REINFORCED CONCRETE BEAMS

Allowable uniformly distributed load in pounds per lineal foot, in excess of weight of beam, for end-supported rectangular reinforced concrete beams. *Tabular values to be multiplied by width of beam in inches to get safe applied load (dead + live + impact).*

Depth of beam, in.	Distance from top to center of reinforcement, in. = d	Area of steel, sq. in.	Moment of resistance, foot pounds	Span in feet							
				6	8	10	12	16	20	25	35
5	4.0	0.031	144	27	13	7	3	....	....	....	...
6	5.0	0.038	224	44	22	12	6	....	....	....	...
7	6.0	0.046	323	64	32	18	10	....	....	....	...
8	7.0	0.054	440	89	46	26	15	5	....	....	...
9	7.75	0.060	539	110	57	33	20	7	....	....	...
10	8.75	0.067	687	142	75	44	27	10	....	....	...
11	9.75	0.075	853	177	95	56	35	15	....	....	...
12	10.75	0.083	1,037	217	117	70	45	19	8	....	...
13	11.5	0.089	1,186	249	134	81	52	23	10	....	...
14	12.5	0.096	1,401	296	160	97	63	29	13	3	...
15	13.5	0.104	1,635	347	188	115	75	35	17	5	...
16	14.5	0.112	1,886	402	219	134	88	42	21	7	...
17	15.5	0.119	2,155	460	251	154	102	49	25	10	...
18	16.5	0.127	2,442	523	286	176	117	57	30	12	...
19	17.0	0.131	2,592	555	304	187	124	61	32	13	...
20	18.0	0.139	2,906	624	342	211	141	70	37	16	...
22	20.0	0.154	3,588	773	425	263	175	88	48	22	...
24	22.0	0.169	4,341	938	517	321	215	109	61	30	...
26	24.0	0.185	5,166	1119	618	385	259	133	75	38	6
28	26.0	0.200	6,063	....	728	455	307	159	91	48	10
30	28.0	0.216	7,032	....	847	531	359	187	109	58	14
36	33.5	0.258	10,070	....	1220	767	521	275	162	90	27
42	39.5	0.304	13,990	....	1704	1074	733	392	235	134	47
48	45.5	0.350	18,570	....	....	1435	981	528	320	187	70

COST. — See section on *Costs* in article on *Concrete*.

BIBLIOGRAPHY. — See *Bibliography* in article on *Concrete*.

[C. M. SPOFFORD.]

CONDENSERS, ELECTRIC. — (*See also Capacity and Charging Current.*) Any two conductors separated from each other by a dielectric form an electric condenser. The capacty C of such a condenser is the quotient of the numerical value of charge Q on either conductor by the difference of potential V between the two conductors *when equal and opposite charges are given the two conductors*, i.e., $C = Q/V$. The usual way of giving the conductors equal and opposite charges is to connect them respectively to the two terminals of a battery or other source of electromotive force. Unless the conductors are close to each other relative to the linear dimensions of their surfaces the capacity is small. Large capacities are usually obtained by using flat plates separated from each other by a thin sheet of dielectric. Unless the two conductors are close to each other relative to their distances from other conductors their capacity is influenced by the presence of the other conductors.

Formulas for Capacity. — See the article on *Capacity and Charging Current.*

Condensers in Series and in Multiple. — See *Electricity and Magnetism, Principles of.*

Energy Stored in a Condenser. — See *Electricity and Magnetism, Principles of.*

Electric Absorption and Dielectric Hysteresis in Condensers. — When an ordinary condenser is charged by connecting it to the terminals of a source of constant e.m.f. the amount of charge which it takes depends upon the time during which the e.m.f. is applied, i.e., there is an apparent absorption of charge by the dielectric. Time is also required for the dielectric to give up this charge when the plates are short-circuited. Absorption is particularly pronounced in such heterogeneous substances as glass, paper, ordinary mica, etc. On account of this absorption both the capacity and apparent resistance of a condenser in general depend upon the time of application of the e.m.f., i.e., upon the time of electrification.

Dielectrics, unless of perfectly uniform structure, when submitted to an alternating electrostatic field also manifest a property similar to magnetic hysteresis, that is, a certain amount of energy is dissipated in the dielectric as heat, over and above that corresponding to its "ohmic" or direct-current resistance. This loss is approximately proportional to the frequency of alternation of the electrostatic field, and consequently a condenser used in a high-frequency circuit does not have zero power factor, which would be the case were there no loss (*see Alternating Currents*).

TYPES OF CONDENSERS AND THEIR APPLICATION. — Condensers may be classified according to the nature of the dielectric used as: (1) air condensers; (2) mica condensers; (3) glass condensers; (4) paper condensers; and (5) electrolytic condensers.

Air Condensers. — Due to the low specific inductive capacity and relatively low dielectric strength of air, air condensers have very low capacity and are therefore seldom used except for standards of small capacity. For the latter purpose they are well suited, since their capacity is not appreciably affected by temperature, time of electrification, or frequency.

Mica Condensers. — On account of the high cost of suitable mica, condensers of this form are but little used except as standards. Mica suitable for such standards is not obtainable, except at prohibitive prices, in sheets larger than 3 inches by 4 inches. A good working thickness is from 0.0015 to 0.002 inch. A single sheet of high-grade mica 3 inches by 4 inches by 0.002 inch will give a capacity of about 0.004 microfarad and will withstand 1000 volts. In order that the insulation shall be high (product of megohms by microfarads not less

than 1000) and the absorption low, only sheets of the finest clear "ruby" mica should be used. The metal "plates" of mica condensers are usually of metal foil, preferably pure tin.

Fig. 1 shows the plan view and connection of a subdivided mica condenser. Since this arrangement permits of series, parallel, or series-parallel connection of the condensers a very large number of values of capacity may be obtained.

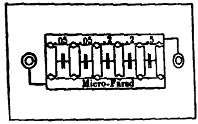

Fig. 1. Subdivided Standard Condenser

The power factor of well-made standard mica condensers ranges from about 0.1 per cent to about 1.75 per cent; see Grover, F. W., *Bulletin of the Bureau of Standards*, Vol. 3, No. 3.

Glass Condensers. — On account of its high dielectric strength and specific inductive capacity glass is particularly well suited for high-tension condensers, such as required for wireless telegraphy and the like. The Leyden jar is a common form of glass condenser. Moscicki's modification of the Leyden jar, shown in Fig. 2, consists of specially formed glass tubes closed at one end and coated inside with silver deposited chemically, the outer coating being sometimes applied in the same way and sometimes dispensed with altogether, the outer electrode in the latter case consisting of a mixture of glycerine and water in which the tubes are immersed.


Fig. 2. Moscicki Condenser

Mordey (*Jour. Inst. Elec. Eng., 1909, Vol. 43, p. 621*) reports a test on a Moscicki condenser consisting of 8 tubes having a total capacity of 0.03 microfarad, designed for 10,000-volt a-c. working. Each tube is 2 inches in diameter and 2 feet 9 inches long, or with connections 3 feet 2 inches long. The power factor of this condenser was approximately constant and equal to 1.0 per cent for frequencies from 40 to 60 cycles per second and for voltages from 5000 to 10,000 volts.

Condensers of this type are used to a considerable extent in Europe as a protective device on high-tension overhead transmission lines, as well as for wireless telegraphy. It has also been suggested by Mordey that they could be used economically for improving the power factor of highly-inductive loads, such as induction motors.

Paper Condensers. — These are made either (1) by building up a stack of alternate sheets of tinfoil (0.0003 inch thick) and tissue paper, two sheets to the layer, each sheet about 0.001 inch thick, or (2) by rolling up alternate strips of foil or "foiled" paper and tissue paper, the roll after being dried and impregnated with paraffin or other wax being pressed into a cubical shape, and suitably mounted in airtight metal boxes. The "foiled" paper is made by depositing very fine flakes of tin on suitable tissue paper which is then run through a press which forces the flakes into contact with each other, resulting in a paper resembling the so-called silver paper used for wrapping tea but having a fairly high conductivity. According to G. F. Mansbridge (*see reference below*) this paper possesses two decided advantages over the ordinary metal foil, (1) it is much lighter, and (2) when punctured it is self-healing, the spark vaporizing the tin in the immediate vicinity of the puncture without destroying the paper itself. For a complete description of the method of making the paper and the condensers see Mansbridge, G. F., *Jour. Inst. Elec. Eng.*, 1908, Vol. 41, p. 535.

Paper condensers are used almost exclusively for telephone and telegraph work. They are usually built to withstand about 400 volts. The following

weights and dimensions are taken from the British Post Office specifications for metal-cased telephone condensers. One-microfarad paper condensers designed for a working pressure of 1000 volts, and having the dimensions 8¾ by 6¼ by ½ inches and weighing 8 ounces each are made by the Western Electric Co.

DIMENSIONS AND WEIGHT OF PAPER CONDENSERS

British Post Office Specifications

Capacity, microfarads	Length, inches	Width, inches	Thickness, inches	Weight, ounces
0.5	4¾	2⅝	5/16	4
1.0	4¾	4¾	5/16	8
2.0	4¾	4¾	9/16	12
4.0	4¾	4¾	1⅛	20
10.0	4¾	4¾	2 11/16	46

The power factor of well-made paper condensers ranges from about 0.2 to 2 per cent, but unless care is taken in their manufacture the power factor may be considerably larger; values as high as 30 per cent have been obtained.

Electrolytic Condensers. — Electrolytic cells of the three following types have been used to a limited extent in Germany as condensers for telephone work: (1) acid cells, consisting of two small electrodes of platinum dipping into an acid solution, (2) sodium cells, in which the electrolyte is a solution of common salt, and (3) aluminum cells, in which the electrodes consist of aluminum and the electrolyte is some kind of basic solution. The capacity action of these cells arises from the formation of an extremely thin insulating layer at the anode of the cell which apparently has an enormously high specific inductive capacity. The anode and electrolyte therefore form two plates of a condenser having this thin layer of dielectric between them, forming a condenser of exceptionally large capacity per unit volume. The power factor of this condenser, however, is very high, due to the leakage current from anode to electrolyte, and consequently these cells are little used strictly as condensers, either in telephone or in power work. However, the aluminum cell is very extensively used as a lightning arrester, on power circuits, its capacity action combined with the self-heating property of the film making it an excellent device for this purpose (*see Lightning Protectors*).

TESTING OF CONDENSERS. — The three important properties of a condenser are its capacity, its insulation resistance and its power factor. The insulation resistance is determined by the use of direct current (*see Resistance and Conductance*); the power factor is measured by employing a source of alternating e.m.f., and may be looked upon as a measure of the effective a-c. resistance. Since this effective resistance is usually many times greater than the d-c. resistance, and since it varies directly as the frequency, the power factor of a condenser is practically constant for a considerable range in frequency.

A great many arrangements of circuits have been devised for measuring the capacity of condensers, cables, etc. Only one or two of the simple methods used in engineering practice can be described here. For a description and comparison of the various methods which have been used see Grover, F. W., *Bulletin of the Bureau of Standards*, Vol. 3, No. 3.

D-C. Versus A-C. Capacity Tests. — Due to the effect of electric absorption above described, the charge taken by a condenser depends upon the time

of electrification; the discharge also depends on the time. Consequently, when the capacity is measured by any direct-current scheme a standard time of electrification and a standard method of determining the discharge must be chosen. The British Post Office call for the capacity to be measured by taking the instantaneous discharge (by ballistic galvanometer) after the condenser has been charged for a period of 10 seconds. As a rule the capacity measured by a-c. methods is less than that measured by d-c. methods, since the charge does not have time to soak in; for the same reason the a-c. capacity also decreases slightly with increase of frequency. Hence a-c. tests should be made at a standard frequency, preferably that of the circuit on which the condenser or cable is to be used. It is also desirable, when possible, to make the capacity test at the same voltage, both in value and wave form, as is to be used on the condenser.

Voltmeter-ammeter Test of Capacity. — Connect the condenser in series with an a-c. ammeter and impress on this circuit the selected a-c. voltage. Measure the voltage drop across the condenser by means of a high-resistance a-c. voltmeter. Let the current read by the ammeter (voltmeter circuit open) be I, the voltmeter reading V, the frequency f, then the capacity is

$$C = \frac{I}{2\pi f V},$$

provided the voltage is a pure sine wave and the power factor of the condenser is negligible. As neither of these conditions are usually fulfilled, such a test as a rule gives only a rough approximation to the true capacity.

Ballistic Galvanometer Method. — Fig. 3 shows the arrangement of circuits for testing the capacity of a cable. B is a battery giving the desired constant e.m.f., K a highly insulated key, G a ballistic galvanometer properly shunted (*see Galvanometers and Shunts*). The sheath of the cable is usually necessarily earthed. It is best to charge the core of the cable positively as shown. If a condenser is to be tested instead of a cable, the plate corresponding to the sheath may or may not be earthed, as desired. Care should be taken that there is no leakage of current directly from the battery into the galvanometer or into the cable.

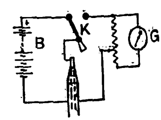

Fig. 3.

The procedure is as follows: Throw the key to the left for the desired length of time, note the position of the galvanometer needle (or spot of light), then throw the key to the right and note the first swing or maximum deflection of the spot of light. Call this D_x. Next, keeping the same battery, substitute for the cable a standard condenser of known capacity C and make a similar observation. Let the deflection in this case be D. Then the unknown capacity is

$$C_x = \frac{D_x}{D} C,$$

provided the damping of the galvanometer is the same in both cases.

While this method is satisfactory when applied to perfect condensers, it fails to a greater or less extent when applied to cables, especially if they are very long, or to any condenser having high absorption. With such condensers the discharge consists of two portions: a sudden rush when the key is first closed, followed by a current gradually diminishing toward zero, due to the release of the "absorbed" charge. Both portions of the discharge are active in producing the deflection, hence the apparent capacity depends to a certain extent on the period of the galvanometer employed, as well as upon the time of electrifi-

Wheatstone Bridge Method. — The following method gives a means of measuring both the capacity and power factor of a condenser, provided there is available a standard condenser whose capacity and power factor is known. The bridge is arranged as shown in Fig. 4. C is the standard condenser and C_x the condenser to be tested. All resistances must be non-inductive and adjustable. A is a source of alternating e.m.f. of pure sine-wave form and of frequency f. T is the detector; it may be either a telephone or, if the frequency be kept perfectly constant, a vibration galvanometer. For ordinary tests the telephone is the more convenient instrument; for good work with it the frequency should be high.

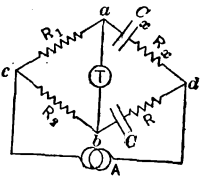

Fig. 4.

The balancing is effected as follows: with R_x and R both zero, adjust R_1 or R_2 until a minimum of sound is obtained; then adjust either R_x or R as the case may require, until the minimum is the best possible; then readjust R_1 or R_2, and so on, thus obtaining the perfect balance by successive adjustments.

$$C_x = \frac{R_2}{R_1} C,$$

$$\tan \phi_x = \tan \phi + 2\pi f (RC - R_x C_x),$$

where f is the frequency, ϕ_x is the angle whose *sine* is equal to the power factor of the condenser under test and ϕ is the angle whose *sine* is equal to the power factor of the standard condenser. The effective alternating-current conductance of the condenser under test is then

$$G_s = 2\pi f C_x \tan \phi_x,$$

which as a rule is many times the reciprocal of its insulation resistance as measured by direct-current methods (*see Resistance and Conductance*).

The value of the power factor for the standard is best determined at the Bureau of Standards in Washington; it is done by indirect reference to an air condenser.

Sources of Error. — The sources of error when the most refined measurements are to be made are the inductance or capacity of the various resistances, error in the ratio of R_1 and R_2, and electrostatic induction between the bridge and its surroundings. These sources of error are fully discussed by Grover above referred to.

COST OF CONDENSERS. — A good standard mica condenser of 1/3 microfarad, guaranteed to stand 300 volts maximum across its terminals costs about \$25. An adjustable mica condenser, such as shown in Fig. 1, having a maximum capacity of 1 microfarad and suitable for voltages up to 300 volts costs about \$60. Paper condensers, such as used on telephone circuits, cost about 70 cents per microfarad. A standard 1000 volt, 1 microfarad paper condenser can be had for about \$3.75.

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[H. PENDER AND H. R. RANKEN.]

CONDENSERS, STEAM.— (*See also Cooling Systems for Power Stations; Power Stations; Pumps; Steam Engines; Turbines, Steam.*) The primary object of a condenser is to reduce the back pressure in the exhaust of a steam-engine or steam turbine, although in cases where the supply of suitable feed water is limited the recovery of the condensed steam is of equal importance. Theoretically the gain in the output of a given engine for the same steam input is proportional to the reduction in back pressure, but practically the gain is usually much less than this, depending upon the type of engine and conditions of operation.

CLASSIFICATION OF CONDENSERS. — Condensers are of two general types, jet condensers and surface condensers.

Jet Condensers. — In a jet condenser cooling water and the exhaust steam mingle together in a closed chamber, the water condensing the steam by direct contact. The cooling or injection water on entering the condenser is broken up into a fine spray or is spread out into a thin sheet. Jet condensers may be classified as follows:

Standard or Ordinary Jet Condensers, in which the cooling water, condensed steam and air are exhausted by a pump.

Barometric Condensers (Siphon Condensers), in which the cooling water and condensed steam are exhausted by a barometric column, the air being exhausted with the water and condensed steam or by means of pump. In certain types of barometric condensers the condensing water is forced into the condenser by atmospheric pressure (if the lift of the condensing water is not over 15 feet), and the condenser is then called a "siphon" condenser.

Ejector Condensers, in which the condensed steam and air are exhausted directly to the atmosphere by the momentum acquired by the cooling water and condensed steam as they pass through the condenser.

Rotary Condenser. — The best known form of this type of condenser is the Leblanc Condenser (made by the Westinghouse Machine Co.), which accomplishes the separate removal of water and air by means of a pair of relatively small turbine-type rotors on a common shaft in a single casing, which is integral with or attached directly to the lower portion of the condensing chamber. The condensing chamber itself is but little more than an enlargement of the exhaust pipe. The injection water is projected downwards through a spray nozzle, and the combined injection water and condensed steam flow downward to a centrifugal discharge pump under a head of 2 or 3 feet, which insures the filling of the pump. The space above the water level in the condensing chamber is occupied by water vapor plus the air which entered with the injection water and with the exhaust steam. This space communicates with the air-pump through a relatively small pipe.

The air-pump differs from pumps of the ejector type in that the vanes in traversing the discharge nozzle at high speed constitute a series of pistons, each one of which forces ahead of it a small pocket of air, the high velocity of which effectually prevents its return to the condenser. A small quantity of water is supplied to the suction side of the air-pump to assist in the performance of its functions. The power required for the pumps is said to approximate 2 to 3 per cent of the power generated by the main engine.

Surface Condensers. — Ordinary surface condensers consist of nests of small brass or copper tubes usually ¾ inch or 1 inch in diameter through which cooling water is forced and which are surrounded by an air-tight shell to which the exhaust steam is admitted. There may be one, two or more sets of tubes through which the steam passes in succession, the condenser being referred to

as a "single-flow," "double-flow" or "multi-flow" respectively. Steam is admitted at the top of the shell and the condensed steam drawn off at the bottom. Water enters the lower tubes and is discharged at the top, securing thus the advantages of the counter-flow principle. The rate of heat transmission depends largely upon the state of the tubes. Transmission is much retarded by a water film coating the exterior of the tubes and by interior coatings of scale, dirt or corrosion. In the so-called "dry-tube" type baffles are arranged to catch the drip from each set of pipes, drain it off at the side and so prevent it from falling on the pipes below.

Evaporative and Air-cooled Surface Condensers are sometimes employed when condensing water is scarce. Brief descriptions and references will be found in Gebhardt's *Steam Power Plant Engineering*.

List of Well-known Condensers. — The following are some of the well-known condensers.

Jet Condensers:
- Ordinary
  - Worthington.
  - Blake.
  - Deane.
- Barometric.
  - Weiss.
  - Alberger.
  - Rotary.
  - Leblanc.

Surface Condensers:
- Baragwanath (single-flow).
- Wheeler (double-flow).
- Wainwright (multi-flow).

CLASSIFICATION OF CONDENSER PUMPS. — (*See also Pumps and Pumping Engines.*) The following names are usually applied to the pumps used in connection with a condenser.

Injection or Circulating Pump, the pump used for injecting the cooling water into a condenser.

Wet-air Pump, the pump used for exhausting the air, condensed steam and hot water from a condenser when these are all exhausted together.

Dry-air Pump, the pump used for exhausting the air and water vapor only. A dry-air pump must be used when high vacua are required.

Hot-well Pump. — This name is applied to the wet-air pump when this pump exhausts to a hot-well. The hot-well is a well provided to hold the condensed steam, which is approximately at the same temperature as that of the exhaust steam in the condenser. If this hot water is to be used over again for boiler feed it is pumped from the hot-well to the boiler by the boiler feed pump (*see Boilers, Steam*).

CHOICE OF TYPE OF CONDENSER. — The chief advantage of the surface condenser over the jet condenser is that the condensed steam does not mingle with the cooling water and therefore the condensed steam may be used over again for boiler feed water, provided any oil which may be carried by the exhaust steam from the cylinders is eliminated before the steam reaches the condenser (*see Separators, Steam*). When there is plenty of cooling water readily handled a higher vacuum is usually obtained by means of a surface condenser than by means of a jet condenser. To offset these advantages, however, the surface condenser is much more expensive than a jet condenser, and requires as a rule a greater amount of cooling water.

Consequently the use of a surface condenser is in general justified only when the cost of obtaining suitable feed water is relatively high (*see also Feed-water Heaters and Purifiers*), and cooling water, unsuitable for feed water however,

relatively cheap (*see also Cooling Towers, Ponds, etc*). Before finally selecting either type of condenser an estimate of the total annual cost, including interest, depreciation, maintenance and all operating charges should be made for each case.

According to Prof. Wickenden the following vacua, based on a 30-inch barometer, have been found most economical in general practice.

Piston engines	26 to 26.5 inches
Turbines at 20 per cent load-factor or less	27 to 27.5 inches
Reaction turbines at high load-factors	28 to 29.5 inches
Impulse turbines at high load-factors	28.5 to 29 inches

In summer months warm cooling water often renders these vacua impracticable. When cooling towers are required to conserve the water supply the economic limits of vacuum are lower, due to the extra investment and the relatively warm state of the water. Jet condensers are cheaper to install and operate for vacua up to 27 inches. They require less water and give somewhat higher hot-well temperatures if closely regulated. For higher vacua the load on the dry-air pump becomes excessive, due to the air entrained with the cooling water. Surface condensers are then preferable and for extreme vacua they are indispensable.

DEGREE OF VACUUM AND BACK PRESSURE. — Let

V = degree of vacuum, i.e., the reading of vacuum gauge, in inches of mercury column,

B = reading of barometer.

Then the back pressure in pounds is

$$p = 0.491\ (B - V).$$

The degree of vacuum is usually referred to a barometric pressure of 30 inches. Calling V_0 the vacuum in inches of mercury corresponding to a barometric pressure of 30 inches, and B and V the observed barometric pressure and gauge reading respectively, then

$$V_0 = V + (30 - B).$$

(If V_0 is the vacuum in inches referred to 760 millimeters or 29.91 inches of mercury, substitute for 30 the number 29.91.)

Vacua from 20 to 29 inches are used in practice, corresponding to back pressures of from 4.5 to 0.5 pounds. The higher the vacuum the more cooling water required and the greater the cost of the air and circulating pumps.

CONDENSING WATER REQUIRED. — Let

T = temperature, in ° F., of dry saturated steam at the pressure corresponding to the desired degree of vacuum,

T_i = temperature, in ° F., of the injection water,

T_s = temperature, in ° F., of the condensed steam at bottom of condenser,

T_w = temperature, in ° F., of the discharge water (for a jet condenser $T_w = T_s$),

H = total heat (above 32° F.) of the exhaust steam, in B.t.u. per pound.

Then the weight of condensing water required per pound of steam is

$$W = \frac{H - T_s + 32}{T_w - T_i}.$$

In applying this formula the values of T_s and T_w must be determined and an allowance must be made for the regulation of the supply of condensing water.

Values of T_s and T_w. — Air is always present in exhaust steam and in the case of jet condensers a certain amount of air also enters the condenser with

the cooling water. The effect of the air is to reduce the temperature of the steam (T_s) below that corresponding to saturated dry steam (T) at the same pressure. The amount of air present depends upon the type of condenser and also upon the amount of air in the cooling water in the case of a jet condenser. The following relations between T_s, T_w and T are found to hold in practice.

	Jet condensers		Surface condensers	
	Parallel-current	Counter-current	Single and double flow	Multi-flow*
	Deg. F.	Deg. F.	Deg. F.	Deg. F.
T_s less than T by....................	10 to 15	5 to 10	5 to 10	0 to 5
T_w less than T by....................	10 to 15	5 to 10	10 to 20	0 to 10

* Cases have been reported where T_s and T_w are both *higher* than T by a few degrees. (*Proc. Inst. Nav. Arch., March*, 1906.) [Probably due to errors of gauges.]

Allowance for Regulation of Injection Water. — It is usual to take for H in the above formula the value corresponding to dry saturated steam at the given pressure in the condenser (*see tables in article on Steam*), and then to increase the value of W so obtained by from 5 to 15 per cent to allow for imperfect regulation of the injection water and for the more or less unknown state of the steam as it enters the condenser.

Example. — A vacuum of 25.85 inches, referred to 29.92-inch barometer, is to be maintained in a surface condenser, with injection water at 60° F. The value of T from the steam tables is 126° F. and H = 1115 B.t.u. Take T_s = 120°, T_w = 115°, T_i = 70°; then

$$W = \frac{H - T_s + 32}{T_w - T_i} = \frac{1115 - 120 + 32}{115 - 70} = 22.8.$$

DIMENSIONS OF CONDENSERS. — There is so great a difference in the design of the various forms of jet condensers that it is impossible to give any average dimensions. See the catalogues of the makers; representative makes of the various types are listed above.

In the case of surface condensers the tube surface exposed to the steam varies inversely as the coefficient of heat transfer (i.e., the number of B.t.u. per hour per ° F. difference in temperature between the steam and water per square foot of cooling surface) and directly as the mean temperature difference between the water and the steam. Let

T_i = temperature, in ° F., of the injection water,
T_s = temperature, in ° F., of the exhaust steam at condenser pressure,
T_w = temperature, in ° F., of the discharge water,
Q = pounds of cooling water required per hour,
U = coefficient of heat transfer.

Then, according to Josse (*Power, Feb. 2, 1909*), the required cooling surface in square feet is

$$S = \frac{2.3\,Q}{U} \log_{10} \frac{T_s - T_i}{T_s - T_w}.$$

The value of U depends upon the metal used for the tubes, the condition of the external surface of the tubes and especially upon the condition of the internal surface of the tubes (scale and corrosion decreasing the value of U very markedly),

and upon the velocity of the water, being greater the higher this velocity. U may be taken as 250 for water velocities of from 25 to 50 feet per minute and 375 for velocities of from 50 to 75 feet per minute. There is, however, considerable difference of opinion regarding the proper formula for S and the proper value to assign to U.

In reciprocating engine plants an allowance of 2 square feet of tube surface per indicated horse-power is customary. In early steam-turbine installations an area of from 2 to 4 square feet per kilowatt was commonly employed, but subsequent improvements in condensers and the water rates of turbines have tended to reduce these areas. In the most modern plants of large capacity condenser surfaces range from 1.2 to 2.5 square feet per kilowatt.

From the relations implied in the above formulas it is seen that the surface may be reduced by forcing large quantities of water at high velocities to pass the tubes; or, with a given surface, the vacuum may be heightened by increasing the flow of cooling water. However, the gain may be more than offset by the cost of pumping this water.

SIZE OF CONDENSER PUMPS. — (*See also articles on Blowers and Compressors; Pumps and Pumping Engines.*) The size of the injection or circulating pump required can be calculated directly from the quantity of injection water to be handled and the head against which it is to be pumped, including of course the friction head. Separate injection pumps for jet condensers are not usually required, as the head does not as a rule exceed that corresponding to the difference between atmospheric and condenser pressure. In the case of surface condensers the intake and discharge tunnels are usually at about the same level, and consequently the head is largely friction head. The friction head of a condenser is not readily calculated, but may be obtained from the manufacturer.

Wet-air Pumps. — The predetermination of the size of wet-air pumps is difficult, owing to the uncertainty in estimating the quantity of water vapor and air which they must handle. In the case of jet condensers the total weight of water (including the vapor) and air is equal to the combined weight of the exhaust steam and cooling water, whereas for a surface condenser only the condensed steam and air in it is handled by the wet-air pump. Hence the wet-air pump for a surface condenser is usually much smaller than for a jet condenser.

Formulas, partly analytical and partly empirical, for calculating the piston displacement of wet-air pumps will be found in Gebhardt's *Steam Power Plant Engineering*. For average practice the volume capacity or piston displacement is equal to the volume of *cooling water* multiplied by the following factors.

For jet condensers, single-acting pump.......................... 3
For jet condensers, double-acting pump.......................... 3.5

Piston speeds are usually about 50 feet per minute at full load.

For surface condensers the volume capacity is equal to the volume of the *condensed steam* multiplied by the following factors.

For reciprocating engines.......................................10
For steam turbines..20

These figures are based on a study of some 200 installations (Gebhardt, 1910 ed.).

Dry-air Pump. — The required volume capacity is found in practice to be equal to the volume of the *condensed steam* multiplied by the following factors:

For vacua under 27 inches............................ 20 to 30
For vacua 28 inches or over.......................... 50

In both cases the barometer is assumed as 30 inches. These figures are given by Gebhardt and are based on an investigation of some 50 installations.

POWER REQUIRED FOR CONDENSER PUMPS. — The power required for the circulating pump is calculated in the ordinary way from the weight of the water delivered and the head, including the friction head (*see article on Pumps and Pumping Engines*).

The power required for the wet-air and dry-air pumps can be accurately determined only when the proportion of air in the exhaust steam and also in the cooling water, in the case of a surface condenser, is known. Surface water under atmospheric pressure contains from 2 to 12 per cent, by volume, of air. To allow for leakage, a liberal factor of 20 per cent may be taken.

Power Required to Exhaust the Air. — Let

p_a = atmospheric pressure, pounds per square inch,
p_c = pressure in condenser, pounds per square inch,
T_i = initial temperature of injection water, in ° F.,
T_f = initial temperature of feed water, in ° F.,
T_c = temperature in condenser in ° F. (approximately the temperature of the exhaust steam),
W_w = weight of cooling water in pounds per *minute*,
W_s = weight of condensed steam in pounds per *minute*.

Then for a jet condenser the volume of air in cubic feet per minute at condenser pressure to be handled by the pump is

$$V_1 = \frac{(W_w + W_s)\, p_a\, (T_c + 460)}{310\, p_c\, (T_i + 460)},$$

and for a surface condenser

$$V_1 = \frac{W_s p_a\, (T_c + 460)}{310\, p_c\, (T_f + 460)},$$

assuming in both cases that there is 20 per cent air by volume in the water at atmospheric pressure. This percentage is probably high when the feed water is taken from a hot-well, since the heating of the water drives out the air. Also, the temperature T_c for a counter-current jet condenser and separate air pump is more nearly the temperature of the injection water than that of the steam.

The horse-power required to exhaust the air is then

$$P_a = \frac{p_c V_1}{66.2\,\epsilon} \left\{ \left(\frac{p_a}{p_c} \right)^{0.29} - 1 \right\},$$

where ϵ is the mechanical efficiency of the pump, which may range from 30 to 60 per cent. (*See article on Blowers and Compressors.*)

Power Required to Exhaust the Condensed Steam and Water. — The power required to exhaust the water and condensed steam from a jet condenser may be calculated in the same manner as for an ordinary pump (*see article on Pumps and Pumping Engines*), using the proper value of the head under which the pump operates. This will depend upon the arrangement of the condenser, location of the pumps and hot-well, etc.

If a wet-air pump only is used, then the total horse-power of this pump will be $P_a + P_w$, where P_w is the horse-power required to remove the condensed steam and water. If a dry-air pump is used in connection with the wet-air pump (which then exhausts the water only) the horse-power of the dry-air pump is P_a, and the horse-power of the wet-air pump P_w.

Per Cent of Total Available Energy Used by Condenser Pumps. — When steam-driven pumps are employed and the exhaust from these pumps is not utilized the steam consumption of the pumps is properly taken as a measure of the energy required for their operation; this may range from 5 to 20 per cent

of the total steam generated by the plant. When the exhaust from the steam cylinders of the pumps is utilized for heating the feed water (see *Feed-water Heaters*), the *heat consumption* of the pumps is properly taken as a measure of the energy required for their operation; this may range from about 1 to 5 per cent of the total B.t.u. utilized by the main engines, for a large part of the heat in the steam used in the pumps is returned to the boiler via the feed water. When electrically-driven pumps are employed, the kilowatt-hour input to the motors driving the pumps is properly taken as a measure of the energy required for operating the pumps; this may range from about 2 to 8 per cent of the output of the main generators.

LOCATION AND ARRANGEMENT OF CONDENSERS. — Where there is only one engine installed the condenser is usually placed close to and just below the engine, so that all condensation may gravitate into it. In some large vertical steam turbine outfits the condenser (surface type) is at the base of the turbine and forms an integral part of the structure. When several engines or turbines are installed in a power house a separate condensing outfit may be used for each engine, or the so-called "central system" may be employed in which one condensing outfit serves several engines. This arrangement is particularly well adapted to plants in which the individual units are subjected to extreme variations in load, as in rolling mills.

Additional data on the location and arrangement of condensers will be found in the article on *Power Stations*.

COST OF CONDENSING EQUIPMENT. — The cost of condensing equipment depends upon the amount of steam to be condensed, the degree of vacuum to be maintained, the type of equipment and the method of driving the pumps. Gebhardt gives the following cost per kw. of generator output for the complete condensing equipments installed and ready for operation.

Siphon condensers without air pump	$2.00-$3.00
Jet condensers (ordinary)	3.00- 4.50
Barometric condensers with dry-air pump	4.00- 6.00
Surface condensers for 26-inch vacuum	3.50- 5.00
High-vacuum surface condensers	3.50-10.00
Leblanc jet condensers and pumps	2.00- 6.00

J. R. Bibbins (*Power, Jan.*, 1905) gives a curve showing the relative cost of high-vacuum surface condensers compared with 26-inch-vacuum surface condensers. The following figures are taken from this curve.

Degree of vacuum, inches	26	26.5	27	27.5	28	28.5
Relative cost, per cent	100	111	123	137	158	205

Maintenance of Condensers. — The cost of maintenance of a surface condenser is subject to wide variations, depending upon the corrosion and deterioration of the condenser tubes, the exact cause of which is not often understood. With clean, fresh water, free from acid, the tubes of a condenser last indefinitely, but where the cooling water contains sulphur, as in drainage from coal mines, or sea water contaminated by sewage, such as harbor water, the deterioration is exceedingly rapid.

The maintenance of a jet condenser is much less affected by impurities in the water.

BIBLIOGRAPHY. — Gebhardt, *Steam Power Plant Engineering*, N. Y., 1909, contains 62 pages on condensers and a very full bibliography of the subject.

[WM. KENT.]

CONDUITS AND CONDUIT LINES, UNDERGROUND. — (*See also Distribution Lines; Transmission Lines; Wires and Cables, Insulated; Wiring of Buildings.*) The following is a brief table of contents of this article:

Underground cables are now almost universally installed in conduits made either of glazed tile, wood or paper fiber. The pipes or conduits are laid so as to form a series of continuous ducts for a length of not over 400 feet, and are terminated in brick or concrete chambers, from which the cables are pulled into the ducts.

TERMINOLOGY. — While there is some confusion in the terminology of conduit lines, the best recent literature sanctions the following definitions.

Conduit, a pipe or tube designed or used for containing electric wires or cables.

Duct, a passage or opening, designed or used for accommodating electric wires or cables.

Conduit Line, a group of installed conduits. (The expression *subway* is largely used for conduit line, but will not be used herein, as it is desired to avoid such expressions as "the subway subway," meaning the conduit lines of an underground railway.)

Splicing Chamber, a chamber built to give access to the ducts of a conduit line; frequently called a manhole. Properly, a manhole is the opening giving access to the spliting chamber.

Manhole, an opening giving access to an underground splicing chamber, from the surface of the ground; frequently, but improperly, used to designate the entire splicing chamber.

Service Box, a part of a splicing chamber with facilities for connecting distributing conductors to the mains. Service boxes are usually made of cast iron and set on the roof of the splicing chamber like a manhole casting.

USE OF CONDUIT LINES. — Conduit lines are used for the distribution of electrical energy wherever the unsightliness, danger or instability of pole lines prohibits the use of the latter. It is therefore in large cities that they have found their principal application. They are used for the transmission and distribution cables of lighting systems, power plants and railways, and for telephone and telegraph lines.

Since the invention of the Pupin loading-coil, the use of conduit lines for telephones has received considerable impetus. Underground telephone lines now extend from Boston to Washington, a distance of nearly 500 miles, creosoted pump-log being used for the conduit the greater part of the way. The most notable installation of conduit for trunk-line railroad electrification is that of the New York Central which comprises over 1,600,000 duct feet, partly in tile conduit and partly in iron pipe.

Advisability of Double-conduit Line. — A conduit line of a large number of ducts should be avoided wherever possible. While a large line may be per-

missible for telephone work, it certainly is not desirable for light or power work. The entire output of a station or substation of considerable size should not be carried out through one conduit line, but should be divided between two or more lines kept well separated.

TYPES OF CONDUITS. — While tile conduit is used far more than any other kind, there are four other kinds in fairly extensive use; namely, fiber conduit, wrought-iron pipe, cement-lined iron and pump-log.

Tile Conduit. — Figs. 1 and 2 show the tile conduits used in the Electric Zone of the N. Y. C. & H. R. R.R. near New York. They represent what is

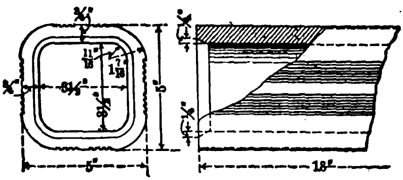


Fig. 1.

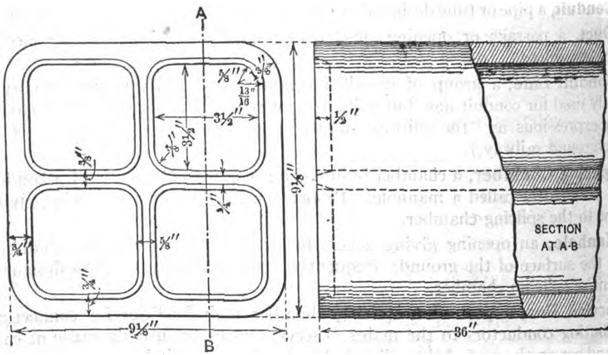


Fig. 2.

probably the best practice up to date. The single-duct conduit weighs 16½ pounds per length of 18 inches, and the four-duct conduit weighs 100 pounds per length of 3 feet.

Fiber Conduit consists of tubes, made by rolling paper saturated with asphalt or bituminous compound around a mandrel. Like iron pipe, which it resembles in appearance, lengths of fiber conduit are usually joined by screw and coupling although it is also made for socket, sleeve and drive joints. It is lighter and easier to handle than iron pipe with which it compares favorably in cost, but is more expensive than tile conduit. It is mechanically inferior to other types of conduit, and difficulty is often experienced in drawing heavy cable around bends without breaking the conduit. For some classes of work this disadvantage is entirely compensated for by superior gas and water tightness.

The dimensions of fiber conduits are given as follows by the Johns-Manville Company and substantially the same by the Fiber Conduit Company.

Socket joint				Sleeve joint			
Inside diameter, inches	Thickness of walls, inches	Approximate average weight per foot, pounds	Length of section, inches	Inside diameter, inches	Thickness of walls, inches	Approximate average weight per foot, pounds	Length of section, inches
1	1/4	0.45	30	1	1/4	0.45	30
1 1/2	1/4	0.75	60	1 1/2	1/4	0.75	60
2	1/4	0.90	60	2	1/4	0.90	60
2 1/2	1/4	1.05	60	2 1/2	1/4	1.05	60
3	1/4	1.30	60	3	1/4	1.30	60
3 1/2	1/4	1.60	60	3 1/2	7/16	2.50	60
4	1/4	1.85	60	4	1/2	3.20	60
Screw joint				**Drive joint**			
1 1/2	5/16	0.85	60	2	1/4	0.90	60
2	3/8	1.35	60	2 1/2	1/4	1.05	60
2 1/2	3/8	1.70	60	3	1/4	1.30	60
3	7/16	2.20	60	3 1/2	1/4	1.60	60
3 1/2	7/16	2.50	60	4	1/4	1.85	60
4	1/2	3.20	60				

Wrought-iron Pipe is used in city streets where the conduit line has to twist about sub-surface obstructions. It is more expensive than tile conduit and does not last as long on account of rusting. The usual sizes are 3-inch and 3 1/2-inch pipe, 20 feet long, provided with threaded ends and couplings. (For weights and dimensions see article on *Pipes*.)

Cement-lined Iron Pipe consists of thin sheet-iron cylinders, like stovepipe, with a lining of hydraulic cement. When the conduit is old the iron usually rusts away without detriment to the construction.

Pump-log Conduit is used only on telephone lines, as its inflammability renders it dangerous for power work. It consists of wooden blocks with the

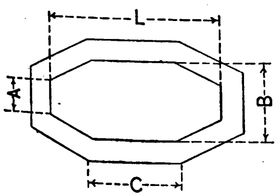


Fig. 3.

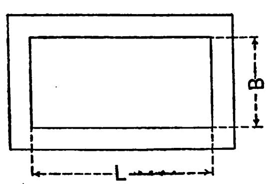


Fig. 4.

duct hole drilled out. A description of a typical pump-log installation is given below under Installation.

SPLICING CHAMBERS. — Splicing chambers, for straight runs, are usually built in the shapes shown in Figs. 3, 4, 5 and 6. That shown in Fig. 3 is usually regarded as ideal, as the excavation is a minimum, and the cables can

follow the walls very closely without being bent to a dangerously small radius, a frequent occurrence in chambers with square corners. The rectangular form shown in Fig. 4 is somewhat more common, but requires more excavation and more wall material than the first form, and is conducive to sharp bends in the

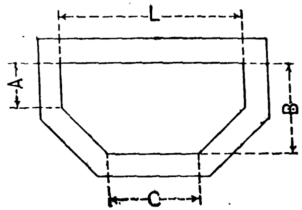


Fig. 5.

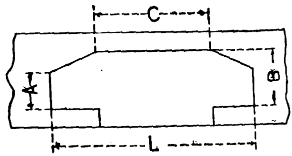


Fig. 6.

cable. The type shown in Fig. 5 is convenient where it is desired to have a wide chamber without occupying much space on one particular side of the duct line as, for example, where two duct lines run parallel and a short distance apart. In this case, economy may be obtained by the use of a common wall on the long side of the chamber.

Fig. 6 shows the side-wall type of chamber, which is entered from the side instead of the top. This form is used in the side walls of tunnels or in retaining walls, and usually takes the form of a long niche in the wall. When built in tunnel walls not far from the surface of the ground, it is desirable to have them open from the street above as well as from the side, in order to avoid the necessity of having the use of a tunnel track when work is to be done in the chambers.

Material of Splicing Chambers. — Monolithic concrete is the cheapest and best material to use where a large number of identical chambers have to be built, as the same form can be used over and over again. Where such uniformity cannot be attained, as in streets congested with sub-surface construction, brick or concrete hollow tile are better and cheaper materials.

Dimensions of Splicing Chambers. — The height of large splicing chambers is usually determined by the height in which a man can stand upright and is seldom less than 6½ feet. The width is similarly influenced by the space required to work in, which is about 4 feet. The length depends upon the length of splice and the space required to curve the cable from the ducts to the supporting shelves or racks, considerations which make a length of 8 feet the practical minimum where there are large cables.

Special chambers, such as those for bare grounded cables, for a small number of telephone or other small cables, may be made of smaller dimensions than stated above, because a man can work on his knees in a chamber 4 feet high and 3½ feet wide, and the length may be reduced to about 4 feet if the cables are small and flexible.

Table I gives the dimensions of standard splicing chambers used by several large traction and lighting companies. The letters refer to the dimensions on Figs. 3, 4, 5 and 6. The average volume of a splicing chamber is about 16 cubic feet per splice, although a maximum of 40 and a minimum of 6 are sometimes found.

Spacing of Splicing Chambers. — It is found that the greatest length of cable that can be pulled through a glazed tile conduit without injuring the cable or requiring special apparatus is about 400 or 500 feet on a straight or slightly curved run. Hence at distances of 400 or 500 feet along a conduit line it is

TABLE I. — DIMENSIONS OF SPLICING CHAMBERS

Name of company	Fig. No.	A	B	C	Height inside	L	No. of splices
		Feet and inches					
Chicago Edison Co.	3	2′ 3″	6′ 0″	3′ 3″	6′ 6″	7′ 6″	16
District Ry., London	5	2 4	5 0	5 6	7 6	14 3	32
I. R. T. Subway, N. Y.	6	1 3	3 0	4 4	14 0	11 4	64
Long Island R.R.	5	1 6	4 0	2 3	6 6	10 0	18
Long Island R.R.	4	......	4 0	....	6 6	9 0	18
Long Island R.R.	3	1 8	4 0	3 8	6 6	9 0	18
Manhattan Ry., N. Y.	4	......	6 0	....	6 6	8 0	16–48
New York Cent. R.R.	3	2 4	5 0	3 6	6 6	11 0	20
New York Cent. R.R.	5	2 6	5 0	3 6	6 6	11 0	20
New York Cent. R.R.	5	1 10	4 0	3 6	6 0	10 0	5
New York Cent. R.R.	6	1 10	3 0	3 6	6′ 6″–8′ 0″	8 6	20–32
Pennsylvania R.R.	4	......	4 0	....	6 4	8 0	24
Philadelphia R.T.	5	1 8	3 6	curved	6 3	7 10	20
P. R.R. Tunnels	(Special shape)				4 9	6 0	

be pulled in and spliced. Local conditions and curves may prevent the full length being attained in all cases, but it is desirable to have no lengths greater than the standard, in order to avoid the necessity of keeping special long pieces of cable in stock.

Service Boxes. — Service boxes are usually iron castings similar in shape and size to an ordinary manhole casting, but provided with a completely waterproof inner cover which screws down on a gasket. The outer cover resembles that of a manhole. Inside the inner cover is a distribution board with copper bus bars, to which the main and feeder cables in the chamber connect through the bottom of the board.

Waterproofing and Drainage. — There is some diversity of opinion with regard to the value of waterproofing splicing chambers, although the modern tendency is to omit waterproofing and provide efficient drainage. Waterproofing is futile unless the duct lines leading to it are waterproofed. This is a very difficult and expensive process which seldom shows good results. Water will also enter chambers from the top and will be retained if the chamber is waterproof.

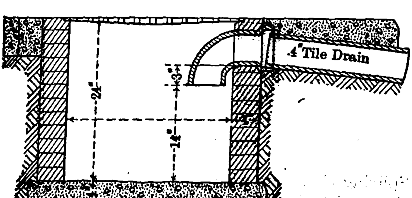


Fig. 7.

Every chamber should be provided with a sump into which the water can drain, Fig. 7. It is desirable to connect the sump to the sewer through a syphon and back-water valve. If this is not practicable the sump may be drained through the manhole by a hand pump.

Where natural drainage cannot be secured, as, for example, where the chamber is below the sewer level, it is good practice to provide a special drain pipe to which all the chambers are connected, the drainage being towards a general sump pit, which is kept

dry by an automatic sump pump. The cost of such a system often compares favorably with that of waterproofing, besides yielding superior results.

The sump of a splicing chamber should be covered with an easily removable grating; wooden ones are often used on account of their property of floating when the chamber is flooded, thereby leaving the sump open for the pipe of a hand pump.

Manhole Castings. — Manhole castings are usually made with two covers (Fig. 8), the outer of which should be made strong enough to stand the weight of the heaviest vehicle, and the inner should be as light as consistent with the rough usage they receive in handling. Both covers should be provided with means to grip them with a hook - bar. Where power cables are used it is essential to provide ventilating holes. It is usual to lock the inner cover with a substantial brass padlock with protected keyhole.

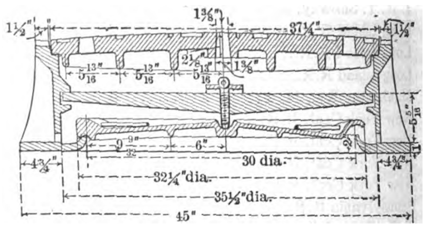


Fig. 8.

Cable Supports. — (*See also article on Wires and Cables, Insulated.*) Cables in splicing chambers are usually supported on iron brackets attached to the chamber walls. This construction is very satisfactory, especially if the power cables are properly wrapped with asbestos, but some engineers consider a shelf to be preferable, as it affords insulation and provides more protection between cables.

Concrete reinforced with expanded metal or wire cloth has proved satisfactory for shelves one inch thick, and is inexpensive. A good support for shelves is a pair of iron pins set in iron pipes, which are sunk in the chamber wall at a slight angle, so as to tilt the shelves toward the chamber wall as shown in Fig. 9.

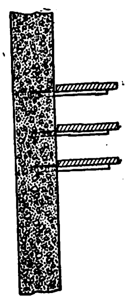


Fig. 9.

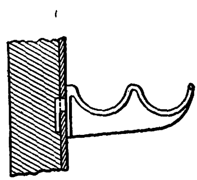


Fig. 10.

Splicing chambers with shelves have to be made larger and more expensive than those with duplex racks (Fig. 10), if an equal number of cables have to be carried.

Open-face conduits (Fig. 11) constitute a type of shelf which may be conveniently run all the way through a chamber to connect duct to duct. This gives excellent protection to the cables, but owing to the large size of the conduits,

used in a moderate-sized chamber is even more limited than where shelves are used.

RODDING AND WIRING OF CONDUIT LINES. — The only test which it is usual to apply to conduit lines is rodding with a mandrel in order to ascertain whether the ducts are continuous and unobstructed.

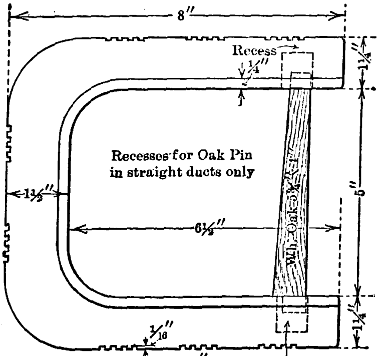


Fig. 11.

The rods used for this purpose are of hickory about one inch in diameter and three or four feet long and are fitted at the ends with steel couplings such as that shown in Fig. 12. The first rod is attached to a mandrel and pushed into the duct. Another rod is coupled to the first and the pair pushed further into the duct. By successively coupling other rods and pushing them into the duct, the mandrel is made to travel from one chamber to another. As soon as the mandrel emerges into the receiving chamber, the rods are pulled through and uncoupled. If an obstruction stops the mandrel an attempt is made to force it through by repeated blows, failing which, it becomes necessary to cut into the conduit line from the side.

Mandrels for Rodding. — Various types of mandrels are used for testing ducts, some hollow and smooth with sharp cutting edges, others fitted with numerous sharp projections and known as hedgehogs (Fig. 13).

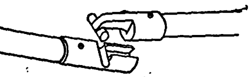


Fig. 12.

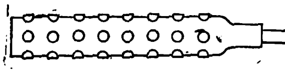


Fig. 13.

"Wiring" the Ducts. — It is usual to attach a galvanized steel wire (No. 10 or 12) to the last rod and leave the wire in the duct after the removal of the rods. The ducts may also be "wired" by the use of a conduit machine. This consists of a reel of steel tape and means for winding and unwinding the tape into the duct at the rate of about ten feet per second. This tape or "snake" having been pushed through the duct is pulled out again with the wire attached to its end.

SPECIFICATIONS FOR CONDUIT LINES. — (*See also article on Specifications.*) By far the greatest number of cable breakdowns occur from injury to the lead sheaths, either by electrolysis or by abrasions during installation. Sharp projections in the ducts should therefore be carefully guarded against, as a cable pulled over such a projection will have a groove cut along its entire length and the effective thickness of the sheath will be thereby materially reduced. The specifications for inspection and rodding given below are typical of the best modern practice, but unfortunately duct inspection is usually so lax that often in a large installation scarcely a single duct conforms in every particular with the specifications. Engineers are now beginning to realize the practical importance of giving each duct a rigid inspection, and rejecting those which have any

roughness or irregularity. Scarcely less important is rigid inspection during installation, in order to assure the most perfect alignment.

Details of Construction. — The following items should be covered:

Conduits. —

Single-way, or four-way.

Holes circular or square with rounded corners.

Maximum outside dimensions.

Minimum inside dimensions.

Minimum thickness of walls.

Shall be free from blisters, cracks and other imperfections which, in the opinion of the Engineer, will tend to injure the cables to be accommodated therein.

Shall be of good quality tile, thoroughly glazed inside and outside.

Shall be straight and true.

Shall be provided with holes for dowel pins if ducts are four-way. Dowell pins may also be called for.

The sides of single duct conduits shall be combed with two (2) sets of three (3) longitudinal combings each, each combing to have a width of one-quarter (¼) inch and a depth of one-sixteenth (1/16) inch. Multiple-duct conduits shall be scored transversely near the ends.

Conduits shall be of stated length. It is usual to call for a certain percentage of shorter lengths (generally 1 per cent), in order to finish runs or stagger joints. The permissible variation from the specified length should be stated.

(If fiber conduit is to be used, substitute the corresponding requirements.)

Cement. —

Shall be of approved brand.

Briquettes made of neat cement and kept one day in air and six days in water shall show a stated tensile strength. A corresponding strength may be similarly required of briquettes made of one part cement and two and one-half parts sand.

Shall be protected from moisture during work.

Sand. —

Shall be clean and free from loam or salt.

Shall be sharp and of stated coarseness.

Stone. —

Shall be of crushed granite, lime stone, trap rock or other approved variety.

Shall pass through a sieve of stated mesh.

Brick. —

Shall be of good commercial hard-burned sewer brick, or other stated variety.

Concrete. —

Stated proportions of cement, sand and stone. (Usually 1 part of cement, 2 parts of sand and 4 parts of stone.)

Maximum size of aggregate. (Usually ¾ inch.)

Sand and cement shall be mixed dry and wetted with only sufficient water to make a stiff paste. The stone having been previously wetted shall be added while wet and thoroughly mixed until all the stones are covered with mortar. It shall then be deposited as rapidly as possible. Machine-mixed concrete will be accepted if made in a

Cement Mortar. —

Stated proportions of cement and sand. (Usually 2½ to 1 of cement.)

Concrete mortar shall not be laid in freezing weather, and shall not be used after initial set has taken place.

Excavation. —

Shall always be of such depth as to leave a stated minimum distance between the top of the concrete over the conduits and the surface of the ground.

Ground on which conduits are laid shall be rammed solid before any concrete is laid.

Refilling Excavations. —

The best part of the material excavated shall be used.

Surplus material shall be carted away by the contractor (or will be carted away by the company).

Filling shall be thoroughly tamped and rolled, or flushed, as seems necessary to the Engineer, and shall be done in a manner to prevent, as far as possible, a settling of the earth after completion.

Obstructions. —

Obstructions encountered in the course of the work shall be overcome in a manner to be approved by the Engineer.

Laying Conduits. —

Shall be laid with ends square so as to leave a tight, well-fitting butt joint.

Joints shall be staggered horizontally and vertically.

Conduits shall be laid in a bed of cement mortar of about ¼-inch thickness.

Each joint shall be wrapped with two strips of burlap 6 inches wide and coated with neat cement mortar, the ends of the wrap to lap 4 inches. (It will insure more careful work if it be specified that the contractor shall supply rubber gloves to the men who lay the burlap.) Where conduits are laid on curves, the wraps shall be doubled if required to protect the openings between the ends of the conduits on the outside of the curve, and to exclude mortar from said openings. (This method of wrapping is not universal. If another method is to be used, corresponding details should be given.)

Conduits shall be laid with a mandrel of specified length and width and provided at the end with a rubber washer for wiping the joints.

Conduits shall be laid on a bed of concrete of stated depth, shall be covered at the top with a stated depth of concrete, and shall have a stated thickness of concrete on each side. Where the conduit line goes under railroad tracks, the concrete shall be suitably thickened and reinforced.

If conduits are four-way, they shall be laid with dowell pins at joints.

The alignment horizontally and vertically shall be satisfactory to the Engineer.

Drainage of Conduit Lines.— The grade of all conduit lines shall be such that water cannot stand in the ducts but shall drain into one or both splicing chambers.

Repairing. — All repairing shall be done in a manner satisfactory to the Engineer and the municipal authorities.

Extras. — No extras will be allowed the contractor for irregular work except

Rock blasting, if necessary.

Removal of pipes or other obstructions, if necessary.

Such work shall not be done without the consent of the Engineer. The contractor shall specify the cost of such work before it is begun.

Details of Terminals of Conduit Lines. — (Whether they go into power stations, etc.)

Splicing Chambers. — Shall be built according to plans supplied, unless local conditions interfere, in which case the suggested modifications shall be approved by the Engineer.

Chambers to be not more than a stated distance apart.

State any details pertaining to the design of the chamber and cable supports which may not be clear from the plans.

Tests. — The contractor shall notify the Engineer sufficiently in advance of the completion of the conduits to enable inspection at the factory to be arranged for.

Rodding, Cleaning and Wiring. — After the conduits are laid and the cement is sufficiently set, they shall be rodded and the contractor shall draw after such rods, wire brushes and a mandrel of specified dimensions. All mortar and other foreign matter shall be removed. If obstructions are found which cannot be removed by cleaners so as to pass the specified mandrel, the ducts shall be removed and relaid. Any expense incurred by such work shall be borne by the contractor. A galvanized wire of stated size shall be left in each conduit from splicing chamber to splicing chamber, and sufficient length shall be left at each end to permit it to be bent in order to prevent it from slipping into the duct.

Inspection. — During the process of construction, the Engineer reserves the right to inspect and the right to reject any and all parts which are not strictly in accordance with this specification.

INSTALLATION OF CONDUIT LINES. — The problems connected with the installation of conduits vary greatly with the local conditions. They involve the choice of trench excavation, drainage, removing or avoiding obstructions, concrete mixing and so on. The following details are gleaned from first-class examples of the various kinds of work described but should in no sense be regarded as standard.

Cross-country Conduit Lines. — The excavation for the conduit line of the American Telegraph and Telephone Company, between New York and Washington, was made partly by a trenching machine and partly by a trench plow. The trenching machine was of the Austin "caterpillar" type and dug a trench 18 inches wide and 3 feet deep at the rate of 3 feet of clean trench for each minute of actual working time. An engineer and two assistants were required to operate the machine, replacing, it is estimated, 50 laborers. Other sections were excavated by a trench plow drawn by two mules or horses, the furrow being made a few inches deeper at each pass, and the loose earth removed by shovelers. The conduit used in this installation was "pump-log" made of southern yellow pine 4½ inches square and 7 feet long with a circular duct 3 inches in diameter. These conduits were creosoted by the pressure process, with 15 pounds dead oil of coal tar per cubic foot of wood. Splicing chambers having concrete-block walls were built every 500 feet.

Conduit Lines for Railroads are usually difficult to construct and operate for the following reasons:

(1) Owing to the right-of-way being usually on made ground, excessive quantities of concrete and reinforcement are required to make a reasonably strong duct construction.

(2) Owing to the width of the right-of-way being usually very restricted it

(3) Owing to the vibration caused by heavy trains, it is necessary to bury the conduits at a greater depth, first to avoid undue stress on the conduit, and second to avoid crystallization of the cable sheaths.

In order to have the conduits below the frost level, the depth of ballast must be neglected, as it has been found that with stone ballast on top of the ground, the frost penetrates the ground about as far as if the ballast were not there, unless the ballast is very dirty.

(4) There is considerable difficulty in obtaining best results from labor where there are continual interruptions from trains. On a busy section a duct construction gang engaged for ten hours can possibly work two full hours.

(5) Owing to the right-of-way being often quite low, in many cases alongside of rivers, duct construction is likely to be seriously impeded by the flooding of trenches.

(6) Where the right-of-way shows signs of settlement, as, for example, on marshy ground, continuous piling is necessary to support the ducts. This involves the use of the track for construction purposes for long periods, and thereby not only impedes, but also endangers traffic.

(7) Duct-line construction generally involves interference with signal and interlocking apparatus, thereby introducing danger and expense.

(8) Bridge abutments, bridges, culverts, and in fact all special right-of-way construction, present complicated problems which can be solved only at great expense.

Conduit Lines in Cities. — The obstructions due to sewer, water and gas pipes, car tracks and foreign conduit lines, also render conduit construction in big cities a complicated problem. Plans made in the office can seldom be followed in the field, as the municipal pipe plans are seldom reliable, and the supervision of an experienced civil engineer is needed to solve the numerous problems which constantly arise. Excavation is almost invariably performed by hand labor, and when the conduits have been laid and covered with concrete, it is usual to lay a plank over them in order that future excavators may not drive picks into them. Obstructions are often avoided by changing the grouping of the conduits.

Laying Conduits. — Conduits must be laid so that joints are mechanically strong and the ducts unobstructed. With this in view joints should be staggered horizontally and vertically, and each joint covered with a wrapping impervious to mortar in bulk. Formerly the wrapping was omitted and the conduits joined with stiff mortar, but this became too expensive when the labor unions insisted upon the employment of masons for this work.

Single-duct Conduits are laid with their joints wrapped in a light-weight fabric saturated with thin cement mortar. In the Pennsylvania Tunnels, canvas weighing 10 ounces per yard was used, and cut into six-inch strips. Cheese cloth doubled has also met with favor. When laying conduits it is necessary to have a long mandrel (Fig. 14) to remove loose cement from the ducts. This mandrel is usually provided with a rubber washer at the rear, and a hook-eye at the front end. The conduit layer is provided with a hook rod by means of which he draws the mandrel after him as he lays the conduit.

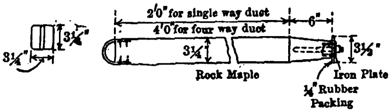
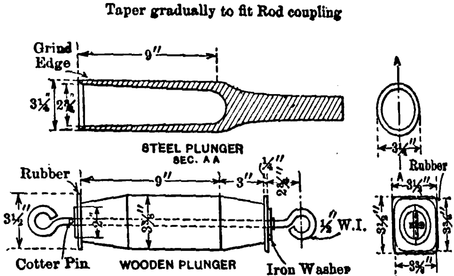


Fig. 14.

Four-duct Conduits are usually provided with holes for dowel pins by means of which the ducts are aligned. The joints are wrapped in burlap soaked in asphalt and afterwards painted with asphalt. No mandrel is used in this type of construction.

Concrete Covering. — However the conduits may be laid, the complete group is always inclosed in concrete to secure rigidity and protection, and the finished ducts are cleared of rubbish and obstructions by pushing a steel plunger through them by means of the rods described above. A steel plunger for this purpose is shown in Fig. 15, in which is also shown a wooden plunger with rubber washers, which should be drawn through the ducts to collect the loose particles left behind the steel plunger.



Steel Plunger, Steel Wire Flue Brush and Wooden Plunger to be passed through ducts in order named.

Fig. 15.

MAINTENANCE OF CONDUIT LINES. — The principal items of conduit-line maintenance are those relating to keeping the line clean and safe, namely, pumping out water, blowing out gas, removing mud, opening and closing manholes for the benefit of cable workers and inspecting the line to guard against theft and injury. Large systems have usually one or more wagons equipped with apparatus required for these purposes, and have men ready to go out with it upon emergency calls.

Removal of Water is usually the most important of operating troubles, especially where no drainage system is installed. When cable accidents occur, it is important to have the chambers accessible without delay and a portable pump is required. For this purpose a small gasoline or electric pump is useful, having a capacity of about 50 gallons per minute. Such a pump which has given good service is of the horizontal centrifugal type with horizontal discharge and costs about $200 complete with electric motor, hand priming pump and starting rheostat.

Ventilation does not occur naturally in conduit lines, because the cold air contained in them has no tendency to rise. Noxious gases therefore tend to accumulate in splicing chambers, endangering workers and making explosions possible. No permanent system of ventilation has proved successful, as it is found that pressure is maintained only at or near the blowing points. When it is necessary to blow out the chambers, it is therefore usual to employ a portable blower in conjunction with an air-tight false manhole cover.

Protection from theft of cable is secured (1) by locking the inner manhole cover and having the lock combination periodically changed; (2) by patrolling the line and (3) by using cable differing in some way from that used by neighboring companies, so that it can be easily identified if stolen.

Electrolysis is more fully treated in the article on *Electrolysis*. It should be noted that the prevention of electrolytic corrosion of cable sheaths depends more upon efficient drainage than anything else.

Maintenance of Conduit Lines Along Railroads present the following peculiar operating difficulties.

(1) Owing to the right-of-way being often on made ground, duct lines settle and crack, injuring the cables in them and preventing the removal and replace-

(2) Owing to the great depth of splicing chambers necessitated by railroad conditions, they are often full of water and cannot be cleared for repairs without pumping water out of as much of the system as is at the same level, a process which may take many hours to complete, possibly interfering with traffic during that time. Drainage is usually out of the question, owing to the absence of any kind of a drainage system below the surface system.

(3) The great depth of chambers requires the use of narrow chimneys connecting the chambers to the surface of the ground. This makes it almost impossible for employees to escape from chambers in case of trouble.

(4) Where improvements are made involving the raising of the right-of-way, as, for example, in eliminating grade crossings, ducts laid previous to the improvements become so deep that they are practically inaccessible for repairs and splicing chambers are correspondingly dangerous on account of their distance from the surface.

(5) The existence of water in low splicing chambers renders the cables particularly liable to electrolytic corrosion. This is a very serious matter where the grounded return is only a few feet away, as is almost invariably the case on a railroad. Electrolytic trouble cannot always be reduced by grounding the cable sheaths to the track rails, as such connections are seldom permissible where electric signals are used.

REPAIRS. — The principal repairs to conduit lines are those due to settlement and to damage done by adjacent building operations, such as the construction of sewers or railway tracks. It is sometimes desirable to replace the conduit line without disturbing the cables they contain. In such cases, the conduits are broken, the utmost care being taken to avoid injuring the cables. New conduits are then relaid on a firm foundation, after having been split longitudinally so as to fit over the cables. The whole construction is then rendered rigid by being inclosed in concrete.

COSTS. — The costs of conduit lines published from time to time are of merely local value, the labor of preparing a dry trench free from obstructions being an item whose variations are so great as to render insignificant the items of constant cost.

Mr. Quimby's table, given below, is probably representative of city practice, but conduit lines along railroads cost from 60 cents to $3.00 per duct foot, an average of 82 cents per duct foot having been obtained on a recent large railroad installation. In this case the labor item was about 80 per cent of the total although the construction was extremely solid and the number of ducts usually 20 or more in each trench.

The items to be considered in making an estimate of the cost of a conduit line are the following, the quantities all being per foot of trench.

Labor: excavating trench; excavating for splicing chambers; boxing and bracing; removing obstacles, such as gas and water pipes; mixing and placing concrete; placing conduit; carting and dumping; repaving; superintendence, outside; office expense.

Material: conduit; manhole and service boxes; covers, shelves, etc; sand, stone and cement; brick for splicing chambers (unless made of concrete); lumber for protecting top of ducts; lumber for bracing; incidentals.

The following table, adapted from that published by E. R. Quimby in the *Elec. World*, 1911, Vol. 57, p. 1294, includes all these items, and is based on actual cost of construction in New York City. Full details will be found in the original article. The figures given are for Belgian block pavement replaced and kept in repair for one year.

COST PER TRENCH FOOT OF CONDUIT LINE UNDER CITY STREETS

Estimates Based on Costs in New York City

Number of ducts	Number of rows of ducts	Trench dimensions in inches, width by depth below pavement (a)	Conduit line exclusive of splicing chambers and service boxes: Labor	Conduit line exclusive of splicing chambers and service boxes: Material (b)	Service boxes (c)	Splicing chambers (d)	Grand total: Per trench foot	Grand total: Cents per duct foot
3-Inch wrought-iron pipe laid in concrete and protected by creosoted wood								
1	1	16×26	$0.507	$0.220	$0.568	.....	$1.295	129.5
2	1	20×30	0.758	0.649	0.568	.....	1.975	98.8
3	1	26×30	0.873	0.943	0.568	.....	2.384	79.5
4	2	30×30	1.006	1.178	0.568	.....	2.752	68.8
4	2	26×35	1.020	1.134	.....	0.384	2.538	63.5
5	2	26×35	1.086	1.389	.....	0.384	2.859	57.2
6	2	35×35	1.148	1.507	.....	0.384	3.039	50.7
7	2	36×35	1.335	1.757	.....	0.420	3.512	50.2
8	2	36×35	1.360	1.930	.....	0.420	3.710	46.4
9	3	30×40	1.405	2.172	.....	0.450	4.027	44.7
10	3	36×40	1.535	2.359	.....	0.450	4.344	43.4
3-Inch tile duct laid in concrete and protected by creosoted wood								
4	1	30×36	1.370	0.615	.....	0.384	2.369	59.2
6	2	25×42	1.485	0.740	.....	0.384	2.609	43.5
8	2	30×42	1.631	0.895	.....	0.420	2.946	36.8
10	3	30×46	1.833	1.080	.....	0.450	3.363	33.6
12	3	30×46	1.942	1.263	.....	0.500	3.705	30.9
14	3	36×46	2.207	1.515	.....	0.500	4.222	30.1
16	4	32×51	2.310	1.620	.....	0.520	4.450	27.8
18	4	36×51	2.585	1.851	.....	0.520	4.956	27.5
20	4	38×56	2.817	1.983	.....	0.520	5.320	26.6
22	5	38×56	2.847	2.133	.....	0.520	5.500	25.0
24	4	43×51	3.043	2.283	.....	0.520	5.846	24.4
26	5	43×56	3.088	2.460	.....	0.550	6.098	23.4
28	4	45×51	3.203	2.648	.....	0.550	6.401	22.9
30	5	43×56	3.278	2.788	.....	0.550	6.616	22.1
32	5	45×56	3.497	2.964	.....	0.580	7.041	22.0
34	5	45×56	3.552	3.119	.....	0.580	7.251	21.3
36	5	49×56	3.736	3.313	.....	0.580	7.629	21.2
38	5	49×56	3.791	3.443	.....	0.580	7.814	20.6
40	5	49×56	3.844	3.599	.....	0.600	8.043	20.1
42	6	49×61	3.905	3.743	.....	0.600	8.248	19.6
44	6	49×61	3.925	3.853	.....	0.600	8.378	19.0

a. Add 8 in. for thickness of pavement. b. Pipe at 18 cents per foot, tile duct at 6 ... feet. c. Service boxes every 55 feet. d. Splicing chambers every 260 feet.

Depreciation. — (*See also article on Depreciation.*) Conduit lines depreciate very slowly from the effects of deterioration or obsolescence. The rates given in the article on depreciation are based more upon guesswork than upon experience, as there are few conduit lines in existence which have lost much of their value from age unless unforseen circumstances have deprived them of some useful association upon which their value depended. The value of conduit lines often increases with age especially where they have been laid in growing districts, where the demand for duct space increases more rapidly than the supply.

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[W. A. Del Mar.]

CONTROL SYSTEMS FOR RAILWAY MOTORS. — (*See also Controllers; Motors; Railways, Electric Traction Systems for.*) The function of the control equipment is to regulate the speed and direction of the motors by certain definite systematic changes in connections. The speed of direct-current railway motors is controlled in two ways: (1) by connecting suitable resistances in series with the motors, which will reduce the voltage across the motors and thereby the current which they will take; (2) by changing the connection of the motors so that they will be connected at first in series, thereby applying half of the line voltage to each motor, and then in parallel across full line voltage. In most control equipments a combination of both methods is used. For the speed control of alternating-current railway motors an auto-transformer, or compensator, is used instead of the resistances.

The direction of rotation of the motors, d-c. or a-c., is changed by changing the direction of the current in either the fields or the armatures; it is customary to connect the terminals of each field coil to a reversing switch in order to accomplish this effect.

TERMINOLOGY. — The following terms are in general use.

Cylinder or Drum Control or Direct Control are names commonly applied to an equipment in which all the connections are made by contacts on a cylinder or drum which is manually operated by the motorman and located on the platform. This may, therefore, be called direct control.

Multiple-unit, Indirect, Remote Control or Train Control are names applied to an equipment in which the changes in connection of the main power circuit are made by switches called "contactors," usually located underneath the floor of the car, and controlled by electric circuits coming from a small master controller on the platform. There are two systems of multiple-unit control in use in this country, viz.,

Sprague-General Electric System. — In this system the contactors are closed by electromagnets which force a plunger against a spring, the latter normally holding the switch open.

Westinghouse "Unit Switch" System. — In this system the contactors are closed by compressed air, from the air-brake cylinders, the air valves at the switches being controlled electrically from the master controller.

Hand Control is a term applied to that method of control in which the motorman has it in his power to regulate the current to any value he pleases by moving the controller handle, the change in connections depending only upon the motion of the latter.

Automatic Control, as distinguished from hand control, is a type of control in which certain automatic devices prevent the motorman from causing the motors to take a current greater than a predetermined value. With this method of control the motors start with a definite current and as soon as the current has decreased to a specified value a change in the connections is automatically made. Thus the rate of acceleration and the current are kept practically uniform throughout the period of control. It is nearly always used in connection with multiple-unit control.

Rheostatic Control consists in connecting a resistance in series with the motor and short-circuiting consecutively parts of this resistance. It is seldom used at present, except on mining locomotives and for single motor operation.

Series-parallel Control, which is used on practically all railway equipments includes the feature of connecting two motors and their resistances in

tively until all resistance is cut out, under which condition the motors will operate efficiently at approximately half speed. On the next step of the controller the two motors with resistance in series are connected in parallel and subjected to full line voltage. There are three methods of accomplishing the change from series to parallel.

Transition with Power Off. — In the so-called type L controller power is entirely cut off from both motors while the change in connection is being made. This was formerly used for large-size motors and locomotives but is not at present much used.

Transition with Series Resistance. — During the transition from series to parallel a resistance is placed in series with one motor and the other motor is first short-circuited, then disconnected from the main circuit, and finally, placed in parallel with the other motor. This method is in general use in equipments of small motors with the so-called type K controller.

Bridge Transition. — The so-called "bridge" method consists in grouping the motors and their resistances like the arms of a Wheatstone bridge, so that after the two motors are in full-series position the resistances may be placed in circuit again in parallel with the motors, without opening the circuit; the two motors are then connected in parallel with each other and each in series with its own resistance. This method is preferable to either of the other two in that both motors are in operation throughout the whole control period. There is no noticeable jerk and it is not necessary to open the circuit, which would cause flashing at the switches. It is used in certain forms of the K control and in most of the multiple-unit control equipments, particularly for motors of large capacities and for locomotives.

Series-parallel Control with Four-motor Equipment. — Whereas the three methods just described apply particularly to two-motor equipments, they are equally applicable to four-motor equipments by connecting two motors permanently in parallel and treating them as a unit.

TYPE K CONTROL. — The type of control as well as the construction of the controllers for ordinary single-car equipments has been practically standardized in this country, the large manufacturers supplying control equipments which are practically identical. This type of control is known as the type K. The various sizes of type K controllers are listed in the table below.

Where type K controllers are used for motors of large capacity they are sometimes adapted with a modification of the remote control by the addition of two electrically-operated main switches placed underneath the floor of the car, the function of which is to open the main power circuit every time it is necessary that it should be opened and thus remove all flashing and arcing from the controller. This expedient makes it possible to use a smaller controller for a given capacity of motors and obviates all danger to the passengers from fire and fright. The scheme is accomplished by substituting for the main power circuit on the controller an auxiliary circuit carrying only one or two amperes and every time this auxiliary circuit is opened in the main controller the main switches underneath the car open the power circuit. When the auxiliary circuit is closed the main switches close the main circuit. By means of an overload trip operated by a coil in the main circuit these switches are also used as circuit breakers and if the current taken by the car exceeds a certain value a relay opens the auxiliary circuit which in turn causes the main switch to open.

Capacity and Weight of Type K Controllers. — The more usual forms of type K controllers and the capacity in motors for which they are adapted are as follows:

TYPE K CONTROLLERS

Designation	Number of motors	Total h.p.	Number of points	Weight, pounds*
K-10-A	2	80	5 series 4 parallel	940
K-10-H	2	80	5 series 4 parallel	
K-11-A	2	120	5 series 4 parallel	1020
K-11-H	2	120	5 series 4 parallel	
K-12-A	4	120	5 series 4 parallel	1175
K-12-D	4	120	5 series 4 parallel	
K-28-B	4	160	5 series 5 parallel	1350
K-28-F	4	160	5 series 5 parallel	
K-34-D	2 or 4	360	6 series 4 parallel	2250
K-34-F	2 or 4	360	6 series 4 parallel	
K-35-G	2 or 4	240	5 series 3 parallel	1800
K-35-M	2 or 4	240	5 series 3 parallel	

* Weight includes cables, etc., but not motors.

The 10 H, 11 H, 12 D, 28 F, 34 F and 35 M have auxiliary contactors for opening the main circuit.

The K 34 and K 35 have "bridge" transition. All others short-circuit one motor during transition.

The letter "B" in the designation of a controller indicates that it has contacts added to it to make it possible to operate electric brakes by causing the motors to act as generators to energize electric brake shoes, either of the axle or rim type.

Method of Operation of Type K Control. — The principle of the type K control for small and moderate-size motors is shown in Fig. 1. The controller has an operating handle which moves the main cylinder and thereby changes the connections, and also a reversing handle which moves the reversing cylinder. The latter merely changes the direction of the current through the fields of all the motors with respect to the armature. These two handles or cylinders are interlocked so that the reversing handle can only be moved when the operating handle is in the off-position, thus preventing reversal with voltage on the motors.

One terminal of one of the motors is grounded throughout. The first three points are known as accelerating steps. As resistance is in circuit for each of

length of time, for there is a considerable power loss in the rheostats and they are not designed for continuous operation. The fourth step, full series, is an efficient "running point," giving about half normal speed. The next two steps are transition steps and are not marked as points on the controller as they must be passed over rapidly. During this period one terminal of the second motor is grounded, thus short-circuiting the motor which has one terminal grounded initially; the connection between the two motors is then opened; finally the two motors are connected in parallel but in series with a part of the rheostat. If the controller is left on the transition point B the short-circuited motor may build up as a series generator and develop an excessive current. However, it takes an appreciable time for this to happen, so that any steady continuous movement of the controller will avoid the trouble. Points 5, 6 and 7 are accelerating steps with motors in parallel and resistance in series and are therefore not to be used continuously. At the last point the motors are in parallel, and all resistance is cut out; it is therefore an efficient high-speed running point.

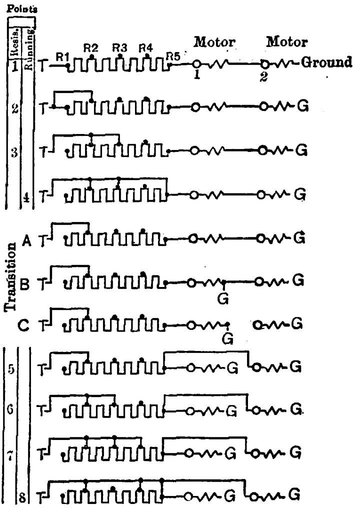


Fig. 1. Type K Control

In all controllers a magnetic blow-out and an "arc chute" are employed to interrupt the current quickly and direct it away from the contacts, in order to prevent short-circuiting other contacts. In the older types of controllers this was obtained from one large magnet coil and a large iron pole piece covering all the contacts. In the later forms each contact has an independent blow-out coil and small pole pieces. This gives a more powerful effect and more accurately directs the arc in the proper direction.

EFFICIENCY OF TWO- AND FOUR-MOTOR CONTROLLERS. — See *Railways, Energy Requirements for.*

REASONS FOR MULTIPLE-UNIT CONTROL. — When the total capacity of the motors on a car or locomotive exceeds 300 horse-power it is advisable, and when the capacity exceeds 400 horse-power it is necessary, to use the indirect or multiple-unit control, for the cylinder type of controllers required to handle the large currents become too bulky and dangerous to place on the platforms of passenger cars. The cylinder control is also inadequate when it is desired to control simultaneously the motors on the several cars in a train, which is necessary in order to obtain the high tractive effort necessary in high-speed service on elevated and underground railways.

For these two reasons the system of multiple-unit or train control was developed. As originally proposed by Sprague this consisted of a large cylinder controller on each car and each controller was actuated by a small motor instead of by hand. These small motors were controlled synchronously from a single point by means of auxiliary control circuits. With the growth in the capacity

of the motors this system became inadequate and was replaced by the systems now in use.

SPRAGUE-GENERAL ELECTRIC CONTROL. — There are two types in use, the type MK and the type MA. The chief difference in these two types is that the type MK is non-automatic whereas the type MA is provided

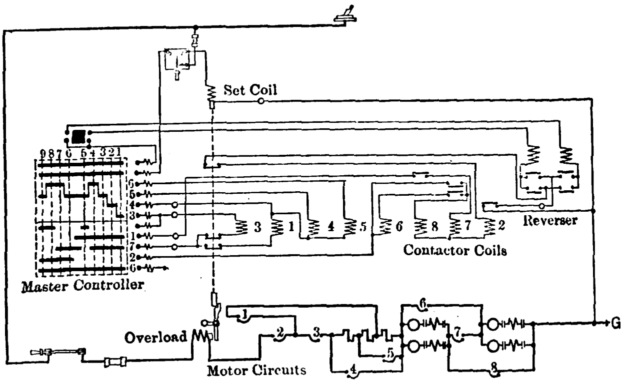


Fig. 2. Sprague-General Electric Type MK Control

with a current-limiting relay. Fig. 2 is a diagram of the type MK, showing the control circuits in light lines and the motor circuits in heavy lines.

The material included in the control equipment of a motor car consists of:

2 master controllers;
1 motor controller containing 8 or 10 contactors and a reverser;
3 master control switches;
1 main switch;
1 main fuse box;
1 set of rheostats;
Cables, Train Couplers.

Master Controller. — This is very similar to an ordinary railway drum-type controller but is much smaller, as the current carried by it is small. Each master controller is equipped with an operating handle, reversing handle, individual magnetic blow-outs and (optionally) a "deadman's handle," which automatically interrupts the current and applies the brakes when the motorman's hand is removed from the button located in the top of the handle.

Motor Controller. — This consists of an iron box lined with asbestos in which the several contactors and the reverser are placed, and is mounted under the car.

Contactors. — Each contactor consists of a powerful magnet operating an arm by means of a toggle joint against a spring pressure. This arm closes and opens the circuit in a strong magnetic field which acts as a blow-out. As one contactor can carry and break currents of several thousand amperes no extra circuit breakers are required. Each contactor is provided with interlocks in the form of relays so that contactor No. 2 cannot be operated until after No. 1 is closed, thus providing the proper sequence of operation under all conditions.

Reverser. — This is a switch with several circuits and contacts and is comparable to the reverser cylinder in an ordinary controller. These contacts are mounted on a rocker-arm actuated by two electromagnets, one for moving the switch to the forward and the other to the reverse position. The electromagnets receive their current from the master controller and are interlocked so that only one can be operated at a time.

Switches and Fuses in Control Circuit. — Motor cut-out switches are located on the reverser to permit cutting out a disabled motor. In the control circuit there is one main control switch and fuse to protect the control circuits, and near each master controller is located a "control and reset" switch, which in one position closes the circuit to the resetting coil of the overload relay in the main controller and in the other position closes the supply circuit for the master controller.

Main Switch. — A knife-blade switch is placed in the power circuit and is intended to disconnect the motor circuits from the trolley when it is desired to test the motor controller.

Train Couplers are provided where cars are to be operated in trains. These couplers or jumpers provide a means of connecting together similar control circuits of the different cars. They contain from 8 to 12 wires and are so designed that it is impossible to couple the cars together improperly.

Current-limiting Relay. — For automatic control, or acceleration at a predetermined current, a current-limit relay is provided on each car and this prevents each successive contactor from operating until the current in the motors has decreased to a predetermined value. On roads having a fairly level profile and operating with frequent stops this refinement is desirable, as it makes it possible for the motorman to accelerate the train every time at the maximum allowable rate and yet never exceed that rate except on a down grade. When this relay is provided the control equipment is known commercially as the type MA control.

WESTINGHOUSE "UNIT SWITCH" SYSTEM. — The Westinghouse multiple-unit control is known as "Unit Switch Control" and is designed either for the operation of several motor cars in a train or of single cars or locomotives, using either large currents or high voltage. There are four types of unit-switch control classified as follows:

Type H. L. — Hand-operated (non-automatic), using line voltage for the control circuits and having as many points on the controller dial as there are steps in the operation; also has a seperate reverser handle. One motor or group is short-circuited during transition from series to parallel.

Type A. L. — Automatically operated, using line voltage for the control circuits. The controller dial shows three points forward and three reverse, corresponding to switching, series running and parallel running, although there are more steps in the operation. Uses bridge transition from series to parallel.

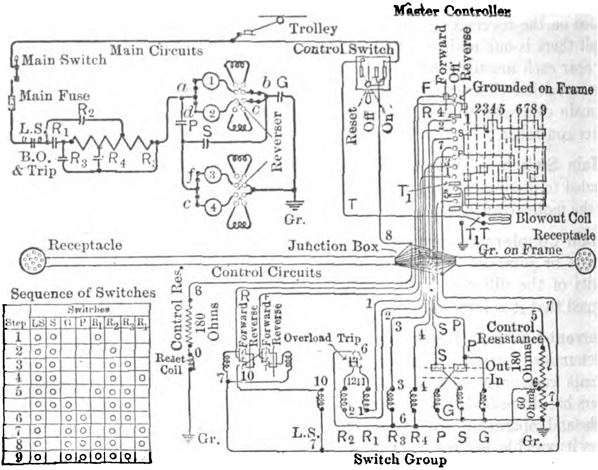
Type H. B. — Hand-operated, using current from a storage battery for the control circuits. Otherwise similar to the H. L.

Type A. B. — Automatically operated, using current from a storage battery for the control circuits. Otherwise similar to the A. L.

Operation of Type H. L. Control. — As typical of all of these classes the diagram in Fig. 3 shows the connections of the H. L. type for a four-motor equipment for either a car or a locomotive (*Cole, Electric Journal, Oct. 1912*),

As many as five motor cars per train may be operated by one man with this control system.

The main-circuit connections are made by means of a number of independent pneumatically-operated switches, known as unit switches, each provided with a strong magnetic blow-out and normally held open by a powerful spring. The overload trip that controls the opening of all switches in case of overload



Sequence of Switches

Step	LS	S	G	P	R_1	R_2	R_3	R_4
	Switches							
1	o	o			o			
2	o	o				o		
3	o	o				o	o	
4	o	o				o		o
5	o	o			o	o	o	
	o	o	o			o	o	
6	o		o	o		o	o	
7	o		o	o		o		o
8	o		o	o	o	o		o
9	o		o	o	o	o	o	

Fig. 3. Westinghouse, Type H. L. Unit Switch Control

or short-circuit is actuated by the magnetic pull produced by one of the blow-out coils, which is in the main circuit. When a predetermined current value is exceeded, a plunger carrying two contact discs breaks the control circuit, which causes certain switches to open in the line and switch group. This trip can only be reset when the controller is in the off-position.

A pneumatically-operated reverser controls the direction of operation of the car. This reverser consists of two pistons similar to those attached to the unit switches, which serve to move the reverser drum to forward or reverse position. The air-brake system furnishes compressed air for operating the reverser and switches.

A multi-conductor train line extends the length of each car and is tapped off at the master controllers and at the unit switch group to form the circuits needed. This train line is made continuous throughout a number of cars by multi-conductor jumpers between cars fitted into receptacles on each car. The current for the electromagnets which operate the valves of the unit switches is taken from low-voltage taps on a resistance connected across line potential.

Operation of Type A. L. Control. — The A. L. control differs from the hand-operated control in that the acceleration of the train is automatic. The master controller has only three positions, switching, series running and parallel running. The "off" position is in the center. The unit switches are provided with interlocks which are electrically connected with the valve magnet in such a manner that the closing of one switch energizes the magnet of the next, thus producing automatic progression of the switches, under the direction of the limit switch. This limit switch controls the rate at which the resistance

is cut out of circuit so as to give uniform accelerating current. The limit switch consists of a solenoid operated by the current of one motor or a pair of motors. When this current exceeds a specified limit for which the switch is adjusted, the circuit is opened through a pair of contacts in the operating circuit. The circuit remains open, so that no more unit switches can close until the accelerating current falls below the predetermined limit, when the control circuit is again closed and allows the unit switches to continue their progression.

SPECIFICATION FOR MULTIPLE UNIT CONTROL EQUIPMENT.* — The following memoranda are intended to assist in writing specifications for a complete equipment for the electrical control for a multiple-unit car or locomotive exclusive of motors and collecting shoes. See also article on *Specifications.*

Give a complete description of the service in which the equipment is to be used, including the maximum number and weight of cars or locomotives per train.

Similar motion of the master-controller handle shall always produce similar train motions.

A device shall be provided (this is optional) which will limit the rate at which the controller increases the motor voltage, and will assure even acceleration without surging, at the changes from series to series-parallel, and series-parallel to parallel, etc.

A relay shall be provided (this is optional) limiting the current to a specified maximum. (The use of such a relay is more usual on heavy, than on light, equipments.)

Provision shall be made so that acceleration can be arrested at any position by the master controller and so that the motor-circuit combinations shall never be beyond the position indicated by the master controller.

The controller shall automatically return to initial or open-circuit position when the general current supply fails, and when current is restored the control shall progress, as specified, to its former advanced position.

The reverser shall be interlocked so that it cannot be thrown when the motors are taking current.

The control apparatus shall operate satisfactory with a maximum line voltage of . . . . and a minimum line voltage of . . . . and shall never take a current exceeding . . . . amperes per car.

In the event of a train breaking in two, provision shall be made so that power shall be cut off from the detached rear portion without affecting the control of the front portion.

The master-controller handle shall be designed (this is optional) so that if the motorman releases it while operating a train, the power will be automatically cut off.

State whether controller is to be interlocked with air brakes.

Main circuit switch (for street cars only) shall be within easy reach of motorman at each end of car.

Each car or locomotive shall be provided with a control-circuit cut-out switch, which will enable the contactors on any car or locomotive to be disconnected from the control circuits.

Each car shall be provided with an automatic circuit breaker and devices for tripping it from any car of the train.

CAPACITY, WEIGHT AND COST OF CONTROL EQUIPMENTS. — The following table gives the capacity and weight and the approximate costs of some typical control equipments.

* By W. A. Del Mar.

Number of motors	H.P. of each motor	Type of control	Weight of control equipment, pounds	Weight of each motor, pounds	Total weight of equipment, pounds	Cost of equipment, dollars
4	25	*K*	1200	1900	8,800	1650
4	50	*K*	2200	2450	12,000	2400
4	75	*K*	2200	3200	15,000	3200
4	75	Multi-unit	2800	3200	15,600	3400
2	125	Multi-unit	2700	4150	11,000	2700
2	200	Multi-unit	3200	6400	16,000	3750

CONTROL OF HIGH-VOLTAGE D-C. MOTORS.—The motors operating on systems of from 1200 to 1500 volts are usually designed to operate two in series, thus each receives normally 600 or 750 volts. However, the insulation of each motor must be designed to withstand the whole line potential, and each motor must be able to withstand momentarily the line voltage across its commutator, for if one motor slips the voltage will be unevenly divided between them. For operation on high voltage two motors in series are normally treated as a unit and the series and parallel connection made with these double units. The multiple-unit control is preferable with these high voltages and contactors or unit switches similar to those for 600 volts are used. To operate the control it is customary to supply a self-starting dynamotor (q. v.) which provides 600 volts for this purpose as well as for the lights and other auxiliary apparatus.

Provision for 600-volt Operation. — Since these equipments usually operate also over 600-volt sections of road, provision has to be made to change the connection of the dynamotor when the transfer is made. If the cars are to operate at reduced speed on the lower voltages, as is usually the case on entering the city districts, no change need be made in the motor connections. But if the cars must operate at 600 volts at high speed over an interurban section, then provision must be made to separate the pairs of motors so that all motors will be in parallel for full-speed operation on 600 volts. This requires a commutating switch with automatic protection in order to provide that it is always changed when the car passes from one section to another.

2400-volt Systems.—For locomotive work 1200-volt motors are constructed to operate two in series on 2400 volts. The control for such a locomotive is similar to the control for a 1200-volt equipment.

CONTROL OF A-C. COMMUTATOR MOTORS.—A transformer or compensator is always used to transform the line voltage (3000, 6000 or 11,000 volts) to a voltage suitable for the motors, which is usually from 400 to 500 volts for two motors in series. Taps on the low-voltage side of the compensator (or auto-transformer) provide the various voltages necessary to start and control the motors and thus there is no need of series-parallel control or rheostats, and the energy lost in the rheostats is obviated. A compensator or auto-transformer is usually preferred to a transformer as it is lighter for a given capacity. It is usually placed in oil in a tank suspended from the bottom of the car body. Fewer steps (5 or 6 in all) are required for the control of a-c. motors on account of the reactance of the circuits. To avoid open circuiting the connection to the motors in changing from tap to tap or short-circuiting the portion of the transformer between the taps a "preventive resistance" is connected in the circuit momentarily during the transition. A reverser is provided to reverse the con-

The control may be either of the cylinder or multiple-unit type. In the former type a standard controller may be adapted for the work. If the multiple-unit control is used the cores of all the magnets and contactors must be of laminated iron and a special design of magnet used on account of the difference in characteristics of a-c. and d-c. magnets. In some a-c. multiple-unit equipments a storage battery is used to supply current for the control circuit, in which case d-c. electromagnets may be used in the contactors.

Provision for D-C. Operation. — For operation on direct current as well as alternating current provision must be made to perform the following operations: (1) cut the transformer out of circuit, (2) connect the motors for series-parallel control, (3) connect rheostats in circuit, (4) change the field connection of the motors, (5) change the connections of the compressor motors, (6) change the connections of the lighting circuits. All this is done by a "commutating switch" which is thrown over at the instant the change is made. This is so arranged that it can only be moved when the controller is at the off-position. The commutating switch is frequently operated automatically, so that when the car reaches a dead section of the trolley between the a-c. and d-c. sections, a no-voltage release throws everything to off-position and when the car reaches the new live section a "selector" coil moves the commutating switch to the proper position.

CONTROL OF THREE-PHASE INDUCTION MOTORS. — Three-phase induction motors for railway work may be controlled by three methods, viz: (1) Changeable pole windings, (2) concatenation of two motors, (3) variable resistance in secondary.

The first two methods were given a considerable trial by German manufacturers some years ago, and have been practically abandoned on account of their complications. In addition to their complications the variable resistance method must also be used with them to provide the smaller gradations of speed.

Variable-resistance Methods. — The secondaries of the motors have a definite winding and the terminals are brought to collector rings by means of which a three-phase starting resistance is connected into the circuit. The speed of the motors is controlled by varying this resistance. Under these conditions the operating characteristics of the motors are similar to those of a d-c. shunt motor. At all fractional speeds a considerable amount of energy is wasted in the rheostats and there is only one efficient running speed. For prolonged running at fractional speed the rheostats must have considerable heat-dissipating capacity. To reverse the direction of the motors a reverser is employed which reverses the connections of two of the three primary leads on each motor. Either the cylinder or multiple-unit control may be used. With induction motors it is desirable to provide a separate set of resistances for each motor to avoid the tendency of the motors to exchange current and "buck," which would occur if the driving wheels were not of exactly the same diameter and one set of resistances were used for all motors. With several induction motors on one car it is desirable to accurately maintain the same diameter of driving wheels on all axles in order to divide the load equally between the motors.

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[W. I. SLICHTER.]

CONTROLLERS. — (*See also Control Systems for Railway Motors; Regulators; Rheostats; Switchgear Equipment for Power Stations.*) Any device for regulating the current or voltage of an electric circuit may be called a controller. Various kinds of controllers have been given different names by users and manufacturers.

DEFINITIONS. — To avoid confusion the various names are used throughout this book as defined below.

Compensator or Induction Starter, an auto-transformer for supplying reduced voltage to the terminals of an induction motor during acceleration; see *Starters, Motor.*

Controller, any device which controls the running speed of a motor. This article deals only with controllers as here defined. A controller frequently combines starting with running features; see *Starters, Motor.*

Regulator, any device for adjusting the voltage of a circuit. A compensator is a special form of regulator. The term regulator is sometimes reserved for devices which utilize inductive action for their control properties; such a device will be designated specially as a potential regulator; see *Regulators.*

Rheostat, the resistance portion of a controller or starter, through which flows the main current of the circuit whose voltage is controlled. A field rheostat is any device employing a variable resistance to control the voltage across the field windings of a generator or motor. The field rheostat of a d-c. motor indirectly controls the speed, because it controls the voltage and current of the field winding; see *Rheostats; Starters, Motor.*

Starters, Motor, a resistance or auto-transformer used with a motor, either d-c. or a-c., to limit the starting current; see *Starters, Motor.*

CONTROLLERS FOR DIRECT-CURRENT MOTORS are usually operated by varying the resistance in the armature or field circuits of the motors, if the motors run separately, or by various groupings of motors in series and parallel, where the motors are in pairs, as in the usual railway practice.

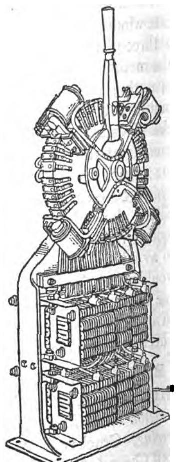

Fig. 1. Grindstone Controller

"Grindstone" Type of Controller (Fig. 1). — One of the simplest types of controllers is the so-called "grindstone" controller shown in Fig. 1, which is ordinarily furnished for series-wound d-c. crane motors. The movable-finger contacts are mounted on a four-arm spider. The connections are so made that moving the arm in one direction will mean clockwise rotation of the motor, and movement from the off position in the opposite direction will mean counter-clockwise rotation. The amount of movement controls the amount of resistance in the armature circuit and consequently the speed of the motor.

In controllers of this type all wearing parts, contacts and finger tips are extremely simple and inexpensive and readily renewable so that the maintenance charge is kept down to a minimum. In the smaller sizes the contacts are mounted on a flat soapstone face plate with a single magnetic blow-out coil centrally located to prevent injurious arcing. For larger sizes, such as illustrated in Fig. 1, four sets of copper contacts are arranged on the periphery of a circular soapstone disk in

is provided with a blow-out coil and in some cases an auxiliary switch is provided for use with a brake magnet when the contact arm passes to the off position.

Drum Type of Controller (Fig. 2). — Another type of controller in common use is the drum controller, such as shown in Fig. 2. Drum controllers are used with machine tools for varying the speed and reversing the direction of rotation of adjustable-speed d-c. motors by means of armature and field resistance. On the larger sizes magnetic blow-outs are used. The controller illustrated is shown with its cover removed and is provided with two dials, one connected with the armature resistor and one with the field resistor. In the smaller sizes these resistors are inside the controller and in larger sizes they are outside.

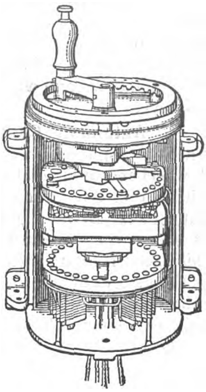

Fig. 2. Drum Controller (cover off)

Series-Parallel Controllers. — A familiar type of drum controller is the series-parallel controller in general use on street cars for the control of two or four d-c. series-wound motors. To keep down the starting current and to provide an even rate of acceleration, the first position of such a controller connects resistance in series with all of the motors, which are themselves connected in series in pairs. The resistance is cut out in steps until the two motors or two pairs of motors are in series without resistance, when the car is running at approximately half speed. The motors are then changed to the parallel arrangement with resistance in series with each motor. The resistance is again cut out in steps until the motors are all operating in parallel and the car is running at maximum speed. Occasionally provision is made for shunting the series fields of the motors for still higher speed. Controllers of this type are almost invariably provided with magnetic blow-outs, reversing switches, motor cut-outs and various mechanical interlocks that prevent passing from series to multiple connection if a motor is cut out, prevent operating the main drum if the reverse handle is in the off position, etc. Such controllers are frequently provided with brake attachments and are made as fool-proof as possible. (*See also Control Systems for Railway Motors.*)

Master Controllers. — Drum controllers are also used as master controllers with contactors to secure starting and speed regulation of large a-c. and d-c. motors, as well as the multiple-unit control of motor cars or locomotives on railway service.

CONTROLLERS FOR INDUCTION MOTORS. — Controllers with squirrel cage secondaries usually operate by connecting the motors to various voltages obtained from transformer taps. If the motors are provided with wound secondaries the controllers frequently operate by varying the resistance in the secondary circuit. The resistors must then be designed to carry the running current of the motor continuously without overheating (*see Motors, Polyphase Induction; Starters, Motor*).

With a reversing motor, as in crane or rolling mill service, a similar method of control is used except that the controller is provided with two drums which are operated by a single handle. The motor will run in one direction when the handle is turned to the right, say, and in the reverse direction when it is turned to the left. In going from the off position the first step connects the primary circuit of the motor to the line and subsequent steps cut out the secondary resist-

ance. In passing from full speed in one direction to full speed in the other the resistance is all cut into circuit before reversing. In the off-position the motor is entirely disconnected from the line.

Use of Contactors with A-C. Controllers. — For reversing, mill or hoisting work using induction motors with wound secondaries many very ingenious and highly satisfactory installations have been put in service which use solenoid-operated magnet switches, or contactors, for the secondary and occasionally for the primary circuits. These are worked from a master controller or similar device, or are operated automatically by the positions of the rolls, hoist, etc. Automatic acceleration can be obtained in the same manner as with direct-current motors and various safeguards, such as dynamic breaking, can also be employed.

Automatic Control of Input to Flywheel Motor-generator Set. — Another application for contactor control with automatic features is with flywheel motor-generator sets which use a very heavy flywheel in connection with a d-c. generator and an a-c. motor with wound secondary. The power put into the flywheel or delivered up by it depends upon the variation in speed of the motor generator. By varying the resistance in the motor secondary this speed regulation can be secured. By the use of suitable relays the input to the motor and consequently the load on the a-c. system can be kept practically constant, while the output of the d-c. generator supplying power to a d-c. hoist or rolling-mill motor is undergoing wide fluctuations, the energy in the flywheel taking care of the difference between the constant input and the variable output. See also article on *Flywheels for Load Equalization.*

COST OF CONTROLLERS. — The following figures will serve as a rough indication of the cost of various sizes and types of controllers. Costs of controllers vary through wide limits depending upon the design, the amount of speed variation desired, etc.

COST OF CONTROLLERS FOR 500-VOLT MOTORS*

Horse-power of motor	5	25	50	100	150
Grindstone type (Fig. 1)	$105	$110	$115	$170	$200
Drum type (Fig. 2)	52	70	100	....	....
Reversing controllers for wound secondary induction motors, 2 to 1 speed range	....	90	140	235	357

* Controllers for 110- or 220-volt motors cost about the same as for 500-volt motors.

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[S. Q. HAYES.]

CONVERTERS, SYNCHRONOUS OR ROTARY. — (*See also Generators, Alternating-current; Generators, Direct-current; Motor Generators; Motors, Synchronous; Transformers; Substations, Railway.*) Since in general it is more economical to transmit electrical energy in the form of alternating currents and more convenient to utilize it in the form of direct currents, some means of converting from one form of electrical energy to the other is desirable. For this purpose synchronous converters and motor-generator sets (q.v.) are available. Synchronous converters are also called "rotary converters."

A synchronous converter is a machine very similar to a d-c. generator in which certain commutator segments, or the conductors connected to them, are connected to 2, 3, 4 or 6 collector rings as the case may be. When the movable member is caused to rotate, the voltage between any two collector rings is alternating. Such a machine, when driven by an engine or motor, may be operated as an a-c. generator or as a "double-current" generator giving alternating current from its collector rings and direct current from its commutator. If the collector rings are connected to a source of alternating currents the machine will run as a synchronous motor and direct current may be obtained from the brushes on the commutator; i.e., the machine, with but one set of windings acts simultaneously as an alternating-current motor and a direct-current generator. It has therefore the friction, core-loss and excitation loss of one machine instead of two, and since the motor and generator currents flow in the same winding and during at least the major part of each cycle are in opposite directions, they more or less balance each other and the armature RI^2 loss is much less than in either a motor or generator alone.

Synchronous Converter versus Motor Generator. — A converter is much more efficient and weighs and costs less than a motor-generator set of the same capacity. It also occupies less space. However, since only one winding is used, there is a definite relation between the e.m.f's. of the a-c. and d-c. terminals. The maximum value of the alternating wave bears a definite relation to the direct e.m.f. (*see below*). It is therefore necessary to supply the converter with a voltage of the same order as the direct voltage and this involves the use of transformers, if a high-voltage transmission line is used to supply the converter. Motors operating at voltages as high as 13,000 volts can be used in motor-generator sets.

Relative Efficiencies. — The efficiency of a converter is in the neighborhood of 93 per cent and of the transformers 97 per cent, thus the efficiency of the combination is about 90 per cent. The efficiency of a synchronous motor is in the neighborhood of 93 per cent and of a d-c. generator 92 per cent, thus the combination motor-generator set has an efficiency of 85.5 per cent. If the supply voltage is greater than 13,000 volts, transformers will also be needed for the motor-generator set and the net efficiency would then be 83 per cent.

Synchronous Converter versus Rectifiers. — A converter differs from a rectifier (q.v.), since the former gives a direct e.m.f. of constant and uniform value and the latter gives a pulsating unidirectional voltage and current. In the former the energy is stored in the form of magnetism for an instant whereas in the latter there is no magnetic field and no storage of energy. A rectifier will not work on an inductive d-c. circuit but a converter will.

APPLICATION OF CONVERTERS. — The most common application of synchronous converters is in electric railway work. The great majority of motors for electric traction are direct-current series type, operating at from 500 to 600 volts. The energy for these motors must be transmitted over long distances, which requires a high voltage a-c. transmission line and converters to link the d-c. distribution with the a-c. transmission.

TERMINOLOGY. — The following terms are used to describe certain characteristic features of the various kinds of synchronous converters.

Phases and Rings. — A single-phase converter has two collector rings and each ring is connected to the windings by as many equally spaced taps as there are pairs of poles. The taps for the two rings alternate at equal spaces. A single-phase converter is therefore a two-ring converter.

A three-phase converter has three rings and three equally spaced taps (one for each ring) for every pair of poles. A four-phase or quarter-phase converter has four rings and four taps for every pair of poles. A six-phase converter has six rings and six taps per pair of poles.

Shunt and Compound-wound Converters. — A converter may be shunt or compound wound, depending upon the service for which it is intended. The series winding is intended to make the converter take leading current when the load increases and thus increase the voltage at the a-c. terminals, but the ratio of the a-c. *terminal* voltage to the d-c. voltage remains unaltered.

Inverted Converter. — Sometimes a converter is operated to convert from d-c. to a-c. It is then called an "inverted converter." The machine will operate satisfactorily in this manner, but its speed depends upon the nature of the a-c. load. An inductive load in the a-c. circuit causes the armature to demagnetize the fields, with a resultant increase in speed. It is therefore dangerous to operate an inverted converter on an inductive load unless it is provided with a speed-limit device. This effect does not occur when the machine is operating as an a-c. motor, since its speed is fixed by the frequency of the supply circuit.

Motor Converter. — This is a combination of an induction motor and converter connected in series or concatenation (*see Motors, Induction*). The converter receives half the power in mechanical form from the shaft and half the power inductively, in the form of alternating current at half frequency, from the secondary of the induction motor. By this means the steadiness of a 30-cycle converter is obtained in a 60-cycle unit.

Split-pole Converter. — The "split-pole" or "regulating-pole" converter is designed to give a variable ratio of alternating e.m.f. to direct e.m.f., for operation in parallel with storage batteries and similar purposes. The field poles are divided into sections which may be assisting or opposing each other magnetically. This changes the shape of the flux distribution, which in turn changes the wave shape of the counter e.m.f. and thus the ratio of d-c. (maximum) to a-c. (effective) voltage is changed.

Converter with Series Booster. — A synchronous converter with series booster is sometimes used for purposes similar to those of the regulating pole converter. It is merely a converter with a separately excited a-c. generator on the same shaft as the converter, and the armature of this generator is connected in series with the converter armature and the line. This generator acts as an a-c. booster and raises the line voltage.

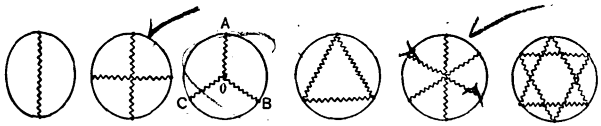
RATING AND PERFORMANCE. — The manufacturing companies have up to the present (1914) divided synchronous converters into classes for purposes of rating according to their overload capacity in the same manner as a-c. generators (q.v.) are classified. Twenty-five-cycle converters will give a considerable overload without injurious sparking at the commutator, but 60-cycle converters are more sensitive to overloads. Certain 25-cycle railway converters will give three times normal output momentarily without damage. For the new A.I.E.E. ratings see *Standardization Rules*.

PRINCIPLES OF CONVERTER ACTION. — In this section are briefly treated those features ... in which the converter

differs in action from an a-c. or d-c. generator; see table on page 284 for a summary of the voltage, current and capacity relations.

Connections and Voltage Ratios. — The ratio of voltage on the a-c. side to that on the d-c. side depends upon the number of rings and type of connection employed.

Two-ring Converter. — The two collector rings are connected by taps to the same winding as the commutator; hence the alternating e.m.f. has the same value as the direct e.m.f. at the instant that the taps pass the brushes. As this is also the maximum value of the alternating e.m.f., the effective value (i.e., the value to be indicated by a voltmeter) will be 0.707 times the maximum or 0.707 times the direct e.m.f. Fig. 1 shows in a simple manner the connection of a single-phase converter. The external circle represents a two-pole arma-



Figs. 1 2 3 4 5 6

Transformer Connections and Vector Relation of Voltages in Synchronous Converter

ture winding and inside is shown the supply transformer connected to two taps diametrically opposite each other. The voltage across this transformer would be 0.707 E, where E is the voltage between the positive and negative brushes on the d-c. side.

Four-ring Converter. — If two additional collector rings are connected to conductors spaced half way between the former taps, there results the quarter-phase converter shown in Fig. 2. The voltage across each supply circuit or transformer is the same as before, but the two voltages will differ in phase by 90 degrees.

Three-ring Converter. — If three collector rings are connected to taps spaced 120 degrees apart the e.m.f. between adjacent collector rings will be the vector sum of AO and OB in Fig. 3. Since AO and OB each equal $0.5 \times 0.707\,E$ the voltage AB will be $\sqrt{3} \times 0.5 \times 0.707\,E = 0.612\,E$. Thus in a three-ring or three-phase converter the voltage between adjacent taps is 0.612 times the direct voltage and the connections of transformer are as in Figs. 3 and 4.

Six-ring Converter — Diametrical and Double Delta Connections. — A six-ring converter may be connected diametrical as in Fig. 5 in which case the voltage of each transformer will be $0.707 \times E$ and the result will be like the combination of three single-phase groups. A six-ring or six-phase converter may also be connected "double delta" as shown in Fig. 6, which is similar to the combination of two groups of three-phase delta transformers. In both cases the voltage between adjacent taps of a six-phase converter is $0.355\,E$.

n-Ring Converter. — In general the voltage between taps of any converter having n equally spaced taps per pair of poles is

$$E_{ac} = \frac{E \sin \frac{\pi}{n}}{\sqrt{2}}.$$

Current Ratios. — To determine the ratio of the continuous current I to the alternating current I_3 per collector ring in a three-phase converter, for example, assume the d-c. output equal to the a-c. input with unity power factor; then

$$\sqrt{3}\, E_3 I_3 = EI,$$

and, from preceding paragraph,

$$E_3 = 0.612\, E,$$

where E_3 = a-c. voltage between lines; I_3 = alternating current per line; E = direct voltage; I = direct current. From these two relations

$$I_3 = 0.94\, I.$$

The ratios of currents for other converters are obtained similarly, and are given in the table on page 284.

In actual practice the current on the input side must be greater than that given by these relations, in order to supply the losses in the converter, and the alternating current will also vary inversely as the power factor, which is taken as unity in the table.

Resultant Coil-Current. — In any machine acting simultaneously as a motor and a generator the two currents must flow in opposite directions, and the current in any particular conductor will be the difference between the two. In any particular coil the direct current is constant in amount and direction from the instant the commutator segment connected to this coil passes the positive brush to the instant it passes the negative brush, and conversely from negative to positive brush. In any coil midway between the a-c. taps the alternating current is a maximum when this coil is half way between brushes (for unity power factor), and the current falls to zero as the coil reaches the interpolar position. Therefore, for the period of time that the direct current in a coil remains constant in amount and direction there is also in it a variable current changing from zero to a maximum and back to zero again. The net or resultant current will therefore have a wave shape and frequency somewhat as shown at R in Fig. 7. The heating in this particular coil will therefore be proportional to the product of the resistance of the coil by the square of this current, and it is readily seen that the power lost is less than that due to either the direct or alternating current alone.

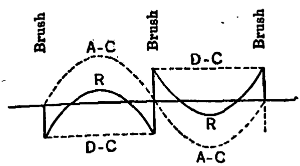


Fig. 7. Current in Coil Midway between Taps

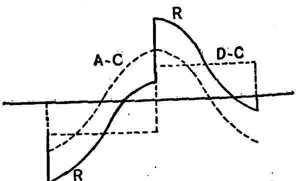
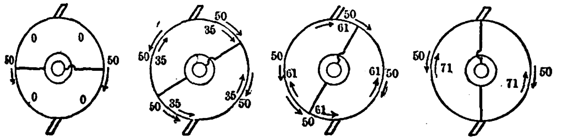


Fig. 8. Current in Coil at Tap in Two-ring Converter

A coil situated very near one of the a-c. taps will carry a direct current subject to the same law as before mentioned, but the alternating current in this coil is the same as that in the middle coil and has its maximum value when the middle coil is at the middle of the poles and not when this particular end coil is at the middle of the pole. The alternating current in this coil may therefore reach its maximum value soon after the coil has passed a brush, as shown in Fig. 8, and the resultant of the two currents is greater than that in the middle coil. The further a coil is situated from the middle coil of a group, the greater is the phase displacement of its alternating current, the greater is the value of

the resultant current and the greater is the heating effect. Thus although the heating effect in a rotary converter winding is less than that of a direct-current machine having the same output it is different in each and every coil, is minimum in the central coil (when the power factor is unity) and is maximum at the coil nearest the tap, and the greater the angle between taps the greater is the total heating effect.

Distribution of Heat Losses in Armature Winding.—In Figs. 9 to 12 inclusive, representing a two-ring converter, the numbers outside the circle



Figs. 9 10 11 12

Component Currents in a Two-ring Converter

represent the direct current in the winding (corresponding to 100 amperes in the external d-c. circuit and unity power factor on a-c. side) and the numbers inside the circle represent the alternating current in the winding. It will be seen that the resultant current in a coil midway between taps, Fig. 9, never exceeds $50 + 0 = 50$ amperes. A coil 30 degrees on one side of the middle coil, Fig. 10, has a maximum current of $50 + 35 = 85$ amperes, a coil 60 degrees from the middle, Fig. 11, has a maximum current of $50 + 61 = 111$ amperes, and a coil 90 degrees from the middle, Fig. 12, has a maximum current of $50 + 71 = 121$ amperes. The heating of the armature winding is therefore not uniformly distributed, though the conduction of heat from one part of the winding to the other tends to equalize the temperature rise.

If the converter operates at a power factor different from unity, the position of minimum resultant current is no longer at the middle coil, but is moved one way with leading current and the opposite way with lagging current. Thus one end coil has improved heating conditions, and the middle coil and the other end coil have much worse heating conditions, the result being that the heating as a whole has increased and is more non-uniformly distributed than with unity power factor.

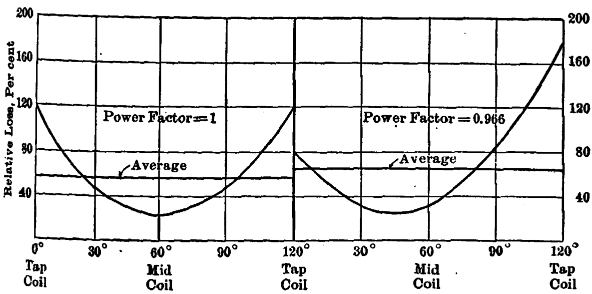


Fig. 13. Relative RI^2 in Individual Coils of Three-ring Converter

The power lost in each individual coil of a converter and the effect of the power factor on this loss is very well shown in Figs. 13 and 14, taken from a

paper by J. E. Woodbridge (*A.I.E.E., 1908*). In Fig. 13 the curved line shows the relative RI^2 loss in a coil having any position throughout 120 degrees of one phase of a three-phase converter when the power factor is unity. The

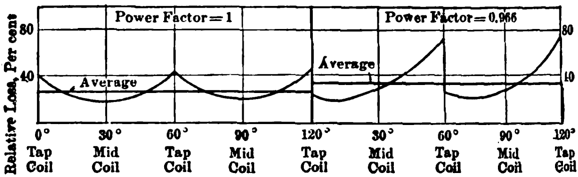


Fig. 14. Relative RI^2 in Individual Coils of Six-ring Converter

curve shows the ratio of the loss compared to the loss in the same machine acting as a d-c. generator of the same capacity. It will be noted that the middle coil has a loss of 22 per cent of that of the generator and the end coils 120 per cent, and that the average loss is 57 per cent. For a power factor of 0.966, representing a phase displacement of current of 15 degrees, the loss in individual coils ranges from a maximum value of 180 per cent at one tap to a minimum value of 22 per cent in the coil shifted 15 degrees to one side of the middle coil. The other end coil has a loss of a little over 80 per cent of the generator loss. The average value has been increased to 65 per cent.

In Fig. 14 the same ratios are shown for a six-phase converter, in which the winding of one phase is distributed over only 60 degrees. Consequently the conditions are better and the maximum loss due to low power factor is less.

Dependence of Output upon Number of Phases. — As a result of these conditions we have the following relations of the capacity of a given armature with various numbers and connections of taps, the capacity being based on an equal total amount of RI^2 loss.

It should be remembered that there are other losses besides coil losses in the converter, and that therefore practical figures are slightly different from those given in the table.

VOLTAGE, CURRENT AND OUTPUT RATIOS

	D-C. generator	Converters					
		2-ring	3-ring	4-ring	6-ring diametrical	6-ring double delta	12-ring
D-C. volts	100	100	100	100	100	100	100
A-C. volts between lines	...	71	61.2	71	71	61.2	71
A-C. volts between rings	...	71	61.2	50	35	35	18
D-C. amperes	100	100	100	100	100	100	100
A-C. amperes in line	...	141	94	71	47	47	24
A-C. amperes in winding	...	71	55	50	47	47	45
Relative RI^2 loss	100	137	55	37	26	26	20
Relative output:							
Unity power factor	100	85	134	165	197	197	224
87 per cent power factor	...	...	99	115	129	129	135

Field Excitation. — The variation of the field excitation of a synchronous converter has much the same effect as in the case of a synchronous motor (*see Motors, Synchronous*); that is, if its field is under-excited the armature will draw a lagging current which assists the field and sets up the necessary flux, whereas if the field is over-excited, the armature will draw a leading current which opposes the field and reduces the flux. By this means the converter may be made to take either a leading or lagging current. A leading current flowing over a line having inductance tends to raise the voltage at the receiving end of the line.

Line Compounding. — If there is sufficient leading current and sufficient inductance in the line, the voltage at the receiving end may be greater than at the sending end in spite of the resistance of the line. This is sometimes called "line compounding." The relation between the voltages at the two ends of the line may be expressed as follows:

Let

E = voltage to neutral at sending end;

V = voltage to neutral at receiving end;

I_1 = component of line current in phase with V, i.e., the power component of the line current;

I_2 = component of line current at 90 degrees to V, i.e., the *leading* reactive component of the line current;

r = resistance of one line;*

x = inductive reactance of one line;*

then, noting that I_2 is to be taken positive when leading,

$$E^2 = (V + rI_1 - xI_2)^2 + (xI_1 + rI_2)^2.$$

Use of Series Field. — In practice a series field is added to the converter and the shunt excitation is adjusted so that the armature current is lagging at no load. As the load increases the series field increases, the adjustment being such that at about ¾ load the proper excitation for unity power factor is given. Hence at all loads over ¾ the field will be over-excited, the current will be leading and the voltage will be raised or compounded.

DESIGN. — The design of a synchronous converter is very similar to the design of a d-c. generator (q.v.), except for certain special conditions due to the fact that the frequency is fixed by the frequency of the supply system, and that greater latitude is allowed in the choice of the nominal value of the d-c. armature reaction and copper density, because the real value of these quantities is the difference between their nominal d-c. values and their a-c. values.

Speed and Number of Poles. — The revolutions per minute and the number of poles are definitely related to the frequency of the supply circuit, in cycles per second, in accordance with the formula:

$$120 \times (\text{frequency}) = (\text{number of poles}) \times (\text{rev. per min.}).$$

The choice of the number of poles usually depends upon the commutator.

Commutator. — The design of the commutator is usually the limiting feature and therefore the first to be considered. This is particularly true of either high-frequency (60 cycle) or high-voltage (600 volt) machines. Three factors must be considered in the design of the commutator to secure successful commutation and life of the commutator, namely, peripheral speed, voltage between bars, thickness of bars. If the peripheral speed or the voltage between

* r and x should include respectively the resistance and reactance not only of the line wire, but also the transformers and reactance coils through which the line current passes.

bars is too high commutation will be bad. If the commutator bars are too thin the commutator will not retain its shape and commutation will be bad. These three limiting factors are very closely related and in a high-voltage machine give very little choice. The diameter of the commutator depends upon the voltage and frequency, and its minimum value is given by the three following relations. Let

s = pitch of commutator bars in inches. This ranges from 0.15 in small machines to 0.20 in high-voltage or high-frequency machines, to 0.40 in liberally designed machines. These values include the width of bar and about 0.03 inch insulation between bars.

V = peripheral speed of commutator in feet per minute. This ranges from 3500 in liberally designed low-voltage, 25-cycle machines to 5000 in 60-cycle machines, and is extended to 6000 under compulsion.

e = average volts per bar = machine voltage divided by the number of bars between brush studs. Normal values are from 8 to 14. The maximum voltage between bars is about 1.57 times this.

f = frequency in cycles per second;

E = voltage between d-c. terminals;

p = number of poles;

n = total number of commutator bars;

d = diameter of commutator in inches.

Then

$$s = \frac{Ve}{10Ef}, \qquad n = \frac{pE}{e}, \qquad d = \frac{sn}{\pi}.$$

Armature Ampere Turns and Current Density. — Since the d-c. and a-c. armature reactions are opposed to each other the nominal value of the d-c. armature reaction ampere turns may be chosen much higher than in generators and values of from 4000 to 8000 ampere turns per pole are in common practice. The nominal or apparent value of the ampere conductors per inch of periphery varies from 500 to 900, and the apparent current density (d-c.) in the armature copper from 2500 to 5000 amperes per square inch.

Diameter of Armature. — The diameter of the armature per pole varies from 3.5 inches to 6 inches, and the diameter is usually from 6 inches to 8 inches greater than the diameter of the commutator, the difference depending upon the possibility of making a good mechanical construction of the end connections. The number of slots in the armature and the number of segments in the commutator must be a multiple of both the number of phases and the number of poles.

Armature Winding. — The winding of the armature is usually of the multiple-drum type, although the series winding may be used. The turns per pair of poles are divided by equally-spaced taps (to the slip rings) into as many groups as there are phases. The number of turns in series between brushes is adjusted for the proper direct e.m.f. and a reasonable value of flux per pole. The alternating e.m.f. bears a definite relation to the direct e.m.f. depending upon the type of connection (*see above*), and also to some extent upon the shape and length of pole arc.

Flux Density in Air Gap. — The air gap or pole-face flux density usually has a value ranging from 40 to 60 kilolines per square inch as in generators.

Damping Copper in Pole Face. — In the pole face of every converter a squirrel-cage winding should be provided to assist in starting and to prevent hunting. The total cross-section of copper per pole ranges approximately

from 1/10 to 1/5 of the armature copper per pole and the end rings must be of reasonable cross-section compared to the bars. The joints between bars and end rings must be carefully made.

Shunt and Series Fields. — The determination of the length of armature, length of commutator and the design of the field follow the same laws as the design of these parts for a d-c. generator. The proportioning of the series field results from the considerations given above under *Field Excitation*, the problem being that a certain amount of leading current is required at a given load and to make the armature take this leading current the field excitation must be increased by a certain number of ampere turns, from which the number of turns in the series field can be determined.

Equalizer Connection. — In large multi-polar machines it is customary to connect to a common ring all commutator bars which are at the same potential; these "equalizer connections" avoid local cross-currents in the armature from flowing through the brushes and causing bad commutation. These cross-currents are caused by unequal or uneven air gap.

Shaft, Bearings, Etc. — Since the transfer of energy is in the conductors themselves, there is no mechanical torque other than that to overcome friction and core-loss. Therefore the shaft, bearings and mechanical housing of a rotary converter are quite light and present no particular mechanical difficulties in mechanical design. For the same reason converters do not require very elaborate foundations. Machines of less than 1000 kilowatt capacity are usually supplied with a base and are complete in one piece, while larger sizes are supplied with foundation plates.

Kw. rating	500	300
Frequency	25	60
Core-loss	1.00	1.75
Armature RI^2	0.55	0.60
Shunt field RI^2	0.70	0.60
Brush RI^2	0.40	0.40
Bearing friction	0.55	1.50
Brush friction	0.30	0.65
Efficiency	96.50	94.50

Efficiency and Losses. — The efficiency and distribution of losses of a typical 25-cycle and 60-cycle converter are shown in the accompanying table. All values are in per cent of input at full load.

EXAMPLES OF DESIGN. — In the following table are given design data on four representative converters of different capacities.

TESTING OF CONVERTERS. — The following are the usual tests made on converters to determine the efficiency, regulation, heating and to show any defects in construction.

1. Resistance of Armature, Shunt Field and Series Field. — The armature resistance is usually measured between points on the commutator diametrically opposite and the equivalent resistance calculated from this value by the equation

$$\text{Equiv. Res.} = \frac{4 \times (\text{diametrical resistance})}{(\text{number of poles})^2}.$$

SYNCHRONOUS CONVERTERS

Design Data

Item	Unit	1	2	3	4
Poles		4	6	6	6
Rating	kw.	500	500	100	300
Speed	rev. per min.	750	500	1000	1200
D-C. volts	volts	600	600	440	600
Frequency	cyc. per sec.	25	25	50	60
Number of phases		6	6	3	3
Armature reaction	amp. turns	15,000	6720	7400	3500
Nominal σ*	amp. cond.	1500	710	675	475
Flux density in gap	kilolines	58	52	23	53
Armature diameter	inches	25.6	36	21	28
Slots per pole		24	24	6 5/6	21
Armature length	inches	11	17.25	7.1	10.5
Commutator diameter	inches	19.7	28	16.5	20
Number of segments		288	288	194	252
Periph. speed of commutator	feet per minute	3800	3650	4300	6280
Volts per bar	volts	8.3	12.5	13.6	14.3
Pitch segments	inches	0.17	0.306	0.266	0.25
Armature diameter per pole	inches	6.4	6	3.5	4.75
Nominal U†	amperes	6000	3600	2630	3400

* σ = ampere conductors (d-c.) per inch periphery.

† U = amperes (d-c.) per square inch.

To obtain the true RI^2 in a converter armature this equivalent resistance is multiplied by the square of the external direct current and by a constant as given in the accompanying table:

	Theoretical	Commercial
Single phase	1.39	1.47
Three phase	0.56	0.59
Quarter phase	0.37	0.39
Six phase	0.26	0.27

These values only hold if the converter is operating at unity power factor. For any other power factor the reactive component of the alternating current per phase must be found and the square of this component times the resistance per phase of the armature gives the additional RI^2 loss due to the lesser power factor.

2. **No-load Saturation Curve.** — The no-load saturation curve, as in a-c. generators and d-c. generators (q.v.) is usually plotted between commutator voltage and ampere turns of shunt field.

3. **Core-loss** as in a-c. and d-c. generators (q.v.).

4. **Phase Characteristic** at no load and at full load as in synchronous motors (q.v.).

5. **Synchronous Impedance** as in a-c. generators (q.v.).

6. **Starting Tests** to determine current and voltage necessary to start the converter on a-c., time to reach full speed, and voltage induced in field windings as in synchronous motors (q.v.).

7. **Heat Run.** — This may be made either with a resistance for load or two similar converters may be tested in parallel by the Hopkinson or "pump back" method; see *Transformers*. In addition to a source of d-c. power of the rated voltage of the converter to supply the losses, either a d-c. booster or an a-c. potential regulator is needed to adjust the load.

8. **Insulation Tests.** — See *Generators, Alternating-current*, and *Standardization Rules of the A.I.E.E.*

9. **Pulsation Test.** — A synchronous converter is very sensitive to any change in impressed voltage or frequency. A sudden change in either of these factors will cause a pulsation which will start the machine hunting, as discussed in the article on *Motors, Synchronous*. This hunting may increase until the machine falls out of step or flashes over. To determine whether a converter has a dangerous tendency of this kind, a test is made in which two similar machines are supplied with power from a common generator. Between each converter and the common connection a resistance is placed in each a-c. line, having a value that will give with full-load current a drop in voltage of 15 per cent of the rated voltage of the machine. Thus there is 15 per cent *RI* drop between each converter and the generator and 30 per cent between the two converters. The two machines are operated at no load and at rated voltage with the shunt fields adjusted for minimum input. The voltage across the commutators is observed for any periodic variation. Then the field of one machine at a time is varied from half normal to twice the normal value and any indications of periodic variations of the direct voltage noted. If the machines have a proper damper winding in the pole faces, they should not develop any dangerous hunting, even under the above unfavorable conditions.

SPECIFICATIONS FOR SYNCHRONOUS CONVERTER.* — The following memoranda are intended to assist in writing specifications. See also article on *Specifications*.

Principal Characteristics and Conditions of Service. — Use to which converter is to be put, such as railway, lighting, motor driving or battery charging. Whether it is to convert from a-c. to d-c. or vice versa. Voltages and number of phases. Nominal rating in kilowatts or Institute rating in kilowatts. Frequency and speed.

Style and Description; Details of Construction. — Whether interpole; whether shunt or compound wound, or whether there is a split-pole field winding; whether rheostat is to be supplied for shunt field; if so, its characteristics. Proposed method of starting and whether starting apparatus is to be supplied. Whether a speed-limit device is required; if so, the shunt field rheostat shall have sufficient resistance to speed the machine for testing the speed-limit device, the latter being set at 15 per cent over rated speed. Whether an end play device (or oscillator) is desired and if so, what type or types are acceptable.

Work to be Done by Other Contractors. — Whether the synchronous converter contractor is to furnish and install the following: Main wiring, field wiring, field-rheostat grids, dial plate and chains, starting panels, starting rheostat or motor generator, foundations.

Performance and Tests. — Temperature rises upon which nominal and Institute ratings are to be based. Overload capacity (*see Standardization Rules of the A.I.E.E.*). Commutation limits. Efficiency at 25, 50, 75, 100, 125 and 150 per cent nominal load. High-potential tests of insulation. Requirements regarding effect of moisture upon insulation. Converters shall operate in parallel without "hunting" from no load to stated (say 200 per cent) overload, provided drop in high-tension lines due to resistance between any converter and

* By W. A. Del Mar.

any other synchronous apparatus in the system does not exceed a stated value (say 20 per cent), and provided that the phase variation does not exceed a stated value (say 2.5°). What voltage regulation is required, how it is to be obtained and whether it is to be hand or automatic. If the converter is shunt wound, state voltage-regulation requirements. What per cent reactance between a-c. bus and converter is required?

OPERATION OF CONVERTERS. — In the operation of synchronous converters the points mentioned below should receive special attention.

Transformer Connections. — The usual methods of connecting transformers to supply converters are shown diagrammatically in Figs. 1 to 6 and are discussed above in the section on *Voltage Ratios* and also in the article on *Transformer Connections*. Of the three-phase to six-phase connections the choice must be made with some care, as all connections are not equally good for each specific use of the converter.

Methods of Starting. — The several methods of starting converters are as follows:

Alternating-current Starting which is the same as the starting of motors; see *Motors, Synchronous*.

Direct-current Starting. — The machine is started as a direct-current shunt motor and synchronized on the a-c. side after it is up to speed. This requires less power, but takes more time and more skill in order to synchronize.

Starting with Auxiliary Motor. — This involves the extra cost and extra continuous loss of the auxiliary induction motor. It is no more efficient than starting by direct current and requires the same amount of time.

Combination Alternating- and Direct-current Starting. — The machine is started up with direct current, then disconnected from the d-c. mains and connected to a low-voltage tap of the a-c. supply and brought up to full speed. This method is more economical of power and time but requires more starting apparatus than either the a-c. or d-c. methods.

Field Break-up Switch. — All converters are supplied with a switch to open the field in several places to avoid the strain of the high potential induced in the field during starting and to reverse the direction of current in the field after the machine is up to speed in order to reverse the polarity in case it should not be right. This is usually a double-throw switch with several poles.

End-play Device. — In order to prevent the brushes from wearing grooves in the commutator and collector rings a device is mounted upon one end of the shaft to move the shaft end-wise back and forth periodically. In small machines this is a mechanical device consisting of a ball running between two warped surfaces. In large machines it consists of an electromagnet which periodically pulls out the armature a short distance. The magnetic pull of the main field poles pulls the armature back.

Speed-limiting Device. — In case a converter should be disconnected from the main a-c. generating circuit and still remain connected so that it would tend to operate from the d-c. side there is danger of its speed becoming dangerously high. To avoid this a centrifugal governor is placed on the shaft and arranged to electrically operate the main d-c. switches of the converter.

VOLTAGE REGULATION. — There are several methods of regulating the voltage delivered by the commutator of a converter.

Compound-wound Converter with External Reactance. — This method is automatic and will give about 10 per cent variation in voltage. It is standard

Regulating or Split-pole Converter. — With this type of converter the voltage regulation is gradual and normally accomplished by hand, but by the addition of an automatic voltage regulator may be made automatic.

Shunt-wound Converter with Induction Regulator. — A large variation in voltage is possible but this method does not respond to quick changes. It is quite generally used in lighting work.

Shunt-wound Converter with Synchronous Booster. — A synchronous generator is carried on the same shaft as the converter and connected in series between the transformers and the collector rings of the converter. The method is good but expensive.

Shunt-wound Converter and Taps on the Transformers. — The voltage ratio of the transformers may be varied. This is usually accomplished by connecting the line to different taps on the primaries, which involves opening the circuit or short-circuiting a portion of the transformer at each change.

WEIGHTS, SPEEDS AND COSTS. — The weights, speeds, over-all dimensions and approximate costs of two commercial lines of converters are given in the following table. The first group is a line of 25-cycle converters and the second a line of 60-cycle converters for railway work. It will be noted that smaller capacities are three-phase machines, whereas the larger capacities are six-phase machines. The six-phase connection makes possible a saving in material at the expense of an increase in manufacturing labor.

The costs given are only approximate, as such figures vary enormously with commercial conditions. There is a tendency at present to reduce both the weight and cost of these machines. All these machines are designed to give their rated output continuously with a rise of 35° C. and an overload of 50 per cent for two hours with a rise of 55° C.

WEIGHTS, SPEEDS AND COSTS

Poles	Kw.	R.p.m.	Volts	Phases	Over-all dimensions			Wt., lb.	Cost per kw. dollars
					Leng., in.	Width in.	Ht., in.		
4	200	750	600	3	91	84	72	19,000	13
4	300	750	600	3	98	84	77	21,000	11.5
6	500	500	600	6	122	95	87	34,000	10
6	750	500	600	6	134	115	95	44,000	9.3
8	1000	375	600	6	148	137	108	63,000	8.3
6	100	1200	600	3	70	61	58	7,000	14
6	200	1200	600	3	79	65	60	11,000	13
8	300	900	600	6	99	72	70	18,000	12
12	500	600	600	6	122	87	80	33,000	11.5
14	750	514	600	6	118	110	87	39,000	11
20	1000	360	600	6	136	145	116	58,000	11

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[W. I. SLICHTER.]

CONVEYORS. — (*See also Power Stations; Telpherage.*) A conveyor is a mechanical device for the continuous handling of materials along a horizontal or inclined plane. There are four general types of conveyors, viz., the scraper or flight, the screw, the bucket and the belt.

THE FLIGHT CONVEYOR consists of a trough of any desired cross section through which are pulled a series of scrapers or "flights" attached to an endless chain. The improved forms of this type of conveyor have sliding shoes or rollers attached to the flights or to the chains, supported on runways.

Capacity. — The following table by S. B. Peck (*Trans. A.S.M.E., 1910*) gives the capacities of flight conveyors in tons of coal per hour when the conveyor is operated at a speed of 100 feet per minute.

CAPACITY OF FLIGHT CONVEYORS

Size of flight, in.	Horizontal				Inclined		
	Flights spaced			Lb. per flight	10°	20°	30°
	18 in.	18 in.	24 in.		24 in.	24 in.	24 in.
	Tons	Tons	Tons		Tons	Tons	Tons
4 by 10	33.75	30	22.5	15	18	14.25	10.5
4 " 12	42.75	38	28.5	19	24.5	18	13.5
5 " 12	51.75	46	34.5	23	28.5	22.5	16.5
5 " 15	69.75	62	46.5	31	40.5	31.5	22.5
6 " 18	.....	80	60	40	49	40.5	31.5
8 " 18	.....	120	90	60	72	57	48
8 " 20	.....	...	105	70	84	66.5	56
8 " 24	.....	...	135	90	120	96	72
10 " 24	.....	...	172.5	115	150	120	90
10 " 30	.....	...	220	147	184	146	116
10 " 36	.....	...	268	179	225	177	142
10 " 42	.....	...	315	210	264	210	167

Power Required. — The following formula gives approximately the horse power at the head wheel required to operate flight conveyors:

$$H.P. = (ATL + BWS) \div 1000.$$

T = tons of coal per hour; L = length of conveyor in feet, center to center; W = weight of chain, flights and shoes (both runs) in pounds; S = speed in feet per minute; A and B constants depending on angle of incline from horizontal and have the following values.

Angle, deg.	*A*	*B*	Angle, deg.	*A*	*B*	Angle, deg.	*A*	*B*
0	0.343	0.01	10	0.50	0.01	30	0.79	0.009
2	0.378	0.01	14	0.57	0.01	34	0.84	0.008
4	0.40	0.01	18	0.63	0.009	38	0.88	0.008
6	0.44	0.01	22	0.69	0.009	42	0.92	0.007
8	0.47	0.01	26	0.74	0.009	46	0.95	0.007

For suspended flight conveyors take B as 0.8 and for roller flights as 0.6 of the values given in the table.

Screw Conveyors. — Screw conveyors consist of a helical steel flight, either in one piece or in sections, mounted on a pipe or shaft, and running in a steel or wooden trough. These conveyors are made from 4 to 18 inches in diameter, and in sections from 8 to 12 feet long. The speed ranges from 20 to 60 r.p.m. and the capacity from 10 to 30 tons of coal per hour. It is not advisable to use this type of conveyor for coal, except the smaller sizes, as the flights are easily damaged by any foreign substance of unusual size or shape.

BUCKET CONVEYORS are of two types, having rigidly connected and pivoted buckets respectively. The buckets are carried by an endless chain driven by sprockets or pawls. Rigid buckets are used to convey coal and other materials over considerable distances when there is no intermediate point of discharge. They are built to carry as much as 2 tons of coal per minute.

Pivoted buckets may be used both as conveyors and elevators, and are particularly well adapted to the handling of coal and ashes in power plants. Their advantages are slow speed, silent operation, adaptability to change of direction without transfer, high efficiency and easy renewal of worn parts. Their disadvantages are danger of buckets sticking or upsetting and jamming in the supports, and the difficulty of preventing spill at the loading and turning points. Spilling in loading may be prevented by the use of special loading devices, by providing overlapping lips on the buckets or by placing small buckets between the main buckets to catch the spill.

Capacity. — Buckets are usually of 2-feet pitch, and range in width from 18 to 48 inches. They run at low speeds, usually not over 50 feet per minute, 40 feet per minute being the usual speed. At the latter speed the capacities when handling coal vary from 40 tons per hour for the 18-inch width to 120 tons per hour for the 48-inch width.

Power Required. — Prof. E. F. Miller gives the following formula for calculating the power required for a bucket conveyor making a rectangular circuit:

$$P = 0.004\,CL,$$

where P is the horse-power required, C is the capacity of the conveyor in tons of coal per hour, and L is the total lift in feet. This is an empirical formula deduced from numerous data on coal conveyors having capacities of from 20 to 50 tons and operating at speeds of from 40 to 55 feet per minute, and making lifts of from 40 to 80 feet. The power given by the above formula is for the conveyor loaded to its full capacity. The conveyor when running empty will require from 40 to 60 per cent of this. The smaller the conveyor the larger the percentage of power empty to power loaded.

BELT CONVEYORS. — Rubber and cotton belt conveyors are used for handling coal, ore, sand, gravel, etc., in all sizes. They combine a high carrying capacity with low power consumption. In the majority of cases the belt is troughed by means of idler pulleys set at an angle from the horizontal and placed at intervals along the length of the belt. Belt conveyors may be used for elevating materials up to about 23° incline. The belt may be run at any speed from 200 to 800 feet per minute, and may be from 12 inches to 60 inches in width. The most serious objection to belt conveyors is the lack of durability of the belts, their liability to destruction from accidental causes and the expense of their frequent renewals.

Link belts (*see Chains and Chain Drive*) are also used for conveying purposes.

Capacity. — The following table gives the capacity of the more common sizes of belt conveyors:

CAPACITY OF BELT CONVEYORS IN TONS OF COAL PER HOUR

Width of belt, in.	Velocity, feet per minute			Width of belt, in.	Velocity, feet per minute				
	300	350	400		300	350	400	450	500
12	34	..	...	20	96	112	128	...	...
14	47	..	...	24	139	162	186	210	...
16	62	72	82	30	218	254	290	326	...
18	78	91	104	36	315	368	420	472	520

For materials other than coal, the figures in the above table should be multiplied by the coefficients given in the table below:

Material	Coefficient	Material	Coefficient
Ashes (damp)	0.86	Earth	1.4
Cement	1.76	Sand	1.8
Clay	1.26	Stone (crushed)	2.0
Coke	0.60		

Power Required to drive a belt conveyor (*C. K. Baldwin, Trans. A.S.M.E., 1908*) depends on a great variety of conditions, as the spacing of idlers, type of drive, thickness of belt, etc. In figuring the power required, the belt should run no faster than is necessary to carry the desired load. If it should be necessary to increase the speed, the load should be increased in proportion and the power figured accordingly.

For level conveyors the horse-power required is

$$P = C \times T \times L \div 1000.$$

For inclined conveyors

$$P = (C \times T \times L \div 1000) + (T \times H \div 1000).$$

C = power constant from table below; T = load, tons per hour; L = length of conveyor, center to center, feet; H = vertical height material is lifted, feet; S = belt speed, feet per minute; B = width of belt, inches.

For each movable or fixed tripper add horse-power in column 3 of table. Add 20 per cent to horse-power for each conveyor under 50 feet long. Add 10 per cent to horse-power for each conveyor between 50 and 100 feet long. The formulas above do not include gear friction, should the conveyor be gear-driven.

	1	2	3	4	5
Width of belt, Inches	*C* for material weighing from 25 lb. to 75 lb. per cu. ft.	*C* for material weighing from 75 lb. to 125 lb. per cu. ft.	H.P. required for each movable or fixed tripper	Minimum plies of belt	Maximum plies of belt
12	0.234	0.147	½	3	4
14	0.226	0.143	½	3	4
16	0.220	0.140	¾	4	5
18	0.209	0.138	1	4	5
20	0.205	0.136	1¼	4	6
22	0.199	0.133	1½	5	6
24	0.195	0.131	1¾	5	7
26	0.187	0.127	2	5	7
28	0.175	0.121	2¼	5	8
30	0.167	0.117	2½	6	8
32	0.163	0.115	2¾	6	9
34	0.161	0.114	3	6	10
36	0.157	0.112	3¼	6	10

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[Wm. Kent.]

COOLING SYSTEMS FOR POWER STATIONS. — When the supply of cooling water is limited or expensive, economy requires that it shall be artificially cooled and used continuously. This cooling is accomplished by natural or forced evaporation in open ponds, spray fountains, a series of artificial cascades or in cooling towers. The latter device is largely used.

COOLING TOWERS. — Cooling towers are usually made in the shape of large cylinders of sheet steel, filled with narrow boards or laths arranged in geometrical forms, or hollow tile, or wire network, so arranged that while the water, which is sprayed over them at the top, trickles down through the spaces it is met by an ascending air column. The air is furnished either by disk fans at the bottom or is drawn in by natural draft. In the latter case the tower is made very high, say 60 to 100 feet, so as to act like a chimney.

Make-up Water. — When used with jet condensers, the water produced by the condensation of the exhaust steam is sufficient to make up for the evaporation in the tower, and therefore only enough water for the boiler feed need be supplied continuously. In fact, the condensed steam is, as a rule, more than sufficient to make up for the loss due to evaporation, and there results a slight overflow, which carries with it the oil from the engine cylinders and tends to clean the system of the oil that would otherwise accumulate in the hot-well.

When a cooling tower is used with a surface condenser make-up water must be added to the cooling water, the amount ranging from 3 to 4 per cent, but in this case the boiler feed water can be used over and over again with but slight loss, provided the oil (there is none with turbines) is eliminated by suitable separators.

Reduction in Temperature Obtainable. — With a properly designed cooling tower the temperature of the hot water (and condensed steam in the case of a jet condenser) can be reduced 40 to 50° F.

Power Required for Forced Draft in Tower. — The power required for the fan, when forced draft is used, averages 2 per cent of that developed by the main engines during the summer months, when maximum volume of air is required, and about 1 per cent during the winter months.

COSTS. — Cooling-tower costs vary greatly with climatic and operating conditions. When these are relatively favorable the cost may be kept as low as $2.50 per kilowatt for natural draft towers and $3.60 for fan draft. Under ordinary central-station conditions the costs will probably average $3.50 and $5.00 per kilowatt respectively for the two types of towers.

BIBLIOGRAPHY. — Additional data on cooling towers, ponds and spray fountains and numerous references to original papers will be found in Gebhardt's *Steam Power Plant Engineering*. See also Kent's *Mechanical Engineers' Pocket-Book*.

[WM. KENT.]

COPPER. — (*See also Electrochemical Processes, Industrial; Wires and Cables, Bare.*) The following discussion applies primarily to copper for electrical conductors.

COMPARISON OF LAKE AND ELECTROLYTIC COPPER. — The copper mined in the vicinity of the Great Lakes is almost pure copper. Copper made from the various ores of the metal must be refined (usually electrolytically) to reduce it to the same degree of purity. Lake copper possesses both high conductivity and excellent mechanical qualities but costs from ⅛ cent to ¾ cent per pound more than electrolytic copper. Why the latter should be inferior to Lake is difficult to explain, but experience shows that the general run of commercial electrolytic copper is by no means uniform in physical qualities and as a general thing is distinctly inferior to Lake for wire-drawing purposes. The lower price of electrolytic copper results in its use by many manufacturers although the wire is generally inferior to a slight extent.

ROLLING-MILL PROCESSES. — The refined copper comes to the rod mill in bars weighing about 200 pounds each. These bars frequently have ridges along the sides, due to faults in castings, and the surface is often covered with a layer of oxide. They are heated in a furnace until sufficiently soft for rolling and are passed through a series of rolls diminishing in size until a rod of the proper diameter is obtained. The rod is then coiled up and immersed in a pickling liquid (10 per cent H_2SO_4) in order to dissolve the oxide formed during rolling. It is then washed and dipped in a fluid tallow mixture.

WIRE-MILL PROCESSES. — The rods having cooled are connected together by brazing and are drawn through a series of dies of decreasing diameter. The dies give the wire a dense hard exterior coating which increases its tenacity. As the strength obtainable is almost a direct factor of the work expended upon the wire, the smaller the size the greater the tensile strength per square inch, so that the strength of the wire is readily varied by changing the size of the rod and the number of dies.

Defects in Wire. — One of the most serious defects occurring to wire at this point is from ridged bars as above described. Ordinarily the bar will not be sufficiently heated to dissolve the copper oxide on the surface so that as the softened bar enters the first passes of the rolls, the ridges are lapped over, inclosing the oxide scale. The subsequent passes and the drawing through the dies obscure this flaw almost entirely, but it remains a serious menace to the toughness and the resistance to wear of the copper.

A second cause of trouble arises at the same point by overheating the copper in the softening furnace, in which case copper oxide is formed on the surface and quickly dissolves through the entire bar, thereby increasing the oxide content and tending toward the production of brittleness. Both of these dangers can be avoided by careful selection of the bars and by proper regulating of the temperature of the softening furnace.

As the production of the hard surface from drawing is at best a rather delicate operation, careless handling, uneven welding of the rods and unequal temperature of the wire while passing through the dies will all produce noticeable effects in the quality of the finished wire, so that care throughout the mill is absolutely necessary for the best results.

It, therefore, appears that the most efficient wire must possess not only high conductivity but the maximum torsion and tensile strength possible in commercial copper and that to obtain this it is first necessary to use high-grade copper and to prevent an excess of cuprous oxide entering it at any stage of the manufacture, and, secondly, to select as perfect bars as possible and to observe extreme

care in every treatment through which they pass. (*Adapted from article by Carl F. Woods, Electric Railway Journal, 1909, Vol. 33, p. 195.*)

ANNEALING. — All wire when first drawn is more or less hard. It may be softened by annealing, i.e., by heating to a high temperature and cooling slowly.

MECHANICAL PROPERTIES. — (*See Wires and Cables, Bare, for tables of tensile strength, etc., of various sizes of wire.*) The more important mechanical properties of copper are discussed in some detail below.

Tensile Strength and Elongation of Soft Annealed Copper. — The tensile strength of soft annealed copper is about 30,000 to 33,000 pounds per square inch with an elongation of about 25 per cent (in 10 inches) at the fracture. It has no true elastic limit, permanent elongation being produced by very small loads.

Tensile Strength and Elongation of Hard-drawn Copper. — (*See Wires and Cables, Bare, for tensile strength of various sizes of wire.*) According to D. R. Pye, hard-drawn copper in wires up to ½ inch diameter varies in tensile strength with the diameter according to a linear law of the form

$$T = 70{,}000 - 45{,}000\,D,$$

where T = tensile strength, pounds per square inch, and D = diameter of wire, inches.

The above constants agree approximately with the tables of the American Society for Testing Materials and represent values somewhat under those usually obtained.

The elongation at fracture is approximately represented by

$$E = 4\sqrt{D},$$

where E = per cent elongation at fracture.

Tests by G. C. Batson on 50-foot lengths of hard-drawn copper showed a tensile strength only ½ per cent less than that of 10-foot lengths, a fact which indicates that the material is very uniform.

Modulus of Elasticity of Hard-drawn Copper. — The modulus of elasticity of hard-drawn copper varies from 12×10^6 to 20×10^6, the higher values applying to small wires. The following tests by G. C. Batson are typical of commercial copper.

Diameter, in.	0.158	0.138	0.112	0.094	0.079	0.066	0.049
Modulus, lb. sq. in. units	17.7×10^6	17.9×10^6	17.5×10^6	17.7×10^6	17.1×10^6	19.5×10^6	19.2×10^6

An apparent modulus of 12×10^6 is often obtained from an initial test, due to straightening-out. (*See also Wires and Cables, Bare.*)

Elastic Limit of Hard-drawn Copper. — The true elastic limit of hard-drawn copper, or load at which permanent set begins, is not the same as the point where the strain begins to increase more rapidly than the stress. The latter point is usually between 30,000 and 35,000 pounds per square inch and the former, somewhat below (*see Wires and Cables, Bare*).

Density. — The density of copper or, for all practical purposes, its specific gravity referred to water, is 8.89 at 20° C. This is the value which has been adopted as standard by the American Institute of Electrical Engineers, and most

the Calumet & Hecla Smelting Works, and the Reichsanstalt have indicated this as a mean.

CONDUCTIVITY AND RESISTIVITY. — (*See also Resistance and Conductance; Wires and Cables, Bare.*) F. A. Wolff and J. H. Dellinger give the resistivities of 89 samples of commercial copper from 14 important refiners and wire manufacturers in this and other countries. The mean for annealed wire is: Resistivity in ohms per meter-gram at 20° C. = 0.15292; per cent conductivity = 100.25. (Per cent conductivity is computed on the basis of 100 per cent conductivity corresponding to the standard resistivity of 0.15328 ohm per meter-gram at 20° C.) The mean result of data furnished by a large wire-manufacturing company, representing tests on more than 100,000,000 pounds of copper, is also given; for example, for annealed samples: Resistivity in ohms per meter-gram at 20° C. = 0.15263; per cent conductivity = 100.42.

Conductivity of Hard-drawn Copper. — The conductivity of hard-drawn No. 12 B. & S. wires was found to be less than the conductivity of annealed wires by a mean value of 2.7 per cent. The difference between the conductivity of annealed and hard-drawn wires increases as the diameter of the wire decreases. The lowest resistivity and highest conductivity found for a hard-drawn wire were: Resistivity in ohms per meter-gram at 20° C. = 0.15386; per cent conductivity = 99.62; and for annealed wire were: resistivity in ohms per meter-gram at 20° C. = 0.15045; per cent conductivity = 101.88.

Effect of Bending on Conductivity. — Copper wire apparently decreases in conductivity when bent, but the greater part of this is caused by local changes in cross-section. This effect is negligible unless the wire is bent to a very small radius.

Effect of Melting. — Electrolytic copper which is drawn without having been melted has a higher conductivity than that which has been melted, J. H. Dellinger recording copper thus drawn as having a conductivity as high as 101.71 per cent after annealing at a dull-red heat. Lake copper likewise drawn without having been melted gave conductivities as high as 101.88 after annealing.

Temperature Coefficient of Resistivity. — It has been found by J. H. Dellinger (*Bulletin No. 147, Bureau of Standards, 1910, Vol. 7, No. 1*) that the temperature coefficient of copper is proportional to the conductivity instead of being virtually a constant, as hitherto assumed. This fact may be expressed by saying that the change of resistivity per degree C. of a sample of copper is 0.000597 ohm per meter-gram or 0.00681 micro-ohm per centimeter cube. (Dellinger's original figures were subsequently changed slightly; *see Circ. No. 31 of Bureau of Standards, 1912 ed.*) The 20° C. temperature coefficient of a sample of copper is found by multiplying the per cent conductivity by 0.00393 and dividing by 100. These rules apply only to copper furnished for electrical use and to the temperature range of 10° C. to 100° C. over which the temperature coefficient was found to be linear. The following table gives the temperature coefficients a_T in the formula:

$$R_t = R_T[1 + a_T(t - T)].$$

Ohms per meter-gram at 20° C.	Per cent conductivity	α_0	α_{15}	α_{20}	α_{25}	α_{30}
0.16134	95	0.00403	0.00380	0.00373	0.00367	0.00360
0.15966	96	0.00408	0.00385	0.00377	0.00370	0.00364
0.15802	97	0.00413	0.00389	0.00381	0.00374	0.00367
0.15753	97.3	0.00414	0.00390	0.00382	0.00375	0.00368
0.15640	98	0.00417	0.00393	0.00385	0.00378	0.00371
0.15482	99	0.00422	0.00397	0.00389	0.00382	0.00374
0.15328	**100**	0.00427	0.00401	**0.00393**	0.00385	0.00378
0.15176	101	0.00431	0.00405	0.00397	0.00389	0.00382

The boldface values in the table have been adopted as standard by the American Institute of Electrical Engineers (*see Standardization Rules*).

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[W. A. Del Mar.]

CORONA, ELECTRIC. — (*See also Electron Theory; Spark Gap; Transmission Lines.*) When the difference of potential between two conductors which are insulated from each other by air is increased from a low to successively higher values, the following sequence of phenomena is observed, provided the conductors are relatively far apart compared with their dimensions: 1. When the potential is but a few volts only an extremely small (practically negligible) current passes between the two conductors, i.e., the insulation resistance is very high. 2. As the potential is increased, the insulation resistance remains practically constant until a certain value of the voltage is reached (depending upon the dimensions, distance apart of the conductors, etc.); when the insulation resistance falls to a much lower value, a bluish light appears around at least those parts of the conductors which are closest together, and a hissing noise is produced; these effects all start at the same value of the voltage. The bluish light is called the "corona" and the accompanying phenomena are called "corona effects." 3. As the voltage is still further increased the sheaths of light extend a greater distance from the conductors until finally a spark passes between them.

If the conductors are near together compared with their dimensions the spark passes without being preceded by the bluish light, i.e., no corona appears.

The appearance of the discharge at the positive conductor is different from the appearance at the negative conductor. If instead of a constant potential difference an alternating one is used the phenomena noticed are very much the same except that now the discharge is not continuous but appears and disappears with a frequency twice that of the potential difference.

CORONA ON TRANSMISSION LINES. — The loss of power accompanying the appearance of the corona on high-tension transmission lines is an appreciable one; in fact, in the present state of the art, the corona loss is the chief obstacle to the use of voltages above 150,000. The most comprehensive study of the "laws" of the corona thus far made is that by F. W. Peek, Jr. (*Trans. A.I.E.E., 1911, Vol. 30, p. 1889; Proc. A.I.E.E., June, 1912*). Peek's investigations were made with alternating voltages on an outdoor transmission line about 500 feet long, under all the variable conditions of spacing, size of conductor, storms, etc., usually met with in practice, these tests being supplemented by extensive laboratory investigations. Peek's results are summarized in the following paragraphs.

Disruptive Potential Gradient. — In his formulas Peek uses a factor which he calls the "disruptive" critical potential gradient. This "disruptive" gradient is not the potential gradient corresponding to the actual formation of the corona on the wires of a transmission line, but is the potential gradient corresponding to the break-down voltage of an air gap of 0.5 cm. or more in length between conductors of very large radius of curvature, i.e., practically plane surfaces (*see article on Spark Gap*). Peek obtains its value by extrapolating his experimentally determined curves, and thus obtains 75.7 kilovolts per inch (29.8 kilovolts per centimeter) maximum, which is equivalent to 53.5 kilovolts per inch effective value for a sine-wave voltage. This value is in close agreement with the values obtained directly for the potential gradient required to break down a spark gap between conductors of large radius of curvature when the gap is greater than 0.5 cm. in length.

The following formulas apply to either a single-phase or symmetrically arranged three-phase line. Sine-wave voltage is assumed and all voltage values are effective or r.m.s. values. Let

a = radius of each conductor in inches.
D = distance, in inches, between centers of conductors.

$H_0 = 75.7 =$ "disruptive" critical potential gradient of air, kilovolts per inch, at 77° F. (25° C.) and 29.92 inches (76 cm.) barometric pressure.

$V_0 =$ kilovolts to neutral corresponding to the potential gradient H_0 at the surface of the wire, assuming a clean, polished cylindrical (circular) surface.

$V_v =$ kilovolts to neutral required to start the corona, called by Peek the "visual" critical voltage.

When the return conductor is so remote that the charge on it produces no appreciable field at the conductor in question *

$$V_0 = 123\, a \log_{10} \frac{D}{a}. \qquad (1)$$

TABLE I. VALUES OF V_0 FOR STANDARD STRANDS †

Effective kilovolts to neutral

Size of wire, cir. mils or A.W.G.	Diam., inches $= 2a$	Feet between wires, center to center $\left(= \frac{D}{12}\right)$							
		3	4	6	8	10	12	15	20
1,000,000	1.152	127.5	136.1	148.9	157.8	164.7	170.3	177.2	186.1
750,000	0.998	114.3	122.0	132.8	140.5	146.4	151.3	157.3	165.3
500,000	0.814	97.7	103.9	112.8	119.0	123.9	127.9	133.2	139.0
350,000	0.681	85.0	90.2	97.6	102.8	107.4	110.2	114.3	119.5
250,000	0.575	74.4	78.8	85.0	89.4	92.9	95.7	99.1	103.5
0000	0.528	69.5	73.5	79.3	83.3	86.5	89.1	92.2	96.3
000	0.470	63.3	66.9	72.0	75.6	78.5	80.7	83.5	87.2
00	0.418	57.6	60.8	65.4	68.6	71.1	73.1	75.6	78.8
0	0.373	52.5	55.4	59.5	62.4	64.6	66.4	68.6	71.5
1	0.332	47.8	50.4	53.9	56.5	58.5	60.1	62.1	64.7
2	0.292	43.1	45.3	48.5	50.7	52.5	53.9	55.6	57.9
4	0.232	35.6	37.4	39.9	41.7	43.1	44.3	45.6	47.4
6	0.184	29.4	30.8	32.8	34.2	35.3	36.2	37.3	38.7

* The potential gradient H, in abvolts per cm., at a point a distance x cm. from the center of a cylindrical wire, due to a charge of Q abcoulombs per cm. on that wire is $2\,Q/x$; the capacity to neutral is

$$C = \frac{1}{2 \log_e \frac{D}{a}} \quad \text{abfarads}$$

and since $Q = CV$, where V is in abvolts, therefore

$$H = \frac{V}{x \log_e \frac{D}{a}} \quad \text{abvolts per cm.}$$

At the surface, $x = a$ and $H =$ the value of H_0 in abvolts per cm., whence the above formula in practical units.

† This table is calculated from the formula $V_0 = 123\, a \log_{10} \frac{D}{a}$ taking for a one-half the over-all diameter of the strand. The values of V_0 given in the table are therefore actually those corresponding to a solid wire of this same diameter; the effect of strand-ing is taken into account by the factor m_0 in Table II.

Visual Critical Voltage in Fair Weather. — Peek's experiments show that the actual voltage, kilovolts to neutral, at which the corona forms* in fair weather may be represented by the formula

$$V_v = m_v \delta \left(1 + \frac{0.189}{\sqrt{a\delta}} \right) V_0, \qquad (2)$$

where m_v ($= 1$ for clean, polished, solid wires) is a factor which takes into account the condition of the surface of the conductors, see Table II, and δ is the specific gravity of the air, referred to air at 77° F. (25° C.) and 29.92 inches (76 cm.) barometric pressure ($\delta = 1$ for these standard conditions). The specific gravity δ is calculated from the formula

$$\delta = \frac{17.95\, B}{460 + t}, \qquad (3)$$

where B = actual barometric pressure in inches, and t = actual temperature in °F.

TABLE II. VALUES OF m_v AND m_0

Surface, etc.	Value of m_v	Value of m_0
Clean, polished wires, solid	1.00	1.00
Weathered or roughened wires, solid	0.93 to 0.98	0.93 to 0.98
7-strand bare cables, decided corona all along conductor	0.82	0.83 to 0.87
7-strand bare cables, local corona all along conductor	0.72	....

Visual Critical Voltage in Foul Weather. — Peek's experiments show that when the wires are thoroughly wet, e.g., in a heavy fog or rain, the visual critical voltage is, to a fair degree of approximation,

$$V'_v = 0.3 \left(1 + \frac{0.512}{\sqrt{a}} \right) V_0. \qquad (4)$$

Smoke, falling sleet, or sleet on the wires also lowers the critical voltage.

Power Loss in Fair Weather. — Peek's experiments show that the power loss in fair weather *in kilowatts per mile of each wire*, when the corona actually appears, may be represented by the formula

$$P = \frac{0.00553}{\delta} f \sqrt{\frac{a}{D}} (V - m_0 \delta V_0)^2, \qquad (5)$$

where

δ = specific gravity of air referred to air at 77° F. (25° C.) and 29.92 inches (76 cm.) barometric pressure, see equation (3). Under standard conditions of pressure and temperature $\delta = 1$.

f = frequency of the impressed voltage.

a = radius of conductor, in inches.

D = distance between conductors, center to center, in inches.

V = actual kilovolts to *neutral*, effective value.

$V_0 = 123\, a \log_{10} \frac{D}{a}$; see Table I.

m_0 = factor depending on the surface of the conductors, see Table II. For clean, polished, solid wires $m_0 = 1$.

* For wires less than 15 diameters apart Peek found that as the voltage was raised a spark passed without being preceded by the corona.

For V less than the visual critical voltage V_v, but greater than $m_0\delta V_0$, the above formula gives a loss which in general is greater than the observed loss. It is probable that were the conductors perfectly clean and their surfaces absolutely uniform, there would be no loss whatever for V less than V_v. The loss that is observed is due to local coronas which form at a lower voltage wherever there is a spot of grease, dirt or other irregularity.

Power Loss in Foul Weather. — The presence of fog or smoke, sleet on the wires or falling sleet, or rain or snow increases the loss. The effect of snow is greater than that of any other weather condition. According to Peek the loss during foul weather may be calculated, to a rough approximation, by using the fair-weather formula taking for m_0 80 per cent of the values given in Table II.

Effect of Humidity, Winds, Frequency, etc. — According to Peek high voltages do not entirely eliminate sleet. Humidity and "vapor products" have no effect on either the critical voltage or on the loss. High winds have no effect upon the loss or critical voltage at ordinary frequencies. The effect of frequency on the visual critical voltage is negligibly small for frequencies between 25 and 60 cycles per second. It should also be noted that the formula for power loss, given above, in which the loss is given as proportional to the frequency, has been tested only for frequencies within the commercial range. It is probable that at extremely low frequencies, the loss decreases less than proportional to the frequency, and approaches a small finite limiting value at continuous impressed voltage.

Ionization of the Air around Charged Wires. — When the corona appears, the air in the neighborhood of the conductors is found to have a conductivity many times larger than its normal value. This conductivity remains for an appreciable length of time after the potential has been cut off. The air is said to be ionized, since, on the assumptions of the electron theory (q.v.), it contains under these conditions a very large number of ions. Whitehead has investigated the relation between the conductivity of the air near the conductors and the voltage between them, and finds that the voltage at which the corona appears is exactly equal to that at which ionization begins. It is now well established that visual corona, power loss and ionization are entirely contemporaneous.

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[W. S. GORTON.]

COUPLINGS, DIRECT.—(*See also Belts and Belting.*) Couplings for connecting electrical apparatus together or to other machines can be divided into two distinct classes, namely, solid and flexible.

SOLID COUPLINGS (Fig. 1) are usually of the flanged type and consist of two steel castings rigidly bolted together. This type should be used where the two machines are mounted on a common iron base and where an exact alignment is possible. Large flanges should be provided for overcoming bending stresses which may occur when the couplings carry part of the load such as, for example, in three bearing sets. See also article on *Machine Tools, Electrical Operation of.*

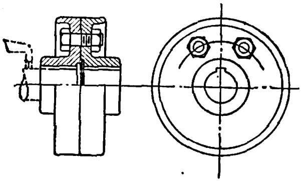

Fig. 1. Solid Flange Coupling

FLEXIBLE COUPLINGS are mainly used to connect machines which cannot be lined up properly or where there is fear that alignment cannot be maintained. Many different kinds of flexible couplings are in use, and they may be divided in two general classes, insulated and uninsulated. The insulated couplings are composed of castings separated by leather or rubber, as a flexible and insulating medium. The principal types of flexible couplings in use are the leather link, the laced belt, the rubber buffer and the mill-type coupling.

Leather-link Flexible Couplings (Fig. 2) consist of two iron castings connected together through leather punchings fastened at the ends by bolts to the alternate halves. For the smaller sizes these links are usually replaced by a single leather disk which connects both halves by means of bolts alternately fastened through opposite halves. The torque stresses are transmitted through these links or disks to the bolts which fasten them to the castings.

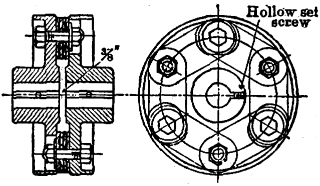

Fig. 2. Leather-link Flexible Coupling

In order to allow sufficient play for the heads of the bolts, alternate holes are bored to a large diameter, the bolts accurately fitting the other holes in the castings. By this means flexibility is obtained and a small amount of end or side play is permissible between the shafts of the two machines to which each half coupling is securely keyed. This type of coupling is recommended for shafts up to 3½ inches in diameter. For shafts between 3½ inches and 5 inches in diameter either this type or the leather-laced type may be used.

Leather-laced Flexible Couplings (Fig. 3) are recommended for shafts above 3½ inches in diameter on account of their structural advantages. They consist of two steel rings, an outer and an inner, with cast-iron hubs bolted to them. Slots are formed in these rings through which an endless leather belt is interwoven.

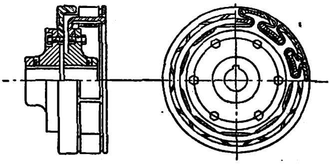

Fig. 3. Leather-laced Flexible Coupling

This construction not only gives great flexibility but, due to the two rings being concentric, it is not subjected to bending strains and therefore is espe-

cially adapted for transmitting a high torque commensurate with the strength of the belt and the size of the coupling employed.

As the outer ring of the coupling is only connected to the shaft of the machine through the coupling hub which is keyed to the shaft, machines using this form of coupling can be readily disconnected without unlacing or interfering with the coupling belt. To do this it is only necessary to remove the bolts, holding the outer ring to the hub. This partial disassembling also aids in the replacing of a worn-out belt without removing the coupling from the machine.

Rubber-buffer Flexible Coupling. — This coupling is made up of two cast-iron spiders, the small interlocking arms of which are separated by cylinders of soft rubber. The rubber cylinders are held in place by projecting plates screwed to the arms of one of the spiders. The construction of this type of coupling affords great flexibility as well as insulating qualities. This type of coupling was once much used on account of its high-insulating qualities but is now to a great extent being superseded by the two previous types.

Mill-type Flexible Coupling (Fig. 4). — Where the conditions are too severe for the belt or rubber of the flexible couplings described above, that is, where there is much grit or hot vapor present and where noise is not objectionable, the mill-type flexible coupling shown in Fig. 4 should be used. This is a rough sturdy coupling consisting of three steel castings, two of which are identical and called the "pods," while the center is called the "box." The most extensive use for this coupling has been in steel mills. Best results are obtained from this coupling when it is used on a constant load, as the noise under these conditions is at a minimum.

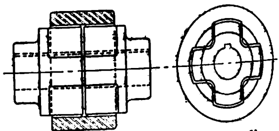

Fig. 4. Mill-type Flexible Coupling

PROPER SIZE OF COUPLING. — The selection of the proper coupling should be determined by calculation and not by the method of choosing the bore corresponding to the shaft diameter. The coupling should have sufficient capacity to take care of the overload capacity of the motor or generator, as the case may be.

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[D. B. RUSHMORE, assisted by E. A. LOF.]

CRANES. — (*See also Blocks and Tackle; Controllers; Hoists, Electric; Telpherage.*) A hoist is a machine for raising and lowering weights. A crane is a hoist with the added capacity of moving the load in a horizontal or lateral direction. That part of a crane carrying the hoist and movable with respect to the main structure of the crane is called the "trolley." Cranes are divided into two classes as to their motions, viz., rotary and rectilinear.

Rotary Cranes may be classified as follows:

Swing-cranes. — Having rotation, but no trolley motion.

Jib-cranes. — Having rotation, and a trolley traveling on the jib. When the jib or boom is movable, carrying a sheave on the end, the device is called a *derrick.*

Column-cranes. — Identical with the jib-cranes, but rotating around a fixed column (which usually supports a floor above).

Pillar-cranes. — Having rotation only; the pillar or column being supported entirely from the foundation.

Pillar Jib-cranes. — Identical with the last, except in having a jib and trolley motion.

Derrick-cranes. — Identical with jib-cranes, except that the head of the mast is held in position by guy-rods, instead of by attachment to a roof or ceiling.

Rotary cranes arranged so that they may be readily moved from place to place may be classified as:

Walking-cranes. — Consisting of a pillar or jib-crane mounted on wheels and arranged to travel longitudinally upon one or more rails.

Locomotive-cranes. — Consisting of a pillar-crane mounted on a truck, and provided with a steam-engine capable of propelling and rotating the crane, and of hoisting and lowering the load.

Rectilinear Cranes may be classified as follows:

Bridge-cranes. — Having a fixed bridge spanning an opening, and a trolley moving across the bridge.

Tram-cranes. — Consisting of a truck, or short bridge, traveling longitudinally on overhead rails, and without trolley motion.

Traveling-cranes. — Consisting of a bridge moving longitudinally on overhead tracks, and a trolley moving transversely on the bridge.

Gantries. — Consisting of an overhead bridge, carried at each end by a trestle traveling on longitudinal tracks on the ground, and having a trolley moving transversely on the bridge.

Rotary Bridge-cranes. — Combining rotary and rectilinear movements and consisting of a bridge pivoted at one end to a central pier or post, and supported at the other end on a circular track, provided with a trolley moving transversely on the bridge.

For descriptions of these several forms of cranes see Towne's *Treatise on Cranes.*

HAND-OPERATED TRAVELING CRANES. — The weight of a hand-operated traveling crane depends not only upon the capacity of the crane and the length of the span, but also upon the particular design. The following approximate formula will serve as a rough guide when actual data are not available:

$$W = KCL,$$

where W is the weight of the crane in tons, C the capacity of the crane in tons, L the length of the span in feet and K a constant ranging from about 0.01 for $L = 10$ feet to 0.012 for $L = 75$ feet.

ELECTRIC OVERHEAD TRAVELING CRANES. — Electric traveling cranes usually have 3 motors, one for the hoist, one for the trolley traveling on the bridge and one for moving the bridge. Cranes of over 15 tons capacity are usually provided with an auxiliary hoist of from 1/10 to 1/3 the capacity of the main hoist. Automatic brakes are used to sustain the load when lifted and to regulate the speed when lowering, the hoist motor usually having to *drive* the load down.

Speeds of Electric Traveling Cranes. — The standard speeds at which the various parts travel in the cranes manufactured by Pawling and Harnischfeger are as follows:

STANDARD SPEEDS OF ELECTRIC CRANES, IN FEET PER MINUTE
(Pawling & Harnischfeger)

Capacity, tons (2000 lb.)	Hoisting speed, ft. per min.	Bridge travel, ft. per min.	Auxiliary hoist	
			Capacity, tons	Speed, ft. per min.
5	25–100	300–450	..	......
10	20–75	300–450	3	30–75
25	10–40	250–350	3 10	50–125 25–60
40	9–30	250–350	5 10	40–100 25–60
50	8–30	200–300	5 10	40–100 25–60
75	6–25	200–250	15	20–50
125	5–15	200–250	25	20–50
150	5–15	200–250	25	20–50

Trolley travel speed from 100 to 150 feet per minute in all cases.

Weight and Dimensions of Electric Traveling Cranes. — Let

C = capacity of crane, in tons (2000 pounds),
W_B = weight of bridge alone, in tons,
W_T = weight of trolley alone, in tons,
W = total weight of crane, in tons.

Then

$$W_B = K_B CL, \qquad W_T = K_T C,$$
$$W = W_B + W_T = C(K_B L + K_T),$$

where K_B and K_T have the following approximate values (*see paper by S. S. Wales, Proc. Eng. Soc. Western Pa., 1902, Vol. 18, p. 146*).

L	K_B	C	K_T
25	0.012	1–25	0.3
50	0.012	25–75	0.4
75	0.013	75–150	0.5
100	0.015		

For example, for a 50-ton crane for a 60-foot span, the bridge alone would weigh $0.013 \times 50 \times 60 = 39$ tons, the trolley would weight $0.4 \times 50 = 20$ tons, and the total weight would be 59 tons, approximately. If the bridge is equipped with 2 wheels at each end, the maximum load on each wheel when the trolley, carrying full load, is at the end of the bridge would be approximately

$$\frac{39}{4} + \frac{20+50}{2} = 45 \text{ tons},$$

or if there are 4 wheels at each end, the maximum wheel load would be 22.5 tons.

The following table is taken from a catalogue of the Alliance Machine Co.

DIMENSIONS AND WHEEL LOADS OF ELECTRIC TRAVELING CRANES

60-foot span and 25-foot lift; wire-rope hoist

Capacity, tons (2000 lb.)	Distance from runway rail to highest point		Distance from center of rail to ends of crane	Wheel base of end truck		Maximum load per wheel; trolley at end of bridge
	Feet	Inches	Inches	Feet	Inches	Pounds
5	6	0	9	9	0	20,000
10	6	6	10	10	0	27,000
25	7	4	12	11	6	51,000
40	8	0	12	12	3	82,000
50	8	9	12	12	6	48,000*

* Has 8 track wheels on bridge.

Standard cranes are built in intermediate sizes, varying by 5 tons up to 40 tons.

Power Required. — The following is adapted from the paper by S. S. Wales above referred to. The frictional losses will of course depend to a certain extent upon the design of the crane, the use of rope or chain, etc., but the constants given below represent fair averages.

The same notation as in the previous section is used. In addition let

R_B = tractive effort in pounds per ton required to move the bridge,
R_T = tractive effort in pounds per ton required to move the trolley,
S_B = speed of bridge in feet per minute,
S_T = speed of trolley in feet per minute,
S_H = speed of hoist in feet per minute.

Then the power required, at shaft of bridge motor, to drive the bridge at speed S_B is

$$P_B = \frac{(C + W_B + W_T)\, R_B S_B}{33{,}000} \text{ horse-power.} \qquad (1)$$

The power required, at shaft of trolley motor, to drive the trolley at speed S_T is

$$P_T = \frac{(C + W_T)\, R_T S_T}{33{,}000} \text{ horse-power.} \qquad (2)$$

The power required, at shaft of hoist motor, to drive the hoist at speed S_H, assuming 60 per cent efficiency of gearing and hoisting tackle, is

$$P_H = \frac{CS_H}{10} \text{ horse-power.} \qquad (3)$$

For *constant-speed running*, Wales gives the following values for R_B and R_T, the former varying with L and the latter with C.

L	R_B	C	R_T
25	30	1–25	30
50	35	25–75	35
75	40	75–150	40
100	45	......	....

If the moving member is *accelerating* at the rate of a ft. per sec. per sec., to the value of R given in the above table should be added an amount

$$R_a = 64\,a.$$

The power as calculated by the above formulas is the power *output* of the motors; to obtain the power input divide by the motor efficiency, which ranges from 80 to 90 per cent.

Motor Equipment.* — (*See also Motors, Industrial Applications of.*) Wales recommends that the bridge motor should have a rating (1-hour rating, the motors being rated on the same basis as railway motors) 1.5 times the calculated power for constant full-load speed, the trolley motor 1.25 times the calculated power for constant full-load speed, whereas the rating of the hoist motor should be taken equal to the calculated power for constant full-load speed. The motors may be designed for 110, 220, or 500–600 volts.

Both direct-current and alternating-current motors are successfully used for crane work. The series direct-current motor has speed-torque characteristics especially well adapted for crane service. At light loads the speed increases and at heavier loads the lifting is slower and the raising effect consequently stronger. This characteristic of the series motor often results in considerable saving of time, due to the fact that the majority of loads in factories are light, so that the increased speed of lift enables work to be performed in a shorter time. For a-c. motors a normal operating speed about 25 to 50 per cent higher than for d-c. motors is therefore generally selected and the slowing down for heavy work is accomplished by inserting resistance in the phase-wound rotor circuit.

Example. — Crane of 50-ton capacity, span 70 feet, bridge speed 200 feet per minute with full load, trolley speed 100 feet per minute with full load, hoist speed 15 feet per minute with full load. Then $C = 50$, $W_B = 45.5$, $W_T = 20$, $R_B = 40$, $R_T = 35$, and

$$P_B = \frac{115.5 \times 40 \times 200}{33{,}000} = 28 \text{ horse-power,}$$

and the motor required for the bridge would be $28 \times 1.5 = 42$ horse-power, or the nearest commercial size over 42, say 50 horse-power.

$$P_T = \frac{70 \times 35 \times 100}{33{,}000} = 7.43 \text{ horse-power,}$$

* By D. B. Rushmore.

and the size motor required for the trolley would be $7.43 \times 1.25 = 9.28$ horse-power, or say 10 horse-power.

$$P_H = \frac{50 \times 15}{10} = 75 \text{ horse-power,}$$

and a motor of this rating would be used for the hoist.

Control of Crane Motors.* — The control equipments for crane motors should be of the regulating and reversible type. The starting and speed regulation of series motors is generally accomplished by inserting resistance in series with the armature and field, and with induction motors, resistance is inserted in the phase-wound secondary or rotor circuit. For cranes which do a large amount of lowering, dynamic control is becoming very generally used, and is very readily accomplished with direct-current motors. It is also occasionally used with alternating-current motors, when direct current is available for excitation.

Drum Versus Magnetic Controllers. — Both hand- and magnetic-control equipments are in general use. The former are satisfactory for small and medium-size motors, and consist simply of a drum-type controller with a set of separate resistances. For large-size motors too much physical effort would be required to move a controller of the necessary size and the magnetic control should then be selected. A magnetic-control equipment consists of a master controller, a contactor panel and the resistances. The contactor panel contains the contactors for cutting in or out the resistances, the interlocks and the current-limit relays for automatically controlling the sequence and rapidity with which the contactors operate. The master-controller handle can be thrown to the full-speed position quickly without causing an overload on the motor, since the current-limit relays automatically prevent the contactors from cutting out the resistances too rapidly.

Dynamic Control. — With dynamic control of d-c. motors the field is separately excited and the armature is connected to the line voltage with one section of a rheostat in series with it, and another section of the rheostat in parallel with it. This is accomplished by means of the controller, whose duty is to make each step on the lowering side keep the speed of the motor under control no matter whether the motor has to drive a light hook downward or hold back against a heavy load. In the former case the current which passes through the series rheostat also passes through the armature and produces the desired torque. The speed is controlled by varying the value of this rheostat just as in hoisting. If the motor has to act as a brake, the power generated in the armature is expended in the parallel rheostat and the speed is controlled by varying this rheostat. In actual practice the two rheostats are varied simultaneously so that if a certain point on the controller tends to cause the motor to drive a load at a high speed it will also cause the motor to hold back at a high speed. The practical result is that an operator always has his load under control, and does not have to worry about dropping his load. On the lowering side of the controller, a motor holds back against its load even when power fails, because the motor then acts as a self-excited series generator, and consequently the solenoid brake alone is not depended on to prevent the load from falling.

In the case of a phase-wound induction motor it is necessary to excite one portion of the primary winding with direct current in order to generate a voltage in the secondary winding. A rheostat is then connected to the secondary windings of the motor and the speed can be controlled as with d-c. motors. As direct current is necessary for excitation dynamic braking is not very often used with a-c. crane motors.

* By D. B. Rushmore.

Solenoid brakes are also provided for holding the load when the hoist motor is stopped. If dynamic braking is not provided, mechanical brakes must be provided in addition to the solenoid brakes to assist in holding the load when the motor is stopped and for regulating the speed when lowering.

Solenoid brakes are, as a rule, applied by gravity when the armature current is shut off and released by the solenoid which is energized when the controller is thrown on the first notch. The solenoids of direct-current motors are generally connected in series with the motor circuit, although with polyphase induction motors they are connected directly across one phase thus receiving the full line voltage.

All crane motors should also be protected by safety-limit switches arranged to cut off the supply current when the limit of motion is reached and thus automatically provide against accidents from over-travel.

SPECIFICATION FOR ELECTRIC CRANE.* — The following memoranda are intended to assist in writing specifications. See also *Specifications.*

Principal Characteristics and Conditions of Service. — Span between crane rails and load to be lifted on main hoist and on auxiliary hoists, if any. Characteristics of current, a-c. or d-c. (phases, cycles and voltage).

Style and Description of Apparatus. — Structural work of open-hearth steel according to some standard steel specifications. Maximum stresses allowable. Whether hand operation is to be provided for. Distance of hook from crane rails, vertically and horizontally, when at extreme limits of its motion. Details of brake. Description and characteristics of motors to be supplied. Whether or not a foot-walk is to be supplied. Whether trolley is to be above bridge or submerged. Hoisting drum to have bronze bearings. Track wheels shall have chilled treads, ground true, with double flanges, and shall be keyed to their axles. Axle bearings shall be bronze. Details of lubrication and bearings. Location of carriage. Details of control. All gearing (except when shrouded) shall be cut, etc. Wiring details. Controller details.

Dimensions. — Supply a diagram of clearances.

Work Done by Other Contractors. — Track rail and weight thereof.

Performance and Tests. — Speeds of main travel, lateral travel and hoists. Time to get up speed or acceleration rates.

COST OF CRANES. — The cost of cranes depends to so great an extent upon the design, motor equipment, etc., that no reliable average figures can be given. As a rough approximation the cost of a hand-operated traveling crane may be taken as from about 5 to 7 cents per pound of total crane weight and the cost of an electric traveling crane as from about 10 to 13 cents per pound of total crane weight.

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* By W. A. Del Mar.

CROSS ARMS. — (*See also Poles for Overhead Lines; Insulator Pins; Insulators.*) Cross arms are usually of wood though sometimes of iron. "Buck arms" or "reverse arms" are cross arms attached to a pole at right angles to the principal arms, and are used for taking off wires at right angles to the line, either at the junction of intersecting lines or at services. "Double arms" are pairs of cross arms attached to opposite sides of a pole so as to act as one compound arm. Double arms are used to increase the strength of an arm and to permit the use of two pins and two insulators for supporting a single wire where additional strength is required.

Cross arms are used principally for supporting pins, insulators and wires, though lightning arresters, transformers, switches and other miscellaneous appliances are often mounted on them, usually for the purpose of keeping the pole free of incumbrances so that it will be more easy to climb.

Alley Arms. — When city distribution lines are located in alleys it is common to locate poles next to the property line. Where it is not permissible to let arms overhang private property special arms may be used which extend on one side of the pole only. These must be well braced and should not be used for dead ending wires. A better construction is obtained by locating two poles on opposite sides of the alley and putting special cross arms across the alley between them.

FORCES ON AND STRESSES IN CROSS ARMS. — The forces which a cross arm resists are:

(a) Vertical forces due to weight of pins, insulators, wires (with sleet) and accidental loads due to linemen standing on arms, etc.

(b) Transverse horizontal forces due to wind pressure on the wires (with sleet) at right angles to the line.

(c) Longitudinal horizontal forces due to the pull of the wires where the pull is unbalanced. Unbalanced pull is usually due to an angle in the line, the ending of the wire at the arm, a change in the size of wire at arm and an unequal tension in the spans on the two sides of the arm.

The principal internal stresses produced in an arm from these forces are:

(1) A bending force in a vertical plane due to vertical forces (a).

(2) A bending force in a horizontal plane due to horizontal forces (c).

(3) A twisting force about the longest axis of the arm due to the "pin leverage" of the horizontal forces (c). The pin leverage is the distance from the center of the wire to the axis of the arm.

Of these stresses the most destructive is probably the twisting stress which tends to split the arm in a vertical plane through the pin holes and along the grain of the wood. On this account the pin and insulator should be no taller than necessary, the pin should extend completely through the arm to give the best distribution of bearing pressure and, where stress is heavy, the arm may be strengthened by two "strengthening bolts" (machine bolts with nut and washers) put horizontally through the arm one on each side of the pin, one being near the top and the other near the bottom of the arm.

The vertical and horizontal bending stresses are of some importance and may be computed by the usual beam formula (*see Structures, Simple*). Data of tests on strength of cross arms for these stresses are given below.

Strength Tests of Cross Arms. — (*Forest Service, Cir. No. 204.*) Tests made on 3¾ in. by 4¼ in. by 6 ft., 6-pin air-dried cross arms with vertical load distributed equally at each pin hole gave the results at top of next page.

The tests showed that for ordinary use the strength, for vertical loads of 6-pin arms, need not be considered in calculations of line construction, except in the rare case of abrupt change of the grade of the line.

Kind of wood	Av'ge max. load in lb.
Longleaf pine, 75 per cent heart	10,180
Longleaf pine, 100 per cent heart	9,780
Shortleaf pine	9,260
Longleaf pine, 50 per cent heart	8,980
Shortleaf pine, creosoted	7,650
Douglas fir	7,590
White cedar	5,200

DIMENSIONS AND RATING. — Cross arms are rated in terms of the number of pins they are designed to carry, 2-pin arms being the minimum and 10-pin arms the maximum ordinarily used.

When arms are made directly from the original logs they are sometimes dimensioned in even inches, i.e., 4 in. by 5 in., 5 in. by 7 in., etc. In most cases they are made from lumber in stock and when finished are a quarter of an inch or more under the even sizes. Several "standard" dimensions have been adopted for distribution arms. In the accompanying table Standard I is that given by Miller (*American Telephone Practice*) and found in various catalogues of dealers in electrical supplies, Standard II is that given in Report of Committee on Overhead Line Construction of the National Electric Light Association, and Standard III is one used by the Stone & Webster Engineering Corporation. Fig. 1 shows a 6-pin cross arm according to Standard III.

Size of arms	Width, inches	Depth, inches
Standard I	3¼	4¼
Standard II	3½	4½
Standard III	3¾	4¾

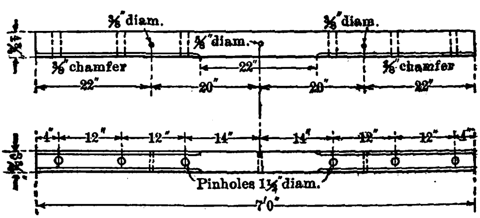

Fig. 1. Standard Cross Arms of Stone & Webster Engineering Corporation

Distance between Pole Pins. — It is desirable to have the space between pole pins (i.e., the two pins on opposite sides of the pole) as large as possible to give a clear space for climbing the pole. It usually varies from 16 in. to 30 in.

Distance between Other Pins. — This usually ranges from 12 in. to 21 in. Under ordinary circumstances 12 in. has been found sufficient to prevent wires on adjacent pins swinging in contact in the middle of the span. Where small wires (such as No. 10 telephone wires) are used on long spans (150 ft. or more) contact is to be expected occasionally where 12 in. spacing is used.

Distance from End Pin to End of Arm. — This is usually 4 in., which is about the minimum that can be used without the arm splitting due to pin leverage.

In order that a line may have a neat systematic appearance it is desirable that the pin spacing be the same for all arms, whether 2 pin, 4 pin, etc. In the tabulations below the arms included in each standard (defined above under *Dimensions and Rating*) have been grouped on this principle which shows that Standard I really consists of one regular group and 5 odd arms while Standard II consists of 2 regular and 1 odd arm. Referring to Standards I and III it will be noted that the length of arm is always a multiple of one foot.

STANDARD DISTANCES BETWEEN PINS

Standard	Group	Number of pins	Length of arm	End pin to end of arm	Between pole pins	Between other pins
			Ft. In.	In.	In.	In.
I	Regular	4	4 0	4	16	12
	Regular	6	6 0	4	16	12
	Regular	8	8 0	4	16	12
	Regular	10	10 0	4	16	12
	Odd	2	3 0	4	28	..
	Odd	4	5 0	4	18	17
	Odd	4	6 0	4	22	21
	Odd	4	6 0	4	24	20*
	Odd	6	8 0	4	18	17½
	Odd	8	10 0	4	17½	15¾
II	Regular	4	5 7	4	30	14½
	Regular	6	8 0	4	30	14½
	Odd	8	9 2	4	30	12
III	Regular	2	3 0	4	28	..
	Regular	4	5 0	4	28	12
	Regular	6	7 0	4	28	12
	Regular	8	9 0	4	28	12
	Regular	10	11 0	4	28	12

* Alternative found in some catalogues.

Pin Holes.—1½ in. diameter is standard for wood pins. N.E.L.A. specifications require that holes shall be tested with steel gauges taking one of the nominal size without forcing but not taking one 1/32 in. larger.

Bolt Holes.—For fastening the arm to the pole there is usually one ⅝-in. bolt hole at the middle of the arm. For fastening cross-arm braces there are two ⅜-in. bolt holes located 38 in. apart (Standard II) or 40 in. apart (Standard III).

High-tension Wooden Cross Arms.—These are not standardized but are specially designed and made as required. They differ from distribution arms in being wider and deeper with larger pin holes and wider pin spacing. The following tabulation will give data on some arms that have been used for different voltages.

Voltage	Number of pins	Width of arm	Depth of arm	Length of arm	Spacing of pins	Pin hole diameter
		Inches	Inches	Inches	Inches	Inches
25,000	2	6	6	64	48	2¼
55,000	2	5	7	88	72	2½

Special high-tension arms are usually unnecessary for circuits of 5500, 6600, 11,000 and 13,200 volts. Standard 2200-volt distribution arms may be used, the wire spacing being increased by not using certain pins. Thus a standard 2200-volt 6-pin arm may be used for a 11,000-volt 4-pin arm by leaving the middle pin holes vacant. For higher voltages the strength of arm and size of pin hole is usually insufficient.

SPECIFICATIONS. — *(See article on Specifications.)* The points to be covered in a set of specifications are:

Material. — Usually pine or fir, sometimes western hemlock. *(See Poles for Overhead Lines.)*

Quality of Material. — Usually should be "straight grained, free from knots (or free from large, loose or unsound knots), sapwood, pitch pockets, shakes, checks, loose heart, rot, worm holes or other defects." The strictness of the specifications, or of the inspection, must, however, be governed in these respects by local conditions. In some localities a quality of material can be obtained cheaply, and should be required, which would be unreasonable to expect where wood is expensive.

Wood should be dried before manufacture. (When arms are dried after drilling, the pin holes become elliptical, i.e., smaller across the grain, due to shrinkage of the wood and the pins do not fit well.)

Dimensions, Pin holes, etc., as noted in preceding section.

COST. — The cost of cross arms is subject to considerable variation and depends largely upon the locality in which they are purchased. The following figures are rough approximations and are exclusive of freight.

APPROXIMATE COST OF CROSS ARMS

Cents per linear foot

Kind	Washington fir, F.O.B., Seattle	Longleaf pine, "75 per cent heart," F.O.B., New Orleans	Commercial yellow pine F.O.B., New York
3¼ by 4¼ inches.....	5	6	5
3½ by 4½ inches.....	7	10	...
3¾ by 4¾ inches.....	8	...	...

In car load lots about 25 per cent less than above.

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[R. A. PHILIP and CABOT STEVENS.]

DAMS. — (*See also Hydraulics; Hydrology; Power Stations; Structures, Simple.*) A dam is a structure built to interrupt a stream's flow and raise the level of the water, thereby impounding water which may be used for power development, water supply, navigation, irrigation or numerous other purposes.

Coffer-dam. — A dam built for the purpose of holding back water from an area, which would otherwise be flooded, and making it accessible for construction purposes, is called a coffer-dam. It is generally a temporary affair.

Diversion Dam. — This name has been applied to dams built for the purpose of diverting the whole or a portion of a stream into a side channel or race by which it is conducted to one or more places where it is used to develop power.

Wing Dam. — In swiftly flowing streams having steep slopes, a diversion dam is sometimes built out from one shore diagonally upstream toward the other, but not to it. This special form of diversion dam is known as a wing dam.

Gravity Dam. — Under this classification fall the many dams, all of which, although differing in form of cross-section, depend upon the force of gravity for resisting the thrust and overturning moment of the water. Earth and loose rock dams, timber cribs and most masonry dams are of this type.

Arch Dam. — These are built curved in plan, resting on the valley floor with the ends terminating in the valley walls which act as abutments to the arch so formed. Theoretically the arch dam should be safer and more economical than the gravity dam, but uncertainties in the theory of its action and design have made its use rare. In a long arch of large radius a certain amount of deformation must take place, before arch action can be developed, due to the elasticity of the materials and the fact that it was built under conditions of no stress. This would be a serious matter in any important dam since it would cause a slipping on the foundations or a rupture on other horizontal planes.

Composite Gravity and Arch Dam. — Often a gravity dam has been built in the form of an arch, using the possible arch action as a factor of safety, and depending wholly on the gravity action to resist the thrust of the water. Many of the highest dams in existence are so constructed and are generally in localities where the stream's valley is relatively narrow and deep.

LOCATION OF DAMS. — In power development the location of the dam depends largely on the topography and geological formation of the ground. In general it should be placed so as to produce the greatest fall at the least expense and insure perfect safety not only to the dam and power house, but to the country round about. A position should not be considered where ample provision cannot be made against possible disaster from log or ice jams and floods.

HEIGHT OF DAMS. — The height of the dam should be as great as possible in order to develop the power to its maximum extent. Care must be taken not to overflow lands not owned or controlled by the building company, neither must the rise in level set the water back on the wheels of another power development above the proposed one. Court cases and heavy damage suits are only too common in this connection.

MATERIALS AND FOUNDATIONS. — Stone masonry, concrete (plain and reinforced), steel, timber, loose rock and earth have all been used for dam construction either singly or in combination. Structural steel has been used only in a very few cases owing to the uncertainty of its durability. It does, however, offer the distinct advantage of permitting stresses to be more accurately computed and taken care of in the design.

Earthern Embankment. — The earthen embankment is the most common form of dam and generally the cheapest It can be safely built on foundations

that would not permit of masonry construction, but offers a decided disadvantage for power developments in that it cannot be over-topped with water without endangering the structure. This makes it in many cases an impossibility, since provision must always be made for flood-flows. When used, it is provided with masonry overflows or waste ways which require most careful design and construction.

Masonry Dams of considerable size require solid rock for a foundation. Small dams have been built on foundations of hard clay or compact sand, timber platforms being used to distribute the load.

Timber Dams may be used on any foundation and generally consist of cribwork filled with rock, sand and clay planked over to make it fairly tight. If placed on soft bottom, piling is sometimes used for a support and sheet piling driven vertically into the bed to prevent seepage. Timber dams are often placed on rock foundations in which case they are bolted in position.

Loose Rock Dams, as the name implies, are embankments of loose rock made partially impervious by a core or facing of earth or other fine material. Since percolation through the dam is possible, the foundation must be safe against scour and, if the dam is of considerable height, should be solid rock.

Rock-filled Dams are not extensively used for power development because of the relatively large loss of water by seepage.

DESIGN AND CONSTRUCTION OF EARTH DAMS.—The materials used are loam, sand, gravel and clay mixed in various proportions, the clay being used to make the other three impervious. The earth dam is generally trapezoidal in section, having a top width and side slopes which are determined by the materials used. It may be built either with or without a core wall, the object of which is to offer an impervious barrier to seepage. When no core wall is used, the whole embankment may be constructed of uniform material mixed and placed so as to be as near water-tight as possible, or the material may be sorted and the best and most impervious placed in the upstream embankment and near the face, with a gradual change to the coarser and more porous material as the lower face is approached. Even with the core wall this method is often followed.

Core and Foundation for Earth Dams.—Since the core is used to obtain imperviousness, its use is imperative only when good material is not at hand or abundant. Cores are most often built of clay puddle, carefully mixed and rammed, but have been constructed of fine earth, and often masonry cores are used. The thickness of a puddle core is ordinarily 4 to 8 feet at the top, and at the bottom about one-third the water depth. Masonry cores can be made thinner in good embankments and generally have a top width of from 2 to 4 feet with side batters of ½ to ¾ inch per foot down to the trench. In all cases the core should reach down into the foundation to a stratum of compact, impervious material and several feet into it. Stiff clay or hardpan makes the best foundation and solid rock is also good if free from cracks and fissures.

Side Slopes and Top Width of Earth Dams.—The slopes given the embankments vary considerably in practice, but in general are about the same for the two faces. The upstream face is usually protected by rip-rap or paving and the slope is about the natural angle of repose. With coarse material this may be 2 horizontal to 1 vertical but with fine material it is sometimes 2½ or 3 to 1. The amount of material in the lower embankment has much to do with the stability of the dam, consequently flatter slopes are used here than the drier, more porous material would necessitate. A slope of 2 to 1 is commonly used. The top width of the embankment is often determined by re-

quirements for a roadway. Where not so fixed it may be made proportional to the height and determined approximately from the relation,

$$w = \frac{h}{5} + 5,$$

where h is the height. High dams with long slopes are liable to rain-scour on the lower face unless provision is made for draining off the water. This can be done by building berms a few feet wide every 30 feet or so on the slope, thereby providing lateral drainage. To protect the upper face from ice and wave action the slope is paved with a hand-laid pavement 12 to 18 inches thick laid on 6 to 12 inches of broken stone or gravel. Heavy rip-rap is often used below the water line.

Procedure in the Construction of Earth Dams. — In preparing the foundation the surface soil is stripped to a depth sufficient to reach sound material, and all loose stones, stumps, roots, etc., removed. If the foundation be rock, the surface is cleaned and roughened to aid in making a good bond with the earth. The core wall (if used) is started in its trench and when it is even with the surface, or a little above it, the embankments are begun. The material is put on in layers 6 to 12 inches thick and rolled with a grooved roller after being sprinkled with water. If a puddle core be used, it is carried up along with the embankments, but a concrete or stone core must be kept several feet above them and the earth should be rammed well against it.

MASONRY DAMS. — Both stone and concrete have been widely used in constructing masonry dams. Recent practice has favored concrete and many of the highest dams have been built of it. It is often cheaper to obtain than stone, may be economically handled, and, if well laid, presents a homogeneous mass with no joint planes or weak spots to offer chances of slipping or crushing. "Cyclopean masonry" is concrete containing large stones or "plums" carefully placed in the wet concrete so as to be entirely surrounded by it. Stones with a content of 1 to 2 cubic yards are not uncommonly used, and economy results. In the last ten years or so, many important dams have been built of reinforced concrete with gain not only in economy but in convenience and in certainty of design.

Foundations for Masonry Dams. — Most masonry dams over 30 feet in height require solid rock foundations. From this all soil and disintegrated rock is removed and the surface scrubbed clean. All cracks and fissures at the site and for some distance above and below it must be located and grouted under pressure. If originally smooth or inclined the rock is roughened or stepped off to offer a good bond between it and the masonry. Often a longitudinal trench is blasted out, into which the masonry may be carried to form a cut-off wall for any water seeking a way under the dam. This is generally placed at the upstream face or heel of the dam. (*See Fig. 1.*)

Where low dams are built on earth foundations, the earth should be hard clay or compact sand of an even texture. Plank foundations on piles are sometimes used to distribute the load, and sheet piling is often driven into the foundation at the upper edge of the dam to prevent seepage. It should reach down to a stratum of impervious material, or deep enough to make percolation under it a slow process.

General Principles of Design of Masonry Dams. — (*See also Structures, Simple.*) The chief forces considered in the design are gravity, water pressure, ice pressure and temperature stresses. If water percolates beneath the dam, upward pressure is developed which, if not taken care of, may ultimately be the cause of the dam's destruction. If considered in the forces acting, it will result in an exaggerated and uneconomical section. Upward pressure is best

avoided by using all possible care in making the foundation work tight and by providing underdrains. *(See below).*

Temperature Stresses generally show their action by a series of vertical cracks in the face of the dam. In massive structures it is probable that the temperature of the interior material changes but little during the year and that only the outer shell is subject to these strains. Little attention is generally paid to temperature stresses in any but large dams, where allowance is made by considering that the first few feet of outer shell may not be so effective in adding to the stability of the whole as the inner core.

Ice Pressure may be very great under some conditions but cannot be well estimated. The thrust from surface ice cannot well exceed its crushing strength, which may be taken at 400 lb. per square inch, yet it is difficult to see how this may be exerted. The chief danger comes from a "shove" of broken cakes piled against the dam and urged over it by a rise in the stream. The forces developed under these circumstances may be very great and the best protection against them is obtained by shaping the top of the spill-way so that the ice may ride easily up and over it.

Fundamental Conditions. — When the following fundamental conditions are fulfilled the stability of any masonry dam will be assured:

(*a*) No pressure to be allowed on any joint * or in any part of the dam greater than the safe working pressure. This varies from 8 to 15 tons per square foot according to the material.

(*b*) No tension to be allowed on any joint. Assuming that the stress varies uniformly over a joint, it can be shown that if the resultant pressure on any joint falls within the middle-third portion of the joint,† no tension can exist. The condition of no tension is, therefore, that on every joint the resultant force must lie within the middle third of the joint.

(*c*) At each joint the resistance to shear or to sliding shall be greater than the shearing force. If the dam be constructed with no continuous horizontal joints, and in accordance with conditions (*a*) and (*b*) above, sliding will be impossible save at the foundation. If the latter be rock it should be roughened or stopped to obviate a continuous plane joint. On other foundations the angle that the resultant force makes with a vertical line should not exceed the angle of repose of masonry on the material composing the foundation.

(*d*) The dam as a whole must resist overturning. If conditions (*a*) and (*b*) be fulfilled the dam will have a factor of safety of at least two against overturning.

Types of Cross-Sections or Profile. — Cross-sections differ in profile principally according to whether or not the dam is to pass water over its top. Where it does not, the type shown in Fig. 1 is fairly representative. If it does, then the rounding top and curved face shown in Fig. 2 is commonly used, although on rock foundation it may not be necessary to add the lower curve to the face, or down-stream side, which is for the purpose of deflecting the water and protecting the foundation. The upper portion of the curve is usually made parabolic to conform to the natural path of the water as it leaves the crest with an initial horizontal velocity and falls freely under the action of gravity. If the descending sheet or nappe does not touch the dam, partial vacuum will occur behind it and the atmospheric pressure on the back, or up-stream side, of the dam will become effective as an overturning force.

* By joint is meant any plane section through the dam, or portion of such a section, whether there exists an actual joint in this plane or not.

† Imagine a horizontal line in any cross-section of the dam, and let this line be divided into three equal parts; the middle part is called the middle third of the joint in which this

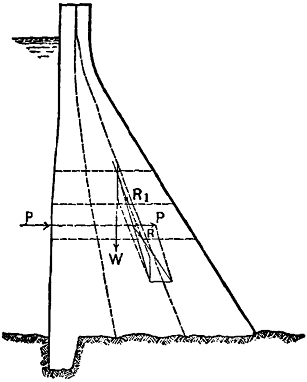

Fig. 1.

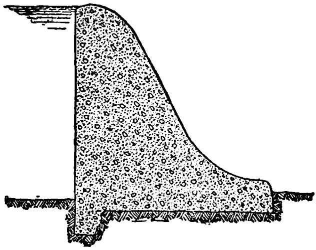

Fig. 2.

Calculation of Dimensions of Cross-Section. — The proportions of the section are possible of mathematical determination once the forces become known. Wegmann, in his *Treatise on Dams*, gives the equations (and their derivation) necessary for the different steps. A more common method is the "cut and try" by which a profile is first assumed and then investigated with regard to the conditions of stability (*a*) to (*c*) as stated above. It is then corrected and again tested until it results in a section satisfactorily strong and yet economical. Fig. 1 shows the steps and result of this method. Starting with a given height, top width and unit weight of masonry (*see Weights of Materials*), a series of horizontal joints are arbitrarily assumed at equal distances through the structure. The position of the resultant force on each joint is graphically found by combining the resultant force on the above, or previous joint, with the weight of the masonry block between joints, and the water pressure on the back face of that block. Starting with the top block, where the weight and the water pressure are the only forces, this is easily accomplished. If the "line of resistance," i.e., the line which connects the points of application of the several resultants falls at any point outside the middle third portion of the section, the profile is changed and a new "line of resistance" drawn.

Calculation of Maximum Pressure on a Joint. — The maximum pressure on any joint (according to the usual assumptions, which are justified by experience) may be found from the relation

$$P_{\max} = \frac{V}{l}\left(1 + \frac{6e}{l}\right) \qquad \text{pounds per square foot,}$$

where V is the vertical component of the resultant force in pounds (per unit length of dam) on that joint, l the length of the joint in feet (the horizontal distance perpendicular to the length of the dam from the face to the back of the dam), and e the distance in feet from the resultant to the centre of the joint. It can be shown that a dam constructed to satisfy condition (*b*) above will be safe against crushing if the height is not much over 100 feet.

Effect of Water on Crest. — When water is passing over the dam extra forces are developed which tend to make the dam less stable. The pressures arising from the dynamic action of the water are generally disregarded and the pressures on the back of the dam are considered to be due to the head of water whose level is that of the water going over the crest. No allowance is

made for the effect of the weight of the water as it flows over the down-stream face.

Underdraining. — In many large and important dams, provision against upward water pressure has been obtained by placing a system of drains in the foundation. They may lead to a main sump in the structure or directly to the outside at a point in the toe of the dam. Their more general use would seem to be a part of good design.

REINFORCED CONCRETE DAMS. — The last few years have seen the rapid development of reinforced concrete in dam construction. Most dams so built consist of an inclined decking of reinforced slabs, resting on a series of parallel buttresses which have their footings in the foundation. Fig. 3 shows the section of such a dam designed to pass water over its crest. With a low dam and rock foundation, the down-stream decking or apron may be omitted and the face made vertical. The great stability of the dam follows from the fact that the water pressure is inclined and, when compounded with the weight of the dam, gives a resultant pressure on the base which is nearly vertical. Fig. 3

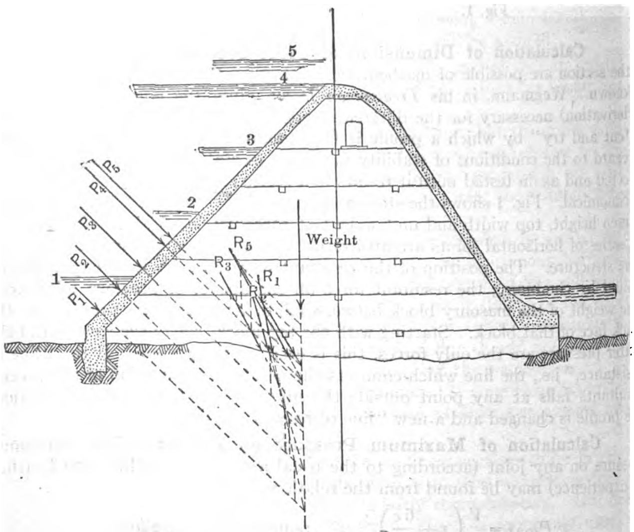

Fig. 3.

shows this resultant to change little in position as the height of water behind the dam changes, a condition far different from that existing in the solid dam as generally designed.

Advantages of Reinforced Concrete Dams. — There is no doubt that this type of structure possesses certain marked advantages. The stresses developed may be calculated with considerable certainty and the structure properly designed to meet them. The temperature stresses are provided for by expansion joints in the deck slabs placed at each or every other buttress. Ice cannot exert its full strength as the face of the dam is inclined and raises the ice. The dam may be easily inspected at all points when in service, and from the manner

of its construction affords simple means for accomplishing the final "closing" of the dam. Not the least of its good points is the practical elimination of chance for upward pressure.

Disadvantages of Reinforced Concrete Dams. — Against all these advantages may be urged the uncertainty of the life of steel reinforcement in concrete under water, and it is this fact that has prevented its wider adoption. A majority of dams so built have been constructed by or from the designs of the Ambursen Hydraulic Construction Co., and to date (1914) no failure has occurred, even of a partial nature. An examination of the steel in a dam 10 years old has revealed no trace of oxidation, or deterioration.

APPENDAGES TO DAMS. — The more common appendages to dams are the following:

Sluice Gates. — As their name implies, these are large gates placed in the dam for the control of the water level, or for relieving the dam in times of flood. A common type of gate is the so-called Tainter gate which is slightly curved and revolves about its axis of curvature. A later improvement on this is the Hall gate which permits of finer regulation.

Flash Boards. — These are light boards, or their equivalent, placed on the crest of the spillway and held in position by light rods or frames. Their object is to temporarily increase the height of the dam during the dry season so as to conserve the water which would otherwise be wasted over the spillway at times when the wheels were not running. They are made light not only to facilitate handling, but also that they may give way quickly in emergency before a dangerous head could be created by sudden flood.

Log Runs. — On any stream where the driving of logs is practiced, it is generally a requirement that provision be made in a dam for the sluicing of logs. This is accomplished by a chute or trough placed generally near the spillway, with its upper end slightly below the pond level and closed by a sluice gate. To this the logs are guided by log booms fastened to the dam and cribs along the shore.

Fish-ways. — A fish-way is a gradual incline connecting the water below the dam with the pond above. A constant stream of water passes through it with a velocity against which the fish may readily swim. This is sometimes attained by using baffles across a portion of the stream, leaving a passageway at the side. The slope of a fish-way should not be greater than 1 in 4 and its ends should be well beneath low water level. Fish-ways are generally more or less covered in order to protect the fish from interference, but should never be made dark, as they would not be used by the fish. Fish-ways are generally required by State law on all natural water ways.

COST OF DAMS. — It is difficult to give items of cost for dam construction, due to the great influence of local conditions. As earth embankments should not be attempted in places where material is not available near the site, the figures for unit cost may be fairly well approximated. Excavation in earth will cost from 20 to 30 cents per cubic yard; embankment 25 to 40 cents; puddle cores 45 to 70 cents; hand-laid paving \$2.00 to \$3.00; rip-rap \$1.50 to \$2.00; sodding 20 to 30 cents per square yard. Figures for masonry dams depend largely on the ease with which the materials may be brought to the site. The following figures serve to show approximately the unit costs under conditions normally favorable. Rock excavation \$1.00 to \$2.00; rubble masonry laid in natural cement \$3.00 to \$5.00; same laid in Portland cement \$4.00 to \$6.00; concrete masonry, Portland cement, \$5.00 to \$7.00; reinforced concrete \$7.00 to \$11.00; rock-faced stone masonry \$10.00 to \$15.00; dimension stone masonry \$15.00 upward.

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[GEO. E. RUSSELL.]

DECIMAL EQUIVALENTS. — The following table will be found useful in converting common fractions into decimals.

8ths	16ths	32nds	64ths	Decimal	8ths	16ths	32nds	64ths	Decimal	8ths	16ths	32nds	64ths	Decimal
			1	0.015625			11	22	0.34375				43	0.671875
		1	2	0.03125				23	0.359375		11	22	44	0.6875
			3	0.046875	3	6	12	24	0.375				45	0.703125
	1	2	4	0.0625				25	0.390625			23	46	0.71875
			5	0.078125			13	26	0.40625				47	0.734375
		3	6	0.09375				27	0.421875	6	12	24	48	0.75
			7	0.109375		7	14	28	0.4375				49	0.765625
1	2	4	8	0.125				29	0.453125			25	50	0.78125
			9	0.140625			15	30	0.46875				51	0.796875
		5	10	0.15625				31	0.484375		13	26	52	0.8125
			11	0.171875	4	8	16	32	0.5				53	0.828125
	3	6	12	0.1875				33	0.515625			27	54	0.84375
			13	0.203125			17	34	0.53125				55	0.859375
		7	14	0.21875				35	0.546875	7	14	28	56	0.875
			15	0.234375		9	18	36	0.5625				57	0.890625
2	4	8	16	0.25				37	0.578125			29	58	0.90625
			17	0.265625			19	38	0.59375				59	0.921875
		9	18	0.28125				39	0.609375		15	30	60	0.9375
			19	0.296875	5	10	20	40	0.625				61	0.953125
	5	10	20	0.3125				41	0.640625			31	62	0.96875
			21	0.328125			21	42	0.65625				63	0.984375

DEMAND INDICATORS. — (*See also Ammeters; Ampere-hour Meters; Watthour Meters.*) For some time past various attempts have been made to inaugurate systems of charging for electric energy which would be more equitable to both the consumer and the central station than a flat kilowatt or kilowatt-hour rate. The so-called "maximum demand system" has found favor with many, and may find a much more general application.

The system is based on the fundamental assumption that the charge to any consumer should be divided into two parts: one part fixed by the maximum power demanded by an individual consumer at any time during a certain definite period, and another part fixed by the total number of kilowatt-hours used during the same period.

Requirements of a Demand Indicator. — The kilowatt-hours supplied to the consumer are readily measured and recorded by a watthour meter (q.v.); to record the maximum power (kilowatts) taken by the consumer various forms of "maximum demand indicators," usually called simply "demand indicators," have been devised. In the case of a practically constant d-c. voltage at the consumer's premises, a device which measures the maximum current is as satisfactory as one which measures maximum power, but when the voltage or power factor (in case of an a-c. system) varies, the maximum current indicator is not suitable.

In any case the device should be one in which the demand measured is not the instantaneous peak of the load demanded by the consumer, but is the average of the power demanded over an appreciable time interval, for the maximum demand recorded should not be influenced by short-circuits, excessive current flow in starting motors, or by any abnormal consumption of energy that covers too short a time to have any real effect on the capacity which must be provided in the central station to take care of the demand.

The time over which the demand should be taken differs with the character of the installation and its relation to the maximum power demanded and to the maximum capacity of the central station. In relatively large consumers' installations the time should be carefully chosen with reference to the time that the central station can endure an overload successfully. In small installations, such as residences, the time interval is not important, provided it is long enough to cover any abnormal and sudden fluctuations of load, but not longer than the ordinary period of sustained maximum load. In some instances times as short as one or two minutes have been selected for large installations, and about fifteen or thirty minutes seems to be quite generally satisfactory for household installations and small industrial plants.

In connection with demand systems of charging it is sometimes of advantage to know at what time of day the maximum demand occurs, for it is sometimes possible to give preferential rates to large consumers if they will draw their heaviest load at periods when the station load-curve is below its maximum.

The general subject of demand indicators has received considerable attention within the past year or two and many new devices have been developed experimentally, and will no doubt soon be on the market. The whole system of charging, including the necessary devices to be used, may be considered to be in a transition state. Some of the demand indicators now in commercial use are briefly described below.

Curve-drawing Wattmeter as a Demand Indicator. — (*See also Wattmeters.*) The most accurate method of obtaining a complete knowledge of the conditions prevailing on a customer's premises is to use a curve-drawing wattmeter. With this device the average demand over any period of time can be taken, and the time at which the maximum occurs may be definitely known. However, curve-drawing wattmeters are too expensive and troublesome for any

but comparatively large installations, and one of the cheaper and simpler devices described below is ordinarily used.

TYPES OF DEMAND INDICATORS. — The devices, which have so far been produced, may be divided into the following classes: (1) Instruments in which the time of reaching the maximum record is delayed, and in which the maximum record and time over which it is averaged depends more or less on the previous load conditions; (2) Instruments in which the time over which the demand is averaged is definite, the period of registration beginning and ending at certain definite times controlled by a clock or equivalent device.

In either of the above types the record may be obtained by automatically pushing a pointer up the scale and leaving it at the maximum indication, in which only the maximum is recorded, or various arrangements of moving charts may be employed on which any number of records may be made. In some cases the time of each record is also indicated, in which case the instrument may be nearly equivalent to a curve-drawing wattmeter, provided the time interval over which the average is taken is short.

Wright Demand Indicator. — This instrument, manufactured by the General Electric Company, records the maximum ampere demand of appreciable duration in either a direct- or alternating-current circuit. It operates on the principle of a differential air thermometer. The instrument consists principally of an enclosed system of a vertical U-tube with unequal arms and with nearly equal bulbs at the ends, Fig. 1. An indicating tube of uniform bore is annealed to the shorter arm of the U-tube just below the bulb b_2. The upper bulb b_1 is closely surrounded by current-carrying heater strips wound non-inductively. The U-tube is filled with sulphuric acid and both bulbs with air, in such a way that the air in the upper bulb b_1, heated by the electric current in the strips, causes some of the liquid to flow into the index tube i_2 and to remain there as a record until the indicator is set back. The system of tubes and bulbs is fastened to a frame which can be turned up slightly more than 90°, so as to allow any liquid contained in the index tube to flow back into the bulb and U-tube. Since the expansion of the air in the heated bulb b_1 is approximately proportional to the final rise of temperature, and hence roughly proportional to the square of the current in the heater coil, the ultimate height of the liquid in the index tube i_2 will be proportional approximately to the square of the current in the heater strips. The scale reads directly in amperes.

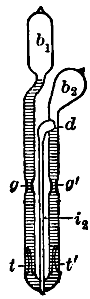

Fig. 1. Wright Demand Indicator

Wright demand indicators of all capacities from 5 to 150 amperes inclusive may be connected directly into alternating-current circuits of any commercial frequency. Indicators of 200 amperes capacity and over are furnished with shunts for direct current and with current transformers for alternating current.

The Wright demand indicator is not suitable for the determination of maximum loads in alternating-current circuits of variable power factors. It leaves no record of the duration of the maximum demand nor of the time at which it takes place. After the instrument has been reset there is no original record of previous maximum demands.

Time Lag and Precision of Wright Demand Indicator. — Each instrument has its individual time lag, and the same instrument may have different time lags at different loads. The time lag is shown by the following average figures: 90 per cent of any increase of load is indicated after approximately 4 minutes' duration, 97 per cent after 10 minutes, 100 per cent after about 40 minutes. Wright demand indicators are sufficiently accurate for most com-

mercial installations, provided they are installed in locations which are not subject to abnormal temperature variations.

Type W Polyphase Demand Indicator. — This instrument, made by the G. E. Co., is suitable for indicating loads and for recording maximum loads on polyphase alternating-current circuits in which these loads have appreciable duration. The time lag, which for this instrument is defined as the number of minutes required for the meter to indicate 90 per cent of any change of load, may be adjusted from one to five minutes. With the time lag adjusted for five minutes, full deflection corresponding to a change of load is reached after about 15 minutes. The instrument is constructed on the principles of a polyphase induction watthour meter (*see Watthour Meters*). Fig. 2 shows the arrangement for a 3-wire, 3-phase instrument. Both current and both potential elements act on the upper disc D_1. A system of controlling springs at S provides a restoring moment, the springs being adjusted so that full-load driving torque will be completely balanced by the restoring moment of the springs at S when the vertical shaft has made three revolutions, corresponding to one revolution of the indicating pointer H_1, driven by means of a system of gearing of 3:1 ratio. The other pointer H_2, driven by the first one, is held at the maximum position reached by H_1 by means of a ratchet and remains there until set back by hand. The propelling torque is proportional to the power in watts and the restoring torque is proportional to the deflection of the moving system, therefore the ultimate deflection of the pointers is directly proportional to the power in watts. The magnets M and M' acting on disc D_2 provide an exceedingly heavy damping system. The indicator is thereby prevented from recording power demands of short duration. The type W polyphase watt demand indicator is made in all standard capacities. For the larger capacities instrument transformers are used in the same way as with the watthour meters. The instrument leaves no record of the duration of the maximum demand nor of the time at which it takes place.

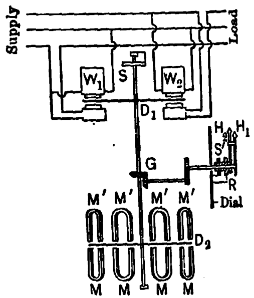

Fig. 2. Type W Demand Indicator

Precision of Type W Demand Meters.— Most of the advantages and the limitations of polyphase induction watthour meters apply to this instrument, and, in addition, it is extremely delicate, and its operation may be affected by mechanical shocks and jarring.

Combined Watthour and Demand Meter, Single Phase. — This instrument, made by the Westinghouse Electric and Manufacturing Co., is a combination of induction watthour meter, induction wattmeter, and an escapement form of time element. The time for this instrument to reach maximum deflection corresponding to any change in load is constant, provided the load remains constant during this time interval. The meter records, besides the kilowatt-hour consumption, the maximum load in watts in the circuit. The instrument leaves no record of the duration of the maximum demand or of the time at which it takes place.

Demand Indicator Attachments to Watthour Meters. — These devices are on the market in various forms and under various names and are used to indicate the average power over a certain predetermined period of time. All of them depend for their operation on a device the speed of which is proportional

to the rate of energy consumption in the circuit, and on a clock or some equivalent constant-speed device by which the time interval is determined over which the average power demand is desired. Consequently these devices may be applied to any circuit in which a watthour meter will correctly measure the energy consumption. The accuracy of any particular device of this kind depends, therefore, on the accuracy of the watthour meter in connection with which the attachment is used, and on the accuracy of the constant-speed device which determines the time interval. Among these demand indicator attachments may be mentioned the Maxicator, Printometer and Graphometer made by the Chicago Electric Meter Co.

Maxicator. — The maxicator is used for the purpose of indicating the largest half-hourly average of power demand. It consists of a register element and a contact making motor or clock. The register element operates as an ordinary gear train of a watthour meter and has connected with it through a special train of gears a driving element which engages with the maxicator hand on a special dial. This hand is carried forward at a rate proportional to the speed of the watthour meter and therefore proportional to the rate of consumption of energy in the circuit. At the end of each interval of thirty minutes, the contact making motor or clock will cause a solenoid, connected to the driving element, to be energized in such a way as to reset the driving element to a zero position. The maxicator hand is thus left at the point which it had reached after the first half-hour interval. The resetting operation is so arranged that it does not affect the regular dial pointers of the register. The driving element after having been reset revolves as before, but does not re-engage and drive the maxicator hand unless the energy consumption in the second half hour exceeds that of the first half hour. Thus the distance moved by the maxicator hand is proportional to the watt-hours for a period of thirty minutes. The maximum half-hourly average of the power demand in watts is obtained by dividing the reading in watt-hours of the maxicator dial by the time interval.

A constant-speed motor is preferable to the clock for making the half-hourly contacts, as the former reduces the cost of inspection and attendance; on the other hand, the clock may be made to operate with greater accuracy than a motor. The maxicator hand is reset by the meter reader, who thereby destroys the original record of the maximum demand. The duration of the demand and the time at which it occurrs cannot be obtained by the maxicator.

Printometer. — This instrument is designed to print on a paper tape at regular time intervals the time of record and the consumption of energy up to that time, the energy consumption being that registered by a watthour meter connected to the circuit. The instrument contains a set of cyclometer type-wheels which are moved forward at a rate proportional to the rate of energy consumption in the circuit. This is accomplished by a solenoid, the energizing circuit of which is closed through a contact wheel fixed to one of the spindles of the gear train of the watthour meter. The reading of the cyclometer is printed, at regular time intervals, on a slowly moving paper tape by the agency of a rubber platen and a copying ribbon, the rubber platen being actuated by a solenoid which is energized at regular time intervals by means of a contact making clock. The time is also printed opposite each cyclometer reading. Thus, the difference between consecutive records on the tape is proportional to the energy consumption in watt-hours in the circuit during a definite time interval. The average demand over a period of time, corresponding to the interval between successive operations of the printing solenoid, is obtained by dividing the watt-hours of energy consumption during a time interval by the length of the interval.

This device, therefore, gives an original record of a permanent nature, which furnishes complete data for a continuous load curve for the circuit to which it is connected.

Graphometer. — The graphometer is designed to draw a curve showing average demands for successive time intervals of equal length. The principle of operation of the graphometer is similar to that of the printometer, described above. The plunger of a solenoid, the energizing circuit of which is closed through a contact wheel fixed to one of the spindles of the gear train of a watt-hour meter, causes a stylus to be moved vertically upward a distance proportional to the number of contacts made by the contact wheel, and hence proportional to the energy consumed in the circuit. The stylus draws a line on a treated paper chart which moves uniformly in a direction at right angles to the direction of motion of the stylus. At the end of the interval for which the contact-making clock is adjusted, the mechanism driving the stylus is caused to become disengaged from the rack carrying the stylus, which is thereby allowed by gravity to drop back to a point corresponding to the zero position on the chart, when the same series of operations begins for the next interval. The various ordinates on the chart represent average power demands for a definite time interval.

The graphometer therefore plots directly the points for a continuous load curve, which serves as a permanent record, and which requires no interpolation, and from which the maximum demand may be read at a glance with the aid of a suitable scale.

Costs. — The net prices of the different types of demand indicators are as follows:

Type	Description	Price
Wright Demand Indicator	5 to 150 amp. for 750 v. and below...	\$5 to \$12
Wright Demand Indicator	D-C. only, 200 to 600 amp. for 750 v. and below........................	10 to 17
Wright Demand Indicator	A-C. only, 200 to 600 amp. with current transformers up to 2300 v....	35 to 40
Type W Demand Indicator	5 to 75 amperes, 110 to 440 v........	75 to 80
Watthour Demand Meter	....	20 to 25
Maxicator.................	15 to 30 min. time interval.........	21
Graphometer..............	....	45
Printometer...............	....	115 to 160

BIBLIOGRAPHY. — Manufacturers' bulletins; *Electrical Meterman's Handbook*, N.E.L.A., 1912; *Massachusetts Board of Gas and Electric Light Commissioners*, 28th Annual Report, Appendix A; Jansky, C. M., *Electrical Meters*, N. Y., 1913. The question of the proper basis for equitable rates for electric energy has been very actively discussed during the last five years. The two earliest papers on this subject are Hopkinson, J., *The Cost of Electric Supply*, Elec. Rev. (London), 1892, Vol. 31, p. 571; Wright, A., *Points of Interest in the Development of Electricity Supply at Brighton*, Elec. Eng. (London), 1895, Vol. 15, p. 537; see also recent articles in the *London Electrician*, *Electrical Review*, London, *Electrical Review*, Chicago, *Electrical World*, *Trans. A.I.E.E.*, *Assoc. Ed. Ill. Co.'s*, *Nat. Elec. Lt. Assoc.*, *Public Service Commission Reports*, Second District of New York and Connecticut.

[L. T. ROBINSON AND O. R. SCHURIG.]

DEPRECIATION. — Depreciation may be defined as a decrease of value due to deterioration, loss of useful association, obsolescence, inadequacy, and a general change in the level of prices.

VALUE. — Value is a ratio or number expressing the relative quantities of commodities or services which are considered equal in exchange. It is not an intrinsic quality of matter like mass or length, but an extrinsic quality or relation like height or electric potential. In other words the value of a property must not be sought in the property itself but in its relations to other properties and services. Value is a function of demand and supply which in turn depend upon utility, scarcity, beauty, cost of production, and many other circumstances. For a discussion of the nature and various elements upon which value depends, see *Science of Money*, by Alex. Del Mar.

The value of public utility corporations often has to be estimated in order to furnish a basis for the determination of rates or charges for services, assessment of taxes, limitation of security issues and fixing prices for purchase or sale.

FACTORS AFFECTING DEPRECIATION. — As already noted the factors affecting depreciation are deterioration, loss of useful association, obsolescence, inadequacy and a general fall in prices.

Deterioration may be defined as any change in a property due to wear and tear or the ravages of the elements, which tends to impair either its usefulness or its life.

Loss of useful association may be defined as any change in the associations of a property, which tends to impair its usefulness or life.

Obsolescence may be defined as loss of commercial utility in any property, due to the advent of superior substitutes.

Inadequacy is loss of commercial utility in a property due to its inability to meet increased business conditions.

Deterioration can be approximately predetermined from the results of previous experience, while loss of useful association, obsolescence and inadequacy are of a more speculative character and less amenable to computation. Variations in the general level of prices are also impossible to predict. While, therefore, it is possible to estimate with some degree of accuracy how much it will cost to overcome deterioration, it is possible only to bet or insure against obsolescence, loss of valuable association and the results of monetary instability.

METHODS OF APPRAISAL. — A property in its entirety can have only one value at a given time and place, although of course different parts have different values, and it may be that for some purposes for which valuations are made only part of the value has to be considered. Thus by omission of certain elements, figures may be obtained called tangible value, intangible value, fair value and so on. They are really partial values, and not different kinds of values. The true value of a property appears only when it is bought or sold, although it exists at all times. There are, however, many occasions when it is necessary to know the value of a property without putting it on the market, and, for this purpose, estimates of value or appraisals have to be made. Considerable difference of opinion exists, however, as to the proper way to estimate value, some writers even referring to the different kinds of estimates as different kinds of values, which is obviously inaccurate.

Market Value. — There are two different ways of estimating the value of a property. One is by taking into account all the elements which would affect its salability, and estimating what a responsible bidder would offer for it.

This gives what is called the market value. It is not a value, but merely an estimate of value by a particular method. The other way of estimating value is by calculating the cost of reproduction of the property. This gives what is called the reproduction value. This again is not the value, but merely an estimate of value by another method.

Reproduction Cost. — The market-value method of appraisal is useless as a basis for rates because the market value itself is determined by the rates. It is therefore usual to estimate value by the reproduction-cost method. Unfortunately this method has been applied in two different ways. By some authorities, cost of reproduction is taken to mean the cost of a substantially identical reproduction of the existing plant. They do not mean that apparatus of antiquated pattern will be exactly duplicated but assume it to be replaced by the nearest modern substitute. According to other authorities, cost of reproduction may mean the cost of a substitute plant of the most modern approved design capable of performing the same service as the existing plant. If the old plant were wiped out, what would it cost at present to construct a plant capable of performing the service now performed by the old plant? In the case of a water plant, perhaps an entirely new source of supply would be used and the distribution system radically changed.

In either case, the present value of the old plant is measured by the cost of an equally efficient new plant less an allowance for the depreciated condition of the old plant. It is obvious that there is no definite distinction between the two interpretations, the difference arising principally in the different degree to which obsolescence is taken into account as an element of depreciation.

Court and Commission decisions in relation to valuation have not yet become uniform and authoritative, showing that the subject of valuation is in a developmental stage. Perhaps the general tendency is toward estimating value by the reproduction-cost method, making the reproduction in the most modern manner.

PRESENT VALUE. — The value of a property can appear only in a purchase or sale, but it exists at all times. Hence some means must be devised to estimate how depreciation affects the value of a property between the time of its purchase and its sale.

In default of a true measure of value during this period, various arbitrary scales have been devised. This can best be explained with the aid of a diagram, Fig. 1, having for ordinates, value in dollars, and for abscissæ, time in years. On this diagram, let the ordinate of the point A represent the replacement cost when the property is in its best physical condition, and the ordinate of the point B the net proceeds resulting from the sale of the property at a time OP years later. It is obvious, that any curve joining A and B, which is confined between the lines AC and DB and never turns back, may be arbitrarily adopted as the curve of value during this interval.

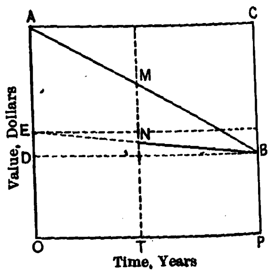

Fig. 1

Various people have suggested different shapes of curves, but the most common practice is to assume A and B to be joined by a straight line. Curves other than straight lines, which have been suggested by various people, have often been curves of utility and misnamed curves of value. Such curves of utility are meaningless, first because utility cannot be measured by money,

and second, because value is not necessarily proportional to utility. Utility is only one of many factors which affect the value of a property.

The shape of the curve of value being arbitrary, it is obviously most simple to assume it to be a straight line, as shown in Fig. 1. The "present value" of the property at any given time is then the corresponding ordinate of this line.

While at any time the curve of value may be taken as a straight line, the same straight line will not necessarily serve throughout the life of the property, especially if obsolescence is allowed as a cause of depreciation in estimating the cost of reproduction. Suppose, for example, that a new invention enables the property to be replaced by one costing half as much. The present value of the old property will then fall from the amount represented by MT, Fig. 1, to that represented by NT, the ordinate EO being the replacement cost (or cost of the new property) and T the time the new property was put on the market. The broken line $AMNB$ will then be the curve of value.

SCRAP VALUES. — The scrap value of a property is its residual value at the end of its useful life after deducting the expense of removal to a market and the expense of selling. If in Fig. 1 the line OP is taken to represent the useful life of the property, the ordinate BP represents the scrap value. Scrap value, therefore, not only depends upon the nature of the property, but also upon the location and the time of the sale. For example, a machine which might have a large scrap value in New York would possibly have a zero or negative scrap value at a mine in Nevada because the cost of removal to a market might cancel the profits from a sale. A property may depreciate to scrap value without deterioration, merely by the loss of valuable association or obsolescence. For example, a miner's stamp mill, in perfectly good condition, might become valueless because of the exhaustion of the mine, its distance from a market making its sale impracticable. It thus appears that data on scrap value should be carefully scrutinized before being adopted for any particular enterprise.

MAINTENANCE AND DEPRECIATION CHARGES AND DEPRECIATION FUND. — On account of deterioration, it is necessary to make replacements and repairs. Those replacements and repairs which occur frequently, or which severally cost a small proportion of the replacement cost of the whole property, should be paid for out of the annual earnings and thus charged to direct operating expenses. Such charges are called maintenance charges. Replacements, which occur infrequently and which cost an important proportion of the whole replacement cost, cannot be paid for out of the annual earnings without causing exaggerated operating charges in given years, unless a separate sum is set aside each year to provide in advance for the impending replacements. Such a sum is called a depreciation charge and the accumulated fund, made up of these annual sums, and the interest thereon, if invested, is called a depreciation fund or depreciation reserve. If the total annual replacements of a property do not fluctuate to such an extent as to affect the financial stability of the enterprise to which it belongs, it is not advisable to create a depreciation fund, while if they do affect the financial stability, a depreciation fund is necessary to avoid disaster.

Thus a large concern, owning property which requires replacements in small units, may dispense with a depreciation fund if care be taken to distribute replacements over several years. Another concern, having to face the replacement of a large proportion of its property, might be seriously embarrassed and would probably have to resort to one of the following expedients: (a) to borrow money on the strength of future earning capacity, such loans being gradually liquidated from the future earnings; (b) to issue stocks or bonds to purchase renewals of property that is already represented in capital, a pro-

cedure which is financially unsound and in many countries illegal; (c) to go into the hands of a receiver. Such a concern should obviously provide a depreciation fund. When the total replacements of a property fluctuate to such an extent that it is advisable to establish a depreciation fund to obtain financial stability, some criterion is desirable wherewith to judge which replacements should be charged to the maintenance account and which to the depreciation account. It is difficult to devise a rational criterion because so many unknown quantities enter into it. Hence it is usual to adopt an arbitrary rule, such as to charge to the depreciation fund the cost of entire renewals and to the maintenance account, partial renewals. Such a rule is obviously open to somewhat free interpretation. This practice is, however, far from being universal, some companies fixing an arbitrary limit, such as $500, and all renewals or repairs less than this are charged against maintenance and all greater than this against the depreciation fund. In several states the Public Service Commissions issue rules on this subject.

AMORTIZATION OR SINKING FUND. — When a property is purchased with borrowed capital, e.g., bonds, it is customary to set aside each year a sum which, with the accumulated interest, will pay off at some stated future time the debt thus incurred. The fund thus accumulated is called an amortization or sinking fund. The depreciation fund should be carefully distinguished from this amortization or sinking fund. The former exists solely for the purpose of equalizing operating expenses from year to year; the latter for the purpose of discharging the debts incurred in purchasing the property.

INVESTMENT OF DEPRECIATION FUND. — The question of what constitutes proper care of a depreciation fund is of considerable importance. Some maintain that such a fund should be invested only in securities with a regular market value and thus salable at any time. Another view is that this fund can be invested as well in the company's own business, provided it be kept in such a way as will render it readily available when needed. It may be urged against the first plan that the investment of these funds in such a manner that they can be turned into cash quickly involves either their deposit with a bank or trust company at a nominal rate of interest or their investment in low-interest-bearing securities. Thus, it appears inconsistent for the company to be borrowing money at high interest on the one hand and loaning it at low interest on the other hand, unless there is some excellent excuse for so doing.

As a matter of fact, what actually occurs in the finances of many companies is something like this: The earnings are sufficient to provide an amount for a depreciation fund. The need of increased capital to meet the growth of business is urgent. The amount which should rightly be put in a depreciation fund is actually invested in additional plant. Now, there does not appear to be a fundamental error about such a procedure, because, by investing this money in additional plant, the company has increased its assets by an amount which offsets the corresponding depreciation in value. The weakness of the plan is that it does not definitely provide money which can be used for replacement. Therefore, what occurs is that, after putting surplus net earnings into additional plant for a number of years, the company effects a reorganization of some kind and issues additional securities equal to the additional value which has been put into the property from year to year. Part of these securities are then used to rehabilitate the property. The effect of this plan is that the cash which is necessary for renewals is secured by the sale of securities at the time of reorganization, instead of by the deposit of the surplus as a depreciation reserve from year to year and the sale of new bonds or stock to pay for the additions to the plant. In the meantime the company has had the use of its own money.

The chance of friction through this plan, especially under commission regula-

tion of the issue of securities, lies in the possibility that the amounts spent out of the depreciation fund for enlarging the plant will not be recognized as an increase in assets against which additional securities may be issued when the company desires. The Massachusetts Gas and Electric Light Commission has decided that in order to keep a company's accounts properly it should pay for additions to plant by the sale of additional securities, rather than by taking the money out of net earnings. In cases where new securities are issued to pay for all increases in plant one course to pursue would be to invest the depreciation fund of the company in its own securities until such time as the cash may be needed for replacements caused by depreciation, when they should be taken from the company's treasury and sold. If such securities can be sold promptly enough or used as collateral for a loan to provide against any embarrassment, such a plan should be sound. Whatever plan is followed, either voluntarily by the companies or by direction of the commissions controlling them, it is equally important that the amount set aside for depreciation and the amount expended in new plant should be distinguishable in the accounts. (*Editorial Elec. W., 1912, Vol. 59, p. 126.*)

In the case of a property made up of a number of elements each having a different period of life, the disbursements from the depreciation fund after a certain number of years will, on the average, equal the accessions to this fund. The fund, however, should not be allowed to fall below an amount sufficient, without additional payments into it, to take care of the replacements for a reasonable period. Three years has been suggested as such a reasonable period, but this will depend largely upon the nature of the property.

ANNUAL DEPRECIATION AND ANNUAL DEPRECIATION CHARGE. — If the depreciation fund is invested, each annual payment draws interest from the time it is made, and, therefore, the annual payments necessary to make the fund equal to the difference between the replacement cost and the scrap value of a given property at the end of the life of the property, i.e., the depreciation charge, will be less than the annual depreciation, or decrease in value. Where the value of the property is assumed to decrease uniformly the depreciation charge is calculated as follows. Let A be the replacement cost of the property and B its scrap value at the end of n years. Then the annual depreciation is

$$D = \frac{A - B}{n} \text{ dollars.}$$

If the annual payments into the depreciation fund are made at the end of each year, and each payment draws compound interest at the rate of $100\,r$ per cent per annum, then the annual depreciation charge to give $A - B$ dollars at the end of n years is

$$D_c = (A - B)\frac{r}{(1 + r)^n - 1} \text{ dollars.}$$

If the annual payments into the depreciation fund are made at the beginning of each year the annual depreciation charge must be

$$D_c = (A - B)\frac{r}{(1 + r)^{n+1} - (1 + r)}.$$

The percentage depreciation, referred to the replacement cost, is $100\,p$ where

$$p = \frac{A - B}{nA} \text{ (a decimal fraction).}$$

The value of the property m years from the time of purchase is

$$V = A - \frac{m}{n}(A - B) \text{ dollars.}$$

Example. — A property costs $100 and its replacement value throughout its life of 8 years remains constant and equal to its first cost; the scrap value at the end of 8 years is $20, (1) what is the annual depreciation D, and (2) what must be the depreciation charge D_c to replace the property at the end of its life if invested at 4 per cent at the end of each year?

$$D = \frac{A - B}{n} = \frac{100 - 20}{8} = \$10. \qquad (1)$$

$$D_c = (100 - 20)\frac{.04}{(1.04)^8 - 1} = \$8.70. \qquad (2)$$

The following methods are also employed for determining the depreciation charge:

(1) **Depreciation Charge a Fixed Percentage of the Value at the Beginning of Each Year.** — The value of the property is assumed to decrease each year by a fixed percentage of its value at the beginning of that year, and the depreciation charge is taken equal to this decrease in value, no account being taken of interest on the annual payments into the depreciation fund. On these assumptions, using the same notation as in preceding paragraph, we have that the percentage annual decrease in value is 100 p where

$$p = 1 - \sqrt[n]{\frac{B}{A}} \text{ (a decimal fraction).}$$

The depreciation charge at the end of the mth year is then

$$D_c = pA\left(\frac{B}{A}\right)^{\frac{m-1}{n}} \text{ dollars}$$

and the value at the end of the mth year is

$$V = A\left(\frac{B}{A}\right)^{\frac{m}{n}} \text{ dollars.}$$

It is, therefore, evident that this scheme results in the annual appropriation of decreasing increments to the fund. This scheme is inapplicable to a property having a definite life and no scrap value. While it is strongly recommended by some engineers, others go to the opposite extreme of recommending a system which makes the depreciation charge grow as the years pass by.

(2) **Depreciation Charge a Fixed Percentage of Gross Earnings.** — The table on the following page, quoted from H. Floy, gives the practice of several companies.

(3) **Depreciation Charge a Fixed Percentage of Net Earnings.** — This scheme is financially unsound.

(4) **Depreciation Charge a Fixed Amount per Kilowatt-hour output,**

Name of company	Per cent of gross revenue expended or appropriated for	
	Maintenance	Depreciation
Milwaukee companies:		
Railway departments	11.3	9.9
Gas, electric-light and steam-heat departments	6.15	8.12
United Railways Company of St. Louis	13.67	10.0
Union Electric Light and Power Co., St. Louis	4.95	16.0
Suburban Electric Light and Power Company	7.10	10.85
Detroit Edison Company and subsidiaries	6.45	10.23
Omaha and Council Bluffs Street Railway Company	....	10.0
Chicago street railways	6.0	8.0

companies. According to H. Floy "the New York Edison Company charges off monthly for renewals and replacements, etc., an amount equal to one cent per kilowatt hour on current sold to general consumers in addition to wear and tear. In Cleveland, 5 cents per car mile is provided to cover both maintenance and other deterioration. In Brooklyn, the subsidiaries of the Brooklyn Rapid Transit System allow amounts varying from 2.7 cents to 4.4 cents per car mile for equipment of surface roads and from 1.4 cents to 2 cents per car mile for equipment of either elevated or partly elevated railways; from 2.2 cents to 2.4 cents per car mile for way and structures for surface roads; from 1.1 cents to 1.8 cents for elevated or partly elevated railways, to cover not only obsolescence, inadequacy, renewals and replacements but also repairs and maintenance."

LIFE OF PROPERTIES. — The expected life of electrical and other machinery and equipment, according to various authorities, is given in the following table. As many electrical machines, which are the first of their kind, are still in service, it is obvious that the lengths of life are really not known in many cases. Obsolescence and inadequacy have moreover been more responsible than deterioration for the replacement of electrical apparatus.

The New York First District Public Service Commission has assumed the life of electric lighting and gas properties as a whole to be about 20 years. The Wisconsin Commission has similarly decided that 18 years is a fair life for electric railway properties and 17½ years for electric lighting properties. (*L. R. Nash, Stone & Webster Journ., 1912.*)

Data on depreciation is usually given in terms of the life of the property or as a percentage depreciation. In the table below some authorities give the life only, some per cent depreciation only, and others both life and per cent depreciation. In all cases where life and per cent depreciation are both given by the same authority, the figures show that zero scrap value has been assumed; that is, the per cent depreciation p is taken equal to $\frac{100}{n}$ where n is the years of life. The following abbreviations are used:

A: Adopted by the Chicago Union Traction Co.
B: Milwaukee Electric Railway and Light Co.
C: J. W. Alvord.

D: Chicago city railways.
E: Engineer of Valuation Staff, of Wisconsin Railroad Commission.
F: W. Preece.
G: R. Hammond.
H: P. Dawson.
I: S. W. Greenland.
J: Cardiff tramways.
K: Glasgow tramways.
L: W. H. Bryan.
M: N. S. Hill.
N: J. Abbott.
O: Third Avenue Railroad (New York City).
P: T. C. Parsons.
Q: Iowa Electric Association.
R: Heidelberg Railway.
S: Recommended by Stone & Webster for the Chicago Union Traction Co.
T: J. I. Beggs.
U: Elberfeld Railway.
V: M. G. Starrett.
W: Wisconsin R.R. Commission.
X: E. Matheson.

LIFE AND DEPRECIATION

	Life in years		Per cent depreciation	
	Life	Authority	Per cent	Authority
Arc lamps	8	Q	..	..
	12 to 15	L	..	..
Boilers, stationary ..	12 to 15	E	10 to 8	H
	12 to 20	I	8.5 to 6.66	E
	15	A, C	8	D
	20	G, S	7.5	B
	25	F	6.66	A
	..	..	5	J, K, S
Buildings, P. H. and S. S.	50	A, E, F, I, N, S	2.5	J, K
	60	G	2	B, E, O, S
Buildings, P. H. and S. S., brick	25 to 40	L	5	D
Cables, underground	25	E	2.5	X
	30	G	4	B, E
	35	F	3.06	K
	..	..	3	J, O
Cars, electric:				
Cars complete	15	U	..	..
	17	R	..	..
Bodies and trucks.	15	T	10	J
	15 to 20	E	7.5	B, K
	20	A, S	7	D
	..	..	6.66	T
	..	..	6.66 to 5	E
	..	..	5	A, O, S.

LIFE AND DEPRECIATION—*Continued*

	Life in years		Per cent depreciation	
	Life	Authority	Per cent	Authority
Electric equipment	12	T	10	J
	..	..	8.5	T
	12 to 15	A, S	8.5 to 6.66	A, S
	..	..	8	D
	..	..	7.5	B, K
Fenders, lights, registers, etc....	..	..	10	B, J
Gear cases...........	..	..	20	D
Trucks only (*See Bodies, above*)...	..	..	8	D
Condensers, steam..	10 to 15	N	7 to 10	M
	12.5	Q	..	..
	15	I	..	..
Conduit lines........	..	..	3.5	M
	..	..	3.06	K
	..	..	3	J, O
	..	..	2	B
Converters, rotary.....	10 to 12.5	H	..	....
Conveyors, coal and ash..............	5	E	20	E
	15	A	6.6	A
	20	S	5	B, J, K, S
Cranes, traveling...	..	..	6.66	A
	15	A	5	B, J, K, S
	20	S	..	..
Cross-arms, wooden...	10 to 12.5	Q	..	..
Engines, steam reciprocating..........	15	A	10 to 5	H
	15 to 20	E, N	6.66	A
	20	I, S	6.66 to 5	E
	25	F	5	B, J, K, S
	27	G, P	4	O
Generators, electric.	12 to 25	I	7 to 12	M
	15	A, L, N	6.66	A
	20	E, S	5	B, E, J, K, S
	25	G, H	3	D
Heaters, economizers and pumps....	15	A	7.5	B
	20	S	6.66	A
	..	..	5	J, K, S
Meters, electric.....	8 to 10	H	..	..
	10 to 12.5	G	..	..
	12	F	..	..
	15	N	..	..
	20	I	..	..
Piping, steam.......	15	A	6.66	A
	20	E, S	5	B, E, J, K, S
	21	I	..	..
	21.3	C	..	..

LIFE AND DEPRECIATION — *Concluded*

	Life in years		Per cent depreciation	
	Life	Authority	Per cent	Authority
Poles, wooden	12 to 15	E	8.33 to 6.66	E
	..	..	8	D
Poles, iron	20	A, S	5	A, S
	40	E, F	4	D
	..	..	2.5	E
Pumping engines	21.3	C	..	..
Switchboards, P. H. and S. S.	15	A	7.5	J
	20	I, S	6.66	A
	50	E	5	B, K, S
	..	..	2	E
Telephone system (railway)	..	..	7.5	B
Tools and machinery (shop)	10 to 30	E	10 to 3.33	E
	20	A, S	7.5	B, J, K
	..	..	5	A, S
Track construction:				
Bonding	..	..	10 to 6	H
	..	..	8	D
Ties	..	..	7	D
Track complete	12	T	13 to 7	H
	12.85	A	9 to 8	V
	13.86	S	8.5	T
	..	..	8	K
	..	..	7.75	A
	..	..	7.5	B
	..	..	7.2	S
	..	..	5.5	D
	..	..	5	J
Transformers, stationary	10	I	5 to 6	H
	15 to 20	N	..	..
	25	G	..	..
Trolley line	..	..	5	D
Turbines, steam	20	G	7 to 9	H
Turbines, water	..	..	3.33	W
Wires, overhead	7 to 10	A, S	14 to 10	A, S
	12	T	10	B
	..	..	8.5	T

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[W. A. Del Mar.]

DERIVATIVES. — (*See also Equations, Differential; Integrals; Maxima and Minima; Series, Mathematical*).

Differentials. — Let y be a function of x, i.e., a quantity which varies continuously with variations of x. If x be increased or decreased by the smallest amount conceivable, y will change by a correspondingly small amount. Such small variations are called "differentials," and are symbolized thus: dy and dx, where d is not a quantity but a symbol meaning "the smallest conceivable value of" y or x.

Derivative or Differential Coefficient. — The ratio $\frac{dy}{dx}$ is the rate of change of y with regard to x and is called the "derivative" or "differential coefficient" of y with respect to x.

Geometrical Meaning of a Derivative. — If a curve be plotted between x and y for any equation $y=f(x)$, and a tangent drawn to the curve at any point, Fig. 1, then the tangent of the angle θ between this tangent and the axis of x is equal to $\frac{dy}{dx}$.

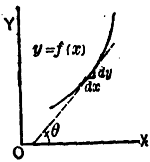

Fig. 1

Differentiation. — Differentiation is the process of obtaining the derivative of a function. If $y=f(x)$

$$\frac{dy}{dx}=\frac{df(x)}{dx}$$
$$=\frac{f(x+dx)-f(x)}{dx}.$$

Example. — If

$$y=ax^2$$
$$\frac{dy}{dx}=\frac{a(x+dx)^2-ax^2}{dx}$$
$$=\frac{a\,dx\,(2x+dx)}{dx}$$
$$=a\,(2x+dx).$$

As dx is the smallest conceivable value of x, it is so small compared to x that it may, with the smallest conceivable error, be neglected. Hence,

$$\frac{dy}{dx}=2ax.$$

Formulas for Differentiation. — (u, v, x and z are variables; a is a constant.)

$$\frac{d}{dx}(u+v)=\frac{du}{dx}+\frac{dv}{dx}$$

$$\frac{d}{dx}(au)=a\frac{du}{dx}$$

$$\frac{d}{dx}(uv)=v\frac{du}{dx}+u\frac{dv}{dx}$$

$$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}.$$

When x is itself a function of some other variable z,

$$\frac{du}{dx} = \frac{du}{dz} \cdot \frac{dz}{dx}.$$

TABLE OF DERIVATIVES

Function $f(x)$	Derivative $\frac{d}{dx}f(x)$	Function $f(x)$	Derivative $\frac{d}{dx}f(x)$
x^n	nx^{n-1}	$\tan^{-1} ax$	$\frac{a}{1+(ax)^2}$
$\sin ax$	$a \cos ax$	$\sinh^{-1} ax$	$\frac{a}{\sqrt{1+(ax)^2}}$
$\cos ax$	$-a \sin ax$	$\cosh^{-1} ax$	$\frac{a}{\sqrt{(ax)^2-1}}$
$\tan ax$	$\frac{a}{\cos^2 ax}$	$\tanh^{-1} ax$	$\frac{a}{1-(ax)^2}$
$\sinh ax$	$a \cosh ax$	$\log_a x$	$\frac{1}{x}\log_a \epsilon$
$\cosh ax$	$a \sinh ax$	$\log_\epsilon x$	$\frac{1}{x}$
$\tanh ax$	$\frac{a}{\cosh^2 ax}$	$\log_{10} x$	$\frac{0.4342944819}{x}$
$\sin^{-1} ax$	$\frac{a}{\sqrt{1-(ax)^2}}$	a^x	$a^x \log_\epsilon a$
$\cos^{-1} ax$	$-\frac{a}{\sqrt{1-(ax)^2}}$	ϵ^x	ϵ^x

Note: See also Table of Integrals in article on *Integrals* noting that the column there headed "Function" is the derivative of the column headed "Integral."

Second Derivative. — The derivative of a derivative, i.e., $\frac{d}{dx}\left(\frac{dy}{dx}\right)$, is usually written $\frac{d^2y}{dx^2}$. Similarly $\frac{d}{dx}\left(\frac{d^2y}{dx^2}\right)$ is called the third derivative and is written $\frac{d^3y}{dx^3}$, and so on.

[W. A. Del Mar.]

DETECTORS, ELECTRIC WAVE. — (*See also Waves, Electromagnetic; Wireless Telegraphy; Wireless Telephony.*) Electric-wave detectors are devices for rendering audible or visible the effects of extremely minute oscillatory electric currents. The detector is usually employed in connection with some auxiliary indicating instrument, such as a sensitive relay, a telephone receiver or a galvanometer.

APPLICATIONS. — The principal application of wave detectors is to wireless telegraphy, either in the actual receipt of messages or in measurements of oscillatory currents of high frequency. However, the thermal detectors, the crystal detectors, the electrolytic detectors and the vacuum detectors are also applicable to the measurement of small alternating currents of commercial frequency. The crystal detectors and also Duddell's thermogalvanometer have been used in measurement of the small alternating currents generated in a magneto telephone by sound waves and have thus served as instruments for the measurement of the distribution of sound in an auditorium.

TYPES OF DETECTORS. — The more important detectors, together with their associated indicating instruments, are briefly described and classified in the following paragraphs. Six different types of detectors are in use; namely, coherers, magnetic detectors, thermal detectors, electrolytic detectors, crystal rectifiers and vacuum detectors.

Sensitiveness of the Various Types. — Coherers are too insensitive to be used except for very short distances. They are so troublesome to adjust that, except where it is required to trip a relay for setting machinery in motion or for operating a call, they cannot compete with the electrolytic, magnetic, crystal or vacuum detectors. The electrolytic detector, which has perhaps not been excelled in sensitiveness, is, however, somewhat more troublesome than some of the others. The crystal detectors are the easiest to construct, maintain and operate. Of these galena, iron pyrite, molybdenite, silicon, and the perikon combination are nearly equal in sensitiveness if made of carefully selected specimens.

Coherers. — Under this title will be included those detectors which employ one or more loose contacts between conducting bodies, and which require to be shaken or moved to restore the contact to its sensitive condition after the receipt of a signal. Under normal conditions the resistance of such a device is high, but upon the receipt of the electric waves, the resistance is lowered. The manner in which this is accomplished is not certain. Some writers have believed that the separating film was removed by the heat effect of the oscillations; others have looked to the electrostatic attraction for a force to draw the particles nearer together and diminish the resistance of the film. It seems to the writer not improbable that the action of the oscillations, apart from the heat action or electrostatic attraction, may produce immediately a motion of electrons into the region of separation between the filings, and that these electrons serve as carriers of the oscillatory currents and of the local battery current.

Branly-Marconi Coherer (Figs. 1 and 2). — The Branly coherer, in the highly improved form devised by Marconi, consists of an insulating tube (Fig. 1) containing metallic filings (preferably 96 per cent nickel and 4 per cent silver) between two plugs *PP* of silver slightly amalgamated. To prevent deterioration by the action of atmospheric gases, the tube, which is usually of glass, is exhausted and sealed up. The plugs should fit accurately within the glass tube and should be within $\frac{1}{50}$ inch of each other. The filings in this space, for high

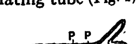

Fig. 1. Coherer

silver, and should be of uniform size, between 80 and 100 mesh. They should be dry and free from grease and dirt. In sealing up the containing tube after exhaustion, care should be taken not to heat the filings. In order that the coherer may not be injured, not more than $\frac{1}{1000}$ ampere should ever be sent through it.

Fig. 2 shows a coherer *Co* of this form in series with an antenna and ground. Under the action of the electric waves, the resistance of the coherer falls to a small value, and a current flows from the battery B_1 through the coherer and through the field coils of a relay *R*. The core of the relay is thus magnetized and attracts its armature so as to close the gap *A*. The closing of this gap, which is in a local circuit containing a battery *B*, a trembler *T* and a sounder *S*, causes the armature of the sounder to strike downward and at the same time starts the trembler *T* into activity. So long as the electric oscillations in the antenna circuit are kept up by the incident electric waves the coherer is repeatedly cohered and decohered and the sounder armature, which possesses considerable inertia, is held down. As soon, however, as the arrival of the waves ceases, the coherer, under the action of the trembler, is restored to its high resistance and the sounder armature flies up. The length of the time that the armature is down is determined by the length of the signal and enables the receiving operator to distinguish dots from dashes. The sounder may be replaced by a Morse tape recorder, so that the signals may be recorded. The zigzag dotted lines *P*, P_1, *q*, *q′* and *h* of Fig. 2 indicate resistances shunted about each electromagnet and each contact point of the local circuits, for the purpose of preventing rise of potential due to the back kick of the several inductances of the local circuits. Such a rise of potential, if not prevented, would operate to cause a coherence of the detector at the time when a decoherence is required.

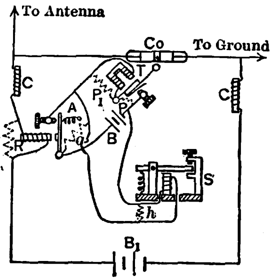

Fig. 2. Coherer Connections

The relay used with the coherer should be a highly sensitive polarized relay of several thousand ohms resistance. The magnet of the trembler and that of the sounder should be wound to equal resistance with each other, and the protective resistances should be of about the same resistance as the coils about which they are placed.

Lodge-Muirhead-Robinson Coherer (Fig. 3). — Another form of coherer, due to Lodge, Muirhead and Robinson and shown in Fig. 3, consists of a steel wheel *A*, rotated by clockwork, in contact with a film of oil on the surface of mercury. A binding post connected to a brush *E*, making contact with the axle of the wheel, is one terminal of the instrument, and a second binding post *H* connected with the mercury is the other terminal. If connected by these terminals to an antenna and ground, the oily contact between the steel disk and the mercury changes resistance under the action of electric oscillations, and this changing resistance is evidenced by a siphon recorder shunted through a battery to the terminals of the detector.

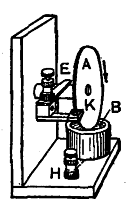

Fig. 3. Lodge-Muirhead Coherer

The Magnetic Detector (Fig. 4). — Of this form of detector the most prominent representative is the continuous magnetic band detector of Marconi.

This instrument, shown in Fig. 4, contains a belt of fine iron wires, insulated from one another, and carried over two wheels *DD*, one of which is slowly driven by a motor. This belt of iron wires passes through a transformer consisting of a primary coil *bb* surrounded by a secondary *C*. Placed near the transformer is a pair of permanent magnets *SNNS* arranged to induce consequent poles in the moving belt of iron. The primary coil *bb* of the transformer is connected in the oscillating circuit *AE*, while the secondary coil *C* of the transformer is connected to a sensitive telephone receiver *R*. Each train of electric oscillations through *bb* modifies the magnetic state of the magnetized band, by suppressing hysteresis, and thereby induces a current in the telephone. A series of trains of waves arriving in uniform sequence gives a uniform sequence of pulls to the diaphragm and results in a musical note. The note continues as long as the transmitting key is pressed and constitutes a long or short note according as a dash or dot is made at the transmitting station. The pitch of the note is the pitch of the spark at the sending station.

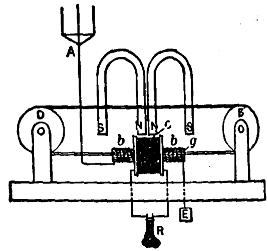

Fig. 4. Magnetic Detector

Thermal Detectors. — Two types of thermal detectors — used chiefly for measuring purposes — have been employed as detectors of electric oscillations. One of these types, of which the "bolometer" of Paalzow and Rubens and the "barretter" of Fessenden are examples, employs the heat-effect of the oscillations to change the resistance of a fine wire; the other type, exemplified in Klemencic's "thermal junction" and Duddell's "thermogalvanometer," employs the heat effect concentrated at a thermal junction to produce thermoelectromotive force at the junction.

Barretter (Fig. 5). — The barretter consists of a very fine platinum wire enclosed in a glass tube. This wire is connected in series with the antenna and is shunted by a local circuit containing a battery and telephone. The change in the resistance of the wire due to the heating effect of the oscillations passing through it to ground causes a variation of the current in the local circuit and thereby produces a click in the telephone. The platinum wire is obtained by dissolving off the silver from a short length of Wollaston wire.

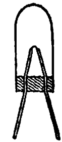
Fig. 5. Barretter

Thermogalvanometer (Fig. 6). — The essential parts of Duddell's thermogalvanometer are shown in Fig. 6. A thermal junction *SbBi* forms a part of a loop *L* of wire suspended between the poles of a magnet *NS*. A stationary heater of fine platinum wire is placed near the thermal junction, and this heater is traversed by the oscillatory currents to be measured. Heat developed in the heater is radiated or convected to the thermal junction and produces a thermoelectromotive force at the junction. This causes a small unidirectional current to flow in the loop, which is, in consequence, deflected by the magnetic field. The deflections are read by the mirror *M* with the aid of a telescope and scale.

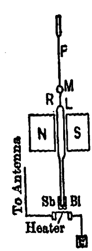

Fig. 6. Thermogalvanometer

Electrolytic Detector (Figs. 7–9). — This detector in its best form, as described by Fessenden and shortly afterwards by Schloemilch, consists of a fine platinum wire dipping into electrolyte contained in a small vessel, so that the fine wire just touches

the electrolyte. The fine wire is attached to one of the terminals. As a second terminal a larger wire entering the electrolyte or a metallic cup containing the electrolyte serves. The fine wire is adjustable as to depth in the liquid or else the fine wire is sealed into a glass tube so that only a very minute portion of the end of the wire protrudes, and this glass tube containing the sealed-in

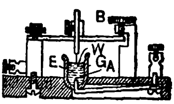

Fig. 7. Electrolytic Detector

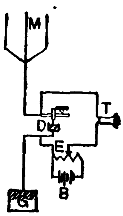

Fig. 8. Circuit with Electrolytic Detector

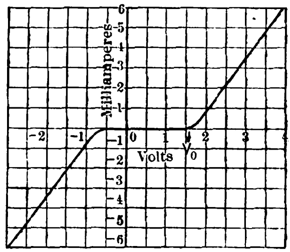

Fig. 9. Current-voltage Curve of Electrolytic Detector

wire is dipped into the liquid. As electrolyte 20-per-cent nitric acid is about the most sensitive liquid, but salt solution, sulphuric acid or any good conducting electrolyte may be used. Fig. 8 shows an electrolytic detector *D* in a position to be traversed by electric oscillations in the circuit *MG*. As indicating instrument a telephone receiver in series with a source of e.m.f. is shunted with the detector.

Theory of Electrolytic Detector. — The electrolytic detector, when used with a polarizing voltage of about 1.6 volts (i.e., when a direct-current voltage of this value is impressed across the terminals of the detector), is a rectifier for alternating electric currents, and this fact accounts for its action as a detector for electric oscillations. Fig. 9 shows a characteristic curve obtained by plotting the direct current through an electrolytic detector against the direct-current voltage applied to produce the current. The curve of the upper quadrant is obtained with the applied voltage in one direction, while the curve of the lower quadrant is obtained with the voltage in the opposite direction. Suppose this polarizing voltage to be V_0 (about 1.6 volts). It is seen from the curve that a steady current of about 0.15 milliampere will flow through the detector. Now let us suppose that an alternating e.m.f. of amplitude 0.5 volt be superposed upon the polarizing voltage. When the alternating voltage is in the direction of the polarizing voltage, we have a total voltage of 2.1 volts and a current of 1.2 milliamperes (from the curve). When the alternating e.m.f. is reversed, the total impressed voltage is 1.1, with a resultant current practically equal to zero (from the curve). Hence the polarized electrolytic cell acts as a rectifier of the oscillations passing through it to ground. The manner in which such a rectifier causes the oscillations to affect the telephone is the same as the action of the crystal rectifiers described below.

Design of Electrolytic Detector. — The fine platinum wire should be 0.0001 or 0.0002 inch. It may be bought as Wollaston wire — platinum core with an exterior of silver — from dealers in platinum. The silver may be removed, after attaching a length of ½ to ¼ inch of the Wollaston wire to a larger wire for support, by dipping the wire a short way into the electrolyte *E* of 20 per cent nitric acid (Fig. 7) and connecting the detector in series with the

receiving telephone and the adjustable source of voltage (potentiometer) that is to be employed as the polarizing voltage in the actual use of the detector. In this connection for clearing off the silver, the fine wire should be anode, and the action may be hastened by raising the polarizing voltage until bubbles begin to escape causing a hissing sound in the telephone. After the silver is removed, the polarizing voltage should be reduced to the value at which the hissing noise just begins; this is the most sensitive adjustment for receiving messages. As a suitable polarizing voltage with the electrolytic detector (platinum point anode), a potentiometer resistance of about 500 ohms should be connected in series with three dry cells, and the polarizing voltage taken as a drop off of this potentiometer wire. The receiving telephones (of the head type) should have a resistance of 2000 or more ohms for both ear-pieces. The resistance of 500 ohms will be sufficient to protect the cells from too rapid use and will not be enough to introduce derogatory resistance in the telephone circuit, as the detector has a high resistance and is also in series with the telephone.

Crystal Rectifiers (Figs. 10–12). — The detectors of this class make use of the high-resistance contact between a crystalline conductor and a metallic electrode or between two crystalline bodies. The parts are all solid. A great

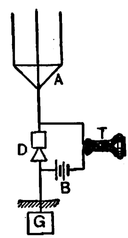

Fig. 10. Crystal Detector Connections

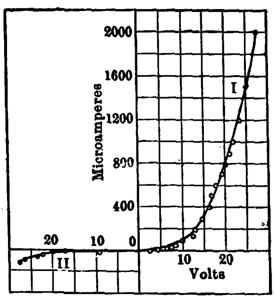

Fig. 11. Current-voltage Curve for Carborundum Detector

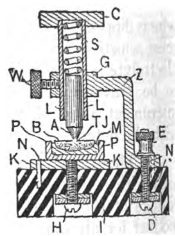

Fig. 12. Mounting of Silicon Detector

variety of substances have been used in detectors of this type; for example, carbon, carborundum, tellurium, silicon, anastase, brookite, octahedrite, molybdenite, chalcopyrite, zincite, iron pyrite, bornite, galena and many others. The contact of any of these substances against a metallic conductor serves as a detector for electric oscillations. Also in some cases the substances enumerated may be used in pairs, one against another, with improvement in the sensitiveness or constancy of action.

Fig. 10 shows diagrammatically the connection of the crystal contact detector into a simple oscillating circuit AG. A telephone T or galvanometer is used as indicating instrument. Some of the detectors are more sensitive when used with a local e.m.f. B in the telephone circuit, while some others are more sensitive without the local battery.

Theory of Crystal Rectifiers. — The detectors making use of a contact with a crystalline substance are all unilaterally conductive; that is to say, they permit the passage of a greater current in one direction than in the opposite direction under the same applied voltage. In addition, these detectors all have a rising current-voltage characteristic. Fig. 11 shows a current-voltage curve for one of these detectors. This curve is taken from experiments on the con-

tact of a brass rod against a carborundum crystal. Although the carborundum detector is not as sensitive as some of the other crystal detectors, this curve, except for magnitudes, is typical of all the crystal detectors.

The fact that the current in Fig. 11 is greater in one direction than in the other under the action of the same impressed voltage makes the detector a rectifier of alternating currents, without an auxiliary battery. Also the fact that the detector has a rising current-voltage characteristic makes it a rectifier when used with an auxiliary battery, as may be shown by a discussion similar to that employed in explaining the action of the electrolytic detector. Whether the detector is a better rectifier with or without the auxiliary battery depends upon the shape of the current-voltage curve.

The action of a rectifier in causing the motion of the telephone diaphragm when oscillations pass through the rectifier is as follows: Let us take the case of the simple form of receiving circuit shown in Fig. 10, with or without a battery in the telephone circuit.

A train of incoming waves produces an alternating e.m.f. in the antenna circuit. This e.m.f., when in one direction, produces a large current through the detector *D* charging the antenna. When the e.m.f. reverses, the current from the antenna to the ground through the detector is smaller, thus leaving the antenna charged with a small quantity of electricity. The effect of the whole train of waves is additive, so that this charge on the antenna is cumulative. The accumulated charge on the antenna escapes through the telephone shunted about the detector, causing the diaphragm to move. Each subsequent train of waves causes a similar motion of the diaphragm, which is evidenced as a note in the telephone with the train frequency of the waves.

It is immaterial whether the detector permits the larger current to flow upward, charging the antenna positive, or permits the larger current in the downward direction, charging the antenna negative. The explanation is the same in both cases.

With very slight change this explanation can be made to apply also to those cases in which the detector is in a condenser circuit coupled inductively or directly with the antenna circuit.*

Design of Crystal Rectifiers. — A typical form of construction of the crystal detectors, which, however, may be greatly varied, is shown in the mounting of Mr. Pickard's silicon detector (Fig. 12).

A rod of brass *A* is pressed down by a spring *S* into contact with a mass of polished silicon *B*, embedded in an easily fusible solder of Wood's metal *M*. The solder in which the silicon is embedded is contained in a metallic cup *P*, which rests upon a metallic plate *K*. Connection to the rod *A* is made by means of the binding post *E*. Connection to the silicon is made by means of a binding post not shown, which connects with the plate *K*. The ability to move the cup containing the embedded silicon is an advantage, because not all parts of the surface of the silicon or other crystalline material of the detector are equally sensitive, and this motion permits the selection of a sensitive place on the material, as the point of contact. Mr. Pickard sometimes uses two of these active materials in the same detector. For example, a contact of zincite with bornite (each held in a bed of Wood's metal) is a highly sensitive electric wave detector.

In the zincite-bornite detector, sold under the trade name of perikon, the crystalline bodies are not polished, but are used as rough fragments.

Another form of highly sensitive detector makes use of galena as a sensitive material. By selection from the galena of various localities, specimens may be

* Quoted from Pierce's "Principles of Wireless Telegraphy," McGraw-Hill, N.Y., 1910.

found that are highly sensitive when used with a very fine copper wire (No. 36 or No. 40 B. & S.) as contacting element. In this case no spring is required, as the stiffness of the fine wire is about sufficient to give the required pressure. Arrangement should, however, be made for displacing the galena so as to make the contact at a sensitive point.

For ruggedness as a detector of electric waves, though rather insensitive, a crystal of carborundum with a metallic rod in contact under a considerable pressure is recommended. Also selected specimens of molybdenite with a metallic contact may be rugged and highly sensitive. Molybdenite is however a soft material and is likely to be injured by unskillful adjustment.

Vacuum Detectors (Figs. 13 and 14). — The wave valve of Fleming and the audion of DeForest make use of the rectifying property of a vacuous space containing electrons produced by an incandescent filament. In Fleming's valve, Fig. 13, a bulb pumped to a high degree of exhaustion contains a filament *b* of carbon, tungsten or tantalum, like that of a low-voltage incandescent lamp.

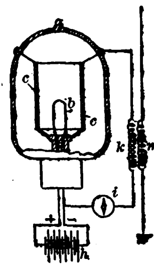

Fig. 13. Fleming's Valve

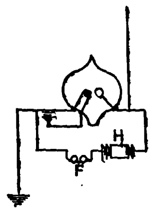

Fig. 14. DeForest's Audion

This filament is heated to incandescence by the current from a local battery *h*. The hot filament serves as one terminal of the detector. The other terminal is in the form of a metallic cylinder *c* surrounding the filament. These two terminals in the circuit shown in the figure are connected with a coil *k* and a galvanometer *i*. The electric oscillations in the antenna circuit, flowing through the coil *m* in the antenna circuit, act inductively on the coil *k* in the detector circuit, giving an alternating e.m.f. at the terminals of the detector. The vacuum valve permits the flow of current in only one direction, so that the alternating e.m.f. produces a direct current through the detector and galvanometer.

DeForest's arrangement, shown in Fig. 12, is a modification permitting the use of a telephone receiver *F* as indicating instrument. For greater sensitiveness a battery *H* is used in the local circuit with the telephone receiver.

BIBLIOGRAPHY. — See references in article on *Wireless Telegraphy*, particularly *Principles of Wireless Telegraphy*, by G. W. Pierce, N. Y., 1910.

[G. W. Pierce.]

DISPATCHING OF TRAINS BY TELEPHONE. — (*See also articles on Telephony.*) Up to 1907 train dispatching was done almost entirely by the telegraph. In that year the telephone began to replace the telegraph for this purpose. At the beginning of 1912, about 60 railroads in the United States, operating over 30,000 miles of line, had abandoned telegraphic train dispatching in favor of the telephonic method.

Telephone train dispatching has many advantages over the former method. The dispatcher's work consists in the main of gathering information and issuing orders for train movements. These operations are much quicker by the use of the telephone and are no less accurate. Special training in the use of the apparatus is not as necessary as when telegraphy is employed. All the agencies are brought into closer personal relations, with the result that there is better co-operation and better discipline. The dispatcher can call and speak at the same time, which is impossible by telegraph.

It now is possible to operate dispatching lines as long and with as many stations as railway conditions may permit. The length of line and number of stations are not limited by the nature of the electrical equipment. Lines of 300 miles length are in operation, and 65 stations on one line are in successful use. More than 65 stations could be operated successfully if desired.

Dispatching circuits are multi-station lines, like the telegraph-dispatching lines they displace. Broadly speaking, they are selective party lines, but are equipped with systems of apparatus which allow the placing of many more stations on a line than is possible with any other party-line system so far developed.

Several train-dispatching systems are on the market, the principal ones being known as the Gill System, the Western Electric System, the Cummings-Wray System, and the Kellogg System. These are in general similar in that the dispatcher has means of calling any station selectively. In some systems he can call several stations at once by setting keys for them in advance and operating a common calling key.

Each way station is equipped with a device called a selector. These selectors can be operated only by the dispatcher. They work on the step-by-step principle and the whole object of the device is to close a contact at the station called and thus to ring a bell. No provision is made for cutting the way-station telephone set on and off the line, as secrecy is not aimed at. Any way station may communicate with the dispatcher at will by lifting the telephone receiver. The dispatcher wears a head telephone constantly and so listens upon the line at all times.

Train orders are written as given and received and are read back to the dispatcher by each station, the dispatcher underscoring each word once for each correct repetition from a way station.

BIBLIOGRAPHY. — See Bibliography in article on *Telephone Instruments and Circuits.*

[S. G. McMEEN.]

DISTRIBUTION LINES. — (*See also Conduits and Conduit Lines; Distribution and Transmission Systems; Transmission Lines; Wires and Cables; Wiring of Buildings.*) Two types of construction are employed, overhead and underground. Overhead construction has the following advantages (1) lower first cost, (2) easier to repair, (3) easier to change, (4) adaptability to higher voltages; while underground construction has the advantages (5) less unsightly, (6) less dangerous to the public, (7) less subject to damage by external agencies. The majority of circuits in use are overhead. Underground circuits are principally used in the central portions of the larger cities. When underground circuits are used a large part of the construction consists of a composite of underground and overhead construction.

MATERIALS FOR LINE CONSTRUCTION. — The principal elements for wood-pole construction are the conducting wires, insulation on the wires, insulators, tie wires, pins, cross arms, cross-arm braces, bolts, poles, guy wires, guy anchors. For underground construction, see *Wires and Cables, Insulated,* and *Conduits and Conduit Lines.*

Wires for Overhead Construction.— The conducting wires are usually of copper, sometimes of aluminum. Iron is not used as it costs more than copper for equal conductivity. Aluminum is used to a much less extent than copper; see articles on *Aluminum* and on *Copper*. Soft-drawn copper is usually employed for ordinary distribution work, as it has the greatest conductivity and is the easiest to work with. Medium-hard-drawn and hard-drawn wires are used in lines where great mechanical strength is necessary.

For overhead city distribution copper wires ranging in size from No. 6 B. & S. (or A. W. G.) to 500,000 circular mils are used; see *Wires and Cables*. Sizes larger than No. 0000 B. & S. are usually stranded. Conductors as small as No. 14 B. & S. have been used for overhead work but are too small as they are frequently broken by wind and sleet.

Standard Sizes of Wire. — Of the gage numbers between No. 6 B. & S. and 500,000 circular mils (*see Gages, Wire*), some are but little used because (1) the difference between consecutive sizes is less than is found necessary in usual practice and (2) the difference is too small to be readily detected by the ordinary lineman or stock keeper. In practice the number of sizes to be used and carried in stock has usually been reduced by omitting certain sizes; No. 5, 3 and 1 are nearly always omitted; practice regarding the omission of larger sizes varies. The most commonly used are No. 6, 4, 2, 0, 0000 B. & S. and 500,000 circular mils.

Wires for Underground Lines. — See *Wires and Cables, Insulated.*

Insulation on Overhead Wires. — Overhead conductors for city distribution are insulated with weatherproof braid. Two thicknesses are recognized, "double braid" and "triple braid," according to the number of coverings, though the actual thicknesses are not the same for different makes. Double braid is considered suitable insulation for use on voltages of 600 or less, and triple braid for voltages up to 2500 constant potential and on series-arc circuits of all voltages. To prevent the possibility of double-braid wire being used for voltages over 600 and to reduce the number of kinds of wire used, it is good practice to have all No. 0000 wires and smaller covered with triple braid. Ordinarily cables larger than No. 0000 are not used on voltages above 600 and may therefore be double braided.

Object of Insulation on Overhead Wires. — The insulation on overhead conductors is solely for the purpose of limiting the short-circuit current due to an accidental cross or grounding. The normal insulation of the line is

maintained by the insulators alone (*see below*); any reinforcement obtained from the insulation on the conductors is neglected in practice. While weatherproof braid is an imperfect insulator, it serves to eliminate the greater proportion of the short-circuits and arcs which would occur, due to momentary contact, were bare wires used. Rubber and other more perfect insulators are not used because of expense and the impossibility of maintaining perfect insulation, due to weakness, against mechanical injury and deterioration.

Bare wire is usually used on circuits operating at 10,000 volts and over to avoid giving a false sense of security. For voltages between 2500 and 10,000 insulated wire is often used, though the protection afforded is doubtful.

Extra Cost and Weight due to Weatherproof Braids.— In estimating the cost of a distribution line, the additional weight and cost of the weatherproof braids should be taken into account. The following is a rough comparison of bare, double-braided and triple-braided copper wire, the cost being based on copper at 15 cents per pound.

COMPARISON OF BARE AND WEATHERPROOF COPPER WIRES

Conductor	Relative weights			Relative Costs		
	Bare	Double braid	Triple Braid	Bare	Double Braid	Triple Braid
No. 6 B. & S., solid	100	123	137	100	127	141
500,000 cir. mil cable	100	112	117	100	115	121

The cost and weight of weatherproof insulation for aluminum wires are greater than for copper of the same conductance on account of the larger cross-section of conductor required.

Insulators for Overhead Lines. — City distribution circuits are ordinarily carried on double-petticoat deep-groove glass (D.P.D.G.) insulators; see article on *Insulators for Overhead Lines*.

Tie Wire. — The conductor is attached to the insulator by a tie wire of the same material as the conductor, though soft wire is usually employed even for hard-drawn conductors; the tie wire is either bare or insulated to correspond to the conductor. The size of tie wire is often the same as that of the conductor; for small wires it is merely a piece of conductor. With large conductors it may be as much as three sizes smaller. See also the section on *Installation* in the article on *Wires and Cables, Bare*.

Pins. — See article on *Insulator Pins*.

Cross Arms. — See article on *Cross Arms*.

Poles. — See article on *Poles for Overhead Lines*.

DESIGN OF CITY DISTRIBUTION LINES. — City distributing systems should be designed so that service may be given to any building in the city and ultimately to every building present and future. On certain streets pole lines may be omitted without defeating this object. The arrangement of lines which will serve scattered customers with the least number of poles will usually contain many poles which should not be used if the ultimate arrange-

ment were immediately constructed. Preliminary studies and designs should be made, first, of arrangements suitable for servicing the initial expected customers, second, of arrangements for ultimately servicing customers on every lot in the city, third, of a plan of extension by which the initial arrangement can be extended to the ultimate with the minimum expense in changing lines and services.

The Pole Line. — The pole lines perform two functions: (1) of carrying feeders from the station to the mains and (2) of carrying the mains supplying services to buildings immediately adjacent. In the old cities of the eastern part of the United States where the streets are crooked general rules for systematic line work cannot be followed far. In the newer cities of the West and South the streets are laid out at regular intervals and at right angles, dividing the city into rectangular blocks of equal size. In such cases the following rules should be followed:

(1) A pole line should continue on the same side of the street throughout its entire length and disconnected lines built in the same street should be on the same side, so that they may be connected when desired without crossing the street.

(2) The spacing between poles should be an exact divisor of the length of a block (including cross street), giving a uniform number of poles per block.

(3) Whenever the line crosses a street where there is, or may be, an intersecting line there should be a corner pole on the proper side of the intersecting street for making a junction.

These rules logically lead to the use of corresponding sides of all parallel streets.

When each block contains lots fronting on all four surrounding streets the servicing of every lot in the city would require poles on all streets, though on the streets in at least one direction the lines may be discontinuous. Often the lots all front on the streets in one direction, which are laid out to be principal streets, in which case no service lines are necessary in cross streets, though at intervals connecting lines are necessary.

Trunk Lines. — Where the location of power house or substation is fixed, trunk lines must be laid out from such point, but often a study of possible trunk-line arrangements made before the location of the power house is fixed will show that other locations are more advantageous. If distributing station (power house or substation) is centrally located, there should be at least four main trunk lines (of poles) from it, say north, east, south and west. A short distance from the station they should be divided into branches, then subdivided into smaller branches and finally merge into the service lines. The trunk lines should be laid out: (1) on back streets where the large poles, numerous and heavy wires and heavy guying will not be conspicuous, (2) on side streets or streets little built up so that interruption to service due to fire in adjacent buildings will be infrequent, (3) on streets where there are few trees, (4) on streets where there are no jogs or offsets to weaken the line and require heavy guys, (5) on streets which lead directly to the section supplied, penetrate its center and intersect the maximum number of service lines. Even when the station location is excellently chosen these desirable conditions will have to be compromised to a serious degree.

Street and Alley Service Lines. —With the symmetrical arrangement of lots described above there is often an alley through each block parallel with the principal streets. Under these conditions there is therefore a choice of two methods of laying out the lines: (1) run the lines on the principal streets servicing the houses from the front and (2) run the lines in the alleys servicing

the houses from the rear. The disadvantages of the first are unsightliness of the poles and wires, difficulty of avoiding or of trimming shade trees; the disadvantages of the second are discontinuity of alleys, proximity of buildings (inflammable barns and out buildings in residence districts and of windows and fire escapes in business districts) and lack of established grade.

Composite Distribution. — The most expensive and difficult part of an underground system is that for taking the current from the conduit to the customer's building. This includes underground secondary mains, transformers, handholes, service pipes and wires. Where it is necessary to remove poles from streets but not to remove overhead wires from private property the pole lines are replaced by conduits which contain primary feeders and mains only. From the conduit in the street a small branch runs into the interior of each block where the ducts (usually iron pipes) come to the surface at the foot of a terminal pole up which the cables run. At the top the cables connect to overhead wires. The interior of each block contains a complete overhead distribution system of poles, transformers, secondary mains and services. Sometimes the servicing is done from a single centrally located pole and at others there may be a pole line of several spans length running longitudinally through the block. The most serious disadvantage of this method is that the poles and wires frequently have to be on private property, where no permanent rights may be obtained. Being dependent on concessions from one or more customers, the company does not have the independent position that a public service company should in order to serve all equally and fairly. The poles and circuits are therefore often put up of insufficient size without proper guying.

The composite system has the advantage of the overhead system in less unsightliness, less accessibility of public to high-voltage wires and less trouble from trees. In thickly built-up blocks its use results in a tangle of wires over roofs and along walls which may be as dangerous in case of fires or high winds as an overhead system. The composite system is used (1) in small cities where the load density is small and (2) in large cities in an annular district between overhead and underground construction; in either case it is usually an intermediate step from overhead to underground construction.

Crossing of Waterways; Submarine Cables. — Where a distributing system is divided by a navigable waterway the connection is usually made by submarine cables laid on the surface of the bottom, or better, below the surface in trenches dredged for the purpose. Cables may be single or multiple conductors, the latter being usually used for alternating currents to avoid reactance due to wire armor of cable. Two conductor cables may conveniently be of concentric type to give a true circular exterior. In laying cables it is desirable to keep them approximately parallel. Where one crosses under another it may be impossible to remove it. Each cable should be in a single length without joints, and the length should be made as short as possible, as repairs are very difficult and often impossible. At ends unimportant cables may be brought up a pole and connected to overhead wires; important cables should land in a cable house with suitable provision for disconnecting the cable or for transferring the overhead circuit to a spare cable in case of trouble, and also with lightning arresters. Submarine cables are weak links in a distribution system and may sometimes be avoided by crossing channels at sufficient height to clear the masts of ships.

Calculation of Size of Wires. — The size to be used depends upon the voltage drop which should be permitted, considering the probable growth of the load. The following table of per cent voltage drop in the various lines is representative of ordinary practice.

	For light
	Per cent
House wiring	2
Service wires	2
Secondary mains	5
Transformers	2
Primary mains	5
Primary feeders	10

The drop in the feeders is usually compensated for by raising the voltage at the substation or power station or by using voltage regulators; see *Distribution and Transmission Systems.*

Formulas for calculating the size of wire for a given length of line, given load and given distribution of load are given in the articles on *Transmission Lines* and *Wiring of Buildings.* Due to the uncertainty regarding the probable increase of load, a close calculation of the size of wire is seldom made, the engineer relying largely on his experience and judgment, making only a rough calculation as a check. For overhead lines 500,000 circular mils is usually the largest size used on account of the difficulty of supporting a larger wire; greater conductance is obtained by installing parallel circuits. For underground lines No. 0000 B. & S. three-conductor cable is the largest size that can be conveniently drawn into the ducts; 1,000,000 c.m. is the usual maximum size of single-conductor cable used.

Effect of Diversity Factor. — It should be noted that in a distribution circuit the maximum load on a feeder is less than the sum of the maximum loads on the mains which it feeds, these in turn are less than the sum of the maximum loads on the transformers connected to these mains, and so on. Therefore, whenever a circuit divides or subdivides, the aggregate sectional areas should ordinarily be greater after division than before. The total drop in voltage from power house or feeding point to a customer's lamp or motor is also usually less than the sum of the maximum drops in the parts of the circuit which are in series (such as house wiring, services, secondary mians, etc.) because these component drops do not have their maximum value simultaneously.

Stresses on Poles. — An overhead line is a framed structure. The poles are struts resisting the weight of wires, including sleet on the conductors, etc., insulators, cross arms and transformers and the downward pull of the guy wires. The horizontal pull of the line wires is balanced by the horizontal component of the pull of the guy wires, which transmit it to the ground. In addition to being struts the poles resist certain bending stresses, but these should be but a small part of the normal horizontal tension of the wires. The principal normal bending stresses are of two classes: (1) constant stresses, due to the unbalanced pull of service and other wires which do not exert sufficient force to require guys, and (2) variable stresses, due to the force of the wind at right angles to the line on both the poles and the wires. Poles also resist a twisting force where the tension on one side of an arm is greater than on the other, due to a difference in the number or weight of wires ending on the two sides. The poles, wires and guys should be so disposed that the bending and twisting forces on the poles are insignificant. The calculation of the strains produced in a pole is given in the article on Poles

Arrangement of Wires on Poles.— The arrangement of wires is governed by mechanical, electrical and practical considerations. For mechanical reasons it is desirable that:

1. The largest wires be on the lowest cross arm, in order to reduce the bending stress on the pole to a minimum.
2. The largest wires be on the pins nearest the pole, in order to reduce the bending stress on the cross arm to a minimum.
3. The wires be arranged symmetrically on the two sides of the pole, especially those which end at the pole, in order to reduce the twisting stress on the pole to a minimum

For electrical reasons (which, however, are of minor importance) it is desirable that:

4. The wires of any one circuit be as close together as practicable (on adjacent pins), in order to reduce the self-inductance of the circuit.
5. The wires of a three-phase circuit be arranged to form the edges of an equilateral prism and the wires of a two-phase circuit be arranged to form the edges of a square prism, in order to render the inductances and capacities of the wires respectively equal.
6. The wires of different circuits be placed as far apart as practicable, in order to reduce their mutual inductance.

For practical reasons it is desirable that:

7. The highest voltage wires be on the top cross arms and on the pins farthest from the pole, in order to reduce the danger of accident to linemen.
8. The mains, which have the greatest number of taps, be on the lowest cross arms, in order to avoid danger of accidental crossing (with contact) of the wires.
9. The arrangement be systematic throughout; this is absolutely essential for safe and economical operation.

As it is impossible to meet all of these conditions the actual arrangements used are compromises and are governed by the relative importance attached to the several desirable conditions. In some cases the electrical requirements (4) to (6) have been considered of most importance, resulting, for example, in an arrangement subordinated to the idea that the three wires of a three-phase circuit must be arranged exactly in an equilateral triangle. It appears, however, that these electrical requirements are really the least important of the considerations and that in most practical cases can be entirely neglected.

Transpositions. — Due to the effect of mutual electromagnetic induction (*see Inductance*) an alternating or varying current flowing in one circuit will induce a voltage, and therefore a current, in any parallel circuit; and due to the effect of mutual electrostatic induction (*see Capacity*) an alternating or varying voltage in one circuit will induce currents in a parallel circuit, even though there is no metallic connection between the two circuits. It is possible, however, by properly transposing equal alternate lengths of the wires forming the two sides of each of the parallel circuits, to neutralize these effects. Fig. 1 shows diagrammatically a scheme of transposition whereby a two-wire circuit can be protected from both electromagnetic and electrostatic induction from a parallel circuit and vice versa, provided neither circuit is grounded. The more frequent the transpositions the more thoroughly are the effects due to inequality in the spacing of wires and poles eliminated. Transpositions cannot be made effective in eliminating inductive effects when either circuit is grounded, otherwise than at the neutral point. See also *Transmission Lines*.

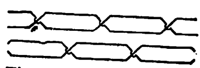

Fig. 1. Transposition of Two Two-wire Circuits

Methods of calculating the induced voltage and induced currents in one line due to currents and voltage in a neighboring line are indicated in the articles on *Inductance* and *Capacity*.

Disturbances in Telephone Circuits due to neighboring power circuits are the most common results of electromagnetic and electrostatic induction. When the power circuit is grounded and carries either (1) fluctuating direct currents (e.g., a d-c. railway) or (2) alternating currents it may produce a noise in the telephone receiver, even though both the power circuit and telephone line are transposed. See also *Telephone Lines*.

Trees. — Trees constitute the most serious obstacle to the proper planning, construction and operation of overhead lines. The principal methods of meeting this difficulty are (1) avoiding them, (2) going over them, (3) going under them, (4) going through them. In most cases a combination of these methods is used. Where trees are a serious factor it is necessary to examine the route of every line in detail and the size and location of the trees may become the determining feature of the whole design. In such cases nearly all rules of good construction and systematic arrangements are violated in the interests of expediency.

The methods by which trees may be avoided are: using alleys instead of streets, or vice versa, choosing streets without trees for important lines, taking side of street with fewest trees, and finally the very bad arrangement of crossing the street back and forth to avoid the trees either individually or in groups.

The plan of going over the trees is the proper one in the case of all small trees and is perfectly satisfactory until the trees grow up and touch the wires. It is therefore only a temporary method, especially where the trees are of tall, quick growing varieties. In going over small trees it is well to have poles tall enough to allow for wires clearing after several years growth. It is usually impracticable to go over large full-grown trees because of cost of poles, unsightliness of very tall poles, and the difficulty of properly guying them to resist wind and the unbalanced pull of wires, which is magnified by the great leverage. It is also difficult or impracticable to take off service wires over the tops of tall trees.

In the case of very large trees it is sometimes practicable to take the wires under the trees on short poles. Where the wires pass the trunks it may be necessary to spread them or necessary to pass between large branches, with insulators fastened to the trees to maintain clearance. While such a line may be kept fairly clear under normal conditions, there is apt to be trouble during storms from branches falling on the wires or from limbs bent by wind or snow touching them.

With trees of moderate size it is usually necessary to take the wires through the trees among the leaves and small branches. Such wires are a constant source of trouble and expense. The branches and leaves should be trimmed from around the wires as much as possible, including not only those in contact with the wires but such as will be brought into contact by wind or which will grow into contact during the season. In addition it is usual to protect the wires in the worse places by tree insulation consisting of split tubes of wood or bamboo.

Pole Transformers. — The primary (1100- or 2200-volt) mains are usually run to transformers mounted on poles and the voltage there stepped down to the lamp or motor voltage (110 or 220 volts), and secondary mains run from the transformer to the buildings in the immediate vicinity. These transformers usually range in size from ½ to 50 kw., but transformers of ¼ kw. were formerly common and transformers larger than 50 kw. are sometimes used in factory

together) does not have to be considered. Very small transformers are objectionable because of their poor regulation, lower reliability and the cost of frequently changing them as the load increases.

Use of Single and Polyphase Transformers. — Transformers are usually of the single-phase type, two being used for motors on two-phase circuits and two for small motors and three for large motors on three-phase circuits. Three-phase transformers are also used for motors on three-phase circuits, and have the advantage of reducing the amount of wiring on the poles. Single-phase transformers have the advantage of being interchangeable between the single-phase lighting and polyphase power circuits where the same voltage is used.

Voltage Ratios of Pole Transformers. — Lighting transformers were early standardized with voltage ratios based on multiples or submultiples of 10 : 1, that is, 1000 and 2000 volts primary to 50, 100 or 200 volts secondary, or 1040 and 2080 primary to 52, 104 and 208 volts secondary. It was found that on account of the voltage drop in the transformers and secondary mains, motors wound for this voltage did not operate well. Instead of remedying this difficulty by winding the motors for a lower voltage or by raising the voltage on the generators and lamps, power transformers were introduced with windings based on a ratio of 9 : 1 thus giving with 1040/2080 volts primary a secondary voltage of approximately 115/230. It was soon found that many complications followed from the use of transformers of the two ratios, so that companies dropped one or the other ratio and made their transformers interchangeable. Of the two ratios the 10 : 1 ratio is preferable, because it is the more extensively used, gives a higher voltage for primary distribution and agrees better with the general principles according to which voltages have been standardized.

Service Wires. — The service wires are those which connect the house wiring with the main on the street. Usually these wires extend in a single span from the nearest pole to the house. At the house they are fastened to insulators similar to those used on the line and mounted on brackets attached to the house. These brackets are often the ordinary wooden bracket used in line work, though the special iron brackets made for the purpose are neater and more secure. The bracket should take the strain off the service span, so that where the service passes through the wall it will not be under the strain of the span. The wires should pass through the wall in porcelain bushings and should have a drip loop between the bracket and bushing so that water will not follow the wire into the building. Where service wires do not conveniently reach the house at point of entrance, they are carried along the wall to such point, being supported at intervals by insulators on brackets. See also *Wiring of Buildings*.

At the pole the service wires are sometimes attached directly to the mains that supply them; while this is the easiest method it has the disadvantage that the strain in the service wire will come on the mains, which may also be injured by the attaching and detaching of numerous service wires. When the service wires do not slope upward or downward at a considerable angle they are liable to become crossed with the main of opposite polarity. Since the general direction of the service wire is at right angles to the main, the service wire should naturally originate on a cross arm at right angles to the arm carrying the mains. In good service work consequently a cross arm is attached to the pole below the main, and all the service wires to both sides of the street run from this. One tap to each of the wires constituting the main can then be used for a number of service wires.

Front and Rear Servicing. — When houses are serviced from a pole line on the street the service wires usually enter the front of the house while if serviced from alley they enter the rear. Service wires can be run around detached houses from front to rear on brackets on the wall, but this is unsightly, expensive and increases the service wire drop. Service entrances can be changed by changing the inside wiring, but the expense is usually heavy and the damage to decoration of rooms sometimes makes it prohibitive. In the wiring of a building the error is frequently made of bringing out services without regard to location of supply lines.

Attic and Basement Servicing. — The service-wire entrance and the center of distribution in the house is generally in the attic for houses serviced overhead and in basement for those serviced underground. Buildings arranged for underground service are sometimes supplied from overhead lines by taking service wires down poles and under sidewalks in iron pipes.

In changing an existing system from a street to an alley distribution, or from overhead to underground service, the center of distribution in the house must be correspondingly changed, or a connection run from the new service entrance to old center of distribution large enough to carry the current without unduly increasing the drop.

Sectionalizing of Distribution Circuits, Fuses and Cut-Outs. — An overhead alternating-current system supplied from a single bus is sectionalized: (1) at the switchboard in power station or substation into circuits having no external interconnection by knife switches or oil switches with fuses or automatic trip; (2) at the poles where long branches leave, by transformer cut-outs, either fused or solid, or by pole-type oil switches; (3) at transformer primaries by fused cut-outs. Switches and fuses are used but sparingly in the primary mains, and in a small compact circuit none are necessary. No switches or fuses are ordinarily used on the secondary side of transformers or on secondary mains. Occasionally transformers have been provided with switches in secondary side so that part of the transformers on a network could be cut out during times of light load to save core-loss. Such sectionalizing has been little used, as the small savings have not justified the complication, care and hazard to service due to mistakes.

When several transformers feed the same secondary main, the opening of a primary cut-out, either by blowing of fuse or by hand, does not make the transformer primary dead, as it is still alive from the secondary side. In case the fuse of one transformer on a secondary network blows the additional load thrown on an adjacent transformer may cause the fuse of that to blow, and all transformers to go out in succession. Under these conditions the fuses cannot be replaced in one transformer at a time. To avoid the interruption to service involved in leaving the fuses out until a time of day when one transformer can carry the whole load temporarily, or in having the whole primary circuit out while the fuses are replaced, it is sometimes possible to have all the transformers on a single secondary network on the same branch of the primary main, this branch being sectionalized from the rest of the primary circuit by a switch.

Lightning Protection. — (*See also article on Lightning Protectors.*) Practically all disturbances from lightning enter a system through the overhead distributing circuits. The most serious effects are not to the distributing circuit itself but to the switchboard and machinery in the station. The effects on the circuit consist of splitting of poles, puncturing of insulators, puncturing of transformers, blowing of transformer fuses. There may also be damage to meters or appliances on the premises of consumers. The protection principally used includes (1) lightning arresters and choke coils in the station (prin-

cipally to protect station apparatus), (2) lightning arresters at intervals on the lines, (3) ground wires over the lines, (4) lightning rods on poles and (5) grounding of the circuit.

Use of Lightning Arresters. — On alternating circuits lightning arresters are used at intervals on the primary but not on the secondary circuits. The amount of line which a lightning arrester will protect is less the more severe the lightning discharge. It is impracticable, and probably impossible, to space the arresters close enough together to absolutely protect a circuit. Theoretically the number of arresters used should be such that the sum of the loss due to the damage by lightning and the expense of providing and maintaining arresters to avoid damage is a minimum. The effects of lightning are too variable and the money loss due to interruption of service is too indeterminate to admit of correct distribution of arresters being determined by calculation. In practice it has been found that when no arresters are used many lightning discharges of considerable intensity do no great amount of damage, also that where the arresters are used much of the damage done is to the arresters themselves, and that their failure is a cause of a considerable number of interruptions to service. It is usually well to begin by using arresters sparingly on the lines and putting additional arresters on if found necessary. The maximum number of arresters would be reached when one was provided for each transformer bank.

The importance of lightning protection is greatest in a composite system, and in most cases underground or submarine cables should be protected by lightning arresters wherever they connect to exposed overhead circuits.

Ground wires are principally used over transmission lines but may be used to advantage over city distribution wires in exposed places. Where adjoining buildings and trees are higher than the pole line, these foreign objects answer the same purpose and little additional screening effect will be obtained from a special ground wire.

RECORDS OF CIRCUITS. — Overhead line construction is constantly changing due to erection, moving and removal of poles, extension of mains, connection and disconnection of service, erection and changing of location and size of transformers. The ease with which changes may be made in the lines necessitates a system of records in such form that any details can be changed at frequent intervals without making a completely new record. The great amount of complicated detail subject to frequent change makes it impracticable to record every feature. In compiling a system of records it is therefore important to determine: (1) what features should be recorded and what omitted, (2) a method of recording information which will be easily corrected. Written records and maps are both used, though for most purposes the latter gives a clearer, more comprehensive and useful record.

Maps of circuits have usually become useless, soon after they have been prepared either because obsolete from lack of correction or unintelligible because of successive erasures and interlineations. The following method of keeping map records has been found to give good results: (1) a map of the circuits is prepared on tracing cloth, giving the circuits on the date of preparation; (2) a blue print is taken from this tracing and all changes marked in pencil or crayon on this print, the print and not the tracing being used for correcting and reference; the tracing is not subject to wear or tear; (3) before the blue print becomes illegible from correction or wear the accumulated corrections are made on the tracing and a new up-to-date blue print substituted, and the process repeated. The corrections on the blue print can be made by line foremen or other unskillful persons, at the time that the line changes are made, while those on the tracing should be made by a draftsman who will make them neatly

and with minimum damage to the cloth. If no colored inks are used on parts subject to change, such a tracing will last a long time and will only have to be redrawn when changes have become so numerous or radical as to amount to a rebuilding of the circuits.

COST OF DISTRIBUTION CIRCUITS. — So many varying items enter into the cost of a complete distribution system that it is impossible to give comprehensive figures of general application. A cost of from $100 to $200 per kilowatt of maximum load at the distributing switchboard may be taken as an indication of the magnitude of the total cost of an overhead distribution system for both light and power, including all outside construction from the power station or the substation to the customer's service inlet, including meters.

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[R. A. Philip.]

DISTRIBUTION AND TRANSMISSION SYSTEMS. — (*See also Alternating Currents; Conduits and Conduit Lines; Distribution Lines; Electricity and Magnetism, Principles of; Ground Connections; Grounding of Electric Circuits; Insulators for Overhead Lines; Insulator Pins; Poles for Overhead Lines; Power Stations; Substations; Transmission Lines; Trolley Systems; Wiring of Buildings; Wires and Cables.*)

This article treats of the following subjects:

The design and construction of pole lines and underground circuits are treated in detail in the articles on *Distribution Lines, Conduits and Conduit Lines, Transmission Lines, and Wiring of Buildings.* Requirements regarding the grounding of circuits and methods of making ground connections are treated in the articles on *Grounding of Electric Circuits* and *Ground Connections.*

Circuits designed for transmitting relatively large amounts of power from one fixed point to another are called transmission lines, while those for delivering small amounts at numerous points are called distribution circuits. Transmission lines usually have no or few branches, while it is characteristic of distribution circuits to have many branches. The various systems of transmitting and distributing electric energy for light and power may be classified under two general heads, viz., constant current or series and constant potential or multiple systems, and each of these may be subdivided into direct-current systems and alternating-current systems.

CONSTANT CURRENT OR SERIES SYSTEM. — The lamps or other devices are connected in series and the current through them is kept constant, the voltages varying automatically to increase or decrease the energy delivered. It is the principal system used in the United States for street lighting, but is now little used for any other purpose. At one time such circuits were extensively used for commercial arc lighting in stores, but the series arc became obsolete for commercial use when the arc lamp was perfected for use in multiple circuits. The value of the current used for street lighting ranges from 1.75 to 9.6 amp., depending upon the type of lamp used.

Source of Current. — Direct current for series circuits is usually obtained from d-c. series generators (*see Generators, Direct-Current*) or from an a-c. circuit by rectifying the alternating current from a constant-current transformer (*see Rectifiers*); adjustable resistance in series in a constant potential d-c. circuit is sometimes used. Alternating current is usually obtained from a constant-current transformer on a constant potential a-c. circuit; other methods sometimes used are an adjustable resistance or reactance in series on constant potential a-c. circuits and by automatic regulating reactance coil on a constant potential a-c. circuit.

Cut-Outs, By-Passes and Transformers. — Switching out of lamps on series circuits is accomplished by short-circuiting them; this leaves the lamps charged to the potential of that point in the circuit. To make them safe to handle "absolute cut-outs" are used which also disconnect both conductors to the lamp, leaving the circuit closed.

To avoid the excessively high voltage which would occur when the circuit opened, due to series incandescent lamps burning out, an automatic by-pass is provided in multiple with the lamps, sometimes consisting of a piece of paper which punctures on a moderate rise of voltage, or of a choke coil which takes but little current at normal lamp voltage. Sockets for series incandescent lamps are arranged to automatically close the circuit when the lamp is withdrawn.

Transformers, one at each lamp, are sometimes used to insulate the secondary circuit containing the lamp from the primary or for reducing the current for low-current lamps used in series with lamps taking higher current.

Advantages and Disadvantages of Series Lighting. — Constant current or series systems have the advantage that low-voltage lamps may be used on high-voltage circuits without the expense, losses or complication of transformation. They have the disadvantage that the lamps are dangerous to handle, the efficiency is low at light loads and it is impracticable to distribute any large amount of power on a single circuit. In the series system the current and consequently the loss in the conductor is the same irrespective of the load, while in the multiple system, the current is proportional to the load, and the watts lost in the line therefore vary as the square of the load and the per cent loss directly as the load. For this reason the series system is not an economical one where the load varies and averages much below full load, which is the case with most commercial loads. For street lighting where all the lamps are turned on and off at once the efficiency at partial loads is of no importance. In constant-current systems the resistance of the circuit does not affect the uniformity of the voltage on the lamps at different parts of the circuit, while with constant-potential systems it does. For a scattering load of lamps, such as street lamps, a uniform light can therefore be obtained from the several lamps, with a much smaller weight of copper and fewer wires by using the series than by using the multiple system.

Thury System. — In Europe the constant-current or series system, with direct current of extra high voltage, has been used for long-distance power transmission under the name "Thury System." The current is obtained by connecting several generators in series, and is utilized by a number of motors also in series. The advantages are simple switchboards, no transformers and minimum strain on line insulators, the latter due first to the fact that in direct current the effective voltage is as high as the maximum voltage, and second, that in a constant-current system the working voltage only remains at its maximum value during the short period when the load is also a maximum. Among the disadvantages are the necessity for insulating frames of generators and motors, need of speed governors on motors, necessity of converting the current by moving machinery in every case where it is used for lighting and in most cases for power. The Thury System is not used in the United States.

CONSTANT-POTENTIAL OR MULTIPLE SYSTEM. — In this system, which is the one principally used for electrical distribution, the voltage between conductors is kept as constant as practicable and the current varies as the load changes. The degree to which the voltage approximates constancy throughout the system is called the "regulation" of the system.

Use of Direct or Alternating Current. — Either direct or alternating current can be used equally well for certain purposes, principally those for which the

heating effect of the current is used, including the lighting of carbon and tungsten incandescent lamps, cooking and heating. For certain purposes, where the current effects chemical or physical changes, such as charging storage batteries and electroplating, direct current is essential; for other purposes, such as operating arc lamps and tantalum incandescent lamps, it is better; whereas for other purposes, such as for Nernst lamps, the alternating current is better. For motive power, the direct current is most favorable where acceleration, variable speed and adjustable speed are desirable, whereas alternating current gives best results where uniform unvarying speed is desired. While the fields of the two kinds overlap to such a large extent that either kind can be used for general distribution, it is found that it is often more advantageous to use both. For low-voltage, underground distribution, direct current is advantageous because the heavy currents can best be carried on single conductor cables; no subway transformers and no small, high tension fuses are required.

Alternating current may be supplied either directly from the generators in the power station or from the secondaries of transformers in substations. Direct current may be supplied either directly from direct-current generators or from converter substations (*see Substations, Railway*) supplied with high-voltage alternating current.

Light and Power Circuits. — For a load consisting of both electric lamps and electric motors the same circuit may be used throughout for both classes of service or entirely or partially independent circuits may be employed. The use of the same circuit throughout for light and power service has the advantages of reduced number of wires, transformers and meters, reduced weight of wire, and reduced capacity of transformers and meters, whereas the use of separate circuits for light and power has the advantage of requiring less capacity of feeder regulators and, sometimes, of less weight of wire for the same perfection of regulation on the lighting circuits and also simplifies the problem of balancing the phases in polyphase distribution (*see below*). In the business districts of many cities the Edison three-wire direct-current system is used, in which case light and power are usually supplied from the same circuit, the motors as a rule being connected to the outside wires. In business districts where the lighting is done by alternating current the lighting is frequently on single-phase circuits and the power on separate polyphase circuits or on 500-volt direct-current circuits. In residential districts, where the lighting is done by single-phase alternating current, small motors are usually put on same services as the lights; whereas larger motors (from 1 to 5 and in some instances as large as 15 horse-power) are put on separate meters, services and transformers, but the same primaries are used for both services. In factory districts when the power is the predominating load and is supplied by polyphase circuits the incidental lighting is sometimes taken off the same services, but usually there are separate meters and transformers. There are a large number of cases where the same polyphase primaries are used for power and light with separate transformers and secondaries; in these cases the lighting transformers are usually distributed as equally as convenient between the several phases, in order to balance the load, though occasionally they are all connected to a single phase. In a few cases the same secondaries have been used for power and light usually on the four-wire three-phase plan.

Circuits with Branches. — There are two methods of providing proper distribution circuits for an extensive area, known respectively as the "tree system" and the feeder and main system. The former is the most extensively used but is not susceptible of providing as good voltage regulation as the latter.

Tree System; Interconnected Networks.— The simplest method of constant-potential distribution is to run one set of wires through the middle of

the district to be served, running branch wires from them wherever convenient for reaching loads at the sides. At the beginning, the single set of wires is large enough to carry the whole current, but the size is diminished as the current branches off. In this system adjacent branches will often have widely different voltages, due to unequal drop because of varying loads. If wires are run connecting the out-lying ends, currents will flow and partially equalize the differences in voltage. This plan, if carried out systematically, develops into a network, usually composed of two sets of parallel mains crossing at right angles and connected at each point of crossing.

Feeder and Main System. — It is practicable to keep the voltage on the customers' electric lamps within 2 to 5 per cent of being constant, even when it is necessary to raise the voltage of the generators from 10 to 20 per cent, from no load to full load, by arranging the distribution circuits on the feeder and main plan. The area to be served is first divided into compact sections; each small enough to be fed from a central point with a voltage drop within the desired percentage. A system of wires called "mains" are run from this central point, called the "feeding point," or "distribution center," to the customers. The mains form a little circuit originating at the feeding point instead of at the power house; they may be arranged on either the tree or interconnected network plan. A set of wires called "feeders" is run from the power house to the feeding point. As no current is taken off at intermediate points, these wires are not tapered and any desired drop of voltage may occur in the feeders without affecting the lamps, provided the power-house voltage is raised proportionally.

Two-wire System. — The simplest multiple system is the two-wire system, where all devices are connected directly in multiple. This system is used very extensively for direct-current light and power, from isolated plants, for power circuits (usually 500 volts) and railway circuits from central stations. It is also used for single-phase alternating-current distribution for both primary and secondary circuits.

Edison Three-wire System. — The three-wire system, Fig. 1, is obtained by replacing the out-going wire of one two-wire system and the return wire of a second two-wire system by a single wire, called the "neutral." The voltage between the outside wires is then double the voltage between the neutral and either outside wire. For example, 110-volt lamps may be connected between the neutral and either outside wire, and 220-volt motors may be connected between the two outside wires, and both the lamps and motors be supplied with their rated voltage. The neutral wire carries a current which depends only upon the *difference* in the loads on the two sides of the system and their distribution. As a rule the neutral of a three-wire main is made equal in cross-section to each outside wire. With perfectly balanced load the three-wire system with all three wires of the same size results in a saving in copper of 62.5 per cent as compared with a two-wire system supplying the same load at the same regulation.

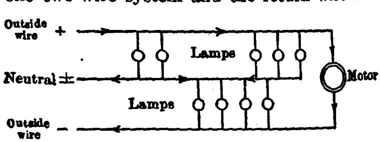

Fig. 1. Three-wire System

The three-wire system is used very extensively for direct-current light and power distribution from central stations, also for large isolated plants and for alternating current for lighting on the secondary circuit. Three-wire systems are usually 110 volts on each side of neutral, or 220 volts between outsides,

though there are several systems using 220 volts on each side and 440 volts between outsides.

The Edison three-wire distribution system, as used in the business sections of large cities, consists of a set of interconnected three-wire mains supplied by two-wire or three-wire feeders from one or more power houses or substations. The different feeders feed into the same set of mains at different points. At the power house or substation end the feeders are often all supplied from a common bus, though where there is a great difference in the length of the feeders there are sometimes two busses which are run in multiple when the load is light but separated when it is heavy, the short feeders on one called the "low bus," and the long feeders on the other called the "high bus," because of the relative voltages. As the feeders from the several substations connect to interconnected mains, the bus voltage in each is raised or lowered in accordance with the drop in its own feeders. All mains which connect feeding points supplied from a given bus are made large enough to allow large equalizing currents to flow through them, without requiring any great drop in the mains, in order to equalize the voltages at these points.

ALTERNATING-CURRENT SYSTEMS. — The simplest form of alternating-current system is the single-phase system, for which the connections are exactly the same as for the direct-current systems. Both two-wire and three-wire circuits are used, the former for primary circuits and small secondary circuits and branches, and the latter for large secondary circuits. The principal use of single-phase circuits is for electric lighting and auxiliary uses, such as heating or cooking, and fan motors. For delivering large amounts of power a two-phase or three-phase system is generally used.

Two-phase Systems. — In this system there are two single-phase currents having a difference in phase of 90°, or a quarter of a cycle. These currents may be distributed on the three-wire, four-wire or five-wire system; see also under *Transformer Connections* in the article on *Transformers*.

Three-wire Two-phase Systems. — Each single-phase current has a separate outgoing wire but unites in a common return wire. Each two-phase motor has two circuits, each connected between an outside wire and the return wire. The voltage between the two outside wires is 41 per cent greater than between outside wire and return wire and the current in the return wire is 41 per cent greater than in each outside wire. This is an unsymmetrical system and has the disadvantage that even a balanced load will cause a distortion and unbalancing of the delivered voltage of the two phases because of the unsymmetrical drop in the common return wire.

Four-wire Two-phase System. — Each of the two single-phase circuits has a complete, independent two-wire circuit. There are two variations, first where the circuits are insulated from each other, in which case a cross between either wire of one circuit with either wire of the other will change the voltage stress to ground, but will not affect the delivered voltage or cause a short circuit; second, when the neutrals of the two circuits are connected. In the latter case, from each wire of one circuit to either wire of the other circuit, the voltage is 71 per cent of the voltage between wires of the same phase. The four-wire system with insulated phases is probably the most extensively used of the two-phase systems.

Five-wire Two-phase System. — This is a modification of the four-wire system, with interconnected neutral in which the common neutral is extended as a fifth wire. Lamps may be connected from each of the four wires to the neutral. The five-wire system may be considered as two three-wire single-phase systems, one for each phase, with a common neutral wire.

Three-phase Systems. — In this system there are three single-phase alternating currents with a phase difference of 120°, or of one-third of a cycle. These currents may be distributed on the three-, four- or six-wire systems.

Three-wire Three-phase System. — Each single-phase current has a separate outgoing wire; the three return currents neutralize so that no return wire is required. The three wires are necessarily interconnected, the voltages are usually the same between any two and the currents equal in each of the three conductors, provided the loads on the three phases are equal, i.e., provided the load is balanced. When equally loaded the voltage drops in the three conductors are equal and symmetrical. This is the most extensively used of the three-phase systems.

Four-wire Three-phase System. — This is a modification of the three-wire system in which a neutral wire is extended as a fourth wire. Lamps or transformers may be connected from each of the three wires to the neutral, which carries only the unbalanced current, due to the differences in loading of the three phases. The voltage between the three outside wires is 73 per cent greater than from each outside wire to neutral.

Six-wire Three-phase System. — If to a three-wire, three-phase system, three wires are added, one with voltage midway between that of each pair of outside wires, lamps may be divided into six groups, between the three outside wires and the three adjacent middle wires. The result is the same as though there were three single-phase three-wire circuits, one for each phase, with the six outside wires combined in pairs giving three common outside wires in place of the three pairs. When connected in this way, the three middle wires cease to be neutrals, as between the three there is a three-phase voltage equal to one-half of that between the three outside wires.

Six-phase System. — This is used for circuits in the interior of substations (q.v.), such as from transformers to rotary converters, but is not used for distribution.

Monocyclic System. — This is a variety of unsymmetrical polyphase system now obsolete. It may be regarded as a species of two-phase system with voltage on one phase one-fourth of that on the other and with the two wires of the larger phase acting as a common return for the small phase.

Star or Y, and Mesh or Δ Connections. — See article on *Alternating Currents*.

Transformer Connections and Phase Transformations. — See article on *Transformers*.

Transformations of Systems. — Any electric system can be transformed to any other electric system by the use of rotating machinery; see *Motor-Generators* and *Converters, Synchronous*. This method is used chiefly for changing from alternating to direct current and for changing the frequency of alternating current. Conversion from alternating to direct current is also accomplished by means of mercury arc rectifiers, particularly for series d-c. arc lighting and for charging storage batteries; see *Rectifiers*. Conversion from any alternating voltage to any other alternating voltage at the same frequency is accomplished by means of transformers (q.v.).

Frequencies in Use in the United States. — At present the two frequencies in most general use and adopted as standards in most new work are:

60 cycles per second or 7200 alternations per minute, used by the majority of companies operating alternating-current lighting systems.

25 cycles per second or 3000 alternations, generally used for alternating-current railway or power work or where the alternating current is to be converted into direct current before final use.

In addition to the above other frequencies ranging from 140 to 25 cycles per second are in use as follows:

140 cycles or 16,800 alternations. Formerly standard frequency of Ft. Wayne electrical works, now obsolete for new apparatus but still used in many small towns.

133⅓ cycles or 16,000 alternations. Formerly a standard frequency of Westinghouse E. & M. Co., and Stanley Co., now obsolete for new apparatus but still used in many small towns.

125 cycles or 15,000 alternations. Formerly a standard frequency of General Electric Co., now obsolete for new apparatus but still used in many small towns.

50 cycles or 6000 alternations. Used extensively in several large systems in Southern California.

40 cycles or 4800 alternations. A compromise frequency at one time standard with the General Electric Co. used on several large systems in that part of New York state around Albany and including the General Electric Co.'s system at Schenectady. Also used by certain mills in New England and elsewhere.

35 cycles or 4200 alternations. Used on the large system of the T.C.R.T. Co., in Minneapolis and St. Paul.

33⅓ cycles or 4000 alternations. Used by the P.G.E. Co., in Portland, Oregon.

16⅔ cycles or 2000 alternations and 15 cycles or 1800 alternations have been proposed for use on single-phase electric railway work and it has even been suggested that they be standardized for that purpose but at the present time practically no progress has been made toward their introduction.

Choice of Frequency. — For new light and power systems, the choice of frequency depends on (1) the frequency of existing systems in same or adjoining territory, and (2) the proportion of load which will be direct current. It is generally advantageous to have a new system of the same frequency as that of existing systems in the same or adjoining territory, so that a physical connection can be made between them or load transferred from one to the other without the use of frequency changers. The choice of frequency now (1914) is in most cases confined to the two standard frequencies, 60 and 25 cycles. Aside from the question of matching adjacent systems, the choice depends principally on the proportion of current to be converted into direct current. Where all load is direct current, 25 cycles has the advantage; where all load is alternating-current lighting, 60 cycles is best.

Use of Two Frequencies. — When the bulk of the load is direct current, but a small though important part is alternating-current lighting, two frequencies, 25 and 60 cycles, are sometimes used. Sometimes the two frequencies are generated by separate prime movers, though sometimes all current is generated at 25 cycles and the 60-cycle current obtained from frequency changers (*see Motor Generators*).

RELATIVE VOLTAGES AND WEIGHTS OF COPPER REQUIRED FOR VARIOUS SYSTEMS. — The comparison of the various systems with respect to the weight of copper required, as tabulated below, is based upon the assumptions (1) that the energy delivered, (2) that the energy loss and (3) that the maximum voltage strain on insulation between any wire and ground are respectively *the same* for all systems.

When the neutral or middle point of the system is grounded, then the maximum voltage strain on the insulation to ground is the maximum instantaneous value of the voltage between any outside wire and neutral. In the case of an alternating sine wave of voltage the maximum instantaneous value is equal to $\sqrt{2}$ times the effective value determined by a voltmeter. When the neutral is not grounded then an accidental ground on any leg will throw full line voltage across the insulation between any other leg and ground.

On this basis of comparison, assuming a sine wave of voltage and 100 per cent power factor for the alternating-current systems, the relative voltages between outside wires and the relative weights of copper are as follows:

RELATIVE VOLTAGES AND RELATIVE WEIGHTS OF COPPER

System*	Relative voltages between outers		Relative weights of copper*	
	Grounded neutral	Insulated neutral	Grounded neutral	Insulated neutral
	Per cent	Per cent	Per cent	Per cent
Direct-current (2-wire)	100	50	100	400
Single-phase (2-wire)	71	35.5	200	800
Two-phase (4-wire)............	71	35.5	200	800
Three-phase (3-wire)	61	35.5	200	600

* Neutral wire not included; when neutral is added increase the figures given in the ratio of weight of neutral to combined weight of the outside wires for the system in question. For example, the addition of a neutral wire equal in size to either outside wire to a d-c. 2-wire system with grounded neutral gives a relative weight of copper of $100 \times 1.5 = 150$ per cent.

Effect of Power Factor. — To correct the above tabulation for alternating-current systems in cases where the power factor is less than unity divide the relative weight of copper given by the square of the power factor (i.e., for a single-phase two-wire system with insulated neutral and 70 per cent power factor the weight of copper will be $800/0.70^2 = 1630$).

VOLTAGES FOR D-C. AND A-C. SYSTEMS. — Unless otherwise specified the "voltage" of a polyphase system refers to the potential difference between adjacent outside wires; see *Alternating Currents* for the relations between the various voltages.

A considerable number of voltages are standard or semi-standard. In general they may be considered as derived from a lamp voltage of 50 or 100 multiplied by the factors 2, 3, 5 and 10, and increased by 4, 5, 10, 15 and 20 per cent for various reasons, principally to allow for line loss. In addition to this system of voltages there is another system which arises from the use of Y and Δ connections on the three-phase system; these voltages are related to the above by the factor $\sqrt{3} = 1.73$, either as a multiplier or a divisor. For example, the voltage between the outside wires of a four-wire three-phase system supplying 115-volt lamps connected between each outside wire and neutral is $\sqrt{3} \times 115 = 198$ volts.

Voltages Commonly Used. — The voltages most used are:

Direct current:

100 to 120 volts	Lighting, small power and field excitation.
200 to 240 volts	Power, lighting and field excitation.
500 to 600 volts	Power, electric railways.

Alternating current, secondary distribution:

100 to 120 volts	Lighting, small power.
200 to 240 volts	Power.
400 to 480 volts	Power.

Alternating current, primary distribution:

1000 to 1200 volts	Lighting (obsolete).
2000 to 2400 volts	Lighting and power.

Alternating current, intermediate distribution:

5000 to 6000 volts	Generating station to substation.
6000 to 7200 volts	Generating station to substation.
10000 to 12000 volts	Generating station to substation.
12000 to 13200 volts	Generating station to substation.

Alternating current, high tension transmission:

20000 to 24000 volts	Generating station to substation.
30000 to 36000 volts	Generating station to substation.
40000 to 48000 volts	Generating station to substation.
50000 to 60000 volts	Generating station to substation.
60000 to 72000 volts	Generating station to substation.
100000 to 110000 volts	Generating station to substation.

CIRCUITS FOR TYPICAL CITY DISTRIBUTION SYSTEM. — For a city covering a space ten miles square, with current generated at one point, a typical distribution would consist of the following lines: —

(1) From generating station to substations; alternating current, 60 cycles, three-phase, 11,000 volts.
(2) From lighting substations in business center; direct current, 110–220 volts, three-wire.
(3) From lighting substations in residence district; alternating current, 60 cycles, one-phase, 2200 volts for house lighting, and alternating current, 60 cycles, one-phase, 6.6 amperes series circuits for street lighting.
(4) From power substation in factory district; alternating current, 60 cycles, three-phase, 2200 volts.
(5) From railway substation; direct current, 550 volts.
(6) From secondaries of lighting transformers; alternating current, 110–220 volts, three-wire for large transformers and alternating current, 110 volts, two-wire for small transformers.
(7) From secondaries of power transformer; alternating current, 220 volts, three-phase.

EFFICIENCY OF DISTRIBUTION. — The ratio of the energy registered by the customers' meters, or which would be so registered if all customers had meters, to the energy supplied by the generator to the bus-bar in the generating station, may be called the over-all efficiency of distribution. The losses may be divided into several kinds: line loss, transformer loss, converter loss, meter loss and error, leakage and unaccounted for. For some purposes it is useful to consider each loss from two points of view: (1) as a loss of energy, and (2) as a loss of power at full load; the first may be called the energy loss and the second the capacity loss. The corresponding efficiencies are usually designated as all-day or energy efficiency and the full-load or capacity efficiency respectively.

Fixed and Variable Losses. — The total energy loss consists of two components, (1) a fixed loss independent of load, including the core-loss of transformers, the core-loss, excitation, friction and windage of rotating converting apparatus, the loss in the shunt coils of meters, the copper loss in constant current circuits, and the loss in the arc of mercury arc rectifiers for constant current circuits; (2) a variable loss proportional to the square of the current, including the copper loss of constant potential circuits and of transformers, and the armature copper loss of rotating converting apparatus. The effect of the fixed loss on the per cent efficiency depends on the load factor of the load, while

that of the variable loss depends both on the load factor and the shape of the load curve.

Representative Losses. — For a lighting system, the full-load losses in primary feeders, primary mains, transformers, secondary mains, services and meters may be expected to be as much as 17.5 per cent of the power generated and the daily energy loss 33.3 per cent of the energy generated, giving 82.5 per cent capacity efficiency and 66.7 per cent energy efficiency respectively.

Effect of Nature of Load. — In making estimates of the efficiencies of particular systems, the effect of the following items should be considered: (1) relation of transformer capacity to maximum loads, (2) load factor, (3) shape of load curve, (4) power factor, (5) diversity factor.

REGULATION. — An ideal constant potential distribution would have one uniform, unvarying voltage, and would be said to have perfect regulation. The greater the variation from such constancy the poorer the regulation. The regulation is usually specified in per cent variation, either "above or below" a standard mentioned. The standard is usually either the nominal voltage desired or the actual average voltage obtained.

Evil Effects of Poor Regulation. — The evil effects of high voltage are short life of electric lamps, excessive speed of direct-current motors, excessive exciting current of induction motors and burning out of motors and other devices; on the other hand, low voltage greatly diminishes both the candle-power and efficiency of electric lamps, decreases the maximum power of motors and increases the current which a motor will take for a fixed horse-power output. As electric lamps are much more sensitive to change of voltage than motors, separate circuits are often used for lighting and power, the former having devices for regulation which are omitted from the latter.

The following figures give roughly the quantitative effect of voltage variation between 5 per cent below and 5 per cent above normal:

Each per cent decrease in voltage decreases candle-power of carbon incandescent lamps	5 Per Cent
decreases torque of induction motor	2 Per Cent
Each per cent increase in voltage decreases the life of carbon incandescent lamps*	13 Per Cent
increases the magnetizing current of induction motors	2 Per Cent

Ordinary Limits of Regulation. — Roughly, the maximum voltage variation at the lamps on a lighting system should never exceed 5 per cent, i.e., the regulation above or below normal should never be greater than 2.5 per cent, and should be as much less as is economically feasible; the voltage variation on power systems is usually 10 per cent (5 per cent above or below normal) and is sometimes considerably more.

Calculation of Regulation. — In order to calculate the variation in voltage at any receiving device or group of such devices from no load to full load, the voltage at the generating or substation or feeding point being assumed constant, the impedance drops in all parts of the distribution system must be calculated and properly combined. The various parts of the system to be considered are the house wiring, service wires (or leads from street mains to the house), secondary mains, transformers, primary mains, primary feeders. See *Distribution Lines; Transmission Lines; Wiring of Buildings.*

In making such calculations account should be taken of the fact that the loads or currents in the several parts of the system seldom have their maximum

* Average for 5 per cent increase in voltage; the first per cent increase in voltage decreases the life 18 per cent, the fifth per cent 8 per cent.

values at the same time. The maximum drop in house wiring will not occur simultaneously in all houses nor will the maximum service drop occur together on all services, etc. Furthermore, the maximum house wiring drop for a given house may not occur at the same time as the maximum drop on the secondary mains, transformer or primary mains to which it is connected.

Effect of Line Reactance. — The regulation of an alternating-current system of unity power factor will be poorer than that of a direct-current system of the same copper efficiency (see tabulation above) because of additional drop due to line reactance.

Effect of Lagging Power Factor. — A lagging power factor usually makes the regulation of an alternating-current system worse than it would be for unity power factor and therefore much worse than for a direct-current system of the same copper efficiency.

Effect of Leading Power Factor. — A leading power factor usually makes the regulation better than it would be for unity power factor and may give even better regulation than can be obtained from a direct-current system of the same copper efficiency.

Effect of Currents in Neutral. — Any current in the neutral wire of a balanced system produces a drop which tends to unbalance the voltage of the two sides. In the case of a three-wire direct-current or single-phase system *one per cent drop in neutral produces two per cent difference in the voltages on the two sides.* Voltage drop in the neutral therefore affects more seriously the regulation than an equal amount in the outside wires. If the currents in the neutral all have the same direction, say toward the station, the voltage drops in the various parts of it will be cumulative, and though the drop in each section may be small and no current may actually reach the station, the aggregate effect may be serious. The individual loads on the two sides of the neutral should be connected so that the unavoidable neutral currents flow alternately in each direction, thus causing drops in alternate directions thereby neutralizing each other over the total length of the circuit.

Feeder Neutral in Three-wire Systems. — When the "feeder and main" system is used with a three-wire system, two-wire feeders should be used only on the outside wires, and only a single "feeder" neutral be used. That is, with respect to the neutral, the "tree" system gives better regulation for the same weight of copper than the "feeder and main" system, but with respect to the outside wires the latter system gives the better results. This is a point which has not always been recognized in laying out three-wire systems.

VOLTAGE CONTROL. — (*See also Controllers.*) The more common methods of controlling the voltage at feeding points when the feeder and main system of distribution is used (*see above*) are the single bus system, the high and low bus system, and feeder regulators.

Single Bus System. — (*See also article on Bus-Bars.*) All the feeders may be connected to a single bus and the bus voltage be raised as load increases so as to compensate for average drop of all the feeders on it. This method gives excessive voltage on such feeders as are comparatively short and low voltage on those that are long. Usually maximum voltage is carried in the evening during the lighting peak and lower voltage during day giving very poor regulation on power circuits having maximum load during the day and low load at night. This method is used on most small direct- and alternating-current distribution systems.

High and Low Bus System. — The bus may consist of two parts which may be separated, and operated at different voltages, feeders of greater average

drop connected to the "high" (voltage) bus and the others to the "low" (voltage) bus. The voltage of each bus is raised or lowered in accordance with average drop of feeders on it as in the single bus system. This method is used extensively on the Edison three-wire direct-current systems, as noted above. The unequal drop of the several feeders on same bus are equalized by heavy interconnection through mains.

Feeder Regulators. — A feeder may have a separate regulator adjustable to compensate for its own drop. This method is sometimes used on direct-current railway feeders and very extensively on alternating-current lighting feeders. See article on *Regulators*. A feeder or other distribution circuit which contains a voltage regulator is frequently referred to as a "boosted circuit." Boosted circuits are not usually interconnected but may be if the boosting is of the same nature in each.

Control of Polyphase Systems. — In a polyphase system the load may be as equally divided among the phases as possible and the voltage regulated with respect to any one phase taken as representing the average of all; or the lighting load may be connected on a single phase and the voltage regulated for this phase alone. The former method is more common, as it permits of full output of all the phases of the generator being used and gives a more equal voltage on the several phases of polyphase motors.

Transformers as Outside Boosters. — An auto-transformer or an ordinary transformer may be connected to a feeder at any point and used as a "booster" to raise or lower the voltage. In the case of an ordinary transformer the primary and secondary are connected in series thus converting the transformer into an auto-transformer; see article on *Auto-Transformers*. Such boosters are not adjustable and have a bad effect on the regulation as they give excessive voltage on the boosted part of line at light load.

To cut such a booster out of service without taking it off the circuit the primary coil must be open circuited and the secondary short circuited. *Caution:* the main circuit must be opened before cutting out booster, because if the primary is opened while current is flowing in the secondary the booster becomes a step-up transformer and may give a dangerously high voltage on the primary. If on the other hand the secondary is short circuited first, a destructive short-circuit current may flow through it and the primary, and when the primary is opened a dangerous arc will form.

Use of Lamps of Different Voltage. — Attempts have been made to compensate for the difference in voltage at the various points of a network by using lamps of different rated voltage, high-voltage lamps being used near the point of supply and low-voltage lamps at the ends of line. This plan has not been successful because the difference in voltage between any two points is not constant. Frequently some of the high-voltage lamps are used by mistake where low-voltage lamps are intended; the results are then poorer than when no difference is made in lamp voltage. While lamps of two or more voltages may be used temporarily as an expedient when regulation is very bad, it is better practice to have all lamps on a system of a single uniform voltage.

Use of Transformers of Different Ratio. — Transformers are also made with taps so that a uniform secondary voltage can be obtained with a varying primary voltage. The plan is not a good one, because if the difference in ratio is correct for uniform voltage at full load it gives unequal voltages at light load. It also has the very serious disadvantage that the haphazard changing of transformer ratios by ignorant linemen to compensate for dim light, due perhaps to lamps already blackened by excessive voltage, makes it impossible to carry out any systematic plan for securing the best average regulation for the system as a whole.

BALANCING OF LOAD. — In three-wire direct-current or single-phase systems and in all polyphase systems supplying single-phase load it is necessary to approximately balance the load between the two sides or the several phases as the case may be. Unbalanced load has two bad effects: (1) it loads the two sides or phases of the system unequally, making it impossible to get full output out of the lightly loaded side or phase without overloading the other; (2) it makes the regulation of the system worse by causing high voltage on the lightly loaded side or phase and low voltage on the other.

The first difficulty is not serious, because through the conduction of heat from the loaded to the unloaded coils of transformers and generators the machine capacity is not reduced in proportion to the unbalance; for moderate unbalancing, say up to 10 per cent greater load on one side or phase than on the other, it is doubtful if any appreciable effect could be discovered. As the total load usually consists of a great number of small parts, a very little foresight in dividing the load in the first place between the sides or phases, and in suitably distributing subsequently connected load, will give a balance good enough for all practical purposes. When polyphase alternators were first installed for supplying existing single-phase lighting circuits, it was supposed to be important to have a close balance of load between the phases, and early switchboards were therefore provided with transfer switches for throwing single-phase feeders from phase to phase. It was later found that instead of being necessary to transfer the load from phase to phase, following the diurnal or annual variations of load, that the circuits could stay on the same phase indefinitely and that transfer switches were unnecessary and undesirable. The time when rebalancing of this kind is necessary is usually when new circuits are established, at which time the changes of connections are best made by changing the taps to the bus-bars.

Motor-generator Balancer. — On direct-current three-wire systems the difference in current on the two sides may be balanced without taking the neutral current to the generator by a balancer consisting of two similar machines mechanically coupled and electrically connected in series. Each machine is wound for the voltage of one side of the system (110 volts for a 110/220-volt system); the common connection between the machines is connected to the neutral and the other two terminals to the two outsides, see Fig. 2 (the field windings are omitted for clearness). The unit acts as a motor generator (q.v.), whichever machine happens to be on the light side being the motor for the time being and the other the generator. By strengthening the field of the one on the side where the voltage is low and weakening that of the other, so as to keep the same total voltage, the voltages on the two sides as well as the currents may be balanced if necessary.

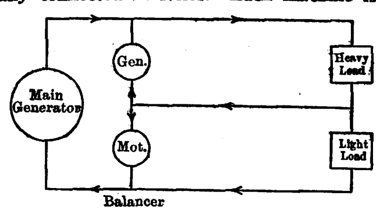

Fig. 2. Motor-generator Balancer

The output of the balancer generator and the input of the balancer motor are practically equal to each other, neglecting the losses in the machines, and each is equal to

$$\frac{P_1 - P_2}{2},$$

where P_1 is the load on the heavily loaded side of the system and P_2 the load on the lightly loaded side. For example, if the load on one side is 110 kw. and on the other side 90 kw., the load on each unit of the balancer is 10 kw.

It would, however, be unsafe to use a balancer as small as such calculations for the normal unbalancing would indicate as correct, because in case of a short-circuit on one side of the system, or of loss of a large amount of load on one side, due say to the blowing of fuses, the balancer would be dangerously overloaded. This may not only destroy the balancer, but may burn out many lamps and motors on one side due to excessive voltage. In small systems, say up to 100-kw. total capacity, the balancer should be able to operate momentarily with any loading up to the capacity of the main generator, and in large systems should have capacity sufficient to burn off a short-circuit on one side of the system without creating an unduly high voltage on the lightly loaded side.

Dynamotor as a Balancer. — A smaller and cheaper balancer is obtained by using a dynamotor (q.v.), which is a single machine with two windings on one armature and two commutators. The armatures are connected in series and balance the currents, but as there is only one field the voltages cannot be balanced.

Alternating-current Balancing Coil. — On alternating-current single-phase three-wire systems it is usual to obtain the neutral from the middle point of the coil of the transformer supplying the current. It can, however, be obtained at any point of the two-wire circuit without going back to the transformer by using a balance coil. A balance coil is a transformer with two similar windings connected in series across the circuit and with the neutral wire connected to the common point between the windings. Whichever coil is on the lightly loaded side acts as the primary and the other as the secondary, thereby balancing the circuit by transferring one-half of the difference in load from the heavily loaded side to the lightly loaded side. Such coils, which are essentially auto-transformers with a 2:1 ratio, may be used in various ways: to obtain the neutral for an unbalanced three-wire circuit from a two-wire circuit; to supply a 110-volt load from a 220-volt circuit; to supply a large 110-volt load from a 110/220-volt circuit without connecting to the neutral. In practice balance coils are not much used as the neutral can more cheaply be obtained from the transformer.

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[R. A. PHILIP.]

DRAFT, MECHANICAL. — (*See also Blowers and Compressors; Chimneys; Fans; Pipes and Piping.*) Any system producing draft either by steam jets or by blowers is called a mechanical draft system. The relative advantages of mechanical draft as compared with chimney draft are: (1) The supply of air may be more readily controlled; (2) a fan permits a lower temperature of flue gases, which increases the boiler efficiency; (3) the boiler may be forced to high overloads in case of emergency; (4) the draft is uninfluenced by climatic conditions; (5) the system is low in first cost. The disadvantages of mechanical draft are: (1) danger of breakdown; (2) cost of maintenance and operation; (3) a stack or other means must be provided to carry off the flue gases wherever their discharge at low levels constitutes a public nuisance. Many plants with tall stacks are provided with mechanical-draft apparatus as a means of forcing the boilers in case of heavy overloads. Mechanical draft is also frequently used in connection with fuel economizers.

Steam Jets for producing artificial draft are simple, inexpensive and easily applied, but are very uneconomical, since they require from 6 to 11 per cent of the total steam generated (A. J. Whitham, *Trans. A.S.M.E.*, Vol. 17).

Forced Draft. — A blower system creating an excess of pressure beneath the fire is known as a forced-draft system. A single fan is commonly employed to supply a number of boilers through a system of ducts beneath the floor. The air is sometimes taken from a chamber built around the breeching, thus utilizing a part of the heat energy in the exhaust gases.

Induced Draft. — In the induced-draft system the fan is placed between the breeching and the stack. All leakage of air is inward, avoiding inconvenience when the doors are opened. This system is frequently used in plants which have high peak loads, being ordinarily installed in connection with fuel economizers. A by-pass directly from breeching to stack is usually provided for use when mechanical draft is not required or in case of accident. A double system of fans is also sometimes provided as a further insurance against accidental interruption.

POWER REQUIRED — SIZE OF FANS. — The power and size of fans required to operate either a forced-draft or induced-draft system may be roughly calculated by the method given in the article on fans. The static pressure may be roughly approximated from the data given in the article on chimneys, the figures there given for loss of draft being taken as the static pressure to be overcome. The quantity of air required per boiler horse power depends upon the quality of the coal, upon the care taken in firing and upon the efficiency of the boiler and grate (*see Boilers*).

Let R be the number of pounds of coal required per boiler-horse-power hour, W the pounds of air required per pound of coal, P the boiler horse power and $w = \frac{1.33\,B}{460 + t}$ = pounds per cubic foot of gas passing through the fan, where B is the barometric pressure in inches of mercury and t the temperature of the gas in degrees Fahrenheit; then the volume of gas passing through the fan per minute is $Q = \frac{RWP}{60\,w}$. This formula and the formula for w are strictly applicable only to the case of a forced-draft fan, but the two formulas may also be used with but a small error for an induced-draft fan, taking for t in the latter case the temperature of the discharged products of combustion (about 500° F., when no economizer is used and about 300° F. with an economizer). The size of fan and the power required to operate it will be greater for induced draft than for forced draft, since in the former case the volume of gas passing through

the fan is greater. According to Gebhardt the power required to operate a forced-draft system for a plant of 1000 boiler horse power or more, will range from 1 to 5 per cent of the plant capacity.

COSTS. — According to Gebhardt the cost of a forced-draft fan, engine and stub stack for a plant of 1000 boiler horse power or more will approximate 20 to 30 per cent of the cost of an equivalent brick chimney. The cost of a single induced fan, engine, stack, etc., will approximate 40 to 50 per cent of the cost of an equivalent brick chimney, and the double-fan outfit will be 50 to 60 per cent of that of an equivalent brick chimney. Two per cent is a liberal allowance for maintenance and depreciation of a brick or concrete chimney, while for a mechanical-draft system an allowance of from 4 to 10 per cent should be made. It costs nothing to operate a chimney, while from 1 to 5 per cent of the cost of operation of a boiler plant is chargeable to a mechanical-draft system.

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[WM. KENT.]

DREDGES, ELECTRICALLY OPERATED. — (*See also Motors, Industrial Applications of.*) Electrically operated dredges are now used very generally, especially in connection with gold mining. This drive has indisputably proved its advantages over steam drive from the standpoint of both operation and economy, as individual motors can be applied to the various units of the dredging machinery, and a larger percentage of the power input can thereby be directly applied in useful work.

The standard form of dredge is of the continuous-chain, close-connected bucket type, ranging in capacity from 3 to 16 cubic feet and with a digging depth of from 40 to 60 feet. As the power is mostly obtained by transmission from some hydroelectric power system or central generating station, alternating current is generally used for driving the dredge machinery. For a modern gold-mining dredge this consists of the digger or bucket line, the winch, the high- and low-pressure pumps, the priming pump, the screen and the stacker.

Bucket Line. — The bucket line is supported on a sheet-steel or girder structure of massive construction, so as to resist the heavy strains while in operation, especially when striking rock. The speed varies from 50 feet (with from 16 to 25 buckets) to 75 feet (with from 35 to 50 buckets) per minute, depending upon the condition of the ground.

For operation and control of the digger a variable-speed induction motor is used. This is located on the lower deck and belted to the driving pulley, which is generally situated in the rear of the pilot house on the upper deck. The duty imposed upon this motor is severe, as it must operate under conditions calling for power varying from 75 per cent overload down to 25 per cent of its rated capacity. A drum-type controller for forward and reverse operation is provided, including the necessary resistance for continuous operation on any notch of the controller from one-half to full speed. The maximum starting torque is required and obtained at about the fourth point of the controller, thus leaving three points on which to bring the motor up to half speed, at which time nearly full rated torque is required. As a result of these conditions the ordinary motor designed for intermittent service cannot be successfully applied.

The raising and lowering of the bucket-line ladder is generally accomplished by a friction clutch which can be connected to the digger motor. For the larger size dredges, however, a separate motor is often provided to perform this duty.

Winch. — To keep the dredge in place and to move it about or hold it against the bank when digging, head lines are used, which are controlled from the forward end and operated by a six- or eight-drum winch driven by a variable-speed motor. The winch motor, though of smaller capacity, is of the same staunch construction as the digger motor and is equipped with a suitable controller and resistance to permit its continuous operation from one-half to full speed. It has been found advisable to equip the motors for this service with solenoid brakes, by means of which the motor can be brought to a standstill almost instantly. It is then ready for the reverse operation without the usual reversing of the motor through the controller, which is not only bad practice but may result in a burn-out due to the heavy strain on the windings.

Pumps.—The high- and low-pressure pumps for supplying water to the screens and sluices are generally connected to individual motors. Constant-speed squirrel-cage motors of compact construction and large overload capacity, with a speed of from 600 to 900 r.p.m., are usually installed for this work.

To prevent the filling up of the basin, in which the dredge floats, when digging in shallow water, it is sometimes found necessary to install a sand pump,

which carries the fine tailings from the sluice boxes to the top of the rock pile by way of the stacker.

Priming Pump. — A priming pump is required for priming the large pumps, or for supplying water on the tables during the "clean-up." It generally consists of a small centrifugal pump driven by a high-speed direct-connected squirrel-cage induction motor.

Screen. — Either a shaking or revolving screen is used to separate the gravel from the clay and permit the fine particles containing the gold to pass through on to the gold tables and sluices below. For this service a constant-speed motor is recommended, which can be placed on the upper deck and belted down to the driving pulley of the screen.

Stacker. — After screening the large rocks are carried on a belt conveyor to the end of the stacker and deposited on the spoil in the rear of the dredge. For operating this conveyor a constant speed motor is installed at the extreme end of the stacker, where it can be readily housed.

Monitor. — When a dredge is working in a high bank of hard material actual experience has demonstrated that it is advisable to lower the water level in the pit and cut the bank down with a "Giant," water being furnished by a three- or four-stage, high-pressure centrifugal pump driven by a phase-wound induction motor. The Giant is mounted on the bow of the boat, and as the dredge swings from one side of the cut to the other, the stream of water is thrown against the bank about two feet above the water level. This undermines the bank, causing the material to cave into the pond where it is brought up by the bucket line. The Giant is also used in localities where the water in the pond cannot be raised high enough to bring the bucket line near the top when the dredge is stepping ahead.

Power Required. — The sizes of motors required for driving the machinery of some dredges of different capacities are given in the following table. The proper speeds will depend on whether the motors are to be belted or direct connected to the machines which they are to drive.

SIZES OF MOTORS FOR DREDGES

Machinery to be driven	3 cu. ft. dredge, 40-ft. digging depth		5 cu. ft. dredge, 40-ft. digging depth		7 cu. ft. dredge, 40-ft. digging depth		13½ cu. ft. dredge, 40–50 ft. digging depth		16 cu. ft. dredge, 55-ft. digging depth	
	H.P.	R.p.m.	H.P.	R.p.m.	H.P.	R.p.m.	H.P.	R.p.m.	H.P.	R.p.m.
Bucket line	75	720	100	720	150	600	300	514	400	360
Winch	15	720	25	720	25	720	35	600	50	600
High-pressure pump	40	900	50	900	50	900	150	720	150	600
Low-pressure pump	25	600	25	600	25	600	75	600	100	600
Priming pump	7½	1200	7½	1200	7½	1200	25	1200	35	1200
Screen	25	900	25	900	25	600	75	600	150	600
Stacker	20	900	20	900	25	600	50	600	50	600

Voltage of Motors. — The power for operating a dredge is generally transmitted from some existing power system, and the connection between the line and the dredge is, as a rule, made by means of armored cable carried on floats. When this transmission can be economically carried out at a voltage of 2200, it is customary to provide the motors for this voltage, thus eliminating the expense of installing step-down transformers. When the transmission voltage is above 2200 volts, it is desirable to step it down to 440 volts and use motors for this voltage. When step-down transformers are required, their capacity is usually taken as two-thirds of the total horse-power load, allowing one kilowatt for each horse-power.

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[D. B. Rushmore, assisted by E. A. Lof.]

DYNAMOTORS. — (*See also Converters, Synchronous; Motor Generators; Transformers.*) A dynamotor is a direct-current device combining both motor and generator action in one magnetic field. It has an armature having two separate windings and two separate commutators, one at each end of the armature. Either winding may be used as the motor winding, and the other as the generator winding. Such a machine performs the same function in a direct-current circuit that a power transformer does in an alternating-current circuit, i.e., serves as a means of transforming high-voltage direct current into low-voltage direct current, or vice versa.

Performance Characteristics. — The device corresponds to two machines in which there is only one core-loss, one friction loss, one excitation loss, but two losses due to RI^2 in the two armature circuits. It is therefore more efficient than a motor-generator set, but less efficient than one machine. Since the currents in the two windings flow in opposite directions their resultant magnetic effect is zero. The machine has therefore no armature reaction (except for the slight amount due to the current to overcome the losses in the machine). It is not subject to the troubles of field distortion and bad commutation that occur in either motors or generators. It is impossible to compound a dynamotor, since any increase in field strength intended to increase the voltage of the generator would decrease the speed of the motor by the same amount and no change would result. The ratio of the two voltages is therefore fixed by the number of turns, and only varies from this by the loss due to RI drop in both windings. These two drops are additive, and therefore the regulation of such a machine is not very good.

Applications. — Dynamotors are used largely to give large currents to start other motors, or to give low voltages or a fractional voltage in a multi-voltage system for speed control. The motor of the combination may be wound for the line voltage and the generator for any fraction of the line voltage. Thus the combination supplies a large current at a low voltage, which will give a good starting torque in motors connected to it with a reasonable consumption of power. They are used as equalizers or "balancers" in three or multi-wire circuits, but are not as desirable for this work as motor-generator sets with compound-wound machines, on account of their poor regulation. They are also used to supply a low voltage for such purposes as telephone and telegraph systems and the low voltage and large currents for electrolytic work.

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[W. I. Slichter.]

ELECTRICITY AND MAGNETISM, PRINCIPLES OF. — In this article are given definitions of the various electric and magnetic quantities, together with a brief statement of the fundamental principles in accord with which all electric and magnetic phenomena take place.

The following is a brief table of contents of this article:

UNITS AND QUANTITATIVE LAWS. — Three different systems of units are in use in terms of which electric and magnetic quantities are expressed. These systems are known as the c.g.s. electrostatic system, the c.g.s. electromagnetic system and the practical system (*see also Units, Practical Electric*). The various units in the practical system have been given short names which are in common use, e.g., ampere, volt, coulomb, etc. For the sake of brevity the corresponding units of the electrostatic system will be designated by these same names with the prefix "stat," and the corresponding units in the c.g.s. electromagnetic system will be designated by these same names with the prefix "ab," the latter prefix arising from the term "absolute" sometimes applied to this system.

The use of the electrostatic and electromagnetic systems of units arises from the manner in which certain of the fundamental quantitative relations, or "laws," were originally formulated. The practical system is related to the electromagnetic system, by even multiples of ten times the units in the latter, e.g., 1 volt = 10^8 abvolts, 1 ampere = 10^{-1} abamperes, 1 ohm = 10^9 abohms.

The quantitative relations given below between *electric* quantities are independent of the system of units employed provided *all the quantities involved are expressed in the same system of units*. (*See Units and Conversion Factors.*) For some of the *magnetic* quantities, certain additional practical units have been introduced which are not related to the c.g.s. electromagnetic units by multiples of 10; consequently, to avoid confusion it is best to reduce all quantities to electromagnetic units before applying any of the formulas given.

WORKING HYPOTHESIS REGARDING THE NATURE OF ELECTRICITY. — The various facts of experience described as electric and magnetic phenomena can best be coördinated by assuming at the outset the existence of electricity as an actual entity, possessing the following properties:

1. Electricity exists in two forms, which for convenience may be called "positive" and "negative" electricity. These names arise from the fact that the two kinds of electricity produce certain effects which are just the opposite of each other, and, therefore, under certain conditions the external effect of one kind of electricity may completely neutralize the effect of the other kind.

A given quantity of electricity is called an "electric charge."

2. All matter contains electricity of both kinds, even when in the neutral or "uncharged" state. An uncharged body contains in every elementary portion (or molecule) equal quantities of positive and negative electricity, so thoroughly

"mixed" that no external effects are produced, unless the two kinds of electricity are constrained to take up a definite position or to move in a definite manner.

3. A body or portion of a body can be charged with electricity of either kind either by adding to it electricity of that kind or by taking away from it electricity of the other kind. In general only the *surface* of a body can be charged, i.e., the excess or deficit of charge exists only at the surface of separation between this body and some other body, the interior elements of the body containing, as in the neutral state, equal amounts of electricity of opposite kinds.

4. It is impossible to charge a body with electricity of one sign without producing on the same or on some other body an equal charge of the opposite sign.

5. Forces can be produced on the electricity in a body by various external agents, without necessarily producing a force on the body as a whole. When such forces are produced the two kinds of electricity are either displaced relatively to each other or are constrained to move through the body in opposite directions. This displacement or motion is always opposed by forces set up within the body itself. The opposing forces are of two kinds, (1) a force analogous to the opposing force set up in an elastic body when it is stretched or compressed, and (2) a force analogous to the resisting force due to mechanical friction produced when one body moves over another.

6. When the electricity in a body is acted upon by an external force, but cannot escape from that body to another, this force is transmitted to the body itself.

7. Electricity possesses inertia, just as ordinary matter possesses inertia; i.e., a force is required to change the motion of a charge of electricity just as a force is required to change the motion of a portion of matter. The "effective" inertia of a given quantity of electricity depends upon the path over which it is caused to move and upon the nature of the surrounding bodies (*see Electron Theory*).

Unit of Charge or Quantity of Electricity. — The unit of electric charge or quantity of electricity in the practical system of units is called the coulomb. For the relation between the coulomb, statcoulomb, abcoulomb, and other units of quantity see *Units and Conversion Factors*.

ELECTRIC FIELDS OF FORCE. — In any portion of a substance in which the electricity is acted upon by a force tending to move it, there is said to be an "electric field of force," or briefly an "electric field." An electric field is also said to exist in any region of free space where a charge, if placed there, would have a force exerted upon it tending to move it.

Intensity of an Electric Field (F). — The "intensity" of an electric field at any point is defined as the force per unit positive charge exerted on a charge at this point by the agent or agents producing the field i.e., by the agent or agents tending to move the charge. The direction of the field intensity, or the direction of the field, is defined as the direction of the force acting on a *positive* charge at this point. A positive charge then moves or tends to move in the direction of the field and a negative charge moves or tends to move in the opposite direction.

Units of Electric Field Intensity. — The unit of electric field intensity has not been given any special name, but since it is of the same nature as electromotive force per unit distance the intensity at any point may be conveniently expressed as so many volts, abvolts or statvolts *per centimeter* or *per inch;* see *Units and Conversion Factors*. Hence electric field intensity is frequently called the potential or voltage "gradient."

Lines of Electric Force. — Lines of Electric Intensity. — A line drawn

with the direction of the field at that point is called a "line of electric force." A line of force is usually a curved line, though in certain special cases it may be straight. Any number of such lines may be drawn in an electric field, but no two of these lines can intersect. The density of these lines, i.e., the number drawn through unit area perpendicular to their direction, may be chosen arbitrarily to represent the value of the field intensity at this area, and when so drawn are preferably called "lines of electric intensity," as distinguished from flux lines and stream lines defined below. The term "lines of force," however, is frequently used to designate any one of these three sets of lines, but this loose use of the term is liable at times to lead to much confusion. The term "line of electric force" will be used in this article to designate merely the *direction* of the field at any point; in any statement involving the *density* of these lines the proper one of the other terms will be employed.

Electric Equipotential Surfaces. — A surface drawn in an electric field in such a manner that it is perpendicular at each point to the line of force through that point is called an "electric equipotential surface." The electric intensity has no component along such a surface, and therefore no work is required to move a charge from one point to another over any path in such a surface.

ELECTROMOTIVE FORCE (E). — The work done by the field intensity F in moving unit positive charge around any *closed* path or circuit (Fig. 1) in an electric field is defined as the "electromotive force," abbreviated "e.m.f.," acting around this path. Electromotive force is not a force in the mechanical sense but is *work per unit charge*. The relation between electromotive force E and field intensity F is analogous to that between work and mechanical force viz.,

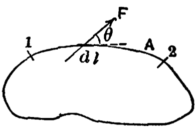


Fig. 1.

$$E = \int (F \cos \theta)\, dl, \qquad (1)$$

where dl represents an elementary length of the path, $(F \cos \theta)$ the *component* of the field intensity along dl and $\int$ represents the integral around this closed path. When the field intensity has the same value F at every point of a path and coincides in direction with the path at every point, the electromotive force acting around the closed loop is

$$E = Fl \qquad (1a)$$

where l is the total length of the path.

Units of Electromotive Force. — The practical unit of electromotive force is the volt; see *Units, Practical Electrical.* See also *Units and Conversion Factors* for the other units and their interrelations.

Sources of Electromotive Force. — The primary sources of e.m.f. are (1) two dissimilar bodies in contact (*see Batteries; Electrochemistry, Principles of*) and (2) a varying magnetic flux linking the circuit (*see the articles on Generators; Transformers, etc.*). The e.m.f. is said to be "located" at the surface of contact in the case of a "contact" e.m.f., or in that portion of the circuit which is linked by the varying magnetic flux in the case of an "induced" e.m.f., for experience shows that the value of the work integral, equation (1), around any circuit passing through a given surface of contact or linking a given varying flux is always the same, irrespective of the shape of the circuit and of the nature of substance through which the path passes, provided there are no other sources of e.m.f. in this path. Contact e.m.f.'s are usually extremely

small (of the order of 0.001 volt) except in the case of metallic substances in contact with electrolytes, in which case they may amount to several volts, see *Batteries; Electrochemistry, Principles of.* E.m.f.'s of 500,000 volts or more can be produced by a varying magnetic flux.

Direction of an Electromotive Force. — The direction of the e.m.f. *in* any portion of a circuit is taken as the direction in which a positive charge would be forced *around* a circuit containing only this one source of e.m.f. A closed circuit may contain several sources of e.m.f.; in this case the resultant e.m.f. acting around in the circuit is the *algebraic* sum of all these e.m.f's, the e.m.f.'s acting around the circuit in one direction being taken as positive and those acting around the circuit in the opposite direction being taken as negative. Those e.m.f.'s which act in the opposite direction to the resultant e.m.f. are called "back" or "counter" e.m.f.'s.

A convenient symbol for a source of constant e.m.f. is shown in Fig. 2; the long light line represents the positive terminal and the short heavy line the

Fig. 2.

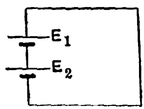


Fig. 3.

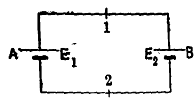


Fig. 4.

negative terminal. Fig. 3 shows two e.m.f.'s acting around a circuit in the same direction, and Fig. 4 shows two e.m.f.'s acting around the circuit in opposite directions. In the first case the resultant e.m.f. is $E_1 + E_2$ and the two e.m.f.'s are said to be "in series," and in the second case the resultant e.m.f. acting in the right-handed direction around the circuit is $E_1 - E_2$ and the two e.m.f.'s are said to be "in opposition." When E_2 is less than E_1 then E_2 is a back or counter e.m.f.

DIFFERENCE OF ELECTRIC POTENTIAL (V). — Consider a portion of a path A between any two points 1 and 2 in an electric field (Fig. 1). The total work done on unit charge, *by the e.m.f.'s external to this portion of the path* when unit charge moves from 1 to 2, is called the "drop of electric potential" from 1 to 2. The term "difference" of electric potential is also commonly used to designate this quantity; the term "drop" is preferable since it signifies the direction of the difference. Electric potential drop is of the same nature as electromotive force and is expressed in the same units. It is frequently abbreviated "p.d." Potential drop may be due either (1) to a "back" electromotive force, analogous to the back pressure of a pump, or (2) to an opposing force analogous to that due to the resistance of a pipe to the flow of water.

Let the path from 1 to 2 be in the direction of the line of force from 1 to 2, let E_{12} be an e.m.f. whose source is between 1 and 2 and whose direction is from 1 to 2, then the general expression for the potential drop from 1 to 2 is

$$V_{12} = \int_1^2 F\,dl - E_{12}. \tag{2}$$

The first term represents the "resistance" drop and $-E_{12} = E_{21}$ represents the "back" pressure. If the field intensity along the path from 1 to 2 is negligible, then $V_{12} = -E_{12}$; hence an e.m.f. in a given direction is equivalent to a negative

(2) with equation (1) it is evident that the total drop of potential around a *closed* circuit is always zero. This relation is conveniently expressed by the formula

$$\Sigma V = 0 \qquad (3)$$

which holds for every closed path or circuit. This is one way of stating Kirchhoff's second "law."

Voltage and Voltage Gradient. — The term "voltage" is commonly used for either an e.m.f. or a potential difference, particularly when these quantities are expressed in volts. Since from equation (2) the field intensity F is equal in volts to the drop in voltage per unit length the electric field intensity is also frequently called the voltage "gradient."

Positive and Negative Terminals. — That terminal of a device which is at the higher potential is called the positive terminal, the other terminal being called the negative terminal. The drop of potential is *always* from the positive to the negative terminal, irrespective of the direction of flow of electricity through the device.

FLOW OF ELECTRICITY. — Whenever an electric field is set up in a substance by any means whatever a displacement of the electricity in that substance always takes place, the nature of the displacement depending upon the nature of the substance. In every case the positive electricity within the substance is displaced in the direction of the field intensity and the negative electricity in the opposite direction, until an opposing force of some kind is set up which just balances the forces due to the impressed field.

Conductors and Insulators or Dielectrics. — A substance in which a *constant* electric field is always accompanied by a *continuous* displacement or *flow* of electricity through the substance is called an "electric conductor;" when there is no flow of electricity through a conductor the field within the conductor is *always* zero. Every substance is a conductor of electricity, at least to a slight extent, but some substances are far better conductors than others. A substance in which a constant electric field produces only a relatively small continuous flow of electricity is called an "electric insulator" or "dielectric." A *perfect* dielectric may be defined as a substance through which it is impossible to set up a continuous flow of electricity; no such substance exists but very poor conductors are approximately such. Free space may be considered a perfect dielectric, but free space differs from a substance in that it does not contain electricity.

Metals are the best conductors, carbon and most moist substances are fair conductors; dry non-metallic bodies such as air and other gases (at normal pressure), porcelain, glass, rubber, and dry paper are very good insulators.

A wire covered with an insulating substance or supported on insulating substances is said to be "insulated," even though its ends are connected to a source of e.m.f.

Electric Current (I). — The displacement of the electricity within a substance cannot be measured directly, but only in terms of some effect produced thereby. Two effects which always occur when a displacement of electricity is produced are (1) a magnetic field is established around the path along which the displacement takes place (but disappears when the electricity comes to rest) and (2) heat is developed in the path of the displacement. The magnetic field produced by a displacement or flow of electricity is usually taken as the measure of the *rate* of flow, i.e., of the quantity of electricity displaced per unit time through a surface perpendicular to the direction of the displacement. This rate of flow is called the "intensity" of the electric current, or simply the "electric current," and is usually represented by the symbol I.

Direction of the Electric Current. — A flow of positive electricity in one direction is equivalent magnetically to a flow of an equal amount of negative electricity in the opposite direction; hence the total flow along a given path is the sum of the positive electricity displaced per unit time in one direction past a point in this path plus the negative electricity displaced per unit time past this point in the opposite direction. The "direction" of the electric current is taken as the direction in which the positive electricity is displaced, and is, therefore, the same as the direction of the field intensity.

Current due to Varying Electric Field. — When the electric field in any substance is *varying*, the total magnetic effect produced is found to depend not only upon the rate of displacement of the electricity within the substance, but also upon the *rate of change* of the electric field. In fact, a magnetic field is produced around a path in *free space* along which the intensity of the electric field is varying, see *Waves, Electromagnetic*. In dealing with varying electric fields it is found convenient to consider the variation of the field intensity as equivalent to an actual flow of electricity, and to take as the total *equivalent* electric current the flow of electricity which would produce the same magnetic field as that actually observed; the *actual* flow of electricity is then in general less than this equivalent current, but the difference is negligible except in substances which are good insulators.

Units of Electric Current. — The practical unit of electric current is the ampere; see *Units, Practical Electrical*. For the relation between this unit and the statampere and abampere see *Units and Conversion Factors*.

Continuity of an Electric Current. — When a varying electric field is considered as equivalent to an electric current, it is found that the total equivalent current coming up to any point or surface in any network of circuits, no matter how complicated, is *always* equal to the total current leaving that point or surface, irrespective of the nature of the substances through which the currents are established and irrespective of whether the currents are constant or are varying. For example, in Fig. 5,

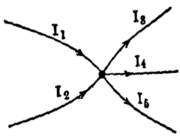


Fig. 5.

$$I_1 + I_2 = I_3 + I_4 + I_5. \tag{4}$$

Or, calling the currents coming up to any point positive, and the currents leaving that point negative, the *algebraic* sum of all the currents at any point in a network of circuits is always zero. This fundamental principle is conveniently expressed by the formula

$$\Sigma I = 0 \tag{5}$$

which holds at every junction point both for variable and for continuous currents. This is Kirchhoff's first "law."

Stream Lines of Electric Current. — Current Density (σ). — As a consequence of the continuity of an electric current, the total current in any substance of any size or shape may be looked upon as made up of a number of small streams of electricity flowing side by side, the strength (quantity of electricity per second) of each stream being constant throughout its length. If the cross-section of each stream at any point is so chosen that each stream represents unit current (unit quantity per second), then the number of these streams crossing unit area of a surface perpendicular to their direction will be equal to the current per unit area of this surface, or to the "current density" at this surface. Each such stream may be represented graphically by a line coinciding with its axis; such lines are called "stream lines." When the stream

the current at this point and the number of these lines per unit area perpendicular to their direction is equal to the current density at this point.

In the case of an insulated wire the stream lines are parallel to the axis of the wire, except in the immediate vicinity of its ends; this non-uniformity at the ends in the case of a long wire is negligible. The stream lines in an ordinary wire may also, in most practical cases, be considered as all coinciding, and the wire may, therefore, be treated as a geometrical line as regards *external* effects. However, in the case of a short rod or strip (such as an ammeter shunt) connected in the circuit by wires attached to its ends, the stream lines are not in general parallel but diverge from one terminal and converge toward the other.

Continuous, Direct, Pulsating, Alternating and Oscillatory Currents.— A "continuous" electric current is defined as a current which does not vary with time. A "direct" current is a current which is always in the same direction but may vary or pulsate in value. The term "direct current" is ordinarily used to designate either a continuous current or a current which varies or pulsates only by an inappreciable amount, such as the current from a battery or direct-current generator (q.v.). A "pulsating" current is a direct current which pulsates by an appreciable amount, such as the current from a rectifier (q.v.). An "alternating" current is a current which reverses in direction, being first positive and then negative, but alternates between *constant* maximum positive and negative values; see article on *Alternating Currents*. An "oscillatory" current is a current which reverses in direction, oscillating between positive and negative values which either decrease or increase with time.

Conduction Current and Displacement Current. — Experience shows that when an electric field is established in any substance, the total equivalent electric current set up depends (1) upon the value of the field intensity, (2) upon the rate of change of the field intensity, and (3) upon the nature of the substance in which the field is established. The current density σ at any point at any instant may in general be expressed by the relation

$$\sigma = \gamma F + \frac{1}{4\pi}\frac{d}{dt}(kF), \tag{6}$$

where F is the field intensity, γ and k are coefficients depending upon the chemical nature and physical condition of the substance at the point in question, and $\frac{d}{dt}$ means the rate of change with respect to time, the factor 4π arising from the historical definition of the quantity k, see below. The total current may then be considered as the sum of two components having respectively the densities

$$\sigma_1 = \gamma F \quad \text{and} \quad \sigma_2 = \frac{1}{4\pi}\frac{d}{dt}(kF). \tag{6a}$$

The first of these components (σ_1) is called the "conduction" current, and the second (σ_2) is commonly called the "displacement" current. The term "displacement" current, however, is somewhat a misnomer, for both components of the total current are probably due, in part at least, to a displacement of electricity; whereas a displacement current is set up by a varying electric field in a *free space*, although there is probably no electricity in free space to be displaced. The conduction current is the only appreciable component in substances usually classed as conductors, and the displacement current is appreciable only in substances ordinarily classed as dielectrics. The conduction current in a dielectric is usually small, though measurable; it is frequently called the "leakage" current. When the electric field in a dielectric is rapidly varying the displacement current may be many times greater than the conduction or leakage current through the dielectric.

Conductivity (γ) and Resistivity (ρ). — The quotient of the density σ_1 of the conduction current by the field intensity F, i.e., the coefficient γ in the expression $\sigma_1 = \gamma F$ is called the "conductivity" of the substance at the point in question. Since in an ordinary conductor the displacement current is inappreciable, the conductivity of an ordinary conductor is also equal to the density of the total current divided by the field intensity, i.e., for a conductor

$$\sigma = \gamma F, \tag{7}$$

where σ represents the density of the total current. Experience shows that for a given conductor at constant temperature (and also at constant pressure in case of a gas) this coefficient γ is a constant irrespective of the strength, distribution or time variation of the current. The value of γ for a dielectric, however, is not in general a constant but depends upon the time variation of the field intensity; see below under *Capacity and Condensers*.

The above relation between σ and F may also be written

$$F = \rho\sigma, \tag{7a}$$

where ρ is the reciprocal of the conductivity γ. The constant ρ is called the "resistivity" or "specific resistance" of the substance. Values of γ and ρ for various conductors are given in the article on *Resistance and Conductance*, and also the methods of measuring these quantities. The values of γ and ρ for some of the more common insulating materials are given in the article on *Insulating Materials, Properties of*. For the units of conductivity and resistivity see *Units and Conversion Factors* and also the next two paragraphs.

Conductance (g) and Resistance (r). — To cause a *continuous* current I to flow through a given conductor in which there is no source of e.m.f. a difference of potential must be established by some external agent between the ends of the conductor. Let V be the potential drop established through the conductor, from any point 1 to any other point 2, then the quotient

$$g = \frac{I}{V} \tag{8}$$

is defined as the "conductance*" from 1 to 2, and the quotient

$$r = \frac{V}{I} \tag{8a}$$

is defined as the "resistance*" from 1 to 2. Conductance is the reciprocal of resistance and vice versa. The practical unit of resistance is called the ohm, and the practical unit of conductance is called the mho; see *Units and Conversion Factors*.

It should be carefully noted that the definitions expressed by equations (8) and (8a) hold only when there is no e.m.f. in the portion of the circuit under consideration; this condition is realized only when the current remains constant in value (i.e., a *continuous* current) and the conductor is of uniform material and at constant temperature throughout. Also, the definition is meaningless unless the same current flows through each cross-section of the conductor and the drop of potential is the same between all points in the two end surfaces; i.e., the end surfaces must be equipotential surfaces.

Factors Upon Which Conductance and Resistance Depend. — The resistance or conductance of a given portion of a conductor included between

* Frequently called the "ohmic" or "d-c." conductance and resistance respectively to distinguish these quantities from the "effective" or "a-c." conductance and resistance; see article on *Resistance and Conductance*.

two equipotential surfaces and bounded laterally by a surface through which no stream line passes, depends upon (*a*) the conductivity of the conductor (*b*) the dimensions of this portion of the conductor, and (*c*) the distribution of the stream lines over each cross-section perpendicular to them. Consider the case of a straight wire or bar over the cross-section of which the current is uniformly distributed. Let l be the length of the bar, A its cross-section and ρ its resistivity. Then from equation (2) the drop of potential along the length l is $V = Fl$; from equation (7a) $F = \rho\sigma$; therefore $V = \rho\sigma l$. The total current $I = \sigma A$; whence the resistance and conductance are

$$r = \rho\frac{l}{A} \quad \text{and} \quad g = \gamma\frac{A}{l}. \tag{9}$$

Hence, the conductivity of a substance is equal to the conductance, and the resistivity is equal to the resistance, of a unit cube of this substance when the current through this cube is parallel to four edges of the cube and is uniformly distributed over the section at right angles to these four edges. Hence conductivity is frequently expressed as mhos per centimeter cube or inch cube; see *Resistance and Conductance; Units and Conversion Factors.*

Equation (9) is applicable only to a conductor in which the stream lines are uniformly distributed and parallel to each other, such as in insulated wires carrying continuous or comparatively slowly varying currents. When the stream lines are not parallel to each other, let dl represent an elementary length along any stream line and σ the current density at dl; then taking the integral along the stream line from one end surface to the other, the resistance is

$$r = \frac{\rho\int\sigma\,dl}{I} \tag{9a}$$

where I is the total current. The calculation of the value of σ at each point along the line from one surface to the other is difficult except in certain special cases of symmetry; see however, equation (22) below.

Ohm's Law and Resistance Drop. — From the relations noted above it is evident that, in a conductor (*a*) which remains at constant temperature, (*b*) in which there is no internal e.m.f., and (*c*) in which the distribution of stream lines remains unaltered *the quotient of the voltage by the current is a constant.* This relation is known as Ohm's Law; it holds only when the three conditions specified above are fulfilled.

The above relation may also be written $V = rI$ which is merely another form of equation (2); i.e., if r is the resistance between any two points 1 and 2 in an electric circuit in which the current is I, then the integral of the electric field intensity along the path from 1 to 2 is equal to rI. This drop of potential due solely to resistance between 1 and 2 is called the "resistance drop" from 1 to 2, and is always in the direction of the current. When there is an e.m.f. in the circuit between 1 and 2 and the distribution of current remains the same,* the total drop from 1 to 2, from equation (2), is

$$V_{12} = rI_{12} - E_{12}, \tag{10}$$

the subscripts indicating the directions of the various quantities. Equation (10) is sometimes referred to as the "modified" Ohm's law.

* In general, the distribution of current depends upon the time variation of the voltage; see article on *Skin Effect.*

Terminal and Impressed Voltages. — The application of this equation is best shown by considering a simple circuit containing a generator and a motor or other receiving device, Fig. 6. Let I be the current in this circuit, let R_g be the internal resistance of the generator, R_m the internal resistance of the motor and R_l the total resistance of the "line" or connecting wires, let E_g be the e.m.f. developed by the generator and E_m the back e.m.f. developed by the motor, and let V_g be the terminal voltage of the generator and V_m the terminal voltage of the motor. Through the generator the current flows from the − to the + terminal, the e.m.f. is from the − to the + terminal, and the drop of potential is from the + to the − terminal. Hence

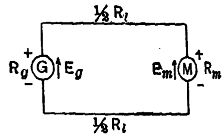


Fig. 6.

$$V_g = E_g - R_g I, \tag{10a}$$

i.e., the terminal voltage is less than the generator e.m.f. by an amount equal to the resistance drop in the armature. Through the motor the current flows from the + to the − terminal, the e.m.f. is from the − to the + terminal, and the drop of potential is from the + to the − terminal. Hence

$$V_m = E_m + R_m I, \tag{10b}$$

i.e., the terminal voltage is greater than the back e.m.f. of the motor by an amount equal to the resistance drop through the motor.

The terminal voltage of the motor is less than that of the generator by an amount equal to the resistance drop in the line, i.e.,

$$V_m = V_g - R_l I. \tag{10c}$$

The expression "impressed" electromotive force is also used to designate the rise of potential from the negative to the positive terminal of any receiving device, whether it be a motor, bank of lamps, or any "straight" resistance, i.e., a resistance containing no source of electromotive force.

Series Circuits. — When several conductors are connected end to end so that the *same current* flows through each of them (Fig. 7), they are said to be connected "in series." Let I_{12} be the current in each conductor in the direction from 1 to 2, let R', R'', R''', etc., be the resistances of the various conductors and E'_{12}, E''_{12}, E'''_{12}, etc., be the e.m.f.'s in the circuit between 1 and 2 in the direction from 1 to 2. Then the potential drop from 1 to 2 is

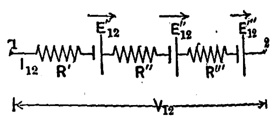


Fig. 7.

$$V_{12} = R'I_{12} - E'_{12} + R''I_{12} - E''_{12} + R'''I_{12} - E'''_{12}, \text{ etc.}$$

Therefore, the resistances between the points 1 and 2 are equivalent to a single resistance

$$R = R' + R'' + R''', \text{ etc.,} \tag{11}$$

and the e.m.f.'s between the points 1 and 2 are equivalent to a single e.m.f.

$$E_{12} = E'_{12} + E''_{12} + E'''_{12}, \text{ etc.} \tag{11a}$$

The equivalent conductance of several conductances G', G'', G''', etc., in series *when there are no e.m.f.'s in the path*, is G where

$$\frac{1}{G} = \frac{1}{G'} + \frac{1}{G''} + \frac{1}{G'''} + \dots \tag{11b}$$

Parallel Circuits. — When several conductors are connected to two common junction points so that the *same potential drop* is established through each (Fig. 8), they are said to be "in parallel." Let the currents, e.m.f.'s and resistances be as designated in the figure. Then from equation (5)

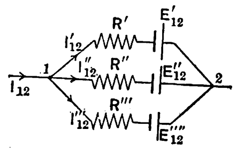


Fig. 8.

$$I_{12} = I'_{12} + I''_{12} + I'''_{12} + \text{etc.}$$

and from equation (10a)

$$V_{12} = R'I'_{12} - E'_{12} = R''I''_{12} - E''_{12} = R'''I'''_{12} - E'''_{12} = \text{etc.},$$

from which relations the currents in the individual branches may be calculated.

When there are no e.m.f.'s in the various branches, the combined resistance of the several branches from 1 to 2 is R where

$$\frac{1}{R} = \frac{1}{R'} + \frac{1}{R''} + \frac{1}{R'''} + \dots, \tag{12}$$

and the combined conductance is

$$G = G' + G'' + G''' + \dots, \tag{12a}$$

where G_1, G_2, G_3, etc., are the individual conductances.

Two Parallel Circuits. — In the special case of two conductors in parallel, and no e.m.f. in either, the combined resistance is

$$R = \frac{R_1 R_2}{R_1 + R_2}. \tag{12b}$$

Series-parallel Circuits. — When a circuit is made up of several conductors some of which are in series and some in parallel, it is called a "series-parallel" circuit. The total resistance or conductance of such a circuit can be calculated from the constants of the several branches by applying successively the formulas for series and for parallel circuits.

Kirchhoff's Network Laws. — The relations given above for conductors in series and in parallel are special cases of two general laws already stated, namely:

1. The algebraic sum of the currents coming up to any junction in a network of conductors is always zero (equation 5).
2. The algebraic sum of the potential drops around any closed loop in a network of conductors is always zero (equation 3).

These two laws are known as Kirchhoff's Laws, from the name of the scientist who first clearly enunciated them. By making use of them one can always predetermine (*a*) the current in each branch of a network when the resistance of each branch and the e.m.f. in each branch are known, or (*b*) the e.m.f. in each branch when the current in each branch and the resistance of each branch are known.

It should be carefully borne in mind in applying these laws that a current leaving a point is equivalent to a negative current *entering* that point, and that

an e.m.f. in any chosen direction is equivalent to a *rise* of potential in that direction. In working out any problem concerning a network of circuits it is convenient to make a diagram of the network and to place on each branch in this diagram a number or symbol to represent the value of the current in this branch and an arrow or subscripts to indicate the direction of the current represented by this number or symbol, and wherever there is an e.m.f. to place a number or symbol to represent the value of this e.m.f. and an arrow or subscripts to indicate its direction. Then at any junction point those currents represented by arrows pointing toward the point are to be considered positive (say) and those represented by arrows pointing away from the point are to be considered negative; and for any closed loop those currents and e.m.f.'s represented by arrows pointing around the loop in the clockwise direction (say) are to be considered positive and those pointing around the loop in the counter-clockwise direction are to be considered negative. With this understanding, these laws may be written

$$\Sigma I = 0 \text{ at every point,} \tag{13a}$$

$$\Sigma E = \Sigma rI \text{ for every closed loop,} \tag{13b}$$

where I, r and E represent the current, the resistance and the e.m.f. respectively in each branch of the loop, and the symbol Σ indicates the algebraic sum of the quantities following it.

These equations enable one to write down a set of simultaneous equations for the given network, but it will be found that at least one of the current equations may be derived directly from the other current equations, and that at least one of the potential equations may be derived from the other potential equations. That is, the number of independent equations of each form will be one less than the number which it is possible to write down. It should also be noted that it is frequently unnecessary to write down formally all the possible independent equations; many of the simpler problems can be solved by writing down two independent expressions for the potential drop between each pair of points and equating these two expressions.*

When the currents and e.m.f.'s are *varying*, the first law applies to the total current (conduction and displacement current), but the second law in the form given in (13b) applies only to closed loops of conduction current; in both cases the instantaneous values of the currents and e.m.f.'s must be taken when the quantities are varying.

ELECTRIC ENERGY AND POWER. — From the definition of potential drop it follows that the total work done by the external agents in forcing Q units of electricity from any point 1 in a circuit to any point 2 is $W = VQ$. From the definition of electric current the quantity of electricity carried from 1 to 2 when the current I is established from 1 to 2 is $Q = It$, where t is the time during which the current exists. Hence when a current I is established through any device for a time t by an impressed voltage V, the energy input to this device is

$$W = VIt \tag{14}$$

and the power input, i.e., energy input per unit time, is

$$P = VI. \tag{15}$$

When V and I are expressed in volts and amperes respectively the power input is in watts; if t is in seconds, the energy input is in joules or watt-seconds. When V and I are in statvolts and statamperes respectively, or in abvolts and abam-

* The application of these laws to the solution of complicated network problems is

peres respectively, the power input is in ergs per second, if t is in seconds the energy input is in ergs. See *Units and Conversion Factors.*

Applying the above relations to the simple circuit shown in Fig. 6 containing a generator and a motor (armature windings only are considered), the power *input* to the motor armature is

$$P_i = E_m I + R_m I^2, \tag{15a}$$

the power *output* of the generator armature is

$$P_0 = E_g I - R_g I^2 \tag{15b}$$

and the power lost in the line is

$$P_l = R_l I^2.$$

The term $R_g I^2$ represents the power lost in heating the armature winding of the generator due to its resistance, and the term $R_m I^2$ represents the power dissipated as heat in the armature winding of the motor. The net electric input to the generator armature is $E_g I$ and the gross mechanical output of the motor armature is $E_m I$. The gross mechanical input to the generator is greater than $E_g I$ and the net mechanical output of the motor is less than $E_m I$ by an amount equal to the friction and "core-loss" in the respective machines; see *Generators and Motors.*

Joule's Law.—That portion of the power input to any device which is equal to the product of the resistance of the conductors forming the winding of the device by the square of the current through this winding is always converted into heat. That is, when a current I flows through a resistance r heat is always "dissipated" in this resistance, and the rate of dissipation is

$$P_h = rI^2. \tag{15c}$$

This experimental fact is known as "Joule's Law." This law applies directly only to continuous or non-varying currents. The relation $P_h = rI^2$ is, however, used as the basis for defining the "effective" resistance of a conductor to an alternating current (q.v.).

DIELECTRIC FLUX (ψ) AND DIELECTRIC FLUX DENSITY (D).—As noted above the displacement current through a dielectric at any point depends upon the rate of change of the electric field intensity and upon the nature of the dielectric, and the density of this displacement current at any point may be expressed by the relation

$$\sigma_2 = \frac{1}{4\pi}\frac{d}{dt}(kF), \tag{16}$$

where F is the field intensity and k a coefficient depending upon the nature of the dielectric. The coefficient k in this expression is the so-called dielectric coefficient or dielectric constant.

The quantity kF, whose rate of change is equal to 4π times the density of the displacement current, is called the "dielectric flux density," and may be represented by the symbol D. Then

$$D = kF. \tag{17}$$

The direction of the dielectric flux density is arbitrarily chosen to be the same as that of the electric field intensity F. Through any surface of area A at each point of which the dielectric flux density has a constant value D and is perpendicular to that surface, there is said to exist a "dielectric flux" equal to DA. The total dielectric flux through a surface may be represented by the symbol ψ.

In general, the total dielectric flux through any surface perpendicular at each point to the direction of the field intensity at that point (i.e., through an equipotential surface) is

$$\psi = \int D\, ds, \tag{18}$$

where ds represents any elementary area of this surface and D the flux density per unit area at ds, and $\int$ represents the sum of all the products $D\,ds$ for that surface. The total displacement current through this surface is then

$$i = \frac{1}{4\pi}\frac{d\psi}{dt}. \tag{19}$$

Lines of Dielectric Flux. — The electrostatic flux through any surface may be represented by lines drawn in the same direction as the lines of electric intensity, but of such a density that *their number per unit area* perpendicular to their direction at any point *is equal* to the *dielectric flux density* at this point. The number of these lines cutting any surface is then equal to the total dielectric flux through this surface. The ratio of the number of flux lines through any surface to the number of lines of electric intensity through that surface is equal to the dielectric coefficient of the substance in which the field exists.

ELECTRIC CHARGE (Q) AND DIELECTRIC FLUX (ψ). — *Within* any substance of uniform structure throughout the dielectric flux lines are continuous lines, i.e., the number of these lines coming up to one side of a surface *within* such a substance is equal to the number of these lines leaving the other side of that surface. Experience shows that it is impossible to produce an appreciable dielectric flux in those substances ordinarily classed as conductors; *hence dielectric flux lines cannot pass through a good conductor*, but terminate at its surface. Every dielectric is a conductor to at least a slight extent, and on account of this fact the dielectric flux lines coming up through one dielectric to the surface of contact between this dielectric and another do not all pass through the second dielectric, but some of them terminate at this surface.

Experience shows that to establish an electric field in the dielectric around a conductor, electricity must be *conducted through the conductor* to the surface of contact between the conductor and the dielectric. For example, consider a good conductor in contact with a perfect dielectric, Fig. 9; a momentary conduction current must flow through the conductor along the stream lines of the conduction current, represented by the dotted lines. While the field is being established (and therefore varying) a displacement current is set up in the dielectric requiring an equal conduction current in

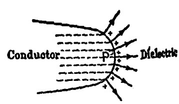


Fig. 9.

the conductor, and consequently $\frac{1}{4\pi}$ times the rate of change of the dielectric flux (ψ) established in the dielectric must be equal to the conduction current (i) flowing up to this surface through the conductor, i.e.,

$$\frac{1}{4\pi}\frac{d\psi}{dt} = i,$$

or

$$\psi = 4\pi \int i\, dt = 4\pi Q, \tag{20}$$

where Q is the quantity of electricity (current multiplied by time) conducted through the conductor to this surface.

This relation is a general one, viz., the total dielectric flux from any area A in the surface of a *conductor* is equal to 4π times the total charge on this area. Hence every flux line originates at a positively charged conducting surface and terminates at a negatively charged conducting surface, 4π of these lines connecting each unit positive charge to each unit negative charge.

The quantity of electricity conducted through a conductor when a momentary current is established through it can be readily measured by means of a ballistic galvanometer (*see Galvanometers*) and consequently the dielectric flux (equal to $4\pi Q$) may be readily determined.

Units of Dielectric Flux. — Dielectric flux may be expressed in the same unit as electric charge, viz., coulombs, statcoulcombs or abcoulombs; see *Units and Conversion Factors*.

Surface Density of Charge (σ_c) and Dielectric Flux Density (D). — When there is no current in a conductor there can be no electric field within it, see equation (7), and therefore *the surface of a conductor in which no current is flowing is always an equipotential surface.* Hence the lines of electrostatic intensity, in the surrounding dielectric, and therefore the dielectric flux lines also, must leave or enter this surface in a direction perpendicular to it. The dielectric flux density in the dielectric just outside a conducting surface in which there is no electric current is perpendicular to this surface and is

$$D = 4\pi\sigma_c, \tag{20a}$$

where σ_c is the charge per unit area of the surface at this point, or the "surface density" of the charge.

Dielectric Flux Density Due to a Number of Charged Conductors. — It can be shown that when any number of charged conductors are surrounded by a *uniform* dielectric, the dielectric flux density at any point in the field may be expressed by considering each elementary surface having a charge q as producing at any point P at a distance r from q a flux density equal to q/r^2, in the direction of the line from q to P when q is positive and in the direction of the line from P to q when q is negative. The total flux density at P due to all the charges is then the *vector summation*

$$D = \sum \frac{q}{r^2}. \tag{20b}$$

Dielectric Flux Density at any Point due to a Uniformly Charged Wire of Circular Cross Section and Infinite Length. — Let q be the charge per unit length, K the dielectric coefficient of the surrounding dielectric, assumed uniform, and r the perpendicular distance from the center of the wire to any point P, then the dielectric flux density at P due to this wire is

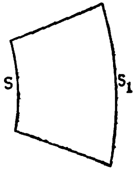
$$D = \frac{2q}{r} \tag{20c}$$

The resultant dielectric flux density at any point due to any number of *uniformly* charged wires is the vector sum of the flux densities due to each separately. However, when two or more wires are close together the distribution of charge on them is not uniform,* but when the wires are more than 10

* The distribution can be calculated readily in the case of two parallel wires; see Pender and Osborne, *Elec. World*, *1910*, *Vol. 56*, *p. 667*.

diameters apart the error introduced by the assumption of uniform charge is practically negligible.

ELECTROSTATIC CAPACITY (C) AND CONDENSERS.—To establish a given dielectric flux ψ through a given dielectric a certain difference of electric potential is always required. Consider any portion of an electric field (Fig. 10) between the two equipotential surfaces S and S_1 bounded laterally by a surface tangent at each point to the flux line through that point. Let V be the drop of potential from S to S_1, and let ψ be the dielectric flux through this region. When there is no source of e.m.f. between S and S_1 the quotient



$$C = \frac{\psi}{4\pi V} \tag{21}$$

Fig. 10.

is defined as the "electrostatic capacity" of this portion of the field. Compare with the definition of conductance, equation (8).

When the equipotential surfaces S and S_1 are the surfaces of two conductors, the two conductors and the dielectric between them are said to form an "electric condenser." Practical forms of condensers are described in the article on *Condensers, Electric*. When all the flux lines from one conductor end on the second conductor (e.g., when they are given equal and opposite charges by connecting them respectively to the two terminals of a source of e.m.f.), then the flux from one to the other is equal to $4\pi Q$ where Q is the numerical value of the total charge on either conductor. The capacity of the condenser may then be written

$$C = \frac{Q}{V}. \tag{21a}$$

When there are several charged conductors in the field the total flux from one conductor does not in general all end on another single conductor, but, some of the flux lines from No. 1, say, may run to No. 2, some to No. 3, etc. Let ψ_{12} be *that portion* of the flux from any conductor 1 which ends on any other conductor 2, and let V_{12} be the drop of potential from 1 to 2, then the capacity between conductor No. 1 and conductor No. 2 is

$$C_{12} = \frac{\psi_{12}}{4\pi V_{12}}. \tag{21b}$$

Or, calling Q_{12} that portion of the charge on No. 1 which is balanced by an equal and opposite charge on No. 2, the capacity between 1 and 2 is

$$C_{12} = \frac{Q_{12}}{V_{12}}. \tag{21c}$$

Units of Capacity.—The unit of capacity in the practical system of units is the farad (*see Units, Practical Electric*), but as this is a very large unit, a unit equal to one-millionth of a farad, called the microfarad, is usually employed. The c.g.s. electrostatic unit may be called the statfarad and the c.g.s. electromagnetic unit the abfarad. See *Units and Conversion Factors.*

Factors Upon Which Capacity Depends.—The capacity of a given portion of a dielectric depends upon (a) the dielectric coefficient k of the dielectric, (b) the length of the dielectric flux lines through it, (c) the cross-section of the dielectric at right angles to the flux lines, and (d) upon the distribution of the flux lines over this cross-section. Compare with electric conductance. In

general, the capacity of any portion of a dielectric bounded laterally by flux lines and at the ends by equipotential surfaces (Fig. 10) can be expressed by the formula

$$C = \frac{k\psi}{4\pi \int D\,dl}, \tag{21d}$$

where k is the dielectric coefficient, ψ the total dielectric flux through the given portion of dielectric, dl any elementary length along one of the flux lines and D the dielectric flux density at this point, the integral being taken along the flux line from one end surface to the other. When the end surfaces are conductors charged with $+Q$ and $-Q$ units respectively, then

$$C = \frac{KQ}{\int D\,dl}. \tag{21e}$$

By the application of this formula the capacity of various practical forms of condensers may be calculated; see the article on *Capacity and Charging Current, Calculation of.* It should be noted that the capacity of a condenser depends upon the *distribution* of the dielectric flux (k being assumed constant), but not upon the absolute value of the flux; i.e., for a given dielectric and given distribution of flux the capacity is a constant. In general, when any conductor or dielectric of a different specific inductive capacity is placed in the electric field set up by the charged plates of a condenser, the distribution of the flux, and therefore the capacity of the condenser, is altered.

Relation Between Conductance and Capacity. — Comparing equations (9a) and (21d), it is apparent that when the dielectric flux lines and the current stream lines have the same distribution in any given region, the ratio of the conductance of this region is $4\pi\gamma/k$ where k is the dielectric coefficient and γ the conductivity of the material in this region. Hence the formulas for the capacity and conductance of the dielectric between the plates of any shape or size of condenser differ only by a constant coefficient. That is, if C is the capacity of any condenser, then

$$g = \frac{4\pi\gamma}{k} \cdot C \tag{22}$$

is the conductance of the dielectric between its plates. Values of C for various cases are given in the article on *Capacity and Charging Current.*

Charge and Discharge of a Condenser. — To charge a condenser a difference of electric potential must be established between its plates. This may be done, as noted above, by connecting the two plates of the condenser respectively to the two terminals of any source of e.m.f., see Fig. 11. If the dielectric has a very high resistance and the source of e.m.f. has a constant value E, the current set up in this circuit will continue only until a difference of potential equal to E has been established across the two plates of the condenser, or until a charge equal to CE has been transferred from the "negative" to the "positive" plate of the condenser. The establishment of the electric flux through the dielectric of the condenser may be looked upon as setting up in the dielectric itself an opposing force analogous to the opposing force set up in a spring when it is compressed. When the opposing force just balances the impressed force a steady state is attained, just as the compressing of a spring ceases

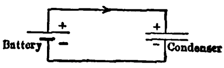


Fig. 11.

when the force producing the compression is just balanced by the opposing force due to the elasticity of the spring.

When a condenser has thus been charged, the wires connecting it to the source of e.m.f. may be removed and the condenser remains charged for a length of time depending upon the resistance of the dielectric separating the plates; the higher this resistance the longer the time that the condenser remains charged. The plates may also be moved apart and they still retain their charges, one plate a positive charge and the other a negative charge, but the distribution of these charges on the plates will in general become altered. Experience shows that a mechanical force is required to separate the charged plates irrespective of whether or not they are connected to the source of e.m.f.

When the two charged plates are "short-circuited" by a wire, as shown in Fig. 12, a momentary current is established through the wire and the electric field between the plates and the charges disappear. The quantity of electricity discharged through the wires is equal to the quantity of charge originally on either plate. A charged condenser, therefore, acts like a source of e.m.f., the direction of this e.m.f. *around* the circuit containing the condenser being in the direction *through* the condenser from its negative to its positive plate. A condenser when it is being charged may also be looked upon as producing a back e.m.f., that is, an e.m.f. opposing the e.m.f. which charges it. When a condenser is considered from this point of view only the *conducting* portion of the circuit is to be considered in applying Kirchhoff's Laws. When the condenser has an appreciable leakage its resistance must be considered to be in *parallel* with its e.m.f.

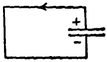


Fig. 12.

Charging or Capacity Current and Leakage Current. — The displacement current through the dielectric of a condenser is frequently called the "charging" or "capacity" current. The conduction current through the dielectric is called the "leakage" current. Let C be the capacity of the condenser, g the conductance of the dielectric, and v the voltage across the condenser, then the total current through the condenser is

$$i = gv + C\frac{dv}{dt}, \tag{23}$$

where $\frac{dv}{dt}$ represents the rate of change of v with time. The component gv of this current is the leakage current and the component $C\frac{dv}{dt}$ is the charging or capacity current.

Capacities in Series. — When several capacities* are connected end to end so that the *same dielectric flux* passes through each of them, they are said to be "in series." The total capacity of any number of individual capacities C_1, C_2, C_3, etc., connected in series is C where

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ \dots \tag{24}$$

Compare with conductances in series, equation (11b).

Capacities in Parallel. — When several capacities* are connected between the same pair of equipotential surfaces so that the *same potential drop* is established through each, they are said to be "in parallel." *When there are no e.m.f.'s*

* Either condensers, or dielectrics of different kinds, sizes or shapes in contact along ...tential surfaces.

in any of the circuits between the two equipotential surfaces the total equivalent capacity of any number of capacities C_1, C_2, C_3, etc., connected in parallel is

$$C = C_1 + C_2 + C_3 \ldots \tag{25}$$

Compare with conductances in parallel, equation (12a).

SPECIFIC INDUCTIVE CAPACITY (K) AND DIELECTRIC COEFFICIENT (k). — From equation (21d) it is evident that when the capacity of a given condenser is measured (1) when a dielectric A is between the plates and (2) when some other dielectric B is between these plates, then the ratio of the two capacities is the same as the ratio of the dielectric coefficients of the two dielectrics. The "specific inductive" capacity of any dielectric is defined as the ratio of the capacity of a condenser having this substance as its dielectric to the capacity of the same condenser when air forms the dielectric between the plates. The specific inductive capacity is, therefore, independent of the system of units employed.

The c.g.s. electrostatic system of units is based on the arbitrary choice of unity as the dielectric coefficient of air; hence, in the c.g.s. electrostatic system of units the specific inductive capacity and the dielectric coefficient are numerically equal. In the c.g.s. electromagnetic system of units the dielectric coefficient of air is not unity but $\frac{1}{9 \times 10^{20}}$ (i.e., the reciprocal of the square of the velocity of light in air). In the practical system of units, when the centimeter is taken as the unit of length, the dielectric coefficient of air is $\frac{1}{9 \times 10^{11}}$. Hence, calling K the specific inductive capacity of any dielectric referred to air as unity, and k its dielectric coefficient, then in the

c.g.s. electrostatic system $\quad k = K,$

c.g.s. electromagnetic system $\quad k = \frac{K}{9 \times 10^{20}},$

Practical system (cm. as unit of length) $\quad k = \frac{K}{9 \times 10^{11}}.$

Values of the specific inductive capacity of various insulating materials are given in the article on *Insulating Materials, Properties of.*

Electric Absorption and Residual Charge. — The value of the dielectric coefficient k of a given dielectric is not strictly a constant unless the dielectric is perfectly homogeneous. In the case of such nonhomogeneous substances as glass, mica, rubber, paper, cloth, etc., the dielectric coefficient is found to depend upon the time of electrification, i.e., upon the length of time that the voltage is applied, its value increasing with the time of electrification (*see Insulating Materials, Testing of*). This phenomenon is sometimes described as "electric absorption," the idea being that the charge from the plates of the condenser soaks into the dielectric, for an increase in the dielectric coefficient for a given impressed voltage means a greater quantity of electricity conducted to the plates. This idea is also in accord with the experimental fact that when such a condenser is discharged by short-circuiting it with a wire, Fig. 12, the wire then being removed, a "residual" charge appears on the plates after a lapse of a few seconds.

Dielectric Hysteresis. — A phenomenon closely associated with electric absorption is the fact that when the electric field in a heterogeneous dielectric is caused to vary rapidly an amount of heat is dissipated in the dielectric greatly in excess of that which can be accounted for in terms of its leakage resistance as determined by continuous-current measurements. This may be due in part to

an actual increase in the resistance of the dielectric with the speed of variation of the field, or may be due to a phenomenon analogous to magnetic hysteresis, i.e., to a lag of the dielectric flux density behind the electric field intensity (*see Magnetic Materials, Properties of*). Whatever may be the cause of this extra loss of power for rapidly varying fields, it is generally described as the loss due to "dielectric hysteresis." The heat developed is in many cases quite appreciable. See also the article on *Condensers, Electric.*

DIELECTRIC STRENGTH. — ELECTRIC SPARK AND ELECTRIC CORONA. — Experience shows that when an electric field is established in a dielectric and the intensity of this field is increased, a point is reached at which the dielectric loses its insulating property and becomes a conductor. This condition of affairs is usually accompanied by a spark which burns a hole through the dielectric, i.e., the dielectric is "punctured." Under other conditions the breakdown may not be permanent, but may result in the acquisition of a high conductivity by the dielectric only while the voltage gradient is maintained above the critical value, the dielectric regaining its insulating property when the field is reduced below this critical value. This latter condition is usually described as the formation of an electric "corona" in the dielectric; in the case of air the formation of corona manifests itself by a bluish light in the air around the conductors between which the field is established. Whether the breakdown produced by a given voltage is of the nature of a puncture or results in the formation of a corona depends chiefly upon the *distribution* of the dielectric flux produced in the dielectric (*see Corona, Electric*).

The critical field intensity or voltage gradient at which breakdown occurs is called the "dielectric strength" of the dielectric. The dielectric strength depends upon the nature of the dielectric, its value for the various dielectrics ordinarily employed in practice depending decidedly upon their chemical and physical nature (*see Insulating Materials, Properties of*). It is also found that for a given dielectric the critical voltage *gradient* at which breakdown occurs depends in general upon (*a*) the distribution of the dielectric flux just prior to breakdown and (*b*) upon the thickness of the dielectric. It is naturally to be expected that the *voltage* (total potential difference) required to produce a breakdown would depend upon the distribution of the dielectric flux and the thickness of the dielectric, for the voltage gradient at all voltages depends upon these factors (*see above under Capacity*), but there is as yet no satisfactory explanation of the dependence of the *critical gradient* upon these factors.

In fact, but little is known regarding the nature of an electric breakdown, and even the values of the dielectric strength are known only approximately in most instances, for in many of the tests made to determine its value the distribution of the electric flux was not known. The values of the dielectric strength given in the article on *Insulating Materials, Properties of*, must, therefore, be considered only as rough approximations except for conditions identical with those under which the tests were made.

ELECTROSTATIC ENERGY. — From the general relations expressed by equations (15) and (23) it is evident that when the potential difference between the plates of a condenser is increased from 0 to V the energy input is

$$\int_0^t gv^2\,dt + \int_0^V Cv\,dv.$$

The energy represented by the first term on the right-hand side of this equation is dissipated as heat in the dielectric, but the energy represented by the second term, which, when C is constant, may be written

$$W = \tfrac{1}{2}CV^2, \qquad (26)$$

does not represent a dissipation of heat; this is a fact of experience. Moreover, when the condenser is discharged, by short-circuiting its plates with a wire, this same amount of energy ½ CV^2 is transferred to the wire. Hence, the energy represented by ½ CV^2 is said to be stored in the condenser, or preferably in the *dielectric* of the condenser, for the electric force F exists only in the dielectric. This stored energy is called the "electrostatic" energy. It is analogous to the energy stored in a spring when the latter is compressed or stretched. Equation (26) may also be written

$$W = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV = \frac{1}{8\pi}\psi V\cdot \tag{26a}$$

The electrostatic energy *per unit volume* of an electric field may be written

$$w = \frac{DF}{8\pi} = \frac{kF^2}{8\pi} = \frac{D^2}{8\pi k}, \tag{26b}$$

where k is the dielectric coefficient, F the electric field intensity or potential drop per unit distance, and D the dielectric flux density or flux per unit area perpendicular to the direction of the drop.

It should be noted that equations (26) to (26b) are based on the assumption that k is a constant, independent of the value of F. When this condition does not hold, the energy required to establish the field is $\int_0^V Cv\,dv$, the evaluation of which depends upon the relation between C and v.

MECHANICAL FORCES IN AN ELECTROSTATIC FIELD.— Experience shows that all bodies (conductors or insulators) in an electric field exert in general mutual mechanical forces upon one another tending to produce such a relative motion as will *decrease* the energy of the field. Let f be the component of the force tending to move any body in the field in a given direction and let dW be the *increase* in the energy of the field due to displacing the body a distance dx in this direction, then

$$f = -\frac{dW}{dx}, \tag{27}$$

provided this displacement does not cause a change in the existing electric charges in the field. As a consequence of this general relation it can be shown that every charged surface exerts a force of repulsion on every other surface charged with electricity of the same sign, and a force of attraction on every surface charged with electricity of the opposite sign.

In the special case of the two conductors forming a condenser the force of attraction exerted by one conductor on the other is

$$f = -\frac{V^2}{2}\frac{dC}{dx}, \tag{27a}$$

where V is the p.d. across the condenser, C the capacity of the condenser, and dC represents the increase in the capacity of the condenser when one conductor moves a distance dx away from the other. This relation results from the substitution of (26a) in (27). For example, the capacity of a parallel plate condenser is approximately,

$$C = \frac{kA}{4\pi x},$$

where k is the dielectric coefficient, A the area of the smaller plate, and x the distance between the plates. Hence the force of *attraction* is

$$f = \frac{V^2kA}{8\pi x^2}.$$

Principle of the Electrometer. — This relation suggests a method of measuring potential difference, for, by transposing,

$$V = x\sqrt{\frac{8\pi f}{kA}}.$$

If the force acting on the upper plate is measured by means of a balance, and if A and x are also measured, and the dielectric between them is air ($k = 1$ in the electrostatic system, by definition), then all the data necessary for the calculation of V is at hand. The above formula for V is approximate only, since the capacity formula is approximate, due to the assumption of a uniform flux density in the dielectric between the plates. As a matter of fact the flux density near the edges of the plate is not uniform, but it is possible to correct for this non-uniformity; see article on *Electrometers*.

MAGNETS AND MAGNETIC SUBSTANCES. — A magnet may be defined as any body which possesses the property of attracting pieces of iron or steel* and which when freely suspended takes up a definite position with respect to the geographical meridian. A magnetic substance is any body which acquires this property when it is placed near a magnet or near a conductor carrying an electric current. A body which is given this property is said to be "magnetized." A magnetic needle is a magnetized needle of iron or steel; the north seeking end of such a needle is called its north pole and the south seeking end its south pole. When such a needle is freely† suspended near a magnet or a conductor carrying an electric current a couple is bound to be exerted upon it which causes it to take up a definite direction. The needle is said to "point" in the direction of a line drawn through it from its south to its north pole.

MAGNETIC FIELD OF FORCE. — Any region in which a magnetic substance (e.g., a piece of soft iron), when placed therein, becomes magnetized is said to be a "magnetic field." A magnetic field exists in and around every magnetized substance and around every stream line of electric current. The direction of the magnetic field at any point P is arbitrarily chosen as the direction in which a small magnetic needle point would point when placed at P without disturbing appreciably the existing conditions.

Magnetic Flux (ϕ). — Consider a small closed turn of wire, Fig. 13, placed in a magnetic field with its plane perpendicular to the direction of the field. Experience shows that when such a turn of wire is removed from the field in any manner whatever (the coil remaining short-circuited on itself or forming part of a closed circuit), or when the magnetic field is caused to disappear in any manner whatever, a momentary electromotive force is set up or "induced" in this coil, which in turn causes a momentary electric current to flow through the coil. This e.m.f. exists only while the coil is *moving across* the field or while the field through the coil is *varying*.

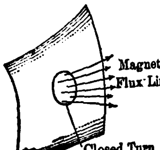


Fig. 13.

* With a force in excess of the gravitational force, which latter is extremely small.

† A needle is said to be freely suspended when there is no controlling force exerted upon it through its suspension tending to make it take up any definite position.

The time integral of the induced e.m.f. when the coil is removed entirely from the magnetic field is taken as the measure of the "magnetic flux" existing through the coil when in its original position. That is, calling e the e.m.f. induced in the coil at any instant due to its motion through the field, and t the time during which the e.m.f. exists in the coil, then the magnetic flux through the coil when in its original position is

$$\phi = \int_0^t e\, dt. \tag{28}$$

This quantity is readily measured by means of a ballistic galvanometer; see *Magnetic Testing*.

Units of Magnetic Flux.—The unit of magnetic flux in the c.g.s. electromagnetic system is frequently called a "maxwell" or simply a "line." The unit of flux in the practical system of units is sometimes called a "weber"; see *Units and Conversion Factors*.

Magnetic Flux Density (B).—Experience shows that the magnetic flux through any closed loop, such as the turn of wire described above, depends upon the area inclosed by this loop. The magnetic flux per unit area through any surface perpendicular to the direction of the field is defined as the "magnetic flux density" at this surface, and is usually represented by the symbol B. By the flux density at any *point* is meant the flux density at an infinitely small surface drawn perpendicular to the field at this point. The direction of the magnetic flux density at any point is the same as that in which a magnetic needle would point if placed at this point; i.e., the direction of the flux density and the direction of the magnetic field are the same. When the flux density has the same value B at every point of a surface of area A and is perpendicular to this surface, then the total flux through this surface is

$$\phi = BA. \tag{29}$$

The total magnetic flux across any surface S may in general be expressed mathematically by the surface integral

$$\phi = \int (B \cos \alpha)\, ds, \tag{29a}$$

where ds represents any elementary area of this surface and $(B \cos \alpha)$ the component of the flux density perpendicular to ds.

Units of Magnetic Flux Density.—The unit of magnetic flux density in the c.g.s. electromagnetic system is called the "gauss"; no name has been given to the corresponding practical unit.

Magnetic Flux Lines. — Continuity of Magnetic Flux.—Magnetic flux can be represented by lines drawn in the field in such a direction that their direction coincides at each point with the direction of the field at that point, and of such a number that their density at each point (number per unit area perpendicular to their direction) is equal to the magnetic flux density at that point. Such lines are called "magnetic flux lines." Experience shows that lines thus drawn in a magnetic field always form *closed* loops, i.e., a magnetic flux line has no ends. As a consequence of this fact the total magnetic flux coming up to any surface in a magnetic field is always equal to the total flux leaving that surface. (Compare with stream lines of electric current.) When the flux

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leaving a surface is considered as equivalent to a negative flux coming up to that surface, this relation may be expressed by the formula

$$\Sigma\phi = 0 \tag{30}$$

at every surface, the summation being an algebraic one. This is analogous to Kirchhoff's first law for electric circuits.

Magnetic Fields Due to Electric Currents. — Experience shows that every stream line of electric current is always accompanied by a magnetic field the flux lines of which *link* the stream line of current. That is, the flux lines thread the loops formed by the stream lines and the stream lines thread the loops formed by the flux lines; see Fig. 14.

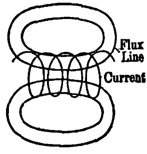

Fig. 14.

Right-handed Screw Law. — The direction of the current flowing around any electric circuit and the direction in which the flux lines due to that current thread this circuit are related to each other in the same manner as the direction of motion of a point on the edge of the head of a right-handed screw placed at the center of the circuit and the direction of advance of the screw. Or, if one faces the electric circuit looking in the direction of the flux lines threading it, the current producing these lines is in the clockwise direction around the circuit. The relative direction of the current and its magnetic flux may be briefly described by saying that the current is in the right-handed screw direction with respect to the flux which it produces.

MAGNETICALLY INDUCED ELECTROMOTIVE FORCE. — The measure of magnetic flux is based on the experimental fact that whenever the magnetic field threading an electric circuit changes, an electromotive force is induced in that circuit. When the circuit is formed by a single turn of wire this induced e.m.f. is, from the definition above, equal to the rate of change of this flux with respect to time, i.e., $e = \frac{d\phi}{dt}$. When the circuit is in the form of a coil each turn of which links the flux, the e.m.f. induced in *each* turn is equal to $\frac{d\phi}{dt}$ where ϕ is the flux which links that particular turn. When each turn links the *same number of flux lines*, then the total induced e.m.f. in a coil of N turns is

$$e = N\frac{d\phi}{dt}. \tag{31}$$

When the change in flux is due to a motion of a circuit or part of a circuit through a magnetic field the induced e.m.f. in any conductor is also equal to the number of flux lines which *cut across* this conductor.

Magnetic Linkages (λ). — The condition that each turn be linked by the same flux ϕ is seldom the case; some of the flux lines link only part of the turns, see Fig. 14. In general, the total e.m.f. is

$$e = \frac{d}{dt}(\phi_1 + \phi_2 + \ldots \phi_n),$$

where ϕ_1, ϕ_2, etc., represent the fluxes linking the various turns. The sum $(\phi_1 + \phi_2 + \ldots \phi_n)$ may be called the total number of "magnetic linkages" and may be conveniently represented by the symbol λ, viz.,

$$\lambda = \phi_1 + \phi_2 + \ldots \phi_n,$$

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and the total induced e.m.f. may then be written

$$e = \frac{d\lambda}{dt}. \qquad (31a)$$

When all the N turns link the same flux, ϕ, then $\lambda = N\phi$.

Direction of the Induced E.M.F. — The direction of this induced e.m.f. around the circuit is found to be in the *left-handed* screw direction with respect to the *increase* of flux; viz., if one faces the circuit looking in the direction of the *increase* of flux, the induced e.m.f. is in the counter-clockwise direction. The current which would be set up by this e.m.f., however, would produce a flux linking the circuit in the right-handed screw direction. Hence a change in the magnetic flux through an electric circuit always sets up an e.m.f. which tends to produce a current around this circuit in such a direction as to set up an *opposing* flux. This fact may be expressed mathematically by writing a minus sign before $\frac{d\phi}{dt}$ in equation (31), i.e., by putting

$$e = -N\frac{d\phi}{dt}. \qquad (31b)$$

The value of $\left(-N\frac{d\phi}{dt}\right)$ is then the e.m.f. induced in the circuit in the right-handed-screw direction with respect to the increase of flux. Or stated in other words $\left(-N\frac{d\phi}{dt}\right)$ represents the *rise* of electric potential and $N\frac{d\phi}{dt}$ represents the *drop* of potential around the circuit in the right-handed screw direction with respect to the increase of flux.

WORK DONE BY A VARYING MAGNETIC FLUX. — Consider a coil A (Fig. 15) of N turns of wire, and let each of these N turns be linked by a flux ϕ due to some external agent, e.g., another coil B in which an electric current is flowing. Let the flux ϕ through A due to B be increasing at any instant at the rate $\frac{d\phi}{dt}$ in the *left-handed* screw direction with respect to the current I in A at this instant. Then there is induced in A at this instant an e.m.f. in the direction of I equal to $E = N\frac{d\phi}{dt}$ and, therefore, the electric

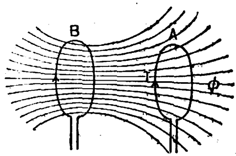

Fig. 15.

power developed in A at this instant is $ei = NI\frac{d\phi}{dt}$. This power is transmitted to the coil A as a result of the varying flux through it; hence the power

$$p = NI\frac{d\phi}{dt} \qquad (32)$$

may be looked upon as the magnetic power input, this power being converted within the coil into electric power.

Magnetic Displacement Current. — The varying flux established by B through A may be looked upon as the means whereby this energy is transferred through the magnetic field, just as an electric current may be looked upon as the means whereby energy is transferred from one point to another in an

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electric circuit. Hence the varying flux may be looked upon as a "magnetic displacement current."

Magnetomotive Force ($\mathfrak{F}$). — The above expression for the rate of transfer of energy by a varying magnetic field may be compared with the rate of transfer of energy, or power input $P = EI$, corresponding to an electric current I flowing in a circuit in which there is *back* electromotive force E. The strength of the "magnetic displacement current" may be chosen arbitrarily* as

$$\text{Magnetic displacement current} = \frac{1}{4\pi}\frac{d\phi}{dt},$$

just as the strength of the electric displacement current is taken as $\frac{1}{4\pi}\frac{d\psi}{dt}$, where ψ is the electrostatic flux density. Then the quantity corresponding to the electromotive force must be chosen numerically equal to

$$\mathfrak{F} = 4\pi NI \qquad (33)$$

and may be called the "magnetomotive force," abbreviated "m.m.f." The magnetic power input may then be written

$$p = \mathfrak{F} \times (\text{Magnetic displacement current}).$$

where $\mathfrak{F}$ is the *back* magnetomotive force.

Direction of the Magnetomotive Force. — The closed path of the magnetic flux lines through a magnetic field is called a "magnetic circuit," just as the path of the stream lines of an electric current is called an electric circuit. In general, a magnetomotive force is produced in every magnetic circuit wherever it is linked by an electric circuit, equal in numerical value to $4\pi NI$, where N is the number of times the current I links this circuit. The magnetomotive force is taken as positive when the current links the flux lines in the right-handed screw direction, for then an increase in the flux corresponds to a magnetic power output. When the current links the flux lines in the left-handed-screw direction the magnetomotive force is taken as negative, i.e., a "back" magnetomotive force, for in this case an increase in flux corresponds to a magnetic power input; see the special case considered above.

Units of Magnetomotive Force. — Ampere-Turns. — The product NI in the expression for the magnetomotive force, equation (33), is called the "current-turns"; when I is expressed in amperes it is called the "ampere-turns." The magnetomotive force as above defined differs from this product only by a constant numerical factor; hence the ampere-turns of a coil may be taken as a measure of the magnetomotive force produced by it. This unit of magnetomotive force, namely, one ampere-turn, is the unit commonly employed in the practical calculation of magnetic circuits. The c.g.s. electromagnetic unit is called the gilbert; 1 ampere-turn = 1.2566 gilberts. See *Units and Conversion Factors*.

MAGNETIZING FORCE OR MAGNETIC FIELD INTENSITY (H). — Experience shows that the magnetic flux density produced at any point by a given magnetomotive force depends (*a*) upon the position of the point with respect to the source of the m.m.f. and (*b*) upon the nature of the substances through which this m.m.f. produces the magnetic flux. Compare with the electric current density or dielectric flux density produced at a given point by

* The factor 4π arises from the manner in which the conceptions of the magnetic field were originally developed.

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a given electromotive force. These facts lead to the conception of the flux density at any point in a magnetic field as being due to a "magnetizing force" H at that point, this magnetizing force H depending solely upon the *magnetomotive forces* producing the field and the *distribution of the flux lines*, as distinguished from the flux density B which depends not only upon these two items but also upon the *nature of the medium at the point in question*.

From analogy with the relation between electromotive force and electric field intensity, the magnetizing force (also called the "magnetic field intensity") at successive points along any closed path in a *magnetic* field may be defined by the relation that its line integral around such a path is equal to the total magnetomotive force acting around this path, viz.,

$$4\pi NI = \int (H \cos \theta) dl, \tag{34}$$

where dl represents any elementary length of this path, see Fig. 16, $(H \cos \theta)$ the value of the component of the magnetizing force at dl in the direction of dl, and NI the total number of current turns linked by the path. Experience shows that such a definition leads to a simple means of expressing in a quantitative manner the interrelations of a number of experimental facts. Magnetizing force may also be expressed as the force in dynes which would act on a "unit magnetic pole," see p. 412.

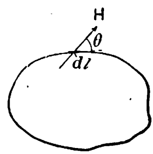

Fig. 16.

When the path coincides in direction with the magnetizing force at each point, equation (34) may be written

$$4\pi NI = \int H dl. \tag{34a}$$

For the application of this formula see the articles on *Magnetic Properties of Iron; Generators; Motors.*

Direction of the Magnetizing Force. — Experience shows that except for points inside a permanent magnet the magnetizing force H and the flux density B are always in the same direction. For points inside a permanent magnet the direction of the magnetic field intensity H, due solely to the magnet itself, is opposite to the direction of the flux lines, i.e., a permanent magnet produces a "demagnetizing force" on itself.

Lines of Magnetizing Force. — The magnetizing force at any point in a magnetic field may be represented by lines drawn in the field in such a direction that their direction coincides with the direction of the magnetizing force at each point, and of such a number per unit area perpendicular to their direction that their density at each point gives the value of the magnetizing force at that point. Such lines are called "lines of magnetizing force" or "lines of magnetic field intensity." In general the lines of magnetizing force and the magnetic flux lines coincide in direction (except within the substance of permanent magnets), *but their densities are different*. Only in non-magnetic substances do the flux lines and lines of magnetizing force coincide. The simple expression "lines of force" is frequently used to designate either the flux lines or the lines of magnetic intensity, but it is evident that such a loose use of this term is liable to lead to confusion when speaking of the magnetic field within a magnetic substance.

Units of Magnetizing Force. — Magnetizing force is of the nature of magnetomotive force per unit length, just as electric field intensity is of the nature

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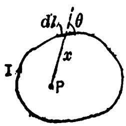
of electric potential difference per unit length. Hence the c.g.s. electromagnetic unit of magnetizing force may be called the "gilbert per centimeter"; compare with volts per centimeter. When the magnetomotive force is expressed in ampere-turns, the magnetizing force is expressed in ampere-turns per centimeter or per inch; see *Units and Conversion Factors.*

Values of the Magnetizing Force in a Uniform Medium. — When the medium surrounding the stream lines of an electric current is of a *uniform magnetic nature throughout,* the magnetizing force at any point may be calculated from the shape and distribution of the stream lines of the current, irrespective of whether the medium be non-magnetic or highly magnetic, e.g., iron. See *Units and Conversion Factors* for numerical multipliers to change them into practical units.

Magnetizing Force at any Point Due to an Element of a Current-Stream Line (Fig. 17). — Consider any closed stream line of electric current and let the surrounding *medium be uniform* in its magnetic properties throughout the region in which the magnetic field produced by this stream line exists. It can be shown that each elementary length dl of this stream line may be considered as contributing to the magnetizing force H at any point P in this region an amount

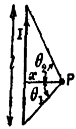


$$dH = \frac{(I \sin\theta)\, dl}{x^2}, \qquad (35)$$

Fig. 17.

where I is the current flowing along this stream line, x the distance from P to dx, and θ the angle between x and dl. The direction of dH is perpendicular to the plane determined by x and dl. The total magnetizing force at P is then the *vector sum* or *vector integral* of dH for all the elementary lengths into which the stream line is divided.

Magnetizing Force Due to a Straight Wire (Fig. 18). — Applying equation (35) to the case of a straight wire of circular cross-section carrying a current I, the magnetizing force at any point P due to a length l of this wire is



$$H = \frac{I}{x}(\sin\theta_1 + \sin\theta_2), \qquad (35a)$$

where x is the perpendicular distance from P to the wire and θ_1 and θ_2 the angles designated in Fig. 18.

Fig. 18.

When the wire is very long compared with x, this becomes

$$H = \frac{2I}{x}. \qquad (35b)$$

This formula also holds approximately for any point outside a wire of any shaped cross-section, provided x is large compared with the maximum diameter of this section. For a point *inside a long wire of circular cross-section* of radius a the magnetizing force is also given by (35b) when I is taken to represent *that part of the current inside the circle through P* concentric with the axis of the wire. When the current density is uniform over the cross-section, as is usually the case (*see, however, the article on Skin Effect*), the magnetizing force inside the wire is

$$H_i = \frac{2xI}{a^2}. \qquad (35c)$$

NOTE. — ALL FORMULAS ON THIS PAGE ARE IN C.G.S. ELECTROMAGNETIC UNITS.

Magnetizing Force on the Axis of a Circular Coil of N Turns.— Let I be the current, r the mean radius of the coil, and x the distance of the point from the center of the circle; then

$$H = \frac{2\pi NIr^2}{(r^2 + x^2)^{3/2}} \tag{35d}$$

Magnetizing Force due to a Solenoid.— A solenoid is a helical coil of wire, each turn having the same radius. Let N = total number of turns, I = current in abamperes, r = mean radius of the helix in centimeters, l = length of helix in centimeters. Then at any point on the axis of the helix (inside or outside) at a distance of x centimeters from its center, the magnetizing force in gilberts per centimeter is

$$H = \frac{2\pi NI}{l}\left[\frac{0.5\,l + x}{\sqrt{r^2 + (0.5\,l + x)^2}} + \frac{0.5\,l - x}{\sqrt{r^2 + (0.5\,l - x)^2}}\right] \tag{35e}$$

This formula holds only when the thickness of the winding is small compared with the mean radius r. When l is large compared with r this reduces to

$$H = \frac{4\pi NI}{l}, \tag{35f}$$

For all points inside the solenoid (whether on the axis or not) at a distance from the ends large compared with r, that is, inside the central portion of a long solenoid, the field is uniform over the cross-section of the solenoid and its value is given by (35f).

Magnetizing Force Inside a Toroid (Fig. 19).— A toroid is a cylinder bent into the form of a closed ring, making a shape like a doughnut. When such a ring is *uniformly* wound with an insulated wire so that the turns of the wire are close together and cover the entire surface of the toroid, the magnetic field is confined entirely within the space inclosed by these turns, and therefore, when the core on which the wire is wound is of uniform magnetic material throughout, both the lines of magnetizing force and the flux lines must be circles concentric with the hole in the "doughnut." The magnetizing force will have the *same* value at every point on the circumference of any *one* of these circles, and, therefore, from equation (34a) the value of H at any point P within the core is

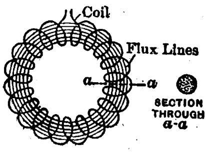


Fig. 19.

$$H = \frac{4\pi NI}{l}, \tag{35g}$$

where N is the total number of turns on the core, I the current in each turn and l the length of the circumference through P. Unless the hole in the "doughnut" has a large radius compared with the radius of the cross-section of the core H will not be uniform over this section, since l for the various points in the cross-section will differ considerably.

It should be noted that the value of H is independent of the material of the core provided only that the core be of uniform material throughout. That is,

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equation (35g) applies to an iron core as well as to an air or wood core, provided the iron is uniform throughout and there is no air gap across the path of the flux lines. Even a mechanically perfect contact between two pieces of iron of the same kind, however, is sufficient to vitiate the above formula.

MAGNETIC PERMEABILITY (μ). — PARAMAGNETIC AND DIAMAGNETIC SUBSTANCES. — The quotient of the magnetic flux density B at any point by the magnetizing force H at that point is defined as the "magnetic permeability" μ of the substance as medium at that point, viz.,

$$\mu = \frac{B}{H}. \tag{36}$$

Compare with the definition of dielectric coefficient, equation (17.) The c.g.s. electromagnetic system of units is based on the arbitrary assumption of unity as the value of the magnetic permeability of air. Any substance which has a magnetic permeability greater than that of air is called a "paramagnetic substance," and any substance which has a permeability less than that of air is called a "diamagnetic substance." The only substances which are strongly paramagnetic, i.e., which have a permeability considerably greater than that of air, are iron, steel, nickel and cobalt, and certain alloys of non-magnetic elements. The only substance which is appreciably diamagnetic is bismuth, which has a permeability of about 0.9998. All other elements are practically non-magnetic, i.e., their permeabilities differ from unity by less than 1 per cent.

The permeability of a given sample of any highly magnetic substance is not a constant, but depends upon the value of the magnetizing force; see the curves in the article on *Magnetic Properties of Iron*. The permeability also depends very largely upon the previous heat treatment and the exact composition of the material, and also upon its previous magnetic history; these relations are also discussed in detail in that article. The methods of measuring permeability are described in the article on *Magnetic Testing*.

North and South Poles. — That portion of the surface of any magnetized body from which the flux lines pass out into the air (or into any substance of lower permeability) is said to be a north magnetic pole, and that portion of the surface at which the flux lines enter the body is said to be a south magnetic pole. A "unit north pole" is a pole from which 4π flux lines emerge into the surrounding air. When a magnetic needle is placed near the surface of a magnetized body its north seeking end points away from this surface when this surface is a north pole and toward the surface when this surface is a south pole.

Difference of Magnetic Potential (U). — Consider any two points 1 and 2 in a magnetic field (Fig. 20) and let the path between them from 1 to 2 pass through an electric circuit producing a magnetomotive force in the direction from 1 to 2, then from analogy with the electric circuit, equation (2), the expression

$$U_{12} = \int_1^2 (H\cos\theta)\, dl - \mathfrak{F}_{12} \tag{37}$$

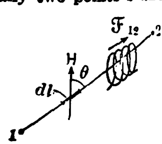

Fig. 20.

is called the "drop of magnetic potential" from 1 to 2. From the definition of magnetizing force, equation (34), it follows that around any *closed* circuit the drop of magnetic potential is always zero. A magnetomotive force $\mathfrak{F}_{12}$ is, therefore, equivalent to a rise of magnetic potential from 1 to 2.

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When there is no source of m.m.f. between 1 and 2 and the path coincides with a line of magnetizing force, the drop of magnetic potential is

$$U_{12} = \int_1^2 H\, dl. \tag{37a}$$

Magnetic potential difference is of the same nature as magnetomotive force and may, therefore, be expressed in the same units, viz., gilberts or ampere turns.

Magnetic Equipotential Surfaces. — A surface drawn in a magnetic field in such a manner that this surface is perpendicular at each point to the magnetizing force at this point (i.e., to the line of magnetizing force through this point) is called a "magnetic equipotential surface," compare with *electric equipotential surface.*

Magnetic Reluctance ($\mathcal{R}$). — To establish a magnetic flux ϕ through a given portion of a substance which is *not itself linked by a source of m.m.f.*, a difference of magnetic potential must always be established between the end surfaces of this substance. Let U be the magnetic potential drop established from one surface to the other, then the quotient

$$\mathcal{R} = \frac{U}{\phi} \tag{38}$$

is defined as the magnetic reluctance of the given portion of the substance. Compare with electric resistance. The c.g.s. electromagnetic unit of reluctance is called the oersted. It should be noted that the above definition is meaningless except when applied to a portion of a substance of which the end surfaces are magnetic equipotential surfaces and through every cross-section of which the same flux passes.

Factors Upon Which Reluctance Depends. — The magnetic reluctance of a given portion of a substance included between two equipotential surfaces and bounded laterally by a surface through which no flux line passes depends upon 1. the magnetic permeability of the substance, 2. the dimensions of this portion of the substance and 3. upon the distribution of the flux lines over each cross-section perpendicular to them. The relations are identical with those which determine the electric al resistance of a conductor, the magnetic permeability taking the place of the electric conductivity. For example, for a straight bar of constant cross-section A and length l, through which the flux lines are straight, parallel and uniformly distributed, the reluctance is

$$\mathcal{R} = \frac{l}{\mu A}. \tag{38a}$$

Magnetic reluctance is not a constant quantity even for a given material and given flux distribution, unless this material is non-magnetic. For all highly magnetic materials μ depends upon the magnetizing force and therefore also upon the flux density. It should also be noted that the magnetic reluctance does not represent a "resistance" in the sense of something which causes a dissipation of energy.

Magnetic Permeance ($\mathcal{P}$). — The reciprocal of magnetic reluctance is called "magnetic permeance." The permeance of a straight bar under the conditions specified above is

$$\mathcal{P} = \frac{\mu A}{l} \tag{39}$$

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The permeability of a substance is, therefore, equal to the permeance of a unit cube of this substance when the flux through the cube is parallel to four edges of the cube and is uniformly distributed over the section at right angles to these four edges.

Magnetic permeance is analogous to electric conductance, except that it is not a factor which affects the *dissipation* of energy in a substance. It does, however, enter into the expression for the energy *stored* in a magnetic field in the same way that the electrostatic capacity of a dielectric is a determining factor in the expression for the energy stored in the electric field.

Kirchhoff's Laws for the Magnetic Circuit. — As already noted, equation (30), the total magnetic flux coming up to any surface in a magnetic field is always zero, provided a flux leaving a surface is considered as a *negative* flux coming up to that surface. This fact may be represented by the formula

$$\Sigma\phi = 0 \tag{40}$$

for every surface in the field. Similarly, from the definition of magnetic potential drop, it follows that the *total* magnetic potential drop around any closed circuit is zero, or that the total magnetomotive force acting around any closed circuit is equal to the sum of the reluctance drops around that circuit, which may be represented by the formula

$$\Sigma\mathfrak{F} = \Sigma\mathfrak{R}\phi. \tag{40a}$$

These two equations are identical in form with those representing Kirchhoff's laws for the electric circuit, equation (13). They are, however, not so easy to use for practical calculations, for the magnetic flux is not confined to approximately geometrical lines like the currents in a network of insulated wires, but in general fills all space surrounding the coils which establish the magnetomotive forces; also, when there is iron or other magnetic material in the circuit, the permeability depends on the flux density and the previous history of the iron. (The distribution of magnetic flux in and around an iron circuit is analogous to the distribution of current in and around an uninsulated mass of copper of the same shape as the iron circuit immersed in a liquid having a conductivity about equal to that of carbon.) Only in the special case of a uniformly wound circular ring or toroid are the lines of induction confined *entirely* to an iron circuit; in general a certain number also exist in the air and in whatever other substances are in the vicinity of the iron circuit.

SELF AND MUTUAL INDUCTION. — When the current in a given electric circuit varies with time the magnetic flux accompanying this current also varies with time, and, since this flux links the current which produces it, an e.m.f. is induced in each turn of the circuit equal to the rate at which the flux through this turn is varying, and in such a direction as to *oppose* the change in the current. That is, an *increasing* electric current is always accompanied by a *back* e.m.f. due to the increase in the magnetic flux which accompanies this increase of current. Or, viewed from another point of view, an increase in the velocity of a stream of electricity develops a back pressure, just as when the velocity of a stream of water is increasing it develops a back pressure or "velocity head" due to the inertia of the water. The magnetic field accompanying an electric current may, therefore, be looked upon as a result of its "electromagnetic inertia," or "electromagnetic mass"; see also the article on *Electron Theory*.

Again, when an electric current *decreases* its accompanying flux decreases and an e.m.f. is set up in the circuit tending to oppose this decrease, i.e., tending

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to keep the current from decreasing. This is again analogous to the effect of inertia in the case of ordinary matter.

Coefficient of Self-Induction or Inductance (L). — In general, the coefficient L by which the rate of change of the current $\left(\frac{di}{dt}\right)$ in any circuit must be multiplied to give the self-induced electromotive force e is called the "coefficient of self-induction" or simply the "inductance" of the electric circuit. In general, then, the self-induced e.m.f. in an electric circuit is

$$e = L\frac{di}{dt}, \qquad (41)$$

where L is the inductance of the electric circuit and $\frac{di}{dt}$ represents the change in the current per unit of time. Since $e = \frac{d\lambda}{dt}$, see equation (31a), where λ is the number of magnetic linkages between the electric circuit and the flux established by the current i, the inductance may also be defined by the relation

$$L = \frac{d\lambda}{di}. \qquad (42)$$

That is, the *inductance is equal to the increase in the number of linkages per unit increase in the current.* When the permeability of the magnetic circuit is *constant* the inductance is also a constant equal to the linkages per unit current. When every flux line is linked by every stream line of electric current, and the permeability of the entire magnetic circuit is constant, then

$$L = \frac{4\pi N^2}{\mathcal{R}}, \qquad (42a)$$

where N is the number of turns forming the electric circuit and $\mathcal{R}$ is the reluctance of the complete magnetic circuit.

Formulas for the inductance of various electric circuits are given in the article on *Inductance and Inductive Reactance.*

Units of Inductance. — The practical unit of inductance is called the henry; for the relation between the henry and the abhenry and millihenry see *Units and Conversion Factors.*

Coefficient of Mutual Induction. — Mutual Inductance (M). — In general, the coefficient M_{ab} by which the rate of change of the current $\left(\frac{di_a}{dt}\right)$ in a circuit A must be multiplied to give the electromotive force e induced by this current in another circuit B, is called the "coefficient of mutual induction" or simply the "mutual inductance" between A and B; see Fig. 15. In general then, the e.m.f. induced in any circuit B by a varying current i_a in any other circuit A, is

$$e_b = M_{ab}\frac{di_a}{dt}, \qquad (43)$$

where M_{ab} is the mutual inductance between A and B.

It can be shown from the principle of the conservation of energy that the mutual inductance of a circuit A with respect to a second circuit B must be equal

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to the mutual inductance of B with respect to A, that is, $M_{ab} = M_{ba}$. Whence, the e.m.f. induced in A when the current in B increases by an amount di_b is

$$e_a = M_{ab}\frac{di_b}{dt}.$$

Since $e_b = \frac{d\lambda_{ab}}{dt}$, where λ_{ab} is the number of linkages between the circuit B and the flux through B due to the current i_a, the mutual inductance may also be defined by the relation

$$M_{ab} = \frac{d\lambda_{ab}}{di_a}. \tag{44}$$

That is, the *mutual inductance between two circuits A and B is equal to the increase in the number of magnetic linkages of the circuit B per unit increase of the current in A*, and vice versa. When the permeability of the magnetic circuit is *constant*, the mutual inductance is also a constant equal to the linkages of B per unit current in A, and vice versa. When every flux line linking *both* A and B is linked by every current stream line in both A and B, then

$$M_{ab} = \frac{4\pi N_a N_b}{\mathcal{R}_{ab}}, \tag{44a}$$

where N_a and N_b are the number of turns forming the circuits A and B respectively and $\mathcal{R}_{ab}$ is the reluctance of that *part* of the magnetic circuit through which the flux from A to B passes.

The units of mutual inductance are the same as those of self-inductance; see *Units and Conversion Factors*.

Formulas for mutual inductance for a few special cases are given in the article on *Inductance, and Inductive Reactance*.

Instantaneous Potential Drop Through a Coil. — Consider a coil of wire which has a resistance r and an inductance L. Then when this coil contains no other source of e.m.f. than its own self-induced e.m.f., the expression for the instantaneous potential drop through the coil is

$$v = ri + L\frac{di}{dt}, \tag{45}$$

where i is the instantaneous value of the current and $\frac{di}{dt}$ the increase in this current per unit of time. Compare with the equation

$$i = gv + C\frac{dv}{dt},$$

for the instantaneous current through a condenser, where g is the conductance of the dielectric in the condenser, v the potential drop across it, and $\frac{dv}{dt}$ the increase in the potential drop per unit of time.

When there is another coil near the first coil, and the two coils have resistances r_1 and r_2 and self-inductances L_1 and L_2 respectively, and a mutual inductance M, then the potential drops through them are respectively

$$\left.\begin{aligned} v_1 &= r_1 i_1 + L_1\frac{di_1}{dt} + M\frac{di_2}{dt} \\ v_2 &= r_2 i_2 + L_2\frac{di_2}{dt} + M\frac{di_1}{dt} \end{aligned}\right\} \tag{46}$$

where i_1 and i_2 are the currents in the two coils in the *same* direction.

Effective or "Total" Inductance. — In certain cases of symmetry, when the algebraic sum of all the currents in the field is zero, the voltage drop along any *portion* of a circuit may be represented by an expression of the form

$$v = ri + L\frac{di}{dt},$$

although there may be several electric circuits in the vicinity in which electric currents are flowing. An example of this is the case of the wires of a three-phase transmission line arranged so that they form the three edges of an equilateral prism; see *Inductance and Inductive Reactance*. The coefficient L in this case is called the "effective self-inductance" of this particular portion of the circuit; it really takes into account not only the self-induction of the particular part of the circuit under consideration but also the mutual induction of the rest of the circuit or circuits with respect to this particular portion. Similarly, when the voltage drop in any portion of a circuit involves not only the current i_1 in this circuit but also a current i_2 in some part of this circuit or in some other circuit, the coefficient M_{12} in the expression

$$v_1 = r_1 i_1 + L_1\frac{di_1}{dt} + M_{12}\frac{di_2}{dt}$$

is called the effective mutual inductance of the part 2 with respect to 1 although L_1 may also be due in part to circuit 2.

Leakage Inductance. — In discussing the action of a transformer (q.v.), which is merely two electric circuits linking the same iron core, it is more convenient to deal with the *resultant* flux due to the currents in *both* electric circuits or windings instead of considering the fluxes due to the two windings separately. Referring to Fig. 21, let ϕ_r represent that portion of the resultant flux due to the currents i_1 and i_2 in the two windings 1 and 2, and let ϕ_1 represent that part of the total flux which links 1 only and ϕ_2 that part of the total flux which links 2 only. Let λ_1 be the linkages between ϕ_1 and circuit 1 and λ_2 be the linkages between ϕ_2 and circuit 2. Then

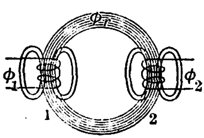

Fig. 21.

$$L_1' = \frac{\lambda_1}{i_1} \quad \text{and} \quad L_2' = \frac{\lambda_2}{i_2},$$

are called the "leakage inductances" of the two windings respectively: the reluctance of the paths of ϕ_1 and ϕ_2 are practically constant since the air portion of these paths forms a greater part of the reluctance in each case.

Let i_1 be the current in winding 1 and i_2 be the current in winding 2 in the *opposite* direction to i_1 (this is the actual relation in a transformer during most of the time) and let e_1 be the impressed e.m.f. across the terminals of the first or primary winding and e_2 the terminal e.m.f. at the terminals of the second or secondary winding when the current i_2 is flowing. Then

$$\left.\begin{aligned} e_1 &= r_1 i_1 + L_1'\frac{di_1}{dt} + N_1\frac{d\phi_r}{dt} \\ e_2 &= N_2\frac{d\phi_r}{dt} - r_2 i_2 - L_2'\frac{di_2}{dt} \end{aligned}\right\} \qquad (47)$$

where r_1 and r_2 are the resistances of the two windings and N_1 and N_2 are the number of turns in the two windings respectively.

Comparing equation (47) with (46) and noting that $e_1 = v_1$ and $e_2 = -v_2$, and i_2 in (47) is taken in the opposite direction from i_2 in (46), it may be shown that*

$$L_1' = L_1 - \frac{N_1}{N_2} M,$$

$$L_2' = L_2 - \frac{N_2}{N_1} M.$$

Whence the leakage inductance of each winding is very much less than the total self-inductance of that winding.

Fundamental Equations of the Transformer. — Comparing equations (47) and (46) it may also be seen that*

$$\phi_r = \frac{M}{N_2}\left(i_1 - \frac{N_2}{N_1} i_2\right).$$

Put $i_2' = \frac{N_2}{N_1} i_2$, that is, i_2' is the current which would be produced in the secondary if it had the same number of turns as the primary. The difference

$$i_m = i_1 - i_2'$$

is called the "magnetizing current," and the quantity $L_m = \frac{M}{N_2}$ may be called the magnetizing inductance. Then

$$\phi_r = L_m i_m.$$

In the above deduction no account is taken of the eddy-current and hysteresis loss in the iron core; these losses (the eddy-current loss in particular) may be looked upon as due to a third current i_3 flowing in a single turn short-circuited on itself and having a resistance r_3 and negligible inductance. The equation for this tertiary circuit is then

$$0 = \frac{d\phi_r}{dt} - r_3 i_3.$$

Whence the corresponding current in the primary winding is $i_3' = \frac{i_3}{N_1}$, which may also be written

$$i_3' = g N_1 \frac{d\phi_r}{dt},$$

where $g = \frac{1}{r_3 N_1^2}$ is called the "leakage conductance" of the transformer. The total primary current is then

$$i_1 = i_2' + i_m + i_3'.$$

From equation (47) and the relations given in the last two paragraphs the complete theory of the transformer may be developed.

ENERGY OF THE MAGNETIC FIELD. — Energy is required to establish a flow of electricity just as energy is required to set a column of water in motion, this "energy of motion" of electricity being analogous to the kinetic energy of a moving body. This energy of motion is most conveniently ex-

* In this equation i_1 and i_2 are taken in the opposite direction.

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pressed in terms of the magnetic field which accompanies the flow of electricity or electric current; the mathematical expression for it may be put into various forms.

Magnetic Energy of Single Electric Circuit, Permeability Constant.—For example, consider a single electric circuit and the magnetic field which is established around this circuit when the current in it increases from zero to a value I. If at any instant the current has the value i and number of magnetic linkages between this current and its own flux at this instant is λ, then when the current increases by di the linkages increase by an amount $d\lambda$ in the right-handed screw direction with respect to the current and therefore the *electric output* of the circuit during this change is, from equation (31a), $id\lambda$, which is also equal to the *magnetic* power *input* into the magnetic circuit which it links. Hence the total energy input into the magnetic circuit or magnetic field is

$$W = \int_0^I id\lambda = \int_0^I Li\,di,$$

since by definition $d\lambda = L\,di$, see equation (42). When the permeability is constant L is also constant, whence for constant permeability

$$W = \frac{1}{2} LI^2. \tag{48}$$

Equation may also be written

$$W = \frac{1}{2}\lambda I = \frac{1}{8\pi}\frac{\mathfrak{F}^2}{\mathcal{R}} = \frac{1}{8\pi}\mathcal{R}\phi^2 \tag{48a}$$

where $\mathfrak{F}$ is the magnetomotive force ($= 4\pi NI$ when the coil has N turns in a concentrated winding), $\mathcal{R}$ is the reluctance of the magnetic circuit and ϕ the total magnetic flux.

Since the impressed m.m.f. per unit length of a magnetic flux line is equal to the magnetizing force H, and the flux per unit area perpendicular to this flux line is equal to the flux density B, the *energy per unit volume* of the magnetic field is

$$w = \frac{HB}{8\pi} = \frac{\mu H^2}{8\pi} = \frac{B^2}{8\pi\mu}. \tag{48b}$$

These various formulas should be compared with the corresponding formulas, equation (26), for electrostatic energy.

Magnetic Energy of Two or More Electric Circuits, Permeability Constant.—It can also be readily shown that the total energy required to establish currents I_1, I_2, etc. in several electric circuits linking one or more magnetic circuits of *constant* reluctance is

$$\begin{aligned} W = {} & \tfrac{1}{2} L_1I_1^2 + \tfrac{1}{2} L_2I_2^2 + \tfrac{1}{2} L_3I_3^2 + \ldots \\ & + M_{12}I_1I_2 + M_{13}I_1I_3 + M_{23}I_2I_3 + \ldots \end{aligned} \tag{49}$$

where the L's and M's represent the self and mutual inductances respectively. This may also be written

$$W = \frac{1}{2}\Sigma\phi I, \tag{49a}$$

where the summation is an *algebraic* one and includes every complete *turn* of the electric circuit, I being the current in this turn and ϕ the magnetic flux linking this turn in the *right-handed* screw direction with respect to the current.

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The energy per unit volume at any point in the magnetic field due to any number of electric circuits is represented by equation (48b), where H and B are taken as the resultant magnetizing force and flux density respectively at this point.

When the permeability is not constant the *energy transferred to unit volume* of the magnetic field due to any number of currents is

$$w = \frac{1}{4\pi}\int_0^B H\,dB \qquad (50)$$

provided the magnetizing force H and the flux density B are either in the same or directly opposite directions, as, for example, in the case of a uniformly magnetized iron toroid. To integrate this expression requires a knowledge of the relation between B and H. Note also that, due to the phenomenon of magnetic hysteresis, part of the energy required to establish a magnetic field in iron or other magnetic substance is dissipated as heat and is not recoverable when the field disappears; therefore, only part of the energy represented by this formula is "stored" in the field in a recoverable form. See article on *Magnetic Properties of Iron.*

MECHANICAL FORCES IN THE MAGNETIC FIELD. (*See also article on Electromagnets.*) — Experience shows that all bodies in which an electric current exists, and all bodies in which a magnetic flux exists, exert in general mutual mechanical forces upon one another tending to produce such a relative motion of these bodies as will *increase* the energy of the magnetic field. Let f be the component of the force tending to move any body in the field in a given direction and let dW be the increase in the energy of the field due to displacing the body a distance dx in this direction, then

$$f = \frac{dW}{dx}, \qquad (51)$$

provided this displacement does not alter the existing magnetomotive forces in the field.

Similarly, calling T the component of the torque tending to turn any body in the field about a given axis, and dW the *increase* in the magnetic energy due to the turning of the body through an angle of $d\alpha$ radians about this axis, then

$$T = \frac{dW}{d\alpha}, \qquad (51a)$$

provided this displacement does not alter the existing magnetomotive forces in the field.

Equations (51) and (51a) also give the actual force and torque respectively during a change in position which *does* cause a change in the magnetomotive forces in the field, provided dW is taken to represent the net increase in the energy of the field. This force and torque may differ greatly from the steady state force and torque; see article on *Electromagnets.*

Force Produced by a Magnetic Field on a Coil Carrying a Current. — From equation (32) the energy *output* of an electric circuit, when the magnetic flux threading it in the right-handed screw direction with respect to the current in it increases by an amount $d\phi$, is $dW = NI\,d\phi$, where N is the number of turns linked by this increase in flux and I is the current in each turn. This is the energy *input* into the magnetic field. Whence if an increase in flux $d\phi$ is

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produced through the coil when it moves a distance dx, the force acting on the coil in the direction of dx is

$$f = NI\frac{d\phi}{dx}; \tag{52}$$

$d\phi$ represents the increase in the flux linking the coil in the *right-handed* screw direction with respect to the current in it or the number of flux lines which cut the coil as the result of its motion. Similarly, when the coil is so mounted that it can move only about a fixed axis, then the value of the torque tending to turn it about this axis is

$$T = NI\frac{d\phi}{d\alpha}, \tag{52a}$$

where $d\phi$ represents the increase in the flux linking the coil in the right-handed screw direction with respect to the current in it when the coil turns through an angle α (in radians).

From these relations it follows that a coil carrying an electric current, when in the magnetic field due to any other agent (current or permanent magnet), always tends to take up that position in which it will embrace the maximum possible flux linking the coil in the right-handed screw direction with respect to the current in it. This accounts for the attraction of two parallel coils when they carry currents in the same direction, and the repulsion of two such coils when they carry currents in opposite directions. This principle is useful in determining the direction of motion of the moving element in such devices as the electric motor, galvanometer, current balance, electrodynamometer (q.v.).

Torque on the Coil When its Plane is Parallel to the Magnetic Field. — When the coil is placed with its plane parallel to the flux lines due to some other agent (e.g., a permanent magnet or another coil carrying a current), the flux linking the coil due to this agent is zero; see Fig. 22. Let the two circles represent sections of the two sides of the coil, its plane being perpendicular to the page, and let the dot in the left-hand circle indicate that the current is up through this side of the coil and the cross in the other circle that it is down through the other side. Let B be the flux density of the field constant for each point along the flux line since the flux lines are parallel, and let A be the area of the coil and let $\phi = BA$, that is, ϕ represents the total flux which *would be produced* through the coil by a uniform field of flux density B at *right angles* to it. Then the torque on the coil when its plane is *parallel* to the field is

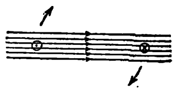

Fig. 22.

$$T = N\phi I. \tag{53}$$

This relation is useful in calculating the torque on the moving element of a galvanometer, ammeter, electrodynamometer, wattmeter, etc.

Average Torque on a Coil Rotating in a Magnetic Field. — Consider a coil which is rotating with an angular velocity ω about a fixed axis in a magnetic field due to some other agent (e.g., an armature coil rotating in the magnetic field produced by the current in the field coils). Let the current in this coil be constant and in the same direction with respect to the coil while the coil turns from the position in which it embraces the maximum flux ϕ in the left-handed screw direction to the position (a half revolution in a 2-pole machine) when it embraces this same maximum flux ϕ in the right-handed

NOTE. — ALL FORMULAS ON THIS PAGE ARE IN C.G.S. ELECTROMAGNETIC UNITS.

screw direction. The total change in the flux while the coil turns through this angle, π radians in the case of a 2-pole machine, is 2ϕ, whence the average torque turning the coil through this half revolution is

$$T = \frac{2N}{\pi} I\phi \cdot \tag{54}$$

That is, the average torque is proportional to the product of the current by the total flux per pole. When a commutator is provided to change the direction of the current every half turn, then the torque is in the same direction for a complete turn. See *Motors*.

When the coil is mounted in a slotted iron core which rotates with the coil as in an ordinary motor, this torque is exerted partly on the wire and partly on the teeth of the core.

Force on a Wire in a Magnetic Field. — Consider a wire of length l forming part of a closed circuit, Fig. 23. Let B be the value of the flux density at the wire, I be the current in the direction indicated, and let the lines representing the flux be perpendicular to the wire in the direction from the eye to the page. When this wire moves a distance dx to the left the flux threading the closed loop formed by the circuit is increased by an amount $d\phi = Bldx$, whence the force acting on the wire is, from equation (51),

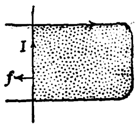

Fig. 23.

$$f = BlI. \tag{55}$$

Left-hand Rule. — The relative directions of this force, the flux density B and the current I may be conveniently determined by pointing the forefinger of the left hand in the direction of the flux and the middle finger in the direction of the current (I), then if the thumb is held perpendicular to these two fingers it will point in the direction in which the force tends to move the wire. Compare with the right-handed rule for e.m.f.

Forces on Magnetic Bodies in a Magnetic Field. — In general the reluctance of a magnetic field to the flux set up by a given magnetomotive force depends upon the relative positions of the various magnetic bodies in the field with respect to one another and with respect to the electric circuit producing this m.m.f. When any magnetic body in the field is displaced the total reluctance will in general be changed due chiefly to the change in the dimensions of the air portion of the circuit. From equations (48a) and (51) it can be shown that the force acting on any magnetic body in the field is in the direction of the flux lines threading it and has the value

$$f = -\frac{1}{8\pi}\phi^2 \frac{d\mathcal{R}}{dx}, \tag{56}$$

provided the magnetomotive force remains constant; where ϕ represents the total flux threading the body and $d\mathcal{R}$ represents the *increase* in the reluctance of the magnetic circuit corresponding to a displacement dx of the body in the direction of the flux lines. The minus sign in this formula indicates that the force is always in the direction in which a motion of the body would *decrease* the reluctance of the circuit. In deducing this expression it is assumed that the permeability of each body in the field is constant. It can also be shown that to a close approximation the same formula holds for actual magnetic bodies, for which the permeability is not a constant.

NOTE. — ALL FORMULAS ON THIS PAGE ARE IN C.G.S. ELECTROMAGNETIC UNITS.

The above relation accounts for the attraction of one magnet for another when their unlike poles are nearer each other than their like poles, and the repulsion of two magnets when their like poles are nearer than their unlike poles. It also accounts for the attraction of iron or other paramagnetic substance by *either* pole of a magnet or by either "face" of an electric circuit, and the repulsion of a diamagnetic substance by either pole of a magnet or either face of an electric circuit; see article on *Electromagnets*.

As a special application of equation (56) consider the electromagnet shown in Fig. 24, which is one form of "permeameter" (q.v.). R is a rod of iron the flat end P of which makes contact with the yoke Y. This rod passes through a hole in the top of the yoke. C is a magnetizing coil. The flux lines through the rod are uniformly distributed and pass perpendicularly into the yoke at the joint P. Let A be the area of the end of the rod and let B be the flux density at this area. When the rod is raised a distance dx, so that an air gap of length dx is formed at P, an increase in the reluctance of the magnetic circuit is produced due to the formation of an air gap at P, and a decrease in reluctance is produced by the shortening of the iron part of the circuit, i.e., by the reluctance of a length dx of the rod. Due to the high permeability of the iron compared with that of the air, the decrease in the reluctance of the iron part of the circuit may be neglected in comparison with the increase in reluctance due to the formation of the air gap. The net increase in reluctance may then be taken as the reluctance of this air gap of length dx, that is, $d\mathcal{R} = dx/A$, since the permeability of air is unity. Whence, from equation (56), the force required to raise the rod is

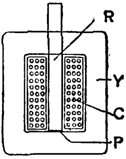

Fig. 24.

$$f = \frac{1}{8\pi}\frac{\phi^2}{A} = \frac{B^2A}{8\pi}. \tag{56a}$$

That is, the tractive force per unit area between the rod and the yoke is proportional to the square of the flux density in the rod.

HYPOTHESIS REGARDING THE NATURE OF MAGNETISM. — The peculiar properties possessed by a magnetic needle or other magnet may be accounted for by assuming that at least some of the molecular charges in a magnetic substance have an orbital motion without friction, forming a kind of "molecular solar system." The magnetization of such a substance is then due to the setting of the molecular currents in planes more or less parallel to the plane of the magnetizing current. In the case of a permanent magnet these molecular currents retain their parallel or "polarized" condition even when the magnetizing current is removed. For example, when a needle is magnetized by a current in a coil placed around it, the molecular currents are set at right angles to the axis of the needle. A magnetic needle placed in a magnetic field, therefore, tends to set itself so that each molecular current embraces maximum flux and hence "points" in the direction of this flux.

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[H. PENDER.]

NOTE. — ALL FORMULAS ON THIS PAGE ARE IN C.G.S. ELECTROMAGNETIC UNITS.

ELECTROCHEMICAL PROCESSES, INDUSTRIAL. — (*See also Electrochemistry, Principles of; Furnaces, Electric.*) The principal industrial electrochemical processes are the following :

Electroplating,
Galvanoplasty, including electrotyping,
Electrolytic refining of metals,
Electrolytic winning of metals,
Electrolytic oxidation and reduction,
Electrolysis of sodium and potassium in chloride solutions; electrolysis of water,
Electric-furnace processes (*see Furnaces, Electric*),
Production of ozone.

Some of the more important of these various processes are briefly described below.

ELECTROPLATING. — Electroplating consists in covering a conducting surface (usually metallic) with a thin, smooth, compact, well-adhering layer of metal, by depositing this metal electrolytically from an aqueous solution of one of its salts. The anode of the electroplating vat consists of rods or plates of the same metal as that of the salt in solution and is connected to the positive terminal of the source of electricity. The object to be plated forms the cathode, and is connected to the negative terminal. The anode dissolves approximately to the same extent that the cathode gains, so that the amount of the metal ions in the bath remains nearly constant.

Suspension of Objects to be Plated. — The cathodes are always suspended in the bath between two rows of anodes, so that they will be plated uniformly on both sides. When the cathode is of irregular shape, or very large, it must be turned frequently during the plating in order to get a uniform deposit. The cathodes and anodes are suspended by copper wires from horizontal metallic tubes, the ends of which rest on the edge of the plating vat. The metallic tubes are permanently connected to the source of the electricity, so that as soon as the cathodes are suspended in the bath, electrolysis begins. Small objects, such as tacks, pins and screws, are suspended in the vat in a wire basket, which is, of course, plated simultaneously. To get a uniform plated surface the objects should be well shaken during electrolysis. The anodes are removed from the bath only when they are nearly used up and have to be replaced.

Construction of Vats. — Large plating vats are made of wood lined with some specially prepared substance resembling pitch, or with lead. Small tanks for silver or gold plating are usually porcelain lined.

Voltage and Current Density. — Electroplating tanks are always connected in parallel, so that they will be electrically independent of each other.

Low-voltage generators of from 5 to 6 volts are, therefore, used in plating, and each tank must be connected directly to the generator through a regulating rheostat in order to regulate the voltage.

The proper current density in any plating process is that density at which a good deposit is formed. This may vary within certain limits for a given solution and temperature; it is a function of the temperature and the nature of the solution.

Washing and Pickling. — In order to make the metal adhere well to the surface to be plated, the surface must be smooth and perfectly clean. It is first polished, and the grease is then removed by dipping it into a hot alkaline bath containing 10 per cent, by weight, of sodium carbonate or sodium hydrate.

After washing off the alkali, the object is dipped into a bath called a "pickle," the purpose of which is to remove any oxide that may have been produced by the alkali, and to give a bright surface. The pickle is then washed off with water and the object is suspended immediately in the electroplating vat.

Pickling Solutions. — The pickle varies with the nature of the metal treated. Cast iron and wrought iron are pickled in a solution consisting of 15 parts, by weight, of water to 1 part of concentrated sulphuric acid. A suitable pickle for zinc is dilute sulphuric or hydrochloric acid. Copper, brass, bronze and German silver are pickled first in a bath consisting of 200 parts, by weight, of nitric acid of specific gravity 1.33, 1 part of common salt and 1 part of lampblack. The lampblack is intended to form some nitrous acid from the nitric acid. The object is then washed in boiling water and is immersed in a "bright dipping bath," to give a bright surface. This bath consists of 75 parts, by weight, of nitric acid of specific gravity 1.38, 100 parts of concentrated sulphuric acid and 1 part of common salt.

After the plating is finished, the object is removed from the plating bath, washed in hot water and placed in warm sawdust to dry.

Plating by Dipping. — A thin film of metal may be deposited on a metal by dipping it into a solution of a salt of a metal which is electronegative (*see Electrochemistry, Principles of*) with respect to the metal to be plated, e.g., by dipping iron into a copper-sulphate solution. A small amount of iron dissolves and an equivalent amount of copper is deposited on the remaining iron. Of course, only a thin film can be produced in this way, for, as soon as the iron is covered with copper, the action ceases. No external electric current is needed in this process.

Plating by Contact. — When the metal to be plated is electronegative with respect to the metal to be deposited on it, the electro-deposition can be obtained by connecting the former to a zinc rod. In this case, the solution must be of a complex salt, in order to reduce the deposition of the metal in solution on the zinc itself. For example, silver may be deposited on copper by connecting a zinc rod or plate to the copper object by a wire and dipping both into a potassium-silver-cyanide solution; the zinc dissolves and silver is deposited on the copper, while some silver is also deposited on the zinc. No external current is needed in this process.

Nickel Plating. — The solution ordinarily used consists of 50 parts, by weight, of the double nickel-ammonium sulphate, $NiSO_4 \cdot (NH_4)_2SO_4 \cdot 6\,H_2O$ with from 25 to 50 parts of ammonium sulphate to 1000 parts of water. The anodes are nickel. The solution is made acid enough to redden litmus slightly, either by addition of a small amount of sulphuric acid or one-half per cent of citric acid. The proper current density on the cathode is about 0.6 ampere per square decimeter (5.5 amperes per square foot) of exposed surface, which requires about 2 volts. The surface should be perceptibly coated with nickel in two or three minutes, and a few bubbles of hydrogen are liberated continuously. If the current is too weak, the surface becomes discolored, and if too strong hydrogen is evolved more rapidly and the surface turns dark.

Iron is sometimes copper plated before it is nickel plated, but this is not necessary, for nickel adheres to iron perfectly well if the surface has been properly cleaned.

A nickel-chloride solution gives good results in plating any metal except iron. Iron always eventually rusts if plated in a chloride bath.

Copper Plating. — The metals on which copper is usually plated, such as zinc, iron and tin, are more electropositive than copper. On dipping any of these metals into an acid copper-sulphate bath, they would become covered with a layer of copper, which in some cases is spongy and does not adhere

well. In order to reduce the velocity with which this reaction takes place a solution of the double cyanide of copper and potassium, $KCu(CN)_2$ is used. This can be made by dissolving cuprous cyanide in potassium cyanide to form a 3 to 8 per cent solution,* with an excess of 0.2 per cent of potassium cyanide. This bath is generally heated to 50° or 60° C. The proper current density at the cathode is about 0.5 ampere per square decimeter (4.6 amperes per square foot) which requires about 3 volts at room temperature.

Surfaces that have already received a thin coating of copper in a cyanide bath are sometimes thickened in an acid copper-sulphate bath. The cyanide must be washed off on transferring to the sulphate bath. A sulphate bath may be made by dissolving 150 grams of copper sulphate $CuSO_4 \cdot 5\,H_2O$ and 50 grams of concentrated sulphuric acid in 1 liter of water. The proper current density is about 0.7 ampere per square decimeter (6.5 amperes per square foot) which requires less than 1 volt.

Small springs are very much weakened by copper plating in a cyanide bath, and are very likely to break while suspended, slightly stretched, in the bath during plating. The reason for this is not known.

Zinc Plating. — Electrolytically deposited zinc is of a dull color and is not as pleasing in appearance as layers obtained by dipping in melted zinc, but electrolytic zinc has been shown to protect iron better for a given thickness of deposit than a coating made from melted zinc. (*Burgess, Electrochem. and Met. Ind., Vol. 3, p. 17, 1905.*) A suitable solution consists of 200 grams of zinc sulphate, $ZnSO_4 \cdot 7\,H_2O$, 40 grams of sodium sulphate, $Na_2SO_4 \cdot 10\,H_2O$ and 10 grams of zinc chloride per liter, slightly acidified with sulphuric acid. The current density in the cathode is from 0.5 to 2 amperes per square decimeter (4.6 to 18 amperes per square foot) which requires from about 1 to 2.5 volts. Zinc anodes are used. A little more zinc is dissolved than is deposited, due to the free acid. The acid must be replaced as it is used up. The resistance may be reduced by warming the bath to 40° or 45° C.

Brass Plating. — If an acid solution of zinc and of copper sulphates were electrolyzed, only copper would be deposited. In a cyanide solution of zinc and copper, however, these metals are deposited simultaneously in the form of an alloy. (*Spitzer, Zeit. f. Electrochemie, Vol. 11, p. 367, 1905.*) The copper is deposited more easily than the zinc, so that at a low current density, 0.1 ampere per square decimeter (0.93 ampere per square foot), only a small amount of zinc is deposited, but at 0.3 ampere per square decimeter (2.8 amperes per square foot), the deposit contains only 80 per cent of copper. Increasing the current density above this amount changes the composition of the deposit only slightly.

A suitable bath for brass plating is made by substituting zinc cyanide for half of the copper cyanide in the solution given above for plating. Brass anodes are used.

Other brass baths that have been found to give good deposits are the following:

1 liter water,
14 grams sodium carbonate, dried,
20 grams sodium sulphate, dried,
20 grams double cyanide of potassium and copper,
20 grams monosodium sulphite,
20 grams double cyanide of potassium and zinc,
1 gram potassium cyanide,
2 grams ammonium chloride.

* An *n* per cent solution of a substance contains *n* parts, by weight, of that substance in 100 parts, by weight, of the solution.

With the electrodes 15 centimeters apart, current density 0.3 ampere per square decimeter (2.78 amperes per square foot) about 3 volts are required.

1 liter water,
15 grams double cyanide of potassium and copper, crystallized,
16.5 grams double cyanide of potassium and zinc,
25.0 grams sodium sulphite,
2.0 grams potassium cyanide, 98 per cent.

Current density 0.3 ampere per square decimeter (2.78 amperes per square foot) requiring 3 volts when the electrodes are 10 centimeters apart. (*Schlötter, Galvanostegie, 1. Teil, p. 238, 1910.*)

Silver Plating. — The double cyanide of potassium and silver is universally used for silver plating on account of the smooth deposit obtained with this solution. The deposit from a nitrate comes down in the form of isolated crystals, which do not cover the surface completely. The solution contains from 1 to 5 per cent silver as potassium-silver cyanide, $KAg(CN)_2$, with 0.5 per cent of free potassium cyanide. Too much or too little free cyanide gives a bad color to the deposit. A good silver plating bath may be made up as follows:

20 grams potassium silver cyanide,
10–12 grams potassium cyanide, 99 per cent,
1 liter water.

The current density is 0.3 ampere per square decimeter, at about 1 volt. (*Schlötter, Galvanostegie, 1. Teil. p. 149, 1910.*) The anodes are silver.

Silver is deposited only on a surface of copper or copper alloy. Other metals must be copper plated before silvering. In order to make the silver adhere well, the copper surface must be amalgamated by dipping the cleaned surface in a "quicking bath," consisting of a solution of 30 grams of potassium-mercury cyanide, $KHg(CN)_3$, and 30 grams of potassium cyanide in 1 liter of water. On removal from the quicking bath, articles are washed and placed immediately in the silvering bath.

Gold Plating. — The solution used in gold plating contains from 0.35 to 1 per cent of gold as the double cyanide of gold and potassium, $KAu(CN)_2$, with twice as much free potassium cyanide. The current density on the cathode is about 0.2 ampere per square decimeter (1.9 amperes per square foot), which requires about 1.5 volts. The anodes are pure gold. The solution may be used hot or cold. The deposit from a hot solution is more dense, more uniform and of a richer color. The color of the gold deposit may be influenced by simultaneously depositing some other metal. Green gilding may be obtained by adding a little silver cyanide to the bath, until the desired tint is obtained. The solution should be cold. To give the deposit a red tint, a little copper cyanide is added to the solution.

Other Plating Processes. — Plating with the following metals is sometimes carried out: platinum, tin, lead, iron, antimony, and arsenic.

Plating on Aluminum frequently does not wear well, on account of the difficulty in getting a perfectly clean aluminum surface on which to plate. This is due to the rapidity with which a thin invisible film of oxide or hydroxide forms on aluminum when exposed to the air or to any solution. One method of overcoming this difficulty is to immerse the aluminum in a solution of potassium hydrate until hydrogen is evolved, and then dip without previous washing in a potassium-silver-cyanide solution. The aluminum is immediately covered with a layer of silver. It is still better to amalgamate by dipping into a 0.5 per cent solution of mercuric chloride immediately after treating with hydrate. The chloride is rinsed off, and the object again treated with potassium hydrate and

then immediately suspended in the silvering bath. (*Langbein, Electrodeposition of Metals, 4th ed., p. 409, 1902.*)

Burgess (*Electrochem. Ind., Vol. 2, p. 85, 1904*) recommends cleaning first with dilute hydrofluoric acid, then with a mixture of 100 parts of sulphuric acid and 75 parts of nitric acid, both concentrated. After rinsing with water the surface is immediately plated with zinc, as this metal is found to adhere better than many others. Starting with the zinc surface, other metals are readily deposited.

GALVANOPLASTY. — Galvanoplasty is the art of reproducing by electrolysis articles of various kinds, or of making finished products, such as set-up type, copper tubes, etc.

Electrotyping. — The object is to make a copper plate which shall be an exact duplicate of type which has been set up ready for printing. First an impression of the type is made in wax, which is then covered with a thin layer of graphite, by dusting the fine powder over the wax surface with soft brushes. A thin layer of copper is then formed by sprinkling the surface with iron filings and pouring over the surface a solution of copper sulphate. The iron goes into solution, depositing copper on itself and on the graphite. The wax plate is then washed in water and suspended in an acid bath of copper sulphate, where copper is deposited electrolytically until a thin sheet that can be stripped from the wax has been formed. After removing the copper sheet from the wax plate, a melted lead-antimony alloy is poured on its reverse side, making a plate approximately 0.5 inch thick. This is then used for printing in place of the original type. The economy of this procedure comes in the saving of wear on the type, and the relatively small amount of type which has to be kept in stock.

The current density ranges from 1 to 2 amperes per square decimeter (0.9 to 1.8 amperes per square foot) and the volts per cell from 0.75 to 1.5.

Copper tubes are made by the Elmore process by depositing copper on a conducting cylinder which rotates in an acid copper-sulphate bath. The surface must be conducting, but the copper must not stick so firmly that the cylinder cannot be slipped out of the tube when finished. In order to keep the outer surface of the tube smooth, it is frequently polished during the deposition of copper. Copper sheets may be made by making tubes of large diameter and cutting them open.

Metallic foil may be made by the electrolytic deposition of a thin metallic layer on a surface from which it can be removed.

Parabolic mirrors are made by depositing copper electrolytically on a parabolic glass surface that has been silvered, and separating the metal from the glass by warming. (*Cowper-Cowles, Electrolytisches Verfahren zur Herstellung parabolischer Spiegel, 1904.*)

ELECTROLYTIC REFINING. — The method of refining metals electrolytically is as follows. The impure metal is made the anode of an electrolytic bath, the electrolyte of which is, at the start, a solution of a pure salt of the metal to be refined, and the cathode is a sheet of the refined metal. On passing the current, metal dissolves, along with certain of the impurities. The impurities which are electropositive with respect to the principal metal dissolve; those electronegative with respect to the principal metal remain adhering to the anode. When the latter finally drop from the anode they may be dissolved by the free acid, in which case they would be precipitated again on coming in contact with the anode or the cathode. Some metals are precipitated as an insoluble salt as soon as they dissolve, and are thus removed from the further action of the current.

When it goes into solution, the principal metal is, therefore, separated from those metals which are electronegative with respect to it. When it is deposited

on the cathode, it is separated from those metals which are electropositive with respect to it. The bath thus becomes contaminated with certain of the impurities in the anode, and these would eventually be deposited on the cathode if the bath is not purified from time to time.

Anode Mud. — The metals which do not dissolve drop to the bottom of the tank forming the "anode mud." It is from this mud that the platinum, gold and silver are recovered in copper refining. For methods of working up anode mud, see Kern, *Met. and Chem. Eng.*, Vol. 9, p. 417, 1911.

Copper Refining. — The object in refining copper electrolytically is to obtain as pure copper as possible for electric conductors and to obtain the precious metals contained in the crude copper. A representative composition of crude copper anodes for American refineries is the following (*Addicks, Electrochem. Met. Ind., Vol. 4, p. 16, 1906*):

Copper	98–99.5 per cent.
Silver	0–300 oz. per ton.
Gold	0–40 oz. per ton.
Arsenic	0–2 per cent.

The refined copper is about 99.95 per cent copper. The electrolyte is a solution of copper sulphate and sulphuric acid containing from 4 to 10 per cent of free acid and from 12 to 20 per cent of copper sulphate. The current density ranges from 0.43 to 4.8 amperes per square decimeter (4 to 45 amperes per square foot) of cathode surface and the volts per cell range from 0.1 to 0.3. (*Ulke, Die Elektrolytische Raffination des Kupfers, p. 42, 1904.*) The electrolyte circulates slowly from tank to tank. The cathodes are thin sheets of refined copper.

Multiple and Series Systems of Arranging the Electrodes. — The tanks for holding the solution and electrodes are made of wood and are frequently lined with lead. There are two methods of arranging the electrodes. In the "multiple system" all the cathodes of one tank are connected, and likewise the anodes, the cathodes having an anode on each side, as shown in Fig. 1.

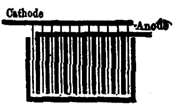

Fig. 1. Multiple System

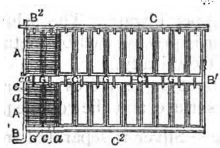

Fig. 2. Walker Multiple System

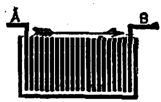

Fig. 3. Series System

The most economical method as regards the use of copper of connecting tanks in the multiple system with each other is that devised by Walker (*U. S. Pat. 687, 800, 1901*). In this system the current flows from tank to tank without being collected in a single bus bar in this passage. The *cathodes* of the first tank rest on a conducting bar *G* (Fig. 2) on which the *anodes* of the second tank also rest. The figure shows two series of tanks with a leading in bus bar *B*, bus bar *B′* connecting the two series, and leading out bus bar B^2.

In the Hayden or "series system," the electrodes are arranged as shown in Fig. 3. The impure copper plates are suspended in the solution at equal distances apart, only the end ones being connected to the dynamo. The current enters at the electrode *A*, which dissolves, and on flowing through the tank passes through the intermediate plates. Pure copper is deposited on the sides of the plates facing *A*, and dissolved from the opposite surfaces facing *B*. Some

conducting preparation is painted over the sides of the intermediate electrodes facing *A*, so that the copper deposited can be separated from the impure copper. When a certain amount of the impure copper has been dissolved, the electrodes are removed, the pure copper is separated from the impure, and the latter is melted and cast into new electrodes. For further details, see Ulke, *Die Elektrolytische Raffination des Kupfers*, 1904.

Comparison of the Two Systems. — The energy required in the series system is about 70 per cent of that required in the multiple system. Contacts give more trouble in the multiple than in the series system. The electrodes in the series system cost more to prepare than in the multiple system. The investment in a series plant of given capacity is less than in a multiple plant. Since lead-lined tanks cannot be used in the series system, the maintenance expense of a series plant is greater than that of a multiple plant.

Nickel Refining. — Nickel may be refined by using a weakly acid solution of nickel chloride or nickel sulphate. To get thick deposits the solution must be heated to from 50° to 90° C. (*Foerster, Zeit. f. Elektroch., Vol. 4, p. 160, 1897*). Nickel and copper may be separated by the David H. Brown process, in which nickel is separated from an alloy of nickel and copper, by depositing the nickel electrolytically. Very pure nickel required for special purposes is refined electrolytically by the Orford Copper Company, by a secret process. (*Electrochem. and Met. Ind., Vol. 4, p. 26, 1906.*) Foerster obtained good results with a current density of 0.5 to 2.5 amperes per square decimeter (4.5 to 23 amperes per square foot) between 50° C. and 90° C. The neutral nickel-sulphate solution contained 30 grams of nickel per liter. Günther obtained good deposits at 60° to 65° C., with 4 to 5 amperes per square decimeter (37 to 45 amperes per square foot) at 3.5 to 4 volts, with nickel sulphate baths. (*Borchers, Elektrometallurgie des Nickels, 1903.*)

Silver Refining. — Two cases arise, (1) the separation of silver from copper in an alloy consisting mainly of these two metals, and (2) the separation of silver from relatively small amounts of gold and platinum.

Dietzel Process. — One method of separating silver and copper is the Dietzel process. This consists in dissolving both of the metals as anode in a weakly acid solution of copper nitrate. The solution is then transferred to another vessel and the silver is precipitated by metallic copper, following which the copper is deposited electrolytically.

The current density is about 1.5 amperes per square decimeter (14 amperes per square foot) of cathode surface, and the voltage per cell from 2.5 to 3.5 volts.

Moebius Process. — Silver is separated from small amounts of other metals by the Moebius process. In this process the anodes consist of the impure silver, the cathodes of thin sheet silver, and the electrolyte is a slightly acid dilute silver-nitrate solution. In the earlier type of apparatus the cathodes were stationary; in the later type the cathode is a rotating sheet of silver. In both the pure silver is scraped off as crystals.

The current density is about 3 amperes per square decimeter (28 amperes per square foot) of cathode surface, and the voltage per cell from 1.4 to 1.5 volts.

Gold refining is carried out in a slightly acid solution of gold chloride. Gold anodes do not dissolve in a solution of gold chloride $AuCl_3$, or of chloroauric acid $HAuCl_4$, but the chlorine liberated comes off in the gaseous form. If some free alkali chloride or hydrochloric acid is present, the gold dissolves. The resulting gold is never less than 999.8 fine, and frequently is 1000 fine.

The baths contain about 3 per cent of free hydrochloric acid with 30 to 40 grams of gold as chloroauric acid per liter. The bath is heated to 60° or 70° C.,

and the current density is about 10 amperes per square decimeter (93 amperes per square foot) at about 1 volt.

Lead refining is carried out by the Betts process. The electrolyte is a solution of lead fluosilicate ($PbSiF_6$) containing 60 to 70 grams of lead and 120 to 130 grams of SiF_6 per liter, and 0.1 per cent of glue. The object in refining lead is to recover the copper, antimony, bismuth, gold and silver. Tin cannot be separated from the lead electrolytically, on account of its proximity to lead in the electrolytic series. (*See article on Electrochemistry, Principles of, and Lead Refining by Electrolysis by A. G. Betts, 1908.*)

The current density ranges from 1.3 to 1.7 amperes per square decimeter (12 to 16 amperes per square foot) of cathode surface, and the voltage per cell from 0.30 to 0.38 volts.

According to Duisberg (*Journ. of Ind. and Eng. Chem. for 1912, p. 752*) iron is now refined electrolytically by the firm Langbein-Pfanhauser & Co., Leipzig. It has valuable magnetic properties, and is used in electromotors. The electrolysis is carried out at 100° to 120° C., but the current density and voltage are not given.

Refining of Other Metals. — A number of other metals may be refined electrolytically, such as mercury, tin, bismuth, and antimony, but these are relatively unimportant.

ELECTROLYTIC WINNING OF METALS. — The attempts that have been made to win metals directly from their ores or from matter are based on the same principle as that underlying metal refining. These attempts have not been successful until recently, however, on account of the large amount of impurity that gets into the bath and on account of mechanical difficulties. The best known of such attempts, are the following.

Marchese Process. — In the Marchese process it was attempted to obtain pure copper from a matte containing principally copper, lead, iron and sulphur, by electrolyzing this matte as an anode in a sulphate bath. The process seemed a success at first, but finally failed.

Siemens and Halske Process. — The Siemens and Halske process for extracting copper consists in electrolyzing a solution of iron sulphate and copper sulphate in a cell in which the anode and cathode are separated by a linen diaphragm. The anode was carbon; the cathode pure copper. In the cathode compartment the solution loses copper by deposition on the cathode. The solution then circulates to the anode compartment, where the ferrous sulphate is oxidized to ferric sulphate. See below, under *Electrolytic Oxidation*. From here the solution circulates to another vat where fresh ore, consisting of copper pyrites ($Cu_2SFe_2S_3$) that has been roasted so as to change the iron sulphide to oxide, is treated. The ferric sulphate dissolves the cuprous sulphide as follows:

$$Cu_2S + 2\,Fe_2(SO_4)_3 = 2\,CuSO_4 + 4\,FeSO_4 + S.$$

This solution, enriched in copper, is then conveyed to the cathode compartment, and the cycle is completed. This process also failed when tried on a commercial scale.

Hoepfner Process. — In the Hoepfner process the copper ore, consisting of copper sulphide, is dissolved in a cupric chloride solution containing a relatively large amount of sodium chloride. The reaction is as follows:

$$Cu_2S + 2\,CuCl_2 = 2\,Cu_2Cl_2 + S.$$

The cuprous chloride, insoluble in water, is held in solution by the sodium chloride. The anode and cathode of the electrolytic cell are separated by a diaphragm. The solution first circulates to the cathode compartment, where part

of the copper is deposited, and then to the anode, where the cuprous chloride is oxidized to cupric chloride. The solution from the anode then circulates to fresh ore.

Copper Hydrometallurgy at Chuquicamata. — A process is now being perfected by the Chile Exploration Company for winning copper directly from the ore of the Chuquicamata copper mine, in Chile. The ore is dissolved in dilute sulphuric acid in leaching tanks. The solution is then conducted to the electrolyzing tanks where the copper is deposited, using insoluble magnetite anodes and their copper cathode starting sheets. The electrolytic refinery will have a capacity of 335,000 pounds of copper per day. (*E. A. Cappelen Smith, Met. & Chem. Eng., 1914, Vol. 12, pp. 291–294.*)

Salom Process. — Lead was at one time extracted from galena (PbS) by the Salom process, consisting in electrolyzing the powdered galena as cathode in sulphuric acid. The hydrogen deposited on the galena combines with the sulphur, forming hydrogen sulphide and leaving the lead as lead sponge. This process was given up, partly at least, on account of the poisonous action of the hydrogen sulphide. (*Trans. Am. Electroch. Soc. Vol. 1, p. 87, 1902; Vol. 4, p. 101, 1903.*)

Hoepfner Zinc Process. — In the winning of zinc, electrolysis may, in the future, take an important part, on account of the fact that in the ordinary metallurgical process there is a large loss. A process devised by Hoepfner consists in roasting the zinc ore if it is insoluble, dissolving the zinc, and depositing it by electrolysis, with insoluble anodes. Very little zinc is refined electrolytically, because it contains no noble metals and there is not much demand for very pure zinc.

Detinning Tin Scrap and Old Cans. — The two principal methods of recovering tin from tin scrap and old tin cans are due to Goldschmidt. The first consists in electrolytically dissolving the tin and depositing it in the metallic state; the second method, which however is not electrolytic, converts the tin into tin tetrachloride by treating with dry chlorine.

In the electrolytic method the solution consists of a sodium hydrate solution, containing a certain amount of carbonate, absorbed from the air, and sodium stannate. The best concentrations are: 10 to 12 per cent alkalinity, not over 7 per cent free alkali; carbonate not over 3 per cent; stannate not over 5 per cent. Temperature, 60° to 70° C. About one-tenth of the solution must be replaced every week.

The tin scrap is suspended in the solution in baskets (39 by 35 by 18 inches) of perforated sheet iron, containing 100 to 130 pounds of scrap. Old cans are cleaned, compressed and then cut up. The cathodes are sheet iron 39 by 35 inches. The tin dissolves from the anode and is deposited as spongy tin on the cathode. The scrap is left in the bath for three hours. The spongy tin is compressed by a hydraulic press into cylinders weighing six pounds; these are subsequently melted in a furnace with sealed tubes. This sponge contains 50 per cent tin and 50 per cent ash. The ash is subsequently reduced with carbon in an open-hearth furnace, yielding 70 per cent of the tin.

Very careful working is necessary to remove the tin so completely that the iron can be sold to open-hearth plants. Even then the iron contains 0.1 to 0.5 per cent tin.

The cathode current density is never over 0.75 ampere per square decimeter (7 amperes per square foot), at 2 to 3 volts per cell. (*Electrochem. and Met. Ind., Vol. 7, p. 79, 1909; Met. and Chem. Eng., Vol. 10, p. 202, 1912.*)

ELECTROLYTIC OXIDATION AND REDUCTION is frequently carried out on substances in solution. One of the advantages of the electrolytic

method is that no other substance has to be added to the solution. In carrying out an electrolytic oxidation or reduction a porous diaphragm is used to separate the anode and cathode compartments. The substance to be reduced is placed in the cathode compartment, where the whole or a part of the hydrogen that would be evolved by the current while in the nascent state, acts on the substance in question. The intensity of the reduction may be varied by varying the potential difference between the solution and the cathode. This is accomplished either by increasing the current density on the cathode, or by making the cathode of different metals on which the overvoltage (*see Electrochemistry, Principles of*) is different. It frequently happens that the metal composing the cathode has a marked accelerating effect on the velocity of the reduction. Thus the reduction may be much more complete on one metal than on another, though the potential difference between the metals and the solution is the same in both cases. The same considerations apply to oxidation on the anode.

Typical Reduction and Oxidation Processes. — Examples where electrolytic reduction has been found useful are: in the manufacture of white lead by corroding a lead anode in a suitable solvent, e.g., the Luckow process (*Zeit. f. Elektroch., Vol. 3, page 482, 1897*); in the preparation of the lower salts of vanadium, molybdenum and titanium. Examples where electrolytic oxidation has been employed are: in the oxidation of the lower cerium sulphate, $Ce(SO_4)$, to $Ce(SO_4)_2$, the latter being useful as an oxidizing agent; in oxidizing potassium manganate, K_2MnO_4, to permanganate, $KMnO_4$; in the oxidation of potassium ferrocyanide, $K_4Fe(CN)_6$, to the ferricyanide, $K_3Fe(CN)_6$; and of chromium sulphate to a chromate. (*See Foerster, Elektrochemie Wässeriger Lösungen, 1905.*)

ELECTROLYSIS OF SODIUM OR POTASSIUM CHLORIDE. — The electrolysis of sodium or potassium chloride may yield several different products, depending on the kind of cell employed. In the following discussion the reactions for sodium choride are given; these reactions also apply to potassium chloride when K is substituted for Na.

Production of Electrolytic Bleach. — If there is no diaphragm and the solution is kept cool, the chlorine and hydrate react on each other to form principally sodium hypochlorite or "electrolytic bleach," according to the equation

$$2\,NaOH + Cl_2 = NaOCl + NaCl + H_2O.$$

As soon as hypochlorite is formed, it begins to change to chlorate, but at first not as rapidly as it is formed. As the concentration of the hypochlorite increases, a larger and larger proportion changes to chlorate, until finally the concentration of the hypochlorite reaches a limit, which depends on the temperature, original salt concentration, current density, and other factors. The hypochlorite then changes to chlorate as rapidly as it is formed.

The reactions by which hypochlorite changes to chlorate are two; (1) the action of the discharged ClO ion on water:

$$6\,ClO + 3\,H_2O = 2\,HClO_3 + 4\,HCl + 1\tfrac{1}{2}\,O_2,$$

and (2) in a slightly acid solution:

$$2\,HOCl + NaOCl = NaClO_3 + 2\,HCl.$$

If the solution is warmed to about 50° C. the conditions are so changed that the limiting concentration of hypochlorite is much lower than when cold. Sodium and potassium hypochlorites are used only in solution, but the chlorates are crystallized out.

By further electrolysis of a cooled sodium (or potassium) chlorate solution, perchlorate, $NaClO_4$, is produced by the action of the liberated ClO_3 ion on water:

$$2\ ClO_3 + H_2O = HClO_4 + HClO_3$$

and

$$HClO_4 + NaOH = NaClO_4 + H_2O.$$

In most of the cells used for producing hypochlorite, a vessel is divided into a number of narrow compartments by bipolar or intermediate electrodes, consisting of carbon plates (Hass and Oettel) or platinum-iridium wire wound on glass plates (the Kellner cell). The solution is cooled by circulation. There is no essential difference between a hypochlorite and a chlorate cell, except that the latter is used at a higher temperature. (*See numerous volumes in the Engelhardt Monographs.*)

Production of Chlorine and Caustic. — If the anode and cathode are separated by a diaphragm of some kind, sodium hydrate is produced at the cathode by the action of the alkali metal on water:

$$2\ Na + 2\ H_2O = 2\ NaOH + H_2,$$

while at the anode chlorine is set free, which may be used to make bleaching powder.

Various devices are used to separate the hydrate and chlorine in cells intended for these products. In the McDonald cell (*Electrochem. Ind., Vol. 1, page 387, 1903*) and others a porous diaphragm is used. In the Hargreaves-Bird cell (*U. S. Pat. 655,343, 1900; 596,157, 1897*), the porous diaphragm is supported by a heavy copper gauge cathode, which is wetted only by the solution percolating through the diaphragm. In the Townsend cell (*Electrochem. and Met. Ind., Vol. 5, page 209, 1907*) the cathode compartment is filled with kerosene oil. During electrolysis the solution in the anode compartment percolates through the porous diaphragm and the perforated cathode, and on coming in contact with the oil forms drops and sinks to the bottom of the cathode compartment where it is collected. In the "bell process" the anode is inside an inverted nonconducting, nonporous bell, and the cathode is a conducting ring outside.

In the Castner cell the compartments are separated by a mercury diaphragm, acting as an intermediate electrode; and in the Solvay and the Whiting cells a mercury cathode is used, which, when charged with sodium in the electrolytic cell, is decomposed by water in a different compartment. The sodium amalgam reacts with water like metallic sodium. (*See the Engelhardt Monographs.*)

ELECTROLYSIS OF WATER. — The electrolysis of water is carried out on a commercial scale for the production of hydrogen and oxygen. The electrolyte is usually a 15-per-cent sodium hydrate solution. A diaphragm of canvas is used to separate the hydrogen and oxygen in the Flamand cell, now in use by the International Oxygen Company at Newark, N. J. The vessel containing the electrolyte is cast iron and acts as the cathode; the anode is steel. In the Siemens and Halske apparatus the gases are separated by a metallic gauze while bubbling to the surface of the solution; the Garuti and Pompeili apparatus is based on a similar principle. (*See Viktor Engelhardt, Die Elektrolyse des Wassers, 1902.*)

ELECTROTHERMAL PROCESSES. — Some of the more important industrial electrothermal processes and products and the various types of electric furnaces are described in the article on ***Furnaces, Electric.***

OZONE (O_3) is made by the silent discharge of electricity through dry air. The electrodes are water cooled, and protected to prevent the passage of sparks. From 8000 to 50,000 volts alternating are applied, this voltage being obtained from a small step-up transformer. For the Siemens and Halske ozonizer the concentration of the ozone is about 2 or 3 grams per cubic meter of the air which passes through it, and the yield is from 18 to 37 grams per kilowatt hour. (*Electrochem. Ind., Vol. 2, p. 67, 1904.*) The General Electric Company manufacture several types of small ozonizers, and are about to put the Siemens and Halske ozonizer on the market in this country.

BIBLIOGRAPHY. — In addition to the references given above, the following books should be consulted for further information regarding industrial electrochemical processes:

Arens, *Handbuch der Elektrochemie*, 1903; Askenasy, editor, *Einführung in der Technische Elektrochemie, Vol. 1, Elektrothermie*, 1910; Blount, *Practical Electrochemistry*, 1901; Engelhardt, editor, *Monographien über angewandte Elektrochemie;* Foerster, *Elektrochemie Wässeriger Lösungen*, 1905; Haber, *Grundriss der Technischen Elektrochemie*, 1898; Pring, *Some Electrochemical Centres*, 1908; Richards, *Metallurgical Calculations;* Billiter, *Die elektrochemischen Verfahren der chemischen Gross-Industrie*, 1911; Thompson, *Applied Electrochemistry*, 1911; Allmand, *Applied Electrochemistry*, 1912. See also the volumes of the *Metallurgical and Chemical Engineering; Transactions of the American Electrochemical Society; Zeitschrift für Elektrochemie; Transactions of the Faraday Society*. See also Bibliography in *Electrochemistry, Principles of.*

[M. DE K. THOMPSON.]

ELECTROCHEMISTRY, PRINCIPLES OF.—(*See also Electricity and Magnetism, Principles of; Elements, Chemical; Electrochemical Processes, Industrial; Furnaces, Electric.*) Electrochemistry is the science which deals with the phenomena resulting from the direct transformation of electrical into chemical energy or the converse transformation of chemical into electrical energy. By general usage the term, especially as applied to industrial processes, has come to include also those thermochemical phenomena which occur at temperatures produced in electric furnaces, although in such processes electrical energy frequently plays no other rôle than that of generating heat.

The following is a brief table of contents of this article:

DEFINITIONS. — The following terms are commonly used:

Element. — An element is a substance which cannot be decomposed into simpler substances by any known means of chemical analysis. (*See Table of Electrochemical Equivalents below, and article on Elements, Chemical.*)

Atoms and Atomic Weights. — An atom is the smallest mass of an element that can enter into chemical combination with another element or itself. The atomic weight of an element is a number which is proportional to the smallest known combining mass of the element, that of oxygen being chosen as the standard and taken as 16. The relative proportions, by mass, of the elements forming any substance are simple multiples of these numbers. For table of the atomic weights of the more common elements see below, and for a complete table see article on *Elements, Chemical.* On the atomic theory the atomic weight is proportional to the mass of the atom.

Molecules and Molecular Weight. — A molecule is the smallest mass of a substance which can exist and preserve its chemical properties. The molecular weight of an element or of a chemical compound is equal to the sum of the atomic weights of the atoms contained in the molecule, and may readily be calculated from the atomic weights when the molecular symbol of the compound is known. Thus the molecular weight of oxygen, O_2, is $2 \times 16.00 = 32.00$ as there are two atoms in the molecule. The molecular weight of silver nitrate, $AgNO_3$, is $107.88 + 14.01 + 3 \times 16.00 = 169.89$.

Radical. — A radical is a combination of two or more elements which persists as a group in chemical reactions, e.g., the SO_4 in H_2SO_4 is a radical, as exemplified by the reaction $H_2SO_4 + Zn = ZnSO_4 + H_2$.

Formula Weight. — The formula weight of an atom, radical or molecule is the sum of the atomic weights of the elements of which it is formed. For example, the formula weight of an oxygen atom, O, is 16; the formula weight

of an oxygen molecule, O_2, is 32; the formula weight of the radical NO_3 is 62.01.

Valency. — An adequate consideration of valency would involve a discussion beyond the limits of this article. As a simple definition, the valency of an acid or acid radical may be taken as the number of replaceable hydrogen atoms which the acid molecule contains, the valency of the hydrogen atom being always equal to unity. Thus hydrochloric acid, HCl, and nitric acid, $H(NO_3)$, are univalent acids and the elements H and Cl and the radical NO_3 are univalent; sulphuric acid, $H_2(SO_4)$, is a bivalent acid and SO_4 a bivalent radical; $H_3(PO_4)$ is a trivalent acid and PO_4 a trivalent radical, etc. Similarly, in a salt like sodium chloride, NaCl, sodium is a univalent metal, since one atom of sodium replaces one atom of hydrogen in the corresponding acid. But in barium chloride, $BaCl_2$, barium is a bivalent metal, as it replaces the hydrogen of two molecules of hydrochloric acid to form the salt. To completely describe the valency of a salt it is therefore necessary to stipulate the valency of the basic and acid constituents; thus sodium chloride, NaCl, is a uni-univalent salt; sodium sulphate, Na_2SO_4, is a uni-bivalent salt, the basic constituent being univalent and the acid constituent bivalent. Barium sulphate, $BaSO_4$, on the other hand, is a bi-bivalent salt. The valency of a number of elements varies according to the compound of which the element forms a part, e.g., copper is bivalent in cupric salts like $CuSO_4$ but univalent in cuprous salts as CuCl; iron is bivalent or trivalent according as it is a constituent of a ferrous or ferric salt respectively.

Bonds or Affinities. — An atom or radical is sometimes said to possess a number of "bonds" or "affinities" equal to its valency in the compound of which it forms a part.

Equivalent Weight — Chemical Equivalent. — The equivalent weight of an atom, ion or radical is defined as its formula weight divided by its valency. The equivalent weight of an element or compound is also referred to as its "chemical equivalent." The equivalent weight of an acid, base or salt is its molecular formula weight divided by the highest valency of either of its ions. As hydrogen exhibits no other valency than unity its equivalent weight and atomic weight are identical. Similarly, the equivalent weight and molecular weight of hydrochloric acid, HCl, are each 36.47. Copper in copper sulphate, $Cu(SO_4)$, and in cupric chloride, $CuCl_2$, has a valency of 2, hence equivalent weights of these salts are one half their molecular weights respectively.

In computations of electrochemical reactions it is frequently convenient to take as the unit of mass (or weight) of a substance, a mass in grams equal to the number expressing the atomic weight, ionic weight, molecular weight or equivalent weight of the substance. The following names have been given these units.

Gram-Atom or Gram-atomic Weight. — A number of grams of an element equal to its atomic weight, e.g., one gram-atom of oxygen is 16 grams.

Gram-Molecule or Mol. — A number of grams of a substance equal to its molecular weight, e.g., one mol of silver nitrate, $AgNO_3$, is 169.89 grams.

Gram-Equivalent. — A number of grams of an atom, radical or molecule equal to its equivalent weight, i.e., to its formula weight divided by its valency. For example, a gram-equivalent of sulphuric acid is $98.09/2 = 49.04$ grams. A gram-equivalent is usually represented by the chemical formula divided by the valency, e.g., a gram-equivalent of sulphuric acid is denoted by $\frac{1}{2} H_2SO_4$.

Electrolyte — Electrolysis. — When an electric current is passed through certain substances a chemical reaction takes place at the places where the current enters or leaves the substance. Such substances are called "electro-

lytes;" if the current effects a decomposition or chemical change in the electrolyte the process is called "electrolysis."

Electrodes — Anode and Cathode. — Electrodes are the conductors by which the current enters and leaves the electrolyte. The electrode at which the current enters the electrolyte is called the "anode," and that by which the current leaves the electrolyte is called the "cathode." The anode is therefore that electrode which is connected to the positive terminal of the generator; the cathode that connected to the negative terminal.

Ions — Anions and Cations. — The constituents of an electrolyte which are *primarily* liberated or deposited from an electrolyte at the electrodes are called "ions." The term also applies to the constituents of an electrolyte which conduct the current. (*See sections below on Dissociation Theory and Theory of Electrolytic Conduction.*) The ions liberated at the anode are called "anions," while those liberated at the cathode are called "cations." For example, when a current enters and leaves an aqueous solution of copper sulphate, $CuSO_4$, at platinum electrodes, the $CuSO_4$ is decomposed, the copper Cu being liberated from the solution and deposited on the cathode, and the SO_4 being liberated at the anode where it reacts with the water to form oxygen gas and sulphuric acid. The copper in the sulphate solution is therefore a cation and the SO_4 an anion. In general the metal atoms in salts and bases and the replaceable hydrogen atom in acids are cations, while the acid radicals (e.g., SO_4) and the basic radicals (e.g. OH in KOH) are anions.

The terms anion and cation are also used to designate specifically the negative and positive constituents of an electrolyte. For example, in dilute aqueous solution, a molecule of hydrochloric acid, HCl, is looked upon as consisting of one hydrogen cation and one chlorine anion; sulphuric acid, H_2SO_4, of two hydrogen cations and one SO_4 anion. There is much evidence to indicate that a negative charge of fixed amount is associated with each anion and a positive charge (or deficit of negative electricity) of equal amount is associated with each cation, irrespective of the chemical nature of the ion, except that the charges carried by any ion are directly proportional to its valency. The charge carried by every univalent ion is estimated as 5×10^{-10} electrostatic units, the charge carried by every bivalent ion is twice this amount, etc., 5×10^{-10} electrostatic units of negative electricity being the charge of an electron. Hence on the basis of the electron theory, a univalent anion as Cl has associated with it one free electron, a divalent anion as SO_4 two electrons, while a univalent cation as H has a deficit of one electron, a divalent cation as Cu has a deficit of two electrons, etc. See *Electron Theory*.

To distinguish cations and anions from neutral atoms or molecules, small plus or minus signs are often written over the symbol, the number of such signs being equal to the valance of the ion. Thus

$\overset{+}{H}$, $\overset{+}{Na}$, $\overset{++}{Cu}$, $\overset{+++}{Fe}$ (ferric iron) represent cations; $\overset{-}{Cl}$, $\overset{-}{NO_3}$, $\overset{--}{SO_4}$, $\overset{---}{PO_4}$ anions, etc.

Ionic Weight. — The formula weight of an ion is called its ionic weight.

Gram-Ion. — A number of grams of an ion equal to its ionic weight, e.g., one gram-ion of hydrogen is 1.008 grams, and one gram-ion of SO_4 is 96.07 grams.

Electrochemical Equivalent. — The electrochemical equivalent of an ion is the mass in grams of the ion liberated or deposited by one coulomb.

Electrochemical Constant or "Faraday." — The electrochemical constant, denoted by F, and called the "Faraday," is the number of coulombs required to liberate one gram-equivalent of any ion. It is a constant for all ions, and its value, as adopted by the International Congress of Applied Chemistry in 1911, is $F = 96,540$ coulombs.

Solvent, Solute and Solution. — When a substance A "dissolves" in another substance B, the substance A which dissolves is called the "solute," and the substance B is called the "solvent." The two substances together form a "solution."

Osmotic Pressure. — The osmotic pressure of a solution may be regarded as the force in virtue of which a solute tends to diffuse from regions of higher to regions of lower concentration. It can be measured by separating the solution and solvent by means of a diaphragm which is permeable to the solvent but not to the solute. (See section below on *Theory of Solutions*.)

Specific Conductance (κ). — The specific conductance of a solution is the reciprocal of its specific resistance, i.e., it is the conductance of a column of liquid one centimeter long and one square centimeter cross section. It is expressed in reciprocal ohms, or mhos, and is denoted by κ.

Concentration (c). — The concentration (c) of a solution is the quantity of the solute contained in unit volume of the solution. Thus it may be expressed as grams per cubic centimeter or grams per liter, etc.

Equivalent Concentration (η). — The equivalent concentration of a solution is the number of gram-equivalents of solute contained in one cubic centimeter of the solution; it is denoted by η. Calling w the equivalent weight of the solute, $\eta = c/w$.

Dilution (ϕ). — The dilution of a solution is the reciprocal of its concentration, i.e., it is the number of cubic centimeters of solution in which one gram-equivalent of solute is dissolved. It is denoted by ϕ. Hence, $\phi = 1/\eta$ and $\eta = 1/\phi$.

Normal Solution. — A normal solution is a solution containing one gram-equivalent of solute per liter. For such a solution $\eta = 10^{-3}$ or $\phi = 1000$.

Equivalent Conductance (Λ). — The equivalent conductance of a solution at the dilution ϕ is the conductance which a volume (in cu. cm.) of the solution containing one gram-equivalent of the solute would have, if placed between parallel plate electrodes one centimeter apart. It is denoted by Λ_ϕ or Λ_η, according as the dilution or concentration of the solution is given. Hence $\Lambda_\phi = \phi\kappa$, or $\Lambda_\eta = \kappa/\eta$.

The dilution or concentration of a solution should always be stated in connection with its equivalent conductance, otherwise the expression is indefinite.

Heat of Reaction. — The heat of reaction is the heat energy given out or absorbed in a chemical reaction. It is usually expressed in calories; see *Heat and Heat Effects*. Ostwald recommends the use of a calorie equal to 100 times the mean gram-calorie for expressing thermochemical data and this will be adopted in the present discussion. It will be denoted in this article by "cal."

Exothermic Reactions. — Reactions in which heat is given out to the surrounding bodies.

Endothermic Reactions. — Reactions in which heat is absorbed from the surrounding bodies.

NOTATION. — The notation used in this article is, in most instances, that proposed by the Bunsen Gesellschaft and adopted by the Fifth International Congress of Applied Chemistry in Berlin, 1903. For convenience of reference the symbols most frequently used are tabulated below.

κ = specific conductance in reciprocal ohms;

η = concentration, expressed as gram-equivalents of solute per cubic centimeter solution;

ϕ = dilution, expressed as cubic centimeters of solution per gram-equivalent solute;

Λ = equivalent conductance;
Λ_∞ = equivalent conductance at infinite dilution;
γ = degree of ionization;
E = electrical potential difference;
I = current strength;
R = resistance [W adopted by International Congress]
e = single potential difference, taken positive in direction of rise of potential;*
e_h = potential difference measured against a normal hydrogen electrode;
e_c = potential difference measured against a normal calomel electrode;
R = gas constant per mol
F = the Faraday = 96,540 coulombs per gram-equivalent
T = absolute temperature in degrees centigrade.
t = centigrade temperature [Θ adopted by International Congress]

CHEMICAL EQUATIONS. — A chemical equation, such, for example, as that representing the solution of zinc in a dilute hydrochloric acid solution (Aq = water),

$$Zn + 2\,HClAq = ZnCl_2Aq + H_2,$$

is to be interpreted as follows: one gram-atom (65.37 grams) of zinc reacting with 2 mols (2 × 36.47 = 72.94 grams) of hydrochloric acid in aqueous solution forms one mol (65.37 + 2 × 35.46 = 136.3 grams) of zinc chloride in solution and one mol (2 × 1.008 = 2.016 grams) of hydrogen gas. Since atomic weights are relative numbers, any unit of weight, as the pound or kilogram, may be substituted for gram in this statement.

If the reaction is intended to express not only the chemical change which takes place, but also the energy change involved, the "heat of the reaction" must be included, thus:

$$Zn + 2\,HClAq = ZnCl_2Aq + H_2 + 342 \text{ cal.}$$

This thermochemical or energy equation signifies that the intrinsic energy represented by the initial system, consisting of 65.37 grams of zinc and 72.94 grams of hydrochloric acid dissolved in enough water to form a dilute solution, is 342 calories (1 gram of water from 0° to 100° C.) greater than that represented by the final system, consisting of 136.3 grams of zinc chloride dissolved in water and 2.016 grams of hydrogen gas; in other words when the substances in the initial system react and form the substances in the final system, an amount of heat energy is evolved equal to 342 cals. When energy is given up by a system to the surroundings (exothermic reactions), as in the above illustration, it is regarded as positive. If a reaction is accompanied by an absorption of heat energy (endothermic reactions), i.e., if the system takes up heat and thereby tends to cool the surroundings, the sign of the heat of the reaction is taken as negative.

FARADAY'S LAWS. — The first quantitative relations between the magnitude of an electric current and its chemical effect were discovered by Faraday in 1834 and are known as Faraday's Laws. They may be stated as follows:

First Law. — The quantity of an electrolyte decomposed by an electric current is directly proportional to the total quantity of electricity which passes through the electrolytic cell; or, the rate of chemical decomposition is directly proportional to the current. The amount of decomposition is independent of the voltage and intensity of the current, size of electrodes and concentration of electrolyte, so long as the total *quantity* of electricity flowing through the

* This notation is employed to harmonize the notation in this article with that used elsewhere in this book. The Berlin Congress recommends the use of the symbol ϵ for a potential difference taken positive in the direction of the *drop* of potential.

circuit remains constant. (These factors, however, do affect the *ultimate products* of an electrolysis.

Second Law.—A given quantity of electricity always decomposes equivalent weights of different electrolytes irrespective of their nature. For example, if three electrolytic cells containing aqueous solutions of silver nitrate $AgNO_3$, copper sulphate $CuSO_4$, and cuprous chloride CuCl (dissolved in sodium chloride), respectively, be connected in series and the same quantity of electricity passed through each, it will be found if *a* grams of silver are deposited in the first cell, *b* grams of copper in the second and *c* grams of copper in the third, that the following proportion holds between these weights: $a : b : c = \frac{107.88}{1} : \frac{63.57}{2} : \frac{63.57}{1}$, where 107.88 and 63.57 are the atomic weights of silver and copper respectively. In copper sulphate the valency of copper is 2, while in cuprous chloride its valency is 1; silver is always univalent and hence the weights *a, b, c* are proportional not to the atomic weights but to the *equivalent weights* of the metals in the compounds from which they are electrolyzed. It follows from this that a given current will deposit in a given time twice the weight of metal from a salt in which it is combined with a valency of 1, as from a salt in which it exists with a valency of 2. Thus, the same current deposits in a given time twice the weight of copper from a cuprous as from a cupric salt solution. In general, if the ion in a compound has a valency *n*, the quantity of electricity required to liberate it will be *n* times as great as that required to liberate the same ion from a compound in which it is univalent.

It is to be noted, since the numbers expressing the atomic weights of the elements, and hence their equivalent weights, are purely relative, that the relative weights of substances decomposed by a given quantity of electricity may be expressed in grams, kilograms, pounds or any other unit of weight. Thus, in the above illustration the quantity of electricity which will deposit $\frac{63.57}{2} = 31.79$ pounds of copper from a copper sulphate solution will deposit 107.88 pounds of silver.

Faraday's Laws are the expression of the results of direct experiment and have been tested to the limit of precision with which physical and chemical measurements are capable of being carried out at the present time. All evidence indicates that they hold rigidly, i.e., that they are exact laws of nature. In cases where exceptions have seemed to exist these have been shown to arise from secondary causes.

Value of the Electrochemical Constant or Faraday. — The determination of the number of coulombs required to deposit one gram-equivalent of an ion, i.e., the constant connecting the quantity of electricity and the mass of a substance liberated, involves two distinct investigations; first, the measurement of the number of coulombs which pass through a given electrolytic cell, and second, the determination of the amount of the chemical decomposition. The importance of the constant has enlisted the skill of a number of the ablest physicists, and its value is still being investigated at the present time.

The various methods which have been employed vary primarily in the means adopted for measuring the current. A primary instrument, the constants of which can be computed from its dimensions, is essential. Tangent galvanometers and various forms of current balances or dynamometers have been constructed for this purpose. The ultimate precision of the measurements with these instruments is thus referred back to instrumental constants and to the accuracy with which the horizontal component of the earth's field, or the value of the acceleration due to gravity, is known at the place where the research is carried out, respectively.

The amount of silver deposited from a silver nitrate solution has been adopted almost universally for the chemical part of the research. Owing, however, to the fact, established by T. W. Richards, that the amount of silver deposited depends to a slight extent upon whether or not the solution at the anode is allowed to diffuse to the cathode, the values obtained in some of the best earlier determinations are somewhat uncertain. To show the degree of precision with which the constant is now known the following résumé of the determinations of the mass of silver in milligrams deposited by one coulomb is given (*Guthe, Bulletin of the Bureau of Standards, 1905, Vol. 1, p. 363*).

TABLE I.—ELECTROCHEMICAL EQUIVALENT OF SILVER IN MILLIGRAMS PER COULOMB

Observer	Year	Found	Electrochemical equivalent corrected by		
			Richards and Heimrod	Van Dijk	Guthe
		mg.	mg.	mg.	mg.
Mascart	1884	1.1156	......	1.1155	1.1153
Fr. and W. Kohlrausch	1884	1.1183	1.1173	1.1182	1.1177
Rayleigh and Sidgwick	1884	1.1179	1.1175	1.1178	1.1176
Gray	1886	1.1183	......	......	1.1178
Koepsel	1887	1.1174	......	......	1.1169
Pellat and Potier	1890	1.1192	......	1.1191	1.1189
Patterson and Guthe	1898	1.1192	1.1175	1.1180	1.1177
Pellat and Leduc	1903	1.1195	......	1.1192	1.1190
Van Dijk and Kunst	1904	1.1182	......	1.1180	1.1178
					Mean 1.1176

If the atomic weight of silver be taken as 107.88 (latest corrected value) the value of the Faraday from the average of Guthe's corrected values is $F = \frac{107.88}{0.0011176} = 96{,}530$ coulombs. This value agrees very closely with the value $F = 96{,}540$ adopted by the International Congress of Applied Chemistry in 1903 and based on the atomic weight of silver = 107.93, and on the definition of the legal ampere as that current which in one second deposits 0.001118 gram of silver. (*See Report of International Electrical Congress, 1893; Definition legalized by the U. S. Government in 1894.*) In view of the above agreement it seems advisable to adhere to the value 96,540 until by international agreement another value shall be adopted.

ELECTROCHEMICAL EQUIVALENTS OF THE ELEMENTS.—From the definitions of the electrochemical equivalent and the Faraday, it follows that the electrochemical equivalent of any ion, expressed in grams per coulomb (one ampere per second) is equal to

$$\frac{\text{gram-equivalent of the ion}}{96{,}540}.$$

Table II contains the values of the electrochemical equivalents of the more common elements which may be found useful in certain calculations. The

electrochemical equivalent of any radical may be readily calculated from its formula; for example, the electrochemical equivalent of SO_4 is

$$(32.07 + 4 \times 16)/2 \times 96{,}540 = 0.0004975 \text{ gram or } 0.4975 \text{ milligram.}$$

TABLE II.—ELECTROCHEMICAL EQUIVALENTS OF THE MORE IMPORTANT ELEMENTS

(Based on atomic weights of 1913, and $F = 96{,}540$ coulombs)

Element	Symbol	Atomic weight	Valence	Milligrams deposited by one ampere in one second	Grams deposited by one ampere in one hour
Aluminum	Al	27.1	3	0.0935	0.3366
Antimony	Sb	120.2	3	0.4152	1.495
Arsenic	As	74.96	3	0.2589	0.9319
Barium	Ba	137.37	2	0.7115	2.562
Bismuth	Bi	208.0	4	0.5387	1.939
Bromine	Br	79.92	1	0.8279	2.981
Cadmium	Cd	112.40	2	0.5821	2.095
Calcium	Ca	40.07	2	0.2076	0.7472
Cerium	Ce	140.25	3	0.4843	1.744
Chlorine	Cl	35.46	1	0.3673	1.322
Chromium	Cr	52.0	2	0.2694	0.9696
Chromium	Cr	52.0	3	0.1795	0.6462
Cobalt	Co	58.97	2	0.3054	1.099
Cobalt	Co	58.97	3	0.2036	0.7331
Copper	Cu	63.57	1	0.6586	2.371
Copper	Cu	63.57	2	0.3293	1.186
Fluorine	Fl	19.0	1	0.1968	0.7086
Gold	Au	197.2	1	2.043	7.353
Gold	Au	197.2	3	0.6810	2.451
Hydrogen	H	1.008	1	0.01043	0.03758
Iodine	I	126.92	1	1.313	4.733
Iron	Fe	55.84	2	0.2894	1.042
Iron	Fe	55.84	3	0.1929	0.6946
Lead	Pb	207.10	2	1.073	3.863
Lithium	Li	6.94	1	0.0725	0.261
Magnesium	Mg	24.32	2	0.1260	0.4534
Manganese	Mn	54.93	2	0.2845	1.024
Manganese	Mn	54.93	3	0.1897	0.6827
Mercury	Hg	200.6	1	2.077	7.479
Mercury	Hg	200.6	2	1.039	3.740
Nickel	Ni	58.68	2	0.3039	1.095
Nickel	Ni	58.68	3	0.2026	0.7290
Oxygen	O	16.00	2	0.08287	0.2984
Platinum	Pt	195.2	2	1.011	3.640
Platinum	Pt	195.2	4	0.5055	1.820
Potassium	K	39.10	1	0.4051	1.458

TABLE II. — *Continued*

Element	Symbol	Atomic weight	Valence	Milligrams deposited by one ampere in one second	Grams deposited by one ampere in one hour
Silver	Ag	107.88	1	1.118	4.025
Sodium	Na	23.00	1	0.2382	0.8576
Strontium	Sr	87.63	2	0.4539	1.634
Sulphur	S	32.07	..	......	......
Thallium	Tl	204.0	1	2.113	7.607
Thallium	Tl	204.0	2	1.057	3.804
Tin	Sn	119.0	4	0.3082	1.109
Titanium	Ti	48.1	4	0.1245	0.448
Tungsten	W	184.0	2	0.9550	3.431
Zinc	Zn	65.37	2	0.3386	1.219

CONDUCTIVITY OF ELECTROLYTES—Solutions. — (*See also article on Resistance and Conductance.*) The specific conductance of an electrolytic solution varies between wide limits. It depends upon the nature of the solute and of the solvent, on the temperature and on the concentration of the solution. The effect of these factors is discussed below. For numerical values see Kohlrausch & Holborn, *Leitvermögen der Electrolyte;* Landolt-Börnstein, *Tabellen.*

Fused Electrolytes. — Most inorganic salts conduct electrolytically when heated to a sufficiently high temperature to cause them to pass into the liquid state. For such electrolytes Faraday's, Ohm's, and Joule's Laws hold as they do in the case of solutions. The conductance increases in general from 1 to 1½ per cent per degree centigrade. An exhaustive résumé of all matters relating to fused electrolytes may be found in Lorenz's *Electrolyze der Geschmolzene Salze.* On account of their enormous concentration (100 per cent) the specific conductance of fused salts is generally very high. This, together with the difficulty of obtaining a non-conducting chemically inert vessel of suitable shape to contain the salt and the difficulty of regulating high temperatures to a fraction of a degree, makes the measurement of the conductivity of fused electrolytes far more difficult than in the case of solutions. Fused silica and natural quartz crystals have been successfully used for conductivity cells. For recent work see Goodwin and Mailey, *Physical Review*, Vol. 25, p. 469, 1907; Vol. 26, p. 28, 1908; Goodwin and Kalmus, *Physical Review*, Vol. 28, p. 1, 1909; Arndt, *Zeit. für Electrochem.*, Vol. 12, p. 337, 1906; Vol. 13, p. 509, 1907; Vol. 14, p. 662, 1908; Foot and Martin, *Am. Chem. J.* 41, 451, 1909; Landolt-Börnstein *Tabellen.*

VOLTAIC AND ELECTROLYTIC CELLS. — When a cell formed by two electrodes and one or more electrolytes gives out electric energy the cell is called a "voltaic" cell. All ordinary chemical batteries are voltaic cells. If across the terminals of a cell an external electromotive force, greater than the electromotive force developed within the cell, is impressed, then a current will flow through the cell in the opposite direction, and the cell will absorb electric energy. A cell which absorbs electric energy is usually referred to as an

"electrolytic cell." The cells used in electrolytic refining and similar processes are electrolytic cells. An electrolytic cell may or may not have an electromotive force on open circuit, but may develop an electromotive force in virtue of the chemical changes or changes in concentration which take place due to the current forced through it by some external source. A voltaic cell, of course, becomes an electrolytic cell when the electromotive force impressed across its terminals exceeds (and opposes) its own electromotive force.

POLARIZATION. — When an electric current passes through either a voltaic or an electrolytic cell there is, in general, developed at the electrodes of the cell, as a result of the chemical actions and changes in concentration which take place, a *back* or *counter-electromotive* force, in addition to its open-circuit electromotive force (if any); and an *increase in the resistance* of the cell, in addition to the change in resistance due to the heating effect of the current. Both of these effects cause a decrease in the strength of the current through the cell. These two phenomena are said to be due to "polarization" and "transition resistance" respectively.

Even in the case where the electrolyte, as a whole, suffers no change in concentration, as in the electrolysis of copper sulphate between copper electrodes, a counter e.m.f. of polarization is produced in consequence of the difference in concentration of the copper ions in the neighborhood of the anode and cathode. Vigorous stirring of the electrolyte tends to reduce this concentration difference and the resulting polarization, but, except for very low current densities, it cannot be completely eliminated. It is for this reason, in refining processes, that energy is required to transfer the metal from anode to cathode in addition to that necessary to overcome the internal resistance of the cell.

A transition resistance results from the formation of a film of poorly conducting material over the electrode and may or may not be present according to the character of the electrolysis.

Measurement of E.M.F. of Polarization and Transition Resistance. — As both transition resistance and polarization tend to cut down the current flowing through the cell, they cannot be distinguished by this effect alone. The transition resistance may be determined by measuring the ohmic resistance of the cell before and after the passage of the current by the usual alternating-current method. The existence of an e.m.f. of polarization in a cell which normally has no open-circuit e.m.f. may be qualitatively demonstrated by short-circuiting the cell through a galvanometer immediately after breaking the primary circuit; if the cell be polarized, a current which diminishes to zero will flow through it in the reverse direction. A voltmeter, or, better, an electrometer connected across the electrolytic cell, will indicate the polarization voltage the instant after the current is broken. This voltage will diminish as the polarization disappears, the rate depending upon the current through the voltmeter and the rate of diffusion of the electrolytic products causing the polarization. Owing to the rapidity with which the e.m.f. of polarization falls off after the exciting cause is removed, special precautions must be observed in its measurement. One of the best methods is to connect up the circuit containing the applied e.m.f., and electrolytic cell with a tuning-fork interrupter and electrometer so that at the instant the battery current is broken at each vibration of the fork, the circuit containing the cell and electrometer or voltmeter is closed, and vice versa. By this arrangement, the e.m.f. of polarization is measured during the fraction of a second that the battery circuit is open, and before it has had time to sensibly diminish. The polarizing current may also be regarded as practically constant.

It is frequently of importance to know, not the polarization of the cell as a whole, but the polarization at each electrode. This is obtained by measuring

the drop in potential at each electrode against a normal hydrogen or calomel electrode while the impressed e.m.f. is acting on the electrolytic cell, or immediately after the current through the cell is interrupted.

REVERSIBLE ELECTRODES — DEPOLARIZERS. — If the chemical and thermal actions which take place at a given electrode when a given quantity of electricity passes in one direction through a cell are exactly the reverse of the actions which take place when the same quantity of electricity passes through it in the opposite direction, the electrode and the solution with which it is in contact are said to form a "reversible electrode." Such an electrode does not polarize provided the concentration of the solution is kept constant. There are two types of reversible metal-liquid electrodes, namely:

Electrodes of the First Type, consisting of a metal in contact with a solution of one of its own salts, e.g., copper in copper sulphate, zinc in zinc sulphate, etc. The two electrodes of a Daniell cell are of this type.

Electrodes of the Second Type, consisting of a metal in contact with a solution containing one of its difficultly soluble salts and a second soluble salt having the same anion of some other metal; the difficultly soluble salt, called the "depolarizer," must be present in excess as solid. Such an electrode is mercury in contact with a solution of zinc sulphate containing mercurous sulphate in excess as solid; this is one of the electrodes of the Clark cell. (*See article on Cells, Standard.*)

Reversible Gas Electrode. — An electrode consisting of "platinum black" saturated with an atmosphere of hydrogen gas and dipping partially into a solution containing hydrogen ions is also a reversible electrode. This is a special form of electrode of the first type in which a gas by being occluded in platinum is made to play the rôle of a metal.

CONTACT POTENTIALS OR ELECTROMOTIVE FORCES. — The electromotive force of any voltaic cell, the back-electromotive force of any electrolytic cell, and the electromotive force of a thermocouple, are all due to differences of potential which always exist at the junction of dissimilar substances. These potential differences are called contact electromotive forces or "contact potentials," the word "difference" in the latter case being understood.

Metal-metal Potentials. — The potential at the junction of two dissimilar metals may be measured either by the Peltier effect, i.e., by the heat developed or absorbed at the junction when a known quantity of electricity is forced across it, or by the thermoelectric force set up in a circuit formed by the two metals in question when the two junctions are kept at different temperatures. In the former case the potential may be calculated from the expression $e = H/nF$ when H is the Peltier heat developed or absorbed at the junction by the passage of nF coulombs. In the latter case $e = T\frac{de}{dT}$, where $\frac{de}{dT}$ is the thermoelectric coefficient of the junction at the absolute temperature T. Both methods give results which prove that the potential difference thus arising is of the order of magnitude of a few millivolts, except in the unusual combination of antimony and bismuth where it amounts to about 0.03 volt. Table III taken from a recent article by Caswell, *Physical Review*, 33, 401, 1911, contains the values of the metal-metal potentials for the more common metals against copper at ordinary temperature. The columns headed A give the values as determined by the first method, columns headed B give the values as determined by the second method. The plus sign (+) indicates that at a junction formed by the given metal and copper the given metal is at the higher potential, the minus sign (−) that it is at a lower potential than the copper.

TABLE III. — CONTACT POTENTIALS BETWEEN COPPER AND OTHER METALS, IN MILLIVOLTS

(Temperature 15–25° C.)

Metal	LeRoux	Jahn		Edlund		Caswell		Other Observers
Against Copper	*A*	*A*	*B*	*A*	*B*	*A*	*B*	*A*
Antimony......	− 5.64	......	......	......	......	......	......	−3.06 Lecher
Iron......	− 2.93	−3.68	−3.07	− 2.96	− 3.08	......	......	......
Cadmium......	− 0.53	−0.72	−0.72	+ 0.16	+ 0.21	......	......	......
Zinc......	− 0.45	−0.68	−0.41	− 0.01	− 0.02	......	......	......
Copper......	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Silver......	......	−0.48	−0.58	+ 0.03	+ 0.04	+ 0.03	+ 0.06	......
Gold......	......	......	......	+ 0.33	+ 0.50	......	......	......
Lead......	......	......	......	+ 0.50	+ 0.57	......	......	......
Tin......	......	......	......	+ 0.56	+ 0.82	......	......	......
Aluminium......	......	......	......	+ 0.70	+ 0.89	+ 0.70	+ 0.89	......
Platinum......	......	+0.37	+0.38	+ 1.02	+ 1.23	+ 0.85	+ 0.66	......
Palladium......	......	......	......	+ 2.17	+ 2.43	......	......	......
Nickel......	......	+5.07	+5.44	......	......	+ 6.0	+ 6.35	+6.75 Barker +8.04 Cermak
Bismuth......	+22.3	......	......	+17.7	+17.6	+16.1	+16.0	......

The contact potential difference between any two of these metals is the *algebraic* difference between the potentials of the two metals against copper; e.g., the contact potential difference between iron and platinum according to Jahn's measurements (*A*) is $+ 0.37 - (- 3.68) = 4.05$ millivolts.

The contact potential difference between any two metals is a function of the temperature, the sign of the potential for a given pair of metals sometimes reversing as the temperature rises. Although of importance in thermopiles, pyrometry and radiometry, metal-metal potentials are of little significance in considerations of the voltaic cell.

Metal-gas Potentials. — The contact potential differences between two metals should be carefully distinguished from potential differences having their origin in the so-called "Volta effect," i.e., potential differences which are manifested when two dissimilar metals in metallic contact are separated by an air or other gaseous gap. The two phenomena are quite distinct, there being no known relation between them.

Values of the potential difference across the air gap between combinations of some of the more common metals are given in the accompanying table. There is a fall of potential from the zinc to the lead through the air. This table is taken from data by Ayrton and Perry, *Phil. Trans.*, *Vol. 171, 1880.*

	Volts
Zinc — air — lead	0.210
Lead — air — tin...........	0.069
Tin — air — iron...........	0.313
Iron — air — copper...........	0.146
Copper — air — platinum......	0.238
Platinum — air — carbon......	0.113

Liquid-liquid Potentials.—When two dissimilar electrolytic solutions are brought into contact the phenom-

enon of diffusion takes place between them until a homogeneous mixture results. At the same time a difference of potential is produced between the solutions, the magnitude of which depends upon the velocity of migration and the relative concentration of the ions taking part in the diffusion, the charge which they carry, the absolute temperature and the gas constant. Liquid junction potentials like metal-metal potentials are, in general, of small magnitude. Thus the potential difference between two solutions of potassium chloride, one solution having 10 times the concentration of the other, is only 0.0004 volt, and diminishes as the ratio of the concentrations approaches unity, being, in fact, proportional to the logarithm of this ratio. Potassium chloride is often used as an intermediate electrolyte when it is desired to reduce the liquid-liquid potentials of a voltaic combination to a minimum because of the small potential to which it gives rise. For further details of eliminating liquid-liquid contact potentials, see Ostwald-Luther's *Physico-Chemische Messungen*, pages 448-449.

Liquid potentials are greatest between acid or between alkali solutions. Thus, between two hydrochloric acid solutions whose concentrations are in the ratio 10 to 1 at a temperature of 18° C., there is a potential difference of about 0.038 volt; between two sodium-hydrate solutions under the same conditions there is a potential difference of 0.034 volt, these voltages also varying as the logarithm of the concentration ratio. For formulas, see page 471.

Metal-liquid Potentials. — From what has been said regarding the magnitude of metal-metal and liquid-liquid potentials, it follows by the process of elimination that the main seat of the electromotive force of a voltaic cell must reside at the junctions between metals and liquids. The *absolute* value of the potential difference between a metal and the solution with which it is in contact can be found only on certain assumptions regarding the nature of surface tension; this point is again referred to below. For practical purposes the value of this potential is expressed as the e.m.f. of a cell constructed with two half elements, one of which is the given metal and solution, and the other some form of "normal" or reference electrode. Two normal electrodes are in common use, namely, the calomel electrode and the hydrogen electrode.

Normal Calomel Electrode. — The normal calomel electrode consists of mercury in contact with a normal solution of potassium chloride saturated with mercurous chloride, the latter being present in excess as a solid.

Normal Hydrogen Electrode. — The normal hydrogen electrode consists of a strip of platinum coated with a thin deposit of platinum black and saturated with hydrogen gas at atmospheric pressure. The electrode is mounted partially surrounded by hydrogen gas and partially dipping into an acid solution of such a concentration that it contains 1 gram-equivalent of hydrogen ion per liter. A sulphuric-acid solution containing 2 gram-equivalents per liter sufficiently fulfills this condition. Hydrogen gas is allowed to bubble through the solution continuously.

Following the recommendation of the International Congress of Applied Chemistry in 1903, all metal-liquid potentials should hereafter be given as directly measured against either the normal calomel or the hydrogen electrode, uncorrected for liquid junction potentials.

From a normal hydrogen electrode to a normal calomel electrode there is a rise of potential through the electrolyte of 0.283 volt, hence, if from a hydrogen electrode to any metal electrode the rise of potential is e_h, and if from a calomel electrode to any metal electrode the rise of potential is e_c, then

$$e_h = e_c + 0.283 \text{ volt.}$$

The difference between any two metal-liquid potentials remains the same irrespective of the normal electrode to which they are referred.

Electrode Potentials of the Common Elements. — Table IV contains the most reliable values known at present of the metal-liquid potentials of the more common elements when in contact with solutions containing one gram-ion of the element per liter. When the concentration of the solution is thus determined the value of the metal-liquid potential is called the "electrode potential" of the metal. This term is however frequently used in the general sense of potential at the junction of a metal and solution irrespective of its concentration. When not otherwise stated the values in the table hold for room temperatures. e_c is the value (measured or computed) against the calomel electrode; e_h the value (measured or computed) against the hydrogen electrode. (*See LeBlanc, Electrochemie, ed. 1911, page 240.*) Values in parentheses are computed from heats of reaction (*see below*). The + sign indicates that the given metal is at a higher potential than the standard electrode, the − sign that it is at a lower potential.

TABLE IV. — ELECTRODE POTENTIALS IN VOLTS

(LeBlanc's Compilation, 1911. See text for the conditions under which these values hold.)

Element	e_c	e_h	Element	e_c	e_h
K	(−3.48)	(−3.20)	H	−0.283	±c
Na	−2.9981	−2.7151	Cu	+0.046	+0.329
Ba	(−3.10)	(−2.82)	As	<+0.01	+0.29
Sr	(−3.05)	(−2.77)	Bi	<+0.11	+0.39
Ca	(−2.84)	(−2.56)	Sb	<+0.18	+0.47
Mg	(−2.82)	(−2.54)	Hg/Hg_2^{++}	+0.492	+0.775
	−1.77?	−1.49?	Hg/Hg^{++}	+0.552	+0.835
Al	−1.56?	−1.28?	Ag, 25°	+0.515	+0.798
Mn	−1.36	−1.07	Pd	<+0.51	+0.79
Zn	−1.053	−0.770	Pt	<+0.58	+0.86
Fe	−0.74	−0.46	Au	<+0.80	+1.08
Cd	−0.703	−0.420	F	+1.7	+2.0
Tl	−0.603	−0.320	Cl 25°	+1.120	+1.400
Co	−0.58	−0.30	Br 25°	+0.812	+1.095
Ni	−0.53	−0.25	J 25°	+0.345	+0.628
Sn	<−0.48	−0.19	O 25°	+0.110	+0.393
Pb	−0.402	−0.119			

Variation of Electrode Potentials with Concentration. — According to Nernst's theory of electrode potentials (*see below*) the *change* in the value of these potentials with concentration may be calculated by the following formula

$$e_1 - e_2 = \frac{0.000198\,T}{n} \log_{10} \frac{c_1}{c_2},$$

where e_1 is the potential at concentration c_1, and e_2 the potential at concentration c_2, T is the absolute temperature, and n the valence of the metal in the solution.

This formula is also justified by experiment. For example, the electrode potential of silver in a normal silver-nitrate solution at 25° C. is 0.798, referred to the normal hydrogen electrode. If the concentration of the silver ions be reduced to 1/1,000,000 its value in a normal solution, the electrode potential of silver will become

$$e_2 = 0.798 - \frac{0.000198 \times 298}{1} \log_{10} \left(\frac{1}{10^{-6}} \right)$$
$$= 0.798 - 0.0590 \times 6 = 0.444 \text{ volt.}$$

Electrochemical Series — Nobility of the Elements. — The elements, as arranged in Table IV, constitute the "Electrochemical Series." Those elements for which e_h is negative are said to be less "noble" than hydrogen, while those for which e_h is positive are said to be more "noble" than hydrogen. The alkali metals having the greatest tendency to form ions in water stand at one end, while the "noble" metals, such as gold, platinum, paladium, having but a very slight tendency to form ions, are at the other end. The halogens and oxygen which go into solution as negative instead of positive ions stand below the metals at the extreme lower end of the series. Other series have been given which differ from the above in that the elements are not compared under similar conditions in regard to the concentration of the solution with which they are in contact. Changing the electrolyte, e.g., to potassium cyanide alters not only the numerical values of the potentials, but may completely change the order in which certain elements occur in the series.

Any metal if placed in a solution of a salt of a metal standing below it in the series will tend to replace it; thus, zinc precipitates iron, copper, silver, etc., from their solutions but will not displace the alkali metals from solutions of their salts. Any metal standing above hydrogen will tend to displace it from an acid solution with evolution of hydrogen. Metals below hydrogen in the series will not dissolve in acid by a simple replacement of hydrogen. From a chemical standpoint the adoption of the hydrogen electrode as a standard has the advantage over the calomel standard that it divides the metals into two groups according to their behavior towards acids.

The approximate electromotive force of a battery consisting of two metals dipping into normal solutions of their respective salts may also be computed at once from the table by taking the difference of the two corresponding electrode potentials. Thus a zinc-copper cell should have an electromotive force equal approximately to $E_{zn-cu} = (e_h)_{zn} - (e_h)_{cu} = -0.770 - 0.329 = -1.099$, the rise of potential being from the zinc to the copper in the cell. As a matter of fact, this combination, the Daniell cell, has an electromotive force of approximately 1.10 volts. Other conclusions to be drawn from the table will be pointed out below.

ELECTROMOTIVE FORCE AND HEAT OF REACTION. — The electromotive force of a reversible cell maintained at constant temperature may be readily calculated from the first and second law of thermodynamics (*see Thermodynamics*). By a "reversible" cell is meant a cell which does not polarize and which operates under conditions such that changes which take place within the cell constitute a thermodynamically reversible process. (*See Thermodynamics, Principles of.*) The discussion given below also applies to a rough degree of approximation to most commercial cells, which, as a rule, are not strictly reversible.

Gibbs-Helmholtz Equation. — In the case of a reversible cell, such, for example, as the Daniell cell,* shown in Fig. 1, the relation between the heat of reaction (H) and the electromotive force (E) of the cell is calculated as follows.

From Faraday's Laws, the quantity of electricity which must flow through the cell in order to deposit or liberate one gram-ion at either electrode is nF coulombs, where n is the valency of the ion of highest valency involved in the chemical reaction and F is the electrochemical constant (*see above*). Hence, the external work done by the cell (when there is no other external work than electrical work) is nFE joules, where E is the e.m.f. of the cell. If the cell is kept at the same temperature (T degrees, absolute scale) as the surrounding bodies, then the external work nFE done by the cell is the maximum external work it can do at this temperature T (*see Thermodynamics, Principles of*). Hence, from equation (8) of the article on thermodynamics,

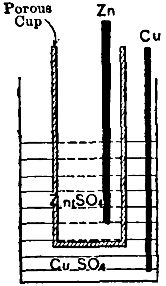

Fig. 1.

$$nFE = H' + nFT\frac{dE}{dT},$$

or

$$E = \frac{H'}{nF} + T\frac{dE}{dT},$$

where H' = the heat of reaction, in joules per gram-ion, corresponding to the chemical change which takes place.* H' gives the change in intrinsic energy corresponding to the chemical changes which take place within the cell. This can be found by an independent calorimetric measurement, by causing the reaction to take place under such conditions (*constant volume*) that heat is the only form of energy produced.

Temperature Coefficient of Electromotive Force. — From the above equation it follows that the energy developed by the cell is equal to the heat energy of the chemical reaction taking place within it plus a certain other quantity of energy equal to $nFT\,dE/dT$. This may be either positive or negative, according as the temperature coefficient of electromotive force dE/dT is plus or minus, i.e., according as the electromotive force of the cell increases or decreases with the temperature. If the electromotive force on open circuit increases with the temperature of the cell, then on closed circuit the cell will tend to cool, i.e., its temperature will fall, and its electromotive force will likewise decrease, the energy $nFT\,dE/dT$ being derived from the heat energy within the cell (or surroundings). If the electromotive force on open circuit decreases with increasing temperature of the cell, then on closed circuit the cell will tend to heat up, i.e., its temperature will rise and its electromotive force will decrease. A cell for which dE/dT is other than zero can work at constant temperature only by absorbing heat from or giving out heat to some surrounding body.

Only for the unique conditions that $dE/dT = 0$, or that $T = 0$, i.e., when the electromotive force is independent of the temperature or the cell works at temperatures approaching absolute zero, is $E = H'/nF$. By a curious chance it happens that the first of these conditions practically obtains in the case of the Daniell cell considered above. The electromotive force of the cell is nearly independent of the temperature. Its observed value is $E = 1.10$ volts, while its value computed by the above equation, putting $dE/dT = 0$, is 1.09 volts.

Thomson's Rule. — In computing the electromotive force of a battery from heats of reaction it is usually necessary to content oneself with a first

* Since 1 large mean calorie = 418.6 joules and $F = 96{,}540$, this may be written, putting H = heat of reaction in large mean calories,

$$E = \frac{H}{230.5\,n} + T\frac{dE}{dT}.$$

approximation and neglect the second term in the above equation, since at present few data are available from which the value of dE/dT can, a priori, be obtained. The approximate equation

$$E = \frac{H'}{nF} = \frac{H}{230.5\, n}$$

is known as Thomson's rule. (H' is in joules, H in large mean calories.)

This rule also applies approximately to most commercial cells, although these are not strictly reversible. Of course, the heat of reaction used must be that corresponding to the actual chemical changes which take place in the cell.

Nernst's Thermodynamic Theorem. — Nernst, by assuming that the values of the intrinsic energy and free energy of a system not only become equal at the absolute zero but also that they approach equality asymptotically at this point, has been able to express the electromotive force (E) of a cell explicitly in terms of the thermal properties of the substances taking place in the reaction. So far as the new formula has been tested a satisfactory agreement has been found between theory and experiment. The principles involved are of fundamental importance and are fully discussed in Nernst's *Silliman Lectures, Applications of Thermodynamics to Chemistry*. See also Nernst's *Theoretical Chemistry*, 6th ed., 1911, pages 709, 745.

Concentration Cells. — All commercial primary cells (*see Batteries, Primary*) are chemical cells, i.e., they derive the major part of their energy from the chemical reactions occurring within them. There is also a class of cells in which the electrical energy developed by the cells at constant temperature is derived from the heat energy of the surroundings, i.e., cells in which $H = 0$, and, therefore,

$$E = T\frac{dE}{dT}.$$

Integration of this equation shows that for such cells the electromotive force is proportional to the absolute temperature. Such cells are very simple of construction and are called concentration cells. They are of considerable importance from a theoretical point of view, and their electromotive force can be readily and exactly computed by an application of Nernst's formula for electrode potentials (*see above*). A pure concentration cell can be formed of any chemical or physical system in which the various parts tend to diffuse into each other without the evolution or absorption of heat, by constructing a cell in which the equalization of concentration, be it gaseous or liquid, is effected electrically. Numerous cases are discussed in Le Blanc's *Electrochemistry*, English translation, page 184, 1907 ed. If the two solutions forming a concentration cell when mixed have an appreciable "heat of dilution" the general Gibbs-Helmholtz formula and not the above special case is applicable.

Carbon Generator. — The problem of converting the energy liberated in the combustion of carbon to carbon dioxide directly into electric energy has not yet been practically solved. The reaction, $C + O_2 = CO_2$, liberates a very large quantity of heat, namely, 961 large calories per gram-atom of carbon consumed. Of this, only about 10 per cent is converted into electrical energy through the agency of a steam engine and dynamo. The difficulties encountered in devising a commercial carbon generator are the following: (1) the velocity of the above reaction is small, except at high temperatures; (2) it is difficult to utilize a gas as an electrode, except through the agency of a conducting medium which occludes it, such as platinum black; (3) a satisfactory high-temperature electrolyte which will not deteriorate in the presence of CO_2, is difficult to obtain; and (4) the slight tendency of carbon to form ions.

ELECTROLYTIC DECOMPOSITION. — Faraday's laws enable one to determine the amount of the substances primarily liberated or deposited at the electrodes of any electrolytic cell, but they do not enable one to predict the nature of the secondary chemical actions which may take place. (*See section on Secondary Reactions.*) The factors which determine what products will be formed in any given case are numerous. The chemical process occurring at the cathode is always of the nature of a reduction; at the anode, of an oxidation, these terms being used in their most general sense. It may be that the electrolyte, taken as a whole, undergoes no change, as is the case, for example, of a copper-sulphate solution when electrolyzed between copper electrodes. Here the chemical reaction at the cathode is the exact reverse of that at the anode. Generally, however, this is not the case; different chemical products are usually formed at the two electrodes, and appear either in the form of gas or as a solid precipitate, or remain dissolved in the electrolyte.

Decomposition Voltage of Electrolytes. — When a gradually increasing difference of potential is applied to the electrodes of an electrolytic cell, electrolysis does not in general begin at once but only after a certain minimum electromotive force is reached, even though the cell has no open-circuit e.m.f. It should be noted that this condition does not apply to the case when the electrodes are such that the electrolyte as a whole undergoes no change as a result of electrolysis. Thus, in the electrolysis of a zinc-sulphate solution between zinc electrodes an exceedingly small electromotive force is sufficient to maintain a steady current through it indefinitely (assuming that concentration changes at the electrodes are completely eliminated by stirring). Although, in this case, electrolysis starts at once, the electrolyte as a whole is not decomposed, for as much zinc dissolves at the anode as is deposited at the cathode. If a platinum anode be substituted for the zinc anode so that as a result of electrolysis zinc is separated at the cathode and oxygen gas is liberated at the anode, an e.m.f. of 2.35 volts is required to start the electrolysis. Again, if the electrolyte is an aqueous solution of sulphuric acid it requires a minimum voltage of 1.67 volts to liberate oxygen and hydrogen gas against smooth platinum electrodes. The minimum voltage necessary to decompose a compound electrolytically is called its "decomposition voltage." It is influenced by a variety of factors and conditions which will now be considered.

Measurement of Decomposition Voltage. — Decomposition voltages have been experimentally determined in several ways. Certain investigators have taken the value necessary to produce visible electrolysis as the criterion; others have followed the polarization at the electrodes until it became constant and assumed this value as that at which electrolysis begins. Results obtained by the so-called ammeter-voltmeter method must be accepted with caution. In this method the value of the electromotive force applied to the cell is gradually increased from zero and plotted as abscissas and the corresponding currents through the cell plotted as ordinates. The resulting curves have the general form shown in Fig. 2.

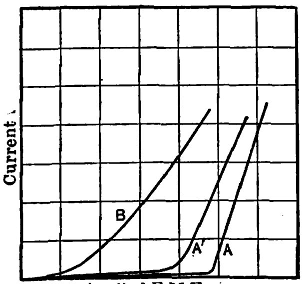

Fig. 2.

The slope of these current-voltage curves depends upon the internal resistance of the cell, the size and the distance apart of electrodes. The critical voltage at which the current suddenly increases in value is, however, quite sharply

defined with some electrolytes, e.g., silver nitrate between platinum electrodes, and other salts from which a metal is deposited. Below the critical voltage very little or no current passes through the cell and no visible decomposition at the electrodes is apparent. When the point *A* is reached, visible electrolysis begins. The sharpness of the bend in the curve and its course below *A* depend in large measure upon the sensitiveness of the galvanometer used for detecting the current, the size of electrodes and the tendency of the products of the electrolysis to go back into solution.

Residual Currents. — With a sensitive reflecting galvanometer, a steady deflection may be observed for days without any apparent electrolysis when an e.m.f. of a few tenths of a volt (far below the "decomposition voltage") is applied to platinum electrodes dipping into acidulated water. It might seem from this that Faraday's law is here violated. These slight currents are to be explained, however, by a diffusion of the products liberated slowly and in minute quantities at the electrodes, back into the solution. Currents produced in this manner are called "residual currents" and play an important rôle in the electrolysis of fused salts and probably also in conduction in solid electrolytes. The larger the residual current the less sharply marked is the decomposition point, see *A′*. For many fused salts the current-voltage curve is of the form shown in *B*, Fig. 2. It would be difficult to say from this curve at what voltage decomposition actually began.

Table V gives the experimentally determined decomposition voltages of some of the more common acids, bases and salts.

TABLE V. — DECOMPOSITION VOLTAGES OF NORMAL SOLUTIONS
(*From Le Blanc's Text-book of Electrochemistry, p. 289.*)

Solution	Voltage	Solution	Voltage
Acids:		Bases:	
Dextrotartaric	1.62	Ammonium hydrate	1.74
Dichloracetic	1.66	½ n. diethylamine	1.68
Hydrazoic	1.29	¼ n. methylamine	1.75
Hydriodic	0.52	Potassium hydrate	1.67
Hydrobromic	0.94	Sodium hydrate	1.69
Hydrochloric	1.31	⅛ n. tetramethyl ammonium hydrate	1.74
Malonic	1.69		
Monochloracetic	1.72	Salts:	
Nitric	1.69	$AgNO_3$	0.70
Oxalic	0.95	$CdCl_2$	1.88
Perchloric	1.65	$Cd(NO_3)_2$	1.98
Phosphoric	1.70	$CdSO_4$	2.03
Pyrotartaric	1.57	$CoCl_2$	1.78
Sulphuric	1.67	$CoSO_4$	1.92
Trichloracetic	1.51	$NiCl_2$	1.85
		$NiSo_4$	2.09
		$Pb(NO_3)_2$	1.52
		$ZnBr_2$	1.80
		$ZnSO_4$	2.35

Calculation of Decomposition Voltage from Heat of Reaction. — If the electrolytic process is reversible in the thermodynamic sense, then Gibbs-Helmholtz's equation (*see above*) is directly applicable. Since however the temperature coefficient of the maximum work (or free energy) is known in but

a few instances, one usually employs Thomson's rule, which gives a fair approximation in the case of most commercial processes, even though these processes are not strictly reversible. That is, calling H the heat of formation of the compound which is decomposed into the substances which are liberated at the electrodes (i.e., H is the heat of the reverse reaction corresponding to the complete chemical actions which take place in the cell), the decomposition voltage is

$$E = \frac{H}{230.5\, n},$$

where n = the valency of the ion of highest valency involved in the reaction, and H is expressed in large mean calories per gram-molecule.

Tables giving the voltages required to liberate a number of elements and radicals from various electrolytes computed by Thomson's rule from heats of formation in dilute solutions are given by J. W. Richards in *Jour. Franklin Institute*, 1906.

Example. — In the formation of one mol of solid lead chloride from metallic lead and chlorine gas 828 large mean calories are liberated. The valency of lead is 2; of chlorine, 1. As solid lead chloride does not conduct electricity appreciably at ordinary temperatures, to decompose it electrolytically, it must be either dissolved or fused. If a saturated solution containing an excess of solid lead chloride be electrolyzed, metallic lead will be deposited at the cathode, chlorine gas liberated at the anode, and an equivalent amount of solid lead chloride will pass into solution, the net result then being a decomposition of solid lead chloride into its elements lead and chlorine. Under these conditions the decomposition voltage by Thomson's rule is

$$E = \frac{828}{230.5 \times 2} = 1.79 \text{ volts},$$

as against 1.612 volts by actual measurement.

It should be pointed out that Nernst has recently applied his new heat theorem (*see above*) to the calculation of the e.m.f. of cells similar to the above, with very satisfactory results. Thus the computed value of the e.m.f. in the above case from thermal data by the formula developed by Nernst is $E = 1.594$ volts, in excellent agreement with the observed value. See Nernst's *Theoretical Chemistry*, page 745, 1911 ed.

Calculation of Decomposition Voltage from Electrode Potentials. — Le Blanc was the first to measure by means of a normal electrode the single potential drop at the cathode when various salts were electrolyzed and found that the cathodic drop in potential necessary to discharge a given cation is numerically equal to the electrode potential which would be established if the separated metal were substituted for the actual cathode. For example, to deposit metallic zinc from a normal zinc-sulphate solution it is necessary to impress a potential from solution to electrode as measured against a normal hydrogen electrode of at least 0.77 volt, to overcome the natural rise of potential from the zinc electrode to the solution (as measured by the normal hydrogen electrode) of this amount. See table page 449.

Before continuous electrolysis can begin, however, not only must the applied e.m.f. be sufficient to establish the necessary cathodic drop e_c, but in addition it must also raise the anode potential to a value e_a, corresponding to the electrode potential of the anion which is liberated. If, as is usually the case, several different anions are present in the electrolyte that one will be liberated first which can part with its negative charge with the least expenditure of energy, i.e., at the lowest voltage. It frequently happens that e_c is established long

before e_a is reached, in which case Le Blanc found that e_c remains constant while the impressed voltage E (and hence e_a) is increased up to the decomposition point. The minimum e.m.f. at which decomposition of a given electrolyte begins may therefore be written $E = e_c + e_a$ where e_c and e_a are the cathodic and anodic polarization values produced by that cation and anion respectively which gives up its charge most readily. In an electrolyte containing several different anions and cations all take part in the *conduction* of a current through the solution, but only specific ions are effective in electrolysis, i.e., in giving up their charges to the electrodes.

Overvoltage. — When a gas is evolved at either electrode the above relations are usually more complicated owing to the phenomenon known as overvoltage. Thus in the electrolysis of an acid between polished platinum or other metal electrodes, hydrogen gas is evolved, to liberate which the cathode potential must be raised to a value in excess of that required to liberate hydrogen at a reversible (platinum black) electrode.

This excess of voltage which is necessary to liberate a gas at any electrode over that required to liberate the same gas against a *reversible* gas electrode is called "overvoltage." Its value depends upon the character of the material of the electrode against which the gas is set free and upon the nature of the gas. It is particularly large in the case of hydrogen. In Table VI are given values of the overvoltage for hydrogen as determined by Caspari when this gas is *visibly* set free. The magnitude of the effect is closely related to the power of the metal surface to occlude the gas in question. Thus, platinum and (to a less extent) iron absorb hydrogen readily and hence these metals exhibit slight overvoltage phenomena. Zinc and mercury, on the other hand, have little or no tendency to absorb hydrogen and hence this gas is liberated with great difficulty against cathodes of these metals, with correspondingly high overvoltages. It is for this reason that chemically pure zinc does not readily displace hydrogen from an acid in spite of its high electrode potential.

For theory of overvoltage phenomenon, see Möller, *Ann. d. Physik*, 27, 566, 1908. Also, Crabtree, *Trans. Faraday Soc.*, 1913, Vol. 9, 125.

TABLE VI. — OVERVOLTAGE OF HYDROGEN FOR VISIBLE ELECTROLYSIS.*

Electrode	Over-voltage in volts	Electrode	Over-voltage in volts
Pt, platinized	0.005	Cu	0.23
Au	0.02	Cd	0.48
Fe	0.08	Sn	0.53
Pt, polished	0.09	Pb	0.64
Ag	0.15	Zn	0.70
Ni	0.21	Hg	0.78

* *Zeit. für Phys. Chem.*, Vol. 30, p. 89, 1899.

Electrolytic Separation of Metals. — The reason that two or more metals can be electrolytically separated from each other by a regulation of voltage follows from the preceding discussion. Consider a solution containing two salts, say silver and copper nitrates, each of normal concentration. If a gradually increasing e.m.f. be applied to inert electrodes dipping into this solution, electrolysis will begin when a voltage is reached sufficient to set free

simultaneously any anions and cations present in the solution. Referring to Table IV, it will be seen that copper stands $0.798 - 0.329 = 0.469$ volt above silver in the electrochemical series, and hence, the latter metal can be deposited from the solution with an e.m.f. nearly half a volt less than copper. No copper ions can separate until the applied e.m.f. has been increased 0.469 volt above that necessary to first separate the silver. The decomposition e.m.f. necessary will, of course, depend on the nature and concentration of the anions present. As electrolysis proceeds, the solution becomes weaker and weaker with respect to silver ions and the voltage necessary to separate them from the solution increases. If the electrolysis is continued until the silver remaining in solution has a concentration only 1/1,000,000 that at the start (the limit of analytical determinations), the change in voltage will be approximately 0.34 volt (*see section above on Variation of Electrode Potentials with Concentration*). This is still insufficient to bring the applied e.m.f. up to a value sufficient to permit the copper to deposit, and hence, silver and copper may be completely, for all practical purposes, separated from each other.

It is to be noted that as the electrolysis proceeds and the silver is removed from the solution a greater and greater percentage of the current is *conducted* by the copper. The solution in the neighborhood of the cathode becomes continually weaker with respect to silver ions. In a quiet electrolysis the silver ions at the cathode are replenished by diffusion from the interior of the electrolyte. This process may be greatly accelerated by violently stirring the solution, as, for example, by the use of a rapidly rotating electrode. This permits good deposits being obtained with higher current density and greatly reduces the time required for the separation.

Secondary Reactions. — In the preceding sections the principles underlying *primary* electrolysis have been explained. The ultimate products which are produced at the anode and cathode are often quite different from the simple ions or ionic complexes liberated by the action of the current. In general, the primary process of discharge of ions at cathode and anode is followed by a secondary reaction of one of the following types:

1. The discharged ions form molecules, e.g.:

At cathode $2\,H = H_2$ gas,
At cathode $x\,Cu = Cu_x$ solid,
At anode $2\,Cl = Cl_2$ gas.

This may occur at both anode and cathode.

2. The discharged ions react with each other or break up, e.g.:

At anode $2\,OH = H_2O + \frac{1}{2}\,O_2$,
At anode $\begin{matrix} COO \\ | \\ COO \end{matrix} = 2\,CO_2$,
At anode $2\,CH_3COO = 2\,CO_2 + C_2H_6$.

This is very characteristic at the anode, particularly of organic anions.

3. The discharged ions react with the solvent, e.g.:

At cathode $Na + H_2O = NaOH + \frac{1}{2}\,H_2$,
At anode $SO_4 + H_2O = H_2SO_4 + \frac{1}{2}\,O_2$.

4. The discharged ions react with the solute, e.g.:

At cathode $8\,H + 2\,H_2SO_4 = H_2SO_3 + S + 5\,H_2O$,
At cathode $K + KAg(CN)_2 = Ag + 2\,KCN$.

5. Discharged ions react with the electrode, e.g.:

At cathode $4\,H + PbO_2 = Pb + 2\,H_2O$,
At anode $2\,Cl + Cu = CuCl_2$.

Which of the above reactions is most likely to occur in any given case depends to some extent upon, (a) current density; (b) temperature; (c) concentration of the solution.

In general, it may be said that at the cathode a reduction takes place, while at the anode oxidation (in its general sense) occurs. The intensity of these reactions depends upon the electrode potential at which the reaction takes place. This can be varied by the use of different electrode materials (thereby increasing the overvoltage). The efficiency of certain secondary reactions has also been found to depend upon the nature of the electrode (catalytic action). The efficacy of electrolytic reduction and oxidation in organic chemistry where the intensity of the reaction must be carefully regulated likewise rests on the above factors.

Passivity. — A metal is said to assume the "passive" state when it comports itself towards acids like a noble metal, i.e., becomes insoluble. Iron affords a very striking example of this phenomenon, although it is exhibited by other metals as well. Thus, if dipped in strong nitric acid, or if anodically polarized, it becomes passive. Two explanations have been advanced to account for the phenomenon, one that it is due to an invisible film of oxide on the surface, and the other that the surface assumes a condition in virtue of which the velocity of reaction between it and the surrounding solution is infinitely slow.

Alternating-current Electrolysis. — The decomposition of an electrolyte, and the solution of metal electrodes by electrolysis, can be effected under certain conditions by an alternating current as well as by a direct current. The chief determining factors are the velocity with which the chemical reaction accompanying the electrolysis takes place and the periodicity of the alternating current.

THEORY OF SOLUTIONS. — To understand the theory of the various phenomena described above, a knowledge of the theory of solutions is necessary.

Osmotic Pressure. — The present theory is based upon Van't Hoff's thermodynamic and experimental investigations published in 1887. When a substance is brought into contact with a second substance in which it is more or less soluble, it dissolves in virtue of a force (the nature of which is imperfectly understood), which is called its "solution tension" or "solution pressure." If the solute is present in excess, the process of solution continues until the solution becomes saturated, i.e., until equilibrium is established between the solute and solution, such that the solution pressure of the former is just balanced by an opposing pressure created within the solution which tends to cause the dissolved substance to separate out. This pressure is called the osmotic pressure of the solution. Under these conditions there are just as many molecules of solute passing into solution as separate out of the solution during any interval of time. A clearer conception of the osmotic pressure of a solution, as above defined, may be obtained from the following considerations.

Semi-permeable Membranes. — If a solution be placed at the bottom of a cylinder and carefully covered with a layer of pure solvent, the process of diffusion will begin at once and continue until a homogeneous dilute solution is formed. If the experiment be repeated with the single modification, that the pure solvent is separated from the solution by a piston consisting of a semi-permeable diaphragm which will permit the solvent to pass freely through it in either direction, but which will not allow the solute to pass through, the process of diffusion will take place as before, provided the piston is movable. Thus the piston will be forced back by the osmotic pressure of the solution while the solvent passes freely through it thereby diluting the solution. Membranes possessing the property of semi-permeability are not only theoretically con-

ceivable but may actually be prepared. Such a membrane is formed by the precipitate of copper ferrocyanide formed within the walls of an unglazed porous cell when copper sulphate and potassium ferrocyanide solutions are allowed to diffuse into the cell from within and without respectively. This membrane is permeable to water, but impermeable to sugar and many other organic and inorganic substances, such as the copper and potassium salts from which it is formed. Most membranes, in fact, possess the property of semi-permeability to a limited degree.

Measurement of Osmotic Pressure. — To prevent the piston being forced up by the diffusing solution, a downward pressure P must be applied to it equal to that exerted upon it from below by the solution. The magnitude of P is, therefore, a measure of the osmotic pressure of the solution. Owing to experimental difficulties, it is impracticable to measure osmotic pressures in precisely the manner just described, but by slightly modifying the arrangement of the apparatus it may readily be done. Thus, if the solution is contained in a closed vessel, A, Fig. 3, the walls of which are made semi-permeable by depositing a membrane of copper ferrocyanide within the pores of the cell, and this be immersed in the solvent B, the osmotic pressure will be exerted as before against the membrane from within. If the latter could stretch like a rubber balloon, it would do so as the solution became diluted; being, however, restrained by the material of the walls of the cell on and in which it is deposited from doing this, dilution of the solution can take place only by the solvent passing through the fixed membrane into the solution cell. This it will do unless the cell A is hermetically sealed. If an open manometer M of small bore be inserted in the top of A, the solvent as it passes into the cell will cause the solution to rise gradually in M until the hydrostatic pressure thus produced prevents further entrance of solvent. When equilibrium is established, the hydrostatic pressure gives a measure of the osmotic pressure of the solution then existing in the cell. As the entrance of an appreciable amount of solvent into the cell reduces the concentration of the solution, the pressure thus measured is less than that of the original solution; hence for quantitative work an open manometer is replaced by a closed mercury manometer.

Fig. 3.

Laws of Osmotic Pressure in Non-electrolytic Solutions. — By an apparatus similar in principle to that just described the following laws have been verified. They hold for aqueous solutions of many organic substances, such as sugar, and for many solutions in organic solvents, all of which possess the common property of being non-electrolytes.

1. The osmotic pressure of a solution is directly proportional to its concentration (and therefore inversely proportional to the volume in which a given mass of the solute is dissolved).

2. It is directly proportional to the absolute temperature.

3. It is independent of the nature of the solute, being a function solely of the number of mols of substance dissolved in unit volume of solution.

4. The numerical magnitude of the osmotic pressure p of one mol of any substance dissolved in a volume v of solution at the absolute temperature T is identical with the gaseous pressure exerted by one mol of a perfect gas at the temperature T and occupying the same volume v.

The gas laws are therefore not only directly applicable to solutions, but the numerical value of the gas constant R is the same for each. Thus the combined laws of Boyle and Gay-Lussac for gases have the same form for solutions, namely

$$pv = RT,$$

where p = the osmotic pressure, v = the volume of the solution containing one mol of solute, and T = the absolute temperature.

The accompanying table gives the numerical value of $\bar{R}$ when the various quantities are expressed in the units designated. In heat units

$$\bar{R} = 1.985 \frac{\text{calories}}{\text{deg. cent.}}.$$

Abs. pressure p	Volume v	Abs. temp. T	Gas constant $\bar{R}$
Atmospheres	Liters	Deg. cent.	0.0821
Grams per sq. cm.	Cu. cm.	Deg. cent.	84,800
Dynes per sq. cm.	Cu. cm.	Deg. cent.	0.83161×10^8

When applied to concentrated solutions of non-electrolytes the above laws suffer deviations analogous to those which they exhibit when applied to highly compressed gases.

Osmotic Pressure in Electrolytic Solutions. — The first of the above laws for non-electrolytic solutions applies to electrolytic solutions only within narrow limits, while the third and fourth laws fail absolutely. The osmotic pressure of this class of solutions is much greater than the equation $pv = \bar{R}T$ would lead one to expect. Thus, a 0.1 normal sodium chloride solution has an osmotic pressure 1.9 times as great as that which it should have on the basis of the above formula. A potassium sulphate solution containing 0.1 gram-equivalent per liter has an osmotic pressure 2.3 times as great as that calculated. To bring these results within the scope of the gas-law formula, Van't Hoff proposed the empirical equation $pv = i\bar{R}T$, where i is a constant having the value unity for non-electrolytes and a value greater than unity for electrolytes.

THE DISSOCIATION THEORY. — Arrhenius, in 1887, gave the first satisfactory explanation of the anomalous behavior of electrolytes, which show not only abnormally high osmotic pressures, but also abnormally high boiling points and abnormally low vapor pressures and freezing points. On thermodynamic principles these phenomena can all be shown to be proportional to the total number of molecular complexes per unit volume and entirely independent of their chemical constitution. Abnormally high osmotic pressures may therefore be explained on the assumption that each molecule, when dissolved, exists in a more or less dissociated condition, i.e., at any given instant there are in the solution more discrete particles or complexes than there would be if each molecule remained continuously intact. The ions are not supposed to be permanently separated, but a continuous process of dissociation and combination is assumed to be going on all the time. If the further assumption be made that the dissolved substance is electrolytically active only during that fraction of time while it exists in a dissociated or ionized state, in other words, that dissociated molecules are alone capable of being acted upon by electric forces, the connection between electrolytes and solutions showing deviations from the gas laws is explained.

On the assumptions of this theory, sodium chloride, NaCl, in aqueous solution is partially dissociated or ionized according to the equation, $NaCl \rightleftarrows \overset{+}{Na} + \overset{-}{Cl}$, and therefore at any instant there are present in the solution not only sodium-chloride molecules, but also positively charged sodium ions and an equal number of negatively charged chlorine ions, the number of which at any instant depends upon the average fraction of a second during which the molecules are dissociated or "ionized." If the salt were "completely, i.e., 100 per cent ionized" there would be no molecules, as such, present in the solution, but instead twice the number of ions as there were molecules originally dissolved. Hence the

from the equation $pv = \bar{R}T$; in equation $pv = i\bar{R}T$, i would be equal to 2. Again, if a given volume of solution contains one mol of potassium sulphate, K_2SO_4, which is 75 per cent ionized according to the equation $K_2SO_4 \rightleftarrows 2\overset{+}{K} + \overset{-}{SO_4}$, then at any instant there will be $1 - 0.75 = 0.25$ mol of K_2SO_4 molecules, 0.75 mol of $\overset{-}{SO_4}$ ions, and $2 \times 0.75 = 1.50$ mol of $\overset{+}{K}$ ions present in the solution, or $0.25 + 0.75 + 1.50 = 2.5$ molecular complexes for every mol of sulphate dissolved; hence $i = 2.5$.

Although the nature of the ions resulting from the ionization of acids, bases and salts is now known to be more complicated in many cases than assumed in such simple ionization reactions as the above, owing to their association with molecules of the solvent, still the above hypothesis of Arrhenius has proved the most fruitful conception which has been introduced into chemistry during the last twenty-five years and it remains to-day, modified somewhat in the light of recent research, the best working hypothesis with which to interpret electrolytic phenomena in aqueous solutions. It is a natural sequel to the free ion theory of Clausius which in turn displaced the old chain theory of Grotthuss.

THEORY OF ELECTROLYTIC CONDUCTION. — The external effects of a current flowing through an electrolyte cannot be distinguished from those produced by a current of the same strength conducted by a metal. Thus the magnetic effect of a current flowing through a helical glass tube filled with electrolyte is the same as that produced by the same current flowing through an equivalent circuit of an equal number of turns of copper wire. A current may be induced in a closed ring of electrolyte just as in a ring of metal. Ohm's Law and Joule's Law hold for conduction in electrolytes as well as in metals. The mechanism of the conduction in the two cases, however, is very different, as shown by the phenomena produced at the junction of two conductors in one of which the conduction is metallic or "electronic" and in the other of which it is electrolytic or "ionic."

Migration of Ions. — Electrical conduction, both metallic and electrolytic, is believed to be a convection phenomenon. In the case of electrolytic conduction there is much evidence to indicate that the convection consists in the transport of electrically charged ions through the electrolyte, both with and against the direction in which the current is assumed to flow. Simultaneous convection of anions and cations in opposite directions may not only be made visible to the eye but their velocities may also be directly measured. On the assumption that the ions are the carriers of charges, the motion of which constitutes the current, the velocity of "migration" can also be calculated from conductivity measurements and the experimentally determined "transport ratios" (*see below*).

Velocity of Migration. — In Table VII are given the results of direct measurements of the migration velocity of various ions and also the calculated velocities, both expressed in centimeters per second and for a potential gradient of one volt per centimeter.

The velocities are directly proportional to the potential gradient. They all increase rapidly with the temperature. They vary but slightly with the concentration of the solution if the concentration is not too great, and for dilute solutions may be regarded as practically constant.

From the table it is seen that hydrogen migrates with the maximum velocity. Hydroxyl $\overset{-}{OH}$ has the next greatest velocity. The high velocity of the hydrogen ion explains the fact that aqueous solutions of the mineral acids are as a class the best electrolytic conductors, and the high velocity of the hydroxyl ion accounts for the relatively good conductivity of the inorganic bases.

As the velocity with which an ion migrates under the action of a given force depends upon the friction which it has to overcome in moving through the solution, an intimate relation should be expected between it and the rate of diffusion and fluidity (reciprocal of viscosity) of the solution. The viscosity of a solution diminishes or its fluidity increases rapidly with an increase in temperature (from one to two per cent per degree) and experiment shows that the migration velocity increases at approximately the same rate. The increased velocity of migration of the ions with rising temperature is the cause of the high temperature coefficients of electrolytes.

TABLE VII. — VELOCITIES OF MIGRATION OF IONS

Ion	Velocities in cm. per sec. per volt per cm.		
	0.1 gram-equivalent per liter, 18° C.		Infinite dilution, 18° C.
	Observed	Calculated	Calculated
$\overset{+}{H}$	0.0026	0.0028	0.003263
$\overset{-}{OH}$	....	....	0.001802
$\overset{-}{Cl}$	....	....	0.000677
$\overset{+}{K}$	....	....	0.000669
$\overset{+}{NH_4}$	....	....	0.000663
$\overset{-}{NO_3}$	....	....	0.000639
$\overset{-}{ClO_3}$	....	....	0.000570
$\overset{+}{Ag}$	....	....	0.000562
$\overset{-}{SO_4}$	0.00045	0.00049	....
$\overset{-}{Cr_2O_7}$	0.00047	0.00047	....
$\overset{+}{Ag}$	0.00049	0.00046	....
$\overset{+}{Ba}$	0.00039	0.00037	....
$\overset{+}{Ca}$	0.00035	0.00029	....

Transport Ratio. — When a current passes through an electrolytic solution a change in concentration is found to take place in the immediate vicinity of the anode and cathode, but practically no change takes place in the middle portion of the solution, provided means are employed to prevent convection and diffusion from the electrodes. Suppose that the solution about the anode and cathode respectively is analyzed after a known current has passed for a known time.

Let

a = number of gram-equivalents of the *solute* which disappear from the region about the anode,

c = number of gram-equivalents of the *solute* which disappear from the region about the cathode,

then $a + c$ = total number of gram-equivalents of the solute which is decom-

The ratio $n_a = c/(a + c)$ is called the transport ratio for the anion and the ratio $n_c = 1 - n_a = a/(a + c)$ is called the transport ratio for the cation. These ratios are also frequently referred to as "Hittorf's transference numbers." For a given electrolyte n_a is proportional to the number of gram-equivalents of the anions which migrate through the solution from the cathode towards the anode and n_c to the number of gram-equivalents of the cations transported from the anode towards the cathode.

On the assumption that the ions are the sole carriers of the charges the motion of which constitutes the electric current, the ratio of the current carried by the anions or cations to the total current is the same as the above transport ratio for these ions, since the charge carried by one gram-equivalent of every ion is constant = 96,540 coulombs.

TABLE VIII.—TRANSPORT RATIOS n_a OF ANIONS IN AQUEOUS SOLUTIONS AT ABOUT 18° C.

(Values in parenthesis are somewhat uncertain. From Le Blanc's *Electrochemistry.*)

Gram-equivalents per liter	0.01	0.05	0.2	1	2	5
Liters per gram-equivalent	100	20	5	1	0.5	0.2
K{Cl, Br, I}, NH_4Cl	0.506	0.506	.....	.....	.....	.....
NaBr, NaCl	0.604	0.604	.....	.....	.....	.....
LiCl	0.670	0.680	0.697	.....	.....	.....
KNO_3	.....	.....	0.496	0.487	0.479	.....
$NaNO_3$	.....	.....	0.614	(0.611)	(0.608)	0.585
$AgNO_3$	0.528	0.528	0.527	0.501	0.476	.....
$KC_2H_3O_2$	.....	.....	0.33	(0.331)	(0.332)	0.335
$NaC_2H_3O_2$	.....	.....	(0.43)	(0.425)	0.421	.....
KOH	.....	.....	0.736	(0.740)	.....	.....
NaOH	.....	(0.81)	(0.82)	0.825	.....	.....
LiOH	.....	.....	0.85	(0.873)	.....	.....
HCl	0.167	0.165	.....	.....	.....	.....
HNO_3	0.170	0.170	0.170	.....	.....	.....
½ $BaCl_2$	0.553	.....	.....	.....	.....	.....
½ $CaCl_2$	(0.58)	(0.61)	(0.66)	0.686	(0.700)	0.737
½ $MgCl_2$	.....	(0.63)	0.68	0.709	(0.729)	0.776
½ $CdCl_2$	0.570	0.570	(0.65)	(0.72)	0.745	0.865
½ CdI_2	0.558	0.606	0.86	.....	.....	.....
½ $Ba(NO_3)_2$ at 25°	.....	0.544	0.545	.....	.....	.....
½ K_2CO_3	.....	(0.39)	(0.41)	0.434	0.413	(0.380)
½ Na_2CO_3	.....	(0.52)	(0.53)	0.548	(0.542)	.....
½ K_2SO_4	0.505	0.512	.....	.....	.....	.....
½ Na_2SO_4	0.610	0.624	.....	.....	.....	.....
½ $CdSO_4$	0.616	0.635	0.672	0.746	.....	.....
½ $MgSO_4$	0.620	0.633	(0.66)	0.74	(0.76)	.....
½ $CuSO_4$	.....	0.625	0.657	.....	.....	.....
½ H_2SO_4	0.176	0.175	.....	.....	.....	.....

Note.—The ½ before the various bivalent electrolytes indicates that 1 gram-equivalent = ½ mol.

Table VIII contains the generally accepted values of the transport ratios for the more common ions. These numbers vary but slightly with the temperature. They tend to approach the value 0.5 with increasing temperature, i.e., as the temperature rises both ions of a given electrolyte tend to conduct equal per cents of the current. With increasing dilution the transport ratios of good electrolytes approach constant limiting values; these are practically reached in solutions whose concentration is 0.01 gram-equivalent per liter.

Relation between Velocity of Migration and Transport Ratios. — On the assumption that the ions are the carriers of the charges the motion of which constitutes the electric current, and on the further assumption that the moving ions do not carry along with them different quantities of the solvent, the ratio of the velocity u_a of the anions to the velocity u_c of the cations in any solution must be equal to the ratio of the transport ratios $n_a = c/(a+c)$ and $n_c = a/(a+c)$ of the anions and cations respectively, i.e.,

$$\frac{u_a}{u_c} = \frac{n_a}{n_c} = \frac{c}{a}.$$

The transport ratios n_a and n_c may be determined experimentally as described above and by the use of this relation the *relative* velocities of migration may be compared with the ratio calculated from the direct determination of the velocities u_a and u_c.

Hydration or Solvation of Ions. — The above relation between the velocities of migration and the transport ratios has been found to hold only approximately in a number of instances. A possible explanation of this lack of agreement between theory and experiment is that the changes in concentration at the two electrodes are due not only to the migration of the ions of the solute from one electrode to the other, but that the ions carry along with them molecules of the solvent, the amount of solvent carried by the anions and cations being different. This "hydration" or "solvation" of the ions has recently been proved by direct experiment (*Washburn, Zeit. für Phys. Chem., Vol. 66, p. 513, 1909*), by adding to the solution an indifferent substance upon which the electric current produces no migratory effect (e.g., sugar or raffinose). See also Lewis, J., *Am. Chem. Soc., Vol. 32, p. 862, 1910.*

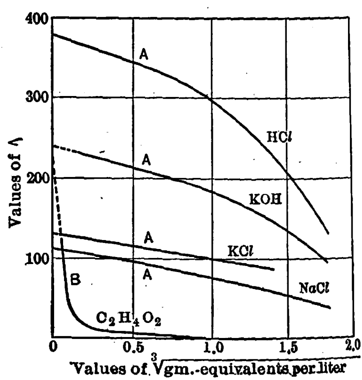

Fig. 4.

Conductance of Electrolytes. — (*See also article on Resistance and Conductance.*) While no simple relations have been found to hold between the *specific* conductances of different electrolytic solutions, very important results follow at once from a comparison of their *equivalent* conductances.

Table IX shows the relation between the equivalent conductance Λ and the concentration m in gram-equivalents per liter for some typical aqueous solutions. In Fig. 4 certain of these data are shown graphically. From a study of these results the following important conclusions may be drawn.

TABLE IX. — EQUIVALENT CONDUCTANCE Λ OF TYPICAL ELECTROLYTES DISSOLVED IN DIFFERENT QUANTITIES OF WATER, AT 18° C.

Concentration in gm.-equivalents per liter $= m = 1000\eta$	Dilution in liters per gm.-equivalent $= v = \phi/1000$	KCl	NaCl	KNO_3	$AgNO_3$	½ $CuSO_4$	½ H_2SO_4	HCl	CH_3COOH	KOH	NH_3
0.0001	10,000	129.07	108.10	125.50	115.01	109.95	......	(378)	107	......	(66)
0.0002	5,000	128.77	107.82	125.18	114.56	107.90	......	(378)	80	......	53
0.0005	2,000	128.11	107.18	124.44	113.88	103.56	(368)	377	57	......	38.0
0.001	1,000	127.34	106.49	123.65	113.14	98.56	361	376	41	(234)	28.0
0.002	500	126.31	105.55	122.60	112.07	91.94	351	375	30.2	(233)	20.6
0.005	200	124.41	103.78	120.47	110.03	80.98	330	373	20.0	230	13.2
0.01	100	122.43	101.95	118.19	107.80	71.74	308	369	14.3	228	9.6
0.02	50	119.96	99.62	115.21	......	62.40	286	366	10.4	225	7.1
0.05	20	115.75	95.71	109.86	99.50	51.16	253	358	6.48	219	4.6
0.1	10	112.03	92.02	104.79	94.33	43.85	225	351	4.60	213	3.3
0.2	5	107.96	87.73	98.74	......	37.66	214	342	3.24	206	2.30
0.5	2	102.41	80.94	89.24	77.5	......	205	327	2.01	197	1.35
1	1	98.27	74.35	80.46	67.6	25.77	198	301	1.32	184	0.89
2	0.5	92.6	64.8	69.4	......	......	183.0	254	0.80	160.8	0.532
3	0.33	88.3	56.5	(61.3)	......	......	166.8	215.0	0.54	140.6	0.364
5	0.2	......	42.7	......	......	......	135.0	152.2	0.285	105.8	0.202

Note. — The ½ before $CuSO_4$ and H_2SO_4 indicates 1 gram-equivalent = ½ mol.

For complete tables consult *Landolt and Börnstein's* or *Kohlrausch and Holborn's Tables.*

1. The Equivalent Conductance of a Solution Increases as the Dilution of the Solution Increases and Approaches a Limiting Maximum Value. — The maximum equivalent conductance, usually denoted by Λ_∞, is different for different solutions. It may be found graphically for good electrolytes (giving curves of the form A, Fig. 4), by extrapolating the curve representing the observed data until it cuts the ordinate of zero concentration. This method fails however for weak electrolytes such as acetic acid, see Curve B.

2. Independent Migration of Ions. Ionic Mobilities. — The product of the equivalent conductance of any solution at infinite dilution Λ_∞ and the transport ratio of either of the ions forming the electrolyte is a constant for that ion and independent of the nature of the electrolyte. That is, calling n_a and n_c the transport ratios of the anions and cations respectively, the product $l_a = n_a\Lambda_\infty$ is a constant irrespective of the nature of the electrolyte of which this anion forms a part; similarly $l_c = n_c\Lambda_\infty$ is a constant for the cation irrespective of the nature of the electrolyte of which it forms a part. For example, the product $n_c\Lambda_\infty$ for the potassium ion is a constant whether the solution be one of potassium chloride, potassium bromide, potassium sulphate, etc. These constants l_a and l_c are called the "mobilities" or "ionic conductivities" of the ions. This relation was discovered by F. Kohlrausch in 1876 and stated as the law of the independent migration of ions.

Since, by definition, $n_a + n_c = 1$, it follows that

$$\Lambda_\infty = l_a + l_c.$$

In other words the maximum equivalent conductivity of a solution (at infinite dilution) is equal to the sum of the mobilities of the anions and cations which form the electrolyte; or, the mobility of an ion depends solely upon the nature of the ion and not upon the nature of the substance of which it originally formed a part.

From this relation the mobility of any ion may be determined directly from the equivalent conductance when the mobility of the ion with which it is associated is known.

The values of the mobilities of the more common ions at 18° C. and at infinite dilution are given in Table X. From this table the equivalent conductance of any given electrolyte at infinite dilution may be readily calculated by the formula $\Lambda_\infty = l_a + l_c$. Thus, for silver nitrate, $AgNO_3$, at 18° C. $\Lambda_\infty = l_{Ag} + l_{NO_3} = 54.3 + 61.7 = 116.0$.

TABLE X. — MOBILITIES OF TYPICAL IONS AT INFINITE DILUTION AND 18° C.

(Values at $t°$ may be computed by the formula $l_t = l_{18}[1 + \alpha(t - 18) + \beta(t - 18)^2]$)
(Kohlrausch)

Anions	l_{18}	α	β	Cations	l_{18}	α	β
F	46.6	0.0232	0.000094	Li	33.4	0.0261	0.000142
Cl	65.5	0.0215	0.000067	Na	43.5	0.0245	0.000116
Br	67.0	......	....	K	64.6	0.0220	0.000075
I	66.5	0.0206	0.000052	Rb	67.5	0.0217	0.000069
SCN	56.6	......	....	Cs	68.0	......	....
ClO_3	55.0	0.0207	0.000054	NH_4	64.0	0.0223	0.000079
BrO_3	46.0	......	....	Tl	66.0	......	....
IO_3	33.9	0.0233	0.000096	Ag	54.3	0.0231	0.000093
ClO_4	64.0	......	....	H	315.0	0.0154	−0.000033
IO_4	48.0	......	....	½ Ba	55.0	0.0239	0.000106
NO_3	61.7	0.0203	0.000047	½ Sr	51.0	......	....
MnO_4	53.4	......	....	½ Ca	51.0	......	....
OH	174.0	0.0179	0.00008	½ Mg	45.0	0.0255	0.000132
CHO_2	47.0	......	....	½ Zn	46.0	0.0256	0.000133
$C_2H_3O_2$	35.0	0.0236	0.000101	½ Cd	46.0	......	....
½ SO_4	68.0	0.0226	0.000084	½ Cu	46.0	0.0240	0.000107
½ Cr_2O_7	72.0	......	....	½ Pb	61.0	0.0244	0.000114
½ CO_3	60.0	0.0269	0.000155				
½ $(COO)_2$	63.0	......	....				

The mobilities at any other dilution ϕ may also be determined in the same way from a measurement of the conductivity Λ_ϕ at this dilution and the corresponding transport ratios n_a and n_c, but the product $n_a\Lambda_\phi$ or $n_c\Lambda_\phi$ for ions in fairly concentrated solutions is not independent of the nature of the electrolyte.

Interpretation of the Laws of Electrolytic Conduction on the Dissociation Theory. — The interpretation of the above experimental facts in the light of the dissociation theory is simple. On this theory, the value of Λ_ϕ is

determined by three factors, any one of which, if zero, will make $\Lambda_\phi = 0$. These are:

1. The fraction of the total number of molecules in one gram-equivalent of the solute which are dissociated at any instant. This fraction, called the "degree of ionization" or "ionization coefficient," is usually denoted by γ_ϕ, the subscript ϕ indicating the dilution to which it refers.

2. The average velocities with which the ions migrate while free. Let u_c and u_a be these velocities, in centimeters per second, for the cations and anions respectively, for a potential gradient of one volt per centimeter.

3. The charge carried by the ions. This is fixed, since one equivalent weight of every ion carries with it $F = 96{,}540$ coulombs.

If a volume ϕ of the solution containing one gram-equiv. of the solute be imagined placed between two parallel plate electrodes 1 cm. apart, and an e.m.f. of one volt be applied, the quantity of electricity conducted across the electrolyte in one second, i.e., the current, will be equal to the equivalent conductance Λ_ϕ (definition of Λ_ϕ). The current is also equal to the total quantity of electricity conducted by the cations plus that conducted by the anions; hence

$$\Lambda_\phi = \gamma_\phi F u_a + \gamma_\phi F u_c = \gamma_\phi F\,(u_a + u_c).$$

The experimental fact that for many electrolytes at infinite dilution Λ_ϕ is equal to the sum of two constants l_a and l_c (the mobilities of the ions) which depend only on the nature of the ions, indicates that the velocity of migration u_a or u_c of any given ion depends only upon the nature of the ion and not upon the electrolyte of which it forms a part, and that the degree of ionization at infinite dilution is the same for all such solutions; there is much evidence to indicate that this is practically 100 per cent (*see below*).

Degree of Ionization. — The fact that Λ_ϕ increases with the dilution may be due either to the velocities u_a and u_c or to γ_ϕ increasing with the dilution, or to both causes combined; F is a constant. In concentrated solutions both the velocities and the degree of ionization undoubtedly decrease as the concentration increases. For dilute solutions, however, there is considerable evidence to show that the frictional resistance opposed to the motion of the ions changes but slightly with increasing dilution, and hence the values of the migration velocities in such solutions may be regarded as nearly constant. If this be assumed, and also that the *nature* of the ions remains unchanged, i.e., that u_c and u_a at the dilution ϕ are practically the same as at infinite dilution, $\phi = \infty$, then the observed increase in Λ_ϕ with the dilution must be due to an increase in γ_ϕ. On these assumptions, the solute becomes more and more ionized as the dilution increases, and at infinitely great dilution all the molecules of the solute are continuously dissociated or ionized, i.e., as ϕ approaches infinity, γ approaches unity.

Hence for $\phi = \infty$, $\Lambda_\infty = F\,(u_c + u_a)$, and therefore

$$\frac{\Lambda_\phi}{\Lambda_\infty} = \frac{\gamma_\phi F\,(u_c + u_a)}{F\,(u_c + u_a)} = \gamma_\phi.$$

This relation affords the simplest way of determining the coefficient of ionization γ_ϕ of an electrolyte, namely, finding the ratio of its equivalent conductance at dilution ϕ to its maximum conductance at infinite dilution. The two assumptions mentioned above should not be lost sight of when applying this formula. The values of γ_ϕ thus computed for a number of typical electrolytes are given in Table XI. The results obtained by this method are in many cases in excellent agreement with the values of ionization coefficients obtained by

methods independent of all electrochemical considerations, such as the lowering of the freezing point of a solution.

TABLE XI. — IONIZATION COEFFICIENTS OF TYPICAL ELECTROLYTES AT VARIOUS DILUTIONS COMPUTED FROM RATIO $\frac{\Lambda_\phi}{\Lambda_\infty} = \gamma_\phi$

(Kohlrausch and Holborn's data)

Liters per gm.-equivalent $= v = \frac{1}{1000}\phi$	Gm.-equivalents per liter $= m = 1000\,\eta$	NaCl	HCl	KOH	½$CuSO_4$	NH_4OH	$C_2H_4O_2$
1	1	0.675	0.784	0.770	0.217	0.0037	0.0036
10	0.1	0.839	0.914	0.891	0.379	0.0139	0.0131
100	0.01	0.933	0.964	0.954	0.608	0.0403	0.0406
1000	0.001	0.978	0.982	0.979	0.856	0.118	0.117

Note.—The ½ before the $CuSO_4$ indicates that 1 gram-equivalent of H_2SO_4 = ½ mol. of H_2SO_4.

Calculation of Velocity of Migration of Ions. — From a knowledge of the mobility of an ion its actual velocity of migration may be easily computed on the assumptions of the dissociation theory. Thus for aqueous solutions at infinite dilution, in which the ionization may be assumed complete, $l_c = Fu_c$ and $l_a = Fu_a$; hence $u_c = \frac{l_c}{F}$ and $u_a = \frac{l_a}{F}$. From Table X we have for the sodium ion $l_{Na} = 43.5$ and for the chlorine ion $l_{Cl} = 65.5$; therefore the velocity in centimeters per second per volt per centimeter with which these ions migrate at 18° C. is $\frac{43.5}{96,540} = 0.000450 \frac{\text{cm.}}{\text{sec.}}$ and $\frac{65.5}{96,540} = 0.000677 \frac{\text{cm.}}{\text{sec.}}$ respectively. For other dilutions the degree of ionization must be taken into account. The values given in Table VII are obtained in this way. For the relation of velocity of migration to diffusion phenomena see Nernst's *Theoretical Chemistry, p. 368; Trans., 6th Ed., 1911.*

Effect of Concentration on Degree of Ionization. — It has been pointed out that the concentration of a solution has relatively little effect on the velocity with which the ions migrate, except at high concentrations. Hence the large increase of the equivalent conductance with the dilution ϕ, see *Fig. 4, and Table IX*, must be due to a change in the degree of ionization. It can be shown on theoretical grounds that the ionization coefficient γ of a binary electrolyte should vary with the dilution ϕ or concentration η according to the equation

$$\frac{\gamma^2}{\phi(1-\gamma)} = K \quad \text{or} \quad \frac{\eta\gamma^2}{1-\gamma} = K,$$

in which K is a constant called "the equilibrium constant of the ionization reaction," for the given temperature and pressure. This formula, known as "Ostwald's Dilution Law," holds exactly only for weak electrolytes, i.e., those which are slightly ionized like the organic acids, and which give rise to dilution curves of the type *B, Fig. 4.* For good electrolytes, giving curves of the type *A*, the formula fails completely. A wholly satisfactory explanation for this discrepancy has not yet been given. (*See discussion in recent address by James*

Walker, British Association Meeting, 1911. Also G. N. Lewis, Zeit. f. physik. Chem., 70, 216, 1910.) Instead of obeying the above law, curves of the type *A* satisfy an exponential equation of the form

$$\frac{(\eta\gamma)^n}{\eta\,(1-\gamma)} = \text{constant},$$

in which n is a constant depending upon the electrolyte and having a value between 1.40 and 1.56. Other "dilution laws" have also been proposed by Kohlrausch, Van't Hoff, Rudolphi, Kraus and Bray.

Effect of Temperature on Ionization. — The per cent to which a substance is ionized in solution may increase or decrease with the temperature. The sign and magnitude of the effect depend upon whether the ionization reaction is accompanied by an absorption or evolution of heat. Substances which dissociate with evolution of heat become less ionized with increasing temperature; substances which dissociate with absorption of heat become more ionized with rising temperature. Highly dissociated substances like sodium chloride, hydrochloric acid, etc., belong to the former class. Water which is very slightly ionized belongs to the latter class. In general a variation of temperature from 0° to 100° C. produces a change in ionization of only a few per cent. Recent investigations by Noyes carried out at temperatures up to 360° C. have confirmed these predictions of theory. The effect of temperature on γ may be computed through its effect on K by means of Van't Hoff's thermodynamic relation

$$\frac{d \log K}{dt} = \frac{Q}{\overline{R}T^2},$$

where K is the above equilibrium constant of the ionization reaction, Q the heat of the corresponding reaction, $\overline{R}$ the gas constant and T the absolute temperature to which the values of K and Q refer. For derivation of this equation see Nernst's *Theoretical Chemistry, page 659, 1911 Ed.*

Negative Temperature Coefficients of Electrical Conductivity. — The fact that certain electrolytes, e.g., a phosphoric-acid solution, may have a negative temperature coefficient is readily explained in terms of the above relations. If the increase in the velocity of migration of the ions with rising temperature is more than offset by a diminution in the average number of free ions, resulting from a decrease in ionization, the conductivity of the solution will diminish. By combining solutions having positive and negative temperature coefficients in suitable proportions, electrolytes have been prepared which have nearly a zero temperature coefficient over quite a range of temperature. The following mixture, proposed by Manganini, has this property: 121 grams mannite, 41 grams boracic acid, 0.06 gram potassium chloride dissolved in sufficient water to make one liter. Its specific conductance at 18° C. is $\kappa = 0.00097$. Such a solution is well adapted for a liquid resistance just as manganine wire is adapted for resistance coils.

Conductivity of Fused Electrolytes. — In Table XII are given the specific conductance (κ) and the equivalent conductance (Λ) of several typical fused salts at their respective melting points; also for comparison the equivalent conductance of these salts in infinitely dilute aqueous solutions at 18° C. It will be seen that the values of the specific conductance (κ) of the salts at their respective melting points are much greater than the values of the specific conductance of the same salts in normal aqueous solutions. On the other hand, if the volume ϕ occupied by one gram-equivalent of the salts in the fused state is determined by specific gravity measurements and the equivalent conductance Λ computed ($\Lambda = \phi\kappa$), it will be seen that these values are less

than the maximum equivalent conductance Λ_∞ of the same salt in aqueous solutions. Thus the conducting power of one equivalent of fused sodium nitrate at 305° C. is less than when the same weight of salt is dissolved in one or more liters of water.

TABLE XII. — SPECIFIC AND EQUIVALENT CONDUCTANCE OF TYPICAL FUSED SALTS AT THEIR MELTING POINTS

Salt	Temperature, melting point, °C.	Specific conductance, κ	Equivalent conductance, Λ	Λ_∞ at 18° C. in aqueous solution
$NaNO_3$	305	0.9510	42.15	105.2
KNO_3	334	0.6225	33.59	126.3
$LiNO_3$	250	0.7886	30.36	95.1
$AgNO_3$	218	0.6815	29.22	116.0
$AgClO_3$	215	0.3676	18.09	109.3

If electrical conduction in fusions is wholly electrolytic it is probable that it is determined by the same three factors which determine the conductance of solutions, and that the equivalent conductance may be expressed by an equation of the general form

$$\Lambda = \gamma F\,(u_c + u_a).$$

As yet, however, all attempts to measure the velocity with which ions migrate in fused salts have led to no satisfactory or conclusive results. Hittorf's transference ratios cannot be determined as in solutions, since in a pure fused salt concentration differences do not occur at the electrodes. Only indirect estimates of the value of γ have been possible. The evidence thus far obtained, however, points to a high rather than to a low state of ionization in molten salts. For a résumé of the present status of this question see Goodwin, *Trans. Am. Electrochem. Soc.*, *1912*.

The equivalent conductance of fused salts increases in general from 1 to 1½ per cent per degree centigrade. As the increase has been found in a number of cases to be almost identical with the increase in the fluidity of one equivalent weight of the fused mass, it is probable that the velocity of migration of the ions increases in this proportion. The ionization of fused salts is certainly not a result of temperature, as in the case of gases; there is evidence, in fact, that the ionization of some fused salts may decrease with increasing temperature as it does in their aqueous solutions.

THEORY OF CONTACT POTENTIALS. — In the case of liquid-liquid junctions and metal-liquid junctions a satisfactory theory, based on the dissociation theory and the theory of solutions, has been worked out.

Theory of Liquid-liquid Potentials. — Nernst (*Zeit. für Phys. Chem.*, *4, page 129, 1889*) was the first to show that in the case of electrolytes a difference of potential necessarily arises between two solutions in contact which gradually diminishes, until the solutions are completely mixed. The general theory, worked out by Planck (*Ann. der Physik, Vol. 40, page 561, 1890*) is complicated. It leads, however, to simple formulæ in the following special cases.

Case I. — The junction is between two dilute solutions of the *same solute at different concentrations* c_1 and c_2, both ions of the solute having the valency n;

for example, c_1-HCl — c_2-HCl or c_1-$ZnSO_4$ — c_2-$ZnSO_4$. The potential rise between solutions 1 and 2 is given by the equation

$$e = \frac{\bar{R}T}{nF}\frac{l_c - l_a}{l_c + l_a}\log_\epsilon\frac{c_1}{c_2}$$
$$= \frac{0.000198}{n}T\frac{l_c - l_a}{l_c + l_a}\log_{10}\frac{c_1}{c_2}\text{ volts,}$$

where $\bar{R}$ = the gas constant; T = absolute temperature; F = the Faraday = 96,540 coulombs; n = valence of ions; l_c and l_a the mobilities of the cations and anions, respectively; and c_1 and c_2 the concentrations (or, more exactly, the corresponding osmotic pressures) of the ions in the two solutions respectively.

It follows from this formula that the potential at a liquid junction approaches zero if the electrolyte is composed of two ions which migrate with nearly equal velocities, and increases as the difference in their velocities increases. Thus $\overset{+}{K}$ and $\overset{-}{Cl}$ ions have nearly identical mobilities, and hence between any two solutions of potassium chloride there exists only a very small potential difference. For a concentration ratio of 10 to 1 the potential difference at 18° C. is only 0.0004 volt. This theory also explains why liquid-liquid potentials are greatest between acids or between alkali solutions, for hydrogen and hydroxyl ions possess relatively great mobilities.

Case II. — The liquid junction is between solutions of two *different solutes*, each containing two ions of the same valence n and at the *same concentration*. Here again the process of diffusion gives rise to potential difference between solution 1 and solution 2, the value of which may be computed by the formula

$$e = \frac{\bar{R}T}{nF}\log_\epsilon\frac{l_{c_2} + l_{a_1}}{l_{c_1} + l_{a_2}}$$
$$= \frac{0.000198\,T}{n}\log_{10}\frac{l_{c_2} + l_{a_1}}{l_{c_1} + l_{a_2}}\text{ volts,}$$

where l_{c_1}, l_{a_1} and l_{c_2}, l_{a_2} are the mobilities or migration velocities of the cations and anions in the two solutions respectively. The above formulæ as well as the general formula for mixed electrolytes have been satisfactorily verified by experiment.

For most recent work see *Trans. Faraday Soc. Vol. 8, p. 86, 1912.*

Theory of Metal-liquid Potentials. — By extending the conception of the process of solution of solids in liquids to the case of metals, which pass into solution only as positively charged ions, Nernst (*Zeit. f. Phys.-Chem.*, 4, 129, *1889*) developed an osmotic theory of metal-liquid potentials. This applies to all kinds of electrodes which are reversible in the thermodynamic sense, or which from an experimental standpoint are non-polarizable. For all such reversible electrodes Nernst has shown that the potential from metal to liquid may be expressed by the relation

$$e = \frac{\bar{R}T}{nF}\log_\epsilon\frac{P_M}{p}$$
$$= \frac{0.000198}{n}T\log_{10}\frac{P_M}{p}\text{ volts,}$$

where again $\bar{R}$ is the gas constant, T the absolute temperature, n the valence of the metal ions, F the Faraday, p the partial osmotic pressure of the metal ions present in the solution and P_M a constant characteristic for each metal, which depends upon the temperature and solvent, but is independent of the nature of the solute.

Electrolytic Solution Pressure. — The constant P_M in the above formula is called the "electrolytic solution pressure of the metal M." Since $e = 0$ when $P_M = p$, the electrolytic solution pressure P_M may be regarded as equal to the osmotic pressure of the metal ions which would be just sufficient to prevent the metal electrode from dissolving or passing into the ionic form, in which case there would be no development of a potential difference at the metal-liquid junction due to this cause. Hence the numerical value of P_M for a given metal and solvent may be considered as a measure of the tendency for the metal to dissolve in the solvent, i.e., to pass from the metallic to the ionic form. Thus the alkali and alkali-earth metals, which pass readily into the form of ions, have a very high electrolytic solution pressure, while the noble metals like gold, silver and platinum have a very low solution pressure. From this point of view the electrolytic solution pressure of a metal may be taken as a measure of its place in the electrochemical series. The only method thus far proposed for obtaining the value of P_M is by computation from Nernst's formula for electrode potentials given above. The value to be assigned to e depends upon the experimental determination of the "absolute" difference in potential (*see next paragraph*) between the electrode and the solution with which it is in contact. At present there is some question as to whether the actual difference of potential between the electrode and solution may not be partially due to the phenomenon of selective absorption of the ions, which is not taken into account in the above formula. For this reason the values of P_M frequently computed from the measured electrode potentials are open to question. See Le Blanc's *Lehrbuch der Elektrochemie, pages 230–235;* Lehfeld, *Phil. Mag. 48, 430, 1899.*

Absolute Electrode Potentials. — On the assumptions that the potential difference between a metal and a solution is zero when the surface tension of the metal is a maximum and that this condition is fulfilled in "dropping electrodes," or may be obtained in a properly adjusted Lippmann capillary electrometer, the absolute potential rise from the solution to the mercury in the normal calomel electrode has been found to be $e = 0.5600 + 0.0006(t - 18° \text{ C.})$ volt. It is in terms of this value for the normal calomel electrode that "absolute" electrode potentials have been expressed. On account of the uncertainty attaching to the interpretation of this value it is better for the present to adopt the arbitrary value zero as the potential drop at the calomel electrode.

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[H. M. GOODWIN.]

ELECTRODYNAMOMETERS. — (*See also Ammeters; Balances, Current; Wattmeters.*) One of the most reliable and accurate laboratory instruments for current and power measurements, particularly a-c. measurements, is the electrodynamometer.

This instrument depends upon the action of one circuit carrying current upon another carrying the same current (*see Electricity and Magnetism, Principles of*). The working parts of the instrument consist of two coils, one fixed and the other movable.

SIEMENS ELECTRODYNAMOMETER. — The simplest form of electrodynamometer is that devised by Siemens, shown diagrammatically in Fig. 1. The coils are arranged at right angles to one another as shown, the heavy lines representing the fixed coil and the light lines representing the movable coil. The force of attraction or repulsion is proportional to the current in the fixed coil times the current in the moving coil, and consequently if the same current flows in the two coils, the deflection is proportional to the *square* of the current. This force is balanced by the torsion of a spring. By turning the torsion head at the top of the instrument the coils are kept at right angles. The torsional force is proportional to the angle D through which the head is turned; hence the current is

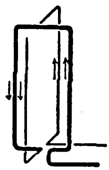

Fig. 1. Siemens Dynamometer

$$I = K\sqrt{D},$$

where K is a constant which is obtained by sending a known current through the instrument. When arranged with separate binding posts for the current or fixed coil and for the potential or swinging coil, as shown in Fig. 2, the instrument may be used as a wattmeter for either direct-current or alternating-current power measurements. The power in watts is then

$$P = K^2R_1D,$$

where K has the same value as before, and R_1 is the total resistance of the potential circuit, including the coil and external resistance R. This power includes the loss in the potential circuit, which is V^2/R_1, where V is the impressed voltage; this correction is usually small.

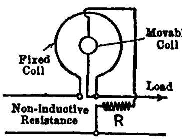

Fig. 2. Dynamometer Wattmeter

Uses of Siemens Electrodynamometer. — The instrument may be calibrated on direct current and may be used to measure either direct or alternating current or power, since when properly designed its readings are independent of frequency and wave form. High-grade standard wattmeters suitable for the precise calibration of commercial indicating wattmeter or watt-hour meters on alternating current and non-unity power-factor circuits are made on the electrodynamometer principle (*see Wattmeters*).

Design of Siemens Electrodynamometer. — The movable coil is usually suspended by a silk thread. Attached to the coil, at the point of suspension, is a spiral spring, the other end of which is attached to a movable collar. This collar carries a pointer which moves over a scale and denotes the angle of torsion of the spring. A second pointer is rigidly attached to the movable coil and registers with a fixed mark on the scale, when the fixed and movable coils are at right angles. The current is led in and out of the movable coil through mercury cups.

Dynamometers and wattmeters for use with alternating current should have as little metal used in their construction as is practical, as eddy currents will be set up in the metal parts that will influence the coil system and change the con-

stant of the instrument. The case and supporting frames should therefore be of insulating material.

Range of Siemens Electrodynamometer. — Commercial instruments of this type for measuring currents are not suitable for measuring currents of less than 0.02 ampere. Precision wattmeters (q.v.) built on the same principle may be used to measure power of the order of 1,000,000 watts on unity power factor.

Sources of Error. — The interaction of the earth's magnetic field and the current in the movable coil may be appreciable when the instrument is used for direct-current measurement unless the movable coil is perpendicular to the earth's field. This action may be detected by sending a direct current through the *movable coil only;* any deflection will be due to the earth's field.

When the instrument is used to measure alternating currents the earth's field has no effect. However, eddy currents induced in the various parts of the windings or case, or in neighboring conductors, may produce a disturbing influence. This may be tested by sending an alternating current through the *movable coil only,* the other coil being open; any deflection will be due to eddy currents. The "zero" position of the pointer should also be such that there is no mutual induction between the two coils when in the "zero" position. This can be tested by short-circuiting one coil and sending an alternating current through the other; any deflection will be due to mutual induction.

The instrument can be used for measuring the power input to circuits of low power factor only when the self induction of each coil and of the external so-called "non-inductive" resistance (*see Fig. 2*) is negligible (*see Fleming, Handbook of the Electrical Laboratory and Testing Room*).

Degree of Precision. — When properly calibrated and properly used the readings of a good Siemens electrodynamometer may be relied upon as accurate to within 1/10 per cent for a full-scale deflection. The smaller the deflection the less the percentage accuracy.

Rowland Electrodynamometer. — This instrument is identical in principle with a Siemens electrodynamometer, except that instead of noting the angle through which a torsion head must be turned, the moving coil is allowed to deflect and the deflection is noted by means of a telescope and scale. Its chief application is for measuring small alternating currents and small amounts of power in alternating-current circuits. It is seldom used except in special investigations requiring a high degree of precision or for the measurement of alternating quantities of small magnitude; e.g., alternating currents of the order of 0.1 ampere or power of from 0.01 to 1 watt. Its uses are fully described in Rowland's "Physical Papers," Johns Hopkins Press, Baltimore, 1902.

COSTS. — A simple Siemens electrodynamometer suitable for measuring currents of from 0.025 to 5 amperes costs about $75. A Rowland electrodynamometer with shunt box costs about $450.

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[H. PENDER AND H. R. RANKEN.]

ELECTROLYSIS OF GROUNDED STRUCTURES. — (*See also Electrochemical Processes Industrial; Electrochemistry, Principles of.*) The following discussion will be limited to the electrolysis of grounded structures, that is, to the corrosion of metal work in the earth, due to the action of stray, vagabond or leakage currents from the grounded rails of electric railroads.

MINIMUM VOLTAGE TO PRODUCE ELECTROLYSIS. — The conduction of current through moist ground is almost entirely electrolytic, ordinary or metallic conduction being almost, if not entirely, negligible. Electrolytic corrosion occurs whenever current flows from metal into the ground, regardless of whatever difference of potential may exist between different parts of the circuit. In order that current may flow, however, it is in general necessary that the difference of potential between anode and cathode exceeds the algebraic sum of the e.m.f.'s of combination and separation of the compounds involved in the electrolytic process. An iron anode in a soil containing iron salts in solution will be attacked when the e.m.f. is infinitesimal. Where hydrogen is liberated, polarization occurs and it is the p.d. between cathode (the metal where the current leaves the ground) and ground that determines the danger of the anode (the metal at which the current enters the ground) being corroded.

CORROSIVE EFFICIENCY AND PASSIVITY. — When an electric current is passed through an electrolyte, the mass of the anode dissolved may be calculated theoretically by Faraday's law (*see index*). In practice this amount may be exceeded, or the reverse. The term "anode efficiency" or "corrosive efficiency" is used to designate the ratio of the actual to the theoretical mass of the anode dissolved. The corrosive efficiency is usually less than 100 per cent owing to the occurrence of secondary reactions. Thus the corrosion of iron, which primarily occurs by the formation of ferrous salts, should theoretically occur at the rate of 1.045 grams per ampere-hour. Owing, however, to the production of ferric salts as well, the corrosive efficiency is usually lower than 50 per cent, especially with weak currents. Under certain conditions corrosion does not take place at all; this is spoken of as a condition of "passivity." Passivity is usually due to the formation of a film of insoluble oxide or other substance, but sometimes to more complex causes.

ALTERNATING-CURRENT ELECTROLYSIS. — (*J. L. R. Hayden, Trans. A.I.E.E., Vol. 26, p. 221.*) Alternating-current electrolysis is not a phenomenon like direct-current electrolysis, on which definite quantitative general laws can be formulated, but is of the character of a secondary effect, that is, the action of the positive half-wave is not quite reversed by the action of the negative half-wave, leaving a small difference rarely exceeding one-half per cent of the electrolytic action of an equal direct current.

Alternating-current electrolysis, when expressed quantitatively, or in per cent of the action of an equal direct current, varies very greatly with the chemical character of the electrolyte. Nitrates tend to increase electrolytic corrosion; carbonates and, in general, alkalis of the soil decrease it. In general, lead is attacked more than iron.

Alternating-current electrolysis, under conditions representing as nearly as seems feasible the conditions of lead cables in the soil, does not appreciably depend upon the current density, but is practically independent thereof, except indirectly, in that very high current densities may, by an increase of temperature, give an increased corrosion.

In general, electrolytic corrosion by alternating current increases with decrease of frequency. This increase with decrease of frequency does not follow a general law, but depends largely upon the chemical character of the electro-

lyte. In general, ammonium salts and nitrates seem to give a very great increase of electrolytic corrosion with decreasing frequency, while carbonates and soils with alkaline reaction, as containing cement, may give little or no increase of corrosion with decreasing frequency.

A direct current of 1.5 per cent of the alternating current gives practically complete protection against the electrolytic action of the alternating current.

DISTRIBUTION OF EARTH CURRENTS. — The distribution of stray currents in the earth depends not only upon the resistance of the earth but also the resistance and distribution of the conducting structures in the earth.

Earth Resistance. — As the earth conducts electrolytically, its specific resistance can be calculated from the concentration of the salt solutions it contains. Such calculations are, however, generally useless, as, even with moderate specific conductance, the extent of the current path in the earth is so great that its resistance is very low, except near the electrodes. The effect of the electrodes is to concentrate the current streams into limited areas, thereby increasing the effective resistance of the earth. It is, therefore, obvious that the resistance of the earth depends more upon the electrode area than anything else.

The resistance of the earth is low when the electrodes consist of considerable lengths of track rails. This is particularly noticeable in the case of single phase lines, due to the skin effect and inductance of the rails. Thus J. Dalziel and J. Sayers (*Inst. Civil Engineers, London, 1909*) found that, on the Midland Railway, the current did not continue along the rails for any considerable length, but within a few hundred yards of the car sank gradually into the earth to re-enter the ballast at a very short distance from the power station. H. F. Parshall found that on a line eight miles long, the earth-return current was 60 per cent of the total. Probably the most exact data of this kind are those due to A. W. Copley (*Trans. A.I.E.E., 1908, Vol. 27, p. 1171*), who stated that on the New York, New Haven & Hartford Railroad the percentage of current in the earth is 25 per cent on the four-track sections and 60 per cent on the single-track sections.

This condition is not confined to single-phase systems; Claude (*L'Eclairage Electrique, p. 141, Vol. 24, 1900*) found that in complicated direct-current networks the stray current was from 25 per cent to 30 per cent of the total.

Electrolytic Zone. — When a drop of potential occurs in a track rail, the current spreads out from it into the soil in much the same way that the leakage lines of magnetic force from a long electromagnet spread out into the air; that is, almost the entire leakage current is confined to a limited zone of the earth, the extent of this zone depending upon the potential drop in the rails themselves. The stream lines of the current will tend to crowd into a pipe or cable sheath (on account of their low resistance) if these conductors are within this zone, but this tendency rapidly diminishes as the distance between these conductors and the rails increases. Compare with the tendency of an iron bar to become magnetized when placed in the leakage field of an electromagnet; the induced magnetization rapidly diminishes as the distance of the bar from the magnet is increased. Conductors at a comparatively short distance from the rails are, therefore, not liable to electrolysis unless the difference of potential between them and the rails is large or the leakage path of the current from the rails to the conductors is of low resistance. Thus in England, where the drop in the grounded return is limited to a total of seven volts, Messrs. Cunliffe found experimentally that the electrolytic zone is confined to within three feet of the track.

Potential Readings. — Random readings between rails and pipes give no definite information about electrolytic conditions. As a matter of fact, a difference of potential between a pipe and rail at any point is often greater the less

the stray currents at that point. To illustrate this, we may assume a pipe to be outside the electrolytic zone and, therefore, safe from electrolytic corrosion. If, at any point, the rail is at the same potential as the pipe, the potential of the rail at every other point will differ from the pipe. Also, if the part of the rail which is at the potential of the pipe happens to be at some distance from the power station, the pipe will be positive to the rail at all points between the earthed point and the power station. Hence the fact that the pipe is positive to the rail does not indicate that it is subject to electrolysis. A more important illustration is the case of a pipe carrying little current itself but connected to some distant pipe which *is* carrying considerable current. In such a case, a pipe may be highly electropositive to the rails and yet be quite innocent of electrolytic tendencies. These illustrations are merely specific cases of the general principle that *potential readings are useless unless taken with reference to the resistance of the earth between rails and conductors, which in turn depends upon the direction and distribution of the current stream lines.* As most electrolytic surveys made in the past consisted of potential readings taken without respect to the direction of the current flow, they gave no indication of the earth currents and afforded no criterion of electrolytic conditions.

The equipotential surfaces in the earth are, of course, perpendicular at all points to the current stream lines. If the earth currents were steady, it would be possible to determine their direction and distribution by testing for equipotential points in the earth and plotting current stream lines therefrom. Unfortunately in most practical cases the equipotential surfaces are in constant motion owing to the fluctuations of current in the track rails.

The inevitable conclusions are, that random potential readings between track rails and pipes, etc., in the earth, cannot be of any use, and that it is generally impossible to plot the current stream lines.

STRUCTURES AFFECTED BY CORROSION.—Stray currents can set up corrosion of the rails, rail spikes, cross-bonds, steel columns, etc., through which they escape into the earth, and they can set up corrosion of gas and water pipes, cable sheaths, building structures, etc., which they may enter and leave in their course through the earth. The protection of the former class of materials is entirely the concern of the railroad company which originates the earth currents; not so with the latter class, which is often of great concern to the municipalities or public-service corporations which own them. Electrolytic surveys should, therefore, be made with a clear understanding of which class of property is under suspicion of danger. As a rule, electrolysis is most destructive to the grounded metal of the railroads; foreign pipes, cable sheaths, etc., being affected in comparatively rare instances. Of all properties foreign to the railroads, the thin sheaths of telephone cables are most susceptible to electrolytic corrosion.

Measurement of Leakage Current in Grounded Return System.—The columns and foundations of elevated structures and subways, rail spikes, bare negative cables and bonds are the parts most likely to be attacked electrolytically.

In the case of the elevated railways of New York City, after about nine years' operation with the steel structure a part of the return system, it was found necessary to remove all metallic connections in the Borough of Manhattan between the track rails and structure and to install negative feeder cables to compensate for the conductivity thereby sacrificed.

The corrosion of rail spikes, while happily rare, is a matter of such grave concern to the railroads and public that it should be carefully watched for. It occurs when ties are old and water-logged or when improperly treated with preservative. Timber is ordinarily classed with the non-conductors, because,

when dry and well-seasoned, it has a high dielectric strength and practically infinite resistance. When green or moist, however, it becomes an electric conductor of comparatively low resistance. The resistance along the grain is much less than across it, and porous woods, such as oak, are better conductors than the non-porous woods such as pine. The conductivity of wood is due primarily to the presence in its pores of electrolytes formed from the salts found in natural timber, from preservatives and from salts originating from coal fumes or ashes. The flow of current, from the rails and spikes into the ties, fills the pores of the wood with iron salts, which add to the electrolytic conductivity and permit the leakage of more current. The effect is, therefore, cumulative, the leakage current increasing until the pores of the wood are completely saturated with electrolyte. Cases have been known where spikes were pitted more than half way through and where the rail flanges were badly corroded. Ties creosoted by the hollow-cell process, which leaves the fibers empty, are particularly likely to acquire high electrolytic conductivity. Red oak treated with zinc chloride is also a bad tie from the electrolytic point of view.

Columns of elevated railroads, subways, passenger stations, etc., are best tested for electrolytic trouble by means of a sensitive galvanometer used in the following way. Iron clamps with pointed tips are fastened to the column at points four or five feet apart, care being taken that the points penetrate the paint and make metallic contact with the steel. Wires are run from these clamps to a galvanometer and the deflection noted. The galvanometer having been calibrated, this gives the drop of potential in the column. The cross-sectional area of the column being known, its resistance may be calculated from the known resistivity of steel (usually 11 times that of copper). The potential drop, divided by the resistance, gives the flow of current in the column. Knowing the direction of flow, its amount and the efficiency of corrosion, the actual damage being done by electrolysis may be calculated as a definite weight of metal per annum. This method is being pursued with great success on the Electric Zone of the New York Central Railroad, the galvanometer employed being a Queen & Co's E-8010 with tube E-8011, a calibration resistance and a tripod. A deflection of one scale division is equivalent to about 0.000003 volt. Considering an average column with a resistance of 0.000,004 ohm for a 4-foot length, a deflection of one scale division corresponds to three-fourths of an ampere. Where the columns are to be encased in concrete, permanent testing terminals should be provided, preferably in the form of small iron pipes screwed into the steel and ending flush with the concrete.

Measurement of Leakage Current in Pipes, Cable Sheaths, etc.—It is not uncommon to find potential readings taken between different systems of pipes, without regard to the location of the connections, the results being recorded as differences of potential between those systems of pipes. The potential of a pipe system, however, may vary from point to point, and consequently such readings have no significance unless the points between which the potential difference is measured are specified. The significance of potential readings between specific points is that they afford an indication of the potential gradient normal to the tracks and thereby help to determine the electrolytic zone.

Making use of all possible connections to the pipe, the potential difference between these points and the anode end of the grounded system should be determined as described below and the limits of the "electrolytic zone" ascertained by noting where the potential gradient in the pipe becomes negligible. For this purpose, water hydrants and water pipes constitute the best connection points.

Potential readings between points on the same pipe line or cable sheath are sometimes made the basis of potential curves showing the drop in the pipe or

sheath. Such potential curves may be very significant when taken from cable sheaths, but are of little use for pipes on account of the variable joint resistance of the latter. Thus in case of a cable sheath of uniform resistance along the entire length tested, where the potential curve is flat, the sheath carries no current; when it is a straight sloping line, the sheath carries current without giving current to or taking current from the earth; where it changes its slope, current is either entering or leaving the sheath.

Hering's Method. — The current in a pipe may also be measured by the following method. (*C. Hering, Trans. A.I.E.E., June, 1912.*) The fundamental principle is as follows. Let P, Fig. 1, be a part of an underground pipe which has been uncovered and through which an unknown current I is flowing. Let D be a sensitive galvanometer, millivoltmeter or any other suitable form of detector; there should preferably be no variable resistance like an unbonded pipe joint between the two contact points. Let A be an ammeter, B a few battery cells and R an adjustable resistance; the shunt circuit containing them is connected as shown, anywhere outside of the points of application of the voltage detector, the farther away the better; they may even be on the other side of a joint.

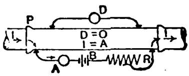

Fig. 1

To find the current flowing in the pipe adjust the resistance R until D reads zero; then the current due to the battery B will be exactly equal and opposite to the current in the pipe. Hence the reading of the ammeter A gives the current in the pipe.

If D is a galvanometer with proportionate deflections, instead of a mere detector, then by taking a deflection immediately after the shunt circuit has been opened a reading proportionate to the drop of voltage for that current will be obtained. The instrument D is thereby calibrated to read the pipe currents directly and can be used for this purpose thereafter; the test with the battery current is therefore merely of the nature of a preliminary calibration, and need be carried out only once for each station.

Instead of attempting to adjust the current in the shunt to bring the voltage D to zero, it is often more convenient to use a regular measuring instrument for D instead of a mere zero detector, and to pass a definite current through the shunt, say 10, 50 or 100 amperes, reading the two deflections of D when this current is on and when it is off; this had best be repeated several times. The difference between these two readings then corresponds to the current in the pipe. The best current to use is that which will reduce the original deflection as much as possible. By thus using the difference between a large and a small deflection the errors due to a loose zero, which are so common with highly sensitive instruments, are reduced.

Methods for overcoming fluctuating currents are given by Hering in the paper cited above.

Having thus calibrated the voltmeter V at each of two neighboring stations, the currents which enter or leave the pipe between them may be determined, with the fluctuating currents, by taking the readings of the two instruments simultaneously by means of visual or telephonic signals, preferably at times when the currents are momentarily steady.

Measurement of Current Leaving Pipe at Any Point. — The strength of the current at the point where it is leaving the sheath may be determined by the following test, which involves the use of a temporary bond in the form of a stout copper cable, electrically connected through an ammeter with both sheath and rail. Let

V = total drop of potential in the rails from *their* anodic center to the point where the temporary bond is connected.

By "anodic center of the rails" is meant the center of gravity of the leakage current *leaving* the rails.

v = difference of potential between the sheath and rail at the bonding point prior to the insertion of the bond.

v_1 = same, after the insertion of a bond which carries I amperes. The current in the bond is read from the ammeter.

Then the earth current, i, after the insertion of the bond will be

$$i = I \cdot \frac{k}{1-k},$$

where

$$k = \frac{v_1}{v} \cdot \frac{V-v}{V-v_1}.$$

The earth current from the pipe, without bonding, would be

$$\frac{V-v}{V-v_1}(I+i).$$

For example, suppose the total drop in the rails to be 100 volts and the differences of potential between the pipe and the rails at the bonding point to be five volts before the bond is inserted, and one volt after. Suppose the current in the bond is found to be 50 amperes.

Then

$$V = 100. \qquad v = 5. \qquad v_1 = 1. \qquad I = 50.$$

$$k = \frac{1}{5} \cdot \frac{100-5}{100-1} = 0.19.$$

$$i = 50 \cdot \frac{0.19}{1-0.19} = 12 \text{ amperes, approximately.}$$

Without bonding, the current would be

$$= \frac{100-5}{100-1}(50+12) = 59 \text{ amperes approximately.}$$

The above formula is based upon the following assumptions:

(1) That the drop in the rails V is not affected by the insertion of the bond.

(2) That the area over which the current from the pipe or sheath enters the earth is short. For proof of formula, see *Elect. World, 1910, Vol. 55, p. 407.*

PREVENTION OF ELECTROLYSIS. — Electrolysis may be partially or wholly prevented by the use of bonds, concrete coverings, paint, tape or braid, or by employing boosters, or by insulating the return circuits.

Protection from Electrolysis by Bonding. — Bonds are not only useful for testing purposes; they may be applied permanently to reduce the earth currents. There are certain effects, however, which may render their use inadvisable.

1. If one piping system is connected to the rails or bus, a difference of potential will be established between it and all other underground metallic structures and it will, therefore, attract current from the latter and expose them to electrolytic danger. A bonded piping system thus becomes a part of the trolley return circuit and the owner may become a party to whatever damage may occur in the others.

2. A considerable current in a gas pipe is a serious fire hazard and in a lead cable sheath is a menace to continuity of service.

3. Electrolysis is promoted at all imperfect joints and connections.

In spite of these objections bonds are very largely used to protect cable sheaths from corrosion. To be most effective, the sheath should be connected to the negative bus by an insulated cable and the bus itself should be connected to the track rails by insulated cables only.

Protection of Steel by Concrete.—The conductivity of concrete depends upon its porosity and wetness. Tests have shown that when wet the specific resistance may be as low as 20 or 30 ohms per yard cube and when dry, as great as 2000 ohms. The extent to which concrete prevents corrosion depends upon its chemical composition and porosity. Some cement protects iron by forming a protective coating of silicate of iron (*E. Noillion, Proc. Inst. Mech. Eng., 1905, p. 485*). The corrosive efficiency of steel in concrete is extremely variable, depending largely upon the nature of the electrolyte with which the concrete is impregnated, and upon the applied e.m.f., being much less with low than with high electromotive forces. For example, a reduction of e.m.f. to one quarter may reduce the corrosive efficiency to one half of one per cent of its former value. This subject is being investigated very thoroughly by the Bureau of Standards. The following tests show values of corrosive efficiency for unpainted iron obtained by some experimenters.

Nature of electrolyte	Thickness concrete protection, inches	Amperes per sq. in. of anode	Volts anode to cathode	Corrosive efficiency, per cent	Time of test, days	Name of experimenter
Lake water...	4	Variable	8	1.05 to 6.88	180	C. F. Burgess
3 per cent salt solution....	2	Variable	8	40 to 80	64	C. F. Burgess
Fresh water..	3½	0.00164	Variable	Over 60	30	A. A. Knudson
Salt (sea).....	3½	0.00164	Variable	Over 41	30	A. A. Knudson
Salt (sea).....	..	..	..	Average 33	..	G. Schaffer

Below 50°C. the efficiency of corrosion is very small, but above that temperature, rises rapidly. Increasing the current density does not materially change the corrosive efficiency at any given temperature. If the heating effect of the current is sufficient to raise the temperature to 50°C. (= 122°F.), or more, active corrosion begins. Below 50°C., the iron remains passive or nearly so, provided that there are present no foreign ingredients of marked corrosive tendencies, such as salt or calcium chloride. The addition of 0.33 per cent by weight of calcium chloride increased the efficiency of corrosion from practically zero to 80 per cent.

The exact cause of passivity at ordinary temperatures is not fully understood, but it is at least partially due to the concentration of calcium hydrate near the cathode surface by the current, where it comes in contact with the carbon dioxide absorbed by the water from the air, with resulting precipitation of calcium carbonate which fills the pores of the concrete, interposing a high resistance.

Briefly stated, concrete affords some protection to steel work, but, in every suspected case, determination should be made of the current flow and corrosive efficiency.

Salt should never be used in concrete if there is the slightest probability of action by electric currents, since the addition of even a fraction of one per cent of chlorine is sufficient to increase the rate of corrosion a hundred fold.

Destruction of Concrete by Electrolysis. — Some experimenters have found that where iron imbedded in concrete becomes an anode, not only is the iron corroded but the concrete also is cracked. This action is due to the internal stress set up by the increase of volume which the iron suffers when it changes to an oxide or salt. The current has no direct effect upon the concrete at the anode even when sufficiently strong to liberate chlorine from the brine used to impregnate it. (*C. E. Magnusson, Trans. A.I.E.E., June 1911*). At the cathode, however, the concrete becomes softened and loses its bond with the electrode. See papers by E. B. Rosa, B. McCallum & A. S. Peters, *Nat. Assn. of Cement Users*, 1912, and abstracted in *Eng. News*, 1912, Vol. 68, p. 1162. No action, however, occurs on concrete through which current flows, except at the electrodes.

Protection of Steel by Paint. — Experiments by M. Toch, C. E. Magnusson, G. H. Smith and others have shown that unpainted steel imbedded in concrete can be electrolytically corroded at the anode, but that a good insulating paint applied to the steel prevents such corrosion. Acid-proof paints with tar or asphalt base such as are commonly used to protect steel imbedded in concrete are usually effective. A typical paint of this kind has the following composition: 16 parts coal-tar paint, 4 parts Portland cement, 3 parts kerosene.

When the p.d. between cathode and ground does not exceed 5 volts the corrosive efficiency of an anode, so protected, is usually less than 1 per cent.

Protection of Sheaths by Tape or Braid. — H. W. Fisher found that electrolysis is not prevented by covering the lead with a weatherproof tape or braid saturated with insulating compound. With such coverings the electrolytic action is apt to be concentrated in spots and thus eat through the lead more quickly. For the same reason, lead-covered cables, laid in wooden boxes filled with pitch or bitumen, deteriorate rapidly under electrolytic action.

The Laclede Gas Company of St. Louis (*Elect. World, 1911, Vol. 57, p. 1103*), however, have had favorable experience with tape protection. Their wrought-iron pipe, in sizes from 3 to ¾ inch, was coated with a tar and pitch mixture, heated and thinned sufficiently to flow easily, and onto this a 4-inch paper ribbon was wrapped spirally, its edges overlapping. This paper covering was then tar painted and again wrapped with paper, the process being repeated until four successive coats were applied. Pieces of pipe thus insulated were placed in the ground under the most distinctive conditions of electrolysis, along with other lengths not so treated. After being taken up at the end of two years the unprotected pipes were badly pitted and almost consumed, while the insulated piping was practically the same as when laid. Although no test has yet been made carrying the insulated pipe to total disintegration, it is believed that pipe so treated will have its life at least doubled, and if this is true an expenditure for insulation equal to that of the cost of the bare pipe is justified. Only service runs are being so treated, the cast-iron mains being less subject to corrosion and electrolysis than the service pipes. The tar and paper coating is very hard when cooled, and the pipe lengths need to be handled with no more care than bare pipe (*J. L. Fitzhugh*).

Protection by Booster. — It is often proposed to render grounded metal work electronegative to the rail return by means of a booster, but such proposals are seldom carried into execution on account of the expense they involve. Thus L. B. Stillwell (*Trans. A.I.E.E., 1907, Vol. 26, Part 1, p. 265*) said: "The plans of the Hudson Company contemplate the use of a booster to prevent possible electrolytic damage to the metallic shells of the tube tunnels destined to connect the Borough of Manhattan and the Jersey Shore. The Interborough Rapid Transit Company is also proposing to use them for a similar purpose in connection with the tubes under the East River." It was, however,

A scheme recently adopted with success at Karlsruhe, Germany, involves the use of a booster which does not have to carry the main current and which is therefore comparatively small. At places where there is danger for gas pipes or water pipes, electrodes are placed in the earth in the neighborhood of the pipes and these electrodes are connected to the positive pole of a low-voltage generator or storage battery, while the pipe to be protected is itself connected with the negative pole. In this way electric current is forced to enter the pipe from the earth so that anodic destruction of the pipe is impossible. The power consumption is said to be insignificant. (*Geppert & Liese, Elek. Kraftb. u. Bahnen, Feb. 14, 1912.*)

Boosters are used to prevent the pitting of condenser tubes, especially where salt water is used for cooling. An example of such an installation is in the Power Station of the Long Island railroad on the East River, New York (*Street Railway Journal, 1906, Vol. 27, p. 545*).

Protection by Insulated Returns. — The only certain way of eliminating all electrolytic trouble is by limiting the drop of potential in the grounded conductors to a moderate amount, as is done in Great Britain by the Board of Trade. This can be most economically effected by the use of insulated negative feeder cables tapped into the rails at intervals, as described under Distribution, and by G. I. Rhodes (*Transactions of the A.I.E.E., 1907, Vol. 26, Part 1, p. 247–263*). Such a system is equivalent to a large number of substations, as far as its effect upon the potential drop in the rails is concerned. The maintenance of efficient bonding is a most important factor in limiting the drop of potential in rails and, therefore, in preventing electrolysis.

BIBLIOGRAPHY. — A complete bibliography of the subject up to 1908 is given in a paper by W. H. Gee, *Electrolytic Corrosion, Jour. Inst. El. Eng., 1908, Vol. 41, p. 425*. In addition to these references and those given in the text, the following more recent papers should be consulted: Burgess, C. F., *Electrolytic Corrosion of Iron in Concrete, West. Soc. Eng., 1911, and Elec. World, 1911, Vol. 57, p. 827;* Chapman, C. M., *The Effect of Electrolysis on Metal Imbedded in Concrete, Eng. Cont., 1911, Vol. 35, p. 99;* Cunliffe, J. G., and M. G., *Vagabond Currents, Jour. Inst. El. Eng., Sept. 1909;* Del Mar, W. A., *Electric Power Conductors, Chap. IX., N. Y., 1914;* Miller, W. H., *Electrolysis of Oil Pipes, Elec. W., 1910, Vol. 55, p. 1667;* Nicholas, N. J., *Tests of the Effect of Electric Currents on Concrete, Eng. News, 1908, Vol. 60, p. 710, and 1910, Vol. 64, p. 590;* Schaffer, G. B., *Corrosion of Iron Imbedded in Concrete, Eng. Rec., 1910, Vol. 62, p. 132;* Toch, M., *The Electrolytic Corrosion of Structural Steel, Jour. Am. Electrochem. Soc., 1905, Vol. 8, p. 133; 1907, Vol. 9, p. 77; 1908, Vol. 14, p. 207; 1909, Vol. 15, p. 351.*

[W. A. Del Mar.]

ELECTROMAGNET WINDINGS. — (*See also Electricity and Magnetism, Principles of; Electromagnets, Lifting and Plunger; Generators; Motors; Transformers.*) Any coil or winding carrying an electric current forms an electromagnet; the term electromagnet is also generally used for such a coil, particularly when it has a magnetic core, whether it is actually carrying a current or not. The winding of most electromagnets is in the form of a cylindrical helix of one or more layers; such a winding is called a solenoid. This winding is usually built up on a bobbin, which may be fixed rigidly to a magnetic core, or the core or part of it may be movable, in which case the electromagnet is called a plunger electromagnet. The electromagnet may also be built upon a horseshoe shaped core, with a movable yoke or "armature" which is attracted to the core when a current is established in the winding. See *Electromagnets, Lifting and Plunger*.

DESIGN AND CONSTRUCTION OF WINDINGS — The problem is to construct a coil which, for a given voltage across its terminals, will produce a given number of ampere-turns, and which will not overheat. Usually one or more of the dimensions of the coil are also fixed by the conditions under which the coil is to be used. The following factors must be taken into account in designing a winding:

Insulation from Core; Design of Bobbin. — When the core is fixed, two washers of hard rubber or vulcanized fiber are forced on at either end of the core. The core is then insulated with a wrapping of paper, mica or oiled linen, and is then ready to be wound. When the core is movable, the two end washers are forced on to the ends of a brass tube or a tube made of the same material as the washers. When a metallic tube is used the washers are sometimes made of the same metal. In the case of a quick-acting plunger magnet a metallic bobbin, if used, should be slotted, in order to avoid eddy currents; this also applies to all forms of alternating-current electromagnets.

Insulation of Wires; Insulation Between Layers; Baking. — The wire may be insulated with a cotton, silk or asbestos wind or by a coating of enamel. In high-voltage solenoids the various layers of wire are insulated from each other by paper, mica or oiled linen wrappings, or the entire winding is divided into several sections separated by vertical washers of insulating material. The wound coil may be further insulated by dipping it into an insulating varnish in a vacuum or at atmospheric pressure, after which it is either air-dried or baked.

Aluminum wires and ribbons are used extensively by some manufacturers, especially abroad. In this case the layer of oxide on the wire is the only insulation which is used, but inasmuch as this insulation cannot be destroyed by heat, the coils can be run at much higher temperatures, which is of special value for lifting magnets and similar devices. The oxide layer will stand a potential stress of about 0.5 volt. See article on *Use of Naked Aluminum Wire in Electromagnets*, by H. F. Stratton, Elec. Wld., 1912, Vol. 60, p. 400.

Temperature Rise of Winding; Watts per Square Inch. — The rise of temperature of the winding will depend primarily upon the average rate at which heat is developed by the electric current and the amount of exposed surface from which this heat can be radiated; the temperature rise will also depend upon the depth of winding, the circulation of the air, etc. The hottest spot in the winding should never reach a higher temperature than 90° C. When the interior of the winding is at 90° C. the temperature of the external surface, as measured by a thermometer, will usually be much less, as will also the average temperature measured by the change of resistance method.

As a rough approximation a solenoid winding should be so designed that the average power developed will not exceed 0.5 watt per square inch of radiating surface for an open winding, and will not exceed 0.7 watt per square inch of radiating surface for an iron-clad solenoid. In figuring the radiating surface of an open winding, the surface of the hole through the solenoid is not included, and the end surfaces are included only when the solenoid is short. By the radiating surface in the case of an iron-clad solenoid is meant the surface of the winding which is in contact with the iron. A radiation of 0.5 watt per square inch and 0.7 watt per square inch for an open and an iron-clad winding respectively corresponds roughly to an average temperature rise of approximately 60° C.; for other rates of radiation the temperature rise will be approximately proportional to the watts per square inch radiated.

For short-time service, i.e., when the solenoid is energized only for short intervals with long intervals between the applications of power, the thermal capacity of the solenoid will permit of a greater dissipation of energy in the winding without overheating it.

Space Factors; Round Versus Square Wire; Layer Versus Haphazard Windings. — By the space factor of a winding is meant the ratio of the space occupied by the conductors to the total space occupied by the conductors, the insulation on the conductors and the voids between conductors. The space factor for strips and square wires is greater than for round wires, but strip and square wires are not extensively used in small sizes because of the increased amount of insulation required for a given section of conductor, and because of the tendency of such wires to twist in winding so that they lie upon their corners instead of upon their faces. However, for conductors of larger section than No. 10 B. & S. gauge, square wire is often used.

In winding wires larger than No. 18 B. & S. it always pays to wind them carefully in smooth layers ("layer" wound), but for smaller sizes used for open solenoids (as distinguished from iron-clad solenoids) the gain in space factor does not as a rule warrant this care and the wires are wound in a more or less haphazard fashion ("haphazard" wound). For iron-clad solenoids, however, a layer winding is always used, for economy of material requires that the winding space be kept as small as possible. The dotted curves A and B in Fig. 1 for haphazard windings are taken from an article by F. A. Willard (*Elec. Wld.*, *1906, Vol. 47, p. 823*).

Round wires are sometimes so wound that the wires of one layer lie in the hollows between the wires of the layer underneath; the wires in this case are said to be embedded. Objections to this procedure, however, are that each layer must be started from the same end and the insulation on the wires becomes tightly compressed and therefore is less effective; in most instances the extra labor and the diminution of insulating quality offset the small gain in space factor, which seldom exceeds 3 per cent.

The space factor s for a layer winding of round wire without embedding, and making no allowance for extra insulation between layers is

$$s = \frac{\pi}{4}\left(\frac{d}{d + 2t}\right)^2, \qquad (1)$$

where d is the diameter of the conductor and t the thickness of insulation. Values of s for various sizes of wire and various thickness of insulation are given by the curves in Fig. 1. The "over-all" space factor, including the allowance for the space occupied by the extra insulation, if any, between layers, is equal to the value of s from these curves multiplied by $(1 - e)$, where e is the ratio of the space occupied by this extra insulation to the total winding space.

Thickness of Insulation. — Magnet wire is usually referred to as "single covered," "double covered" and "triple covered," depending upon the number of layers of insulating threads wrapped around it. Different manufacturers

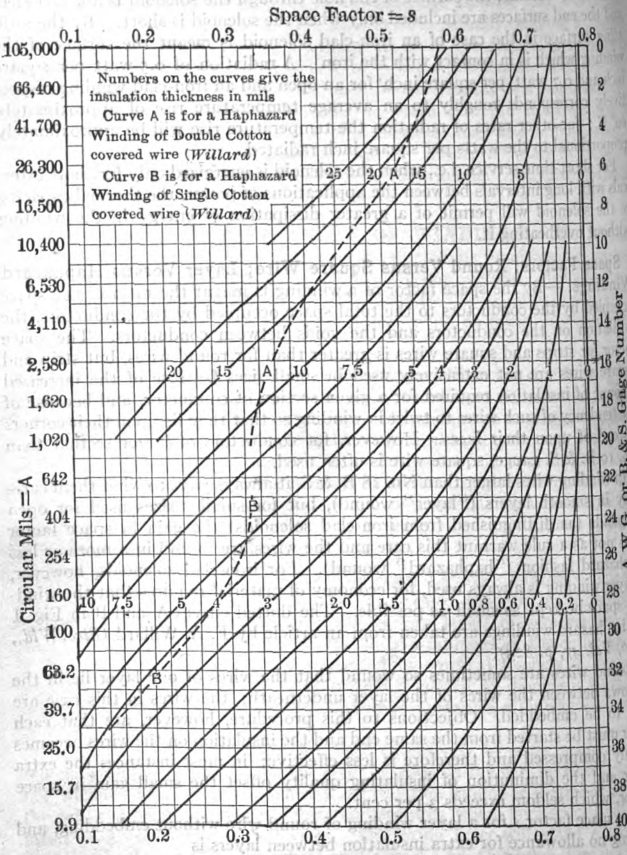

Fig. 1. Space Factor Curves

use different thicknesses for these layers, with the result that the thickness of the insulation on a "double cotton covered" wire of a given size or gauge number depends upon the manufacturer of the wire. In the following table is given the range in insulation thickness, taken from the catalogues of several large manufacturers.

INSULATION THICKNESS IN MILS

Size of wire A.W.G. or B. & S.	Single* cotton covered	Single* silk covered	Asbestos covered (Deltabeston)	Enamel covered
0000 - 00	4.5 - 7.5	....	....	....
0 - 3	4.5 - 7.5	....	18	....
4 - 7	4.5 - 7.5	....	16	....
8 - 10	4.5 - 7.5	....	14	....
11 - 12	2.5 - 5.0	....	12	....
13 - 15	2.5 - 5.0	....	10	1.5
16 - 19	2.5 - 5.0	1 - 2	10	1.1
20	2.0 - 4.0	1 - 2	10	1.1
21 - 23	2.0 - 4.0	1 - 2	....	1.0 - 1.0
24 - 28	2.0 - 4.0	1 - 2	....	0.8 - 1.1
29 - 33	2.0 - 4.0	1 - 2	....	0.7 - 1.0
34 - 36	2.0 - 4.0	1 - 2	....	0.4 - 0.7
37 - 40	2.0 - 4.0	1 - 2	....	0.3 - 0.6

* "For double covered" multiply these thicknesses by 2, and for triple covered multiply by 3.

Winding Calculations for Direct-current Solenoids. — Round solid wires are assumed throughout. Let

s = space factor, from Fig. 1.

k = ratio of specific resistance of conductor used to that of standard annealed copper at 20° C. For copper of 100 per cent conductivity at 20° C., $k = 1$; for copper of any other per cent conductivity, say C per cent, at any other temperature, say t° C.,

$$k = \frac{100}{C} + 0.004\,(t - 20).$$ See also *Wires and Cables, Bare*, and *Wires, Resistor*.

A = cross section of wire in circular mils (= square of diameter in thousandths of an inch).

$n = \frac{1{,}270{,}000\, s}{A}$ = number of conductors per square inch, the square inch being taken perpendicular to the direction in which the wire is wound.

$\rho = \frac{1{,}100{,}000\, sk}{A^2}$ = resistance of the winding per cubic inch of the winding space, excluding the space, if any, occupied by extra insulation between layers; ρ is in ohms.

$w = 0.271\, s + 0.040$ = weight of the winding (copper and cotton insulation) per cubic inch of the winding space, exclusive of the space, if any, occupied by extra insulation between layers; w is in pounds.*

E = impressed volts.

(NI) = ampere-turns, where N is the total number of turns and I the current in amperes,

* This formula also holds approximately for other kinds of insulation and for most alloy resistance wires. Calling g_c the specific gravity of the conductor and g_i the specific gravity of the insulation, the exact formula is

$$w = 0.0362\,(g_c - g_i)\, s + 0.0284\, g_i.$$

Underhill gives g_i as 1.6 for asbestos, 1.4 for cotton, and 1.0 for silk.

p = allowable watts per square inch of radiating surface (may be taken approximately as 0.5 for open and 0.7 for iron-clad solenoids, see above).

S = radiating surface, in square inches, calculated as described above under *Temperature Rise of Winding*.

l = mean length of turn in inches (see Fig. 2 for a simple solenoid).

T = depth of winding space in inches (see Fig. 2), excluding the space occupied by the extra insulation, if any, between layers.

L = length of winding space in inches (see Fig. 2).

$V = LlT$ = volume of winding space in cubic inches, excluding space occupied by the extra insulation, if any, between layers.

The problem is usually to find the size of wire and necessary winding space for a coil which will give, without overheating, a required number of ampere-turns (NI) at a given impressed voltage E. The diameter of the core or spool is also usually known, or at least must not exceed certain fairly well-defined limits, depending upon the service for which it is to be used. The procedure is then to assume a reasonable value for the mean length of turn l. The size of wire is then immediately fixed by the relation

$$A = \frac{kl\,(NI)}{1.16\,E}. \tag{2}$$

From Fig. 1 the corresponding size of wire (A. W. G. or B. & S. gage) and the corresponding space factor s may then be found.

The volume V and radiating surface S of the coil must then satisfy the relation

$$\frac{LTS}{l} = \frac{k\,(NI)^2}{1{,}470{,}000\,ps}. \tag{3}$$

The length L and depth T of the winding space must be so chosen that this relation will be satisfied; as a rule this can be done only by cut and try. Note that changing the depth T of the winding space will also change the mean length of turn l, unless the diameter of the core or spool is so altered as to keep l constant. The cross section of the wire varies directly as l, as shown by equation (2), and therefore the value of the space factor s to be used in (3) will depend upon l, but only to a slight extent except in the case of very small wires.

Having determined the cross section of the wire (A), the mean length of turn (l) and the dimensions (L and T) of the winding space so that both (2) and (3) are satisfied, the total number of turns in the winding will be

$$N = nLT = \frac{1{,}270{,}000\,sLT}{A}, \tag{4}$$

and the current is then equal to the given number of ampere-turns divided by N and the total length of wire is equal to Nl.

The total resistance R and total weight W (including insulation) of the wire may then be found directly from a wire table, or may be calculated from the formulas

$$R = \rho V = \frac{klN}{1.16\,A} = \frac{1{,}100{,}000\,skLTl}{A^2}, \tag{5}$$

$$W = wV = (0.271\,s + 0.040)\,LTl. \tag{6}$$

See note at bottom of preceding page regarding w.

Calculation of Open Solenoid of Circular Cross Section. — In the case of a coil wound on a spool of diameter D, see Fig. 2, the mean length of turn is $l = \pi\ (D + T)$. Assuming that the outside cylindrical surface of the winding is the only radiating surface, which is only approximately true, as pointed out above, $S = \pi\ (D + 2\ T)L$. Whence, putting

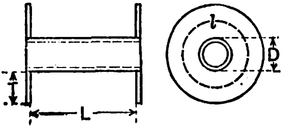

Fig. 2.

$$Q = \frac{k(NI)^2}{1{,}470{,}000\ ps}, \qquad (7)$$

the required length of coil for a given diameter of core D and depth of winding T is

$$L = \sqrt{\frac{(D + T)Q}{(D + 2\ T)T}}. \qquad (8)$$

When L is given instead of T, then the required depth of winding is

$$T = \frac{1}{4}\left[\left(\frac{Q}{L^2} - D\right) + \sqrt{\left(\frac{Q}{L^2} - D\right)^2 + \frac{8\ QD}{L^2}}\right]. \qquad (9)$$

In either case, the number of turns, total resistance, weight, etc., are found as described above for the general case.

Example. — Required to design an open solenoid 10 inches long and having an internal diameter of 1.5 inches, to give 12,000 ampere-turns at 110 volts, the heat developed not to exceed 0.5 watt per square inch of radiating surface (taken as the outside cylindrical surface of the coil). Assume a mean length of turn equal to 10 inches, then from (2) the required cross section of the wire, assuming copper at 70° C., is $A = 1130$ cir. mils. From Fig. 1 the space factor is then $s = 0.62$, assuming single cotton-covered wire with 2-mil insulation wound in layers. From (7) the value of Q is then $Q = 380$, whence from (9) the required depth of winding is $T = 2.36$ inches. The actual mean length of turn is then $l = 12.1$ inches which substituted in (2) gives for the cross section of the wire required, $A = 1368$. This wire section gives a space factor, see Fig. 1, of $s = 0.63$, which agrees practically with the value $s = 0.62$ used above. The nearest commercial size of wire is No. 19 B. & S., having a cross section of 1288 cir. mils. If this size of wire is used the actual ampere-turns will be $NI = 11{,}300$. From (4) the total number of turns will then be $N = 14{,}700$, and from (5) the total resistance will be $R = 143$ ohms. The current is then $I = 0.77$ amperes. The total weight of the winding is then from (6),

$$W = 60 \text{ pounds.}$$

BIBLIOGRAPHY. — See *Bibliography* in article on *Electromagnets, Lifting and Plunger.*

[H. PENDER AND R. G. HUDSON.]

ELECTROMAGNETS, LIFTING AND PLUNGER. — (*See also Electricity and Magnetism, Principles of; Electromagnet Windings.*) A solenoid provided with a movable magnetic core, or with a fixed core and movable "armature," serves as a very convenient means of causing an electric current to produce a direct mechanical pull. This principle is utilized in various forms of lifting magnets, relays (q.v.), contactors (*see Switches*), electric brakes, clutches, etc. In the paragraphs following are given the formulas required in calculating the pull of various kinds of electromagnets in terms of their dimensions and ampere-turns, and also a brief statement of the applications of the various types.

Approximation of Formulas; Leakage Factor. — In applying the formulas given below, it should be noted that in general the effect of magnetic leakage is neglected. The leakage factor varies so greatly with the different forms of electromagnets that it is impossible to go into this matter in detail in the limited space available for this article. The designer, in making an allowance for leakage, has to rely chiefly upon his previous experience with other magnets of the same general form and dimensions. Merely as a rough guide to the designer who has not had this experience, it may be stated that for magnets of reasonable shape and dimensions, the formulas for pull given below may be relied upon to give the actual pull with an error of less than $\pm$ 10 per cent, the actual pull usually being less than the calculated pull. Under certain conditions the agreement between the actual pull and calculated pull may be much closer than the difference just stated.

SIMPLE SOLENOID AND PLUNGER. — The simplest type of electromagnet is a simple solenoid, as shown in Fig. 1, consisting of a cylindrical coil of circular or rectangular section and an iron plunger which fits into the inside of this coil. When current passes through the coil the plunger is attracted.

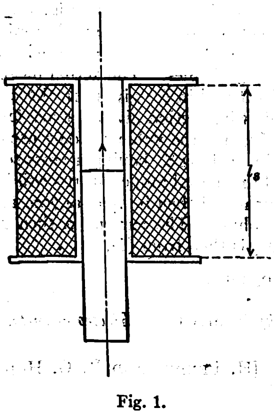

Fig. 1.

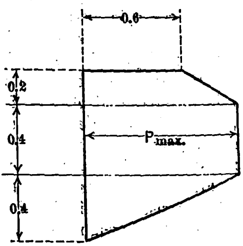

Fig. 2.

Variation of Pull of Simple Solenoid During the Stroke. — The pull of the simple solenoid varies between approximately zero when the plunger is at the lower end of the coil and a maximum which is nearly constant over approximately 40 per cent of the length of the coil. The maximum is reached when the plunger has entered the coil a distance of approximately 40 per cent. When the plunger has entered the coil 80 per cent of its length, the pull de-

creases again, reaching a value of about 0.6 of the maximum pull when the plunger is even with the top of the coil. The approximate pull variation is shown in the diagram, Fig. 2; of course the actual variation is a smooth curve, such as shown by curve A in Fig. 6.

Calculation of Pull of Simple Solenoid. — The calculation of the exact pull of such a magnet is very complicated, being dependent not only upon the ampere-turns, but also upon the shape of the coil, the size and length of the plunger and the induction in same. For practical purposes the maximum pull, when the plunger is at least as long as the coil, can be represented by the formula

$$P_1 = \frac{cANI}{l_s} \qquad \text{pounds,} \qquad (1)$$

where c = the pull in pounds per square inch per ampere-turn per inch of coil length,
A = the area of the cross-section of plunger, in square inches,
I = the current in the coil, in amperes,
N = the number of turns in the coil,
l_s = the length of the coil, in inches.

It has been found by the author that c varies between 9×10^{-3} and 10.5×10^{-3}. For practical purposes sufficiently close results are obtained if c is taken equal to 10^{-2}. The formula shows that the maximum pull is directly proportional to the current, which makes this type of magnet especially suitable for relays and instruments which should be very sensitive.

The pull at any point in the stroke may then be expressed by the formula

$$P_1 = \frac{10^{-2} ANI}{l_s} \cdot k \qquad \text{pounds,} \qquad (2)$$

where k is a factor which gives the ratio of the pull at any point of the stroke to the maximum pull; Fig. 2 gives the approximate value of k at various points of the stroke.

IRON-CLAD ELECTROMAGNET WITH FLAT-END PLUNGER. — The simple solenoid is a very inefficient type of magnet, because a large percentage of the reluctance of the magnetic path is found in the long air path outside the coil. Therefore, the plunger magnet is modified by putting an iron return circuit around the outside of the coil, thus reducing considerably the reluctance of the path and increasing the work which can be obtained with a certain amount of power, and with a certain expenditure of energy in the coil. Fig. 3 shows this type of "iron-clad" magnet. In its highest position the plunger strikes against the frame, so that in reality the magnet represented is a special form of magnet with stop, which is described below. If it is desired to have the pull decrease towards the end of the stroke, a hole similar to the one on the lower end is drilled in the upper end of the frame, and the plunger permitted to protrude through it.

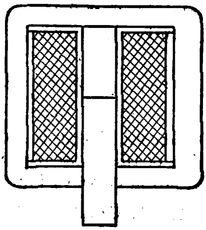

Fig. 3.

Use of Stop to Increase Pull. — The total pull on a plunger depends on the total number of flux lines which pass through it. For a given number of ampere-turns the total number of flux lines may be increased by decreasing the reluctance of the magnetic circuit. This reluctance may be still further reduced by using

a plug or stop in the upper part of the solenoid of such a length that the air-gap is central to the coil for the maximum travel which is required. This form is shown in Fig. 4.

Pull of Stop on Plunger; "Air-gap Pull." — The pull P_2 between the stop and plunger may be expressed by the formula

$$P_2 = \frac{B^2 A}{72 \times 10^6} \qquad \text{pounds,} \qquad (3)$$

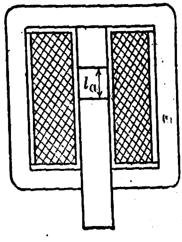

Fig. 4.

where B is the flux density in the air-gap* perpendicular to the end surface of the plunger, in lines (maxwells) per square inch, and A the area of the end of the plunger, in square inches (*see Electricity and Magnetism, Principles of*). The value of B may be calculated from the ampere-turns of the coil and the dimensions of the magnetic circuit in the same manner as the flux due to the field coils of a generator is calculated; see *Generators, Direct-current*. Or, neglecting leakage and the reluctance of the iron part of the path, B may be calculated approximately from the formula

$$B = \frac{3.19\, NI}{l_a} \qquad \text{lines per sq. in.,} \qquad (4)$$

where NI is the total ampere-turns of the coil and l_a is the length of the air-gap in inches. Combining (3) and (4) gives

$$P_2 = 1.4 \times 10^{-7} A \left(\frac{NI}{l_a}\right)^2 \qquad \text{pounds.} \qquad (5)$$

This pull of the stop on the plunger is usually referred to as the "air-gap pull." The actual variation of this air-gap pull with the length of the air-gap, for a particular magnet, is shown in curve B, Fig. 6.

Total Pull of Iron-clad Solenoid with Stop. — The total pull of the iron-clad solenoid with stop may be looked upon as due to two components, the solenoid effect P_1 given by equation (2) and the pull P_2 between stop and plunger given by equation (5); hence the total pull is

$$P = 10^{-2} A NI \left(\frac{k}{l_s} + \frac{1.4 \times 10^{-5}\, NI}{l_a^2}\right) \qquad \text{pounds.} \qquad (6)$$

When the air-gap is short and at the center of the solenoid $k = 1$. The variation of this total pull with length of air-gap for a particular electromagnet is shown in curve C, Fig. 6.

IRON-CLAD SOLENOID WITH CONED PLUNGER. — It will be noted that for a given air-gap pull P_2, the ampere-turns required are directly proportional to the length of the air-gap. If, therefore, for the same stroke the length of the path for the magnetic lines through the air-gap can be reduced, the pull for the same ampere-turns will be increased, or if the pull remains constant the ampere-turns to obtain it can be decreased. This is accomplished by

* Strictly, B in this formula is the actual air-gap flux density less the flux density which would be produced by the solenoid were there no iron whatever in its magnetic circuit. This correction, however is smaller than the probable error in calculating B and may therefore be neglected.

coning the end of the plunger, as shown in Fig. 5. For such a magnet the stroke should not be much in excess of the plunger diameter, as the leakage increases considerably with longer strokes, and this defeats the object of the coning. As the induction in the iron plunger is greater than the induction in the air-gap, there is a limit to the practicable value which may be given to the cone angle; this limit is about reached for cast iron for a cone angle of 28° and for cast steel for a cone angle of 19 degrees.

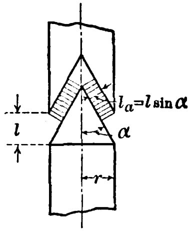

Fig. 5.

"Air-gap Pull" on Coned Plunger. — In the following it is assumed for simplicity's sake that a uniform flux at right angle to and distributed over the entire surface of the cones passes between plunger and plug. This is not strictly correct, especially for long strokes and the pull calculation is therefore only approximately correct.

Let

l = length of stroke, in inches,
A = total cross-section of plunger, in square inches, $= \pi r^2$ where r is the radius of the plunger, see Fig. 5,
α = angle of the cone, in degrees, see Fig. 5,
NI = total ampere-turns of coil.

Then the flux density in the plunger is

$$B_i = \frac{3.19\, NI}{l \sin^2 \alpha} \quad \text{lines per sq. in.} \tag{7}$$

and the air-gap pull in the direction of the stroke is

$$P_2 = 1.4 \times 10^{-7} A \left(\frac{NI}{l \sin \alpha}\right)^2 \quad \text{pounds.} \tag{8}$$

The solenoid pull, as found by experiment, is practically the same as for a flat-end plunger, equation (1). Whence the total pull on the coned plunger is

$$P = 10^{-2} ANI \left(\frac{1}{l_s} + \frac{1.4 \times 10^{-5} NI}{l^2 \sin^2 \alpha}\right) \quad \text{pounds,} \tag{9}$$

where l_s is the length of the solenoid winding assuming the gap at the center of the solenoid ($k = 1$).

Note that $l \sin \alpha = l_a$ is the "effective" length of the air-gap, i.e., the length perpendicular to the surface of the cone. Hence, comparing with equation (6), it is seen that the pull on a coned plunger for the same effective air-gap is the same as on a flat-end plunger, but the length of the stroke is increased in the ratio of $\frac{1}{\sin \alpha}$. Comparing equation (7) with equation (4), it is seen that this advantage is gained by increasing the flux density in the plunger by the square of $\frac{1}{\sin \alpha}$. For small air-gaps, therefore, the coned plunger becomes saturated much more quickly for the same current than does the flat-end plunger, and the leakage and the reluctance of the iron part of the path produces an appreciable effect, causing the pull to become almost constant for small air-gaps instead of increasing as shown by the approximate equation (9). This effect is clearly shown by the curves D and E in Fig. 6.

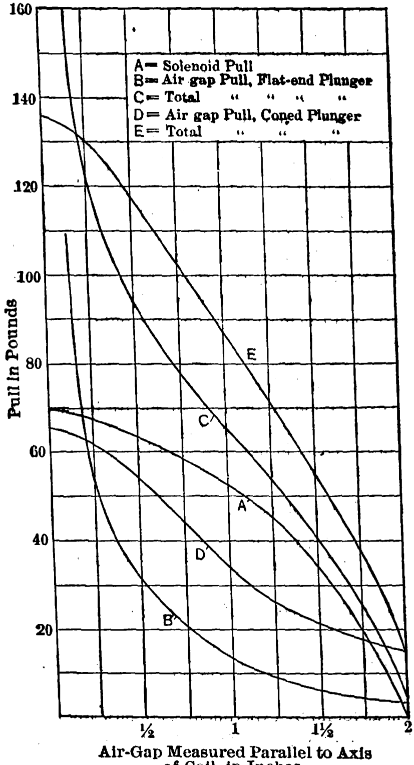

Fig. 6. Pull Curves of D-C. Magnet. —Length of solenoid 3 inches; internal diameter of solenoid 1¾ inches; external diameter 2¾ inches; diameter of plunger 1¼ inches; angle of cone 20°; stop projects 1 inch inside coil; number of turns 300; current 57 amperes.

HORSESHOE-TYPE ELECTROMAGNETS. — Another type of electromagnet which is used quite extensively is the horseshoe-type electromagnet, as shown in Fig. 7. This magnet is not suitable for very long air-gaps, because the leakage increases very rapidly with increasing distance between armature and poles. The total pull on the "armature" K of such a magnet, neglecting the leakage, may be expressed approximately by the relation

$$P = 2.8 \times 10^{-7} A \left(\frac{NI}{2\, l_a + l'} \right)^2 \qquad \text{pounds}, \qquad (10)$$

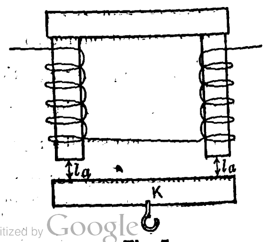

Fig. 7.

where A = cross-section of each pole, in square inches, NI = total ampere-turns,

l_a = length of each air-gap in inches, and l' = length of air-gap equivalent to the reluctance of the iron. This last depends on the permeability and dimensions of the iron part of the circuit; calling l_i the mean length of this iron circuit and μ its permeability, and assuming the mean cross-section of this path to be the same as the cross-section A of each pole, then $l' = l_i/\mu$. A more exact calculation of the pull may be made by calculating, as for a generator field (*see Generators, Direct-current*), the ampere-turns required to establish a given flux density of B lines per square inch in the gap, and then applying equation (3) to determine the pull.

Applications of Horseshoe Type. — This type of electromagnet is frequently used where it is only desired to hold a weight after the armature has come into contact with the poles. A modified form of this magnet whereby the winding is put on the yoke connecting the two poles is used extensively as a no-voltage release on hand starters for direct-current motors (*see Starters, Motor*) and in larger sizes this type of magnet is used for lifting rails, tubes and similar material.

Concentric Lifting Magnets. — A still further modification of this horseshoe magnet has two concentric poles and the winding is arranged concentrically between the two poles. This type of magnet is used as a lifting magnet, either in circular or rectangular shape.

These lifting magnets are especially suitable for lifting pig iron, billets, skull crackers and other magnetic material. The material to be lifted forms in this case the armature of the magnet. The magnetic frame consists of a bell-shaped structure forming the outer pole with a central projection forming the inner pole, the coil being arranged ring-shaped and located concentrically between the inner and outer pole. A circular plate of non-magnetic material mounted between the inner and the outer pole, protects the coil from injury. The voids inside the coil winding are usually filled up with impregnating material, which also serves to make the magnet waterproof.

Magnetic Disc Brakes. — A similar magnet is used for magnetic disc brakes. The magnet is usually mounted on the end plate of the motor and attracts a disc-shaped armature which operates against a spring. When the disc is under the influence of the spring pressure and the magnet deënergized, it presses against a series of stationary and movable discs, the latter being connected with the motor shaft, and causes friction between these parts, which tends to retard and stop the motor. The movement of the armature on these disc brakes is very small, being of the order of ¼ inch.

Magnetic Clutches. — Of similar construction are magnetic clutches. In this case the magnet is mounted on the end of the shaft of the driving machine, whereas the armature, which consists of a circular ring, mounted to a hub by means of a somewhat flexible connection, is connected with the driven member. When the magnet is energized the disc is attracted and takes part in the rotation, thereby driving the second shaft. A pair of collector rings are provided to convey the current from the stationary wires to the rotating magnet. In this case also the movement of the armature is very slight. In order to make the brake release quickly a non-magnetic material is often provided between the magnet and the armature which prevents sticking, due to residual magnetism.

SPEED OF MOVEMENT OF PLUNGER. — In order to obtain quick action, the flux, upon closing the circuit, should reach its full value in as short a time as possible. The flux being a function of the current, the speed depends upon the rapidity with which the current reaches its full value. The time required for the current to reach its full value depends upon the quotient of the inductance L divided by the "effective" resistance R of the circuit. The larger

this ratio $\frac{L}{R}$, which is called the "time constant" of the circuit (*see Transient Electric Phenomena*), the longer the time required for the current to reach its full value. The inductance L is proportional to the square of the number of turns in the solenoid winding and inversely proportional to the total reluctance of the circuit. The effective resistance R depends not only upon the d-c. or ohmic resistance of the winding, but also upon the eddy currents and hysteresis loss set up when the current is changing.

In Fig. 8 is given current-time curve showing the change of the current during the switching-in period of a direct-current magnet. It will be seen that, after the closure of the circuit, the current first rises to a certain value which corresponds to a flux just sufficient to cause a movement of the armature and lift the plunger. As the plunger moves the flux increases, thereby causing a counter electromotive force, which tends to reduce the current. This counter e.m.f. depends upon the speed of the plunger. In the case shown the current drops off continuously until the plunger strikes against the stop, at which moment it has a value of approximately one-third of the value which started the motion of the plunger. After the plunger has come to rest, the current again increases and gradually reaches the value which is dependent upon the terminal voltage and the resistance of the coil.

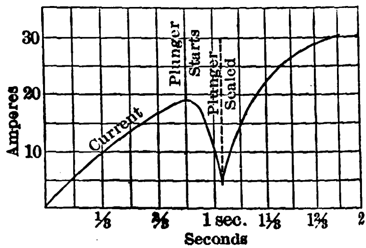

Fig. 8. Current-time Characteristics of D-C. Electromagnet

Methods of Obtaining Quick Action. — To reduce the eddy-current effect, the cross-section of the magnet frame should be as small as consistent with other considerations. Where very quick action is required it is sometimes advisable to slot this frame at right angles to the direction of the eddy currents or laminate it similar to transformers. In such cases it is also advisable to eliminate the brass tube on which the coil is very frequently wound and which acts as a guide for the plunger, or to slot this brass tube parallel to its axis. It is also advisable to slot or laminate the plunger.

Another method of obtaining quick action is to impress at the start a high voltage on the coil and insert resistance into the circuit of the coil as the plunger rises, in order to protect the magnet from overheating. This reduces the ampere-turns at the end of the stroke, which is permissible in most cases, because usually the pull of the magnet increases very rapidly toward the end of the stroke, as is indicated by the formulas given above.

ALTERNATING-CURRENT ELECTROMAGNETS.—Electromagnets for producing a mechanical pull may also be designed to operate on alternating current. The flux in an alternating-current magnet passes through zero twice per cycle. The pull, which varies with the square of the current, therefore becomes zero twice every cycle, and it can be shown that it also varies according to a sine curve when the current is sinusoidal. The average effective pull is one-half of the maximum pull. Whenever the pull is less than the load, there is a tendency for the plunger to move away from the stop, and this causes rattling or humming of the magnet. This humming may be overcome in different ways, one method being to use a "shading coil," described below.

Polyphase Electromagnets. — It can be shown that if three magnets are used and each is supplied with current from one phase of a three-phase source, or if two magnets are used and each is supplied with current from one phase of a two-phase source, then the resultant pull will be constant at any moment, and if the plungers are rigidly connected there will then be no chattering. The most common form of three-phase magnet is shown in Fig. 9. This consists of a core having three poles and a plunger of similar construction. Over each pole there is wound a coil which is supplied from one of the three phases of the circuit. There are various modifications of the polyphase magnet, but their general principle is the same. In calculating the total pull, the pull of each pole is figured separately and the several pulls, which are equal to one another, are combined vectorially, since they differ in time phase (*see Alternating Currents*).

Calculation of Pull of Single-phase Electromagnets, or of One Phase of Polyphase Electromagnets.—The formulas for pull given above for direct-current electromagnets also hold for alternating-current magnets, provided I is taken as the effective value of the current, B as the effective value of the flux density and P as the average or effective value of the pull.

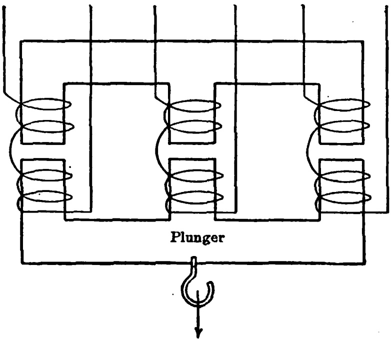

Fig. 9.

In contrast to the d-c. electromagnet, the flux in an a-c. electromagnet for a given impressed e.m.f. is approximately constant irrespective of the length of the air-gap. This is due to the fact that the opposition to the flow of current through the winding is due almost entirely to the back e.m.f., due to the alternation of the flux, and only to a very small extent to the resistance of the winding, the action in this respect being similar to that of a transformer (q.v.) or induction motor (*see Motors*). The back e.m.f. being practically equal to the impressed e.m.f., the flux producing this back e.m.f. is also proportional to the impressed e.m.f., and is therefore practically constant when the impressed e.m.f. is constant.

Since the flux remains practically constant, the current I, as may be seen from equation (4), must vary approximately proportionally to the length of the air-gap l_a; this proportionality holds only approximately, since equation (4) neglects the magnetic leakage and the reluctance of the iron part of the magnetic circuit. The actual variation of the current with the length of the air-gap for a particular a-c. electromagnet is shown in Fig. 10.

Using the same notation as used above for d-c. electromagnets and in addition putting E = effective value of impressed voltage per phase, and f = frequency of impressed voltage in cycles per second, the current taken by each phase of the magnet, neglecting the iron losses, leakage and reluctance of the iron part of the magnetic circuit, will be

$$I = \frac{10^7 E l_a}{2 f N^2 A} \quad \text{amperes.} \tag{11}$$

A more accurate calculation of the current, taking into account the losses and

the magnetic leakage, may be effected by the method used for calculating the current in a transformer (*see Transformers*).

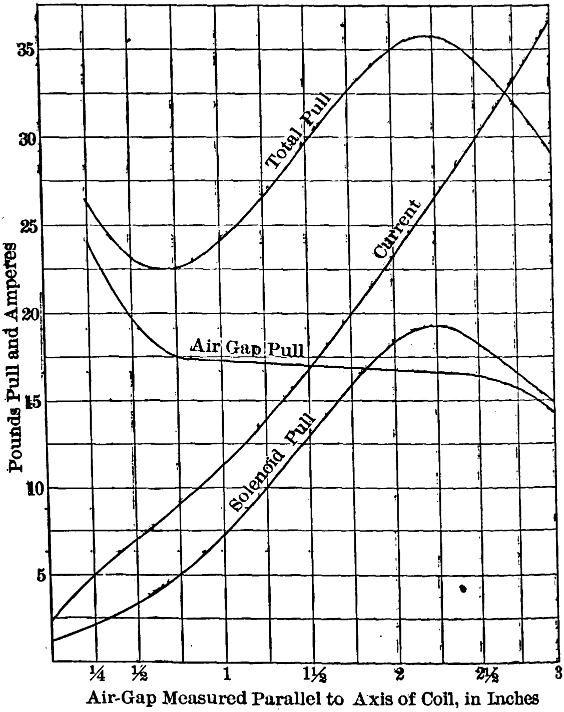

Fig. 10. Pull and Current Curves of A-C. Magnet.—Length of solenoid 5 inches; internal diameter of solenoid 3 inches; external diameter 4½ inches; diameter of plunger 2⅜ inches; stop projects 1½ inches inside of coil; turns 144; voltage 220; frequency 60 cycles per second.

Substitution of the value of the current given by (11) in equation (6) for the total pull on a plunger with a flat end gives the approximate formula

$$P = \frac{10^6 E}{2fN}\left(\frac{kl_a}{l_s} + \frac{72\,E}{fNA}\right) \qquad \text{pounds.} \tag{12}$$

This equation also applies to a coned plunger, when l_a is taken equal to the length of the air-gap perpendicular to the face of the cone.

Fig. 10 shows the pull curve of an alternating-current magnet with flat-end stop and plunger. It will be noted that the current is roughly proportional to the air-gap. The solenoid pull reaches a maximum at an air-gap of about 2¼ inches and then falls again approximately proportional to the air-gap; the air-gap pull is constant over a wide range of the travel. The result is a total pull curve which has a maximum at approximately 2¼ inches and which drops until an air-gap of approximately ¾ inch is reached, and from there on it increases

again, due to the increase of the air-gap pull, the latter being caused by the diminution of the leakage flux as the air-gap decreases.

Limiting Flux Density and Losses in A-C. Electromagnets. — Attention must be paid to the fact that the iron losses, similar to those of the transformer increase with the flux density and therefore the design must be such that the flux density is not too high. The flux density in the iron in the case of a flat-end plunger, from equation (4), which neglects the magnetic leakage and reluctance of the iron part of the magnetic path, is

$$B = \frac{1.6 \times 10^7 E}{fNA} \quad \text{maxwells per sq. in.} \qquad (13)$$

and in the case of a coned plunger, under the same assumptions, from equation (7), is

$$B = \frac{1.6 \times 10^7 E}{fNA \sin \alpha} \quad \text{maxwells per sq. in.} \qquad (14)$$

where α is the cone angle, see Fig. 5.

These relations are approximate only. The magnetizing component of the exciting current, and from it the flux density can be more accurately calculated by the methods employed in the design of transformers or induction motors. See *Transformers* and *Motors, Polyphase Induction.*

The iron and copper losses can also be calculated by similar methods, and from these losses the energy component of the exciting current can be determined. The total current and power factor of the electromagnet can then be deduced.

Shading Coil for Single-phase Electromagnets.—The humming of single-phase magnets may be greatly reduced by introducing a so-called "shading coil" in the pole face. This shading coil is nothing more than a short-circuited secondary winding, consisting of one or more turns, which encloses only part of the total flux passing through the plunger. Due to the leakage reactance of this turn, the current induced in it is out of phase with the inducing flux, so that at the moment when the inducing flux, due to the main winding, is zero, there still remains a flux due to the current in the shading coil, which flux produces a pull. The result is that the combined pull from the main flux and the shading coil flux never becomes zero. Fig. 11 shows the arrangement of the shading coil. This coil is mounted in the plunger or in the plug close to the pole face, in order to reduce the length of the path for the magnetic lines which are interlinked with the shading coil. Naturally the shading coil has no effect with long air-gaps, and it is therefore imperative that a good magnetic contact be obtained when the plunger is in the sealed position to get the greatest possible effect of the shading coil.

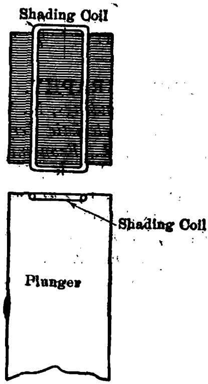

Fig. 11. Shading Coil.

Incidentally the shading coil also increases considerably the maximum pull for a given impressed e.m.f., as the minimum pull is also due to the combined main and local flux. As an example, a plunger magnet without shading coil, which gave a minimum pull of zero and a maximum pull of 28 pounds, had, after the introduction of the shading coil, a minimum pull of 18 pounds and a maximum pull of 143 pounds.

COSTS, WEIGHTS AND DIMENSIONS OF ELECTROMAGNETS. — Due to the great variety in the designs, it is not possible to give unit costs

of electromagnets. The subject of the most economical magnet design has been discussed by Wikander (*Trans. A.I.E.E., 1911, Vol. 30, p. 2019*), but unfortunately the most economical design will usually be found not to be suitable for practical purposes, because it results in a magnet which is too long compared to its diameter, and which usually cannot be incorporated in the machine with which it is to be used. Therefore, magnets as they are found in practical application deviate greatly from the most economical design. Also for the same energy output (usually expressed as inch-pounds or foot-pounds) there is as much as a 1 to 3 variation, depending upon the service conditions as to speed of operation, stroke, etc., which they have to meet. The following table of costs, weights and dimensions of some typical electromagnets is given merely as a rough guide.

Use for which designed	Over-all length, inches	Over-all diameter, inches	Total weight of active iron and copper, pounds	Length of stroke, inches	Pull at beginning of stroke, pounds	Voltage	Watts input at end of stroke	Power factor at end of stroke	Factory cost
1. Relay, D.C.	3¼	2⅛	2.9	⅛	0.15	110–500	9	....	$5.10
2. Relay, A.C.	3¼	2⅛	2.9	⅛	0.10	110–550	15	0.39	6.95
3. Lifting, D.C.	5	3	8	1	8	110–550	50	....	6.40
4. Lifting, D.C.	11	7	80	3	20	110–500	210	....	33.00
5. Lifting, A.C.	5½	4	10	⅞	7	110–550	30	0.30	8.15
6. Lifting, A.C.	14	6¼	90	2½	50	110–550	400	0.31	32.00
7. Brake, D.C.	5¼	8½	72	⅛	160	110–220	50	....	21.00
8. Brake, D.C.	6	15	320	5/32	1260	110–220	210	....	39.00
9. Clutch, D.C.	6½	10	82	⅛	60	110–220	52	....	19.00
10. Clutch, D.C.	7	18	210	⅛	400	110–220	115	....	35.00

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[Arthur Simon.]

ELECTROMETERS. — (*See also Electrodynamometers; Voltmeters: Wattmeters.*) An electrometer is primarily an instrument for measuring potential differences, but under certain conditions may also be used as a wattmeter; in the latter case it is usually called an electrostatic wattmeter. The deflection of the instrument is due to the attraction or repulsion of electrostatic charges. It may be used for either direct- or alternating-current measurements.

There are a great variety of forms of electrometers. On account of the care required in its use, the ordinary quadrant electrometer is seldom employed except for laboratory purposes, but various modifications of the electrometer provided with pointer and scale so as to be direct reading are used commercially for high voltage measurements. Such instruments are known as electrostatic voltmeters; for description see article on *Voltmeters*.

Electrometer versus Galvanometer or Electrodynamometer. — The advantage of the electrometer over the galvanometer or electrodynamometer is that it takes no current when used for constant (d-c.) voltage measurements. Due to its electrostatic capacity, however, it does take a certain amount of charge which should always be allowed for if the capacity of the electrometer is appreciable compared with any other capacity which affects its reading. Also, when used for a-c. measurements, the charging current taken by the electrometer should be allowed for, if this charging current is appreciable. This charging current is usually considerably less than would be taken by an electrodynamometer of the same degree of sensitiveness.

KELVIN QUADRANT ELECTROMETER (Fig. 1). — Two brass quadrants a and a' are connected together and two quadrants b and b' are connected together and these respective pairs are well insulated from one another. The "needle" n (a light aluminum vane) is suspended by a silk, silvered quartz or other fiber and insulated from the quadrants.

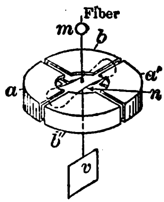

Fig. 1. Quadrant Electrometer

A light aluminum vane v is suspended from the needle by a fine conducting wire and dips into a conducting solution, usually 60 per cent sulphuric acid, which serves as a means of connecting the needle to any external source of p.d. or to the ground. If a conducting suspension is used this may be employed to connect the needle to the external source of potential. The sulphuric acid also keeps the air dry in the case containing the quadrants, and the motion of the vane in the acid damps the vibrations of the needle. The suspension carries a mirror m by means of which the deflection of the needle may be read.

Formulas for Quadrant Electrometer. — In order to obtain a straight line calibration curve the needle must be properly shaped and must be suspended midway (vertically) between the two quadrants and must have a symmetrical zero position with respect to the two quadrants. Under these conditions when the two pairs of quadrants are charged to potentials V_a and V_b and the needle to the potential V_n above any fixed potential, say that of the ground, the angular deflection is

$$D = \frac{1}{K}(V_a - V_b)\left(V_n - \frac{V_a + V_b}{2}\right),$$

where K is a factor, which is practically a constant. The value and constancy of the factor K should be determined by calibration.

There are three ways of using the instrument, as indicated in the diagram;

Fig. 2. E is a known voltage, V the p.d. to be measured, a and b the two pairs of quadrants and n the needle.

Case I. $V = E \pm \sqrt{E^2 - 2KD}$. (*See footnote.*)

Case II. $V = \dfrac{KD}{E}$

Case III. $V = \sqrt{2KD}$.

Cases I and II are applicable to measurements of constant (d-c.) potentials only. Case III may be used for the measurement of either alternating or direct potentials. In the case of alternating potentials V is the *effective* value.

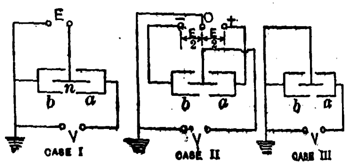

Fig. 2.

Range of Quadrant Electrometers.—The range of a quadrant electrometer when used as in Case I or II depends upon the maximum voltage E which can be impressed between the two pairs of quadrants and between the quadrants and the needle, and also upon the fineness and material of the suspension fiber. Case III is not applicable to the measurement of very low voltages. Electrometers can be purchased suitable for measuring direct voltages as low as about 2 volts, and as high as 50,000 volts, and for alternating voltages as low as about 2 volts and as high as 25,000 volts.

Extension of Range by Use of Auxiliary Condensers (Fig. 3).—By connecting the p.d. to be measured across two condensers in series and measuring the voltage across only one of them, the instrument may be used for the measurement of high voltages of practically any magnitude. The connections are as shown in Fig. 3. If the condensers have no leakage, then

$$V = V_1 \frac{C_1 + C_2 + c}{C_2},$$

where V_1 is the voltage read by the electrometer, C_1 and C_2 the capacities of the two condensers and c the capacity of the condenser formed by the electrom-

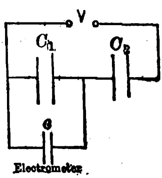

Fig. 3. Electrometer Range Extended by Condenser

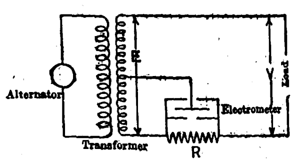

Fig. 4. Electrostatic Wattmeter

eter quadrants and needle. If C_1 and C_2 are large compared with c, then c may be neglected, but the larger C_1 and C_2 the greater will be the charge (and therefore the charging current, in case of an a-c. measurement) taken by the measuring circuit.

Quadrant Electrometer as Electrostatic Wattmeter (Fig. 4).—The quadrant electrometer may be used to measure with a fair degree of precision

* Use the − sign if V is less than E, the + sign if V is greater than E, and call the deflection positive in every case.

very small amounts of power, of the order of 1 watt, when the voltage giving this power is 5000 volts or more. It therefore serves as a very convenient means of measuring the power loss in small samples of insulating materials at high voltages. See paper by E. H. Rayner, *Jour. Inst. Elec. Eng.*, 1912, Vol. 49, p. 3.

The connections used by Rayner are shown in Fig. 4. R is a non-inductive resistance. The needle of the electrometer is connected to the middle point of the high-tension winding of the transformer. *On the assumption that the charging current of the electrometer and the difference in phase between* V *and* E *may be neglected*, the power supplied to the load is

$$P = \frac{2K}{R}D,$$

where K is the instrument constant and D the deflection. The range of the instrument as thus used depends upon the value of R, the higher R the smaller the amount of power which may be read.

However, the higher the value of R the greater the phase difference between E and V if the load current is out of phase with V, and therefore for small power measurements when the load has a low power factor *e.g.*, when it is a condenser, an allowance must be made for this difference in phase angle. Also, when the charging current of the electrometer is comparable in magnitude with the load current a correction must also be applied on this account. These corrections are discussed in detail in Rayner's paper.

COST. — An ordinary Kelvin quadrant electrometer suitable for measuring potentials of from 400 to 1300 volts costs approximately $150.

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[H. Pender and H. R. Ranken.]

ELECTRON THEORY. — (*See also Electricity and Magnetism, Principles of; Electrochemistry, Principles of.*) Maxwell's assumption of a displacement current producing magnetic effects similar to those due to an ordinary conduction current is fully justified by the existence of electromagnetic waves (q.v.) whose properties are now well known. In free space these waves travel with the velocity of light; this fact leads to the conception of light waves as being the same as electromagnetic waves, only of shorter wave lengths. However, when this theory is applied to waves in material media, the optical constants as calculated from the theory are found not to agree with the values determined experimentally. The electron theory, namely, the assumption that *in every body, whether charged or uncharged, there exists electricity of both signs distributed not continuously but on discrete particles*, was first introduced* to account for these discrepancies.

Not only does this theory account for these discrepancies in a satisfactory manner, but it also leads to a simple explanation of such phenomena as metallic conduction, the discharge of electricity through gases, the corona effect, etc.

NATURE OF THE ELECTRONS. — The smallest negatively charged particle that it has been found possible to detect is called an electron; the smallest positively charged particle was also originally called an electron, but this name is now usually reserved for the smallest negatively charged particle.

Properties of the (Negative) Electron. — The results of all experiments justify the following assumptions regarding the electron.

1. The charge carried by a single electron is the smallest possible charge which can exist in nature and no charge of electricity can be produced which is not an integral multiple of this charge. The value of the charge carried by a single electron is approximately

$$e = 4.9 \times 10^{-10} \qquad \text{c.g.s. electrostatic units}$$

($= 1.6 \times 10^{-20}$ c.g.s. electromagnetic units). This charge is also equal to that carried by the hydrogen atom in electrolysis. The charge on an electron may be looked upon as the "atom" of electricity. (There is also considerable evidence indicating that the electron *is* electricity pure and simple, having no mass in the ordinary sense; *see below.*)

2. The "effective" mass of an electron may be defined as the force required to give it unit acceleration.† All experiments indicate that the "effective" mass of an electron, as thus defined, is approximately

$$m = 8.9 \times 10^{-28} \qquad \text{grams,}$$

provided the electron is not moving with a velocity greater than one-tenth the velocity of light; for velocities greater than one-tenth that of light the "effective" mass of the electron increases with increase in velocity (*see below*). The mass 8.9×10^{-28} is about one-seventeen hundredth ($1/1700$) that of a hydrogen atom.

Unless otherwise stated, wherever the expression "mass of an electron" is used in this article, it is to be understood that this *effective* mass is meant.

3. An electron having a charge e and moving with velocity v produces the same magnetic field as an elementary length ds of a conduction current of strength i, where $ev = ids$. (*See Electricity and Magnetism, Principles of.*)

4. Every neutral atom of matter contains at least one electron which is held in position by forces analogous to elastic forces, that is, an electron may oscillate

* H. A. Lorentz: Proceedings of the Amsterdam Academy, 1878.

† That is, $f = ma$, where f is the force, a the acceleration, and $m = \frac{f}{a}$ is the "effective" mass.

within the atom, or may be displaced by an impressed electrostatic field. The electrons are always of the same nature irrespective of the substance in which they exist.

5. An electron may be forced from the atom by the influence (mutual repulsion) of a "free" electron moving at a high velocity in its immediate vicinity. This action is usually spoken of as a bombardment, or collision, although it is not necessary to assume that the free electron actually hits the atom.

6. When an electron is expelled from an atom the atom manifests the properties of a positively charged body.

7. In every substance there exists in addition to the electrons within the atoms a certain number of "free" electrons, i.e., electrons which can move freely in the inter-atomic spaces; these free electrons may also pass from one substance to another. Under certain conditions, e.g., in a gas at ordinary pressures, a free electron may attach to itself one or more atoms or molecules.

Properties of the Positive Particle. — Experiment also justifies the following assumptions regarding the positively charged particles:

1. The smallest positive charge that it has been found possible to produce is numerically equal to that of an electron, viz., 4.9×10^{-10} c.g.s. electrostatic units.

2. The smallest possible positive charge is always associated with a particle having a mass of the same order of magnitude as that of an atom, this mass being never less than that of a hydrogen atom.

3. The positively charged particle may be looked upon as an atom from which one or more electrons have been expelled; its mass therefore depends upon the nature of the atom. A "free" positively charged particle may also attach to itself one or more neutral atoms or molecules.

Relation between Electrons and Ions. — The term "ion" is usually reserved to designate any charged body having a mass of the order of magnitude of that of a molecule or atom. In this sense an electron is not an ion, but the positively charged particle is an ion. If, however, the electron becomes attached to an atom or molecule, then this combination forms an ion. In the case of the discharge of electricity through gases at low pressure, the electron, although it is not attached to an atom or molecule, is sometimes called an ion. See also *Electrochemistry, Principles of.*

APPLICATIONS OF THE ELECTRON THEORY. — The hypothesis of electrons, as sketched above, leads to a simple explanation of many of the electrical and optical properties of matter which are otherwise inexplicable. In the following paragraphs will be taken up briefly some of the applications which best illustrate the utility of the electron theory.

Variation of Optical Properties with Wave Length. — The change of refractive index with wave length and the occurrence of selective absorption, emission and reflection can be explained on the assumption that in general there exist in a given molecule several kinds of electrical systems each capable of vibrating with a definite period. The simplest sort of such electrical system is an electron which is subject to forces of restraint varying directly as the displacement of the electron from its position of equilibrium and also to forces of an energy-dissipating character. If, in any one substance, the assumption is made that there exist electrons of as many different periods of vibration as there are wave lengths for which the substance absorbs selectively, it is possible to account for the variation with wave length of the refractive index in a very satisfactory manner and for selective absorption, emission and reflection.

Zeeman Effect. — If a body capable of emitting radiation selectively, for example a sodium flame, is placed in a magnetic field and the light given out is examined by means of a spectroscope, the spectrum lines are found to be

broken up into three or more components if viewed perpendicularly to the lines of magnetic force, and into two or more components if viewed along the lines of force. In the former case, when there are but three component lines, the lines are plane polarized, the plane of polarization of the middle component (which has the same wave length as the original line) being at right angles to the planes of polarization of the other two components; in the latter case, when there are but two component lines, the two components are circularly polarized in opposite directions. These phenomena are known as the Zeeman effect.

If it is assumed that the electrons in the molecules of a substance normally vibrate harmonically, the simple type of Zeeman effect (i.e., a line broken up into only three or two components) can be readily accounted for. Moreover, the observed distance between the component lines when a given spectrum line is split up under the action of a magnetic field is found to agree with the distance as calculated on the assumptions of the electron theory. Or, stated otherwise, quantitative measurements of the Zeeman effect give a means of determining the ratio $\frac{e}{m}$, that is, of the charge carried by an electron to its mass, and the ratio as thus determined is found to agree with the determinations of this ratio by other methods.

By making proper assumptions as to the influence upon any electron of its neighbors it is possible to explain satisfactorily many of the complicated types of Zeeman effect.

Faraday Effect. — When a ray of plane-polarized light is passed through a transparent substance in a magnetic field, the field being parallel to the ray, the plane of polarization is found to be rotated. This phenomenon is known as the Faraday effect. The explanation of this effect in terms of vibrating electrons offers no difficulties. See Campbell's *Modern Electrical Theory*.

Electrical and Thermal Conductivity. — One of the fundamental assumptions of the electron theory is that in conductors there are many electrons which are free to move about among the molecules besides those electrons having fixed positions of equilibrium. It is likely that these electrons are not always the same ones; the molecules are probably continually gaining and losing electrons. It is reasonable to assume that the more free electrons (on the average) in a body the better electrical conductor it is. According to the kinetic theory of matter, the molecules of all substances not at the absolute zero of temperature are in a state of ceaseless agitation; if the assumption is made that the *free* electrons share in the motion of the molecules, then the more free electrons there are in a metal the more rapidly will kinetic energy be communicated from any molecule to neighboring ones, that is, the better conductor of heat the metal will be.

In order to deal quantitatively with the matter some assumption as to numerical values must be made; the one that has been made is that the mean kinetic energy of the electrons is equal to the mean kinetic energy of the molecules. Using the above assumption and also making one as to the relation between the number of electrons per unit volume and the temperature, it has been found possible to explain: 1. Joule's law for the production of heat in a conductor carrying an electric current; 2. the Wiedemann-Franz law, which states that for pure metals the ratio of the thermal to the electrical conductivity is independent of the nature of the metal and is proportional to the absolute temperature; 3. the Peltier and Thomson effects. The theory also throws much light on the nature of the Hall effect.

Electrolysis. — Faraday's laws of electrolysis (*see Electrochemistry, Principles of*) are readily explicable in terms of the electron theory. It is only necessary to assume that each positive univalent ion in the solution is an atom from which

one electron has been expelled, each positive bivalent ion is an atom from which two electrons have been expelled, etc., and that each negative univalent ion is a neutral atom to which one electron has attached itself, each negative bivalent ion is a neutral atom to which two electrons have attached themselves, etc.

Magnetic Properties. — The magnetic properties of bodies may be explained in terms of the electron theory by assuming that the electrons in the molecules of magnetic substances revolve in orbits within the molecule, thus making each molecule produce a magnetic field similar to that produced by a single turn of wire carrying an electric current.

Cathode Rays. — When an electric discharge is passing in a vacuum tube, at a certain value of the pressure of the gas contained in the tube it is noticed that a greenish phosphorescence occurs on the walls of the tube. By placing solid bodies in the tube it is found that the phosphorescence is due to something proceeding in straight lines perpendicularly from the surface of the cathode. The name "cathode rays" has been given to this agent which produces the phosphorescence. These rays are deflected by both electric and magnetic fields and communicate a negative electric charge to an insulated conductor. The assumption that these "rays" consist of a rapidly moving stream of electrons leads to a simple explanation of their properties, both qualitatively and quantitatively. The amount of the deflection of the rays, under the action of magnetic and electrostatic fields, can be calculated on the assumptions of the electron theory, and a comparison of the calculated with the observed deflections shows the two to be in close agreement. Or, stated otherwise, the ratio $\frac{e}{m}$ as calculated from the observed deflections is found to agree with the value of this ratio determined from other data.

Kanalstrahlen. — If the cathode of a tube producing cathode rays is perforated, faint luminous streaks are seen to proceed through these perforations in a direction opposite to that of the cathode rays. The name "kanalstrahlen" has been given to this phenomenon (the German name is very generally retained in English). These rays are also deflected by both electric and magnetic fields but to a lesser extent and in relatively the opposite direction from the deflection of cathode rays, and communicate a *positive* charge to an insulated conductor. In terms of the electron theory these rays are positively charged *ions*. Calculations from actual measurements, making this assumption, show that the charge carried by each of these ions is numerically equal to that of an electron, and that the mass of each ion is never less than the mass of a hydrogen atom, but may be greater, depending upon the nature of the gas and the pressure in the tube. That is, the cathode particle is an electron traveling in one direction and the kanalstrahlen particle is what is left of the atom which has lost an electron, traveling in the opposite direction.

Ionization of a Gas. — Gases at or near atmospheric pressure are normally very good insulators, i.e., have an extremely high insulation resistance. However, a gas may be rendered a fairly good conductor in a number of ways, viz., by passing X-rays through it, letting ultra-violet light fall upon a metal plate immersed in it, subjecting it to the influence of the rays given off by radium, passing it close to a flame or a hot piece of metal or by establishing in it a brush discharge or corona (q.v.). A gas rendered conducting in any of these ways is said to be "ionized."

In terms of the electron theory the conductivity thus given to a gas is attributed to the breaking up of some of the molecules of the gas into positive and negative ions. To account for the observed facts it is necessary to assume that, at pressures of 20 cm. or over, by barometer, the electrons and positive ions thus formed immediately attract molecules to themselves just as an electrified rod

attracts dust particles. Since the force of attraction is independent of the sign of the charge both the positively and negatively charged particles thus produced would act in very much the same manner. In gases at low pressures, however, the electron, or the positive ion, would rarely come within attracting distance of a molecule and even if it did collect molecules about itself a *collision* with a molecule would probably reduce it to its simplest form at once. A collision in gases at high pressure probably has the same effect but the ion very soon reforms owing to the greater number of molecules in its vicinity.

Ionization by Collision—Theory of the Corona.—The electron theory assumes that at least a few free ions (positively charged particles or electrons with one or more molecules attached) exist in every substance. The fewer these free ions, the better the insulating properties of the substance. Under the action of an electric field these ions are given a velocity which depends upon their mass and upon the distance which they can travel before colliding with other molecules or ions. In the case of a gas the "mean free path" is relatively large, being greater the less the density.

The ions present in a gas may therefore acquire, under the action of an electric field, a very considerable velocity. When one of these rapidly moving ions comes very close to a molecule of the gas it exercises a large force of repulsion on the electrons in the molecule and may thereby expel one or more electrons from the atom. The electric field therefore causes a large increase in the number of ions present in the gas and consequently the conductivity of the gas is largely increased.

This is the probable explanation of the formation of the corona (q.v.), granting the existence of a few free ions in the gas to start with. The existence of these ions is rendered extremely probable by the fact that all gases in the natural state have an appreciable conductivity.

Formation of Clouds in Dust-free Ionized Gas.—If a dust-free gas rendered conducting by any of the methods described above is saturated with water vapor and then subjected to an adiabatic expansion, it is found that a cloud of small drops will form and that the drops will be charged. By observing the rate of fall of these drops under the action of gravity alone and also under the action of gravity and a known electric field, the charge on each drop can readily be calculated. (*See J. J. Thomson, Conduction of Electricity through Gases.*) This charge is found to have approximately the value 4.9×10^{-10} electrostatic units or some *exact multiple* of this charge. Small drops carrying electric charges are also very commonly produced in the process of spraying a liquid. The charges carried by such drops have been investigated by numerous observers and have always been found to be exact multiples of the charge 4.9×10^{-10} electrostatic units. These experiments are the justification for assuming that the charge of an electron is 4.9×10^{-10} c.g.s. electrostatic units.

MASS OF THE ELECTRON.—Since to account for the various observed facts noted above it is necessary to assume that a moving electron produces a magnetic field (which assumption is in accord with the experimental fact that a moving charged body of finite dimensions produces a magnetic field), it follows that the total energy of motion of an electron is

$$W = \tfrac{1}{2}\, m_0 v^2 + W_e,$$

where the term $\tfrac{1}{2}\, m_0 v^2$ represents the energy which would be required to give the electron a velocity v *if it had no electric charge whatever*, and W_e is the energy stored in the magnetic field established by the motion of the charge carried by the electron. This expression may also be written

$$\tfrac{1}{2}\,(m_0 + m_e)\, v^2,$$

where $m_e = 2\,W_e/v^2$. The quantity m_0 is the ordinary or mechanical mass of the electron; the quantity m_e is called the "electromagnetic mass" of the electron. The latter, however, is more analogous to the coefficient of self-induction than it is to mechanical mass, since it is part of the expression for the energy of the *magnetic field* established by the moving electron.

The total "effective" mass of the electron is then

$$m = m_0 + m_e.$$

Since it is the total "effective" mass that enters into all calculations of $\frac{e}{m}$ from experimental data, it is impossible by direct experiment to determine m_0 and m_e separately. However, m_e can be calculated in terms of the charge e and the velocity, provided certain assumptions are made regarding the distribution of charge in the electron. Several different formulas have been proposed, based on different assumptions, regarding the distribution of the charge. All these contain a *constant* term independent of the velocity, and *this constant term, within the limits of experimental error, is found to be equal to the total mass m calculated from experimental determinations of the ratio* $\frac{e}{m}$ *and the charge e*, provided the velocity of the electron is less than one-tenth that of light (see next paragraph). Hence the mechanical mass m_0 of the electron is negligible, within the limits of experimental error, compared with the electromagnetic mass m_e.

The above considerations are the basis of the assumption, now generally made by physicists, that the mass of the electron is *entirely* electromagnetic i.e., that the electron is electricity pure and simple. In fact, some physicists go farther and try to explain all mass in terms of electricity. A satisfactory theory of this kind, however, cannot be developed until the properties of the positively charged particle are better understood. See J. J. Thomson, *The Corpuscular Theory of Matter*.

Variation of the Mass of the Electron with Velocity. — When the velocity of the electron approaches that of light both theory and experiment indicate that its electromagnetic mass increases. The formula which best satisfies the experimental facts is that deduced by Lorentz, viz.,

$$m_e = \frac{8.9 \times 10^{-28}}{\sqrt{1 - \left(\frac{v}{V}\right)^2}}$$

where v is the velocity of the electron and V the velocity of light. For $\frac{v}{V}$ less than 0.1 this formula gives a value for m_e constant to within an error of 1 per cent. Since the mechanical mass of the electron (if there is such) is negligible compared with m_e, the above formula also represents, within the limits of experimental error, the variation of the total effective mass with velocity.

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[W. S. GORTON.]

ELECTROSTATIC PRECIPITATION OF SUSPENDED PARTICLES. — (*See also Electricity and Magnetism, Principles of.*) Gases containing particles of matter suspended in them, like smoke or smelter fumes, may be purified electrically, the particles being deposited and recovered. The process has been applied with success to a number of installations, by the International Precipitation Co. of San Francisco, operating under the patents of F. G. Cottrell.

Principle. — The essential features of the process are as follows: Two insulated electrodes are placed in a gas containing suspended particles, one electrode being a flat plate of iron or lead and the other a wire covered with a fuzzy semi-conductor such as asbestos. In some plants, the charging electrode is made of wire gauze through which the particle-laden gas is forced to escape. In passing through the gauze the solid particles become electrified and are drawn to the flat electrode. The electrodes are connected to the terminals of a source of direct current, between which is maintained a difference of potential of 20,000 or more volts. The discharge from the fuzzy surface of one electrode electrifies the suspended particles with charges of the same sign. The result is that the particles are repelled from the fuzzy electrode and attracted to the flat plate, on which they are deposited. The source of current is a transformer and a commutator driven by a synchronous motor.

Typical Installations. — This process is in use for the deposition of fumes from smelter plants, powder factories, cement plants, etc. The first commercial installation was at the works of the Du Pont de Nemours Powder Co. at Pinole, Cal., where sulphur trioxide fumes were deposited electrically. One kilowatt at a transformer voltage of 6600 volts was consumed in depositing the fumes from 100 to 200 cubic feet of flue gas per minute. The next installation was at the Selby Smelting and Lead Co. (Vallejo Junct., Cal.), where several tons of lead fumes mixed with arsenic and sulphur trioxide issued daily from the stacks. A stack, which gave forth about 5000 cubic feet of gas per minute, was treated by suspending lead plates 4 inches wide and 5 feet long, four inches apart in several rows, the flue being 4 feet square. Between the plates were suspended lead-covered iron wires covered with a fuzzy coating of mica. These conductors were supported on two sets of insulators. The power consumption was 2 kilowatts. An installation at the Balaklala Smelter Works (Coram, Cal.) was tested with the result that 85 per cent of the solids in the gases were shown to be deposited. At the Coulton Cement Works, Cal., 20 tons of cement dust per day were deposited from flue gas at 450°C.

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[W. A. Del Mar.]

ELEMENTS, CHEMICAL. — (*See also Electrochemistry, Principles of; Weights of Materials; Heat and Heat Effects.*) An element is a substance which cannot be further decomposed into simpler substances by any known means of chemical analysis. At the present time about 100 elements (including the radioactive elements) have been discovered. Recent investigations in radioactivity have proved that substances now regarded as stable elements may be undergoing a process of transformation, the rate of which is, however, too slow to be detected by any known means. In the case of other highly radioactive elements the rate of spontaneous transformation is relatively great; thus it has been shown that helium is an element resulting from the disintegration of radium and the unstable elements formed in the process of its breaking up. The "life" of these unstable elements varies from a few minutes to many years.

Atomic Weights. — Elements always combine in definite proportions (or simple multiples of these proportions). It is possible to assign to each element a number such that the relative proportions (by mass) in which two or more elements combine are simple multiples of these numbers. The minimum combining mass or weight of oxygen has been chosen as 16, for certain practical reasons, and in terms of this standard the smallest combining weights of the other elements are determined by experiment. The numbers thus obtained are called "atomic masses" or more commonly "atomic weights." Formerly the atomic weight of hydrogen was taken as unity, in which case the atomic weight of oxygen equalled 15.98. On the atomic theory the atomic weight is proportional to the mass of the atom. The values of the atomic weights of the elements, as revised to 1913 by the International Committee on Atomic Weights, are given in the following table.

The Periodic Law. — The most important relation between the atomic weights was discovered by Mendelejeff and Lothar Meyer and is known as the Periodic Law. This states that "*the properties of the chemical elements are periodic functions of their atomic weights.*" Thus the elements may be arranged according to ascending values of their atomic weights in groups or families in each of which the chemical and physical properties of the elements are related. The existence and properties of several elements have been predicted by the aid of this law before their discovery.

INTERNATIONAL ATOMIC WEIGHTS, 1913.

Element	Symbol	Atomic weight	Element	Symbol	Atomic weight
Aluminium	Al	27.1	Molybdenum	Mo	96.0
Antimony	Sb	120.2	Neodymium	Nd	144.3
Argon	A	39.88	Neon	Ne	20.2
Arsenic	As	74.96	Nickel	Ni	58.68
Barium	Ba	137.37	Nitron (radium emanation)	Nt	222.4
Bismuth	Bi	208.0	Nitrogen	N	14.01
Boron	B	11.0	Osmium	Os	190.9
Bromine	Br	79.92	Oxygen	O	16.00
Cadmium	Cd	112.40	Palladium	Pd	106.7
Caesium	Cs	132.81	Phosphorus	P	31.04
Calcium	Ca	40.07	Platinum	Pt	195.2
Carbon	C	12.00	Potassium	K	39.10
Cerium	Ce	140.25	Praseodymium	Pr	140.6
Chlorine	Cl	35.46	Radium	Ra	226.4
Chromium	Cr	52.0	Rhodium	Rh	102.9
Cobalt	Co	58.97	Rubidium	Rb	85.45
Columbium	Cb	93.5	Ruthenium	Ru	101.7
Copper	Cu	63.57	Samarium	Sa	150.4
Dysprosium	Dy	162.5	Scandium	Sc	44.1
Erbium	Er	167.7	Selenium	Se	79.2
Europium	Eu	152.0	Silicon	Si	28.3
Fluorine	F	19.0	Silver	Ag	107.88
Gadolinium	Gd	157.3	Sodium	Na	23.00
Gallium	Ga	69.9	Strontium	Sr	87.63
Germanium	Ge	72.5	Sulphur	S	32.07
Glucinum	Gl	9.1	Tantalum	Ta	181.5
Gold	Au	197.2	Tellurium	Te	127.5
Helium	He	3.99	Terbium	Tb	159.2
Holmium	Ho	163.5	Thallium	Tl	204.0
Hydrogen	H	1.008	Thorium	Th	232.4
Indium	In	114.8	Thulium	Tm	168.5
Iodine	I	126.92	Tin	Sn	119.0
Iridium	Ir	193.1	Titanium	Ti	48.1
Iron	Fe	55.84	Tungsten	W	184.0
Krypton	Kr	82.92	Uranium	U	238.5
Lanthanum	La	139.0	Vanadium	V	51.0
Lead	Pb	207.10	Xenon	Xe	130.2
Lithium	Li	6.94	Ytterbium (Neoytterbium)	Yb	172.0
Lutecium	Lu	174.0	Yttrium	Yt	89.0
Magnesium	Mg	24.32	Zinc	Zn	65.37
Manganese	Mn	54.93	Zirconium	Zr	90.6
Mercury	Hg	200.6			

[H. M. GOODWIN.]

ELEVATORS, ELECTRIC.—(*See also Hoists; Motors, Industrial Applications of.*) Elevators may be divided in two general classes, freight and passenger. The former requires a comparatively slow speed, about 75 feet per minute and, as a rule, only an infrequent starting and stopping. The passenger elevator, on the other hand, requires a speed of from 250 to 500 feet per minute. The lower speed is used for local elevators which may be called upon to stop at every floor. Ability to start and stop the elevator quickly and economically is here of the greatest importance, and the hoisting speed is of minor importance. Express elevators require a much higher speed, but as they generally must operate as locals for a number of the upper floors they are often of the two-speed type.

Power Required. — The horse-power of an elevator motor depends on the load to be raised, the speed of travel and the elevator efficiency, that is, the combined efficiency of the motor, the gear and the drive. It is customary to counterbalance the weight of the cage, and if it is under or overbalanced this amount must be added or deducted from the live load to get the actual load.

$$\text{Horse-power} = \frac{(\text{Unbalanced load in lb.}) \times (\text{Speed of elevator in ft. per min.})}{33{,}000 \times (\text{Efficiency})}$$

An efficiency of 0.50 is generally used in problems of this kind.

As the load is intermittent the motors are generally rated on the basis of 55° C. rise, with full load for one-half hour. The starting torque should be from 2 to 2½ times full-load torque.

Direct-current and Alternating-current Systems are both in general use for elevator service. With the direct-current system the adjustable-speed motor having a small percentage of compound winding is best adapted for obtaining the best control and a smooth acceleration. The series winding is cut out by the controller at normal speed, but is necessary in starting, insuring a smooth quick start, and it is furthermore of great value in the subsequent control of the motor. The best results will be obtained with about 10 to 15 per cent series-field winding.

Very successful service is also obtained with the alternating-current system. Squirrel-cage induction motors should, however, only be selected for slow- and constant-speed freight elevators and where the impairment of the line regulation, caused by the high-starting currents, can be overlooked. For large capacity elevators the squirrel-cage motor is entirely unsuitable, as it cannot be thrown directly on the line without inserting a primary starting resistance. This, however, cuts down the applied voltage and decreases the starting torque. Where squirrel-cage induction motors are used they must be of the high-resistance rotor type so as to provide for a high starting torque.

The polyphase-slip-ring type of induction motor has proven to be very satisfactory for elevator service for speeds up to about 250 or 300 feet per minute. This type of motor has a high starting torque characteristic, and suitable secondary control can readily be provided that will insure a smooth acceleration. For higher speeds two-speed machines are as a rule desirable, which eliminates the alternating-current motor. Dynamic braking (*see Cranes*) is also desirable with high-speed elevators, and in order to accomplish this with alternating-current motors, it would be necessary to provide a small motor-generator set to supply a direct-current excitation to the primary winding of the motor. Therefore, modern high-speed elevators are, as a rule, of the direct-current type.

Elevator Controllers must fulfill the most exacting service requirements. They must insure a smooth acceleration, afford positive speed control under widely varying loads, bring the elevator quickly, but smoothly, to a stop, or

from high to low speeds, regardless of load. Accurate stops at the landings must be made and devices should be provided to protect against slack cable, overtravel, overloads, failure of voltage, etc. Controllers may be of the full-magnet type, the semi-magnet type or the mechanically operated type. High-speed passenger elevators running from about 200 to 600 feet per minute are practically always installed with full-magnet controllers.

Direct-current Control Equipment. — With a direct-current equipment the car is started by moving the car switch or car-controller handle directly to the full on position. This releases the solenoid brake, after which the motor starts with full field strength and with resistance in the armature circuit. After the motor has started, the contactors on the contactor panel automatically cut out the dynamic brake resistance, the armature resistance and the series field, when the motor reaches its normal speed. Current relays limit the current taken during the acceleration to a predetermined value. An overload circuit breaker is provided which opens the circuit, while running, in case an abnormal current would tend to flow. This overload circuit breaker is generally interlocked with the car switch in such a way that should the motor be stopped by the action of the circuit breaker relay, this can be reset from the car by merely moving the control lever to the off position. The control equipment is generally arranged to give two speeds — one, a range of from rest up to the normal speed of the motor, secured by cutting out the armature resistance step by step, and a higher range, up to twice normal speed, secured by inserting resistance in the shunt field of the motor.

In stopping, the controller handle can be brought back to the second point, giving half speed, and then to the first point, which introduces the resistance in series with the armature and shunts the armature with the dynamic brake resistance which is relatively low. In this way it is possible to give a slow landing speed of about 25 per cent of the normal speed and which does not vary very much with the load in the car. Moving the control handle to the off position stops the car. It causes the motor to be disconnected from the circuit at the same time applying the solenoid brake, which is actuated by a spring or weight. The coil of the dynamic brake is connected to the motor armature so that this circuit is kept closed in stopping until the motor comes practically to rest, and the braking effect of the current generated in this local circuit is available to assist the mechanical brake.

Limit switches actuated by the elevator winding drum are provided to stop the elevator at predetermined limits of travel. In addition to these, overtravel switches are also ordinarily provided in the elevator hatchway as an extra precaution. These switches are arranged to open the control circuit on overtravel of the car. The first-named switches are generally connected so that after the car has been automatically stopped at either limit of travel, it is possible for the operator to start the car in the reverse direction only. When the hatchway-limit switches open, it is, however, generally impossible for the operator to reverse the car without first going to the elevator machine, thus insuring attention to the cause which made it necessary for the hatchway-limit switch to operate the car.

An auxiliary switch which will automatically stop the motor in case the cable slackens from any cause, such as the breaking of the cable or the catching of the cage, is also generally provided. It is mounted on one side of the winding drum and operated by a cross arm extending across the drum. Should the cable become slack the movement of the cross arm will serve to trip the switch, opening the control system and stopping the elevator. The slack-cable switch cannot be automatically reset, but after having tripped out must be reset by hand thus also obliging the operator to go to the source of trouble before the elevator can be again operated.

Alternating-current Control Equipment. — As with the direct-current system there are also many types of controllers made for the alternating-current system. The best types are those known as the full-magnet controllers, which are automatic in operation and controlled from a car switch in the same manner as the direct-current type. As stated above, alternating-current motors should preferably not be used for speeds over 250 or 300 r.p.m., because it is not feasible to employ dynamic braking. This means that the car must be slowed down and stopped by the application of the solenoid brake alone, and the speed must therefore be one that will permit of this being done with safety and comfort under all the widely varying conditions met with in elevator service.

A complete alternating-current control equipment generally comprises a contactor panel, a car switch, one or more limit switches, a slack cable switch and a solenoid brake. The solenoid brake is designed to be connected directly to the motor terminals, and when the circuit to the motor is closed the solenoid is energized and the brake retracted. Upon the opening of the main-line circuit, whether this is done intentionally in operating the elevator or is the result of accident, the solenoid is deënergized and the brake is instantly applied. The brake should be designed so as to permit of a gradual rather than an instantaneous braking effect, so as to avoid jarring the car by stopping it too suddenly.

The acceleration is accomplished by a number of alternating-current contactors, these being so connected as to cut a section of resistance out of each of the three phases of the rotor circuit simultaneously, the rate of acceleration being governed automatically by a number of current-limit relays in the rotor circuit.

Energy Consumption. — In all elevators, particularly in those electrically operated, the use of power increases in proportion to the number of starts and stops. This is a natural result of the use of the electric motor, in which the largest part of the energy is required for the process of acceleration, the next largest for electrical and mechanical retardation, and a lesser amount for the actual running speed, with a small proportion devoted to the continuous excitation of the field. It therefore follows that in this form of elevator, which is now so widely used, the main part of the consumption of power is directly related to the number of stops and starts which result from the number of persons carried.. The diversities in results which have been reported from time to time as to the consumption of current by electrical elevators may therefore be attributed to variations in traffic conditions. The results of a series of tests made upon the operation of a one-to-one traction elevator in a ten-story building as reported by R. P. Bolton (*A.S.M.E., 1910*) are given in the table on the following page.

SPECIFICATIONS FOR ELECTRIC ELEVATORS.* — The following memoranda are intended to assist in writing specifications. See also article on *Specifications*.

Performance.—Where elevator is to be situated. Purpose, i.e., whether for passenger or freight service. Characteristics of current supply. Length of run and number of stops. Full running speed and number of feet traveled from rest to full speed. Noiselessness outside of spaces occupied by machinery. Maximum kilowatt-hours per car mile. (This is often supplied by bidder for a given schedule.)

Machinery.—Type of hoisting machinery. Who supplies supports and framing for machines. Details of drum and shaft materials, and minimum diameter. (Drums are usually of grey cast iron and shaft of forged open-hearth steel.) Worm and gear details. The worm shall be of open-hearth steel and the gear of phosphor bronze. Motor details (*see articles on Motors.*) Method of taking up thrust of shaft and drum. Details of brake.

* By W. A. Del Mar.

ENERGY CONSUMED BY AN ELECTRIC ELEVATOR AT VARIOUS LOADS AND STOPS

Machine: One-to-one Electric Traction.
Total Weight of Car: 3956 pounds; Overbalance: 1060 pounds.
Capacity: 2500 pounds at a speed of 500 feet per minute.

Stops at floors No.													
	Up	1 and 9							1, 5 and 9				
	Down	9 and 1							9, 5 and 1				
Distance ..		101 ft. 6 in. one way or per trip = 52 trips per car mile											
Load, lb...		Operator	666	1060	1360	2010	2360	2660	Operator	666	1060	2010	2660
Kw-hr. per car mile..		2.345	2.075	1.945	1.87	2.15	2.495	3.22	3.09	2.855	2.52	2.915	3.85

Stops at floors No.									
	Up	1, 3, 5, 7 and 9				1, 2, 3, 4, 5, 6, 7, 8 and 9			
	Down	9, 7, 5, 3 and 1				9, 8, 7, 6, 5, 4, 3, 2 and 1			
Distance..........		101 ft. 6 in. one way or per trip = 52 trips per car mile.							
Load, lb...........		Operator	666	1060	2010	Operator	666	1060	2010
Kw-hr. per car mile		4.91	4.185	3.975	4.25	7.285	6.75	6.7	7.425

Safety Devices.—(a) Centrifugal governor: When the speed exceeds a stated speed, the car shall be gradually retarded. (b) Machine governor: When speed of car reaches a certain amount, greater than that at which the centrifugal governor acts, and less than that required to throw the safety clutches on the car, the circuit breaker shall open. (c) Limit stops, upper and lower: When the car approaches the limits, the controller circuit shall be automatically opened, and the brake applied. (d) Circuit-breaker details.

Cables.—Description, diameter and minimum number. The number of cables shall be sufficient to prevent any slipping of the cables on the drum when the car is operated under maximum load and contract speed.

Switchboard and Wiring.—Details of switchboard. Details of controller cable, which should be extra flexible and insulated with the best quality rubber.

Shaft Equipment.—Description of shaftway inclosures, with drawings. Elevator pit; general description; whether elevator contractor will find it ready for use and whether any machinery may be installed below the pit. Pent-house; general description; whether elevator contractor will find it ready for use, and whether any machinery may be installed outside or above penthouse

inclosure. Beams for support of sheaves, guides, governors, bumpers, etc. Description and drawings. Whether to be supplied by elevator contractor. Buffers under elevator, description. Counterweights, description; the material is usually grey cast iron. Shaftway fittings, including supports, oil buffers, grating for overhead sheaves, material of sheaves (usually tough grey cast iron), bearing boxes, drip pans, guide rails. The guide rails are usually steel tees of a stated weight per foot. The details of their attachment to the shaft structure should be specified. Doors, details.

Car. — Details of construction, including statement of sides where door or doors are required. Safety clutch, details. Slack-cable device, details. Counterbalance chains shall be attached to bottom of car to compensate for weight of elevator cables.

Cage. — Details of construction, fireproofing, floor details, including finish of floors at edges, folding gate, lighting, emergency door, etc. There shall be no strain on the cage when the safety devices operate. Operating equipment, including controller, emergency switch, lighting switch, safety lever, annunciator in connection with signal system, etc.

Control System. — Control mechanism, description. Number of speeds by controller. Car to stop upon release of lever. Controller shall be out of way of passengers, yet allow operator to keep one hand on the gate while the other is on the switch. Provision shall be made to operate the car from the switchboard when not being operated from within. If the operator reverses while the car is running, the reversal of direction shall be gradual and without shock to passengers. The following operations shall be accomplished by controller: (a) Make and break circuit and reverse motor without destructible arcing at contacts or flashing at brushes. (b) Give easy acceleration of car independently of operator; stopping or starting without shock or jar. (c) Lift brake, cut out starting resistance quietly and operate fields by successive steps in starting. (d) Apply brake and cut in special field resistance in series with armature in stopping in order to attain a retardation of the car prior to the pressure of the brake shoes upon the brake pulley. (e) Other control will be considered.

Signaling and Indicator System. — Details of signaling system. Signals shall indicate correctly the position of the car whatever the motions of the latter. The indicator system shall give the operator in the car a clear signal showing what floor to stop at and whether passengers want to go up or down.

General Matters. — Details of test. Instruction by contractor in use of equipment. During the operation of the cars in the shaft and before its final completion, the contractor shall install on the cars a 6-inch gong, which will ring automatically at each floor up and down the shaft as a warning signal. Lubrication details. Painting details. Tools to be supplied.

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[D. B. Rushmore, assisted by E. A. Lof.]

ENDOSMOSE OR ELECTRIC OSMOSIS. — (*See also Electrochemistry, Principles of.*) An electric current flowing through a porous partition, or capillary opening, filled with conducting liquid causes a flow of the liquid through the partition or opening. This phenomenon is known as endosmose or electric osmosis (*see Electrochemistry, Principles of*).

The quantity of a given liquid which is carried through a given porous diaphragm in a definite time varies directly with the current strength and is independent of the area and thickness of the diaphragm. The quantity varies with the nature of the solution, being greater with liquids of high specific resistance. The direction of flow is from positive to negative electrode.

Applications of Endosmose. — Endosmose may be utilized to remove water from wet substances. Thus if a wooden box be equipped with perforated metallic plates at opposite ends and be filled with wet turf and current circulated through the turf from one end to the other, water will ooze out of the perforations of the cathode plate. Endosmose is utilized in tanning processes to accelerate the passage of tanning fluids through the hides, and efforts are being made to utilize it for drying peat, extracting sap and similar purposes.

Endosmose of Negative Feeders. — If electric conductors covered with a saturated braid or a number of such braids be made the cathode of a water bath and say 110 volts applied between anode and cathode, the braid will blister in a few minutes and the blisters will finally burst, grounding the conductor to the water. The same action goes on at lower voltages at a proportionately lower rate, but with equal certainty. Consequently weatherproof conductors otherwise uninsulated are useless for negative feeders on railway circuits. No action is observable if the wire be made the anode instead of the cathode. The same phenomenon is said to be observable with rubber insulation, but the writer has tried several samples of high-grade American compounds made with 30 per cent of Para rubber and obtained no signs of endosmose after subjecting them for a week to a pressure of 110 volts. C. H. Wordingham (*Elect. Engineer, London, Vol. 26, 1900, p. 93*) found, however, that certain makes of rubber are porous and water may enter and eventually destroy the whole of the rubber on the negative conductors. For this reason insulation of low specific resistance should never be used on negative feeders. For the same reason it is not practical to maintain a negative contact rail on electric railways, below earth potential, as the insulators soon become saturated with moisture and thereby become conductors. Thus S. B. Fortenbaugh, relating the experience of the Tube Railways in London, says:

1. That the difference of potential between the positive conductor and earth is always normally considerably greater than the potential existing between the negative conductor and earth.

2. That a reversal of polarity is always instantly accompanied by a considerable increase in the normal leakage current between the positive and negative conductor.

3. The insulation of the negative conductor to earth cannot be indefinitely maintained.

Effect of Endosmose on Insulation Measurements. — When testing wires having insulation of low specific resistance, endosmose may further lower the resistance if the wire is negative to the water bath.

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[W. A. Del Mar.]

EQUATIONS. — (*See also Complex Numbers; Errors of Observation; Logarithms; Vectors.*) The subjects treated in this article are

QUADRATIC EQUATION. — The solution of

$$ax^2 + bx + c = 0$$

is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

CUBIC EQUATION. — The solution of the general cubic equation of the form

$$ax^3 + 3bx^2 + 3cx + d = 0 \qquad (1)$$

is obtained as follows: Put

$$y = ax + b$$

$$H = b^2 - ac$$

$$G = a^2d - 3abc + 2b^3$$

then (1) becomes

$$y^3 - 3Hy + G = O. \qquad (2)$$

The nature of the roots of this equation depends upon the algebraic sign of H and the relative numerical magnitudes of G and H.

Case I. H positive and G numerically less than $2H\sqrt{H}$. (G may be either positive or negative.) Put

$$\phi = \frac{1}{3}\sin^{-1}\left[\frac{G}{2H\sqrt{H}}\right]$$

then the three roots of (2) are

$$\begin{aligned} y_1 &= 2\sqrt{H}\sin\phi \\ y_2 &= 2\sqrt{H}\sin(\phi + 120) \\ y_3 &= 2\sqrt{H}\sin(\phi - 120) \end{aligned} \qquad (3)$$

and all three roots are real.

Case II. H positive and G numerically greater than $2H\sqrt{H}$. Put

$$u = \frac{1}{3}\cosh^{-1}\left[\frac{\pm G}{2H\sqrt{H}}\right]$$

the plus sign to be used when G is positive, the negative sign when G is negative thus making $\cosh 3u$ positive in either case.

Then the three roots of (2) are

$$\begin{aligned} y_1 &= \mp 2\sqrt{H}\cosh u \\ y_2 &= \pm\sqrt{H}\cosh u + \sqrt{-3H}\sinh u \\ y_3 &= \pm\sqrt{H}\cosh u - \sqrt{-3H}\sinh u \end{aligned} \qquad (4)$$

the upper signs to be used when G is positive, the lower when G is negative. The first root in this case is real, the other two being complex quantities. See articles on *Complex Quantities* and *Hyperbolic Functions.*

Case III. H negative and G any value. Put

$$u = \frac{1}{3} \sinh^{-1} \left[\frac{\mp G}{2 H \sqrt{-H}} \right]$$

the negative sign to be used when G is positive, the positive sign when G is negative, thus making $\sinh 3u$ positive in either case. Then the three roots of (2) are:

$$\begin{aligned} y_1 &= \mp 2 \sqrt{-H} \sinh u \\ y_2 &= \pm \sqrt{-H} \sinh u + \sqrt{3H} \cosh u \\ y_3 &= \pm \sqrt{-H} \sinh u - \sqrt{3H} \cosh u \end{aligned} \tag{5}$$

the upper sign to be used when G is positive, the lower sign when G is negative. The first root in this case is real, the other two complex quantities.

Note that in each case the sum of the three roots is zero. Also, in the special case when $G = 2H\sqrt{H}$, the angle ϕ is 30°, and therefore from (3) the three roots are $\sqrt{H}$, $\sqrt{H}$ and $-2\sqrt{H}$; this also follows from (4).

Examples. — *Case I.*

$$y^3 - 6y + 2 = 0.$$

$H = 2$ and is positive and $G = 2$ and is less than $2H\sqrt{H}$.

$$\phi = \frac{1}{3} \sin^{-1} \left[\frac{2}{4\sqrt{2}} \right] = 6.90°$$

whence $y_1 = +0.3399$; $y_2 = 2.2618$; $y_3 = -2.6018$.

Case II.

$$y^3 - 3y + 4 = 0.$$

$H = 1$ and is positive and $G = 4$ and is greater than $2H\sqrt{H}$.

$$u = \frac{1}{3} \cosh^{-1} 2 = 0.439$$

whence $y_1 = -2.196$; $y_2 = 1.098 + 0.785\sqrt{-1}$; $y_3 = 1.098 - 0.785\sqrt{-1}$.

Case III.

$$y^3 + 6y + 2 = 0.$$

In this case, $H = -2$ and $G = 2$

$$u = \frac{1}{3} \sinh^{-1} \frac{-2}{-4\sqrt{2}} = 0.1155$$

$$\begin{aligned} y_1 &= -0.3275 \\ y_2 &= 0.1638 + 2.467\sqrt{-1} \\ y_3 &= 0.1638 - 2.467\sqrt{-1} \end{aligned}$$

SIMULTANEOUS EQUATIONS. — Given n independent equations in n unknowns, these n equations fix the values of each of the n unknowns. To solve such a set of simultaneous equations in x, y and z, say, solve each of

the three equations for x in terms of y and z. Equating these three values for x gives two equations in y and z. Solving each of these two equations for y in terms of z and equating these two values of y gives a single equation in z. The solution of this last equation then gives the value of z. Then substitute this value of z in either of the equations in y and z, and solve for y. Then substitute these values of y and z in any one of the original equations and solve for x.

Determinants. — In the case of linear simultaneous equations (i.e., when x, y and z occur only to the first power), the equations may be solved by determinants. This method is a considerable time saver when the number of unknowns is greater than three, but when the number of unknowns is three or less the straight substitution method is preferable.

The determinant of a set of simultaneous equations is formed by writing the equations one below the other with the same unknown in the same relative position in each. The block of numbers forming the coefficients of the unknowns is called the determinant. For example, the determinant of the equations

$$\begin{aligned} w + x + y + z &= 6 \\ w + y + 3z &= 4 \\ w + 2x + 3y &= 1 \\ w + 3x + z &= 3 \end{aligned}$$

is

$$D = \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 3 \\ 1 & 2 & 3 & 0 \\ 1 & 3 & 0 & 1 \end{vmatrix}$$

The values of any one of the unknowns, say y, is found by writing a second determinant, D_y, exactly like the determinant D, except that the constants forming the right-hand members of these equations are substituted for the coefficients of y in the determinant, that is

$$D_y = \begin{vmatrix} 1 & 1 & 6 & 1 \\ 1 & 0 & 4 & 3 \\ 1 & 2 & 1 & 0 \\ 1 & 3 & 3 & 1 \end{vmatrix}$$

Then
$$y = \frac{D_y}{D}$$

and similarly for the other unknowns.

The value of any determinant is found by making use of the following rules:

(1) If a determinant has two equal rows or columns, it is equal to zero.

(2) To any row or column one may add or subtract any number of times any other row or column without altering the value of the determinant.

(3) To multiply any row or column by a number is the same as multiplying the determinant by that number.

(4) If all the terms in a row or column except one are zero, the determinant reduces to one of a lower order which may be obtained by striking out the row and column which intersect at the term in question, and multiplying the whole by that term, changing the sign of this term, however, if it is removed by an odd number of terms from the principal diagonal. The principal diagonal is

the line of terms beginning at the upper left-hand corner and ending at the lower right-hand corner. Thus,

$$\begin{vmatrix} 1 & 2 & 8 & 5 \\ 3 & 4 & 6 & 9 \\ 0 & 3 & 0 & 0 \\ 6 & 7 & 4 & 3 \end{vmatrix} = -3 \begin{vmatrix} 1 & 8 & 5 \\ 3 & 6 & 9 \\ 6 & 4 & 3 \end{vmatrix}$$

the principal diagonal being that with the figures 1, 4, 0 and 3. It is immaterial whether the distance from the diagonal is counted along a row or a column.

(5) The value of a determinant of the second order is

$$\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1 b_2 - a_2 b_1$$

The reduction of determinants is effected by altering the terms according to the above rules until a row or column is obtained in which all terms but one are zero. This enables a reduction of order to be effected in accordance with rule 4. Reductions are continued until one of the second order is obtained.

Example. — By rule (2), the determinant D given above may be written as follows, the top row being subtracted from each of the others.

$$D = \begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & -1 & 0 & 2 \\ 0 & 1 & 2 & -1 \\ 0 & 2 & -1 & 0 \end{vmatrix}$$

By rule (4) this reduces to

$$D = \begin{vmatrix} -1 & 0 & 2 \\ 1 & 2 & -1 \\ 2 & -1 & 0 \end{vmatrix}$$

By rule (3) this may be written

$$D = \frac{1}{2 \times 2} \begin{vmatrix} -2 & 0 & 4 \\ 2 & 4 & -2 \\ 2 & -1 & 0 \end{vmatrix}$$

By rule (2), adding the top row to each of the others, this reduces to

$$D = \frac{1}{2 \times 2} \begin{vmatrix} -2 & 0 & 4 \\ 0 & 4 & 2 \\ 0 & -1 & 4 \end{vmatrix}$$

which by rule (4) reduces to

$$D = \frac{-2}{2 \times 2} \begin{vmatrix} 4 & 2 \\ -1 & 4 \end{vmatrix}$$

whence by rule (5) $\qquad D = -\frac{1}{2}(4 \times 4 + 1 \times 2) = -9.$

Similarly, $\qquad D_y = 21.$

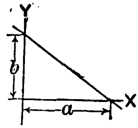
and therefore $\qquad y = -\frac{21}{9} = -2.333.$

Similarly, $\qquad w = 13.33, \quad x = -2.667, \quad z = +2.333.$

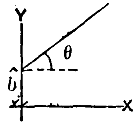
EQUATIONS OF COMMON CURVES.

Straight Line. —

$$\frac{x}{a} + \frac{y}{b} = 1$$

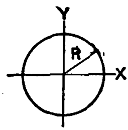


or $\quad y = x \tan\theta + b.$



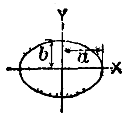
Circle. —

$$x^2 + y^2 = R^2.$$



Ellipse. —

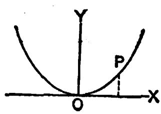
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$



Parabola (Vertical). —

$$y = kx^2$$

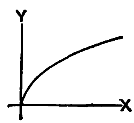
where k is a constant.



Parabola (Horizontal). —

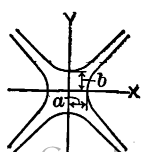
$$y = k\sqrt{x}$$

where k is a constant.



Hyperbola. —

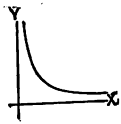
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1.$$



Rectangular or Equilateral Hyperbola.—

$$y = \frac{k}{x}$$

where k is a constant.

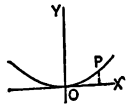


Catenary.—

$$y = \frac{1}{k}\left[\cosh kx - 1\right]$$

where k is a constant. The length of arc from O to P is

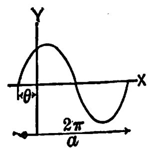
$$\lambda = \frac{1}{k}\sinh (kx).$$



See tables in article on *Hyperbolic Functions*.

Sinusoid.—

$$y = A \sin (ax + \theta).$$



DIFFERENTIAL EQUATIONS.—Differential equations of the following forms are met with in the theory of alternating and transient currents.

The following notation is used: $\epsilon = 2.7183 \ldots$ = base of natural system of logarithms; x, y, u, w are variables. A, ϕ, γ, and θ are integration constants. Other letters represent known constants.

$$\frac{dy}{dx} = ay. \qquad (1)$$

Solution: $$y = A\epsilon^{ax}.$$

$$\frac{dy}{dx} + ay = 0. \qquad (2)$$

Solution: $$y = A\epsilon^{-ax}.$$

$$\frac{dy}{dx} + ay = b. \qquad (3)$$

Solution: $$y = \frac{b}{a}[1 - A\epsilon^{-ax}].$$

$$\frac{d^2y}{dx^2} = -a^2y. \qquad (4)$$

Solution: $$y = \sin (ax + \phi).$$

$$\frac{d^2y}{dx^2} = a^2y. \qquad (5)$$

Solution: $$y = A \sinh (ax + \phi).$$

$$\frac{d^2y}{dx^2} + 2u\frac{dy}{dx} + (u^2 + a^2)\,y = 0. \tag{6}$$

Solution:

Case I. a^2 positive: — $y = A\epsilon^{-ux}\sin(ax + \phi)$.
Case II. a^2 negative: — $y = A\epsilon^{-ux}\sinh(ax + \phi)$.
Case III. $a^2 = 0$: — $y = A\,(x + \phi)\,\epsilon^{-ux}$.

$$\frac{d^2y}{dx^2} + 2a\frac{dy}{dx} + (u^2 + a^2)\,y = B\sin(\omega x + \theta). \tag{7}$$

The complete solution of this equation consists of the solution of No. 6 plus the term

$$\left(\frac{B\sin\delta}{2\,uw}\right)\sin(\omega x + \theta - \delta), \tag{a}$$

where

$$\delta = \tan^{-1}\frac{2\,u\omega}{a^2 + u^2 - \omega^2}.$$

For each additional sine term added to the right-hand member of the equation, there will be a corresponding term of the same form as (*a*) in the solution.

$$\frac{d^ny}{dx^n} + a_{n-1}\frac{d^{n-1}y}{dx^{n-1}} + \ldots a_1\frac{dy}{dx} + a_0y = B\sin(\omega x + \theta). \tag{8}$$

Solution:

$$y = A_1\epsilon^{m_1x} + A_2\epsilon^{m_2x} + \ldots A_n\epsilon^{m_nx} + KB\sin(\omega x + \theta + \delta),$$

where m_1, m_2, etc., are the n roots of the equation

$$m^n + a_{n-1}m^{n-1} + \ldots a_1m + a_0 = 0,$$

and k and δ are found by substituting the $KB\sin(\omega x + \theta + \delta)$ by itself in the given differential equation and equating the coefficients of $\sin(\omega x + \theta)$ and $\cos(\omega x + \theta)$ respectively on the two sides of the resulting equation. When the second member of the differential equation is a constant, B, the sine term in the solution becomes simply $\frac{B}{a_0}$.

Note that all of the preceding equations are merely special cases of the general equation (8).

$$\frac{d^2y}{dx^2} + 2u\frac{dy}{dx} + (u^2 - q^2)\,y = \frac{1}{c^2}\frac{d^2y}{dz^2}. \tag{9}$$

The complete solution of this equation contains an infinite number of terms of the form

$$y = \epsilon^{-(u-s)x}\left[A_1\epsilon^{mz}\sin(\omega x + nz + \phi_1) + A_2\epsilon^{-mz}\sin(\omega x - nz + \phi_2)\right], \tag{a}$$

where A_1, ϕ_1, A_2, ϕ_2 and two of the four constants ω, s, m and n are integration constants (fixed by the terminal conditions). The values of m and n in terms of ω and s are

$$m = c\sqrt{ab}\cos\frac{\eta + \epsilon}{2},$$

$$n = c\sqrt{ab}\sin\frac{\eta + \epsilon}{2},$$

where

$$a = \sqrt{(s+q)^2 + \omega^2}, \qquad \epsilon = \tan^{-1}\left(\frac{\omega}{s+q}\right),$$

$$b = \sqrt{(s-q)^2 + \omega^2}, \qquad \eta = \tan^{-1}\left(\frac{\omega}{s-q}\right).$$

The values of ω and s in terms of m and n are

$$\omega = \frac{\sqrt{FG}}{c} \cdot \cos\frac{\alpha+\beta}{2},$$

$$s = \frac{\sqrt{FG}}{c} \sin\frac{\alpha+\beta}{2},$$

where

$$F = \sqrt{(n+cq)^2 + m^2}, \qquad \alpha = \tan^{-1}\left(\frac{m}{n+cq}\right),$$

$$G = \sqrt{(n-cq)^2 + m^2}, \qquad \beta = \tan^{-1}\left(\frac{m}{n-cq}\right).$$

The solution of equation (9) may also be written as a series of terms of the form

$$y = M\epsilon^{-(u-s)x} \sin(\omega x + \phi + \mu), \tag{b}$$

where

$$M = \frac{A}{\sqrt{2}} \sqrt{\cosh 2(mz+\gamma) + \cos 2(nz+\theta)},$$

$$\tan \mu = \tanh(mz+\gamma) \tan(nz+\theta),$$

where A, ϕ, γ and θ are integration constants, and the relations between the other constants ω, s, m and n are the same as above.

In the special case when $q = 0$, the solution of equation (9) is

$$y = \epsilon^{-ux}[f_1(\omega x + nz) + f_2(\omega x - nz)], \tag{c}$$

where f_1 and f_2 are any two arbitrary functions and w and n are connected by the relation

$$\frac{\omega}{n} = \frac{1}{c}.$$

$$\frac{d^2y}{dx^2} + \frac{1}{x}\frac{dy}{dx} = 4\,a^2y. \tag{10}$$

Solution is an infinite series:

$$y = A\left[1 + (ax)^2 + \frac{(ax)^4}{2^2} + \frac{(ax)^6}{(3!)^2} + \frac{(ax)^8}{(4!)^2} + \ldots\right]. \tag{a}$$

This series is absolutely convergent for all values of x, but for $ax > 1$ the following approximate series is more convenient and sufficiently accurate.

$$y = \frac{B\epsilon^{2ax}}{\sqrt{2\,ax}}\left[1 + \frac{1}{16\,ax} + \frac{3^2}{2\,(16\,ax)^2} + \frac{3^2.5^2}{3!\,(16\,ax)^3} + \frac{3^2.5^2.7^2}{4!\,(16\,ax)^4}\right]. \tag{b}$$

[W. A. Del Mar.]

ERRORS OF OBSERVATION. — When a quantity is measured with all possible accuracy many times in succession, the numbers expressing the results are found to differ by amounts, which, although generally small, are occasionally considerable in comparison with the quantity measured. Though these differences may be decreased by improved methods, better instruments or greater skill, they can never be entirely removed. They are known as the errors of observation. The following formulas, which are derived from the Theory of Least Squares, apply to such errors and not to errors which can be eliminated by correcting mistakes of the observer or defects of instruments or methods of observation. *That is, they apply only to errors which may be either positive or negative, the chance of a positive error occurring being exactly the same as the chance of a negative error occurring.*

Weighted Observations. — Sometimes in spite of the care with which observations are taken, there are reasons for believing that some observations are better than others. In this case the observations are given different "weights" or numbers expressing their relative practical worth. A weighted observation is an observation multiplied by its weight.

PROBABLE VALUE OF SEVERAL OBSERVATIONS. — The most probable value of a quantity which is observed directly several times with equal care is the arithmetical mean of the measurements.

Example. — The length of a room (in feet) is measured ten times with the following results:

20.05	20.06	19.98	19.99	20.00
20.07	20.01	20.05	19.95	19.98

The arithmetical mean of these lengths is 20.01 feet, which is accordingly the probable length of the room.

The most probable value of a quantity which is observed directly several times, but where the observations have different weights, is equal to the sum of the weighted observations divided by the sum of the weights.

Example. — In the above example if the fourth and fifth observations are twice as reliable as the others, and the seventh three times as reliable, the most probable value would be the sum of 20.05, 20.06, 19.98, (19.99 × 2), (20.00 × 2), 20.07, (20.01 × 3), 20.05, 19.95 and 19.98, divided by 14, which is equal to 20.01, which happens in this instance to agree with the unweighted average.

PROBABLE ERROR OF ANY ONE OF SEVERAL OBSERVATIONS. — The probable error of a number of direct observations made with equal care is found by the following formula:

$$r = 0.6745 \sqrt{\frac{\Sigma v^2}{n-1}},$$

where

n = number of observations,
r = probable error of a single observation,
v = residuals found by subtracting each measurement from the arithmetical mean.

Example. — In the example cited above, $n = 10$. The values of v in the order of the observations given above are as follows:

+0.04	+0.05	−0.03	−0.02	−0.01
+0.06	0	+0.04	−0.06	−0.03

The sum of the squares of these quantities is 0.0152. The probable error in any one of the ten observations is

$$r = 0.6745 \sqrt{\frac{0.0152}{9}} = 0.0277.$$

The probable error of each of a number of direct observations, where the observations have different weight, is found by the following formula, in which p represents the weight of an observation.

$$r_1 = 0.6745 \sqrt{\frac{\Sigma pv^2}{n-1}}.$$

Example. — In the example cited above for weighted observations, the values of v are the same as the values of v in the preceding example.

The values of pv^2 are as follows:

0.0016	0.0025	0.0009	0.0008	0.0002
0.0036	.0	0.0016	0.0036	0.0009

and their sum is 0.0157.

Hence the probable error of a single observation

$$r_1 = 0.6745 \sqrt{\frac{0.0157}{9}} = 0.0282.$$

PROBABLE ERROR OF THE ARITHMETICAL MEAN. — If

r = probable error of a single observation,
n = number of observations,
r_0 = probable error of the arithmetical mean,

$r_0 = \dfrac{r}{\sqrt{n}}$ for observations of equal weight, or

$r_0 = \dfrac{r_1}{\sqrt{\Sigma p}}$ for unequal weight.

Example. — In the example cited above for observations of equal weight, $r = 0.0277$ and $n = 10$, and $r_0 = 0.0277/\sqrt{10} = 0.0088$.

In the example cited above for observations of unequal weight, $r_1 = 0.0282$ and $\Sigma p = 14$, and $r_0 = 0.0282/\sqrt{14} = 0.0075$.

It should be noted that the probable error of the mean decreases inversely as the square root of the number of observations.

Example. — The probable error in making a single reading of a watt-hour meter on an electric locomotive is 5 per cent, after all constant sources of error have been eliminated. If 25 locomotives are used, what will be the probable error in the total energy consumption obtained, by adding the watt-hour meter readings? Answer, one per cent.

$$r_0 = \frac{5}{\sqrt{25}} = 1.$$

PROBABLE ERROR IN A RESULT CALCULATED FROM THE MEANS OF SEVERAL OBSERVED QUANTITIES. — Let Z = a sum or difference of several independent quantities.

Let r_1, r_2, r_3, etc., be the probable errors in these quantities. Then the probable error of Z is equal to

$$\sqrt{r_1^2 + r_2^2 + r_3^2 + \text{etc.}}$$

Let $Z = Az$, where z is an observed quantity, and A, a known number. Let be the probable error in z. Then the probable error in Z is Ar.

Let Z be the product of two independently observed quantities z_1 and z_2 whose probable errors are r_1 and r_2 respectively. Then the error in Z is equal to

$$\sqrt{z_1^2 r_2^2 + z_2^2 r_1^2}.$$

Let Z be any function of the independently observed quantities z_1, z_2, z_3, etc., whose probable errors are r_1, r_2, r_3, etc. Then the probable error in Z is equal to

$$\sqrt{\left(\frac{dZ}{dz_1}\right)^2 r_1^2 + \left(\frac{dZ}{dz_2}\right)^2 r_2^2 + \left(\frac{dZ}{dz_3}\right)^2 r_3^2 + \text{etc.}}$$

METHOD OF LEAST SQUARES. — A set of simultaneous equations containing more equations than unknowns cannot have any exact solution unless the superfluous equations are deducible from the others. They may, however, have a most probable solution which is such that the sum of the errors remaining after corrections for the deduced values of the unknowns shall be a minimum. This result is obtained as shown in the following example:

$$\begin{aligned} s - t + 2u &= 3, \\ 3s + 2t - 5u &= 5, \\ 4s + t + 4u &= 21, \\ -s + 3t + 3u &= 14. \end{aligned}$$

The coefficients of s are 1, 3, 4 and -1. Multiplying by these coefficients:

$$\begin{aligned} s - t + 2u &= 3, \\ 9s + 6t - 15u &= 15, \\ 16s + 4t + 16u &= 84, \\ s - 3t - 3u &= -14. \end{aligned}$$

Add these equations and obtain $27s + 6t = 88$.

The coefficients of t are -1, 2, 1 and 3. Multiply the original equations by these coefficients.

$$\begin{aligned} -s + t - 2u &= -3, \\ 6s + 4t - 10u &= 10, \\ 4s + t + 4u &= 21, \\ -3s + 9t + 9u &= 42. \end{aligned}$$

Add these equations and obtain $6s + 15t + u = 70$.

The coefficients of u are 2, -5, 4 and 3. Multiply the original equations by these coefficients.

$$\begin{aligned} 2s - 2t + 4u &= 6, \\ -15s - 10t + 25u &= -25, \\ 16s + 4t + 16u &= 84, \\ -3s + 9t + 9u &= 42. \end{aligned}$$

Add these equations and obtain $t + 54u = 107$.

The equations

$$\begin{aligned} 27s + 6t \quad\quad &= 88, \\ 6s + 15t + u &= 70, \\ t + 54u &= 107, \end{aligned}$$

are then solved in the ordinary way, the solution being

$$s = 2.47, \quad t = 3.56, \quad \text{and} \quad u = 1.92.$$

BIBLIOGRAPHY. — The subject of error of observation is very fully treated in *A Text Book on the Method of Least Squares*, by Mansfield Merriman.

[W. A. DEL MAR.]

EXPLOSIVES. — Explosives may be divided into two classes: progressive and detonating explosives. A progressive explosive is one which burns quickly but not instantaneously. The inner parts of progressive explosives do not burn until the outer parts are consumed so that the combustion takes place gradually. The common progressive explosives are the charcoal and nitrocellulose powders. In detonating explosives the explosive reaction takes place almost simultaneously throughout the substance. The explosive substance is then converted instantaneously into a gas of great volume. The common detonating explosives are guncotton, nitroglycerine, dynamite, explosive gelatin, picric acid derivatives and fulminate of mercury.

Black Powder consists of 75 parts potassium nitrate, 15 parts charcoal and 10 parts sulphur by weight. After being ground to a powder, water is added and the mixture is pressed into cakes and dried. The cakes are then broken into grains and sifted through screens. Prismatic perforated powder is made by pressing the granulated powder in moulds, the mixture being moistened in the process.

Brown Powder consists of 80 parts potassium nitrate, 16 parts charcoal, 3 parts sulphur and 1 part water by weight. In black powder the charcoal is made from wood, while in the brown powder the charcoal is made from rye-straw, the resultant powder being slower burning than black powder. The process of manufacture is the same as with black powder.

Smokeless Powders. — These are commonly made of nitrocellulose powders, which differ from the black or brown charcoal powders in that the mixture is a chemical instead of a mechanical one. Nitrocellulose is obtained by treating cellulose, usually cotton waste, with nitric acid. The nitrocellulose is mixed with ether, being thus converted into a "colloid," and assumes a form devoid of cellular structure. This colloided nitrocellulose, when pressed into perforated prismatic grains, forms smokeless powder. The English smokeless powder, Cordite, is composed of 37 parts colloided nitrocellulose, 58 parts nitroglycerine and 5 parts vaselin.

Guncotton is an uncolloided nitrocellulose containing about 12.9 per cent of nitrogen. It retains the complete cotton structure and can only be distinguished from cotton by its rough feeling. It is compressed into cakes and when stored it is kept wet, as it is then non-explosive except with a powerful detonator.

Nitroglycerine is a nitrate of glycerine and is made by treating pure glycerine with nitric acid. Commercial nitroglycerine is a yellowish oily liquid without odor. It has a sweet taste and burns the tongue. It is poisonous when absorbed through the mouth, nostrils or skin. It is exploded by shock, heating or friction. Nitroglycerine is seldom used in the pure state because of the danger in handling it.

Dynamite is a mixture of nitroglycerine and some solid substance, which may be either inert or active. The solid substances commonly used are kieselguhr, wood-fiber, wood-pulp, charcoal and cork. The mixture is pressed into "sticks" about one inch in diameter and 8 inches in length and is then wrapped in paraffined paper. It is usually light brown in color. Dynamite freezes at 4° C. and is very insensitive to shock in the frozen state.

Explosive Gelatin, a mixture of colloided guncotton and nitroglycerine, is the most powerful explosive known, and at ordinary temperatures it is less sensitive to shock than dynamite. At low temperatures it is more sensitive than dynamite. It is a soft, gelatinous substance, yellowish brown in color and is prepared in the form of cartridges like dynamite.

Picric Acid Derivatives form the base of many common explosives, among which may be mentioned Mellinite (French), Lyddite (English) and Maximite. Picric acid is a crystalline substance possessing a very bitter taste. These explosives are generally used for charging shells, as they are not detonated when fired from a gun.

Mercury Fulminate, formed by treating metallic mercury with nitric acid and alcohol, is extensively used as an exploder with dynamite, guncotton, picric acid, etc. It is a gray crystalline substance of specific gravity 4.42. It is easily exploded by heat, an electric spark usually being employed.

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[R. G. HUDSON.]

EXPONENTIAL FUNCTIONS. — (*See also Roots and Powers.*) When the relation between any variable y and another variable x is such that x occurs as an exponent of one or more terms, y is said to be an exponential function of x. Of particular importance in connection with electric circuits are the exponential functions ϵ^x and ϵ^{-x}, where ϵ is the base of the natural logarithms (*see Logarithms*).

ϵ^x may be obtained directly from the table of common logarithms (*see Logarithms*) from the relation that ϵ^x is equal to the number whose logarithm to the base 10 is $0.4343\,x$ (or more exactly $0.4342945\,x$), which may be written symbolically:

$$\epsilon^x = \log_{10}^{-1}(0.4343\,x).$$

Similarly,

$$\epsilon^{-x} = \frac{1}{\log_{10}^{-1}(0.4343\,x)}.$$

Example. — Find the value of ϵ^{10}. We have $0.4343 \times 10 = 4.343$, whence $\epsilon^{10} = 22{,}050$.

For convenience, however, the following table giving the value of ϵ^x and ϵ^{-x} from $x = 0$ to $x = 6$ is given. For larger values of x use the above relations. Also note that for $x < 0.1$ (*see Series*).

$$\epsilon^x = 1 + x \text{ with an error of less than 0.47 per cent,}$$

$$\epsilon^{-x} = 1 - x \text{ with an error of less than 0.54 per cent,}$$

$$\epsilon^x = 1 + x + \frac{x^2}{2} \text{ with an error of less than 0.016 per cent,}$$

$$\epsilon^{-x} = 1 - x + \frac{x^2}{2} \text{ with an error of less than 0.018 per cent.}$$

EXPONENTIAL FUNCTIONS ϵ^x AND ϵ^{-x} — 0.00 - 0.89

x	Function	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	ϵ^x	1.0000	1.0101	1.0202	1.0305	1.0408	1.0513	1.0618	1.0725	1.0833	1.0942
	ϵ^{-x}	1.0000	0.9900	0.9802	0.9704	0.9608	0.9512	0.9418	0.9324	0.9231	0.9139
0.1	ϵ^x	1.1052	1.1163	1.1275	1.1388	1.1503	1.1618	1.1735	1.1853	1.1972	1.2093
	ϵ^{-x}	0.9048	0.8958	0.8869	0.8781	0.8694	0.8607	0.8521	0.8437	0.8353	0.8270
0.2	ϵ^x	1.2214	1.2337	1.2461	1.2586	1.2712	1.2840	1.2969	1.3100	1.3231	1.3364
	ϵ^{-x}	0.8187	0.8106	0.8025	0.7945	0.7866	0.7788	0.7711	0.7634	0.7558	0.7483
0.3	ϵ^x	1.3499	1.3634	1.3771	1.3910	1.4049	1.4191	1.4333	1.4477	1.4623	1.4770
	ϵ^{-x}	0.7408	0.7334	0.7261	0.7189	0.7118	0.7047	0.6977	0.6907	0.6839	0.6771
0.4	ϵ^x	1.4918	1.5068	1.5220	1.5373	1.5527	1.5683	1.5841	1.6000	1.6161	1.6323
	ϵ^{-x}	0.6703	0.6637	0.6570	0.6505	0.6440	0.6376	0.6313	0.6250	0.6188	0.6126
0.5	ϵ^x	1.6487	1.6653	1.6820	1.6989	1.7160	1.7333	1.7507	1.7683	1.7860	1.8040
	ϵ^{-x}	0.6065	0.6005	0.5945	0.5886	0.5827	0.5769	0.5712	0.5655	0.5599	0.5543
0.6	ϵ^x	1.8221	1.8404	1.8589	1.8776	1.8965	1.9155	1.9348	1.9542	1.9739	1.9939
	ϵ^{-x}	0.5488	0.5434	0.5379	0.5326	0.5273	0.5220	0.5169	0.5117	0.5066	0.5017
0.7	ϵ^x	2.0138	2.0340	2.0544	2.0751	2.0959	2.1170	2.1383	2.1598	2.1815	2.2034
	ϵ^{-x}	0.4966	0.4916	0.4868	0.4819	0.4771	0.4724	0.4677	0.4630	0.4584	0.4538
0.8	ϵ^x	2.2255	2.2479	2.2705	2.2933	2.3164	2.3396	2.3632	2.3869	2.4109	2.4351
	ϵ^{-x}	0.4493	0.4449	0.4404	0.4360	0.4317	0.4274	0.4232	0.4190	0.4148	0.4107

EXPONENTIAL FUNCTIONS ϵ^x AND ϵ^{-x} 0.90-3.09

x	Function	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.9	ϵ^x	2.4596	2.4843	2.5093	2.5345	2.5600	2.5857	2.6117	2.6379	2.6645	2.6912
	ϵ^{-x}	0.4066	0.4025	0.3985	0.3946	0.3906	0.3867	0.3829	0.3791	0.3753	0.3716
1.0	ϵ^x	2.7183	2.7456	2.7732	2.8011	2.8292	2.8577	2.8864	2.9154	2.9447	2.9743
	ϵ^{-x}	0.3679	0.3642	0.3606	0.3570	0.3535	0.3499	0.3465	0.3430	0.3396	0.3362
1.1	ϵ^x	3.0042	3.0344	3.0649	3.0957	3.1268	3.1582	3.1899	3.2220	3.2544	3.2871
	ϵ^{-x}	0.3329	0.3296	0.3263	0.3230	0.3198	0.3166	0.3135	0.3104	0.3073	0.3042
1.2	ϵ^x	3.3201	3.3535	3.3872	3.4212	3.4556	3.4903	3.5254	3.5609	3.5966	3.6328
	ϵ^{-x}	0.3012	0.2982	0.2952	0.2923	0.2894	0.2865	0.2837	0.2808	0.2780	0.2753
1.3	ϵ^x	3.6693	3.7062	3.7434	3.7810	3.8190	3.8574	3.8962	3.9354	3.9749	4.0149
	ϵ^{-x}	0.2725	0.2698	0.2671	0.2645	0.2618	0.2592	0.2567	0.2541	0.2516	0.2491
1.4	ϵ^x	4.0552	4.0960	4.1371	4.1787	4.2207	4.2631	4.3060	4.3492	4.3929	4.4371
	ϵ^{-x}	0.2466	0.2441	0.2417	0.2393	0.2369	0.2346	0.2322	0.2299	0.2276	0.2254
1.5	ϵ^x	4.4817	4.5267	4.5722	4.6182	4.6646	4.7115	4.7588	4.8066	4.8550	4.9037
	ϵ^{-x}	0.2231	0.2209	0.2187	0.2165	0.2144	0.2122	0.2101	0.2080	0.2060	0.2039
1.6	ϵ^x	4.9530	5.0028	5.0531	5.1039	5.1552	5.2070	5.2593	5.3122	5.3656	5.4195
	ϵ^{-x}	0.2019	0.1999	0.1979	0.1959	0.1940	0.1920	0.1901	0.1882	0.1864	0.1845
1.7	ϵ^x	5.4739	5.5290	5.5845	5.6407	5.6973	5.7546	5.8124	5.8709	5.9299	5.9895
	ϵ^{-x}	0.1827	0.1809	0.1791	0.1773	0.1755	0.1738	0.1720	0.1703	0.1686	0.1670
1.8	ϵ^x	6.0496	6.1104	6.1719	6.2339	6.2965	6.3598	6.4237	6.4883	6.5535	6.6194
	ϵ^{-x}	0.1653	0.1637	0.1620	0.1604	0.1588	0.1572	0.1557	0.1541	0.1526	0.1511
1.9	ϵ^x	6.6859	6.7531	6.8210	6.8895	6.9588	7.0287	7.0993	7.1707	7.2427	7.3155
	ϵ^{-x}	0.1496	0.1481	0.1466	0.1451	0.1437	0.1423	0.1409	0.1395	0.1381	0.1367
2.0	ϵ^x	7.3891	7.4633	7.5383	7.6141	7.6906	7.7679	7.8460	7.9248	8.0045	8.0849
	ϵ^{-x}	0.1353	0.1340	0.1327	0.1313	0.1300	0.1287	0.1275	0.1262	0.1249	0.1237
2.1	ϵ^x	8.1662	8.2482	8.3311	8.4149	8.4994	8.5849	8.6711	8.7583	8.8463	8.9352
	ϵ^{-x}	0.1225	0.1212	0.1200	0.1188	0.1177	0.1165	0.1153	0.1142	0.1130	0.1119
2.2	ϵ^x	9.0250	9.1157	9.2073	9.2999	9.3933	9.4877	9.5831	9.6794	9.7767	9.8749
	ϵ^{-x}	0.1108	0.1097	0.1086	0.1075	0.1065	0.1054	0.1044	0.1033	0.1023	0.1013
2.3	ϵ^x	9.9742	10.074	10.176	10.278	10.381	10.486	10.591	10.697	10.805	10.913
	ϵ^{-x}	0.1003	0.0993	0.0983	0.0973	0.0963	0.0954	0.0944	0.0935	0.0926	0.0916
2.4	ϵ^x	11.023	11.134	11.246	11.359	11.473	11.588	11.705	11.822	11.941	12.061
	ϵ^{-x}	0.0907	0.0898	0.0889	0.0880	0.0872	0.0863	0.0854	0.0846	0.0837	0.0829
2.5	ϵ^x	12.182	12.305	12.429	12.554	12.680	12.807	12.936	13.066	13.197	13.330
	ϵ^{-x}	0.0821	0.0813	0.0805	0.0797	0.0789	0.0781	0.0773	0.0765	0.0758	0.0750
2.6	ϵ^x	13.464	13.599	13.736	13.874	14.013	14.154	14.296	14.440	14.585	14.732
	ϵ^{-x}	0.0743	0.0735	0.0728	0.0721	0.0714	0.0707	0.0699	0.0693	0.0686	0.0679
2.7	ϵ^x	14.880	15.029	15.180	15.333	15.487	15.643	15.800	15.959	16.119	16.281
	ϵ^{-x}	0.0672	0.0665	0.0659	0.0652	0.0646	0.0639	0.0633	0.0627	0.0620	0.0614
2.8	ϵ^x	16.445	16.610	16.777	16.945	17.116	17.288	17.462	17.637	17.814	17.993
	ϵ^{-x}	0.0608	0.0602	0.0596	0.0590	0.0584	0.0578	0.0573	0.0567	0.0561	0.0556
2.9	ϵ^x	18.174	18.357	18.541	18.728	18.916	19.106	19.298	19.492	19.688	19.886
	ϵ^{-x}	0.0550	0.0545	0.0539	0.0534	0.0529	0.0523	0.0518	0.0513	0.0508	0.0503
3.0	ϵ^x	20.086	20.287	20.491	20.697	20.905	21.115	21.328	21.542	21.758	21.977
	ϵ^{-x}	0.0498	0.0493	0.0488	0.0483	0.0478	0.0474	0.0469	0.0464	0.0460	0.0455

EXPONENTIAL FUNCTIONS ϵ^x AND ϵ^{-x}

x	Function	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
3.1	ϵ^x	22.198	22.421	22.646	22.874	23.104	23.336	23.571	23.807	24.047	24.288
	ϵ^{-x}	0.0450	0.0446	0.0442	0.0437	0.0433	0.0429	0.0424	0.0420	0.0416	0.0412
3.2	ϵ^x	24.533	24.779	25.028	25.280	25.534	25.790	26.050	26.311	26.576	26.843
	ϵ^{-x}	0.0408	0.0404	0.0400	0.0396	0.0392	0.0388	0.0384	0.0380	0.0376	0.0373
3.3	ϵ^x	27.113	27.385	27.660	27.938	28.219	28.503	28.789	29.079	29.371	29.666
	ϵ^{-x}	0.0369	0.0365	0.0362	0.0358	0.0354	0.0351	0.0347	0.0344	0.0340	0.0337
3.4	ϵ^x	29.964	30.265	30.569	30.877	31.187	31.500	31.817	32.137	32.460	32.786
	ϵ^{-x}	0.0334	0.0330	0.0327	0.0324	0.0321	0.0317	0.0314	0.0311	0.0308	0.0305
3.5	ϵ^x	33.115	33.448	33.784	34.124	34.467	34.813	35.163	35.517	35.874	36.234
	ϵ^{-x}	0.0302	0.0299	0.0296	0.0293	0.0290	0.0287	0.0284	0.0282	0.0279	0.0276
3.6	ϵ^x	36.598	36.966	37.338	37.713	38.092	38.475	38.861	39.252	39.646	40.045
	ϵ^{-x}	0.0273	0.0271	0.0268	0.0265	0.0263	0.0260	0.0257	0.0255	0.0252	0.0250
3.7	ϵ^x	40.447	40.854	41.264	41.679	42.098	42.521	42.948	43.380	43.816	44.256
	ϵ^{-x}	0.0247	0.0245	0.0242	0.0240	0.0238	0.0235	0.0233	0.0231	0.0228	0.0226
3.8	ϵ^x	44.701	45.150	45.604	46.063	46.525	46.993	47.465	47.942	48.424	48.911
	ϵ^{-x}	0.0224	0.0221	0.0219	0.0217	0.0215	0.0213	0.0211	0.0209	0.0207	0.0204
3.9	ϵ^x	49.402	49.899	50.400	50.907	51.419	51.935	52.457	52.985	53.517	54.055
	ϵ^{-x}	0.0202	0.0200	0.0198	0.0196	0.0195	0.0193	0.0191	0.0189	0.0187	0.0185
4.0	ϵ^x	54.598	55.147	55.701	56.261	56.826	57.397	57.974	58.557	59.145	59.740
	ϵ^{-x}	0.0183	0.0181	0.0180	0.0178	0.0176	0.0174	0.0172	0.0171	0.0169	0.0167
4.1	ϵ^x	60.340	60.947	61.559	62.178	62.803	63.434	64.072	64.715	65.366	66.023
	ϵ^{-x}	0.0166	0.0164	0.0162	0.0161	0.0159	0.0158	0.0156	0.0155	0.0153	0.0151
4.2	ϵ^x	66.686	67.357	68.033	68.717	69.408	70.105	70.810	71.522	72.240	72.966
	ϵ^{-x}	0.0150	0.0148	0.0147	0.0146	0.0144	0.0143	0.0141	0.0140	0.0138	0.0137
4.3	ϵ^x	73.700	74.440	75.189	75.944	76.708	77.478	78.257	79.044	79.838	80.640
	ϵ^{-x}	0.0136	0.0134	0.0133	0.0132	0.0130	0.0129	0.0128	0.0127	0.0125	0.0124
4.4	ϵ^x	81.451	82.269	83.096	83.931	84.775	85.627	86.488	87.357	88.235	89.121
	ϵ^{-x}	0.0123	0.0122	0.0120	0.0119	0.0118	0.0117	0.0116	0.0114	0.0113	0.0112
4.5	ϵ^x	90.017	90.922	91.836	92.759	93.691	94.632	95.583	96.544	97.514	98.494
	ϵ^{-x}	0.0111	0.0110	0.0109	0.0108	0.0107	0.0106	0.0105	0.0104	0.0103	0.0102
4.6	ϵ^x	99.484	100.48	101.49	102.51	103.54	104.58	105.64	106.70	107.77	108.85
	ϵ^{-x}	0.0101	0.0100	0.0099	0.0098	0.0097	0.0096	0.0095	0.0094	0.0093	0.0092
4.7	ϵ^x	109.95	111.05	112.17	113.30	114.43	115.58	116.75	117.92	119.10	120.30
	ϵ^{-x}	0.0091	0.0090	0.0089	0.0088	0.0087	0.0087	0.0086	0.0085	0.0084	0.0083
4.8	ϵ^x	121.51	122.73	123.97	125.21	126.47	127.74	129.02	130.32	131.63	132.95
	ϵ^{-x}	0.0082	0.0081	0.0081	0.0080	0.0079	0.0078	0.0078	0.0077	0.0076	0.0075
4.9	ϵ^x	134.29	135.64	137.00	138.38	139.77	141.17	142.59	144.03	145.47	146.94
	ϵ^{-x}	0.0074	0.0074	0.0073	0.0072	0.0072	0.0071	0.0070	0.0069	0.0069	0.0068
5.0	ϵ^x	148.41	149.90	151.41	152.93	154.47	156.02	157.59	159.17	160.77	162.39
	ϵ^{-x}	0.0067	0.0067	0.0066	0.0065	0.0065	0.0064	0.0063	0.0063	0.0062	0.0062
5.1	ϵ^x	164.02	165.67	167.34	169.02	170.72	172.43	174.16	175.91	177.68	179.47
	ϵ^{-x}	0.0061	0.0060	0.0060	0.0059	0.0059	0.0058	0.0057	0.0057	0.0056	0.0056
5.2	ϵ^x	181.27	183.09	184.93	186.79	188.67	190.57	192.48	194.42	196.37	198.34
	ϵ^{-x}	0.0055	0.0055	0.0054	0.0054	0.0053	0.0052	0.0052	0.0051	0.0051	0.0050

EXPONENTIAL FUNCTIONS ϵ^x AND ϵ^{-x} 5.30–5.99

x	Function	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
5.3	ϵ^x	200.34	202.35	204.38	206.44	208.51	210.61	212.72	214.86	217.02	219.20
	ϵ^{-x}	0.0050	0.0049	0.0049	0.0048	0.0048	0.0047	0.0047	0.0047	0.0046	0.0046
5.4	ϵ^x	221.41	223.63	225.88	228.15	230.44	232.76	235.10	237.46	239.85	242.26
	ϵ^{-x}	0.0045	0.0045	0.0044	0.0044	0.0043	0.0043	0.0043	0.0042	0.0042	0.0041
5.5	ϵ^x	244.69	247.15	249.64	252.14	254.68	257.24	259.82	262.43	265.07	267.74
	ϵ^{-x}	0.0041	0.0040	0.0040	0.0040	0.0039	0.0039	0.0038	0.0038	0.0038	0.0037
5.6	ϵ^x	270.43	273.14	275.89	278.66	281.46	284.29	287.15	290.03	292.95	295.89
	ϵ^{-x}	0.0037	0.0037	0.0036	0.0036	0.0036	0.0035	0.0035	0.0034	0.0034	0.0034
5.7	ϵ^x	298.87	301.87	304.90	307.97	311.06	314.19	317.35	320.54	323.76	327.01
	ϵ^{-x}	0.0033	0.0033	0.0033	0.0032	0.0032	0.0032	0.0032	0.0031	0.0031	0.0031
5.8	ϵ^x	330.30	333.62	336.97	340.36	343.78	347.23	350.72	354.25	357.81	361.41
	ϵ^{-x}	0.0030	0.0030	0.0030	0.0029	0.0029	0.0029	0.0029	0.0028	0.0028	0.0028
5.9	ϵ^x	365.04	368.71	372.41	376.15	379.93	383.75	387.61	391.51	395.44	399.41
	ϵ^{-x}	0.0027	0.0027	0.0027	0.0027	0.0026	0.0026	0.0026	0.0026	0.0025	0.0025

A more complete table is given in the *Smithsonian Mathematical Tables, Hyperbolic Functions* (*No. 1871*), by G. F. Becker and C. E. Van Orstrand, Washington, 1909. Semi-exponentials, i.e., values of $\frac{1}{2}\epsilon^x$ and $\log_{10} \frac{1}{2}\epsilon^x$, are given in *Tables of Complex Hyperbolic and Circular Functions*, by A. E. Kennelly, Cambridge, Mass., 1914.

[W. A. Del Mar.]

FACTORIALS. — The multiple product represented by

$$n(n-1)(n-2)\ldots 3 \times 2 \times 1$$

is called "*n* factorial," and is represented either by the symbol $n!$ or $\underline{n|}$. The following table gives the value of $n!$ up to $n = 10$.

1! =	1.	6! =	720.
2! =	2.	7! =	5040.
3! =	6.	8! =	40320.
4! =	24.	9! =	362880.
5! =	120.	10! =	3628800.

[W. A. Del Mar.]

FANS. — (*See also Blowers and Compressors; Draft, Mechanical.*) For ventilating and drying purposes and for producing mechanical draft, two kinds of fans are used, the disk fan and the centrifugal fan.

Disk Fans. — A windmill reversed, that is, driven by power applied to the shaft, may be used to blow air in large volume at low pressure, and it is then called a disk fan. Such fans are in common use for blowing air into or out of a room for the purpose of ventilation. Usually they consist simply of a rapidly rotating shaft carrying four or more nearly flat blades slightly inclined to the plane of rotation. Sometimes these blades are given various curved forms for the purpose of increasing the efficiency of the fan.

Centrifugal Fans. — The older form of centrifugal fan is that of a paddle wheel, a rotating shaft carrying from four to twelve radial arms, to which are attached flat blades in axial planes. In modified forms the blades are bent backward or forward, with reference to the direction of rotation, from an axial plane. Very large centrifugal fans up to 25 feet diameter and 10 feet width have been built for ventilation of mines, and in many cases without an external casing, but usually a spiral casing is used, with a tangential discharge outlet, with two "eyes," or circular inlets, concentric with the shaft, one on each side of the casing. In a recent modification of the centrifugal fan known as the "multivane" or "Sirocco" fan, the number of blades is increased to from 30 to 80; the blades or vanes are made relatively very long, axially, and narrow, radially, the ratio of length to width being as much as 9 to 1; they are made of curved form, the outer edge tipped forward in the direction of rotation and the diameter of the eyes in the casing and the radial distance of the vanes from the shaft are greatly increased. The pressure produced by centrifugal fans varies practically as the square of the speed of the tips of the blades and with the forms of the blades, and reaches a maximum commercially of about 1 pound per square inch for ordinary types of centrifugal or multivane fans. Much higher pressures have recently been obtained from special forms of fans built on the principles of steam turbines (*see Blowers and Compressors*).

POWER REQUIRED. — In the article on *Blowers and Compressors* is given the formula for "useful" work done by a blowing machine. Since an ordinary fan seldom produces a pressure of more than 1 per cent of normal atmospheric pressure, the change in the volume and static pressure of the air as it passes through a fan is negligibly small in comparison with the initial volume and pressure, and the formula for the theoretical horse power reduces to

$$P = \frac{1}{33,000}\left[\frac{dQ^3}{2\,g\,(60\,A)^2} + 144\,p_sQ\right],$$

where

p_s = the excess of static pressure at discharge over the initial static pressure, in pounds per square inch,

Q = cubic feet of air displaced per minute,

$d = \dfrac{1.33\,B}{460 + t}$ = weight of one cubic foot of air at the barometric pressure B (inches of mercury) and at $t°$ F.,

A = cross section of full outlet opening in square feet,

g = acceleration due to gravity in feet per second (= 32.2),

$p_v = \dfrac{d}{2 \times 144\,g}\left(\dfrac{Q}{60\,A}\right)^2$ = pressure in pounds per square inch necessary to give an average linear velocity $V_a = \dfrac{Q}{A}$ feet per minute to the air.

This pressure p_v is called the "velocity" pressure.

Useful or "Air Horse Power" of the fan may then be written

$$P = \frac{(p_v + p_s)\,Q}{229}.$$

The total pressure $p_v + p_s$ developed by the fan is called the "impact" or "dynamic" pressure. The pressures developed by a fan are usually expressed in inches of water column; hence putting H_v and H_s for the corresponding inches of water column required to balance p_v and p_s, the above formulas become

$$H_v = 0.832 \times 10^{-6} \frac{d\,Q^2}{A^2},$$

$$P = \frac{(H_v + H_s)\,Q}{6360}.$$

When the discharge outlet is restricted (e.g., by partially closing the discharge valve), the cross section A refers to the area of the discharge pipe between the restriction and the fan, not to the cross section of the restricted opening.

Actual Horse Power. — The actual horse power required to drive the fan is equal to the value given by the last formula divided by the efficiency. The fan efficiency depends not only upon the design of the fan, but also upon the relative values of the velocity and static pressures. The maximum efficiency of a centrifugal fan ranges from 40 to 70 per cent. The maximum efficiency of disk fans usually ranges from 30 to 40 per cent. The table below, taken from curves published by F. R. Still of the American Blower Co., in the *Jour. Wes. Soc. Eng.*, 1902, is fairly representative of the performance of ordinary steel-plate centrifugal fans. For a given type of fan and given ratio of velocity pressure within the casing on the outlet side to the static pressure, there is a fixed relation between the linear velocity $V_a = \frac{Q}{A}$ of the air within the casing on the outlet side and the peripheral velocity V_f of the fan; the ratio $\frac{V_a}{V_f}$, as calculated from Still's curves, is also given in the table below.

$\frac{H_v}{H_s}$	Efficiency	$\frac{V_a}{V_f}$	$\frac{H_v}{H_s}$	Efficiency	$\frac{V_a}{V_f}$
0.00	0.00	0.00	0.4	0.38	0.45
0.05	0.30	0.22	0.5	0.37	0.47
0.10	0.43	0.30	1.0	0.32	0.51
0.15	0.43	0.35	1.5	0.30	0.53
0.20	0.42	0.39	2.0	0.28	0.54
0.30	0.41	0.42	∞	0.22	0.58

More recent fans of the multivane or "Sirocco" type, when properly proportioned, have given efficiencies of 60 per cent and upwards.

THE VOLUME OF AIR DISCHARGED per minute is $Q = V_a A$ where V_a is the linear velocity of the air in feet per minute. When the speed is kept constant and the outlet pipe is partially closed, the linear velocity of the air through the full cross section A of the outlet decreases and the power required

to drive the fan decreases. A fan running at constant speed requires maximum power when the discharge outlet is wide open; the volume discharge is also a maximum under these conditions.

For an ordinary steel-plate fan running at a given speed the maximum volume of air per minute which it can deliver is roughly $\frac{1}{3}DWV_f = \frac{\pi}{3}D^2WN$, where D is the diameter of the fan in feet, W its width (at periphery) in feet, V_f the peripheral velocity of the fan in feet per minute and N the velocity in revolutions per minute.

The capacity of disk fans working against very low resistances is directly proportional to the area of the circle inclosing the fan and to the number of revolutions per minute, and varies with the form and inclination of the vanes. An ordinary type of propeller fan of diameter D feet will deliver $Q = KD^3N$ cubic feet per minute, where K ranges from 0.4 to 0.7 (depending on the design of the fan) when the resistance is small, the delivery decreasing rapidly as the resistance is increased.

DIMENSIONS AND SPEED.— The full discharge outlet A of an ordinary steel plate fan is usually about two-thirds the product of the width W by the diameter D of the fan, that is $A = \frac{2}{3}DW$. The width of the fan ranges from one-third to one-half the diameter. Using the latter figure, we have $D = \sqrt{3A}$. From this formula and the table given above a rough estimate of the dimensions of a fan for a given service may be made. For example, 20,000 cubic feet of air per minute at 60° F. is to be discharged against a static head of 1 inch of water; what must be the approximate dimensions and speed of the fan and what will be the approximate horse power required? The size of fan and the horse power required will depend upon the linear velocity of the air through the fan, the greater this velocity the smaller the fan and the greater the horse power. Using the symbols as defined above we have $Q = 20{,}000$, $H_s = 1$, $d = 0.0766$. A reasonable value of the linear velocity V_a is 2000 feet per minute. Then $A = 10$, $D = 5.48$, $W = 2.74$, $H_v = 0.255$, $\frac{H_v}{H_s} = 0.255$, $\frac{V_a}{V_f} = 0.41$, $V_f = 4880$, $N = 284$, efficiency $= 0.415$, H.P. $= 9.6$. That is, the diameter of the fan should be approximately 5 feet 6 inches, its width 2 feet 9 inches, its speed 285 r.p.m., and 9.6 horse power will be required to operate it. If V_a is taken as 4000 feet per minute, then the required diameter will be approximately 3 feet 10 inches, its width 1 foot 11 inches, its speed 650 r.p.m., and 20 horse power will be required to operate it.

The above method of arriving at the size of fan is extremely rough at best; for more accurate data the reader is referred to manufacturers' catalogues.

SPECIFICATIONS FOR FANS.*— The following memoranda are intended to assist in writing specifications. See also article on *Blowers* and article on *Specifications*.

General description of fan and service for which it is to be used. Rating, cubic feet of air per minute, under specified conditions of test. Whether exhaust or pressure. How supported. Whether fixed or oscillating. Details of motor, i.e., whether a-c. or d-c., frequency, phases and voltage.

MOTOR DRIVE.† — (*See also Motors, Industrial Applications of.*) The load on a fan motor increases as the speed rises and unless some sort of shuttering is provided the motor will be fully loaded when it comes up to speed. Shuttering is essential and the starting conditions should be carefully considered, when a synchronous motor is intended for driving the fan. As the torque

* By W. A. Del Mar. † By D. B. Rushmore.

required to start a fan is usually small, a motor with low-resistance starting winding may be used without requiring a very heavy current. To keep the current down, however, a low tap on the starting compensator must be used and an intermediate step may be very desirable.

Fig. 1 shows the starting-torque curve of a shuttered centrifugal fan from rest to full speed, and also a speed-torque and a speed-current curve of a synchronous motor with a squirrel-cage starting winding of fairly low resistance plotted for 50, 70 and 100 per cent of normal voltage. If the motor is started from the 50 per cent tap of the compensator it will bring the fan to about 80 per cent of full speed. If the motor is then thrown over to the full-line voltage it may exert an excessive torque, and the current, as shown by the current curve, will rise to a very high value. If an intermediate tap is provided — say a 70 per cent tap — the torque exerted will be less but still enough to bring the motor to about 95 per cent of synchronous speed and the current will be much less. Application of the field would then pull the machine into step after which it could be thrown over to the line with a very little rush of current. For a variable-speed service the synchronous motor is of course not suitable and a phase-wound induction motor must be used.

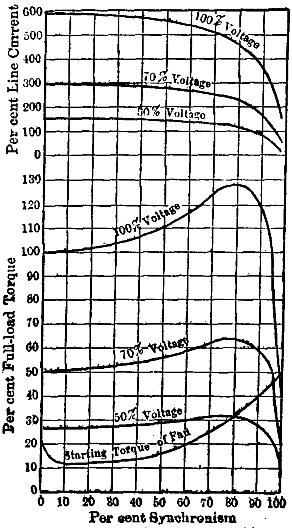

Fig. 1. Curves of Synchronous Motor with Low-resistance Starting Winding

For direct-current installations shunt motors are preferred, the speed adjustment being accomplished either by field control, armature control or a combination of the two. Field control is generally used where the motor operates for long intervals at speeds between 50 and 100 per cent of maximum speed; the combination control is used if the motor operates for long intervals at speeds between 75 and 100 per cent of maximum speed, and is occasionally required to operate at speeds below 75 per cent; and armature control is used advantageously for motors operating at long intervals near full speed, but where occasionally a lower speed is required. Ventilating fans offer one of the best applications for armature control because the torque and current decreases with the cube of the speed.

With very large mine fans, having a high inertia, the time required for acceleration is comparatively long — one minute or more — and this must be considered when the starting resistances are selected. Starting resistances for motors driving such large fans should therefore as a rule be of a larger size than would ordinarily be the case.

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[Wm. Kent.]

FEED-WATER HEATERS AND PURIFIERS.—(*See also Boilers; Power Stations, Steam Electric.*) Impurities in the water supplied to a boiler may cause: (1) the formation of scale on the heating surface of the boiler, resulting in a decrease of boiler efficiency and overheating and consequent weakening of the tubes and plates; (2) the corrosion and consequent weakening of the tubes and plates; (3) an increase in the amount of suspended moisture carried over by the steam from boiler to engine. Sulphates of lime and magnesia, soluble salts of silica, iron and aluminum and suspended matter are the chief scale-forming impurities. Acids, organic matter, magnesium chloride and sulphate cause corrosion, while priming is induced by the presence of organic matter, sodium carbonate and other alkalies.

By the introduction into the feed water of various chemicals, as, for example, carbonate of soda, the scale-forming impurities may be changed into harmless substances by the reaction between the impurities and the "boiler compounds" introduced. Again, the adhesion of the scale to the heating surfaces may be prevented by the introduction into the feed water or boiler of such substances as kerosene oil. Suspended matter, such as sand, mud, insoluble organic matter, etc., may be eliminated by mechanical filtration or by allowing the water to stand in settling tanks. Organic impurities usually float on top of the water when the boiler is making steam and may be blown out through a "surface blow-out." Precipitated matter may be ejected by frequent blowing off before it has time to form a crust. Scale, when it has once formed, can be removed only by cleaning the tubes with some form of scraper. Very hard sulphate-of-lime scale may be softened so as to be more easily scraped, by dissolving a considerable quantity, say 50 to 100 pounds, of caustic soda in the boiler and slowly boiling it at atmospheric pressure for from 12 to 24 hours. Carbonates of lime and magnesia, two of the chief scale-forming impurities, may be almost completely precipitated by raising the temperature of the water to 290° F. (= boiling point for a pressure of about 60 pounds absolute).

SAVING DUE TO PREHEATING FEED WATER.—Although the heating and subsequent filtering or settling of the feed water results in the elimination of certain of the impurities, the primary object of preheating the feed water is to reduce the coal consumption. The heat required to raise the temperature of 1 pound of feed water 11° F. is approximately 1 per cent of the total heat required to convert the water into saturated steam at ordinary pressure. Hence, for every 11° F. the temperature of the feed water is raised by the application of heat which would otherwise be wasted, a saving of approximately 1 per cent will be effected in the amount of coal required. Let H = total heat of the steam at the given pressure, h_0 = heat of the water entering the heater, h = heat of the water leaving the heater, then the saving, expressed as a percentage of the heat that would be required by a boiler without a superheater, is

$$S = 100\frac{h-h_0}{H-h_0}.$$

The values of H, h and h_0 may be taken from steam tables (*see Steam*). This formula also gives the saving in fuel, provided the boiler efficiency is the same with or without the feed-water heater. However, if the boiler is overdriven, the installation of a feed-water heater will effect a greater saving than that given by the formula, since when the work to be done by the boiler is reduced the boiler efficiency will also increase.

TYPES OF FEED-WATER HEATERS.—The heat used for preheating the feed water may be derived: (1) from the exhaust steam; (2) from the flue

gases, in which case the heater is usually called a "fuel economizer"; (3) from live steam taken directly from the boilers, in which case the primary function of the heater is to purify the feed water, and it is therefore usually called a "live-steam purifier." The heat may be transmitted from the steam to the water either (1) by allowing the steam to mingle with the water and give up its heat by condensation, or (2) by passing the water and steam through separate chambers arranged so that the heat is conducted to the water through the walls of the chambers. These two types of heaters are referred to respectively as "open" and "closed" heaters. Closed heaters may be either of the water-tube or steam-tube type; in the former the steam surrounds a set of tubes through which the water is passed, while in the latter the steam is passed through a set of tubes surrounded by water. A closed heater in which the steam pressure is less than atmospheric is called a "vacuum" or "primary" heater; such heaters are frequently used in the exhaust of condensing engines. "Atmospheric" or "secondary" heaters are those in which the steam pressure is that corresponding to the back pressure of the engines or pumps. A heater in which the steam pressure is the same as that in the boiler is called a "pressure" heater.

Gebhardt gives the following list of typical heaters used in this country:

- Exhaust steam
  - Open — Atmospheric: Cochrane. Hoppes. Stillwell. Webster.
  - Closed { Atmospheric, Vacuum, or Pressure:
    - Wainwright, Wheeler — Water Tube.
    - Otis, Berryman — Steam Tube.
- Flue gas: Green. American. Sturtevant.
- Live steam — Open — Pressure: Hoppes. Baragwanath.

Open Heaters. — The essential parts of an open heater are: (1) a shell containing (2) a set of trays or pans to catch the scale-forming elements precipitated from the water; (3) a filter to take out suspended impurities; and (4) an oil separator to extract the oil from the steam before it enters the superheater.

Dimensions. — C. L. Hubbard (*Practical Engineer, Jan. 1, 1909*) gives the following:

Exhaust heaters should be proportioned according to the quality of the water to be used, the size being increased with the amount of mud or scale-producing properties which the water contains, regardless of the quantity of water to be heated. The general proportions of an open heater will depend somewhat upon the arrangement of the trays or pans, but an approximation of the size of shell for a cylindrical heater is as follows: $A = H \div aL$; $L = H \div aA$; in which A = sectional area of shell in square feet; L = length of shell in linear feet; H = total weight of water to be heated per hour divided by the weight of steam used per horse power per hour by the engine; a = 2.15 for very muddy water, 6.0 for slightly muddy water, and 8.0 for clear water.

The pan or tray surface varies according to the quality of the water, both as regards the amount of mud and the scale-making ingredients. The surface

in square feet for each 1000 pounds of water heated per hour may be taken as follows, for the vertical and horizontal types respectively:

Very bad water	8.5 and 9.1
Medium muddy water	6 and 6.5
Clear and little scale	2 and 2.2

The space between the pans is made not less than 0.1 of the width for rectangular and 0.25 of the diameter for round pans. Under ordinary circumstances it is not customary to use more than six pans in a tier, in order to obtain a low velocity over each pan. The size of the storage or settling chamber in the horizontal type varies from 0.25 to 0.4 of the volume of the shell, depending on the quality of the water; 0.33 is about the average. In the case of vertical heaters, this varies from 0.4 to 0.6 of the volume of the shell. Filters occupy from 10 to 15 per cent of the volume of the shell in the horizontal type and from 15 to 20 per cent in the vertical.

Temperature Elevation of Feed Water. — Let H = total heat of steam in B.t.u. above 32° F. at the pressure of the steam in the heater; I = initial temperature of the water entering the heater; F = final temperature of the water leaving the heater; K = ratio of weight of exhaust steam condensed per hour in the heater to weight of the feed water per hour; then, neglecting loss due to radiation,

$$F = \frac{I + K(H + 32)}{1 + K}.$$

Closed Heaters. — (*H. L. Hepburn, Power, April, 1902.*) Let W = pounds feed water per hour; A = sq. ft. of heating surface between steam and water; T_s = temperature of the steam; I = initial temperature of the water; F = final temperature of the water; U = B.t.u. transmitted per sq. ft. per hr. per deg. difference of temperature between the steam and the water; then

$$A = \frac{2.30\,W}{U} \log_{10} \frac{T_s - I}{T_s - F}.$$

The value of U varies widely according to the condition of the surface and with the velocity of the water, and also depends upon whether the heater is of the water-tube or steam-tube type. Gebhardt gives the following values of U: for multi-flow water-tube heaters, 250 for plain copper or brass tubes and 300 for corrugated tubes with a water velocity of 50 feet per minute; for single-flow water-tube heaters, 175 for plain brass with a water velocity of 12.5 feet per minute; for coil water-tube heaters, 300 for copper tubes with a water velocity of 150 feet per minute; for steam-tube heaters, 120 for iron tubes.

Economizers. — Economizers for boiler plants are usually made of vertical cast-iron tubes contained in a long rectangular chamber of brickwork. The feed water enters the bank of tubes at one end, while the hot gases enter the chamber at the other end and travel in the opposite direction to the water. The tubes are made of cast iron because it is more non-corrosive than wrought iron or steel when exposed to gases of combustion at low temperatures. An automatic scraping device is usually provided for the purpose of removing dust from the outer surface of the tubes.

The amount of saving of fuel that may be made by an economizer varies greatly according to the conditions of operation. With a given quantity of chimney gases to be passed through it, its economy will be greater (1) the higher the temperature of these gases; (2) the lower the temperature of the water fed into it; and (3) the greater the amount of its heating surface.

The maximum saving of fuel which may be made by the use of an economizer when attached to boilers that are working with reasonable economy is about 15 per cent. If the boilers are not working with fair economy on account of being overdriven, then the saving made by the addition of an economizer may be much greater.

Temperature Elevation of Feed Water. — Let A = square feet of economizer heating surface per boiler horse power, T_1 = temperature (Fahrenheit) of flue gases entering economizer (ranges from about 450 to 700° F.); I = temperature of feed water entering economizer; F = temperature of feed water leaving economizer; W = pounds feed water per boiler horse power per hour; G = pounds flue gas per pound coal; C = pounds coal per boiler horse-power hour; S = specific heat of flue gases; U = B.t.u. transmitted per sq. ft. per hr. per deg. difference of temperature between flue gas and water; then

$$F - I = \frac{A\,(T_1 - I)}{\dfrac{W}{U} + \dfrac{(W + CGS)\,A}{2\,CGS}}.$$

A varies from 3.5 to 5, and U from 2.25 to 3.3, depending upon the conditions of operation. Putting $W = 30$, $S = 0.2$ and $U = 3.3$, the above formula reduces to

$$F - I = \frac{A\,(T_1 - I)}{9.1 + \dfrac{(5\,W + GC)\,A}{2\,GC}},$$

which is the formula advocated by the Green Economizer Co.

Rating and Cost. — Economizers are sometimes rated by tubes, the usual area per tube being 15 square feet. From 3.5 to 5 square feet are usually installed per boiler horse power. Economizers cost approximately $1.25 per square foot of heating surface, this figure including erection and brick setting.

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[WM. KENT.]

FIRE-ALARM TELEGRAPH. — (*See also Telegraph Instruments and Apparatus; Telegraph Systems; Wiring of Buildings for Miscellaneous Devices.*) The object of fire-alarm telegraph systems is to notify the fire-fighting forces of a community promptly of the existence and location of a fire, and also to afford a means of communication between the various branches of the fire-fighting organization whether at a fire or in quarters.

The simplest form of fire-alarm system is shown in Fig. 1, where several fire alarm boxes are arranged in series in the alarm or "box" circuit extending from the fire-alarm headquarters throughout the district covered by the boxes. These boxes are provided with a clockwork mechanism, so that when the box is started or "pulled" by the person sending the alarm, a break wheel, carrying notches

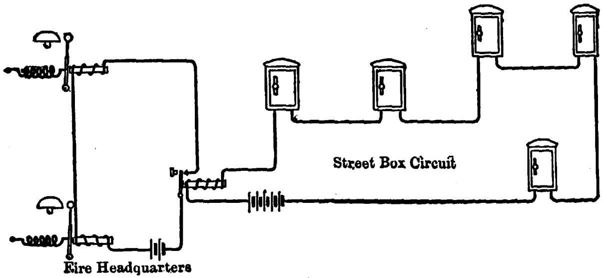

Fig. 1. Fire-Alarm System

corresponding to the code number of the box, is caused to revolve, thus making and breaking the circuit in accordance with the code. These makes and breaks cause the operation of the relay at headquarters and by means of a local circuit the gong sounds the alarm at headquarters. As many gongs as may be desired may be operated by the same relay, these being placed in the engine-house, men's quarters, or wherever else it is desired to give the alarm.

Types of Boxes. — The placing of a number of boxes on the same circuit gives rise to the possibility of confusion, due to two or more boxes being pulled at once. This is called "interference." A box which makes no provision against this confusion is called an "interfering box." A common type of box, termed the "non-interfering box," is so arranged that if two or more boxes on the same circuit are pulled at once the signal of the first box to be started will be transmitted without interference from the others, but the signals of the others will not be transmitted. In a later development, and one that is largely used, the arrangement is such that if any number of boxes up to four on the same circuit are simultaneously pulled, no interference and no loss of signals will be entailed, each box securing possession of the line in succession, the deferred boxes waiting their turn, so that the signals of all the boxes will be transmitted without confusion. These are called "non-interference succession boxes."

Gong Circuits. — In fire departments which have a number of engine and other apparatus houses, it is customary to establish a single fire-alarm headquarters, at which all box circuits center. From this headquarters other circuits, termed "gong circuits," extend to the apparatus houses. An alarm received over a box circuit may be transmitted to the gong circuits either manually or automatically.

Manual Systems. — In manual systems the box alarm is received at headquarters and then is set up on a transmitter, which, when started, automatically repeats the alarm over the desired gong circuits. The manual part of this operation may consist in placing a disc, notched on its periphery to accord with the desired box number, in the transmitter, and then starting the mechanism, the notches on the disc effecting the desired makes and breaks in the gong circuits. This method is employed in the fire-alarm system of the Borough of Manhattan, New York.

Another manual method of transmission is to set up the desired box number on a dial transmitter, which, when set in motion, transmits the alarm to the gong circuits. Still another method is to actually re-transmit the box numbers manually by means of a Morse key.

Automatic Systems. — In these the relays at headquarters, which are operated by the box circuits, automatically perform the making and breaking of the gong circuits, and thus re-transmit the box alarm to the gong circuits. This is advantageous in point of time saving, but has certain objectionable features in removing all discretionary power from the operators at headquarters.

Confusion in Fire-alarm Nomenclature. — Considerable confusion exists in the nomenclature of the fire-alarm art in various parts of the country. For instance, the term "automatic system," instead of applying to the automatic re-transmission of the alarm at fire headquarters, is frequently applied to those systems where the original alarm is automatically sent, as by the operation of a thermostat in a building under the influence of undue heat.

BIBLIOGRAPHY. — See *Bibliography* in article on *Telegraph Systems*.

[S. G. McMeen.]

FLUXMETER. — (*See also Galvanometers.*) The Grassot fluxmeter is essentially a dead-beat galvanometer, the suspension fiber of which is designed to exert no appreciable torsional force on the moving system when the latter is displaced.

Construction. — Fig. 1 is a diagram showing its construction. The coil *C* swings in the uniform air gap between the pole pieces *NS* of a permanent magnet and the soft iron core *A*. The system is supported at the top by a single cocoon fiber, the upper end of this fiber being attached to a flat spiral spring *R*, to minimize the effect of shocks. The torsional force exerted by such a fiber is extremely small. *S* and S_1 are very thin silver strips, serving to lead the current to and from the coil *C*. These strips are in the form of springs, which, however, are extremely weak and therefore exert but a small theoretically inappreciable torsional force on the coil. An index or pointer is attached to the instrument, this pointer swinging over a calibrated scale. The fluxmeter is also provided with a mirror in addition to the pointer so that it may be used in conjunction with a lamp and scale or with a telescope and scale.

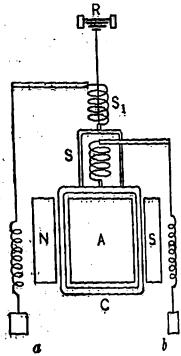

Fig. 1. Grassot Fluxmeter System

Principle of Operation. — When a given quantity of electricity is discharged through the moving coil, for example by changing the flux threading an exploring coil connected to the terminals *a* and *b*, a force is exerted upon the coil tending to deflect it. The only opposing forces (neglecting the small and theoretically inappreciable torsional forces) are the mechanical friction to motion and the back e.m.f. induced in the coil due to its motion through the field of the permanent magnet. The latter is proportional at any instant to the velocity of the coil, and the frictional force is also approximately proportional to this velocity. If both forces are directly proportional to the velocity, when a given quantity of electricity is discharged through the circuit, the coil comes to rest at a definite point, depending only upon its initial or "zero" position and the total quantity discharged through it. As the quantity of electricity discharged through an exploring coil, when the magnetic flux threading it is changed, depends solely upon the change in flux and the resistance of the coil and the rest of the circuit in series with it, the instrument may be calibrated to read directly the change in the flux density produced in the region occupied by the coil. In practice the friction is not exactly proportional to the velocity and the fiber and leading in springs usually exert an appreciable force on the coil. The instrument therefore has not proved altogether satisfactory.

Applications. — In motor or in dynamo work an exploring coil, consisting of one or more turns of wire, may be fixed or wound in position and the change in flux of induction observed on exciting the field magnet. Even in the largest work, where some minutes may be taken to reach the limit of magnetization, no error is thus introduced. The fluxmeter can also be used for the measurement of magnetic field strength, pole strength and the distribution of magnetism in bar magnets. It is also adapted to the measurement of permeability (q.v.) and hysteresis (q.v.).

Cost. — A direct reading fluxmeter with one exploring coil costs about $75. Additional exploring coils cost from $8 to $10 each.

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FLYWHEELS FOR LOAD EQUALIZATION. — (*See also Hoists; Motors, Industrial Application of; Steel Mills, Electric Drive of.*) There are two methods in general use for equalizing a fluctuating load, the storage battery and the flywheel. The former is mostly used in connection with a central service where the peaks are of considerable duration and for which service the flywheel is entirely unsuitable. For short fluctuating loads, such as steel-mill service, mine hoists, etc., the flywheel is particularly suitable, and it may be applied either to the driving motor direct or to an intermediate motor-generator equalizing set, depending on the operating conditions.

Selection of Flywheel. — The problem of selecting a flywheel for a given service is not an easy one, and each case must be treated separately. The general problem is to determine what effect a flywheel will have on smoothing out the load fluctuations; what effect it will have on the motor and supply system; whether a flywheel is warranted; and what size flywheel will result in maximum economy.

Flywheel Effect. — The effectiveness of a flywheel, rotating at a given number of revolutions per minute, in equalizing the load, depends not only upon its weight but also upon the square of its radius of gyration (q.v.). The product of the weight of a flywheel by the square of its radius of gyration is commonly called the "flywheel effect"; it is proportional to the moment of inertia of the flywheel. A factor which limits the usefulness of a given flywheel is the allowable variation in speed, as the energy which a flywheel is able to give up is proportional to the difference between the squares of the initial and final speeds. Another point to be considered is the fact that a flywheel is not an inexhaustible source of energy. The time during which it can supply a certain number of horse-power is limited, and it can, with a given drop of speed, only supply a given number of horse-power seconds. A certain flywheel, for example, may easily take care of a peak amounting to 100 per cent overload for one second, but it may be entirely inadequate to handle a 50 per cent overload lasting for five seconds.

Induction Motor with Flywheel. — Let the motor at any instant be developing a torque T, and let the opposing torque of the load at this instant be T_1. Then $T = T_1 + \frac{I}{32.2}\frac{dw}{dt}$, where I is the moment of inertia of all the revolving parts and w their angular velocity. In the case of an induction motor the torque T is practically proportional to the slip (*see Motors, Induction*). By making use of this relation and the above equation the following working formula is readily deduced for the change in the torque developed by an induction motor for an interval during which the opposing torque of the load remains constant. Let

T_0 = torque, in pound-feet, developed by motor at a given instant.
T = torque, in pound-feet, developed by motor *t seconds later.*
T_1 = opposing torque, in pound-feet, due to the load, *assumed constant.*
R = ratio of torque (in pound-feet) to slip for the given motor, obtained from characteristic curves of the motor.
N = synchronous speed of motor, in r.p.m.
k = radius of gyration of flywheel, in feet (the inertia of the other rotating parts is usually negligible in comparison with that of flywheel).
W = weight of flywheel in pounds.

Calculate the constant

$$A = \frac{308\,R}{k^2WN}.$$

Then the torque developed by the motor t seconds from the time that its torque was T_0 is

$$T = T_1 - \frac{T_1 - T_0}{\epsilon^{At}},$$

where ϵ is the base of the natural system of logarithms (*see Exponential Functions*).

This equation shows that if the load torque suddenly increases from the value T_0 to a new constant value T_1, then the torque developed by the motor does not change immediately to T_1, but builds up to this value at a rate depending upon the value of the constant A. The smaller A the more slowly does the torque of the motor change; if A is sufficiently small and the load represented by the torque T_1 is of short duration, then the torque of the motor may increase only a relatively small amount before the load drops back to its original value. The constant A can be made small: (1) by having a flywheel with a large flywheel effect; (2) by using a high-speed motor, or (3) by designing the motor so that for a given torque the slip is large, i.e., with high secondary resistance.

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[D. B. RUSHMORE, assisted by E. A. LOF.]

FREQUENCY INDICATORS. — (*See also Ammeters; Voltmeters; Wattmeters.*) The frequency of the current supplied by a generator may be determined directly from its speed, N revolutions per minute, and number of poles p, from the formula

$$f = \frac{Np}{120}.$$

The same formula also applies to a synchronous motor, N and p being the speed and number of poles of the motor. The speed may be measured by a revolution counter and stop watch or by means of a tachometer, or speed indicator. The latter may be calibrated to read the frequency directly for a given generator or motor.

INDICATING FREQUENCY METERS. — Indicating frequency meters are made in a variety of types, the most common being (1) the moving-vane or moving-coil type, (2) the induction type, and (3) those operating on the principle of the mechanical resonance of an iron reed acted upon by the magnetic field produced by a current from the given source.

Moving-vane Type of Frequency Indicator. — A common form of frequency indicator of this type consists of a moving vane, with pointer attached, so mounted that it is acted upon by two coils set at right angles to each other. One coil is connected in series with a non-inductive resistance and the other in series with a comparatively large inductance. The vane tends to set itself in the direction of the resultant field due to the currents in two coils, there being no controlling spring, and consequently for a fixed ratio of these currents the disc takes up a definite position irrespective of the values of these currents. The two circuits are so connected to the source of supply that when the voltage across the mains varies, the voltages across the two circuits of the instrument vary proportionally to each other, and therefore the position of the vane and pointer is unaffected by voltage variations. An increase in the frequency, however, decreases the current through the inductive circuit relative to that through the non-inductive circuit, and consequently causes a deflection of the vane and pointer; similarly a decrease in the frequency increases the current through the inductive circuit, producing a deflection in the opposite direction. The resistance and inductance of each circuit is so adjusted that for the standard frequency, e.g., 60 cycles, the pointer stands in the middle of the scale. A given instrument is suitable for a range ot frequency, from about 33 per cent below to 50 per cent above normal.

Effect of Wave Form. — Such instruments are affected to a very small extent by the wave form of the voltage on the mains to which they are connected. Numerous expedients have been devised to overcome this error by various combinations of resistive and inductive circuits, and by providing a small adjustable resistance in series with the inductance, this resistance being set to correspond with the wave form on which the instrument is to be used.

Moving-coil Type of Frequency Indicator.— This type is similar to the moving-vane type except that two moving coils, rigidly fastened together, constitute the moving element and the stationary element is a single coil.

Resonant-Circuit Frequency Indicator. — A new form of instrument of this type, in which the sensitiveness is greatly increased, has recently been developed (*Pratt and Price, Trans. A.I.E.E., 1912, Vol. 31, p. 1595*). The two moving coils or armatures are connected in series with suitable resistances, inductances and condensers, so adjusted that the two circuits are nearly in electrical resonance (*see Alternating Currents*) with the impressed frequency. A 6-inch deflection of the pointer over the scale of the

instrument is readily obtained for a change of frequency from 55 to 65 cycles per second, and by changing the constants of the circuits a 6-inch deflection can also be obtained for a change of frequency from 60 to 61 cycles per second. This instrument shows only a trace of variation due to wave form, voltage or temperature.

Induction-type Frequency Indicator. — In these instruments a disk is subjected to torque in opposite directions by two small shaded pole motors, the construction being similar to that of an induction voltmeter. The circuit of one motor is inductive and in the other is practical non-inductive. The disk or rotating member is provided with one edge not concentric, or some equivalent device is used, so that the torque developed on the disk varies with its position relative to the motor elements. The disk then finds a position in which the torques are balanced, which position will depend on the relative currents in the two motor elements. The current in the inductive element depends upon the frequency as in the types described above, and consequently the deflection of the disk is a measure of the frequency.

Vibrating-reed Type of Frequency Indicator. — In these instruments a series of vibrating reeds is provided, each tuned to vibrate at a frequency corresponding to a certain number of cycles per second, or sometimes the tuning is done for differences as small as ¼ cycle per second or less. Some arrangement is provided by means of which the frequency of the alternating circuit is communicated to these vibrating reeds. In one arrangement all of the reeds are mounted on a common support which is vibrated by a single electromagnet through which the alternating current whose frequency is to be measured passes. In another arrangement the reeds are attached to a rigid support but are constructed of magnetic materials and acted on by one or more electromagnets directly. These instruments are quite accurate in service if the voltage is not varied beyond that for which the instruments are intended, in which case the wrong reed may be made responsive.

COST OF FREQUENCY INDICATORS. — Switchboard frequency meters of the moving-vane or moving-coil type (not tuned), cost from $45 to $65. The tuned, or electrical resonance, switchboard type cost from $75 to $85. Vibrating-reed frequency indicators for switchboard service cost from $35 to $95; portable indicators of this type cost from $110 to $125; duty included in both cases. The various types are made for standard frequencies of 25, 40 and 60 cycles per second and for circuits of 110 or 220 volts. For higher voltages potential transformers should be used.

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[L. T. ROBINSON.]

FRICTION. — (*See also Automobiles, Electric; Bearings; Belts and Belting; Brakes and Braking Systems; Friction Drive; Gears and Gearing; Hydraulics, Principles of; Lubricants and Lubrication; Pipes and Piping; Railways, Energy Requirements for; Ropes and Rope Drive; Valves; also under name of machine in question.*) Friction is the tangential force set up whenever an external force is applied to a body tending to move it, or actually moving it, over the surface of another. There are two distinct types of friction, namely, "sliding friction" and "rolling friction." Again, the force required to start a body sliding or rolling over the surface of another body (in addition to the force required to overcome the inertia of the first body) is in general greater than the force required to keep it sliding or rolling at slow speed after it is once started. The force required to start a body is called the "static friction" or the "friction of rest" and the force required to keep it in motion is called the "kinetic friction" or the "friction of motion." There is evidence to indicate that there is no abrupt change in the value of the friction from rest to motion, but that the change is a continuous one, varying rapidly with the speed at low speeds. At first the friction decreases with the speed, at moderate speeds it is nearly constant, and at higher speeds increases rapidly with the speed. Air friction must also be taken into account at very high speeds.

COEFFICIENT OF SLIDING FRICTION (f) AND ANGLE OF REPOSE (θ). — The coefficient of sliding friction is defined as the ratio of the force required to move a body along a horizontal plane surface to the normal component of the force pressing the body against this surface. If the body is resting on an inclined plane and the force normal to this plane is due only to the weight of the body, the angle of inclination of this plane to the horizontal required to start the body is called the angle of repose. The tangent of this angle is equal to the coefficient of static friction, i.e.,

$$f = \tan \theta.$$

The friction of a bearing is a special case of sliding friction (*see Bearings*).

Factors Affecting the Coefficient of Sliding Friction. — For a given normal pressure (force per unit area perpendicular to surface of contact) the coefficient of friction is approximately independent of the area of contact, all other conditions being the same, except in the case of fibrous materials, in which case the coefficient increases with extent of surface.

The coefficient of friction, however, is as a rule materially affected by the pressure, speed, degree of smoothness of the surfaces in contact, the condition of these surfaces (whether dry, moist, greasy or oily), temperature in case of lubricated surfaces, etc.

Friction of Fluids Against Solids. — Thurston states that for all fluids, whether liquid or gaseous, the resistance is: (1) independent of the pressure between the masses in contact; (2) directly proportional to the area of rubbing-surface; (3) proportional to the square of the relative velocity at moderate and high speeds, and to the velocity nearly at low speeds; (4) independent of the nature of the surfaces of the solid against which the stream may flow, but dependent to some extent upon their degree of roughness; (5) proportional to the density of the fluid, and related in some way to its viscosity (*see Pipes and Piping; Hydraulics, Principles of*).

Friction of Lubricated Surfaces approximates more closely the laws of fluid friction the more thoroughly the surface is lubricated (*see also Bearings; Lubricants and Lubrication*).

Values of the Coefficient of Sliding Friction. — Due to the numerous factors which affect the coefficient of friction, the values of this coefficient given

by various authorities are found to differ widely. The table below will serve to indicate the order of magnitude of the coefficient in the cases stated, but it should be kept in mind that these values are only rough approximations (*see also Belts and Belting, Brakes and Braking Systems; Ropes and Rope Drive*).

COEFFICIENT OF STATIC FRICTION BETWEEN DRY, SMOOTH SURFACES

Materials	Pressure, lb. per sq. in.	*f*	Authority
Wrought iron on wrought iron	187–560	0.25–0.41	Rennie.
Wrought iron on cast iron	187–672	0.28–0.38	Rennie.
Steel on cast iron	187–672	0.30–0.40	Rennie.
Brass on cast iron	187–784	0.21–0.23	Rennie.
Yellow pine on yellow pine	100–1500	0.25–0.32	Messiter and Hanson.
Spruce on spruce	100–800	0.18–0.53	Messiter and Hanson.
Metals on metals	....	0.15–0.25	Rankine.

COEFFICIENT OF STATIC FRICTION AND ANGLES OF REPOSE OF BUILDING MATERIALS

Materials	Angle of repose, degrees	*f*	Authority
Dry masonry and brickwork	31–35	0.6–0.7	Rankine.
Masonry and brickwork with damp mortar	36.5	0.74	"
Masonry on dry clay	27	0.51	"
Masonry on moist clay	18.25	0.33	"
Timber on stone	22	0.4	"
Timber on timber	11.3–26.5	0.2–0.5	"
Timber on metals	11.3–31	0.2–0.6	"
Iron on stone	16.7–35	0.3–0.7	"
Earth on earth	14–45	0.25–1.0	"

COEFFICIENT OF KINETIC FRICTION (*Rankine*)

Materials	Dry surface	Wet surface	Soapy surface	Oily or greasy surface
Wood on wood	0.25–0.5	....	0.04–0.2	....
Metals on oak	0.5–0.6	0.24–0.26	0.2	....
Metals on elm	0.2–0.25	....	....	....
Metals on metals	0.15–0.2	0.3	....	0.03–0.08
Hemp on oak	0.53	0.33	....	....
Leather on oak	0.27–0.38	....	....	....
Leather on metals	0.56	0.36	0.15–0.23	....
Bronze on lignum vitæ	....	0.05	....	....

Power Lost Due to Sliding Friction. — Let

f = coefficient of friction,
W = total force acting normal to surface of contact,
v = velocity in feet per second,

then the power lost is

$$fWv \text{ foot-pounds per second} = \frac{fWv}{550} \text{ h.p.}$$

COEFFICIENT OF ROLLING FRICTION. — Let

F = resisting force in pounds tangent to circumference of wheel,
r = radius of wheel in feet,
W = load on wheel in pounds,

then the coefficient of rolling friction (f) is defined by the relation

$$F = \frac{fW}{r}.$$

Note that the value of f depends upon the unit in which r is expressed. If r is expressed in inches instead of feet f will be 12 times as great.

Factors Affecting the Coefficient of Rolling Friction. — Rolling friction is a consequence of the irregularities of form and the roughness of surface of bodies rolling one over the other. Its laws are not yet definitely established in consequence of the uncertainty which exists in experiment as to how much of the resistance is due to roughness of surface, how much to original and permanent irregularity of form, and how much to distortion under the load. (*Thurston.*)

Values of Rolling Friction. — The following are some reported values of rolling friction, when r is expressed in feet,

Lignum-vitæ roller on oak track	0.0016
Elm roller on oak track	0.0027
Car wheel on iron or steel rail	0.0015–0.002
Steel-tired wagon wheel on soft soil	0.065
Steel-tired wagon wheel on smooth hard road	0.02
Steel-tired wagon wheel on wood	0.0185
Steel-tired wagon wheel on asphalt	0.012

Power Lost Due to Rolling Friction. — Let

f = coefficient of friction, in feet,
W = total vertical load on wheel, in pounds,
r = radius of wheel, in feet,
n = number of revolutions of wheel per second,
v = linear speed in feet per second,

then the power lost in friction is

$$\frac{fWv}{r} = 6.28\, fWn \text{ foot-pounds per second}$$

$$= \frac{fWv}{550\, r} = \frac{fWn}{87.5} \text{ h.p.}$$

BIBLIOGRAPHY. — Numerous references will be found in *Kent's Mechanical Engineers' Pocket-Book.*

[H. PENDER AND R. G. HUDSON.]

FRICTION DRIVE. — (*Adapted from paper by W. F. M. Goss, Trans. A.S.M.E., 1907.*) A friction drive consists of a fibrous or somewhat yielding driving wheel working in rolling contact with a metallic driven wheel. Such a drive may consist of a pair of plain cylinder wheels mounted upon parallel shafts, or a pair of beveled wheels, or of any other arrangement which will serve in the transmission of motion by rolling contact.

Suitable fibrous materials for the driving wheel are straw fiber, leather fiber, tarred fiber, sulphite fiber, or leather; suitable materials for the driven wheel are iron, aluminum and type metal.

Crushing Strength and Safe Load for Fiber Wheels. — The crushing strength of each fibrous material, as determined by finding the load under which the wheel failed before 15,000 revolutions had been made, is given in the following table, together with the safe working load, taken as one-third the crushing load.

Material	Crushing load, lb. per in. width	Safe load, lb. per in. width
Straw fiber........	750	250
Leather fiber......	1200	400
Tarred fiber.......	1200	400
Sulphite fiber......	700	233
Leather...........	750	250

Coefficient of Friction. — The coefficients of friction between the various fibrous materials and the three metals are given in the table below, these being maximum values, corresponding to a slip of about 2 per cent. The friction at constant slip was found to be practically independent of the pressure between the limits of 150 and 400 pounds per inch width of face in contact.

Horse-power Transmitted. — Goss gives the following formula for the maximum horse-power which can be safely transmitted by a friction drive

$$P = \frac{\pi d}{12} \times \frac{WPN \times 0.6 f}{33{,}000} = kdWN,$$

in which d = diameter of driving wheel in inches, W = width of face in inches, P = safe working pressure in pounds per inch of width, N = revolutions per minute, f = coefficient of friction, 0.6 a factor for the decrease of the coefficient in service and for the loss in journal friction, k a coefficient including P, f and the numerical constants.

COEFFICIENTS OF FRICTION AND HORSE-POWER OF FRICTION DRIVES

Surface of Driving Pulley	On iron		On aluminum		On type metal	
	f	k	f	k	f	k
Straw fiber..........	0.255	0.00030	0.273	0.00033	0.186	0.00022
Leather fiber.........	0.309	0.00059	0.297	0.00057	0.183	0.00035
Tarred fiber..........	0.150	0.00029	0.183	0.00035	0.165	0.00031
Sulphite fiber........	0.330	0.00037	0.318	0.00035	0.309	0.00034
Leather..............	0.135	0.00016	0.216	0.00026	0.246	0.00029

BIBLIOGRAPHY. — Additional data on friction drive will be found in Kent's *Mechanical Engineers' Pocket-Book.* [WM. KENT.]

FUELS.—(*See also Boilers; Calorimeters, Fuel; Gas; Gas Producers; Power Stations.*) Commercial fuels are wood, peat, coal, charcoal, coke, petroleum and gas. These fuels in the raw state all contain carbon and hydrogen as the heat-producing elements, together with oxygen, nitrogen, sulphur, earthy matter and moisture, which are undesirable and detract from the value of the fuel. Sulphur in the form of sulphide of iron, which frequently exists in coal, tends to cause spontaneous combustion.

Wood is rarely used for the production of large amounts of energy. Peat, which is formed in bogs or marshes by the partial decomposition or destructive distillation of vegetable materials, is unsuitable for fuel until dried. To compete with coal, it must also be especially prepared and compressed into briquettes; such briquettes are extensively used in Europe, but up to the present peat has not been employed commercially in this country. Charcoal is the solid material left after evaporating the major portion of the volatile ingredients of wood or peat, or other vegetable matter. Coke is similarly the solid material left after evaporating the volatile ingredients of coal. Coke is of dark gray color, with slightly metallic luster, porous, brittle and hard. It is hygroscopic, i.e., absorbs and retains moisture when exposed to the air. One pound of coal yields from 0.35 to 0.90 pound of dry coke, depending on the kind of coal from which it is made. Coke is used chiefly in blast furnaces and foundries; its high cost prevents its use for the production of power.

In power plants, the fuels used are coal, oil and gas. Coal and oil will be treated in detail below; for a discussion of gas see the article on *Gas*.

DEFINITIONS.—When a fuel is heated to red heat in a non-oxidizing atmosphere, the carbon in the solid residue is called "fixed carbon." The hydrocarbons and other gaseous compounds which distil off are called "volatile matter." When a fuel burns or oxidizes, the solid mineral matter left is the ash. Fuels in the raw state also contain a certain amount of water or "moisture." The fixed carbon and volatile matter together are called the "combustible" though the nitrogen and oxygen in the volatile matter are not actually combustible. The determination of the fixed carbon, volatile matter, moisture and ash in a fuel is called the "proximate analysis." The determination of the moisture and ash of the fuel and the constituent elements of the combustible, i.e., the carbon, hydrogen, oxygen, nitrogen and sulphur, is called the "ultimate analysis." The "heating" or "calorific" value of a fuel is the number of units of heat energy developed as the result of the complete combustion of a unit weight (or mass) of the fuel. In this country and in England, the heat energy is expressed in British thermal units, abbreviated B.t.u. (1 B.t.u. = 777.5 ft.-lb. = 0.2928 watt-hour = 0.3927×10^{-4} hp.-hr.) and the unit of weight is 1 pound. The heating or calorific value should be expressed as so many B.t.u. per pound of *combustible*, although it is sometimes given in B.t.u. per pound of *dry fuel* or per pound of *fuel as fired* (including ash and moisture).

MOISTURE IN FUELS.—The analyses of the solid fuels given above are of perfectly dried fuels. Wood when freshly felled contains on an average about 40 per cent of moisture, varying with different species. After eight to twelve months drying in the open air, the moisture is reduced to 20 or 25 per cent. If dried in an oven to greater dryness, it will, on exposure to the atmosphere, absorb moisture again, and the percentage it will then contain will vary with the dryness or dampness of the air; that is, wood is hygroscopic.

The moisture in coal may be surface moisture, received from rain while being transported or in storage, and which may be dried out on exposure to the atmosphere, or hygroscopic moisture, contained inside of the lumps, which cannot be

dried out without subjecting the coal for some time to a temperature considerably above 212° F. The anthracite, the semi-bituminous coals and the bituminous coals of the Appalachian coal field seldom contain hygroscopic moisture in excess of 1 or 2 per cent, but it is a characteristic of the western bituminous coals and lignites that the hygroscopic moisture is much higher; thus in some Illinois coals it is as high as 14 per cent. When this moisture is dried out, the coal will again absorb it on being exposed to the atmosphere. Some lignites contain as much as 30 per cent moisture.

ULTIMATE ANALYSES OF VARIOUS FUELS. — Typical ultimate analyses of various fuels together with the percentage of ash are given below; the analyses are for fuels from which all moisture has been expelled.

	C	H	O	N	S	Ash
Wood	50	6	41	1	..	2
Peat	59	6	30	1	..	4
Lignite	69	5	16	1	1	8
Bituminous coal	76	5	10	1	1	7
Semi-bituminous coal	86.5	4.5	3	1	0.5	4.5
Semi-anthracite	78.5	3.5	2	1	2	12
Anthracite	77	2.5	2	1	1	16.5
Charcoal	82	2	..	1	..	15
Coke	89	..	..	..	1	10
Petroleum (Texas)	84.5	11	3	..	1.5	..
Natural gas	70	25	1	4	..	..

In the progression from wood to anthracite, the chief change in chemical constitution, as shown in the above table, is a decrease in the oxygen from 41 per cent to 2 per cent. The hydrogen also decreases, but less rapidly. In all the varieties of coal, the ash is exceedingly variable, ranging from as low as 2 per cent up to 25 per cent or more. Coals high in ash are usually also high in sulphur. The ash and sulphur may be removed to a considerable extent by crushing and washing.

AIR REQUIRED FOR COMBUSTION. — The theoretical weight of air A required for complete combustion of 1 lb. of fuel may be determined from the formula,

$$A = 0.115\,C + 0.346\left(H - \frac{O}{8}\right) + 0.043\,S,$$

where C, H, O and S are the percentages (by weight) of carbon, hydrogen, oxygen and sulphur in the fuel. This gives about 12 lb. of air per pound of combustible. In the best boiler practice, the weight of air required is from 16 to 20 lb. per pound of coal, but in actual practice, the air supplied varies between much wider limits. See article on *Boilers*.

COAL. — **The proximate analysis** of coal is made by separating a sample by successive heatings at different temperatures, into moisture, volatile matter, fixed carbon and ash. For the determination of moisture, a rather large sample, say 2 ounces or 60 grams, should be taken, crushed to about ¼-inch size, and heated for two hours to 140° C. (284° F.) or until the coal ceases to decrease in weight. If a finely crushed small sample is taken, much of the moisture may be lost by air-drying while crushing and weighing. For the

determination of the other constituents, a sample of about 1 gram of finely crushed coal is taken, dried for an hour at 105° C. (221° F.) and weighed, then heated at a red heat in a covered crucible until all the volatile matter is driven off, weighed, heated with a blast lamp to a white heat, the cover being off the crucible, until all the carbon is burned away, leaving the ash, which is weighed. The analysis should be reported as in the example below, the percentages being percentages of weight.

	Moist coal	Dry coal	Combustible
Moisture	10.0	.....	.....
Volatile matter	30.0	33.33	57.14
Fixed carbon	40.0	44.45	42.86
Ash	20.0	22.22	.....
	100.0	100.0	100.0

Classification of Coal. — The percentages of fixed carbon and volatile matter in the combustible furnish a means of classifying different kinds of coal. The writer's classification is as follows:

	Fixed carbon	Volatile matter	Heating value per lb. of combustible, B.t.u.
Anthracite	97 to 90	3 to 10	14,600 to 15,000
Semi-anthracite	90 to 85	10 to 15	14,700 to 15,500
Semi-bituminous	85 to 75	15 to 25	15,500 to 16,000
Bituminous, Eastern	75 to 60	25 to 40	14,800 to 15,500
Bituminous, Western	65 to 50	35 to 50	13,500 to 14,800
Lignite	Under 50	Over 50	11,000 to 13,500

Anthracite is hard, shiny, burns with little or no smoke, is slow to ignite, burns slowly and breaks into small pieces when rapidly heated. Anthracite is crushed at the mine and the lumps separated into different sizes by passing them over screens or parallel bars. The sizes are given the following names:

Lump, over bars set $3\frac{1}{2}$ to 5 inches apart;
Steamboat, over $3\frac{1}{2}$-inch mesh and out of screen;
Broken, over $2\frac{3}{4}$-inch mesh, through $3\frac{1}{2}$-inch mesh;
Egg, over 2-inch mesh, through $2\frac{3}{4}$-inch mesh;
Stove, over $1\frac{3}{8}$-inch mesh, through 2-inch mesh;
Chestnut, over $\frac{3}{4}$-inch mesh, through $1\frac{3}{8}$-inch mesh;
Pea, over $\frac{1}{2}$-inch mesh, through $\frac{3}{4}$-inch mesh;
Buckwheat, over $\frac{3}{8}$-inch mesh, through $\frac{1}{2}$-inch mesh;
Rice, over $\frac{3}{16}$-inch mesh, through $\frac{3}{8}$-inch mesh;
Culm, slack or screenings, through $\frac{3}{16}$-inch mesh.

Sizes larger than "pea coal" are prohibitive in price for power-plant use.

Semi-anthracite is similar to anthracite, but is less hard, less shiny and burns more rapidly. It can usually be distinguished by its tendency to soil the hands, while true anthracite does not.

Semi-bituminous coal is softer than semi-anthracite and has a greater tendency to smoke. It is one of the best steam coals.

Bituminous coal is also a soft coal. It is distinguished by high percentage of volatile matter, which causes it to give off dense volumes of smoke when heated. It requires careful firing and furnaces especially adapted for it to prevent smoke. There is a wide variation in its physical properties.

Lignite, or brown coal, is intermediate between coal and peat. It is brown in color, fragile and rapidly splits into fine pieces upon exposure to air.

The U. S. Geological Survey classifies coals into six groups, as follows: (1) anthracite; (2) semi-anthracite; (3) semi-bituminous; (4) bituminous; (5) sub-bituminous, or black lignite; and (6) lignite.

Classes 5 and 6 are described as follows:

Sub-bituminous coal is commonly known as "lignite," "lignitic coal," "black lignite," "brown coal," etc. It is generally black and shining, closely resembling bituminous coal, but it "weathers" or disintegrates more rapidly on exposure and lacks the prismatic structure of bituminous coal. Its calorific value is generally less than that of bituminous coal.

Lignite is commonly known as "lignite," "brown lignite" or "brown coal." It usually has a woody structure and is distinctly brown in color, even on a fresh fracture. It carries a higher percentage of moisture than any other class of coals, its mine samples showing from 30 to 40 per cent of moisture.

The following analyses of representative coals of the six classes are given by Prof. N. W. Lord:

Class 1. Anthracite Culm. Penn.
Class 2. Semi-anthracite. Arkansas.
Class 3. Semi-bituminous. W. Va.
Class 4(*a*). Bituminous coking. Connellsville, Pa.
Class 4(*b*). Bituminous non-coking. Hocking Valley, Ohio.
Class 5. Sub-bituminous. Wyoming, black lignite.
Class 6. Lignite. Texas.

COMPOSITION OF ILLUSTRATIVE COALS—CARLOAD SAMPLES

Proximate Analysis of "Air-dried" Sample							
Class	1	2	3	4a	4b	5	6
Moisture	2.08	1.28	0.65	0.97	7.55	8.68	9.88
Vol. comb.	7.27	12.82	18.80	29.09	34.03	41.31	36.17
Fixed carbon	74.32	73.69	75.92	60.85	52.57	46.49	43.65
Ash	16.33	12.21	4.63	9.09	5.85	3.52	10.30
Loss on air-drying	3.40	1.10	1.10	4.20	Undet.	11.30	23.50
Ultimate Analysis of Coal Dried at 105° C.							
Hydrogen	2.63	3.63	4.54	4.57	5.06	5.31	4.47
Carbon	76.86	78.32	86.47	77.10	75.82	73.31	64.84
Oxygen	2.27	2.25	2.68	6.67	10.47	15.72	16.52
Nitrogen	0.82	1.41	1.08	1.58	1.50	1.21	1.30
Sulphur	0.78	2.03	0.57	0.90	0.82	0.60	1.44
Ash	16.64	12.36	4.66	9.18	6.33	3.85	11.43

COMPOSITION OF ILLUSTRATIVE COALS — *Continued.*

Results Calculated to an Ash and Moisture Free Basis							
Class	1	2	3	4a	4b	5	6
Volatile comb.......	8.91	14.82	19.85	32.34	39.30	47.05	45.31
Fixed carbon........	91.09	85.18	80.15	67.66	60.70	52.95	54.69
Ultimate Analysis							
Hydrogen...........	3.16	4.14	4.76	5.03	5.41	5.50	5.05
Carbon.............	92.20	89.36	90.70	84.89	80.93	76.35	73.21
Oxygen.............	2.72	2.57	2.81	7.34	11.18	16.28	18.65
Nitrogen...........	0.98	1.61	1.13	1.74	1.61	1.25	1.47
Sulphur............	0.94	2.32	0.60	1.00	0.87	0.62	1.62
Calorific Value in B.t.u. per pound of Combustible by Dulong's Formula							
Air-dried coal.......	12,472	13,406	15,190	13,951	12,510	11,620	10,288
Combustible........	15,286	15,496	16,037	15,511	14,446	13,235	12,889

Heating Value of Coal. — The heating value of coal depends on its percentage of total combustible and on the heating value per pound of that combustible. The latter differs in different districts and bears a relation to the percentage of volatile matter as shown by the above table.

For coals in which the volatile matter is less than 40 per cent of the combustible, the heating value per pound of combustible may be approximately determined (within about 2 per cent) by means of the following table. When the volatile matter is in excess of 40 per cent, the figures in the table may have a plus or minus error of as much as 4 per cent, since the coals high in volatile matter differ greatly in their content of oxygen.

Volatile matter, per cent of combustible	Heating value, B.t.u. per pound of combustible	Volatile matter, per cent of combustible	Heating value, B.t.u. per pound of combustible	Volatile matter, per cent of combustible	Heating value, B.t.u. per pound of combustible
3	14,940	28	15,660	43	14,220
6	15,210	32	15,480	45	13,860
10	15,480	37	15,120	47	13,320
13	15,660	40	14,760	49	12,420
20	15,840				

The heating value of all the semi-bituminous coals, containing from 15 to 25 per cent of volatile matter in the combustible, is within $1\frac{1}{2}$ per cent of 15,750 B.t.u. per pound. This is within the limits of error of sampling, of chemical analysis or of calorimetric determination.

The heading value of any coal may also be calculated from the ultimate analysis by means of Dulong's formula,

$$\text{B.t.u. per pound} = 146\,C + 620\left(H - \frac{O}{8}\right) + 40\,S,$$

in which C, H, O and S are respectively the percentages by weight of carbon hydrogen, oxygen and sulphur. The probable error of this formula is not over 2 per cent for any coal containing less than 40 per cent volatile matter in the combustible. For coals higher in volatile matter, the error is sometimes larger. When the percentages are expressed as percentages of the various elements in the *combustible*, the formula gives the B.t.u. per pound of combustible; when the percentages are percentages of the various elements in a given weight of **coal**, the formula gives the B.t.u. per pound of coal, which is, of course, less than the B.t.u. per pound of combustible in proportion to the per cent of moisture and ash present. In general, if K be the per cent of combustible in a given sample, and H the heating value per pound of combustible, the heating value per pound of coal is $\frac{KH}{100}$.

For the direct determination of the heating value of coal or other solid fuel, a bomb calorimeter, in which the fuel is burned in an atmosphere of compressed oxygen, is the most accurate instrument. (*See Calorimeters, Fuel.*)

The Commercial Value of a Coal is not always in direct proportion to its heating value. Excessive moisture causes a reduction in its temperature of combustion, which reduction, in steam boiler practice, decreases the efficiency; excessive ash tends to obstruct the draft and thus cause imperfect combustion. High percentages of volatile matter tend to cause smoke and soot, and require the use of special furnaces.

Specifications for Coal. — For the reasons just stated, contracts for the purchase of coal on specifications of quality should penalize excess of moisture, ash and volatile matter above certain stated percentages. The author's specifications are as follows:

Anthracite and Semi-anthracite. — The standard is a coal containing 5 per cent volatile matter, not over 2 per cent moisture and not over 10 per cent ash. A premium of 0.5 per cent on the price will be given for each per cent of volatile matter above 5 per cent up to and including 15 per cent, and a reduction of 2 per cent on the price will be made for each 1 per cent of moisture and ash above the standard.

Semi-bituminous and Eastern Bituminous. — The standard is a semi-bituminous coal containing not over 20 per cent volatile matter, 2 per cent moisture and 6 per cent ash. A reduction of 1 per cent in the price will be made for each 1 per cent of volatile matter in excess of 25 per cent, and of 2 per cent for each 1 per cent of ash and moisture in excess of the standard.

Western Coals. — For western coals in which the volatile matter differs greatly in its percentage of oxygen, the above specification based on proximate analysis may not be sufficiently accurate, and it is well to introduce the heating value, as determined either by a calorimeter or by calculation from the ultimate analysis below:

The standard is a coal containing not over 6 per cent moisture and 10 per cent ash in an air-dried sample, and having a heating value of 14,500 B.t.u. per pound of pure coal (coal free from moisture and ash). For lower heating value per pound of pure coal, the price shall be reduced proportionally, and for every 1 per cent increase in ash or moisture above the specified figures, 2 per cent on the price shall be deducted.

Space Required for Storage.—The space occupied by a ton of coal depends both upon the quality of the coal and the size of the lumps. A ton of 2240 pounds of anthracite of pea size, or smaller, occupies a space of from 36 to 45 cubic feet. A ton or 2240 pounds of bituminous coal occupies a space of from 40 to 50 cubic feet. In estimating the space required for storage, 45 cubic feet per ton (2240 pounds) is usually assumed.

Weathering of Coal.—Anthracite coal when exposed to the weather undergoes practically no change except the oxidation of the sulphur content, which is small. Bituminous coal contains a larger percentage of sulphur, the oxidation of which, if present in sufficient amount, may develop sufficient heat to cause spontaneous combustion. Some lignites are rapidly disintegrated when exposed to the air. Experiments on carload lots of Illinois coal (*F. W. Wheeler, Trans. A.S.M.E.*, 1908) showed that the screenings and 3-inch nut coal lost 1.3 per cent of its heating value in one month, and 2 per cent in six months. Pillar coal in the mine, exposed underground twenty-two to twenty-seven years, showed only 3 per cent less heating value than the fresh face coal from the same mine.

Cost of Coal.—The cost of coal delivered to any power plant depends not only upon the quality of the coal and the size of the lumps, but also upon the cost of transportation, the railroad or water facilities for delivering the coal, and upon various conditions affecting the cost of mining. In making an estimate involving the cost of coal, one should obtain quotations from the dealers for the specific locality and time under consideration.

LIQUID FUEL.—(*See also Boilers; Gas Engines.*) Crude petroleum and various distillates of petroleum, such as gasoline, kerosene, etc., are largely used as fuel, the extent of their use in any locality depending chiefly on their relative cost as compared with coal. In Texas, California, Russia and other places near to oil wells and where coal is relatively expensive, petroleum has largely replaced coal as a fuel for steam boilers. Gasoline, kerosene and heavier oils are also extensively used in various types of internal combustion engines. Crude petroleum is composed chiefly of hydrocarbons, which distil at different temperatures, the lightest vapors being driven off as low as 113° F., and heavier vapors and oils at temperatures rising to 600°, above which waxes and residuum are formed. The crude oil contains small percentage of water, sulphur and oxygen as impurities. The specific gravity, weight and heating value of California oil are given as follows by J. N. LeConte (*Jour. A.S.M.E.*, *Aug.*, 1911):

CALIFORNIA OIL

Degree Baumé	Specific gravity	Weight per barrel, pounds	B.t.u. per pound	B.t.u. per barrel (42 gallons)
10	1.000	350	18,280	6,398,600
12	0.986	345	18,400	6,349,800
14	0.972	340	18,520	6,302,400
16	0.959	336	18,640	6,256,500
18	0.946	331	18,760	6,211,400
20	0.933	327	18,880	6,167,900
22	0.921	322	19,000	6,126,000
24	0.909	318	19,120	6,084,000
25	0.903	316	19,180	6,063,800

Oils from other sources have different densities and heating values, thus Lima, O., crude is reported to have a sp. gr. of 0.792; Beaumont (Texas) oil, sp. gr. 0.92, B.t.u. per pound 19,060; Pennsylvania heavy crude, sp. gr. 0.886, B.t.u. 20,736; Caucasian light crude, sp. gr. 0.884, B.t.u. 22,027. California oil, six lots, used in a boiler test at Redondo, Cal., contained moisture 1.82 to 2.70 per cent; sulphur 2.17 to 2.607; B.t.u. per pound 17,717 to 17,966.

The following table shows the relative heating values of crude petroleum and coal, based on oil of sp. gr. 0.885; B.t.u. per pound 20,000; 1 barrel, 42 gallons = 310 pounds.

Coal B.t.u. per pound	1 pound oil = pounds coal	1 barrel oil = pounds coal	1 ton (2240 pounds) coal = barrels oil
10,000	2	620	3.61
11,000	1.818	564	3.97
12,000	1.667	517	4.33
13,000	1.538	477	4.69
14,000	1.429	443	5.05
15,000	1.333	413	5.42

Cost. — The price of oil fluctuates much more than the price of coal. Prices should be obtained from local dealers at the time under consideration.

GASEOUS FUEL. — The use of gas as a fuel in power plants is confined almost entirely to its use in gas engines. (*See articles on Gas and Gas Engines.*)

BIBLIOGRAPHY. — *Steam*, published by the Babcock & Wilcox Co.; Kent, Wm., *Steam Boiler Economy*, 1914; numerous Bulletins and Technical Papers published by the U.S. Geological Survey and the Bureau of Mines, 1906–1912. A list of these papers may be obtained from the Bureau of Mines, Washington, D.C.

[WM. KENT.]

FURNACES, ELECTRIC, AND ELECTRIC FURNACE PRODUCTS. — (*See also Electrochemical Processes, Industrial.*) Electric furnaces may be classified as follows:

Arc furnaces.
Induction furnaces.
Resistance furnaces.
- *a.* Current conducted by the materials heated:
  1. With electrolysis.
  2. Without electrolysis.
- *b.* Current conducted by a special resistor.

In the arc furnace the heat is produced by an electric arc (*see Arc, Electric*) usually between carbon electrodes. The induction furnace is essentially a static transformer (*see Transformers*) with the low-tension "winding" formed by the material to be heated. In the resistance furnace the current is supplied to the material to be heated (i.e., to the furnace "charge"), or to the special resistor, by connecting the charge or resistor directly to the source of the current supply. The heat developed in both the induction and resistance furnaces arises from the passage of the current through the resistance offered by the charge or special resistor.

The arc furnace may be considered a resistance furnace in which the resistor is a gas. Since the resistance of a gas at atmospheric pressure is greater than that of any solid resistor of the same dimensions as the arc, the amount of heat that can be produced in a small space will be greatest with an arc furnace.

TEMPERATURE AND DISTRIBUTION OF HEAT. — The advantage in electric heating is that a higher temperature can be produced than by using fuel, that the heat is produced inside the furnace where it is needed, and that the heat can be easily and accurately regulated. In the arc the hottest part of the positive carbon is estimated to be between 3900° C. and 4000° C. absolute. (*Waidner and Burgess, Bull. Bureau Stds., Vol. 1, p. 123, 1905.*) The temperature of the arc itself increases with the current. (*Kayser, Handbuch der Spektroscopie, Vol. 1, pp. 154–160, 1900.*)

The electric energy delivered to the furnace as heat is used as follows: (1) to heat the charge to the desired temperature, which involves heating up the furnace walls, if cold at the start. This energy is equal to the mass of the charge times the temperature rise times the specific heat. If the charge is melted or vaporized in the furnace, additional heat must also be supplied. This item may be reduced by delivering the charge to the furnace already hot as in steel refining. (2) To supply the energy needed for the reaction; the energy so required cannot be reduced. (3) To supply the loss due to conduction and radiation through the walls and electrodes, and the heat carried off by hot gases. A part of the heat carried off by hot gases may be recovered in heating water in boilers, so that it is not a complete loss.

MAXIMUM SIZE OF ELECTRIC FURNACES. — The largest workable capacity of an open-arc single hearth with a compact bundle of electrodes has been found in practice to be from 2500 to 3000 kilowatts at from 30,000 to 40,000 amperes and from 75 to 90 volts. These large sizes are always used with the three-phase system, so that the maximum total power absorption of a furnace is from 7500 to 9000 kilowatts. In carbide furnaces double three-phase furnaces are used with six electrodes, in place of three, in the same shaft, and the power absorbed is from 15,000 to 18,000 kilowatts. Great progress has been made in electric-furnace construction, by closing the furnace at the top and by having special means of feeding in the charge, thereby avoiding the dust nuisance and protecting the workmen from the heat, as well as distributing the

charge more uniformly. (*Taussig, VII Int. Cong. App. Chem., Sec. 10, p. 24, 1910; Trans. Faraday Soc., Vol. 5, p. 254, 1909; VIII Int. Cong. App. Chem., Vol. 21, p. 105, 1912.*)

DESIGN OF FURNACE WALLS. — To reduce the conduction of heat through the walls and electrodes to a minimum amount, these must be properly designed. The heat flow through the walls of three different shapes may be computed by the formulas below. In all cases

k = thermal conductivity of the walls, at the mean temperature $(t_1 + t_2)/2$. This coefficient k may be expressed in any convenient unit, e.g., gram calories per centimeter cube per second per ° C., or watts per inch cube per ° C. See article on *Heat and Thermal Properties* for values.

H = total heat conducted per second through the walls.

t_2 = temperature of the inside surface of the wall.

t_1 = temperature of the outside surface of the wall.

Hollow Rectangular Parallelopiped. —

$$H = \left(\frac{A}{\vartheta} + 0.54\,\Sigma l + 0.15\,n\vartheta\right)k(t_2 - t_1),$$

where A = the area of the six inner surfaces, ϑ = thickness of wall, Σl = the total length of all the inner edges, n = the number of corners. This applies where all three inner dimensions are greater than $\frac{1}{5}\vartheta$. (Langmuir, Adams, and Meikle, *Trans. Am. Electrochem. Soc., Vol. 14, p. 53, 1914.*)

Hollow Sphere. —

$$H = \frac{\pi k D d\,(t_2 - t_1)}{l},$$

where D = outside diameter of sphere, d = inside diameter, l = thickness of wall, all in the same unit.

Hollow Cylinder. —

$$H = \frac{2\pi k L\,(t_2 - t_1)}{2.3\log_{10}\dfrac{D}{d}} + \frac{\pi k D d\,(t_2 - t_1)}{2l},$$

where L = mean height of inner and outer walls, D = outside diameter, d = inside diameter, and l = thickness of top and bottom walls, all in the same unit. The first term gives the flow of heat through the cylindrical walls, the second the flow of heat through the top and bottom. (Hering, *Trans. Am. Electroch. Soc., Vol. 14, p. 215, 1908.*)

Linings and Composite Walls. — The most refractory substances do not have the lowest thermal conductivities. Consequently, it is advantageous to use a highly refractory substance only for the inner part or lining of the walls, using only such a thickness that the drop in temperature will be sufficient to permit of a less refractory substance of a lower conductivity being used for the next layer. "Graded" walls of several layers may be employed.

Refractories for Furnace Walls. — The most refractory substance is carbon, which however is a good heat conductor. Some of the products of the electric furnace, as silicon carbide and siloxicon (a substance containing varying amounts of carbon, silicon and oxygen) stand next to carbon as refractories, and do not have such high heat conductivities. (*See Fitzgerald, Electrochem. Ind., Vol. 2, p. 439, 1904.*) The numerical values of heat conductivities of refractories at high temperatures are known for only a few substances and then only approximately. See article on *Heat and Thermal Properties;* also article by Hering in *Met. and Chem. Eng.*, Vol. 9, p. 625, 1911, and *the table below for graphite and carbon.* For electrical conductivity see article on *Resistance and Conductance.*

DESIGN OF ELECTRODES FOR ELECTRIC FURNACES. — The electrodes of an electric furnace should be so designed that the energy will be carried into the furnace with a minimum energy loss. The loss due to the electrical resistance is directly proportional to the electrode's length; this should therefore be made as short as convenient. The loss due to electrical resistance will be smaller the greater the cross-section of the electrode, but the heat loss from the furnace through the electrode will be directly proportional to the cross-section. It is therefore possible to find a cross-section of a given material which will give a minimum total loss for a given length. The cross-section that would give the minimum loss on certain assumptions, only approximately true, is found from the equation

$$S = 0.346\, LI \sqrt{\frac{r}{k(t_2 - t_1)}},$$

and the loss itself, in gram calories, is

$$h = 2.89\, I \sqrt{kr}\,(t_2 - t_1),$$

where S = cross-section of electrode, L = its length, I = the current in amperes carried by the electrode, t_2 and t_1 the temperatures, in °C., of the hot and cold ends of the electrode respectively, r = its mean electrical resistivity in ohms per unit cube and k = its mean heat conductivity for the mean temperature $(t_1 + t_2)/2$. If S and L are in centimeters, r and k must be per cm.^{3}; if S and L are in inches, r and k must be per in.^{3}.

The values of k and r for carbon and graphite are not known accurately at high temperatures. The following values have been computed from measurements of Hering (*Trans. Am. Electroch. Soc., Vol. 17, p. 166, 1910*).

Material	Temperature °C.		Thermal cond. g-cal. per cm.^{3} per °C. per sec.	Electrical resistivity, ohms per cm.^{3}
	Hot end	Cold end		
Carbon	300	40	0.0891	0.00422
	701	50	0.124	0.00381
	902	60	0.130	0.00377
Graphite	355	66	0.399	0.000837
	516	70	0.325	0.000827
	707	87	0.309	0.000802

SMALL LABORATORY FURNACES. — A great variety of electric furnaces have been devised. A few typical laboratory furnaces will be described in this section. Some industrial furnaces are described in the next section:

Moissan's Furnace (Fig. 1). — Moissan's work was carried out in a furnace consisting of two horizontal electrodes, mounted so that the distance between the two ends could be adjusted longitudinally by a screw thread. An arc was formed between these electrodes in a cavity formed by some refractory material, such as lime. The substance to be heated was placed in a crucible under the arc as shown in Fig. 1. When the substance was not to be exposed to the gases of the arc, a furnace was made with a carbon tube passing through it at right angles to the electrodes and immediately below the arc. The substance to be heated was placed inside this tube.

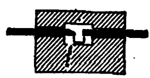


Fig. 1.

Borchers' Furnace (Fig. 2). — A type of furnace due to Borchers consists in a carbon rod placed between two larger electrodes. The charge is either packed around the small rod or placed under it. This type of furnace is convenient where a temperature below that of the arc is desired.

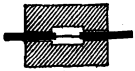


Fig. 2.

Héroult Furnace (Fig. 3). — A furnace that takes its name from Héroult consists in a crucible with one or more electrodes connected together above the crucible. The crucible is represented in Fig. 3 packed in carbon in an iron container. Graphite crucibles may be turned out from graphite electrodes. The charge in the crucible is usually melted by forming an arc between the crucible and the electrode above with an adjustable resistance in series with the arc. After the charge has melted the electrode may be partly immersed in the bath. In case it is desired to melt a substance in a crucible without the use of an arc, a smaller piece of carbon may be placed between the crucible and the electrode as in Borchers' furnace. If after the substance has been melted it is desired to pass the current through the bath itself, as, for example, in case a salt is to be electrolyzed, the upper electrode may be raised, the thin rod removed with a pair of tongs and the electrode then lowered into the bath. The salt will not solidify during this operation.

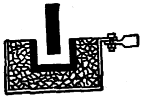


Fig. 3.

Arsem Furnace (Fig. 4). — It frequently happens in the laboratory that it is desired to heat a substance to a high temperature in a vacuum or in some pure gas, such as hydrogen or nitrogen. A very convenient furnace for this purpose has been designed by Arsem (*Trans. Am. Electroch. Soc., Vol. 9, p. 153, 1906*), and has been extensively employed in research work. This furnace may be obtained from the General Electric Company in more than one size. A vertical section is shown in Fig. 4. It consists of a chamber *A* and cover *B* made of a gun-metal casting turned true at the joint. A lead gasket *C*, 1/16 inch thick, forms an air-tight joint when the cover is fastened down by the cap-screws *D*. The tube *J* through which the air is removed from the furnace is soldered into the cover. The window *E* is a disk of clear white mica 0.005 inch thick clamped between lead washers *F*.

The electrodes *W* are brass tubing which are insulated from the cover. The clamps *UU* for holding the heater are copper. The heater *L* is a helix, usually of graphite, which is made by boring out a graphite electrode and cutting it along a spiral as shown. Metallic heaters may also be used. The lower end of the heater rests in the graphite cup which also holds the crucible support insulated from it by a lava ring. The screen for preventing radiation is a double-walled cylindrical box of Acheson graphite filled with graphite powder.

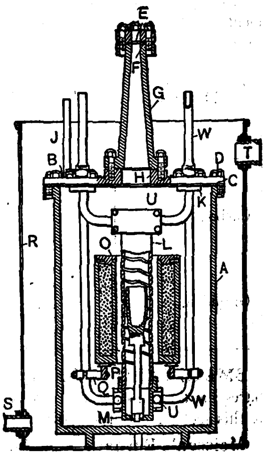


Fig. 4.

The water jacket *R* is a galvanized-iron tank provided with an inlet *S* and an outlet *T*. In a vacuum in the small size of furnace 9 to 10 kilowatts produce a temperature of 2500° C.

High-pressure Furnaces. — The Arsem furnace may be of course used with the internal pressure above an atmosphere, but it is not designed for high pressures. A furnace for working up to 200 atmospheres has been designed by Hutton and Petavel. (*Phil. Trans., Series A, Vol. 207, p. 421, 1908, and Electrochem. and Met. Ind., Vol. 6, p. 97.*) For a modified form see *Pring and Fairlie,* (*VIII Int. Cong. App. Chem., Vol. 21, p. 79, 1912*). For a furnace for spectroscopic work on gases at pressures up to 200 atmospheres, see *King, A. S., Astrophysical Journal, Vol. 28, p. 300, 1908.*

Heræus Furnace (Fig. 5). — A very useful type of furnace is due to the firm of Heræus. This consists in a tube wound in its middle portion with an electrical resistor. The ends of the tube are cooled by the air, or may be cooled by a coil of copper pipe through which water flows. The tube may be of some non-conducting substance, such as porcelain, in which case a ribbon of metal may be wound directly on the tube. Furnace tubes with grooves for winding with wire are now made by the Norton Company of Worcester, Mass., from fused alumina. These, however, are porous and cannot be used for a vacuum furnace. Glazed German porcelain may be heated up to 1180° C. and a vacuum maintained. At temperatures higher than this the glazing melts and air leaks into the tube. A nickel tube may be used for higher temperatures.

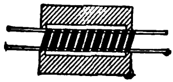


Fig. 5.

The metallic winding for carrying the current may be of platinum, nickel, nichrome, tungsten, molybdenum or any other suitable high-resistance material (*see Wires, Resistance*).

These furnaces are particularly useful when the temperature is to be held constant over a long period of time. For most purposes a constant current will keep the temperature sufficiently constant. For greater constancy some kind of regulator must be used (*see Bodenstein and Kranendieck, Z. f. Elektroch., Vol. 18, p. 417, 1912*). The Heræus type of furnace is usually more satisfactory when homemade, as a greater choice of materials is then possible.

Hoskins Furnace. — A convenient furnace for heating rather large crucibles or masses of material is made by the Hoskins Manufacturing Company of Detroit, Michigan. The heater consists of two rows of narrow, thin carbon plates which extend over two sides of the cavity which receives the substance to be heated. The contact resistance between the plates may be varied by pressure. This furnace of course requires a very large current at a low voltage.

Granular Carbon Furnace (Fig. 6). — A carbon or graphite crucible may be easily heated by placing it in a trough surrounded by granular gas carbon; this conducts better than coke. The current is passed through the trough, into which it is conducted by carbon rods. Clay crucibles should not be used in this way for temperatures over 1000° C., as at a high temperature they are attacked by the carbon.

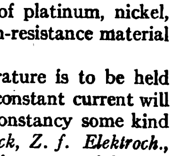


Fig. 6.

ELECTRIC-FURNACE PRODUCTS AND INDUSTRIAL FURNACES. — Some typical industrial furnaces and their products are described below.

Calcium Carbide. — Willson and Horry Furnaces. Calcium carbide was first made on a large scale by Willson in 1892 at Spray, N. C., by heating lime and carbon in an arc furnace. The reaction that takes place is

$$CaO + 3\,C = CaC_2 + CO.$$

The furnace consisted of a carbon plate 3 by 2.5 feet with a carbon electrode suspended above it. The whole was surrounded by brick walls. Furnaces similar to this were at first used at Niagara Falls, with the lower electrodes

mounted on a car which was removed when filled with an ingot of carbide. Later the Horry rotary furnace was used (*U. S. Pat. 656,156*). The Carbide Company keeps the style of their present furnace secret. In Europe the Willson type of furnace is used. Carbide may be either formed in an ingot in the crucible of the furnace and removed solid, or it may be drawn off in the liquid state. When an ingot is formed it has been found better to have the current flow between two electrodes suspended over the crucible, in place of having the crucible form one electrode. (*Conrad, Electrochem. and Met. Ind., Vol. 6, p. 397, 1908.*) The purity of the carbide is in the neighborhood of 80 per cent. The yield of 80 per cent carbide is about 5 kilograms per 24 kilowatt hours.

Carborundum. — Acheson Process. — Silicon carbide or carborundum was first made on a large scale by Acheson. It is produced from quartz and carbon when these substances are heated in an electric-resistance furnace, according to the reaction

$$SiO_2 + 3\,C = CSi + 2\,CO.$$

The furnace has a granular carbon core around which the charge, consisting of quartz, carbon, sawdust, and sodium chloride, is packed. The latest furnaces are 9.15 meters long by 3.67 meters wide, and absorb 1600 kilowatts. The current is 20,000 amperes (*Min. Industry, Vol. 16, p. 155, 1907; Vol. 17, p. 112, 1908*). From measurements on a 750-kilowatt furnace it was found that the carbide is formed at 1840° C. and decomposes when heated above 2240° C. (*Saunders, Trans. Am. Electroch. Soc., Vol. 21, p. 425, 1912.*) The yield is about one kilogram of crystallized carbide for 8.5 kilowatt hours. Silicon carbide is used as an abrasive, as furnace linings and as a substitute for ferro-silicon in the manufacture of steel. (*Fitzgerald, Carborundum, Vol. 13, in the Engelhardt Monographien über angewandte Elektrochemie.*)

Silundum is the trade name for silicon carbide made by exposing hot carbon rods to silicon vapor. These rods are used for electric heating.

Siloxicon is a product of the incomplete reduction of silica, and may be represented by the formula SiCO, though compounds with varying proportions of these elements are found. Siloxicon is used for crucible linings. It is made by heating silica and an insufficient quantity of carbon for complete reduction of the silica.

Silicon is made in an arc furnace from coke and sand. At the high temperature produced the silica is completely reduced and the melted metal is drawn off in amounts weighing from 600 to 800 pounds. It varies from 90 to 97 per cent in purity (*F. J. Tone, Electrochem. and Met. Ind., Vol. 7, p. 192, 1900; Min. Industry, Vol. 17, p. 768, 1908*).

Graphite. — Berthelot (*Ann. de Chem. et de Phys., Series 4, Vol. 19, p. 393, 1870*) defines graphite as that allotropic form of carbon which when oxidized at low temperature with powerful oxidizing agents (potassium chlorate and nitric acid) gives graphite oxide. Arsem suggests as a definition for graphite that it is that allotropic form of carbon whose density lies between 2.25 and 2.26. (*Trans. Am. Electroch. Soc., Vol. 20, p. 105, 1911.*)

Graphite is made by heating carbon, containing a small amount of impurity, to a high temperature in an electric resistance furnace. Anthracite coal is graphitized in bulk; electrodes and crucibles are also graphitized after moulding. Acheson's theory of the formation of graphite is that the carbon first forms a carbide, which decomposes at a higher temperature, leaving the carbon in the form of graphite. This theory is not confirmed by Arsem's experiments. (*See Fitzgerald, Künstlicher Graphit, Vol. 15 of the Engelhardt Monographien über angewandte Elecktrochemie.*)

Carbon Bisulphide is made in a specially designed furnace by heating together sulphur and carbon. Most of the disagreeable features encountered in the manu-

facture of this substance are thus avoided. (*Taylor, Trans. Am. Electroch. Soc., Vol. 1, p. 115, 1902; Vol. 2, p. 185, 1902.*)

Phosphorus is a substance the production of which the use of the electric furnace has much simplified. It is made in the Readman-Parker furnace according to Wöhler's process:

$$Ca_3(PO_4)_2 + 3\,SiO_2 + 5\,C = 3\,CaSiO_3 + 5\,CO + 2\,P.$$

(*Min. Industry, Vol. 6, p. 537, 1897; Vol. 7, p. 557, 1898.*)

Alundum is the trade name of fused aluminum oxide, which is made by the Norton Company of Worcester. Aluminum oxide is fused in an arc furnace. (*Min. Industry, Vol. 19, p. 28, 1910.*) Fused aluminum is used as an abrasive, as a refractory substance for furnace linings, and porous crucibles of this substance are used in analytical laboratories.

Aluminum. — Hall and Héroult Processes. — Aluminum is now produced by the electrolysis of a solution of alumina in fused cryolite ($AlF_3.3\,NaF$) to which other fluorides, such as those of aluminum and of sodium, are added in some factories. The aluminum sinks to the bottom of the crucible and is drawn off. This process was discovered nearly simultaneously by C. M. Hall and Héroult. The heat developed by the current in passing through the solution is sufficient to keep the bath melted. The cathode is an iron trough lined with carbon, and the anode consists of a number of carbon rods suspended over the crucible. The Aluminum Company of America uses as anode for one crucible 48 carbon rods 3 inches in diameter and 15 inches long. The electromotive force applied to each crucible is 5.5 volts; the current is 10,000 amperes. The yield is 1.75 pounds of aluminum per horse-power day. For further information see *Min. Industry, Vols. 6, 14, 15, 17, 20.*

Sodium, Potassium. — Castner Process. — Sodium and potassium are obtained by the electrolysis of their fused hydrates, usually in the cell designed by Castner (*U. S. Pat. 453,030, filed 1890; see also Becker, Die Elektromettallurgie der Alkalimettale*). At Holcomb's Rock, Va., sodium is made by the electrolysis of fused sodium chloride. (*Mineral Ind., Vol. 19, p. 614, 1910.*)

Calcium is made by the electrolysis of fused calcium chloride, to which calcium fluoride is added to lower its melting point. Calcium is made at Holcomb's Rock, Va., probably in a cell devised by Seward and Von Kügelgen (*U. S. Pat. 880,760, 1908; Min. Industry, Vol. 16, p. 131, 1907; Vol. 17, p. 99, 1908*); and abroad by the method of Rathenau (*Z. f. Elektroch., Vol. 10, p. 508, 1907*). In the Ratheneau method the cathode is an iron rod which just touches the surface of the melted calcium chloride. The calcium solidifies when deposited on the cathode by electrolysis. As the calcium grows the rod is withdrawn so that a rod of calcium is produced.

Zinc may be made by the electrolysis of fused zinc chloride. In the Swinburne-Ashcroft process sodium chloride is added to zinc chloride in such quantity that the resulting mixture contains 28 per cent zinc. The cell is a brick-lined, sheet-iron vessel. The anode is carbon, the cathode, melted zinc. Each vat takes 4.5 volts, with a cathode current density of 400 amperes per sq. ft. The temperature of the fused salt is 450° C. (*Electroch. and Met. Ind., Vol. 3, p. 65, 1905.*)

Magnesium is made by the electrolysis of fused carnallite ($MgCl_2.KCl.6\,H_2O$) and floats to the surface of the salt, which is, of course, anhydrous when melted.

ELECTRIC FURNACES IN METALLURGY. — Under special local conditions, where iron ore is plentiful, where coke is expensive and where power is cheap, the electric reduction of iron ore is carried out commercially, as at Trollhätten and Domnarfvet, Sweden, and at Héroult, California. The furnaces

have a shaft resembling a blast furnace, with a crucible at the base into which the electrodes project in a slanting position. (*For detailed accounts, see volumes of the Met. and Chem. Eng.*) More recently it has been found better to have the electrodes vertical. (*Taussig, VIII Int. Congress App. Chem., Vol. 21, p. 105, 1912.*)

Electric tin smelting has been tried on a commercial scale with apparent success (*Met. and Chem. Eng., Vol. 9, p. 453, 1911*) as well as the smelting of copper and nickel (*Met. and Chem. Eng., Vol. 11, p. 22, 1913*), and zinc (*see volumes of the Met. and Chem. Eng.*). The use of electric furnaces in steel refining and in the production of ferro-alloys is much more extensive for in this case power does not need to be so cheap as in the reduction of iron ore.

Steel Refining. — Usually the steel which is refined in electric furnaces is taken from Bessemer converters or open-hearth furnaces and poured directly into the electric furnace. The advantages of the electric furnaces are (*Walker, Met. and Chem. Eng., Vol. 10, p. 371, 1912*):

1. Complete removal of oxygen,
2. Absence of oxides caused by additions, such as silicon manganese,
3. Production of electric steel ingots of 8 tons and less that are practically free from segregation,
4. Reduction of sulphur to 0.005 per cent if desired,
5. Reduction of phosphorus to 0.005 per cent as in the basic open-hearth process, but with complete removal of oxygen.

There are in operation over 70 electric steel furnaces of different types in Europe and America (*Walker*).

The recent progress in electric steel refining consists in an improvement in existing methods and in a reduction of costs. While in 1911 it was considered good practice to melt and refine steel scrap in six hours at 750 kilowatt hours per ton, the same operation is now carried out in four hours with 600 kilowatt hours. (*Héroult, VIII Int. Cong. of App. Chem., Vol. 21, p. 59, 1912.*)

Some of the types of furnaces used in steel refining are the following:

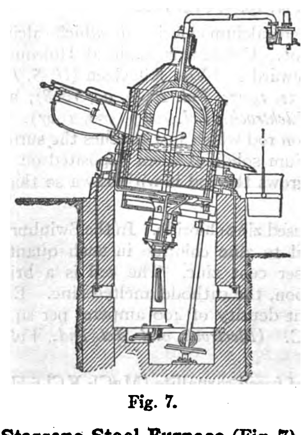


Fig. 7.

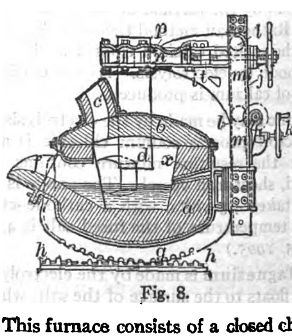


Fig. 8.

Stassano Steel Furnace (Fig. 7). — This furnace consists of a closed chamber with three electrodes connected to a three-phase system above the slag. The furnace rotates so as to stir all of the metal. This furnace is used in Italy, Odessa, and Newcastle-on-Tyne (*Met. and Chem. Eng., Vol. 10, p. 66, 1912.*)

Héroult Steel Furnace (Fig. 8). — This furnace consists of a crucible lined with refractory material. Carbon electrodes project into it through the roof. An arc is formed where the current passes from each electrode into the slag. The power is regulated by an electrical automatic regulator which moves the electrodes up and down as required.

Girod Steel Furnace. — This furnace consists of a crucible with several soft-steel rods projecting through the base. These form one electrode; the other is one or more carbon rods suspended from above. The steel electrodes of course melt several inches below the surface of the refractory lining of the crucible. The furnaces are sold by C. W. Leavitt and Co., 30 Church St., New York, from whom the following data have been obtained.

These furnaces require from 65 to 70 volts at frequencies from 25 to 50 cycles. The power factor is about 80 per cent and the duration of heat when the metal is charged in the melted state is 1½ to 2½ hours. The number of electrodes are from 1 to 3, according to the capacity of the furnace. For a furnace of about 12 tons capacity, the maximum power required is 1200 kilowatts measured at the terminals of the electric generator; the energy consumption when the metal is charged cold is 900 kilowatt hours per ton of steel; with a melted charge, from 150 to 250 kilowatt hours. The electrodes are so designed that the current does not exceed 5 amperes per square inch of cross-section. The number of consecutive heats possible without repairs is:

	Linings	Cover	Electrodes
With cold charge....................	30 to 40	20 to 30	10
With melted charge..................	60 to 90	40 to 50	20

To handle the furnace 6 persons are necessary: a melter, 2 assistant melters, 2 workmen and a boy.

Other Resistance Furnaces more or less similar to the Héroult furnace are the Keller furnace (*Trans. Am. Electroch. Soc., Vol. 15, p. 96, 1909*) and the Nathusius furnace (*Met. and Chem. Eng., Vol. 10, p. 227, 1912*).

Induction Furnaces. — Induction furnaces are transformers in which a melted ring of steel is the secondary. The Kjellin furnace, Fig. 9, consists of a single deep ring of metal. It has only a small area of contact between the metal and slag, and the slag is not easily heated. The use of this furnace is therefore restricted in its application (*Kjellin, Trans. Am. Electroch. Soc., Vol. 15, p. 175, 1909*).

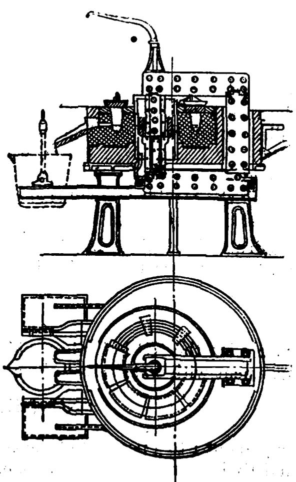


Fig. 9.

A modified form of induction furnace is the Röchling-Rodenhauser furnace. This furnace has two annular rings combined in the form of a figure 8. The central portion carries the currents induced in both circuits, as well as a current from electrodes supplied by extra secondary coils. This current passes through the lining of the furnace, which has sufficient conductivity when hot.

These furnaces, as well as another modification, known as the Frick furnace, can be obtained from Siemens and Halske, represented in this country by Dr. G. K. Frank, 80 West St., New York. Another design of induction furnace is due to Hiorth (*Trans. Am. Electroch. Soc., Vol. 20, p. 293, 1911*).

Pinch Effect in Induction Furnaces. — The magnitude of the current which can be sent through a trough of melted metal is limited by the so-called pinch effect (*Trans. Am. Electroch. Soc., Vol. 11, p. 329, 1907*). On account of the attraction of the current elements for each other, a compressing force is exerted on the metal which causes a decrease in the cross-section at some point. If the current is too great the metal may be entirely separated and the circuit broken.

Ferro-Alloys. — Ferro-alloys were originally made from iron ore, the oxide of other metal, carbon, and flux, but on account of impurities, scrap iron and steel shavings are now used in place of iron ore (*Met. and Chem. Eng., Vol. 8, p. 133, 1910*). Arc and resistance furnaces similar to those used for steel refining are used.

Ferro-Silicon is the most important of the ferro-alloys. It is made from iron, quartzite, and carbon. At the Keller-Leleux works at Livet it has been found practicable to turn out 20 tons of 30 per cent ferro-silicon with 4000 horse-power during a day. (*See preceding reference.*) For an account of the uses of the other ferro-alloys, see *Electrochem. Ind., Vol. 1, p. 583, 1903; Electrochem. and Met. Ind., Vol. 4, p. 247, 1906.*

FIXATION OF ATMOSPHERIC NITROGEN. — One of the most important applications of the electric furnace is the fixation of atmospheric nitrogen. The various processes employed are described below.

Carbide Method. — In this method calcium carbide is heated in pure nitrogen, forming calcium cyanide according to the reversible reaction

$$CaC_2 + N_2 \rightleftarrows CaCN_2 + C.$$

The carbide is heated in iron drums by a thin carbon conductor running through the center of the drum. Heat is evolved by the reaction. The product is called "nitro-lime" or "lime-nitrogen," and contains 12 to 15 per cent nitrogen. (*Met. and Chem. Eng., Vol. 5, p. 78, 1907.*) It is used directly as a fertilizer, or may be converted into ammonia by superheated steam. The yield in nitrogen by the carbide method is about 51.6 grams per kilowatt hour, including the manufacture of the carbide.

Direct-Oxidation Method. — In this method the nitrogen and oxygen in air are caused to combine in a high-voltage arc according to the reversible reaction

$$N_2 + O_2 \rightleftarrows 2\ NO.$$

On cooling the NO is further oxidized to nitric dioxide. The nitric dioxide on treatment with water gives nitric and nitrous acids:

$$2\ NO_2 + H_2O = HNO_3 + HNO_2.$$

Only about 1 to 3 per cent of the air treated is oxidized. The yield in the Birkeland-Eyde furnace is 12.7 grams of nitrogen fixed per kilowatt hour. (*Trans. Faraday Soc., Vol. 2, p. 98, 1906.*) Furnaces of three different designs are now in operation for oxidizing nitrogen: that of Birkeland and Eyde, that of Schönherr, and that of H. and G. Pauling.

Direct Synthesis of Ammonia. — In this method, due to Haber, ammonia is formed directly from a mixture of nitrogen and hydrogen by passing over a

catalyzer between 500° C. and 700° C. at 200 atmospheres, according to the reversible reaction

$$\tfrac{1}{2} N_2 + \tfrac{3}{2} H_2 \rightleftarrows NH_3.$$

(*Zeit. f. Elektroch., Vol. 16, p. 244, 1910; Vol. 19, p. 53, 1913.*) The efficiency of this process has not been made public.

Serpek Process. — In this process aluminum nitride is made by heating aluminum oxide, carbon, and nitrogen together. On heating the aluminum nitride with water, ammonia and aluminum hydrate are formed. The product obtained from the furnace is said to contain 20 to 24 per cent of nitrogen, and the power required per unit of nitrogen is said to be only one-half of that used in the calcium-carbide method. (*Bull. de la Soc. ind. de Mulhouse, Vol. 79, p. 39, 1909. U.S. Pat. 996,032, 1911.*)

BIBLIOGRAPHY. — Borchers, *Electric Smelting and Refining*, 1904; Kershaw, *Electrometallurgy*, 1908; Macmillan and Cooper, *Electrometallurgy*, 1910. See also references in text above, and bibliography in article on *Electrochemical Processes, Industrial*.

[M. DeK. Thompson.]

FUSES. — (*See also Circuit Breakers; Wiring of Buildings.*) Metal strips or wires which open electric circuits by melting or fusing when the current reaches a predetermined value are called fuses. They were used in the earliest electric plants to furnish automatic protection for feeder and generator circuits. Their use is now confined largely to low-voltage distribution circuits and to the protection of small pieces of apparatus such as motors and small-capacity transformers.

All fuses have an inherent time-element feature due to the fact that the current must heat the fuse metal up to its melting temperature. This time lag varies with the size and type of fuse, the large ones in general taking longer to reach their fusing temperature than the smaller sizes, due to their thermal capacities. There are three designs of fuses in general use, namely, the open or link, the expulsion, and the enclosed or cartridge types. The choice of type depends upon the service for which the fuse is intended.

RATING OF A FUSE.—The rating of a fuse depends somewhat on its type and general design. Fuses will, as a rule, carry their normal rated current indefinitely but will blow at a certain overload varying from 15 per cent in an enclosed fuse to about 80 per cent in an open fuse if the overload continues a sufficient length of time. For greater overloads the length of time required for blowing diminishes rapidly.

OPEN OR LINK FUSES were the original type of automatic protection. The early ones were small copper wires and had the drawback of forming copper globules and of possessing a high fusion temperature. In order to reduce the temperature of the molten metal there were employed alloys of low fusing points made of lead, tin, or other metals. These fuses were soft and were easily damaged when tightening up the contact nuts. The next step was to use alloy fuses with copper tips, and these are still used to some extent.

As the price of aluminum was reduced this material was used largely for fuses as it has a high conductivity (thus reducing the amount of metal to be fused), a fairly low melting point and almost complete vaporization of the metal fused. By using wide strips of aluminum cut to form two or more bridges, fairly reliable open fuses can be made up to 1200 amperes capacity.

Any metal strip which is exposed to drafts is apt to be very erratic in its behavior as a fuse. To protect link fuses from drafts, and also to remove the danger from molten metal which may be thrown at the time of blowing, it is desirable to install them in porcelain fuse boxes.

EXPULSION FUSES consist essentially of open fuse wires or strips placed in a holder, so designed that the expulsion of the gases formed by the melting of the fuse blows out the arc. The earliest designs of this type comprised a removable fuse holder of lignum vitæ or similar tough, close-grained wood, equipped with terminals which fitted into suitable blocks. Later types have the fuse placed in a fiber tube and arranged to blow out through one end like a bomb.

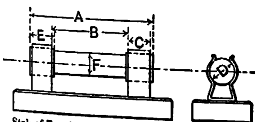
The fuses used for the protection of distributing transformers mounted on poles or houses are usually of the expulsion type. For such outdoor service a combination fuse and disconnecting switch is frequently used. Modifications of the expulsion-type fuse were formerly used to advantage in railway service, but in recent car equipments circuit breakers have practically replaced fuses and cut-outs.

INCLOSED OR CARTRIDGE FUSE.—This type of fuse consists essentially of a fusible wire, strip or sets of wires and strips inclosed within a tube or cartridge usually of fiber. The tube is filled with a material to exclude the

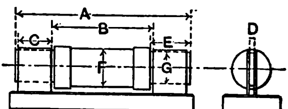
air and to facilitate the opening of the circuit when the fuse blows by absorbing the gases formed and chilling out the arc. Suitable terminals are provided so that the fuse may be mounted in a fuse block.

National Electrical Code Fuses. — When inclosed fuses were first put on the market each manufacturer developed his own spacings and designs of terminals, so that there was no uniformity and the fuse of one manufacturer could not be used in the fuse holder of another manufacturer. To avoid this confusion the representatives of the fuse builders and the National Board of Fire Underwriters finally adopted the standard dimensions and types of contacts given in the accompanying table. Up to 60 amperes ferrule type contacts (i.e., cylindrical metal ends) are used and from 61 to 600 amperes knife-blade contacts are employed. One set of dimensions are used for fuses up to 250 volts and another for fuses up to 600 volts. Fuses that correspond to the accepted dimensions and that meet other requirements agreed on are known as National Electrical Code (N. E. C.) fuses and are interchangeable. Fuses are made for higher voltages than 600 volts and larger currents then 600 amperes, but they have not been accepted by the National Board of Fire Underwriters.

DIMENSIONS OF N. E. C. CARTRIDGE FUSES



Style of Terminal for Cartridge Fuses
0-60 Amperes



Style of Terminal for Cartridge Fuses
61-600 Amperes

			A	B	C	D	E	F	G
Voltage	Rated capacity		Length over terminals	Distance between contact clips	Width of contact clips	Diameter of ferrules or thickness of terminal blades	Min. length of ferrules or of terminal blades outside of tube	Diameter of tube	Width of terminal blades
	Amp.		Inches	Inches	Inches	Inches	Inches	Inches	Inches
0–250	0–30	Form 1	2	1	½	9/16	½	½	. . .
	31–60	Form 1	3	1¾	⅝	13/16	⅝	¾	. . .
	61–100	Form 2	5⅞	4	⅞	⅛	1	1	¾
	101–200	Form 2	7⅛	4½	1¼	3/16	1⅜	1½	1⅛
	201–400	Form 2	8⅝	5	1¾	¼	1⅞	2	1⅝
	401–600	Form 2	10⅜	6	2⅛	¼	2¼	2½	2
251–600	0–30	Form 1	5	4	½	13/16	½	¾	. . .
	31–60	Form 1	5½	4¼	⅝	1 1/16	⅝	1	. . .
	61–100	Form 2	7⅞	6	⅞	⅛	1	1¼	¾
	101–200	Form 2	9⅝	7	1¼	3/16	1⅜	1¾	1⅛
	201–400	Form 2	11⅝	8	1¾	¼	1⅞	2½	1⅝

Cost of N. E. C. Fuses. — In moderate quantities of standard packages the price of 30-ampere 250-volt fuses is about 15 cents each, and of 400-ampere 600-volt fuses about $3.30 each. The others fall between these limits.

Plug Fuses. — A plug fuse is a fuse mounted in a plug of the standard Edison-lamp base dimensions. They are in common use on lighting and power circuits up to 30 amperes capacity. The outer face of the plug usually has a mica covering which is discolored by the blowing of the fuse. A brass cap is also sometimes used as a cover.

SPECIFICATIONS FOR FUSES. — The following memoranda are intended to assist in writing specifications. See also articles on *Specifications*.

Open or inclosed. Wire or ribbon. Current to blow fuse in a specified time. To blow without violence at a stated current. Voltage of circuit above ground.

BIBLIOGRAPHY. — *Manufacturers' Catalogues and Circulars;* Harvey, D., *Elec. Jour.*, Mch., 1908; Downes, L. W., *Trans. A.I.E.E.*, 1909, Vol. 28, p. 947.

[S. Q. Hayes.]

GAGES, SHEET-METAL. — (*See also Copper; Gages, Wire; Iron, Wrought; Steel.*) There are two principal gages for sheet iron and steel used in the United States, one established by Act of Congress in 1893 and the other recommended by a joint committee of the American Society of Mechanical Engineers and the American Railway Master Mechanics Association.

U. S. Standard Gage. — The former gage, which is known as the U. S. Standard Gage, is based upon the fact that a cubic foot of iron weighs 480 pounds. The scale of this gage has been arranged so that each gage number represents a certain number of ounces in weight, or an equal number of 640ths of an inch in thickness. This gage is used for determining duties and taxes levied on sheet and plate iron and steel, and in its application a variation of 2½ per cent either way is allowable.

Decimal Gage. — The latter gage, which is known as the Decimal Gage, is not based upon the weight but upon the thickness of the metal. Each number of the gage is the number of thousandths of an inch of thickness. This gage has been adopted recently by the Association of American Steel Manufacturers, the American Railway Master Mechanics' Association, and by most of the principal railroads of the United States, Canada and Mexico. The decimal system of gaging was recommended by the American Institute of Mining Engineers in 1877 and by the American Society of Mechanical Engineers in 1895.

Brown and Sharpe Gage. — Copper and brass plates are rated by the Brown and Sharpe gage. The plate thicknesses on this gage correspond to the diameters on the American Wire Gage. (*See Gages, Wire.*)

The following tables give the dimensions and weights of sheet metal according to gages described above.

U. S. STANDARD GAGE FOR SHEET AND PLATE, IRON AND STEEL

No. of gage	Approximate thickness in decimal parts of an inch	Approximate thickness in millimeters	Weight per square foot in pounds avoirdupois	Weight per square meter in kilograms
0000000	0.5000	12.7000	20.00	97.65
000000	0.4687	11.9062	18.75	91.55
00000	0.4375	11.1125	17.50	85.44
0000	0.4062	10.3187	16.25	79.33
000	0.3750	9.5250	15.00	73.24
00	0.3437	8.7312	13.75	67.13
0	0.3125	7.9375	12.50	61.03
1	0.2812	7.1437	11.25	54.93
2	0.2656	6.7469	10.62	51.88
3	0.2500	6.3500	10.00	48.82
4	0.2344	5.9531	9.375	45.77
5	0.2187	5.5562	8.750	42.72
6	0.2031	5.1594	8.125	39.67
7	0.1875	4.7625	7.500	36.62
8	0.1719	4.3656	6.875	33.57
9	0.1562	3.9687	6.250	30.52
10	0.1406	3.5719	5.625	27.46
11	0.1250	3.1750	5.000	24.41
12	0.1094	2.7781	4.375	21.36
13	0.0937	2.3812	3.750	18.31
14	0.07812	1.9844	3.125	15.26
15	0.07031	1.7859	2.812	13.73
16	0.06250	1.5875	2.500	12.21
17	0.05625	1.4287	2.250	10.99
18	0.05000	1.2700	2.000	9.765
19	0.04375	1.1112	1.750	8.544
20	0.03750	0.9525	1.500	7.324
21	0.03437	0.8731	1.375	6.713
22	0.03125	0.7937	1.250	6.103
23	0.02812	0.7144	1.125	5.490
24	0.02500	0.6350	1.000	4.882
25	0.02187	0.5556	0.875	4.272
26	0.01875	0.4762	0.750	3.662
27	0.01719	0.4366	0.687	3.357
28	0.01562	0.3969	0.625	3.052
29	0.01406	0.3572	0.5625	2.746
30	0.01250	0.3175	0.5000	2.441
31	0.01094	0.2778	0.4375	2.136
32	0.01016	0.2580	0.4062	1.983
33	0.009375	0.2381	0.3750	1.831
34	0.008594	0.2183	0.3437	1.678
36	0.007031	0.1786	0.2812	1.373
38	0.006250	0.1587	0.2500	1.221

STANDARD DECIMAL GAGE FOR SHEET AND PLATE, IRON AND STEEL

Standard decimal gage in inches	Approximate thickness in millimeters	Weights per square foot in pounds avoirdupois	
		Iron, Basis: 480 pounds per cubic foot	Steel, Basis: 489.6 pounds per cubic foot
0.002	0.0508	0.08	0.0816
0.004	0.1016	0.16	0.1632
0.006	0.1524	0.24	0.2448
0.008	0.2032	0.32	0.3264
0.010	0.2540	0.40	0.4080
0.012	0.3048	0.48	0.4896
0.014	0.3556	0.56	0.5712
0.016	0.4064	0.64	0.6528
0.018	0.4572	0.72	0.7344
0.020	0.5080	0.80	0.8160
0.022	0.5588	0.88	0.8976
0.025	0.6350	1.00	1.0200
0.028	0.7112	1.12	1.1424
0.032	0.8128	1.28	1.3056
0.036	0.9144	1.44	1.4688
0.040	1.0160	1.60	1.6320
0.045	1.1430	1.80	1.8360
0.050	1.2700	2.00	2.0400
0.055	1.3970	2.20	2.2440
0.060	1.5240	2.40	2.4480
0.065	1.6510	2.60	2.6520
0.070	1.7780	2.80	2.8560
0.075	1.9050	3.00	3.0600
0.080	2.0320	3.20	3.2640
0.085	2.1590	3.40	3.4680
0.090	2.2860	3.60	3.6720
0.095	2.4130	3.80	3.8760
0.100	2.5400	4.00	4.0800
0.110	2.7940	4.40	4.4880
0.125	3.1750	5.00	5.1000
0.135	3.4290	5.40	5.5080
0.150	3.8100	6.00	6.1200
0.165	4.1910	6.60	6.7320
0.180	4.5720	7.20	7.3440
0.200	5.0800	8.00	8.1600
0.220	5.5880	8.80	8.9760
0.240	6.0960	9.60	9.7920
0.250	6.3500	10.00	10.2000

BROWN AND SHARPE GAGE FOR COPPER AND BRASS PLATES*

B. & S. Gage No.	Thickness, inches	Weight, pounds per sq. ft.	
		Copper	Brass
0000	0.4600	20.84	19.69
000	0.4096	18.56	17.53
00	0.3648	16.53	15.61
0	0.3249	14.72	13.90
1	0.2893	13.11	12.38
2	0.2576	11.67	11.03
3	0.2294	10.39	9.82
4	0.2043	9.26	8.74
5	0.1819	8.24	7.79
6	0.1620	7.34	6.93
7	0.1443	6.54	6.18
8	0.1285	5.82	5.50
9	0.1144	5.18	4.90
10	0.1019	4.62	4.36
11	0.09074	4.11	3.88
12	0.08081	3.66	3.46
13	0.07196	3.26	3.08
14	0.06408	2.90	2.74
15	0.05707	2.59	2.44
16	0.05082	2.30	2.18
17	0.04526	2.05	1.94
18	0.04030	1.83	1.73
19	0.03589	1.63	1.54
20	0.03196	1.45	1.38
21	0.02846	1.29	1.22
22	0.02535	1.15	1.08
23	0.02257	1.02	0.966
24	0.02010	0.911	0.860
25	0.01790	0.811	0.766
26	0.01594	0.722	0.682
27	0.01420	0.643	0.608
28	0.01264	0.573	0.541
29	0.01126	0.510	0.482
30	0.01003	0.454	0.429
31	0.008928	0.404	0.382
32	0.007950	0.360	0.340
33	0.007080	0.321	0.303
34	0.006304	0.286	0.270
36	0.005000	0.226	0.214
38	0.003965	0.180	0.170
40	0.003145	0.142	0.135

* The specific gravity of copper plate is taken as 8.88 and of brass plate as 8.39.

BIBLIOGRAPHY.—*Report of Committee on Standard Thickness Gage for Metals*, Trans. A.S.M.E., 1895, Vol. 16, p. 641; *Report of Committee on Standard Gages*, Proc. Am. Ry. Master Mech. Assn., 1895, p. 149.

[WM. A. DEL MAR.]

GAGES, WIRE. — (*See also Gages, Sheet Metal.*) The sizes of wires having a diameter less than ½ inch are usually stated in terms of certain arbitrary scales called "gages." The size or gage number of a solid wire refers to the cross-section of the wire perpendicular to its length; the size or gage number of a stranded wire refers to the total cross-section of the constituent wires, irrespective of the pitch of the spiraling. Larger wires are usually described in terms of their area expressed in circular mils. A circular mil is the area of a circle 1 mil in diameter, and the area of any circle in circular mils is equal to the square of its diameter in mils. See *Units and Conversion Factors.*

There are a number of wire gages in use, the principal ones being the following.

AMERICAN OR BROWN AND SHARPE WIRE GAGE. — This gage is the one commonly used in the United States for copper, aluminum and resistance wires. The gage is designated by either of the abbreviations A. W. G. or B. & S.

Basis of the A. W. G. or B. & S. Gage. — The diameter of wires having successive numbers on this gage are in the ratio of $\sqrt[39]{92}$ (= 1.1229 approx.) to 1, and the No. 36 wire has a diameter of 5 mils. No. 35 A. W. G., therefore, has a diameter of $5 \times 1.1229 = 5.61$ mils and so on until No. 0000 is reached, having a diameter of 460 mils.

The ratio $\sqrt[39]{92}$ is approximately equal to $\sqrt[6]{2}$, which is 1.1225. This circumstance makes it possible to have a group of wires of regular gage size with an aggregate area approximately equal to that of another regular gage size.

The following approximate relations are also useful:

An increase of 1 in the number increases the resistance 25 per cent.
An increase of 2 in the number increases the resistance 60 per cent.
An increase of 3 in the number increases the resistance 100 per cent.
An increase of 10 in the number increases the resistance 10 times.

A No. 10 A. W. G. copper wire has the following approximate characteristics:

Ohms per 1000 feet	1
Circular mils area	10,000
Weight, pounds per 1000 feet	32

A No. 10 A. W. G. aluminum wire has the following approximate characteristics:

Ohms per 1000 feet	1.6
Circular mils area	10,000
Weight, pounds per 1000 feet	9.5

Remembering these rules it is easy to find the approximate size, resistance, area, or weight of any size wire. For example, a No. 12 A.W. G. copper wire has a resistance of 1 plus 60 per cent = 1.6 ohms per 1000 feet approximately. Its area, being inversely as its resistance, is $\frac{10,000}{1.6} = 6250$ circular mils; its diameter is therefore $\sqrt{6250} = 79$ mils and its weight, $\frac{32}{1.6} = 20$ pounds per 1000 feet.

U. S. STEEL WIRE GAGE. — This gage, known also as the "Washburn and Moen," "Roebling," "American Steel and Wire Co.'s" gage, is the one usually employed in the United States for steel and iron wire. It is frequently abbreviated "S. W. G.," but to avoid confusion with the British Standard Wire Gage (*see below*) it should be abbreviated "Stl. W. G." or "A. (steel) W.G."

BIRMINGHAM (OR STUBS') WIRE GAGE.—This gage is still used in the United States for some purposes, e.g., to designate the size of brass wire, and is also employed to a limited extent in Great Britain. It is usually abbreviated "B. W. G." It is sometimes referred to as the "Stubs'" gage, but it should not be confused with the Stubs' Steel Wire Gage.

BRITISH STANDARD WIRE GAGE.—This gage, usually called simply the "Standard Wire Gage," and abbreviated "S. W. G.," is also known as the "New British Standard" (abbreviated "N. B. S."), the English Legal Standard, or the Imperial Wire Gage, and is the legal standard of Great Britain for all wires, as fixed by order in Council, August 23, 1883. It was constructed by modifying the Birmingham Wire Gage, so that the differences between successive diameters were the same for short ranges, i.e., so that a graph representing the diameters consists of a series of a few straight lines.

EDISON WIRE GAGE.—The size of a wire on this gage is equal to its cross-sectional area in circular mils divided by 1000. For example, a solid wire 0.2 inch in diameter has the number $(200)^2/1000 = 40$. This gage is now rarely used.

OTHER GAGES.—In addition wire sizes are sometimes specified in terms of the "Old English Wire Gage," known also as the "London Gage," and the "Stubs' Steel Wire Gage."

COMPARISON OF WIRE GAGES.—A comparison of the different gages, in terms of the diameters (in mils or thousandths of an inch) of solid wires corresponding to the various numbers, is given below. The cross-section in circular mils is the square of the diameter in mils.

COMPARISON OF WIRE GAGES

Diameters in Mils

(*Bureau of Standards, Circular No. 31*)

Gage No.	American wire gage (B. & S.)	Steel wire gage	Birmingham wire gage (Stubs')	Old English wire gage (London)	Stubs' steel wire gage	(British) Standard wire gage	Gage No.
7-0	.....	490.0	...	......	...	500	7-0
6-0	.....	461.5	...	......	...	464	6-0
5-0	.....	430.5	...	......	...	432	5-0
4-0	460	393.8	454	454	...	400	4-0
3-0	410	362.5	425	425	...	372	3-0
2-0	365	331.0	380	380	...	348	2-0
0	325	306.5	340	340	...	324	0
1	289	283.0	300	300	227	300	1
2	258	262.5	284	284	219	276	2
3	229	243.7	259	259	212	252	3
4	204	225.3	238	238	207	232	4
5	182	207.0	220	220	204	212	5
6	162	192.0	203	203	201	192	6
7	144	177.0	180	180	199	176	7
8	128	162.0	165	165	197	160	8
9	114	148.3	148	148	194	144	9

COMPARISON OF WIRE GAGES — *Continued*

Diameter in Mils

(Bureau of Standards, Circular No. 31)

Gage No.	American wire gage (B. & S.)	Steel wire gage	Birmingham wire gage (Stubs')	Old English wire gage (London)	Stubs' steel wire gage	(British) Standard wire gage	Gage No.
10	102	135.0	134	134	191	128	10
11	91	120.5	120	120	188	116	11
12	81	105.5	109	109	185	104	12
13	72	91.5	95	95	182	92	13
14	64	80.0	83	83	180	80	14
15	57	72.0	72	72	178	72	15
16	51	62.5	65	65	175	64	16
17	45	54.0	58	58	172	56	17
18	40	47.5	49	49	168	48	18
19	36	41.0	42	40	164	40	19
20	32	34.8	35	35	161	36	20
21	28.5	31.7	32	31.5	157	32	21
22	25.3	28.6	28	29.5	155	28	22
23	22.6	25.8	25	27.0	153	24	23
24	20.1	23.0	22	25.0	151	22	24
25	17.9	20.4	20	23.0	148	20	25
26	15.9	18.1	18	20.5	146	18	26
27	14.2	17.3	16	18.75	143	16.4	27
28	12.6	16.2	14	16.50	139	14.8	28
29	11.3	15.0	13	15.50	134	13.6	29
30	10.0	14.0	12	13.75	127	12.4	30
31	8.9	13.2	10	12.25	120	11.6	31
32	8.0	12.8	9	11.25	115	10.8	32
33	7.1	11.8	8	10.25	112	10.0	33
34	6.3	10.4	7	9.50	110	9.2	34
35	5.6	9.5	5	9.00	108	8.4	35
36	5.0	9.0	4	7.50	106	7.6	36
37	4.5	8.5	...	6.50	103	6.8	37
38	4.0	8.0	...	5.75	101	6.0	38
39	3.5	7.5	...	5.00	99	5.2	39
40	3.1	7.0	...	4.50	97	4.8	40
41	.....	6.6	...	......	95	4.4	41
42	.....	6.2	...	......	92	4.0	42
43	.....	6.0	...	......	88	3.6	43
44	.....	5.8	...	......	85	3.2	44
45	.....	5.5	...	......	81	2.8	45
46	.....	5.2	...	......	79	2.4	46
47	.....	5.0	...	......	77	2.0	47
48	.....	4.8	...	......	75	1.6	48
49	.....	4.6	...	......	72	1.2	49
50	.....	4.4	...	......	69	1.0	50

[W. A. Del Mar.]

GALVANIZING FOR IRON OR STEEL, Specification for Acceptance Test. — These specifications give in detail the test to be applied to galvanized material, as recommended by the Electric Railway Engineering Association, the National Electric Light Association, etc. All specimens shall be capable of withstanding these tests.

(a) **Coating.** — The galvanizing shall consist of a continuous coating of pure zinc of uniform thickness, and so applied that it adheres firmly to the surface of the iron or steel. The finished product shall be smooth.

(b) **Cleaning.** — The samples shall be cleaned before testing, first with carbona, benzine or turpentine, and cotton waste (not with a brush), and then thoroughly rinsed in clean water and wiped dry with clean cotton waste.

The sample shall be clean and dry before each immersion in the solution.

(c) **Solution.** — The standard solution of copper sulphate shall consist of commercial copper sulphate crystals dissolved in cold water, about in the proportion of 36 parts, by weight, of crystals to 100 parts, by weight, of water. The solution shall be neutralized by the addition of an excess of chemically pure cupric oxide (CuO). The presence of an excess of cupric oxide will be shown by the sediment of this reagent at the bottom of the containing vessel.

The neutralized solution shall be filtered before using by passing through filter paper. The filtered solution shall have a specific gravity of 1.186 at 65° F. (reading the scale at the level of the solution) at the beginning of each test. In case the filtered solution is high in specific gravity, clean water shall be added to reduce the specific gravity to 1.186 at 65° F. In case the filtered solution is low in specific gravity, filtered solution of a higher specific gravity shall be added to make the specific gravity 1.186 at 65° F.

As soon as the stronger solution is taken from the vessel containing the unfiltered neutralized stock solution, additional crystals and water must be added to the stock solution. An excess of cupric oxide shall always be kept in the unfiltered stock solution.

(d) **Quantity of Solution.** — Wire samples shall be tested in a glass jar of at least two (2) inches inside diameter. The jar without the wire samples shall be filled with standard solution to a depth of at least four (4) inches. Hardware samples shall be tested in a glass or earthenware jar containing at least one-half (½) pint of standard solution for each hardware sample. Solution shall not be used for more than one series of four immersions.

(e) **Samples.** — Not more than seven wires shall be simultaneously immersed, and not more than one sample of galvanized material other than wire shall be immersed in the specified quantity of solution.

The samples shall not be grouped or twisted together, but shall be well separated so as to permit the action of the solution to be uniform upon all immersed portions of the samples.

(f) **Test.** — Clean and dry samples shall be immersed in the required quantity of standard solution in accordance with the following cycle of immersions.

The temperature of the solution shall be maintained between 62° and 68° F. at all times during the following test.

First. Immerse for one minute, wash and wipe dry.
Second. Immerse for one minute, wash and wipe dry.
Third. Immerse for one minute, wash and wipe dry.
Fourth. Immerse for one minute, wash and wipe dry.

After each immersion the samples shall be immediately washed in clean water having a temperature between 62° and 68° F., and wiped dry with cotton waste.

In the case of No. 14 galvanized iron or steel wire, the time of the fourth immersion shall be reduced to one-half minute.

(g) **Rejection.** — If after the test described in section "f" there should be a bright metallic copper deposit upon the samples, the lot represented by the samples shall be rejected.

Copper deposits on zinc or within one inch of the cut end shall not be considered causes for rejection.

In the case of a failure of only one wire in a group of seven wires immersed together, or if there is a reasonable doubt as to the copper deposit, two check tests shall be made on these seven wires and the lot reported in accordance with the majority of the sets of tests.

[W. A. Del Mar.]

GALVANOMETERS. — (*See also Ammeters; Electrodynamometers; Fluxmeter; Voltmeters.*) The essential parts of a galvanometer are a permanent magnet (electromagnet in Einthoven String Galvanometer, see below) and a coil or wire designed to carry an electric current, the magnet and coil being so mounted that the passage of the current through the coil causes a deflection of either the coil or the magnet, as a result of the mechanical force with which the current and magnet act on each other. See *Electricity and Magnetism, Principles of.* Ordinary direct-current ammeters and voltmeters are operated on the same principle, but the term galvanometer is usually reserved for instruments designed for measuring relatively small currents, the deflection being read either by means of a telescope and scale or by means of a lamp and scale. In portable test set galvanometers the deflection of a pointer is read.

Moving-needle versus Moving-coil Galvanometers. — Galvanometers in which the moving element is the permanent magnet, in the form of a light needle, are called needle galvanometers, whereas those in which the moving element is the coil are called moving-coil galvanometers. The former type of galvanometer is required when extreme sensitiveness is desired, but cannot as a rule be used in a commercial laboratory, on account of the disturbing influences of the magnetic fields due to the lighting and power circuits in the laboratory and in neighboring buildings. Railway circuits, even though a mile or more away, may produce a disturbing field sufficient to render impossible the use of a sensitive needle galvanometer. For ordinary laboratory work, however, the moving-coil galvanometer can be made sufficiently sensitive, and since the permanent magnets used to produce the controlling field can be made very strong, the effect of stray fields can be rendered relatively negligible.

Astatic Galvanometer. — The disturbing effect of stray magnetic fields on a needle galvanometer can be partially prevented by the use of two sets of stationary coils and two needles, or the equivalent, the two needles being so arranged that any external field will produce opposing torques, and the coils so connected that their torques are additive. The Broca galvanometer, Fig. 1, described below, is a modern type of astatic instrument.

Damping and Logarithmic Decrement. — Due to the mechanical friction to motion and the induced currents set up in the adjacent metal or coils of the instrument as a result of the motion of the moving element, successive swings of the moving element when it is once set in motion decrease in amplitude. This effect is known as damping. Successive swings from the zero or equilibrium position decrease approximately in geometric ratio. Hence the ratio of the logarithm of one swing to the logarithm of the next swing, to the other side of the zero, is approximately a constant. This constant, which is a measure of the damping, is called the logarithmic decrement of the instrument.

Ballistic Galvanometer. — A ballistic galvanometer is one in which the moving element has a relatively long period and small damping. It can be shown that when a quantity of electricity Q is discharged through such a galvanometer in an interval of time *very short* compared with the period of the galvanometer, then the first throw or swing of the moving element is approximately proportional to the quantity of electricity Q. Such an instrument therefore serves as a very convenient means of measuring the capacity of a condenser (*see Capacity and Charging Current; Condensers, Electric*) as well as for measuring the quantity of electricity discharged through a circuit by transient induced electromotive forces (*see Magnetic Testing*).

Effect of External Resistance on Damping. — Since the damping of a galvanometer is due in part to the currents induced in the coils of the instrument by the motion of the moving element, and since these currents depend upon

the total resistance in series with these coils, it follows that the damping depends upon the constants of the external circuit. In particular, a low-resistance shunt around the galvanometer terminals may reduce the damping very considerably. Consequently, in any measurements involving the comparison of initial throws or swings of the moving elements, care must be taken that the same external resistance is kept in the external circuit, or that a universal shunt (*see Shunts*) connected to give a high multiplying power (100 or more) be used.

Dead-beat Galvanometers. — When the damping of a galvanometer is very large, the free motion of the moving element ceases to be periodic, and for a constant current sent through the coils the moving element swings out to its new position and comes to rest without oscillation. Such an instrument is said to be dead-beat. This dead-beat feature is very desirable when steady deflections are to be observed; it may be obtained mechanically by mounting a light vane of mica on the moving element, or by placing a solid piece of metal close to the moving element so that the motion of the latter will induce currents in the metal.

Aperiodic Galvanometer. — When the damping is *just sufficient* to render the instrument dead-beat, the galvanometer is said to be aperiodic.

Alternating-current Galvanometers. — The ordinary type of galvanometer cannot be used for measuring alternating currents, since the twisting moment depends upon the *direction* of the current. An ordinary galvanometer may, however, be used indirectly by connecting in series with it a thermo-couple, the latter being heated by a coil carrying the alternating current to be measured; this is the principle of Duddell's thermo-galvanometer. The "twisted-strip" galvanometer in which the deflection is caused by the untwisting of a strip, due to the heating action of the current, has also been used as an alternating-current galvanometer. These instruments, however, are both decidedly inferior to the Einthoven "vibration" galvanometer (Fig. 3) described below.

Tangent and Sine Galvanometers are of historical interest only, and need not be described here.

Sensitiveness of Galvanometers. — The sensitiveness or deflectional constant of a galvanometer may be defined as the scale deflection in millimeters produced by a current of one micro-ampere passing through the galvanometer coil, the scale being at one meter distance from the mirror; or, if the instrument has a scale attached, as the number of smallest scale divisions per micro-ampere, it may also be stated in terms of the potential difference in micro-volts, which must be applied to the terminals of the galvanometer to produce unit deflection, or as the deflection in millimeters, at meter distance of scale, per micro-volt on the terminals. The sensitiveness of a galvanometer must be accompanied by good *zero-keeping quality*.

TYPICAL MODERN GALVANOMETERS. — Only a few typical forms of modern galvanometers can be described here.

The Broca Galvanometer. — This galvanometer is one of the best of the moving-needle type. It owes its sensitiveness and freedom from disturbance by external magnetic fields to the unique construction of its system. This system, shown in Fig. 1, consists of two vertical steel wires placed side by side and magnetized to have consequent poles at their centers.

This construction makes a system that is very astatic and though magnets of considerable size are used, they are placed so close together that the system has a very small moment of inertia. A small mirror is located below the magnet system and below this an aluminum damping vane that moves in a chamber, the size of which can be varied, thus changing the damping. The whole system is suspended by a quartz fiber which serves as a control for the system, although

the main control is obtained by a movable permanent magnet on the base of the instrument. The stationary coils are made interchangeable and removable and vary in resistance from 10 ohms to 1000 ohms per pair.

Either lamp and scale or telescope and scale can be used to read the deflections of this galvanometer. In the latter case the scale must be brilliantly illuminated, owing to the small size of the mirror.

The resistance of this galvanometer can be varied by connecting the fixed coils in parallel or series or by substituting coils of different resistances. To vary the damping two rods on either side of the instrument case are pushed in

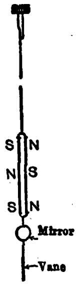


Fig. 1. Broca Galvanometer System

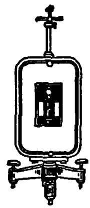


Fig. 2. D'Arsonval Galvanometer

or pulled out as required, to increase or decrease the size of the damping chamber. The period is changed by changing the position of the control magnet in reference to the system.

In the following table is given the sensitiveness of the instrument for various periods and resistances.

Resistance of coils in series in ohms	Period in seconds	Deflection in mm. at 1 m.	
		For 1 micro-ampere	For 1 micro-volt
10.9	8	218	19.9
107	12	1190	11.1
935	11	2450	2.62

D'Arsonval Galvanometers. — For the great majority of electrical measurements requiring galvanometers the D'Arsonval types, of which there are many forms, are the most satisfactory. Fig. 2 shows a typical form of construction, suitable either for mounting on the wall or on a tripod. This galvanometer has for its frame an iron casting which also serves as the permanent magnet and to hold the upper suspension tube. In the wall type this magnet casting is attached to a backboard holding the telescope and scale and the soft-iron core.

The coil is wound with copper wire, free from iron and suspended from above by a wire or strip of phosphor bronze, silver or steel, which also serves as one terminal of the coil. The other terminal is in the form of a spiral of phosphor bronze or silver strip brought off from the bottom of the coil. The coil carries a light mirror and the entire moving system is visible through a large window in

a metal plate that covers the front of the magnet casting, this front plate being removable. The deflection is read by a telescope and scale, the scale curved so that the scale reading will be closely proportional to the deflection.

The following table gives the approximate sensitiveness of galvanometers of this type for different coil resistance and period.

MILLIMETERS PER MICRO-AMPERE

Periods in seconds	Galvanometer resistance			
	50 ohms	300 ohms	500 ohms	1000 ohms
7	50	150	200	300
10	100	300	450	600
12	150	500	650	800

D'Arsonval galvanometers of a slightly different form, having a sensitiveness 50 per cent greater than the above, are on the market. Portable galvanometers, or ammeters of high sensitiveness (0.5 to 10 micro-amperes per smallest scale division) are also used in connection with thermo-couples and for measuring insulation resistance.

Einthoven String Galvanometer. — The Einthoven string galvanometer, a diagrammatic view of which is shown in Fig. 3, is essentially a moving-coil galvanometer, though the conductor which moves is but a single filament. *NS* are the poles of a powerful electromagnet, and the fine conducting "string" *AB* stretched in the narrow air gap between the poles forms the moving element. When a current is passed through this conducting string, it deflects at right angles to the field of the electromagnet as indicated by the solid arrows. This deflection is observed by means of a microscope or by projecting an enlarged image of the string on a screen by means of a projection lantern, or the deflection may be recorded on a photographic plate.

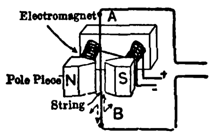


Fig. 3. Einthoven String Galvanometer

The electromagnet is relatively large and the pole faces are so shaped that an exceedingly high flux density is secured in the narrow air gap in which the string is stretched. The magnetic circuit is always fully saturated so that small variations in the exciting current do not affect the flux density in the air gap. The string, which may be an extremely fine silver wire or a silvered quartz fiber, is held under tension, which is adjustable by means of a micrometer screw. The resistance may vary from 5 ohms to 10,000 ohms, depending on the material of the string. The period depends on the tension on the string and may be reduced to less than 0.01 second or may be increased to 8 or 10 seconds with corresponding increase in sensibility.

The following table gives the characteristics of some Einthoven string galvanometers built by the Cambridge Scientific Instrument Company, when used on direct current.

Einthoven Galvanometer as an A-C. Detector, or Vibration Galvanometer. — If an alternating current is sent through the string and the tension on the latter is adjusted so that the free period of the string is the same as the period of the current, then the string will be set in vibration, the amplitude of the vibration depending upon the strength of the current. Consequently when the instrument is thus tuned, it may be used as a detector of extremely

CHARACTERISTICS OF EINTHOVEN STRING GALVANOMETERS

Material of string	Diameter of string	Resistance in ohms	Period in seconds	Magnifying power of microscope	Deflection* in mm.	
					Per micro-amp.	Per micro-volt
Silver wire.......	0.02 mm.	4.7	......	500	4.4	0.94
Silvered quartz fiber........	0.002 mm.	20,000	......	500	62,500	3.13
	0.003 mm.	6,600	......	650	333,000	50.5
	0.002 mm.	5,800	0.008	750	30	0.005
	0.002 mm.	3,890	0.005	750	9.7	0.003

* These are apparent deflections as observed through the microscope. For example, in the case of the galvanometer having the constants given in the third line, the actual deflection is $333000/650 = 512$ mm. per micro-ampere. An apparent deflection of about ⅛ mm. can be detected by the eye, consequently with this particular galvanometer a current of $10^{-6}/3 \times 333000 = 10^{-12}$ amperes, approximately, may be detected.

minute alternating currents. When thus used the instrument is capable of detecting an alternating current of 10^{-12} amperes, effective value.

Marine Galvanometers. — Any galvanometer that will give deflections that are not affected by the rolling of a ship and consequently can be used on shipboard may be called a marine galvanometer. A common form of marine galvanometer is of the D'Arsonval type with the coil held by suspensions, both above and below, that are stretched under considerable tension.

The moving system of the galvanometer is carefully balanced, so that a 30-degree inclination will not cause a deflection of more than 3 mm. with the scale at 1 meter distance. It has a very stable zero and a sensibility of 10 to 20 micro-amperes per smallest scale division with a 3-second period.

USE AND CALIBRATION OF GALVANOMETERS. — Galvanometers, other than ballistic, are used in engineering work primarily as current *detectors* in various bridge (q.v.) and potentiometer (q.v.) measurements, and therefore careful calibration is usually unnecessary. In order to determine the degree of precision of such measurements, however, the sensitiveness of the galvanometer must be known. This may be determined by connecting in series with a battery of known e.m.f. a fairly high resistance, and connecting the terminals of galvanometer across a known small fraction of this resistance. If the galvanometer resistance is known or previously measured, the current through the latter may then be calculated by Kirchhoff's laws (*see Electricity and Magnetism, Principles of*), and consequently the millimeters deflection per micro-ampere determined. If the sensitiveness of the galvanometer is too great for any particular measurement, it may be reduced by properly shunting it (*see Shunts*).

Calibration of Ballistic Galvanometer. — The ballistic galvanometer is frequently used for capacity measurements and for the determination of the magnetization curves of samples of iron or steel (*see Magnetic Testing*). One method of calibration is to charge a standard condenser to a known voltage, and discharge it through the galvanometer, noting the deflection, and repeating the test for several known voltages. As the capacity of ordinary standard condensers is dependent upon the time of charging (*see Condensers, Electric*), this method is not very accurate.

Use of Standard Solenoid. — A better method is to use a standard solenoid. Such a solenoid should have a length about 50 times its diameter. A standard used in a number of experiments conducted under the auspices of the Standardization Committee of the American Society for Testing Materials is made of 1520 turns of No. 18 A.W.G. double-cotton-covered copper wire carefully wound in a single layer upon a cylindrical core of red fiber approximately 75 inches long and 1.5 inches in diameter. (A convenient way of winding such a coil uniformly is to cut a shallow thread in the core of such a pitch that the wire when wound in the groove between the threads will lie up snug.) A secondary coil of 2000 turns of No. 36 A.W.G. double-silk-covered wire was wound in 2 layers over a length of 6 inches, at the middle of this coil.

Connect the primary coil of this solenoid in series with an adjustable resistance, reversing switch and source of constant e.m.f. Connect the secondary coil in series with the galvanometer. Let

n_1 = the number of turns *per inch length* of the primary coil,
I_1 = the strength of the current in amperes in the primary coil,
N_2 = the number of turns in the secondary coil,
R = the total resistance in ohms of the secondary coil, galvanometer and any extra resistance which may be in the secondary circuit,
A = the area in square inches of the mean cross-section of the primary coil.

Then if the current I in the primary coil is reversed a quantity of electricity

$$Q = \frac{2.54 \times 8\pi}{10^9} \frac{n_1 I_1 A N_2}{R} \quad \text{coulombs}$$

$$= 0.0638 \frac{n_1 I_1 A N_2}{R} \quad \text{microcoulombs}$$

is discharged through the galvanometer. Hence, by calculating Q and noting the galvanometer swings when currents of various strengths are reversed in the primary coil, a curve can be plotted giving the quantity of electricity corresponding to a swing of any value; such a curve will be approximately a straight line.

COSTS. — A Broca galvanometer costs from \$40 to \$50 depending upon its sensitiveness. A D'Arsonval galvanometer costs from \$18 to \$100, including telescope and scale. An Einthoven galvanometer costs from \$400 to \$500.

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[H. PENDER AND H. R. RANKEN.]

GAS. — (*See also Gas Engines; Gas Producers.*) The word gas in its most general sense means a substance which is capable of indefinite expansion. The word is also used in a restricted sense to mean any mixture of gases suitable for illumination or for fuel. It is in this sense that the word will be used in this article.

Gas is usually measured in thousands of cubic feet, at 60° F. and at a pressure of 30 inches of mercury. The term "permanent" or "fixed" gas is used to designate a gas which will not precipitate any of its constituents when cooled.

KINDS OF GAS. — In addition to natural gas, which exists in limited quantities, in restricted localities (in this country, chiefly in western Pennsylvania, West Virginia, Ohio, Indiana, Kansas, Texas and California), there are four types of manufactured gas. These various types all contain the same constituent gases, but in different proportions, depending upon the process of manufacture and the fuel from which they are made. Gas may be made from practically any kind of solid or liquid fuel, but coal is the fuel usually employed.

Coal-Gas. — This name is given to the mixture of permanent gases resulting from the distillation in closed retorts of the volatile matter in coal. The heavy hydrocarbons and various impurities are extracted by cooling and "washing" the gas and passing it through purifiers containing hydrated lime or sesquioxide of iron. The residue left in the retorts is coke; the condensed hydrocarbons form coal-tar; ammonia, sulphur compounds and cyanogen are absorbed in the washers and purifiers; all these by-products are of value.

Coal-gas is used chiefly for illumination, but in this country carburetted water-gas (see below) is more extensively employed for this purpose. Coal-gas is seldom used for power purposes on account of its high manufacturing cost as compared with producer-gas (see below), which, though unsuitable for illumination, makes a very satisfactory fuel for gas engines.

Water-Gas. — This name is given to the mixture of permanent gases resulting from the reactions which take place when steam is passed through a body of coal, coke or charcoal heated to redness or beyond, the hot gas being condensed and purified in much the same manner as coal-gas. The chief reaction in the formation of this gas is the decomposition of the steam by the heated carbon, forming hydrogen and carbon monoxide, the reaction being $C + H_2O = CO + 2\,H$. This reaction is a cooling process, hence the production of water-gas is intermittent, consisting of two stages: (1) the "blowing up" or heating the fuel in the producer by blowing air into it, producing CO_2 by the reaction $C + 2\,O = CO_2$, and (2) the production of water-gas by blowing steam through the fuel, the supply of air having been shut off. Water-gas is generally made from anthracite coal and contains not only hydrogen and carbon monoxide, but also small amounts of oxygen, nitrogen and light hydrocarbons. Pure water-gas is not suitable for illumination, since it is deficient in the heavy hydrocarbons which render coal-gas a good illuminant. This defect is readily supplied by adding naphtha vapor to the water-gas as it comes from the producer and superheating the mixture thus formed; the naphtha vapor and the superheated steam in the gas react to form permanent gases of the hydrocarbon group. Water-gas thus heated is called "carburetted water-gas" and forms an excellent illuminant. In this country, by far the greater proportion of illuminating gas is carburetted water-gas. Water-gas, pure or carburetted, is seldom used for power purposes, on account of its high cost.

Siemens Gas. — This name is given to the mixture of permanent gases resulting from the chemical reactions which take place when air is blown or drawn into a deep bed of coal. At the bottom of the bed, where the air is in excess, CO_2 is formed and this is reduced on passing through the upper part of

the bed to CO by the reaction $CO_2 + C = 2\,CO$. The resulting gas also contains hydrogen, resulting from the reaction between the moisture in the coal and the hot carbon, this reaction being the same as in the production of water-gas. The nitrogen of the air is also present in the gas, forming more than half its volume, thereby making a very dilute or "lean" gas. This process has been superseded by the producer-gas process described below.

Producer Gas. — This name is given to the mixture of permanent gases resulting from the chemical reactions which take place when both steam and air are blown through a bed of hot coals. The gas is essentially a mixture of water-gas and Siemens gas. The process is a continuous one, enough air being supplied with the steam to maintain the producer bed at the proper temperature, thus eliminating the "blowing up" operation in the water-gas process; also, due to the presence of the steam and the reactions which take place between it and the carbon, the proportion of hydrogen is increased and the proportion of nitrogen decreased, thus making a gas "richer" than Siemens gas. The exact composition of producer gas depends upon the kind of fuel used and the type of producer, of which there are numerous forms (*see Gas Producers*). The gas made in any particular form of producer is frequently called by the name or type of the producer, for example, "Mond gas," "Dowson gas," etc. Producer gas is used in power plants employing large gas engines; it is also used for firing ceramic kilns and metallurgical furnaces, and, to a limited extent, for firing steam boilers.

The thoroughness with which the gas must be cleaned depends upon the purpose for which it is to be used. For gas-engine work, it is particularly important that no tar should be present. On this account, anthracite coal is frequently used for the production of the gas for this purpose, in spite of the relatively high cost of such coal.

Other Kinds of Gas. — "Oil-gas" is a mixture of gases resulting from the vaporization of crude oil and superheating the vapor; it is used to improve the illuminating qualities of water-gas. "Coke-oven gas" is the gas formed as a by-product in coke ovens. Its properties are intermediate between those of coal-gas and Siemens gas; the composition of the gas, however, varies with the kind of coal and with the time, changing from the beginning to the end of the process. "Oil-water gas" is produced by the chemical reactions between steam and oil heated to a high temperature in closed retorts. The cost of manufacture is low where oil is cheap. "Blast-furnace gas" is a waste product of blast furnaces, and is a mixture of CO, CO_2 and N, the latter being the largest constituent. The relative proportions of CO and CO_2 vary with the conditions of the furnace. An average analysis of blast-furnace gas cooled to atmospheric temperature is 25 per cent CO, 10 per cent CO_2, 65 per cent N. Coke-oven gas, oil-water gas and blast-furnace gas are all suitable for use in gas engines.

CALORIFIC OR HEATING VALUE. — The calorific or heating value (*see Heat and Thermal Properties*) of a given quantity of gas is the heat energy developed by its complete combustion. In this country, the calorific value of a gas is usually expressed in B.t.u. per cubic foot, the volume being measured at 60° F. and at a pressure of 30 inches of mercury. The total heating value of the gas produced from a given quantity of coal is less than that of the coal (or other fuel) from which it is made. In the case of coal-gas, the fixed carbon in the coal is not "gasified," and, therefore, its calorific value contributes nothing to that of the gas. In the case of water-gas and producer-gas, the carbon in the coal is converted into carbon monoxide, but the heating value of carbon monoxide is less than that of the carbon in it, since this carbon is already partially oxidized. Heat energy is also required to distil the hydrocarbons in the coal and in addition there are losses due to radiation. The calorific value of pro-

ducer-gas made from a given quantity of coal varies from 55 to 85 per cent of the calorific value of the fuel from which it is made, depending upon the character of the fuel, the size of the producer and the manner of operating it.

EFFECTIVE OR NET HEATING VALUE. — The discharge from a gas engine is at a temperature considerably greater than the boiling point of water, and, consequently, the latent heat of condensation of the water vapor in the products of combustion is not available for the production of power, whereas in a calorimetric determination of the heating value of a gas this latent heat of condensation is included. The difference between the heating value, as determined by calorimetric measurement and the latent heat of condensation of the water vapor in the products of combustion, is called the effective or net heating value of the gas. The net heating value of producer-gas is usually about 5 per cent lower than the gross heating value.

COMPARISON OF DIFFERENT KINDS OF GAS. — The following table shows what may be considered the average composition (percentage by volume), weight, calorific value and moisture content of the different types of gases used for illumination and for fuel. It should be noted, however, that the characteristics of any particular gas may vary considerably from the figures given in the table, depending upon the quality of the coal used and the size and type of producer.

COMPARISON OF DIFFERENT KINDS OF GASES

Item	Natural gas	Coal-gas	Water-gas	Producer-gas	
				Anthracite	Bituminous
CO	0.50	6.0	45.0	27.0	27.0
H	2.18	46.0	45.0	12.0	12.0
CH_4	92.6	40.0	2.0	1.2	2.5
C_2H_4	0.31	4.0	....	....	0.4
CO_2	0.26	0.5	4.0	2.5	2.5
N	3.61	1.5	2.0	57.0	56.2
O	0.34	0.5	0.5	0.3	0.3
Vapor	....	1.5	1.5	....	....
Lb. gas in 1000 cu. ft.	45.6	32.0	45.6	65.6	65.9
B.t.u. in 1 cu. ft. gas	1100	735	322	137	157

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[WM. KENT.]

GAS ENGINES AND OTHER INTERNAL-COMBUSTION ENGINES. — (*See also Gas; Gas Producers, Power Stations, Gas-Electric.*) An internal-combustion engine is an engine in which combustible gas, vapor, or oil is burned in a cylinder, generating a high temperature and high pressure in the gases of combustion, which expand behind a piston, driving it forward. (Rotary gas engines or gas turbines are still, 1914, in the experimental stage.)

Gas Engine. — A gas engine in its simplest form is similar to a reciprocating single-acting steam engine, comprising a cylinder, a piston, crank, flywheel, etc. The pressure to move the piston is obtained by exploding or burning with great rapidity a compressed mixture of air and combustible gas.

Oil and Gasoline Engines. — The lighter distillates of petroleum, such as gasoline, are easily vaporized at moderate temperatures, and a gasoline engine differs from a gas engine only in having an atomizer attached, for spraying a fine jet of the liquid into the air-admission pipe. With kerosene and other heavier distillates, or crude oils, it is necessary to provide some method of atomizing and vaporizing the oil at a high temperature, such as injecting it into a hot vaporizing chamber at the end of the cylinder, or into a chamber heated by the exhaust gases.

Diesel Oil Engine. — The distinguishing features of the Diesel engine are: It compresses air only, to a predetermined temperature above the firing point of the fuel. This fuel is blown as a cloud of vapor (by air from a separate small compressor) into the cylinder when compression has been completed, ignites spontaneously without explosion, solely by reason of the heat of the air generated by the compression, and burns steadily with no essential rise in pressure. The temperature of gases, developed and rejected, is much lower than with engines of the explosive type. The engine uses crude oil and residual petroleum products. Guarantees of fuel consumption are made as low as 8 gallons of oil (not heavier than 19° Baumé) for each 100 brake horse-power hours at any load between half- and full-rated load.

American Diesel engines are built for stationary purposes, in sizes of 120, 170 and 225 horse-power in three cylinders, and in "double units" (six cylinders) of 240, 340 and 450 horse-power. Much larger sizes have been built in Europe.

Alcohol Engines. — Due to the relatively high cost of alcohol, even when "denatured," alcohol engines are not yet in commercial use in this country.

CLASSIFICATION OF INTERNAL-COMBUSTION ENGINES. — Internal-combustion engines are classified as four-cycle and two-cycle engines, single- and double-acting engines, and single- and multi-cylinder engines.

Four-cycle Engines. — In what is known as a four-cycle engine, one ignition of gas takes place in one end of the cylinder every two revolutions of the flywheel, or every two double strokes. The following sequence of operations takes place during four consecutive strokes: (*a*) inspiration of a mixture of gas and air during an entire stroke; (*b*) compression during the second (return) stroke; (*c*) ignition at or near the dead-point and expansion during the third stroke; (*d*) expulsion of the burned gas during the fourth (return) stroke.

Fig. 1 is an indicator diagram of a four-cycle engine. *AB*, the lower line, shows the admission of the mixture, at a pressure slightly below the atmosphere on account of the resistance of the inlet valve. *BC* is the compression into the clearance space, ignition taking place at *C* and combustion with increase of pressure continuing from *C* to *D*. The gradual termination of the combustion

is shown by the rounded corner at D. DE is the expansion line, EF the line of pressure drop as the exhaust valve opens, and FA the line of expulsion of the burned gases, the pressure being slightly above the atmosphere on account of the resistance of the exhaust valve.

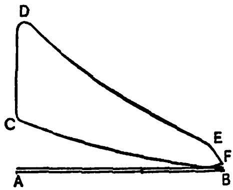

Fig. 1. Indicator Diagram of Four-cycle Engine

Two-cycle Engine. — In a two-cycle single-acting engine an explosion takes place with every revolution, or with each forward stroke of the piston. Referring to the diagram, Fig. 1, and beginning at E, when the exhaust port begins to open to allow the burned gases to escape, the pressure drops rapidly to F. Before the end of the stroke is reached an inlet port opens, admitting a mixture of gas and air from a reservoir in which it has been compressed. This mixture being under pressure assists in driving the burned gases out through the exhaust port. The inlet port and the exhaust port close early in the return stroke and during the remainder of the stroke BC the mixture, which may include some of the burned gas, is compressed and the ignition takes place at C, as in the four-cycle engine.

In one form of the two-cycle engine only compressed air is admitted while the exhaust port is open, the fuel gas being admitted under pressure after the exhaust port is closed. By this means a greater proportion of the burned gases is swept out of the cylinder. This operation is known as "scavenging."

Single- and Double-acting Engines. — These terms have the same significance as in the case of steam engines (q.v.). Comparatively few large engines are single-acting.

Multi-cylinder Engines. — In small multi-cylinder engines the cylinders are in "parallel," i.e., each has a separate piston connected to the crank shaft. In large engines two cylinders are frequently arranged in tandem. The term "twin cylinder" is used to designate two cylinders operating in parallel as contrasted with the "two-cylinder tandem" arrangement, in which the two cylinders are in "series." A "twin-tandem" engine has four cylinders in all, or two pairs of tandem cylinders.

RATING. — In contrast to steam prime movers the gas engine has a definite limit of power with the usual methods of governing and its economy improves until this limit is almost reached. This characteristic makes a maximum rating more significant than a normal rating with overload capacity. As ordinarily rated, however, engines can develop 10 to 15 per cent overload for brief periods and will care for momentary swings much greater. Due to their low mechanical efficiency (75 to 85 per cent) gas engines are preferably rated in terms of their brake horse-power instead of indicated horse-power.

Rating of Automobile Engines. — Automobile engines are single acting and as a rule multi-cylinder, the cylinders all being in parallel. The American Licensed Automobile Manufacturers Association have adopted the following arbitrary formula for the rating of automobile engines

$$\text{Brake horse-power} = \frac{D^2N}{2.5},$$

where D = diameter of each cylinder in inches, N = number of cylinders. This rating is usually referred to as the A.L.A.M. rating. It corresponds approximately to the actual brake horse-power when the piston speed is 1000 feet per minute and the mean effective pressure is 55 pounds per square inch.

The following ratings are derived from the formula:

Bore, in.	2½	3	3½	4	4½	5	5½	6
Bore, mm.	64	76	89	102	114	127	140	152
Horse-power, 1 cylinder	2½	3.6	4.9	6.4	8.1	10	12.1	14.4
Horse-power, 2 cylinders	5	7.2	9.8	12.8	16.2	20	24.2	28.8
Horse-power, 4 cylinders	10	14.4	19.6	25.6	32.4	40	48.4	57.6
Horse-power, 6 cylinders	15	21.6	29.4	38.4	48.6	60	72.6	86.4

DESIGN AND CONSTRUCTION. — Only a few of the more important features in the design and construction of internal-combustion engines can be noted here. See *Bibliography* at end of this article for references to various treatises on the subject.

Arrangement of Cylinders in Large Engines. — The number and arrangement of cylinders depends upon the space, the power required and the uniformity of rotation to be produced. Vertical-cylinder engines are built in sizes up to 750 horse-power. They require less floor space than the horizontal types but the superior mechanical properties of the latter warrant their selection when space economy is not important. The tandem double-acting arrangement is standard in horizontal engines. As it is not practicable to develop more than 1500 horse-power in a single cylinder, engines of the largest sizes are twin, tandem, double-acting.

Cooling is one of the most important problems of gas-engine design. In the small sizes provision for air cooling or a simple water jacket about the cylinder walls suffices, but the large sizes require the cooling of the piston, piston rod, and valve stems. The superficial area of the cylinders increases less rapidly than their cubical contents and this fact tends to aggravate the cooling problem as the size of the cylinders is increased. Cooling is more difficult with rich gases than with lean. One practical result of these difficulties is found in the fact that engines of large capacity are little if any cheaper per horse-power than small ones and that maintenance is apt to be proportionately larger.

Ignition. — The "hot-tube" method of igniting the compressed mixture of gas and air in the cylinder is practically obsolete, and electric systems are used instead. Of these the "make-and-break" and the "jump-spark" systems are in common use. In the former two insulated contact pieces are located in the end of the cylinder, and through them an electric current passes while they are in contact. A spark coil is included in the circuit, and when the circuit is suddenly broken at the proper time for ignition, by mechanism operated from the valve-gear shaft, a spark is made at the contacts, which ignites the gas. In the "jump-spark" system two insulated terminals separated about 0.03 inch apart are located in the cylinder, and the secondary or high-tension current of an induction coil causes a spark to jump across the space between them when the circuit of the primary current is closed by mechanism operated by the engine.

In some oil engines the mixture of air and oil vapor is ignited automatically, by the temperature generated by compression of the vapor, in a chamber at the end of the cylinder, called the vaporizer, which is not water-jacketed and therefore is kept hot by the repeated ignitions. Before starting the engine the vaporizer is heated by a Bunsen burner or other means.

Timing. — By adjusting the cam or other mechanism operated by the valve-gear shaft for causing ignition, the time at which the ignition takes place, with reference to the end of the compression stroke, can be regulated. The mixture

is usually ignited before the end of the stroke, the advance depending upon the inflammability of the mixture and on the speed of the engine.

Governing. — Two methods of governing the speed of an engine are in common use, the "hit-and-miss" and the throttling methods. In the former the engine receives its usual charge of air and gas only when the engine is running at or below its normal speed; at higher speeds the admission of the charge is suspended until the engine regains its normal speed. One method of accomplishing this is to interpose between the valve rod and its cam or other operating mechanism, a push rod, or other piece, the position of which with reference to the end of the valve rod is controlled by a centrifugal governor so that it hits the valve rod if the speed is at or below normal and misses it if the speed is above normal.

The throttling method of regulating is similar to that used in throttling steam engines; the quantity of mixture admitted at each charge being varied by varying the position of a butterfly valve in the inlet pipe. Cut-off methods of governing are also used, such as varying the time of closing the admission valve during the suction stroke, or varying the time of admission of the gas alone, or "quality regulation."

Lubrication of cylinders is best provided by a time force-feed system injecting oil between the piston rings at dead-center positions. The supply should be closely regulated, as an excess carbonizes on the cylinder walls and becomes a source of trouble. Bearings are commonly ring-oiled or flooded.

Starting Devices. — In large engines starting is now universally accomplished by compressed air. In some small stationary engines an explosive cartridge is inserted into the cylinder head and set off by percussion.

Self-starting devices, using compressed air or a small electric motor and storage battery, are coming into extensive use for starting automobile engines.

Speeds. — Small gasoline engines are usually designed for speeds of from 300 to 1000 revolutions per minute, Large gas engines, for central-station service, usually have speeds about as follows:

100 h.p.	275 rev. per min.
300 h.p.	200 rev. per min.
500 h.p.	150 rev. per min.
1000 h.p.	100 rev. per min.

POWER DEVELOPED BY AN INTERNAL COMBUSTION ENGINE. — In order to calculate the power developed by an internal-combustion engine it is necessary to know the mean effective pressure in the engine cylinder, the dimensions of the cylinder and either (1) the length of stroke and number of explosions per minute, or (2) the piston speed and number of explosions per revolution.

Mean Effective Pressure. — The mean effective pressure can be obtained from the indicator diagram (*see Steam Engines*), or if a test diagram is not available the mean effective pressure either has to be assumed from a knowledge of that found in other engines of the same type and working under the same conditions as those of the design, or it may be calculated from the ideal air diagram and modified by the use of a coefficient or diagram factor depending on the kind of fuel used and the compression pressure.

The following figures are given by C. P. Poole as a rough approximate guide to the mean effective pressures obtained with different fuels in a four-cycle engine. In a two-cycle engine the mean effective pressure of the pump diagram should be subtracted. The delivery pressure is usually from 4 to 8 pounds per square inch above the atmosphere, and the corresponding mean effective pressure of the pump about 3.8 to 7.

PROBABLE MEAN EFFECTIVE PRESSURE, POUNDS PER SQUARE INCH, FOUR-CYCLE ENGINES

Kind of fuel	Compression pressure	Engine horse-power						
		5	10	25	50	100	250	500
Anthracite Producer gas	100	..	55	60	65	70	75	80
	130	..	65	70	75	80	85	90
	160	..	..	..	80	85	90	90
Mond Producer gas	100	..	..	60	65	65	70	75
	130	..	65	65	70	75	80	85
	160	..	..	75	80	85	90	90
Natural and Illuminating gas	65	..	60	65	70	80	85	..
	100	..	75	80	90	90	95	100
	130	..	..	..	..	100	105	110
Gasoline vapor	65	70	75	80	85	..	..	..
	100	85	90	90	95	..	..	..
Kerosene spray	65	50	55	60	65	..	..	..
	115	70	75	80	85	..	..	..

Number of Explosions per Revolution. — The number of explosions per revolution depends upon the type of engine, as given in the accompanying table.

Formulas for Horse-power Output. — Let

p = mean effective pressure, lb. per sq. in.
d = diameter of cylinder bore, inches,
N = number of revolutions per minute,
e = total number of explosions per revolution,
S = piston speed, feet per minute,
L = length of stroke, feet,
ϵ = mechanical efficiency, as a fraction,
P_i = indicated horse-power.

Type of engine	Explosions per rev. per cylinder
Single-acting, 4-cycle..	0.5
Single-acting, 2-cycle..	1.0
Double-acting, 4-cycle.	1.0
Double-acting, 2-cycle.	2.0

Then

$$P_i = \frac{pLd^2eN}{42{,}000} = \frac{pd^2eS}{84{,}000}.$$

The brake horse-power is

$$\text{Brake h.p.} = \epsilon P_i.$$

The brake horse-power is frequently expressed by the formula

$$\text{Brake h.p.} = Cd^2,$$

where $C = pe S\epsilon/84{,}000$ is called the "engine constant." For $\epsilon = 0.84$ and 2 explosions per revolution the values of C for various mean effective pressures are as follows:

VALUES OF *C* FOR 2 EXPLOSIONS PER REVOLUTION

M.E.P., lb. per sq. in.	Piston speed, feet per minute					
	500	600	700	800	900	1000
50	0.50	0.60	0.70	0.80	0.90	1.00
60	0.60	0.72	0.84	0.96	1.08	1.20
70	0.70	0.84	0.98	1.12	1.26	1.40
80	0.80	0.96	1.12	1.28	1.44	1.60
90	0.90	1.08	1.26	1.44	1.62	1.80
100	1.00	1.20	1.40	1.60	1.80	2.00
110	1.10	1.32	1.54	1.76	1.98	2.20

These values of *C* apply to 4-cylinders, 4-cycle, single-acting, to 2-cylinders, 2-cycle, single-acting, and to 1-cylinder, 2-cycle, double-acting. For single cylinders, 4-cycle, single-acting, divide by 4; for single cylinders, 4-cycle, double-acting, or 2-cycle, single-acting, divide by 2.

TESTING INTERNAL-COMBUSTION ENGINES. — The test of a gas or oil engine includes the measurement of its power by a friction brake, of the number of cubic feet of gas or pounds of oil used per brake horse-power hour, and of the calorific value of the gas or oil. In connection with the tests indicator diagrams may be taken, the air supply and the jacket cooling water may be measured, the exhaust gas analyzed, the temperatures of air, water and gas taken, and a heat balance calculated showing the thermal efficiency and the several sources of loss of energy. For details see *Code of 1914 of the A.S.M.E.*

EFFICIENCY AND FUEL CONSUMPTION. — The thermal efficiency of an internal-combustion engine is considerably higher than that of the best reciprocating engine or steam turbine; its mechanical efficiency is lower, ranging from 70 to 85 per cent as against 90 to 95 per cent for a high-class reciprocating engine and 92 to 97 per cent for steam turbines.

Thermal Efficiency. — The conditions which appear to give the highest thermal efficiency in gas and oil engines are: (1) high temperature of cooling water in the jackets; (2) high pressure at the end of compression; (3) lean mixture; (4) proper timing of the ignition; (5) maximum load. The higher economy of a lean mixture may be due to the fact that high compressions may be used with such a mixture, while with rich mixtures high-compression pressures cannot be used without danger of preignition.

Other things being equal, the hotter the walls of the cylinder the less heat is transferred into them from the hot gases, and therefore the higher the efficiency. Cool walls, however, allow of higher compression without preignition, and high compression is a cause of high efficiency. Cool walls also tend to give the engine greater capacity, since with hot walls the fuel mixture expands more on entering the cylinder, reducing the weight of charge admitted in the suction stroke.

Distribution of Heat Losses. — The heat losses are: (1) the heat carried away in the jacket water, (2) that carried away in the waste gases, and (3) that lost by radiation. The relative amounts of these three losses vary greatly, depending on the size of the engine and on the amount of water used for cooling. Carpenter and Diederichs quote the following,

Ratio of compression	R.p.m.	M.E.P., lb. per sq. in.	Ratio air to gas	Heating value of charge, B.t.u.	Work done by 1 B.t.u., ft. lb.	Exhaust temp., deg. F.	Distribution of heat losses, per cent		
							Work	Jacket water	Exhaust
2.67	187	54.3	7.11	18.5	140	1022	18.0	51.2	30.8
2.67	247	51.5	7.35	17.4	141	1137	18.1	45.6	36.3
4.32	187	69.3	7.43	17.0	190	867	24.4	53.8	21.8
4.32	247	65.2	7.40	16.8	184	992	23.7	49.5	26.8

showing that the distribution of the heat losses varies with the rate of compression and with the speed.

Over-all Efficiency and Fuel per Brake Horse-Power. — The over-all efficiency of internal-combustion engines is the product of the thermal efficiency by the mechanical efficiency. A more convenient expression is

$$\text{Over-all efficiency} = \frac{2546}{HQ} \quad \text{as decimal fraction}$$

where H = heat of combustion, B.t.u. in one cubic foot, and Q = number of cubic feet of gas supplied per brake horse-power hour. In the case of a liquid fuel it is more convenient to take H = heat of combustion, B.t.u. per pound, and Q = pounds of liquid supplied per brake horse-power hour. See articles on *Gas and Fuels*. The product HQ in either case is the number of B.t.u. per brake horse-power.

In Fig. 2 are given the results of tests on several internal-combustion engines using different kinds of fuels, and also, for comparison, the performance of three steam turbines. These curves were supplied by Prof. W. E. Wickenden. These curves bring out very strikingly the relatively poor performance of internal-combustion engines at light loads.

Coal per Brake Horse-power Hour. — Figures giving the coal consumption of a producer per brake horse-power hour of the gas engine supplied from it are of little value unless the type of producer and type of engine as well as the quality of the coal are stated. Let H = B.t.u. per pound coal, e_1 = efficiency of producer (*see Gas Producers*), e_2 = over-all efficiency of gas engine. Then

$$\text{pounds coal per brake horse-power hour} = \frac{2546}{He_1e_2}.$$

For example, if the heating value of the coal used is 14,000, the over-all efficiency of the gas engine 30 per cent and the efficiency of the producer 80 per cent, then $2546/14{,}000 \times 0.3 \times 0.8 = 0.76$ pound coal will be required per brake horse-power hour. Figures as low as 0.7 pound coal per brake horse-power hour have been obtained, but they are exceptional.

GAS AND OIL ENGINE TROUBLES. — Among the causes of troubles are: the variable composition of the fuel; too much or too little air supply; compression ratio not right for the kind of fuel; ignition timer set too late or too early; preignition; backfiring; electrical and mechanical troubles with the igniting system; carbon deposits in the cylinder and on the igniting contacts.

COST OF GAS ENGINES. — Differences in the ratings of gas engines, the partial development of the art and the close competition of builders make

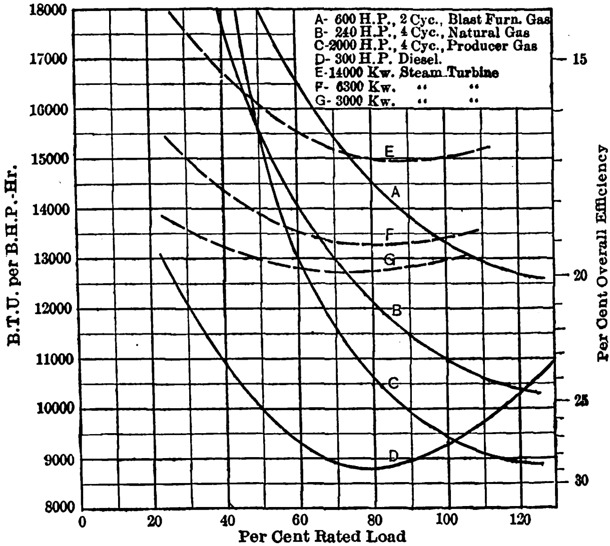

Fig. 2. Fuel Consumption of Gas and Oil Engines

costs quite variable. The following methods of estimating are derived from a number of installations and probably represent average conditions for electrical generation. (*W. E. Wickenden.*) Let P = brake horse-power rating; then

50 to 125 horse-power, cost in dollars: ....400+40 P
125 to 250 horse-power, cost in dollars.......2500+20 P
250 horse-power and larger, cost in dollars... 32 P

BIBLIOGRAPHY. — For theory of the internal-combustion engine, see paper by Dugald Clerk, *Proc. Inst. C. E.*, 1882, Vol. 69; and Van Nostrand's *Science Series No. 62*. See also Wood's *Thermodynamics*. Standard works on gas engines are: *A Text-book on Gas, Air, and Oil Engines*, by Bryan Donkin; *The Gas and Oil Engine*, by Dugald Clerk; *Internal Combustion Engines*, by Carpenter and Diederichs; *Gas Engine Design*, by C. E. Lucke; *Gas and Petroleum Engines*, by W. Robinson; *The Modern Gas Engine and the Gas Producer*, by A. M. Levin; *The Gas Engine*, by C. P. Poole; and *Internal Combustion Engines*, by Hugo Guldner, translated by H. Diederichs, N. Y., 1910. For practical operation of gas and oil engines, see *The Gas Engine*, by F. R. Jones, and *The Gas Engine Handbook*, by E. W. Roberts. For descriptions of large gas engines using blast-furnace gas, see papers in *Proc. Iron and Steel Inst.*, 1906, and *Trans. A.S.M.E.*, 1906. *Trans. A.S.M.E.*, 1905 to 1913.

[Wm. Kent.]

GAS LIGHTING. — (*See also Gas; Gas Producers; Illumination, Laws of.*) Illuminating gases are rated in terms of "calorific value," expressed in B.t.u.'s per cubic foot, and in "illuminating power," or the candle-power of a standard luminous flame burner, in its most efficient state, consuming 5 cubic feet of the given gas per hour. Measurements of volume are reduced to standard conditions of temperature and pressure, viz., 60° F., and a 30-inch barometric height. Gas pressures are sometimes stated in atmospheres and in pounds per square inch, but more commonly by the inches of water manometer column which the gas sustains against atmospheric pressure (*see Units and Conversion Factors*). The supply pressure ranges from 1.5 to 4 inches in low-pressure systems and from 40 to 60 inches in high-pressure systems. Most of the gas systems in the United States are of the low-pressure type, but high-pressure systems are common in Europe. The chief commercial varieties of gas and their more important properties are as follows:

Coal Gas, produced by distillation from bituminous coal, consists chiefly of hydrogen, methane and richer hydrocarbons. It varies greatly in quality with the grade of coal and the stage of distillation. Values of 16 candle-power and 600 B.t.u.'s are representative of the usual commercial supply. The use of coal gas is diminishing.

Water Gas, produced by the reaction of steam and incandescent coal or coke, consists chiefly of hydrogen and carbon monoxide, and is usually enriched with oil gas to raise its calorific and luminous value. The properties of water gas supplied in different cities range from 18 to 22 candle-power and from 580 to 700 B.t.u.'s per cubic foot. Many plants supply a mixture of coal gas and water gas.

Oil Gas is a mixture of rich hydrocarbon gases produced by "cracking" petroleum oils by heat. Its candle-power is from 40 to 60 and its calorific value 1200 B.t.u.'s or more. On account of its richness oil gas is often diluted with air or mixed with water gas for commercial delivery.

Pintsch Gas is a type of oil gas stored under a pressure of 150 pounds per square inch in steel tanks for self-contained and portable systems of lighting. It is rated at approximately 900 B.t.u. and 36 candle-power. An essential feature of the Pintsch system is an automatic reducing valve which supplies the gas at a constant low pressure adapted to flame and mantle illuminants.

Acetylene, produced by the reaction of water and calcium carbide, is a simple rich gas with a rating of about 1470 B.t.u.'s and 240 candle-power. For use in portable and self-contained systems acetylene is dissolved in acetone under a pressure of about 150 pounds per square inch. The usual container has a storage capacity of 10,000 volumes at this pressure.

Gasoline-air Gas is a mixture of air with gasoline vapor produced by forced evaporation. The generator commonly used comprises a blower operated by weights or water power, a carburretor and a mixing chamber. In the most widely used type of carburretor air is forced through sheets of cotton cloth which dip in gasoline and a rich mixture is produced. In the mixer the gas is diluted with air to a mixture containing from 2 to 5 gallons of gasoline to 1000 cubic feet, which is well above the explosive limit. Self-contained lamps in which the fuel mixture is locally generated by heat from the lamp have a wide use in street lighting. The luminous value of this gas in flames is very low, but it is well adapted to incandescent burners.

Natural Gas consists chiefly of methane, has a calorific value of about 1000 B.t.u.'s and little luminous value in open flames. It is excellently adapted to incandescent burners.

PERFORMANCE OF FLAME ILLUMINANTS. — Light from a flame is due to minute incandescent carbon particles liberated from the gas by heat. The highest efficiency is obtained when the migration of these particles is slow; hence the pressure at the flame should not exceed two inches of water and the flame should be set but little above the smoking point. The horizontal light distribution from flames is sensibly uniform. The spherical reduction factor ranges from 85 to 92.5 per cent, depending on the occlusion of light by the burner. The batswing and fishtail burners used with ordinary gas should give fully 90 per cent of the rated illuminating power of the gas when properly adjusted. Acetylene is commonly burned in duplex burners with air holes giving slight pre-aeration. The usual one-foot acetylene burner gives 40 candle-power.

INCANDESCENT MANTLES AND BURNERS. — The incandescent or Welsbach mantle is the ashy residue of a woven structure of vegetable fibre, such as cotton, ramie or artificial silk, impregnated with oxides of cerium and thorium, with traces of other rare earths. The strength of mantle varies greatly with the material and weave of the fibrous base. Its luminous properties depend on the purity and proportions of the oxides used. Mantles are graded in quality chiefly by strength to resist shock, but the better grades are greatly superior in the maintenance of their candle-power.

The incandescent burner is made in two general types, upright and inverted. Each has a Bunsen tube with adjustable gas cock and air inlet. The entraining action and flame projection of the upright type is fairly self-regulating for a wide range of gas pressure and consumption. The inverted type admits of a narrow range of adjustment. The conditions required to obtain the highest efficiency from mantle burners are: (*a*) all gas and air ports must be clear and adjusted to give the proper air-gas mixture and best projection pressure at the flame; (*b*) the flame should burn at the highest possible temperature and the mantle should be completely immersed in the hottest or outer zone of the flame.

Performance of Mantle Burners. — Candle-power and efficiency ratings are based on lamps fitted with new mantles and tuned up by a fitter's adjustment to the most efficient conditions. The average performance of mantle burners in service is always somewhat below the rated values and depends to a marked degree on maintenance conditions. Service deterioration is due to (*a*) the fouling of the burner with dust, (*b*) the improper adjustment of air and gas ports, (*c*) the shrinkage of mantles and loss of their active materials, and (*d*) the collection of dirt on glassware. Low-grade mantles show very rapid deterioration, often amounting to 40 per cent in 500 hours. R. F. Pierce (*Trans. Ill. Eng. Soc., Vol. VII, p. 686*) reports from reliable tests under favorable conditions the inherent deterioration in candle-power of well-adjusted mantle lamps of the best grade to be as follows: Duration of test, 1000 hours; loss due to mantle alone, 2.5 per cent; loss due to burner alone, 2.5 per cent; loss due to dirt on glassware, 10 per cent; total reduction in candle-power, 15 per cent. Tests of commercial installations receiving skilled maintenance show an average performance from 15 to 25 per cent below the rated candle-power and efficiency. Without skilled maintenance the average performance is generally from 30 to 50 per cent below rated candle-power and efficiency. (*Proc. Nat. Elec. Light Assn., 1911, Vol. I, p. 809.*)

The efficiency of incandescent gas lamps varies with the pressure and the calorific value of the gas. At a given pressure the lumen-hours per cubic foot of gas are closely proportional to the calorific value of the gas. The pressure variations common to most gas systems have little effect on lamp efficiency, as the candle-power and rate of gas consumption are about equally affected.

PERFORMANCE OF GAS ILLUMINANTS. — The curves of Figs. 1 to 3 show the light distribution of the gas illuminants in general use.

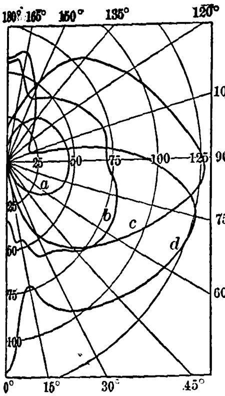

Fig. 1. Light Distribution of Upright, Single-mantle Gas Lamps

a. Junior type, mica chimney, 1.66 cu. ft. per hour
b. Standard type in opal globe, 3.8 cu. ft. per hour
c. Standard type in clear chimney, 4.66 cu. ft. per hour
d. Standard type with opal dome, 4.66 cu. ft. per hour

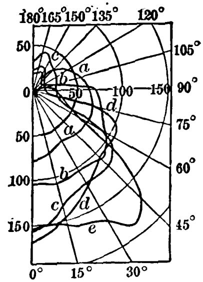

Fig. 3. Light Distribution of Single-mantle Inverted Gas Lamps

a. In roughed glass globe, 3.31 cu. ft. per hour
b. With flat opal reflector, 3.31 cu. ft. per hour
c. With opal dome reflector, 3.31 cu. ft. per hour
d. With enamelled cone reflector, 3.31 cu. ft. per hour
e. With holophane reflector No. 6321, 3.65 cu. ft. per hour
All tests with water gas of 20.5 to 22 c.p.

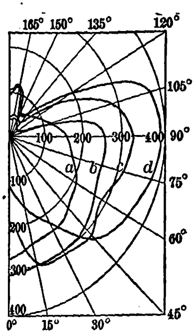

Fig. 2. Light Distribution of Gas Arcs

a. 4-mantle inverted, alabaster globe, 13.03 cu. ft. per hour
b. 4-mantle inverted, clear globe, 13.03 cu. ft. per hour
c. 4-mantle upright, alabaster globe, opal reflector, 18.8 cu. ft. per hour
d. 4-mantle upright, clear globe, opal reflector, 18.8 cu. ft. per hour

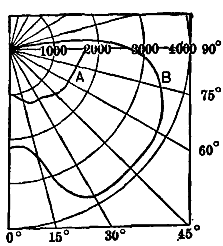

Fig. 4. Light Distribution of High-pressure Inverted Gas Lamps

A. 1-mantle, 31.6 cu. ft. per hour
B. 3-mantle, 70.3 cu. ft. per hour at 72.1 inches-pressure

The following table gives performance data of low-pressure gas illuminants under favorable test conditions.

PERFORMANCE OF GAS ILLUMINANTS

Illuminant	Glassware	Gas conditions			Light production			
		C.P.	Pressure, in.	Cu. ft. per hour	L.H.C.P.	M.S.C.P.	Lumens	Lumen-hours per cu. ft.
1-Mantle upright...	Clear chimney......	21.5	2.5	5.12	90.5	96.3	1210	236
" "	Opal dome..........	21.5	2.5	5.12	130.2	85.7	1078	211
" "	Opal globe..........	23.6	2.0	3.8	71.1	65.3	821	216
4-Mantle upright...	Alabaster globe.....	23.3	2.0	18.8	263.6	259.5	3261	173
" "	Clear globe, opal reflector..........	23.3	2.0	18.8	332.2	279.4	3511	187
1-Mantle inverted..	Ground glass ball...	21.9	2.5	3.31	68.8	54.7	688	208
" "	Green flashed reflector............	21.9	2.5	3.31	91.5	47.4	596	180
" "	Satin holophane reflector............	21.7	2.5	3.31	71.4	46.6	586	177
" "	Alba reflector.......	18.13	2.00	3.65	65	46.1	579	159
4-Mantle inverted..	Alabaster globe.....	21.9	2.5	13.03	207.5	150.8	1895	146
" "	Clear globe.........	21.9	2.5	13.03	273.6	174.3	2187	168
5-Mantle inverted..	Clear globe and enamelled ref........	23.0	2.5	17.07	516	266	3343	196

High-pressure Gas Systems. — The amount of air entrained in the mantle burner at ordinary low pressures is insufficient to produce the highest flame temperature and mantle efficiency. Three methods of gas supply are designed to improve this condition, viz., (*a*) to provide a gas pressure at the lamps of from 40 to 60 inches of water, (*b*) to provide gas at ordinary pressure and compressed air from a separate system of piping and (*c*) to supply both gas and air under pressure. The results produced by the three systems are approximately on a par. Results of the operation in Germany of inverted high-pressure lamps as cited by Wrightington (*Lectures on Illuminating Engineering, Johns Hopkins University, 1911, Vol. 2, p. 872*) may be summarized as follows:

Mantle life, 110 hours. Globe life, 2230 hours. Energy for gas compression, 1 horse-power-hour for 1400 cubic feet of gas. Consumption of 3-burner lamp, 65 cubic feet per hour. Costs per lamp-hour of 3-burner inverted lamp: compression, 0.31¢; maintenance and renewals, 0.58¢; gas at 61¢ per 1000 cubic feet, 3.98¢; total per lamp-hour, 4.87¢; total per lamp-year of 3675 hours burning, $178.97.

The light output of the high-pressure inverted single-mantle lamp is approximately 1500 mean lower hemispherical candle-power for a gas consumption of from 25 to 30 cubic feet per hour. The 3-mantle inverted lamp gives approximately 3500 m.l.h.c.p. at 70 cubic feet per hour. These efficiencies can be sustained only by the most careful attention to maintenance conditions. Fig. 4 shows typical light distribution curves of high-pressure lamps.

Methods of Automatic Ignition. — The automatic ignition of gas lamps may be accomplished by the following methods:

(*a*) A pilot flame consuming from 0.05 to 0.10 cubic foot per hour is fitted to each burner. As the main cock is opened a momentary jet of gas is supplied to the pilot tube, causing the ignition of the main burners. On some remote-controlled street-lighting systems a momentary extra pressure is applied as the gas is turned on to produce this jet action at the local burners.

(*b*) A high-tension spark is produced in the gas issuing at each burner by means of an induction coil. The primary circuit contains a battery, ignition switch and vibrating interrupter. The secondary coil is in series with the several spark terminals and each end of the wire is grounded. Good insulation of the secondary wiring is essential.

(*c*) The mantle may be provided with a small spot of sponge platinum which is raised to incandescence by catalytic action due to the rapid absorption of gas. Such self-lighting mantles are apt to be short-lived.

A clock-controlled ignition cock is frequently employed with show-window lamps and street lamps operated during definite time intervals on a flat rate system of charges.

COST OF GAS LIGHTING. — The following table shows the approximate cost of operation of various gas lamps. The assumed cost of gas is $1.00 per thousand cubic feet and the assumed average light output as 80 per cent of the nominal ratings with clear glassware.

Type of lamp	Cost per 1000 lamp-hours			Cost per 100,000 lumen-hours
	Renewals and maintenance	Gas	Total	
1-Mantle upright	$0.60	$5.00	$ 5.60	$0.65
1-Mantle miniature	0.40	2.00	2.40	0.70
1-Mantle inverted	0.50	3.30	3.80	0.63
4-Mantle upright	1.65	18.80	20.45	0.74
4-Mantle inverted	1.65	13.00	14.65	0.80

Gas vs. Electricity. — For comparison with the above it may be noted that the operating costs of 100-watt tungsten lamps with energy at 5¢ and 10¢ per kw-hr. are respectively $0.75 and $1.40 per 100,000 lumen-hours. Corresponding costs for 250-watt lamps are $0.61 and $1.15 respectively. The advantages of electric lighting are chiefly convenience, flexibility in the size and location of illuminants, non-vitiation of air, small heat output, and freedom from the dangers of explosion and asphyxiation. The chief disadvantages of electric lighting are danger of shock from abnormal voltages, the fire risk of faulty wiring and the greater likelihood of service interruption. The color of high-grade incandescent mantle lamps is a closer approach to daylight than that of the tungsten lamp, but is considered by many as less pleasing.

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[W. E. WICKENDEN.]

GAS METERS. — (*See also Gas.*) To determine the volume of gas flowing through a pipe in a given time, either the "dry" or the "wet" type of meter may be used. The "wet" meter, in which a cylindrical drum divided into four equal compartments revolves with its lower half submerged in water, has a limited commercial use because of the liability of the freezing of the water in cold climates and because of the necessary attention required to keep the water level constant. The wet meter, however, forms the most satisfactory standard for testing purposes.

DRY METERS. — The "dry" meter, which is almost universally used, is constructed as shown in Fig. 1. The sheet-iron case is divided into three parts: *A*, *B* and *C*. On the partition separating *A* and *B* are mounted two leather bellows *DD*, the faces of the bellows *EE* consisting of circular plates of sheet iron. In the compartment *C* are placed the gas valves *FF* and the recording dial *G*. The motion of the bellows is communicated to the valves and the gear train of the dial through the arms *HH*, the spindles *II* and the shaft *J*. As the bellows fill and empty, the spindles *II* oscillate and acting together upon the shaft *J* cause it to revolve. The gear train of the dial is then set in motion and the gas valves are opened and closed in synchronism with the motion of the bellows. The inlet and outlet pipes are shown at *K*, the two pipes overlapping in the side elevation shown. The pipe connections between the inlet and outlet pipes *K*, the valves *FF*, and the compartments *DD* and *EE* are omitted from the figure.

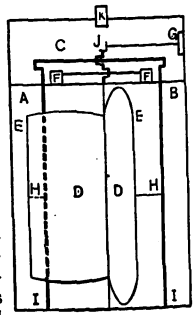

Fig. 1. Dry Meter

Dimensions of Dry Meters. — In the following table are given the standard sizes of "dry" meters, with their capacities and over-all dimensions.

Size, Number of Lights	Capacity, cubic feet per hour	Height, inches	Width, inches	Depth, inches
3	80	15	10	9
5	145	17	12	10
10	210	18	13	11
20	330	22	17	14
30	420	25	19	15
50	510	30	23	20
60	600	32	25	20
80	720	37	27	23
100	900	39	29	25
150	1500	49	35	27
200	2000	55	41	29
250	2500	57	44	30
300	3000	60	46	33
500	5000	73	50	33

[R. G. Hudson.]

GAS PRODUCERS. — (*See also Gas; Gas Engines; Power Stations, Gas-Electric.*) The name gas producer is usually applied to the apparatus used for making and purifying producer-gas (*see Gas*). The name, however, is sometimes limited to the apparatus in which the heating of the fuel takes place, and again it is used to designate the apparatus used in the manufacture of any kind of gas. In this article, the name gas producer will be used as a general term for all the apparatus used in making and purifying producer-gas, including the generator in which the heating of the fuel takes place, the evaporator or boiler for producing the necessary steam, and various filters and scrubbers.

The Generator. — The simplest form of generator is merely a cylindrical structure of fire-brick, provided with a grate, a roof and an outlet for the gas. A bed of coal from three to five feet thick is maintained in it and air and steam are forced through this bed. The oxygen of the air combines with some of the fixed carbon in the coal, forming carbon monoxide, the steam reacts with the heated carbon, forming hydrogen and carbon monoxide, and the hydrocarbons in the coal are driven off by the heat. In addition to adding combustible gases to the gas by its decomposition, the steam diminishes the formation of clinker, reduces the temperature of the escaping gases and makes a gas lower in nitrogen, and, therefore, of higher heating value, than would be produced if air alone were used. For facilitating the removal of ashes, revolving grates are used, or else the grate is dispensed with and the bottom of the producer rests in a shallow pit filled with water, from which the ashes are removed from time to time. The coal is usually fed by filling a hopper set in the roof of the producer, and opening its bottom so as to drop the coal, but sometimes special forms of rotating hoppers or feeders are used which distribute the coal evenly over the bed. To avoid choking of the producer by the "caking" of some varieties of soft coal, stirring apparatus is sometimes used in the upper part of the producer to break up the cake.

Evaporator. — The evaporator is essentially a steam boiler, the water being heated by causing the hot gases from the generator to circulate around the pipes or other container holding the water. The steam thus formed is fed into the generator with the proper proportion of air.

Filters and Scrubbers. — In order to remove the dust, tar and other harmful ingredients, the gas is passed through various devices designed to absorb or precipitate these substances. The filter consists of some porous material, such as shavings, excelsior or sawdust, in a suitable retainer. Liquid "scrubbers" are for the purpose of bringing the gas into intimate contact with water, which absorbs or precipitates the various impurities. This is accomplished by causing the gas to bubble through water, by passing the gas through a chamber into which water is sprayed, or by passing the gas over surfaces continually kept moist by a film of water. The spray type of scrubber is usually a large cylindrical shell filled with coke, the latter aiding in absorbing the tar from the gas as it passes through. Rotating scrubbers are also employed, which may be designed either to mix thoroughly the gas and water, or to drive out the impurities by centrifugal force. Rotating scrubbers are particularly effective in removing tar, a large amount of which is present in gas made from bituminous coal.

TYPES OF PRODUCERS. — The various forms of gas producers, of which there are a great number, differ chiefly in details of construction and in the arrangement of the various parts of the apparatus. There are, however, two distinct types of producers, the "suction" producer and the "pressure" producer. In the former type, the draft through the producer is produced by the suction of the gas engines which are fed from it. This type of producer requires no gas-holder and is very compact. It requires a high-grade fuel, such as coke

or anthracite. An auxiliary starting fan is also necessary, as a draft must be set up through the producer before the engine can be started. The Körting producer is of this type. For large installations, the pressure type of producer is nearly always employed. In this type, air and steam are forced into the producer under pressure, and the gas is collected in a suitable holder. This type of producer can be adapted to the use of practically any kind of fuel. The Mond producer and the Morgan producer are of the pressure type.

UTILIZATION OF BY-PRODUCTS. MOND PROCESS.—Ammonia is about the only by-product of producer gas which has enough commercial value to justify the additional expense required to save it. This saving is accomplished by the Mond process, which is the only one used to any extent in this country, as follows: An excess of steam is injected with the air, lowering the temperature and thereby preventing the destruction of the ammonia in the coal. (The proportion of hydrogen and carbon dioxide in the gas is also increased and the proportion of nitrogen decreased.) The gas, after passing through a mechanical scrubber, is caused to pass through a tower where it is thoroughly washed with dilute sulphuric acid. The ammonia in the gas combines with the sulphuric acid, forming ammonium sulphate. This acid solution of ammonium sulphate circulates through the tower over and over again until it reaches a given degree of saturation; then a certain amount of it is bypassed out of the system and fresh acid is added. In this system, the heat energy lost by the gas in cooling is also used to preheat the air which is used in generating the gas, effecting a considerable increase in the economy of the process.

RATING AND PERFORMANCE. — A gas producer is usually rated in horse-power, the rating being the number of horse-power which can be developed by gas engines supplied from the producer when the latter is gasifying coal at a maximum rate. Such a rating is, of course, indefinite, as the power developed by the engines depends upon their design and the percentage of load which they are carrying, and the rate of gasification depends upon the quality of coal and the care taken in firing. The following data are the results of a series of tests on bituminous coal and lignites made by the U. S. Geological Survey at St. Louis on a 250-horse-power Taylor producer and a three-cylinder vertical Westinghouse 235-horse-power gas engine. These tests are summarized in a paper by R. H. Fernald in the *Jour. West. Soc. Eng., 1907, Vol. 12, p. 551*. The following table is derived from the curves there given.

DATA ON 250 H.-P. GAS PRODUCER

B.t.u. per lb. of fuel as fired	9000	10,000	11,000	12,000	13,000	14,000	15,000
Lb. of fuel per brake hp.-hr	1.9	1.7	1.5	1.3	1.1	1.0	0.95
Lb. fuel per hr. per sq. ft. of fuel bed	11.6	10.2	8.9	7.8	7.0	6.2	5.5
Cu. ft. of gas per lb. of fuel as fired	38	45	52	57	65	74	87
Lb. steam per lb. of fuel as fired	0.30	0.35	0.40	0.45	0.50	0.55	0.60
Efficiency — Per cent of B.t.u. in fuel in gas produced	66	69	70	72	75	80	85

All items in this table affected by load factor are given only for loads ranging from 90 to 100 per cent of full load (235 brake horse-power). The weight of coal per horse-power-hour does not include the coal required to produce the steam injected into the fuel bed. The efficiency is based on calorimetric measurements, no deduction being made for the latent heat of condensation of the water vapor in the products of combustion of the gas.

Dimensions. — F. C. Tryon (*Power, Dec. 1, 1908*) gives the following rules for determining roughly the dimensions of the various parts of a producer plant:

1. Fuel-bed cross-section 0.125 square foot per horse-power capacity. This gives the formula

$$d = \frac{\sqrt{P}}{2.5},$$

where d is the diameter of fuel bed in feet and P the horse-power capacity.

2. For sizes smaller than 100 horse-power, the walls of the generator should be 9 inches thick; for larger sizes, 12 inches thick. Hence, for sizes smaller than 100 horse-power, the external diameter of the generator in feet is $d_0 = d + 1.5$ and for larger sizes $d_0 = d + 2$.

3. Height of generator approximately twice its internal diameter; for sizes under 100 horse-power the height is usually greater than twice the internal diameter while for larger sizes the height is usually slightly less than twice.

4. Diameter of wet scrubber three-fourths of the internal diameter of generator.

5. Height of wet scrubber one and one-half times the height of generator.

6. Diameter of dry scrubbers, of which there should be two, equal to internal diameter of generator.

7. Height of each scrubber 3 to 4 feet for plants ranging from 25 to 200 horse-power.

COSTS. — The following approximate formulas for costs of producer-plant equipments have been derived by Prof. W. E. Wickenden from a large number of actual installations. They include the purchase price of generators, evaporators and the necessary scrubbers and filters, and the cost of erection, but do not include buildings and foundations; P is the horse-power rating of the producer.

Suction producers	$650 + $11 × *P*.
Pressure producers up to 200 horse-power	$1000 + $16 × *P*.
Pressure producers over 200 horse-power	$2000 + $15 × *P*.
Gas holders	$1000 + $0.26 × *P*.

Detailed manufacturers' estimates of investment cost and cost of operation of twenty-six producer plants are given in the paper by Mr. Fernald in the *Jour. Soc. Wes. Eng., 1907, Vol. 12, p. 551.*

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[Wm. Kent.]

GEARS AND GEARING. — The following terms are commonly employed.

Gear Wheel or Gear. — A wheel with teeth cut in its periphery, designed to mesh with the teeth of a similar wheel or with a screw or worm.

Pitch Circle or Pitch Line and Pitch Diameter. — Referring to Fig. 1, the two tangent circles *A* and *B* with their centers on the axes of the two gears, are called the pitch circles or pitch lines of the two gears. The diameter of the pitch circle is called the pitch diameter.

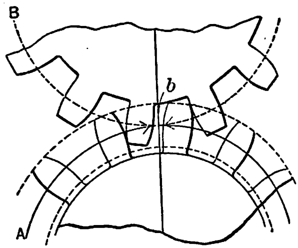

Fig. 1. Spur Gear

Backlash. — Referring to Fig. 1, the distance *b*, which is the difference in the width of the tooth space and the width of the tooth, measured along the circumference of the pitch circle, is called the backlash.

Pitch of a Gear. — The *circular pitch* of a gear is the distance in inches, measured along the *arc* of the pitch circle, between the center lines of two successive teeth; it is also

$$\text{Circular pitch} = \frac{\pi \times (\text{diameter})}{\text{Number of teeth}}.$$

The *chordal pitch* is the distance in inches, measured along the *chord* of the pitch circle between the center lines of two successive teeth; let N be the number of teeth, D the diameter of pitch circle, then

$$\text{Chordal pitch} = D \sin\left(\frac{180°}{N}\right).$$

The *diametrical pitch* is the quotient of the number of teeth by the diameter of the pitch circle.

Pinion. — The smaller wheel of a gearing consisting of two gear wheels is called the pinion; e.g., the gear wheel on the shaft of a railway motor is called the pinion. The name pinion is also used for the smaller of two gears mounted on the same shaft.

Sprocket. — A gear wheel driven by a chain.

Gear Ratio or Velocity Ratio. — The ratio of the number of teeth in the larger wheel of a gearing to the number of teeth in the smaller wheel; this is also equal to "velocity ratio," or the number of revolutions of the smaller wheel corresponding to one revolution of the larger wheel.

Spur Gear. — A gear wheel in which the external surfaces of the teeth lie on a cylinder and the center planes of the teeth pass through the axis of the gear — the common type of gear wheel.

Bevel Gears. — A gear wheel in which the external surfaces of the teeth lie on a cone, so that the gear may mesh with another gear having its axis inclined to the axis of the first.

Stepped Gears. — Two gears of the same pitch and diameter mounted side by side on the same shaft will act as a single gear. If one gear is keyed on the shaft so that the teeth of the two wheels are not in line, but the teeth of one wheel slightly in advance of the other, the two gears form a stepped gear. If mated with a similar stepped gear on a parallel shaft the number of teeth in

contact will be twice as great as in an ordinary gear, which will increase the strength of the gear and its smoothness of action.

Twisted Teeth. — If a great number of very thin gears were placed together, one slightly in advance of the other, they would still act as a stepped gear. Continuing the subdivision until the thickness of each separate gear is infinitesimal, the faces of the teeth instead of being in steps take the form of a spiral or twisted surface, and we have a twisted gear. The twist may take any shape, and if it is in one direction for half the width of the gear and in the opposite direction for the other half, we have what is known as the herring-bone or double helical tooth. This form of tooth is much used in heavy rolling-mill practice, where great strength and resistance to shocks are necessary. They are frequently made of steel castings. The angle of the tooth with a line parallel to the axis of the gear is usually 30 degrees.

Spiral or Helical Gears. — If a twisted gear has a uniform twist it becomes what is commonly called a spiral gear (properly a helical gear). The line in which the pitch surface intersects the face of the tooth is part of a helix drawn on the pitch surface.

Pitch Angle. — The pitch angle of a helical gear is the angle between the center line of the tooth and a line perpendicular to the axis of the gear.

Screw or Worm. — A spiral wheel may be made with only one helical tooth wrapped around the cylinder several times, in which it becomes a screw or worm. If it has two or three teeth so wrapped, it is a double- or triple-threaded screw or worm.

Worm-Gearing. — When the axes of two spiral gears are at right angles and a wheel of one, two or three threads works with a larger wheel of many threads, it becomes a worm gear, or endless screw, the smaller wheel or driver being called the worm, and the larger, or driven wheel, the worm wheel. With this arrangement a high velocity ratio may be obtained with a single pair of wheels. For a one-threaded wheel the velocity ratio is equal to the number of teeth in the worm wheel. The worm and wheel are commonly so constructed that the worm will drive the wheel, but the wheel will not drive the worm.

Differential Gearing. — Various forms of differential gears are used. The object is to devise a motion such that the velocity ratio is equal to the ratio of the number of teeth in one gear wheel to the *difference* between the number of teeth in this gear wheel and the number of teeth in the other wheel. In one form of differential gearing one wheel meshes with a set of teeth on the inside of a second wheel. The second wheel is so arranged that it cannot turn about its own axis but its center is caused to move in a circle concentric with the first wheel.

But for the limitation that the difference between the wheels must not be too small, the possible ratio of speed might be increased almost indefinitely, and one pair of differential gears made to do the service of a whole train of wheels. If the problem is properly worked out with bevel gears this limitation may be completely set aside, and external and internal bevel gears, differing by but a single tooth if need be, made to mesh perfectly with each other.

DESIGN OF GEARS. — In order that the teeth of wheels and pinions may run together smoothly and with a constant relative velocity, it is necessary that their working faces shall be formed of certain curves called odontoids. The essential property of these curves is that when two teeth are in contact the common normal to the tooth curves at their point of contact must pass through the pitch point, or point of contact of the two pitch circles. Two such curves are in common use — the cycloid and the involute.

The design of gears is treated in detail in Kent's *Mechanical Engineers' Pocket-Book* and in Halsey's *Handbook for Machine Designers.*

Materials and Construction.* — Gears are usually made of cast iron or steel. To obtain the most satisfactory results the teeth should be machined. Rawhide pinions, cambric-cloth pinions, and composite gears, consisting of alternate sheets of rawhide or fiber and steel or bronze, are also used where smooth and quiet running are necessary. Of this latter form of gearing the cloth pinions have the advantages of being strong, not liable to damage from cold or hot oil, unaffected by atmospheric changes, and vermin proof.

POWER TRANSMITTED BY GEARING. — The strength of gear-teeth and the horse-power that may be transmitted by them depend upon so many variable and uncertain factors that the formulas and rules given by different writers show a wide variation. The pitches indicated in the accompanying table* are recommended as representing safely conservative practice and, although somewhat smaller teeth may afford sufficient strength in some cases, these standards should be adhered to with as few exceptions as possible.

Maximum horse-power transmitted	Diametrical pitch	Maximum horse-power transmitted	Diametrical pitch
1/6	12	10	4
1/4	10	15	3
1/2	10	20	3
3/4	10	25	3
1	8	50	2½
2	8	60	2½
3	6	75	2
5	6	100	2
7½	5	125	1½
		150	1½

EFFICIENCY OF GEARING. — An extensive series of experiments on the efficiency of gearing, chiefly worm and spiral gearing, is described by Wilfred Lewis in *Trans. A.S.M.E.*, Vol. 7, p. 273. The average results are shown in a diagram, from which the approximate average figures in the table on the following page are taken.

The experiments showed the advantage of spur gearing over all other kinds in both durability and efficiency. The variation from the mean results rarely exceeded 5 per cent in either direction, so long as no cutting occurred, but the variation became much greater and very irregular as soon as cutting began. The loss of power varies with the speed, the pressure, the temperature and the condition of the surfaces.

The excessive friction of worm and spiral gearing is largely due to the end thrust on the collars of the shaft. This may be considerably reduced by roller bearings for the collars. When two worms with opposite spirals run in two spiral worm gears that also work with each other, and the pressure on one gear is opposite that on the other, there is no thrust on the shaft. A low efficiency

* By D. B. Rushmore.

for a worm gear means more than the loss of power, since the power which is lost reappears as heat and may cause the rapid destruction of the worm.

PER CENT EFFICIENCY OF SPUR, SPIRAL AND WORM GEARING

Gearing	Pitch angle, degrees	Velocity at pitch line in feet per min.				
		3	10	40	100	200
Spur pinion	..	90	93.5	97	98	98.5
Spiral pinion	45	81	87	93	95.5	96.5
Spiral pinion	30	75	81.5	89	93	94.5
Spiral pinion	20	67	75	84.5	90	92
Spiral pinion	15	61	70	80.5	87	90
Spiral pinion or worm	10	51	61.5	74	82	86
Spiral pinion or worm	7	43	53	72	76.5	81.5
Spiral pinion or worm	5	34	43	60	70	76.5

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[WM. KENT.]

GENERATORS, ALTERNATING-CURRENT. — (*See also Alternating Currents; Electricity and Magnetism, Principles of; Generators, Direct Current-Motors; Standardization Rules.*) The following is a brief table of contents of this article:

In all dynamo-electric machines of the usual forms (excepting the homopolar) the electromotive force induced in any one conductor or turn varies in value and alternates in direction. If the terminals of any coil or group of coils are brought out to an external circuit by means of revolving contacts, such as slip rings, an alternating current will flow in the circuit. In order to obtain a direct or continuous current it is necessary to add a commutator or rectifier. Thus the most simple and elementary form of electric generator is an alternating-current generator and the direct-current generator is a special form adapted by the addition of a commutator to give a continuous or unidirectional current.

CLASSIFICATION. — Alternating-current generators may be classified according to several different distinguishing characteristics. The following classification considers these characteristics in the order of their prominence.

Synchronous Generators. — In the synchronous type the action of inducing the electromotive force results from the relative motion of the armature conductors and a constant magnetic field produced by exciting coils in which a continuous or direct current flows. The frequency of the alternating electromotive force depends directly on the number of field poles and the angular velocity of the revolving part.

Induction Generators. — In the induction type the magnetic field is of the rotating polyphase type and is produced by polyphase alternating currents flowing in the same windings with the load current. The mechanical construction is usually that of an induction motor with a short-circuited polyphase winding on the revolving member. The frequency depends upon the characteristics of the external circuit. In practice the frequency is determined or "set" by a synchronous machine, either generator or motor, in the external circuit. This "frequency setter" is necessary to supply the exciting current of the induction generator; that is, by means of the frequency setter the power factor of the total load is adjusted to equal the inherent power factor of the generator.

Advantages and Disadvantages of Induction Type. — A minority of the units of a station may be of the induction type with advantage as they will cause less disastrous effects in case of a short circuit on the system. Their instantaneous short-circuit current is less. A certain number of the units must be of the synchronous type to "set" the frequency and supply the exciting current for the induction machines. Consequently, if there is a large proportion of induction machines the synchronous machines operate at a poor power factor unless there is much capacity effect in the load system, such as synchronous motors or line capacity.

Revolving-field and Revolving-armature Types. — In the revolving-field or revolving-armature type there are two, or some multiple of two, poles, each pole having its own coil for the direct-current excitation, and the flux in each pole is to all intents and purposes constant in value. The relative movement of the poles and conductors causes the variation in flux interlinkages both in direction and intensity.

The early machines were constructed with an armature revolving inside a stationary field. As sizes and voltages increased it was found that a more effective use of the material could be obtained by making the armature the external member, and that the insulation of the high-voltage member was better preserved if that member was kept stationary.

At the present time all generators of large capacity (500 kilowatts and greater) and for high voltages (600 volts and up) are made of the revolving-field type.

Inductor Type. — In the inductor type the direct-current excitation is concentrated in one (usually stationary) coil and the variation in magnetic flux is obtained by revolving a spider with bare projecting poles which alters the reluctance of the magnetic path. Thus the flux threading any particular armature coil is always in the same direction, but varies in intensity or quantity.

Comparison of the Inductor Type with Revolving Field or Revolving-armature Type. — There are no windings or insulation on the moving member of the inductor type, which is, therefore, adapted to high speeds. The shorter and more compact magnetic circuit of the revolving-field or revolving-armature type, however, makes for a better voltage regulation. This type is also better adapted to operation in parallel with other generators. It is the type in by far the most general use.

Single Phase, Two Phase or Quarter Phase and Three Phase. — The single-phase generator is usually about 30 per cent heavier and more costly than a polyphase generator of the same rating. By changing the internal armature connections a polyphase machine may frequently be reconnected to be a three-, two- or single-phase machine.

As transmission by three-phase currents is more economical of copper than by two-phase or single-phase currents, all power transmission lines are three phase. Consequently unless there is some local condition requiring single or two phase, the three-phase generator is preferable.

Two-phase and three-phase generators of the same capacity and voltage strain are of practically the same dimensions, weight and cost.

Trade Terms. — It has become almost universally the custom to classify an alternating-current generator as *ASB*, *AQB* and *ATB*; the *A* standing for alternator, *S* for single phase, *Q* for quarter phase, *T* for three phase and *B* for revolving field.

Separately-excited and Self-excited Types. —For years it was attempted to develop a satisfactory and simple self-excited alternator and some were very successful but not simple. The object was to obtain a constant voltage on the load by means of automatic self-excitation. However, with the perfection of the automatic voltage regulator the need for automatic self-regulation ceased, and there is now very little demand for self-excited alternators.

METHODS OF RATING. — While generators have been built to meet many different special specifications, it has generally been found more economical to adopt one of the three ratings standardized by the large manufacturing companies. These rating specifications are summarized below. However, it should be noted that **in the standardization rules issued by the A.I.E.E. in 1914 a number of radical changes in methods of rating are recommended;** see article on *Standardization Rules of the A.I.E.E.*

A-Rating. — Generator designed to give the rated load in kilowatts at 100 per cent power factor continuously with a maximum rise in temperature by thermometer of 45° C. above the surrounding air at 25° C. Allowances for other temperatures of the air to be made in accordance with instructions given in the *Standardization Rules of the A.I.E.E.*, Edition of 1911. Also required to give for 2 hours an output in kilowatts, at 100 per cent power factor, 25 per cent greater than the rating, with a maximum rise in temperature by thermometer of 55° C. above surrounding air at 25° C., the overload condition occurring immediately after the machine has been running for some hours at the rated load.

The A-rating has been most generally used, particularly for lighting and general power-station installations.

B-Rating. — Generator designed to give the rated output in kilowatts at 100 per cent power factor continuously with a rise in temperature by thermometer not to exceed 35° C. above the surrounding air at 25° C., and to give 50 per cent more than rated load in kilowatts for 2 hours with a rise in temperature by thermometer not to exceed 55° C. above the surrounding air at 25° C., the overload occurring immediately after a run for a stated number of hours at rated load.

The B-rating was designed particularly for railway work and is desirable in cases of varying loads, loads varying from moment to moment as well as loads with pronounced peaks for an hour or two.

Continuous or "Maximum" Rating. — Generator designed to give continuously the rated load in kilowatts at 100 per cent power factor with a maximum rise in temperature of any part of 50° C. above the surrounding air at 25° C. (or above the incoming air at 25° C. when forced ventilation is used) corrected as per 1911 edition of the *A.I.E.E. Standardization Rules*. This type is not designed to give any overloads except momentarily.

The maximum rating has been developed particularly to harmonize the characteristics of the electric generator with the steam turbine. The rating of the generator corresponds to the maximum economical rating of the turbine. If the generator is driven by a water wheel the maximum rating corresponds to the maximum continuous rating of the wheel at full gate opening.

VOLTAGE. — Alternators are now built to generate voltages up to 13,000 and 15,000 volts between lines, either single phase or polyphase. Above that voltage the extra cost of the insulation and the danger of damage from discharges cause it to be less expensive to install transformers with a lower voltage alternator. Some engineers consider the limit of economical voltage of generators to be even lower than 13,000 volts.

FREQUENCY. — The frequency depends upon the speed of rotation and the number of poles. If the speed of rotation of the revolving part is given in revolutions per minute, the frequency is

$$f = \frac{\text{r.p.m.}}{60} \times \frac{\text{number of poles}}{2}.$$

In the early alternators it was found much more economical to run at high speeds, and thus high frequencies, such as 133 and 125 cycles per second, were customary; but, as systems increased in size and complexity, electrical difficulties arose as a result of these high frequencies and the tendency has been to reduce the frequency till now we have 60, 50, 40 and 25 cycles per second as usual frequencies, of which 25 and 60 are standard in this country. In Europe 50 cycles is used instead of 60. Of late there has been an attempt to have 15 or 16 cycles standardized for railway work.

A frequency of 25 cycles is preferable where there is a very long transmission line, on account of the lower inductive voltage drop, and where there is much synchronous machinery, such as synchronous motors and rotary converters, as these are more stable and better adapted to parallel operation at low frequencies.

A higher frequency (60 cycles in U. S. A., and 50 in Europe) is preferable for electric lighting, as the light is steadier. The higher frequency is also preferable where many transformers are used, as these are cheaper and more efficient at the higher frequencies.

PHASE AND LINE VOLTAGES AND CURRENTS. — For moderate voltages, up to 3000 between lines, the method of connection is decided by such details, as which will give a convenient number for the conductors per slot for the required voltage, but for higher voltages the Y connection is usually chosen as it causes a lesser strain on the machine insulation for a given voltage between lines. See also p. 645.

Single Phase. — In a single-phase generator the voltage per phase is the same as the voltage between lines and the current per phase is the same as the current per line, the product of voltage and current giving the volt-ampere rating of the machine.

Two Phase or Quarter Phase. — In a properly designed machine of this type each phase supplies half the rating, thus the voltage and current per phase in the machine are respectively the same as the voltage and current per phase on the line. The current is equal to $1000 \times$ (kv-a.) $\div 2 \times$ volts per phase.

Three Phase. — Machines of this type may be connected either Y or Δ. In a Y-connected machine the current per phase is the same as the current in each line and is equal to $1000 \times$ (kv-a.) $\div \sqrt{3} \times$ (volts between lines), while the voltage per phase is (volts between lines) $\div \sqrt{3}$.

In a Δ-connected machine the line current is equal to $1000 \times$ (kv-a.) $\div \sqrt{3} \times$ (volts between lines) and the current per phase is equal to (line current) $\div \sqrt{3}$, while the voltage per phase is equal to the voltage between lines.

DESIGN. — The procedure in designing an alternating-current generator for a given power output, voltage and frequency to fulfill given requirements regarding regulation, efficiency, etc., is partly analytical and partly empirical. The data of four specific designs are given below in the section on *Examples of Design and Performance*. The method of procedure is to lay out a preliminary design from rough calculations, calculate the performance of this design, modify the preliminary design where this calculation indicates, recalculate the performance, etc., until a design is arrived at which meets the given requirements.

Definitions. — The following terminology is used in the discussion of the design of generators:

Pole Arc. — The arc subtended by one pole face, measured along the periphery of the armature; in the following discussion it will be expressed in inches.

Pole Pitch. — The arc measured along the periphery of the armature from the center of one field pole (N-pole, say) to the center of the next field pole (S-pole); in the following discussion it will be expressed in inches.

Ampere Conductors per Inch of Armature Periphery. — The product of the number of conductors per inch measured along the periphery of the armature by the effective amperes flowing in each conductor.

Slot Pitch. — The distance in inches measured along the armature periphery between the centers of two adjacent slots. The slot pitch is equal to the pole pitch divided by the number of slots per pole.

Coil Pitch. — The number of slots spanned by a coil; that is, if a coil has one side in slot number 1, and the other side in slot number 7, the pitch of the coil is $7 - 1 = 6$. A coil which spans a distance exactly equal to the pole pitch is said to have a "full pitch," but if it spans a lesser distance it is said to have a "fractional pitch." For example, if there are six slots per pole and a coil has one side in slot number 1, and the other side in slot number 5, it is said to have a fractional pitch of $\frac{4}{6}$ or $\frac{2}{3}$.

Leakage Factor. —The ratio of the flux per pole which enters the armature core to the total flux (including the leakage flux) which would be produced by the field winding.

Preliminary Calculation of Main Dimensions. — Let

P_0 = output in kilovolt amperes (kv-a),
ρ = ratio; pole arc divided by pole pitch,
B = average magnetic flux density per square inch in air gap,
σ = ampere-conductors per inch of periphery,
V = peripheral velocity in feet per minute at gap,
D = diameter of armature in inches,
L = length of armature along shaft in inches,
N = revolutions per minute,
p = number of poles,
f = frequency in cycles per second,
$A = D \div p$.

Then the following relation holds for either a single-phase or a polyphase machine:

$$P_0 = \frac{\rho B \sigma V D L}{144 \times 10^9}. \quad * \qquad (1)$$

Five of the six design constants in the right-hand member of this equation are subject to choice; the equation then fixes the sixth constant. The choice of the values of the various constants should be based upon modern practice, an idea of which is given below.

Ratio of Pole Arc to Pole Pitch (ρ) is governed by two antagonistic phenomena; for the sake of small magnetic leakage and good form factor of e.m.f. wave a low value is desired; for the sake of low reluctance to the main flux and economy of material a high value is desired. The usual values are ranged from 0.6 to 0.7.

Magnetic Flux Density (B) is limited by the exciting ampere turns required to produce it, length of gap and sometimes by the heating resulting from core loss. Usual values B are given in the accompanying table.

Cycles	Lines per sq. in.
25	44,000 to 58,000
60	32,000 to 48,000
125	30,000 to 40,000

Ampere-Conductors per Inch of Periphery (σ) is limited by the heating resulting from high current densities. If ventilation is good, insulation thin, or if the slots are very deep, as in armatures of large diameters, the values of σ may be high. Usual values of σ for continuous rating and for a rise in temperature of 40° C. are:

* For an exact check the constants of p. 629 must be introduced.

Diameter of armature, in.	Less than 500 volts	500 to 2000 volts	2000 to 7000 volts	Above 7000
0 to 20	250 to 350	200	...	...
20 to 30	350 to 600	400	300	250
30 up	600 to 900	700	500	400

Naturally, with special means of ventilation, σ may be much greater than the values here given, as, for example, in turbo-alternators with forced ventilation.

Peripheral Velocity (V) is altogether a matter of mechanical design and values may be found from 2000 feet per minute to 10,000. Usual values are from 3000 to 6000.

Diameter and Length of Armature. — The diameter of armature (D) is related to the number of revolutions per minute (N) by the formula

$$D = \frac{12\,V}{\pi N}.$$

Hence, for a given value of N, the diameter D is fixed when V is chosen. This value of D together with the chosen values of ρ, B, σ and V then fixes the value of L by equation (1).

If, however, the value of N is not given, reasonable values for D and L may be found by assuming that, in accordance with normal and economical conditions, L is approximately equal to the pole pitch. This gives, putting A = diameter divided by number of poles, $L = \pi A$, and equation (1) reduces to

$$P_0 = \frac{\pi\rho B\sigma VAD}{144 \times 10^9}.$$

This fixes D when ρ, B, σ, V and A are chosen. The value of D must, of course, be adjusted to the frequency and an even number of poles (*see next paragraph*). Usual values of the diameter per pole are given in the accompanying table.

Cycles	A = diameter per pole, in.
25	5 to 6
60	2.5 to 3
125	1.75

Number of Poles (p) **and Revolutions per Minute** (N). — The number of field poles is related to revolutions per minute N and the frequency f by the formula

$$f = \frac{Np}{120}, \quad \text{or} \quad p = \frac{120f}{N}.$$

Hence when the frequency and speed are both given, the number of poles is fixed. When N is not given, but L is assumed approximately equal to πA as above, the number of poles is found, when D has been determined, from the relation

$$p = \frac{D}{A}.$$

A value of A must of course be chosen such that p is an even number. The speed N is then obtained from the relation

$$N = \frac{120f}{p}.$$

Armature Windings. — While there are many forms of armature windings which may be used, there are practically only two forms that are in general use in this country. These are the "Chain Winding" and the "Lap Winding."

Chain Winding (Fig. 1). — This winding is characterized by having a number of coils equal to half the number of slots; that is, there is only one side of a coil in each slot. The coils may be either form wound and insulated, in which case slots with open faces are required; or they may be wound by hand in place, in which case partly or entirely closed slots may be used.

There are at least two kinds of coils in each machine. These are characterized by the shape of their end connections, as these end connections lie some in one plane and some in another. If there is more than one slot per pole per phase, there must be four different shapes of coils, having two different pitches; the coils of any one phase per pole are placed concentrically. The chain winding is most generally used in high-voltage machines, 2000 volts or higher.

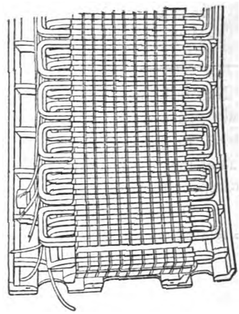

Fig. 1. Chain Winding

Lap Winding (Fig. 2). — This winding contains a much larger number of coils, all of the same form and size. There may be two, four, six, etc., sides of coils in each slot, the coils being placed side by side two coils deep. This winding is similar to the multiple-drum winding of a direct-current armature. With two coil sides per slot the slots must be open, but with more than two coil sides per slot the slots may be partly closed. This type of winding is satisfactory for low voltages and very convenient if there are a large number of slots per pole.

The coils are connected in groups of 2, 3, etc., per pole per phase, depending upon the slots per pole per phase. Successive groups or poles of the same phase are connected in series or multiple by "pole connections."

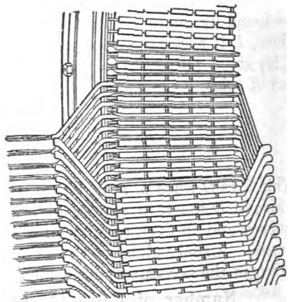

Fig. 2. Lap Winding

In both forms of winding each coil may contain from one to many turns in series.

Turns in Series per Phase (S). — Let

σ = ampere-conductors per inch of periphery,
D = diameter of armature at gap,
Q = number of phases,
I = effective amperes per phase at rated load,
S = turns in series per phase.

Then, for a uniformly distributed winding, such as is used in two- or three-phase machines, and all conductors of each phase in series,

$$S = \frac{\pi \sigma D}{2\,QI}.$$

If the conductors are arranged in 2, 3, or more parallel paths, the factor 2 in the denominator should be changed to 4, 6, etc., respectively. For single-phase machines *see special treatment* below.

Number of Slots. Pitch of Slots (s). — The number of slots per pole in the armature of a two-phase generator may be any multiple of 2, and in a three-phase generator any multiple of 3. The pitch of the slots for any assumed number of slots per pole is

$$s = \frac{\pi D}{p \times (\text{slots per pole})},$$

where D is the diameter of the armature at the gap and p the number of poles.

Number of Conductors per Slot is $\sigma s/I$, where σ is the number of ampere-conductors per inch of periphery, s the slot pitch, and I the effective full-load amperes per phase.

Size of Conductors. — The size of each conductor is a compromise between a size which will give a reasonable current density in the conductor, and a size which will properly fill a reasonable slot (*see discussion of magnetic circuit below*). The current density in the copper conductor should be from 2500 amperes per square inch in small machines to 2000 in medium and moderate voltage and 1600 in high-voltage machines.

The conductors are arranged in the slots so that the slots are usually about four times as deep as they are wide. The size of the slots should be such as to allow a space for insulation over and above the space occupied by the conductors and such cotton covering as they may have. The space required for this extra insulation in straight slots is as follows:

Voltage	Coil sides per slot	Insulation, inches	
		Vertical	Horizontal
500	2	0.350	0.150
2,000	2	0.430	0.160
3,000	2	0.50	0.30
5,000	1	0.75	0.375
6,600	1	0.835	0.50
13,000	1	1.125	0.75

For example, the necessary vertical depth of slot may be found in a machine insulated for 2000 volts with 2 coil sides per slot by multiplying the over-all diameter of cotton-covered wire, or depth of bar, by the number of conductors in a vertical row and adding 0.43 to the product.

Slot Factor. — The ratio of cross section of copper in a slot to cross section of slot is known as the slot factor. Normal values of the slot factor for a 1000-kilowatt machine are given in the accompanying table. For smaller machines the slot factor is slightly less and for larger machines slightly greater.

Voltage	Slot factor
500	0.5
1,000	0.46
2,000	0.42
4,000	0.35
6,000	0.33
10,000	0.30

Preliminary Dimensions of Magnetic Circuit. — The diameter of the armature and the axial length of the pole face for the preliminary design are fixed by the above calculations. There remain to be determined the air gap, the

dimensions of the armature teeth, the length and radial depth of the armature iron, the dimensions of the field poles, and the dimensions of the field yoke. Preliminary values for these quantities may be arrived at as indicated below.

Air Gap. — The minimum clearance between armature core and field poles in modern generators is given in the accompanying table. The average length of the air gap, due to the chamfer on the poles, is about 25 per cent greater.

Armature Diameter, in.	Minimum air gap, in.
40	0.125
80	0.160
120	0.200
160	0.312
200	0.440
240	0.500

Armature Teeth. — The dimensions of the armature teeth depend upon the number and dimensions of the slots, which in turn must be such as to contain the necessary conductors. Another important limitation to the width of the teeth, however, is the value of the magnetic flux density therein. Let s = pitch of slots, a = pole arc, g = average radial depth of air gap. Then the effective number of teeth under each pole is $(a + 2g) \div s$. The effective width of the tooth is its width at a section one-third the distance from its minimum width towards its maximum width. The effective axial length of the teeth is approximately equal to 0.9 of the axial length of the pole face L determined above. From these data the effective cross section of the teeth perpendicular to the flux lines is determined, and this divided into the total flux per pole ϕ gives the average flux density in the teeth. The value of ϕ from the preliminary calculations above is $\pi \rho LDB \div p$, where ρ = ratio of pole arc to pole pitch, D = diameter of armature at gap, B = average magnetic flux density in gap, and p = number of poles. The average flux density in the teeth in modern alternating-current generators ranges from 90,000 to 110,000 lines per square inch.

Armature Core. — The axial length of the armature core is usually slightly greater than the axial length of the pole face. A fair allowance is twice the radial depth of the air gap. The core is usually built up of sheet-steel punchings, 14 mils thick, the punchings having received previously a thin coat of varnish. Ventilating ducts from ⅜ to ½ inch in width (measured along the shaft) are usually provided, these ducts dividing the core into sections from 2.5 to 3 inches thick (measured along the shaft). The effective axial length of the armature iron is therefore the total length of the armature core less the space occupied by the air ducts and insulation between punchings. This latter is about 10 per cent of the net length after deducting the space occupied by the air ducts.

One-half the total flux per pole passes through each section of the armature core. The flux density in the armature core of modern alternating-current generators ranges from 50,000 to 70,000 lines per square inch. Let B_a = the value of the flux density chosen, l = effective axial length of armature iron, ϕ = flux per pole. Then the necessary radial depth of the armature core is $\phi \div 2 B_a l$.

Field Poles. — The axial length of the pole face f is about 0.5 in. less than L, the length of armature. The pole arc a has already been determined. The radial length of the magnet core (c in Fig. 3, see below) and the width of the magnet core (b in Fig. 3) have yet to be determined. The width of the magnet core is usually from 65 to 75 per cent of the pole arc (see Fig. 3), and the radial length in modern machines from 50 to 150 per cent of the width. The dimensions chosen must be such that the core will accommodate the field spool, the approximate size of which is determined as follows:

The number of ampere turns required per pole to force the flux through the

air gap is: $0.313\ \phi g \div af$, where ϕ = flux per pole; g = average radial depth of gap; a = pole arc, and f = axial length of pole face. The ampere turns required for the rest of the magnetic circuit is about 40 per cent of this for most 25-cycle machines and 20 per cent of this for most 60-cycle machines. Hence the total net ampere turns required to produce the flux ϕ is approximately $0.44\ \phi g \div af$ for 25-cycle machines, and $0.38\ \phi g \div af$ for 60-cycle machines. The actual ampere turns required on each field spool at full load, however, will be greater than this, on account of the armature reaction, by an amount depending upon the armature ampere turns per pole and the power factor. For preliminary calculations of the field winding space, however, it is sufficient to increase the net ampere turns as above calculated by about 50 per cent, giving as the approximate total number of ampere turns per pole $F = 0.66\ \phi g/aL$ for 25 cycles and $F = 0.57\ \phi g \div aL$ for 60 cycles.

It is usual to allow a current density of 1000 amperes per square inch in the field copper. The cross section of the copper in the winding is then $F \div 1000$. The total cross section of the winding including the cotton insulation and interstices will be 40 per cent greater than the cross section of the copper when wire is used, and 15 per cent greater when strip is employed. The field pole may then be laid out to accommodate this winding, which is usually wound on a spool, allowing ⅜ inch for the insulating collar at each end, and 0.25 inch for the thickness of the spool and protecting material.

In deciding upon the width of the core, the flux density in the core should also be considered. This should not exceed 100,000 lines per square inch in a laminated steel core. Let ϕ = useful flux per pole, ν = leakage factor, f = axial length of core, b = width of core, then the flux density is $B_c = \nu\phi \div bf$. The value of ν in modern generators ranges from 1.1 to 1.5.

Field Yoke. — The diameter of the armature and the radial length of air gap and field magnet cores fixes the internal diameter of the field yoke (or external diameter in case of revolving-armature type). The radial depth of the yoke, which is usually of cast iron or cast steel, is determined by the flux density desired. Let ϕ = useful flux per pole, ν = leakage factor, e = axial length of yoke, and B_y the flux density in the yoke, then the radial depth is $\nu\phi \div 2\ B_y e$. In modern machines B_y ranges from 25,000 to 30,000 lines per square inch in cast-iron yokes, and from 70,000 to 80,000 lines per square inch in cast-steel yokes.

Field Winding. — It is unnecessary to attempt an accurate calculation of the field winding until a design of the armature and magnetic circuit has been adopted which will give the required regulation. The method of calculating the regulation and determining the number of field ampere turns is given in the next section.

After the excitation in ampere turns per pole required for various loads and power factors desired (according to the specifications or the service required of the generator) has been determined, that condition of operation which requires the greatest number of ampere-turns F is selected. A field winding which will give this excitation and a margin for safety, with the exciter voltage specified, is then designed. Using this value of F and a current density of 1000 amperes per square inch, the cross section of the field winding is $1.4\ F \div 1000$ when made of insulated wire, or $1.15\ F \div 1000$ when made of insulated strip. The length of the coil will be $c - 0.75$ (see Fig. 3), allowing ⅜ inch at each end for the collar and insulation. Therefore the depth of winding will be for wire $\delta = 1.4\ F \div 1000\ (c - 0.75)$ + thickness of spool on which the wire is wound, or $\delta = 1.15\ F \div 1000\ (c - 0.75)$ + thickness of spool. The spool is usually 0.25 inch thick.

The radiating surface of the coil is then

$$A = 2\,c\,(f + b + 4\,\delta).$$

Resistance and Temperature Rise of Field Winding. — The rise in temperature per watt radiated per square inch of surface of a field coil depends on the peripheral speed of the revolving field or revolving armature. The accompanying table gives the temperature rise in degrees centigrade per watt radiated per square inch. For other rates of radiation the temperature rise is proportional. Let w = watts radiated per square inch for the assumed temperature rise (usually 45° C.), A = area of radiating surface, then the allowable loss per pole is wA, and the field current is then $I_f = wA \div v$, where v = volts per pole. The volts per pole should be taken as 80 per cent of the exciter voltage divided by the number of poles (all field spools in series), allowing 20 per cent for emergencies (low speed, etc.). The number of field turns per pole is then $t = F \div I_f + 0.5$, the half turn being added for convenience in connecting up. The proper resistance (r) of the field winding is then $wA \div I_f^2$, and the necessary cross section (q) of the conductor in square inches is

	Peripheral speed, ft. per min.	Temp. rise ° C. per watt per sq. in.
Revolving field	1,000	45
	2,500	40
	5,000	25
	10,000	15
Revolving armature	1,000	80
	2,500	60
	5,000	45
	10,000	30

$$q = \frac{0.0186\, t\, (f + b + 2\,\delta)}{12{,}000\, r}.$$

(The resistance of 1000 feet of copper 1 square inch in cross section at 60° C. is 0.0093 ohm and $2\,(f + b + 2\,\delta)$ is the mean length of turn.) From this cross section the nearest size of wire may be chosen and r, w and volts per pole calculated by reversing the above procedure.

The cross section of the winding should now be recalculated, and the necessary adjustments made to make it fit the winding space available on the field poles.

Bearings, Shafts, Bedplates, etc. — Alternating-current generators may be belt-driven or direct-connected and the latter type may have either a horizontal or vertical shaft.

Belt-driven generators are only used in the smaller sizes up to 500 kilowatts. The smallest sizes usually have two bearings and an overhanging pulley. The larger sizes frequently have three bearings and the weight of the pulley and pull of the belt are taken by two of the bearings.

Generators with horizontal shafts for direct connection to water wheels are usually built with two bearings and a coupling and are complete in themselves, i.e., they have bedplates, shafts and bearings.

Generators with horizontal shafts for direct connection to steam engines are usually built without bedplates, shafts or bearings. The generator frame is supported on the foundation by a suitable sole plate. The engine bearing serves for one of the generator bearings. The revolving field is carried by an extension of the engine shaft. The shaft and bearings are usually furnished by the engine builder and the revolving member of the generator is pressed on the shaft at the factory where the generator is built.

Vertical shafts are sometimes used with water-wheel-driven generators, and in this case the weight of the revolving part of the generator may be balanced by the upward thrust of the water jet.

In some machines of the revolving-field type the magnetic yoke is made

extremely large to serve as a flywheel. Such machines are known as flywheel generators.

For the purpose of preparing the foundations for the generator, the manufacturer usually supplies in advance a template showing the shape and size of bedplate, number, size and spacing of bolt holes.

Design of Turbo-Generators. — In turbine-driven generators the steam turbine is usually built by the same company that builds the generator and the two machines are practically one unit. The steam turbine has developed to such an extent that for a given capacity in power it weighs less than a reciprocating engine, occupies much less space, costs less and is fully as economical of steam if not more economical. Good economy in steam turbines, however, is obtained only with high angular velocities. This has made it very difficult to apply the turbine to useful purposes. The electric generator has shown itself to be the most natural device to absorb the power of the turbine and make it available for various applications in a convenient form. However, it required many years of experience to develop an electric generator which would operate successfully at the high angular velocities suitable for direct connection to the steam turbine. The principal difficulty was the design of a construction which would withstand the enormous stresses in the revolving member which resulted from the centrifugal force. Thus the design of the machine as a whole turns upon the type and construction of the rotor, which is usually in the form of a revolving field.

Normal Weights and Speeds. — Most turbo-generators have much more copper on the fields and less on the armature than do slow-speed machines, although the total amount of copper is not far different in the two classes of machines. It is quite normal to build machines of from 1000 to 5000 kw. capacity to operate at 3600 r.p.m. and machines of 10,000 kw. at 1800 r.p.m. To do this involves the use of peripheral speeds of from 15,000 to 25,000 feet per minute. At these high speeds the amount of material per kilowatt is much reduced. Increasing the speed ten fold reduces the weight of material to about one-quarter. Thus these machines have a lesser cost per kilowatt of capacity. But this low weight is somewhat offset by the higher cost of the construction necessary to withstand the enormous centrifugal forces. Most of the machines, even the large sizes, have two or four poles or at most six poles. This necessitates the use of a pole pitch of from 25 to 30 in. in 60-cycle machines and 50 to 60 in. in 25-cycle machines. Since such large powers are concentrated in such small bulk a great deal of energy in the form of losses must be dissipated from a small space. Thus special means of ventilation must be provided such as numerous air ducts, fan blades on the rotor or a separate blower and air ducts. Usually from 15 to 20 per cent of the whole length is occupied by air ducts.

Methods of Rotor Construction. — The centrifugal force in the revolving member is from 1000 to 1500 lb. for every pound of material. A method for determining the centrifugal force in every part is given in *Electric Machine Design* by Gray and *High-speed Dynamo-electric Machinery* by Hobart and Ellis. The rotor must be of substantial and rigid construction so that the critical speed of vibration is above the normal speed of operation. There are several methods of construction, prominent among which are:

a. Steel punchings with radial slots, assembled on a forged steel shaft with the distributed windings held in place by wedges and bronze end rings.

b. A steel forging with a distributed winding in parallel slots milled in the periphery, the shaft either being in one piece with the forging (4 poles) or bolted on at the ends (2 poles).

c. Laminated pole pieces attached to the spider by keys and holding four or more concentrated field coils.

In these revolving fields the diameter is not much greater than the length of core and the air gap is quite large compared to the diameter; for instance, diameter 48 in., length 40 in., gap 1.25 in. A symmetrical air gap is necessary to prevent noise and undue strains. A sine distribution of flux is desirable to reduce magnetic losses and give a good wave form to the electromotive force of the armature. This is obtained by using a distributed field winding of concentric coils. The pole arc is usually from 60 to 65 per cent of the pole pitch, the flux per pole very high, and the average gap density lower than in slow-speed machines, since due to the sine distribution the maximum density in the gap is 1.57 times the average (usual value of maximum density 50,000 lines per square inch).

Construction of Stator. — Since the pole pitch is large, a large number of slots per pole are used (4, 6 and 8 slots per pole per phase) and this gives a low armature self-inductance and high short-circuit current. Due to the long pole pitch the end connections are long and subjected to considerable mechanical forces as a result of the leakage flux. They must, therefore, be well held in place by non-magnetic supports. For armature reaction in ampere turns per pole values from 6000 to 8000 are common and the field winding is usually of a capacity three times the armature ampere turns, namely, 18,000 to 24,000, giving a regulation of from 4 to 6 per cent. The current density in the copper (2000 amperes per square inch) is lower than in slow-speed machines. The constant, "ampere conductors per inch periphery," has values from 500 to 800.

Provision for Ventilation. — The ventilation of these machines is a problem similar to that of air-blast transformers. Thus there must be provided: (*a*) Sufficient air to carry off the heat generated with a reasonable rise in temperature of the machine and the air. (*b*) Ample duct capacity to prevent too high velocity of the air. (*c*) Proper spacing of the ducts so that there is no part very far from an air duct. (*d*) Proper precautions to prevent the air from carrying dirt and moisture into the machine.

Lamme (*Trans. A.I.E.E.*) states that 100 cu. ft. of air per minute will carry off 1 kw. with a rise (of the air) of 18° C. Velocities of the air of from 5000 to 6000 feet per minute are common in the machine proper. The surface cooled will give off 4 or 5 watts per square inch with a rise of 35 to 40° C. above the cooling air.

PREDETERMINATION OF PERFORMANCE OF A GENERATOR FROM ITS DIMENSIONS. — From the above calculations a preliminary drawing to scale of the machine may be laid out. The next step is to calculate its performance, i.e., predetermine what will be the regulation, the efficiency and the temperature rise in the various parts.

The calculations for a quarter-phase or a three-phase machine are very similar and are given together in the paragraphs immediately following. The special features of a single-phase machine are described below. Examples of specific designs and the tested performance are also given below.

Magnetization Curve. — The first step is the calculation of the magnetization curve, i.e., a curve showing the relation between the voltage per phase at no load (at normal speed) and the field ampere turns. To do this the useful flux per pole corresponding to any given value of the no-load voltage is first calculated, and then the field ampere turns per pole required to produce this flux are determined. A sufficient number of points to give a magnetization curve up to 20 per cent above rated voltage should be calculated.

Useful Flux (i.e., the flux cut by the armature conductors). — Let

E = a given value of the terminal voltage per phase with zero armature current,

f = frequency in cycles per second,

S = number of turns in series per phase,
k_1 = a constant, depending on the shape of the pole shoe, which may be called the "pole shoe constant,"
k_2 = a constant, depending on the distribution of the armature winding, which may be called the "winding-distribution constant,"
k_3 = a constant, depending on the pitch of the armature coils, which may be called the "pitch constant."

Then the useful flux per pole entering the armature is

$$\phi = \frac{E \times 10^8}{4.44\, k_1 k_2 k_3 f S}.$$

Pole Shoe Constant (k_1). — This constant is proportional to the form factor of the flux distribution around the periphery of the armature. In all modern machines the pole faces are so shaped that this distribution is practically a sine wave, and therefore $k_1 = 1$. This is usually done by making the air gap at the pole tips greater than at the center of the pole.

One method of doing this is to make the outline of the pole face a portion of a circle of such a radius (less than the radius of the armature in revolving field machines) that the gap at the tips is twice the gap at the center.

For very accurate predeterminations the distribution of the flux is carefully calculated. (*See S. P. Thompson, Dynamo Electric Machinery, Vol. II, p. 206; C. A. Adams, Trans. A.I.E.E., Vol. 33.*)

Winding Distribution Constant (k_2). — This constant allows for the fact that if there is more than one slot per pole per phase, the conductors in the various slots of one phase under a pole do not generate e.m.f.'s of exactly the same phase. Since the slots pass under the pole consecutively the e.m.f.'s of the conductors reach a maximum consecutively. These e.m.f.'s must therefore be combined vectorially and not merely added together. k_2 is different in single-phase, two-phase and three-phase machines.

The following table gives the values of k_2 for uniformly distributed windings with equally spaced slots.

Slots per pole	Value of k_2		
	1 phase	2 phase	3 phase
1	1.000	.....	.....
2	0.707	1.000	.....
3	0.666	.....	1.000
4	0.65	0.925	.....
6	0.64	0.912	0.966
8	0.64	0.905	.....
9	0.64	.....	.....
12	0.64	0.90	0.960
18	0.64	0.90	0.960
24	0.635	0.90	0.958
∞	0.632	0.90	0.958

These constants apply to a winding uniformly distributed around the armature periphery. This is always the condition in a two- or three-phase machine. A single-phase machine usually has its working winding irregularly distributed,

so the above constants are only of theoretical interest. (*See special treatment of single-phase machines below.*)

Pitch Constant (k_3). — It is sometimes desirable to use coils having a fractional pitch, particularly in machines of large pole pitch, in order to save copper and I^2R loss, and also in any machine to give a particularly good wave shape. When this is done each turn connects in series two conductors generating e.m.f.'s which are not in phase and their resultant is therefore not as great as their arithmetic sum. The constant k_3 is the ratio of this resultant or vector sum to the arithmetic sum of the two e.m.f.'s. The relation between k_3 and winding pitch expressed as a percentage of the pole pitch is given in the accompanying table.

Per cent pitch	k_3
100	1.00
83	0.97
80	0.95
75	0.93
67	0.87
50	0.71

Leakage Factor (ν). — The leakage factor, i.e., the ratio of the total flux produced by the field to the flux which enters the armature, may be determined by calculating the permeance of the path of the armature flux and the permeance of the various leakage paths. The sum of the permeances of the main and leakage paths, divided by the permeance of the main path, is then the value of ν.

Referring to Fig. 3, the average radial depth of the air gap is approximately 1.25 g, and the reluctance of the gap is therefore 1.25 g/af. The reluctance to the useful flux of the iron part of the magnetic circuit is about 20 per cent of that of the gap in 60-cycle machines, and 40 per cent in 25-cycle machines. Hence the approximate value of the permeance of the path of the main flux is

$$P_0 = \frac{af}{1.5\,g} \text{ for 60 cycles, and } P_0 = \frac{af}{1.75\,g} \text{ for 25 cycles.}$$

The flux emanating from or entering each side of a pole has a path l_1 inches long and cf square inches in cross section. The permeance of this path to the plane midway between a pair of poles is $2\,cf/l_1$. There are two of these paths in multiple, one in each direction, from opposite sides of each pole. The total permeance of this path per pole is therefore $4\,cf/l_1$. If a uniform m.m.f. acted on this path at all points the flux would be proportional to this permeance, but since the m.m.f. varies from 0 at the yoke to the full m.m.f. per pole at the pole shoe, the average m.m.f. is one-half the m.m.f. per pole. The leakage flux through this path is therefore proportional to the m.m.f. per pole and to one-half this permeance. Hence the "effective permeance" of this path is

$$P_1 = \frac{2\,cf}{l_1}.$$

The same reasoning applies to the flux which leaks out from the end surface bc (see Fig. 3) of the poles, giving as the effective permeance of this path

$$P_2 = \frac{2\,bc}{l_1 + \frac{b}{2}} = \frac{2\,bc}{l_2}.$$

The leakage between the pole tips is due to the total m.m.f. per pole, and the permeance of this path is therefore

$$P_3 = \frac{4\,c_2 f}{l_3}.$$

The leakage from the faces of the poles at the chamfer is also due to the total m.m.f., and the permeance of this path is

$$P_4 = \frac{4\,c_3 f}{l_4}.$$

The leakage factor is then

$$\nu = 1 + \frac{P_1 + P_2 + P_3 + P_4}{P_0}. \qquad (3)$$

Ampere Turns. — The ampere turns required to produce the useful flux ϕ may now be calculated by the following systematized procedure. The symbols are: ϕ = useful flux per pole; ν = leakage factor; T = pole pitch; l = effective length of armature iron; s = pitch of slots at gap; D = diameter of armature at gap; D_1 = outside diameter of armature iron. Other dimensions as shown in Fig. 3. All dimensions are in inches.

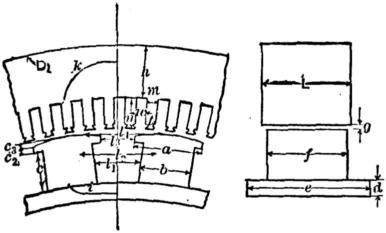

Fig. 3. Dimensions of Magnetic Circuit

Prepare a table like the following:

Part	Flux	Cross section, A	Flux density = flux/A	Ampere turns per inch, m *	Length of path, λ	Total ampere turns $= m\lambda$
Field yoke....	$0.5\,\nu\phi$	de	......	......	i	......
Field pole.....	$\nu\phi$	$0.95\,bf$	......	......	c	......
Air gap........	ϕ	af	......	......	See below	......
Arm. teeth....	ϕ	See below	......	......	n	......
Arm. core.....	$0.5\,\phi$	hl	......	......	k	......

* The value of m is found from Fig. 4 when the flux density has been calculated.

Field Yoke. — The field yoke carries only half as much flux as the pole piece, as the flux divides at this point. The material is usually cast iron or cast steel. Find the magnetic density as indicated and refer to the proper magnetization curve to find the ampere-turns magnetizing force per inch for this density (see Fig. 4). The length of path is one-half the distance from the center of one pole to the center of the adjacent pole and is shown by i in Fig. 3. Find the total ampere turns as indicated.

Field Pole or Magnet Core. — This carries all the flux. The material is usually sheet steel of high permeability, but sometimes solid steel (formerly). The factor 0.95 is used for laminated steel poles; for solid poles this factor is of course unity. The length of this path is usually taken as c, the length of the space for the field spool. This is not strictly accurate, but the density in the pole shoe (c_2) is so low that the excitation required for this part is negligible.

Air Gap. — All the flux in the pole piece does not cross the gap, as some leaks across the interpolar space. The value of the flux in the gap is taken the same as the useful flux in the armature. For the area of gap section the cross

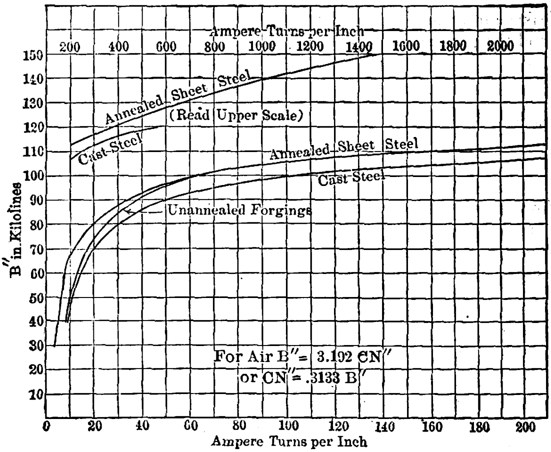

Fig. 4. Typical Saturation Curves

section (*af*) of the pole shoe is taken. The length of the path in the air gap is taken as the mean of the gap length. As the maximum length (at the pole tips) is usually made twice the minimum, and the outline of the pole face is made the arc of a circle, the average length is usually 1.25 times the minimum. The ampere turns per inch are 0.313 × (density per square inch).

The total ampere turns calculated as above indicated is sometimes multiplied by a constant (0.9 to 1.1), to allow for the spreading of the flux into the slots.

Armature Teeth. — The flux is confined at any one time to a certain portion of the teeth per pole known as "teeth under one pole." Since the flux spreads somewhat on leaving the pole piece it is logical to assume that it takes up a peripheral length equal to the pole arc plus twice the length of the air gap. If, therefore, this length is divided by the pitch of teeth at the periphery of the armature, the quotient is the average number of teeth carrying flux at any given instant. This figure may quite properly contain a fractional number of teeth. The teeth being wedge shaped or sectors of a circle, that cross section (not the mean cross section) which will give the average excitation must be chosen, since saturation increases more rapidly than the cross section decreases. A good approximate value is found at a point one-third the distance from the minimum width towards the maximum width.

The effective cross section of the teeth is then equal to this width multiplied by the product of the effective length of the core by the number of the "teeth under one pole." The effective length of core is the net length of iron in the core after deducting the space occupied by air ducts and insulation between sheets of steel. This latter is usually 10 per cent of the measurable length of iron. The effective length (l) = 0.9 × (total length of armature core less space occupied by ducts).

Armature Core. — The flux divides again in the armature core, one-half the useful flux being in each section of the armature core. The core is made of annealed sheet punchings. The cross section of core is equal to the radial depth of core back of the slots multiplied by the effective length of iron in the core. The length of path is a little greater than one-half the pitch of poles at this radius, and is as shown at (k), Fig. 3.

The total ampere turns per pole as thus calculated, corresponding to the chosen value of the voltage per phase give one point on the magnetization curve. As noted above, a sufficient number of points should be calculated in the same manner to enable one to extend the saturation curve up to about 120 per cent of rated voltage per phase.

Armature Resistance. — The length of wire in the armature winding is estimated from the mean length of one turn and the number of turns. Let L = total length of armature core in inches; γ = per cent pitch of coils; D = diameter of armature at gap; p = number of poles; S = number of turns in series per phase; a = cross section of conductor* (wire or strip); m = number of parallel paths per phase; k = 10 for low-voltage machines, and 12 for high-voltage machines (k allows for the curves in the ends of the coils). Then the mean length of turn is

$$l = 2L + \frac{k\gamma D}{p},$$

and the resistance *per phase* is

$$R = \frac{0.0093\, lS}{12{,}000\, am},$$

where 0.0093 is the resistance at 60° C. of 1000 feet of conductor having a cross section of 1 square inch. 60° C. is the approximate temperature of the armature conductors at full load.

This is the resistance per phase to a direct current. The alternating-current resistance, due to eddy currents and hysteresis, is about 15 per cent greater; see paragraph on *Load Loss*, below.

In a single-phase or two-phase machine this resistance is the same as the resistance between terminals. In a Y-connected three-phase machine the resistance between terminals is twice the resistance per phase. In a Δ-connected three-phase machine the resistance between terminals is two-thirds of the resistance per phase.

Armature Leakage Reactance. — The load current in flowing through the armature conductors sets up a local magnetic flux which interlinks with the armature conductors, producing inductance. This inductance causes a loss of voltage and a "dephasing" effect, or lag of current behind the e.m.f.

The inductance L of any circuit, expressed in henries, is

$$L = 1.016\pi S^2 P \times 10^{-8},$$

where S = number of turns in series,
P = permeance of flux path in inches.

The corresponding reactance is $2\pi fL$, where f is the frequency in cycles per second.

There are several paths for this so-called armature leakage flux, each path surrounding one or more slots, namely, the path around each slot, the path around a group of slots of one phase, the path around the end connections. The effect of the flux of one phase on the conductors of another (mutual inductance) must also be considered.

The slots lying under a pole have a path of greater permeance than those lying opposite the interpolar space or between poles, as the former are more completely surrounded by iron.

* If pressed cable is used, the cross section of the copper in the cable is approximately 82 per cent of the cross section of the cable, exclusive of insulation. The length of each of the wires forming the cable is about 7 per cent greater than the length of the cable.

With the exception of the path around the end connections, the permeance of each path is readily calculated by the same process as used above in calculating the leakage factor. The path around the end connections is so complex and the permeance so small compared with that in the core proper, that it is convenient and sufficiently accurate to add a percentage for this flux. This is done by allowing for every inch of the projecting length of the end connections one-tenth as much flux or inductance as for an inch of the embedded portion of the conductors.

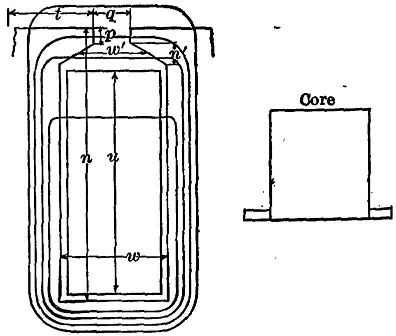

Fig. 5. Leakage Flux

In calculating the permeance of the various paths only the length of the path in air is considered, as the reluctance of the path in iron is so small compared to the path in air as to be negligible.

Let l = effective length of armature core; l' = length of end connections (one end); P_1 = the total permeance of all the leakage paths for a slot under a pole piece; P_2 = the total permeance of all the leakage paths for a slot midway between two poles, and the other quantities as in Fig. 5. Then

$$P_1 = \left(\frac{u}{3w} + \frac{n'}{w'} + \frac{p}{q} + \frac{t}{2g}\right)\left(l + 0.1\, l'\right)$$

$$P_2 = \left(\frac{u}{3w} + \frac{n'}{w'} + \frac{p}{q} + \frac{t}{t+q}\right)\left(l + 0.1\, l'\right)$$

In addition to the above symbols, let f = frequency in cycles per second; p' = the slots per pole per phase; s' = slots in series per phase; c = effective conductors per slot = $2 \times$ (turns per phase)/slots per phase; k_3 = "pitch constant" (*see above*). Then the effective reactance in ohms per phase corresponding to the permeance P_1 is

$$x_1 = 20.1\, f p' c^2 s' k_3 P_1 \times 10^{-8},$$

and corresponding to the permeance P_2 is

$$x_2 = 20.1\, f p' c^2 s' k_3 P_2 \times 10^{-8}.$$

Reactance Drop. — For preliminary or approximate calculations it is better and more conservative to use the value found for the inductance of the slots "under the poles," as this gives a greater reactance and voltage loss, and, as the poles usually cover about two-thirds of the armature periphery, this is nearer correct. That is, the armature reactance drop per phase is taken as Ix_1, where I is the armature current per phase.

For more accurate calculations both values must be used, and the power component and reactive component of the current considered separately. Let θ be the power-factor angle and I the armature current per phase. Then the reactive drop per phase is

$$IX = I\sqrt{(x_1 \cos\theta)^2 + (x_2 \sin\theta)^2},$$

which is therefore equivalent to taking for the effective armature reactance

$$X = \sqrt{(x_1 \cos\theta)^2 + (x_2 \sin\theta)^2}.$$

Armature Reaction. — When current flows through the armature the armature winding becomes the seat of a magnetomotive force which reacts on the field m.m.f. and either distorts or diminishes the useful flux. If the current in the armature is in phase with the generated e.m.f. it causes a "cross magnetizing" force acting along an axis passing midway between any pair of poles.

As the phase of the current in the armature changes, the direction of the magnetizing force due to the armature m.m.f. shifts, and a component is introduced either opposed to the field magnetizing force (for lagging current) or assisting the field magnetizing force (for leading current). In a polyphase generator the magnetizing force due to the armature m.m.f. for a given armature current is constant in magnitude and has a fixed direction with respect to the field magnetizing force, depending on the phase of the current.

If the field iron had no polar projections nor interpolar spaces, the direction of the armature magnetizing force would be such that the angle between this magnetizing force and a line perpendicular to the field magnetizing force would be equal to the phase angle between the induced voltage and current.

With the usual type of alternator having interpolar spaces the reluctance of the path offered to the armature m.m.f. is intentionally much greater than that offered to the field m.m.f. The result of this is that the effect of armature reaction is minimized. The non-uniformity of the field iron, however, changes the relative directions of the two magnetizing forces and renders accurate calculations difficult. As a rule, however, the approximation resulting from the assumption of uniform distribution of field iron is sufficiently accurate. The error introduced by this assumption is on the safe side, since the armature reaction as thus calculated is greater than its actual value.

Armature-Reaction Ampere Turns per Pole. — Let S = number of turns in series per phase; I = effective value of armature current per phase; p = number of poles, and k_3 = pitch constant of the winding (*see above under Magnetization Curve*). Then the armature ampere turns per pole effective in producing armature reaction are

$$\frac{\sqrt{2}\,k_3SI}{p}, \qquad \text{for two-phase machine,}$$

$$\frac{1.5\sqrt{2}\,k_3SI}{p}, \qquad \text{for three-phase machine.}$$

As noted above the armature reaction for a given effective value of the current is constant in magnitude. The armature reaction in single-phase generators is given in the discussion of these machines below.

The armature-reaction ampere turns of different machines at full rated load is greater for a high than for low pole pitch, and is less for high frequencies than for low frequencies. The permissible armature-reaction ampere turns depend, of course, upon the desired regulation.

A reasonable value for armature reaction would be between 1500 and 5000 ampere turns per pole for a 25 cycle machine and between 1000 and 2000 for a 60 cycle machine.

Synchronous Reactance. — The effect of the armature leakage reactance and armature reaction upon the terminal voltage of a generator at given field excitation are of like character, since the voltage induced in the armature due to each of these causes is in quadrature with the current. The "synchronous reactance" of a generator is the equivalent reactance which would produce the same effect as the armature leakage reactance and armature reaction combined.

The synchronous reactance may be predetermined by finding the excitation ampere-turns (from magnetization curve) corresponding to a voltage equal to

the drop due to leakage reactance and adding to these ampere turns the armature reaction ampere turns. The voltage from the magnetization curve corresponding to this sum divided by the armature current per phase gives the synchronous reactance per phase. In the calculation of regulation by the "magnetomotive force method" (see below), however, it is more convenient to express the synchronous reactance in terms of the excitation ampere turns to overcome it.

Regulation. — The regulation of a generator machine is defined as the ratio of the difference in terminal voltage at no load and at full load to the full-load voltage, the field excitation being kept constant at its full-load value. Expressed as a percentage the regulation is 100 $(V_0 - V) \div V$ where V is the full-load terminal voltage and V_0 the no-load terminal voltage at full-load field excitation.

Two different methods have been employed for the calculation of regulation, one known as the "magnetomotive-force" method, and the other as the "electromotive-force" method. Both are approximations, the first giving a value lower than the actual value, and the second a value higher than the actual value. More accurate methods have recently (1914) been recommended by the American Institute of Electrical Engineers (*see Standardization Rules*).

Magnetomotive-force Method. — One way in which this method has been applied is the following: Let V = terminal voltage per phase at full load; R = alternating-current resistance per phase in ohms (= 1.15 times direct-current resistance); I = full load amperes per phase; $\cos\theta$ = power factor of the load; X = leakage reactance per phase in ohms. Calculate

$$V' = \sqrt{(V\cos\theta + RI)^2 + (V\sin\theta + XI)^2} \quad \text{and} \quad \theta' = \tan^{-1}\left[\frac{V\sin\theta + XI}{V\cos\theta + RI}\right]$$

taking θ' positive for I lagging behind V'. From magnetization curve find m = excitation ampere turns corresponding to V' and let n = armature reaction ampere turns per pole for current I (see p. 635). The total field ampere-turns is then

$$F = \sqrt{m^2 + n^2 + 2\,mn\sin\theta'}.$$

Let V_0 = voltage per phase from the magnetization curve corresponding to this excitation F; then the regulation is

$$100\,\frac{V_0 - V}{V}.$$

The difference between V' and V and between θ' and θ is usually quite small, and in preliminary calculations m may be taken as the excitation ampere turns corresponding to V, and θ' may be taken equal to θ.

Electromotive-force Method. — Let V = terminal voltage per phase at full load; R = alternating-current resistance per phase in ohms (see preceding paragraph); XI = volts per phase from magnetization curve corresponding to the field excitation required to send full-load current through armature on synchronous impedance test; I = full-load amperes per phase; $\cos\theta$ = power factor. Then the voltage per phase E at no load, corresponding to the full-load excitation, is the vector sum of V, RI and XI, or

$$E = \sqrt{(V\cos\theta + RI)^2 + (V\sin\theta + XI)^2}.$$

The regulation is, as before, 100 $(E - V)/V$, and the full-load field ampere turns F is taken from the point corresponding to E on the magnetization curve.

Losses. — The losses in a synchronous alternating-current generator or motor are:

(a) Friction, bearing and windage.
(b) Excitation or field copper loss.
(c) Core loss.
(d) Armature copper loss.
(e) Load loss.

Of these the first three are approximately constant for various loads, but the last two vary as the square of the load current.

Friction and Windage depend in magnitude upon the details of the physical or mechanical construction, the amount of induced ventilation and speed. It is impossible to give a method of predetermining this quantity which will apply to various designs and makes. Each manufacturer has an empirical formula for each line of machines. This loss ranges from 2.5 per cent in 100 kv-a. machines to 0.5 per cent in 10,000 kv-a. machines. These values are for complete machines with their own bearings (two in number). Some machines are designed with only one bearing, the other bearing being a part of the prime mover (hydraulic or steam) in which case the friction chargeable to the generator is less. Some machines with devices to produce ventilation, such as fan blades attached to the revolving part, have greater friction losses.

Core Loss. — The core loss is made up of hysteresis and eddy-current losses. These losses are principally in the armature core and teeth, but if proper care is not taken there may be a considerable loss in the frame of the machine and the pole shoes.

It is a simple matter to calculate the magnitude of these losses in the armature core proper, as the frequency and flux density are definite in this part, but the losses in the teeth are due not only to the fundamental frequency and main flux, but also to pulsations due to the passage of pole tips past the teeth and the leakage flux of the armature. The total core loss is the sum of the hysteresis and eddy-current losses. See *Magnetic Properties of Iron and Other Metals* for curves of hysteresis and eddy-current losses and formulas for their calculation.

Excitation Loss. — The calculation of the field current and of the resistance per pole of the field winding is given above in the section on *Field Winding.* The total power required for excitation will be the product of this resistance, the square of the current and the number of poles.

If the machine is separately excited, which is usually the case, the losses in the field rheostat are, by convention of the Am. Inst. of E. E., not chargeable against the generator.

Armature Copper Loss. — The calculation of the direct-current resistance per phase of the armature winding is given above in the section on *Armature Resistance.* The total armature copper loss is equal to the number of phases times this direct-current resistance times the square of the current per phase.

Load Loss. — When a current flows in the armature conductors a local flux is set up which will cause eddy currents in these conductors, if they are not well subdivided, and in the surrounding iron, as well as a hysteresis loss in the surrounding iron. The loss due to this load flux is called the "load loss."

The load loss is a function of the leakage flux and the subdivision of the conductors. A large number of turns of fine wire will involve a very small loss. If the conductors must be large, they may be made of stranded cable pressed to shape. It is almost impossible to calculate this loss, and very difficult to measure it. A rough method is to assume the resistance per phase increased by 15 per cent, as this loss, like the true copper loss, is proportional to the square of the armature current. The total amount of the loss is less than 1 per cent of the input in well-constructed machines.

Efficiency. — Let P = total output in kilowatts; R_a = resistance per phase of armature; I_a = armature amperes per phase; R_f = resistance per pole of field winding; I_f = field current; q = number of phases; p = number of poles; C = total core loss in kilowatts; and F = friction and windage loss in kilowatts. Then the per-cent efficiency is

$$\frac{100\,P}{P+C+F+(1.15\,R_a q I_a^2+pR_f I_f^2)\times 10^{-3}}.$$

This assumes the load loss equivalent to increasing the armature resistance (as calculated or measured by direct current) by 15 per cent. If the load loss is determined from the short-circuit core loss the formula for efficiency is

$$\frac{100\,P}{P+C+F+L+(R_a q I_a^2+pR_f I_f^2)\times 10^{-3}},$$

where L is the load loss in kilowatts.

The efficiency is a maximum for that load at which the constant losses are equal to the variable losses.

Customary values for the efficiency at full load and each of the losses at full load for various sizes of generators are given in the following table. These values are merely indications and vary with the frequency, voltage, speed, power factor, etc. A 60-cycle low-voltage machine will be likely to have a better efficiency than a machine of the same rating for 25 cycles or high voltage.

EFFICIENCY AND LOSSES, USUAL VALUES

Rating, kv-a.	Efficiency, per cent	Friction, per cent	Excitation, per cent	Core, per cent	Armature,* per cent
100	91	2.5	2.5	2.4	1.6
500	94	1.4	1.5	2.2	1.2
1,000	95	0.9	0.9	2.1	1.0
2,000	96	0.6	0.7	1.8	0.9
3,000	96.5	0.6	0.6	1.7	0.8
5,000	97	0.55	0.4	1.6	0.5
10,000	97.2	0.5	0.35	1.5	0.45

* Copper and load loss.

Heating. — The rise in temperature of the field coils is ascertained as an incidental step in the calculation of the field winding as explained above.

The rise in temperature of the armature is best determined by a method given by Arnold (*see Wechselstromteknik, Vol. IV, p. 141*). In this calculation the losses in the projecting portions of the end windings are assumed to be radiated by the end windings while the core loss and copper loss in that portion of the winding embedded in the slots are radiated by the surface of the armature core. Let L = length of armature iron in inches; λ = mean length of armature turn in inches (*see Armature Resistance, below*); R_a = armature resistance per phase in ohms; I_a = armature amperes per phase; D_1 = outside diameter of armature punchings in inches; D = inside diameter of armature punchings; z = number of ventilating ducts; q = number of phases; t = rise in temperature in degrees centigrade per watt radiated per square inch of surface. Then the watts to be radiated are

$$P = \text{core loss} + \frac{2\,LqR_aI_a^2}{\lambda},$$

and the radiating surface, including only one-half the area in the air ducts, as the sides of the air ducts are not as effective as the rest of the surface, is

$$A = \pi L (D_1 + D) + \frac{\pi}{4} (D_1^2 - D^2)(2 + z).$$

The rise in temperature by resistance in degrees centigrade is then

$$T = \frac{Pt}{A}.$$

The value of t for a well-ventilated stationary armature ranges from 30 to 40, depending upon the thickness of coil insulation, and the peripheral speed of the field. For a stationary field and revolving armature the value of t given above for the rise in a revolving field may be used.

SINGLE-PHASE GENERATOR. — Single-phase generators do not make as effective use of the material as do polyphase generators, for the reason that the armature winding can occupy effectively only about one-half of the peripheral surface of the armature; if it occupies more than this there will be voltages generated in the windings which are so out of phase with each other that the resultant voltage is only from 63 to 70 per cent of the sum of all the voltages generated.

Therefore, if a polyphase generator is used as a single-phase machine with the same magnetic densities and the same copper densities, the output will be much less on account of the lesser voltage available. It is, therefore, customary to overload the magnetic elements of the machine somewhat to raise the voltage and thus reduce the overload on the copper which would be necessary to get the desired output. However, if both the iron and copper densities are increased until the machine gives as much output single phase as it is intended to give polyphase, there will be an increased heating. Thus, for the same heating, obtained by a readjustment of the iron and copper losses, a machine of a given first cost will give about 75 per cent as much output single phase as may be obtained polyphase.

In addition to the disadvantage of a single-phase generator that it cannot make use of all the periphery of the armature, it also labors under the disadvantage that its armature reaction is pulsating instead of constant, and this introduces an additional loss in the form of eddy currents.

By using two phases in series of a three-phase machine, or one phase of a two-phase machine, a fairly good single-phase machine is obtained. By arranging the winding slightly differently the same number and arrangement of inductors (or coil sides) can be connected up to give a simpler winding requiring no crossings of coils, that is, a winding "in one plane."

The formula for the calculation of the e.m.f. of a single-phase generator is $E = 4.44\ k_2k_3fS\phi\ 10^{-8}$, where the symbols, with the exception of k_2, have the same significance as in the formula for polyphase machines, p. 629. The value of k_2 depends upon the portion of the armature periphery (including the teeth between slots) occupied by the main winding. Let A = this fraction of the armature surface, then the corresponding values of k_2 are given in the accompanying table. The value of A corresponding to a uniformly distributed winding is unity.

A	k_2
1	0.63
¾	0.785
⅔	0.83
½	0.90
⅓	0.956

The average value of the armature-reaction ampere turns per pole of a single-phase machine is SI/p, where S = the number of turns, I = the

armature current per phase, and p = the number of poles, but this quantity pulsates between 0 and twice the above value. The evil effects of this pulsating may be reduced by employing a short-circuited winding having its axis at 90 degrees to the main field winding. A squirrel cage or "amortisseur" winding in the pole faces is frequently employed for this purpose.

Due to this pulsating action the load losses are greater and this should be taken into account in calculating the efficiency and regulation by considering the effective armature resistance as 1.15 to 1.5 times the direct-current resistance.

The leakage reactance of a single-phase machine pulsates between a value equal to that obtained by the formula above in the paragraph on *Armature Leakage Reactance*, and a value two-thirds as great. Satisfactory results are obtained by multiplying the value obtained from the formula by 0.85 for the effective single-phase value. In this case the whole winding is considered as one phase.

CHECKING CALCULATIONS.—Substitution in the following formula gives an excellent check on the above calculations:

$$\frac{DL_f}{Q} = \frac{KA \times 10^9}{fB_g nak_2k_3},$$

where D = diameter of armature at gap in inches; L_f = length of pole face parallel to shaft; Q = total kv-a. of generator; f = frequency in cycles per second; n = armature-reaction ampere turns; a = pole arc in inches; k_2 and k_3 = constants (given in the tables on pp. 629 and 639); A = diameter per pole in inches; B_g = flux density, lines per sq. in. in air gap; and K = 22.5 for single-phase and 15.9 for two-phase or three-phase generators.

TESTS OF ALTERNATING-CURRENT GENERATORS.—(*See also Standardization Rules of the A.I.E.E.*) The principal tests are

- Magnetization or saturation test;
- Core-loss and friction tests;
- Synchronous-impedance test;
- Load-loss test;
- Resistance measurements;
- Heat runs;
- Insulation tests.

Examples of test results are given below; typical test curves are given in Fig. 6.

Magnetization Curve.—The magnetization curve, popularly named the "no-load saturation curve," shows the relation between the no-load voltage and the current in the field. Fig. 7 shows the connections for making this test. On account of the existence of the hysteresis loop (*see article on Magnetic Properties of Iron*), it is necessary, in running this test, to increase the field current gradually from point to point and never reduce the value at any step until the highest excitation has been obtained. The curve differs in shape from the magnetization curve of a closed sample of iron because the magnetic circuit of the alternator contains a considerable air gap, the magnetization curve of which is a straight line. Therefore, if the saturation curve of a machine contains a portion which is very straight, the indications are that the air gap is of considerable magnitude. If the machine is operated at very high magnetic densities, this is indicated by the fact that the point corresponding to rated voltage is found at a point on the curve where it is nearly horizontal.

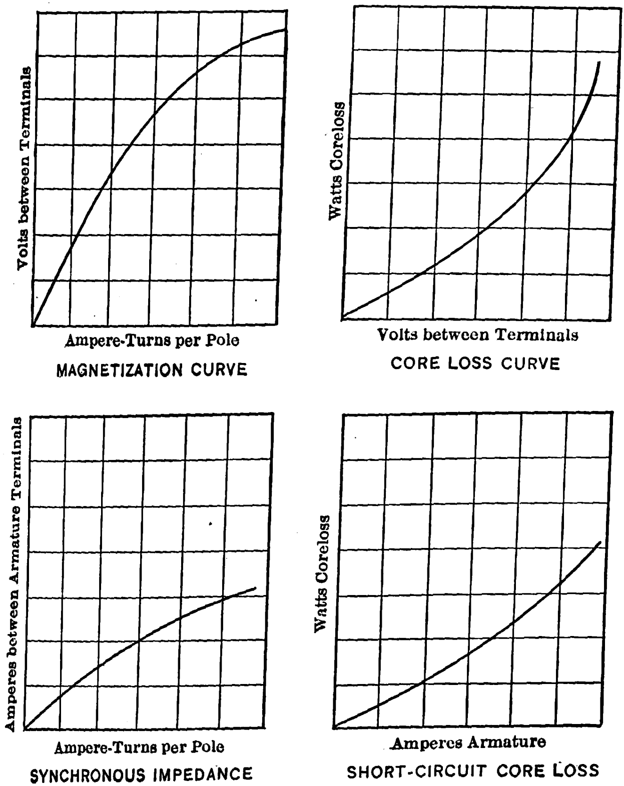

Fig. 6. Typical Test Curves

Some machines obtain good regulation by operating at high saturation, as this is a cheaper method than by using a low value of armature reaction and leakage reactance.

The magnetization curve is usually plotted in terms of the volts between terminals. In comparing the observed and calculated magnetization curve it should be noted that the calculations are expressed in terms of the volts per phase.

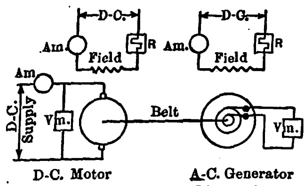

Fig. 7. Connections for Magnetization Curve, Core Loss and Friction Tests

Core Loss and Friction. — The open-circuit, or true core loss curve shows the core loss in watts for each value of the no-load terminal voltage. It is made by driving the generator at rated speed by means of a small motor, the efficiency of which is known, and varying the generator excitation. Fig. 7 shows the connections for this test. The voltage and mechanical power *P*

(= input to motor multiplied by its efficiency) required for each value of excitation are noted. The power P_0 for zero excitation is the friction loss in the machine. The core loss for any excitation is $P - P_0$. The no-load saturation curve can be made at the same time as the core-loss test.

Synchronous Impedance. — The synchronous-impedance curve shows the relation between various values of field current, or excitation ampere turns, and the current that flows in the armature on short circuit. It is made by short circuiting the armature through ammeters and operating at full frequency with various values of field current. Fig. 8 shows the connections for this test. Of course only fractional values of normal excitation are used, or the current in the armature would be so great as to cause damage.

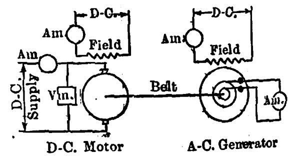

Fig. 8. Connections for Synchronous Impedance and Load-loss Tests

The synchronous-impedance curve gives approximately the ampere turns corresponding to armature reaction and the leakage reactance drop in the armature combined, i.e., the ampere turns corresponding to the synchronous reactance. The approximation arises from the effect of the armature resistance and the low saturation or magnetization of the magnetic circuit under the short-circuit conditions.

Load Loss. — The power dissipated in the core and armature conductors under short-circuit conditions, called the "short-circuit core loss," may be made at the same time as the preceding test, by measuring the power required to drive the machine under the same condition, proper allowance being made for friction. From this test the load losses for normal load conditions are assumed, as recommended in the *Standardization Rules of the A.I.E.E., 1911 Edition,* to be, for a given armature current, one-third of the value found with the same current in the short-circuit test. This is reasonable, since the armature current is considerably out of phase, which condition magnifies the losses. Another approximation to the load loss is to assume the "effective" resistance of the armature to be 1.15 times the d-c. resistance. In the standardization rules of the *A.I.E.E.* issued in 1914 the allowance for load loss (called the "stray load loss") is differently treated; see *Standardization Rules of the A.I.E.E.*

Resistance Measurements must be made when all parts of the machine are at some known temperature. They are also made after the heat run, when the machine is hot, to check the temperature as measured by thermometers (*see article on Resistance and Conductance*).

Field Resistance is measured by the simple voltmeter-ammeter method (*see article on Resistance and Conductance*).

Armature Resistance. — The armature resistance between terminals is also measured by the simple voltmeter-ammeter method, by connecting two of the terminals to a source of direct current (the other terminal being free), the rotor (field or armature) of course being at rest. In the case of a single-phase or two-phase machine the resistance as thus measured is the resistance per phase. In the case of a three-phase machine, let R_t = the resistance as thus measured and R_p = the resistance per phase. Then for a Y-connected armature $R_p = R_t \div 2$, and for a Δ-connected armature $R_p = 3\,R_t \div 2$. If the connection is not known, the calculation of the resistance drop and copper loss may be figured correctly assuming it either Y or Δ connected, provided the resistance per phase and current per phase are both calculated on the same assumption regarding the connection. In the case of a two-phase machine the

resistance between terminals A and B and between B and C (B being the common terminal) should each be measured, and the average taken. In the case of a three-phase machine the resistance between A and B, B and C, and A and C should be measured and the average taken.

Calculation of Regulation and Efficiency from Tests. — From the results of the preceding observations, the regulation and efficiency of the machine at various loads may be calculated by the methods given above, in the discussion of *Predetermination of Performance*, using the test data instead of the calculated quantities. The synchronous-reactance ampere turns may be taken as equal to the synchronous-impedance ampere turns, since the resistance drop in the armature on short circuit and reduced excitation is seldom over 10 per cent of the synchronous-reactance drop, and as the two are in quadrature the synchronous-impedance drop differs from the synchronous reactance drop by less than one per cent. In using the magnetization curve and the synchronous-impedance curve one must keep in mind whether they are plotted in terms of voltage and current per phase or in terms of volts between lines and line current.

Full-load Saturation Curve. — This curve shows the relation between the terminal voltage and field current with full-load current in the armature. It may be plotted from tests or calculated from the magnetization curve and the synchronous impedance in the same manner as the regulation is calculated.

Armature Leakage Reactance. — There are three methods by which the leakage reactance may be determined:

(1) **Synchronous-impedance Method.** — This gives only approximate results but is very generally used as a synchronous-impedance test is made on every generator. Let n be the field ampere turns per pole required to force full-load current through the short-circuited armature. Let a be the calculated armature-reaction ampere turns per pole (see above), with rated current. Then $n - a$ is the excitation necessary to induce the leakage reactance voltage (IX) in the armature. From the open-circuit saturation curve find the voltage corresponding to $(n - a)$ ampere turns. Reducing to volts per phase, if necessary, and dividing by the current per phase, the result is the leakage reactance X per phase in ohms.

(2) **Full-load Characteristic Method.** — This method is also approximate. Let the excitation in ampere turns per pole required to give rated voltage at full non-inductive load be q. Let the ampere turns to give rated voltage without load be m. Then $n = \sqrt{q^2 - m^2}$ is taken as the synchronous-reactance ampere turns, and the leakage reactance is calculated as described in the preceding paragraph.

(3) **Inductance-measurement Method.** — This method gives exact values, but is not very often employed. From an external source of proper frequency, full-load current is sent through the armature of the machine to be tested, with the fields excited to the normal value but not rotating. The voltage drop across the armature terminals is measured for several different positions of the coils with respect to the poles, ranging through an arc of about one-half the pole pitch. Let Z = the voltage per phase divided by the current, R = the resistance per phase corrected for load loss (i.e., 1.15 times the direct-current resistance), then the leakage reactance per phase is $X = \sqrt{Z^2 - R^2}$. This reactance will vary with the position of the coils and the maximum and minimum may be found by making this calculation for the different positions of the coils with respect to the fields.

Heat runs are made at rated load and various other specified loads. The parts in which the rise in temperature is of interest are:

Armature core surface;
Armature core ventilating ducts;
Armature conductors;
Collector rings;
Both pole tips;
Field winding;
Bearings;
Frame;
Room.

The temperature of the field and armature winding should be measured both by thermometers and by the resistance method. In taking the temperature of a hot surface by thermometer a small pad of waste should be placed over the bulb after the thermometer is put in place. The pad should not be too large or it will prevent the normal radiation from the surface.

With large machines it is inconvenient and expensive to test under full-load conditions. For determining the heating without actually developing the full power of the machine there are several methods available.

Reversed-field Method. — The field circuit may be tapped and the full-load field current sent through a portion of the field coils in a direction opposed to that for normal operation. For example, in a 24-pole machine the current in 8 of the field coils may be directed in such a manner that they are reversed with respect to the remaining 16. The armature winding will therefore generate a voltage due to the 8 poles that are not neutralized and therefore of approximately one-third rated value. If the armature is short circuited a current will flow due to this reduced voltage, and this current can be made to approximate closely the full-load value by making a proper division of the poles. If the machine is operated under these conditions the friction, excitation, core loss and armature copper loss will be very nearly equal to their value under normal operating conditions, yet to drive the machine only an amount of power approximately equal to the sum of the losses is required.

Synchronous-motor Loading. — If two machines are available they may be connected up as a generator and synchronous motor and the field excitation of the motor adjusted so that a current of full-load value, but having a large reactive component, will flow in the armature, the power factor being nearly zero. The power required to drive the generator at full-load excitation and at full-load current will then be small.

Direct-current Loading. — Full-load losses may be simulated on a three-phase generator by connecting the three phases in delta and leaving the delta open at one point, to which a direct-current ammeter and a source of direct current may be connected in series. The delta is first closed through an alternating-current ammeter and the triple-frequency current in the delta (*see Alternating Currents*) is measured. The direct-current ammeter and source of direct current are then connected in series with the delta and the direct current increased until the sum of the squares of the local current of triple frequency and the direct current is equal to the square of the rated current per phase.

Open-circuit Short-circuit Method. — Another ingenious method, advocated by Hobart, consists in alternating open-circuit and short-circuit tests, each under exaggerated conditions, so that at the end of each hour the total watt hours lost in each part are equal to the watt hours that would be lost in normal operation. For example, let the core and field copper loss be 4 kilowatts, and armature copper loss 1 kilowatt. If, now, the machine is operated for 20 minutes with armature short circuited and the excitation adjusted to give 3 kilowatts copper loss, and for 40 minutes on open circuit with excitation

adjusted to give 6 kilowatts core and field copper loss, then the loss per hour in the armature copper is 1 kw.- hr., and in the field winding and core 4 kw.- hr., the same as under normal conditions. For an exact division of the time between the open-circuit and short-circuit conditions, the core and field copper loss during the short-circuit run must be taken into consideration. After several hours of this relaying the machine will have reached a temperature corresponding to full-load operation.

Insulation Tests. — The insulation of a machine is usually tested by applying a given voltage between the conductors and the part of the machine from which they are insulated. An insulation resistance test is also made.

Voltage Tests. — During manufacture the coils are tested separately, and after the machine is assembled a voltage test between the completed windings and frame is made. After the heat run, while the machine is still warm, a third test of the insulation between windings and frame is made.

For this third test an alternating voltage is applied between armature and frame, according to the *Standardization Rules of the A.I.E.E.* (q.v.)

The voltage should be increased gradually and the final value should be applied for one minute. In this test the machine acts as a condenser and therefore care should be taken that the frequency is not so high or the inductance of the supply circuit so great as to set up resonance. It is advisable to have considerable resistance in the supply circuit.

To prevent an uneven distribution of voltage, all the terminals of the winding should be connected together by fine wires and one terminal of the high-potential circuit connected to the common connection. The high potential should be measured by a spark gap connected in shunt to the machine.

Insulation Resistance. — The insulation resistance is measured by connecting one terminal of a 500-volt direct-current supply to the windings of the machine through a voltmeter with a 500 scale, and connecting the other 500-volt terminal to the frame. If the resistance of the voltmeter is R_v ohms and the voltmeter deflection x, the insulation resistance is $R = \frac{R_v}{x}(500 - x)$.

EXAMPLES OF DESIGN AND PERFORMANCE. — In the tables on pp. 646 and 647 will be found the essential data both of the mechanical and electrical features of four representative alternators. The list of items will be found useful as a guide in collecting data on various machines. Performance data are deduced from tests.

OPERATION. — In the operation of an alternating-current generator the following factors should be considered:

Phase Connections and Grounding. — When a third harmonic is present in the e.m.f. wave of a three-phase generator, the triple frequency e.m.f.'s in the three phases (or windings) are additive when the three phases are connected in Δ, that is, the Δ forms a short circuit to the third harmonic e.m.f.'s, and a large triple frequency current may therefore be set up in the windings irrespective of the load on the machine. On this account, large three-phase generators are usually Y-connected, since with this connection the third harmonic e.m.f.'s between any two terminals of the machine neutralize each other. However, when two or more Y-connected machines are operated in parallel with their neutrals grounded, a triple frequency cross-current of considerable magnitude may be set up between the machines, unless the wave form of the e.m.f.'s of the various generators are exactly the same, which is practically never the case. To prevent such cross-currents with Y-connected machines with grounded neutral, it is the usual practice to ground the neutral of but one generator at a time. Provision must of course be made to shift this ground connection from one ma-

MECHANICAL DATA ON TYPICAL A-C. GENERATORS

Dimensions in Inches, Weights in Pounds

Type	1 *ATB*	2 *ATB*	3 *ATB*	4 *ATB*
Poles	40	8	54	48
Kv-a. rating	2500	2500	2500	5000
R.p.m.	75	375	133	150
Voltage between terminals	6500	6600	2300	4000
Frequency	25	25	60	60
Connection	Y	Y	Y	Y
Armature diam. at face	200	92	170	192
Armature diam. at back	220	120	180	204
Armature, total length	22	34	20	30
Armature air ducts				
Number	7	7	8	11
Width	½	½	½	½
Slots, number	240	144	648	432
Slots, dimensions	3⅝×1.55	2⅝×0.9	2⅛×0.4	2⅛×⅝
Conductor size	0.485×0.275	0.75×0.17	0.75×3/16	0.6×0.15
Conductors, no. in mult.	2	1	2	4
Conductors, no. per slot	18	4	2	4
Pitch of connection	1	⅔	⅔	1
Air gap, minimum length	0.3125	0.5	0.375	0.313
Air gap, average length	0.39	0.625	0.47	0.415
Field pole arc	10	23	6	8
Field pole, length along shaft	21.5	33.5	19.5	29
Magnet core, width	10	15	4	5.5
Magnet core, radial depth	9	8	7	9
Spool, no. turns	42.5	96.5	33.5	49.5
Spool, size of conductor	1⅝×0.17	2¼×0.06	1.5×0.16	1⅛×0.14
Yoke, length	29	36	24	40
Yoke, radial depth	5	4	4	5
Total weight	209,000	....	....	....

ELECTRICAL DATA ON TYPICAL A-C. GENERATORS

Percentages are all in terms of rated or full-load values

		1 *ATB*	2 *ATB*	3 *ATB*	4 *ATB*
Rating	kv-a.	2500	2500	2500	5000
Rating	kw.	2500	2000	2500	5000
Rated volts per phase	volts	3760	3820	1350	2300
Full-load current per phase	amperes	222	219	628	723
Flux per pole	maxwells	9.5×10^6	43.3×10^6	5.6×10^6	12.6×10^6
Arm. res. per phase at 25° C.	ohms	0.13	0.072	0.018	0.14
Field resistance at 25° C.	ohms	0.31	0.43	0.37	0.84
Excitation amp. turns, no load	amp. turns	8100	12,770	7230	7700
Excitation amp. turns, full load	amp. turns	9660	13,470	7830	8280

ELECTRICAL DATA ON TYPICAL A-C. GENERATORS — *Continued*

Percentages are all in terms of rated or full-load values

		1 ATB	2 ATB	3 ATB	4 ATB
Friction loss	kw.	10	29	14	25
Core loss at rated volts	kw.	42	40	43	125
Syn. imp., volts between term.	volts	5100	3070	1000	1700
Syn. imp., total amp. turns	amp. turns	5100	5300	2800	3070
Short-circuit core loss, full-load current	kw.	....	3.4	5.3	19.2
Leakage reactance drop	per cent	14	23	7.8	7.3
Regulation at full load	per cent	7.85	16.7	5.8	4.75
at power factor of	per cent	100	80	100	100
Friction loss	per cent	0.4	1.4	0.54	0.48
Core loss	per cent	1.62	1.9	1.66	2.4
Field copper loss	per cent	0.62	0.65	0.77	0.45
Armature copper loss	per cent	0.86	0.52	0.90	0.46
Load loss	per cent	....	....	0.20	0.12
Efficiency at full load	per cent	96.5	95.5	96.0	96.1
Rise in temp. by thermometer:					
Armature	°C.	35	35	45	40
Field	°C.	35	35	45	40

chine to any other, so that a ground connection can always be maintained irrespective of which machine or group of machines may be running.

The advantages and disadvantages of grounding the neutral are discussed in the article on *Grounding of Electric Circuits*.

Division of Load between Alternators in Parallel. — The division of load between two or more alternators operating in parallel cannot be changed by altering their field excitation, as is the case with direct-current generators. Changes in the load taken by any alternator of a group can be effected only by admitting more or less steam to the driving engine or turbine (or water to a water-wheel). In order that the various alternators shall share the combined load properly, it is therefore necessary that the governors on the several prime movers give the same speed-load characteristics.

Although the field excitation has no effect on the distribution of the load among the alternators, it does affect the power factor of the load delivered by each machine. The excitation of each alternator should be so adjusted that it delivers its load at the same power factor as the others.

Starting a Single Generator. — Before a generator is started up its bearings must be inspected and cleaned and filled with oil if necessary. The machine is then brought up to the proper speed and the bearings again inspected to see that the oil-rings are running properly. The exciters or excitation circuit is then put in readiness and the rheostat in the alternator field circuit adjusted for maximum resistance. Before exciting the field the armature insulation must be thoroughly dry. If it is not the armature is short-circuited through an ammeter and run for several hours at a partial excitation to give about rated current in the short-circuited armature. When the insulation is thoroughly dry the short-circuit is removed and the excitation adjusted to give rated voltage at the armature terminals with correct speed. To shut down the machine the load is first removed by opening the circuit breaker; then the field rheostat is turned to

maximum resistance as is also the rheostat in the exciter field if there is an individual exciter. Then the field circuit is opened.

Paralleling of Generators. — Before connecting a generator to bus bars to which one or more other generators are connected, the following conditions must be satisfied:

1. The frequency of the generator must be the same as that of the bus bars.
2. The frequency of the generator, and therefore its speed, must be constant for an appreciable interval of time.
3. The voltage of the generator must be the same as the voltage of the bus bars.
4. The generator and bus-bar voltage must be in phase.

If the two machines have not the same frequency or if the frequency is not constant, a condition will occur intermittently in which the two voltages are 180° apart or the two machines are in series on a short circuit, and a dangerous current will flow. If the voltages are not equal, a large "wattless" or reactive current may flow, and if the two voltages are not in phase, a large power current will flow which will cause a mechanical shock. To indicate when these conditions are fulfilled any one of several "synchronizing" devices may be employed, as described in the article on *Synchronizers and Synchroscopes*.

Synchronizing with Lamps. — The simplest method of synchronizing small machines is to use incandescent lamps as shown in Fig. 9. In Fig. 9A, the connections are such that the lamps remain dark when the above conditions are satisfied, while in Fig. 9B they will remain bright under these conditions. If the frequencies are wrong the lamps will flicker (the slower the flicker the nearer the two frequencies). If the voltages are wrong the lamps in 9A will glow slightly but steadily. Transformers should be used with the lamps in case of high-voltage machines.

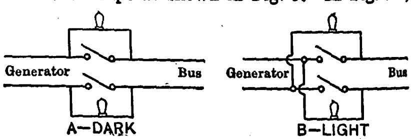

Fig. 9. Connections of Lamps for Synchronizing

Hunting. — Unless the angular velocities of two machines which are connected in parallel remain the same, either both constant or both varying together, a cross current will flow, due to the phase displacement between them. This current will tend to drag ahead the machine which is lagging, but due to the inertia of the rotating parts the machine which is at first lagging will "overreach" and become leading, and under certain conditions a cumulative see-saw action will be set up. When this takes place the machines are said to "hunt." The value of this current is proportional to the short-circuit current of the machines and to the angular displacement expressed in electrical degrees. In a machine having a large number of poles (40), a very small variation in angular velocity of the prime mover may cause a considerable (20-fold) phase displacement in electrical degrees.

To prevent hunting it is necessary to have:

1. A prime mover giving reasonably constant tangential effort and angular velocity. In general, the maximum variation in angular displacement between any machine and the bus bars should not exceed 2.5 electrical degrees.

To secure this condition the machines should have constants such that they are not especially sensitive as pendulums to the strokes or impulses of the particular engines used.

2. A governor which is not too sensitive to slight and sudden variations in load, i.e., a damped governor.

3. A low-resistance drop (usually not over 10 per cent) in the connections between the machines. This refers particularly to groups of machines in power houses miles apart.

4. Short-circuited or "Amortisseur" windings on the fields of the machines. Currents induced in these windings cause them to act as electrical brakes.

Short Circuits. — An alternator when suddenly short circuited will deliver for an instant a current many times as great as will flow after conditions have become constant. This is due to the fact that it takes a finite period of time for the increased armature reaction to weaken the magnetic field. During this short period much damage may be done to circuit breakers, etc. This is especially true of large turbo-alternators, as these, due to their construction, have a very large short-circuit current.

Use of External Reactance. — It is therefore frequently the case that reactance or choke coils are connected in circuit with these machines to prevent a dangerous current flowing in case of sudden short circuit. See *Reactance Coils.*

Induction Generators on Short Circuit. — Induction generators are free from this fault, since they lose their excitation almost immediately on short circuit, and are therefore becoming popular in large central stations.

Use of Imbedded Thermometers. — In large generators thermocouples or resistance thermometers (*see Pyrometers*) are sometimes embedded in the estimated hottest spot of the winding, and connected to a suitable indicating device to show at all times the maximum temperature of the machine. See also *Standardization Rules of the A.I.E.E.*

SPECIFICATION FOR A-C. GENERATOR.* — The following memoranda are intended to assist in writing specifications. See also article on *Specifications.*

Principal Characteristics and Conditions of Service. — Service for which generator is to be used, such as a-c. lighting, single-phase railway service, operating railway or lighting synchronous converters, etc. Voltage and number of phases. Rated output in kilowatts or kilovolt amperes at stated power factor. Frequency and speed.

Style and Description; Details of Construction. — Type of generator, revolving field, induction type, etc. Details of speed, governing of prime mover, and how generator is connected to prime mover, e.g., direct or belt-driven by steam turbine, reciprocating engine, water wheel, water turbine, gas engine, oil engine, etc., with vertical or horizontal shaft. Whether compensated. Whether exciter is to be supplied; if so, its characteristics. If field rheostat is to be supplied, its characteristics, including the effect upon the generator voltage of each step and of all steps; whether to be controlled by hand or automatically. Restriction of excitation current and voltage and requirements respecting carrying capacity of slip rings. Accessibility of armature. Maximum length of armature conductor projecting from slot. Windings shall be clamped securely to prevent any vibration of overhanging parts. Mechanical protection of armature conductors if exposed. If belt driven, specify pulley details; whether bed plate is desired.

Work to be Done by Other Contractors. — Whether Contractor is to furnish and install the following. Main wiring, field wiring, field rheostat grids, dial plate and chains. Point of division between engine and generator contracts.

* By W. A. Del Mar.

Performance and Tests. — (*See Standardization Rules of the A.I.E.E.*). Temperature rises upon which ratings are to be based. Details of overload capacity. Efficiency at 25, 50, 75, 100, and 125 per cent load. High-potential tests of insulation. Requirements regarding effects of moisture upon insulation. Requirements for parallel operation, i.e., whether the machine is to operate in parallel with similar machines or different ones. It is usual to specify that the terminal voltage shall vary according to a sine law. Regulation with 100 per cent power factor and normal speed; the load may be varied from zero to 150 per cent of rated load without causing more than a stated variation of voltage, the exciter field being kept constant.

DIMENSIONS, WEIGHT AND COST. — While generators vary widely in their specific weights and costs, that is weight per kv-a. rating, and cost per kv-a. rating, they may be divided into classes in which these characteristics are fairly definite.

The conditions primarily affecting the specific weight are: method of rating, speed, frequency, voltage and size. In addition there are the peripheral speed and mechanical construction which cannot be easily classified. The method of rating is fundamental. For purposes of comparison the rating may be taken as the output in kv-a. which each machine will give continuously with a rise in temperature not exceeding 45° C. On this basis the specific weight decreases as the speed, frequency or capacity increases, and increases with increase of voltage. The cost per pound decreases as the frequency decreases, and as the capacity increases. The cost per pound increases as the voltage and peripheral speed increase.

Alternating-current generators may be divided into three classes according to their speeds and purposes:

High speed, as turbine-driven generators.
Medium speed, as belt-driven and water-wheel-driven.
Slow speed, as engine-driven generators.

Approximate over-all dimensions of typical 60-cycle three-phase generators of various ratings for each of the above classes are given in the table below. The dimensions and weights for the turbo-generators include both the turbine and the generator. The voltages given in this table are the highest values for which the given size of a standard line is wound. Lower standard voltages are obtainable readily.

Approximate weights, costs per kv-a. and suitable speeds are indicated in Fig. 10 for engine-driven and in Fig. 11 for water-wheel-driven 60-cycle, 2300-volt, polyphase machines. The weight and cost of 25-cycle generators are from 10 to 20 per cent greater than for 60-cycle generators. The cost of machines for less than 2300 volts is practically the same as for 2300-volt equipment. Two-phase generators weigh and cost practically the same as three-phase. Single-phase generators weigh and cost from 25 to 30 per cent more. These data are of course suitable only for preliminary estimates; the costs, particularly, are subject to large variations due to changes in commercial conditions from year to year. For final estimates exact dimensions and quotations should be obtained from the manufacturers.

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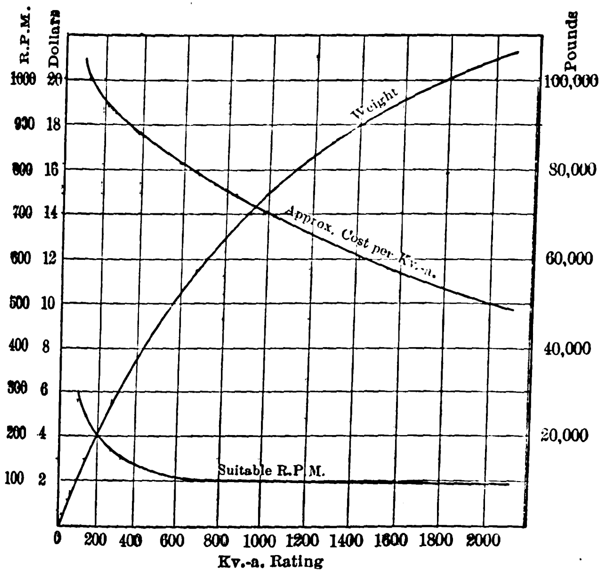

Fig. 10. Cost, Weight and Suitable Speed of Engine-driven A-C. Generators for 60-cycle, 2300-volt, Polyphase Service

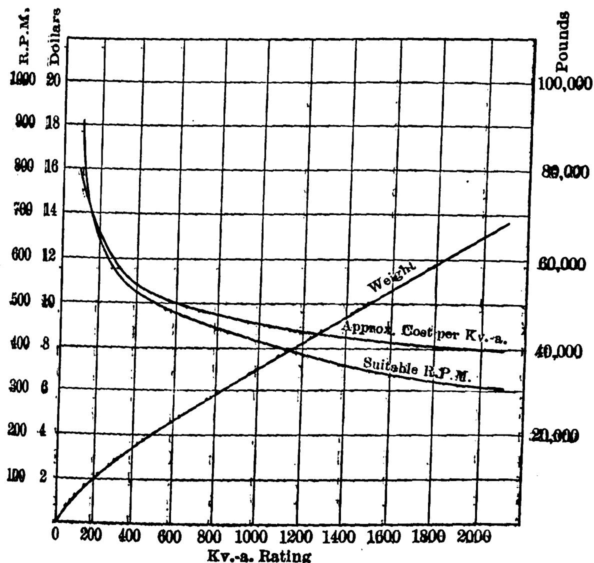

Fig. 11. Cost, Weight and Suitable Speed of Water-wheel-driven A-C. Generators for 60-cycle, 2300-volt, Polyphase Service

APPROXIMATE DIMENSIONS OF A-C. GENERATORS

Three-Phase, 60 Cycles

	Rating, kv-a.	Volts between terminals	Speed, r.p.m.	Over-all dimensions, inches Length	Width	Height	Total weight, pounds
Water-wheel driven	100	2300	900	60	45	50	4,800
	250	2300	600	73	56	55	9,200
	500	2300	514	96	87	88	21,000
	1,000	2300	400	113	120	96	34,000
	3,000	2300	277	153	162	133	100,500
	3,000	2300	225	166	224	152	151,000
	5,000	2300	400	167	179	122	119,000
	10,000	6600	360	208	208	126	278,000
Engine driven	125	2300	276	34	74	64	5,700
	250	2300	200	40	100	80	12,000
	500	2300	120	51	179	125	38,000
	1,000	2300	100	66	215	140	64,000
	1,250	2300	100	60	255	160	78,500
	2,000	4000	100	66	255	160	95,000
Turbo-generators, including turbine and generator	500	2300	3600	168	78	87	39,000
	1,000	2300	3600	168	82	85	43,000
	3,000	2300	1800	293	120	115	125,000
	5,000	6600	1800	322	153	144	237,000
	7,500	6600	1800	364	147	125	248,000
	10,000	6600	1800	435	210	156	397,000

p. 379; 1914, Vol. 17, p. 478; *Elec. Jour.*, 1906, Vol. 3, p. 545, p. 631, p. 668; 1911, Vol. 6, p. 672; *Trans. A.I.E.E.*, 1909, Vol. 28, p. 871, p. 932, p. 980; 1912, Vol. 31, p. 1657; June, 1913; May 1914. **Wave Form and Harmonics:** *Elec. Jour.*, 1907, Vol. 4, p. 189, p. 382; *Elec. World*, 1909, Vol. 54, p. 909, p. 910; *Trans. A.I.E.E.*, Feb. 1913. **Testing:** *Elec. Jour.*, 1905, Vol. 2, p. 11; 1906, Vol. 4, p. 610; *G. E. Rev.*, 1911, Vol. 14, p. 505; *Trans. A.I.E.E.*, Feb. 1913. **Regulation:** *El. Rev. and West. Elec.*, 1911, Vol. 59, p. 1072; *Elec. World*, 1912, Vol. 59, p. 46; *Trans. A.I.E.E.*, Feb. 1913. **Parallel Operation:** *Elec. World*, 1907, Vol. 50, p. 1243; 1908, Vol. 51, p. 1100; *Trans. A.I.E.E.*, 1910, Vol. 29, p. 765. **Selection:** *Elec. Jour.*, 1909, Vol. 6, p. 583. **Installation:** *Elec. World*, 1907, Vol. 49, p. 110, p. 357. **Troubles:** *Elec. World*, 1907, Vol. 49. p. 442; *G. E. Rev.*, Feb. 1909.

[W. I. SLICHTER.]

GENERATORS, DIRECT-CURRENT. — (*See also Alternating Currents; Electricity and Magnetism, Principles of; Generators, Alternating-Current; Motors, Direct-Current; Standardization Rules.*)

The following is a brief outline of the contents of this article:

APPLICATIONS. — Direct-current, or, as it is sometimes called, continuous-current apparatus was in very general use before the alternating-current type was introduced, and wherever the distance to which energy is to be transmitted is not a factor, there is no doubt that the direct-current system is to be preferred. However, if energy is to be transmitted over long distances the alternating-current system with its transformer has unequivocal advantages. The result of these two conditions is that a compromise is adopted. The majority of the generating stations provide alternating currents and many of the motors use direct currents provided by rotary converters. Thus the direct-current generators are becoming of less relative importance while the direct-current motors maintain a very important position.

Series Machines. — A series machine, generator or motor, is one in which the entire armature current flows through the field winding. Series generators are usually built to supply a constant current to an external circuit irrespective of the effective resistance of that circuit.

DEFINITIONS. — The fundamental principle involved in the construction and operation of any type of dynamo-electric machine is the production of an electromotive force in one or more conductors by the relative motion of these conductors and a magnetic field. Such a machine may, as a rule, be used either as a generator or motor.

Shunt Machine. — A shunt machine, either generator or motor, is one in which the entire field excitation is derived from a circuit of many turns and high resistance connected in "shunt" or multiple with the armature circuit. The characteristic of a shunt machine is poor regulation; that is, the voltage of a shunt generator decreases as the load increases. This is so marked that some shunt generators may be short-circuited, their terminal voltage dropping to zero, without resultant harm.

Separately Excited Machine. — This type of machine is sometimes used in large stations where the exciting current is readily obtained from separately excited bus-bars and a voltage regulator is used.

Compound-wound Machine. — This type of machine has on each field pole in addition to its shunt winding a few turns of thick wire which carry the load

current and are known as the series winding. This winding causes the excitation to increase as the load increases and tends to keep the terminal voltage constant or even to increase it. If the field windings are proportioned to cause the voltage at full load to be higher than the voltage at no load the machine is said to be "over-compounded." A "flat compounded" machine has the same voltage at full load and at no load; an "under compounded" machine has a lower voltage at full load than at no load.

Short- and Long-shunt Connections. — A compound-wound machine may be connected short shunt as in Fig. 1 or long shunt as in Fig. 2. The choice is merely a matter of convenience of station wiring.

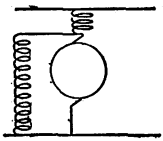

Fig. 1. Short-shunt Connections

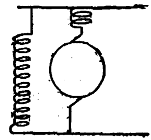

Fig. 2. Long-shunt Connections

Commutating Pole or Interpole Machines. — These machines have small auxiliary poles alternately placed with respect to the main poles and excited by a few turns in series with the load. The effect of these poles is to improve the operation of the machine in the matter of commutation; see below.

Bi-polar Machines. — Small continuous-current machines are usually of the bi-polar or two-pole type with a more or less inclosed frame of cylindrical shape.

Multi-polar Machines. — Large direct-current machines are of the multipolar type, that is, have a large number of radial pole pieces.

Belt-driven and Direct-connected Types. — Direct-current generators may be of either the "belt-driven" or the "direct-connected" type, as determined by the method of connecting to the driving unit. Belt-driven generators are characterized by a higher angular velocity than direct-connected.

RATINGS. — It has been common practice (up to 1914) to rate direct-current machines on the basis of the output in kilowatts which they will give continuously with a maximum rise in temperature by thermometer of 50° C. above the surrounding air at 25° C. See however the recent recommendations of the A.I.E.E. in the article on *Standardization Rules*.

VOLTAGE. — The standard voltages for which continuous-current machines are built are:

80 volts for use on shipboard.
110–125 volts for lighting purposes and incidental small power motors, fan motors, cooking utensils.
220 volts for three-wire systems with lighting and power combined.
500–600 volts for power alone and particularly for railway service.
1200–2400 volts for special railway service and heavy traction.
2000–6000 volts for series-arc-light circuits fed by series machines.

ELECTROMOTIVE FORCE INDUCED IN ARMATURE. — Let

Z = total number of armature conductors or sides of coils.
p = number of field poles.
m = number of parallel conducting paths between the positive and negative brush sets; that is $\frac{Z}{m}$ is the number of armature conductors in series between positive and negative brush sets. (*See below under Armature Windings.*)

ϕ = total useful magnetic flux per pole.
N = revolutions of armature per minute.
f = frequency in cycles per second.

If a coil of wire be revolved about an axis in a magnetic field, as shown in Fig. 3, each length of conductor, or side of the coil, will pass entirely around the armature in $\frac{60}{N}$ seconds. The time taken for a conductor to pass through the magnetic field under each pole, or from a to b, is $\frac{60}{Np}$. Hence the average value of the electromotive force induced in each armature conductor as it passes under each pole is $\frac{Np\phi}{60 \times 10^8}$ volts, since a cutting of 10^8 lines per second gives one volt. Since in an actual machine the conductors are uniformly distributed around the surface of the armature, this is also the average voltage per conductor in each of the conductors between a and b at any instant. Since there are $\frac{Z}{m}$ conductors in series between the positive and negative brush sets, the average value of the total electromotive force between the brushes, when a suitable commutator (see below) is provided, is

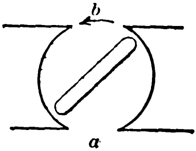

Fig. 3. Elementary Generator

$$E = \frac{p\phi ZN}{60 \times 10^8 m} \text{ volts.}$$

The electromotive force in each conductor alternates (i.e., passes through a complete cycle of positive and negative values) with a frequency of $f = \frac{pN}{120}$ cycles per second. Hence the formula for the average e.m.f. between brushes is

$$E = \frac{2 f\phi Z}{10^8 m} \text{ volts.}$$

The number of turns (S) in series between the positive and negative brushes is $Z/2m$, whence E may also be expressed as

$$E = 4 f\phi S \times 10^{-8} \text{ volts.}$$

Commutator. — A continuous electromotive force can be obtained from the machine if provision is made for reversing the connection from each conductor to the external circuit at the same time that the direction of e.m.f. induced in the conductor reverses. This is accomplished by the commutator. Taps leading from the front connections of the armature windings are connected to the segments of the commutator, so that as the segments come alternately under positive and negative brushes the current delivered to the circuit is always in the same direction.

ARMATURE WINDINGS. — Though there are a large variety of types of armature windings the practical man and even the designing engineer seldom meets types other than (1) the multiple-drum or lap winding and (2) the two-circuit series drum or wave winding. The other types may be used in a few special and exceptional machines but a designer may work for many years without finding any necessity for using them.

Multiple-drum or Lap Winding. — This type of winding is very common in direct-current machines, rotary converters, induction motors and is somewhat used in alternating-current generators. Its chief advantage is that it affords a

very free choice in the number of coils and slots, and is very simple to lay out and connect up. Its disadvantage is that it is not easily adapted to high voltages. Its chief characteristic is that there are always as many circuits in multiple and as many studs of brushes as there are poles. The distinguishing feature in appearance is the direction of bending of the end connections, as represented by Fig. 4. The characteristic form of the coils is shown in Fig. 7.

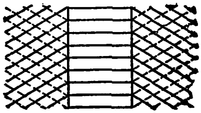

Fig. 4. Lap Winding

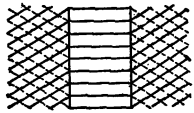

Fig. 5. Wave Winding

Conditions for Lap Winding.—The conditions to be fulfilled in laying out a simplex multiple-drum winding are expressed in the formula

$$Z = pz = sb,$$

where

Z = total number of coil-sides or bars,
p = number of poles,
z = average pitch* $= \dfrac{\text{front pitch} + \text{back pitch}}{2}$,
s = total number of slots,
b = number of coil sides or bars per slot.

The total number of inductors (Z) is double the number of coils, as each coil has two sides. Z must be a multiple of the number of slots (s) and of the number of poles (p).

The average pitch (z) must be an even number so that the front and back pitches may be different and odd ($z - 1$) and ($z + 1$), respectively. There are always two layers of coil-sides, top and bottom. One side of each coil lies in the top layer in one slot and in the bottom layer in another slot. The actual front and back pitches of a coil are always odd because they are made from a coil-side in the top layer (odd numbers) to one in the lower layer (even numbers) as in Fig. 6. Here the pitch is $14 - 1 = 13$.

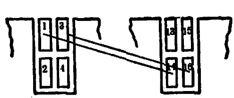

Fig. 6. Pitch of Connection

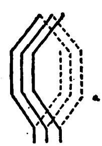

Fig. 7. Lap Winding

In order that the winding should progress continuously the front pitch, or pitch of commutator connections, must differ by one coil (or two coil-sides) from the back pitch, or actual pitch of coils. The actual spread of a formed coil (Fig. 7) is the same front and back and is equal to the "back pitch."

The total number of coils $Q = Z/2$ is equal to the number of commutator segments and must be a multiple of the number of slots. In general if Q is a common multiple of the number of poles and the number of slots, a multiple-

* Let the coil-sides be numbered consecutively from slot to slot, and let a point travel through the conductors in the order in which they are connected; if this point starting at conductor No. 1, say, passes through the point or (commutator) connection to conductor No. 8 and thus through the back connection to No. 3, then the front pitch is $8 - 1 = 7$ and the back pitch is $8 - 3 = 5$, and the average pitch is 6.

drum winding is possible. It is desirable that the number of slots should not be a multiple of the number of poles. In order to group the coils in polycoils z should be a multiple of the coil-sides per slot.

Series-drum or Wave Windings, Simplex. — This type of winding is used in direct-current armatures where the ordinary multiple drum would give either too low a voltage or require too many turns of fine wire in each coil. Its advantage is that it gives an armature that is better balanced magnetically and that only two brush studs are necessary. A greater number of studs may, however, be used. There are always two circuits in multiple, regardless of the number of poles. The characteristic reverse bends of its face conductors are shown in Fig. 5. The characteristic form of the coils is represented in Fig. 8.

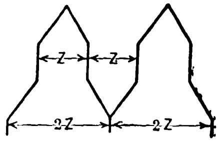

Fig. 8. Wave Winding

Conditions for Wave Winding. — The conditions to be fulfilled in laying out a series-drum winding are expressed in the formula

$$Z = pz \pm 2 = sb,$$

where

Z = total number of coil-sides or bars,
p = number of poles,
z = average pitch of end connections,
s = total number of slots,
b = number of coil-sides or bars per slot.

The total number of coil-sides (Z) is double the number of coils and must also be a multiple of the number of slots. In the formula $+ 2$ is preferable to $- 2$, as the positive sign gives shorter end connections.

The average pitch z may be odd or even. If it is odd the front and back pitches are both equal to z. If it is even the front and back pitches must be $(z-1)$ and $(z+1)$. For a wire winding the back pitch must be one greater than a multiple of the coil-sides per slot b in order to fit the coils into the slots in groups.

The total number of slots (s) is very much restricted and is intimately connected with the number of poles unless the expedient of using a dead coil is used (*see below*).

The bars or coil-sides per slot b must be even and cannot be 4 or 8 in a four-pole machine and cannot be 6 or 12 in a six-pole machine, unless a dead coil is used.

Use of Dead Coils. — By leaving one of the coils dead or out of circuit, a greater choice of slots and conductors is available. Four coil-sides per slot for 4 poles and 6 coil-sides for six poles may then be employed. In general $s \times b$ may be any number divisible by the number of poles. The formula is then

$$Z = pz - 2 = sb - 2.$$

Two bars or one coil are not connected in at all and the number of commutator segments is equal to the number of active coils or one less than the total coils. The total number of active coils is $Z/2$.

Usual Arrangements for 4- and 6-pole machines with no dead coil are:

Four Poles. — For $b = 6$, then z may be 1, 7, 13, etc., and usual values fulfilling all conditions are $s = 17, 21, 25, 29, 33$, etc. For $b = 10$, then z may be 1, 11, 21, etc., and s may be 19, 23, 27, 31, etc.

Six Poles. — For $b = 4$, then z may be 1, 5, 9, etc., and s may be 26, 32, 38, 44, etc.

Multiplex Wave Windings. — It sometimes becomes desirable to provide a winding which has more than two circuits in multiple but not as many as the number of poles. In such a case a multiplex wave winding would be selected. These are windings in which the circuit passes completely around the armature more than once. If to do this we use two entirely separate electric circuits we have a duplex doubly reëntrant winding, denoted by the symbol O O. If, on the other hand, the winding closes on itself after passing twice around the armature we have a duplex singly reëntrant winding, denoted by the symbol ◎. These types of windings are sometimes used when it is desired to have a number of circuits in parallel different from the number of poles. Thus we may have a six-pole machine with four circuits in multiple which with a given number of inductors would give a greater voltage than a multiple-drum winding and lesser voltage than a series drum. This would be a duplex winding.

Conditions for Multiplex Winding. — The conditions are imposed by the formula

$$Z = pz \pm 2m = sb,$$

where the symbols have the same meaning as before and m is the number of multiple windings and $2m$ the number of circuits in multiple. If m and z are prime to each other we have a singly reëntrant winding. The greatest common factor of m and z gives the number of times the winding reënters or in other words the number of independent windings.

The multiplicity and complexity may be carried to a very extreme limit. (*See under Armature Windings in any standard textbook.*) These multiplex windings are hardly ever used in the United States and England and only occasionally in Germany and France.

ARMATURE REACTION (OR ARMATURE INTERFERENCE). — The armature reaction of a d-c. machine has a very important influence on the commutation and regulation of both generators and motors. It is the effect of the magnetomotive force of the current in the armature conductors on the magnetic field set up by the field coils.

Separate Fluxes by Field and Armature Currents. — When current flows in the field coils and no current flows in the armature, a flux is set up following a path directly across the armature from pole to pole as in Fig. 9. On the other hand when current flows in the armature and there is no current in the field a flux is set up in the armature across the axis of the poles, as in Fig. 10.

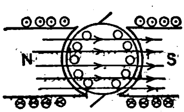

Fig. 9. Field Flux

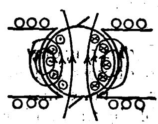

Fig. 10. Armature Flux

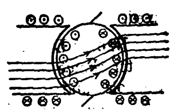

Fig. 11. Distorted Flux

Resultant Flux. — As a result of this action the flux is shifted around so that one tip of each pole has its density increased and one tip has the density decreased as in Fig. 11. If the brushes are at the geometrical neutral they are no longer on an axis at right angles to the flux; that is, they are no longer at the neutral point with respect to the resultant flux. With the brushes in this position the coil underneath a brush is cutting flux and generating a voltage, which is short-circuited by the brush and causes sparking. The brushes must, therefore, be moved a small angle in the direction of rotation in a generator (in the opposite direction in a motor) until they are on the neutral axis. The shift of the brushes aggravates the conditions, but nevertheless, unless the armature

strength is too great, a position can be found in which a brush short-circuits a coil that is in the real neutral position and is inactive.

Relations Shown Vectorally. — In Fig. 12 F_1 and A_1 are the m.m.f's. of the field and armature with the brushes on the geometrical or apparent neutral axis. R_1 is the resultant of these. The brushes which are in line with A_1 are not at right angles to R_1. If the brushes are moved to an axis at right angles to R_1 as at A_2, then the resultant becomes R_2 and the desired conditions have not been attained. By moving the brushes still further to A_3 giving the resultant R_3 we are able to get R_3 and A_3 at right angles to each other. The stronger the armature flux as compared to the field flux the greater will be the angle through which the brushes must be moved.

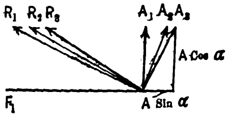

Fig. 12. Vector Relations of M.M.F.'s

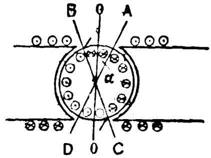

Fig. 13. Demagnetizing and Cross-magnetizing Turns

When the brushes are moved through an angle α to the position A_3 there is one component of the armature m.m.f., A sin α, directly opposed to the field m.m.f. Another component, A cos α, is at right angles to the field m.m.f. This is better understood by a reference to Fig. 13.

Demagnetizing Action and Cross-magnetizing Action. — Let the movement of the brush from A_1 to A_3 be represented in Fig. 13 by the movement from 0 to B, or through the angle α. Then the conductors in the angle 2α, between A and B and between C and D, carry currents whose m.m.f. directly opposes the magnetic strength of the field coils; these are known as back or demagnetizing conductors. The remainder of the armature conductors, in A–C and B–D, carry currents which give a m.m.f. at right angles to the field and cause a distortion of the flux; these are known as cross-magnetizing conductors.

It should be noted that while all the conductors in the sections A–B and C–D subtending an angle 4α constitute back ampere-turns, these are opposed to the strength of two poles. When we come to our quantitative treatment in which our unit of design is a pole we make use of the turns of 2α as the back ampere-turns per pole.

Effect of Interpoles on Armature Reaction. — The demagnetizing effect of armature reaction may be almost entirely overcome by placing the brushes in the neutral axis O–O. To prevent sparking under these conditions interpoles must be used. The use of interpoles does not in itself prevent armature reaction, but makes possible sparkless commutation when the brushes are set in such a position as to reduce armature demagnetization to a minimum.

ARMATURE INDUCTANCE. — Each coil of the armature carries a current flowing in one direction as it travels from a positive brush to a negative brush and in the opposite direction as the coil travels from a negative to a positive brush. Thus the direction of the current in each coil reverses during the time the commutator bars to which the coil is connected pass under the brush. To reverse the current in any circuit it is necessary to take out the energy stored up in the form of magnetic flux linked with the circuit, and put back an equal amount represented by a flux in the opposite direction. In doing this there is induced in the circuit a voltage which is proportional to the time

rate of change of the flux and which opposes any change in the value of the current. This voltage is the e.m.f. of self-inductance of the coil.

Methods of Improving Commutation. — To minimize the voltage of self-inductance it is necessary to keep the number of ampere-turns in each coil as low as possible and to cause the reversal to take place as slowly as possible. It is possible to neutralize the voltage of self-inductance by introducing an opposing voltage, which is accomplished in practical machines by giving the brushes a forward lead or by using interpoles.

Forward Lead of Brushes. — The brushes are moved in the direction of rotation in a generator (opposite direction in a motor) away from the true neutral until the coil undergoing reversal is moving in a flux coming from an adjacent pole tip of such a density that it induces by rotation in that particular coil a voltage that opposes and neutralizes the voltage of self-inductance.

Effect of Interpoles on Commutation. — Interpoles or commutating poles are placed between the main poles over the neutral space. These poles are excited by the load current until they give a flux of the proper direction and value to induce the desired neutralizing voltage. (*See also above under Armature Reaction.*)

Increasing Resistance in Path of Short-circuit Current also aids commutation. This changes the time constant of this circuit, that is, some of the stored energy of the magnetism is dissipated in I^2R loss in this resistance instead of in sparking at the brush. The usual method of accomplishing this is to use carbon brushes which have a higher resistance of contact than metal brushes. Another method is to introduce a high resistance in the connection between the winding and the commutator segment.

DESIGN. — The steps in the systematic procedure in the design of a d-c. machine of given voltage and power rating are given below. Simplex windings are assumed.

1. Statement of problem.
2. Suitable speed.
3. Number of poles.
4. Diameter and length of armature.
5. Length of air gap.
6. Number and size of slots.
7. Total flux, preliminary.
8. Number of turns on armature, preliminary.
9. Armature reaction, ampere-turns.
10. Number of conductors per slot.
11. Form of armature winding.
12. Size of armature conductors.
13. Exact size of slots.
14. Armature resistance and heating.
15. Design of commutator and brushes.
16. Reactance voltage.
17. Exact value of flux.
18. Cross-section of magnetic path; usual flux densities.
19. Excitation; calculation of magnetic circuit.
20. Stability factor.
21. Armature reaction or interference.
22. Field winding, series, shunt.
23. Interpoles or commutating poles.

The performance of the machine should then be calculated from the preliminary design to determine whether the design meets the imposed conditions

and such modifications as are necessary should then be made. The steps in calculating the performance are the calculations of:

24. Core-loss.
25. Friction.
26. Excitation loss.
27. Armature circuit loss.
28. Efficiency.
29. Regulation.
30. Heating.

Symbols. — The following notation is employed uniformly throughout this article; other symbols, and the same symbols with primes or subscripts, are defined in the paragraphs in which they are used.

(AR) = armature reaction ampere-turns per pole.
B = flux density in air gap, lines per sq. inch.
b = number of coil sides in series per slot.
C = total field ampere-turns per pole.
c = number of conductors per slot.
D = outside diameter of armature, in inches.
E = terminal armature voltage.
e = reactance voltage of short-circuited coil.
$f = \frac{pN}{120}$ = frequency of voltage induced in armature.
I = full-load line current, in amperes.
L = length of armature, in inches.
m = number of parallel paths between brushes.
N = revolutions per minute.
P = power output in kilowatts in case of generator; input in case of motor.
p = number of poles.
$\mathcal{P}$ = permeance of local magnetic path of short-circuited coil.
q = cross-section of one armature conductor, in square inches.
R_a = effective resistance of armature between brushes, in ohms.
$S = \frac{Z}{2m}$ = number of turns in series between brushes.
s = total number of slots.
V = peripheral velocity of armature in feet per minute.
V_c = peripheral velocity of commutator in feet per minute.
Z = total number of coil sides.
$z = \frac{Z}{p} = \frac{bs}{p}$ = average pitch of end connections.
ν = leakage coefficient.
$\rho = \frac{\text{pole arc}}{\text{pole pitch}}$; pole pitch is the arc from pole center to center of adjacent pole, measured at air gap; pole arc is the arc covered by the pole face.
σ = ampere-conductors per inch of armature periphery.
ϕ = total useful flux per pole.

1. Statement of Problem. — The rating in kilowatts or horse-power and the voltage are always given. The current and speed may or may not be given. The line current at full load is found as follows:

For a generator $$I = \frac{1000\,P}{E}.$$

For a motor $$I = \frac{746 \times (\text{Horse-powet})}{E \times (\text{Efficiency})}.$$

The proper efficiency to assume may be taken from the table of usual values given below in section 28.

2. Suitable Speed (N).—Machines are classified in commercial practice as high speed, moderate speed and low speed. The proper speed for a machine of a given size for each class is given in the curves in the section below on *Cost, Weight, and Speed.*

3. Number of Poles (p) depends upon the size, speed and character of the machine. There is, however, much variation depending upon specific conditions. Reasonable arrangements are indicated in the accompanying table.

Kw. rating	Number of poles		
	High speed	Moderate speed	Low speed
0–10	2	4	4
10–50	4	4	4
50–100	4	4–6	6
100–300	6	6	8
300–500	6	8	10
500 and greater	..	..	12 and more

4. Diameter and Length of Armature are related by the following formula:

$$D^2L = \frac{550\,P \times 10^9}{\sigma \rho BN}.$$

Usual Values of the Constants in the above formula are:

Magnetic Density in Air Gap (B) has values from 40,000 lines per square inch in small machines, 55,000 in medium and 70,000 in large machines. In general, the density in the air gap is taken the same as the density at the pole face.

Ampere-Conductors per Inch of Periphery, or the Specific Loading (σ) is the product of the total conductors around the armature times the current in each, divided by the periphery of the armature. The usual values for σ are given in accompanying table.

Kw. rating	σ-amp.-cond. per inch of periphery	
	For continuous rating	For intermittent rating
0–150	250–400	700
150–400	400–600	1000
400 and greater	600–900	1200

Ratio of Pole Arc to Pole Pitch (ρ) varies from 0.6 to 0.85 with an average value of 0.7.

Determination of D and L.—Having found D^2L the two factors may be separated by two methods. Assuming a square pole face (a desirable proportion for economy of armature and field copper), then

$$L = \frac{\pi \rho D}{p},$$

and substituting in D^2L we can solve for D.

Another method of determining separate values for D and L is by a consideration of the peripheral speed. The peripheral speed is fairly uniform in d-c. machines, having values of from 3000 to 5000 feet per minute. Assume an average value near 4000. The diameter of armature is given by the formula

$$D = \frac{(\text{Peripheral speed}) \times 12}{\pi N}.$$

5. Length of Air Gap in inches for a given diameter has a value given approximately by the table in the next paragraph. This gives a good value

for preliminary work but it may be found advisable to change it later in the design.

6. Number and Size of Armature Slots. — A rough approximation to practice gives the number of slots as four times the diameter in inches. General practice is shown in the table.

D = arm. diam., in.	Air gap, in.	No. of slots	Depth of slots, in.
5	0.08	25- 40	0.6
10	0.10	30- 60	0.75
15	0.12	40- 80	1.00
20	0.125	60-100	1.25
30	0.15	80-150	1.50
50	0.19	100-200	1.75
100	0.25	150-300	2.50
150	0.35	200-400	3.25

7. Total Flux in the Armature (ϕ) is fixed by the assumed gap density B and the length L and diameter D of the armature:

$$\phi = \frac{\pi \rho L D B}{p}.$$

8. Number of Turns in Series (S) between brushes is

$$S = \frac{E \times 10^8}{4 f \phi}.$$

9. Armature Reaction Ampere-Turns per Pole are

$$(AR) = \frac{IS}{p}.$$

The armature reaction ampere-turns per pole (AR) should not exceed certain values and should properly check as explained in the accompanying table.

If the (AR) comes out higher than advisable it is reduced by increasing the value of the flux by increasing either B, L or D.

Kw. rating	Armature reactive ampere turns per pole (AR)
0-100	1500-3500
100-300	3500-6000
300-500	6000-8000

10. Number of Conductors per Slot (c). — If s is the total number of slots on the armature, the proper number of conductors per slot (c) must fulfill two conditions which are sometimes antagonistic and therefore the solution is a compromise.

For a multiple-drum winding:

$$c = \frac{\pi \alpha p D}{Is} \quad \text{and} \quad c = \frac{2pS}{s}.$$

For a series-drum winding:

$$c = \frac{2\pi\sigma D}{Is} \quad \text{and} \quad c = \frac{4S}{s}.$$

c must be an even number and give a number of conductors which can be conveniently arranged in a slot having a depth of from 3 to 4 times its width. If c comes out very large for a multiple winding we then choose a series winding.

11. Form of Armature Winding (for general discussion of forms of windings see above).

Multiple-drum Winding. — If the number of conductors per slot (c) indicates the desirability of a multiple-drum winding, we choose an arrangement having an even number of coil sides per slot and assume the conductors per slot at a value the nearest multiple of the coil sides per slot, thus grouping the conductors into coils.

Series-drum Winding. — If a series-drum winding is indicated:

For a four-pole machine choose a number of conductors per slot (c) divisible by 6 and a number of slots (s) in the series 29, 33, 37, etc., endeavoring to obtain

that value of $\frac{sc}{4}$ that comes nearest to our preliminary value of S, the number of turns in series between brushes. Or for a four-pole machine choose a number of conductors divisible by 10 and a number of slots in the series 27, 31, 35, that gives a value of $\frac{sc}{4}$ the nearest the preliminary value of S.

For a six-pole machine choose a value of c divisible by 4 and a number of slots (s) in the series 26, 32, 38, that gives a value of $\frac{sc}{4}$ nearest the desired value of S.

The preceding values of s give the best winding arrangement but there are other combinations giving unequal front and back pitches and longer connections.

Revised Value of Turns in Series (S). — Having chosen arbitrarily values for s and c to fit the form of winding desired we must revise our preliminary assumption for the turns in series, S, to accord with the actual winding. Thus for a multiple drum $S = \frac{sc}{2p}$, and for a series drum $S = \frac{sc}{4}$.

12. Size of Armature Conductors.— The current in each conductor is equal to the line current I divided by the number of poles in a multiple-drum winding, and to $I/2$ in a series-drum winding of the simple type. The cross-section of each conductor is equal to the current per conductor divided by the allowable amperes per square inch, which varies from 3000 in small machines and 2000 in intermediate size machines to 1500 in large machines.

13. Exact Size of Slots, Slot Factor. — The total number of slots is definitely set by the style of winding. The maximum allowable depth depends upon the diameter of the armature. The width of each slot should be from 0.4 to 0.6 of the pitch of the slots, which is the quotient of the periphery of the armature divided by the number of slots. The ratio of the total cross-section of copper in a slot to the cross-section of the slot is known as the slot factor and has more or less consistent values.

Kw. rating	Slot factor	
	For 125 volts	For 500 volts
0– 10	0.28	0.20
10– 50	0.35	0.25
50– 100	0.40	0.30
100– 400	0.50	0.40
400–1000	0.60	0.50

The actual dimensions of the slots are found by adding to the width and depth of the cotton covered conductors, suitably arranged, an allowance for the coil and slot insulation. The allowances in the accompanying table are for voltages of 500 and less.

Type of slot	No. of bars or coil-sides per slot	Allowance for coil and slot insulation, in.	
		Width	Depth.
Open	2	0.09	0.30
Open	4	0.14	0.35
Open	6	0.15	0.35
Partly closed	4	0.15	0.40

14. Armature Resistance and Heating. — The mean length of turn is obtained from a drawing or is roughly estimated as

$$l = 2L + \frac{10D}{p}.$$

The equivalent resistance of the armature from brush to brush is

$$R_a = \frac{0.0093\,Sl}{12{,}000\,qm}.$$

The voltage drop in the armature is R_aI.

The loss in armature copper is R_aI^2.

The rise in temperature of the copper in ° C. is given approximately for preliminary purposes by

$$T_c = \frac{100\,R_aI^2}{(a+b)\,ls},$$

where

a = width of coils in slot, in inches.
b = depth of coils in slot, in inches.

15. Design of Commutator and Brushes. — The diameter of the commutator must be less than the diameter of the armature. The peripheral speed should be about 3000 feet per minute and should not exceed 4000 unless a special construction is used. The number of segments should be such that the average volts per bar should be less than 15.

$$\text{Volts per bar} = \frac{(\text{Rated voltage}) \times (\text{Poles})}{\text{Number of segments}}.$$

To avoid a weak construction the pitch of segments should not be less than 0.20 inch. Of this amount about 30 mils is occupied by insulation.

Dimensions of Brushes. — The width of the brushes is limited by the number of commutator segments which it is permissible to cover and the width of these segments. The number of segments covered is limited by the reactance voltage, as explained below. In general, the width of the brushes is from 2 to 3 times the pitch of segments. The area of the brush surface is such as to allow from 40 to 50 amperes per square inch of brush contact surface at full load. This surface is distributed equally over a number of studs equal to one-half the number of poles with a multiple-wound armature and may be all on one stud or disposed at pleasure with a series-wound armature.

Total Length of Commutator. — The total length of brushes per stud or the active length of the commutator is

$$l_c = \frac{2 \times (\text{Total brush surface})}{(\text{Width of brushes}) \times (\text{No. of studs})}.$$

The total length of commutator is greater than this by about 0.5 inch to allow for clearances between brushes and at the ends. It is also customary to allow an extra space at the end of the commutator for insulation to prevent creepage. Commutators in general have a gross superficial area of from 0.67 to 1.0 square inch per ampere of total current.

Resistance of Brushes and Brush Contact is a variable quantity. Some designers allow 2 volts total drop in positive and negative brushes under all conditions of load.

Another method is to use the values for the resistance per square inch of each brush and brush contact as given in the accompanying table.

Peripheral speed, ft. per min.	Res. of each brush, ohm		
	20 amp. per sq. in.	40 amp. per sq. in.	60 amp. per sq. in.
2400	0.035	0.024	0.02
2800	0.036	0.024	0.021
3200	0.037	0.025	0.022
3600	0.038	0.026	0.023

Friction of Brushes on Commutator. — The value of this loss is

$$\text{Friction in watts} = \frac{(\text{No. of brushes}) \times (\text{Pressure per brush}) \times V_c \times k}{45},$$

where

V_c = peripheral speed of commutator, feet per minute.
k = coefficient of friction = 0.2 to 0.3.

The usual value of pressure on the brushes is from 1.5 to 2 pounds per square inch of contact surface.

Rise in Temperature of Commutator is

$$T = t \times \frac{(\text{Brush } I^2R) + (\text{Brush friction})}{\text{Surface of commutator}}$$

where t has the value indicated in the accompanying table; t is the temperature rise in °C. for a total commutator loss of 1 watt per sq. in.

Peripheral speed of commutator, ft. per min.	Value of t.
2000	30
3000	20
4000	10

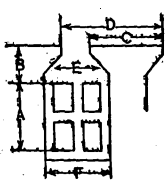

Fig. 14. Slot Diagram

16. Reactance Voltage. — The permeance of the local magnetic path surrounding each coil is

$$\mathcal{P} = 2\left(\frac{A}{3F} + \frac{B}{E} + \frac{C}{D}\right)(l_1 + 0.1\, l_2),$$

where the distances, expressed in inches (see Fig. 14), are as follows:

A = from top of upper conductor to bottom of lower,
B = from top of upper conductor to surface of armature,
C = width of tooth at face,
D = pitch of slots at surface,

E = mean width of wedge portion of slot,
F = width of slot proper,
l_1 = length of embedded portion of a coil side,
l_2 = length of free portion of coil side.

The coefficient 0.1 for l_2 allows for the fact that the flux surrounding the end connections is about one-tenth as great as that in the embedded portion of the coil.

The flux set up by each ampere flowing per coil is $3.2\,\mathcal{P}C$, where C is the number of conductors undergoing commutation simultaneously.

$$C = (\text{Turns per coil}) \times \frac{(\text{Brush width}) \times k}{\text{Segment pitch}}.$$

If the brush covers two segments there are 2 coils undergoing commutation at that brush and 2 under the brush of opposite polarity. All these coils will have conductors lying in the same slot if the windings have full or normal pitch. Hence $k = 2$, for full pitch, less for other pitches.

The inductance of a short-circuited coil is

$$L_c = 3.2\,\mathcal{P}Ct \times 10^{-8} \text{ henries},$$

where

t = turns in series short-circuited by the brush
= $k_2 \times$ (turns per coil)
k_2 = 1 for all multiple drum-windings
= number of poles in series in armature for series drum-windings.

The reactance voltage is then

$$e = 2\pi f_c L_c I_c,$$

where

$I_c = \dfrac{I}{m}$ = current per circuit,

f_c = frequency of commutation $= \dfrac{12\,V_c}{120 \times (\text{Brush width in inches})}$,

V_c = peripheral speed of commutator, feet per minute.

17. **Exact Value of Flux Leakage Coefficient** (v). — The exact number of turns was determined under *Form of Armature Winding*, section 11, above. The useful flux in the armature must then be

$$\phi = \frac{(E + R_a I) \times 10^8}{4fS}.$$

The flux in the pole or magnet core and the field yoke is greater than this, due to the leakage between poles. The ratio of the flux in the field to that in the armature is known as the leakage coefficient. It may be easily determined for each particular machine by the method given in the article on *Generators, Alternating-Current;* or if ample margin in the strength of field is always allowed it may be assumed without great error.

Rating in kw.	Leakage coefficient
2.5	1.2 –1.5
5	1.18–1.4
10	1.16–1.35
25	1.15–1.30
50	1.14–1.28
100	1.12–1.20
500	1.08–1.15

18. **Cross-Section of Magnetic Path. — Usual Flux Densities.** — Usual magnetic densities for determining the proportions of the magnetic circuit are given in the accompanying table.

Part	Material	Lines per sq. in.
Field yoke	Steel	70,000–100,000
Field yoke	Cast iron	35,000– 70,000
Pole core	Steel	70,000–100,000
Air gap	Air	40,000– 70,000
Armature teeth	Steel laminations	90,000–125,000
Armature core	Steel laminations	60,000– 90,000

The proper cross-section of each part is found by dividing the flux in that part by an assumed density. (*See also following paragraph.*) In most cases one dimension, as the length along the shaft, is known and the other is thus fixed.

19. Excitation. — Calculation of Magnetic Circuit. — It is advisable to calculate the densities and excitation for a value of flux corresponding to $E + R_a I$ at the speed at full load, as this is the most important condition. It is common practice to assume a loss of voltage of 5 per cent at full load instead of accurately predetermining the value.

Values of Flux in Different Parts of Magnetic Circuit. — In the armature teeth and air gap the total flux is equal to the calculated value of the useful flux. In the armature core the flux divides and we have one-half the useful flux in each branch. The field flux or flux in pole cores is greater than the useful flux and is equal to the useful flux multiplied by the leakage coefficient (ν). (*See section* 17.) In the yoke the flux divides and each branch contains one-half as much as the pole core.

Ampere-Turns. — The number of ampere-turns required for excitation is best calculated by the assistance of a tabulation like the following, in which the symbols refer to the dimensions shown in Fig. 15.

1	2	3	4	5	6	7
Part	Flux	Cross-Section	Flux density	Amp.-turns per in.	Length of path	No load amp.-turns per pole
Field yoke....	$\frac{\nu\phi}{2}$	de	......	....	i	......
Pole core......	$\nu\phi$	$0.785 f^2$ or fb	......	....	c	......
Air gap........	ϕ	aL_p	......	$0.313\ B$	g	......
Arm. teeth....	ϕ	$Tt_0\ l_i$	......	....	n	......
Arm. core.....	$\frac{\phi}{2}$	hl_i	......	....	k	......

Net ampere-turns for flux: .....

The dimensions in column 3 can be taken directly from a sketch similar to Fig. 15. If the pole is circular its area is $0.785\ f^2$ and if it is square its cross-

section is fb. The length of pole shoe L_p is usually about 0.5 inch less than L, the length of armature core.

The effective cross-section of the armature teeth is found as follows: At any given time only a portion of the teeth are carrying flux. These are known as the teeth under one pole, T, and are slightly greater than those actually under the pole, on account of the tendency of the flux to spread.

$$T = \frac{a + 2g}{p_1},$$

where a = pole arc, g = length of gap, p_1 = pitch of slots at face.

The effective or equivalent cross-section of the teeth is not that which gives the average density but that which gives the mean excitation required and is

$$\frac{(t_1 + 2t_2)}{3} l_i = t_0 l_i.$$

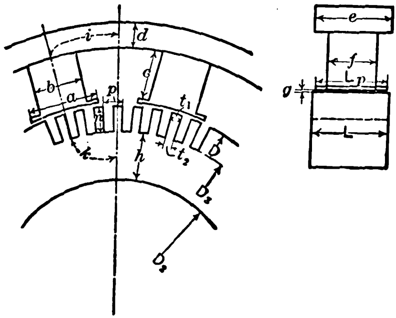

Fig. 15. Dimensions of Magnetic Circuit

The factor l_i is introduced to give the effective length of iron, excluding that space occupied by the insulation between laminations.

$$l_i = 0.9 \times (L - \text{space of air ducts}).$$

Thus the equivalent cross-section of armature teeth = $T t_0 l_i$.

Column 4 is the flux (column 2) divided by the cross-section (column 3).

Column 5 contains the ampere-turns per inch which are to be taken from saturation curves for the respective materials as given in the article on *Magnetic Properties of Iron.* The magnetizing force for a one-inch gap is always 0.313 B.

Column 6 gives the lengths of the respective paths as shown on Fig. 15.

Column 7 contains the ampere-turns required for each part of the path and is obtained by multiplying the values in column 5 by the respective lengths in column 6. The sum of all values in column 7 is the excitation in ampere-turns required on *each pole* of the field to establish the necessary flux in the machine. With no current flowing in the armature this would give a voltage about 5 per cent greater than the rated voltage of the machine. See section 21 below for compensation for armature reaction.

20. **Stability Factor.**—It is necessary at this point in the design to check the relations to see if the machine is liable to be unstable or require an excessive shift of the brushes. If interpoles are to be used this is not important, but if interpoles are not to be used certain conditions must be fulfilled. The criterion of these conditions is the "stability factor" which is the quotient of the field ampere-turns required for gap and teeth divided by the armature ampere-turns beneath the pole.

The ampere-turns beneath the pole are equal to

$$\frac{(\text{Armature reaction ampere-turns}) \times (\text{Pole arc})}{\text{Pole pitch}}.$$

The conditions to be filled are that at maximum current in the armature the armature ampere-turns beneath the pole shall not exceed the field ampere-turns required for gap and teeth. This is covered by the custom of making the stability

factor at full load have a value of from 1.3 to 1.5 in generators and 1.5 to 3 in motors.

21. Armature Reaction or Interference.—The current in the armature conductors sets up a magnetomotive force which reacts on the magnetomotive force of the field coils and prevents them from setting up as much flux as when there is no current in the armature. It is, therefore, necessary to have the field strength at any load greater than at no load by an amount sufficient to overcome or neutralize the armature reaction, or to provide special auxiliary poles for the purpose. There are four methods of estimating the effect of armature interference and overcoming it.

(*a*) It is common practice in the manufacture of d-c. machines to provide a field coil of such ample capacity that it is sure to be more than sufficient to overcome armature interference and set up the flux desired. This field is then adjusted in practice by putting an adjustable resistance in series with the shunt field and a suitable resistance in multiple with the series field. For this practice it is usual to provide a total number of field ampere-turns per pole (C) equal to the net ampere-turns per pole at full load required for flux, as calculated under item No. 19, plus a number of field ampere-turns equal to 40 per cent of the armature reaction ampere-turns as calculated under item No. 9.

(*b*) The effect of armature reaction is divided into back ampere-turns and cross ampere-turns. The excitation ampere-turns to overcome the back ampere-turns (caused by shifting the brushes) is

$$A' = \frac{(\text{Armature reaction}) \times (\text{Brush shift in segments}) \times p}{\text{Total commutator segments}}.$$

The excitation to overcome the effect of the cross ampere-turns depends upon many variables but in machines designed in accordance with standard practice it bears a fairly uniform relation to the cross ampere-turns, depending upon the value of the ampere-turns in gap and teeth, as follows:

Cross ampere-turns = (armature reaction ampere-turns) − (back ampere-turns).

Field ampere-turns to compensate for cross ampere-turns is then $B' = K \times$ (cross ampere-turns).

Where K has values as shown in Fig. 16 (*given by Cramp, see Bibliography*). Thus the total field ampere-turns at full load is then equal to

$$C = (\text{Net ampere-turns for flux, section 19}) + A' + B'.$$

(*c*) It is possible by working with the saturation curve of the machine to actually determine the effect of the cross ampere-turns on the saturation in the teeth and pole face. This method is too elaborate and complicated to be discussed here and is of more value for educational than for practical purposes. It is discussed very fully in the books by S. P. Thompson and E. Arnold (*see Bibliography, below*).

(*d*) Interpoles may be used as described in section 23.

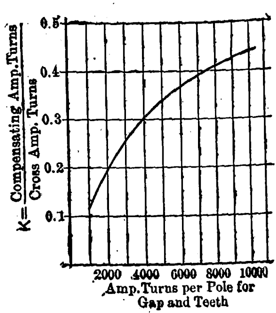

Fig. 16. Constant for Cross Ampere-turns

Excitation for Various Loads.—At no load the excitation required would be only that necessary to produce a flux corresponding to the rated terminal voltage. At any load corresponding to a current I the excitation in

ampere-turns must be equal to the sum of: (1) that required to produce a flux corresponding to $E + RI$, where R is the total resistance of the armature between brushes, the brush resistance and the resistance of the series field; (2) that required to overcome the back ampere-turns caused by I, and (3) that required to overcome the cross ampere-turns due to I. The process of calculation indicated in sections 19, 20 and 21 for full-load conditions is, therefore, repeated for other load conditions.

22. Field Winding. — If the machine has a series field this should be laid out first, since the shunt-field coils are sometimes placed outside of, and concentric with, the series-field coils.

Series Field. — For a compound-wound machine the ampere-turns required in the series field are equal to the difference between the full-load ampere-turns and the no-load ampere-turns. As it is usual to shunt about 25 per cent of the current through a shunt resistance the number of series turns per pole for each coil is

$$t_1 = \frac{\text{Ampere-turns required}}{0.75 \times \text{Load current}}.$$

The cross-section of each turn in square inches is taken equal to the full-load current divided by 1000.

This winding is usually wound in copper strap like a ribbon, the width being equal to the total length of field spool if it is wound under the shunt winding, or is arbitrarily chosen if the two coils are placed side by side. The thickness of each turn is the quotient of the cross-section divided by the width.

The coils are wound with an extra odd half turn to facilitate connection. Thus the depth of winding is

$$(\text{Thickness} + 0.016 \text{ for insulation})\ (t_1 + 1).$$

Allowance for a layer of insulation inside and outside adds about 0.375 inch to the depth of winding.

The mean length of turn l_f' is (see Fig. 16):

For square pole $l_f' =$ (periphery of pole) + 4 (depth of winding).

For round pole $l_f' = \pi\ (f +$ depth of winding).

The resistance is

$$r_1 = \frac{0.0093\, t_1 l_f'}{12{,}000\, q_1},$$

where q_1 is the cross-section of a turn.

The total drop in voltage in the series field is $0.75\, r_1 I$ which should be between 0.5 per cent and 1 per cent of the rated voltage.

Shunt Field. — The shunt field is required to give a certain number of ampere-turns, usually equal to the no-load ampere-turns. Let this number of ampere-turns per pole be F. At 1000 amperes per square inch the cross-section of copper in a coil would be $F/1000$. The total cross-section of the coil, including insulation, is determined by means of the space factor f'. Then the cross-section of the coil $= \dfrac{F}{1000 f'}$, where f' has a value of 0.5 if No. 18 B. & S. wire is used, 0.55 for No. 12, 0.60 for No. 6, and from 0.6 to 0.8 where copper ribbon is used. Where very small wires are used f' becomes as small as 0.25 (*see article on Electromagnet Windings*).

The depth of winding will be

$$b_2 = \frac{F}{1000 f' c_2},$$

where $c_2 =$ length of coil (*see Fig. 17*).

The superficial area will be (see Fig. 17) $A_2 = 2\,(f + b + 4\,b_2)c_2$.

The rise in temperature in ° C. of the coil will be from 60° to 80° per watt per square inch of A_2; thus for 40° rise on the spool the watts (w_2) per spool should be from $0.5\,A_2$ to $0.6\,A_2$ depending upon the ventilating effect of the armature.

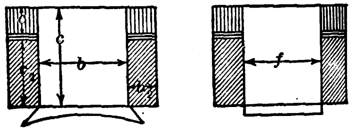

Fig. 17. Dimensions of Field Coils

Voltage per spool under normal conditions, allowing 25 per cent to spare for rheostat, will be

$$e_2 = \frac{0.75\,E}{p}.$$

$$\text{Current in shunt field} = i = \frac{w_2}{e_2}.$$

$$\text{Turns per spool} = t_2 = \frac{F}{i}.$$

$$\text{Mean length of turn, in in.} = l_f = 2\,(f + b + 2\,b_2) \text{ or } = \pi\,(f + b_2).$$

$$\text{Resistance per spool} = r_2 = \frac{e_2}{i}.$$

Cross-section of conductor in sq. in. =

$$q_2 = \frac{0.0093\,l_f t_2}{12{,}000\,r_2}.$$

The size of wire having a cross-section as nearly equal to q_2 as possible is chosen and arranged in layers. Single cotton-covered wire is used. The number of turns per layer is found by dividing c_2 by the outside diameter of the single cotton-covered wire.

Certain allowances must be made for the thickness of the spool and of the collars or flanges.

After these details of arrangement are settled it is advisable to recalculate the mean length of turn, the resistance, and the heating to be sure that the practical details have not interfered with the preliminary assumptions of watts per square inch.

23. **Interpoles or Commutating Poles** are used where either the armature reaction or the reactance voltage is very great. If the armature reaction at full load is greater than 80 per cent of the field ampere-turns at no load, then interpoles should be considered. If the reactance voltage is greater than 2.5 volts, interpoles should be considered.

The design of the interpoles may be roughly made in accordance with the practice in some of the large companies by making its width equal to the width of two slots and two teeth, and proportioning the winding to have 1.4 times as many ampere-turns as the armature reaction ampere-turns (AR). The axial length of the pole is frequently less than that of the armature, the gap density should be about 40,000 and the iron or steel of the pole should not be saturated when 1.5 times the full-load current is flowing through its windings.

In practice a shunt having inductance is connected across the terminals of the interpole winding and adjusted for sparkless commutation. When the interpoles are used on a compound-wound generator it is possible to decrease the strength of the series coils to such a value as will produce the proper flux for the desired $E + rI$, letting the interpoles take care of the armature reaction. Series coils having 20 per cent as many ampere-turns as the armature will be satisfactory.

Exact Design of Interpoles. — A more exact method starts with the value of the reactance voltage, e, and an assumed density in the air gap of $B_1 = 40{,}000$. The axial length of the interpole is then

$$L_1 = \frac{10^8 \times e}{kB_1 v_a},$$

where

v_a = the peripheral velocity of armature in inches per second,
k = number of inductors in series in short-circuited coil = $2\,t$ (*see formula for reactance voltage, section 16*).

The width (a_1) of pole at face is made equal to or a little greater than twice the pitch of armature slots. The flux per pole in gap is $\phi_1 = B_1L_1a_1$.

The leakage coefficient is higher for interpoles than for the main poles and is in the neighborhood of 1.6 to 1.8.

The magnetic density is kept low in the pole core so that there will be margin for overload.

The necessary exciting ampere-turns to force this flux through the gap and to take care of the leakage and increased densities in the field and armature core are calculated. Let this value be A''.

Let the armature reaction ampere-turns per pole be B''.

Then the total ampere-turns on the interpoles should be $A'' + B''$.

The number of turns per pole is $(A'' + B'')$ divided by the full-load current of the machine, and the winding itself is laid out in the manner employed in laying out the series field.

PREDETERMINATION OF PERFORMANCE FROM DESIGN. — The steps involved in predetermining what will be the performance of a given design are the following:

24. **Core-Loss.** — The core-loss consists of hysteresis and eddy current losses in the armature teeth and core. It is best calculated by taking from the curves in the article on *Magnetic Properties of Iron* the hysteresis loss per cubic inch at one cycle per second for the proper density and the eddy loss per cubic inch at one cycle per second for the proper density and thickness of sheet. These values are substituted in the following formulæ.

Total loss in core $= V_cC_hf + V_cC_ef^2$,
Total loss in teeth $= V_tC_hf + V_tC_ef^2$,

where

V_c and V_t = volume in cubic inches of core and teeth respectively,
C_h = constant from hysteresis loss curve for proper density,
C_e = constant from eddy loss curve for proper density, and thickness of sheet.
f = frequency.

A rough value for V_c and V_t can be conveniently obtained from the values in the table for the calculation of the excitation, section 19. Then

$V_t = p \times$ (area path) $\times$ (length of path),
$V_c = 2\,p \times$ (area path) $\times$ (length of path).

The calculation of the core-loss is one of the most unreliable and unsatisfactory steps in the design, as the value calculated by theoretical principles has to be multiplied by a factor having a value between 1.5 and 3 to obtain the value of the core-loss that will be found in the finished machine, on account of the pulsations of flux caused by the teeth. This factor is obtained in commercial practice from tests on machines similar to the one being designed.

25. **Friction and Windage** can be calculated by formula for any particular line of similar machines but no method applicable in general can be given.

Machines are designed so that the value of the friction loss very closely approximates those given in the table in section 28. The true friction varies directly as the speed, and the windage as the square of the speed. In shunt machines the speed does not change much with the load so this loss may be assumed constant in this class of machines (*for series motors see article on that subject*).

Stray Power Loss. — This term is sometimes used to include both the core-loss and friction and windage.

26. Excitation Loss or RI^2 in Shunt Field. — Since in a self-excited machine the loss in the field rheostat is charged against the machine the total loss is equal to the terminal voltage multiplied by the current in the shunt field. The loss is usually constant and only subject to the arbitrary variations due to hand regulation.

27. RI^2 Loss in Armature Circuit, including the brushes, series field and connections, varies as the square of the armature current, and, therefore, approximately as the square of the load, and also irregularly to a small extent due to the variation of brush contact resistance with the value of the current. The temperature of the copper must also be considered. (*See Resistance and Conductance.*)

28. Efficiency. — The efficiency is the ratio of the output to the sum of the output and all losses. It is predetermined by estimating the value of each loss for the particular condition of load under consideration, as described in the preceding paragraphs.

If P = the output in watts and the symbols A, B, C and D represent respectively the values in *watts* of the various losses described in sections 24, 25, 26 and 27 for the load P, then

$$\text{Per cent efficiency} = \frac{10^2 \times P}{P + A + B + C + D}.$$

The efficiency is a maximum for that load at which the variable loss D is equal to the sum of the constant losses $A + B + C$. Hence the desirability of making the constant losses small in a machine which is to be operated most of the time at a small load.

Usual Efficiencies and Losses. — An idea of a reasonable value for the efficiency of d-c. machines of various sizes and the distribution of the losses is given in the following table. It must be remembered that the efficiency of a machine of any given size may vary throughout a wide range depending upon the speed, weight and cost.

Rating, kw.	Efficiency, per cent	Friction (total), per cent	Excitation, per cent	Core-loss, per cent	Arm. RI^2, per cent
1	80	6	6	4	4
5	84	5	4.2	3.2	3.6
10	86	4	3.6	3.0	3.4
20	88	3	3.0	2.8	3.2
50	90	2.6	2.2	2.2	3.0
100	91.4	2.3	2.0	1.7	2.6
200	92	2.2	1.8	1.6	2.4
500	93	2	1.6	1.4	2.0

29. **Regulation** is not very important in a d-c. generator as compounding provides a means of obtaining any desired voltage at full load. Most generators are designed with a series field which will give 15 per cent overcompounding at full load. If less compounding effect is desired a certain portion of the load current is shunted by the series field, which adjustment is made by trial.

The inherent regulation of a shunt generator may be predetermined as follows: Let C (*see section 19*) be the total excitation in ampere-turns per pole to give a voltage E at the terminals at full load. C is made up of the m.m.f. to produce flux for $E + RI$ and the m.m.f. to overcome armature reaction. Let E_0 be the voltage at no load for an excitation C as shown by the no-load saturation curve. Then

$$\text{Per cent regulation} = 100\,\frac{E_0 - E}{E}.$$

30. **Heating.**—The rise in temperature of the field coils was found in section 22. The rise in temperature of the armature conductors was roughly checked in section 14, but it is necessary to determine more accurately the probable rise in temperature of the armature conductors as the heat due to the core-loss affects the rise in temperature of the armature windings. The following method given by Arnold takes into account all the variables in this complex problem and with a correct choice of constants gives very reliable results. The dimensions are as indicated in Fig. 18.

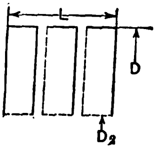

Fig. 18. Radiating Surface of Core

The surface of the armature core itself is taken as the sum of the outer cylindrical surface, the two annular surfaces at the ends, and one-half of the annular surfaces in the air ducts. Only one-half the surface in the ducts is used since this surface is not as effective as the external surfaces. It is assumed that the energy dissipated by this surface consists of the core-loss and that portion of the armature R_aI^2 which occurs in the portion of the conductors embedded in the slots.

A part of the heat due to R_aI^2 in the conductors flows from the copper through the slot insulation to the iron, if the temperature of the copper is higher than that of the iron. Sometimes, however, at light loads for instance, the temperature of the iron is greater than that of the copper and the flow of heat is reversed. This condition is taken care of in the following method.

The remainder of the heat due to R_aI^2 is dissipated through the insulation on the end connections to the surrounding air which is in motion due to the peripheral speed of the armature.

Power in watts dissipated by surface of armature core is

$$P_1 = \text{core-loss} + R_aI^2\left(\frac{2\,L}{l}\right),$$

I = full-load current; R_a = resistance of armature winding; L = total length of core; l = mean length of armature turns.

Effective radiating surface of armature core in square inches is

$$A_i = \pi DL + \frac{\pi}{4}(D^2 - D_2^2)(2 + d).$$

D = outside diameter of armature (Fig. 15); D_2 = inside diameter of armature; d = number of air ducts.

Rise in temperature in ° C. of armature core above air is

$$T_i = \frac{P_1}{A_i}\,t.$$

$t = 30$ for narrow, or high-speed armature; $= 40$ for long or slow-speed armature.

Permeance of heat path from copper to iron through slot is

$$M = \frac{U_s L s}{k_1 d_1},$$

where

U_s = perimeter of slot insulation; s = total number of slots; $k_1 = 200$ to 250; d_1 = thickness of insulation in slot; L = length of core.

Heat power in watts transferred from copper to iron or vice versa is

$$Q_1 = M\ (T_c - T_i),$$

where

T_c = rise in temperature of copper above air

Permeance of heat path from copper to air at end connections is

$$N = \frac{U_c l_c Z}{k_2 d_2 + k_3},$$

where

U_c = perimeter of end connection; l_c = length of end connection; Z = total number of bars or coil sides; $k_2 = 400$ to 500; d_2 = thickness of insulation on end connection; $k_3 = \dfrac{170}{1 + 0.00025\ V}$; V = periphery speed of armature, feet per minute.

Heat power in watts transferred from copper to air by end connections is

$$Q_2 = NT_c.$$

Total power of $R_a I^2$ in armature winding is

$$P_2 = Q_1 + Q_2 = M\ (T_c - T_i) + NT_c.$$

Whence, by transposal, the temperature rise, in °C., of copper of conductor above surrounding air is

$$T_c = \frac{P_2 + MT_i}{M + N}.$$

Explanation of Constants and Choice of Values:

t is the rise in temperature per watt per square inch at the surface of the armature or the specific drop in thermal potential from iron to air. It is affected by the peripheral speed and by the length of the armature.

k_1 is the drop in thermal potential per watt per square inch for slot insulation having a thickness of one inch. For any other thickness d_1, the drop is $k_1 d_1$.

k_2 is the drop in thermal potential per watt per square inch for insulation on end connections when the thickness is one inch, $k_2 d_2$ for any thickness d_2.

k_3 is the drop in thermal potential per watt per square inch between the surface of the insulation and the surrounding air. For stationary conditions $k_3 = 170$. As the peripheral velocity increases k_3 decreases.

Usually four-coil sides are bound together in the slot but separate on leaving the slot. Hence the reason for using s and Z in the different equations.

TESTING OF DIRECT-CURRENT MACHINES. — Direct-current machines are judged by four characteristics: efficiency, regulation, commutation and heating. It is necessary to subject a machine to actual full-load conditions to determine its heating and commutation and it is desirable to do so to determine the regulation, but the efficiency is determined more accurately by the "separate loss method" which does not involve loading the machine.

The customary tests on d-c. machines are: (*see also Standardization Rules.*)

(1). Resistance measurements, cold and hot. (2). Saturation curve. (3). Core-loss and friction test. (4.) Load run for commutation and regulation. (5). Heat runs. (6). Compounding test. (7). Insulation test.

1. **Resistance Measurements** — The first measurements are made when the machine is at the same temperature as the room, that is, after it has been idle from 12 to 24 hours depending upon its size. This is in order that the relation between the resistance and temperature may be accurately known. The resistance of the shunt-field, series-field and armature winding proper are measured.

Resistance of Lap Winding. — In measuring the resistance of the armature winding it is preferable to measure between two diametrically opposite points of the commutator of a multiple-wound armature. The effective resistance of the armature is calculated from this by the formula

$$R_a = \frac{4 R'}{p^2},$$

where R' = resistance measured as above, and p = number of poles.

Resistance of Wave Winding. — In measuring two-circuit series-drum windings it is necessary to measure between two commutator segments separated by a distance equal to the periphery of the commutator divided by the number of poles. This will give the effective resistance of the armature.

Brush Contact Resistance. — The resistance of the brushes and brush contact is calculated from data such as given in section 15 in the above section on *Design;* a measurement made with the actual circuit is not very satisfactory. However, it is sometimes the practice to measure the resistance of the entire armature circuit from brush to brush by the drop in potential method for several values of current, the armature being at rest. This is not accurate, however, as the resistance of brush contacts depends upon the speed.

2. **Saturation Curve.** — The saturation curve is not of immediate interest in determining the quality of a machine but is more particularly of interest to the designing engineer. It is made by driving the armature at the proper speed and supplying current to the shunt fields from a separate source of potential. As the current in the shunt field increases the potential generated by the armature varies and this is measured by a voltmeter.

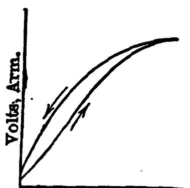

Fig. 19. Saturation Curve

On account of the existence of the hysteresis loop (*see Magnetic Properties of Iron*) it is necessary to increase the field current gradually and never to reduce the value at any step until the maximum value has been reached. The curve is plotted with volts as ordinates, and field current or ampere turns as abscissæ and shows the typical knee curve of all saturation curves. Different curves are obtained with increasing and decreasing excitation as shown in Fig. 19.

3. **Core-Loss and Friction, or Stray Power.** — These tests are made with the same arrangement as used in determining the saturation curve, and may be made at the same time. The machine to be tested is driven by a small motor having a capacity about 10 per cent of the rating of the machine under test. All the losses and constants of this motor must be known.

First the machine under test is driven at the desired speed with no current in the fields and the power taken by the driving motor noted. The mechanical output of the driving motor is calculated and this gives the friction loss of the large machine.

Then the large machine is excited and the power to drive it is determined. This power represents the core-loss plus the friction loss. Deducting the friction loss already determined, the values of the core-loss for various values of excitation are found. See also *Magnetic Properties of Iron*.

The combined core-loss and friction loss constitute the stray power loss.

4. **Load Runs.** — The machine is run and the excitation adjusted to give the proper voltage at no load. The load is then added and the terminal voltage noted. If E_0 = voltage at no load and E = voltage at full load, the regulation is $\frac{E_0 - E}{E}$.

If the machine has not commutating poles it is necessary to make a preliminary test in order to set the brushes at that position which gives the best compromise in sparking at no load and full load. Commutation is judged by observing the action of the brushes at full load and 150 per cent load. The conclusions are a matter of judgment and experience, although degrees of sparking have been arbitrarily agreed upon and are represented in a chart or series of pictures.

5. **Heat Runs** are made by operating the machine at full load for a period of time until the temperatures of the various parts that can be noted during operation have become constant. The greater the capacity of the machine the longer will be the time necessary.

Dead-load and Pumping-back Methods. — Heat runs may be made either by the "Dead Load" method in which a resistance load, such as a water-rheostat, is connected to the terminals and full rated load power is required to drive the machine; or by the Hopkinson or "Pumping-back" method in which two machines having similar characteristics are run together, one acting as a generator to supply electrical power to the motor which in turn drives the first by means of a belt or similar mechanical connection. For this test a "loss-supply" is required which may consist of a source of either electrical power or mechanical power. The amount of power required is from 10 to 20 per cent of the rating of one of the machines. The connections are shown in Fig. 20, for electrical loss supply and Fig. 21 for mechanical loss supply.

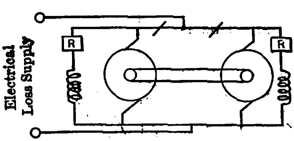

Fig. 20. Hopkinson Load Test, Electrical Supply of Losses

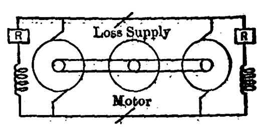

Fig. 21. Hopkinson Load Test, Mechanical Supply of Losses

Thermometer Readings. — During the heat run there are taken at stated periods, readings of thermometers which show the temperature of the frame, field coils, bearings and surrounding air. After the heat run the thermometers are placed on various parts of the machine, the bulbs being protected from radiation by small pads of cotton, and the following temperatures are noted:

Armature-core surface;	Field coils;
Armature-core ventilating ducts;	Bearings;
Armature conductors or winding;	Frame;
Commutator surface;	Room.
Pole tips;	

The resistance of all electrical circuits should be measured and the average temperature of the copper calculated from the formula,

$$t_1 = \frac{R_1}{R_0}(239 + t_0) - 239,$$

where R_1 and R_0 are the hot and cold resistances respectively, and t_1 and t_0 the hot and cold temperatures respectively, 98 per cent conductivity copper being assumed. See article on *Copper.*

6. **Compounding Test.** — In order to adjust the current in the series field so that a compound-wound generator will give specified voltages at no load and full load, the machine is first operated at no load and the current in the shunt field is adjusted to give the desired no-load voltage. The load is then put on and usually it will be found that at full load the terminal voltage is too great. Strips of German silver or other resistance metal are then connected in multiple with the series field until by shunting current from the series field the voltage is reduced to such value as is desired. This shunt resistance is then made up in permanent form and connected in circuit. Before making the no-load adjustment it is desirable to overexcite the shunt field for a moment in order to over come the hysteresis. See also section on *Parallel Operation* below.

7. **Insulation Tests.** — After the heat runs it is customary to apply a high-potential test to the insulation of the machine. This consists of applying an alternating potential between each electrical circuit and the frame of the machine for one minute. The value of the potential depends upon the capacity and rated voltage of the machine under test and is definitely specified in the Standardization Rules of the A. I. E. E. (*q. v.*).

EXAMPLES OF DESIGN AND PERFORMANCE. — In the tables on pp. 680 and 681 will be found the essential data including mechanical, electrical and magnetic characteristics of four examples of direct-current machines. In these data are included all the factors necessary to determine the efficiency, regulation, commutation and heating of the machines. Dimensions are in inches and weights in pounds. Performance data are deduced from tests.

SERIES ARC-LIGHT GENERATORS. — These machines are intended to give a constant value of current in the external circuit irrespective of the resistance or counter e.m.f. in that circuit. This is accomplished by causing the voltage impressed on the external circuit to decrease whenever there is a tendency of the current to increase. The load usually consists of a number (20 to 50) of arc lamps connected in series, each taking about 40 volts. Each lamp requires the same value of current and the total voltage is proportional to the number of lamps in use. If a lamp is no longer needed it is short-circuited by a switch and the current in the remainder of the lamps remains the same because the generator automatically reduces the voltage supplied. This regulation of the current is obtained by two or more of the following methods: (1) high armature reaction which tends to weaken the field whenever the current increases; (2) movable brushes to vary the number of turns in series between brushes; (3) a series field arranged so that turns may be shunted, short-circuited or cut out; and (4) a high degree of saturation in the magnetic circuit of the field so that the flux changes very slightly for considerable change of m.m.f.

The pole face density is usually quite high and the path of the flux in the field is of high reluctance, whereas the path of the armature flux is of low reluctance; thus there is much field distortion. The machines are usually wound for 2000 to 4000 volts. The distorting effect of armature reaction is usually relied upon to take care of sudden and small variations of the load, and large and lasting variations of the load are taken care of either by moving the brushes to another

MECHANICAL DATA ON D-C. MACHINES

Dimensions in Inches, Weights in Pounds

Type	1 Motor	2 Generator	3 Generator	4 Generator
Poles	2	6	6	8
Rating, kw	2.25	35	90	250
Revolutions per minute	1200	1050	750	150
Volts	115	125	125	250
Current	23.4	280	720	1000
Armature diameter at face	7.13	20.25	24	45
Armature diameter at back	1.63	13.5	14.5	32
Armature, total length	4.69	3.38	7	21.5
Air ducts	....	1×0.38	1×0.5	7×0.5
Slots, number	34	109	81	128
Slots, dimensions	0.95×0.22	1×0.28	1.16×0.49	1.32×0.52
Conductors per slot	28	2	4	6
Conductors, size	d=0.054	0.16×0.32	0.4×0.15	0.5×0.125
Conductors in multiple	4	2	6	8
Type winding	Drum	S.D.	M.D.	M.D.
Pitch of coils	1	1	1	1
Air-gap length	0.06	0.156	0.25	0.312
Pole arc	6.83	7.38	9.5	13.25
Pole length	4.25	3.13	6.75	21
Magnet-core length	4.25	3.13	6.75	21
Magnet, width	3.94	4.83	5.87	8.13
Magnet, radial length	3.44	5.25	6.38	10
Yoke length	6	8	9	21
Yoke, radial length	2.75	2.5	5.88	5
Shunt spool, turns	2210	481	436	372
Shunt spool, size conductors	d=0.018	d=0.083	d=0.12	0.144×0.156
Series spool, turns	28	9.5	4.5	4.5
Series spool, size conductors	0.14×0.13	5.06×0.05	6.12×0.05	9.25×0.075
Commutator diameter	5	13.5	18	30
Commutator length	3.13	6.25	10.62	14.5
Commutator, number segments	34	109	162	384
Studs × brushes	2×2	6×3	6×6	8×8
Dimensions each brush	0.75×1	0.75×1.38	0.75×1.63	0.75×1.25
Interpole arc	0.87	....	....	....
Interpole dimensions	0.87×3	....	....	....
Interpole, size conductors	0.09×0.09	....	....	....

ELECTRICAL AND MAGNETIC DATA ON TYPICAL D-C. MACHINES

Percentages are all in Terms of Rated or Full-load Values

Type		1 Motor	2 Generator	3 Generator	4 Generator
Rating	kw.	2.25	35	90	250
Rated voltage	volts	115	125	125	250
Rated current	amperes	23.4	280	720	1000
Flux per pole	maxwells	1.12×10^6	1.19×10^6	3.19×10^6	13.5×10^6
Leakage coefficient	....	1.2	1.13	1.13	1.13
Excitation amp. turns, no load	amp. turns	1040	3470	5610	6525
Excitation amp. turns, full load	amp. turns	....	5270	7670	9140
Armature reaction	amp. turns	1390	2540	3240	6000
Stability factor	....	0.9	1.55	1.5	1.36
Excitation to balance arm. reaction	amp. turns	....	1090	920	1150
Excitation at 110% volts	amp. turns	1145	4200	7000	7800
Volts per bar	volts	6.8	6.8	4.6	5.2
Reactance volts	volts	5.7	1.6	1.3	1.01
Friction loss, brush	watts	60	460	860	485
Friction loss, other	watts	100	600	1090	....
Core-loss	watts	60	1680	2710	2815
Armature res. at 25° C.	ohms	0.27	0.013	0.0025	0.006
Brush res. at full load, 25° C.	ohms	0.044	....	....	....
Series-field res. at 25° C.	ohms	0.05	0.0033	0.00104	0.00137
Interpole res. at 25° C.	ohms	0.123	....	....	....
Shunt-field res. at 25° C.	ohms	150	9	7.1	9.2
Friction loss	per cent	6	2.6	2	0.2
Core-loss	per cent	2	4.1	2.8	1
Shunt-field loss	per cent	3	2.2	1.3	1.6
Arm. copper loss	per cent	6.5	4.9	3.5	4.1
Efficiency at full load	per cent	82.5	86.2	90.4	93.1
Rise in temp. by thermometer:					
Armature	° C.	....	....	....	18
Field	° C.	....	....	31	15
Rise in temp. by resistance:					
Armature	° C.	....	....	....	....
Field	° C.	....	....	....	....

position giving either a lower or higher voltage, or by commutating or shunting some of the turns of the series field.

Brush Arc Machine. — This was the first arc-light machine to be brought out. It consists of a ring armature having a winding of the open-circuit type. There are a number of spool-wound coils on the armature, diametrically opposite coils being connected in series to an independent pair of commutator segments. The brushes make a series connection of the various groups of coils, the coils of highest e.m.f. being connected in series, those of medium e.m.f. in multiple and those of low e.m.f. being left out of circuit. The numerous field poles are on both sides of the ring armature. The two poles facing each other are of the same polarity so that the flux flows along the ring of the armature from one pole to the next pole on the same side of the ring. Regulation is obtained by shunting field turns and shifting the brushes for good commutation. A rotary oil pump is used as a regulator to move the handle of the field regulator and to move the brushes.

Thomson-Houston Arc Dynamo. — This machine contains a spherical-shaped armature with a three-part open-circuit winding. The three windings being spaced at 120° on a ring armature and connected in "Y," the terminals going to the segments of a three-part commutator. The commutator has air spaces between segments and a jet of air to blow out any arcs between brushes and segments. There are four brushes on the commutator, connected in pairs. At some part of each revolution two coils are in multiple, at other positions only one. A relay moves the brushes so that as the load increases the positive and negative brushes move farther apart thus giving more voltage. As one brush of a pair moves forward the other moves backward thus keeping them symmetrical with respect to the neutral axis. The field structure is of a hollow, cage-like construction.

Wood Arc-light Machine. — This generator has a closed-circuit Gramme ring armature with a commutator having a large number of segments. Regulation is obtained by moving the brushes so as to vary the voltage available and by the use of a high armature reaction balancing the field m.m.f.

HOMO-POLAR GENERATORS. — This type of machine is also sometimes called "acyclic" and was formerly incorrectly called uni-polar. Its method of operation is based on the principle of the Faraday disk, which consisted of a copper disk revolving about an axis and projecting between the poles of a magnet. By this rotation in a magnetic field an e.m.f. is set up between the axis and the periphery of the disk, and if brushes bearing on these two parts are connected to an external circuit a current will flow. The peculiar characteristic of a homo polar machine is that each conductor always cuts the flux in the same direction, consequently the e.m.f. induced in it is always in the same direction and is not alternating as in the usual direct-current machine. Thus no commutator is required.

This absence of a commutator is the feature which makes the homo-polar machine attractive. The commutator presents many difficulties in high-speed machines to be driven by steam turbines. It is for this application that recent attempts to develop a successful homo-polar machine have been directed. Instead of a commutator, collector rings with brushes are used to collect the current from the moving conductors. These collector rings, however, present difficulties in construction and operation on account of the high peripheral speed at which they must run. The rings are subject to a considerable centrifugal force and there is a tendency of the current to arc between the brush and the collector on account of the high rubbing speed.

Voltage. — The voltage of such a machine is not only unidirectional as in all direct-current machines but is really constant. But since there can be only a

few conductors in series, the voltage generated is very low. The voltage generated per disk or inductor is

$$E = Blv\,10^{-8},$$

where B = magnetic lines per sq. cm., l = length of conductor in cm., v = velocity of conductor in cm. per sec.,
This is more conveniently expressed,

$$E = \frac{NZ\phi 10^{-8}}{60} \text{ volts},$$

where N = revolutions per minute, Z = conductors in series, ϕ = total flux traversing gap.

Radial and Axial Types. — There are two types of homo-polar machines, the radial and the axial. The radial type (Fig. 22) is like the Faraday disc and consists of a disc revolving between the two poles of a cylindrical magnet. Brushes bear on the outer rim and the shaft to collect the current. The disc may be made of steel to reduce the reluctance of the magnetic path. The voltage of such a machine is limited to 10 or 15 volts but the current may be

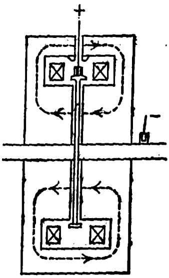

Fig. 22. Radial Type of Homo-polar Generator

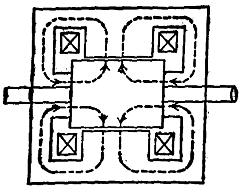

Fig. 23. Axial Type of Homo-polar Generator

very large. A variation of this type has two discs on the same shaft and the magnetic path so arranged that the voltage of the two discs may be added in series by brushes bearing on the peripheries of the two discs. The axial type (Fig. 23) consists of a cylindrical steel armature with copper bars in the surface, the whole revolving in a cylindrical field so arranged that the magnetic flux flows outward from the armature in a radial direction at all points. The several conductors on the armature are connected to slip-rings at both ends, and by means of brushes and stationary conductors, these conductors are connected in series. The voltage of such a machine may be from 40 to 50 volts per conductor.

Data on Large Axial Type Machine. — In the *Trans. A.I.E.E., Vol. 24,* Noeggerath describes a machine of the axial type rated at 300 kw. for 500 volts at 3000 r.p.m. The armature has 12 conductors connected in series for 500 volts. The diameter of the armature is 19 inches and the length 12 inches. The peripheral velocity is 15,000 ft. per min. The armature is of cast steel and has 24 cast steel collector-rings on it. The stationary conductors connecting the collector-rings together are placed in the face of the pole and thus their m.m.f. may be used to balance the armature reaction.

Excitation. — The armature reaction of such machines is very high and has only a distorting effect. However, it weakens the field as the cross magnetiza-

tion weakens one part more than it strengthens another due to saturation. By a proper arrangement of the movable connections between the collector-rings and stationary conductors a m.m.f. may be set up which will strengthen the field and thus the machine may be compounded. These machines may be made self-exciting but the resistance of the shunt fields must be very low in order that the machine may pick up on starting as the resistance in the brush contacts is high. The drop in voltage in each brush contact is about 0.8 volt at full load but is higher before the current flows. It sometimes requires from 10 to 20 times the normal voltage of brush contact to start the current.

Losses. — Such a machine as described above has an efficiency of approximately 90 per cent at full load. The losses are made up principally of friction and I^2R in the brushes. The field I^2R is low as the air gap is small for mechanical considerations. The armature I^2R is almost negligible due to the few turns. The eddy losses are low if the flux density is constant in one zone around the armature but the density may vary along an element of the cylinder. The total weight of the machine is about the same as for the usual d-c. machine but the proportion of copper is much lower than in the usual machines.

SPECIFICATIONS FOR D-C. GENERATORS.* — The following memoranda are intended to assist in writing specifications. See also article on *Specifications*.

Principal Characteristics and Conditions of Service. — Service for which it is to be used, such as railway, lighting, etc. Voltage. Rated output, kilowatts. Speed.

Style and Description, Details of Construction. — Type, whether shunt or compound wound. Whether interpole. Details of speed and governing of prime mover and how generator is connected to prime mover, e.g., direct or belt driven by steam turbine, reciprocating engine, water wheel, water turbine, gas engine, oil engine, etc., with vertical or horizontal shaft. If field rheostat is to be supplied, its characteristics, including the effect upon the generator voltage of each step and of all steps; accessibility of armature. If belt driven, specify pulley details.

Work to be Done by Other Contractors. — *See Generators, Alternating-Current*, under same heading.

Performance and Tests. — (*See Standardization Rules of the A.I.E.E.*). Temperature rises upon which ratings are to be based. Details of overload capacity. Efficiency at 1/4, 1/2, 3/4, 1, and 1 1/4 loads. (Whether rheostat losses to be included in calculating efficiencies.) High potential tests of insulation. Requirements regarding effect of moisture upon insulation. When run at constant rated speed, the load may be varied from zero to a stated per cent of rated load without causing more than a stated variation of voltage, the field rheostat being kept constant.

INSTALLATION. — In installing a d-c. machine the following precautions must be observed:

1. For large machines with foundations the foundation bolts must be provided in accordance with drawings.

2. Bearings must be lined up and well cleaned before being filled with oil.

3. The armature must be properly centered so that the air gap is correct at all points. Taper wedges are used to measure the gap. The magnet frame should be bolted to the base.

4. The field coils must be properly connected. Test for polarity with a compass in order to make sure that no field coils are reversed. For a self-

* By W. A. Del Mar.

exciting generator there is one particular connection of the field to the armature for each direction of rotation.

5. The commutator must be smooth and polished; use sandpaper to polish the commutator, never use emery cloth.

6. The brushes must be properly and accurately spaced around the commutator. They must be sandpapered and fitted to the curvature of the commutator. The pressure on the brushes must be adjusted to the correct value which is usually 1.5 to 2 pounds per square inch of contact surface.

7. The machine must be thoroughly dried out by heating and the insulation measured as a check.

OPERATION. — In starting up a single generator it is sometimes necessary to "charge" the field by separately exciting the shunt fields for a moment to set up residual magnetism.

To cause the machine to "pick up" or generate voltage by self-excitation it is necessary to cut out or short-circuit most of the resistance of the regulating rheostat connected in series with the shunt field. If the total resistance of the shunt-field circuit exceeds a certain critical value the machine will not "pick up," however much time is allowed.

Parallel Operation. — In order to operate a power station under economical conditions it is necessary to have a number of machines whose aggregate capacity is equal to the maximum demand on the station. As the demand varies the number of machines in operation is adjusted so that the machines running are operating at a load near their rating, and, therefore, at a good efficiency.

Shunt Generators. — In order to operate shunt generators in parallel, that is, feeding the same bus-bars, it is only necessary to adjust them all to the same polarity and voltage, connect them to the bus-bars, and adjust the division of load by strengthening the field of the underloaded machine if the voltage of the bus-bars is low. If the voltage of the bus-bars is high, weaken the field of the overloaded machine.

Compound Generators, Equalizer Connection. — In order to operate compound-wound machines in parallel it is necessary to provide an equalizer connection which makes a common connection on all the machines at the point between the armature and the series field as shown in Fig. 24.

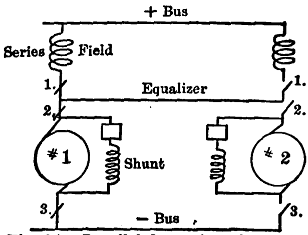

Fig. 24. Parallel Operation of Compound-wound Generators

The function of the equalizer is to divide the load current at all times in the proper proportion between the series fields of the different generators. This prevents the machines from acting as series generators or differential motors which would cause short-circuits.

For compound-wound machines to operate successfully in multiple all machines connected to one set of bus-bars must have the same amount of compounding as well as the same voltage at no load. Due to saturation in the magnetic circuit it is not always possible to make the compounding curve a straight line, i.e., the increase in voltage directly proportional to the load. It is, therefore, necessary to investigate the compounding curves of machines before they are operated in parallel. With unlike compounding curves one machine may become overloaded while another is under-loaded, unless the field current of one machine is adjusted.

In connecting a machine in parallel with those in operation it is necessary to see that it has the proper polarity and voltage, that the equalizer circuit is made,

and that the switches are closed in the order 1, 2, 3 as shown in Fig. 24. If any other order is used the effect is the same as having no equalizer.

In shutting down one machine switch No. 3 must be opened first, and then No. 2 and No. 1.

COST, WEIGHT AND SPEED. — In Figs. 25, 26 and 27 will be found the weight and cost per kilowatt of direct-current generators for usual or standard speeds. Fig. 25 gives the weight and cost of a line of high-speed machines for the suitable speeds shown by the curve. Fig. 26 gives the same information for a line of moderate-speed machines, generators or motors, and Fig. 27 for a line of slow-speed generators designed for direct connection to steam engines.

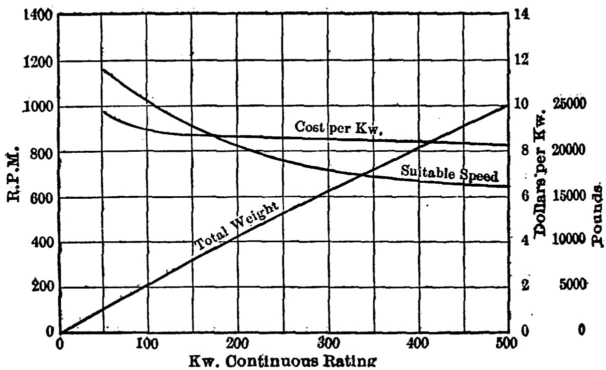

Fig. 25. Data on High-speed D-C. Machines

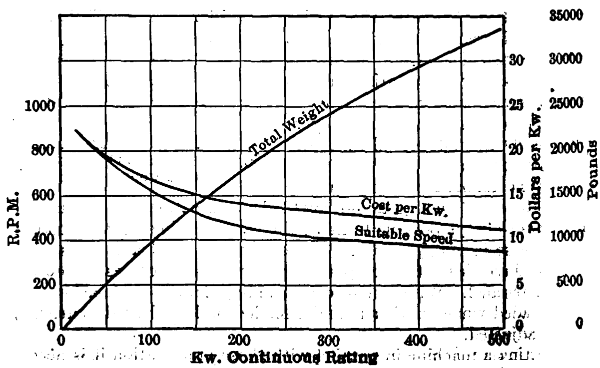

Fig. 26. Data on Moderate-speed D-C. Machines

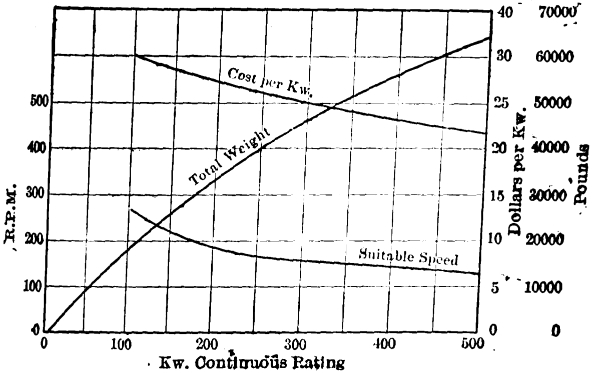

Fig. 27. Data on Slow-speed D-C. Machines

BIBLIOGRAPHY. — The following references will be found valuable in studying the design of direct-current machinery and as reference books to be used in seeking solutions to special intricate problems:

Arnold, E., *Gleichstromtechnik*, Vol. I and II, Berlin; Cramp, W., *C. C. Machine Design*, N. Y.; Crocker, F. B., *Design of D. C. Machines*, N. Y.; Fischer-Hinnen, *Gleichstrom Maschinen*, N. Y.; Gray, *Electric Machine Design*, N. Y.; Hawkins & Wallis, *The Dynamo*, London; Hobart, H. M., *Dynamo Design*, London; Kapp, G., *Dynamo Maschinen*, Berlin; Parshall & Hobart, *Electric Machine Design*, London; Thompson, S. P., *Dynamo-Electric Machinery*, Vol. I, London.

[W. L. Slichter.]

GENERATORS, STATIC. — (*See also Generators, Alternating-current; X-Rays.*) In electro-therapeutic and X-ray work a unidirectional high-frequency current is desired and is often obtained by a kind of electric generator called a static generator. There are two types of static generators available for the purpose, the frictional machine and the influence machine.

Frictional Machine. — In the frictional machine two dissimilar substances are rubbed together in some form of rotating apparatus as shown in Fig. 1. A rotating glass plate *A* is rubbed on both sides at *BB* by two pieces of leather which are greased and covered with the amalgam 1 Zn, 1 Sn, 2 Hg. The glass is electrified positively and the charge is drawn off by the metal points *CC* which almost touch the glass. Two silk aprons *DD* cover the glass between the rubbers and the metal points, to prevent the leakage of the charge into the air. The rubbers are connected to ground to remove the negative charge which accumulates upon them.

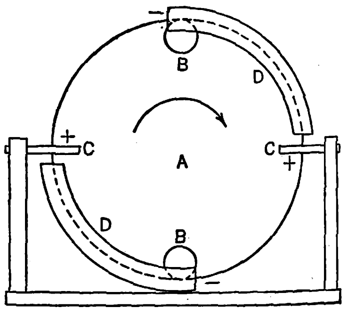

Fig. 1. Frictional Machine

Influence Machine. — The influence machine generates an e.m.f. by electrostatic induction, the principle of the action being clearly shown (see Fig. 2) in a machine invented by G. Belli in 1831. Two spheres or disks *AA*. called carriers and normally insulated from each other, are rotated about the shaft *B*. The two fixed plates *CC* are charged initially from some external source. When the carriers *AA* take the position shown in the figure, a momentary connection is made to the neutralizing wire *E* at the spring contacts *DD*. As a result of this connection between *A* and *A*, the carriers are charged by induction, the charge on each being opposite to that of the adjacent plate. When the carriers rotate still further, the connections at *DD* are broken and the charges induced on the carriers become isolated. When the carriers are rotated through half a revolution from the position shown, they strike the springs *FF* and add their charges to the field plates *CC*.

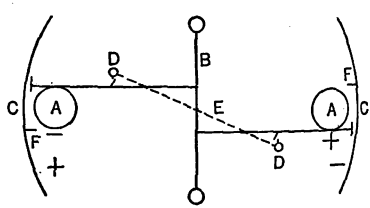

Fig. 2. Influence Machine

Wimshurst Machine. — Many forms of influence machines based upon the above principles have been devised. The most successful designs were made by Toepler, Holtz and Wimshurst. The Wimshurst machine, shown in Fig. 3, has superseded most of the other forms because of its self-exciting powers and suitability for work in all conditions of atmosphere. Two glass disks, *A* and *B*, are rotated close together and in opposite directions. A certain number of tin-foil carriers *CCC*, are mounted upon the outside surface of each plate. Neutralizing conductors *DD* for each disk are placed at right angles to each other. The collecting combs *EE* are connected to a condenser *F* and a spark gap *G* across which the discharge takes place. If two diametrically opposite

carriers on the back plate are charged positively and negatively respectively, opposite charges will be induced upon the adjacent carriers on the front plate if they are connected by the neutralizing wire. The induced charges are isolated when this connection is broken and are drawn off at the collecting combs *EE*. In the Wimshurst machine induced charges in moving from the point of electrification to the collecting combs also serve to induce other charges on the back plate.

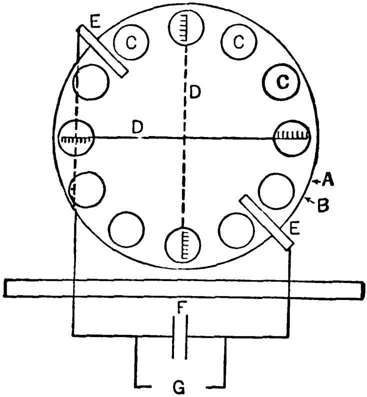

Fig. 3. Wimshurst Machine

After a few revolutions of the disks, half of the carriers on each plate are charged negatively and half positively, giving a machine of reliable service and high power. Unlike the Holtz machine the Wimshurst machine does not reverse its polarity when the distance between the terminals is made greater than the sparking distance. Large influence machines are constructed with several plates revolving in opposite directions and connected in parallel, the whole being contained in a glass case to protect the plates from dust. An eight-plate Wimshurst machine gives a spark of eight inches and a twelve-plate machine will give a spark of 13⅝ inches. The plates in these machines are approximately 30 inches in diameter and the machines give 6 sparks for each revolution of the disks.

BIBLIOGRAPHY. — Gray, J., *Electrical Influence Machines*, London, 1903; Mason, H., *Static Electricity*, N. Y., 1904; Holtz, Dr. W., *The Priority of Invention of the Double-plate Influence Machine*, Elec. Tech. Zeit., 1904, Vol. 25. p. 728.

[R. G. Hudson.]

GREEK ALPHABET.

Α	α	Alpha.	Ν	ν	Nu.
Β	β	Beta.	Ξ	ξ	Xi.
Γ	γ	Gamma.	Ο	ο	Omicron.
Δ	δ	Delta.	Π	π	Pi.
Ε	ε	Epsilon.	Ρ	ρ	Rho.
Ζ	ζ	Zeta.	Σ	σ	Sigma.
Η	η	Eta.	Τ	τ	Tau.
Θ	θ	Theta.	Υ	υ	Upsilon.
Ι	ι	Iota.	Φ	φ	Phi.
Κ	κ	Kappa.	Χ	χ	Chi.
Λ	λ	Lambda.	Ψ	ψ	Psi.
Μ	μ	Mu.	Ω	ω	Omega.

GROUND CONNECTIONS. — (*See also Grounding of Electric Circuits.*) A ground connection is a conductor buried in the earth and connected by conductors to other conductors or apparatus which are to be maintained at earth potential, irrespective of the current which may flow through it. Usually a circuit is not grounded except at such points that under normal conditions of operation no appreciable current flows to ground. A current may, however, be established through a ground connection due (1) to a lightning discharge, (2) to an accidental grounding of a live wire of a grounded circuit, and (3) to a cross between the grounded circuit and a high voltage ungrounded circuit; in the last instance the current through the ground connection is only the relatively small charging current between the other wires of the high voltage system and the ground (*see Capacity and Charging Current*).

USE OF GROUND CONNECTIONS. — Ground connections are used for the following purposes: (1) for lightning arresters, to afford a path to ground for the lightning; (2) for lightning rods on buildings, for the same purpose; (3) for steel transmission poles to give the current a short circuit to ground in case of insulator failure. Also as a lightning protection; (4) to bring the neutral points of circuits to ground potential and thus reduce the potential stresses of the system and minimize the danger of accident from shock; (5) to bring transformer cases, instrument cases, conduits, etc. to ground potential and thereby reduce the danger from shock; (6) to obtain an earth return for telegraph circuits; (7) for wireless telegraph systems to obtain the use of the earth's electrostatic capacity.

GROUNDING TO CONDUCTORS USED FOR OTHER PURPOSES. — Where an extensive system of water pipes buried in the earth is available, a satisfactory ground connection can be made directly to the pipes. Gas pipes are not suitable for ground connections, as in case of broken joints the resulting arc might ignite the escaping gas. Steam heat pipes are heat insulated (and therefore partially electrically insulated) from the earth. Where there are two ground connections, one to an electric railway track and the other to a water pipe, there is danger that railway current may thereby be introduced into the water piping system and produce electrolysis. Sheaths of cables are not good ground as the cables might be injured by the currents in their sheaths, either from electrolysis or unforseen heating. In fact, cable sheaths, like other buried conductors, have a tendency to collect stray currents from electric railways and special ground connections sometimes have to be made to permit these currents to escape without producing electrolysis. Steel frames of buildings present only a limited surface to ground at the foundations and are therefore only good when the current to be discharged will be proportionally small.

Connections to Piping System. — The National Electrical Code contains the following requirements:

Size of Ground Wires. — Not to be smaller than No. 6 B. & S. copper for lightning arresters (rule 5-c), or for grounding low potential circuits (rule 15-d), also not to be smaller than neutral for grounding three wire direct-current systems (rule 15-d), or less in carrying capacity than any one of the three mains for grounding three-phase systems (rule 15-d).

Method of Making Ground Connection. — In connecting a ground wire to a piping system, the wire should be sweated into a lug attached to an approved clamp, and the clamp firmly bolted to the water pipe after all rust and scale have been removed; or be soldered into a brass plug and the plug forcibly screwed into a pipe fitting, or where the pipes are cast iron, into a hole tapped into the pipe itself. For large stations, when connecting to underground pipes,

with bell and spigot joints, it is well to connect to several lengths, as the pipe joints may be of rather high resistance.

GROUNDING TO SPECIAL PLATES, PIPES, ETC. — When suitable buried conductors are not already available, direct connection to the ground may be made (1) by burying a plate in the earth, (2) by burying a wire or ribbon in the earth, or (3) by driving a pipe vertically into the earth. The last form of ground connection is now (1914) generally accepted as the best. These three methods of direct grounding are described in detail in the section below on *Design*.

Factors Affecting the Resistance of Ground Connections. — The resistance opposing the flow of current into the ground through a ground connection depends (1) upon the extent of metal surface in contact with the earth, (2) the thoroughness of this contact, (3) the wetness of the earth, (4) the distance between this ground connection and the ground connections (if any*) at which the current leaves the earth.

The effect of the distance upon the resistance between the electrodes is an important one, but is frequently overlooked. If the electrodes are close together, the resistance will be low on account of the shortness of the current path. If the distance between electrodes be increased, the resistance will be increased until a certain point is reached, after which it will steadily decrease on account of the increase in the cross-sectional area of the earth path. In the experiments the Cunliffes (*Jour. I.E.E., 1909, Vol. 43, p. 449*) the resistance reached a maximum when the ground plates were about 25 feet apart, the decrease in resistance being quite sudden for smaller or greater distances, as shown in Fig. 1. A curve by E. E. F. Creighton (*G. E. Rev., 1912, Vol. 15, p. 69*) shows the resistance between pipe grounds at various distances apart, the maximum resistance being again attained at 25 feet.

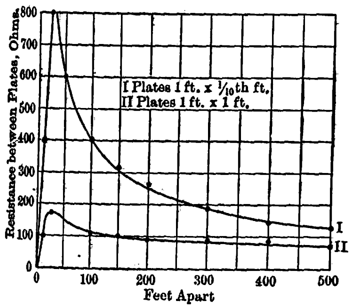

Fig. 1

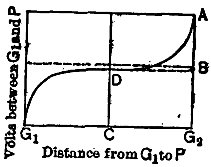

Fig. 2.

Definition of the Measure of the Resistance of a Single Ground Connection. — If the distance between ground connections be fixed and the drop of potential from one of them G_1 and a point P between them be plotted against the distance of P from G_1 a curve such as G_1DA, Fig. 2, will be obtained, if the two ground connections G_1 and G_2 are similar. If, on the other hand, the ground connection G_2 has very much more perfect and extensive contact with the earth, the curve of potential drop will have the shape G_1DB. Both curves have the peculiarity of being steep near the electrodes and of becoming flat between them, indicating that the greater part of the resistance occurs near the electrodes,

* The circuit may be completed by a displacement current through the air from the surface of the earth.

where the current is restricted to a limited area, practically no drop occurring where the current spreads out.

The resistance of the ground connection G_1 is then defined as the ratio of the potential drop CD, to the current entering the ground at G_1, and similarly for the ground connection G_2.

Resistance of Some Typical Ground Connections. — The resistance of ground connections varies so greatly with the condition and nature of the soil, that data on resistance have only local application. The following tests are presented merely to give a general idea of the results which are obtained in practice.

Authority	Time of year	Description of soil	Type of ground connection	Dimensions	Depth	Ohms
					ft.	
Hoxie.....	Feb.–June	Surface loam	Plate	1 ft.×1 ft.	..	1940
	Feb.–June	Moist black loam	Plate	6 ft.×2 ft. 3 in.	6	113
	Feb.–June	River bottom	Plate	1 ft.×1 ft.	..	132
	Feb.–June	Gravelly soil	Pipe	1¼ in.	5	630
	Feb.–June	Gravelly soil	30 Pipes	1¼ in. each	5	13
	Feb.–June	Swampy soil	7 Pipes	1¼ in each	5	15
Del Mar...	Summer	Rock & loam fill	Plate	15 in.×15 in.	6	155
Hayden...	Aug.–Sept.	Clay loam	Pipe	2½ in.	3¾	26.1
Hayden...	Feb.–Mch.	Clay loam	Pipe	2½ in.	3¾	120
Hayden...	June–July	Clay loam	Pipe	2½ in.	3.1	35.4
	Mch.–Apr.	Clay loam	Pipe	2½ in.	3.1	240

E. E. F. Creighton (*G. E. Review, 1912, Vol. 15, p. 67*) says: "If an iron pipe one inch in diameter is driven into normally moist earth to a depth of 6 or 8 feet, it will usually have a resistance of about 15 ohms. Eight ohms may be considered unusually low, while dry soils may give a resistance of 56 ohms and upwards." Creighton also states that after a depth of several feet in the conducting stratum has been reached, each additional foot decreases the total resistance by the factor $1/d$ where d is the depth in feet.

DESIGN OF GROUND CONNECTIONS. — It is as a rule impracticable to install a ground connection of sufficiently low conductivity to allow the passage of the maximum current it may have to carry without producing a considerable rise of voltage. The conductivity which a ground connection should have is therefore largely a matter of experience and judgment rather than a matter of exact calculation. Cheapness and durability usually determine the material, usually iron or copper, and the thickness.

Copper Plates. — Until recently the most common form of ground connection consisted of a copper plate embedded in charcoal or coke in moist earth. The use of coke is questionable, for the reason that the sulphur in the coke is likely to corrode the copper plate and thus increase its resistance. Some engineers are of the opinion that neither charcoal nor coke is of any particular use in connection with a ground plate. The corrosion of copper ground plates is greatly reduced by coating with tin. As the conductance of a ground connection is not proportional to the area of the metal plate, the use of a single large plate is an uneconomical method of obtaining a low resistance.

Copper Wire or Ribbon. — The wire or ribbon has the advantage of a lower resistance than the plate for the same area. The wire has the further advantage that no special material or connection is required. The wire may be laid out in a straight line or coiled with turns wide apart into a plane spiral. The latter form is sometimes made by wrapping a number of turns of wire around the butt of a pole before setting it.

Pipes Driven into the Ground. — The driven pipe or rod is probably the best of all forms of direct ground connections. The pipe is usually an iron one, often galvanized or sherardized, from ¾ inch to 2 inches in nominal size, driven 6 feet or more into the moistest ground available. The diameter of a pipe has little effect upon its earth resistance; thus a pipe 2 inches in diameter has a resistance only 6 to 12 per cent less than a pipe 1 inch in diameter (E. E. F. Creighton, *G. E. Rev., 1912, Vol. 15, p. 14*). To facilitate driving, the pipe is sometimes provided with a pointed casting at the lower end and cap at the upper end. The pipe has the advantage that it is cheap to install, as no hole is dug, and being driven vertically, it can tap any conducting stratum of the earth which is practically accessible. It is highly efficient as regards conductivity per square foot of surface exposed to the earth, and while the resistance of a single pipe will exceed that of a large plate, a higher and more permanent conductivity can be obtained from several pipes driven at some distance apart and connected in multiple, than from an equal expenditure in money on a single large ground plate. Ground pipes or plates in multiple should be not less than 6 feet, and preferably 10 feet, apart, in order to avoid superimposing two zones of high current density.

If the ground available is naturally dry the conductivity may be improved by cupping out the soil around the pipe at the surface and adding a few pounds of salt and water. A neater form of earth unit is shown in Fig. 3. A cylinder of metal or earthenware of any available diameter is set around the pipe at the surface of the ground and covered by a lid. This receptacle will hold the salt. Its advantages lie in the easy construction of the connection and protection of surrounding vegetation from the saline water.

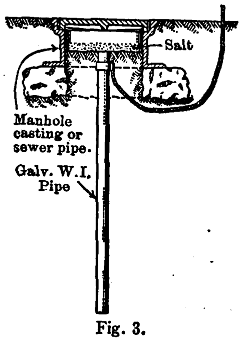

Fig. 3.

Pipe Grounds for Lightning Arresters. — The General Electric Company recommends a large number of grounds at each installation. These numerous ground connections are joined together by a copper connection. It sometimes happens that a good ground cannot be conveniently made near the arrester, or that a better one can be made at a more distant point. In this case it is recommended that the principal ground be made at the more distant point, but that a ground of some sort, the best possible under the conditions, be made directly underneath the arrester.

Connections to Ground Plates and Pipes. — The wire connecting the apparatus to be grounded to the buried conductor should lead directly to the latter with as few bends as possible; this is particularly important in grounding lightning arresters, in which case a flat strip is also better than a round wire. All exposed joints and those buried in the ground should be made in such a way that they will not rust off. The buried connection from a ground plate to the surface should be so protected that it will not rust off or be cut off when excavations are made for other purposes. It is also necessary that the connections be protected from mechanical injury and theft, from the surface of the ground

to above the level at which it can be reached. To insure reliability, connections should as far as possible be made so that their continuity can be determined by inspection.

Dimensions of Some Typical Ground Connections. —

PIPE GROUND CONNECTIONS

Name	Company	Iron pipe, galvanized	Diameter of pipe, inches	Depth in ground, feet	Connected to track rails also
F. B. Musser*.....	Central Penn. Tract. Co......	Yes	3/4	6	Yes
J. R. McFarlin*.. .	Electric Service Supply Co....	..	..	about 8	Yes
E. J. Cook*.......	N. Y. State Railways........	..	..	10	Yes
F. H. Miller*......	Louisville Ry. Co.	..	5/8 or 3/4 rod	8	Yes
R. McCulloch*....	United Rys. Co., St. Louis...	Yes	..	10	..
E. E. F. Creighton.	General Electric Co..........	..	1 to 2	6 or more	..

* Am. Elect. Railway Assn., 1911.

COPPER PLATE GROUND CONNECTIONS

Name	Company	Thickness, Stubs gauge	Size, Inches	Depth of charcoal above, inches	Depth of charcoal below, inches
C. F. Hewitt.	E. St. Louis & Suburban Ry. Co...............	16	24×24	6 to 8	6
....	New York Central R. R.	14	15×15	6	6

TESTS OF GROUND CONNECTIONS. — Ground connections should be frequently tested to determine their continuity and occasionally to determine their resistance. Alternating current should be used, in order to avoid the effects of polarization. In order to pass a current through a ground connection it is necessary to have a second connection in order to complete the circuit. Where a large water piping system is available for the return connection, an approximate measure of the resistance can be obtained by passing a current from the ground connection to an accessible part of the piping, say a hydrant, and computing the resistance by dividing the volts by the amperes. The resistance thus obtained includes both the resistance to be measured and that of the connection through the piping system. As the latter is probably much the smaller and may be insignificant, the result gives an approximate idea of the resistance and locates a discontinuity as an infinite resistance, that is, no current flowing. Where there are two similar grounds, current may be passed from one to another and the sum of the two resistances, or the average resistance, obtained.

When a more accurate determination of the individual resistances of two or more ground connections is desired, the connections shown in Fig. 4 may be employed and a curve obtained by test, similar to that shown in Fig. 2. The apparatus required comprises a source of alternating current, a rod P which may be driven into the earth, a high resistance wire BD stretched between the two ground connections G_1 and G_2, an ammeter A and a telephone receiver T. The contact C is slid along the resistance wire until the telephone receiver is silent. The drop of potential between G_1 and the rod is then the same as between G_1 and the contact C. The drop between G_1 and P is therefore equal to

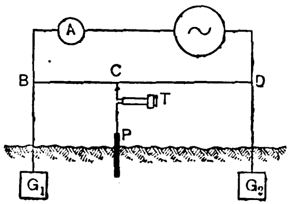

Fig. 4.

$$\frac{(\text{length of wire } BC)}{(\text{length of wire } BD)} \times (\text{total drop from } G_1 \text{ to } G_2).$$

By taking a series of observations with P driven into the ground at various points the curve of potential drop is readily plotted and the resistance of each ground connection, as defined above, readily obtained.

CARE OF GROUND PLATES AND PIPES. — The greatest trouble with ground plates and pipes arises from electrolysis, and periodic resistance measurements should be made as described above. When a ground connection shows an abnormally high resistance, it is usual to supplement it with a new ground connection, as it seldom pays to remove the old ones. Ground connections deteriorate rapidly if equipped with salt boxes; but even under the worst conditions they are likely to last many years. Practically no accurate measurements of the resistance of ground connections were made until quite recently, so that little is known about their life performance. J. L. R. Hayden cites the case of some pipe grounds which maintained the same average resistance for three years.

SPECIFICATIONS FOR GROUND PIPES AND PLATES. — (*See also article on Specifications.*) In the case of ground plates, the following data should be specified: Material of plate (usually copper); size of plate, area and thickness; depth at which the plate shall be buried; amount of charcoal below and above it; method of connecting the plate to its cable or wire (usually soldered and bolted); whether the plate shall also be connected to the track rails and if so, how.

In the case of pipe ground connections, the following data should be specified: Diameter of pipe; whether galvanized, sherardized, etc.; depth to which the pipe shall be driven; method of finishing the top of the pipe; height the pipe shall project above the ground; whether the pipe shall be connected to the track rails and if so, how.

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[W. A. DEL MAR.]

GROUND DETECTORS AND ARCING-GROUND SUPPRESSORS. — A ground detector is a device for indicating an accidental ground on a transmission line; an arcing-ground suppressor is an arrangement of switches and relays by means of which an arc tending to maintain an accidental ground is automatically extinguished

GROUND DETECTORS FOR NORMALLY UNGROUNDED SYSTEMS. — On ungrounded two-wire direct-current systems a voltmeter of the central-zero type connected between the ground and the middle point of a resistance joining the two line wires serves as a satisfactory ground detector. The direction of the deflection will indicate on which wire the ground has taken place, and the magnitude will indicate the nature of the ground. For alternating-current circuits, this method is unsatisfactory on account of capacity effects and also because of the higher voltages commonly involved.

Electrostatic Ground Detectors. — For single-phase ungrounded systems the simplest form of ground detector consists of two electrostatic voltmeters, each connected between one line wire and the earth. A ground on either wire causes an inequality in the readings of the two meters.

The more common form of ground detector operates as an electrostatic differential voltmeter. Fig. 1 shows diagrammatically the arrangement for a single-phase instrument. The movable vane V is connected to the earth, the fixed plates S_1 and S_2 are connected one to each of the two mains. Vane V will be equally attracted by the two plates unless the insulation resistances of the two line wires to ground are not equal, but will deflect from its normal position to one side or the other when a ground occurs.

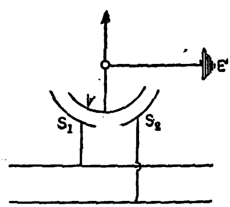

Fig. 1. Single-phase Electrostatic Ground Detector

The three-phase electrostatic ground detector for ungrounded circuits is made in two principal forms. One form consists of three separate elements, each similar to that used in the single-phase instrument, all three elements being in one case but having individual scales. Another form consists of a movable spherical vane, connected to the earth, and three fixed vanes, each of which is connected to one line wire. The movable vane in its normal position is equally distant from each one of the fixed vanes, but, in case of a ground on one line wire, it moves away from the fixed vane which is connected to the line wire on which the ground occurs. The principal parts of this instrument are clearly visible through the glass front of the instrument cover; hence no pointer and scale are needed. Two single-phase detectors will work satisfactorily on three-phase circuits, if the junction point of the two instruments is connected through a condenser to the third line wire.

Voltage Range. — Commercial sizes of electrostatic ground detectors range from 650 volts to 22,000 volts for single-phase circuits, and from 1150 volts to 22,000 volts for three-phase circuits.

Connections. — Electrostatic ground detectors are usually not connected directly to the line wires, especially if the voltage is above 3300. Some types are furnished with special terminal studs containing high-resistance rods of graphite through which the instrument is connected to the line wires. Other types are connected to the circuit through condensers, commonly of tubular form. Although, in this case, the condensers practically insulate the instrument from the line, the leads connecting the detector and the condensers must be treated as high-tension conductors, i.e., they should be properly separated and

far enough from neighboring metal in order that disturbing capacity effects may not be introduced. Therefore, the leads should be fairly short and must not be enclosed in metal conduit.

GROUND DETECTORS FOR NORMALLY GROUNDED SYSTEMS. — The methods outlined above, depending on the variable potential difference between mains and earth, do not apply to earthed systems, e.g., three-wire circuits with grounded neutral, or four-wire, three-phase circuits with grounded neutral. The most usual method for earthed circuits is to connect an ammeter in the earth circuit between the earth plate and the system; hence any leakage current will have to pass through the ammeter. A low-range instrument is desired if slight grounds are to be detected. In order to protect the instrument from excessive currents, an arrangement may be used by which the instrument is short-circuited as soon as the current reaches a certain value, or a resistance may be inserted in the circuit to reduce the current to a safe value. By the second method a short-circuit due to a ground would be suppressed without interruption of service on the line in question. At the same time, however, the pressure on the other line wires with respect to earth would be raised to a higher value.

ARCING-GROUND SUPPRESSORS. — Interruptions to service and damage to apparatus are frequently caused by grounds which tend to persist as arcs, due to the burning away of the conductor. Arcs of this kind may be intermittent, due, for example, to the swinging of the conductor back and forth. Arcing grounds may give rise to disastrous surges in the system. The arcing-ground suppressor, placed on the main bus at the generating station, removes arcs to ground on a line wire by automatically grounding that line wire at the power station, thereby short-circuiting the arc. It is obvious, from the nature of the device, that it cannot be used on earthed systems.

A three-phase arcing-ground suppressor consists of the following parts:

(*a*) Three single-pole, motor-operated, oil switches each connecting one phase of the power house main bus to the ground. These switches are provided with interlocking relays, each of which has three contacts, one of which closes the tripping circuit of its own oil switch while the other two open the tripping circuits of the other two oil switches. Hence it is impossible to have more than one oil switch closed at the same time. For use with overhead systems each switch is equipped with the second-stroke lock device described later. The oil switches are provided with the usual remote-control switches with red and green lights to be placed on the station switchboard.

(*b*) Three single-pole, single-throw disconnecting switches for disconnecting the oil-switches.

(*c*) An electromagnetic selective relay for detecting the grounded phase and operating the proper oil switch. This selective relay is connected to the main bus through three Y-connected transformers of which the neutral on the high-tension side is grounded.

Operation when used with Overhead Lines. — The arcing-ground suppressor is suitable for overhead lines only when metal towers or poles of relatively low resistance are used, as on a wooden pole line the resistance of the pole is liable to prevent sufficient current flowing to ground to reduce the potential sufficiently to operate the phase selective relay.

The operation is as follows: If a ground occurs on one wire in the system, the unbalancing in the potentials of the grounded Y-connected transformers causes the phase selective relay to be operated in such a way that the proper interlocking relay is thereby energized. This closes the corresponding oil switch, thereby grounding the line wire, on which the arcing ground has occurred, through a

metallic circuit. Thus the arcing ground is almost instantly cleared, and the oil switch opens automatically after a fractional part of a second, and remains open, provided the normal state of insulation of the line is reëstablished after the suppression of the arc to ground. Whenever an arc is established or broken, high-frequency oscillations are set up, the duration of which depends on the amount of damping resistance in the circuit (*see Transmission Lines*). In order to eliminate the dangers from such oscillations, a damping resistance placed in the switch pot is thrown in series before the switch rod closes the main contact to ground and after it has opened the main contact.

If the line insulation is permanently broken down by the arc, the line potential will establish a second arc to ground as soon as the metallic ground through the oil switch is opened. The second-stroke lock device then operates; it locks the oil switch as it closes, provided the second closing of the switch occurs immediately after it has been opened. In this case the system remains grounded on one side, and can continue to operate with the metallic ground. After the trouble has been cleared, the oil switch is opened by the attendant.

Operation when used with Underground Cables. — When the arcing-ground suppressor is used for the protection of cables, the second-stroke lock device is omitted, as it is desirable for the oil switch not to be opened after it has once been closed until the feeder has been cleared, the reason being that in a cable the distance of a conductor from the sheath is usually so small that puncture of the insulation produces a permanent fault, and the normal difference of potential is sufficient to reëstablish an arc, even if it were automatically extinguished. The arcing-ground suppressor on cable systems, therefore, minimizes the injury due to a puncture in the cable and prevents the trouble from spreading, and suppresses dangerous high-frequency surges due to arcing grounds. It is not intended to clear short-circuits between line wires.

Location of Arcing-ground Suppressor. — The best location for the arcing-ground suppressor is at the power house, and directly connected to the high-tension bus, where it may be under the immediate observation of the station operator, where a direct-current supply is available, and where the more important switching operations in case of grounds are usually performed.

Costs. — A single-phase electrostatic ground detector costs from $30 to $60, depending upon the voltage, and a three-phase electrostatic ground detector costs from $50 to $150. A three-phase arcing-ground suppressor for voltages from 11,000 to 110,000 costs approximately from $2200 to $6000.

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[O. R. SCHURIG.]

GROUNDING OF ELECTRIC CIRCUITS. — (*See also Generators, Alternating-current; Ground Connections; Transformers.*) Some electric circuits are operated insulated, that is with no intentional electrical connection from any point of the circuit to the ground, while others are grounded by one or more conductors provided for the purpose.

Among the considerations influencing the decision of this point are: (1) danger of shock to persons touching the circuit; (2) danger of fire from escaping current, spark, or arc; (3) elimination of electrical oscillations set up in a circuit due to the sudden changes in current resulting from the making and breaking of an arc, usually to ground, usually referred to as an "arcing ground;" such grounds may give abnormal voltages particularly on a-c. circuits where the arc forms and is extinguished twice every cycle; (4) decreased strain on circuit insulation in case of accidental ground; (5) the utilization of the ground and conductors on or in the ground for carrying return currents; (6) the danger of electrolysis and disturbance of telegraph and telephone systems by currents escaping to the ground.

CURRENT (1914) PRACTICE REGARDING GROUNDING. — While the practice of different companies is not consistent with respect to insulating or grounding of various classes of circuits, the following rules probably cover the best practice at the present time.

(1) Low-voltage circuits should be insulated where they are in no way exposed to direct or indirect crossing with other circuits of higher voltage.

(2) Low-voltage circuits should be grounded if there is any danger of crossing with circuits of higher voltage.

(3) Intermediate voltage circuits should be insulated when practicable.

(4) Intermediate voltage circuits if grounded (600-volt railway circuits, for example) should be handled only by experienced people and should be inaccessible to the public.

(5) High-voltage circuits are always dangerous and may be insulated or grounded according to other conditions than safety from shock.

The reasons for the above practice are discussed below.

ELECTRIC SHOCK FROM GROUNDED AND UNGROUNDED CIRCUITS. — Of the various considerations affecting the desirability of grounding a circuit, that of shock is perhaps the most important. (*See also article on Shock, Electric.*) A person touching a circuit at any two points between which there is a difference of potential will receive a shock. The danger or severity of shock from touching simultaneously two line wires of a circuit is not affected by grounding the circuit, but the danger or severity of shock from touching simultaneously either wire and the ground does depend upon whether the circuit is grounded or not.

It should be noted that the sensitiveness of different people to shock varies a great deal and that the severity of a shock depends as much on the surface resistance of the skin (whether dry or wet), and on the parts and organs of the body through which the current passes, as on the voltage. It is therefore impossible to say that any voltage used in practice is so low that under no condition can it give a dangerous shock. However, considering ordinary conditions and the result of a majority of the shocks, circuits of voltages up to and including 220-volts may be considered as not liable to cause a serious shock, whether grounded or ungrounded, and may be referred to as "low-voltage" circuits.

Voltage to Ground of Grounded Circuit. — When a circuit is intentionally grounded, the voltage of the grounded point is made permanently that of the

earth, and the potential of every other point of the circuit becomes fixed with respect to the ground and may be readily calculated (*see Kirchhoff's Laws in index*). Each conductor then carries a definite, predetermined risk, instead of an indefinite risk, which may be nothing or may be very great.

Voltage to Ground of Ungrounded Circuit. — Under normal conditions the average potential of an insulated circuit is the same as that of the ground, the positive parts of a circuit having a potential above and the negative parts below that of the ground. These differences of potential cause the positive and negative wires to be charged like the plates of condensers, the positive wire and earth forming one condenser and the earth and negative wire the other. The positive and negative charges on the wires will be equal and the resultant charge on the earth will be zero in all cases. These condensers are in series and if the circuit is symmetrical the two condensers will have equal capacity. Then the voltage of the positive wire will be as much higher than that of the earth as that of the negative wire is below earth potential. In an unsymmetrical circuit the potentials of the positive and negative wires with respect to earth will be inversely as the capacities.

Current Through Ground Connection. — In case of a ground, either intentional or accidental, the grounded point of the circuit is brought to ground potential and a change is made in the voltage to ground of the positive and negative parts of the circuit, and the resultant charge on the earth is no longer zero. This charge must come by conduction from the circuit and must therefore enter the earth through the ground connection. As long as there is a change in voltage of the wires of the circuit due to the ground connection a current will flow through the ground connection. Such a current is only momentary in the case of a d-c. circuit, but flows as long as the ground continues with alternating currents. See also article on *Capacity and Charging Current.*

The number of amperes which flows through such a ground connection is roughly proportional to the voltage of the system and its electrostatic capacity to ground. The electrostatic capacity is proportional to the extent of the circuit (miles of line). As the miles of line of commercial circuits ordinarily increase about in proportion to the voltage, it follows that the amperes to ground will be about as the square of the voltage of the circuit. The energy which can be liberated at the point of accidental ground will be proportional to this current multiplied by the voltage, that is, will vary as the cube of the voltage. In addition to current due to the capacity to ground there may be a leakage current due to imperfect insulation.

Shock from Single Contact with High-voltage Circuits. — Circuits having a normal line voltage of 11,000 volts or over are generally of sufficient extent (and therefore have sufficient electrostatic capacity) so that a fatal shock may be received from wire to ground, due to the current passing to ground through the body, even though the circuit be otherwise perfectly insulated from ground.

Shock from Intermediate Voltage Circuits. — In general the power available for shock in the current escaping from an insulated circuit to ground is small compared to that in case of contact with the two wires of the circuit. Consequently, there is a class of circuits whose voltages are high enough to give serious or fatal shocks where contact is made with two points of the circuit, but not high enough to give serious (or sometimes even appreciable) shocks from wire to ground when the circuit is ungrounded. These circuits are intermediate between the low-voltage (220-volt) and high-voltage (11,000-volt) circuits above mentioned. This class includes 440- and 550-volt power circuits and usually 1100- and 2200-volt primary circuits, if leakage is small.

Shock from Low-voltage Circuit Crossed (in Contact) With High-voltage Circuit. — When an insulated circuit becomes crossed with a circuit of higher voltage it becomes charged to the voltage of the latter circuit at the point of contact. It becomes practically a part of the higher voltage circuit and is equally dangerous to touch. As its insulation is not designed to stand such abnormally high voltage, there is danger of shock even through the insulation of the conductor.

When a grounded circuit is crossed with one wire of a higher voltage circuit, this wire tends to come to ground potential, and the voltage of the several parts of the low-voltage circuit with respect to the ground remains as before. In cases where a very large amount of current escapes from the high-voltage circuit, this will be appreciably modified by the addition of a voltage due to the resistance or impedance drop in the low-voltage circuit due to the current from the high-voltage circuit from point of contact to ground. If, for example, an outside wire of a three-wire, 110/220-volt, alternating-current secondary circuit, with grounded neutral, should become crossed with a 11,000-volt grounded circuit, a large amount of current may flow into it, which can only escape to the ground after passing through the lamps on one side of the circuit, or through the transformer secondary. The impedance of the path through the lamps and transformer may be high enough so that the voltage of the crossed outside wire of the secondary may be raised to a dangerous voltage above that of the grounded neutral.

Conclusions regarding Effect of Grounding with respect to Shock. — The considerations on which above classification is based lead to the conclusions:

(1) The insulating of low-voltage (under 220 volts) circuits decreases the danger of shock under normal conditions, but as such shocks would rarely be serious, there is little gain in safety through the insulation of the circuit.

(2) The grounding of low-voltage (under 220 volts) circuits greatly decreases the danger of fatal shocks in cases where the low-voltage circuits become crossed with one of higher voltage.

(3) The insulating of intermediate voltage circuits (440 to 2200 volts) greatly decreases the danger of shock under ordinary conditions because most shocks are obtained from touching one conductor, and in such circuits the electrostatic capacity is ordinarily so small that no appreciable current flows to ground.

(4) The grounding of intermediate voltage circuits (440 to 2200 volts) does not greatly decrease the danger of fatal shock in case the circuit becomes crossed with one of higher voltage because the normal voltage of the circuit when grounded is itself dangerous.

(5) Neither the insulating nor grounding of a high-voltage circuit (above 11,000 volts) materially changes the danger of shock which is very great in either case.

DANGER OF FIRE FROM GROUNDED AND UNGROUNDED CIRCUITS. — In general, conditions which diminish the danger of electric shocks also diminish the danger of fire.

The Rules and Requirements of the National Board of Underwriters is the principal authority on the subject of grounding from the standpoint of fire hazard. These rules, given in the National Electrical Code (1911 edition), provide (rule 15) for the grounding of "low-potential systems" (550 volts and under), provided the ground is made on conductor mentioned below:
The rules state that the direct-current system "may be grounded," and that the alternating system "should preferably be grounded." There is no requirement for grounding or insulating of systems having voltages in excess of 550.

	Direct current	Alternating current
2 wire	Not to be grounded	One side for voltage not exceeding 250
3 wire	Neutral only	Neutral only

The rules provide (rule 15) that the grounding of low-potential circuits is only allowed when such circuits are so arranged that under normal conditions of service, there will be no flow of current through the ground connection.

The Committee of the N. E. L. A. on Grounding Secondaries has presented reports at the annual conventions since 1907. These reports discuss the reasons for, and progress in, grounding of secondary distribution circuits.

This committee reported in 1912 the results of a conference with committees from the American Institute of Electrical Engineers, Association of Edison Illuminating Companies and National Inspectors Association, at which it was unanimously voted to recommend to the Electrical Committee of the National Fire Protection Association changes in Rule 15, National Electrical Code (1911 edition), so as to require that transformer secondaries of distributing systems *must* be grounded, provided the maximum difference of potential between the grounded point and any other point in the circuit does not exceed 150 volts, and that where the maximum difference of potential between the grounded point and any other point in the circuit exceeds 150 volts, grounding *may be permitted*.

GROUNDING OF HIGH-VOLTAGE CIRCUITS. — The grounding of low-voltage circuits is governed by considerations of danger of shock and fire, while that of high-voltage circuits is for the purpose of increasing the reliability (continuity of service over the line), or of decreasing the cost of transmission by decreasing or limiting the voltage strain on the line insulator and transformer insulation; see also article on *Generators, Alternating-current* and section on *Transformer Connections* in the article on *Transformers*.

Eliminating Arcing Grounds by Metallic Grounding. — In normally ungrounded high-voltage circuits the current to ground is sometimes sufficient to maintain an arc between one of the wires and a grounded conductor near it but not quite in contact with it, producing a so-called "arcing" ground. Such grounds have been found to produce destructive voltages on the circuit. By metallically grounding the neutral or one wire of the circuit the rise of voltage is reduced. This is perhaps the main reason why many high-voltage circuits are grounded.

Limiting Insulation Strain by Grounding the Neutral. — In high-voltage circuits the cost of insulators is an important element. In an ungrounded circuit each insulator must be large enough to stand full line voltage (between wires), for any phase may become grounded accidentally. Where the neutral of the circuit is grounded the maximum voltage on any insulator can never exceed the normal voltage to ground. The voltages to neutral of a single- or two-phase system is 50 per cent of the voltage between lines, and the voltage to neutral of a three-phase system is 58 per cent of the voltage between lines. A circuit with grounded neutral therefore requires insulators of from 50 to 58 per cent of those for same circuit with insulated neutral, or for the same size insulator the line voltage can be from 73 to 100 per cent higher with grounded neutral than without. These figures neglect the fact that the maximum strain is constant for grounded neutral, and occurs only for a short time at irregular intervals with insulated neutral. In practice the same insulators are usually

used whether the neutral is grounded or not, the factor of safety being higher when the neutral is grounded.

Limiting Short-circuit Current by Resistance.— In a grounded circuit every accidental ground becomes a short-circuit. As such short-circuits may be destructive to generating machinery, the current is sometimes limited by a resistance in the ground connection (usually made at the neutral). The amount of resistance required depends on the per cent of short-circuit current which is permissable. The resistance has, however, the disadvantage that it increases the strain on the insulation, so that as the short-circuit current is reduced, the benefit to the insulation is also reduced. Where the circuit is grounded as a remedy for arcing grounds and no increased factor of safety on the insulator for normal condition of operation is desired, the use of resistance in the ground connection is allowable.

Use of Ground for Return Circuit. — Connections to ground are little used as return paths for the normal current of the circuit, except for railway work. Formerly many 500-volt direct-current power circuits, operated from railway circuits which were necessarily grounded, used the ground as a return circuit, though usually only for small or outlying motors. Such connections may cause electrolysis in underground pipes and arcs, in cases where the pipes or other foreign conductors over which the currents are returning, are broken. Present practice on light and power distribution is to provide complete metallic circuits for all currents which will flow in a circuit under normal operating conditions. Where one conductor is to be intentionally grounded the escape of current into the ground may be prevented by grounding it at only one point, or where, on account of great extent of circuit, grounds on same conductor are necessary at several points, the escape of current will be reduced to a minimum, by making the ground connection on the neutral instead of an outside wire.

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[R. A. PHILIP.]

GUTTA-PERCHA. — (*See also Insulating Materials; Telegraph Instruments and Circuits; Wires and Cables, Insulated.*) Gutta-percha is derived from the milky secretion or latex of the bark of certain trees of the order of Sapotaceæ, especially the Dichopsia Gutta, found chiefly in the Straits Settlements and Malaccan Archipelago. The trees are felled immediately after the rainy season, and the gutta or gum collected as it exudes from incisions in the bark. Latex is also extracted from the leaves by digesting them in toluol. However it may be extracted, the latex is boiled in water and it is then ready for export.

The chemical composition of gutta-percha is represented by the formula $C_{10}H_{16}$. It resembles dark brown leather at temperatures between 0° C. and 27° C. At higher temperatures it softens, and at 65° C. it is plastic and capable of being molded or rolled. On cooling it returns to the non-plastic condition.

Gutta-percha oxidizes when exposed to the air, changing from dark brown or black to yellowish grey and becoming brittle.

PREPARATION OF GUTTA-PERCHA INSULATION. — For insulating purposes gutta-percha is shredded and squeezed in warm water. It is then kneaded and strained through fine-wire gauze and rolled into sheets. Its further refinement is carried on differently by various manufacturers, the processes being more or less trade secrets. Like rubber it is applied to the wire either by a tubing machine or by strips. Unlike rubber it is used in the pure state without mixture with minerals. Gutta-percha is less porous than rubber and therefore more waterproof, a quality which makes it the best material for submarine cables. Its specific gravity is just above unity.

SPECIFIC RESISTANCE. — The constant K in the formula

$$M = K \log \frac{D}{d}$$

has the value 900 approximately, at 75° F. after one minute electrification. See also article on *Rubber*.

Temperature Coefficient of Resistance. — The temperature coefficient of resistance of gutta-percha is of the same nature as that of rubber (*see article on Rubber*), i.e.,

$$R_T = R_{75}\epsilon^{(75-T)C},$$

where R_{75} is the resistance at 75° F., R_T the resistance at T° F. and C a constant which varies from 0.065 to 0.085. For values of ϵ^x see *Exponential Functions.*

Effect of Pressure upon Resistance. — Gutta-percha being used principally for submarine cables, the effect of pressure upon its resistance is important. Let R = its resistance at atmospheric pressure, R_p = resistance under pressure of p pound per square inch.
Then

$$R_p = R\,(1 - 0.00023\,p).$$

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[W. A. DEL MAR.]

HEAT AND THERMAL PROPERTIES. — (*See also Temperature and Thermometers; Thermodynamics, Principles of.*)

The following is a brief table of contents of this article:

Heat is said to be added to a body, or the body is said to absorb heat, (1) whenever its temperature rises, (2) whenever it passes from a solid to a liquid state, or (3) whenever it passes from a liquid to a gaseous state; when the reverse of these changes takes place the body is said to lose or to give out heat. A body can also absorb (or give out) heat without any of these changes taking place, provided it gives out (or absorbs) at each instant an amount of energy of some other form equivalent to that absorbed (or given out) as heat. The heat absorbed or given out by a body in virtue of a change in its temperature is called "sensible" heat; heat absorbed or given out in passing from one state to another is called "latent" heat. Experiment shows that heat may be considered as a form of energy.

Symbol for Heat (H). — Both H and Q are commonly used to designate quantity of heat. The symbol H is used throughout this article.

UNITS OF HEAT. — There are several arbitrarily chosen units of heat, viz.,

15° Gram-Calorie or Small Calorie. — The heat necessary to raise the temperature of 1 gram of water from 14.5° C. to 15.5° C. This is the unit commonly employed in scientific work.

Mean Gram-Calorie or Small Calorie. — The $\frac{1}{100}$th part of the heat required to raise the temperature of 1 gram of *water* from 0° C. to 100° C., the latent heat of fusion and boiling not being included. According to Marks and Davis (*Steam Tables and Diagrams, N. Y., 1912*), the 15° calorie and the mean small calorie differ by less than one-tenth of one per cent.

Kilogram-Calorie or Large Calorie. — The heat required to raise the temperature of 1 kilogram of water from 14.5° C. to 15.5° or the $\frac{1}{100}$th part of the heat required to raise the temperature of 1 kilogram of water from 0° C. to 100° C. The relation between the gram-calorie and the kilogram-calorie is then

$$1 \text{ kilogram-calorie} = 1000 \text{ gram-calories.}$$

Ostwald Calorie. — The heat required to raise the temperature of 1 gram of water from 0° C. to 100° C. This unit is frequently used by electrochemists.

$$1 \text{ Ostwald calorie} = 100 \text{ mean gram-calories.}$$

British Thermal Unit (B.t.u.). — The heat required to raise the temperature of 1 pound of water 1° F. There is no general agreement as to *which* degree of temperature shall be used; Peabody uses the degree from 62° F. to 63° F. Marks and Davis define the British thermal unit as the $\frac{1}{180}$th part of the heat required to raise the temperature of 1 pound of water from 32° F. to 212° F. The B.t.u. as defined by Marks and Davis is about 0.13 per cent greater than the B.t.u. as defined by Peabody. Marks and Davis's definition is adopted throughout

this book; the difference between the two is negligible for ordinary practical work. The relation between the B.t.u. and the mean kilogram-calorie is

$$1 \text{ B.t.u.} = 0.25200 \text{ kilogram-calories.}$$

Mechanical Equivalent of Heat. — This is the name given to the experimentally determined conversion factor between any heat unit and any unit of mechanical work; see *Units and Conversion Factors*. The fundamental relation is

$$1 \text{ mean gram-calorie} = 4.1834 \times 10^7 \text{ ergs.}$$

This is the value given by Marks and Davis.

THERMAL CAPACITY AND SPECIFIC HEAT. — The "thermal capacity" of a body is defined as the heat absorbed by the body per unit increase in its temperature, there being during this change in temperature no change of state (e.g., no change from solid to liquid or from liquid to gaseous form or no chemical change) and no transfer of heat energy from the body in question to other bodies. The thermal capacity *per unit mass* of a substance is approximately constant, but increases slightly with increase in temperature; in the case of iron the increase with temperature is quite marked. Calling C the thermal capacity per unit mass of a substance the heat absorbed by a homogeneous mass M when its temperature increases from t_1 to t_2 is

$$H = CM\,(t_2 - t_1)$$

provided C is constant.

The mean thermal capacity per unit mass of water (between 0° C. and 100° C.), when expressed in mean gram-calories per gram per degree centigrade, is numerically equal to unity. The ratio of the thermal capacity per unit mass of any substance to the mean thermal capacity of water is called the "specific heat" of the substance. The specific heat of a substance does not depend upon the units in which the various quantities are measured; its thermal capacity per unit mass does. When heat is expressed in mean gram-calories, mass in grams and temperature in degrees centigrade, the thermal capacity per unit mass is equal to its specific heat; compare with density and specific gravity.

Calculation of Heat Absorbed or Given Out. —

C = specific heat (gram-calories per gram per °C.),
M = mass heated,
$t_2 - t_1$ = rise of temperature.

Then for any set of units the heat absorbed is

$$H = kCM\,(t_2 - t_1),$$

where k has the following values:

VALUES OF k

Unit of heat or energy	Unit of mass	Temperature scale	Value of k
Gram-calorie	Gram	Centigrade	1.000
Kilogram-calorie	Kilogram	Centigrade	1.000
B.t.u	Pound	Centigrade	1.800
B.t.u	Pound	Fahrenheit	1.000
Watt-second (joule)	Gram	Centigrade	4.183
Watt-second (joule)	Pound	Fahrenheit	1054
Kilowatt-hour	Kilogram	Centigrade	1.162×10^{-3}
Kilowatt-hour	Pound	Fahrenheit	2.928×10^{-4}

Values of Specific Heat. — In the table below are given the values of the specific heat for the more common substances used in engineering work. These numbers are also equal to the thermal capacity per unit mass, when mass is expressed in grams, temperature in degrees centigrade and heat in gram-calories.

TABLE I. — SPECIFIC HEAT OF SOME COMMON SUBSTANCES

(From Landolt-Börnstein Tables; see also article on Pyrometers.)

Substance	Temperature, °C.	Specific heat C.	Substance	Temperature, °C.	Specific heat C.
Air (*a*)	–102 to 440	0.237	Lead	17 to 100	0.031
Aluminum	15 to 435	0.236	Manganin (*e*)	18	0.097
Ammonia	23 to 216	0.520	Manganin (*e*)	100	0.100
Antimony	22 to 600	0.052	Marble	0 to 100	0.206
Asbestos	20 to 98	0.195	Mercury	0	0.0335
Bismuth	–79 to 100	0.029	Mercury	100	0.0326
Brass (*b*)	20 to 100	0.092	Mica	20 to 98	0.208
Bronze (*c*)	20 to 100	0.104	Molybdenum	20 to 550	0.072
Carbon (gas carbon)	20 to 1040	0.315	Nickel	0 to 105	0.108
Carbon (graphite)	0 to 3000	0.535	Nitrogen (*a*)	0 to 200	0.244
Carbon dioxide (*a*)	– 78 to 7	0.184	Nitrous oxide (*a*)	13 to 172	0.231
Carbon dioxide (*a*)	0 to 200	0.215	Oxygen (*a*)	20 to 440	0.224
Carbon monoxide (*a*)	23 to 198	0.243	Osmium	19 to 98	0.031
Cement (Portland)	28 to 30	0.271	Palladium	0 to 100	0.059
Chlorine	13 to 202	0.124	Palladium	0 to 1265	0.071
Cobalt	15 to 350	0.109	Paraffine	25 to 30	0.589
Constantan (*d*)	18	0.098	Petroleum	21 to 58	0.511
Constantan (*d*)	100	0.102	Petroleum	18 to 99	0.498
Copper	–188 to 20	0.080	Platinum	0 to 100	0.032
Copper	0 to 100	0.094	Rhodium	10 to 97	0.058
Copper	300	0.098	Silver	0 to 260	0.057
Copper	900	0.126	Steam (*g*)	....	.....
Cork	....	0.485	Steel	20 to 100	0.118
Cotton	0 to 100	0.362	Tantalum	–185 to 20	0.033
Ebonite	....	0.339	Tin	17 to 100	0.056
German silver	0 to 100	0.095	Tungsten	20 to 100	0.034
Glass	0 to 19	0.171	Wax (yellow)	26 to 42	0.820
Glass	56 to 78	0.192	Wood's metal (*f*)	5 to 50	0.035
Gold	0 to 100	0.032	Wool	....	0.393
Hydrogen (*a*)	– 28 to 198	3.41	Zinc	20 to 100	0.093
Ice	– 78 to –18	0.463			
Iridium	0 to 100	0.032			
Iron, cast	18 to 100	0.113			

(*a*) At constant pressure of 1 atmosphere.
(*b*) 60 Cu + 40 Zn.
(*c*) 88.7 Cu + 11.3 Al.
(*d*) 60 Cu + 40 Ni.
(*e*) 84 Cu + 4Ni + 12 Mn.
(*f*) 25.85 Pb + 6.99 Cd + 52.43 Bi + 14.73 Sn.
(*g*) See article on *Steam*.

Specific Heats at Constant Volume and Constant Pressure.—In general, when a body absorbs heat an expansion (contraction in a few cases) results and the body does work on whatever opposes this expansion, part of the heat absorbed being thus converted into mechanical work; see *Thermodynamics, Principles of*. In the case of solids and liquids the external work done is practically negligible. In the case of gases and vapors, however, the external work done is appreciable. The specific heat of a gas or vapor when kept at constant volume, so that it can do no external work, is called the specific heat at constant volume, and is usually designated by the symbol C_v. The specific heat of a gas or vapor when it is allowed to expand at constant pressure is called the specific heat at constant pressure, and is usually designated by the symbol C_p. The ratio

$$\gamma = \frac{C_p}{C_v}$$

has very nearly the same value, 1.40 approximately, for all ordinary (diatomic) gases; see Table II.

TABLE II.—VALUE OF $\gamma = \frac{C_p}{C_v}$ FOR SOME GASES AT ATMOSPHERIC PRESSURE

(*From Landolt-Börnstein Tables.*)

Gas	Temperature, °C.	$\gamma = \frac{C_p}{C_v}$	Gas	Temperature, °C.	$\gamma = \frac{C_p}{C_v}$
Air	−181	1.34	Chlorine	20–340	1.32
Air	0	1.40	Hydrogen	......	1.40
Air	900	1.39	Nitrogen	......	1.40
Ammonia	0–100	1.30	Nitrous oxide	0–100	1.29
Carbon dioxide	0–100	1.30	Oxygen	......	1.40
Carbon monoxide	0–100	1.40			

Recalescence.—When heat is supplied at a uniform rate to a piece of iron or steel it is found that the rate of increase of temperature gradually increases (i.e., uniform increase of specific heat) until a certain temperature is reached at which the rise of temperature is suddenly and in most cases greatly retarded or even completely arrested. The reverse of this effect occurs when the sample is cooled down from a temperature above this point, and under certain conditions there occurs a spontaneous reheating during the cooling. Any point at which there is an abrupt change in the slope of a heating or cooling curve is called a "recalescence" point; there is a very marked recalescence point for most irons and steels and also one or more points at which the same effect occurs but to a lesser degree. The major recalescence point of ordinary iron or steel is usually between 750° C. and 850° C.

MELTING OR FREEZING POINT AND HEAT OF FUSION.—Certain chemically simple substances when heated to a definite temperature pass from the solid to the liquid state with no increase in temperature during this change in state, provided the solid and liquid are kept thoroughly mixed, but the change is accompanied by a considerable absorption of heat. The temperature at which the change takes place is called the melting point or freezing point (the reverse change takes place at the same temperature), and the heat

absorbed per unit mass is called the heat of fusion or heat of liquefaction; this same amount of heat is given out when the body solidifies. In the case of many substances, however, there is no definite melting point, the change from one state to the other being gradual; such substances begin to melt at a lower temperature than that at which solidification begins during cooling. The melting points and heats of fusion for some common substances are given in Table III. The values printed in bold face type are recommended by the Bureau of Standards as suitable for pyrometer calibration.

TABLE III. — FUSION AND VAPORIZATION

(At Atmospheric Pressure, i.e., 760 mm. Mercury.)

Substance and References	Melting point, ° C. *	Heat of fusion, gr-cal. per gr.†	Boiling pt., ° C.*	Heat of vap't'n gr-cal. per gr.†
Aluminum (1, 2, 3)	**658.7**	76.8	1800–2200	.....
Ammonia (3)	−77.7	108.1	−33.5	321.3
Antimony (1)	**630.0**	.....	1440	.....
Bismuth (1, 3)	271	12.64	1420	.....
Brass	900±	.....	.....	.....
Bronze	900±	.....	.....	.....
Cadmium (1, 2, 3)	**320.9**	13.66	770	.....
Carbon (1, 3)	Over 3600	.....	Over 3600	.....
Carbon dioxide (3)	−79	.....	−79	.....
Carbon monoxide (2, 3)	−203	.....	−190	51.2
Chlorine (1, 3)	−101.5	22.96	−33.7	61.9
Chromium (1, 3)	510	.....	2200	.....
Cobalt (1)	1490	.....	.....	.....
Copper (1, 3)	**1083.0**	43.0	2100–2310	.....
German silver	1100±	.....	.....	.....
Glass, flint	1300	.....	.....	.....
Gold (1, 3)	**1063.0**	.....	2200	.....
Gutta percha	100	.....	.....	.....
Hydrogen (1, 3)	−259	.....	−252.6	.....
Iridium (1, —)	2300	.....	2535	.....
Iron (1, 3)	1520	23.0 to 34.0	2450	.....
Lead (1, 2, 3)	**327.4**	5.86	1525	.....
Manganese (1, 3)	1225	.....	1900	.....
Marble	2500±	.....	.....	.....
Mercury (1, 3)	−38.7	2.85	357.2	62.0

* Let t_c be the value in °C.; then the value in °F. is $t_f = 32 + 1.8\,t_c$.

† Let H be the value in gram-calories per gram; then the corresponding heat of fusion or of vaporization

In kg-cal. per kg. is	$1.000\,H$,
In watt-seconds per gram is	$4.183\,H$,
In kw-hr. per kg. is	$1.162 \times 10^{-3}\,H$,
In kw-hr. per lb. is	$5.271 \times 10^{-4}\,H$,
In kw-hr. per ton (2000 lbs.) is	$1.054\,H$.

References: (1) Bureau of Standards, Cir. No. 35; (2) Smithsonian Physical Tables, 1910; (3) Landolt-Börnstein-Roth, Physikalisch-Chemische Tabellen, 1912.

TABLE III.—FUSION AND VAPORIZATION — *Continued.*

(*At Atmospheric Pressure, i.e., 760 mm. Mercury.*)

Substance and References	Melting point, ° C.*	Heat of fusion, gr-cal. per. gr.†	Boiling pt.,° C.*	Heat of vap't'n gr-cal. per gr.†
Molybdenum (1)	2500	.....	.....	.....
Nickel (1, 2, —)	1452	4.64	2325	.....
Nitrogen (1, 3, 2)	−210	.....	−195.7	47.6
Nitric oxide (3)	160.6	.....	153	.....
Oxygen (1, 3)	−218	.....	−182.9	51.0
Osmium (1, —)	2700	.....	2600	.....
Palladium (1, 3)	1549	36.3	2535	.....
Paraffine	52.4	35.1	.....	.....
Platinum (1, 3, —)	1755	27.2	2450	.....
Rhodium (1, —)	1940	.....	2500	.....
Rubber	100	.....	.....	.....
Selenium (1, 3)	217 to 220	.....	688	.....
Silicon (1)	1420	.....	.....	.....
Silver (1, 3)	960.5	21.1	1955	.....
Steel	1300 to 1475	.....	.....	.....
Sulphur (1, 3)	107 to 119	93.7	444.6	362.0
Tantalum (1)	2850	.....	.....	.....
Tin (1, 3)	231.9	14.25	2270	.....
Tungsten (1)	3000	.....	3700	.....
Vanadium (1)	1730	.....	.....	.....
Wax, bees (2)	61.8	42.3	.....	.....
Wood's metal (3)	75.5	7.63	.....	.....
Zinc (1, 3)	419.4	28.1	930	.....

* Let t_c be the value in ° C.; then the value in ° F. is $t_f = 32 + 1.8\, t_c$.

† Let H be the value in gram-calories per gram; then the corresponding heat of fusion or of vaporization

In kg-cal. per kg. is	$1.000\ H$,
In watt-seconds per gram is	$4.183\ H$,
In kw-hr. per kg. is	$1.162 \times 10^{-3}\ H$,
In kw-hr. per lb. is	$5.271 \times 10^{-4}\ H$,
In kw-hr. per ton (2000 lbs.) is	$1.054\ H$.

References: (1) Bureau of Standards, Cir. No. 35; (2) Smithsonian Physical Tables, 1910; (3) Landolt-Börnstein-Roth, Physikalisch-Chemische Tabellen, 1912.

Freezing Mixtures. — The addition of an impurity to a liquid lowers the freezing point, a common example of which is the lowering of the freezing point of water by the addition of salt. Also, when certain substances go into solution the temperature of the solution is lowered. Some common freezing mixtures are the following, taken from *Hütte.*

FREEZING MIXTURES

Mixture	Parts by weight	Decrease of temp., °C. From	Decrease of temp., °C. To	Parts by weight	Decrease of temp., °C. From	Decrease of temp., °C. To
Snow Common salt (NaCl)	3 1	0	−17.7	1 1	0	−18
Snow Calcium chloride ($CaCl_2$)	2 3	0	−33	1 2	0	−42
Snow Potassium hydrate (KOH)	3 4	0	−37	..	..	....
Water Ammonium nitrate (NH_4NO_3)	1 1	+10	−16	..	..	....
Water Sal ammoniac (NH_4Cl) Saltpeter (KNO_3)	16 5 5	+10	−12	1 1 1	+8	−24

VAPORIZATION. — Above the surface of any liquid there always exists a certain amount of the substance in a gaseous form, i.e., as a vapor, the amount of which depends upon the nature of the substance and upon the temperature and the pressure in the space occupied by the vapor and such other gases (e.g., air) as may be present. In the case of a simple liquid evaporating into a space from which all other gases and vapors have been removed, the evaporation ceases when a definite pressure is established in this space, this equilibrium pressure depending only upon the temperature at which this space is maintained. This statement is true only when there always remains some unevaporated liquid; if all the liquid evaporates, then the equilibrium pressure also depends upon the mass of the vapor and the space which it occupies; as long as some liquid remains, the equilibrium pressure depends only upon the temperature and is independent of the volume of the space and the mass of the vapor which occupies it. This equilibrium pressure for any given temperature is called the (normal) "vapor pressure" or "vapor tension" at that temperature, and the vapor is said to be "saturated," i.e., each unit volume of the space contains the greatest possible mass of vapor which can occupy it at this particular temperature. Diminishing the volume of the space (at constant temperature) occupied by a saturated vapor causes some of the vapor to condense, and what is left remains saturated at the same pressure and temperature. Increasing the volume of the space (at constant temperature) causes more of the liquid to evaporate and the vapor still remains saturated at the same pressure and temperature, provided always that some liquid is left.

Boiling Point and Heat of Vaporization. — The temperature corresponding to any given pressure at which a vapor is completely saturated is called the "boiling point" of the liquid at this pressure. The temperature of a liquid which is "boiling" in the ordinary sense of the term is in general greater than the temperature of the saturated vapor above it, since the vapor in the bubbles formed is under a greater pressure than the vapor above the surface. The quantity of heat required to convert unit mass of a liquid into vapor at a given pressure is called the heat of vaporization or heat of evaporation of the liquid at this pressure; the same quantity of heat is given out by unit mass of the vapor when it condenses at this same pressure. The boiling points and heats of vaporization generally given are for normal atmospheric pressure, viz., 76 cm.

mercury. Values of these two quantities for some common substances are given in Table III.

Unsaturated or Superheated Vapors; Gases.—When the pressure exerted by a vapor against the walls of the containing vessel is less than the saturation or "normal" vapor pressure, the vapor is said to be unsaturated or superheated. This state of affairs may be brought about either (1) by increasing the volume of a saturated vapor when there is no longer any liquid left to evaporate, keeping the temperature constant, or (2) by raising the temperature of such a saturated vapor, keeping the pressure constant, or (3) by a proper combination of (1) and (2). The ordinary so-called "permanent" gases are superheated vapors, the degree of superheat being very great. The distinction ordinarily made between a gas and a vapor is that a gas is far removed, with respect to temperature and pressure, from the saturated state, whereas a comparatively small increase in the pressure or decrease in the temperature of a vapor will saturate it.

Laws of Perfect Gases. — Ordinary gases, such as air and superheated vapors (the greater the superheat the more nearly do the relations hold), are found to obey *approximately* the following "law,"

$$pV = \frac{MRT}{\mu},$$

where p = absolute pressure of the gas,
V = volume occupied by it,
M = mass of the gas,
T = absolute temperature; see Table V below,
μ = molecular weight of the gas;* see Table IV below,
R = a constant for all gases, called the "gas constant," whose value depends only upon the units in which the various quantities are expressed; see Table V.

TABLE IV. — MOLECULAR WEIGHTS OF GASES

Gas	μ	Gas	μ
Air (75.5 N+23.2 O+1.3 A)...	28.98	Chlorine (Cl_2)...............	70.92
Acetylene (C_2H_2)	26.02	Hydrogen (H_2)...............	2.016
Ammonia (NH_3)............	17.03	Nitrogen (N_2)...............	28.02
Carbon dioxide (CO_2).......	44.00	Nitrous oxide (N_2O).........	44.02
Carbon monoxide (CO).....	28.00	Oxygen (O_2).................	32.00

The above relation can be deduced from purely thermodynamic relations (*see Thermodynamics, Principles of*) on the assumptions: (1) that the product pV at constant temperature is a constant for any particular gas (*Boyle's or Mariotte's Law*), (2) that the intrinsic energy per unit mass of a gas depends only on its temperature, being independent of the volume and nature of the gas (*Joule's Gas Law*), and (3) that at constant volume the specific heat of the gas is inde-

* For a mixture (without chemical reaction) of perfect gases of different molecular weights, the equivalent molecular weight of the mixture is

$$\mu = \frac{M_1 + M_2 + M_3 + \ldots}{\frac{M_1}{\mu_1} + \frac{M_2}{\mu_2} + \frac{M_3}{\mu_3} + \ldots},$$

where M_1, M_2, M_3, etc., are the masses of the individual constituents and μ_1, μ_2, μ_3, etc., their molecular weights.

pendent of its temperature. A gas which satisfies the above conditions is called a "perfect" gas; all ordinary gases and highly superheated vapors satisfy these conditions approximately.

As a consequence of the above law the following relations must hold for a perfect gas; they also apply approximately to ordinary gases. In addition to the symbols above, let

C_v = specific heat at constant volume,
C_p = specific heat at constant pressure,
$\gamma = \frac{C_p}{C_v}$,
W_{12} = work done on the gas when its pressure changes from p_1 to p_2
H_{12} = heat absorbed by the gas when its pressure changes from p_1 to p_2
J = mechanical equivalent of heat.

Then for a perfect gas, for an *isothermal change*, i.e., a change at constant temperature ($T_1 = T_2$)

$$W_{12} = p_1 V_1 \log_e \left(\frac{p_2}{p_1}\right), \quad H_{12} = \frac{W_{12}}{J},$$

and for an *adiabatic change*, i.e., no heat passes in or out of the gas ($H_{12} = 0$)

$$\frac{p_1}{p_2} = \left(\frac{V_2}{V_1}\right)^{\gamma}, \quad \frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma-1} = \left(\frac{p_1}{p_2}\right)^{\frac{\gamma-1}{\gamma}}.$$

$$W_{12} = JC_v\,(t_1 - t_2) = \frac{p_1 V_1\,(t_1 - t_2)}{T_1\,(\gamma - 1)}.$$

Any one of the four sets of units given in the following table (or any other *consistent* set) may be used in these formulas.

TABLE V.—VALUES OF THE GAS CONSTANT R AND MECHANICAL EQUIVALENT J

Notation	Units and constants			
M=mass	kg.	grams	lb.	gram
V=volume	cu. meter	cu. cm.	cu. ft.	cu. cm.
p=absolute press.	kg. per sq. m.	dyne per sq. cm.	lb. per sq. ft.	cm. of Hg.
t=temperature	° C.	° C.	° F.	° C.
H=quantity of heat	kg-cal.	gm-cal.	B.t.u.	gram-cal.
W=work	m-kg.	erg	ft-lb.	meter-gram
T=absolute temp.=	t+273	t+273	t+460	t+273
R=gas constant=	849	0.832×10^8	1547	0.624×10^4
J=mechan. equiv.=	427	4.19×10^7	778	427

Gas Saturated with Vapor.—Experience shows that when a substance evaporates into a space already occupied by a gas (e.g., water evaporating into the atmosphere), evaporation ceases for any given temperature when a definite pressure (depending upon the temperature and the nature of the vapor and the gas) is established in the mixture; or, if the pressure and temperature are maintained constant, then evaporation ceases when a definite amount of vapor has been produced. When a mixture of a gas and a vapor contains this maximum mass of vapor per unit volume of the mixture, the gas is said to be saturated with the vapor. Such a mixture may contain less of the vapor per unit

volume than this maximum amount; the ratio of the mass of vapor per unit volume actually present to the maximum possible mass per unit volume at any given pressure and temperature is called the "relative humidity" of the mixture. The relative humidity of a mixture (e.g., air and water vapor) can be determined by the use of a dry- and wet- bulb thermometer. (*See the Hygrometric Tables of the U.S. Weather Bureau.*)

Calculation of the Amount of Moisture in Saturated Air.— The relations given below are approximate but sufficiently accurate for practical work; they apply to a mixture of any gas and vapor which do not act chemically upon each other. Let

t = temperature of the mixture,
p = absolute pressure of the mixture,
T = absolute temperature corresponding to t, see Table V above,
R = gas constant, from Table V above,
29 = molecular weight of air; see Table IV above,
p_w = normal vapor pressure of steam at the temperature t, to be taken directly from steam tables, see article on *Steam*,
δ_w = mass of unit volume of steam at the pressure p_w; also to be taken from steam tables,
$\delta_a = \dfrac{29\,(p - p_w)}{RT}$ = mass of unit volume of air at the pressure $(p - p_w)$ and temperature t,
$\delta_m = \delta_w + \delta_a$ = mass of unit volume of the mixture.

Example. — (1) What is the weight (mass) in lb. per cu. ft. of air saturated at normal atmospheric pressure (14.70 lb. per sq. in.) with water vapor at 100° F.? (2) What is the weight in lb. of the water contained in 1 cu. ft. of the mixture? From the above formulas $p = 14.70 \times 144$ lb. per sq. ft.; $p_w = 0.946 \times 144$ lb. per sq. ft.; $\delta_w = 0.002851$ lb. per cu. ft.; $T = 460 + 100 = 560°$ F.; $R = 1547$; $\delta_a = \dfrac{29\,(14.7 - 0.946) \times 144}{1547 \times 560} = 0.06630$ lb. per cu. ft.; $\delta_m = 0.002851 + 0.06630 = 0.06915$ lb. per cu. ft. Answer: The mixture weighs 0.06915 lb. per cu. ft. and contains 0.002851 lb. of moisture per cu. ft.

Dalton's Law; Partial Pressures. — The above calculation is based upon the experimentally determined relation, known as Dalton's Law, that, in a mixture of several gases or vapors which do not react chemically upon each other, the total pressure for a given volume of the mixture is approximately equal to the sum of the individual pressures which each gas or vapor would produce if it alone filled this volume. For example, if a mixture of three gases or vapors A, B and C fill, when mixed, a volume V and produce a pressure p, then if the gas or vapor A by itself would exert a pressure p_a when it alone occupied this volume V, and gas B by itself would exert a pressure p_b when it alone occupied this volume V, and similarly for gas or vapor C, then

$$p = p_a + p_b + p_c.$$

The pressures p_a, p_b and p_c are called "partial" pressures. Dalton's Law holds only approximately, but is sufficiently accurate for most practical calculations.

Sublimation. — When a substance passes directly from a solid to a gaseous state the phenomenon is called sublimation. A common instance of this is the sublimation of solid carbonic acid, which passes from solid to gaseous form at atmospheric pressure at a temperature of − 79° C. and absorbs 140 gram-calories per gram or kilogram-calories per kilogram.

TRANSFER OF HEAT. — When a body is at a higher temperature than the surrounding bodies, energy is transferred from the hotter to the colder

bodies, as is manifested by the changes in temperature or state which tend to, or actually do, take place, even though the intervening space is entirely void of matter. The energy thus transferred from one body to another through empty space is called "radiant energy," or "radiant heat," and is similar in nature to the energy radiated in the form of light waves and electromagnetic waves. The waves of radiant heat have a length greater than that of light waves and less than that of the ordinary electromagnetic waves used in wireless telegraphy. Radiant heat is absorbed by, transmitted through and reflected by, ordinary matter in much the same way that light waves are absorbed, transmitted and reflected. Matter which is transparent to light waves, however, may be practically opaque to heat waves; e.g., water absorbs practically all the heat waves which fall upon it.

When a hot and a cold body are separated by a fluid which is free to circulate, heat is transferred from the hot to the cold body by currents of the fluid itself flowing from one to the other; similarly, all parts of a fluid which is being heated quickly come to approximately the same temperature. This transfer of heat by currents of the fluid itself is called "convection."

In the case of a hot and a cold body separated by a solid the transfer of heat, which may be very rapid, particularly when the separating medium is a metal, is probably due to an extremely rapid to-and-fro motion of the molecules which constitute the medium. In any event, the process is essentially different from the transfer of heat either by radiation or by convection; it is described by the term "conduction" of heat.

Radiation, Absorption and Reflection of Heat. — The rate at which a hotter body radiates heat, and a colder body absorbs heat, depends upon the state of the surfaces of the bodies as well as on their temperatures. The rate of radiation and of absorption are increased by darkness and roughness of the surfaces of the bodies, and diminished by smoothness and polish. For this reason the covering of steam pipes and boilers should be smooth and of a light color; uncovered pipes and steam-cylinder covers should be polished.

The heat radiated by a body at a given temperature T to surrounding bodies at a lower temperature is equal to the heat which this body would absorb at this same temperature T from surrounding bodies at a higher temperature. When a given quantity of radiant heat strikes a body, only part of the heat is, as a rule, absorbed, the rest being reflected.

Let H_i be the incident heat, H_r the reflected heat, and H_a the absorbed heat, at temperature t, and let H_e be the heat which the body would emit at this same temperature to bodies at a lower temperature; then

$$H_i = H_r + H_a,$$
$$H_a = H_e.$$

Definition of "Black Body"; Stefan-Boltzman Law. — A "black body" is defined as one that absorbs all radiations falling upon it, neither reflecting nor transmitting any. The radiation of such a body is a function of the temperature alone, and is identical with the radiation inside an inclosure all parts of which have the same temperature. By heating the walls of an inclosure as uniformly as possible and observing the radiation through a very small opening, a practical realization of a black body is obtained.

The radiation from such a body is found to obey the following law

$$E = K\,(T^4 - T_0^4)\,At,$$

where E is the total energy radiated in time t from such a body at an *absolute* temperature T to surrounding bodies maintained at an *absolute* temperature T_0. A is the area of the surface of the body and K is a constant. This relation

is known as the Stefan-Boltzman Law. When the *absolute* temperature T is over three times the absolute temperature T_0, this relation may be written

$$E = KT^4At.$$

The mean of the best determinations of the value of K is 1.28×10^{-12} when E is expressed in gram-calories and T in centigrade degrees. For other units K has the following values

VALUES OF K

Unit of energy (E)	Unit of area (A)	Unit of time (t)	Temperature scale	Absolute zero	K=
Gram-calorie.......	sq. cm.	second	Centigrade	−273	1.28×10^{-12}
Kg-calories	sq. meter	hour	Centigrade	−273	4.61×10^{-8}
B.t.u..............	sq. ft.	hour	Fahrenheit	−460	1.62×10^{-9}
Watt-seconds.......	sq. cm.	second	Centigrade	−273	5.35×10^{-12}
Kw-hr	sq. in.	hour	Centigrade	−273	3.45×10^{-14}

Radiating and Reflecting Powers. — The ratio of the heat radiated per unit area by any surface at a given temperature t to the heat radiated per unit area by an absolutely black surface at this same temperature t is called the radiating power of the surface at that temperature. The difference between this ratio and unity is a measure of the heat which would be reflected by this surface at the same temperature, and is defined as the reflecting power of the surface. The radiating power of a surface depends upon the temperature of the surface; at very high temperatures the radiating power of every surface approaches the value of unity, i.e., at high temperatures the total energy radiated by any surface approaches in value the total energy radiated by an absolutely black body. The following table from Kent's *Mechanical Engineer's Pocketbook* gives the approximate value of the radiating power of some common surfaces at ordinary temperatures.

TABLE VI. — APPROXIMATE RADIATING AND ABSORBING POWERS

Surface	Radiating or absorbing power	Surface	Radiating or absorbing power
Lampblack..................	100	Zinc, polished..............	19
Water......................	100	Steel, polished..............	17
Carbonate of lead..........	100	Platinum, polished..........	24
Writing-paper..............	98	Platinum in sheet...........	17
Ivory, jet, marble..........	93–98	Tin.........................	15
Ordinary glass.............	90	Brass, cast, dead polished...	11
Ice........................	85	Brass, bright polished.......	7
Gum lac....................	72	Copper, varnished...........	14
Silver-leaf on glass..........	27	Copper, hammered..........	7
Cast iron, bright polished...	25	Gold, plated................	5
Mercury, about.............	23	Gold on polished steel.......	3
Wrought iron, polished......	23	Silver, polished bright.......	3

Oiling a polished surface may increase its radiating power from 2 to 3 times, but oiling does not seriously affect the radiating power of a rough surface.

Conduction of Heat. — Whenever a difference of temperature is maintained between any two parts of the same body there is a transfer of heat from the hotter to the colder part by the process described by the term "conduction," as distinguished from radiation and convection; also when two bodies at different temperatures are in direct contact there is a transfer of heat across the surface contact from the hotter to the colder body, the process being similar in nature to the conduction of heat from a hotter to a colder part of the same body. The first type of conduction is called "internal" conduction; the second type "external" conduction.

Internal Conduction. — Consider a flat layer *within* a substance, the two sides of the layer being parallel and its thickness small compared with its area. Let one side of this be maintained at a constant temperature T, the same at all points of this surface, and the other side of the layer be maintained at a constant temperature T_1; let A be the area of the layer (i.e., of one of its flat surfaces) and x the thickness of the layer. Then the amount of heat transferred through the layer in time t is,

$$H = \frac{K A (T - T_1) t}{x},$$

where K, called the "thermal conductivity," is approximately a constant for a given material, and is independent of the temperature difference $T - T_1$, when this difference is small; K is not constant, however, for wide temperature variations. Values of K are given in Tables VII and VIII. The reciprocal of the thermal conductivity, viz., $\rho = 1/K$, is called the thermal resistivity.

The values of K given in the tables below are the values of this factor when H is expressed in gram-calories, A in sq. cm., x in cm., $(T - T_1)$ in ° C., and t in seconds; i.e., K is the number of gram-calories transmitted per second through a cube 1 cm. on each edge when a difference of temperature of 1° C. is maintained between opposite faces of this cube. For other units the value of K as given should be multiplied by the factor noted in the following table:

Unit of heat	Unit of area	Unit of thickness	Unit of time	Unit of temp. diff.	Multiply K by
Gram-calorie	sq. cm.	cm.	second	° C.	1.000
Kg-calorie	sq. m.	cm.	hour	° C.	3.600×10^4
B.t.u.	sq. ft.	inch	hour	° F.	2902
Watt-seconds	sq. cm.	cm.	second	° C.	4.183
Watt-seconds	sq. ft.	inch	second	° F.	850.8
Kw-hr.	sq. m.	cm.	hour	° C.	41.83
Kw-hr.	sq. ft.	inch	hour	° F.	0.8508

Conduction through Other than Thin Layers. — The formula for H given above is applicable only *when the lines of flow are straight and perpendicular to A and the temperatures T and T_1 are temperatures of the surfaces of the substance itself*, not the temperatures of a fluid, for example, in contact with these surfaces. Failure to take these facts into consideration accounts for some of the inconsistencies in the reported values of thermal conductivity from tests. To calculate the flow of heat in other cases, formulas analogous to those for electric resistance and conductance must be employed; see ***Resistance and Conductance.***

Temperature Coefficient of the Internal Thermal Conductivity.—In the case of metals the variation of the internal thermal conductivity with temperature may be expressed with a fair degree of approximation by the relation

$$K = K_0 (1 + at),$$

where K_0 is the conductivity at 0° C., say, and K is the conductivity at any other temperature of t °C., and a is a constant. The coefficient a may be either positive or negative; its value for some of the common metals is given in Table VII.

Values of the Internal Thermal Conductivity (K) of Materials.—In the following tables, VII and VIII, are given the thermal conductivity of certain common non-metallic and metallic substances respectively. The data are from the following sources: Landolt-Börnstein, *Physikalisch-Chemische Tabellen*, 1912; Nusselt, W., *Zeit. Ver. Deutch. Eng.*, June, 1908; Hering, Carl, *Trans., A.I.E.E.*, 1910; Randolph, C. P., *Trans. Am. Electrochem. Society*, 1912; Ordway, *Trans. Am. Soc. Mech. Eng.*, 1884-85; Coleman, J. J., *Engineering*, Sept. 5, 1884; Scott, H. G., *Power*, 1902; Wolff, *Jour. Frank. Inst.*, 1893; Peclet, *Practical Treatise on Heat*; *Met. & Chem. Eng.*, Feb. 1909, p. 72; Brill, G. M., *Trans. Am. Soc. Mech. Eng.*, Vol. 16, p. 827; *Smithsonian Physical Tables*.

TABLE VII.—INTERNAL THERMAL CONDUCTIVITY OF NON-METALLIC SUBSTANCES

Substance	Temp. range, °C.		Therm. conduct. K**		Temperature coefficient per °C. a
	From*	To	From†	To†	
Air	0	....	0.000568	....	+0.0019 to +0.0039
Asbestos	100	500	0.000172	0.000219	....
Brick, building	15 to 30	....	0.00149	....	....
Brick, dust	15 to 30	....	0.000461	....	....
Brick, fire	0	1300	0.00140	0.00419	....
Carborundum	....	....	0.005	....	....
Cardboard	Below 0	....	0.000394	....	....
Cement, Portland	35	90	0.000712	0.00217	....
Chalk	....	....	0.00219	....	....
Concrete, slag	50	....	0.000528	....	....
Cork	20	200	0.000153	0.000201	....
Ebonite	6	90	0.00038	....	−0.0019
Eiderdown	100	150	0.0000471	0.000112	....
Feathers	20	155	0.000163	....	....
Felt	21	175	0.000125	....	....
Flannel	50	....	0.0000355	....	....
Hair	20	155	0.000148	....	....
Horn	......	....	0.0000870	....	....
Ice	......	....	0.00213	0.00570	....
Infusorial earth	20	450	0.000216	0.000354	....
Lampblack	100	500	0.0000756	0.000109	....

TABLE VII.—INTERNAL THERMAL CONDUCTIVITY OF NON-METALLIC SUBSTANCES—*Continued*

Substance	Temp. range, °C.		Therm. conduct. K**		Temperature coefficient per °C. a
	From*	To	From†	To†	
Leather	......	....	0.00015	0.00042	....
Linen	......	....	0.00021	....	....
Liquids, hydrocarbons, oils, etc.	......	....	about 0.000300	....	....
Magnesia, carb.	20	188	0.000175	....	....
Magnesia, calcined	20	155	0.000165	0.000173	....
Magnesia, asbestos	100	400	0.000162	0.000178	....
Marble	15 to 30	....	0.00770	0.00910	−0.0005
Oil, olive	6.6	....	0.000392	....	....
Oil, castor	......	....	0.000425	....	....
Oil, petroleum	0	34	0.000355	0.000382	+0.0110
Oil, turpentine	13	....	0.000325	....	+0.0067
Paraffin	0	34	0.000473	....	+0.0634
Pasteboard	......	....	0.000450	....	....
Plaster	......	....	0.00130	....	....
Plaster of Paris	20	155	0.000425	....	....
Plumbago	20	155	0.00100	....	....
Poplox, made from Na_2SiO_3	200	500	0.0000920	0.000162	....
Porcelain	95	....	0.00249	....	....
Pumice stone	20	155	0.000428	....	....
Quartz	500	....	0.0000247	0.000718	−0.0019
Rubber, vulcanized	0	49	0.0000890	0.000340	....
Sand	20	155	0.000855	0.000867	....
Sawdust	......	....	0.000123	0.000152	....
Silk	50	100	0.000124	0.000141	....
Slag	50	....	0.00264	....	....
Slate	94	....	0.00360	....	....
Snow	......	....	0.000060	0.00115	....
Stone, calcareous	15 to 30	....	0.00470	0.00570	....
Strawboard	......	....	0.000330	....	....
Water	0	....	0.0012	0.0015	−0.0055
Water	30	....	0.00158	....	....
Wood	......	....	0.0000878	0.000300	....
Wool, sheep's	20	100	0.000126	0.000152	....
Wool, mineral	0	175	0.0000930	0.000128	....
Wool, steel	100	....	0.000192	0.000216	....
Woolen	100	....	0.0000553	0.000119	....

* When only one temperature is given the measurement was made at that temperature.
† Range of determinations by different experimenters.
** In gram-calories per centimeter-cube per degree centigrade per second; see table on page 717 for multiplying factors when other units are employed.

TABLE VIII. — INTERNAL THERMAL CONDUCTIVITY OF METALS AND VARIOUS FORMS OF CARBON

Substance	Temp. range, °C. From *	Temp. range, °C. To	Therm. conduct. K** From†	Therm. conduct. K** To†	Temperature coefficient per °C. a
Aluminum	0	100	0.344	0.362	+0.00054
Brass, yellow	0	100	0.204	0.254	+0.0024
Brass, red	0	100	0.246	0.283	+0.0015
Carbon	100	360	0.089	......	....
Carbon	100	942	0.130	......	....
Carbon, Ach. Graph	100	390	0.340	......	....
Carbon, Ach. Graph	100	914	0.290	......	....
Charcoal	20	155	0.00019	......	....
Coal	....	....	0.00030	......	....
Constantan	18	100	0.054	0.064	....
Copper	−54	14	0.921	1.059	−0.00053 to
Copper	−74	167	0.914	1.024	+0.00047
German silver	0	100	0.070	0.089	+0.0027
Gold	100	....	0.703	......	....
Iron	0	....	0.167	0.207	−0.00023 to
Iron	100	....	0.142	0.163	0.00061
Iron	200	....	0.136	......	....
Lead	0	100	0.0764	0.0834	−0.00086 to −0.00016
Manganin	18	100	0.052	0.063	+0.0027
Mercury	0	50	0.0148	0.0189	−0.0013 (a)
Nickel	18	....	0.142	......	−0.00031 (b)
Platinum	18	100	0.166	0.173	+0.00053
Platinoid	18	....	0.060	......	....
Silver	0	100	1.096	0.992	−0.00017
Steel	....	....	0.062	0.111	−0.0006
Tin	0	100	0.153	0.142	−0.0007
Zinc	18	100	0.265	0.262	−0.00015

* When only one temperature is given the measurement was made at that temperature.
† Range of determinations by different experimenters.
** In gram-calories per centimeter-cube per degree centigrade per second; see table on page 717 for multiplying factors when other units are employed.
(a) Range from this value to − 0.00045.
(b) Range from this value to − 0.000066.

External Thermal Conduction — Thermal "Contact" Resistance. —A "current" or flow of heat in passing from one substance to another, e.g., from air to a metal or from water to a metal and vice versa, experiences a thermal resistance analogous to the contact resistance at the junction of two dissimilar conductors, e.g., a carbon brush on a commutator. Data on this point are meagre. For thin plates Peclet gives the following formula for small temperature differences,

$$R_c = \frac{1}{A[1 + B(T - T_1)]},$$

where R_c represents the *total* contact resistance for *both* surfaces *per unit area* perpendicular to the direction of flow, T and T_1 the temperatures of the air or water in contact with the two sides of the plate, and A and B are coefficients having the following values when the quantities involved are expressed in B.t.u.-hour-sq. ft.-Fahrenheit units.

VALUES OF A AND B

	A	B
Air on both sides:		
Polished metal surfaces	0.90	0.0028
Glassy or varnished surfaces	1.34	0.0037
Dull metallic surfaces	1.58	0.0037
Lampblack	1.78	......
Water on both sides:		
Metal surfaces	8.8	0.058

For a large difference of temperature, the term $B(T - T_1)$ becomes large in comparison with unity, whence the total flow of heat H from the fire box to the water through an area S is

$$H = \frac{(T - T_1)^2 S}{a}.$$

Results from tests on boiler plates and tubes give values of a ranging from 160 to over 200, when the quantities involved are expressed in B.t.u.-hour-sq. ft.-Fahrenheit units.

Total Thermal Resistance of a Thin Plate. — Let x be the thickness of the plate, S the area of its surface, T and T_1 the temperatures of the *fluid* in contact with the two surfaces of the plate, ρ the internal thermal resistivity, and r_c and r_c' the thermal contact resistances per unit area between the fluid and the surface of the plate on the two sides respectively. Then the total thermal resistance from fluid to fluid is

$$R = \frac{(r_c + r_c' + \rho x)}{S}$$

and the total flow of heat from fluid to fluid in time t is

$$H = \frac{(T - T_1)t}{R} = \frac{(T - T_1)St}{r_c + r_c' + \rho x}.$$

For boiler plates and tubes with a large difference of temperature between the fire box and the water, the term ρx is small, and $r_c + r_c' = \frac{a}{T - T_1}$, where $a = 1/AB$ is approximately a constant, as noted above.

EXPANSION DUE TO HEATING. — Most substances expand when heated, but water between 0° degrees and 4° C., quartz glass below − 84° C. and a few other substances contract with increase of temperature. When the temperature of a solid is changed by rapid cooling, slow changes in its dimensions continue long after it has attained the same uniform temperature throughout. This effect, which is particularly marked in glass, is known as "thermal hysteresis." It can be largely eliminated by prolonged heating at a high temperature followed by a very gradual cooling, i.e., by annealing.

The volume of a physically homogeneous substance which is not subjected to any treatment causing more or less permanent changes in its structure, is a definite function of its temperature and the pressure or tension in it. In general, under constant pressure or tension the amount of expansion caused by a given change in temperature depends upon both the material of the body and its initial temperature.

Coefficient of Expansion of Gases. — In the case of gases, however, there is a remarkable uniformity, all the so-called permanent gases expanding about $\frac{1}{273}$ part of their *initial volume at 0° C.* per degree centigrade increase of temperature, irrespective of the pressure, provided this remains constant throughout. That is, for any of the ordinary gases,

$$V = V_0\left(1 + \frac{t_c}{273}\right),$$

where V_0 is the volume at 0° C., and V the volume at any other temperature t_c° C., the pressure being the same at both temperatures. If the temperature is expressed in Fahrenheit degrees and V_0 is the volume at 0° F., then

$$V = V_0\left(1 + \frac{t_f}{460}\right).$$

Note that − 273 and − 460 are the absolute zeros on the centigrade and Fahrenheit scales respectively.

Coefficients of Linear Expansion of Liquids and Solids. — Let l_0 be length of a rod, or of column of liquid, at any standard temperature, say 0° C., and l be the length at any other temperature t. Then the exact relation between l and l_0 may be expressed as a series of the form

$$l = l_0\,(1 + at + bt^2 + \ldots),$$

where a, b, etc., are constant coefficients for any given material and *fixed reference temperature*. As a rule the coefficients of the powers of t above the first are much smaller than the first coefficient, and for small ranges of temperature the approximate formula

$$l = l_0\,(1 + at)$$

is usually employed. The coefficient a in this formula is practically a constant for any range of temperature not exceeding 100° C.; it is sometimes called the "mean coefficient of linear expansion" between the limits of temperature chosen. For example, between 0° C. and 100° C., the mean coefficient of linear expansion is defined by the relation

$$a_0 = \frac{l_{100} - l_0}{100\, l_0},$$

where $l_{100} - l_0$ is equal to the change in length produced by increasing the temperature from 0° C., to 100° C.

It should be noted that for each value of the standard or reference temperature a different value of a must be used. For example, if a rod has a length l_1

TABLE IX. — COEFFICIENTS OF LINEAR EXPANSION

$l = l_0 (1 + at + bt^2)$, temperature in °C.*
a_{20} = "true" coefficient at 20°C.

(*From Landolt-Börnstein's Tables, 1912 Edition.*)

Substance	Temp., °C. From	Temp., °C. To	a	b	a_{20}
Aluminum	0	610	0.235×10^{-4}	0.707×10^{-8}	0.238×10^{-4}
Brass (73.7 Cu+24.2 Zn +1.5 Sn+0.6 Pb)	0	80	0.179×10^{-4}	0.456×10^{-8}	0.181×10^{-4}
Bronze (81.2 Cu+8.6 Zn +9.9 Sn+0.2 Pb)	0	80	0.176×10^{-4}	0.469×10^{-8}	0.177×10^{-4}
Carbon, gas-carbon	40	....	0.054×10^{-4}	....	....
Carbon, graphite	40	....	0.079×10^{-4}	....	....
Constantan (60 Cu+40 Ni)	0	500	0.148×10^{-4}	0.402×10^{-8}	0.150×10^{-4}
Copper	0	625	0.167×10^{-4}	0.403×10^{-8}	0.169×10^{-4}
Glass, Jena	0	100	0.077×10^{-4}	0.350×10^{-8}	0.079×10^{-4}
Glass, French	2	100	0.072×10^{-4}	0.544×10^{-8}	0.075×10^{-4}
Gold	9	95	0.136×10^{-4}	1.12×10^{-8}	0.140×10^{-4}
German silver	0	100	0.184×10^{-4}	....	....
Ice	−27	−2	0.514×10^{-4}	....	....
Iron, cast	0	625	0.098×10^{-4}	0.566×10^{-8}	0.102×10^{-4}
Iron, wrought	0	500	0.117×10^{-4}	0.525×10^{-8}	0.119×10^{-4}
Lead	14	94	0.273×10^{-4}	0.74×10^{-8}	0.276×10^{-4}
Marble, white	15	100	0.117×10^{-4}	....	....
Mica, parallel to cleavage	5	80	0.077×10^{-4}	1.200×10^{-8}	0.082×10^{-4}
Mica, perpendicular to cleavage	4	82	0.076×10^{-4}	0.490×10^{-8}	0.079×10^{-4}
Nickel	0	1000	0.135×10^{-4}	0.332×10^{-8}	0.136×10^{-4}
Nickel steel (24% Ni)	0	38	0.175×10^{-4}	0.711×10^{-8}	0.178×10^{-4}
Phosphor bronze (97.6 Cu +2.2 Sn+0.2 P)	0	80	0.167×10^{-4}	0.462×10^{-8}	0.168×10^{-4}
Platinum	0	1000	0.0887×10^{-4}	0.1324×10^{-8}	0.0892×10^{-4}
Porcelain, Berlin	20	100	0.027×10^{-4}	0.306×10^{-8}	0.028×10^{-4}
Porcelain, Bayeux	0	600	0.034×10^{-4}	0.107×10^{-8}	0.035×10^{-4}
Rubber, hard	17	25	0.77×10^{-4}	....	....
Rubber, hard	25	35	0.84×10^{-4}	....	....
Silver	0	750	0.1827×10^{-4}	0.4793×10^{-8}	0.1846×10^{-4}
Steel	0	300	0.092×10^{-4}	0.336×10^{-8}	0.093×10^{-4}
Tin	8	95	0.203×10^{-4}	2.63×10^{-8}	0.214×10^{-4}
Vulcanite	0	18	0.636×10^{-4}	....	....
Zinc	9	96	0.274×10^{-4}	2.34×10^{-8}	0.284×10^{-4}

* When the temperature is expressed in Fahrenheit degrees, the formulas become

$$l = l_{32}\left[1 + \frac{a(t_f - 32)}{1.8} + \frac{b\,(t_f - 32)^2}{3.24}\right],$$

$$a_{68} = \frac{a_{20}}{1.8},$$

where a, b and a_{20} have the values given in the above table.

at t_1 degrees and a length l at t degrees, and a_0 is the mean coefficient of linear expansion referred to 0° C., then $l_1 = l_0(1 + a_1t_1)$ and $l = l_0(1 + a_0t)$, whence

$$l = l_1[1 + a_1(t - t_1)],$$

where

$$a_1 = a_0/(1 + a_0t_1).$$

If t_1 is so large that a_0t_1 is appreciable compared with unity then a_1 is not equal to a_0.

True Coefficient of Linear Expansion. — The "true" coefficient of linear expansion at any temperature t is defined as the *rate* of increase of length with increase in temperature divided by the length at this temperature t, or

$$a = \frac{1}{l} \cdot \frac{dl}{dt}.$$

The "true" coefficient a_t any temperature depends on the temperature.

Coefficients of Cubical Expansion of Liquids and Solids. — The volume or cubical expansion of liquids and solids may be expressed in exactly the same manner as the linear expansion, viz.,

$$V = V_0(1 + \alpha t + \beta t^2 + \ldots)$$

or approximately

$$V = V_0(1 + \alpha t),$$

and the true coefficient at any temperature t is defined as

$$\alpha_t = \frac{1}{V}\frac{dV}{dt},$$

where V and V_0 are the volumes at t degrees and at the reference temperature respectively.

As a first approximation, when the coefficient of linear expansion is small and the temperature rise not excessive, the volume coefficient may be taken equal to 3 times the linear coefficient.

A few volume coefficients, determined from direct experiment, are given in Table X. For Water see article on *Weights of Materials.*

TABLE X. — COEFFICIENTS OF CUBICAL EXPANSION

$V = V_0(1 + \alpha t + \beta t^2)$, temperatures in ° C.
α_{20} = "true" coefficient at 20° C.
(*From Landolt-Börnstein's Tables, 1912 edition.*)

Substance	Temp., ° C. From	Temp., ° C. To	α	β	α_{20}
Caoutchouc, crude gray..	0	75	6.62×10^{-4}	24.2×10^{-8}	6.80×10^{-4}
Gutta-percha, pure rolled.	0	40	4.96×10^{-4}	496×10^{-8}	6.94×10^{-4}
Paraffin.................	0	33	5.84×10^{-4}	99.2×10^{-8}	5.88×10^{-4}
Petroleum, sp. gr. 0.8467..	24	120	8.99×10^{-4}	140×10^{-8}	9.55×10^{-4}
Wax, white solid..........	10	57	10.7×10^{-4}	-5580×10^{-8}	3.06×10^{-4}

Mercury (− 10 to 300° C.):

$$V = V_0(1 + 1.805553 \times 10^{-4}t + 1.2444 \times 10^{-8}t^2 + 2.539 \times 10^{-11}t^3)$$

FLASH POINT OF OILS.—The flash point of an oil is the temperature to which the oil must be raised before the vapor immediately above it will take fire upon the application of a flame. This temperature depends to an appreciable extent upon the size of the flame, the method of applying it, and the shape and dimensions of the containing vessel; see *Oil, Transformer*. The following values of the flash point for various oils are taken from J. Lewkowitisch, *Chemical Technology and Analysis of Oils, Fats and Waxes*.

TABLE XI.—FLASH POINT OF OILS

Oils	Spec. grav. at 60° F.	Flash point ° F.
Mineral oils:		
Refined American	0.875 to 0.920	325 to 425
Refined Russian	0.895 to 0.915	300 to 425
Scotch	0.875 to 0.895	300 to 350
Natural (dark) American	0.880 to 0.895	325 to 425
Natural (dark) Russian	0.910 to 0.915	250 to 300
Natural filtered American	0.885 to 0.905	450 to 575
Animal oils:		
Sperm	0.8804 to 0.8807	446 to 457
Lard	0.9172	494
Tallow	0.951	265
Neat's foot	0.9178	470
White whale	0.9207	476
Vegetable oils:		
Castor	0.963	275
Linseed	0.930	285
Olive	0.914	305
Rape, crude	0.920	265
Rape, refined	0.911	305

HEATS OF FORMATION, COMBUSTION AND SOLUTION.*—When two or more substances react chemically heat is generally either given out or absorbed. When heat is given out the reaction is called "exothermic" and when heat is absorbed the reaction is called "endothermic." The following tables give the heats of reaction for some of the more important industrial processes; see *Landolt-Börnstein's Tables* for more complete tables. A minus sign indicates an endothermic reaction; i.e., an absorption of heat. When two values are given these refer to independent measurements.

The values given are the total amounts of heat in kilogram-calories given out or absorbed per mol (*see Electrochemistry, Principles of*) of the product, starting with the proportions of the reacting substances represented by the left-hand member of the reaction equation, the temperature at the beginning and at the end of the reaction being the same, i.e., room temperature. For example, starting with 1 gram-atom of oxygen (16 kilograms, say) and 1 gram-atom of calcium (40.07 kilograms) both at room temperature, then the net amount of heat given out in the formation of one mol ($16 + 40.07 = 56.07$ kilograms) of calcium oxide when its temperature is brought back to room temperature is 130.9 kilogram-calories or 2.33 kilogram-calories per kilogram of calcium oxide.

* By M. De K. Thompson.

TABLE XII. — HEATS OF FORMATION OF IMPORTANT COMPOUNDS

(At room temperature unless otherwise stated.)

Product	Equation	Kilogram-Calories* evolved per mol of product
Acetylene	$2C+2H=C_2H_2$	−47.8 to −58.1
Aluminium hydroxide	$2Al+3O+3H_2O=2Al(OH)_3$	194.5 to 196.5
" "	$Al+3O+3H=Al(OH)_3$	297.0
" oxide	$2Al+3O=Al_2O_3$	380.2
Ammonia gas	$N+3H=NH_3$	11.9 to 12.2
Barium dioxide	$BaO+O=BaO_2$	18.4
" "	$Ba+2O=BaO_2$	145.5
" hydroxide	$Ba+2O+2H=Ba(OH)_2$	217.0
" monoxide	$Ba+O=BaO$	133.4 to 126.4
Calcium carbide	$Ca+2C=CaC_2$	−7.25
" oxide	$Ca+O=CaO$	145.0 to 151.9
Carbon dioxide	C amorphous$+2O=CO_2$	96.96-97.65
" "	C graphite$+2O=CO_2$	94.8
" monoxide	C amorphous$+O=CO$	29.0
Copper chloride	$Cu+2Cl=CuCl_2$	51.63-51.4
" sulphate	$Cu+S+4O=CuSO_4$	182.6
Cupric oxide	$Cu+O=CuO$	37.2
Cuprous oxide	$2Cu+O=Cu_2O$	40.8-43.8
Ferric hydroxide	$2Fe+3O+3H_2O=2Fe(OH)_3$	95.6
" oxide	$2Fe+3O=Fe_2O_3$ (dried at 400°)	65.2
	$2Fe+3O=Fe_2O_3$ (heated to 1000°)	64.8
Ferrous chloride	$Fe+2Cl=FeCl_2$	82.2
" hydroxide	$Fe+O+H_2O=Fe(OH)_2$	68.3
" oxide	$Fe+O=FeO$	65.7
" sulphate	$Fe+S+4O+7H_2O=FeSO_4\cdot 7H_2O$	240.1
Lead dioxide	$PbO+O=PbO_2$	12.6
" monoxide	$Pb+O=PbO$	50.3
" sulphate	$Pb+S+4O=PbSO_4$	216.2
Magnesium chloride	$Mg+2Cl=MgCl_2$	151.0
" hydroxide	$Mg+2O+2H=Mg(OH)_2$	217.5
" oxide	$Mg+O=MgO$	143.3
Nickel chloride	$Ni+2Cl=NiCl_2$	74.5
" hydroxide	$Ni+O+H_2O=Ni(OH)_2$	60.8
" oxide	$Ni+O=NiO$	57.9
" sulphate	$Ni+S+4O+7H_2O=NiSO_4\cdot 7H_2O$	233.6
Nitric oxide	$N+O=NO$	−21.6
Nitric acid	$N+H+3O=HNO_3$ liquid	41.5
	$N+H+3O=HNO_3$ dissolved	49.1

* 1 Kg-cal. = 1000 gram-cal. = 3.968 B.t.u. = 4183 watt-seconds (joules) = 1.162×10^{-3} kw-hr.

TABLE XII. — HEATS OF FORMATION OF IMPORTANT COMPOUNDS — *Continued*

(At room temperature unless otherwise stated.)

Product	Equation	Kilogram-Calories* evolved per mol of product
Nitrogen dioxide	$N+2O=NO_2$ at 22° C.	−1.7
" "	$N + 2O = NO_2$ at 200°	− 7.9
Ozone	$1\frac{1}{2} O_2=O_3$	−34.1
Potassium chlorate	$K+Cl+3O=KClO_3$	95.8– 93.8
" chloride	$K+Cl=KCl$	105.6
" hydroxide	$K_2O+H_2O+Aq.=2\,KOH\,(Aq.)$	67.4
" "	$K+O+H=KOH$	103.2 to 104.6
" hypochlorite	$K+Cl+O+Aq.=KClO\,(Aq.)$	89.4 to 88.0
" oxide	$2\,K+O=K_2O$	97.1
" perchlorate	$K+Cl+4O=KClO_4$	113.5
Silver nitrate	$Ag+N+3O=AgNO_3$	28.7
" oxide	$2\,Ag+O=Ag_2O$	5.9– 7.0
Sodium chlorate	$Na+Cl+3O=NaClO_3$	86.7 to 84.8
" chloride	$Na+Cl=NaCl$	97.8
" hydroxide	$Na_2O+H_2O+Aq.=2\,NaOH\,(Aq.)$	63.9 to 56.5
" "	$Na+O+H=NaOH$	101.9 to 102.7
" hypochlorite	$Na+Cl+O+Aq.=NaClO\,(Aq.)$	83.4 to 84.7
" oxide	$2\,Na+O=Na_2O$	100.3 to 91.0
" perchlorate	$Na+Cl+4\,O=NaClO_4$	100.3
Sulphuric acid	$S+4\,O+2\,H=H_2SO_4$ liquid	192.9
Sulphurous oxide	S solid$+2O=SO_2$ (gas)	71.1
" "	S solid$+2\,O=SO_2$ (dissolved)	78.8
Tin (stannous) chloride	$Sn+2\,Cl=SnCl_2$	80.8
" " oxide	$Sn+O=SnO$	67.6 to 66.2
Water	$2\,H$ gas$+O$ gas$=H_2O$ at 18° C.	68.36
"	$2\,H$ gas$+O$ gas$=H_2O$ at 0° C.	68.25 to 69.0
Zinc chloride and H	$Zn+2\,HCl(aq.)=ZnCl_2(aq.)+H_2$	34.2
" oxide	$Zn+O=ZnO$	85.0 to 85.4
" sulphate	$Zn+S+4\,O=ZnSO_4$	231.1 to 229.6

* 1 Kg-cal. = 1000 gram-cal. = 3.968 B.t.u. = 4183 watt-seconds (joules) = 1.162×10^{-3} kw.-hr.

TABLE XIII. — HEATS OF COMBUSTION (COMPLETE) OF SOME IMPORTANT INORGANIC SUBSTANCES*

(*At temperature 18° C. and constant pressure.*)

Name	Formula	Heat of combustion in kilogram-calories per mol
Methane	CH_4	213.5
Acetylene	C_2H_2	315.7
Methyl alcohol	CH_3OH	170.7
Ethyl alcohol	C_2H_5OH	326.1
Nitroglycerine	$C_3H_5N_3O_9$	361.2
Acetone	$CH_3-CO-CH_3$	427.3

* See also article on *Fuels*.

TABLE XIV. — HEATS OF SOLUTION

(*Room temperature; anhydrous salts unless otherwise noted.*)

Compound	Formula	Mols of water per mol of compound dissolved	Kilogram-calories evolved per mol of compound dissolved
Copper chloride	$CuCl_2$	600	11.1
" sulphate	$CuSO_4$	400	15.8
" " (crystals)	$CuSO_4 \cdot 5H_2O$	400	−2.75
Ferrous chloride	$FeCl_2$	350	17.9
" sulphate (crystals)	$FeSO_4 \cdot 7H_2O$	400	−4.5
Magnesium chloride	$MgCl_2$	800	35.9
Nickel chloride	$NiCl_2$	400	19.2
" " (crystals)	$NiCl_2 \cdot 6H_2O$	400	−1.16
" sulphate (crystals)	$NiSO_4 \cdot 7H_2O$	800	−4.25
Potassium chloride	KCl	200	−4.4
" chlorate	$KClO_3$	400	−10.02
" perchlorate	$KClO_4$	200–400	−12.1
Sodium chloride	$NaCl$	100	−1.03
" chlorate	$NaClO_3$	180–360	−5.6
" perchlorate	$NaClO_4$	200–400	−3.5
Silver nitrate	$AgNO_3$	200	−5.44
Zinc chloride	$ZnCl_2$	300	15.6
" sulphate	$ZnSO_4$	400	18.4
" " (crystals)	$ZnSO_4 \cdot 7H_2O$	400	−4.26

BIBLIOGRAPHY. — Preston, T., *The Theory of Heat*, London, 1904; Burgess and Le Chatelier, *Measurement of High Temperatures*, New York, 1912; Landolt-Börnstein-Roth, *Physikalisch-Chemische Tabellen*, Berlin, 1912; *Smithsonian Physical Tables*, Wash., 1910.

[H. PENDER and R. G. HUDSON.]

HEATING OF BUILDINGS. — (*See also Railways, Energy Requirements for.*) The problem of determining the amount of heat necessary to maintain a room at a given temperature above the out-door temperature resolves itself into the determination of the amount of heat radiated through the walls and the amount of heat necessary to raise the incoming ventilating air to the temperature of the air in the room.

HEAT REQUIRED TO WARM INCOMING AIR. — Authorities differ as to the amount of air required for ventilating purposes, but a commonly accepted figure is 30 cubic feet of air per person per minute, or 1800 cubic feet per person per hour. 1/56 B.t.u. is required to heat 1 cu. ft. of air 1° F. Let

t = temperature of incoming air, ° F.,
T = temperature of room, ° F.,
N = number of people in room.

Then, assuming 1800 cubic feet of air per person, the B.t.u. per hour required to raise the temperature of the incoming air to the room temperature is

$$H_A = 32.1\,N\,(T - t)$$

For any other assumed quantity of air per person take H_A in proportion.

Heat Required to Supply Radiation Losses. — Let

S = radiating surface, in square feet,
t = temperature, ° F., of outside air,
T = temperature, ° F., of inside air,
K = B.t.u. lost per hour per square foot of radiating surface per ° F. difference in temperature (*see table below*).

Then the heat lost due to radiation is

$$H_R = \Sigma\, KS\,(T - t),$$

where the summation includes all the radiating surfaces. Certain arbitrary additions are usually made to the value of H_R as thus calculated for the nature of the exposure, etc., see below.

The following values of K are taken from two papers, one by Wolff (*Jour. Frank. Inst., 1893*), the other by Hauss (*Trans. A.S.H.V.E., 1904*).

VALUES OF K FOR WALLS (*Hauss's Values*)

Thickness, inches	Brick walls	Solid sand-stone	Limestone
4¾	0.48	....	....
10	0.34	....	....
12	....	0.45	0.50
15	0.26	....	....
16	....	0.39	0.43
20	0.22	0.35	0.39
24	....	0.32	0.35
25	0.18	....	....
28	....	0.29	0.32
30	0.16	....	....
32	....	0.26	0.29
35	0.13	....	....
36	....	0.24	0.26
40	0.12	0.22	0.24
44	....	0.20	0.22
45	0.11	....	....
48	....	0.19	0.21

VALUES OF K FOR WINDOWS, DOORS, PARTITIONS, FLOORS, CEILINGS

	Wolff	Hauss		Wolff	Hauss
Glass Surfaces			*Floors*		
Vault light........	1.42	....	Joists with double floor..	0.10	0.07
Single window.....	1.20	1.00	Concrete floor...........	0.31	....
Double window....	0.56	0.46	Fireproof construction, planked over..........	0.124	....
Single skylight....	1.03	1.06	Wooden beam construction, planked over.....	0.083	....
Double skylight...	0.50	0.48	Concrete floor on brick arch.................	....	0.22
Doors			Stone floor on arches.....	....	0.20
Door..............	....	0.40	Planks laid on earth.....	....	0.16
1-in. pine..........	0.40	....	Planks laid on asphalt...	....	0.20
2-in. pine..........	0.28	....	Arch with air space......	....	0.09
Partitions			Stones laid on earth.....	....	0.08
Solid plaster, 1¾ to 2¼ in......	....	0.60			
2½ to 3¼ in......	....	0.48	*Ceilings*		
Fireproof..........	0.30	....	Joists with single floor...	....	0.10
2-in. pine board....	0.28	....	Arches with air space.....	....	0.14

Allowances for Exposure, etc. — Hauss makes the following additions to the values of H_R as calculated from the above formula and constants:

	Per cent
Rooms with unusual exposure................	5
North, east, northeast, northwest and west exposures..............................	10
Ceiling between 13 and 15 ft. above floor.....	3⅓
Ceiling between 15 and 18 ft. above floor.....	6⅔
Ceiling over 18 ft. above floor................	10

For rooms heated daily, but where heating is interrupted at night, add

$$\frac{0.25\,(N-1)}{Z}\text{ per cent,}$$

where N = hours from cessation of heating to time of starting again, Z = hours from starting heating again until required room temperature is reached.

TOTAL HEAT REQUIRED. — If there are no sources of heat already in the room the total number of B.t.u. which must be supplied per hour by the heating device installed is

$$H = H_A + H_R,$$

where H_A and H_R have the values given above, H_R including the allowances for exposure, etc. If there are a number of people in the room, or a number of lamps, or electric machinery (as in a power house or substation) the additional B.t.u. required in excess of the heat given off by the people or apparatus is

$$H - H_M,$$

where H_M is the B.t.u. given out by the people and apparatus, and is calculated as follows:

Heat Given Out by Electric Machinery, Lamps and People. — A loss of 1 kilowatt for one hour in any piece of electric machinery or lamp corresponds to 3415 B.t.u. per hour. A person gives out about 400 B.t.u. per hour. Hence if L is the average number of kilowatts loss per hour due to all the lamps and electric machinery in the room and N the number of people in the room, then the total B.t.u. per hour supplied to the room by the machinery, lamps and people in it is

$$H_M = 3415\,L + 400\,N.$$

HEATING APPARATUS. — For dwellings, shops, etc., hot-air, hot-water or steam-heating systems are usually employed; for large buildings the last two only. In substations or power houses (e.g., a hydro-electric station) where the heat losses are not sufficient to provide enough heat, either steam or electric heaters are used.

J. K. Allen (*Trans. A.S.H.V.E.*, *1908*) gives the following values of K (= B.t.u. radiated per square foot per hour per ° F. difference in temperature between steam and outside air) for cast-iron radiators, from which the necessary square feet of radiating surface may be calculated when the number of B.t.u. required have been determined as outlined above.

VALUES OF K FOR CAST-IRON RADIATORS

Diff. in temp. bet. steam and air, ° F.	2-column radiator	3-column radiator	Diff. in temp. bet. steam and air, ° F.	2-column radiator	3-column radiator
110	1.71	1.65	170	1.93	1.93
120	1.745	1.695	180	1.965	1.98
130	1.76	1.745	200	2.04	2.075
140	1.82	1.79	220	2.11	2.165
150	1.855	1.835	240	2.185	2.26
160	1.895	1.885	260	2.265	2.36

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[Wm. Kent.]

HEATING AND COOKING BY ELECTRICITY. — The essential element of an electric heating device is the "heating unit," which consists of a coil or plate of high-resistance wire or ribbon capable of supporting a high temperature (about 500° F. to 2200° F.) without deterioration. Data on a number of such resistor metals are given in the article on *Wires, Resistance*. When the heating unit is exposed directly to the air a non-oxidizing resistor must be used; nickel-chromium is a very satisfactory alloy for this purpose. German silver and nickel-steel alloys deteriorate rapidly at high temperatures when exposed to air.

Types of Heating Units. — A common form of heating unit used in flat-irons, chafing dishes, percolators, disc stoves, etc., is the encased disc, the resistor being either a ribbon wound on a mica disc or a grid stamped from a thin sheet of the alloy and mounted between thin sheets of mica; this type of unit is sometimes called a "monoplane heater." The radiant open-coil type of heating unit is used for toasters, grills, etc., mica or porcelain being used as the supporting insulator. Electric radiators for heating rooms are variously constructed, one type (G.E.) consisting of wire wound on asbestos tubes two inches in diameter and 22 inches long and covered with a coating of fire-proof cementing compound. When air is thus excluded German silver may be used as the resistor. A form of construction, called from its shape a "cartridge unit," is made of a high-resistance ribbon wound on a mica cylinder and coated with insulating cement; this unit is inserted in a hole in the casting which is to transfer the heat to the point where it is wanted. Cartridge units are used for disc stoves, grids and broilers. Where a comparatively low operating temperture is required, as in flexible heating pads, asbestos insulated wire woven into a sort of honeycombed mat is used.

Heating of Rooms. — The use of electric heaters for heating rooms is practically limited in this country to bathrooms, ticket booths, and intermittent auxiliary service in dwellings and offices. The power required to keep an ordinary sized room warm when the outside air is near the freezing point, ranges from about 1 to 2 watts per cubic foot; for more exact data see the article on *Heating of Buildings*. Were cost the only factor, electric heating, with electric energy at the present rates (2 to 10 cents per kilowatt-hour), would be out of the question. However, electric heating possesses the following advantages, which, in many instances, offset the higher cost, viz., (1) electric heaters may be fitted into places where coal or gas stoves could not be used, (2) the fire risk is less, (3) greater cleanliness is secured, and (4) the hygienic conditions are better.

Heating of Cars. — See article on *Railways, Energy Requirements for*.

Flatirons. — An internally-heated gas flatiron of household size burns about 5 cu. ft. of gas per hour. For continuous service with an externally-heated iron three irons are required, two heating while one is being used; for such service about 16 cu. ft. of gas are used per hour by the burner. An electric flatiron of household size takes about 550 watts. Hence, assuming gas to cost $1.00 per 1000 cu. ft. and electricity to cost 10 cents per kilowatt-hour, the energy cost per hour for each of the three types would be:

	Cents
Internally-heated gas flatiron	0.5
Externally-heated gas flatiron	1.6
Electric flatiron	5.5

However, the evident advantages of cleanliness, convenience, safety and comfort bring about a very extensive use of the electric flatiron, even though the the actual cost is greater than for coal or gas heating.

Electric Cooking. — Two types of electric "cookers" are in general use, the electric fireless cooker and the electric range.

Electric Fireless Cookers. — A number of designs of electric fireless cookers have been put on the market. An ingenious type is that made by the Berkeley Electric Cooker Co. This device consists of a food compartment, around which is wound a 550-watt heating element of nichrome ribbon, which in turn is surrounded by an air-tight water jacket partly filled with water and from which the air is exhausted, and this again is surrounded by a suitable thickness of heat-insulating material. Soon after the current is switched on, the water boils and the pressure produced in the water jacket forces out a metal diaphragm, which opens the main switch when the temperature of the water reaches about 212° F.; when the temperature falls below this value, the diaphragm automatically closes the switch again. Actual tests have shown that the temperature may thus be kept practically constant and that the current is on only about one-fifth of the time. Such a device takes care of that cooking which is the most expensive, viz., boiling, stewing and steaming.

Electric Ranges. — There are several types of electric ranges on the market designed to do all the cooking for from two persons up to two hundred. In one type (Simplex) "hot plates" in the top of the range are used, over which fit special flat-bottomed utensils with an apron or flange around the rim; good contact between the utensil and plate is secured by a lock or clamp with wedging action. In another type (G.E.) the cooking utensil is set into a chamber in the top of the range and covered up by an ordinary stove lid. The ovens are built of light sheet steel and have double walls, the space between being filled with a non-conductor of heat. Pilot lights are used to indicate when the current is on or off.

Advantages of Electric Cooking. — The following advantages of electric cooking over coal or gas are claimed: (1) cleanliness, (2) safety, (3) convenience, (4) a cool kitchen, (5) better cooking, especially broiled meats, (6) less shrinkage of meats, (7) no odors, (8) exact reproducibility of conditions, (9) actual saving in cost under certain conditions.

Cost of Electric Cooking. — A great amount of data on the kilowatt-hour consumption and cost per meal has appeared of late in the various technical journals. The average consumption per person per meal ranges from about 0.2 to 0.8 kw-hr., which at 3 cents per kilowatt-hour corresponds to a cost per person per meal of from 0.6 to 2.4 cent. The actual cost in any particular case of course depends upon the number of persons served, the food cooked and the kilowatt-hour cost.

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[A. P. KITCHEN.]

HOISTS, ELECTRIC. — (*See also Cranes; Elevators; Motors, Industrial Applications of; Telpherage.*) A hoist is a machine for raising and lowering weights. The most common application is in mines, to which service the following article is devoted.

The hoists most generally used are of the drum type, the drums being either cylindrical or conical or a combination of the two. As a rule two drums operating in balance are used for each hoist. The weight of the skip, or cage and car carrying the ore, is balanced by a similar empty skip which is lowered in a second compartment simultaneously with the hoisting of the loaded skip in the first, the loaded skip being dumped at the top and the empty one loaded at the bottom, and the cycle being repeated. The difference between balanced and unbalanced hoisting is that the static load when working unbalanced is greater than when working balanced, and the change of load due to the rope is only half as great.

Preliminary Data. — In dealing with hoisting problems the following data are required:

Present and ultimate depth of shaft.
If inclined, give angle of inclination with horizontal, or grade, in per cent.
Weight of material per trip.
Weight of skip or cage and car.
Diameter and weight of rope.
Size of tail rope, if used.
Rope speed.
Number of trips per hour.
Time required for loading and unloading.
Double or single drum.
Diameter and weight of drums.
Motor geared or direct connected.
Hoist balanced or unbalanced.
If balanced, will unbalanced operation ever be necessary?

Besides the above, the ordinary data of the power supply system must be known; also the restrictions as to permissible peak loads, so as to make it possible to decide whether a load equalizing set will be required.

For cylindrical hoists the power required for the various periods of the duty cycle can be figured directly in horse-power. For conical drums the problem is somewhat more complicated and a moment diagram is usually first figured and plotted, and then converted into horse-power.

HOIST FORMULAS FOR CYLINDRICAL DRUMS. — Let

w = weight of ore in pounds,
w_r = weight of one rope in pounds,
w_a = weight of rope wound on drum during acceleration period,
w_d = weight of rope wound on drum during retardation period,
w_s = weight of one skip or cage and car,
W = weight of revolving parts* (except motor armature) $+ w + 2w_r + 2w_s$ (balanced operation),
W' = weight of revolving parts $+ w + w_r + w_s$ (unbalanced hoisting),
W'' = weight of revolving parts $+ w_r + w_s$ (unbalanced lowering),
V = maximum rope speed in feet per second,
t_1 = period of acceleration in seconds,
t_2 = period of full-speed running in seconds,

* The weight of the motor armature should not be included in the weight of revolving parts W, as the horse-power obtained from the formulas is the output and not the input.

t_3 = period of retardation in seconds,
t_0 = period of rest in seconds,
T = actual hoisting time in seconds (not including rest),
L = depth of shaft in feet (total travel of rope),
θ = angle of incline with horizontal,

$$w_a = \left(\frac{V}{2}t_1\right) \times \text{(weight of rope in pounds per foot)},$$

$$w_d = \left(\frac{V}{2}t_3\right) \times \text{(weight of rope in pounds per foot)},$$

$$t_{eqv.} = \frac{L}{V}$$

$$t_1 + t_3 = 2\,(t_{eqv.} - t_2)$$

$$t_2 = \frac{2\,t_{eqv.} - (t_1 + t_3)}{2},$$

$$V = \frac{L}{T - \frac{t_1 + t_3}{2}},$$

$$t_1 + t_3 = 2\left(T - \frac{L}{V}\right).$$

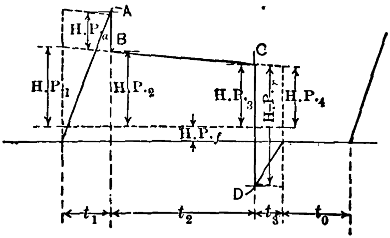

Fig. 1. Duty Cycle Diagram

The power required for each portion of a typical duty cycle, a diagram of which is given in Fig. 1, is indicated in the accompanying table.

Horse-power output at point $A = (HP)_a + (HP)_f + \left[\frac{(HP)_1 + (HP)_2}{2}.\right]$

Horse-power output at point $B = (HP)_2 + (HP)_f$.

Horse-power output at point $C = (HP)_3 + (HP)_f$.

Horse-power output at point $D = (HP)_r + (HP)_f + \left[\frac{(HP)_3 + (HP)_4}{2}.\right]$

$(HP)_r$ is always negative. If D is negative, power must be absorbed by either the motor or the brakes during the period of retardation. If D is positive, power must be delivered by the motor during retardation.

Motor Rating. — The rating of the motor can be obtained from the following formulas:

For Alternating Current.

$$(HP) = \sqrt{\frac{A^2 t_1 + \frac{B^2 + C^2 + BC}{3} \times t_2 + D^2 t_3}{0.50\,t_1 + t_2 + 0.50\,t_3 + 0.25\,t_0}}.$$

If D is negative the factor $(D^2 \times t_3)$ should be eliminated from the equation.

For Direct Current.

$$(HP) = \sqrt{\frac{A^2 t_1 + \frac{B^2 + C^2 + BC}{3} \times t_2 + D^2 t_3}{0.75\,t_1 + t_2 + 0.75\,t_3 + 0.5\,t_0}}.$$

SYSTEMS OF ELECTRIC HOISTING. — Of the large number of systems of electric hoisting which have been proposed, by far the greater majority can be included in the three following systems:

1. Those driven by induction motors.

HORSE-POWER FOR DUTY CYCLE

Period	Symbol (see Fig. 1)	Balanced hoisting	Unbalanced Hoisting: Hoisting	Unbalanced Hoisting: Lowering
Acceleration (max. value)	$(HP)_a$	$+\frac{WV^2}{32.2\times 550\,t_1}$	$+\frac{W'V^2}{32.2\times 550\,t_1}$	$+\frac{W''V^2}{32.2\times 550\,t_1}$
Retardation (max. value)	$(HP)_r$	$-\frac{WV^2}{32.2\times 550\,t_3}$	$-\frac{W'V^2}{32.2\times 550\,t_3}$	$-\frac{W''V^2}{32.2\times 550\,t_3}$
Hoisting (eqv.) at beginning of acceleration	$(HP)_1$	$+\frac{(w+w_r)\,V\sin\theta}{550}$	$+\frac{(w+w_r+w_s)\,V\sin\theta}{550}$	$-\frac{w_s\,V\sin\theta}{550}$
Hoisting, full speed at end of acceleration	$(HP)_2$	$+\frac{(w+w_r-2w_a)\,V\sin\theta}{550}$	$+\frac{(w+w_r+w_s-w_a)\,V\sin\theta}{550}$	$-\frac{(w_s+w_a)\,V\sin\theta}{550}$
Hoisting, full speed at end of full-speed run	$(HP)_3$	$+\frac{(w-w_r+2w_d)\,V\sin\theta}{550}$	$+\frac{(w+w_s+w_d)\,V\sin\theta}{550}$	$-\frac{(w_s+w_r-w_d)\,V\sin\theta}{550}$
Hoisting (eqv.) at end of retardation	$(HP)_4$	$+\frac{(w-w_r)\,V\sin\theta}{550}$	$+\frac{(w+w_s)\,V\sin\theta}{550}$	$-\frac{(w_s+w_r)\,V\sin\theta}{550}$
H.P. friction	$(HP)_f$	$+[(HP)_{f1}+(HP)_{f2}+(HP)_{f3}]$	$+[(HP)_{f1}+(HP)_{f2}+(HP)_{f3}]$	$+[(HP)_{f1}+(HP)_{f2}+(HP)_{f3}]$
H.P. mechanism friction (including head sheave)	$(HP)_{f1}$	$\frac{\frac{(HP)_1+(HP)_4}{2}}{\text{Eff.*}}-\frac{(HP)_1+(HP)_4}{2}$	Approximately equal to 60%-75% of $(HP)_{f1}$ for balanced hoisting.	Approximately equal to 60%-75% of $(HP)_{f1}$ for balanced hoisting.
Skip or car friction (slope only) (Coefficient of friction = 0.02)	$(HP)_{f2}$	$\frac{0.02\,(w+2w_s)\,V\cos\theta}{550}$	$\frac{0.02\,(w+w_s)\,V\cos\theta}{550}$	$\frac{0.02\,w_s\,V\cos\theta}{550}$
Rope (slope only) (Coefficient of friction = 0.10)	$(HP)_{f3}$	$\frac{0.10\,w_r\,V\cos\theta}{550}$	$\frac{0.10\,w_r\,V\cos\theta}{550}$	$\frac{0.10\,w_r\,V\cos\theta}{550}$

Efficiency for direct connection 0.88
Efficiency for single-gear reduction 0.85
Efficiency for double-gear reduction 0.80

2. Those driven by direct-current motors, power for which is supplied by a motor-generator set.
3. Those driven by direct-current motors, power for which is supplied by a flywheel motor-generator set.

1. Induction-motor System. — In the first and simplest system the speed of the direct-connected induction motor is controlled by a variable resistance, usually a variable resistance in the rotor circuit, which resistance is usually some sort of a water rheostat. The advantages of this system are: simplicity, low first cost, high motor efficiency when the hoist is running at full speed (approximately 90 per cent), no power consumed when the hoist is at rest. The disadvantages are: low motor efficiency during acceleration (approximately 45 per cent), no power is returned to supply system during retardation, large power consumption for small movements of cage, fluctuation of power demand.

2. Direct-current Motor System. — In the second system the hoist is driven by a separately excited direct-current motor, receiving power from the alternating-current supply system through a synchronous or induction motor-generator set. The hoist motor is controlled by varying the voltage of the generator, which is separately excited, one generator being used for each motor. The advantages of the system are: high motor efficiency during acceleration (approximately 80 per cent), return of large portion of energy during retardation. The disadvantages are: low motor efficiency when running at full speed (approximately 82 per cent), loss in motor-generator set when hoist is at rest, high initial and operating expense, fluctuation of power demand.

3. Direct-current System with Flywheel. — The third system takes advantage of the low first cost and efficiency of the flywheel (*see Flywheels for Load Equalization*) as a means for storing and returning large quantities of power for short intervals. This system is similar to the second, except for the addition of a flywheel to the induction-motor-generator set, and an automatic regulator for varying its speed. In its most common form this regulator consists of a water rheostat connected in series with the induction-motor armature. The resistance is varied by means of movable electrodes suspended from an arm mounted on the shaft of a small induction motor, which is connected in series either directly or through series transformers, with the induction motor of the flywheel set. The regulator motor is so connected that its torque opposes the weight of the electrodes, which are partially counterbalanced to reduce the size of the *regulator* motor to a minimum, and to permit of an adjustment of the regulator for different values of line current. When the line current exceeds the value for which the regulator is adjusted, the torque of the motor overbalances the weight of the electrodes, lifting them and inserting resistance in the armature circuit of the induction motor. This causes it to slow down, and allows the flywheel to assist in driving the generator during the peak loads.

Steam vs. Electric Systems. — A comparison between a steam system and these three electrical systems is given in the following table, in which the fuel and ore ratios for each are given for a small installation hoisting from a 2000-foot level, and a large installation hoisting from a 6000-foot level.

In addition to the saving in fuel which may be realized by the use of electric hoists instead of steam hoists, there is a very material reduction in the labor, the cost of which is chargeable against the hoist. This may amount to the wages of one or two men in the boiler house if power is developed by the mining company, or of the whole boiler house force if power is purchased, and frequently the wages of one man in the hoist house.

Power Consumption. — So many factors enter into the cost of electric hoisting that each individual case must be analyzed separately. An approximate

Installation	Hoisting system	Coal burned, tons per day	Ore hoisted, tons per day	Ratio tons ore to tons coal
Small, hoisting from 2000-foot level.	Steam	47	1780	40
	Electric, first system	13	1780	137
	Electric, second system	15	1780	119
	Electric, third system	16	1780	110
Large, hoisting from 6000-foot level.	Steam	65.5	1580	24
	Electric, first system	23	1580	69
	Electric, second system	24	1580	66
	Electric, third system	25	1580	63

estimate of the power consumption would be from 1¼ to 2¼ kilowatt-hours per 1000 ton-feet, the tonnage, in the case of unbalanced hoisting, including the weight of the ore and skip, while for balanced hoisting only the ore.

SPECIFICATIONS FOR ELECTRIC HOIST.* — The following memoranda are intended to assist in writing specifications. See also article on *Elevators, Electric* and the article on *Specifications*.

Principal Characteristics and Conditions of Service. — General description and use of hoist. Motor voltage and frequency (*see articles on Motors*). Mechanical rating, i.e., load to be lifted a specified height in a specified time. Proportion of time above load will be carried and of time machine is at rest.

Details of Construction. — Length of hoisting rope and whether or not rope is to be supplied. If so, details of rope and of hook or other device. Brake requirements. Lubrication details. Materials of principal parts.

Performance and Tests. — Maximum load to be lifted. Maximum speed descending. How tests are to be conducted.

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[D. B. Rushmore, assisted by E. A. Lof.]

* By W. A. Del Mar.

HYDRAULICS. — (*See also Dams; Hydrology; Pipes and Piping; Power Stations, Hydro-Electric; Water Wheels; Water Wheels, Speed Regulation of.*) Hydraulics comprises the principles and laws governing the motion and mechanical reactions of liquid bodies. There are two general divisions of the subject: Hydrostatics, which refers to liquids at rest; and Hydrodynamics, which refers to liquids in motion. This article will be confined to that portion of the subject which relates more directly to the problems met with in the study of water-powers.

Head and Pressure. — In water power terminology the word *head* means the difference in level of water between two points. It is usually expressed in feet. The pressure of water in a pipe, for instance, is usually given as so many feet head and not as so many pounds per square inch. The relation between head in feet, h, and pressure in pounds per square inch, p, is as follows:

$$h = 2.31\,p,$$
$$p = 0.433\,h.$$

These formulas assume 62.4 lb. as the weight of 1 cu. ft. of water. See also *Units and Conversion Factors.*

POWER OF WATER. — Water exerts its power by falling through some distance and its action is exactly analogous to that of a falling weight. The total number of foot-pounds of work per second which a stream is capable of performing is equal to the weight of a cubic foot of water, multiplied by the number of cubic feet flowing per second, and by the head or difference in level of the stream at the two places under consideration. If the horse-power is desired, divide the number of foot-pounds of work done per second by 550. This gives the "theoretical" horse-power, i.e., it includes not only the power available for useful work but also the power lost in friction in the various water passages, in the wheel, and in giving velocity to the water discharged from the wheel.

Let Q = number of cubic feet of water flowing per second;
h = total head or difference in elevation between pond and tail water;
ϵ = over-all efficiency, as a decimal, including all losses up to wheel shaft.

Then the net power available at the wheel shaft is

$$P = 0.114\,Qh\epsilon \qquad \text{horse-power.}$$

The over-all efficiency (pond to wheel shaft) in a well-designed water-power station should not vary greatly from 80 per cent. This does not include the losses in electric machinery nor the losses in long pipe lines.

Friction and Velocity Head. — Neglecting any leakage of water (i.e., assuming no water to pass into the tail race except through the wheel) the losses of energy due to friction and to giving velocity to the water may each be represented by a loss of head. The total loss of head due to friction is called the "friction head," and may be represented by the symbol h_f; the loss of head due to giving velocity to the water is called the "velocity head," and may be represented by the symbol h_v. The net useful head is the $h_u = h - (h_f + h_v)$, where h is the total head, and the efficiency is

$$\epsilon = 1 - \frac{h_f + h_v}{h}.$$

The friction head depends upon the cross section, shape and roughness of the water passages, see below, and in the case of a water wheel the loss due to the friction in the bearings may also be considered as contributing to the friction head. The friction head which can be economically allowed in the intake and discharge passages depends so greatly upon local conditions that no general rules can be given here.

The net velocity head may be expressed as

$$h_v = \frac{v^2}{2g},$$

where v is the linear velocity, in feet per second, of the tail water where it comes out at atmospheric pressure, and g is the gravitational acceleration (32.16 feet per second per second). The importance of a low discharge velocity is therefore apparent.

FLOW THROUGH PIPES. — See articles on *Pipes and Piping* and *Water Wheels, Speed Regulation.*

FLOW THROUGH OPEN CHANNELS. — The term channel includes all forms of closed conduits when flowing partly full as well as ordinary stream beds or canals. The rate of flow in such a case depends upon the slope given to the water surface. It is difficult to treat analytically the flow in natural channels, because of the wide divergence in shape of cross-section and nature of sides and bottom of channel. Only artificial channels are considered here, although the same formulas apply to any section of natural channel which has the same cross-section throughout.

Chezy's Formula. — This formula gives the relation between the linear velocity of the water, hydraulic radius and the slope and a coefficient which must be determined experimentally, viz.,

$$v = C\sqrt{rs},$$

where v = linear velocity of water in feet per second, r = hydraulic radius in feet, s = slope of stream (i.e., the difference in elevation between two points in the water surface divided by the slope distance between the two points measured along the surface) and C is an experimentally determined coefficient which may also be expressed by the empirical formulas given below. These formulas, however, may give results from 5 to 10 per cent in error.

The values of C which may be expected in ordinary cases are as follows:

	Values of C
Very smooth cement or timber	125 to 150
Ordinary timber, concrete, brickwork, etc	100 " 130
Rough masonry or firm gravel	70 " 90
Earth bed in good condition	50 " 75
Earth bed in bad condition	25 " 50

Hydraulic Radius. — This term is used to designate the quotient of the area of a cross-section, e.g., $ABCD$ in Fig. 1, by the *wetted* perimeter of the channel walls and bottom, $AB + BC + CD$ in Fig. 1.

Fig. 1.

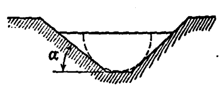

Fig. 2.

Best Section to Use. — The most advantageous cross-section to use is that having the maximum value of the hydraulic radius r. The semi-circle is therefore the best but is the most difficult to construct and to maintain. Of trapezoidal cross-sections the half hexagon is the best and of rectangular cross-sections the half square. In unlined earth channels the trapezoidal cross-section must be used. Of these sections the best to use is shown in Fig. 2. The angle α is determined from the character of the soil. The sides and bottom are made tangent to a semi-circle having its center at the water surface.

Kutter's Formula. — This is one of two formulas in common use for expressing the coefficient C in Chezy's formula in terms of the dimensions of the cross-section of the channel. Kutter's formula (published in 1869) is

$$C = \frac{41.65 + \frac{0.00281}{s} + \frac{1.811}{n}}{1 + \left(41.65 + \frac{0.00281}{s}\right)\frac{n}{\sqrt{r}}},$$

where r and s are as above, and n is the "coefficient of roughness," values of which for various channel linings are as follows:

	Values of n
Planed timber, glazed or enameled surfaces	0.009
Smooth clean cement	0.010
Unplaned timber, new well-laid brickwork	0.012
Smooth stonework, ordinary brickwork, iron	0.013
Rough ashlar and good rubble masonry	0.017
Firm gravel	0.020
Earth in ordinary condition	0.025
Earth with stones, weeds, etc	0.030
Earth or gravel in bad condition strewn with detritus	0.035

Bazin's Formula. — The second formula commonly used for expressing the coefficient C in Chezy's formula is that given by Bazin in 1897, viz.,

$$C = \frac{87}{0.552 + \frac{m}{\sqrt{r}}},$$

where r is as above and m is a "coefficient of roughness" values of which are as follows:

	Values of m
Very smooth cement surfaces or planed boards	0.06
Concrete, well-laid brick, unplaned boards	0.16
Ashlar, good rubble masonry, poor brickwork	0.46
Earth beds in perfect condition	0.85
Earth beds in fair or ordinary condition	1.30
Earth beds in bad condition covered with debris	1.75

Bazin's formula is probably the best, and it certainly is the simpler one to use. Many engineers, however, prefer to use Kutter's formula, which has been graphically solved in a book of diagrams by Prof. Church (Wiley & Sons).

FLOW OVER WEIRS.—In hydraulics the term "weir" is used in general to designate any kind of a dam across a stream, and in particular to designate a vertical notch, usually rectangular, by means of which the quantity of water passing a given point may be determined, Fig. 3.

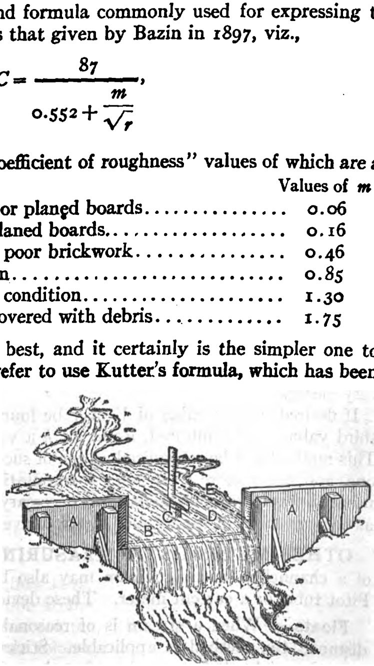

Fig. 3.

Definitions Regarding Weirs. —The "crest" is the horizontal edge of the weir. The "head" on a weir is not the depth of water over the crest, but is the difference in level between the crest and the reservoir surface.

A "fully contracted" weir is one in which each end of the weir is at a distance of at least 3 times the head from the sides of the channel. A "suppressed" weir is one in which the sides of the channel form the end of the weir. The "velocity of approach" is the velocity of the water in the channel leading to the weir. For a given head on the weir the quantity of water passing the weir depends somewhat upon the velocity of approach.

Weir Formulas. — The formulas below are applicable only when the following conditions are fulfilled: (1) edges of weir sharp, smooth and beveled on the down stream side as shown in Fig. 3 (or a thin metal plate bolted to a wooden board may be used for the edge of the weir), (2) the crest of the weir must be at least 3 times the head above the bottom of channel, (3) the crest of the weir must have a length at least 3 times the head on it. In all the formulas

Q = cu. ft. of water per sec. flowing over weir,
H = measured head on weir, in ft.,
h = head in ft., due to velocity of approach,
H_1 = "effective" head on weir, in ft.,
b = breadth of weir (length of crest) in ft.

Then for a *fully contracted* weir

$$Q = 3.33\,(b - 0.2\,H)\,H_1^{\frac{3}{2}}$$

and for a *suppressed* weir

$$Q = 3.33\,bH_1^{\frac{3}{2}},$$

when the cross-section of the stream immediately above the weir is more than $6\,bH$ the effect of the velocity of approach is negligible and H_1 may be taken equal to H. When the cross-section of the stream is less than $6\,bH$, then

$$H_1^{\frac{3}{2}} = (H + h)^{\frac{3}{2}} - h^{\frac{3}{2}}.$$

It should be carefully noted that the above formulas hold only under the conditions specified; when applied to weirs of other shape (e.g., a dam with a flat or curved surface) a coefficient other than 3.33 must be used (*see Dams*).

Determination of Velocity Head (h). — First take $H_1 = H$, and from the above formula calculate Q. This value of Q divided by the cross-section A of the channel immediately above the weir gives an approximate value for the velocity of approach V. The head due to the velocity of approach is then approximately $h = \dfrac{V^2}{64.32}$. Recompute the discharge Q, calculating $H_1^{\frac{3}{2}}$ from the measured head H and this value of h. This will usually give the discharge very closely.

If desired, a new value of V may be found from this new value of Q and a third value of Q computed, using for h a value based on the new value of V. This method will be recognized as one of successive approximations. Generally one, and never more than two, recomputations are all that are necessary. In using the weir formulas given it is necessary to adhere to this method strictly, as the constants in the formulas were derived by using this method.

OTHER METHODS OF MEASURING FLOW. — The rate of discharge of a channel or natural stream may also be measured by means of floats, a Pitot tube, or a current meter. These devices are briefly described below.

Floats. — When a stream is of reasonably regular cross-section for a little distance, this method is applicable. Sticks about 1¼ inches in diameter, sufficiently weighted at one end so that they will float upright in the water, should be used. Surface floats are unreliable because of the influence of wind, skin friction of the water, etc. The floats are started at the beginning of a measured course and the time which they take to reach the end of the course is accu-

rately taken. The velocity of the stream is thus obtained; this multiplied by the area of the cross-section gives the rate of discharge.

The floats should be started from various points in the width of the stream in order to obtain the average velocity as nearly as possible. They should be long enough to come as close as practicable to the bottom of the stream without touching, in order to average the velocities, which are variable throughout the depth of the stream. This method is less accurate than the use of the current meter.

Current Meters. — The ordinary current meter is essentially a wheel revolved by the water and held with its axis perpendicular to the direction of flow by means of vanes. Some means of recording the revolutions are provided. The meter must be rated by drawing it through still water at various measured speeds. Elaborate apparatus for meter rating is maintained by the Government and by various universities. Measurements of the velocity of the stream are made at frequent intervals across the stream and at varying depths, and the average velocity determined; this multiplied by the cross-sectional area of the stream for the height of water existing at the time gives the rate of discharge.

Pitot Tube. — This is merely an open tube having a right-angle bend, the tube is placed in the stream in the position shown in Fig. 4, the lower end being directed against the current. The height h to which the water rises in the tube above the surface of the stream is equal to the velocity head, or the linear velocity of the stream is

$$v = \sqrt{2gh},$$

where h is in ft., $g = 32.16$ ft. per sec. per sec., and v in ft. per sec. This formula does not apply to the flow through a pipe, since the pressure head must also be taken into account. For measuring the flow in pipes a differential gage of this type may be used, but the Venturi meter (see below) is better suited to such measurements.

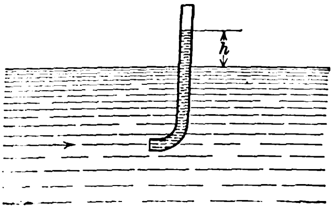

Fig. 4. Pitot Tube

Venturi Meter. — This meter, Fig. 5, serves as a simple and accurate means of measuring the discharge through a pipe. The meter consists essentially of an hour-glass-shaped section of pipe, with smoothly rounded internal walls, into which are fitted two gages as shown. The planes of the internal openings of gage pipes are at right angles to the direction of the current. Referring to Fig. 5, let

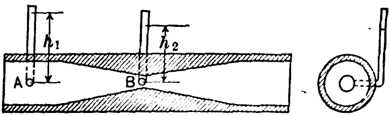

Fig. 5. Venturi Meter

a_1 = area, in sq. ft., of the cross-section at A,
a_2 = area in sq. ft., of the cross-section at B,
h_1 = reading of gage at A, ft. of water column,
h_2 = reading of gage at B, ft. of water column,
g = acceleration due to gravity, ft. per sec. per sec.,
C = a coefficient, ranging from 0.94 to 1.00, which takes into account the friction in the meter.

Then the discharge through the pipe in cu. ft. per sec. is

$$Q = C \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \sqrt{2 g (h_1 - h_2)}.$$

Ordinary pressure gages may be used at A and B, and their reading converted to feet of head, with a correction added for height of gage above center line of meter.

FLOW THROUGH NOZZLES. — Nozzles for practical purposes are made either of a conical shape with a cylindrical tip of sufficient length so that the water will touch it, or are bell-shaped with the sides parallel at the point of exit. The discharge through such nozzles in cubic feet per second is found by multiplying the product of the area of the nozzle in square feet and its "coefficient of discharge" by $\sqrt{64.32\,H}$, where H is the sum of the pressure head and velocity head at the base of the nozzle, in feet. The value of the coefficient of discharge for nozzles averages about 0.97.

When the nozzle is used for the purpose of measuring water a pressure gage is usually attached to the base of the nozzle. This gage should be at the same height as the base of the nozzle. If from necessity it is located at a higher point, the difference in elevation between the center of the gage and the center of the nozzle base must be added to the gage reading. If it is below, the difference in elevation must be subtracted. This gage measures only the pressure head at its point of attachment. There is also at this point a considerable velocity in the pipe which adds materially to the discharge. Instead of using the total head H, the discharge in cubic feet per second may be expressed in terms of the pressure head only by the formula

$$Q = a \sqrt{\frac{2 gh}{\left(\frac{1}{C}\right)^2 - \left(\frac{a}{A}\right)^2}},$$

where h = pressure head in feet at base of nozzle (from gage), a = cross-sectional area of nozzle tip in square feet, A = cross-sectional area of pipe in square feet at point of attachment of gage, C = coefficient of nozzle, g = gravitational acceleration in feet per second per second. When $C = 0.97$ and $g = 32.16$,

$$Q = 8.02\, a \sqrt{\frac{h}{1.063 - \left(\frac{a}{A}\right)^2}}.$$

BIBLIOGRAPHY. — A complete bibliography up to 1908 is given in Mead's *Water Power Engineering*. Among the references there given may be noted Merriman, M., *Treatise on Hydraulics*, N. Y., 1903; Bellasis, E. D., *Hydraulics with Tables*, N. Y., 1911. Some more recent references are Russell, G. E., *Text Book on Hydraulics*, N. Y., 1912; Hughes and Safford, *A Treatise on Hydraulics*, N. Y., 1911.

[L. E. MOORE.]

HYDROLOGY. — (*See also Hydraulics; Power Stations, Hydro-Electric.*) In the broadest sense, hydrology is the science of water, including its properties, the phenomena and natural laws associated with it, its distribution over the earth's surface, and in fact everything relating to water as a physical entity. As ordinarily employed, however, the term is limited to include only those laws and properties which have to do directly with its distribution over the earth's surface, particularly the relations between rain-fall, natural drainage, and the flow of water in rivers.

Hydrological Data Required in Water-power Engineering. — To determine whether a particular stream is capable of development and the economical extent of its possible development, it is necessary to know not only the *average* rate of flow of the water in the stream, but also the *variation* in flow from day to day and from year to year. In those rare cases where gaging stations have been maintained for a number of years on a stream these data may be directly determined. In most cases, however, it is necessary to estimate the flow from the rain-fall over the water shed drained by the stream. An estimate of this kind requires careful judgment, because the intensity and distribution of the rain-fall have a very important influence. The character of the drainage area, whether sandy, rocky, or clayey, the character of the topography and the character and extent of the vegetation are all important. While it is difficult to reduce these different elements to a mathematical formula, the general principles involved are discussed below.

RAIN-FALL. — Rain-fall is usually expressed in inches, the number of inches being the depth of rain water which would be caught in a vessel with vertical sides set out in the open. The records of rain-fall may be obtained from the United States Weather Bureau in Washington, D.C. Records covering as extended a period and as wide a portion of the watershed under consideration as is practicable should be obtained. Too absolute dependence should not be placed on these results, as the intensity of rain-fall at different places in the drainage area of the stream under consideration may be quite variable, not only in general, but also in particular storms. It is conceivable that a station for recording rain-fall might be located at a point in an area where the annual precipitation is far above or below the average, although such a condition is not likely to occur.

The average rain-fall for an average year over an area, multiplied by the area, will give the total average rain-fall. For power purposes it is important to know whether this average is fairly representative, or whether some years may fall far below the average. If the latter be the case it may be financially practicable to develop only on the basis of minimum rain-fall, if at all. Where records for an area do not exist, it may be possible to obtain records for an area at a similar altitude for a similar locality. These records may be used for an estimate, but such an estimate should not be relied upon too implicitly.

RUN-OFF. — By the "run-off" from any area is meant the amount of rain water falling on this area which ultimately finds its way into the stream or streams draining this area. Run-off may be expressed either in inches or in per cent of the rain-fall on the given area.

Factors Affecting Run-Off. — The primary factor is of course the rain-fall. For a given rain-fall, the nature of the soil, topography, climate and vegetation all affect the run-off. A porous, sandy soil may absorb nearly all the rain-fall, whereas a rocky one will absorb little or none. The run-off from steep slopes is much more rapid than from gentle ones. The climate has a considerable effect. In this connection the effect of temperature conditions should be noted. Rain, falling on frozen ground, runs off almost at once. Trees and

vegetation absorb more or less moisture and their roots obstruct the flow of ground-water to an extent which governs very largely the rapidity with which the water reaches the stream. Forests on a water-shed are the best means of equalizing stream flow throughout the seasons and preventing floods. The shade which forests cast has an extremely important effect on evaporation, reducing it very considerably, and consequently making available a much larger proportion of the rain-fall for power purposes. Fig. 1 shows the probable extreme variations in the relation between rain-fall and run-off (*from Proc. Eng. Club, of Phila., 1895*).

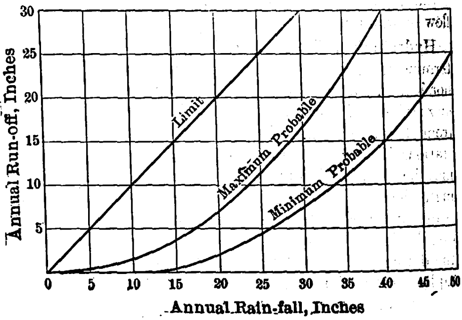

Fig. 1.

Formula for Run-Off in Terms of Rain-Fall. — J. D. Justin (*Proc. A.S.C.E., Vol. 39, p. 1211*) gives the following formula for the *annual* run-off, which applied to a number of drainage areas in the Eastern States gave results which checked with the actual run-off to within 10 per cent. It should be applied with caution to other parts of the country as both the coefficient 0.934 and the exponent 0.155 may vary under other climatic conditions. It should be applied to give the annual, not the monthly, run-off. Let C = annual run-off, in inches, on the watershed, R = annual rain-fall, in inches, on the watershed, S = "slope" of the watershed, defined as the difference in elevation in feet between highest and lowest point divided by the square-root of the area in square feet, T = mean annual temperature on watershed, in degrees Fahrenheit (*see next paragraph*); then

$$C = \frac{0.934\, S^{0.155} R^2}{T}.$$

Determination of Mean Annual Temperature (T). — Mean annual temperature can be obtained from the *Summary of Climatological Data for U.S. by Sections* issued by the U.S. Weather Bureau. If the watershed is mountainous and the observations of temperature are taken only in the valleys, the mean annual temperature over the whole watershed will be lower than that at the stations. To obtain the actual mean temperature (item 6 below) over the entire watershed from the observations at the stations, proceed as follows: (1) Take the average temperature at the stations; (2) Take the average elevation for the same stations; (3) Find the "average" elevation of watershed, i.e., the sum of the elevations above sea level of the highest and lowest points divided by 2; (4) Take the difference between the average elevation of the stations and the average elevation of the watershed; (5) Find the mean difference in temperature per foot of increase in elevation, by comparing records and elevations at the several stations; (6) Multiply (4) by (5) and subtract the result from (1).

STREAM FLOW. — The most satisfactory way of determining the water available from a given stream is to make a direct measurement of the rate of discharge. When the stream is small the number of cubic feet per second may be readily and cheaply measured by constructing a weir across it; see article on *Hydraulics*. When the stream is large it is necessary to measure the linear

velocity of the water at various points in a given cross-section, using for this purpose some form of current meter, or Pitot tube, or floats; see *Hydraulics*. From these velocity measurements at various intervals across the stream and at various depths, the average linear velocity of the water is determined; this, in ft. per sec., multiplied by the cross-section of the stream in sq. ft. gives the number of cu. ft. per second (frequently called "second-feet") flowing.

Rating Curve. — To obtain daily records of the stream flow at the chosen section a permanent post (called a gage), usually graduated in feet and inches, may be set up, and the height of water noted corresponding to the discharge under various stream conditions. A curve, called a "rating curve," see Fig. 2, may then be plotted, giving the discharge in cu. ft. per sec. as abscissa and the gage height as ordinate. Such a curve will in general be irregular due to the irregular shape of the cross-section of the stream for various depths of water. After a sufficient number of velocity measurements for various gage heights have been taken to give the shape of the rating curve, it is only necessary to make daily observations of the gage height in order to obtain a daily record of the stream flow.

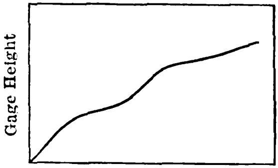

Fig. 2.

U.S. Government Records of Stream Flow.— The U.S. Government has maintained for a number of years gaging stations at one or more points on the larger rivers and their important tributaries. These records are published in the Annual Reports of the U.S. Geological Survey (1888 to 1900) and in the Water Supply and Irrigation Papers (1896 to date) also issued by the U.S. Geological Survey. Even though there are no government records for the particular stream under consideration, the records for a neighboring stream will be found of value in arriving at the probable relation between run-off and rain-fall for the stream in question. If the power project be a large one, it may be desirable to establish a gaging station and maintain it for at least one year.

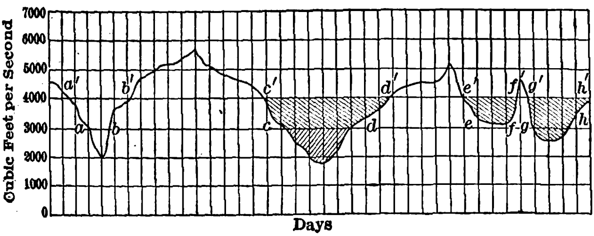

Fig. 3.

Hydrograph. — The hydrograph is a curve showing rate of flow of water as ordinates and time as abscissas; see Fig. 3. The quantities may be expressed in any units, but usually the rate of flow is expressed in cubic feet per second and the time in days. The average flow in cubic feet per second is plotted on the day during which the flow occurred. This is done for each successive day. A curve drawn through the points so obtained is the hydrograph. It must be clearly understood that the word day refers to some definite date, as Oct. 3, 1913, and not day in the abstract.

It should be noted that the hydrograph for different stations along the course of a stream will in general differ not only in the heights of the ordinates (average

rate of flow) but will also differ in shape, due to the difference in run-off from the different portions of the watershed.

Mass Curve. — The mass curve is a graphical representation of the total available amount of water during any period. Abscissas represent time and ordinates total net quantity available up to the date represented by the abscissa allowing for evaporation, seepage, etc. The time scale is usually months, while the water scale may be (1) inches on the drainage area, (2) total cubic feet, (3) acre-feet, or (4) second-feet-months (i.e., an average flow of Q cu. ft. per sec. for M months is MQ second-feet-months). The slope of the curve at any point indicates the rate of flow at that time.

STORAGE AND PONDAGE. — In water-power engineering the term "storage" is used when referring to a reservoir of sufficient capacity to equalize the flow of a stream from month to month, and the term "pondage" is used when referring to a reservoir (usually the pond above the dam at the power-house site) of a capacity only sufficient to equalize the flow of the stream for a short period, say, from one to three days. For the complete utilization of the power possibilities of a stream there should be storage enough to equalize the flow throughout the year. In making storage and pondage calculations the following relation is useful: 1 cubic foot per second for 24 hours will cover an acre with water 2 feet deep; that is, 1 cubic foot per second for 12 hours requires a storage of 1 acre-foot, or, to be precise, 0.992 acre-feet.

Calculation of Size of Storage Reservoir. — The following example illustrates the method recommended by Mead (*Water Power Engineering, N.Y., 1908*). Let Fig. 3 represent the hydrograph of a stream during the low-water period of the driest year. The area of each rectangular space represents 86,400,000 cu. ft. or 2000 acre-ft. Let it be required to find the capacity of the storage reservoir required to maintain a flow of 3000 sec.-ft. or over throughout the year. The maximum deficiency below 3000 sec.-ft. occurs between c and d and amounts to 3.25 rectangles or 6500 acre-ft., as shown by the cross-hatched area. A reservoir capable of supplying this quantity of water will also be able to supply the deficiency between a and b and between g and h, since the amount drawn from it during the interval ab will be restored during the interval bc and the amount drawn out during the interval cd will be restored during the interval dg, giving an ample supply of water to supply the deficiency during the interval gh. The actual capacity of the reservoir should exceed the 6500 acre-ft. by an allowance for evaporation, this allowance depending on the duration of the low-water periods, the time of the year, and the surface area of the reservoir.

Again, let it be required to find the capacity of the reservoir required to maintain a flow of 4000 sec.-ft. or over. In this case the entire interval from c' to h' must be considered, since the small amount of water stored during the intervals $d'e'$ and $f'g'$ is not sufficient to maintain the flow during the intervals $e'f'$ and $g'h'$. The total deficiency during the period $c'h'$ is 18 rectangles, shown by the cross-hatched area. The water stored during this interval, the unshaded portion above the 4000 line, is 4 rectangles. The difference, $18 - 4 = 14$ rectangles or 28,000 acre-ft., represents the net capacity of the reservoir, to which should be added an allowance for evaporation.

BIBLIOGRAPHY. — A complete bibliography up to 1908 is given in Mead's *Water Power Engineering*. Among the references there given may be noted the *Annual Reports of the U. S. Geological Survey; Water Supply and Irrigation Papers of the U. S. Geological Survey;* Hoyt and Grover, *River Discharge*, N. Y., 1907; numerous papers in the *Engineering News*. Some more recent references are: Parker, P.A.M., *The Control of Water*, London, 1913; Turneaure and Russell, *Water Supply*, N. Y., 1913.

[L. E. MOORE.]

HYPERBOLIC FUNCTIONS. — (*See also Derivatives; Integrals; Series, Mathematical.*) Hyperbolic functions are an extension of the trigonometric functions to those cases where the use of the latter gives rise to imaginary or complex angles. From the relations,

$$\cos x = \frac{\epsilon^{jx} + \epsilon^{-jx}}{2}$$

$$\sin x = \frac{\epsilon^{jx} - \epsilon^{-jx}}{2j}$$

where $j = \sqrt{-1}$, it follows that, putting $x = jz$:

$$\cos jz = \frac{\epsilon^{z} + \epsilon^{-z}}{2} \qquad (1)$$

$$-j \sin jz = \frac{\epsilon^{z} - \epsilon^{-z}}{2} \qquad (2)$$

Expressions (1) and (2) are both real quantities when z is real, that is, when the angle jz is imaginary. The first expression is called the hyperbolic cosine of z, abbreviated and pronounced "cosh"; the second expression is called the hyperbolic sine of z, abbreviated sinh and pronounced "shin." Hence, using x for the variable,

$$\sinh x = \frac{\epsilon^{x} - \epsilon^{-x}}{2}$$

$$\cosh x = \frac{\epsilon^{x} + \epsilon^{-x}}{2}$$

The hyperbolic tangent, cotangent, secant and cosecant are defined as follows:

$$\tanh x = \frac{\sinh x}{\cosh x}$$

$$\coth x = \frac{\cosh x}{\sinh x}$$

$$\operatorname{sech} x = \frac{1}{\cosh x}$$

$$\operatorname{csch} x = \frac{1}{\sinh x}$$

The hyperbolic angle x is a number analogous to radians in circular measure; it is never expressed in degrees.

Period of the Hyperbolic Functions. — Adding 2π to an angle does not change the value of the trigonometric functions; they are therefore said to have a period equal to 2π radians. Hyperbolic functions, however, have no true period, but adding $2\pi j$ to the hyperbolic angle does not change the values of the functions; hence these functions have an imaginary period, $2\pi j$.

Table of Hyperbolic Functions.* — Below is given a table of hyperbolic functions.

* More complete tables of hyperbolic functions may be found in the *Smithsonian Mathematical Tables*, by G. F. Becker and C. E. Van Orstrand, Washington, 1909 and in *Tables of Complex Hyperbolic and Circular Functions*, by A. E. Kennelly, Harvard Univ. Press, 1914.

HYPERBOLIC FUNCTIONS

Angle		0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
	sinh	0.0000	0.0100	0.0200	0.0300	0.0400	0.0500	0.0600	0.0701	0.0801	0.0901
0.0	cosh	1.0000	1.0001	1.0002	1.0005	1.0008	1.0013	1.0018	1.0025	1.0032	1.0041
	tanh	0.0000	0.0100	0.0200	0.0300	0.0400	0.0500	0.0599	0.0699	0.0798	0.0898
	sinh	0.1002	0.1102	0.1203	0.1304	0.1405	0.1506	0.1607	0.1708	0.1810	0.1911
0.1	cosh	1.0050	1.0061	1.0072	1.0085	1.0098	1.0113	1.0128	1.0145	1.0162	1.0181
	tanh	0.0997	0.1096	0.1194	0.1293	0.1391	0.1489	0.1587	0.1684	0.1781	0.1878
	sinh	0.2013	0.2115	0.2218	0.2320	0.2423	0.2526	0.2629	0.2733	0.2837	0.2941
0.2	cosh	1.0201	1.0221	1.0243	1.0266	1.0289	1.0314	1.0340	1.0367	1.0395	1.0423
	tanh	0.1974	0.2070	0.2165	0.2260	0.2355	0.2449	0.2543	0.2636	0.2729	0.2821
	sinh	0.3045	0.3150	0.3255	0.3360	0.3466	0.3572	0.3678	0.3785	0.3892	0.4000
0.3	cosh	1.0453	1.0484	1.0516	1.0549	1.0584	1.0619	1.0655	1.0692	1.0731	1.0770
	tanh	0.2913	0.3004	0.3095	0.3185	0.3275	0.3364	0.3452	0.3540	0.3627	0.3714
	sinh	0.4108	0.4216	0.4325	0.4434	0.4543	0.4653	0.4764	0.4875	0.4986	0.5098
0.4	cosh	1.0811	1.0852	1.0895	1.0939	1.0984	1.1030	1.1077	1.1125	1.1174	1.1225
	tanh	0.3800	0.3885	0.3969	0.4053	0.4136	0.4219	0.4301	0.4382	0.4462	0.4542
	sinh	0.5211	0.5324	0.5438	0.5552	0.5666	0.5782	0.5897	0.6014	0.6131	0.6248
0.5	cosh	1.1276	1.1329	1.1383	1.1438	1.1494	1.1551	1.1609	1.1669	1.1730	1.1792
	tanh	0.4621	0.4700	0.4777	0.4854	0.4930	0.5005	0.5080	0.5154	0.5227	0.5299
	sinh	0.6367	0.6485	0.6605	0.6725	0.6846	0.6967	0.7090	0.7213	0.7336	0.7461
0.6	cosh	1.1855	1.1919	1.1984	1.2051	1.2119	1.2188	1.2258	1.2330	1.2402	1.2476
	tanh	0.5370	0.5441	0.5511	0.5581	0.5649	0.5717	0.5784	0.5850	0.5915	0.5980
	sinh	0.7586	0.7712	0.7838	0.7966	0.8094	0.8223	0.8353	0.8484	0.8615	0.8748
0.7	cosh	1.2552	1.2628	1.2706	1.2785	1.2865	1.2947	1.3030	1.3114	1.3199	1.3286
	tanh	0.6044	0.6107	0.6169	0.6231	0.6292	0.6352	0.6411	0.6469	0.6527	0.6584
	sinh	0.8881	0.9015	0.9150	0.9286	0.9423	0.9561	0.9700	0.9840	0.9981	1.0122
0.8	cosh	1.3374	1.3464	1.3555	1.3647	1.3740	1.3835	1.3932	1.4029	1.4128	1.4229
	tanh	0.6640	0.6696	0.6751	0.6805	0.6858	0.6911	0.6963	0.7014	0.7064	0.7114
	sinh	1.0265	1.0409	1.0554	1.0700	1.0847	1.0995	1.1144	1.1294	1.1446	1.1598
0.9	cosh	1.4331	1.4434	1.4539	1.4645	1.4753	1.4862	1.4973	1.5085	1.5199	1.5314
	tanh	0.7163	0.7211	0.7259	0.7306	0.7352	0.7398	0.7443	0.7487	0.7531	0.7574
	sinh	1.1752	1.1907	1.2063	1.2220	1.2379	1.2539	1.2700	1.2862	1.3025	1.3190
1.0	cosh	1.5431	1.5549	1.5669	1.5790	1.5913	1.6038	1.6164	1.6292	1.6421	1.6552
	tanh	0.7616	0.7658	0.7699	0.7739	0.7779	0.7818	0.7857	0.7895	0.7932	0.7969
	sinh	1.3356	1.3524	1.3693	1.3863	1.4035	1.4208	1.4382	1.4558	1.4735	1.4914
1.1	cosh	1.6685	1.6820	1.6956	1.7093	1.7233	1.7374	1.7517	1.7662	1.7808	1.7956
	tanh	0.8005	0.8041	0.8076	0.8110	0.8144	0.8178	0.8210	0.8243	0.8275	0.8306
	sinh	1.5095	1.5276	1.5460	1.5645	1.5831	1.6019	1.6209	1.6400	1.6593	1.6788
1.2	cosh	1.8107	1.8258	1.8412	1.8568	1.8725	1.8884	1.9045	1.9208	1.9373	1.9540
	tanh	0.8337	0.8367	0.8397	0.8426	0.8455	0.8483	0.8511	0.8538	0.8565	0.8591
	sinh	1.6984	1.7182	1.7381	1.7583	1.7786	1.7991	1.8198	1.8406	1.8617	1.8829
1.3	cosh	1.9709	1.9880	2.0053	2.0028	2.0404	2.0583	2.0764	2.0947	2.1132	2.1320
	tanh	0.8617	0.8643	0.8668	0.8693	0.8717	0.8741	0.8764	0.8787	0.8810	0.8832
	sinh	1.9043	1.9259	1.9477	1.9697	1.9919	2.0143	2.0369	2.0597	2.0827	2.1059
1.4	cosh	2.1509	2.1700	2.1894	2.2090	2.2288	2.2488	2.2691	2.2896	2.3103	2.3312
	tanh	0.8854	0.8875	0.8896	0.8917	0.8937	0.8957	0.8977	0.8996	0.9015	0.9033

HYPERBOLIC FUNCTIONS 1.50–2.99

Angle		0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
1.5	sinh	2.1293	2.1529	2.1768	2.2008	2.2251	2.2496	2.2743	2.2993	2.3245	2.3499
	cosh	2.3524	2.3738	2.3955	2.4174	2.4395	2.4619	2.4845	2.5074	2.5305	2.5538
	tanh	0.9052	0.9069	0.9087	0.9104	0.9121	0.9138	0.9154	0.9170	0.9186	0.9202
1.6	sinh	2.3756	2.4015	2.4276	2.4540	2.4806	2.5075	2.5346	2.5620	2.5896	2.6175
	cosh	2.5775	2.6013	2.6255	2.6499	2.6746	2.6995	2.7247	2.7502	2.7760	2.8020
	tanh	0.9217	0.9232	0.9246	0.9261	0.9275	0.9289	0.9302	0.9316	0.9329	0.9342
1.7	sinh	2.6456	2.6740	2.7027	2.7317	2.7609	2.7904	2.8202	2.8503	2.8806	2.9112
	cosh	2.8283	2.8549	2.8818	2.9090	2.9364	2.9642	2.9922	3.0206	3.0493	3.0782
	tanh	0.9354	0.9367	0.9379	0.9391	0.9402	0.9414	0.9425	0.9436	0.9447	0.9458
1.8	sinh	2.9422	2.9734	3.0049	3.0367	3.0689	3.1013	3.1340	3.1671	3.2005	3.2341
	cosh	3.1075	3.1371	3.1669	3.1972	3.2277	3.2585	3.2897	3.3212	3.3530	3.3852
	tanh	0.9468	0.9478	0.9488	0.9498	0.9508	0.9518	0.9527	0.9536	0.9545	0.9554
1.9	sinh	3.2682	3.3025	3.3372	3.3722	3.4075	3.4432	3.4792	3.5156	3.5523	3.5894
	cosh	3.4177	3.4506	3.4838	3.5173	3.5512	3.5855	3.6201	3.6551	3.6904	3.7261
	tanh	0.9562	0.9571	0.9579	0.9587	0.9595	0.9603	0.9611	0.9619	0.9626	0.9633
2.0	sinh	3.6269	3.6647	3.7028	3.7414	3.7803	3.8196	3.8593	3.8993	3.9398	3.9806
	cosh	3.7622	3.7987	3.8355	3.8727	3.9103	3.9483	3.9867	4.0255	4.0647	4.1043
	tanh	0.9640	0.9647	0.9654	0.9661	0.9668	0.6974	0.9680	0.9686	0.9693	0.9699
2.1	sinh	4.0219	4.0635	4.1056	4.1480	4.1909	4.2342	4.2779	4.3221	4.3666	4.4117
	cosh	4.1443	4.1847	4.2256	4.2668	4.3085	4.3507	4.3932	4.4362	4.4797	4.5236
	tanh	0.9705	0.9710	0.9716	0.9722	0.9727	0.9732	0.9738	0.9743	0.9748	0.9752
2.2	sinh	4.4571	4.5030	4.5494	4.5962	4.6434	4.6912	4.7394	4.7880	4.8372	4.8868
	cosh	4.5679	4.6127	4.6580	4.7037	4.7499	4.7966	4.8437	4.8914	4.9395	4.9881
	tanh	0.9757	0.9762	0.9767	0.9771	0.9776	0.9780	0.9785	0.9789	0.9793	0.9797
2.3	sinh	4.9370	4.9876	5.0387	5.0903	5.1425	5.1951	5.2483	5.3020	5.3562	5.4109
	cosh	5.0372	5.0868	5.1370	5.1876	5.2388	5.2905	5.3427	5.3954	5.4487	5.5026
	tanh	0.9801	0.9805	0.9809	0.9812	0.9816	0.9820	0.9823	0.9827	0.9830	0.9834
2.4	sinh	5.4662	5.5221	5.5785	5.6354	5.6929	5.7510	5.8097	5.8689	5.9288	5.9892
	cosh	5.5569	5.6119	5.6674	5.7235	5.7801	5.8373	5.8951	5.9535	6.0125	6.0721
	tanh	0.9837	0.9840	0.9843	0.9846	0.9849	0.9852	0.9855	0.9858	0.9861	0.9864
2.5	sinh	6.0502	6.1118	6.1741	6.2369	6.3004	6.3645	6.4293	6.4946	6.5607	6.6274
	cosh	6.1323	6.1931	6.2545	6.3166	6.3793	6.4426	6.5066	6.5712	6.6365	6.7024
	tanh	0.9866	0.9869	0.9871	0.9874	0.9876	0.9879	0.9881	0.9884	0.9886	0.9888
2.6	sinh	6.6947	6.7628	6.8315	6.9009	6.9709	7.0417	7.1132	7.1854	7.2583	7.3319
	cosh	6.7690	6.8363	6.9043	6.9729	7.0423	7.1123	7.1831	7.2546	7.3268	7.3998
	tanh	0.9890	0.9892	0.9895	0.9897	0.9899	0.9801	0.9903	0.9905	0.9906	0.9908
2.7	sinh	7.4063	7.4814	7.5572	7.6338	7.7112	7.7894	7.8683	7.9480	8.0285	8.1098
	cosh	7.4735	7.5479	7.6231	7.6991	7.7758	7.8533	7.9316	8.0106	8.0905	8.1712
	tanh	0.9910	0.9912	0.9914	0.9915	0.9917	0.9919	0.9920	0.9922	0.9923	0.9925
2.8	sinh	8.1919	8.2749	8.3586	8.4432	8.5287	8.6150	8.7021	8.7902	8.8791	8.9689
	cosh	8.2527	8.3351	8.4182	8.5022	8.5871	8.6728	8.7594	8.8469	8.9352	9.0244
	tanh	0.9926	0.9928	0.9929	0.9931	0.9932	0.9933	0.9935	0.9936	0.9937	0.9938
2.9	sinh	9.0596	9.1512	9.2437	9.3371	9.4315	9.5268	9.6231	9.7203	9.8185	9.9177
	cosh	9.1146	9.2056	9.2976	9.3905	9.4844	9.5792	9.6749	9.7716	9.8693	9.9680
	tanh	0.9940	0.9941	0.9942	0.9953	0.9944	0.9945	0.9946	0.9948	0.9949	0.9950

HYPERBOLIC FUNCTIONS 3.00–4.49

Angle		0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
	sinh	10.018	10.119	10.221	10.324	10.429	10.534	10.640	10.748	10.856	10.966
3.0	cosh	10.068	10.168	10.270	10.373	10.476	10.581	10.687	10.794	10.902	11.011
	tanh	0.9951	0.9952	0.9953	0.9953	0.9954	0.9955	0.9956	0.9957	0.9958	0.9959
	sinh	11.076	11.188	11.301	11.415	11.530	11.647	11.764	11.883	12.003	12.124
3.1	cosh	11.121	11.233	11.345	11.459	11.574	11.689	11.806	11.925	12.044	12.165
	tanh	0.9960	0.9960	0.9961	0.9962	0.9963	0.9963	0.9964	0.9965	0.9966	0.9966
	sinh	12.246	12.369	12.494	12.620	12.747	12.876	13.006	13.137	13.269	13.403
3.2	cosh	12.287	12.410	12.534	12.660	12.786	12.915	13.044	13.175	13.307	13.440
	tanh	0.9967	0.9968	0.9968	0.9969	0.9969	0.9970	0.9971	0.9971	0.9972	0.9972
	sinh	13.538	13.674	13.812	13.951	14.092	14.234	14.377	14.522	14.668	14.816
3.3	cosh	13.575	13.711	13.848	13.987	14.127	14.269	14.412	14.556	14.702	14.850
	tanh	0.9973	0.9973	0.9974	0.9974	0.9975	0.9975	0.9976	0.9976	0.9977	0.9977
	sinh	14.965	15.116	15.268	15.422	15.577	15.734	15.893	16.053	16.215	16.378
3.4	cosh	14.999	15.149	15.301	15.455	15.610	15.766	15.924	16.084	16.245	16.408
	tanh	0.9978	0.9978	0.9979	0.9979	0.9979	0.9980	0.9980	0.9981	0.9981	0.9981
	sinh	16.543	16.709	16.877	17.047	17.219	17.392	17.567	17.744	17.923	18.103
3.5	cosh	16.573	16.739	16.907	17.077	17.248	17.421	17.596	17.772	17.951	18.131
	tanh	0.9982	0.9982	0.9983	0.9983	0.9983	0.9984	0.9984	0.9984	0.9985	0.9985
	sinh	18.285	18.470	18.655	18.843	19.033	19.224	19.418	19.613	19.811	20.010
3.6	cosh	18.313	18.497	18.682	18.870	19.059	19.250	19.444	19.639	19.836	20.035
	tanh	0.9985	0.9985	0.9986	0.9986	0.9986	0.9987	0.9987	0.9987	0.9987	0.9988
	sinh	20.211	20.415	20.620	20.828	21.037	21.249	21.463	21.679	21.897	22.117
3.7	cosh	20.236	20.439	20.644	20.852	21.061	21.272	21.486	21.702	21.919	22.139
	tanh	0.9988	0.9988	0.9988	0.9989	0.9989	0.9989	0.9989	0.9989	0.9990	0.9990
	sinh	22.339	22.564	22.791	23.020	23.252	23.486	23.722	23.961	24.202	24.445
3.8	cosh	22.362	22.586	22.813	23.042	23.273	23.507	23.743	23.982	24.222	24.466
	tanh	0.9990	0.9990	0.9990	0.9991	0.9991	0.9991	0.9991	0.9991	0.9992	0.9992
	sinh	24.691	24.939	25.190	25.444	25.700	25.958	26.219	26.483	26.749	27.018
3.9	cosh	24.711	24.959	25.210	25.463	25.719	25.977	26.238	26.502	26.768	27.037
	tanh	0.9992	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993	0.9993	0.9993	0.9993
	sinh	27.290	27.564	27.842	28.122	28.404	28.690	28.979	29.270	29.564	29.862
4.0	cosh	27.308	27.583	27.860	28.139	28.422	28.707	28.996	29.287	29.581	29.878
	tanh	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9994	0.9994	0.9994
	sinh	30.162	30.465	30.772	31.081	31.393	31.709	32.028	32.350	32.675	33.004
4.1	cosh	30.178	30.482	30.788	31.097	31.409	31.725	32.044	32.365	32.691	33.019
	tanh	0.9995	0.9995	0.9995	0.9995	0.9995	0.9995	0.9995	0.9995	0.9995	0.9995
	sinh	33.336	33.671	34.009	34.351	34.697	35.046	35.398	35.754	36.113	36.476
4.2	cosh	33.351	33.686	34.024	34.366	34.711	35.060	35.412	35.768	36.127	36.490
	tanh	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996
	sinh	36.843	37.214	37.588	37.965	38.347	38.733	39.122	39.515	39.913	40.314
4.3	cosh	36.857	37.227	37.601	37.979	38.360	38.746	39.135	39.528	39.925	40.326
	tanh	0.9996	0.9996	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997
	sinh	40.719	41.129	41.542	41.960	42.382	42.808	43.238	43.673	44.112	44.555
4.4	cosh	40.732	41.141	41.554	41.972	42.393	42.819	43.250	43.684	44.123	44.566
	tanh	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

HYPERBOLIC FUNCTIONS 4.50–5.99

Angle		0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
	sinh	45.003	45.455	45.912	46.374	46.840	47.311	47.787	48.267	48.752	49.242
4.5	cosh	45.014	45.466	45.923	46.385	46.851	47.321	47.797	48.277	48.762	49.252
	tanh	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
	sinh	49.737	50.237	50.742	51.252	51.767	52.288	52.813	53.344	53.880	54.422
4.6	cosh	49.747	50.247	50.752	51.262	51.777	52.297	52.823	53.354	53.890	54.431
	tanh	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
	sinh	54.969	55.522	56.080	56.643	57.213	57.788	58.369	58.955	59.548	60.147
4.7	cosh	54.978	55.531	56.089	56.652	57.221	57.796	58.377	58.964	59.556	60.155
	tanh	0.9998	0.9998	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
	sinh	60.751	61.362	61.979	62.601	63.231	63.866	64.508	65.157	65.812	66.473
4.8	cosh	60.759	61.370	61.987	62.609	63.239	63.874	64.516	65.164	65.819	66.481
	tanh	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
	sinh	67.141	67.816	68.498	69.186	69.882	70.584	71.293	72.010	72.734	73.465
4.9	cosh	67.149	67.823	68.505	69.193	69.889	70.591	71.300	72.017	72.741	73.472
	tanh	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
	sinh	74.203	74.949	75.702	76.463	77.232	78.008	78.792	79.584	80.384	81.192
5.0	cosh	74.210	74.956	75.709	76.470	77.238	78.014	78.798	79.590	80.390	81.198
	tanh	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
	sinh	82.008	82.832	83.665	84.506	85.355	86.213	87.079	87.955	88.839	89.732
5.1	cosh	82.014	82.838	83.671	84.512	85.361	86.219	87.085	87.960	88.844	89.737
	tanh	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
	sinh	90.633	91.544	92.464	93.394	94.332	95.281	96.238	97.205	98.182	99.169
5.2	cosh	90.639	91.550	92.470	93.399	94.338	95.286	96.243	97.211	98.188	99.174
	tanh	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	1.0000	1.0000	1.0000	1.0000
	sinh	100.17	101.17	102.19	103.22	104.25	105.30	106.36	107.43	108.51	109.60
5.3	cosh	100.17	101.18	102.19	103.22	104.26	105.31	106.67	107.43	108.51	109.60
	tanh	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	sinh	110.70	111.81	112.94	114.07	115.22	116.38	117.55	118.73	119.92	121.13
5.4	cosh	110.71	111.82	112.94	114.08	115.22	116.38	117.55	118.73	119.93	121.13
	tanh	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	sinh	122.34	123.57	124.82	126.07	127.34	128.62	129.91	131.22	132.53	133.87
5.5	cosh	122.35	123.58	124.82	126.07	127.34	128.62	129.91	131.22	132.54	133.87
	tanh	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	sinh	135.21	136.57	137.94	139.33	140.73	142.14	143.57	145.02	146.47	147.95
5.6	cosh	135.22	136.57	137.95	139.33	140.73	142.15	143.58	145.02	146.48	147.95
	tanh	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	sinh	149.43	150.93	152.45	153.98	155.53	157.09	158.67	160.27	161.88	163.51
5.7	cosh	149.44	150.94	152.45	153.99	155.53	157.10	158.68	160.27	161.88	163.51
	tanh	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	sinh	165.15	166.81	168.48	170.18	171.89	173.62	175.36	177.12	178.90	180.70
5.8	cosh	165.15	166.81	168.49	170.18	171.89	173.62	175.36	177.13	178.91	180.70
	tanh	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	sinh	182.52	184.35	186.20	188.08	189.97	191.88	193.80	195.75	197.72	199.71
5.9	cosh	182.52	184.35	186.21	188.08	189.97	191.88	193.81	195.75	197.72	199.71
	tanh	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Example. — sinh 0.83 = 0.9286, cosh 0.83 = 1.3647, tanh 0.83 = 0.6805.

Approximate Formulas. — Note that for x less than 0.1,

$$\sinh x = x \text{ with an error of less than 0.2 per cent.}$$

$$\cosh x = 1 + \frac{x^2}{2} \text{ with an error of less than 0.09 per cent.}$$

For x greater than 6,

$$\sinh x = \cosh x = \frac{\epsilon^x}{2} = \frac{1}{2} \log_{10}{}^{-1} (0.43429\, x)$$

with an error of less than 0.01 per cent.

Anti-Functions. — If $a = \sinh x$, then x is the angle whose hyperbolic sine is a; this may be expressed symbolically

$$x = \sinh^{-1} a$$

which is read "x equals the angle whose hyperbolic sine is a." The angle x is also called the "anti-hyperbolic sine" or the "inverse hyperbolic sine" of a. Similarly for the other hyperbolic functions. (See *Trigonometric Functions, sub-heading Anti-functions.*) The following relations exist between the anti-hyperbolic functions and the natural logarithms:

$$\sinh^{-1} x = \log \left(x + \sqrt{x^2 + 1}\right)$$

$$\cosh^{-1} x = \log \left(x + \sqrt{x^2 - 1}\right)$$

$$\tanh^{-1} x = \frac{1}{2} \log \left(\frac{1 + x}{1 - x}\right)$$

Relations among Functions of the Same Angle. —

$$\cosh^2 x - \sinh^2 x = 1$$

$$1 - \tanh^2 x = \frac{1}{\cosh^2 x}$$

$$\coth^2 x - 1 = \frac{1}{\sinh^2 x}$$

$$\sinh(-x) = -\sinh x$$

$$\cosh(-x) = \cosh x$$

$$\tanh(-x) = -\tanh x.$$

See also the definitions given above.

Sum and Difference of Two Angles. —

$$\sinh(x + y) = \sinh x \cosh y + \cosh x \sinh y$$

$$\cosh(x + y) = \cosh x \cosh y + \sinh x \sinh y$$

$$\tanh(x + y) = \frac{\tanh x + \tanh y}{1 + \tanh x \tanh y}$$

$$\sinh(x - y) = \sinh x \cosh y - \cosh x \sinh y$$

$$\cosh(x - y) = \cosh x \cosh y - \sinh x \sinh y$$

$$\tanh(x - y) = \frac{\tanh x - \tanh y}{1 - \tanh x \tanh y}$$

Product of the Functions of Two Angles. —

$$\sinh x \sinh y = \tfrac{1}{2} [\cosh(x + y) - \cosh(x - y)]$$

$$\sinh x \cosh y = \tfrac{1}{2} [\sinh(x + y) + \sinh(x - y)]$$

$$\cosh x \sinh y = \tfrac{1}{2} [\sinh(x + y) - \sinh(x - y)]$$

$$\cosh x \cosh y = \tfrac{1}{2} [\cosh(x + y) + \cosh(x - y)]$$

Functions of Twice an Angle.—

$$\sinh 2x = 2 \sinh x \cosh x$$
$$\cosh 2x = \sinh^2 x + \cosh^2 x$$
$$\tanh 2x = \frac{2 \tanh x}{1 + \tanh^2 x}$$

Functions of Half an Angle.—

$$\sinh \frac{x}{2} = \sqrt{\frac{\cosh x - 1}{2}}$$
$$\cosh \frac{x}{2} = \sqrt{\frac{\cosh x + 1}{2}}$$
$$\tanh \frac{x}{2} = \sqrt{\frac{\cosh x - 1}{\cosh x + 1}}$$

Functions of Three Times an Angle.—

$$\sinh 3x = 3 \sinh x + 4 \sinh^3 x$$
$$\cosh 3x = 4 \cosh^3 x - 3 \cosh x$$
$$\tanh 3x = \frac{3 \tanh x + \tanh^3 x}{1 + 3 \tanh^2 x}$$

Relations between Hyperbolic and Trigonometric Functions.—

$$\sinh(jx) = j \sin x \qquad \sin(jx) = j \sinh x$$
$$\cosh(jx) = \cos x \qquad \cos(jx) = \cosh x$$
$$\tanh(jx) = j \tan x \qquad \tan(jx) = j \tanh x$$
$$\sinh^{-1} jx = j \sin^{-1} x \qquad \sin^{-1} jx = j \sinh^{-1} x$$
$$\tanh^{-1} jx = j \tan^{-1} x \qquad \tan^{-1} jx = j \tanh^{-1} x.$$

$$\cosh^{-1} jx = j \cos^{-1} jx = \log\left(x + \sqrt{1 + x^2}\right) - j\frac{\pi}{2}$$

Hyperbolic Functions of a Complex Angle.—

$$\sinh(x + jy) = \sinh x \cos y + j \cosh x \sin y = M\epsilon^{j\theta}$$

where $M = \sqrt{\dfrac{\cosh 2x - \cos 2y}{2}}$ and $\tan\theta = \dfrac{\tan y}{\tanh x}$.

$$\cosh(x + jy) = \cosh x \cos y + j \sinh x \sin y = N\epsilon^{j\phi}$$

where $N = \sqrt{\dfrac{\cosh 2x + \cos 2y}{2}}$ and $\tan\phi = \tanh x \cdot \tan y$.

$$\tanh(x + jy) = \frac{\sinh x \cos y + j \cosh x \sin y}{\cosh x \cos y + j \sinh x \sin y} = P\epsilon^{j\psi}$$

where $P = \sqrt{\dfrac{\cosh 2x - \cos 2y}{\cosh 2x + \cos 2y}}$ and $\psi = \tan^{-1}\left[\dfrac{\sin 2y}{\sinh 2x}\right]$.

$$\tanh^{-1}(A\epsilon^{j\alpha}) = B_1 + jB_2,$$

where $B_1 = \frac{1}{2}\tanh^{-1}\left[\dfrac{2A\cos\alpha}{1 + A^2}\right]$ and $B_2 = \frac{1}{2}\tan^{-1}\left[\dfrac{2A\sin\alpha}{1 - A^2}\right]$.

[W. A. Del Mar.]

ILLUMINATION, INTERIOR. — (*See also Illumination, Laws of; Illumination, Street; Vision, Laws of.*) The chief problems in interior illumination are: (1) to render visible certain objects or surfaces so as to avoid fatigue and injury of the eyes, and to enhance in effectiveness all operations dependent on good vision; (2) to reveal in their true values the architecture and decoration of interiors; and (3) to produce intrinsic artistic effects through the control of the color, intensity and direction of light. Engineering practice has been developed mainly with reference to the first of these problems, though the other two are often of chief importance and must at all times be considered. For a discussion of the hygienic aspects of illumination, see *Vision, Laws of.*

Reference Surfaces. — Illumination problems are usually worked out by reference to one or more planes or surfaces whose illumination can be taken as an index to the meeting of the general requirements. Unless special considerations dictate otherwise the plane selected is usually horizontal and at a height of from 2½ to 3 feet above the floor, corresponding with that of table tops, desks, counters, work benches, etc. In many cases other surfaces, as the floor, walls, faces of shelving, stacks of books, etc., are more significant than an arbitrarily chosen horizontal plane. The proper reference surface for the illumination of show windows is usually a surface inclined or curving upward from the base of the window. In many art galleries and on the stage of a theatre the proper reference planes are vertical. The most significant reference planes for any problem should be selected by a study of the special requirements.

Efficiency of Utilization. — The ratio of the luminous flux received by a reference plane to that produced by the light sources used for its illumination is known as the efficiency of utilization, or the utilization factor of the system. The light received by a reference plane is often termed useful flux to distinguish it from the total flux output of illuminants. The above terms may be very misleading if used indiscriminately, for the flux received by other surfaces than a reference plane is usually essential to the effectiveness of a system of illumination.

DIRECT, INDIRECT AND SEMI-INDIRECT ILLUMINATION. — Methods of illumination may be classified according to the manner in which light is delivered to the working surfaces. Direct light is that received directly from lamps and their accessory shades and reflectors. Indirect light is that received by reflection from some extended diffusing surface, as a ceiling or wall. Systems of illumination are either direct, indirect or semi-indirect. In the direct system the illuminants, including their reflectors and shades, are exposed and deliver a considerable part of their light in the direction of the reference areas. Indirect light from the walls and ceilings is received as an auxiliary component. In the direct system the lower surfaces of a room have the highest illumination. The illuminants produce shadows, the sharpness of which vary inversely with the diffusion of light. Polished or glazed surfaces may produce an annoying glare in certain lines of vision, due to the specular reflection of the exposed light sources. Unshaded lamps are a source of great annoyance and fatigue. The shadows of direct lighting intensify and in some cases exaggerate relief effects, and may be made to serve as aids to vision where color differences are slight.

The indirect system of lighting conceals all primary sources and distributes light from large diffusing surfaces, usually white ceilings and upper walls. The upper surfaces of the room have the highest illumination. Shadows are eliminated or greatly reduced in intensity. Glare from visible light sources is avoided and the specular reflection of glazed surfaces is generally inappreciable. The uniformity of illumination on the lower surfaces of a room is very great. The great diffusion of light in the indirect system is frequently stated to closely reproduce daylight. This is not strictly true, however, for daylight though diffused is directed and gives a diminishing gradation of illumination from the

lower to the upper part of the room. The faintness of shadows with indirect lighting tends to flatten relief effects.

Semi-indirect lighting mediates between the two extremes. The light sources are shielded from direct vision by inverted bowl reflectors of translucent glass, which transmit downward a moderate amount of direct light and reflect the remainder to the ceiling. There is an absence of agreement as to the most favorable ratio of direct to indirect light. It is desirable that the brightness of the translucent bowls shall equal or slightly exceed that of the ceiling. The direct component should be adequate to create a normal relief effect and to bring up the illumination of working surfaces to a value above that of the background. The experiments of T. W. Rolph (*Trans. Ill. Eng. Soc., Vol. 7, pp. 234 and 549*) indicate that very satisfactory results are secured with a direct component of about 15 per cent.

In comparing the various systems outlined a large allowance must be made for differences of psychological effect. Critics of indirect lighting assert that the complete elimination of shadows and the reversal of the customary gradation of brightness are very annoying, distract attention, distort the sense of distance and promote fatigue. Others find the same aspects advantageous and hold that the elimination of glare by specular reflection outweighs other considerations. The unfavorable qualities of direct lighting avoided by the indirect method are largely abuses due to unshaded and poorly-located lamps rather than inherent faults. Several investigators have studied the relative illumination intensities required for equally effective vision by the different systems. The observed differences are inconsistent and are probably due to incidental conditions, such as unequal contrast between working surfaces and backgrounds and to direct glare from visible light sources. It is agreed that with equal skill in design direct lighting ranks first, semi-indirect lighting second and indirect third in efficiency of utilization.

Desirable Color and Direction of Illumination. — There is no well-defined color standard in illumination. Daylight white is desirable for exact color matching and for color printing, but is not essential for other purposes. The Moore carbon-dioxide tube reproduces daylight white with great fidelity and also closely approaches daylight in diffusion. True white may also be obtained from arc and incandescent lamps by the aid of color screens to filter out the hues present to excess. Magnetite arcs and nitrogen-filled tungsten lamps are best adapted to this process. Of the illuminants in common use the white flame arc and the intensified carbon arc give the nearest approach to white. The nitrogen-filled tungsten lamp surpasses other incandescent lamps in whiteness. The ordinary tungsten lamp and the best grade of Welsbach mantles are somewhat unlike in tone, but differ about equally from white. Light having a predominant hue, as that of the mercury arc, is superior for the revelation of fine detail. Light of amber tint is regarded as softer and warmer than light of bluish or greenish tint. The tint of indirect light is modified by the color of the reflecting walls and ceilings.

In determining the direction of light the avoidance of glare and shadows in the visual field is most important. In the display of art objects and architectural details the dominant direction should be carefully chosen to produce a correct sense of relief. An inversion of light and shade by artificial light as compared with daylight is especially to be avoided. Head and hand shadows are especially to be avoided in reading and in mechanical operations. The old rule of light from above the left shoulder is an excellent one. It has been found advantageous in schoolrooms, offices and drafting rooms to locate desks and tables with the windows to the left of those at work and to displace the artificial light sources from symmetrical positions toward the windows in order to give a directive effect similar to daylight.

QUANTITY OF LIGHT AND NUMBER OF ILLUMINANTS FOR DIRECT LIGHTING. — The intensities of illumination required for various purposes are given in the article on *Vision, Laws of*. The simplest illumination problems are those in which a uniform illumination of E foot candles is desired on a horizontal plane of A square feet. The useful flux required is then A times E. If a factor of utilization K is known or assumed for the conditions of the room the total flux F, in lumens, to be produced by the illuminants is

$$F = \frac{AE}{K}.$$

The total watts of electric power or the cubic feet per hour of gas consumption to produce the required flux is found by dividing the required flux by the lumens per watt or per cubic foot per hour given by the type of light source under consideration.

For detailed data on the lumens per watt or per unit of gas consumption given by various illuminants, see *Lamps, Incandescent; Lamps, Arc;* and *Gas Lighting.*

Factors of Utilization vary with the shape of the room, the absorption in reflectors and shades, the form of light distribution of the illuminants and the reflecting efficiency of the walls and ceilings. Their values cannot be reduced analytically. When the ratio of room height to smallest floor dimension is great the absorption light by walls and ceilings is evidently greater than under the opposite conditions. Light-colored walls and ceilings and a strong downward light concentration by efficient reflectors are factors which increase utilization efficiency. Cleanliness of lamps, reflectors, globes and walls is of great importance to the maintenance of utilization efficiency. The following values have been compiled empirically by J. R. Cravath from the data of numerous reliable illumination tests.

FACTORS OF UTILIZATION

Equipment	Ratio of shortest floor dimension to height of room	Utilization factor, per cent		
		Light ceiling, light walls	Light ceiling, dark walls	Dark ceiling, dark walls
Direct system	3 or more	55 to 65	50 to 60	....
Prismatic or opal reflectors near ceiling	1 to 2.5	50 to 60	45 to 55	30 to 40
Direct system	3 or more	35 to 45	30 to 40	....
Frosted globes near the ceiling	1 to 2.5	30 to 40	25 to 35	15 to 25
Direct system	3 or more	40 to 50	37 to 47	....
Bare lamps at ceiling	1 to 2.5	38 to 48	30 to 40	20 to 30
Indirect system	3 or more	15 to 25	....	....
Lamps in enamelled coves	1 to 2.5	10 to 20	....	....
Indirect system	3 or more	34 to 44	32 to 42	....
Lamps in inverted bowl reflectors	1 to 2.5	30 to 40	25 to 35	....

C. E. Clewell, *Factory Lighting*, p. 17, reports average utilization efficiencies found by him in extensive tests of direct lighting as follows:

	Per cent
Low offices	27.1
Fairly high factory offices	27.4
Low factory space	27.0
Medium-high factory space	30.8
Fairly high factory space	29.1

The illuminants were tungsten lamps in standard reflectors. The measurements were taken in intervals between cleaning and renewals.

Estimates of Effective Flux. — The preliminary estimates for most problems in interior illumination can be made with fair accuracy from the following tables, which give empirical values based on laboratory and service tests. (*See Trans. Ill. Eng. Soc., Vol. 3, p. 518 and Vol. 4, pp. 321, 849, 885.*)

LUMENS ON REFERENCE PLANE PER WATT, ELECTRIC LAMPS

(Lamps and glassware are assumed to be clean and at a height above the reference plane not exceeding 15 feet.)

Lamp	Reflector or globe	Ceiling	Walls	Effective lumens per watt
Tungsten	Silvered reflector	Light	Light	6.1
Tungsten	Prismatic reflector	Light	Light	5.0
Tungsten	Prismatic reflector	Light	Dark	4.0
Tungsten	Enamelled reflector	Light	Light	3.5
Tungsten	Enamelled reflector	Light	Dark	3.0
Gem	Prismatic reflector	Light	Light	2.2
Gem	Prismatic reflector	Light	Dark	1.8
5-amp. d-c. arc	Opal inner globe	Light	Medium	2.0
Mercury arc	Enamelled reflector	Medium	Medium	5.5

LUMENS ON REFERENCE PLANE PER CUBIC FOOT OF GAS PER HOUR

(Lamps and glassware assumed to be clean; new mantles; height above reference plane not exceeding 15 feet; gas, 700 B.t.u. per cu. ft.)

Lamp	Reflector or globe	Ceiling	Walls	Effective lumens per cu. ft. per hour
Upright mantle	Opal globe	Light	Light	49
Upright mantle	Opal globe	Light	Dark	27
Upright mantle	Opal reflector	Light	Light	85
Upright mantle	Opal reflector	Light	Dark	50
Inverted mantle	Prismatic reflector	Light	Light	140
Inverted mantle	Prismatic reflector	Light	Dark	128
Inverted mantle	Roughed ball	Light	Light	101
Inverted mantle	Roughed ball	Light	Dark	75
4-mantle upright arc	Alabaster globe	Light	Light	66
4-mantle upright arc	Alabaster globe	Light	Dark	48
5-mantle inverted arc	Alabaster globe	Light	Light	87
5-mantle inverted arc	Alabaster globe	Light	Dark	65

Location of Illuminants. — Having found the total wattage or rate of gas consumption to be provided the next step is to select a proper number and arrangement of lamps. It is seldom possible to produce a uniform illumination from a single source along a radial distance greater than the height of the lamp above the plane. It is practically impossible to avoid shadows from a single central unit. A symmetrical and uniform spacing of several lamps is necessary for uniform illumination by direct lighting. The use of many small units with close spacing promotes uniformity and reduces shadows. The use of few large units widely spaced promotes economy. In many cases a compromise must be made between the two plans. The various spacing schemes in vogue are largely reducible to three types, viz.: (*a*) a long and narrow room with a single row of lamps on the center line, the spacing about equal to the width of the room; (*b*) a rectangular room divided into squares with lamps located at the center of each, the width of each square preferably not greater than twice the height of suspension of the lamps above the reference plane; (*c*) lamps at corners of equilateral triangles and spaced at not more than twice suspension height, spacing to walls being half the spacing from lamp to lamp. Plans of these schemes are shown in Fig. 1.

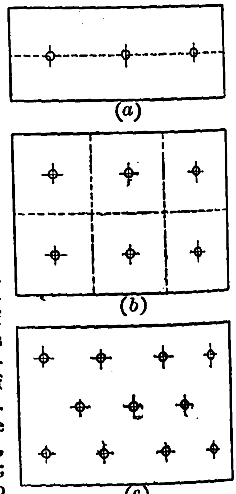

Fig. 1. Spacing Schemes for Direct Lighting

In determining the spacing of lamps very careful attention should be given to the structural divisions of ceiling space. Each bay or division may properly have a symmetrical spacing as this will greatly reduce the shadows cast by pillars and beams. It is possible to secure a very uniform direct illumination from any unit giving a symmetrical distribution if a certain critical ratio of height to spacing is exceeded. The value of this critical ratio depends on the form of light distribution, as shown by the prototype curves of Fig. 16 in the article on *Illumination, Laws of*. Holophane reflectors for tungsten lamps are made in three types, viz., extensive, for wide spacing, the critical ratio of height to spacing being 0.5; intensive, for medium spacing, the critical ratio being 0.67; focussing, for high suspension and close spacing, the critical ratio being 1.33. A spacing chart for Holophane and for X-ray reflectors is shown in Fig. 2. With a ratio of height to spacing below the critical value the light is spotted. With a higher ratio the illumination remains uniform, but with some loss of utilization efficiency, due to the increased absorption of light by the walls. The efficiency does not follow an inverse square relation to the height in any case, and is but slightly affected when the walls are of light color and the reflectors give a strong downward distribution. In very large rooms the effect of suspension height on utilization efficiency is trifling. The suspension height should if possible be sufficient to remove the light sources from direct lines of vision.

Single-lamp Units vs. Clusters. — A single-lamp electric unit is less expensive to install and maintain and is usually more efficient than a cluster of several lamps having the same total candle-power. The cross-absorption of light in clusters may be as high as 10 per cent. Gas clusters of the so-called "arc" type have an advantage over single-mantle lamps in heat conservation and facility of remote control. In fact, single-mantle lamps offer a small range candle-power.

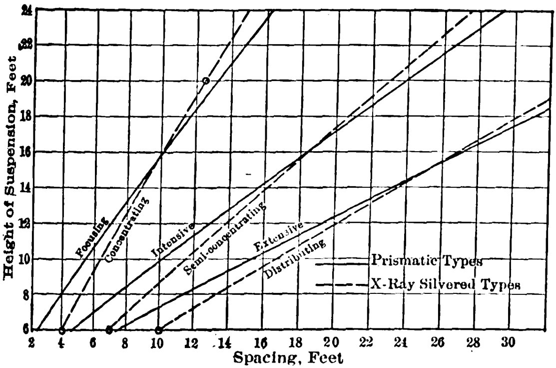

Fig. 2. Maximum Spacing of Reflectors for Uniform Illumination

Point-by-point Calculations of Illumination. — Such calculations are very useful in checking up the distribution of illumination. The method of calculating is described in the article on *Illumination, Laws of.*

METHODS OF INDIRECT LIGHTING. — The process of determining the power required to produce a given illumination of any area by indirect means is the same as that outlined above for direct lighting, a suitable value of the efficiency of illumination being used. There are two types of indirect lighting, the bowl and the cove. Indirect lighting from suspended bowls affords greater flexibility in light distribution than cove lighting and is capable of producing more uniform and efficient effects. The efficiencies of utilization obtainable with indirect bowl lighting are given by the National X-ray Reflector Co. as ranging from 0.20 to 0.32 with dark walls and from 0.24 to 0.34 with light walls for ratios of minimum floor dimension to ceiling height of from 1.0 to 3.5; the ceilings are assumed to be painted white and an allowance of 20 per cent for loss of light by dust and lamp aging is included in these values. The proper suspension height for indirect bowl units is approximately three-quarters the height of the room. In spacing the units a symmetrical arrangement at the centers of ceiling squares is highly desirable. The maximum dimension of such a spacing square or rectangle should not exceed a certain ratio to the ceiling height for the best results, viz., for ceiling heights below 12 feet, the maximum ratio is 1.5; for ceiling heights from 12 to 17 feet, the maximum ratio is 1.75; above 17 feet, 2.0.

In the cove type of indirect lighting the lamps are placed in the trough of the cove with axes horizontal, and are backed with a trough reflector of high efficiency. Light is thus thrown to the upper surface of the cove and from it reflected into the room. Cove lighting about the base of a flat-domed ceiling is somewhat more effective than with a flat ceiling. Dust tends to collect on lamps and reflecting surfaces and seriously reduces the average efficiency of the system.

Illuminants for Indirect and Semi-indirect Lighting. — The enclosed carbon arc has been extensively used in semi-indirect and indirect lighting systems. The arc should be arranged with the positive carbon below and the enclosing globe should be of clear glass. The reflector for wholly indirect arc lighting is an inverted flat cone of enamelled metal. A translucent opal glass reflector resembling an inverted bell is placed beneath the arc for semi-indirect arc lighting. The latter type of unit is usually provided with an enamelled

diffusing reflector above the lamp. This reflector has a surface of concentric ripples, which serve to improve the symmetry of light distribution from the wandering crater of the arc. While somewhat less efficient than direct lighting these methods of using arcs produce superior results, due to the adequate diffusion and improved steadiness of the light. Indirect lighting with carbon arcs is inferior in efficiency to similar systems using tungsten lamps, especially those of the nitrogen type. The nitrogen-filled tungsten lamp is excellently adapted to use in large indirect lighting fixtures, but its extreme brilliancy renders it unsuited to direct lighting without a diffusing envelope.

MAINTENANCE OF EFFICIENCY. — Loss of efficiency in lighting systems may be due to the aging of lamps, failure to maintain proper lamp voltage, and the collection of dirt on lamps, reflectors, shades and the reflecting surfaces of the room. Systematic inspection and cleaning are essential if the most economical results are to be secured from large systems. Lamps should be replaced when the bulbs become badly blackened. Globes and reflectors should be cleaned at least once a month for most effective service. C. E. Clewell, *Factory Lighting*, p. 48, reports that a depreciation test of an office installation of tungsten lamps in glass reflectors showed a gradual loss of efficiency which reached a steady value of 19 per cent in 30 days. A similar test in a factory showed a gradual reduction of efficiency which reached a steady value of 48 per cent in 24 days. Arc lamps depreciate in efficiency between trimmings and cleaning from 15 to 30 per cent due to the collection of ash on the inner globes. Arcs are relatively little affected by outside dirt and are quite generally preferred to incandescent lamps in very smoky and dusty locations. Careful attention should be given to the ceilings of rooms with indirect lighting to prevent the accumulation of dust and the loss of whiteness.

SPECIAL LIGHTING PROBLEMS. — In art galleries for the exhibition of paintings a moderate general illumination should be provided from sources giving good diffusion. Paintings should be illuminated by direct light received from concealed sources several feet in front of, and slightly above, the level of the paintings. The direction of this light should be carefully studied to avoid specular reflection from glass or glossy portions of paint. For good color effects the light should approach as near as possible the color of daylight. Sculpture is effectively lighted by indirect or semi-indirect methods, but the illuminants may properly be suspended along the side of the room containing the windows to obtain directed illumination sufficient to reproduce the relief effects of daylight.

In ritualistic churches it is important to produce a brilliant illumination of the sanctuary with a large vertical component by means of concealed lamps. The distribution of lamps in the nave should be chosen to reveal the architectural effects of the structure. Large and low-hanging pendant fixtures in the axis of the room are generally undesirable. Bracket clusters, groups of small incandescent lamps worked into the capitals of pillars, and small pendant fixtures in the arcades can be used with excellent effect. In non-ritualistic churches adequate reading light is desired at all pews. Brilliant light sources in the field of vision are especially to be avoided. If a balcony is used the light sources should be hung high and thoroughly diffused. The platform should be brightly lighted, but by light sources not visible to the auditors.

Stages and platforms in theatres and public halls should have high illumination, especially in vertical planes, but the light sources should be entirely concealed.

Office desks should be lighted with a view to preventing head and hand shadows and glare from glossy surfaces. Well-diffused general lighting with a dominant component from above the left shoulder is perhaps the best solution of the

problem. When local lighting is required a pendant lamp in a deep conical reflector of green flashed opal glass suspended at a height above the desk of 2 to 2.5 feet and near the left edge of the desk will fulfill all requirements.

Show windows should be lighted by entirely concealed light sources placed at or above the upper edge of the window. Illumination should be designed for a surface which is inclined or concave upward from the lower edge of the window. On brilliantly-lighted business streets a very high illumination of 20 foot-candles or more is necessary to attract attention. On less brilliantly-lighted streets the illumination may be reduced in proportion. The lamps should be backed by highly efficient reflectors. The form of light distribution desired depends on the dimensions of the window, the degree of concentration increasing with the ratio of the height to the depth of the window space. Tests reported by H. B. Wheeler (*Ill. Eng. Soc., Vol. 8, p. 555*) show that utilization efficiencies on the trim surface depend on the ratio of height to depth of the window space. The results cited range from 58 per cent for a height to depth ratio of 1, to 42 per cent for a ratio of 2. Show cases in the aisles of stores afford a problem akin to show-window lighting. In brightly-lighted rooms the illumination of the interior of the case must be very high to gain attention. Entirely concealed lamps are required and these may properly be placed in shallow trough reflectors in the upper dihedral angles of the case. Tubular and linolite lamps are appropriate, due to their space economy.

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[W. E. WICKENDEN.]

ILLUMINATION, LAWS OF. — (*See also Angles; Photometric Quantities; Vision, Laws of.*) Light may come from either a point source or a finite source. Serious errors may result from the application of the laws of point sources to the illumination from finite areas, due to the non-parallel rays. The errors are negligible, however, when the distance separating the light source and the illuminated surface exceeds ten times the greatest projected dimension of either in a plane normal to the mean direction of light. For definitions of quantities see *Photometric Quantities*.

LIGHT FROM A POINT SOURCE. — When the light comes from a point source the "inverse square" and the "cosine" laws apply:

Inverse Square Law. — The illumination from a point source varies inversely as the square of the distance from the source. This law rests on the geometrical fact that the spherical area intercepting light flux varies as the square of its radius.

Cosine Law. — The illumination received by an element of surface varies as the cosine of the angle of incidence. The basis of this law is also geometrical. These two laws may be stated thus:

$$E = \frac{I \cos \alpha}{l^2},$$

where E = intensity of illumination in foot-candles; I = intensity of the light source in candle-power; α = angle of incidence of light on the illuminated area; and l = distance in feet from the light source to the point illuminated.

Calculation of Light Flux and Mean Intensities from Point Sources. — Let I_θ be the mean intensity of a light source at an angle θ from the axis about which the light distribution is symmetrical, usually the vertical axis. The flux within an elementary zone at this angle of inclination is then

$$dF = I_\theta \, d\omega = 2 \pi I_\theta \sin \theta \, d\theta.$$

The flux within any finite zone about the axis of distribution may be found by integration when the law of distribution is known. In general, however, it is necessary to employ graphical means of integration for this purpose. The mean intensity within any finite zone cannot be found by direct average, since each beam bears a weight proportional to $\sin \theta$. The graphical methods described below are the most serviceable.

Protractor Methods. — This method is adapted to quick approximations of mean zonal, mean hemispherical and mean spherical intensities. The method assumes a sphere of light distribution divided into zones of equal solid angular content, i.e., of equal altitude. A direct average of the mean intensities of the several zones gives the general mean for all the zones included, since each component zone bears equal weight. For convenience it may be assumed generally that the mean intensity in any zone agrees closely with that at the angle which bisects the area of the zone. Fig. 1 shows a convenient subdivision of the sphere based on this assumption. To obtain the mean spherical candle-power the intensities at all the angles indicated by radial lines are averaged. This result multiplied by 2π gives the total flux from the source. Similarly the average of the intensities at the angles indicated in either hemisphere gives the mean hemispherical candle-power, which may be multiplied by π to obtain the hemispherical light flux. The average taken at the three lowest positions gives the mean intensity

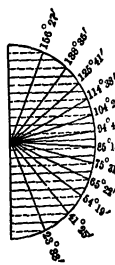
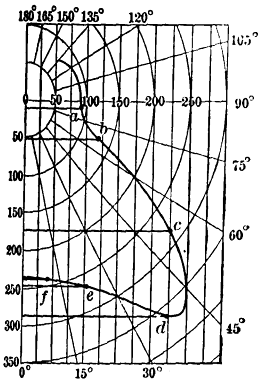


Fig. 1.

below 60° and this value, multiplied by $\pi/2$, gives the flux within the $0° - 60°$ zone. For convenience in practice a transparent protractor may be prepared with the reference angles shown by black lines. This protractor may be placed over the polar distribution curve and the values for averaging read off directly. The accuracy of the method depends on the regularity of the distribution curve.

Flux-o-lite Diagram. — (Fig. 2.) This diagram is constructed from the polar distribution curve by drawing vertical reference lines tangent to the candle-power circles, with uniform intermediate divisions as desired. To find the flux within any zone the horizontal projection of the mid-zone intensity is measured on the scale created by the vertical reference lines. Multiply this value by the constant which corresponds to the arc of the zone in the accompanying table.



$a = 90$ $b = 118$ $c = 225$

$d = 223$ $e = 103$ $f = 41$

Sum $= 800$ $F_{lh} = 800 \times 1.64 = 1312$

$I_{mlh} = 1312 \div 6.2832 = 208.8$

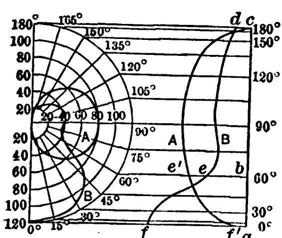
Fig. 2. Flux-o-lite Diagram

The flux within any angular limits is found by summing the components from the several zones included. For accuracy a large number of small component zones should be taken. For proof, it is readily shown that the solid angular content of any zone is equal to

Arc of zone, degrees	Constant
5	0.548
10	1.098
15	1.64
20	2.18
25	2.72
30	3.25

$$4\pi \sin\theta \sin\frac{n}{2},$$

where n is the arc of the zone in degrees and θ the bisecting angle measured from the vertical axis. The above constants in each case equal $4\pi \sin n/2$ and the horizontal projections read from the diagram are in each case equal to the assumed mean intensity of the zone times the sine of the bisecting angle.



A = Bare 80 c.p. Lamp B = Same with Reflector

Area $abce'f' = 15000$ For A, $F_s = 785.4$ $I_s = 62.5$

Area $abcdef = 12080$ For B, $F_s = 632.9$ $I_s = 50.3$

Absorption of Reflector $= 152.5 = 19.52\%$

Fig. 3. The Rousseau Diagram

Rousseau Diagram. — (Fig. 3.) The Rousseau diagram admits of considerable accuracy, but involves the measurement of area. The arcs of the several zones of distribution are projected horizontally on the line abc. The intercepts equal the altitudes of the several zones and are therefore proportional to their several solid angular contents, with a total abc equal to 4π. On each projection line is laid off from abc a length equal to the radius of the polar

curve at the corresponding angle. The terminal points are then connected by the smooth curve *def*. The area *abcdef* so inclosed represents the total flux, since

$$\text{Area } abcdef = \int_0^{4\pi} I\, d\omega = 4\pi I_s.$$

The total flux is

$$F_s = \frac{4\pi \times \text{area } abcdef}{\text{length } ac}.$$

The flux in any zone, e.g., between 0° and 60°, is

$$F_z = \frac{4\pi \times \text{area } abcf}{\text{length } ac}.$$

The mean intensity between any angular limits is equal to the portion of the area *abcdef* between those limits divided by the corresponding portion of the base line *abc*. This method is to be preferred to the others outlined where accuracy of a high order is desired, as in the measurement of the light absorption of reflectors, globes, shades, etc. In such cases the number of points of the photometric distribution curve determined by direct measure and referred by projection to the line *def* should be as large as practicable.

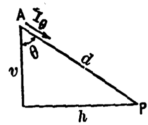


Fig. 4.

Calculation of Illumination at Points.—In Fig. 4 let A be the location of an illuminant and P a point at which the illumination from A is to be determined. The three most important cases of this problem refer respectively to the illumination of elements of surface at P located in horizontal and vertical planes and in a plane normal to the light path AP. From the laws of inverse squares and of cosines (*see above*) the horizontal illumination at P is

$$E_h = \frac{I_\theta \cos\theta}{d^2} = \frac{I_\theta \cos^3\theta}{v^2}.$$

The vertical illumination at P is

$$E_v = \frac{I_\theta \sin\theta}{d^2} = \frac{I_\theta \sin^3\theta}{h^2}.$$

The normal illumination at P is

$$E_n = \frac{I_\theta}{d^2} = \frac{I_\theta \cos^2\theta}{v^2} = \frac{I_\theta \sin^2\theta}{h^2}.$$

Calculations of this type are facilitated by a chart (Fig. 5) showing the value of θ for various values of v and h, and by tables giving the values of the illumination constants $\cos^3\theta/v^2$, $\sin^3\theta/h^2$, and $\cos^2\theta/v^2$ for various values of v and h. Such tables are given below.

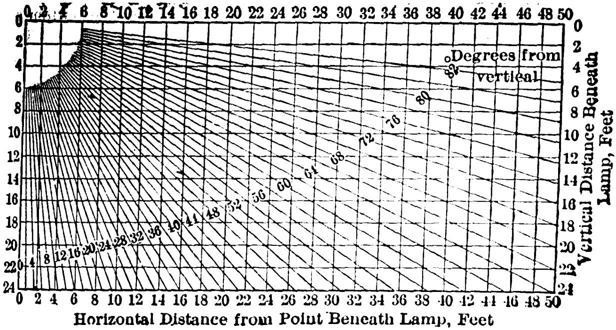


Fig. 5. Angle of Effective Beams for Point-by-point Calculations

TABLE OF ILLUMINATION CONSTANTS

For Horizontal and Vertical Illumination

Horizontal distance from point beneath lamp, feet	Vertical distance below lamp, feet									
	6	7	8	9	10	12	14	16	18	20
0	0.0278	0.0204	0.0156	0.0127	0.0100	0.00695	0.00510	0.00391	0.00309	0.00250
1	0.0266	0.0198	0.0152	0.0122	0.0099	0.00687	0.00505	0.00388	0.00307	0.00249
2	0.0236	0.0182	0.0143	0.0115	0.0094	0.00665	0.00491	0.00382	0.00303	0.00246
3	0.0200	0.0158	0.0129	0.0106	0.0088	0.00635	0.00476	0.00371	0.00296	0.00241
4	0.0160	0.0134	0.0112	0.0094	0.0080	0.00592	0.00453	0.00357	0.00288	0.00235
5	0.0126	0.0111	0.0094	0.0082	0.0072	0.00547	0.00426	0.00340	0.00278	0.00228
6	0.0098	0.0089	0.0080	0.0071	0.0063	0.00496	0.00397	0.00320	0.00264	0.00220
8	0.0060	0.0058	0.0055	0.00515	0.00476	0.00400	0.00333	0.00279	0.00235	0.00200
10	0.0038	0.00385	0.00382	0.00369	0.00353	0.00312	0.00274	0.00238	0.00211	0.00179
12	0.0025	0.00261	0.00266	0.00268	0.00263	0.00245	0.00222	0.00200	0.00178	0.00158
14	0.0017	0.00182	0.00191	0.00195	0.00196	0.00190	0.00180	0.00167	0.00152	0.00138
16	0.0012	0.00131	0.00140	0.00146	0.00149	0.00151	0.00145	0.00138	0.00129	0.00119
18	0.00088	0.00097	0.00105	0.00110	0.00114	0.00118	0.00118	0.00114	0.00109	0.00103
20	0.00066	0.00074	0.00080	0.00085	0.00089	0.00095	0.00097	0.00095	0.00093	0.00088
24	0.00038	0.00044	0.00050	0.00053	0.00057	0.00062	0.00065	0.00067	0.00067	0.00066
28	......	0.00028	0.00032	0.00036	0.00039	0.00043	0.00046	0.00048	0.00049	0.00049
32	......	......	0.00022	0.00024	0.00027	0.00030	0.00033	0.00035	0.00036	0.00037
36	......	......	......	0.00017	0.00019	0.00022	0.00024	0.00026	0.00028	0.00029
40	......	......	......	......	0.00014	0.00017	0.00019	0.00020	0.00021	0.00022
45	......	......	......	......	......	0.00012	0.00014	0.00015	0.00016	0.00017
50	......	......	......	......	......	......	0.00010	0.00011	0.00012	0.00013

The horizontal illumination at any point equals the intensity of the light source in its direction (see Fig. 5) multiplied by the constant in the above table

corresponding to its location. With a few exceptions near the bottom of the table, direct interpolations are correct to within 2 per cent. To compute vertical illumination at any point, exchange the vertical and horizontal distance components and use the constant so found.

TABLE OF ILLUMINATION CONSTANTS

For Normal Illumination

Horizontal distance from point beneath lamp, feet	Vertical distance below lamp, feet									
	6	7	8	9	10	12	14	16	18	20
0	0.0278	0.0204	0.0156	0.0127	0.0100	0.00695	0.00510	0.00391	0.00309	0.00250
1	0.0270	0.0200	0.0154	0.0122	0.00990	0.00690	0.00507	0.00389	0.00308	0.00249
2	0.0250	0.0189	0.0147	0.0118	0.00962	0.00675	0.00500	0.00384	0.00304	0.00247
3	0.0222	0.0172	0.0137	0.0111	0.00917	0.00653	0.00487	0.00377	0.00300	0.00244
4	0.0200	0.0154	0.0125	0.0103	0.00842	0.00625	0.00472	0.00367	0.00294	0.00240
5	0.0164	0.0135	0.0112	0.00943	0.00800	0.00592	0.00452	0.00356	0.00286	0.00235
6	0.0139	0.01175	0.01000	0.00855	0.00735	0.00555	0.00431	0.00342	0.00278	0.00229
8	0.0100	0.00885	0.00780	0.00689	0.00610	0.00480	0.00385	0.00312	0.00258	0.00215
10	0.00735	0.00672	0.00610	0.00552	0.00500	0.00410	0.00338	0.00281	0.00235	0.00200
12	0.00556	0.00518	0.00480	0.00444	0.00410	0.00347	0.00294	0.00250	0.00214	0.00184
14	0.00431	0.00408	0.00385	0.00361	0.00338	0.00294	0.00255	0.00221	0.00192	0.00168
16	0.00342	0.00327	0.00312	0.00297	0.00281	0.00250	0.00221	0.00195	0.00172	0.00152
18	0.00278	0.00268	0.00258	0.00247	0.00235	0.00214	0.00192	0.00172	0.00154	0.00138
20	0.00229	0.00222	0.00215	0.00208	0.00200	0.00184	0.00168	0.00152	0.00138	0.00125
24	0.00163	0.00160	0.00156	0.00152	0.00148	0.00139	0.00129	0.00120	0.00111	0.00102
28	0.00122	0.00120	0.00118	0.00115	0.00113	0.00108	0.00102	0.00096	0.00090	0.00084
32	0.00094	0.00093	0.00092	0.00090	0.00089	0.00086	0.00082	0.00078	0.00074	0.00070
36	0.00075	0.00074	0.00074	0.00073	0.00072	0.00069	0.00067	0.00064	0.00062	0.00059
40	0.00061	0.00061	0.00060	0.00059	0.00058	0.00057	0.00056	0.00054	0.00052	0.00050
45	0.00049	0.00048	0.00048	0.00047	0.00047	0.00046	0.00045	0.00044	0.00043	0.00041
50	0.00039	0.00039	0.00039	0.00039	0.00038	0.00038	0.00037	0.00036	0.00035	0.00034

The normal illumination at any point equals the intensity of the light source in its direction (see Fig. 5) multiplied by the constant in the above table corresponding to its location.

Calculation of Solid Angle Subtended by Illuminated Area. — In many cases it is desired to compute the total flux of light which falls upon a plane area from an approximate point source. This problem resolves itself into two elements, (1) to determine the mean intensity acting toward the illuminated plane, to which the methods previously described apply, and (2) to determine the solid angle subtended by the area illuminated at the source of light. The following theorems apply to the latter problem.

The solid angle subtended by a circle with the light source in its axis (Fig. 6) is

$$\omega = 2\pi\,(1 - \cos\alpha).$$

The solid angle subtended by a rectangle, one corner of which is directly beneath the source of light (Fig. 7), is

$$\omega = \tan^{-1}\frac{ac}{hd}.$$

When the projection of S falls on one edge of the area, the latter may be divided into two rectangles to meet the above theorem. When the projection falls within the area, the area may be divided into four such sections. When the projection falls outside ac in Fig. 7, the surface may be extended to meet the theorem and the solid angle subtended by the extension substracted from that of the combined areas.

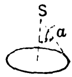


Fig. 6.

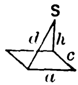


Fig. 7.

LIGHT FROM FINITE SOURCES. — Under conditions pointed out in connection with point sources, finite sources may be treated as point sources without appreciable error. When those conditions are not met, illumination problems may be solved by the aid of the following theorems.

Lambert's Cosine Law. — A true diffusing surface emits light in any direction with an intensity proportional to the cosine of the angle of emission, measured from the normal to the source. Unless especially prepared, reflecting and transmitting surfaces deviate from this law to some degree, depending on surface glaze and the nature of the translucent medium. In the following theorems true diffuse emission is assumed for each element of surface.

Total Light Emitted by a Plane Area A, having a surface brightness of b units of intensity in the normal direction, is

$$F = \pi A b.$$

Illumination Due to a Disk Source, having a brightness of b.

(a) At a point P under the disk in a plane parallel to the disk (Fig. 8) the illumination is

$$E = \frac{\pi b}{2}\cot\theta\,(\beta + 2\theta - \alpha).$$

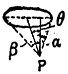


Fig. 8.

Fig. 9.

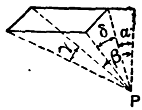


Fig. 10.

(b) In the special case where point P is in the axis of the disk (Fig. 9) the illumination is

$$F = \pi b \sin^2\alpha.$$

Illumination Due to a Rectangular Source, having a brightness of b.

(a) The illumination in a plane parallel to the source at a point under one corner of the source or one edge extended (Fig. 10) is

$$E = \frac{b}{2}[\delta\cos\alpha - \gamma\cos\beta].$$

(b) When the projection of the point P lies within the source, the source may be subdivided into rectangles and each solved according to case (a).

(c) When the projection of P lies without the area and not on one edge extended, a solution is made by enlarging the rectangle to meet case (a) and subtracting the solution for the part added from that for the entire extended area.

(d) The illumination in a plane normal to the source at a point under one corner of the source (Fig. 11) is

$$E = \frac{b}{2}\left[\tan^{-1}\frac{c}{h} - \frac{h}{d}\tan^{-1}\frac{c}{d}\right].$$

Extensions of this theorem may be made in a manner analogous to that suggested for cases (b) and (c) above.

Illumination Due to the Hollow Interior of a Dome, whose lower boundary is a circle and whose surface brightness is b. The solution of this case is

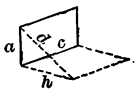


Fig. 11.

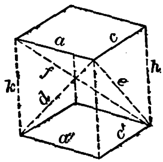


Fig. 12.

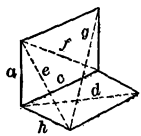


Fig. 13.

identical with that for a circular disk equal in area to the base of the dome and having the same brightness; see Fig. 9.

Flux Received by a Rectangular Area from a Similar and Parallel Area, having a brightness of b (Fig. 12).

$$F = 2\,b\left[ac\tan^{-1}\frac{a}{e} - ah\tan^{-1}\frac{a}{h} + cd\tan^{-1}\frac{c}{d} - ch\tan^{-1}\frac{c}{h} + \frac{h^2}{2}\log_e\frac{e^2d^2}{h^2f^2}\right],$$

when $$a = c = h,\ F = \frac{a^2\pi b}{5} = 0.6283\ a^2b.$$

Flux Received by a Rectangular Area from an Adjacent Normal Area of the same length, having a brightness of b (Fig. 13).

$$F = b\left[ah\tan^{-1}\frac{a}{h} - ad\tan^{-1}\frac{a}{d} + ac\tan^{-1}\frac{a}{c} + \frac{a^2}{2}\log_e\frac{e^2f^2}{a^2g^2} + \frac{e^2}{4}\log_e\frac{g^2}{e^2}\right.$$
$$\left. + \frac{a^2}{4}\log_e\frac{a^2}{f^2} + \frac{c^2}{4}\log_e\frac{c^2}{f^2} + \frac{h^2}{4}\log_e\frac{h^2}{d^2} + \frac{c^2}{4}\log_e\frac{g^2}{d^2}\right],$$

when $$a = c = h,\ F = \frac{a^2\pi b}{5} = 0.6283\ a^2b.$$

Light Flux Within an Inclosure. — If an entirely inclosed space is surrounded by walls having a reflection coefficient of r and a flux of light equal to F is emitted within this space by a light source, the total flux within the space due to direct emission and multiple reflection is

$$\Sigma F = F + Fr + Fr^2 + Fr^3 + Fr^4 + \ldots = F/(1-r).$$

In computing the total illumination of areas in rooms due to direct and indirect illumination this expression is of limited application due to the unequal reflection coefficients of the various surfaces. McAllister (*see Elec. Wld., Vol. 52, p. 1158*) has suggested a method of calculation based on the absorption of light by various surfaces, which rests upon the principle that the light absorbed by each surface equals the total flux received by it multiplied by its absorption coefficient, and that the total flux so absorbed equals the total flux emitted by the light sources. That is, given a room bounded by six surfaces whose areas are respectively A_1, A_2, A_3, etc., whose absorption coefficients are respectively a_1, a_2, a_3, etc., and

which are to be illuminated to respective intensities of E_1, E_2, E_3, etc., the total flux required of light sources is

$$\Sigma F = E_1 A_1 a_1 + E_2 A_2 a_2 + E_3 A_3 a_3 + \text{etc.}$$

STANDARD FORMS OF LIGHT DISTRIBUTION FOR UNIFORM ILLUMINATION. — The most important phase of this problem, viz., the uniform illumination of a horizontal plane by one and by many illuminants, will be outlined. The form of light distribution required to obtain uniform horizontal illumination from a single lamp is found by reversing the expression for horizontal illumination,

$$E_h = \frac{I\theta \cos^3\theta}{v^2} \qquad \text{to} \qquad I\theta = \frac{E_h v^2}{\cos^3\theta},$$

taking E_h and v as constants. The resulting form of polar curve for various ratios of the limiting horizontal distance h' to v are shown in Fig. 14. It is apparent that practical difficulties limit the range of application of this method. The more general case must be solved by the use of a considerable number of lamps uniformly spaced above the area illuminated. The type of horizontal illumination curve shown in Fig. 15 adequately meets this condition, as shown by the resultant illumination curves along the side and diagonal of the square included by the points directly under four equally spaced lamps. To obtain the form of illumination curve shown for various ratios of horizontal spacing s to vertical distance to the lamps v, the forms of light distribution shown in Fig. 16 are required.

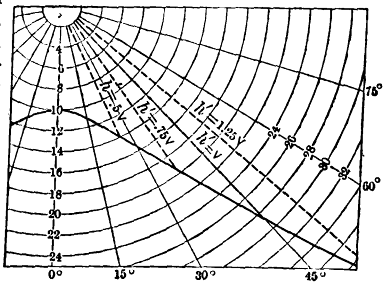


Fig. 14. Light Distribution Required to Obtain Uniform Horizontal Illumination from One Source

The ratios of spacing to height indicated in the figure are to be considered as the maximum ratios with which uniform illumination is obtained. With smaller ratios the uniform condition still exists. With a fixed spacing varying the height within the limit imposed by the ratio affects neither the uniformity nor the intensity of illumination, except in that portion of the room lying outside the outer row of lamps. By aid of well-designed reflectors it is possible to closely approximate the typical distribution curves for certain ratios of spacing to height and this ratio is important as a criterion in the selection of reflecting devices.

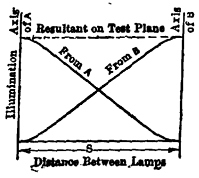


Fig. 15.

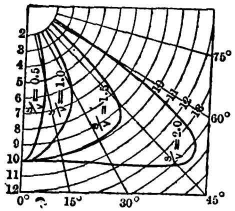


Fig. 16. Curves of Light Distribution for Uniform Illumination from many Sources

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[W. E. WICKENDEN.]

ILLUMINATION, STREET. — (*See also Illumination, Interior; Illumination, Laws of; Lamps, Arc; Lamps, Incandescent; Lighting Plants.*) Street lighting is intended to promote public safety, facilitate travel and business, and reveal architectural effects during hours of natural darkness. Experience shows that good street lighting attracts traffic and stimulates retail trade. The most important factors affecting the degree of street illumination required are the density of traffic, prevalence of retail business, degree of police supervision needed and the architectural importance of the thoroughfare. In moderate and dim light vision depends primarily on differences of brightness and is but slightly assisted by color distinctions. Objects may be seen directly and in some detail against a darker background, but are seen in mass or silhouette against a brighter background.

Requirements of Street Lighting. — The most important streets require sufficient illumination for direct and detail vision to clearly reveal vehicles, persons, obstructions, irregularities of the pavement, and to permit the easy reading of timepieces and addresses, which calls for an average of about 0.1 foot-candle and a minimum of 0.05 foot-candle on the most important working surfaces. Uniform illumination without deep shadows, which may be obtained by well-diffused light sources with fairly close spacing, is highly desirable on important streets.

Secondary streets with moderate evening traffic and orderly conditions require greater illumination at intersections than at intermediate points. The silhouette aspect of vision is very important in such streets and emphasizes the need of fairly bright and even roadway illumination without spots of deep shade from foliage. An average illumination of 0.05 foot-candle and a minimum of 0.02 meet the reasonable requirements of such streets. In this range of intensities the eye is highly susceptible to glare. Brilliant light sources without diffusing globes and suspended at a low level often largely defeat their purpose under these conditions.

Minor streets with scattered buildings and infrequent travel require mainly beacon lighting, equivalent in intensity to moderate moonlight, or normal illumination varying from a minimum of 0.01 foot-candle to an average of 0.03. At such low intensities the need of good diffusion at light sources is most acute, but is quite generally neglected for reasons of supposed economy.

Reference Planes — Street illumination in America is usually referred to normal reference planes (i.e., perpendicular to the light rays), and to horizontal planes in Europe. There is wide diversity in the elevation of reference planes, the most common levels being that of the pavement and that of a plane four feet above the pavement. The use of normal planes of reference takes satisfactory account of the light from but one source, while it is possible to sum up the light received from all directions on horizontal planes. It is evident from the cosine law of incidence that the horizontal illumination from low and distant lamps is less in magnitude and more difficult of exact measurement than the normal component. Normal illumination is a more useful index to the visibility of upright objects. Small objects as cards and timepieces are instinctively held normal to the rays of the nearest lamp. The vertical component of street illumination should have due consideration for its importance in revealing the architecture of buildings and assisting in the recognition of persons.

TYPES OF ILLUMINANTS. — The illuminants most extensively used in street lighting are as follows: (*For detailed descriptions and data see Lamps, Arc; Lamps, Incandescent; and Gas Lighting.*)

Electric Arc Lamps on Series Circuits, including (*a*) carbon arcs of open d-c., enclosed d-c. and enclosed a-c. types, with standard currents of 6.6,

7.5 and 9.6 amperes; (*b*) flame carbon arcs of open d-c., enclosed d-c. and enclosed a-c. types, with standard currents of 6.6, 7.5 and 9.6 amperes; and (*c*) metallic or luminous arcs of open d-c. type with standard currents 4.0, 5.5 and 6.6 amperes, arranged for suspension and pedestal mounting.

Electric Arc Lamps on Multiple Circuits at or near 110 volts, including (*a*) carbon arcs of open d-c., enclosed d-c. and enclosed a-c. types; (*b*) flame carbon of open d-c., enclosed d-c. and enclosed a-c. types; and (*c*) quartz tube mercury arcs.

Electric Incandescent Lamps, Series Type, including (*a*) vacuum lamps with standard currents of 3.5, 4.0, 5.5, 6.6. and 7.5 amperes and range of candle-power from 32 to 350; and (*b*) gas-filled lamps of from 57 to 500 watts, and from 80 to 1000 candle-power.

Electric Incandescent Lamps, Multiple Type at or near 110 volts of from 25 to 1000 watts and from 21.5 to 1670 candle-power.

High-pressure Gas Lamps of upright and inverted types, including (*a*) compressed gas, (*b*) compressed air and (*c*) compressed gas and air types.

Low-pressure Gas Lamps of upright and inverted types, including (*a*) single mantle lamps and (*b*) clusters of 3, 4 and 5 mantles.

Naphtha Vapor Mantle Lamps of self-contained, upright type with nominal candle-power of from 45 to 60.

Relative Advantages of Various Street Illuminants. — The advantages of electric lighting over gas lighting are (1) superior flexibility in sizes and possible locations, (2) greater ease of maintenance, (3) availability of white color, and (4) ease of control from central points with series circuits. The advantages of gas over electricity are (1) lower probability of interruption by accident, (2) steadiness of light and (3) the low intrinsic brilliancy of mantles.

Both electric and gas lamps suffer fluctuations due to unsteady pressure, but these conditions are more easily regulated in electric systems. Skillful maintenance is necessary to insure the proper efficiency of all street lamps, but affords a simpler problem in the case of electric lamps. High-pressure gas lamps require maintenance of an exceptionally high order.

Gas lamps are usually lighted by hand or by local automatic clock devices, though a few methods of central control have been devised. Naphtha vapor lamps are difficult to maintain in good efficiency and require costly attention, as each lamp must be filled by hand and heated by a blast torch before lighting. Such lamps should be used only where electric or gas lighting from central systems is not available.

Series electric lamps tend to economy in power distribution and to simplicity of central control. Separate mains must be used for all circuits requiring central control. Lamps operated in multiple from regular mains are lighted and extinguished by hand or by special clock switches. The high voltage of series circuits involves elements of danger and liabilities to interruption by accidents not found in parallel systems. Parallel systems can employ metal poles and grounded supports with greater safety. Incandescent electric lamps provide a very wide range of candle-power and involve smaller expense for maintenance and renewals than arc or gas lamps. On well-regulated circuits incandescent lamps surpass all other street illuminants in steadiness. The effectiveness of incandescent lamps as commonly installed without diffusing shades is often badly impaired by glare.

The color of light is relatively less important in street lighting than in interiors. White is generally preferred, as satisfying a sense of naturalness. Yellow and green have superior penetrating power in fog and smoke.

Small vs. Large Units. — Large units are generally more efficient than small units and involve relatively less expense for installation, maintenance

and operation per unit of light production. With proper spacing the light of small units can be more completely utilized on roadways and produces a more uniform illumination. Small units lend themselves more readily to decorative schemes and to the lighting of shady streets and curving roadways. The effect of numerous small units at low levels, especially when unshaded, is distinctly more obtrusive and glaring than a small number of high power sources hung fairly high and well diffused.

Ratings of Street Lamps.—Street lamps are variously rated in candle-power, watts and rate of gas consumption. Much confusion is caused by loose usage of the term candle-power, which in different cases refers to mean horizontal, maximum, mean spherical, mean lower hemispherical and merely nominal values. Horizontal and maximum candle-power or that at any specified angle are significant only when comparing lamps having a definite form of light distribution in common. Mean spherical candle-power gives the gross light output of a lamp without any index to the effectiveness with which it may be utilized. It is an appropriate rating for a bare luminous element without accessories. The mean lower hemispherical candle-power is perhaps the best single index to the available light output of a lamp, but it cannot be conveniently measured in service. Candle-power ratings are sometimes of a purely nominal nature, e.g., the 1200 and 2000 candle-power ratings applied to carbon arc lamps denote lamps consuming 330 and 480 watts respectively.

The rated candle-power of gas and gasoline mantle lamps usually represents the maximum intensity which a given type of lamp can produce commercially. This rating almost invariably exceeds the average results in service by a wide margin due to imperfect maintenance, and can serve only as an index to the type of lamp referred to.

Incandescent electric lamps are rated in horizontal candle-power on the basis of actual initial performance. The candle-power rating of incandescent lamps installed on streets with reflectors is often taken to imply the actual average intensity at some angle between 15° and 25° below horizontal. A test of 130 incandescent lamps with flat enameled metal reflectors in service, which was made by the writer in 1913, showed the average candle-power of such units at 25° below horizontal to be 111.6 per cent of the rated horizontal candle-power of the lamps.

Indefinite candle-power specifications in lighting contracts have caused much litigation. Arc lamps of all types are preferably rated in watts, as affording the only accurately measurable index to their performance. The specification that a reasonable average of candle-power as indicated by a prescribed photometric test shall be maintained is recognized as a valuable check on the quality of maintenance and service.

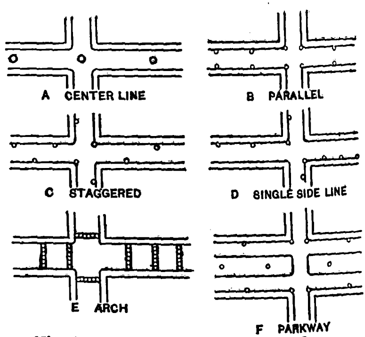


Fig. 1. Spacing Schemes for Street Lamps

SPACING OF LAMPS.—The plans of lamp spacing most widely used are shown diagrammatically in Fig. 1. Plan A, or single center line spacing, is the most effective method of using high-power units. Lamps may be hung from span wires between poles or buildings or from mast arms projecting over roadways. In some cases lamps

are mounted directly on poles or standards in a central parkway or series of safety isles. Center suspension is especially desirable when only one lamp is placed at an intersection. Plan B, or parallel spacing, and Plan C, or staggered spacing, are best adapted to the brilliant lighting of main thoroughfares with high-power units and the lighting of ordinary streets with small units. The staggered arrangement favors the meeting of a definite minimum requirement of roadway illumination with the smallest number of lamps per mile. Plan D, or single side line spacing, sacrifices symmetry and uniformity to the convenience of running electric circuits or gas piping on but one side of the street. Plan E, or arch lighting, is adapted only to incandescent electric lamps; it tends to give a street a festive appearance, but is not an effective method of illumination. Plan F, or parkway lighting, is of obvious value where the roadway is divided as shown.

Side vs. Center Mounting. — Fig. 2 shows the effect of center and side mounting on the per cent of the total light which falls within the limits of streets

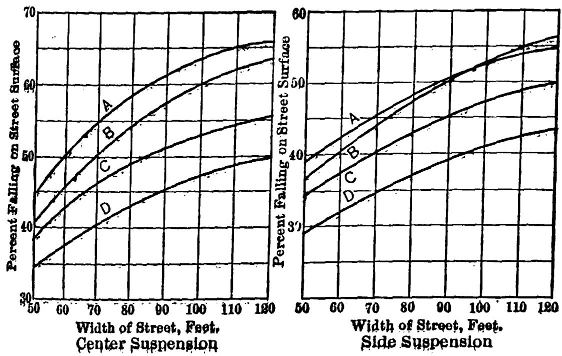


Fig. 2. Per Cent of Light from Arc Lamps which falls within Limits of the Street

A. 9.6 amp. Open Carbon Arc, Clear Globe
B. 4 amp. Magnetite Arc, Clear Globe
C. 6.6 amp. Magnetite Arc, Opal Globe
D. 6.6 amp. Enclosed Carbon Arc, Opal Globe

of various widths. These curves all refer to a mounting height of 20 feet above street level. The lamps with side mounting are assumed to be over a curb line which is distant from the nearer street line by an amount equal to one-fifth the total width of the street. The higher efficiency of center mounting is apparent. (*Data by P. S. Millar, Trans. Ill. Eng. Soc., Vol. 5, p. 658.*)

Various attempts have been made to increase the percentage of light thrown on the street by reflectors of special design which deflect lengthwise of the street light which would otherwise be thrown to the sides. These have had a very limited acceptance.

Excellent use of large units at intersections and small units at intermediate points may be made on secondary streets, especially where foliage interferes with the distribution of light at distances. On curved roadways it is preferable to locate the lamps on the outer sides of the curves, as they are then visible at greater distances.

Height of Suspension. — The spacing of lamps should be laid out with due regard to the height of suspension of the lamps and the degree of minimum illumination to be provided. In many cities a standard height of mounting is em-

ployed with each type of lamp. When such is the case, spacing problems are readily solved by the aid of diagrams such as shown in Fig. 3. Taking account of the suspension height and the light distribution curve of each lamp in its normal service condition, the horizontal distances from the axis of the lamp at

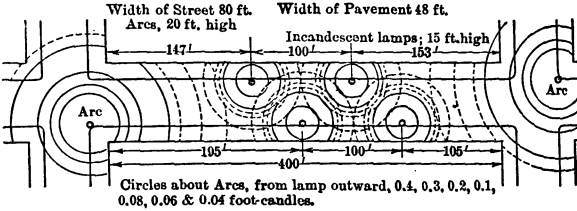


Fig. 3. Spacing and Illumination Plan for a Minimum Normal Illumination of 0.04 Foot-candle

which various normal intensities of illumination are produced at an appropriate level, as that of the street or at a height four feet above the street, are computed by the point-by-point method. Using these distances as radii, circles are drawn to scale about a center representing the axis of the lamp and are marked with the corresponding intensity of illumination. These diagrams are preferably made on translucent tracing cloth and several of each type should be prepared. A plan of the space to be lighted is drawn to the same scale on a separate paper and the circle diagrams are used as templates for various trial spacings until the desired result is obtained. The circles then furnish points for the plotting of illumination profiles or contours at the center line and curb lines, from which the approximate average illumination is readily computed. The illustration shows an appropriate location for 4-ampere magnetite arcs, with globes at street intersections, and 60-candle-power series incandescent lamps with flat enameled metal reflectors at intermediate points, to produce a minimum normal illumination of 0.04 f.c. on a section of street 80 feet wide and 480 feet long lying between centers of intersecting streets.

Selection of Height of Suspension. — The most appropriate suspension height for a street lamp depends on its light output, form of distribution and intrinsic brilliancy, and upon the degree of minimum illumination required. It is practically impossible to realize uniform illumination from large arcs and high-pressure gas lamps without higher suspension than that commonly employed. The usual result is a bright spot immediately about the lamp and low illumination at mid-points. The contrast so produced reduces the effectiveness of the light.

A high ratio of suspension height to horizontal spacing tends to improve the uniformity of light distribution and to increase the distance over which a single lamp can provide illumination above a specified minimum value. A high ratio of height to distance is especially desirable when the maximum light intensity of the lamp is more than 30° below the horizontal.

The following example may be taken. In Fig. 4 polar curve (*a*) refers to a 510-watt magnetite arc with clear globe and (*b*) to a 450-watt d-c. flame arc with vertical electrodes and a light opal globe. The illumination curves show the normal illumination at various distances for various suspension heights. In case (*a*) the maximum intensity is but 10° below the horizontal and little gain at distant points is obtained by employing a suspension height exceeding 25 feet. In case (*b*), however, the maximum intensity is much lower on the polar curve and a much higher suspension could advantageously be employed. Fig. 5 shows

the effect of height of suspension of these two lamps on the normal illumination at various distances.

The actual suspension height is often determined by the exigencies of trimming, by the length of wooden poles available, and by the necessity of distributing light below the foliage of trees. It must be recognized that a lamp with high mounting throws a smaller total percentage of its light

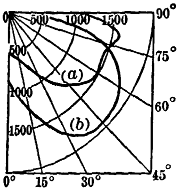
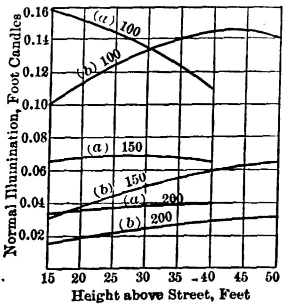


Fig. 4. Polar Light Distribution Curves of (a) 6.6-ampere Magnetite Arc with Clear Globe and (b) 450-watt Enclosed Flame Arc



Numbers on curves are horizontal distances (feet) from source

Fig. 5. Effect of Height of Suspension of Arc Lamps of Fig. 4 on Normal Illumination at Various Distances

on the street, and that the gain is from the better distribution to distant points.

LIGHTING SCHEDULES. — All-night schedules provide for a yearly total of from 3830 to 4000 hours of operation. The former total results from lighting 30 minutes after sunset and extinguishing an hour before sunrise. The latter involves one half-hour longer burning each night. The "Philadelphia moonlight schedule" allows lamps to remain unlighted on nights when the moon is full or nearly so and on other nights the lamps are lighted one hour before moonset and extinguished one hour after moonrise. This schedule calls for an annual total of 2000 hours of operation. The "Frund system" ignores the moon until midnight, after which the provisions of the moonlight schedule apply, making an annual total of 3000 hours. As a large portion of the cost of street lighting is due to fixed charges the saving of reduced schedules as compared with an all-night schedule is much less in proportion than the reduction of hours. Each year the *Electrical World* issues a detailed set of lighting schedules for the ensuing year, separate tables being furnished for northern, middle and southern latitudes of the United States.

COST OF STREET LIGHTING. — The cost of street lighting is usually based on an annual price per lamp or fixture for a specified number of hours of burning. This cost may properly include all fixed charges on the lamps and their accessories as well as a due proportion of the charges on poles, lines, transformers and other appliances assignable to the service, all costs of maintenance and renewals involved in the service, and a reasonable charge for the energy supplied. These costs naturally vary with the character of the distributing system, value of poles, rate of depreciation of lamps and the cost of producing electrical energy. Depreciation rates on street lamps and special appliances used in supplying them are apt to reach or exceed 10 per cent, due to rapid obsolescence. The average prices charged per year for various types of street lamps operated on all-night schedules in a representative group of cities are given below:

PRICES PER YEAR FOR ELECTRIC STREET LAMPS; ALL-NIGHT SCHEDULE

Type of lamp	Cities averaged	Average rate	Maximum rate	Minimum rate
4-amp. magnetite arc.	27	$63.00	$80.00	$45.00
6.6-amp. enc. carbon arc.........	40	73.00	100.00	74.00
7.5-amp. enc. a-c. carbon arc....	16	70.00	85.00	60.00
80-c.p. series tungsten lamp......	6	29.50	39.00	18.50
60-c.p. series tungsten lamp......	16	23.25	28.00	15.00
40-c.p. series tungsten lamp......	4	20.00	28.00	18.50
32-c.p. series tungsten lamp	16	18.00	22.50	12.50

Single-mantle gas lamps are usually furnished on an all-night schedule for prices ranging from $20 to $25 per annum. In many of the smaller cities special display systems of incandescent electric lamps are operated on main business streets, the expense being assumed jointly by the city and the occupants of abutting property. The lamps are usually arranged in arches, festoons or in clusters on pedestals, and are often operated from constant-potential mains to permit the turning off of part of the lamps after midnight. These systems are usually maintained by the operating company and charged for on a flat rate basis.

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[W. E. WICKENDEN.]

INDETERMINATE FORMS. — (*See also Derivatives.*) Let $f(x)$ and $F(x)$ be any two functions of x and let $y = \frac{f(x)}{F(x)}$. For certain values of x, both $f(x)$ and $F(x)$ may be zero, making y equal to $\frac{0}{0}$, an expression which, considered alone, may be anything between 0 and ∞. Its true value may, however, be determined from the nature of the functions $f(x)$ and $F(x)$ by the following process, which involves finding the derivatives of the two functions. If $y_1 = \frac{0}{0}$ when $x = x_1$, then

$$y_1 = \left.\frac{\frac{d}{dx}f(x)}{\frac{d}{dx}F(x)}\right|_{x=x_1}$$

which expression may have a perfectly determinate value. For example if

$$y = \frac{x^2 - 4}{x^3 - 8}$$

and $x = 2$, then

$$y_1 = \frac{0}{0} = \left.\frac{2x}{3x^2}\right|_{x_1=2} = \frac{4}{12} = \frac{1}{3}.$$

If the ratio of the derivatives is still indeterminate, differentiate numerator and denominator again and, if necessary, repeat the process until a determinate form is obtained.

When $y = \frac{f(x)}{F(x)}$ reduces to the indeterminate form $\frac{\infty}{\infty}$ for any particular value x_1 of x, the corresponding value of y is

$$y_1 = \left.\frac{\frac{d}{dx}\left(\frac{1}{F(x)}\right)}{\frac{d}{dx}\left(\frac{1}{f(x)}\right)}\right|_{x=x_1}$$

When $y = f(x) \times F(x)$ reduces to the indeterminate form $0 \times \infty$ for any particular value x_1 of x, the corresponding value of y is

$$y_1 = \left.\frac{\frac{d}{dx}f(x)}{\frac{d}{dx}\left(\frac{1}{F(x)}\right)}\right|_{x=x_1}$$

[W. A. Del Mar.]

INDUCTANCE AND INDUCTIVE REACTANCE. — (*See also Alternating Currents; Electricity and Magnetism, Principles of; Skin Effect; Transmission Lines.*) The phenomena of self and mutual inductance are described in the article on *Electricity and Magnetism, Principles of*. In general, when the current i_1 in a circuit, No. 1 say, is varying at the rate $\frac{di_1}{dt}$, a potential drop, in the direction of the current (or *back* e.m.f.) is induced in the circuit, which may be written

$$v_{11} = L_1 \frac{di_1}{dt},$$

where L_1 is called the coefficient of self induction, or self inductance, or simply the inductance, of the circuit. Similarly, when a second circuit, No. 2 say, is in the vicinity of No. 1 and the current i_2 in No. 2,* is varying at the rate $\frac{di_2}{dt}$, an additional potential drop is induced in circuit No. 1 equal to

$$v_{12} = M_{12} \frac{di_2}{dt},$$

where M_{12} is called the coefficient of mutual induction, or simply the mutual inductance, of one circuit with respect to the other.

When the currents in both instances are sine-wave currents of effective values I_1 and I_2 respectively and of frequency f, the effective values of these potential drops are

$$V_{11} = x_1 I_1 \qquad \text{and} \qquad V_{12} = x_{12} I_2 \tag{1}$$

and V_{11} leads I_1 by 90 degrees, and V_{12} leads i_2 by 90 degrees, and

$$x_1 = 2\pi f L_1 \qquad \text{and} \qquad x_{12} = 2\pi f M_{12}. \tag{1a}$$

x_1 is called the inductive self-reactance, or simply the inductive reactance of circuit No. 1. (*see Alternating Currents*), and x_{12} is called the inductive mutual reactance of one circuit with respect to the other. The mutual inductance, and therefore the mutual reactance, between any two circuits is the same for No. 1 with respect to No. 2 as for No. 2 with respect to No. 1, i.e., $M_{12} = M_{21}$ and $x_{12} = x_{21}$.

The total inductive† drop in any circuit due to the *variation of the current* in this and in any number of neighboring circuits is

$$v_1 = L_1 \frac{di_1}{dt} + M_{12} \frac{di_2}{dt} + \mathrm{M}_{13} \frac{di_3}{dt} + \text{etc.} \tag{2}$$

For sine-wave currents of frequency f the total inductive* drop is, in *vector notation* (*see Alternating Currents*),

$$V_1 = j\,(x_1 I_1 + x_{12} I_2 + x_{13} I_3 + \ldots). \tag{2a}$$

Units of Inductance and Reactance. — The practical unit of inductance is the henry, the c.g.s. electromagnetic unit the abhenry (also called a "centimeter"), and the c.g.s. electrostatic unit the stathenry. Inductances are fre-

* i_2 is to be considered positive with respect to i_1 when the flux lines threading No 1 due to the current in No. 2 thread through No. 1 in the same direction as the flux lines due to the current in No. 1.

† In addition to these drops there are also the resistance drop and such other back e.m f.'s as may be present. See *Alternating Currents*.

quently expressed in thousandths of a henry, i.e., in millihenrys. The units of reactance are the same as the units of resistance (q.v.) See *Units and Conversion Factors* for the interrelations of the various units.

Total Inductance of Two or More Circuits in Series. — When several circuits, having the coefficients L_1, L_2, L_3, etc., and M_{12}, M_{13}, M_{23}, etc., are connected in series the currents $i_1 = i_2 = i_3 = \text{etc.} = i$, say, and the total induced e.m.f. is $v = v_1 + v_2 + v_3$, etc., or

$$v = \left[(L_1 + L_2 + L_3 + \text{etc.}) + 2\,(M_{12} + M_{13} + M_{23} + \text{etc.})\right]\frac{di}{dt},$$

whence the resultant or total inductance is

$$L = (L_1 + L_2 + L_3 + \text{etc}) + 2\,(M_{12} + M_{13} + M_{23} + \text{etc.}). \tag{3}$$

This relation makes possible the accurate calculation of the self inductance of a coil of a number of turns when the self inductance of each turn and the mutual inductance of each pair of turns are known.

RELATION BETWEEN FLUX, CURRENT AND INDUCTANCE. — When the permeability of the conductors and of the medium between and surrounding them is constant (e.g., when the conductors and medium are non-magnetic substances), the self and mutual inductances are constants, independent of the values of the currents, for any given arrangement, shape and size of the circuits. Under these conditions the total number of linkages (*see Electricity and Magnetism, Principles of*) between the flux lines linking any particular circuit, No. 1, and the turns forming that circuit may be expressed by the relation

$$\lambda_1 = L_1 i_1 + M_{12} i_2 + M_{13} i_3 + \text{etc.}, \tag{4}$$

where

$L_1 i_1 = \lambda_{11} =$ the linkages between the flux lines due to the current i_1 in circuit No. 1 and the turns of circuit No. 1,

$M_{12} i_2 = \lambda_{12} =$ the linkages between the flux lines due to the current i_2 in circuit No. 2 and the turns of circuit No. 1, etc.

Hence the common definition of inductance as "linkages per unit current." In certain simple cases the linkages per unit current may be calculated from the configuration of the circuit or circuits, starting from the fundamental relations given by equations (35), (36) and (29) in the article on *Electricity and Magnetism, Principles of.*

Internal Flux, Internal Inductance and Internal Reactance. — In the case of a wire of finite cross-section the various "filaments" of which the wire may be considered as made up are not all linked by the same number of flux lines, and consequently the back e.m.f.'s induced in the various filaments are different, tending to produce a non-uniform distribution of current. This variation in the induced e.m.f. from filament to filament is due only to that portion of the total flux which actually cuts the wire in question; it may therefore be called the "internal flux," and that portion of the inductance or reactance corresponding to this internal flux may be designated as the "internal inductance" and "internal reactance" respectively. Although the internal flux tends to produce a non-uniformity in the distribution of current, this effect is counteracted by the resistance of the wire and for ordinary non-magnetic wires at frequencies under 60 cycles per second the current remains practically uniformly distributed over the cross-section of the wire. See article on *Skin Effect.*

FORMULAS FOR SELF INDUCTANCE. — The following formulas are taken from a very comprehensive paper on the calculation of inductance

by Rosa and Grover in the *Bull. Bur. Stand.*, Vol. 8, No. 1, p. 1, 1912, in which the accuracy of the various formulas is thoroughly discussed and many others given, as well as tables to minimize the labor of calculation.

Unless otherwise stated *all formulas are in c.g.s. electromagnetic units;* the conversion factors are given in the article on *Units and Conversion Factors.*

Self Inductance of a Single Circular Turn formed by a Wire of Circular Cross-Section. — Let a = mean radius of the turn, in centimeters, r = radius of the wire, in centimeters; then the self inductance is

$$L = 4\pi a\left[\left(1+\frac{r^2}{8a^2}\right)\log_e\frac{8a}{r}+\frac{r^2}{24a^2}-1.75\right]. \qquad (5)$$

This formula is derived from an infinite series in $\frac{r}{a}$, hence for $\frac{r}{a}$ large it is approximate only; however, for $\frac{r}{a}$ less than 0.1 which covers all ordinary cases, the error is less than 1 part in 100,000.

Mutual Inductance of Two Co-axial Circles. — Dimensions as in Fig. 1, all in centimeters. Put

$$k=\sqrt{1-\left(\frac{m_2}{m_1}\right)^2}, \qquad k_1=\frac{m_1-m_2}{m_1+m_2}.$$

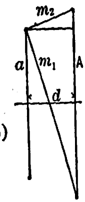

Fig. 1.

For $k < 0.2$ use the formula

$$M=\frac{\pi^2k^3}{4}\sqrt{Aa}\left[1+\frac{3}{4}k^2+\frac{75}{128}k^4+\frac{245}{512}k^6+\ldots\right], \qquad (6)$$

the general term in the brackets being

$$\left(\frac{3\cdot5\cdot7\ldots(2n+1)}{4\cdot6\cdot8\ldots(2n+2)}\right)^2\frac{(2n+2)}{(2n-1)}k^{2n}.$$

For $k > 0.2$ use the formula

$$M=2\pi^2k_1^{3/2}\sqrt{Aa}\left[1+\frac{3}{8}k_1^2+\frac{15}{64}k_1^4+\frac{175}{1024}k_1^6+\ldots\right], \qquad (6a)$$

the general term in the brackets being

$$\left(\frac{n+1}{2n+1}\right)\left[\frac{3\cdot5\cdot7\ldots(2n+1)}{4\cdot6\cdot8\ldots(2n+2)}\right]^2k^{2n}.$$

Self Inductance of a Long Solenoid* of Circular Cross-Section. — Let

l = axial length of solenoid in centimeters,
n_1 = number of turns per centimeter length, i.e., total number of turns = n_1l,
a = mean radius of the solenoid in centimeters

Then for $\frac{a}{l}$ small, the self inductance is, to a first approximation,

$$L=4\pi^2n_1^2a^2l. \qquad (7)$$

There is a considerable error in this formula, due to the end effect, but the variations in L due to changes in l are almost exactly proportional to the changes in

* By a solenoid is meant a coil in which the wire forms a uniform, straight, cylindrical helix.

l, and hence this formula may be used for calculating the corresponding variations in L as long as $\frac{a}{l}$ remains small.

Self Inductance of a Single-layer Short Solenoid of Circular Cross-Section. — For such a solenoid (see Fig. 2) the summation formula, equation (3), becomes when there are n turns,

$$L = nL_1 + 2(n-1)M_{12} + 2(n-2)M_{13} + 2(n-3)M_{14} + \ldots + 2M_{1n}, \tag{8}$$

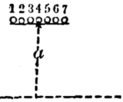

Fig. 2.

where L_1 is the self inductance of a single turn, M_{12} is the mutual inductance of the first and second turns or any two adjacent turns, M_{13} is the mutual inductance of the first and third or of any two turns separated by one, etc., and M_{1n} is the mutual inductance of the first and last turns. L_1 may be calculated from equation (5) and the M's from equation (6) or (6a). The general equation (3) may also be used to calculate the self inductance of a coil of any number of layers, but the calculation becomes tedious.

Self Inductance of Circular Coil of Rectangular Section (Fig. 3). — Dimensions as in Fig. 3, all in centimeters, also

N = total number of turns,
$R = 0.2235\,(b + c)$.

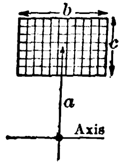

Fig. 3.

Then to a degree of approximation sufficient for most practical purposes

$$L = 4\pi a N^2 \left\{ \log_e \frac{8a}{R}\left(1 + \frac{3R^2}{16a^2}\right) - \left(2 + \frac{R^2}{16a^2}\right)\right\}. \tag{9}$$

Mutual Inductance of Two Concentric Co-axial Solenoids.* — All dimensions being in centimeters, let

l = one-half the length of the *shorter* coil,
x = one-half the length of the *longer* coil,
A = radius of *outer* coil (which may be either the shorter or longer coil),
a = radius of the *inner* coil,

$$d = x\sqrt{1 + \left(\frac{A}{x}\right)^2},$$

N_1 and N_2 = the *total* number of turns in the first and second coils respectively.

Then as a first approximation, for $\frac{A}{x}$ small, the mutual inductance between the two coils is

$$M = \frac{2\pi^2 a^2 N_1 N_2}{d}. \tag{10}$$

By calculating a sufficient number of terms the mutual inductance may be obtained exactly from the formula

$$M = \frac{2\pi^2 a^2 N_1 N_2}{d}\left[1 - \frac{A^2}{2d^4}\,\frac{4l^2 - 3a^2}{4} - \frac{A^2(4x^2 - 3A^2)}{8d^8}\cdot\frac{8l^4 - 20l^2a^2 + 5a^4}{8} - \frac{A^2(8x^4 - 20x^2A^2 + 5A^4)}{16d^{12}}\cdot\frac{(64l^6 - 336l^4a^2 + 280l^2a^4 - 35a^6)}{64} - \ldots\right]. \tag{10a}$$

* One solenoid inside the other with their centers coinciding.

Mutual Induction of Two Co-axial Circular Coils Each of Rectangular Section (Fig. 4). — Dimensions as in Fig. 4, all in centimeters. N_1 and N_2 represent the total number of turns on the two coils respectively. The following formula, known as the "formula of quadratures," is sufficiently exact for most practical cases:

$$M = \frac{N_1 N_2}{6}(M_1 + M_2 + M_3 + M_4 + M_5 + M_6 + M_7 + M_8 - 2M_0), \tag{11}$$

where M_0 is the mutual inductance of the two central turns o_1 and o_2 in Fig. 4. M_1 is the mutual inductance of the circle through o_2 and the circle through 1, M_5 is the mutual inductance of the circle through o_1 and the circle through 5, etc. These mutual inductances M_0, M_1 . . . M_8 may be calculated from equation (6) or (6a). When the sections of the two coils are small compared with their distance apart, $M_0 = M_1 = \ldots M_8$ and equation (11) becomes

$$M = N_1 N_2 M_0. \tag{11a}$$

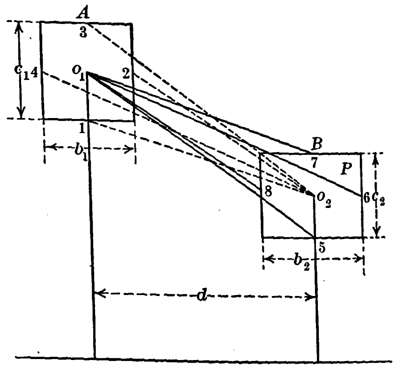

Fig. 4.

INDUCTANCE OF STRAIGHT CONDUCTORS. — The inductance of a circuit formed by straight conductors, such as the wires of a transmission line, may be considered from two points of view, viz., (1) each conductor may be looked upon as having a certain *self* inductance independent of the position of the other conductors forming the circuit or circuits and a *mutual* inductance with each of these conductors or (2) when the conductors are symmetrically arranged and the currents in them bear a fixed relation to one-another, as in a two-wire single-phase or symmetrical three-wire three-phase transmission line, each conductor may be looked upon as having a *total* inductance which takes into account both its self inductance and its mutual inductance with respect to the other wires. In the case of a 2-wire line the total inductance of the loop formed by the two conductors is twice the total inductance of *each* conductor; in the case of more than two wires the "loop inductance" has no simple physical meaning.

In the following paragraphs are given the formulas for *self*, *mutual* and *total* inductance *per conductor* for straight conductors of various sections and for various arrangements of such conductors. *The formulas are all in c.g.s. electromagnetic units* unless otherwise specified; see *Units and Conversion Factors.*

Self Inductance of a Single Straight Round Wire, Return Neglected. — Let

l = length of wire, in centimeters,
d = diameter of round wire, in centimeters,
$r = \frac{d}{2}$ = radius of round wire, in centimeters.

Then for a round wire the self inductance is

$$L' = 2\left[l \log_e \frac{l + \sqrt{l^2 + r^2}}{r} - \sqrt{l^2 + r^2} + \frac{l}{4} + r\right] \tag{12}$$

$$= 2l\left[\log_\epsilon \frac{2l}{r} - \frac{3}{4}\right] \text{ approximately.} \qquad (12a)$$

Where the permeability of the wire is μ, and that of the medium outside is unity and the frequency low (12a) appears in the form

$$L' = 2l\left[\log_\epsilon \frac{2l}{r} - 1 + \frac{\mu}{4}\right]. \qquad (12b)$$

This last formula is of theoretical interest only, as the value to assign to μ is doubtful and when such wires are used even for currents of moderate frequencies, the skin effect is appreciable; see article on *Skin Effect*.

External and Internal Self Inductance of a Round Wire. — The self inductance of a round wire may be considered as made up of two parts, viz., the inductance due to the flux *external* to the wire and that due to the flux *within* the wire. The first or "external" inductance is

$$L_e = 2l\left[\log_\epsilon \frac{2l}{r} - 1\right], \qquad (13)$$

and the "internal" self inductance is

$$L_i = l\frac{\mu}{2}. \qquad (13a)$$

Self Inductance of a Hollow Tube of Circular Section, Return Neglected. — The *external* inductance is the same as for a solid wire, i.e., equation (13), taking for r the external radius of the tube. The *internal* inductance of the tube, putting r_2 = external radius and r_1 = internal radius, is

$$L_i = 2\mu l\left[\frac{r_1^4}{(r_2^2 - r_1^2)^2}\log_\epsilon \frac{r_2}{r_1} - \frac{1}{4}\,\frac{3r_1^2 - r_2^2}{r_2^2 - r_1^2}\right]. \qquad (14)$$

The term in the square brackets is always less than ¼, i.e., the internal inductance of a hollow tube is always less than the internal inductance of a solid wire; see equation (13a). In the limit, where $r_1 = r_2$ (tube with infinitely thin walls), the *internal* inductance is zero.

Self Inductance of a Straight Bar or Strip, Return Neglected. — For a straight bar of a non-magnetic substance and of rectangular cross-section the self inductance, neglecting the return circuit, is

$$L' = 2l\left[\log_\epsilon \frac{2l}{a+b} + \frac{1}{2} + \frac{0.2235\,(a+b)}{l}\right], \qquad (15)$$

where a and b are the lengths of the two edges, in centimeters.

Mutual Inductance of Two Parallel Straight Wires or Bars. — See Fig. 5. Same notation as above and in addition let

D = distance between centers of the two wires, in centimeters.

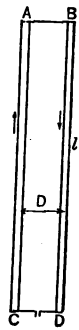

Fig. 5.

Then the mutual inductance between the two is

$$M = 2\left[l\log_\epsilon \frac{l + \sqrt{l^2 + D^2}}{D} - \sqrt{l^2 + D^2} + D\right] \qquad (16)$$

$$= 2l\left[\log_\epsilon \frac{2l}{D} - 1 + \frac{D}{l}\right] \text{approximately,} \qquad (16a)$$

when the length l is great in comparison with D.

Equation (16), which is an exact expression when the wires have no appreciable cross-section, is not an exact expression for the mutual inductance of two parallel cylindrical wires, but is not appreciably in error even when the section is large and D is small if l is great compared with D.

Equation (16) is also applicable, with a practically negligible error to bars of rectangular section and in fact to the mutual inductance between any two parallel conductors of any section and external to each other, e.g., between an overhead wire and a rail, the distance D being the distance* between the center of gravity of the two sections.

Mutual Inductance Between a Tube and an Interior Wire. — Using the same notation as for equation (14) above the mutual inductance in this case is

$$M = 2l\left[\log_\epsilon 2l - \frac{r_2^2 \log_\epsilon r_2 - r_1^2 \log_\epsilon r_1}{r_2^2 - r_1^2} - \frac{1}{2}\right]. \qquad (16b)$$

Total Inductance of a Two-wire Transmission Line. — Let the two wires (Fig. 5) be designated as No. 1 and No. 2 and the currents as i_1 and i_2. Since $i_2 = -i_1$, from equation (2) the total inductive drop in each wire is

$$v_1 = (L_1 - M_{12})\frac{di_1}{dt}.$$

The total inductance of each wire is then $L = L_1 - M_{12}$, where L_1 is given by equation (12) or (14) and M_{12} by equation (16). For a length so great that d and D are negligible compared with l, this total inductance per wire for equal round wires becomes

$$L = 2l\left[\log_\epsilon \frac{2D}{d} + \frac{\mu}{4}\right],$$

where d is the diameter of each wire. The total inductance of the line per unit length of *wire*† is

$$\left.\begin{aligned} L &= \frac{\mu}{2} + 2\log_\epsilon \frac{2D}{d} && \text{abhenries per centimeter} \\ &= 0.01524\mu + 0.14037 \log_{10}\frac{2D}{d} && \text{millihenries per 1000 feet} \\ &= 0.08047\mu + 0.74113 \log_{10}\frac{2D}{d} && \text{millihenries per mile,} \end{aligned}\right\} \qquad (17)$$

where D and d may be expressed in any units of length provided they are both expressed in the *same* units. The formulas given in equation (17) also apply approximately to stranded wires, provided d is taken as the diameter of the solid wire having a cross-section equal to that of the copper in the stranded wire, i.e., the inductance of a No. 0000 stranded wire on a given spacing is approximately the same as that of a No. 0000 solid wire on the same spacing.

Tables of L and the corresponding reactances for 25 and 60 cycles for various sizes of wires and various spacings are given below.

Total Inductance of a Rectangle. — Let L_a = self inductance of long side, L_b = self inductance of short side (calculated from the proper formulas, 12 to 15) and let M_a = mutual inductance between the two long sides and M_b = the mutual inductance between the two short sides (calculated from formula 16); then the total inductance of the rectangle is

$$L = 2(L_a - M_a) + 2(L_b - M_b)$$

* Accurately, the geometrical mean distance between the two areas; see *Bull. Bur. Stand.*, 1912, Vol. 8, pp. 125 and 166. For round wires, solid or tubular, the geometrical mean distance between them is exactly the distance between their centers.

† To obtain the total inductance of both wires multiply by *twice* the length of the line.

SELF INDUCTANCE OF SOLID NON-MAGNETIC WIRES*

Millihenries per 1000 FEET of each wire of a single-phase or of a symmetrical three-phase line

Size of wire, cir. mils. or A.W.G.	Diam. of wire, inches	Inches between wires, center to center							
		1	3	6	9	12	18	24	30
1,000,000	1.0000	0.05750	0.1245	0.1667	0.1915	0.2090	0.2337	0.2512	0.2648
750,000	0.8660	0.06627	0.1332	0.1755	0.2002	0.2178	0.2425	0.2600	0.2736
500,000	0.7071	0.07863	0.1456	0.1879	0.2126	0.2301	0.2548	0.2724	0.2860
350,000	0.5916	0.08950	0.1565	0.1987	0.2235	0.2410	0.2657	0.2832	0.2968
250,000	0.5000	0.09976	0.1667	0.2090	0.2337	0.2512	0.2760	0.2935	0.3071
0000	0.4600	0.1048	0.1718	0.2141	0.2388	0.2563	0.2810	0.2986	0.3122
000	0.4096	0.1119	0.1789	0.2211	0.2459	0.2634	0.2881	0.3057	0.3193
00	0.3648	0.1190	0.1860	0.2282	0.2529	0.2705	0.2952	0.3127	0.3263
0	0.3249	0.1260	0.1930	0.2353	0.2600	0.2775	0.3022	0.3198	0.3334
1	0.2893	0.1331	0.2001	0.2423	0.2671	0.2846	0.3093	0.3269	0.3405
2	0.2576	0.1402	0.2072	0.2494	0.2741	0.2917	0.3164	0.3339	0.3475
4	0.2043	0.1543	0.2213	0.2635	0.2883	0.3058	0.3305	0.3481	0.3617
6	0.1620	0.1685	0.2354	0.2777	0.3024	0.3199	0.3447	0.3622	0.3758
8	0.1285	0.1826	0.2496	0.2918	0.3165	0.3341	0.3588	0.3763	0.3899
10	0.1019	0.1967	0.2637	0.3060	0.3307	0.3482	0.3729	0.3905	0.4041
12	0.08081	0.2109	0.2778	0.3201	0.3448	0.3623	0.3871	0.4046	0.4182
14	0.06408	0.2250	0.2920	0.3342	0.3590	0.3765	0.4012	0.4187	0.4323
16	0.05082	0.2391	0.3061	0.3484	0.3731	0.3906	0.4153	0.4329	0.4465

Size of wire, cir. mils. or A.W.G.	Feet between wires, center to center								
	3	4	5	6	8	10	15	20	25
1,000,000	0.2760	0.2935	0.3071	0.3182	0.3358	0.3494	0.3741	0.3916	0.4052
750,000	0.2847	0.3023	0.3159	0.3270	0.3445	0.3581	0.3828	0.4004	0.4140
500,000	0.2971	0.3146	0.3282	0.3393	0.3569	0.3705	0.3952	0.4127	0.4263
350,000	0.3080	0.3255	0.3391	0.3502	0.3678	0.3814	0.4061	0.4236	0.4372
250,000	0.3182	0.3358	0.3494	0.3605	0.3780	0.3916	0.4163	0.4339	0.4475
0000	0.3233	0.3408	0.3544	0.3656	0.3831	0.3967	0.4214	0.4390	0.4526
000	0.3304	0.3479	0.3615	0.3726	0.3902	0.4038	0.4285	0.4460	0.4596
00	0.3374	0.3550	0.3686	0.3797	0.3972	0.4108	0.4356	0.4531	0.4667
0	0.3445	0.3620	0.3756	0.3867	0.4043	0.4179	0.4426	0.4601	0.4737
1	0.3516	0.3691	0.3827	0.3938	0.4114	0.4250	0.4497	0.4672	0.4808
2	0.3586	0.3762	0.3898	0.4009	0.4184	0.4320	0.4568	0.4743	0.4879
4	0.3728	0.3903	0.4039	0.4150	0.4326	0.4462	0.4709	0.4884	0.5020
6	0.3869	0.4045	0.4181	0.4292	0.4467	0.4603	0.4850	0.5026	0.5162
8	0.4011	0.4186	0.4322	0.4433	0.4608	0.4744	0.4992	0.5167	0.5303
10	0.4152	0.4327	0.4463	0.4574	0.4750	0.4886	0.5133	0.5308	0.5444
12	0.4293	0.4469	0.4605	0.4716	0.4891	0.5027	0.5274	0.5450	0.5586
14	0.4435	0.4610	0.4746	0.4857	0.5033	0.5169	0.5416	0.5591	0.5727
16	0.4576	0.4751	0.4887	0.4998	0.5174	0.5310	0.5557	0.5732	0.5868

* The inductances given in this table also apply, with a practically negligible error, to ordinary stranded wires of the *same cross-section*.

SELF INDUCTANCE OF SOLID NON-MAGNETIC WIRES*

Millihenries per MILE of each wire of a single-phase or of a symmetrical three-phase line

Size of wire, cir. mils. or A.W.G.	Diam. of wire, inches	Inches between wires, center to center							
		1	3	6	9	12	18	24	30
1,000,000	1.0000	0.3036	0.6572	0.8803	1.011	1.103	1.234	1.327	1.398
750,000	0.8660	0.3499	0.7035	0.9266	1.057	1.150	1.280	1.373	1.445
500,000	0.7071	0.4152	0.7688	0.9919	1.122	1.215	1.346	1.438	1.510
350,000	0.5916	0.4726	0.8262	1.049	1.180	1.272	1.403	1.496	1.567
250,000	0.5000	0.5267	0.8803	1.103	1.234	1.327	1.457	1.550	1.622
0000	0.4600	0.5536	0.9072	1.130	1.261	1.353	1.484	1.577	1.648
000	0.4096	0.5909	0.9445	1.168	1.298	1.391	1.521	1.614	1.686
00	0.3648	0.6282	0.9818	1.205	1.335	1.428	1.559	1.651	1.723
0	0.3249	0.6654	1.019	1.242	1.373	1.465	1.596	1.688	1.760
1	0.2893	0.7029	1.057	1.280	1.410	1.503	1.633	1.726	1.798
2	0.2576	0.7402	1.094	1.317	1.447	1.540	1.671	1.763	1.835
4	0.2043	0.8148	1.168	1.392	1.522	1.615	1.745	1.838	1.910
6	0.1620	0.8894	1.243	1.466	1.597	1.689	1.820	1.912	1.984
8	0.1285	0.9641	1.318	1.541	1.671	1.764	1.894	1.987	2.059
10	0.1019	1.039	1.392	1.615	1.746	1.839	1.969	2.062	2.134
12	0.08081	1.113	1.467	1.690	1.821	1.913	2.044	2.136	2.208
14	0.06408	1.188	1.542	1.765	1.895	1.988	2.118	2.211	2.283
16	0.05082	1.263	1.616	1.839	1.970	2.062	2.193	2.286	2.357

Size of wire, cir. mils. or A.W.G.	Feet between wires, center to center								
	3	4	5	6	8	10	15	20	25
1,000,000	1.457	1.550	1.622	1.680	1.773	1.845	1.975	2.068	2.140
750,000	1.503	1.596	1.668	1.726	1.819	1.891	2.021	2.114	2.186
500,000	1.569	1.661	1.733	1.792	1.884	1.956	2.087	2.179	2.251
350,000	1.626	1.719	1.791	1.849	1.942	2.014	2.144	2.237	2.309
250,000	1.680	1.773	1.845	1.903	1.996	2.068	2.198	2.291	2.363
0000	1.707	1.800	1.872	1.930	2.023	2.095	2.225	2.318	2.390
000	1.744	1.837	1.909	1.967	2.060	2.132	2.262	2.355	2.427
00	1.782	1.874	1.946	2.005	2.097	2.169	2.300	2.392	2.464
0	1.819	1.911	1.983	2.042	2.135	2.206	2.337	2.430	2.501
1	1.856	1.949	2.021	2.079	2.172	2.244	2.374	2.467	2.539
2	1.894	1.986	2.058	2.117	2.209	2.281	2.412	2.504	2.576
4	1.968	2.061	2.133	2.191	2.284	2.356	2.486	2.579	2.651
6	2.043	2.135	2.207	2.266	2.359	2.430	2.561	2.654	2.725
8	2.118	2.210	2.282	2.341	2.433	2.505	2.636	2.728	2.800
10	2.192	2.285	2.357	2.415	2.508	2.580	2.710	2.803	2.875
12	2.267	2.359	2.431	2.490	2.582	2.654	2.785	2.877	2.949
14	2.341	2.434	2.506	2.565	2.657	2.729	2.860	2.952	3.024
16	2.416	2.509	2.581	2.639	2.732	2.804	2.934	3.027	3.099

* The inductances given in this table also apply, with a practically negligible error, to ordinary stranded wires of the *same cross-section.*

25-CYCLE REACTANCE OF SOLID NON-MAGNETIC WIRES*

Ohms per MILE of each wire of a single-phase or of a symmetrical three-phase line

Size of wire, cir. mils. or A.W.G.	Diam. of wire, inches	Inches between wires, center to center							
		1	3	6	9	12	18	24	30
1,000,000	1.0000	0.04770	0.1032	0.1383	0.1588	0.1733	0.1939	0.2085	0.2196
750,000	0.8660	0.05497	0.1105	0.1456	0.1661	0.1807	0.2011	0.2157	0.2270
500,000	0.7071	0.06523	0.1208	0.1558	0.1763	0.1909	0.2115	0.2259	0.2372
350,000	0.5916	0.07425	0.1298	0.1648	0.1854	0.1998	0.2204	0.2350	0.2462
250,000	0.5000	0.08274	0.1383	0.1733	0.1939	0.2085	0.2289	0.2435	0.2548
0000	0.4600	0.08697	0.1425	0.1775	0.1981	0.2126	0.2331	0.2477	0.2589
000	0.4096	0.09283	0.1484	0.1835	0.2039	0.2185	0.2389	0.2536	0.2649
00	0.3648	0.09869	0.1542	0.1893	0.2097	0.2243	0.2449	0.2594	0.2707
0	0.3249	0.1045	0.1601	0.1951	0.2157	0.2302	0.2507	0.2652	0.2765
1	0.2893	0.1101	0.1661	0.2011	0.2215	0.2361	0.2565	0.2712	0.2825
2	0.2576	0.1163	0.1719	0.2069	0.2273	0.2419	0.2625	0.2770	0.2883
4	0.2043	0.1280	0.1835	0.2187	0.2391	0.2537	0.2741	0.2887	0.3001
6	0.1620	0.1397	0.1953	0.2303	0.2509	0.2653	0.2859	0.3004	0.3117
8	0.1285	0.1515	0.2071	0.2421	0.2625	0.2771	0.2975	0.3122	0.3235
10	0.1019	0.1632	0.2187	0.2537	0.2743	0.2889	0.3093	0.3239	0.3353
12	0.08081	0.1749	0.2305	0.2655	0.2861	0.3005	0.3211	0.3356	0.3469
14	0.06408	0.1866	0.2422	0.2773	0.2977	0.3123	0.3327	0.3473	0.3587
16	0.05082	0.1984	0.2539	0.2889	0.3095	0.3239	0.3445	0.3591	0.3703

Size of wire, cir. mils. or A.W.G.	Feet between wires, center to center								
	3	4	5	6	8	10	15	20	25
1,000,000	0.2289	0.2435	0.2548	0.2639	0.2785	0.2898	0.3103	0.3249	0.3362
750,000	0.2361	0.2507	0.2620	0.2712	0.2858	0.2971	0.3175	0.3321	0.3434
500 000	0.2465	0.2609	0.2723	0.2815	0.2960	0.3073	0.3279	0.3423	0.3536
350,000	0.2554	0.2701	0.2814	0.2905	0.3051	0.3164	0.3368	0.3514	0.3627
250,000	0.2639	0.2785	0.2898	0.2990	0.3136	0.3249	0.3453	0.3599	0.3712
0000	0.2682	0.2828	0.2941	0.3032	0.3178	0.3291	0.3495	0.3642	0.3755
000	0.2740	0.2886	0.2999	0.3090	0.3236	0.3349	0.3554	0.3700	0.3813
00	0.2800	0.2944	0.3057	0.3150	0.3294	0.3407	0.3613	0.3758	0.3871
0	0.2858	0.3002	0.3115	0.3208	0.3354	0.3466	0.3671	0.3818	0.3929
1	0.2916	0.3062	0.3175	0.3266	0.3412	0.3525	0.3730	0.3876	0.3989
2	0.2975	0.3120	0.3233	0.3326	0.3470	0.3583	0.3789	0.3934	0.4047
4	0.3092	0.3238	0.3351	0.3442	0.3588	0.3701	0.3906	0.4052	0.4165
6	0.3210	0.3354	0.3467	0.3560	0.3706	0.3818	0.4023	0.4169	0.4281
8	0.3327	0.3472	0.3585	0.3678	0.3822	0.3935	0.4141	0.4286	0.4399
10	0.3444	0.3590	0.3703	0.3794	0.3940	0.4053	0.4257	0.4404	0.4517
12	0.3561	0.3706	0.3819	0.3912	0.4056	0.4169	0.4375	0.4520	0.4633
14	0.3678	0.3824	0.3937	0.4030	0.4174	0.4287	0.4493	0.4638	0.4751
16	0.3796	0.3942	0.4055	0.4146	0.4292	0.4405	0.4609	0.4755	0.4869

* The reactances given in this table also apply, with a practically negligible error, to ordinary stranded wires of the *same cross-section.*

60-CYCLE REACTANCE OF SOLID NON-MAGNETIC WIRES*

Ohms per MILE of each wire of a single-phase or of a symmetrical three-phase line

Size of wire, cir. mils. or A.W.G.	Diam. of wire, inches	Inches between wires, center to center							
		1	3	6	9	12	18	24	30
1,000,000	1.0000	0.1145	0.2478	0.3319	0.3811	0.4158	0.4652	0.5003	0.5270
750,000	0.8660	0.1319	0.2652	0.3493	0.3985	0.4336	0.4826	0.5176	0.5448
500,000	0.7071	0.1565	0.2898	0.3739	0.4230	0.4581	0.5074	0.5421	0.5693
350,000	0.5916	0.1782	0.3115	0.3955	0.4449	0.4795	0.5289	0.5640	0.5908
250,000	0.5000	0.1986	0.3319	0.4158	0.4652	0.5003	0.5493	0.5844	0.6115
0000	0.4600	0.2087	0.3420	0.4260	0.4754	0.5101	0.5595	0.5945	0.6213
000	0.4096	0.2228	0.3561	0.4403	0.4893	0.5244	0.5734	0.6085	0.6356
00	0.3648	0.2368	0.3701	0.4543	0.5033	0.5384	0.5877	0.6224	0.6496
0	0.3249	0.2509	0.3842	0.4682	0.5176	0.5523	0.6017	0.6364	0.6635
1	0.2893	0.2650	0.3985	0.4826	0.5316	0.5666	0.6156	0.6507	0.6778
2	0.2576	0.2791	0.4124	0.4965	0.5455	0.5806	0.6300	0.6647	0.6918
4	0.2043	0.3072	0.4403	0.5248	0.5738	0.6089	0.6579	0.6929	0.7201
6	0.1620	0.3353	0.4686	0.5527	0.6021	0.6368	0.6861	0.7208	0.7480
8	0.1285	0.3635	0.4969	0.5810	0.6300	0.6650	0.7140	0.7491	0.7762
10	0.1019	0.3917	0.5248	0.6089	0.6582	0.6933	0.7423	0.7774	0.8045
12	0.08081	0.4196	0.5531	0.6371	0.6865	0.7212	0.7706	0.8053	0.8324
14	0.06408	0.4479	0.5813	0.6654	0.7144	0.7495	0.7985	0.8335	0.8607
16	0.05082	0.4762	0.6092	0.6933	0.7427	0.7774	0.8268	0.8618	0.8886

Size of wire, cir. mils. or A.W.G.	Feet between wires, center to center								
	3	4	5	6	8	10	15	20	25
1,000,000	0.5493	0.5844	0.6115	0.6334	0.6684	0.6956	0.7446	0.7796	0.8068
750,000	0.5666	0.6017	0.6288	0.6507	0.6858	0.7129	0.7619	0.7970	0.8241
500,000	0.5915	0.6262	0.6533	0.6756	0.7103	0.7374	0.7868	0.8215	0.8486
350,000	0.6130	0.6481	0.6752	0.6971	0.7321	0.7593	0.8083	0.8433	0.8705
250,000	0.6334	0.6684	0.6956	0.7174	0.7525	0.7796	0.8286	0.8637	0.8909
0000	0.6435	0.6786	0.7057	0.7276	0.7627	0.7898	0.8388	0.8739	0.9010
000	0.6575	0.6925	0.7196	0.7416	0.7766	0.8038	0.8528	0.8878	0.9150
00	0.6718	0.7065	0.7336	0.7559	0.7906	0.8177	0.8671	0.9018	0.9289
0	0.6858	0.7204	0.7476	0.7698	0.8049	0.8317	0.8810	0.9161	0.9429
1	0.6997	0.7348	0.7619	0.7838	0.8188	0.8460	0.8950	0.9301	0.9572
2	0.7140	0.7487	0.7759	0.7981	0.8328	0.8599	0.9093	0.9440	0.9712
4	0.7419	0.7770	0.8041	0.8260	0.8611	0.8882	0.9372	0.9723	0.9994
6	0.7702	0.8049	0.8320	0.8543	0.8893	0.9161	0.9655	1.001	1.027
8	0.7985	0.8332	0.8603	0.8826	0.9172	0.9444	0.9938	1.028	1.056
10	0.8264	0.8614	0.8886	0.9105	0.9455	0.9727	1.022	1.057	1.084
12	0.8547	0.8893	0.9165	0.9387	0.9734	1.001	1.050	1.085	1.112
14	0.8826	0.9176	0.9448	0.9670	1.002	1.029	1.078	1.113	1.140
16	0.9108	0.9459	0.9730	0.9949	1.030	1.057	1.106	1.141	1.168

* The reactances given in this table also apply, with a practically negligible error, to ordinary stranded wires of the *same cross-section.*

Total Self Inductance of a Concentric Main. — In a similar manner the total inductance of a concentric main may be found from equations (3), (13), (14) and (16b), which give for the total inductance per unit length of *cable*

$$L = 2\left[\log_\epsilon \frac{d_2}{d_1} + \frac{d_3^4}{(d_3^2 - d_2^2)^2}\log_\epsilon \frac{d_3}{d_2} - \frac{d_3^2}{2\,(d_3^2 - d_2^2)}\right] \text{ abhenries per centimter,}$$

where d_1 = diameter of internal conductor, assumed solid, d_2 = internal diameter of outer conductor and d_3 = external diameter of outer conductor.

Inductive Drop in a Three-wire Transmission Line. — Let the wires be designated as Nos. 1, 2, and 3; and the three currents in the direction away from the generator as i_1, i_2 and i_3. From equation (1) the inductive drop in each wire* is

$$\left.\begin{aligned} v_1 &= L_1\frac{di_1}{dt} + M_{12}\frac{di_2}{dt} + M_{13}\frac{di_3}{dt},\\ v_2 &= M_{12}\frac{di_1}{dt} + L_2\frac{di_2}{dt} + M_{23}\frac{di_3}{dt},\\ v_3 &= M_{13}\frac{di_1}{dt} + M_{23}\frac{di_2}{dt} + L_3\frac{di_3}{dt}.\end{aligned}\right\} \qquad (18)$$

The values of the L's and M's in terms of the radii and distances apart of the three wires are given in equations (12) or (14) and (16).

Equilateral Triangle Arrangement. — For ordinary three-phase work when there is no neutral current, the sum of the three line currents at any instant is equal to zero, whence

$$(i_2 + i_3) = -i_1,\ (i_1 + i_3) = -i_2, \text{ and } (i_1 + i_2) = -i_3.$$

When the three wires are all of the same size and are arranged so that they form the three edges of an equilateral prism, a common arrangement, $L_1 = L_2 = L_3$ and $M_{12} = M_{13} = M_{23}$, and the inductive drops per unit length in the three wires are respectively

$$v_1 = L\frac{di_1}{dt},\quad v_2 = L\frac{di_2}{dt},\quad v_3 = L\frac{di_3}{dt}, \qquad (19)$$

where L has here the same value as in equation (17), numerical values of which are given in the accompanying tables.

For a sine-wave current of effective value I and frequency f, the inductive drop in each wire per unit length of line has the effective value

$$V = 2\pi fLI \qquad (19a)$$

and leads the current in this particular wire by 90 degrees, irrespective of whether the load be balanced or not, the only condition being that no current returns to the generator through any other conductor than the three line wires.

Three Parallel Wires in the Same Plane. — Let the three wires of the three-phase system have equal diameters d, and let No. 2 be the middle wire, and let Nos. 1 and 3 be at equal distances (D between centers) from No. 2 and on opposite sides of No. 2. Under these conditions, considering unit length of line, put

L = same values as in (17), taking D as the distance between either outer and the middle wire,

and $M = 2\log_\epsilon 2 = 1.3863$ abhenries per centimeter
$= 0.04225$ millihenries per 1000 feet
$= 0.2231$ millihenries per mile.

*Note that in vector notation $\frac{di}{dt} = j\,2\pi fI$ where f is the frequency, I the current and $j = \sqrt{-1}$.

Then equations (18) for the inductive drops in the three wires become, provided only that $i_1 + i_2 + i_3 = 0$,

$$\left.\begin{aligned} v_1 &= L\frac{di_1}{dt} - M\frac{di_3}{dt}, \\ v_2 &= L\frac{di_2}{dt}, \\ v_3 &= L\frac{di_3}{dt} - M\frac{di_1}{dt}. \end{aligned}\right\} \qquad (20)$$

Such an arrangement, therefore, causes an unbalancing of the system, but when the wires are far apart, so that L is large compared with M, the effect is slight. It can be avoided by transposing the wires at intervals.

Inductance of Overhead Wires With Earth Return. — This case is not susceptible of a definite solution, since the inductance depends upon the distribution of the return current in the earth. When the current returns through one or more rails immediately below the wire, the leakage current to the earth may be neglected and the wire and rails* treated as linear conductors, applying the formulas given above. When there is no metallic return circuit, an approximate solution may be obtained by considering the earth as equivalent to the "images" of the overhead wires in the plane of the earth's surface, i.e., considering the return circuit as consisting of the same number of wires as there are overhead, these fictitious return wires being the same distance below the earth as the actual wires are above it. The value of the inductances as thus calculated can never be greater than the actual inductances but will usually be slightly less than the actual values.

EFFECT OF FREQUENCY ON INDUCTANCE. — See article on *Skin Effect.*

BIBLIOGRAPHY. — Rosa and Grover, *Formulas and Tables for the Calculation of Mutual and Self-Inductance,* Bull. Bur. Stand. 1912, Vol. 8, p. 1; Russell, A., *Alternating Currents,* London, 1904.

[H. Pender.]

*The *external* self-inductance of a rail is practically the same as that of a round wire, taking for r the perimeter of the rail divided by 2π; see equation (13). The *internal* self-inductance depends upon the permeability μ and the frequency of the current. The internal inductance, however, is small compared with the external inductance and for approximate calculations may be neglected. See *Rails, Track and Third; Trolley Systems, Overhead.*

INDUCTION COILS. — (*See also Electricity and Magnetism, Principles of; Electromagnet Windings.*) An induction coil is a device which transforms a low direct e.m.f. to a high alternating e.m.f. of unsymmetrical form. The single-winding coil, called a primary coil, is used extensively in gas-engine ignition and in automatic gas lighting, and the double-winding coil, called a secondary coil, is used for the excitation of X-ray tubes, gas-engine ignition, automatic gas lighting, electrotherapeutics and wireless telegraphy.

PRIMARY INDUCTION COIL. — The primary induction coil consists of a single coil wound upon an iron core made up of a compact bundle of soft-iron wires. To obtain a spark from such a coil it is connected in series with a battery and some kind of "make-and-break" contact. When the circuit is closed, the current increases gradually according to the expression,

$$i_t = \frac{E}{R}\left(1 - \epsilon^{-\frac{Rt}{L}}\right),$$

where i_t equals the current at any time t seconds after the circuit is closed, E equals the e.m.f. of the battery, R equals the resistance in ohms and L equals the inductance in henries of the entire circuit, and ϵ the base of the natural system of logarithms. The shape of the current curve at "make" is shown in the curve AB (Fig. 1),* which is an oscillograph record taken when an e.m.f. of 4 volts is impressed upon a primary coil of 1 ohm resistance and 0.01 henry inductance.

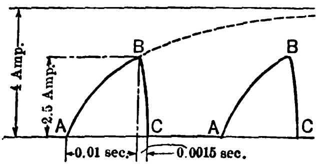

Fig. 1.

If the circuit is opened when the current reaches some value, such as B (Fig. 1), the current decreases rapidly to zero as shown in the curve BC. Since the e.m.f. induced in such a coil at any instant equals $L\frac{di}{dt}$ when the current falls from B to C, the e.m.f. induced in the coil will be many times that of the battery e.m.f. and a spark will be established between the open contacts at "break." For use in connection with gas engines the "make and break" contacts are located within the cylinder. Reliable ignition is obtained when about 0.02 joule is dissipated in the spark at "break." Since the energy dissipated as heat in the spark is converted from the electromagnetic energy stored up in the coil, it follows that the coil must have an inductance and carry a current at "break" such that the energy stored up, i.e., $\frac{1}{2}LI^2$, will exceed the required value of 0.02 joule. In practice it is customary to design the coil and time of contact so that $\frac{1}{2}LI^2$ equals about 0.04 joule, such coils having an efficiency of about 50 per cent. One-half of the energy put into the coil is then used up in heating the conductors and metallic parts by I^2R and hysteresis losses.

SECONDARY INDUCTION COIL. — The secondary coil has two separate windings wound about an iron core, one of few turns called the primary winding and the other of many turns called the secondary winding. The primary winding is connected in series with a battery and an interrupter (described below). The spark is produced between the terminals of the secondary winding. In most cases a condenser is shunted across the interrupter to prevent sparking at

*This and the following oscillograph records are due to Bailey, B. F., *Elec. W. 1910, Vol. LV., p. 943.*

that point. Since the e.m.f. induced in each turn linked by the magnetic flux is the same, the e.m.f. induced in the secondary winding, neglecting leakage, will equal $\frac{N_s}{N_p}$ times that induced in the primary winding, where N_s and N_p equal the respective number of turns on the secondary and primary windings. Secondary coils which give sparks as long as 5 feet have been constructed in this manner.

Effect of Shunting Interrupter with Capacity. — When the interrupter is shunted by a capacity the e.m.f. induced in the secondary winding becomes oscillatory. This effect of the capacity is well shown in the oscillograph records given in Figs. 2 and 3. In each case a high *non-inductive* resistance is connected

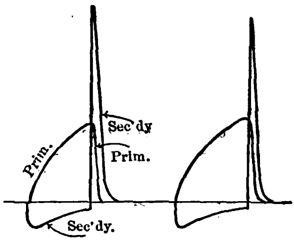

Fig. 2.

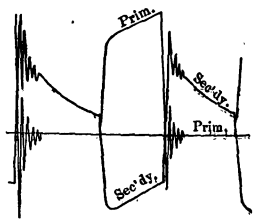

Fig. 3.

across the secondary terminals, and the curves indicate the variation of the primary current and the secondary e.m.f. In Fig. 2 the interrupter is unshunted by a condenser, and as a result the secondary e.m.f. increases at "break" and decreases to zero without oscillation. In Fig. 3 the interrupter is shunted by a condenser and the secondary e.m.f. becomes oscillatory. The form of the e.m.f. curve in any actual case is not shown accurately by Fig. 3 except at the time of "break," since the resistance between the terminals of a spark gap decreases rapidly after the spark is established. In Fig. 4 a low resistance is connected across the secondary terminals and shows the general form of the e.m.f. curve after the spark is established.

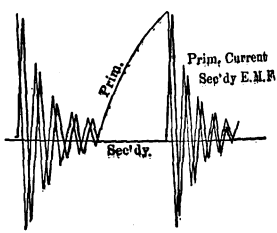

Fig. 4.

Although not shown by the curves owing to difference in scale, the capacity shunted around the interrupter also increases the secondary e.m.f. above the value obtained without the capacity. Assuming that no spark is formed at the interrupter, it may be shown that the e.m.f. of the secondary equals $\frac{N_s}{N_p}\sqrt{\frac{L}{C}}I_b$, where L equals the inductance of the primary in henries, C equals the capacity of the condenser in farads and I_b equals the primary current at "break" in amperes. If C is made so low that a spark appears at the interrupter, the above formula no longer holds and the secondary e.m.f. will be greatly reduced. C should then be made as small as possible but of sufficient size to suppress the spark at the interrupter.

Insulation. — The conductors in the secondary winding must be heavily insulated to withstand the very high e.m.f. induced in that winding. In most

cases a double-covered silk insulation impregnated with some insulating compound is used. In large induction coils the secondary winding is built up of several flat coils insulated from each other by ebonite or fiber disks. The coils are so wound that the electrical connections are made alternately at the top and at the bottom of the respective coils.

Dimensions of Three-inch Coil. — The general dimensions of parts of a 3-inch induction coil, as reported by S. R. Bottone, *Radiography (London, 1898)*, are given following: Iron, bundle of No. 20 (B. & S. G.) annealed iron wire, 1¼ in. diameter, 13 in. long. Primary winding, four layers of No. 12 double silk-covered copper wire, about 4¼ lb. Ebonite tube over primary, 12 in. long, 2 in. inside diameter, 2½ in. outside. Two ebonite heads, 5 in. square, ½ in. thick. Seven vulcanized fiber circlets (for sections), 4½ in. diameter, ⅛ in. thick, 2½ in. central aperture. Secondary winding, 4 lb. No. 36 double-silk-covered wire. Platinum-tip contact breaker, height from base to center of iron hammer, 2½ in., size of iron head of hammer ¾ in. diameter, ¾ in. long. Base (fitted with false bottom to contain condenser), 18 in. long by 9 in. wide by 2¾ in. deep. Condenser, 144 sheets of tinfoil, size 6 by 12 in., interleaved with 144 sheets of paraffined paper 8 by 13 in.

Dimensions of Six-inch Coil.—Similar dimensions for a 6-inch induction coil, as reported by Bottone, are given following: Iron, bundle of No. 16 (B. & S. G.) annealed iron wire, 1½ in. diameter, 15 in. long. Primary winding, four layer of No. 12 double silk-covered copper wire, about 5 lb. Ebonite tube over primary 14 in. long, 2¼ in. inside diameter, 2¾ in. outside. Ebonite heads 6 in. square, ¾ in. thick. Seven vulcanized fiber circlets 5¼ in. diameter, ⅛ in. thick, with 2¾ in. central hole. Secondary winding, 7 lb. No. 38 double-silk-covered copper wire. Platinum-tip contact breaker, height from base to center of hammer, 3 in. size of hammer head, 1 in. diameter, 1 in. long. Base (fitted with false bottom to contain condenser), 20 in. long by 10 in. wide by 3½ in. deep. Condenser, 144 sheets of tinfoil, size 6 by 12 in., interleaved with 144 sheets of paraffined paper, 8 by 13 in.

Interrupters. — An interrupter should be so designed that it will close the circuit for a definite time by easy adjustment and will open the circuit at the end of that time as quickly as possible. Two distinct types, the mechanical and the electrolytic interrupter, are in common use. Mechanical interrupters may be divided into the following forms: hammer, atonic, commutator and mercury interrupters. Of the electrolytic interrupters the Wehnelt type is the most popular.

Hammer Interrupter. — In the hammer interrupter, shown in Fig. 5, the circuit is opened at *A* when the core *B* attracts the iron mass *C* mounted at the free end of the spring blade *D*. Since the core attracts the iron mass only when the circuit is closed and loses its attraction when the circuit is opened, the spring blade is set in vibration and opens and closes the circuit in rapid succession. This type is used extensively in connection with small coils. The contacts at *A* are usually tipped with platinum to withstand the intense heat developed by sparking. The rapidity of the "break" is not great enough for large coils, and when a large current must be broken, trouble is experienced in keeping the contact points in good condition.

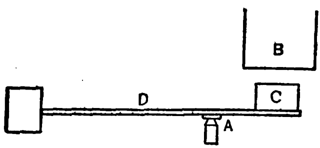

Fig. 5. Hammer Interrupter

Atonic Interrupter. — In the atonic interrupter, shown in Fig. 6, the free end of the iron strip *P* is attracted as in the hammer interrupter and is

returned to its original position by the spring *R*. The circuit is opened at the contacts *ab* when the free end of *P* strikes the blade *L*. Interrupters constructed in this manner open the circuit quicker than do the hammer type since the attracted member is moving faster at the instant of "break." The period of the atonic interrupter may be varied within wide limits by regulating the tension on the spring *R* by means of the thumb-screw *M*.

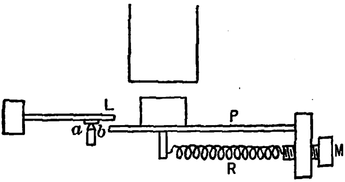

Fig. 6. Atonic Interrupter

Commutator Interrupters are often used when the primary current is too large to be broken by the hammer or atonic interrupters. A brush bearing upon a revolving disk, built up of conducting and insulating segments, makes and breaks the circuit at any desired rate, depending upon the speed of the motor which drives the disk.

Mercury Interrupters are specially adapted to circuits of high e.m.f. In the *plunger* type a pointed electrode is alternately immersed and withdrawn from a cup of mercury, the moving electrode being set in vibration magnetically as in the hammer interrupter, or by the positive action of a cam driven by a motor. The mercury cup is covered with a layer of oil so that the spark is quickly extinguished when the moving electrode is withdrawn into the oil. In the *turbine* type a small stream of mercury is directed upon the periphery of a revolving toothed wheel. The circuit is made when the mercury stream strikes a tooth and is broken when the mercury stream passes through a slot between the teeth. The toothed wheel is rotated at high speed by a motor which also drives a small mercury pump.

Wehnelt Interrupter. — The Wehnelt interrupter has two fixed electrodes which are immersed in dilute sulphuric acid. The anode consists of a small platinum wire insulated by glass except at its tip end. The cathode usually consists of a sheet of lead. If the electrolyte is well circulated, an interrupter of this kind will give about 450 interruptions per second with 24 volts impressed upon the primary circuit. When used with large coils the electrolyte heats up quickly, and a cooling coil is sometimes immersed in the electrolyte to control the temperature.

Tesla Coil. — In some cases, as in wireless telegraphy, electro-therapeutics, etc., where a unidirectional e.m.f. is not required, but an e.m.f. of high frequency is desired, a Tesla coil may be used in place of an induction coil. The construction of a simple Tesla coil is shown in Fig. 7. In the primary circuit a primary winding of few turns and a small spark gap are shunted by a condenser. Secondary and primary windings are wound together upon an air core. If an alternating e.m.f. of from 5,000 to 10,000 volts is impressed upon the primary the condenser is charged until the voltage across the primary gap breaks it down. The condenser then discharges through the gap, producing an oscillating current in the primary winding. The oscillating current in the primary induces a high-frequency e.m.f. in the secondary, which is sufficient to break down the long secondary spark gap.

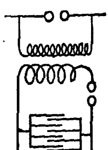

Fig. 7. Tesla Coil

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[R. G. Hudson.]

INSULATING MATERIALS, MISCELLANEOUS. — (*See also Cambric, Varnished; Electricity and Magnetism, Principles of; Gutta Percha; Insulating Materials, Testing of; Paper, Impregnated; Rubber; Wires and Cables, Insulated.*) Materials used in the insulation of electrical apparatus may be classified as follows (*see also Standardization Rules of A.I.E.E.*):

1. Vitreous, including glass, enamel, etc., 2. Stony, such as slate, marble, mica, asbestos, porcelain, etc., 3. Osseus, such as bone and ivory, 4. Resinous, including shellac, resins, copal and other gum, 5. Bituminous, as bitumen, asphaltum, pitch, etc., 6. Waxy, including bees-wax, paraffin, etc., 7. Elastic, such as india-rubber, ebonite, gutta-percha, etc., 8. Oily, including various oils and fats of animal and vegetable origin as mineral petroleum, 9. Cellulose, including dry wood and paper, cotton, celluloid, etc., 10. Silk and allied animal tissue such as catgut, 11. Sulphur.

SELECTION OF INSULATING MATERIAL. — In selecting an insulating material for any special purpose and in predetermining accurately the behavior of such a material under working conditions, it is desirable that the following electrical, physical and chemical properties of the material should be known as far as they apply to the material in question.

Electrical. —

Dielectric strength (voltage per mm. or per in. at puncture).
Relative dielectric hysteresis.
Specific inductive capacity; Direct-current; Alternating-current.
Conductivity or insulation resistance. Surface leakage.

Physical. —

Specific gravity and specific weight.
Hardness.
Toughness.
Brittleness.
Ductility.
Workability.
Flexibility (with reference to varnished cloths, etc.).
Strength (tensile, compressive and shearing).
Fracture, (fibrous, crystalline or amorphous).
Ability to take polish.
Melting point.
Shrinkage.
Adhesiveness.
Effect of high temperature (electric arc).
Effect of low temperature.
Artificial aging test.
Porosity.
Viscosity.
Flash and fire test.
Film-making power.
Rate of drying.
Penetration.
Absorption of moisture.
Amount of volatile matter given off at prescribed temperatures.

Chemical. —

Proximate composition.
Solubility in water, oil, etc.
Effect of moisture and moist air.
Weathering qualities.
Chemical effect upon metals in contact.
Presence of acid.

As the testing of insulating materials (*see Insulating Materials, Testing of*) is not standardized and few authorities have investigated the subject, the existing data pertaining to insulating materials are scanty, disconnected, and in many cases unreliable. In the following pages available data are given concerning the important insulating materials, the materials being listed alphabetically. The puncturing voltage and the dielectric strength reported are presumably effective a-c. values for a sine wave; the reports of tests, however, are not always specific on this point.

Adit is a form of papier-maché impregnated and covered with a special insulating compound. It may be moulded accurately in any form and possesses a tensile strength of about 130 kilograms per sq. cm. Certain grades of Adit withstand heat up to 60° C., others up to 120° C. It does not support combustion and its insulating properties are stated to be unaffected by dampness. The voltages required to puncture sheets of various thicknesses are given as follows (Hobart and Turner):

Thickness in mm.	2	3	4	5
Volts at puncture	1000	1800	3000	4000

Aetna Material is a composite material, used principally for strain insulators. Symons gives the following results of tests made upon a strain insulator constructed of Aetna material: puncturing voltage, 11,000; insulation resistance 20,000 megohms; tensile strength, 2.46 tons per square inch; a sample immersed in water at 120° F. absorbed 3.17 per cent of its own weight in 1.5 hours. Aetna material will withstand great heat without disintegration but is inclined to be brittle at high temperatures.

Air has a low conductivity and specific inductive capacity, and a dielectric strength sufficient for most purposes except very high voltage apparatus and transmission lines; see articles on *Corona* and *Spark Gap*.

Ambroin is made by baking in a vacuum silicate of sodium, asbestos, fossil copals, etc., mixed with alcohol. It is moulded in heated moulds, under high pressure. Certain grades of Ambroin will resist very high temperatures and are suitable for use in arc shields. The tensile strength of grade A. F. is 2140 pounds per square inch and the compressive strength is 2680 pounds per square inch, both tests being made at room temperatures. Ambroin is only slightly hygroscopic as compared with other insulators of the same nature. (Adapted from Hobart and Turner.)

In tests made by the German Reichsanstalt the voltages required to puncture specimens of Ambroin of grade A. F. of various thicknesses were found to be as follows:

Thickness in millimeters	0.33	0.84	5.0
Volts at puncture	3500	over 5000	over 36,000

The insulation resistance of a specimen 3 mm. thick and 25 sq. cm. in area was found to be 200,000 megohms.

Asbestos is a mineral consisting chiefly of silica, magnesia, lime, alumina, water and oxide of iron. The fibrous variety, called amianthus, is used in the manufacture of asbestos paper, cardboard, yarn and cloth. It is unaffected by oils, acids and alkalies and withstands very high temperatures. As a non-conductor of heat, it has an extensive use as an insulator in heating devices.

According to Stifler its insulating qualities break down at 1000° C. but recover after the asbestos is cooled. At temperatures above 1000° C., asbestos loses its mechanical strength and melts at about 1300° C. Steinmetz gives the following puncturing voltages for asbestos paper:

Thickness in millimeters	0.6	1.2
Volts at puncture	2700	5000

Whittaker's Pocket Book gives the insulation resistance as 16×10^4 megohms per centimeter cube.

Asphalt is a mineral pitch found in geological formation in various parts of the world. Asphalt is used (1) in the manufacture of insulating varnishes and japans, (2) for the impregnation of hygroscopic non-waterproof insulating materials and (3) as an insulating covering for cables. The various grades of

asphalt used for electrical work are called Trinidad, Bitumen, Elaterite, Gilsonite, Byerlyte and Manjak. Asphalt insulators possess a high insulation resistance, dielectric strength, flexibility and mechanical toughness, are very cheap, and are unaffected by moisture.

Pure asphalt (solid bitumen) softens at from 90° to 100° C. and is not recommended for use for immersion in hot oils, or where it would be subjected to centrifugal stress. Certain grades of Byerlyte melt at temperatures ranging from 200° to 350° F., while the melting point of Gilsonite varies from 230° to 400° F. Symons states that the dielectric strength of pure asphalt is 30,000 volts per inch (test made on sheet 1/16th inch thick). The manufacturers (Byerley & Sons, Cleveland, O.) state the dielectric strength of Byerlyte to be as follows:

Thickness in mils	140	215	430
Time of application	10 sec.	4 min.	5 min.
Dielectric strength, volts per inch	50,000	50,000	50,000

Pirani gives the value 2.68 for the specific inductive capacity of pure asphalt.

Bakelite is a synthetic organic substance resulting from the chemical condensation of phenols and formaldehyde (General Bakelite Co., N. Y.). It may be applied as a liquid or used as a solid. The liquid Bakelite is useful for the impregnation of porous materials, for enameling under heat and pressure and as a binding agent for moulded compounds. The solid Bakelite is unaffected by water, steam, oils and almost all chemicals. It does not melt or soften at ordinary machine temperatures; it is destroyed only at temperatures in the vicinity of 300° C. It is easily and accurately moulded and may be made to take any color It compares favorably with rubber and gutta-percha in every way except that it is not as flexible as those substances. The following data regarding Bakelite are given by the manufacturers:

PROPERTIES OF BAKELITE

Composition	Thickness of testing piece in millimeters	Volts at puncture	Volts per millimeter at puncture*
Different varieties of transparent "C" made especially for electrical purposes by hardening liquid "A" on a glass plate in a stove.	1.78	......	17,400
	2.18	......	14,200
	2.60	......	13,400
	0.43	16,750	38,800
	0.80	23,300	28,000
	0.95	25,600	26,900
Moulded composition, 70% Asbestos, 30 % Bakelite	8.0	......	8,500
Moulded composition: wood, flour and Bakelite	7.9	......	11,000
Impregnated blotting paper, hardened in a hot hydraulic press	3.66	102,000	27,880
Pressed paper impregnated with Bakelite	1.6	53,700	33,500

* Tested with a 50-kilowatt transformer, raising voltage gradually so that puncture occurred within 20 or 30 seconds.

Item	Bakelite and wood pulp	Bakelite and asbestos
Specific resistance in megohms per cm. cube	35×10^6	10^4
Tensile strength, lb. per square inch	650	1200
Compression strength, lb. per square inch	4000	18,000
Maximum working temperature, ° C.	90	450

The specific inductive capacity ranges from 5.60 to 8.85, depending upon the composition.

Berrite is a gum used for impregnating paper and cloth. Materials impregnated with it are tough but crack easily. It runs freely at low temperatures and is unaffected by high temperatures. The dielectric strength is given by Hobart and Turner as 5000 volts per millimeter.

Bitumen. — See section on *Asphalt.*

Byerlyte. — See section on *Asphalt.*

Cambric, Varnished. — See separate article on *Cambric, Varnished.*

Celluloid (xylonite) is a dried solution of gun-cotton (pyrolin) and oil. It may be machined or moulded into any form by softening in boiling water. Its dielectric strength is very much reduced at high temperatures, and it is very combustible. It is only slightly hygroscopic. Hobart and Turner give the following values for its dielectric strength:

	Volts per mm.
Clear samples, 0.25 millimeter thick, 20° C.	12,000-28,000
Clear samples, 0.25 millimeter thick, 100° C.	4,000-12,000
Colored samples	10,200-18,900

Tests made at the Stadt. Lab., Munich, give for the insulation resistance 71×10^3 megohms per centimeter cube.

Coal-tar Pitch is the residue remaining after the fractional distillation of coal tar. It flows at low temperature and when cold is quite brittle. Symons gives its dielectric strength as 2000 volts per millimeter.

Copal is a resinous substance, which makes a colorless varnish when dissolved in alcohol, oil of turpentine or linseed oil. Copal is easily fused, is very inflammable and is quite brittle when cold. Steinmetz gives the following puncturing voltages: 9700 volts at 3.0 millimeters and 20,400 volts at 6.0 millimeters.

Condensite is made by the Condensite Co. of America, Glen Ridge, N. J., and by the Dickinson Mfg. Co., Springfield, Mass. It is a hard infusible substance, the chief constituent of which is a resinous gum, made by the reaction between phenol and formaldehyde. Condensite is produced by combining this gum under heat with a hardening agent. It is supplied in three grades; for plastic moulding, for impregnating and as a cement. The moulded Condensite has a compressive strength of 25,600 pounds per square inch and tensile strength of 4270 pounds per square inch.

It is non-flammable and infusable, insoluble in oils, most acids and other solvents and shrinks but one-fifth of one per cent in moulding. The impregnating material is used in connection with metal, wood, paper, cardboard, rubber, leather, etc. The plastic cement is used for fastening together the parts of porcelain insulators, for sealing terminals in porcelain bases, etc.

Results of tests made upon Condensite by the Elec. Test. Lab. of N. Y. City, follow:

Thickness in mils	189	152	147
Temperature, degrees Fahrenheit	Room temp.	75	170
Dielectric strength, volts per inch	65,600	60,500	27,900

Ebonite. — See section on *Rubber, Hard*.

Eburin is a mixture of infusorial earth, asbestos, or cotton and gums, and may be moulded into any form. As it is not hygroscopic and is very strong, it is used frequently for out-door strain insulators (*Hobart and Turner*).

Elaterite. — See section on *Asphalt*.

Empire Cloth consists of a closely-woven cambric coated with two or more films of an oxidized oil, prepared by a secret process. Canvas, duck, linen, silk, and paper are also coated by the same process.

The Mica Insulator Co. give the puncturing voltages for various grades and thicknesses of Empire cloth as follows:

	Canvas	Linen	Muslin		Silk	
Thickness in mils	16	6	4	15	4	6
Volts at puncture	7800	7800	4200	15,000	4500	7225

Fiber. — Horn fiber (red, black, white and grey) has a very great tensile and dielectric strength. While not soluble in water, most acids or alkalies, it swells when immersed in water. The dielectric strength may be greatly increased by impregnating the material with oil or varnish. It will not melt at any temperature.

Vulcanized fiber, made of chemically-treated paper fiber, is made into sheets, tubes, rods and special shapes. It warps and shrinks badly and absorbs moisture. It has a tensile strength of 12,000 to 14,000 pounds per square inch and a compressive strength of 38,000 to 40,000 pounds per square inch. The dielectric strength is claimed to be independent of the thickness. Vulcanized fiber is insoluble in water or natural oils. Some of the strong acids will destroy it or make it brittle.

Miscellaneous fibers are manufactured by secret processes, many of them containing wood-pulp or asbestos.

PUNCTURING VOLTAGE OF VARIOUS FIBERS

Grade of fiber	Thickness	Volts at puncture	Authority
Dry wood fiber	0.22 mm.	2,800	Steinmetz
Dry wood fiber	1.69 mm.	21,600	Steinmetz
Red fiber	30.9 mils	8,000 to 9,500	Canfield and Robinson
Black fiber	70.9 mils	7,200	Canfield and Robinson
Vulcanized fiber	0.58 mm.	2,200	Steinmetz
Vulcanized fiber	12.8 mm.	22,500	Steinmetz
Vulcanized fiber	187 mils	10,000 to 12,000	Am. Vulc. Fiber Co.
Vulcanized fiber	375 mils	25,000 to 90,000	Am. Vulc. Fiber Co.
Horn (undried) fiber	0.56 mm.	5,990	Hobart and Turner
Horn (dried) fiber	0.56 mm.	6,880	Hobart and Turner
Horn (varnished) fiber	0.66 mm.	8,120	Hobart and Turner

Hendricks gives the curve in Fig. 1 for the puncturing voltage of hard fiber, the conditions being as follows:

Curves. — Measured on single thicknesses.

Dimensions. — 0.031-inch to 1-inch sheets.

Composition. — Chemical hard fiber.

Treatment. — Dried before testing.

Method of test. — Between flat, square cornered disks under oil.

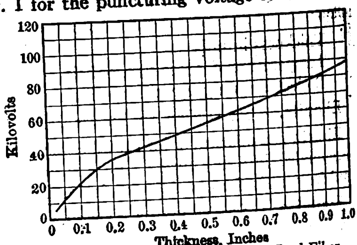

Fig. 1. Puncturing Voltage for Hard Fiber

Temperature. — 20 to 25° C. *Time.* — One minute. *Frequency.* — 60. *Wave.* — Sine.

Accuracy of curve. — About 10 per cent plus or minus.

Characteristics. — Results depend largely on dryness of fiber.

Notes. — Results on fiber of different colors seem to be identical.

Fuller-Board. — See section on *Pressboard*.

Galalith is made by heating the residue of skimmed milk after the water has been extracted. It is easily moulded or bent but is highly hygroscopic. Walter gives the dielectric strength as 6000 to 8500 volts per millimeter.

Gilsonite. — See section on *Asphalt*.

Glass is a silicate of soda or potash combined with certain metallic oxides, usually lead oxide. Owing to the condensation of moisture upon its surface, glass has a very large surface leakage. Rain water roughens its surface and the resulting accumulation of dirt reduces its insulating qualities. According to Gray and Dobbie the specific resistance of potash glass is higher than that of soda glass. Annealed glass has in general a higher specific resistance than unannealed glass. As compared with porcelain, glass is inferior in mechanical strength; whereas porcelain is only chipped by a blow, glass is easily cracked or shattered.

DIELECTRIC STRENGTH AND SPECIFIC RESISTANCE OF GLASS

Grade	Volts per mm. (*a*)	Megohms per cm. cube (*b*)
Common glass	8,000 to 9,000	99×10^7
Lead glass	5,500	25×10^5
White alabaster glass	11,500	33×10^8
Black alabaster glass	8,500	84×10^6 to 85×10^4

(*a*) Walter. (*b*) Foussereau (*see Bibliography*).

In various samples tested by Löwe and v. Pirani, the specific inductive capacity varied from 5.5 to 9.1.

Gohmak, made by the Vulcanized Products Co., Muskegon, Mich., is a moulded insulation of specific gravity 1.37 to 1.75, depending upon the compounding formula. The tensile strength varies from 9000 to 12,000 pounds per square inch. It will soften slightly at a temperature of 100° C. It is claimed

to be unaffected by immersion in water, oils, dilute acids and alkalies. The manufacturers state that sheets ⅛-inch thick are punctured by 50,000 volts and that the insulation resistance of Gohmak in megohms per centimeter cube is 17×10^3 at 28° C. and 9×10^3 at 50° C.

Gummon, made by the Dickinson Mfg. Co., Springfield, Mass., is made by a secret process. It is heatproof up to 700° F. and can be moulded when cold. It is not easily sawed or drilled as it takes the temper out of steel.

Gutta-Percha. — See article on *Gutta-Percha.*

Isolit, like ambroin, is a form of papier-maché, impregnated and covered with a special insulating compound. See section on *Ambroin.*

Jute is a fiber obtained from the inner bark of certain trees growing in India. Commercial jute is usually softened and rendered less brittle by impregnating it with paraffin or some similar mineral oil. It is used extensively as a filler in lead-covered cables and as a constituent of certain kinds of paper and press-board. Baur gives the following values of the puncturing voltage for impregnated jute:

Thickness in millimeters	3	6	12	24
Volts at puncture	4,800	7,200	12,000	19,000

Coyne and Howe give the specific inductive capacity of impregnated jute as varying from 3.0 to 4.0.

Lava, made by the Colonial Mfg. Co., Am. Lava Co., and Stewart Mfg. Co. of Chattanooga, Tenn., is a mineral talc, machined in its natural condition and then baked at a temperature of 1100° C. to a condition of extreme hardness. It is then unaffected by any subsequent temperature short of its baking temperature. It is slowly attacked by hydrochloric acid but is not affected by other acids or alkalies. Its dimensions are unchanged by absorption of moisture and it has a negligible coefficient of expansion with temperature. The dielectric strength varies from 250 to 75 volts per mil depending upon the thickness of the sample tested.

Lavite. — See section on *Lava.*

Leatheroid. — See section on *Fiber.*

Litholite is an insulating material resembling psychiloid in structure. It is made in sheets and is pressed into any form. It is softened by sulphuric acid and distilled water and is reduced to a pulp if immersed in caustic soda. It is tough but inflammable. It is used for commutator rings, bushings, washers, formers, bobbins and for square and round tubing. Symons states that 20,000 volts is required to puncture a thickness of 0.445 centimeter.

Marble is the name given to any limestone which is sufficiently compact to admit of a polish. While pure marble is white, the presence of iron oxide or other impurities give it different colors. It is used principally for switchboard work and should not contain metallic veins, which reduce its insulating qualities. According to Walter the dielectric strength of marble is 6500 volts per millimeter. Schmidt gives the specific inductive capacity of Carrara marble as 8.3. The insulation resistance, from tests made at Stadt. Lab., Munich, is stated to be 435 to 510 megohms per centimeter cube.

Megohmit is a form of reconstructed mica in which the adhesive matter is reduced to a minimum and does not exceed 1.25 per cent of the finished product. The hard megohmit plates soften at about 80° C. It is used for slot insulation, commutator rings, collars, etc. Hobart and Turner give the following values for the puncturing voltage:

Thickness in millimeters	0.25	0.6	1.0
Volts at puncture	8,000	20,500	36,000

Mica is an anhydrous silicate of aluminum and potash or sodium. It crystallizes in a laminated mass, some grades of which may be subdivided down to a thickness of 0.006 millimeter. It is useful as an insulator because of its high insulating qualities and its ability to withstand high temperatures. Owing to its impurity, lack of flexibility and excessive surface leakage in the natural state, the laminæ are separated and sorted into various grades of purity and are then cemented together to form plate or flexible reconstructed mica of any thickness or purity. Moulded mica is used in the manufacture of overhead line material and in the making of moulded pieces as a substitute for hard rubber. Moulded mica softens at 136° F. It is insoluble in water, but is affected by certain oils and all acids and alkalies to some extent. Mica loses its insulating qualities at a temperature of 1000° C., but recovers them after cooling. Andrews showed that the dielectric strength of mica is greatly reduced when immersed in or coated with oil.

DIELECTRIC STRENGTH OF MICA

Grade	Thickness, mils	Volts per millimeter	Volts at puncture
	...	28,000 (a)	....
German E. Africa	...	17,500	....
Calcutta	...	24,500	....
Madras	...	21,000	....
Russia	...	20,000	....
Ceylon	...	....	5000 (b)
....	1.7	....	5400
....	2.8	....	6200
....	3.2	....	6100
....	5.1	....	

(a) Walter, B. (b) Canfield and Robinson (*see Bibliography*).

Curie states that the insulation resistance of mica at 20° C. is 88×10^8 megohms per centimeter cube. Starke states that the specific inductive capacity varies from 5.8 to 7.7.

Micabond Cloth, Paper and Plate, made by the Chicago Mica Co., Valparaiso, Ind., are forms of reconstructed mica. Micabond cloth is made of muslin, India mica and paper, the binding material being gutta-percha tissue. It is made in thicknesses of 12, 15 and 18 mils respectively for use in armature slots, cores, coils, etc. The dielectric strength is 435 volts per mil. Micabond paper is made of Japanese paper, mica and rubber. It is made in thicknesses of 6, 9 and 11 mils respectively for use in armature slots, coils, etc. Its dielectric strength is 1012 volts per mil. Numbers 101 and 111 are easily moulded when heated, whereas numbers 102, 104, 122 and 144 cannot be moulded, particular attention being given to these grades to secure a uniform thickness for commutator use.

Micanite, made by the Mica Insulator Co. of New York, is a form of reconstructed mica. The puncturing voltage for micanite plate, used for commutator segments, ranges from 9540 volts for a thickness of 10 mils to 119,000 volts for a thickness of 125 mils. The puncturing voltage for flexible micanite, used for armature slots, armature, magnet and commutator cores, transformers, etc., ranges from 2940 volts for a thickness of 5 mils to 73,500 volts for a thickness of

125 mils. Micanite cloth is constructed of layers of mica, muslin and Japanese paper and the average puncturing voltage for thicknesses of from 12 to 18 mils is 4350 volts. Another grade of micanite cloth is constructed of mica, rubber tissue and Japanese paper. Its average puncturing voltage for thicknesses of from 12 to 18 mils is 4000 volts. Micanite paper, constructed of mica and Japanese paper, has an average puncturing voltage of 3700 volts for thicknesses of 6 to 11 mils. When constructed of mica, rubber tissue and Japanese papers the average puncturing voltage for thicknesses of 6 to 11 mils is 3675 volts. Rope paper and mica, 7 to 15 mils thick, breaks down at about 4800 volts and pressboard and mica, 7 to 15 mils thick, breaks down at 5500 volts.

Minerallac, made by the Minerallac Elec. Co., Chicago, Ill., is an insulating compound, made in four grades as follows: No. 1, a fluid of the consistency of molasses, No. 2, a semi-solid compound resembling hard rubber, No. 3, a heavy dark-brown impregnating liquid, and No. 5, a capping compound used for sealing apparatus after impregnation. The following results were obtained from tests made upon Minerallac No. 2 by Elec. Test. Lab. of N. Y.: dielectric strength, approximately 1000 volts per mil; insulating resistance 1.3×10^9 megohms per inch cube; specific inductive capacity, about 2.1 at 75° F.; softening point, 118° F.; melting point, 146° F.; flash-point, 395° F.; fire-point, 425° F.; acid or alkali reactions, none.

Oil. — See article on *Oil, Transformer.*

Paper. — See article on *Paper, Impregnated.* Unheated paper is suitable for insulation only when inclosed in an air-tight envelope (e.g., a telephone cable), for paper is very hygroscopic. Unheated dry paper from 3.5 to 4.5 mils thick punctures at about 800 volts according to Canfield and Robinson. The specific resistance depends largely upon the quality of the paper, being of the order of 5×10^4 megohms per centimeter cube. The specific inductive capacity of the paper used in telephone cables, as given by various authorities, ranges from 1.7 to 3.8.

Paraffin is a product obtained by the destructive distillation of petroleum shale. The dielectric strength of cloth, wood and paper is increased by impregnation with paraffin and the materials are rendered less hygroscopic. Paraffin is acidproof but is very inflammable and has a low melting point, about 65° C. The dielectric strength of solid paraffin is given by Walter as 11,500 volts per millimeter. Steinmetz gives 7690 volts per millimeter as the dielectric strength of melted paraffin. The insulation resistance according to Braun is 284×10^{10} megohms per centimeter cube. According to Zietkowsky the specific inductive capacity ranges from 2.105 at 44° C. to 2.165 at 76° C.

Porcelain. — (*See also article on Insulators, for Overhead Lines.*) Porcelain for electrical purposes has a specific gravity of 2.65. It is unaffected by oils, acids or alkalies. The Locke Insulator Mfg. Co. gives the tensile strength as 1500 pounds per square inch and the compressive strength 15,000 pounds per square inch. It is quite brittle and cannot be moulded accurately enough for use in some cases. Ink placed upon good porcelain at a fracture will not flow from the part initially inked.

The puncturing voltage is as follows;

Thickness in millimeters	10	20	30
Volts at puncture	115,000	160,000	190,000

The insulation resistance is as follows:

Temperature, °C.	50	400	1000
Megohms per cm. cube	215×10^7 (*a*)	20 (*b*)	1 (*b*)

(*a*) Foussereau. (*b*) Goodwin and Mailey.

The Locke Insulator Mfg. Co. gives the specific inductive capacity as 4.4.

Pressboard is similar to paper in its construction, except that it is thicker and less flexible. It is usually hygroscopic unless impregnated with some moisture repellant. It is pressed into many useful forms, and is used as an insulator

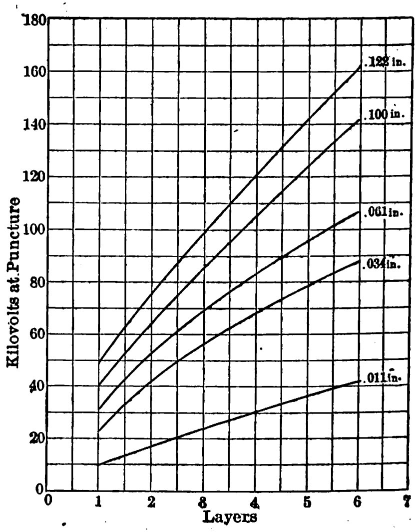

Fig. 2. Puncturing Voltage for Oiled Pressboard

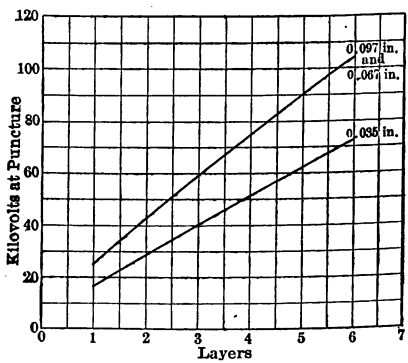

Fig. 3. Puncturing Voltage for Varnished Pressboard

principally in connection with low voltages. See sections on *Fiber*, *Paper*, etc., for other properties.

Figs. 2 and 3 from Hendrick's paper show the dielectric strength of oiled and varnished pressboard respectively.

OILED PRESSBOARD

Dimensions. — 0.011 inch to 0.122 inch thick — one to six layers.

Composition. — Cotton rags and paper clippings.

Treatment. — Dried and boiled in transformer oil.

Method of test. — Between square-edge flat disks 4 inches in diameter under oil.

Temperature. — 20 to 25° C. *Time.* — 1 minute. *Frequency.* — 60. *Wave.* — Sine.

No. of trials. — Each point, one to four.

Accuracy of curve. — 10 per cent plus or minus. Curve is based on but a few trials, hence is not very reliable, but shows typical results.

Characteristics. — Material is variable; results depend largely on time of application of stress.

Specific capacity. — 4.9 at 20 to 25° C. under oil.

Notes. — Total time of test is about 5 minutes (average) giving rather low results.

VARNISHED PRESSBOARD

Dimensions. — 0.035 inch, 0.067 inch and 0.097 inch thick — one to six layers.

Composition. — Cotton rags and paper clippings.

Treatment. — Dried and given two coats of varnish.

Method of test. — Between square-edge flat disks 4 inches in diameter under oil.

Temperature. — 20 to 25° C. *Time.* — 1 minute. *Frequency.* — 60. *Wave.* — Sine.

No. of trials. — Each point, one to four.

Accuracy of curve. — 10 per cent plus or minus. Curve is based on but a few trials hence is not very reliable but shows typical results.

Characteristics.—Dielectric strength low but fairly uniform, depending largely on varnish film; nearly proportional to total thickness within limits of tests.

Specific capacity. — 2.9 at 20 to 25° C. on 0.097-inch board.

Presspahn is a type of pressboard made in Germany. It is strong and can be easily bent. It is usually impregnated and boiled in pure linseed oil, thinned with benzine.

Kinzbrunner gives the dielectric strength of presspahn in volts per millimeter as ranging from 9300 to 5200, the puncturing voltage decreasing as the radius of the test electrodes increases. The insulation resistance from tests at the Stadt. Lab. Munich is given as 11×10^3 megohms per centimeter cube.

Psychiloid consists of a paper pulp, which is cured, dried and then treated chemically. According to Hobart and Turner it may be machined and worked into any form and is pressed into sheets varying from 1/16 to 1½ inches in thickness. It is stated to be non-absorbent and unaffected by oils. Symons gives the volts at puncture as 25,000 for a thickness of 3.2 millimeters.

Rubber. — See article on *Rubber*.

Resin is the oxidized exudation of certain trees or plants and may be of recent origin or exist as a fossil. Some of the common resins which are used for insulating purposes are mastic, shellac, jalap, turpentine, storax, amber, hartite, ozokerite, copal, kauri-gum, etc. Most resins are insoluble in water but dissolve in alcohol, ether, etc., forming varnishes of high dielectric strength. The resin varnishes are limited in their use by the fact that they become brittle at high temperatures, and will disintegrate if under vibration. Resins are hygroscopic and in general possess undesirable acid properties. Tests made by various authorities of the specific inductive capacity of the various resins indicate a value between 3.9 and 4.0.

Slate is hygroscopic and should be boiled in paraffin. It is often permeated by metallic veins, making it unfit for use unless the electrical connections are

insulated by bushings. It is useful for switchboard and switch-base work owing to its desirable mechanical and fireproof qualities. Its dielectric strength decreases rapidly as the temperature increases, and at a high temperature slate becomes a conductor. Where a high voltage is impressed upon a piece of slate for some time, the slate usually is not punctured but, due to the consequent rise in temperature, the slate acts as a short-circuit to the impressed voltage.

The breakdown is thus only apparent as the specimens regain their dielectric properties after cooling. Values of this breakdown voltage, as determined at the Massachusetts Institute of Technology in 1913, are as follows:

Thickness in mm.	3.7	7.2	10.3	13.5	25.3
Volts at breakdown	11,300	10,800	13,700	12,200	22,500

Schulze gives the specific inductive capacity of slate as varying from 6.60 to 7.37. The insulation resistance of slate, as determined at the Stadt. Lab., Munich, is 78×10^3 megohms per centimeter cube.

Varnished Cambric. — See article on *Cambric, Varnished.*

Varnishes, Insulating, are made in two classes: (1) baking varnishes, which harden by oxidation when subjected to artificial heat and (2) air-drying varnishes and similar compounds, which harden or set by evaporation of the solvent. Varnishes and similar compounds are manufactured in a number of grades adapted for use in connection with armature, field, and transformer coils, magnet and arc lamp spools, transformer bushings, insulating cloth, paper and fiber, armature and transformer stampings, asbestos and magnesia board, cut-out boxes, switch and panel boards, mica plate, flexible mica, mica cloth and paper, storage batteries, weatherproof wire, iron and metal fittings, moulding, potheads, vacuum impregnating, taped connections, etc. Some of the trade designations are P. and B., S. P. C., S. V. W., G. P., H. R., Voltalac, W. P., Glinolac, Benolite, Dolph's, Ohmlac, Minerallac, Nico, M. C., Insullac, Enamelac, Armalac, Chatterton, Linolac, M. I. C., etc.

Vulcabeston is a compound of asbestos and rubber, the asbestos predominating. It is used in the form of moulded insulating pieces in connection with electrical machinery where a material is desired which is not affected by oils and which will withstand a high degree of heat. Although insoluble in water and unaffected by oils, it is slightly disintegrated by acid and alkaline solutions. It softens at temperatures ranging from 300 to 600° F., depending upon the grade. The above data are given by Holitscher. The puncturing voltage of vulcabeston, according to Canfield and Robinson, is 4000 volts for a thickness of 42.8 mils and 6000 volts for a thickness of 75.4 mils.

Vulcanite. — See article on *Rubber.*

Wood. — Thoroughly dried hard wood, if impregnated with an insulating material, makes a good insulator. The woods commonly used are maple, cherry, ash and yellow pine. For use in transformers, wood is usually impregnated with transformer oil, and when used in air, the wood is impregnated with paraffin or rosin. Fig. 4 from Hendrick's paper gives the puncturing voltage of oiled wood. The conditions under which the tests were made are as follows:

Material. — Hard maple.

Dimensions. — ½ inch to 1 inch across grain; 1 inch to 6 inches with grain.

Treatment. — Across grain, boiled in transformer oil under vacuum; with grain, dried under vacuum, boiled at atmospheric temperature.

Method of test. — Between square-cornered flat disks under oil.

Temperature. — 20 to 25 °C. *Time.* — 1 minute. *Frequency.* — 60. *Wave.* — Sine.

No. of trials. — Each point, one to three; three points across grain; five points with grain.

Accuracy of curve. — 10 per cent plus or minus.

Characteristics. — Dielectric strength across grain increases much slower than thickness but with the grain is proportional to thickness.

Specific capacity. — Across grain = 4.1 at 20 to 25° C. under oil.

Notes.—Test with and across grain on samples treated by different methods a long time apart. Wood seems to be identical in quality however.

The insulation resistance and specific inductive capacity of various woods are as follows; the specific inductive capacities are from tests by Starke; the insulation resistance from tests by various experimenters.

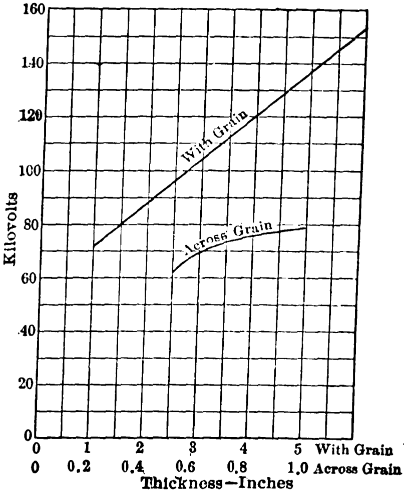

Fig. 4. Puncturing Voltage for Wood

Kind of wood	Megohms per cm. cube	Kind of wood	Specific inductive capacity
Beech	5×10^6–6×10^4	Red beech (parallel to fiber)	2.51–4.83
Pine (parallel to fiber)	35×10^9	Red beech (perpendicular to fiber)	3.63–7.73
Pine (perpendicular to fiber)	10×10^{10}	Oak (parallel to fiber)	2.46–4.22
Walnut (dry)	53–133	Oak (perpendicular to fiber)	3.64–6.84
Walnut (paraffined)	8×10^2–124×10^2		

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[R. G. Hudson.]

INSULATING MATERIALS, TESTING OF. — (*See also Insulating Materials, Properties of.*) The most important electrical properties of an insulating material are its dielectric strength, its specific inductive capacity, its insulation resistance and its effective a-c. conductance. Some of the more common methods of measuring these quantities are described below.

DIELECTRIC STRENGTH. — The dielectric strength of an insulating material is defined as the puncturing voltage per unit thickness, the thickness usually being measured in millimeters or mils. The voltage required is in general higher than can be obtained with direct-current sources of e.m.f. and alternating e.m.f.'s are therefore used in such tests. The materials to be tested are placed between electrodes connected to the high-tension terminals of a transformer, which receives power from a low-voltage alternator.

Testing Transformers and Alternators. — Transformers for such use are made to deliver voltages up to 500,000 volts. For the purpose of comparison and computation of the maximum e.m.f. from the effective value, it is essential that the alternator should give a sine-wave voltage at all loads. Both transformer and alternator should be large enough to operate with good voltage regulation at all testing loads, so that no distortion of the wave-form will be produced by the charging current. Skinner suggests the following ratings of testing transformers:

Max. test voltage	Cap. of trans. in kw.	Max. test voltage	Cap. of trans. in kw.
2,000	1	50,000	50
6,000	3	100,000	100
10,000	5	150,000	150
30,000	30	250,000	250

The alternator should have a distributed field winding and in order that the combination of alternator and transformer may respond to any instantaneous load, the impedance of each should be low; that is, the speed of the alternator should be high and the e.m.f. induced per turn in the transformer should be high.

Form of Electrodes. — Many investigators have demonstrated that the form of the electrodes used is an important factor of the test. If the edges of the electrodes are rounded, the resulting brush discharge heats the specimen so that it breaks down at a low voltage. If the edges of the electrodes are not rounded, the increased flux density at the edges produces an excessive strain on the dielectric under the edges of the electrodes. Most authorities favor the use of flat electrodes with slightly rounded edges. Hendricks suggests flat electrodes with corners rounded to a radius equal to one-tenth of the diameter of the flat face. Kinzbrunner has discussed in detail the effects of various shapes of electrodes.

For testing oils or other liquids two standards composed of definitely shaped terminals supported in a liquid container are commonly used. In one the testing terminals consist of brass balls ½ inch in diameter fastened to rods $\frac{3}{16}$ inch in diameter and placed vertically in a glass tube. The distance between the balls is adjustable, a distance of 0.15 inch, however, being considered standard. In the other standard the terminals consist of brass disks ½ inch in diameter mounted on rods $\frac{3}{16}$ inch in diameter and placed horizontally in a small box made of wood

or some other insulating material. The spacing of the disks in this case is also adjustable, although 0.2 inch is adopted as a standard. Average dry oil should not break down in the ball electrode standard at less than 30,000 volts and in the disk electrode standard at less than 45,000 volts.

Time of Electrification. — Materials may be tested by impressing the puncturing voltage, 1. instantaneously, 2. in steps of one-minute duration or 3. in several instantaneous applications of decreasing voltage and increasing time of electrification. The question of the proper time of electrification is at present undecided, although many investigators favor the one-minute electrification. For arguments against this standard, see paper by Kinzbrunner (reference in Bibliography at end of this article).

Control of Voltage. — The voltage impressed between the test electrodes may be controlled 1. by varying the excitation of the alternator, 2. by inserting resistance in series with the low-voltage terminals of the transformer or 3. by varying the number of active turns on the low-tension side of the transformer. A combination of the first and third methods is most commonly used. In the second method the voltage wave is made more peaked as resistance is introduced; in the third method, if used alone, the circuit must be broken as connections are made to the tap leads. By varying the field excitation of the alternator and changing the tap connections between tests, the shape of the voltage wave is kept constant and the variation of field saturation in the alternator is reduced to a minimum.

Measurement of Voltage. — The voltage impressed upon the material under test may be determined by 1. a variable spark gap shunted across the electrodes, 2. by a voltmeter and an extension coil or multiplier connected across the electrodes, 3. by an electrostatic voltmeter connected across the electrodes, 4. by a voltmeter connected across the low-tension terminals of the transformer, the reading of which is to be multiplied by the ratio of transformation of the transformer, 5. by a potential transformer and a voltmeter connected across the terminals and 6. by a special voltmeter winding placed in the middle of the high-tension winding of the power transformer. The needle spark-gap method, while convenient because of its indication of the maximum rather than the effective value of the voltage, is tedious in use, and its readings are dependent upon the time of electrification, circulation of air, the condition of the needles, etc. The sphere spark gap is preferable; see *Spark Gap*. The use of the voltmeter and extension coil, while flexible and convenient, is not recommended because of the load which is placed upon the transformer and the possibility of leakage in the extension coil. The use of an electrostatic voltmeter is desirable, except that at high voltages the moving element must be immersed in oil, making the instrument sluggish in action, and may frequently break down. In any method involving the ratio of transformation of the transformer, the results are questionable owing to the assumption of a constant ratio of transformation at all loads. The use of the potential transformer and voltmeter is prohibited in most cases by the cost of the instrument transformer and the excessive distorting load introduced by such an arrangement.

A voltmeter winding in the power transformer furnishes as a rule the most accurate and efficient method of measurement, the chief source of error being due to leakage flux and impedance drop in the transformer.

Connections for Test. — In Fig. 1 is shown the complete arrangement suggested by Hendricks for measurements of dielectric strength, energy loss in the dielectric and specific inductive capacity. The explanation of the figure follows: *A* exciter, *B* field switch, *C* generator field rheostat, preferably motor driven so as to vary voltage at constant rate, *D* main switch with auxiliary contact for lighting red lamp as a danger signal, *E* automatic circuit breaker, *F* series-paral-

lel switches on low-tension side of transformers, G ground connection, H instrument loop, I voltmeter winding with taps, J voltmeter, wattmeter and ammeter, K choke coils, L electrodes and M film cut-out.

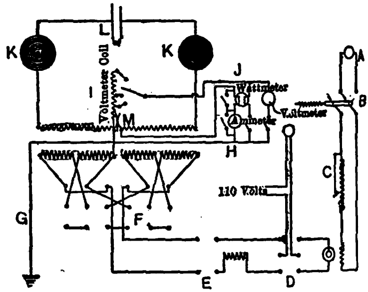

Fig. 1. Arrangement and Connections of Apparatus for High-tension Tests of Insulators

Causes of Variations in Results.—The puncturing voltage of an insulating material is affected by its previous history, precise condition when tested, size, thickness, form, uniformity of electrostatic field, temperature, time of electrification, frequency and the surrounding medium. The puncturing voltage is affected more by temperature than any other factor, and since the temperature of a specimen is increased greatly under the action of the corona discharge (*see Corona*) it is desirable that corona discharge should be eliminated. To prevent the heating action of the corona, 1. guard rings may be applied, as described by Ryan, Norris and Hoxie, 2. the electrodes may be imbedded in the dielectric when possible, 3. the "picein drop" method described by Walter may be used on the specimen, or 4. the electrodes and test specimen may be immersed in oil as recommended by Hendricks. In the last case materials easily permeated by oils should be glazed over with a varnish before testing. While the temperature of insulation in practice may range from − 25° C. to + 125° C., tests between + 25° C. and + 100° C. should suffice to indicate average working conditions.

SPECIFIC INDUCTIVE CAPACITY OR DIELECTRIC CONSTANTS. — (*See also Condensers.*) The specific inductive capacity of any substance is the ratio of the capacity of a condenser, in which the substance in question fills the space between the plates, to the capacity of the same condenser when the space between the plates is filled with air. This ratio, designated by K, varies with the temperature of the substance, the time of electrification used in the test and the frequency, when tested with alternating currents. The specific inductive capacity of most substances decreases as the temperature increases, except in the case of a few substances for which the specific inductive capacity increases with increased temperatures. The specific inductive capacity in general increases with the time of electrification and decreases as the frequency increases. The determination of the specific inductive capacity from test involves the determination of the capacity of a condenser of known dimensions. A plate or cylindrical condenser is commonly used for solid material and a spherical condenser for liquids. Having determined the capacity of any condenser of known dimensions from test, the specific inductive capacity of the dielectric used can be obtained by substituting the known constants in the condenser formulas given in the article on *Capacity and Charging Current*.

Direct-deflection Method. — The unknown condenser C_x and a ballistic galvanometer are connected in series with a battery of about 500 volts e.m.f. and the deflection D_x is noted. A standard condenser C_s is then substituted for the unknown condenser and capacity is inserted until the deflection D_s upon charging is the same as in the first case. Then

$$C_x = \frac{D_x}{D_s} C_s.$$

As most condensers possess leakage and absorption, the method gives erroneous results, since no distinction is made between charging current and leakage current. Zelleny gives a modification of this method in which, by regulating the time of charge, insulation and discharge, a constant deflection may be obtained.

Resonance Method. — The condenser and a variable inductance are connected in series in a circuit of high frequency and low resistance. A modification of this method is given by Rohman. Resonance is obtained and detected by noting the maximum value of the current as the inductance is varied. Then

$$C = \frac{1}{4\pi^2 f^2 L},$$

provided the effective conductance of the condenser is negligible and the impressed voltage is a pure sine wave.* The accuracy of this method depends principally upon the flexibility and precision of the variable inductance, upon the actual shape of the voltage wave and upon the leakage resistance of the condenser.

Alternating-current Bridge. — The unknown condenser C_x and the standard condenser C_s are connected in the arms of a Wheatstone bridge (q.v.) as shown in Fig. 2. The resistances R_1, R_2 and R_3 must be wound non-inductively. The detector D consists of a telephone receiver or a vibration galvanometer. After connecting the bridge to a high-frequency source of sinusoidal e.m.f., R_1, R_2 and R_3 are adjusted until a minimum current passes through the detector. Then, assuming the effective conductance of the dielectric to be negligible,

$$C_x = C_s \frac{R_2}{R_1}.$$

Sources of error are: inductance or capacity of R_1, R_2 and R_3, error in ratio of R_1 and R_2, and electrostatic induction between the bridge and its surroundings.

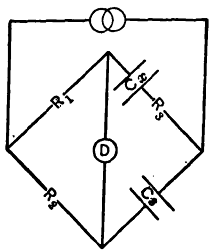

Fig. 2.

INSULATION RESISTANCE. — The method commonly employed is one of substitution. A standard resistance R_s (usually 1/10 megohm) connected in series with a D'Arsonval galvanometer is connected across a direct-current source of e.m.f. of about 500 volts and the deflection D_s of the galvanometer noted. The galvanometer must have a sensitiveness of about 1×10^9 and the deflection should be directly proportional to the current. If the galvanometer is shunted, an Ayrton universal shunt should be used (*see Shunts*) so that the damping of the galvanometer will be independent of the multiplying power. The deflection D_x is then noted when the unknown resistance R_x is connected in place of the standard resistance. Then

$$R_x = \frac{D_s}{D_x} R_s.$$

The usual errors of this method are due to leakage and absorption. Leakage may be reduced considerably by the use of a guard ring, so connected as to shunt the leakage current around the galvanometer. The deflection of the

* The exact formula for a sine-wave voltage is

$$2\pi f l = \frac{2\pi f C}{(2\pi f C)^2 + g^2},$$

where g is the effective leakage conductance (*see Alternating Currents*).

galvanometer will vary with time due to absorption, and the resistance of the material will apparently vary with the time of electrification. To facilitate comparison of the insulation resistance of dielectrics, it has become common practice to take the resistance obtained after one minute's electrification. Owing to the large negative resistivity temperature coefficient of most dielectrics, it is essential that the temperature of the material be noted when tested.

ENERGY LOSS IN THE DIELECTRIC — EFFECTIVE CONDUCTANCE. — If the transformer losses can be determined accurately under the testing conditions, the energy loss in the dielectric may be determined at any voltage by measuring the input to the transformer and then subtracting the transformer losses. Since the transformer losses are usually large compared with the dielectric losses, it is preferable to adopt the arrangement suggested by Hendricks, Fig. 1, and connect the wattmeter in the high-tension winding. The wattmeter then reads directly the dielectric losses at any voltage, the only error being that due to copper losses in the high-tension winding, which may be corrected for. The watts lost divided by the square of the voltage gives the effective conductance of the dielectric.

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[R. G. Hudson.]

INSULATOR PINS. — (*See also Cross Arms; Insulators for Overhead Lines.*) Insulator pins are made of wood, iron and various combinations of wood, iron and porcelain.

WOODEN PINS. — Wooden pins are usually made of locust but sometimes of oak, birch, maple or eucalyptus.

The forest service reports: "Black locust is admitted to be the best of all woods used for insulator pins, but of late years other woods are being brought into use. Among those which will probably prove satisfactory as substitutes for the black locust are Osage orange, various oaks, yellow birch, gum, hard maple, elm, etc." Also: "The value of eucalyptus, particularly blue gum, for insulator pins has been thoroughly demonstrated. After fifteen years' service, sound pins are still in use."

Standard Distribution Pins (Fig. 1). — The standard wooden pin for distribution lines is of locust, 9 inches long, 1½-inch diameter shank, and 1 inch diameter thread, as shown in Fig. 1. It is often described simply as a "1½-in. by 9-in. pin." It is standard for use with double-petticoat deep-groove glass insulators and is also used for some insulators designed for higher voltages.

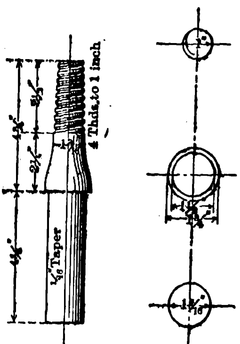

Fig. 1. Standard Locust Pin

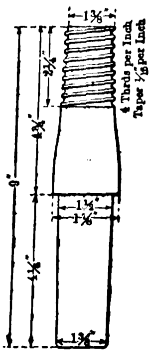

Fig. 2. Insulator Pin for 13,000 Volts

High-voltage Distribution Pins (Fig. 2). — For voltages above 2200 and not exceeding 13,200 it is desirable to have pins with 1½ inches diameter shanks to fit standard cross arms, but with larger threads, as shown in Fig. 2.

High-voltage Transmission Pins. — For voltages above 13,200 and not exceeding 25,000 wooden pins are used to a considerable extent. These pins are special and their length must be sufficient to give the clearance from insulator to cross arm required by the design of insulator used. The shanks should be larger than those of standard pins.

Paraffined Pins for High Voltage. — On high voltages wooden pins are too weak for the high stress which usually results from the great pin leverage of large insulators; such pins also rapidly deteriorate from a chemical action called "digestion," especially if insulators are too small for the voltage. Paraffined wooden pins have been used to reinforce the insulation in some cases where the insulators have been too small. Paraffined pins are useless as insulation unless the wood is first *thoroughly* dry and is then *thoroughly* impregnated. It is almost impossible to accomplish this in practice and the paraffining of commercial pins should be considered only as a preservative treatment.

Porcelain Bases for High-voltage Pins. — Porcelain bases are used to a small extent and for certain types of insulators. These extend from bottom of thread to top of arm. They are used for the purpose of preventing arc from striking pin when insulator arcs over. They act as a reinforcement of the insulation and may be used to assist an insulator which has inadequate insulation, due to incorrect design, or to enable it to be used on a higher voltage than intended. In general it is better to put all of the insulation in the insulator than to divide it between the insulator and the pin.

IRON DISTRIBUTION PINS. — Iron pins (malleable iron and drop-forged iron) are made with 1½-inch shanks and 1-inch threads so as to be interchangeable with standard wooden pins. They are used at points of heavy strain.

Pins with Iron Shanks consisting of ½-inch, ⅝-inch or ¾-inch iron bolts are also used. These have a nut and washer on the lower end used for pulling the base of the pin firmly into contact with the top of the arm. This type of pin has the advantage that the smaller hole removes less wood from the arm. For heavy stresses the shank of the pin should be stiff, or the base of the pin broad to prevent local crushing of wooden cross arms due to the pin leverage.

ATTACHMENT OF INSULATORS TO PINS. — Insulators are usually attached to wooden pins by a screw thread having a pitch of 4 threads per inch. The threaded portion of the pin is tapered, the diameter increasing downward from the top about 1/16 inch per inch of length. Standard thread diameters are 1 inch for distribution insulators and 1⅜ inches for high-tension insulators measured at the small end over the thread. Good fit in the thread between pins and insulators is not commonly attained because: (1) the threads are often imperfectly cut on the pins, (2) the wood of the pin shrinks so that it ceases to be circular or warps so that the axis of the pin is not truly straight, (3) the porcelain of the insulator warps in manufacture so that the hole ceases to be circular or the surface of the thread may not be smooth. Where a pin does not screw into the insulator to full depth intended, or where it is loose or bears unequally, the strength of the attachment is much reduced. The weakest point is usually at the root of the bottom thread.

Attachment of Insulators to Iron Pins. — Insulators are sometimes attached to iron pins by a screw thread similar to that used for wooden pins. The thread on the iron pin is often made of lead to get a more uniform bearing. High-tension insulators are frequently cemented to iron pins. The pin hole of the insulator may be threaded as usual. The end of the pin is usually corrugated or fluted instead of being threaded. The pin is grouted to insulator with neat Portland cement.

ATTACHMENT OF PINS TO ARMS. — The standard wooden distribution pin has a shank of 1½-inch nominal diameter. The actual diameter at the top is the same as the nominal diameter, the shank tapering so that the bottom is about 1/16 inch smaller. The pin hole in the cross arm is bored to the nominal diameter but often becomes smaller (across the grain) due to shrinkage of the wood, so that a standard pin, although smaller than nominal size of the pin hole except at extreme top, will often make a good driving fit. Pins which are too small (or pin holes which are too large) will make a loose fit and should not be used. Pins are often made considerably oversize at the top and undersize at the bottom to allow for inaccuracy of workmanship. Such pins will not drive completely in, and they weaken the construction by increasing the pin leverage and decreasing the bearing surface between the pin and cross arm. At the top of the shank a shoulder should be provided to limit the distance the pin can be driven in. For maximum strength the shank should be long enough to go completely through the arm and the taper should be small enough so that it will

bear against the arm for the full depth without leaning far to one side. The standard 1½-inch by 9-inch pin has a 4¼-inch shank length corresponding to the old "standard" depth of cross arm, and, therefore, does not develop the full strength of the later standard arms which are 4½ inches or 4¾ inches deep.

Pins are prevented from pulling out of the arm (especially where the wire exerts an uplift on the insulator and pin) by driving a nail (usually six-penny) through arm and pin.

Wooden pins for high-tension insulators are made with larger diameter shanks, 2 inch, 2¼ inch and 2½ inch being used.

Comparative Strength of Some Special High-tension Pins.—The following are the results of tests made on pins fitted into wooden cross arms. The pins broke at the top of the shank.

Material	Dimensions of shank, inches		Pin leverage, inches from center of stress to top of arm	Breaking strength, pounds
	Diameter	Length		
Oak	2¼	6	12	140
Oak	2¼	6	16	120
Oak	2½	6	12	205
Oak	2½	6	16	120
Eucalyptus	2¼	6	about 10	260
Red oak	2½	7¼	12½	495
White oak	2½	7¼	12½	738
Locust	2½	7¼	12½	1121

COSTS.—The following figures, giving the cost, are rough approximations.

Material	Dimensions in inches			Approximate cost per 1000
	Diam., Shank	Overall length	Diam., Top	
Standard Oak (Fig. 1)	1½	9	1	$ 7.50
Standard Locust (Fig. 1)	1½	9	1	14.00
Special Locust	1½	10¾	1⅜	22.00
Solid Drop-forged Galvanized Iron	1½	9	1⅜	500.00

BIBLIOGRAPHY.—*Report of Committee on Overhead Line Construction*, Trans. N.E.L.A., 1911; Lindquist, R. A., *Transmission Line Construction*, N. Y., 1912; *Forest Service Circular, No. 179;* Ohio Brass Co., *Catalog No. 12;* Western Electric Co., *Catalog Bulletin No. 74.*

[R. A. PHILIP AND CABOT STEVENS.]

INSULATORS FOR OVERHEAD LINES. — (*See also Distribution Lines; Insulator Pins; Transmission Lines.*) Insulators for overhead lines may be classified as insulators for distribution lines, pin insulators for transmission lines and suspension insulators for transmission lines. The general features of design common to all classes of line insulators will first be considered.

DESIGN OF LINE INSULATORS. — Glass is commonly used for making small low-voltage insulators and porcelain for large insulators. Porcelain has greater mechanical strength, condenses less moisture on its surface and parts may be cemented together, but faults cannot be as readily detected as in glass. Glass insulators are often completely shattered by a blow while porcelain insulators usually are only chipped under similar circumstances. Various compositions are also used to a limited extent for line insulators.

Properties of Porcelain and Glass. — Following are some of the more important characteristics of porcelain and glass:

Property	Glass	Porcelain
Tensile strength, lb. per sq. in.	2,500–9,000	1,500–2,200
Crushing strength, lb. per sq. in.	6,000–10,000	14,000–16,000
Elastic limit, lb. per sq. in.	3,200 approx.	....
Modulus of elasticity, lbs. per sq. in.	8,000,000	2,500,000 (*b*)
Coeff. of expansion per °F.	0.0000046	0.00000585 (*b*)
Coeff. of expansion per °C.	....	0.00000413 (*a*)
Weight per cubic foot in lbs.	160	155
Puncturing strength in volts per inch	300,000	400,000
Specific inductive capacity (air = 1)	5 to 10 (*a*)	4.38 (*a*)

(*a*) Smithsonian Physical Tables. (*b*) Austin, A. O., *Proc. N.E.L.A.*, 1913.

In American practice porcelain parts are made from ¼ inch to ⅝ inch in thickness. In European practice porcelain is used in thicknesses up to 1 inch. The working voltage usually averages from 10,000 to 20,000 volts for each thickness of porcelain used. 15,000 volts is about an average figure, so that a 30,000-volt insulator usually has two parts, a 40,000-volt insulator 3 parts and a 60,000-volt insulator 4 parts.

Requirements to be Met in a Satisfactory Design. — Shapes that are very long or very large in diameter are not economical to manufacture. The design of high-tension insulators is a "cut and try" process and many of the principles have not yet been reduced to a scientific basis. An insulator must be designed to stand extreme and sudden temperature changes, sleet and rain as well as smoke, dust and sometimes special conditions such as salt storms and salt-water spray without deterioration from chemical action, breakage from mechanical strain or electrical failure.

Some of the principal points in the electrical design of high-voltage insulators are:

Thickness to Resist Puncture. — The porcelain must be thick enough to resist puncture. If this thickness is greater than is desirable from a manufacturing standpoint, two or more pieces are used to give the proper aggregate thickness.

Arcing Distance; Free Arcing. — The porcelain must extend beyond the charged conducting connections (i.e., tie wire or cap at the top and pin at

the bottom) sufficiently so that the distance between the connections through the air around the porcelain is greater than the arcing distance of the maximum voltage to be carried. The arcing distance required for a given voltage may be determined roughly (but only very roughly) from the tables of arcing distances between needle points; see article on *Spark Gap*. The greater radius of curvature (compared with needle points) of such metal parts as the insulator pin decreases the potential gradient at the terminals. Also the porcelain has a much greater specific inductive capacity than the air and its proximity to the arcing path disturbs the electrostatic field through the air. Surface charges on the porcelain because of surface leakage or corona also modify the field.

Free arcing is the property of arcing over along a line which does not touch the porcelain body from the point where the arc leaves the metal cap to where it strikes the metal pin. Where the arc touches any part of the porcelain the great heat fractures the porcelain in a few seconds; hence the desirability of designing the insulator so that it is free arcing. A properly designed insulator will arc over as a whole before any individual part (i.e., shell or unit) arcs over. In many defective designs the insulator will fail by some parts arcing over, thereby increasing the voltage on others which then fail by puncture or arcing over.

Factor of Safety Against Puncture. — The thickness of a porcelain part must be so related to the distance around it that it will arc over before it will puncture. The ratio of puncture strength to arc over voltage is the factor of safety of the part, or of the insulator, against puncture.

Guard Rings and Rods. — Where an insulator is not naturally "free arcing" from cap to pin, properly located rings or rods will divert the arc so as to accomplish an equivalent result. Rings used for this purpose are known as Nicholson guard rings. Where a pin insulator is improperly designed so that it is liable to puncture before flashing over, a Nicholson ring around the base may be used to redistribute the potential or to reduce the arcing distance so as to reduce trouble from puncture. Such rings and rods are also sometimes used in connection with suspension insulators.

Rated voltage	Length of leakage surface, inches
5,000	4
6,600	5½
10,000	6¾
23,000	12
44,000	29
66,000	53

Leakage Surface. — Leakage surface is ordinarily measured as the number of (linear) inches from cap (or equivalent) to pin taken radially along the surface; see Fig. 3. This, however, is only a rough measure as it neglects the varying width of leakage path. The figures in the accompanying table on pin insulators from a manufacturers' catalog give approximately the total amount of leakage surface used in practice. The amount of surface allowed varies considerably between different designs and makes.

Spread of Petticoats of Pin Insulators. — In two concentric shells the two surfaces which lie opposite to each other are at different potentials except where they are cemented together. The difference in potential between two points on opposite surfaces is greater the further they are removed from the joint. Unless the shells diverge correspondingly so as to increase the distance between the shells as the potential increases the air will break down and part of the leakage surface will be short-circuited by a corona discharge. This divergence is shown in Fig. 3, where the top is a disc made slightly con-

vex to shed water and the inner shells are a series of cones which radiate from the top of the pin.

Color and Glazing. — Brown, slate and white are the common colors used in glazing porcelain. Brown is the most common color since it is more of an aid in determining faults. Slate colored glazing of the same color as galvanizing on towers makes insulators a less conspicuous target for malicious destruction.

Cementing of Insulators. — Insulator parts are cemented together with neat freshly burned and finely-ground Portland cement which is usually allowed to set under water from ten to fourteen days before testing mechanically. The more freshly burned and finely ground the cement and the higher the temperature, the less time required for setting. The cemented surfaces of porcelain are unglazed and corrugated to obtain good bond.

Faults in Insulators. — The more common faults in porcelain are folds and flaws in moulding and the development of checks and hair cracks in process of drying, incomplete and non-uniform glazing, warping, air bubbles, conducting impurities, under- and over-firing and chipping of edges. Only 50 to 75 per cent of moulded shapes ordinarily pass final test and even a less number of the more difficult shapes. Inspection and testing are essential to eliminate faults in both design and manufacture.

DISTRIBUTION-LINE INSULATORS, STANDARD (Fig. 1). — For ordinary distribution circuits, including constant-potential circuits up to 2200 volts nominal and series-arc circuits of all voltages, the double-petticoat deep-groove (D.P.D.G.) insulator, of the type shown in Fig. 1, is standard.* Such insulators are usually of glass, though occasionally of porcelain, and in cases where there are heavy stresses, as at dead ends where iron pins are required, the insulators are made of moulded mica. The dimensions vary with different manufacturers.

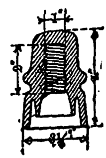
Fig. 1.

These insulators are used with the standard pin shown in Fig. 1 in the article on *Insulator Pins*.

Pony Insulators. — These are small insulators used for telegraph and telephone wires. They are unsuitable for electric light and power wires. The groove is too small (sometimes only ¼ in.) to properly support the larger wires used for lighting and the leakage surface is insufficient for proper insulation.

Insulators with Top Groove. — The standard distribution insulator has a groove in the side to which the wire is tied. For very heavy cables special insulators with groove in top are used. Insulators for higher voltage usually have both side and top groove, the former used where there is horizontal stress due to angle in line, and the latter on straight parts of line where the weight is the principal force.

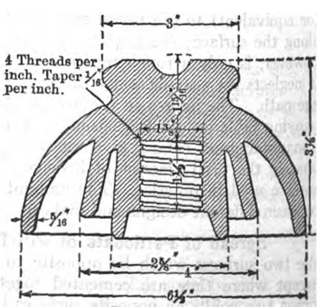

Fig. 2.

DISTRIBUTION-LINE INSULATORS, HIGH-VOLTAGE (Fig. 2). — For circuits exceeding 2200 volts

* The same size insulator is used for 110-volt and 2200-volt circuit. This has the advantage that only one kind of insulator is kept in stock, and a wire can be transferred from use on a low-voltage circuit to one of higher voltage without reinsulating it.

but not exceeding 13,200 volts nominal, it is convenient to have insulators which can be used on the standard distribution pins. A number of insulators are made with 1-in. threads to fit the standard 1½-in. by 9-in. pin. It is better, however, to use stronger special pins with 1⅜-in. top; see article on *Insulator Pins*. Fig. 2 shows an insulator which has been used on these voltages. It is used with the pin shown in Fig. 2 in the article on *Insulator Pins*. The manufacturer rates this insulator at 23,000 volts with a factor of safety (wet) of 2.

TRANSMISSION-LINE INSULATORS, PIN-TYPE (Fig. 3).—The pin-type insulators are similar in mechanical construction to the low-voltage insulators used for distribution lines, but are larger and designed for heavier mechanical stresses as well as higher voltages. The conductor is supported on a top groove except where an angle in the line causes a lateral pressure, in which case the side groove is used. For voltages up to about 10,000 volts the insulator usually consists of but a single porcelain part. For higher voltages from two to five parts are cemented together. The size and expense of pin-type insulators increase rapidly with the voltage so that while they may be used for operating potentials of 70,000 and even more, their use is generally restricted to potentials of not over 50,000. A typical three-part insulator is shown in Fig. 3. The separate parts are called parts, shells or cones and are numbered in order beginning with the top.

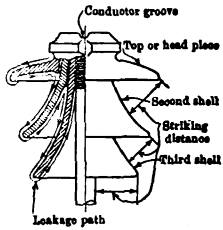

Fig. 3.

Tie Wires and Clamps.—Tie wires consisting of the same material as conductors are commonly used for fastening conductor to insulator. Two common types of ties are shown in Figs. 4 (single tie wire) and 5 (double tie

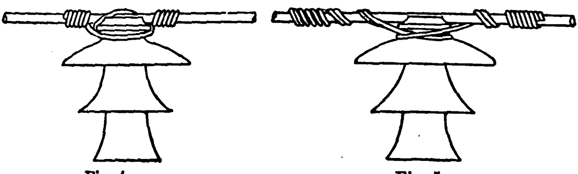
Fig. 4. Fig. 5.

wire). For copper conductors the tie wires are usually soft-copper wires which are three sizes on the A.W.G. (B. & S. gage) smaller than the conductor. Metal caps carrying clamps for conductor are cemented to tops of insulators when more strength is required than can be obtained by tie wires.

TRANSMISSION-LINE INSULATORS, SUSPENSION TYPE (Figs. 6 to 8).—The suspension-type insulator is always used in tension, the connections at the two ends being made so that the insulator is free to swing in any direction; the insulator takes such a position that its axis coincides with the direction of the mechanical stress. This type is used hanging below the cross arm with axis vertical as a suspension insulator for sustaining the weight of cable at points where there is little horizontal force, and also with axis approximately horizontal as a "strain" or "dead-end" insulator at points where the horizontal force predominates.

The suspension type consists of one or more complete insulators, called "units," connected in a string. Each unit consists of one or more insulating

parts usually with a metal cap above and a metal pin below, all cemented together. Typical designs of suspension units are shown in Figs. 6 to 8.

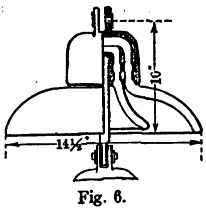

Fig. 6.

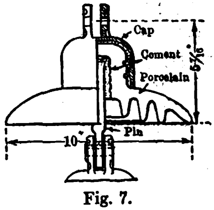

Fig. 7.

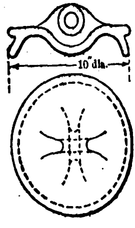

Fig. 8.

Relation Between Electrical Strength of Single Unit and a String of Units.—The potential per unit required to flash-over suspension insulator composed of units of the same design decreases with an increasing number of units as shown in Fig. 9. This is probably due to an unequal potential gradient.

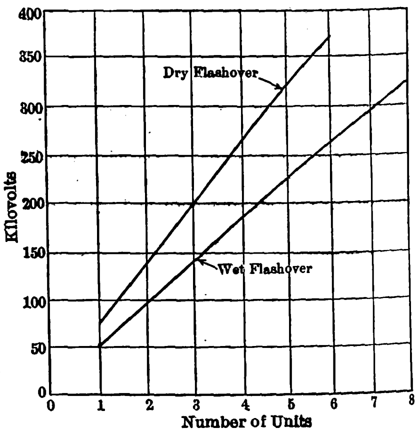

Fig. 9.

Connections Between Suspension Units.—The caps and pins of suspension insulators are commonly made of galvanized malleable iron castings, but where strength is required the pins are steel drop forgings. The common types of connection between the pin of one insulator and the cap of the one next below are ball and socket, clevis and pin (Figs. 6 and 7) and hook and eye. The hook and eye is the simplest type, but there is danger of its becoming unhooked and, in order to obtain strength, the design requires more space between units than other types of connection. The clevis and pin has the disadvantage of loose parts. The ball-and-socket connection requires special terminal fittings and a rigid connection to prevent the clamp from turning when used on dead-end insulators for conductors which tend to untwist. It can, however, be designed with less space between units than other types.

A type of suspension insulator which does not require caps and pins is shown in Fig. 8. These units are connected together by cable loops. This was one of the first types of suspension insulators. It is not commonly used on account of cost of manufacturing. The holes for cables require filling with cement or other material to prevent the insulators breaking by the collection and freezing of water in them in cold climates. A very close inspection is often required to find punctured units that are not shattered by the failure.

Conductor Clamps for Suspension Insulators. — A common form of insulator clamp is shown in Fig. 8 in the article on *Wires and Cables, Bare*. These clamps are commonly made of malleable iron castings. Fig. 10 shows a pressed steel clamp designed for and used on the Keokuk-St. Louis transmission line. This clamp is used on both suspension and "dead-end" or strain insulators. Clamps should have an ultimate holding power of not less than 50 per cent in excess of the elastic limit of the cable for which they are used.

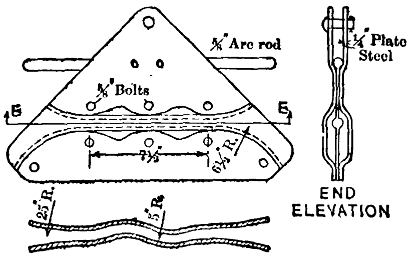

Fig. 10.

TESTS OF INSULATORS. — Tests of insulators may be classified as (1) design tests, made on a very few insulators to determine the characteristics of new designs, and (2) routine tests made on each insulator manufactured to detect defects of material or workmanship.

Design Tests. — Design tests are quite expensive for very high-voltage insulators because of the difficulty of securing suitable testing apparatus.

Mechanical Tests should be made to determine the strength. With pin-type insulators the important point is the strength of pin and insulator combined against a force (representing a horizontal wire pull) applied at the wire groove in a direction at right angles to the axis of the pin. For suspension-type insulators the tensile strength is the important mechanical consideration. These tests should be carried to the point of destruction of a number of samples.

Electrical Tests should be made for puncture strength, arc-over voltage dry and wet, free arcing properties and corona formation. The electrodes used should conform in shape to the cap and pin used in practice so that all surface exposed to puncture when the insulator is in use will be tested. Design tests of the electrical properties of suspension insulators should be made on complete strings of insulators as well as on single units.

Puncture Test. — As the air surrounding a properly designed insulator acts as a safety valve, it is necessary to immerse the insulator in oil in order to test the material of which the insulator is composed up to its puncture strength at normal frequency. Such a test may be used to ascertain the margin by which the puncture strength exceeds the flash-over voltage and to verify the dielectric strength of the porcelain. High frequencies are found to produce puncture in air in insulators which arc over before they puncture at normal operating frequencies. Standard methods of using high frequencies as a means of testing have not been developed. The frequencies used for normal tests include all commercial frequencies or say from 25 to 125 cycles per second with 60 cycles the most common.

Arcing Tests. — Arcing tests, both dry and wet, are made to determine the voltage at which an insulator will arc over when the voltage is raised with the insulator in its normally dry condition and when wet as in rainy weather. The dry arcing test is the most common one made because of its simplicity, the voltage merely being raised on cap and pin until arc over occurs. Wet test in new designs is very important as there is danger of puncture or arc over of individual parts which appear safe on dry test. The standard precipitation

for a wet test is 1/5 inch of distilled water per minute inclined at an angle of 45° with the axis of the insulator. The wet arcing test usually gives a lower arc-over voltage and hence gives a measure of the electrical factor of safety of the line (ratio of voltage at which failure occurs to working voltage) under the weather conditions when the factor is usually the lowest.

Free Arcing Tests. — In order to test the free-arcing properties the testing transformer must be large enough so that the flash at time of arc over will continue as a true power arc; even then the phenomena of fracture will rarely occur, even if the insulator is not free-arcing, because of the comparative feebleness of the power used in testing.

Corona Formation is indicated by the emission of a sound (i.e., the insulator is not "quiet") and may be still more accurately detected by the light from the corona when insulator is tested in the dark. Corona is considered an evidence of defective design and it is considered desirable that the voltage of corona formation should not be much below the arc-over voltage and certainly not below working voltage. However, many and perhaps most, high-voltage insulators show some corona at working voltage, though the amount is so small that it is difficult to detect.

Routine Tests. — No high-voltage insulator should be used without having been tested at the factory. Mechanical tests, except on a few selected samples, are not ordinarily made on pin-type insulators. On suspension insulators a tension test on each unit is desirable, but should not exceed about one-half the expected ultimate strength, for this is sufficient to eliminate defective ones, and an insulator is permanently injured at a point somewhat below its ultimate strength.

Flash-over or arcing tests should be made on each porcelain part before assembly (so that no poor part will get into any insulator) and on the whole insulator when assembled. Each part should be tested to flash-over potential for five minutes. The cup-shaped parts (also complete pin-type insulators) may be tested by setting them inverted in a pan of water used as one electrode, and partly filling the cup with water for the other. A large number of similar parts are ordinarily tested simultaneously in this way. Assembled suspension units are tested by using pin and cap as electrodes. The string of units forming a complete suspension insulator is not tested, except for a few selected samples (design tests). Factory tests of parts can be made at the factory with a transformer which will give a moderately high voltage, and no special measuring instruments are necessary, as the fact that parts are tested to arc-over voltage is evidenced by the fact that the arc-over is visible.

SPECIFICATIONS FOR INSULATORS. — (*See also article on Specifications.*) Several specifications for high-voltage insulators have recently been suggested as suitable for "Standard Specifications"; see papers by Peek, Sanford and Thomas in *Transactions of A.I.E.E.*, July, 1913. Specifications covering the inspection and tests of porcelain high-tension line insulators for over 25,000 volts, which had been prepared by the High Tension Transmission Committee of the A.I.E.E., were presented at the 1914 annual convention of the Institute. The reader is referred to the above-mentioned papers for details.

INSTALLATION OF INSULATORS. — See the articles on *Distribution Lines* and *Transmission Lines*.

WEIGHTS AND COSTS. — The approximate dimensions, weights and costs of glass and porcelain pin-type insulators and for porcelain suspension units for different operating and test voltages are as follows:

PIN TYPE

Material	Operating voltage, volts	Test voltage		Diameter, inches	Height, inches	Number of parts	Weight, pounds	Cost, dollars
		Wet, volts	Dry, volts					
Glass.......	110-2,200	......	......	3¼	4	1	1¼	0.03
Porcelain...	13,200	40,000	80,000	6½	3¾	2	3⅜	0.18
Porcelain...	22,000	45,000	72,000	7	5	2	5	0.50
Porcelain...	33,000	60,000	90,000	9	8	2 or 3	8	0.75
Porcelain...	44,000	80,000	110,000	10½	10	3	13	1.20
Porcelain...	50,000	95,000	120,000	12	11	3	18	1.50
Porcelain...	60,000	115,000	150,000	14	13	4	27	2.00

SUSPENSION TYPE UNITS

Diameter, inches	Number of parts	Spacing, inches	Test voltage		Ultimate strength, pounds	Working stress, pounds	Weight, pounds	Cost, dollars
			Wet, volts	Dry, volts				
10	1	5½	50,000	75,000	8,000	4,000	11	1.00
12	1	6½	50,000	75,000	9,000	4,500	13	1.40
14	2	9	65,000	90,000	12,000	6,000	20	2.00

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[R. A. PHILIP, CABOT STEVENS and E. A. EKERN.]

INTEGRALS. — (*See also Derivatives; Equations, Differential.*) If the function $y = f(x)$ be plotted as a curve (see Fig. 1) and the axis of x be divided into a number of very small sections, each of width dx, the area between the curve and the axis of x may be conceived as the sum of the products obtained by multiplying each length dx by the corresponding altitude or ordinate y. Then

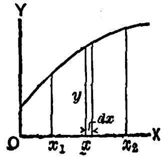

Fig. 1.

$$\text{Area} = \int y\,dx,$$

where $\int$ stands for "sum."

The Definite Integral. — The area between the values x_1 and x_2 is written

$$\int_{x_1}^{x_2} y\,dx.$$

Such an expression is called a "definite integral."

It is evident from the figure that the area of the curve between x_1 and x_2 is equal to the difference between the area from o to x_2 and the area from o to x_1. Hence calling the former area $F(x_2)$ and the latter area $F(x_1)$, we have

$$\int_{x_1}^{x_2} y \cdot dx = F(x_2) - F(x_1).$$

This equation is the general expression for any definite integral.

Indefinite Integrals. — The above equation may also be written

$$F(x_2) = \int_{x_1}^{x_2} y\,dx + F(x_1)$$

and if x_2 is considered a variable, the subscripts are omitted, as follows:

$$F(x) = \int y\,dx + A,$$

where A is a constant for any given reference point x_1. This constant A is an arbitrary constant, since any point x_1 may be chosen as the reference point, but when a point is once chosen, A is fixed. The expression $\int y\,dx$ is called an "indefinite integral" and A is called the "integration constant."

From the definition of a derivative (*see Derivatives*), the derivative of $F(x)$ is

$$\frac{dF(x)}{dx} = \frac{\int_{x_1}^{x+dx} y dx - \int_{x_1}^{x} y\,dx}{dx}$$

and from the figure, it is evident that the difference between the area from x_1 to $x + dx$ and the area from x_1 to x is simply $y\,dx$. Hence

$$\frac{dF(x)}{dx} = y,$$

that is, the integral $F(x)$ of any function y, with respect to x, must be such a function that when differentiated with respect to x, the result is the function y.

Formulas for Integration. — u, v, x and z are variables; a, m and n are constants.

$$\int (u+v)\,dx = \int u\,dx + \int v\,dx$$

$$\int y\,dx = \int y\,\frac{dx}{dz}\,dz$$

$$\int u\,dv = uv - \int v\,du$$

$$\int \frac{F'(x)}{F(x)} = \log F(x)$$

where $F'(x)$ stands for $\dfrac{dF(x)}{dx}$.

TABLE OF INTEGRALS

Function $f(x)$	Integral $\int f(x)dx$	Function $f(x)$	Integral $\int f(x)dx$
x^m	$\frac{1}{m+1}x^{m+1}$	$\frac{1}{\sqrt{a^2-b^2x^2}}$	$\frac{1}{b}\sin^{-1}\frac{b}{a}x$
$\frac{1}{ax}$	$\frac{1}{a}\log_\epsilon ax$	$\frac{1}{\sqrt{a^2+b^2x^2}}$	$\frac{1}{b}\sinh^{-1}\frac{b}{a}x$
ϵ^{ax}	$\frac{1}{a}\epsilon^{ax}$	$\frac{1}{a^2+b^2x^2}$	$\frac{1}{ab}\tan^{-1}\frac{bx}{a}$
a^{bx}	$\frac{1}{b\log a}a^{bx}$	$\frac{1}{(a^2-b^2x^2)_{bx<a}}$	$\frac{1}{ab}\tanh^{-1}\frac{bx}{a}$
$\cos ax$	$\frac{1}{a}\sin ax$	$\frac{1}{(a^2-b^2x^2)_{bx>a}}$	$\frac{1}{ab}\tanh^{-1}\frac{a}{bx}$
$\sin ax$	$-\frac{1}{a}\cos ax$	$\frac{1}{(x-a)(x-b)}$	$\frac{1}{a-b}\log\frac{x-a}{x-b}$
$\tan ax$	$-\frac{1}{a}\log(\cos ax)$	$\frac{1}{\sqrt{2ax-x^2}}$	$2\sin^{-1}\sqrt{\frac{x}{2a}}$
$\cosh ax$	$\frac{1}{a}\sinh ax$	$\frac{1}{x\sqrt{x^2-a^2}}$	$-\frac{1}{a}\sin^{-1}\frac{a}{x}$
$\sinh ax$	$\frac{1}{a}\cosh ax$	$\frac{1}{x\sqrt{x^2-a^2}}$	$\frac{1}{a}\cos^{-1}\frac{a}{x}$
$\tanh ax$	$\frac{1}{a}\log(\cosh ax)$	$\frac{1}{x\sqrt{a^2+x^2}}$	$-\frac{1}{a}\sinh^{-1}\frac{a}{x}$
$\tan x\sec x$	$\sec x$	$\frac{1}{x\sqrt{a^2-x^2}}$	$-\frac{1}{a}\cosh^{-1}\frac{a}{x}$
$\sec^2 ax$	$\frac{1}{a}\tan ax$	$\frac{x}{\sqrt{a^2\pm x^2}}$	$\pm\sqrt{a^2\pm x^2}$
$\frac{1}{\cos^2 ax}$	$\frac{1}{a}\tan ax$	$\frac{x}{\sqrt{x^2-a^2}}$	$\sqrt{x^2-a^2}$
$\frac{1}{\sin^2 ax}$	$-\frac{1}{a}\cot ax$		

Function $f(x)$	Integral $\int f(x)dx$
$\sqrt{a^2-x^2}$	$\frac{1}{2}\left(x\sqrt{a^2-x^2}+a^2\sin^{-1}\frac{x}{a}\right)$
$\sqrt{x^2+a^2}$	$\frac{1}{2}\left[x\sqrt{x^2+a^2}+a^2\sinh^{-1}\frac{x}{a}\right]$
$\sqrt{x^2-a^2}$	$\frac{1}{2}\left[x\sqrt{x^2-a^2}-a^2\cosh^{-1}\frac{x}{a}\right]$
$\sin ax \sin bx$	$\frac{\sin(a-b)x}{2(a-b)}-\frac{\sin(a+b)x}{2(a+b)}$
$\cos ax \cos bx$	$\frac{\sin(a-b)x}{2(a-b)}+\frac{\sin(a+b)x}{2(a+b)}$
$\sin^2 ax$	$\frac{1}{2}x-\frac{1}{4a}\sin 2ax$
$\cos^2 ax$	$\frac{1}{2}x+\frac{1}{4a}\sin 2ax$
$\sinh ax \sinh bx$	$\frac{\sinh(a-b)x}{2(a-b)}-\frac{\sinh(a+b)x}{2(a+b)}$
$\cosh ax \cosh bx$	$\frac{\sinh(a-b)x}{2(a-b)}+\frac{\sinh(a+b)x}{2(a+b)}$
$\sinh^2 ax$	$\frac{1}{2}x-\frac{1}{4a}\sinh 2ax$
$\cosh^2 ax$	$\frac{1}{2}x+\frac{1}{4a}\sinh 2ax$
$\epsilon^{ax}\sin bx$	$\frac{\epsilon^{ax}(a\sin bx-b\cos bx)}{a^2+b^2}$
$\epsilon^{ax}\cos bx$	$\frac{\epsilon^{ax}(a\cos bx+b\sin bx)}{a^2+b^2}$
$\frac{\epsilon^{mx}}{x}$	$\log_\epsilon x+\frac{mx}{1}+\frac{m^2x^2}{2!}+\frac{m^3x^3}{3!}+\text{etc.}$
$\sin^m x\cos^n x$	$-\int \sin^{m-1}\cos^n x\, d\cos x$

Double Integrals. — Just as the area of a surface may be represented by the expression

$$\int y \cdot dx$$

so the volume of a solid may be represented by

$$\int\left[\int y \cdot dx\right]dz$$

or adopting the usual notation, by

$$\int\int y\, dx \cdot dz$$

taken between limits determined by the data of the problem.

[W. A. Del Mar.]

INTEREST, ANNUITIES AND SINKING FUND. — Let

P = principal invested in dollars;
n = number of years principal is invested;
r = rate of interest, per cent per annum;
A = total amount of principal and interest at end of n years.

Note. — The following relations hold irrespective of the unit of time selected, provided r is taken as the interest earned per $100 during that time.

Simple Interest. — If the principal is invested at simple interest, then

$$A = \left(1 + \frac{nr}{100}\right)P.$$

Compound Interest. — If the principal is invested at compound interest, i.e., if the interest earned each year is invested at the end of that year at the same rate as the original principal, then the total amount due at the end of n years is

$$A = PR^n,$$

where

$$R = 1 + \frac{r}{100}.$$

R^n is the amount of the principal of 1 dollar and interest at the end of n years. The following table gives the value of R^n for n ranging from 1 to 50 years and r from 3 to 6 per cent.

VALUES OF $R^n = \left(1 + \frac{r}{100}\right)^n$

Years n	Per cent interest = r				Years n	Per cent interest = r			
	3	4	5	6		3	4	5	6
1	1.03	1.04	1.05	1.06	16	1.6047	1.8730	2.1829	2.5403
2	1.0609	1.0816	1.1025	1.1236	17	1.6528	1.9479	2.2920	2.6928
3	1.0927	1.1249	1.1576	1.1910	18	1.7024	2.0258	2.4066	2.8543
4	1.1255	1.1699	1.2155	1.2625	19	1.7535	2.1068	2.5269	3.0256
5	1.1593	1.2166	1.2763	1.3382	20	1.8061	2.1911	2.6533	3.2071
6	1.1941	1.2653	1.3401	1.4185	21	1.8603	2.2787	2.7859	3.3995
7	1.2299	1.3159	1.4071	1.5036	22	1.9161	2.3699	2.9252	3.6035
8	1.2668	1.3686	1.4774	1.5938	23	1.9736	2.4647	3.0715	3.8197
9	1.3048	1.4233	1.5513	1.6895	24	2.0328	2.5633	3.2251	4.0487
10	1.3439	1.4802	1.6289	1.7908	25	2.0937	2.6658	3.3863	4.2919
11	1.3842	1.5394	1.7103	1.8983	30	2.4272	3.2433	4.3219	5.7435
12	1.4258	1.6010	1.7958	2.0122	35	2.8138	3.9460	5.5159	7.6862
13	1.4685	1.6651	1.8856	2.1329	40	3.2620	4.8009	7.0398	10.2858
14	1.5126	1.7317	1.9799	2.2609	45	3.7815	5.8410	8.9847	13.7648
15	1.5580	1.8009	2.0789	2.3965	50	4.3838	7.1064	11.4670	18.4204

The following table gives the number of years required for a given principal to double itself at compound interest.

Interest Rate	3	4	5	6
Years to Double	23.5	17.7	14.2	11.9

Annuities. — An annuity is a fixed sum of money paid yearly, or at other equal times agreed upon.

One dollar invested at interest at r per cent at the beginning of every year will at the end of n years amount to

$$R \cdot \frac{R^n - 1}{R - 1} \text{ dollars,}$$

where $R = 1 + r/100$, the interest being compounded at the end of each year.

One dollar invested at the beginning of a period of n years will yield at the end of each year an annuity of

$$\frac{R^n (R - 1)}{R^n - 1} \text{ dollars,}$$

where $R = 1 + r/100$, the interest being compounded at the end of each year.

Sinking Fund. — A sinking fund is a fund built up from fixed yearly payments or annuities. Sinking funds are usually provided to retire bonds, which are issued for a given number of years. The annuity required to retire a bond of $1000 at the end of n years is

$$1000 \left(\frac{R - 1}{R^n - 1} \right).$$

Values of this annuity for various rates of interest and for various values of n are given in the following table.

ANNUITY REQUIRED TO REDEEM $1000

At end of years	Rate of interest, per cent								
	2	2½	3	3½	4	4½	5	5½	6
2	495.05	493.78	492.69	491.42	490.20	489.00	487.80	486.62	485.43
3	326.72	325.14	323.56	321.94	320.36	318.77	317.21	315.63	314.10
4	242.63	240.84	239.02	237.26	235.50	233.74	232.01	230.29	228.60
5	192.16	190.24	188.35	186.49	184.63	182.79	180.98	179.13	177.39
6	158.53	156.56	154.61	152.67	150.79	148.88	147.02	145.18	143.36
7	134.52	132.49	130.51	128.57	126.61	124.67	122.82	120.96	119.13
8	116.51	114.47	112.46	110.48	108.53	106.60	104.72	102.86	101.03
9	102.52	100.46	98.44	96.44	94.49	92.57	90.69	88.83	87.02
10	91.33	89.25	87.24	85.24	83.29	81.38	79.50	77.67	75.87
11	82.18	80.11	78.07	76.09	74.15	72.25	70.39	68.57	66.79
12	74.56	72.49	70.46	68.48	66.55	64.67	62.83	61.03	59.28
13	68.12	66.05	64.03	62.06	60.14	58.27	56.45	54.68	52.96
14	62.60	60.54	58.53	56.57	54.67	52.82	51.02	49.28	47.58
15	57.83	55.77	53.77	51.82	49.94	48.11	46.34	44.62	42.96
16	53.65	51.60	49.61	47.68	45.82	44.01	42.27	40.58	38.95
17	49.97	47.93	45.95	44.04	42.20	40.42	38.70	37.04	35.44
18	46.70	44.67	42.71	40.82	38.99	37.24	35.54	33.92	32.36
19	43.78	41.76	39.81	37.94	36.14	34.40	32.75	31.15	29.62
20	41.15	39.14	37.22	35.36	33.58	31.87	30.24	28.68	27.18
25	31.22	29.27	27.43	25.67	24.01	22.44	20.95	19.55	18.23
30	24.65	22.78	21.02	19.37	17.83	16.39	15.05	13.80	12.65
35	20.00	18.20	16.54	15.00	13.58	12.27	11.07	9.97	8.97
40	16.55	14.84	13.26	11.83	10.52	9.34	8.28	7.32	6.46
45	13.91	12.27	10.78	9.45	8.26	7.20	6.26	5.43	4.70
50	11.82	10.26	8.87	7.63	6.55	5.00	4.78	4.06	3.44

[W. A. Del Mar.]

IRON, PIG AND CAST. — (*See also Iron, Wrought; Castings, Iron and Steel; Magnetic Properties of Iron; Steel.*) Chemically pure iron is not a commercial product. Iron of a very high degree of purity may be obtained, however, by electrolytic deposition. The microscopical constituent of pure iron is called ferrite, and pure iron is said by metallographists to be composed of polyhedral crystalline grains of ferrite. Commercial products are pig iron, cast iron, malleable cast iron, wrought iron and steel.

PIG IRON. — Pig iron is the crude product obtained from iron ore by smelting in a blast furnace. The name is applied either to the molten material or to rough castings varying in length from 30 inches to 36 inches and in cross-section from about 10 sq. in. to 30 sq. in. A mass of piled pig iron 8 by 10 by 12 ft. weighs approximately 100 tons. Pig iron is made by burning a mixture of iron ore, coke or other fuel and limestone in a high furnace through which a blast of heated air is forced under pressure. The liquid iron when drawn from the furnace is either run into sand molds formed on the ground near the furnace, or into cast-iron molds forming a part of a conveyor chain which receives the molten iron, chills it by contact, and delivers it at the car. A pig is the bar of iron formed in either of these types of molds. The molten iron is also occasionally conveyed to the steel furnace without being allowed to cool.

Classification. — Pig iron may be classified as follows:

a. Gray, white or mottled as determined by the color of its fracture and its chemical composition.

b. Coke, anthracite or charcoal iron as determined by the fuel used in reducing the ore.

c. Foundry iron, Bessemer iron, etc., according to the use to which it is to be put.

Use of Pig Iron. — The product of the blast furnace is irregular in composition and physical properties and is not used directly. By forming it into pigs it is possible to grade it, and also transport it to foundries and iron and steel furnaces. Pig iron is sometimes used by engineers because of its great weight to form counterweights in unbalanced structures such as movable bridges, but concrete is more economical for this purpose.

Strength. — See below in the section on *Cast Iron*.

Specifications for Pig Iron. — See Standard Specifications for Foundry Pig Iron in Year Book of American Society for Testing Materials.

Cost. — The cost of pig iron is an important factor in commerce. Quotations are given weekly in *The Iron Age* and in the first issue of each month of the *Engineering News*. The fluctuations in price are very considerable, Bessemer pig at Pittsburg varying during 1912 from $14.90 to $18.15 per long ton of 2240 pounds.

CAST IRON. — Cast iron may be defined as iron containing so much carbon that it is not malleable at any temperature, specifically, iron cast into articles of specific form and purpose as distinguished from pig iron. Except for special cases castings are made of gray iron. Cast iron is made from pig iron by melting the latter and casting in sand or iron (chill) molds. The furnace commonly used is a cupola furnace although reverberatory furnaces are sometimes employed.

Composition of Cast Iron. — The composition of an iron casting is identical with that of the pig iron from which it is made and usually consists of metallic iron (ferrite) accompanied by from 2.5 per cent to 4 per cent (by weight) of carbon, together with silicon, phosphorus, manganese and other impurities.

The carbon may be chemically combined with the iron, giving homogeneous white iron, or some of it may be precipitated in cooling in the form of graphite, making gray iron or mottled iron. The influence of carbon upon the properties of the cast iron is of great importance and may be summed up as follows:

Uncombined Carbon or Graphite. — Porosity and workability increase and shrinkage decreases, with an increase in the percentage of graphite. Strength also generally decreases with an increase in graphite, although it should be noted that an iron may be weak by having too much combined carbon and that treatment which changes some of this combined carbon to graphite increases its strength.

Combined Carbon. — In slowly-cooled white iron all the carbon occurs in the form of an alloy, called cementite, which forms in cooling, and the iron partakes of the characteristics of cementite, being hard and brittle, the latter characteristic increasing with an increase in the percentage of the cementite. The carbon in this condition is called combined carbon.

Silicon. — This acts as a precipitant of carbon, drawing it out of combination into graphitic form. The maximum precipitation occurs with from 2.5 per cent to 3.5 per cent of silicon. Beyond this percentage the opposite effect is noted. Increase of silicon up to 3.5 per cent therefore softens the iron, decreases shrinkage, imparts fluidity and reduces the strength. The maximum density of gray iron occurs with about 1 per cent of silicon; more than 2 per cent causes porous iron.

Sulphur. — Sulphur increases the amount of combined carbon, its effect in this direction being far greater than that of silicon in reducing the combined carbon, it being generally considered that 0.01 per cent S will neutralize 0.15 per cent Si. The amount of sulphur should be restricted to a very low percentage. In the Standard Specifications of the A.S.T.M. the sulphur content is limited to not over 0.08 per cent for light castings, not over 0.10 per cent for medium castings and not over 0.12 per cent for heavy castings.

Manganese. — This increases the total carbon and the proportion of carbon in combined form, although by combining with sulphur and reducing the effect of the latter, the net result may be a decrease in the combined carbon. It strengthens the iron if below 1 per cent, strengthens but increases brittleness if between 1 per cent and 1.5 per cent, and if over 1.5 per cent decreases strength and toughness and increases hardness and shrinkage.

Phosphorus. — This causes expansion after solidification, therefore making the iron useful for very thin castings. It, however, tends to make the iron weak and brittle and should be kept below the following values:

	Per cent
Chilled castings	0.3
Malleable castings	0.2
Gray castings	0.7

Use of Cast Iron. — Cast iron should in general not be used where tensile or bending strength is required. Its cheapness and high crushing strength make it useful for heavy parts of machinery, and for pieces where intricate patterns not easily made by tools are needed. It is frequently used for water pipes, despite its low tensile strength, and often for columns in buildings, although cast-iron columns have been superseded to a considerable extent by steel columns, except for the simplest forms of construction. Car wheels of cast iron with the outer surface chilled by iron molds are much used.

Compressive Strength. — The ultimate compressive strength of cast iron is from 60,000 to 200,000 lb. per sq. in. A safe working value for cases where

column action cannot occur is 16,000 lb. per sq. in. For columns the following formula may be used if the applied load is not eccentric.*

$$\frac{P}{A} = 6100 - 32\frac{l}{d},$$

in which

P = total allowable load in pounds,
A = cross-section area in square inches,
l = length unrestrained against lateral deflection in inches,
d = diameter, or shortest side of rectangular column, in inches.

Tensile Strength. — Ultimate strength may vary from 15,000 to 35,000 lb. per sq. in., but is ordinarily from 18,000 to 22,000 lb. per sq. in. For cast-iron water pipes, it is usual to specify 3,300 lb. per sq. in. or 1/5 of the tensile strength, assuming the latter to be 16,500 lb. per sq. in.

Modulus of Elasticity varies from 10,000,000 to 30,000,000 lb. per sq. in. For ordinary foundry iron the modulus of elasticity may be taken as from 12,000,000 to 15,000,000 lb. per sq. in.

Elastic Limit.— Cast iron has no well-defined elastic limit.

Specifications for Cast Iron. — See *Year Book of the American Society for Testing Materials.*

Cost of Cast Iron. — See *Castings, Iron and Steel.*

MALLEABLE CAST IRON. — This is the name given to cast iron which has had a portion of the combined carbon changed to graphitic carbon in the form of a fine powder by reheating white iron to a temperature somewhat below the melting point. The process used in the United States generally eliminates the carbon entirely from the outer layer. Malleable cast iron is used for parts of agricultural machinery, pipe fittings which have to be threaded, plow shares, etc. It has a much higher tensile strength (from 40,000 to 50,000 lb. per sq. in.) than ordinary cast iron and is more ductile. It is frequently sold as some special form of steel, and should be carefully guarded against by purchasers of small articles such as bevel gears, hammers and automobile drop forgings.

Specifications for Malleable Cast Iron. — See Standard Specifications for Malleable Castings in Year Book of the American Society for Testing Materials.

Cost of Malleable Iron Castings. — See *Castings, Iron and Steel.*

BIBLIOGRAPHY. — See *Bibliography* in article on *Steel.*

[C. M. SPOFFORD.]

* See article on *Structures, Simple* for effect of eccentricity on building columns. It should be noted that the allowable loads on cast-iron columns as given in the building laws of various cities, and summarized in the article on *Buildings, Allowable Unit Stresses in*, are in many instances too high for conservative practice.

IRON, WROUGHT. — (*See also Iron, Pig and Cast; Magnetic Properties of Iron; Steel.*) If commercial iron is mechanically mixed with a suitable amount of slag (*see article on Steel*) there results a malleable material called wrought iron which does not harden when suddenly cooled. It melts at a full white heat, but becomes pasty at a lower temperature, in which condition it can be readily welded. It is ductile when cold.

PROCESS OF MANUFACTURE. — Practically all wrought iron is produced from pig iron by indirect processes although direct processes for production from the ore exist. These indirect processes may be divided into two general classes based upon the type of furnace used: (*a*) reverberatory or puddling furnaces; and (*b*) charcoal hearths. The best iron is made upon hearths, but puddling furnaces produce the larger quantity. The essential difference between the two processes is that in hearths the chief source of oxidation is atmospheric air, and the fuel is burned in contact with the iron, while in puddling furnaces the chief source of oxygen is magnetic oxide of iron, and the fuel is burned in a chamber separate from that containing the iron. A description of these processes follows.

Puddling. — This method consists of melting pig iron in a reverberatory furnace heated either by coal or natural gas. The furnace hearth is lined with oxide of iron. The pig is exposed for about two hours to the continuous action of a flame hot enough to melt it and to remove most of the impurities, but not hot enough to keep pure iron in a molten state. By the action of the flame the molten iron becomes less fusible and finally pasty. After reaching this condition it is puddled by being worked into balls by hand labor. It is then taken from the furnace and squeezed or hammered into blooms, and then rolled into small bars about ¾ inch thick and from 2 to 6 inches wide, called "muck bars." After cooling, these muck bars are cut into short pieces about 2 feet in length, piled into bundles, fastened by iron wire, reheated to welding heat and re-rolled into merchant bars. If the iron is subjected to a second piling, heating and re-rolling, it is called "double refined iron."

Charcoal Hearths. — The following are the more important hearth processes.

1. **Finery Process.** — Charcoal fineries produce "knobbled" iron of a high degree of softness which is much used for boiler tubes.

In the finery process the pig iron is first melted down in a coke or charcoal refinery to remove the silicon, phosphorus, and sulphur, and is then transferred in a molten condition to a charcoal hearth which is still hot from its previous charge. Damp charcoal is thrown in, a low-pressure, unheated blast turned on and the metal agitated to keep it in contact with the blast. After an hour or more the metal is collected into a ball and hammered to remove some of the slag, cut up and reheated in piles. Gray iron may be used in this process.

2. **Walloon Process.** — The Walloon process is used in Sweden for producing wrought iron from Dannemora pig iron, the resulting product being shipped to England, particularly to Sheffield, for conversion into blister steel for use in fine toolmaking.

In the Walloon process long pigs are melted gradually by being pushed forward into a charcoal fire. The molten iron drops through the blast, becoming decarburized, and collects in a pasty mass at the bottom of the furnace. The partially refined iron is then raised to the top of the charcoal fire, and melted down with the addition of rich slag and hammer scale. The metal is then balled, reheated and hammered.

3. Lancashire Process. — The Lancashire process is used principally in Sweden, but is also used in the United States.

The Lancashire process somewhat resembles the Walloon process. Pig iron is melted between two layers of charcoal, the liquid dropping down through the blast and becoming oxidized. The molten metal collects in a pasty mass at the furnace bottom where it is allowed to remain for twenty or twenty-five minutes; it is then mixed with decarburizing slag and remelted in a similar manner. Finally the pasty mass is removed from the hearth and hammered.

Busheled Iron. — "Busheled iron" is made of scrap instead of pig iron. The scrap is heated in a furnace, squeezed and rolled into bars. The resulting product is of inferior quality.

COMPOSITION. — Wrought iron consists of a mass of ferrite interspersed with elongated particles of slag. Commercial shapes of wrought iron, such as plates and rods, are made from piles of muck bars, and have a fibrous character since they consist of a series of welds.

USE. — Wrought iron has been gradually replaced by steel for most structural purposes. Good quality of iron, however, is still in demand where toughness and ductility are necessary, and where welding or other blacksmith work is to be done. Iron of a high degree of purity is sometimes specified for use where a non-corrosive material is needed.

GRADES. — Wrought iron is on sale in the eastern states under the following classification:.

Norway (or Swedish Iron). — Best grade; used for fine wrought work and machine work. It is particularly fibrous.

Double Refined or Best Refined. — This is the best domestic iron. It is used for forging, welding or machine work.

Common Iron.— This is the cheapest grade. Used for nails, horseshoes, etc. It does not weld as readily as other grades.

STRENGTH AND ELASTICITY; WEIGHT. — The values stated in the specifications which follow indicate the tensile strength of wrought iron of the grades specified. The following values are from tests made upon merchant iron at the Massachusetts Institute of Technology, and show the variations that may occur between the various grades bought in the market without specifications. The specifications which follow show the strength which may be obtained in material of the grades specified therein.

TENSILE STRENGTH, MODULUS OF ELASTICITY, ELONGATION, ETC., OF WROUGHT IRON

(M.I.T. Tests)

Item	Single Refined	Double Refined	Swedish
Ultimate strength, lb. per sq. in.....	47,000	51,700	40,600
Elastic limit........................	32,000	25,800	20,900
Yield Point..........................	35,000		
Modulus of Elasticity...............	27,700,000	29,700,000	29,100,000
Per cent of elongation in 10 inches..	....	22.7	26.4
Reduction per cent..................	20.6	37.5	75.0

Compressive Strength. — For all practical purposes this may be taken as the yield point, or from 3000 lb. to 4000 lb. above the elastic limit.

Weight. — Wrought iron weighs 480 lb. per cu. ft. A bar 1 yard in length and 1 sq. in. in cross-section weighs 10 lb.

STANDARD SPECIFICATIONS FOR WROUGHT IRON. — See *Year Book of the American Society for Testing Materials.*

COST. — The following quotations are taken from *The Iron Age*, Jan. 2, 1913.

Iron bars, Philadelphia......................1.675 cents per lb.
Pittsburgh........................1.70 cents per lb.
Chicago..........................1.575 cents per lb.

On the same date small quantities were for sale in Eastern cities at the following prices:

Norway iron, 3.00 cents to 3.60 cents per lb.
Double Refined or Best Refined, 2.50 cents per lb.
Common Refined, 2.00 cents to 2.10 cents per lb.
Borden's Best Bar Iron, 3.35 cents per lb.

Prices vary considerably, weekly quotations being given in *The Iron Age.*

BIBLIOGRAPHY. — See bibliography in article on *Steel.*

[C. M. Spofford.]

LAMPS, ARC. — (*See also Arc, Electric; Distribution and Transmission Systems; Lamps, Incandescent; Illumination, Street; Photometric Quantities; Photometry; Rectifiers.*) The theory of the electric arc is discussed in detail in the article on *Arc, Electric*. Arc rectifiers are discussed in the article on *Rectifiers*. This article deals with the use of the electric arc as a source of illumination.

The positive and negative electrodes are termed respectively anode and cathode. The active conducting medium of the arc is supplied by the electro-vaporization of the cathode. The wasting of the anode is due only to its oxidation and can be prevented without interrupting the arc by cooling or by isolation from air. With the exception of the carbon arc, it is more difficult to maintain an arc with alternating than with direct current.

Voltage Drop Across an Arc. — The voltage drop across an arc comprises three elements, (1) a sensibly constant drop at the anode, (2) a similar but much smaller drop at the cathode and (3) a variable drop in the arc stream. The electrode drops depend only on the nature of the electrodes. The drop in the arc stream varies with the current according to Ohm's law, but the relation is greatly complicated by the changes in the resistance of the arc stream. Steinmetz (*Radiation, Light and Illumination, p. 139*) proposes the following general expression for arc conduction in air:

$$E = E_0 + \frac{k\,(l + l_1)}{\sqrt{I}},$$

where E is the total voltage, E_0 the electrode drop, l the arc length in inches, I the current, and k and l_1 constants depending on the cathode material.

For carbon electrodes, $E_0 = 36$, $k = 130$, $l_1 = 0.33$.
For magnetite-copper electrodes, $E_0 = 30$, $k = 123$, $l_1 = 0.05$.

Steadying Resistance. — When the arc is employed in a series circuit its instability is overcome by supplying it with a constant current. Arcs in multiple on constant-potential circuits must each be compensated by a considerable ballast of stable series resistance or reactance; see article on *Arc, Electric*.

Power Factor. — Due to the distortion of the current wave by the varying resistance of the arc, the alternating-current carbon arc has a power factor of about 85 per cent, although the current and voltage pass through their zero values simultaneously; see *Alternating Currents*. Reactive ballast coils used in constant-potential lamps reduce the over-all power factor to values between 60 and 75 per cent.

Sources of Luminosity. — There are three distinct modes of light production by electric arcs, viz., (1) by incandescence of the electrodes due to their high temperature, (2) by the luminescence in the arc of salts derived from mineralized carbon electrodes, and (3) by the luminescence in the arc of the conducting vapors from the cathode. The first is exemplified by the ordinary carbon arc. The second mode is exemplified by the flame arc. The color of the light and its efficiency depend on the nature of the luminescent salts and the temperature attained in the arc. Carbon electrodes heavily mineralized with calcium salts yield a yellow light of high efficiency. Barium salts yield white light and strontium red, but the efficiencies are lower than those attained with yellow light. The third mode is exemplified in the metallic arcs, the chief practical representatives being the magnetite and the mercury arcs. Here the color of the light depends solely on the natural spectrum of the cathode vapor and the efficiency depends on the richness of that spectrum in highly luminous components.

Steadiness of Light. — Light from arc lamps is inferior to that from incandescent lamps in the matter of steadiness. The conditions requisite for

steady light are constant arc length, constant current, fixity of arc position, uniform feed, homogeneous electrodes and complete protection from drafts. When well ballasted the mercury arc in the vacuum tube best meets these conditions. The alternating-current arc displays instantaneous variations in intensity which are very annoying at low frequencies. Arcs should not be operated at less than 40 cycles per second where considerations of hygiene are important. The three-phase arc is free from this defect.

CARBON ARC LAMPS.— Unless used for indirect light the upper electrode of a direct-current carbon arc should be the anode in order to gain the fullest advantage of downward light distribution from the crater. In the alternating-current arc semi-craters of lower temperature are formed on both electrodes, resulting in lower efficiency of light production than in the direct-current arc. The performance of typical carbon arc lamps is given in Table I and Fig. 1.

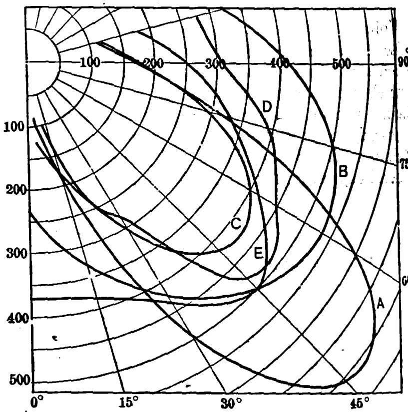

Fig. 1. Light Distribution of Typical Carbon Arcs.

A. 6.6-amp. series D-C. open arc, clear globe.

B. 6.6-amp. series D-C. enclosed arc, opal inner, and clear outer globe.

C. 7.5-amp. series A-C. enclosed arc, opal inner, and clear outer globe, small reflector.

D. 5.5-amp. multiple D-C. enclosed arc, 110 volts, opal inner, clear outer globe.

E. 5-amp. multiple D-C. intensified arc, 110 volts, opal inner, and clear outer globe.

Open Carbon Arcs. — The open arc is characterized by the rapid oxidation of the electrodes due to the free access of air. The anode of a direct-current open arc wastes at a rate between 1 and 2 inches per hour and the cathode at about half the rate of the anode. The electrodes of open alternating-current arcs burn from 1 to 1.5 inches per hour. A life per trim sufficient for all-night operation may be obtained by the use of a single pair of very long, heavy carbons or by providing two sets for successive consumption. The open arc is very sensitive to drafts. It is operated with short arc length, low voltage, usually 45 to 55, and heavy current, usually 6.6 to 10 amperes, to improve the steadiness of the light. The short arc length impedes downward light distribution and the intense crater casts harsh shadows. The erratic travel of the arc about the electrodes makes the light very unsteady. The use of the open arc is now confined to series circuits.

Inclosed Carbon Arcs. — The inclosed carbon arc with closely restricted air supply affords a greatly increased electrode life over the open type. It is also much less liable to extinction by wind, and hence can effectively employ greater arc length, higher voltage and lower current than the open arc. The crater of the inclosed arc is less pronounced, entailing some loss of efficiency compared with the open arc, but the greater length of the arc reduces the interference of the lower carbon with downward distribution. The light is much steadier than

TABLE I. PERFORMANCE OF TYPICAL CARBON ARC LAMPS

Item	Multiple d-c. inclosed	Multiple d-c. inclosed	Intensified d-c. inclosed	Multiple 220-volt d-c. inclosed	Multiple a-c. inclosed
Terminal volts	110	110	110	220	110
Volts at arc	80	80	80	150	72
Amperes	5	6.5	5	3.25	6
Watts	550	715	550	715	430
Power factor	....	....	....	....	0.65
Diameter of electrodes, inches	½	½	Two ¼, One ⅜	½	⅓
Reflectors and glassware.	Porcelain reflector. Opal inner, no outer globe.	Porcelain reflector. Opal inner, no outer globe.	Porcelain reflector. Opal inner; no outer globe.	Porcelain reflector. Opal inner, no outer globe.	Porcelain reflector. Opal inner, no outer globe.
Life per trim	150 hours	100 hours	75 hours	150 hours	125 hours
Maximum C.P.	400 at 60°	580 at 60°	600 at 35°	240 at 60°	310 at 60°
M.S.C.P.	215	318	225	160	160
Watts per M.S.C.P.	2.56	2.25	2.44	4.44	2.68
M.L.H.C.P.	379	559	414	215	276
Watts per M.L.H.C.P.	1.45	1.28	1.33	3.33	1.56

Item	Multiple a-c. inclosed	Series d-c. open	Series d-c. open	Series d-c. inclosed	Series a-c. inclosed	Series a-c. inclosed
Terminal volts	110	50	50	75	77	77
Volts at arc	72	48	48	73	72	72
Amperes	7.5	6.6	9.6	6.6	6.6	7.5
Watts	540	330	480	495	425	480
Power factor	0.65	.....	.....	......	0.84	0.84
Diameter of electrodes, inches	½	½ 4 pairs	⅝ 2 pairs	½	½	½
Reflectors and glassware.	Porcelain reflector. Opal inner, no outer globe	Clear globe.	Clear globe.	Porcelain enamel reflector, Clear globes.	Porcelain enamel reflector, Clear globes.	Porcelain enamel reflector. Clear globes.
Life per trim	100 hours	18 hours	18 hours	125 hours	125 hours	100 hours
Maximum C.P.	410 at 60°	720 at 45°	1250 at 45°	530 at 60°	250 at 70°	305 at 70°
M.S.C.P.	215	265	460	290	144	173
Watts per M.S.C.P.	2.51	1.25	1.02	1.71	2.95	2.77
M.L.H.C.P.	371	395	690	479	232	291
Watts per M.L.H.C.P.	1.45	0.82	0.71	1.03	1.83	1.65

that from the open arc. It is bluer in color, due to the greater proportion derived from the arc proper. Electrodes of moderate length afford a life per trim of from 80 to 100 hours in alternating-current lamps and of 100 to 150 hours in direct-current lamps. The range of currents in common practice is from 3 to 7.5 amperes.

Intensified Carbon Arc. — The intensified carbon arc, used only on direct-current circuits, employs a short, thick vertical lower carbon as cathode and two long, thin, inclined carbons as anode. The upper carbons converge just above the cathode and feed downward at the rate of consumption without the aid of any regulating mechanism. The lower carbon is controlled by a feeding solenoid and tends to maintain an arc of constant length at a fixed level. The high current density in the positives produces a very intense dual crater which emits light of somewhat whiter quality and of higher efficiency than that of the ordinary inclosed arc. The fixed level of the arc and the absence of crater travel aid in the effective use of reflectors and diffusing globes and in the maintenance of a definite form of light distribution. Under standard conditions of use the 5-ampere arc has a life per trim of from 80 to 100 hours.

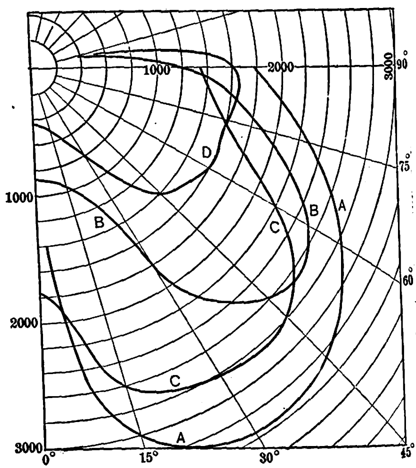

Fig. 2. Light Distribution of Typical Flame Arcs

A. 10-amp., 55-volt, D-C. yellow flame arc, clear globe, inclined electrodes.

B. 10-amp., 63-volt, A-C. yellow flame arc, clear inner, opalescent outer globe, vertical electrodes, enclosed type.

C. 10-amp., 55-volt, D-C. yellow flame arc, opal globe, inclined electrodes, open type.

D. 10-amp., 55-volt, A-C. white flame arc, clear inner and outer globes, vertical electrodes, enclosed type.

FLAME ARC LAMPS. — The flame arc depends for light production on the luminescence of salts impregnated in carbon electrodes. In its electrical characteristics it is essentially a carbon arc. The evolution of this type of lamp has been along two structural lines, one having vertical electrodes, introduced by Blondel, and the other having converging electrodes, introduced by Bremer. In both types both electrodes are fed by the regulating mechanism and the arc is maintained at a fixed level beneath a vitreous canopy or economizer which serves to conserve the heat of the arc, deflect the fumes to the ports at the periphery of the housing, and to assist in the downward reflection of light. Flame arcs may be either open or inclosed. The performance of typical flame arcs is given in Table II and Fig. 2.

Open-flame Arcs. — The open type provides positive ventilation to sweep the fumes of the arc from the arc inclosure and to prevent their deposit on the inclosing globe. The unrestricted air supply causes a rapid wasting of the electrodes. To obtain a practicable life per trim the electrodes are made either

TABLE II.—PERFORMANCE OF TYPICAL FLAME ARCS

Item	Open d-c. inclined carbons	Open a-c. inclined carbons	Open d-c. vertical carbons	Open d-c. vertical carbons
Terminal volts	55	55	110	78
Volts at arc	45	45	75	75
Amperes	12	12	6.5	6.5
Watts	660	500	715	510
Power factor	......	0.75	......	......
Diameter of electrodes inches.	1/4	1/4	1/2	1/2
Kind of electrodes	Impregnated yellow.	Impregnated yellow.	Lower impregnated.	Lower impregnated.
Glassware	Light opal globe.	Light opal globe.	Alba globe.	Alba globe.
Life per trim, hours	17	17	20	20
Maximum C.P.	2150 at 0°	1350 at 0°	2425 at 60°	2350 at 60°
M.L.H.C.P.	1890	1270	2058	2050
Watts per M.L.H.C.P.	0.35	0.39	0.35	0.25

Item	Inclosed d-c. multiple vert. carbons	Inclosed a-c. series vert. carbons	Inclosed a-c. multiple vert. carbons
Terminal volts	110	60	110
Volts at arc	70	55	70
Amperes	6.5	10	7.5
Watts	715	450	510
Power factor	....	0.75	0.62
Diameter of electrodes, inches	7/8	7/8	7/8
Kind of electrodes	Impregnated yellow.	Impregnated yellow.	Impregnated yellow.
Glassware	Clear inner, opalescent outer globe.	Clear inner, opalescent outer globe.	Clear inner, opalescent outer globe.
Life per trim, hours	100–125	100–125	100–125
Maximum C.P.	....	....	....
M.L.H.C.P.	1740	1595	1600
Watts per M.L.H.C.P.	0.41	0.28	0.32

very long or of very great cross-section. The co-axial arrangement of very long electrodes is obviously impracticable; hence the converging arrangement. The slender electrodes afford additional advantages through the high current density at the arc, absence of arc travel and low heat loss by conduction. A magnetic blow coil is employed to hold the arc in its proper position below the electrode tips. Lamps of this type have a maximum luminous intensity in the downward direction, and hence must be hung very high to produce uniform illumination. The open-flame arc with converging electrodes has a life per trim of from 10 to 15 hours. The open type with vertical electrodes has a life per trim of from 12 to 18 hours, but produces a less steady light than the converging type, due to arc travel.

The ventilating system of open-flame arcs requires very careful design. Unless the fumes are positively removed from the inclosing globe, the accumulation of powder on the globe during the life of a single pair of electrodes may reduce the light transmitted by as much as 40 per cent.

Lamps of the open type are greatly handicapped by the high cost of maintenance and electrodes.

Inclosed Flame Arcs. — The inclosed type of flame arc is provided with a condensing chamber into which the fumes of the arc are carried by convection, as shown in Fig. 3. The gases are thus cooled and freed from their solids without rapid ingress of air. The life of the electrodes is thus prolonged about ten times, giving a life per trim of from 100 to 150 hours. The electrodes used in inclosed flame arcs are less highly mineralized than those used in open arcs and the gain in life is accompanied by some loss in efficiency. The converging carbon type of flame arc has not been developed in the inclosed form.

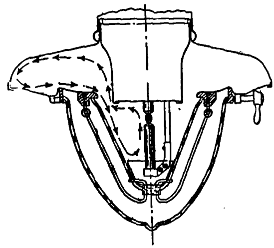

Fig. 3. Method of Condensation of Gas in Inclosed Flame Arc

MAGNETITE ARC LAMPS. — The magnetite arc depends for light production solely on the luminescence of the conducting vapors produced at the cathode. The cathode consists of a thin iron tube closely packed with a uniform powdered mixture of magnetite, oxide of titanium and oxide of chromium. The positive electrode is a block of copper of large heat-radiating capacity. The magnetite produces an arc of excellent conductivity, but of relatively lean spectrum. The titanium oxide enriches the spectrum of the arc, improving its color and efficiency. The oxide of chromium serves to restrain the rate of vaporization of the active elements and so prolong their life. The resulting light is of excellently balanced white color. The copper anode is not consumed and requires infrequent replacement. The cathode has a life of from 100 to 250 hours depending on the current and arc length. It is not practicable to operate the magnetite arc in an inclosure with limited air access, due to the need of ventilation to remove the copious brown fumes. The light of the magnetite arc is relatively unsteady, a feature which is aggravated by the intermittent method of feeding employed.

The two commercial types of magnetite arc differ principally in the electrode arrangement. In the General Electric type the positive is the upper; in the Westinghouse type the negative is the upper. The advantages claimed for each type conflict sharply.

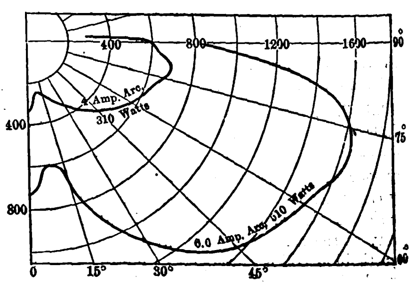

Fig. 4. Light Distribution of Magnetic Arc in Clear Globes

Unlike other types of arc lamps the magnetite arc is poorly adapted to interior use on constant potential circuits. Its field is therefore largely confined to street lighting on constant-current circuits. While the magnetite arc is solely a direct-current device, it is readily adapted to use in connection with an

alternating-current supply by the use of the mercury arc rectifier combined with the constant-current transformer; see *Rectifiers*.

The voltage range per lamp on series circuits is from 75 to 80. The standard currents are 4, 5 and 6.6 amperes. The performance of typical magnetite arcs is given in Table III and Fig. 4.

TABLE III.—PERFORMANCE OF TYPICAL MAGNETITE ARCS

Item	Series type	Series type	Multiple type
Terminal volts	78	78	110
Volts at arc	75	75	75
Amperes	4	6.6	6.5
Watts	312	515	715
Glassware	Clear globe.	Clear globe.	Clear globe.
Life per trim, hours	175	125	80
Maximum C.P.	700 at 80°	1650 at 80°	1650 at 80°
M.L.H.C.P.	545	1340	1340
Watts per M.L.H.C.P.	0.59	0.38	0.53

LOSS OF CANDLE POWER OF CARBON AND FLAME ARCS.—The light output of all types of arc lamps falls off gradually during the life of each set of electrodes, due to the accumulation of ash and fumes on the glassware. The amount of this loss varies greatly with the conditions. Matthews (*Nat. Elec. Light Assn., 1901, p. 296*) reported tests of inclosed carbon arcs of which the following results are typical, viz., reduction of light output in 100 hours, best grades of carbons 5 per cent, low-grade carbons 30 per cent. In carbon arcs this loss depends chiefly on the purity of the carbons employed. The loss in flame arcs depends on the scheme of ventilation and the extent to which the carbons are mineralized. Composite results of many tests on inclosed flame arcs reported in *Good Lighting*, Vol. 7, p. 515, show a loss in 100 hours burning of 23.5 per cent of the initial mean lower hemispherical candle-power. Tests on General Electric inclosed white flame arcs made at the Massachusetts Institute of Technology (*Wright and Sprowls, 1912*) showed the average reduction of light in 150 hours burning to be 18 per cent. The loss of light in magnetite arcs is relatively small, due to the positive character of the ventilation. The importance of very careful cleaning of glassware of arc lamps at each trimming is apparent from the above data.

EFFECTS OF DIFFUSING GLOBES.—The intrinsic brilliancy of all arcs is sufficiently high to seriously interfere with good vision when in the direct field of view. Diffusing globes reduce the light by from 25 to 40 per cent through absorption, but generally serve to increase the effectiveness of vision to a degree which fully compensates for the loss of light. Diffusing globes tend to modify light distribution toward greater uniformity at various vertical angles; hence pronounced side-wise distribution must be attained by a properly designed reflector.

REGULATING MECHANISMS FOR CARBON AND FLAME ARCS.—The regulating mechanism of an arc lamp serves the following functions: (1) to strike the arc upon lighting, (2) to feed one or both electrodes at the rate of consumption and (3) to maintain approximately constant the arc length, voltage and current. Feeding systems are of two general classes. In the first the electrodes are fed by gravity upon the releasing of a clutch controlled by solenoids. As the feeding steps are small and frequent the process approaches a uniform continuous feed. In the second type the arc is restruck at each feeding, as in the magnetite arc.

Gravity-feed Mechanism for Multiple Carbon Arcs.—The mechanisms of constant-potential lamps are practically all of the gravity-feed type. The commonest type has a ballast of resistance or reactance in series with a solenoid and with the arc itself; see Fig. 5. When the arc is not lighted the carbons are in contact. When the switch is closed the solenoid is energized and draws its plunger, tightens the clutch and strikes the arc by drawing the carbons apart. As the arc lengthens the current falls until the forces on the plunger are balanced. As the arc lengthens by burning away, the current tends to fall, releasing the clutch and permitting the carbon to feed until checked by the rising of the plunger. In a well-adjusted lamp this process is practically continuous. In alternating-current mechanisms of this type the plungers are laminated and the ballast consists of a reactive coil.

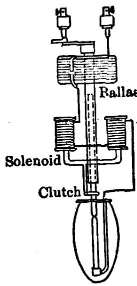

Fig. 5. Multiple Carbon Arc Lamp

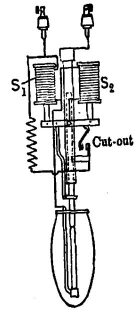

Fig. 6. Series Carbon Arc Lamp

Gravity-feed Mechanism for Series Carbon Arcs.—The commonest type of mechanism for gravity-feed lamps of the series class employs the differential action of two solenoids to control the feeding clutch. Fig. 6 shows this type of mechanism as applied to the inclosed carbon arc. It provides a solenoid S_1 in series with the arc, a solenoid S_2 and resistance coil in parallel with the arc, and a short-circuiting cut-out. The solenoids act differentially on the clutch. The shunt solenoid when strengthened tends to release the clutch, the series solenoid to lift it.

When the lamp is idle the carbons are normally in contact. When current is switched on, the series solenoid is energized and draws out the arc. The cut-out opens, S_2 is energized and tends to oppose the drawing action of S_1, equilibrium being attained when the arc has the proper length. As the carbons burn apart S_2 is strengthened and releases the clutch momentarily, allowing the upper carbon to feed by gravity. If the circuit through the arc is accidentally broken, S_2 alone is energized and the cut-out is immediately closed, preventing the interruption of the circuit. Alternating-current mechanisms have laminated plungers, but differ in no essentials from those in direct-current lamps.

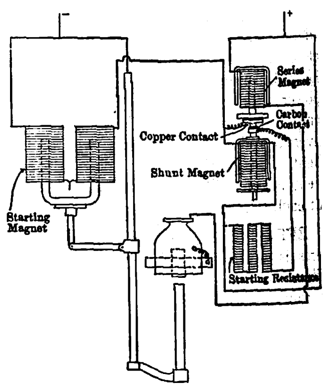

Fig. 7. Mechanism and Circuits of G. E. 4-amp. Arc Lamp

Intermittent Feed Mechanism.—The intermittent type of feeding mechanism for series lamps is shown in Fig. 7. When the current is switched on, the starting magnets are energized, bring the electrodes together and cause the arc to strike. As the electrode burns away the arc gradually increases in length

and potential drop until the shunt coil is sufficiently energized to momentarily short-circuit the arc and cause the striking process to be repeated. As the electrodes come together with considerable force the accumulated slag is broken from the end of the cathode.

Feed Mechanisms for Flame Arcs. — The mechanisms of flame arcs with vertical carbons differ only in details from those of carbon arcs of analogous type, with the exception that the feeding device is dual and acts on both carbons, tending to maintain the arc at a fixed level. Mechanisms for converging carbon flame arcs exist in great variety. The most generally-used type for constant potential operation has a series coil whose plunger acts laterally on one electrode to draw out the arc and gravity feed. In many cases an escapement clutch controlled by a shunt solenoid is used to regulate the rate of feed. In another type the rate of feed is controlled by a metal fin attached to each electrode. The base of this fin rests against a refractory lug. The fin is vaporized away by the heat of the arc at a rate exactly corresponding to the required rate of electrode feed.

MERCURY-VAPOR LAMPS. — (*See also Rectifiers.*) The luminous element of this type of lamp is a luminescent arc, in a highly evacuated tube of glass or quartz, formed between a mercury cathode and an anode of mercury or other metal not attacked by it. The voltage required to sustain such an arc, when the cathode (negative electrode) is mercury, consists of a constant drop at the electrode of about 13 volts and an arc stream voltage which is directly proportional to the length. The arc voltage per inch increases with the vapor pressure. At low vapor pressures the arc is an unstable conductor and the arc voltage per inch falls with a rising current. At high pressures, as in the quartz-tube lamp, the arc voltage rises very rapidly with the current.

The large output of actinic radiation from mercury-arc lamps gives them special advantages in the fields of photography and blue-printing. The large ultra-violet radiation from quartz-tube lamps not inclosed in glass may be utilized in a wide variety of processes for bleaching, sterilization, etc.

Power Factor of Mercury-arc Lamps. — The mercury arc can be sustained at ordinary voltages only when the mercury electrode is the cathode; this principle is utilized in the mercury-arc rectifier (*see Rectifiers*).

To operate a mercury-arc lamp with alternating current the tube is provided with two anodes which are attached to the respective terminals of a reactance coil bridging the line. The cathode is attached to the middle point of this reactance bridge. The power factor of this type of arrangement, including the wave distortion and the lag produced by the reactance coil, is from 50 to 60 per cent.

Glass-tube Mercury-arc Lamps. — The glass-tube type of lamp operates at a low vapor pressure and temperature. Its spectrum is that characteristic of the mercury arc which is entirely deficient in red lines and has a great preponderence of yellow, green and violet.

The very low intrinsic brilliancy of this type of lamp renders diffusing glassware unnecessary. Its simple color composition tends to enhance the acuity of vision. For general use the spectrum is perceptibly improved by combining with the arc a parabolic reflector coated with rhodamine enamel, which by fluorescence converts part of the blue and violet radiation incident upon it to red and orange rays.

Methods of Starting. — The mercury arc is started either by tilting the tube until mercury from the cathodes makes metallic contact with the anode and strikes the arc or by breaking down the gap by a high-potential spark discharge. As installed for general lighting the mechanism usually in-

cludes an automatic starter operating on one of the two methods referred to. The tilting mechanism is operated by a solenoid which is cut out as the arc is struck. The spark-discharge device is essentially an automatic quick-break switch which opens the circuit of an induction coil so as to cause it to discharge through the tube. The mechanism also includes a ballast coil to compensate for the instability of the arc. The tube is mounted in an inclined or vertical position. The mercury vapor condenses at the top in a condensing chamber and is restored to the cathode by gravity.

Performance of Glass-tube Mercury-arc Lamps. — Performance data on several typical lamps of this class are given in Table IV.

TABLE IV. — DATA ON GLASS-TUBE MERCURY-VAPOR LAMPS

Type	K (*D-C.*)	H (*D-C.*)	P (*D-C.*)	F (*A-C.*)
Volts	100 to 124	50 to 62*	100 to 124	95 to 250†
Amperes	3.5	3.5	3.5	....
Watts, at rated volts	385	193	385	350–430
Length of tube, inches	52	27	57	55.5
M.H.C.P.	700	300	800	670–850
Watts per M.L.H.C.P.	0.55	0.64	0.48	0.52

* To be operated as one or more pairs in series.
† Adjustable to circuit voltage by auto-compensator.

Life of Glass Tubes. — Tubes fail ultimately through breakage or through deterioration of vacuum. The life per tube varies considerably, but is usually several thousand hours of operation.

Loss of Candle Power with Age. — The light output of glass-tube lamps deteriorates quite rapidly during the early portion of tube life. Tests reported by Harrison (*Trans. Ill. Eng. Soc., Vol. 6, p. 545*) give the following average value on two type *H* tubes operated at 3.3 amperes:

Per cent initial candle power....	100	83	80	77	73	70.5	67
Hours burning................	0	250	500	1000	2000	3000	4000

Quartz-tube Mercury-arc Lamps. — The quartz-tube lamp operates at a high temperature and pressure. Its luminescent spectrum is richer than that of the low-pressure lamp and has superposed upon it a continuous spectrum of incandescence including all luminous elements. The light of the high-pressure lamp is therefore much nearer white than that of the low-pressure lamp. On account of the high pressure of the arc in the quartz tube the length required for circuits of commercial voltage is very much less than the length of glass-tube arcs. The quartz-tube lamp emits a considerable amount of invisible ultra-violet radiation which is capable of doing serious injury to the eye. Ample protection is afforded by surrounding the tube by an envelope of clear glass. Quartz tubes are nominally indestructible, but may require occasional repumping.

The performance of several typical quartz-tube mercury-arc lamps is given in Table V.

COST OF ARC LAMPS. — Open carbon-arc lamps are no longer sold. Inclosed carbon-arc lamps cost from $16 to $20 a piece. Magnetite and metallic oxide arc lamps cost from $23 to $27. The first cost of flame-arc lamps is from $33 to $45. Glass-tube mercury-arc lamps cost complete, including reactance coil, from $33 to $35, and quartz-tube mercury-arc lamps cost complete from $65 to $75; the latter, however, are not yet used to any great extent. The renewal cost of short glass tubes is approximately $5.50, that of long glass tubes $10.50 to $12.

TABLE V.—PERFORMANCE OF QUARTZ-TUBE MERCURY-VAPOR LAMPS

Type	X	Y	Z
Voltage range	100 to 125	200 to 250	450 to 625
Average current	3.8	3.3	2.0
Maximum arc voltage	90	170	345
Mean L.H.C.P., clear glass globe	1000	2400	3500
Watts per M.L.H.C.P.	0.42	0.30	0.31

Annual Costs.—The following comparative costs of arc-lamp operation under street lighting conditions for a total of 4000 hours (one year, all night service) are given by Friedman (*Elec. World, Vol. 55, p. 1071*).

ANNUAL COST OF OPERATING ARC LAMPS

Type / Amperes	Open d-c. 9.6	Open d-c. 6.6	Enc. a-c. 7.5	Enc. d-c. 6.6	Enc. a-c. 6.6	Magnetite 4.0	Magnetite 6.6
Electrodes	$5.50	$5.50	$1.50	$1.20	$1.20	$1.55	$2.85
Trimming	6.00	6.00	2.00	2.00	2.00	1.00	2.00
Repairs	2.50	2.50	1.00	1.00	1.00	0.75	0.75
Inner globes	......	......	0.45	0.45	0.45	......	......
Outer globes	0.30	0.30	0.30	0.30	0.30	0.50	0.50
Renewals of station equipment	1.50	1.50	......	1.50	......	2.00	3.00
Energy, 1.5 cts. per kw-hr	50.00	32.70	34.50	42.90	30.30	22.80	38.82
Totals	$65.80	$48.50	$39.75	$49.35	$35.25	$28.60	$47.92

The above costs do not include fixed charges. Interest may properly be taken at 6 per cent and depreciation at from 7.5 to 10 per cent.

Comparative operating costs of flame arcs are given by Blake (*Gen. Elec. Rev., Dec., 1911*) for 1000 hours of operation as follows:

Type	Open flame	Inclosed flame
Life per trim	17 hours	100 hours
Cost of electrodes per trim	$0.15	$0.20
Cost of labor per trim	0.04	0.04
Maintenance per 1000 hours:		
Electrodes	8.81	2.00
Trimming	2.35	0.40
Globes	0.09	0.22
Totals	$11.25	$2.62

In addition to these costs allowance may be made for repairs at $1 per annum and for energy at a suitable cost per kw-hr. Interest and depreciation must be allowed as before stated.

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[W. E. Wickenden.]

LAMPS, INCANDESCENT ELECTRIC. — (*See also Illumination, Laws of; Illumination, Interior; Illumination, Street.*) Incandescence denotes the emission of light by a solid or liquid body due to its temperature elevation. The performance of incandescent bodies is referred to a standard *black body,* or one which, at any temperature, emits the maximum possible intensity of radiation at each wave-length of the spectrum. The percentage of luminous radiation from incandescence is low, but increases markedly with the temperature. Hyde computes the luminous radiation from a black body as 1.9 per cent at 2000° C. Abs., 8.8 per cent at 2500° C., 16.9 per cent at 3000° C., 32.5 per cent at 4000° C., 44 per cent at 5000° C. and a maximum of 50 per cent at 6500° C. Certain materials excel the black body in relative light radiation due to *selectivity* or the relative depression of heat radiation as compared with light.

The efficiency of the incandescent lamp depends on: (a) the temperature which its filament can sustain without an undue rate of decay; (b) the degree of selectivity manifested; and (c) the prevention of thermal leakage by conduction and convection. These conditions require a filament of the highest obtainable melting point and lowest vapor tension, which must be isolated in a vacuum and supported by a system of low thermal conductivity. So fully are the two latter conditions met that the thermal leakage of modern lamps is less than 5 per cent.

Electrical and Mechanical Properties of Lamps. — The specific resistance of the filament material should be relatively high to insure favorable dimensions in units of low power and high voltage. A relatively large positive temperature coefficient of resistance is desirable as this tends to lessen the variations in performance caused by voltage fluctuations. The filament should be of great mechanical strength, especially in its resistance to fracture. The material required should be obtainable in ample quantities, readily workable and uniform in its finished state. Experience indicates that alloys are distinctly inferior to pure metals and metalloids in all essential features.

Vacuum. — Incandescent lamps require the highest practicable vacuum for the chemical and thermal protection of the filament. In the manufacture of lamps the first stage of evacuation is mechanically produced. Final exhaustion is produced chemically by burning within the inclosure a small amount of phosphorous compound. During the latter stage the filament is raised to incandescence to drive off residual gases. To prevent the loss of vacuum through the unequal expansion of the glass and the leading-in wires the latter are made of platinum.

TYPES OF LAMPS. — There are described in the following paragraphs the types of incandescent lamps which have become of commercial importance. Of the several types the gem (metallized carbon filament) and the tungsten filament are at present the most widely used.

Carbon Lamps. — Carbon vaporizes at about 3900° C. and exists in a variety of forms which vary greatly in other physical properties. *Base carbon,* prepared as lamp filaments by carbonizing squirted threads of dissolved cellulose, is of low conductivity, high vapor tension and of negative temperature coefficient of resistance. *Graphitic carbon,* which is deposited from hydrocarbon vapors on incandescent filaments of base carbon, has a high conductivity, low vapor tension and a small positive temperature coefficient. *Metallized carbon,* prepared from the two preceding types by heat treatment in an electric furnace, has a greatly increased conductivity, a pronounced positive temperature coefficient, low vapor tension and a fair degree of selectivity. The common carbon lamp has a core of base carbon on which a shell of graphitic

carbon has been deposited by the process of flashing. The hot resistance of this type is approximately half the value cold. The chief advantage of this type is its low cost and great ruggedness. It is being rapidly superseded by the metallized-carbon type, commercially known as the gem lamp, which has a higher efficiency and better color. The hot resistance of the gem lamp is 2.6 times its value cold.

Tantalum Lamps. — Tantalum melts at about 2800 deg. C; has a very low vapor tension, its atomic weight being 183; a high conductivity, 6.5 microhms per cm. cube at 25° C., and of 38.8 microhms at its operating temperature; a mean temperature coefficient between 0° and 100°C. of + 0.00234; and a valuable degree of selectivity. Tantalum is readily drawn into filaments of minute diameter. Tantalum filaments undergo a gradual deterioration in structure with use, especially when operated by alternating current. This change reduces the life of the lamp and limits its general usefulness. The tantalum lamp is now largely superseded by the tungsten lamp.'

Tungsten Lamps. — Tungsten melts at about 3200 deg. C.; has a low vapor tension, its atomic weight being 184; has a conductivity of 6.2 microhms per cm. cube at 25° C. for the hard-drawn variety and 5 microhms for the annealed; has a mean temperature coefficient of + 0.0051 between 0° and 170° C. and is selective in radiation to a valuable degree. The hot resistance of the filament is from 10.8 to 13 times the cold resistance, depending on the type of lamp. Tungsten is exceedingly hard and is worked with difficulty. Early filaments were produced by sintering finely divided tungsten reduced from tungstic oxide; they were extremely fragile and were uncertain in performance. Methods of drawing continuous filaments are now in vogue, giving a product of great strength and uniformity. Prior to 1912 much difficulty was experienced with the erratic blackening of bulbs. This fault is now (1914) largely overcome by the introduction into the bulb of a special chemical. The tungsten lamp of American manufacture is commercially known as the Mazda lamp.

Nitrogen-filled Lamps. — This type of lamp has a closely coiled helical filament of drawn tungsten wire mounted in a glass chamber filled with nitrogen or other inert gas. The pressure of the gas retards the decay of the filament so that it may be operated with a satisfactory life at a higher temperature than is practicable in a vacuum. The gain in radiant efficiency so obtained is offset in part by the convection of heat from the filament by the gas. When the diameter of the filament is minute there is little or no net gain in efficiency. When the filament is relatively heavy the net efficiency may be doubled. The helical coiling of the filament increases its effective diameter as a radiant and simplifies the problem of its support, for the filament is distinctly soft when incandescent. The gas-filled lamp has an elongated bulb, the upper portion of which serves as a cooling chamber. The walls of this chamber receive the black deposit from the filament, but are so placed that they absorb but little of the useful light. The gas-filled lamp is designed for operation in a pendant position. Such lamps are much more brilliant than vacuum lamps and should be fully shaded. The light of the gas-filled lamp is decidedly whiter than that of the vacuum tungsten lamp.

Nernst Lamps. — This type of lamp enjoyed considerable vogue prior to the development of the drawn-wire tungsten lamp but is rapidly becoming obsolete in America. Its luminous element consists of a glower of refractory rare earth oxides operating in the atmosphere. The glower is a non-conductor when cold but its resistance decreases greatly with rise of temperature. A separate heater element is employed to bring the glower to incandescence and a series ballast resistance is used to correct the instability of its conduction. After the

period of ignition the heater is disconnected by a solenoid cut-out. It is impossible to operate the glower in a vacuum owing to the electrolytic nature of its conduction. Its performance on direct current is inferior to that on alternating current. The lamp has the advantage of an inherently good light distribution downward, but its disadvantages are marked, viz., complexity, slow ignition, costly renewals, non-adaptation to ordinary sockets, and efficiency inferior to metal-filament **lamps.** Under the best conditions with clear glass globes, a performance of from 1.4 watts per mean lower hemispherical candle for the smallest to 1.10 watts per m.l.h.c.p. for the largest types is realized. The alabaster globes usually employed with Nernst lamps increase the watts per candle performances by about 10 per cent.

Use of Various Types of Lamps. — The following table shows the relative use of each of the four types of lamps produced for domestic sale in recent years as reported by the Lamp Committee of the Nat. El. Lt. Assoc. in 1913.

Type of Lamp	1907	1908	1909	1910	1911	1912
	Per cent	Per cent	Per cent	Per cent	Per cent	Per cent
Carbon	93.27	84.12	68.98	63.08	52.90	25.47
Gem	5.88	8.58	15.07	14.88	19.00	33.50
Tantalum	0.75	1.78	2.12	3.57	2.74	1.00
Tungsten	0.10	5.52	13.83	18.47	25.30	39.94

There is a marked tendency away from the carbon lamp. Many electric light companies have substituted the gem type for the carbon in their free renewal service. The total domestic consumption of incandescent lamps was approximately 85,000,000 in 1911 and 90,000,000 in 1912.

LAMP RATINGS AND PERFORMANCES. — Constant potential lamps are rated primarily by watts and normal voltage, constant current lamps by amperes and candle-power. Secondary ratings of mean horizontal candle-power, mean spherical candle-power, total lumens, watts per mean horizontal candle-power, lumens per watt, and average life at rated efficiency are usually given by makers. Constant potential lamps bear a rating tag giving normal volts and watts. The rating tag of gem lamps has three voltages differing by steps of two volts, and the normal watts for the highest voltage. This type of rating enables the user to vary efficiency and life so as to secure the most economic conditions (*see paragraph below on Cost of Light*). The following table gives approximate data on the relative performance at the three voltages.

PERFORMANCE OF GEM LAMPS AT THE THREE RATED VOLTAGES

Voltage	C. P. Per cent	W. P. C. Per cent	Watts Per cent	Life Per cent
High	100	100	100.0	100
Medium	92	105.2	96.8	140
Low	84	111.5	93.7	190

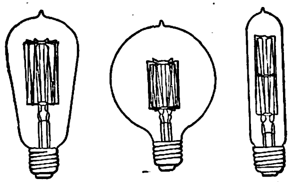
A similar method of rating other types of incandescent lamps was in use before 1914.

DATA ON CONSTANT POTENTIAL LAMPS

100 to 130 Volts

	Watts	M.H.C.P.	W. P. C.	Lumens	L. P. W.	Life, hours	Bulb
Mazda (Vacuum Type)	10	7.1	1.40	70	7.0	1500	S
	15	11.5	1.30	113	7.54	1000	S
	15	11.5	1.30	113	7.54	500	G
	20	16.0	1.25	157	7.84	1000	S
	25	21.4	1.17	210	8.40	1000	S G
	25	21.4	1.17	210	8.40	500	T
	40	34.2	1.17	336	8.40	1000	S G
	60	53.6	1.12	526	8.77	1000	S G
	100	92.6	1.08	908	9.08	1000	S G
	150	146.0	1.03	1,430	9.53	1000	S G
	250	250.0	1.00	2,450	9.80	1000	S
	400	400.0	1.00	4,070	10.18	1000	G
	500	500.	1.00	5,089	10.18	1000	G
Mazda (Gas-filled Type)	750	1150	0 65	13,000	1.74	....	Special
	1000	1670	0.60	18,900	1.89	....	Special
Gem (At top voltage)	20	5.0	4.00	52	2.60	1000	S
	30	10.0	3.00	104	3.46	1050	S
	40	15.6	2.56	162	4.05	600	S
	50	20.0	2.50	207	4.15	700	S
	50	20.0	2.50	207	4.15	500	G
	50	16.7	3.00	173	3.46	500	T
	60	24.0	2.50	249	4.15	700	S
	80	32.5	2.46	337	4.21	700	S
	100	40.7	2.46	422	4.22	650	S
Carbon	20	4.8	4.15	50.3	2.52	2000	S
	25	8.1	3.10	83.6	3.34	500	S
	30	9.3	3.23	96.4	3.21	1050	S
	50	16.8	2.97	174	3.49	700	S
	60	20.2	2.97	208	3.49	700	S
	100	33.6	2.97	349	3.49	600	S
	120	40.4	2.97	419	3.49	600	S

M.H.C.P. = Mean Horizontal Candle-Power.
W.P.C. = Watts per Mean Horizontal Candle-Power.
L.P.W. = Lumens per Watt.
S denotes Straight-side Bulb, see Fig. 1 (S).
G denotes Round Bulb, see Fig. 1 (G.)
T denotes Tubular Bulb, see Fig. 1 (T).



(S) (G) (T)

Fig. 1. Standard Types of Lamp Bulbs

DATA ON MAZDA SERIES STREET LAMPS

Vacuum Type *

C. P.	Lumens	4-ampere type			5.5-ampere type			6.6-ampere type		
		Watts	W. P. C.	L. P. W	Watts	W. P. C.	L. P. W.	Watts	W. P. C.	L. P. W.
32	318	34.6	1.08	9.20	38.1	1.19	8.34	41.6	1.30	7.63
40	397	42.0	1.05	9.46	45.6	1.14	8.71	48.8	1.22	8.14
60	596	61.2	1.02	9.74	63.0	1.05	9.46	65.4	1.09	9.11
80	794	80.0	1.00	9.93	81.6	1.02	9.74	81.6	1.02	9.74
100	993	100.0	1.00	9.93	100.0	1.00	9.93	100.0	1.00	9.93
200	1986	200.0	1.00	9.93	200.0	1.00	9.93	198.0	0.99	10.03
350	3563	350.0	1.00	10.18	350.0	1.00	10.18	346.5	0.99	10.28

* In addition to the lamps listed in the table 32-, 40-, 60- and 80-candle-power lamps of the vacuum type are also made for 3.5 and 7.5 amperes. 200-candle-power vacuum lamps are made for 7.5 amperes. The life of all the above vacuum lamps on constant current at rated initial watts per candle is 1350 hours.

Gas-filled Type

C. P.	6.6-ampere type			7.5-ampere type			20-ampere type		
	Watts	Volts	W. P. C	Watts	Volts	W. P. C	Watts	Volts	W. P. C.
80	57	8.6	0.71	57	7.6	0.71	...	..	....
100	70	10.6	0.70	...	....	....	...	..	....
250	170	25.7	0.68	170	22.6	0.68	...	..	....
400	264	40.0	0.66	...	....	....	200	10	0.50
600	396	60.0	0.66	...	....	....	300	15	0.50
1000	...	....	....	...	....	....	500	25	0.50

W.P.C. = Watts per Mean Horizontal Candle-Power. L.P.W. = Lumens per Watt.

DATA ON OTHER TYPES OF LAMPS

Type	Volts range	Watts range	Filament	Base
High voltage	100–130	25–80	Tantalum	MS
High voltage	200–260	25–500	Tungsten	MS
High voltage	200–260	50, 80	Tantalum	MS
High voltage	200–275	35–120	Carbon	MS
Street railway	105–130	23–94	Tungsten	MS
Street railway	100–130	42–200	Carbon	MS
Train lighting	25–34	10–50	Tungsten	MS
Train lighting	50–65	10–50	Tungsten	MS
Sign	10–13	2.5, 5	Tungsten	MS
Sign	50–65	5	Tungsten	MS
Sign	100–130	10	Tungsten	MS
Sign	100–130	10, 20	Carbon	MS
Sign	200–275	30	Carbon	MS
Electric vehicle	21–90	15, 25	Tungsten	CB
Low volt	4–20	2.5–30	Tungsten	MS
Miniature	1.5–8	0.45–24	Tungsten	Min. S; Min. CS; Min. CB

MS denotes medium screw base. CB denotes candelabra bayonet base.
Min.S denotes miniature screw base. CS denotes candelabra screw base.

Performance with Voltage Variations. — The very high rate of change of light produced and of filament life with temperature causes incandescent lamps to be extremely sensitive to variations in voltage. A high positive temperature coefficient of resistance, as in the tungsten filament, affords a partial compensation to voltage variations and increases the stability of the lamp's performance. The relations of candle-power, watts, watts per candle and life to voltage are shown for modern types of lamps in Fig. 2. High voltage increases the candle-power and efficiency, but greatly reduces the life of the lamp. Operating conditions are always a compromise between these three factors. Under ordinary conditions circuit voltage and lamp voltage should agree closely. The effect of low voltage on candle-power is marked. Satisfactory service requires that the voltage be maintained within 3 per cent of the rated value for the lamp. The sensitiveness of life to voltage emphasizes the utter futility of life tests with imperfect voltage regulation.

The relations between lamp performance and voltage may be very completely expressed by a series of simple proportions and exponents:

$$\frac{I_1}{I_2} = \left[\frac{V_1}{V_2}\right]^a; \frac{W_1}{W_2} = \left[\frac{V_1}{V_2}\right]^b; \frac{E_1}{E_2} = \left[\frac{V_2}{V_1}\right]^c; \frac{L_1}{L_2} = \left[\frac{V_2}{V_1}\right]^d,$$

where V = voltage, I = candle-power, W = watts, E = watts per candle, L = life.

The values of the exponents in the above expressions, as determined by the Nat. El. Lamp Assoc., are as follows:

	a	b	c	d
Carbon	5.55	2.05	3.51	20.5
Gem	4.80	1.75	3.06	17.6
Tantalum	4.35	1.74	2.60	14.7
Tungsten	3.68	1.59	2.10	13.8

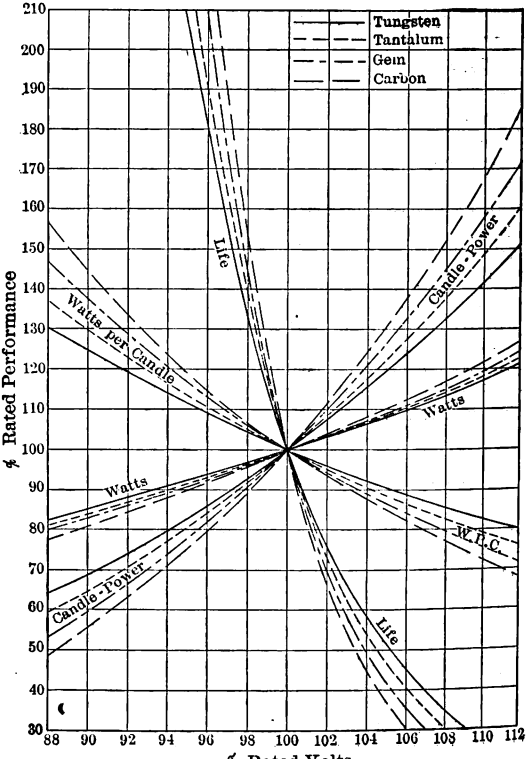


Fig. 2. Characteristic Curves of Incandescent Electric Lamps

Life Performance. — The rated life of incandescent lamps applies to the average of a very large number and not to individual performance. All lamps deteriorate with continued burning in candle-power, efficiency and strength. This decay is partly due to the slow vaporization of the filament and partly to growing light absorption in a black deposit on the inner side of the bulb. The frosting of bulbs, while not accelerating filament decay, hastens the decline of candle-power and efficiency due to the added absorption by the bulb deposit of internally reflected light and the greater tendency of the lamp to accumulate exterior dirt. The effect of bowl frosting is practically negligible. The life curves of Fig. 3 are typical of present-day lamps.

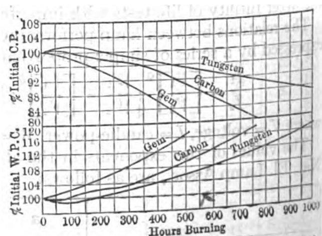


Fig. 3. Life Curves of Typical Incandescent Lamps

Effect of Frequency. — Incandescent lamps operated by alternating current of low frequency produce a flickering light due to cyclic variations of temperature. The flicker is greatest in filaments of small diameter and low thermal capacity, and increases in magnitude as the frequency is reduced. 110-volt carbon lamps of low efficiency have been successfully used with a frequency of 25 cycles. Tungsten street series lamps are satisfactory at 25 cycles. Constant potential tungsten and gem lamps are not as a rule satisfactory at frequencies below 40 cycles.

Smashing Point. — The term "smashing point" denotes the percentage of initial candle-power to which a lamp should be allowed to decline in service to obtain the most economical service. It is apparent from Fig. 3 that continued loss of efficiency ultimately leads to the condition where it is more economical to replace a lamp than to continue it in service. This occurs when the ratio of the total light output, lumen-hours, to date to the total expenditure, lamp cost plus energy cost, has reached its maximum. The smashing point has often been arbitrarily stated as 80 per cent. Recent determinations (*see Bulletin 101 of the Nat. El. Lamp Assoc.*) give the values in the accompanying table, which indicate that in most cases lamps may be economically operated to the point of failure.

Watts	Smashing point, per cent		
	Carbon	Gem	Mazda
25	59.2	....	67.5
40	....	62.0	71.6
50	59.0	62.5	....
60	....	62.0	72.3
80	....	64.0	....
100	67.5	62.5	75.0
150	....	....	75.5
250	....	....	74.0
400	....	....	73.2
500	....	....	73.0

COST OF LIGHT BY INCANDESCENT LAMPS. — The main factors in the cost of light are the renewals of lamps and the cost of energy. Minor factors are the labor of inspection, the cost of renewals and the interest on investment in lamps and fixtures. The longer the lamp remains in service the less is the renewal cost chargeable to each lumen-hour and the greater the energy cost. Operating the lamp at better efficiency increases the renewal cost and reduces the energy cost per lumen-hour; at poorer efficiency the reverse is true. The efficiency giving the minimum ultimate cost with any type of lamp is a function of the ratio of the cost of the kilowatt-hour to the lamp cost.

A comparison of Fig. 4 with the table of rated performances shows that the rated efficiencies of commercial lamps agree quite closely with the most economical conditions of use at current lamp and energy prices. The data of Fig. 5 were compiled by A. G. Rakestraw (*Southern Electrician, Oct. 1912*) and include the cost of energy, renewals, interest, repairs and depreciation.

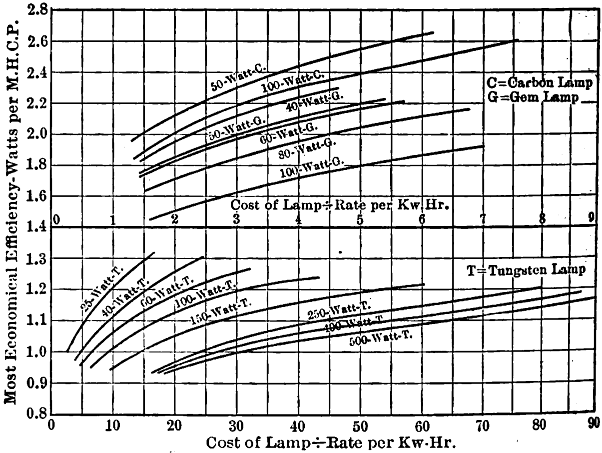


Fig. 4. Most Economical Efficiencies of Carbon, Gem and Tungsten Lamps

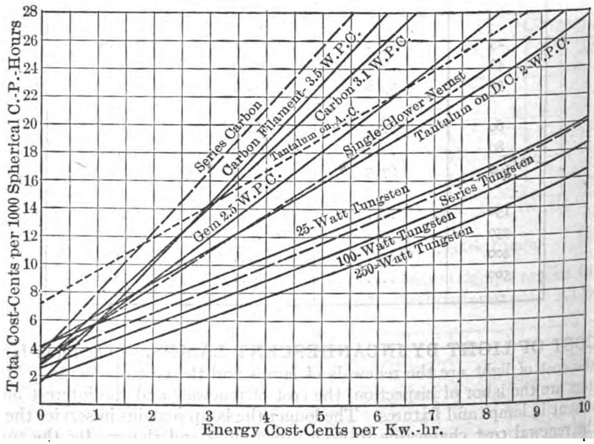


Fig. 5. Cost of Light from Incandescent Electric Lamps

DIFFUSERS FOR INCANDESCENT LAMPS.—Diffusers soften light and reduce its brilliancy by the increase in the area of its apparent source. This is at the expense of considerable absorption (*see table on Absorption of Light by Glassware in article on Illumination, Laws of*). The chief diffusing media are: ground glass, produced by sand-blasting or acid etching; translucent glass of various types, such as alabaster, opaline, opal, milk, etc., which has in

its structure minutely-divided mineral oxides; and prismatic glass. These media are used in frosted bulbs, globes and bowl reflectors partially covering the lamp. The diffusion from ground glass is inferior to that from other media, as the light source is partially visible as a bright spot in high contrast to the surrounding area. Opaline and alabaster transmit a subdued image of the filament but diffuse a considerable portion of the light very effectively. Opal and milk glass are truly diffusing, but are high in absorption. Prismatic globes transmit by refraction innumerable images of small elements of the filament distributed over the entire surface of the globe. When viewed directly the diffusion is imperfect, but to averted vision the effect is excellent. The refracting prisms enable the light distribution to be modified throughout a considerable range. Other diffusing globes tend to equalize light intensity in all directions.

REFLECTORS FOR INCANDESCENT LAMPS. — Reflectors serve primarily to modify light distribution and incidentally to shade the illuminant, to diffuse its light partially and to produce artistic effects. The chief reflecting agencies are polished metal, silvered glass, prismatic glass, opal glass, enamelled metal and aluminum paint. Polished metals, silvered glass and prismatic glass afford a wide latitude in the distribution of light, depending on the contour and depth of the bowl. Diffusely-reflecting surfaces afford less range of control and are not capable of producing strong downward concentration. Prismatic and opal glass reflectors transmit a moderate amount of diffused light and afford some direct illumination for surfaces not receiving the directed beams from the unit. Practically all reflectors are designed for mounting at a definite position with reference to the luminous center of the filament and produce their normal distribution only in this position. The table below gives data on the mean performance of representative groups of reflectors for incandescent lamps.

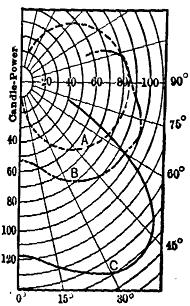


Fig. 6. Light Distribution from a 100-watt Tungsten Lamp with Enamelled Metal Reflectors

A. Bare lamp
B. Radially-fluted reflector
C. Bowl reflector

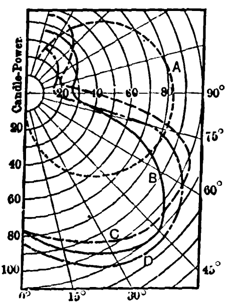


Fig. 7. Light Distribution from a 100-watt Tungsten Lamp with Opalescent Reflectors

A. Clear lamp
B. With blown opalescent bowl
C. With pressed flared reflector
D. With blown flared reflector

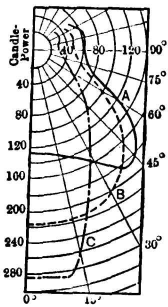


Fig. 8. Light Distribution from a 100-watt Tungsten Lamp with Prismatic Glass Reflectors

A. Extensive type
B. Intensive type
C. Focussing type

Curves of the light distribution from various types of reflectors are given in Figs. 6, 7, and 8.

MEAN PERFORMANCE OF TYPICAL GROUPS OF REFLECTORS FOR INCANDESCENT ELECTRIC LAMPS *

	Clear lamp	Industrial types, metal				Opalescent glass types				
		Flat dome enamel, 5 makes, 7 types	Radially fluted enamel, 2 makes, 2 types	Bowl enameled, 2 makes, 2 types	Bowl aluminized, 4 makes, 5 types	Blown bowl, 6 makes, 9 types	Pressed bowl, 8 makes, 16 types	Blown flared, 5 makes, 7 types	Pressed flared, 7 makes, 9 types	Prismatic bowl, 4 makes, 11 types
Mean lower hemispherical c-p.	70.7	104.0	92.6	81.1	75.5	72.3	73.0	88.3	85.7	80.6
Mean spherical c-p.	67.3	54.0	61.7	40.7	37.8	56.0	57.6	58.5	60.0	58.1
Per cent total flux absorbed	1	20	9	40	44	17	14	13	11	13
Lumens, 0°–60° zone	177	398	287	442	419	303	304	349	295	331
Lumens, 0°–90° zone	444	654	583	510	475	454	458	553	537	506
Total lumens	846	677	774	510	475	703	723	735	755	732

* Table adapted from data by A. L. Powell, *Gen. Elec. Review*, Vol. 15, p. 717.

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[W. E. WICKENDEN.]

LIGHTING PLANTS. — (*See also Distribution and Transmission Systems; Illumination, Interior; Illumination, Street; Lamps, Arc; Lamps, Incandescent; Power Stations; Substations.*) Electric lighting systems divide broadly into two classes, viz., series or constant-current, and parallel or constant-potential. The former system is appropriate where a large number of similar lamps in scattered locations are to be controlled simultaneously and where a high voltage can be safely applied to the circuit, as in street lighting. The parallel system is extremely flexible, is adapted only to low voltages and permits the independent control of single lamps and groups.

LOAD CONDITIONS. — The load of a lighting plant varies greatly with the season and with the hours of the day. There is usually a sharply-defined peak load in the early evening which reaches its annual maximum in mid-winter. Storms attended by darkness often give rise to sudden and heavy increases of load. Such conditions demand generating equipment of great flexibility in operation. Sufficient generating capacity must be available to carry the maximum peak load and to provide an ample reserve against accidents. The brief duration of the peak load in many plants makes desirable the selection of types of equipment having large temporary overload capacity in order to decrease the reserve needed. Typical load curves are shown in Figs. 1 and 2.

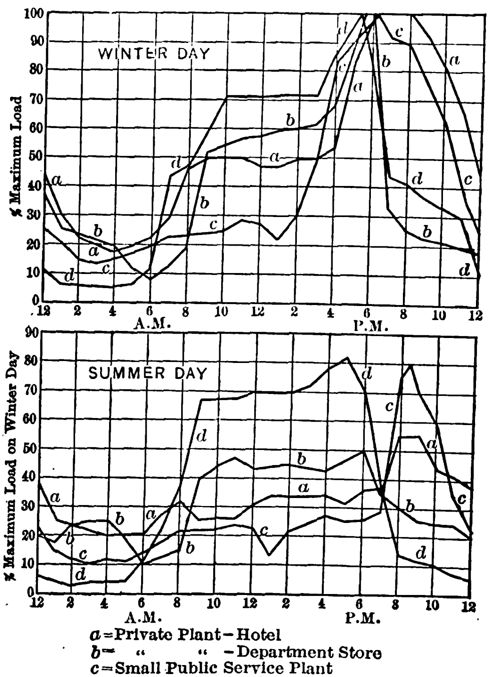

Fig. 1. Load Curves of Lighting Plants

Load Factor and Diversity Factor.— (*See also Standardization Rules of the A.I.E.E.*) The *load factor* of a machine, plant or system is the ratio of the average power to the maximum power during a certain period of time. The *diversity factor* is the ratio of the sum of the maximum power demands of the subdivisions of any system or part of a system to the maximum demand of the whole system or the part of the system under consideration, measured at the point of supply. The investment in a lighting system is largely determined by the peak load to be provided for. The annual charges on this investment are practically independent of the output; hence

the investment expense per kilowatt-hour varies in nearly inverse ratio to the annual load factor. The average annual load factors of lighting installations, of various types, as given by E. W. Lloyd (*Nat. El. Lt. Assoc., 1909, Vol. 2, p. 586*) range from 5 to 29 per cent, with the general average of complete systems less than 20 per cent. The economic advantages of high load factor can be secured in lighting service only by combining it with loads having non-coincident peaks, as commercial power and railway systems. Fig. 2 shows typical load curves and load factors for a typical winter and a typical summer day of (*a*) a city system of lighting and power, (*b*) a city street and elevated railway system and (*c*) the combined load of (*a*) and (*b*). (*Trans. A.I.E.E., 1912, Vol. 31, p. 240.*) The advantage of the diversity factor of these loads is apparent.

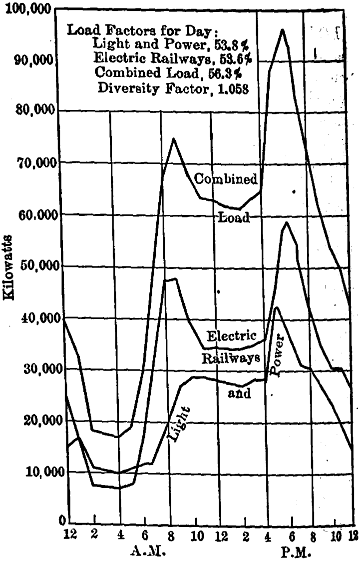

Fig. 2. Load Curves of Composite System on Winter Day. Daily Load Factor: Light and Power 53.8%, Electric Railways, 53.6%, Combined Load 56.3%, Diversity Factor, 1.058

In a general sense an appropriate scale of charges for electric energy should be graded according to the degree to which the load contributes to the system's peak as well as according to the energy consumed. Off-peak loads may appropriately receive low rates. The gain in load factor from diversity often makes it possible for a lighting station to sell energy at wholesale to a railroad system at a cost less than that at which the latter system could generate it independently.

Diversity factors exist between the various subdivisions of a lighting system and have an important influence on the rated capacity and investment required in the several elements of a system. H. B. Gear (*Trans. A.I.E.E., 1910, Vol. 29, p. 375*) reports the diversity factors of various classes of service and branches of the system of the Commonwealth Edison Company of Chicago to be:

DIVERSITY FACTORS OF A LIGHTING SYSTEM

Subdivision of the System	Residence lighting	Commercial power	Scattered power	Large users
Substation to feeders..........	1.15	1.15	1.15	1.15
Feeders to transformers.......	1.8	1.25	2.0	1.25
Transformers to meters........	3.0	1.6	1.1	1.0
Total diversity factor.........	6.20	2.30	2.53	1.44

DIRECT- VS. ALTERNATING-CURRENT SYSTEMS.—(*See also Distribution and Transmission Systems.*) Direct current has the following

advantages for lighting: (1) safety, since the lines are in no way associated with high-voltage conductors; (2) freedom from power factor, reactance and skin effect, which results in superior voltage regulation in heavily loaded low-voltage circuits; (3) the direct availability of the storage battery as a reserve and a load regulator; (4) the self-exciting and self-regulating features of direct-current generators; (5) the superiority of direct-current motors for adjustable speed service and for the operation of elevators and cranes; and (6) the marked superiority of direct-current arc lamps. Direct current is generally used in isolated plants and in congested city districts because of the greater ease with which good voltage regulation is maintained.

The advantages of alternating current are chiefly those incidental to its flexibility of voltage transformation and control, which makes possible an extended range of economical transmission and the independent regulation of separate feeders and lines. Alternating-current generators are less expensive than direct-current machines, being free from commutator limitations and adaptable to much higher speeds.

Voltage. — The range of voltage from 100 to 125 is standard for electric lighting in America. In England and to some extent on the continent the 200- to 250-volt range has been extensively used. The higher range lends itself to more economical distribution of power, the lower range to the superior construction of incandescent and arc lamps. The Edison 3-wire system of distribution is largely used in direct-current systems. The voltage generated for alternating-current systems is usually the same as that used for primary distribution when the latter is below 15,000 volts, although steam turbo-alternators are often provided with raising compensators having a ratio of 1 to 2 when the line voltage supplied is in the range from 6000 to 15,000. In most systems of small and medium capacity the a-c. line voltage is 2300. The secondary distribution system is supplied by transformers and is usually at 110 to 115 volts between lines.

Small self-contained lighting outfits for train-lighting and similar service where a storage battery is used for regulating purposes are usually designed for a voltage range from 25 to 35 or from 50 to 65, and use lamps of the train-lighting and compensator types. See also *Lighting of Trains by Electricity*.

Phases. — The polyphase system affords distinct advantages in the first cost and voltage regulation of both generating equipment and lines. Primary distribution in all modern a-c. systems is either 2-phase or 3-phase, the latter predominating largely. Secondary distribution from transformers is either single-phase, 2-wire or 3-wire; 2-phase, 3-wire or 4-wire; or 3-phase, 3-wire or 4-wire. 6-phase connections are used only as a link between high voltage systems and synchronous converters. See also *Converters, Synchronous; Transformers*.

Frequency. — The standard lighting frequency in America is 60 cycles, and that in Europe 50 cycles. A frequency of 25 cycles has many advantages for overhead transmission and for conversion to direct current by synchronous converters. Power transmitted at 25 cycles is frequently converted to 60 cycles for lighting purposes by the use of frequency changing motor generators. In a few cases where a large load of induction motors is associated with a lighting system the compromise frequency of 40 cycles is employed. The performance of arc lamps and of incandescent lamps for interior lighting is not satisfactory below 40 cycles, due to an obvious and annoying flicker.

PRIME MOVERS. — (*See also Gas Engines; Steam Engines; Steam Turbines; Water Wheels.*) Prime movers for lighting plants should provide approximately constant speed without variation in angular velocity during each revolution. The steam turbine has a great advantage over the reciprocating engine for large alternators, due to its lower first cost and superior operating

economy. It has marked incidental advantages due to lighter foundations, smaller space requirements, elimination of fly-wheel, ease of attendance, and lower cost of maintenance. The piston engine is preferred for the driving of direct-current generators, because of its more appropriate speed.

Internal combustion engines surpass steam power in thermal efficiency, but are handicapped by higher first cost and small range of overload capacity. Their net advantage over steam power may be large when the cost of coal is very high, when a supply of natural or by-product gas is available at low cost and when the load factor of the system is unusually good. Internal combustion engines do not compete successfully with steam units in plants furnishing both heating and lighting service where a supply of exhaust steam has large value.

The crude-oil engine involves a high investment and heavy maintenance expense, but is in other respects an excellent type of motive power where oil is a more economical fuel than coal. Gas from a public source of supply and gasoline are used to a large extent in very small lighting plants to operate internal combustion engines. They are relatively expensive fuels, but may afford a saving by displacing a gas-producing plant on the premises. Gas-engines in lighting plants should be controlled by close-regulating governors and should be supplied with fly-wheels of considerable inertia.

Waterwheels of all types are well adapted to the driving of lighting generators if controlled by close-regulating governors.

Overload Capacities. — Steam prime movers are capable of carrying continuous overload capacities of 50 per cent or more whereas gas engines and waterwheels are very limited in this respect. A large overload capacity enables a plant to carry a smaller reserve equipment for emergencies. Current practice favors the use of a sufficient boiler or gas producer capacity to carry the average load with high efficiency and resort is had to forcing during peaks of short duration. In the average of a large number of modern stations of medium capacity 0.4 boiler horse-power is provided for each kilowatt of generating capacity.

GENERATORS. — (*See also separate articles on Generators.*) Direct-current generators are usually compound-wound and are operated at the voltage of the distributing system. A heavy direct current can usually be more economically secured by the conversion of alternating current than by direct generation. This method is an economic necessity when the direct-current load is more than one mile distant from the station. Edison 3-wire systems may be supplied by sets of two generators in series, by special 3-wire generators or by standard 2-wire generators operated in conjunction with voltage balancing sets.

Polyphase alternating-current generators are chosen for low cost and good regulation. Small low-speed alternators are usually provided with individual exciters, often on the same shaft, but large alternators are supplied with exciting current from a central system comprising several exciters in parallel.

Constant-current d-c. generators are now practically obsolete. Power for series circuits is now most economically obtained from constant-potential a-c. generating sources through constant-current transformers. Direct current for series circuits is obtained by operating the mercury arc rectifier in series with the secondary circuit of the constant-current transformer.

DISTRIBUTION. — (*See also Distribution and Transmission Systems.*) Distribution systems are designed with a view to the greatest possible reliability of service and the closest regulation of voltage consistent with reasonable investment. The 3-wire system is standard for local d-c. service. An interconnected network of mains is used in extensive 3-wire systems and is fed at many points by feeders from stations or substations. Alternating current is distributed at 2300 volts or more, 3-phase, to transformers, from which secondary distribution is made to consumers by 2-wire and 3-wire systems. To obtain good voltage

regulation on polyphase systems it is necessary to maintain a close balance of loads.

SUBSTATIONS. — (*See also Substations, Lighting.*) Substations are used for the following purposes: (*a*) to receive alternating current from high voltage transmission lines or feeders and reduce it in voltage for local distribution; (*b*) to convert alternating current from feeders into direct current for use in the adjacent territory; (*c*) to convert alternating current received from feeders at 25 cycles to 60 cycles for distribution by local lighting circuits; (*d*) transform alternating current from constant potential sources to constant current, either direct or alternating, for the supply of arc lighting circuits; (*e*) to house storage batteries used as load regulators or reserves on d-c. systems; and (*f*) to house feeder regulators for the control of voltage on lighting circuits. Conversion to direct current is accomplished either by motor-generator sets or by synchronous converters. Motor-generator sets may be either of the synchronous or the induction type and are usually operated without step-down transformers. Synchronous converters are usually 3-phase or 6-phase and are always associated with voltage-lowering transformers. Converters may be of the split-pole type, the synchronous booster type or the compound type to provide automatic control of d-c. voltage, or may be operated in conjunction with voltage regulators similar to those used on a-c. feeders.

Motor Generators versus Synchronous Converters. — (*See also Motor Generators; Converters, Synchronous.*) Motor-generator equipment is more expensive and less efficient than converter equipment, but occupies less space, affords more synchronous condenser capacity for power factor correction and is more reliable in its operation, especially at an a-c. frequency of 60 cycles. Synchronous converters are used almost exclusively for 25-cycle conversion and are gaining in use at 60 cycles. Each converter is usually supplied with its own bank of lowering transformers. Converters of larger capacity than 500 kw. are usually operated 6-phase. Converters supplying 3-wire d-c. systems are adjusted to give from 220 to 250 volts, d-c., and the neutral wire is derived from the neutral connection of the transformers.

Storage Batteries. — (*See also articles on Batteries, Storage.*) Storage batteries are extensively used in stations and substations as a reserve against the failure of the primary source of direct current. It is difficult to operate the battery as a load regulator on constant-voltage systems, especially by a floating connection. A booster or end-cell arrangement is required to compensate for the sloping volt-ampere characteristic. The substation battery commonly floats on the bus bars fully charged and assumes the load if the voltage of the bus bars falls a certain amount. The battery falls in voltage as its discharge proceeds and additional cells must be added in series to sustain the line voltage. The end-cell switch for this purpose may be either hand-operated or automatic. If automatic, it is controlled by a relay switch operated by a solenoid connected across the main bus bars.

PROTECTIVE EQUIPMENT. — (*See also Switchgear Equipment for Power Stations; Lightning Protectors.*) As lighting systems are required to render practically uninterrupted service the protective appliances are intended to disconnect apparatus and circuits only in extreme emergencies and then only those in serious danger. Generators and exciters are not as a rule provided with automatic circuit breakers. Radial feeders and transmission lines have circuit breakers which open with extreme overloads, often with inverse time elements. Feeders and transformers operated in parallel at both ends are provided with selective relays which open the circuit breakers at both ends in case of trouble, and so prevent the disturbance of other circuits.

VOLTAGE REGULATION. — Aside from the condition in which the illuminants are installed and maintained, the quality of lighting service is almost solely a matter of continuity of service and close regulation of voltage. The best service standards require that the variation of voltage at the lamps shall at no time exceed 2 per cent on either side of the mean. Service with less than 3.5 per cent variation may be considered fair and with more than 5 per cent decidedly poor.

In direct-current systems fairly constant voltage at the loads may be obtained by the use of generators properly over-compounded. A much more sensitive method of control is afforded by the Tirrill regulator (*see Regulators*), which automatically corrects all but the most transient voltage fluctuations of the generators. Synchronous converters have fixed ratios of voltage conversion, but may be regulated to provide approximately constant voltage at load centers by methods cited above in the paragraph on *Substations*. A plan of voltage regulation commonly used in 3-wire networks is to provide in the station or substation two sets of bus bars differing somewhat in voltage. All feeders terminate in selector switches, by which they may be connected with either set of bus bars as desired. The more heavily loaded feeders are connected to the higher bus, the others to the lower bus, with the result of nearly uniform voltage throughout the network of mains.

The following voltage-control methods are employed in a-c. systems: (1) to regulate all generators by hand or by Tirrill regulators to constant voltage or to a voltage rising in proportion to the load; (2) to provide not only approximate regulation of generators but to equip each outgoing line with feeder regulators of the switch or induction type, to be automatically controlled if desired, so that each feeder is caused to supply approximately constant voltage at its center of load. The latter system is the more flexible and economical for extensive distribution systems, since each feeder is independently regulated and may be proportioned in cross-section with regard to economy rather than inherent voltage regulation. Single-phase regulators are preferred for close control if the loads are subject to unbalancing.

As checks on voltage regulation at load centers voltmeters should be provided at stations or substations to indicate conditions at these centers. The voltmeters may be connected to the feeders at the distributing centers by small pressure leads or to the feeders at the station through compensators which allow for the voltage drop in the feeders. When a power load of low power factor is associated with an a-c. lighting system, voltage regulation and economy in transmission can often be assisted by the use of the synchronous condenser to compensate the lagging current; see *Converters, Synchronous*.

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[W. E. Wickenden.]

LIGHTING OF TRAINS BY ELECTRICITY. — (*See also Batteries, Storage, Alkaline and Lead Types; Generators, Direct-current.*) The electric lighting of railway trains on steam lines has become a very important industry. According to H. A. Currie and B. F. Wood, whose comprehensive paper is the basis of this article, there were in use or contracted for on June 30th, 1912, a total of over 15,000 cars lighted by electricity on the railways of the United States and Canada.

Systems Used. — Three systems are in general use, which are known as the straight storage system, the head-end system and the axle generator system. In the straight storage system, each car is provided with a storage battery which must be charged at terminals during the lay-over period. The head-end system consists essentially of a steam-driven generator located on the locomotive or baggage car and with connections through the various cars. The axle generator system, which is rapidly superseding the others, consists of a generator belted to the axle of each car, a storage battery for supplying current when the speed of the cars falls below a certain value, a regulator device for controlling the generator output and voltage at all train speeds, and a regulator for controlling voltage on the lamp circuits. Each of these systems is described in detail below.

Standard Lamp Voltages. — The standard lamp voltage was 60 volts for all three systems when carbon lamps were used, but it has been changed in the axle generator system to 30 volts since the introduction of tungsten lamps. The usual lamp is a 25-watt drawn-wire filament tungsten lamp with a round bulb, although lamps of 10, 15, 20, 25 and 50 watts are available for this service. The standards of the Association of Railway Electrical Engineers are accepted as representing the best American train-lighting practice.

Limiting Size of Battery. — The batteries in general use in train-lighting service have a rated capacity of approximately 300 ampere-hours. This is about the maximum limit of capacity for Planté type batteries having weight low enough for convenience in handling.

STRAIGHT STORAGE SYSTEM. — In this system, each car is provided with a storage battery, which must be charged at terminals during the lay-over period. The fundamental requirements are:

1. The capacity of the battery must be in excess of the demand for current to operate lamps, fans, etc. for the longest run between charging periods.
2. The power plant, or other outside source of power, must be of sufficient capacity to meet the maximum demand for charging current.
3. The lay-over time at terminals must be sufficient to cover all necessary shifting and charging of the batteries at the proper rate.
4. The yard must have a sufficient number of tracks provided with charging outlets, so arranged that the charging of batteries will not interfere with shifting operations.

Battery Equipment. — The great majority of cars on which this system is used are equipped with a 64-volt lead or nickel-iron battery with cells connected in series. A few cars are operated at other voltages, viz. 26, 30, 32 and 110.

Two battery boxes are generally provided for 64 volt batteries and secured to the under side of the car, one on each side equidistant from the ends of the car, and with the front or door side slightly back of the line of outside finish. The cells are put up in double compartment lead-lined wood tanks provided with handles, rollers, etc. for convenience in handling. The two halves of the battery are connected in series and leads are run to the switchboard in the end of the car.

Taps are taken off these leads at the battery terminals and run to charging receptacles, conveniently located on each side of the car.

Charging of Batteries. — The charging voltage provided is usually 50 per cent higher than the normal voltage of the battery. Hand or automatically operated resistance devices, usually carbon rheostats, are provided for reducing the voltage of the individual charging lines to the proper point. The batteries are charged while on the car under normal conditions. When the lay-over period is short it is sometimes necessary to exchange a discharged battery for one fully charged.

HEAD-END SYSTEM. — This system requires the following apparatus:

1. A generator, usually steam turbine driven, placed in the baggage car or on the locomotive, and furnished with steam from the locomotive.

2. The necessary indicating, regulating and controlling apparatus placed near the generator and in an accessible position.

3. Train line wires of the proper size on each car and running the entire length of the train, flexible connections being made between cars, in the vestibule.

4. Batteries, consisting of a suitable number of cells connected in series and placed in battery boxes attached to the under side of each car.

5. Lamp regulators are sometimes installed in the cars to compensate for the line drop and to maintain constant voltage at the lamps.

Head-end systems are generally operated at 64 or 110 volts, although the introduction of tungsten lamps has to a great extent eliminated the need of the high voltage equipments, and comparatively few railroads are now using 110 volts.

Another system has a 64 volt 200 kw. axle-driven generator mounted on the baggage car truck but projecting into the car body. It is operated through a chain drive.

Requirements for Successful Operation. — The successful operation of this system requires that:

1. A sufficient amount of steam at the proper pressure be provided when lighting is necessary. As it is the object of the transportation department to get trains to their destination on time, lack of steam is felt first by the lighting system, the pressure being reduced or steam cut off entirely so that the schedule may be maintained.

2. When the train is broken en route, each section must either be equipped with a battery to insure light until the train is again made up, or provided with some auxiliary light.

3. A member of the train crew must be capable of operating the generating apparatus and of making running repairs and adjustments en route.

AXLE GENERATOR SYSTEM. — The axle generator systems used in this country comprise the following principal parts:

1. An axle-driven generator mounted on the car truck. (Abroad where rigid trucks are used the axle generator is frequently secured to the under side of the car body.)

2. A suspension by which the axle generator is supported from the truck frame.

3. A drive, connecting the armature shaft to the axle.

4. A regulator for controlling the voltage and output of the generator at all train speeds.

5. An automatic switch designed to open on reverse current for the purpose of preventing discharge of the battery through the generator.

6. A regulator for controlling the voltage impressed on the lamp circuits.

7. A battery of a suitable number of cells to supply current when generator current is not available.

Three European systems which do not require complicated regulator mechanisms are being introduced into the United States.

Requirements for Successful Operation. — For the successful operation of the system, the following requirements must be met:

1. The polarity of the generator terminals must remain unchanged with a movement of the car in either direction.

2. At all train speeds, from the cutting-in speed of the generator to the maximum, the generator output and voltage must be maintained within the desired working limits.

3. The generator must be automatically connected and disconnected from the battery circuit as the train speed rises above or falls below the critical range of speed.

4. The lights may be burned at any time and the transfer of this load from the battery to the generator and vice-versa must result in no appreciable change in the candle power of the lamps.

5. The voltage impressed on the lamp circuit must be maintained within such limits as will give satisfactory illumination and reasonable life of lamps.

Suspension of Generator. — In this country it is general practice to support the axle generator from the truck frame. When first applied, the generator was placed between the axle and the truck end sill, this arrangement being known as "inside suspension." The generator was not easily accessible for inspection and repairs, and at the present time it is placed outside of the truck frame, this arrangement being known as "outside suspension." There are four general methods of carrying the generator from the suspension framing, viz. bottom pivoted, top pivoted, parallel link and sliding. The bottom pivoted was first used but at the present time the parallel link suspension is in more general use.

Transmission. — The most usual form of transmission for axle generators consists of a rubber-filled canvas belt running on pulleys on the axle and the armature shaft. The axle pulley as first used was cast iron mounted directly on the car axle, the bore of the pulley conforming to the taper of the axle, but on account of inequalities in the axle which was hammered or rough-turned, it had to be wrapped with tarred paper. The axle pulley at present in use is of pressed steel, mounted on a steel bushing, the bushing being secured independently to a turned seat on the axle, and the pulley mounted thereon. Belt-tension is provided by means of springs which also afford relief to the belt due to the movement of the car axle with respect to the truck frame. One spring is generally used when the generator has top, bottom or sliding suspension and two springs with the parallel link suspension. Chains of the silent type have also been tried and have the advantage of positive action and decrease in bearing pressure, but the wear of the links both on the face and the pivot sprockets has been excessive. Belts of V section have been tried and would seem to have the same advantages as the chains, but it is found that in winter the bottom of the V groove in the sheaves packs with ice and snow, and driving power is lost. Neither the chain nor V belt requires tension device.

Lubrication. — The generator bearings are a great source of trouble, and some method of lubrication is required which will confine the oil in the bearings under the severe conditions of service to which these machines are subjected. Ball bearings have been very successful when used with grease free from acid or alkali, which does not oxidize, evaporate, become gummy or lose its body.

Method of Controlling Voltage and Current. — The usual modern regulating mechanism keeps the current output of the generator constant regard-

less of the demand as long as the battery is not fully charged. As the battery becomes charged, the generator voltage gradually increases and at a predetermined voltage, a change in control is effected whereby the current is caused to decrease until the battery is floating on the line. When the speed of the train (and therefore the generator voltage) falls below a predetermined value the generator is cut out of circuit and the lamps receive their current from the battery.

As the voltage on the battery on charge is approximately 30 per cent higher than on discharge, it is necessary to provide some means of lamp regulation in order to keep a constant voltage on the lamps. This is accomplished by means of a regulating resistance, a series of carbon blocks, the resistance being varied by varying the pressure on these blocks, the variation of pressure being determined by a pilot voltage coil connected across the lamp mains. (The preceding part of this article is abstracted from H. A. Currie and B. F. Wood, *Trans. A.S.M.E.*, 1912.)

Costs. — The cost of axle lighting includes the following items: fixed charges on equipment, maintenance, depreciation, haulage and energy. The first cost and maintenance cost of axle-lighting equipments are given in the following table.

Type of car	Kw. rating of generator	First cost including battery, dollars	Maintenance cost per car, dollars	
			Per 1000 miles	Per month
Baggage................	1	600	0.70–0.75	5.00– 5.50
Passenger coach or Sleeping car............	2	1000	1.50–2.25	10.75–16.00
Dining car.............	3	1200	4.00–5.00	31.50–40.00

The cost of hauling an increment of weight of the order of magnitude of axle-lighting equipments is about one mil per ton-mile. The energy supplied to the lighting generator costs about one mil per rated kilowatt capacity per mile. The depreciation should be figured on the cost of equipment less that of the battery renewals which are included under maintenance. No reliable data exist as to the relative cost of operating axle generator and other systems.

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[W. A. Del Mar.]

LIGHTNING PROTECTORS: GROUND WIRES, ARRESTERS, CHOKE COILS. — (*See also Distribution Lines; Ground Connections; Power Stations; Substations; Switch Gear Equipment for Power Stations; Transmission Lines.*) Abnormal rises of voltage in transmission and distribution circuits may be set up either by lightning discharges or by electrical oscillations caused by switching loads on and off the lines. The latter type of disturbances have been called "internal lightning." High-frequency oscillations are usually of small power and are generally called "statics." The term "surge" is used for any kind of oscillation.

A "ground wire" is a grounded wire which is run above and parallel to the main wires of an aërial line. A "lightning arrester" is any device shunted between wires or from wire to ground, which device under normal voltage permits practically no current to flow, but which becomes a fairly good conductor, usually by the formation of an arc, when the voltage rises a given amount above normal. A "choke coil" is a coil of wire of comparatively low resistance and small inductance, which is placed in series with the line. The coil has a very small impedance to normal line frequencies, but a high impedance to high-frequency oscillations.

GROUND WIRES. — A line completely inclosed in a grounded metallic sheath, as, for example, a lead-covered cable, is completely protected from induced electrostatic charges (*see Electricity and Magnetism, Principles of*). The ground wire used over aërial transmission lines acts as a partial screen and is one of the best means of protecting aërial lines against lightning. The protection is not perfect, however, since the wire forms only a partial screen.

Where systems operate with a thoroughly grounded neutral this neutral wire can form the overhead grounded conductor and the system becomes practically a three-phase four-wire system allowing the use of single-phase transformers of 58 per cent normal line voltage to be connected between any phase wire and the neutral. See articles on *Distribution Circuits; Transmission Lines.*

LIGHTNING ARRESTERS. — A lightning arrester consists essentially of a spark gap so set that excess voltage will cause the gap to arc over, allowing the charge due to this voltage to pass to ground. There is combined with the gap some means of suppressing the power arc which follows and which tends to continue after the abnormal voltage has ceased.

An ideal lightning arrester should take no current at the ordinary operating potential, but at any potential much higher than ordinary there should pass enough current to limit the abnormal potential to some fixed safe value. When the abnormal potential ceases the arrester should stop taking current from the line. The closer an arrester approaches these ideal conditions the better the arrester.

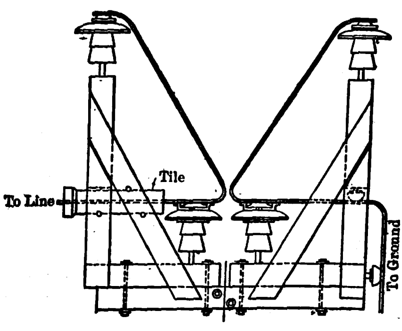

Fig. 1. Horn-gap Arrester

Horn-gap Arrester (Fig. 1). This arrester consists of two knee-shaped horns mounted on regular line insulators. The distance between the knees depends largely upon the voltage between line and ground, and varies from about 1/16 in. to 3/32 in. for 2200

volts to about 6½ in. to 9⅞ in. for 110,000 volts. One of the horns is connected directly to the line and the other through a resistance, usually water, and a choke coil, to the ground. Practice differs as to whether the water should have salt added, but it should always be covered by a layer of oil about one-eighth inch deep to prevent evaporation.

The operation of the horn gap is based on the fact that a short-circuit once started at the base travels upward due to the heated gases and to the force exerted on the arc by the magnetic field of the current until the arc is ruptured by attenuation. On circuits of high voltage this rupture sometimes takes a second or two, but seems to act with but little disturbance of the line. The angle between the horizontal and the straight portion of the knee ranges from 55 to 60 degrees. The curvature of the knee should have a radius of from 3 to 5 inches.

Multipath Arresters have been developed for a-c. and d-c. service for voltages not exceeding 1000 by the use of a carborundum block fastened between two terminal plates, thus allowing the static discharge to spread itself over a number of minute discharge paths. The normal voltage between the line and the ground is divided into so many minute gaps that the voltage across each gap is too small to maintain an arc after the discharge has passed.

Nonarcing Arresters (Fig. 2) based on the discovery of "nonarcing metal" by Mr. A. J. Wurts, formed the first successful high-voltage arresters. The peculiar property of this metal is that an alternating current will not maintain an arc between adjacent cylinders of the metal provided the voltage is not too high and that the power current which follows the lightning discharge does not vaporize too much of the metal. The first condition is met by having a fairly large number of very small gaps in series, and the second condition gives no trouble where the amount of power current is comparatively small as was the case on the early high-voltage installations. For large amounts of power it becomes necessary to use resistances in series with the spark gaps to limit the current, and these resistances reduce the effectiveness of the arresters. For very high voltages different schemes are used to reduce the number of gaps required. It has been found that by shunting a certain number of these gaps by a non-inductive resistance the effectiveness of the arrester is increased.

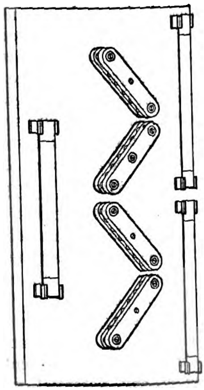

Fig. 2. Nonarching Lightning Arrester

Fig. 2 shows an arrester of this design intended for service on 6600-volt lines where the capacity does not exceed 2000 kv-a. The nonarcing cylinders are held between porcelain insulators in such a way that there is an air gap of about 1/32 inch between adjacent cylinders in each one of the four sets of seven cylinders. The marble slab forming the base of the arrester also has mounted on it three graphite resistance rods shunting some of the gaps. Modifications of this scheme were used for the "low equivalent," "multigap," "multiphase" and similar "shunted gap" arresters that were installed before the electrolytic arresters were brought out and are still giving good satisfaction in many plants operating at voltages as high as 88,000 volts.

Electrolytic Arresters (Fig. 3) have been found after experimental research and operating experience of many years to be the best suited for high voltages up to the highest in actual service. See also *Power Stations, Hydroelectric.*

These lightning arresters are usually provided with a horn-gap device mounted on a framework and arranged so that the length of gaps can be adjusted easily while the breaker is in service. The arrester itself consists essentially of a system of nested aluminum cup-shaped trays as shown in Fig. 3, suitably supported and arranged in a steel tank containing a liquid electrolyte which forms insulating films on the surfaces of these trays. These films prevent passage of current at normal voltages and break down at abnormal voltages; on the cessation of the abnormal stress the film regains its original resistance. When these arresters are arranged for outdoor service, the aluminum trays filled with electrolyte are completely immersed in transformer oil contained in the steel tanks. This oil provides an insulation and cooling medium, and prevents evaporation of the electrolyte. The volume of oil in the tanks, which are nearly filled, is great enough to absorb the heat due to a continuous discharge for a long period.

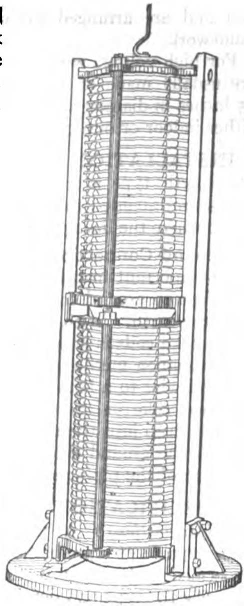

Fig. 3. Column of Aluminum Trays for Electrolytic Arrester

It is usually necessary to charge the arrester periodically for the purpose of maintaining the film of oxide on the aluminum plates. The horn gaps are readily adjustable so that in one position the line current will arc across the gaps and charge the cells. In a second position the gaps are so set as to discharge only at a predetermined point slightly above the operating voltage. In the third position of the horn gaps, the gap is so large as to prevent the possibility of arcing across and in this manner acts as a disconnecting switch when it is necessary to inspect the tanks containing the electrolytic cells. See also *Power Stations, Hydroelectric.*

CHOKE COILS (Fig. 4) are an important element in the protection of circuits against static disturbances, in addition to the lightning arrester itself. The inductance of the choke coil acts as a reflector to high-frequency waves and prevents the potential to which the leads of the generator or transformer coil are subjected from undergoing excessive or abrupt changes. As at the operating frequency the value of volts per turn in a choke coil is very small, a surge may cause a spark to pass momentarily between turns but no arc will be formed. This is not usually true of a generator or transformer. Although extra insulation for the end turns of generators and transformers is desirable, it cannot entirely take the place of choke coils but frequently permits the use of coils of a smaller choking power.

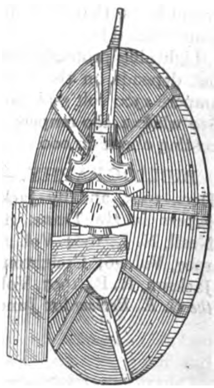

Fig. 4. Choke Coil

Fig. 4 shows a type of choke coil built in normal capacities up to 260 amperes at 25,000 volts. These coils are made by winding wire or copper strap on circular or elliptical center blocks with heavy insulation between turns. The coils are without

iron and are arranged for mounting on the station wall or on a suitable framework.

For high-voltage work one or more open helical coils are used. These coils are usually made of aluminum wire wound in a helix of about 20 turns and 15 inches in diameter. They are arranged for supporting on insulators and for either indoor or outdoor mountings.

INSTALLATION OF ARRESTERS is dependent on their design. The electrolytic type is frequently made suitable for outdoor service; the other types are intended to be mounted indoors and are connected in circuit at the point where the lines pass from the building.

Ground Connection. — (*See also article on Ground Connections.*) One of the most important features of a lightning-arrester installation is the securing of a satisfactory ground connection to enable the static electricity to pass readily into the earth. With a poor ground connection the value of lightning arresters and choke coils is greatly reduced.

A common method of securing a ground connection is to solder or rivet the ground wire to a large tinned copper plate which is buried in several layers of crushed coke or charcoal in permanently damp earth. Wrought-iron pipes driven deep into the moist earth will also make a good ground. In hydraulic plants the ground should include a connection to the penstock or some other portion of the piping system.

SPECIFICATION FOR LIGHTNING ARRESTER. — (*See also article on Specifications.*) As the services to which lightning arresters are subjected are not capable of reproduction for testing purposes, lightning arresters cannot be specified in terms of performance. Hence it is necessary for the engineer to specify the type and details of construction after an examination of the various types. When calling for proposals, the following details should be stated: station or out-of-door service, voltage and frequency of circuit; details as to where arresters are to be located.

DIMENSIONS, WEIGHTS AND COSTS vary so much for the different voltages, class of service, whether grounded or ungrounded neutral, indoors or outdoors, that it is impossible to tabulate this information in the limited space available.

Lightning protection in a power house is almost independent of its capacity but depends on the voltage and number of feeders to be protected. A 200 ampere 2300 volt 3-phase feeder would require lightning protection costing about $20 while a 200 ampere 110,000 volt feeder would require lightning protection costing about $2,000.

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[S. Q. HAYES.]

LOCOMOTIVES, ELECTRIC. — (*See also Cars, Electric; Collectors, Current; Locomotives, Steam; Railways, Energy Requirements and Motor Capacity for.*) For many years a new type of electric locomotive was designed for nearly every new proposition. However, with the very numerous applications of electric locomotives of from 30 to 50 tons weight for slow freight and switching service on interurban roads and in terminals, the double-truck bogie type finally demonstrated its superior fitness and came to be adopted almost universally in America for all work involving speeds less than 45 m.p.h. For higher speeds special provision must be made for guiding the locomotive around curves by the addition of guiding trucks or axles, for placing as much of the weight of the motors as possible on springs and for raising the center of gravity to a reasonable height (five feet or over). With a low center of gravity every sidewise movement of the mass of the locomotive strikes a blow side-wise on the track, but with a high center of gravity a side swaying is transformed into a downward thrust on the track. As the track is not usually designed to withstand great side thrusts it is better to avoid a low center of gravity in high-speed locomotives.

The coefficient of adhesion for electric locomotives has usual and safe values of from 18 to 22 per cent. It is higher for electric than for steam locomotives on account of the uniform torque of the electric motor. With clean dry rails the coefficient for electric locomotives may be as high as from 30 to 40 per cent.

CLASSIFICATION. — Locomotives are usually classified by the arrangement of their wheels and the subdivision of the wheels into driving wheels and guiding wheels. A series of numerals is used, each numeral representing a group of wheels of one form usually on one truck. Thus 4–4–0 designates a locomotive having four wheels on a guiding truck, four driving wheels connected and no trailing wheels or truck. This is the common "American" type of steam passenger locomotive. An ordinary double-truck four-motor freight locomotive or motor car would be designated as 4–0–4. (*See also Locomotives, Steam.*)

TYPES OF MOTORS.— Locomotives are built with various types of motors and operate from various systems of electrical distribution, e. g.,

a. Direct-current Motors at 600, 1200, 1500 or 2400 volts; for the higher voltages two motors are operated in series.

b. Single-phase Motors for 250 volts and 15 or 25 cycles, connected to the secondary of a transformer which receives its power from a 6000- or 11,000-volt trolley line.

c. Three-phase Induction Motors operating at about 500 volts supplied by the secondary of a transformer whose primary is connected to a 6000- or 11,000-volt three-phase trolley.

CONTROL SYSTEMS. — The control of all modern electric locomotives is by the multiple-unit system (q.v.), as the currents required are too large or the voltage too high for a drum control (q.v.). The possibility of one motorman operating and controlling two or three locomotive units at the same time is also advantageous.

TYPES OF ELECTRIC LOCOMOTIVES.— The simplest form of electric locomotive is a box car with a motor geared to each of the four axles on two bogie trucks. Such a locomotive would be geared for a very low speed and hence high tractive effort. To prevent slipping of the wheels the car would be weighted down with ballast or a load of freight.

The most common type of electric locomotive consists of a cab and framing carried on two heavy four-wheel trucks, each axle carrying a geared motor. Extra strength is provided in the cab framing to transmit the tractive effort of

the motors to the couplers. The complete locomotive may weigh from 30 to 50 tons and is equipped with motors having an aggregate capacity of approximately 500 horse-power. It will haul trains of about 20 cars, weighing about 500 tons, at speeds from 20 to 30 miles per hour. The usual tractive effort at the one-hour rating of the motors is from 10,000 to 15,000 lb. Such locomotives are well adapted for switching purposes in terminal freight yards and for hauling freight trains on interurban electric railways.

Detroit River Tunnel Locomotives. — A further development of this type of locomotive is exemplified in the large locomotive used for pusher and grade service in the Detroit River Tunnel of the Michigan Central R.R. and other similar installations. Each of these locomotives weighs from 100 to 120 tons, all on drivers, and consists of two four-wheel trucks carrying geared motors of from 300 to 500 horse-power each. The two trucks are coupled together by a pin or hinge which causes them to guide each other. This is called the "Articulated" type. The cab, containing all the control and auxiliary apparatus, is mounted on the trucks. The Detroit River Tunnel locomotives operate on 600 volts d-c. Locomotives of this type are now in operation also on 2400 volts d-c. and on 6000 volts three-phase a-c. They are limited in speed to about 40 miles per hr. on account of their lack of guiding trucks.

New York Central R.R. Locomotives. — The locomotives constructed in 1910 have a leading and a trailing two-axle guiding truck and in the middle four driving axles with gearless motors. The armature of each motor is mounted directly on the driving axle and the bi-polar field of the motor forms a part of the mechanical frame work of the locomotive. The magnetic flux passes through all four motors in series and returns by the side frames. The motors are wound for 600 volts d-c. and have very large air gaps to allow for the play between the armature and the field as the wheels pass over irregularities in the tracks. The field structure is spring borne but the armatures are not.

The locomotives constructed in 1913 consist of two sections articulated, with two two-axle trucks on each section. One truck on each section is rigid and the other is a bogie or guiding truck. Every axle carries a motor and all wheels are of the same size. The single cab is carried on a king pin on each section.

New Haven R.R. Locomotives. — The type of electric locomotive adopted by the N. Y., N. H., & H. R.R. in 1911 has four gearless single-phase series motors, the armatures being mounted on quills concentric with the driving axles and driving the wheels by means of springs. The locomotive has two trucks, each having four large driving wheels and two smaller guiding wheels. The motors are designed to operate at from 250 to 300 volts either a-c. or d-c., two in series on the line for d-c. and on the secondary of a transformer for a-c. The line voltage is 11,000 at 25 cycles.

Pennsylvania R.R. Locomotives. — The locomotives of the Pennsylvania R.R. consist of two similar units coupled back to back. Each unit has a two-axle guiding truck and has two driving axles rigid with the body frame. There is one large motor per unit, mounted in the cab and driving the wheels by means of an inclined connecting rod from the motor to a jack shaft and horizontal side rods from the jack shaft to the drivers. The connecting rods on opposite sides are placed at right angles to avoid dead centers.

SPECIFICATION FOR ELECTRIC LOCOMOTIVE.* — The following memoranda are intended to assist in writing specifications. See also article on *Specifications*.

On important high-speed systems it is usual for the design of the locomotive to be worked out by the purchaser and manufacturer working in collaboration,

* By W. A. Del Mar.

and in such cases, the design is usually specified in detail. In other cases, it is more usual to specify the operating characteristics and leave the design to the manufacturer. The following memoranda are to assist in the preparation of a specification of the latter type.

General Description of Service. — Whether for direct- or alternating-current, single-phase or three-phase, freight or passenger hauling, overhead trolley or third rail. Line voltage, etc.

Specific Details of Work to be Performed by Locomotive. — Weight of cars loaded and empty. Maximum train weight. Average train weight. Time to make typical run of stated length. Number and duration of stops in typical run. Ton miles per day per locomotive. Maximum speed on level with average load. Maximum speed on maximum grade with maximum load. Acceleration (miles per hour per second), with maximum load. Hours per day in regular service. Amount of time in shifting and yard service.

Profile and Plan of Line. — Grades and curves.

Clearances and Limiting Dimensions.—Gauge of track, clearance diagram of right-of-way. Maximum and minimum height of trolley wire or third-rail location. Height of coupler. Wheels, tread and flange (M.C.B. or special). Weight of rail. Minimum radius of curve. Wheel diameter. Maximum permissible weight per running foot of right-of-way.

Operating Characteristics. — Absence of nosing or lateral swing. Absence of rail pounding. Temperature-rise limitations. Efficiency.

Control. — See specifications under *Control Systems for Railway Motors*.

Motors. — See specifications in articles on *Motors*.

Air Brake. — Straight, automatic or combined. General characteristics.

COST, WEIGHT AND DIMENSIONS. — Electric locomotives cost about $400 per ton weight, which is considerably more than steam locomotives, but they are cheaper to operate and maintain as they can make more mileage per day. It costs, roughly, between 5 and 10 cents per locomotive mile to operate electric locomotives.

The characteristics of the most prominent types of electric locomotives are given in the following table:

CHARACTERISTICS OF ELECTRIC LOCOMOTIVES

Item	Bush terminal	Piedmont R.R.	Butte A. & P. R.R.	Detroit River Tunnel
System	D-C.	D-C.	D-C.	D-C.
Trolley voltage	500	1500	2400	600
Service	Freight	Freight	Freight and passenger	Freight and passenger
Total weight, tons	40	55	80	100
Number of motors	4	4	4	4
Horse-power per motor	90*	185*	300†	275*
Weight of electrical equipment, tons	14	19	30	27
Weight on drivers, tons	40	55	80	100
Diameter of drivers, inches	36	37	46	48
Rigid wheel base	6 ft. 6 in.	7 ft. 4 in.	8 ft. 8 in.	9 ft. 6 in.
Total wheel base	22 ft.	25 ft.	26 ft.	27 ft. 6 in.
Rated tractive effort, pounds	17,000	13,700	30,000	35,000
Rated speed, miles per hour	8	20	15.5	12
Classification	404	0440	0440	0440

Item	New Haven R.R.	N. Y. Central R.R.	Great Northern R.R.	Pennsylvania R.R.
System	A-C.	D-C.	3-phase	D-C.
Trolley voltage	11,000	600	6,000	600
Service	Passenger	Passenger	Freight and passenger	Passenger
Total weight, tons	102	115	115	166
Number of motors	4	4	4	2
Horse-power per motor	250*	550*	375*	1250*
Weight of electrical equipment	55	30	55	64
Weight on drivers, tons	81	71	115	104
Diameter of drivers, inches	62	44	60	72
Rigid wheel base	13 ft. 9 in.	13 ft.	11 ft.	8 ft.
Total wheel base	33 ft. 6 in.	36 ft.	31 ft. 9 in.	55 ft. 11 in.
Rated tractive effort, pounds	9,000	20,000	38,000	25,000
Rated speed, miles per hour	41	40	15	32
Classification	240042	484	0440	'4444

* At one-hour rating of motors. † At continuous rating of motors.

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[W. I. SLICHTER.]

LOCOMOTIVES, STEAM. — (*See also Locomotives, Electric; Railways, Energy Requirements and Motor Capacity for.*) Locomotives are classified broadly by the number of truck and driving wheels. Each arrangement of wheels has a definite name but the various wheel arrangements have multiplied so rapidly of late that the "Whyte classification" has largely come into use.

Whyte Classification. — According to this scheme of classification three numbers are used; the first indicates the total number of leading or truck wheels in front of the drivers, the second indicates the total number of drivers, and the third indicates the total number of wheels behind the drivers and under the locomotive proper. Tender wheels are not included. This classification may be extended, by the use of more than three numbers, to cover engines of the articulated, or Mallet, type. The classification and names of some of the more important types are given below, the weights being in short tons.

DESIGN AND PERFORMANCE. — A steam locomotive is essentially a moving power plant and as such it must transport its fuel and water with it. Its limitations as to size and weight are determined by roadway clearances, the curves upon which it must run, and the strength of the track supporting it.

The load upon a driving axle is generally limited to 30 tons. This weight is sometimes exceeded, but as a rule is not often equaled.

Maximum Power. — The limit of power of a steam locomotive is measured by the size and steam-producing capacity of its boiler (*see Boilers*). Owing to the necessarily small grate area, coal must be burned very rapidly under an induced draft. The rate of combustion is quite commonly as high as 100 lb., and is sometimes 150 lb. per sq. ft. of grate area per hour. The use of brick arches and superheaters largely increases the power, adding from 30 to 40 per cent in some cases. On account of difficulties of lubrication, wear of valves, etc., piston valves must be used with superheaters.

The maximum horse-power which can be exerted by a steam locomotive is in some cases 4000, although this is unusual; from 2000 to 2500 is a fairer maximum figure for ordinary cases. The largest tenders have a capacity of 9000 gallons of water and 15 tons of coal.

Tractive Effort. — The tractive effort of a single expansion steam locomotive at slow speed is computed from the following formula:

$$\text{Tractive effort} = \frac{P \times D^2 \times S}{d},$$

where P = mean effective pressure in cylinders, in lb. per sq. in. (usually taken as 0.8 of boiler pressure),

D = diameter of the piston, in inches,

S = stroke, in inches,

d = diameter of driving wheels, in inches.

Tonnage Rating. — By the tonnage rating of a locomotive is meant the weight, in tons, of the train which it can pull, exclusive of the weight of the locomotive, but including the weight of the tender. Short tons (2000 lb.) are used. Let:

F = maximum tractive effort, usually taken as the tractive effort corresponding to a mean effective pressure equal to 0.8 of boiler pressure,

G = maximum grade, per cent,

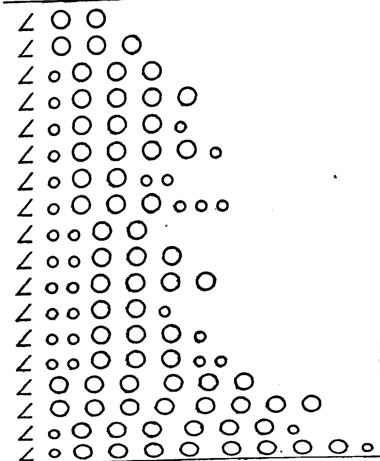
L = weight of locomotive, in tons,

R = locomotive resistance, lb. per ton,

r = train resistance, lb. per ton,

T = "tonnage rating," i.e., weight behind locomotive.

CLASSIFICATION OF STEAM LOCOMOTIVES

Type	Wheel arrangement	Name	Total wheels	Kind of service	Approximate weights,* tons, engine only	Load on drivers, tons per axle
0-4-0		4-wheel switcher	4	Switching	36	....
0-6-0		6-wheel switcher	6	Switching	83	....
2-6-0		Mogul	8	Freight	93.5	....
2-8-0		Consolidation	10	Freight	126	28
2-6-2		Prairie	10	Passenger & freight	122.5	28.3
2-8-2		Mikado	12	Freight	141	....
2-4-4		4-coupled double ender	10	Suburban	83	....
2-6-6		6-coupled double ender	14	Suburban	90	....
4-4-0		8-wheel or "American"	8	Passenger	60	....
4-6-0		10 wheel	10	Passenger & freight	109	28.3
4-8-0		12 wheel	12	Freight	111	....
4-4-2		Atlantic	10	Passenger	101	29
4-6-2		Pacific	12	Passenger & freight	158	33.5
4-6-4		6-coupled double ender	14	Suburban	78	....
0-6-6-0		Articulated	12	Freight	167.3	27.9
0-8-8-0		Articulated	16	Freight	222.5	27.8
2-6-6-2		Articulated	16	Freight	196	27
2-8-8-2		Articulated	20	Freight	231	25

* The weight of the tender loaded ranges from 58 to 92 per cent of the weight of the engine, the average being 74 per cent.

Then

$$T = \frac{F - (R + 20\,G)\,L}{r + 20\,G}.$$

See article on *Railways, Energy Requirements and Motor Capacity for*, for values of r and R. On heavy grades this value of the tonnage rating will be limited by the ability of the fireman to keep up full steam pressure.

Reduction of Tonnage Rating in Stormy Weather. — Weather conditions also affect the steam locomotive's capacity. The worst condition is cold weather with a heavy wind at right angles to the track. This side wind always makes engines steam badly and increases flange friction against the rails. It is a common cause of delays.

Tonnage ratings are reduced a variable amount under severe weather conditions. Some roads have fixed rules and some have not. The maximum reduction, taken from a table published in Bulletin No. 59 of the University of Illinois Engineering Experiment Station, is 30 per cent for temperature around 15 degrees below zero. On lines having heavy grades the grade resistance is such a large proportion of the total work to be done that the train resistance becomes a very small factor. Consequently the tonnage ratings for roads having very heavy grades are practically the same for all seasons and weather conditions.

Unbalanced Forces. — The necessity of partially balancing the heavy reciprocating parts by weights located on the wheels produces unbalanced vertical forces which are hard on roadbed and bridges.

Center of Gravity and Stability. — The boiler is necessarily placed almost wholly above the wheels and the center of gravity of the whole engine is therefore a considerable distance above the rails. Careful tests have shown this to be a positive advantage so far as ease of maintenance of track is concerned. By proper elevation of outer rails on curves, the overturning tendency of the centrifugal force may be wholly or sufficiently neutralized. This overturning tendency is much less than most engineers suppose. It is safe to state that it is impossible to *overturn* a locomotive at any practicable speed upon any ordinary curve by the action of centrifugal force unless the conditions are such (as in a cross-over, for instance) as to set up a rolling or oscillatory motion of the engine about its longitudinal axis. Generally speaking, the tender is the part of the train most likely to leave the rails. This is due to its varying and shifting load of coal and water.

Typical Large Locomotives. — The first three locomotives listed in the following table are perhaps the highest development of the steam locomotive up to the present date. The data for these locomotives, which are in use on the Pennsylvania R.R., was furnished through the kindness of T. R. Cook, Assistant Engineer of Motive Power, Pennsylvania Lines. These engines are all equipped with superheaters. The external heating surface of the superheaters is taken as 50 per cent greater than the heat surface of boiler in obtaining the total heating surface. In a test these locomotives produced an indicated horse-power of from 2 to 4 lb. of dry fuel per hour. The lowest rate represents a fuel consumption of 3500 to 4000 lb. of coal per hour, the locomotive having sufficient fire-box capacity to burn up to 9,000 or 10,000 lb. of coal per hour. This is a consumption of from 65 lb. to 180 lb. per sq. ft. of grate per hour.

The fourth locomotive is of the Mikado type, is not equipped with a superheater, and was built by the Philadelphia and Reading. It is designed to burn a mixture of buckwheat (anthracite) and soft coal. The information concerning this engine was taken from the Ry. Age. Gazette of Aug. 13, 1913.

TYPICAL STEAM LOCOMOTIVES

Item	Atlantic (4–4–2)	Pacific (4–6–2)	Consolidation (2–8–0)	Mikado (2–8–2)
Service	Passenger	Passenger	Freight	Freight
Maximum weight on any pair of drivers, lb	....	63,000	59,725	62,250 approx.
Weight of engine in working order, lb	....	289,000	250,500	331,000
Total weight on drivers, lb	....	187,000	226,000	249,000
Weight of tender loaded, lb	158,000	170,000	183,000	160,000
Total tractive effort, lb. (a)	25,797	36,031	50,069	54,000
Ratio weight on drivers to tractive power	....	5.18	4.45	4.35
Diameter of driving wheels	80″	80″	62″	61½″
Length of driving wheel base	7′ 5″	13′ 10″	17′ 0½″	16′ 6″
Total wheel base of engine	29′ 7½″	35′ 2½″	25′ 9½″	35′ 0″
Total wheel base of engine and tender	63′ 8″	67′ 0¾″	62′ 4⅞″	68′ 4¼″
Size of cylinders	22″×26″	26″×26″	26″×28″	24″×32″
Type of boiler	Belpaire wide fire box	Belpaire wide fire box	Belpaire wide fire box	Wooten fire box
Minimum internal boiler diameter	76¾″	78″	76¾″	82¼″
Number of tubes	242–36–144	202–32–128	265–36–144	504
Outside diameters of tubes	2″–5⅜″–1½″	2¼″–5½″–1½″	2″–5⅜″–1½″	2¼″
Size of fire box, inside	72″×110¼″	72″×110¾″	72″×110¼″	108″×144″
Grate area, sq. ft	55.13	55.37	55.13	108
External heating surface of tubes, sq. ft	2433.2	3453.3	2841.2	5210
Heating surface, superheater, sq. ft	653.4	1005	782.2	....
Heating surface, fire box, sq. ft	218	203.3	187	298
Total heating surface of boiler, sq. ft	3631	5164	4202	5508
Steam pressure, lb. per sq. in	205	205	205	225
Ratio of heating surface to grate surface	65.87	93.27	76.21	51
Ratio of external flue heat surface to fire-box heat surface	15.66	17.08	15.19	17.50
Tractive effort per lb. of mean effective pressure, lb	157.30	219.7	305.3	300

(a) Mean effective pressure taken as 0.8 of boiler pressure. Piston valve and Walschaert valve gear used on all four locomotives.

OPERATION OF STEAM LOCOMOTIVES. — Below are given some of the more important facts in regard to steam locomotive operation.

Location of Coal and Water Stations. — Coal and water stations for replenishing the tender are required at intervals, depending upon the topography of the line and volume of traffic. On fairly level roads coaling stations are required at intervals of 50 or 60 miles for freight engines and 120 miles for passenger engines. Water stations are needed at about 50-mile intervals for passenger and 25-mile intervals for freight service. These distances may be much reduced on heavy grades. The location of water stations is also a matter of available water supply. Where possible water should be taken at regular stops.

Fuel and Water Consumption. — This is very variable depending upon topography and alignment of road, kind of service, frequency of stops, whether single or double track, climatic conditions, weight of trains, etc. The number of pounds of coal burned per locomotive mile averages about 104 for passenger, 208 for freight, 130 for mixed, 108 for switching and 150 for all types of service. The actual water evaporated varies from 4.5 to 6.5 lb. per pound of coal burned. The first figure is for a coal consumption of 200 lb. and the last for 65 lb. per sq. ft. of grate per hour.

In some parts of the U.S. oil is used for fuel on account of the high cost of coal. It results in a greater expense for upkeep of boilers on account of intense heat, but in the far west and southwest is economical on the whole.

Use of Different Kinds of Coal. — Exhaust nozzles and draft appliances can be arranged to suit any one of various kinds of coal, and arrangements should be made to use only the kind of coal for which the engine is "drafted" as other kinds are burned at a less efficiency and consequently at what is probably a larger expense even though the cost per ton of the improper coal is less.

Idle Steaming. — Whenever a locomotive is standing idle under steam, coal is being consumed. This idle time should be reduced to a minimum by avoiding firing up a long while before an engine is needed, and by keeping trains moving while they are on the road. Roads having traffic largely of one kind can show better results in this particular than those having a mixed traffic composed of equal parts of all kinds.

Lubrication. — The friction in a locomotive is large, ranging from about 20 per cent of the total power of the engine to a maximum of 50 per cent or more under certain conditions of passenger service. Probably 35 per cent is a fair average. In many cases an appreciable economy can be instituted by increasing the quantity of oil allowed for an engine. Most motive power officials are strangely blind in this respect. Generally oil consumption is kept at a minimum, and enough coal is burned to overcome the resulting friction. The proper method is to so arrange the ratio of oil to coal that the total expense for the two is a minimum.

Blowing-off. — Boilers must be blown off and washed out at intervals depending upon the quality of water used.

COSTS.—The first cost of a steam locomotive is from 6 to 7 cents per pound of weight, including tender.

Annual Costs. — The cost of maintenance and repairs ranges from $2000 to $3500 per locomotive per year, an average figure being $2600.

The average life of a locomotive is about 20 years; hence in addition to the cost of maintenance and repairs an annual depreciation of 5 per cent of the first cost should be charged against the locomotive. In addition, an interest charge of, say, 5 per cent of the first cost should be included.

The operating costs are as follows, the figures being for ordinary mixed service on a trunk line with moderate grades:

Wages of crew,	10.6 cents per loco.-mile
Coal, at $3.00 per ton,	18.9 cents per loco.-mile
Oil, waste, etc.,	0.3 cent per loco.-mile
Wipers,	0.5 cent per loco.-mile
Repairs,	11.3 cents per loco.-mile
Total operating cost,	41.6 cents per loco.-mile

The average cost of operation of a locomotive, including wages of conductor and flagman, is about $50 per day, on the basis of 100 miles being a day's work.

The total annual cost of a locomotive costing initially $20,000 and covering an average of 100 miles per day would then be roughly:

Operation	$10,000
Maintenance and repairs	2,600
Depreciation	1,000
Interest	1,000
Total	$14,600

BIBLIOGRAPHY. — Henderson, G. R., *The Cost of Locomotive Operation*, N. Y., 1906; Goss, W. F. M., *Locomotive Performance*, N. Y., 1907; Interstate Commerce Commission, *Statistics of Railways in the United States, 1910*, Washington; Pettigrew, W. F., *A Manual of Locomotive Engineering*, London, 1909; numerous articles in *Railway and Locomotive Engineering;* and the *Railway Age Gazette*.

[L. E. Moore.]

LOGARITHMS. — The logarithm of a number A, to a given base b, is the power n to which that base b must be raised in order to equal the number A. Thus, if $b^n = A$, then n is the logarithm of A to the base b, which may be written $n = \log_b A$. From this definition the following properties are readily deduced, where A and B are any two numbers.

$$\log AB = \log A + \log B.$$

$$\log \frac{A}{B} = \log A - \log B.$$

$$\log A^n = n \log A.$$

Characteristic and Mantissa. — These three equations hold irrespective of what is chosen as the base of the logarithms, provided the same base is used for each logarithm. In the common or Brigg's system of logarithms the number 10 is chosen as the base. In such a system the logarithm of any number may be expressed directly in terms of the logarithm of a number (including decimal fractions as numbers) between 1 and 10. For example:

$$\begin{aligned} \log_{10} 376.42 &= \log (100 \times 3.7642) \\ &= \log 100 + \log 3.7642 \\ &= 2 + \log 3.7642, \end{aligned}$$

since the power to which 10 must be raised to give 100 is 2. Similarly, $\log_{10} 3764.2 = 3 + \log 3.7642$. The logarithms of all numbers between 1 and 10 are less than unity. The whole number or integer part of a logarithm is called its "characteristic" and the fractional part its "mantissa." The characteristic of the logarithm of a number less than unity is negative. For example:

$$\begin{aligned} \log 0.037642 &= \log (\tfrac{1}{100} \times 3.7642) \\ &= \log 1 - \log 100 + \log 3.7642 \\ &= -2 + \log 3.7642. \end{aligned}$$

In general, *the characteristic of a number greater than unity is positive and is one less than the number of figures to the left of the decimal, while the characteristic of a number less than unity is negative and is one greater than the number of ciphers between the decimal and the first significant figure.* A table of logarithms gives the mantissas only, the characteristics being determined by the above rule. Such a table is given below.

Antilogarithms. — If $n = \log A$, then A is the number whose logarithm is n. This may be written symbolically

$$A = \log^{-1} n.$$

A is then called the antilogarithm or inverse logarithm of n. The antilogarithm of a number (i.e., the number which has the given number for its logarithm) is found from a table of logarithms by finding the number in the margin corresponding to the decimal point of the given number in the table, and fixing the decimal point by the rule given above. Example:

$$\log^{-1} 1.6464 = 44.3.$$

Use of Logarithms. — Logarithms are used in the processes of multiplication, division, raising to powers and taking roots. For example, to find the product of two numbers take from the table the logarithms of the two numbers,

COMMON LOGARITHMS OF NUMBERS

No.	0	1	2	3	4	5	6	7	8	9
10	00000	00432	00860	01283	01703	02119	02531	02938	03342	03743
11	04139	04532	04922	05308	05690	06070	06446	06819	07188	07555
12	07918	08279	08636	08991	09342	09691	10037	10380	10721	11059
13	11394	11727	12057	12385	12710	13033	13354	13672	13988	14301
14	14613	14921	15229	15534	15836	16137	16435	16732	17026	17319
15	17609	17897	18184	18469	18752	19033	19312	19590	19866	20140
16	20412	20683	20952	21218	21484	21748	22010	22271	22530	22788
17	23045	23299	23552	23804	24054	24303	24551	24797	25042	25285
18	25527	25767	26007	26245	26481	26717	26951	27184	27415	27646
19	27875	28103	28330	28555	28780	29003	29225	29446	29666	29885
20	30103	30319	30535	30749	30963	31175	31386	31597	31806	32014
21	32222	32428	32633	32838	33041	33243	33445	33646	33845	34044
22	34242	34439	34635	34830	35024	35218	35410	35602	35793	35983
23	36173	36361	36548	36735	36921	37106	37291	37474	37657	37839
24	38021	38201	38381	38560	38739	38916	39093	39269	39445	39619
25	39794	39967	40140	40312	40483	40654	40824	40993	41162	41330
26	41497	41664	41830	41995	42160	42324	42488	42651	42813	42975
27	43136	43296	43456	43616	43775	43933	44090	44248	44404	44560
28	44716	44870	45024	45178	45331	45484	45636	45788	45939	46089
29	46240	46389	46538	46686	46834	46982	47129	47275	47421	47567
30	47712	47856	48000	48144	48287	48430	48572	48713	48855	48995
31	49136	49276	49415	49554	49693	49831	49968	50105	50242	50379
32	50515	50650	50785	50920	51054	51188	51321	51454	51587	51719
33	51851	51982	52113	52244	52374	52504	52633	52763	52891	53020
34	53148	53275	53402	53529	53655	53781	53907	54033	54157	54282
35	54407	54530	54654	54777	54900	55022	55145	55266	55388	55509
36	55630	55750	55870	55990	56110	56229	56348	56466	56584	56702
37	56820	56937	57054	57170	57287	57403	57518	57634	57749	57863
38	57978	58092	58206	58319	58433	58546	58658	58771	58883	58995
39	59106	59217	59328	59439	59549	59659	59769	59879	59988	60097
40	60206	60314	60422	60530	60638	60745	60852	60959	61066	61172
41	61278	61384	61489	61595	61700	61804	61909	62013	62118	62221
42	62325	62428	62531	62634	62736	62838	62941	63042	63144	63245
43	63347	63447	63548	63648	63749	63848	63948	64048	64147	64246
44	64345	64443	64542	64640	64738	64836	64933	65030	65127	65224
45	65321	65417	65513	65609	65705	65801	65896	65991	66086	66181
46	66276	66370	66464	66558	66651	66745	66838	66931	67024	67117
47	67210	67302	67394	67486	67577	67669	67760	67851	67942	68033
48	68124	68214	68304	68394	68484	68574	68663	68752	68842	68930
49	69020	69108	69196	69284	69372	69460	69548	69635	69722	69810
50	69897	69983	70070	70156	70243	70329	70415	70500	70586	70671
51	70757	70842	70927	71011	71096	71180	71265	71349	71433	71516
52	71600	71683	71767	71850	71933	72015	72098	72181	72263	72345
53	72428	72509	72591	72672	72754	72835	72916	72997	73078	73158
54	73239	73319	73399	73480	73559	73639	73719	73798	73878	73957

COMMON LOGARITHMS OF NUMBERS

No.	0	1	2	3	4	5	6	7	8	9
55	74036	74115	74193	74272	74351	74429	74507	74585	74663	74741
56	74818	74896	74973	75050	75127	75204	75281	75358	75434	75511
57	75587	75663	75739	75815	75891	75966	76042	76117	76192	76267
58	76342	76417	76492	76566	76641	76715	76789	76863	76937	77011
59	77085	77158	77232	77305	77378	77451	77524	77597	77670	77742
60	77815	77887	77959	78031	78103	78175	78247	78318	78390	78461
61	78533	78604	78675	78746	78816	78887	78958	79028	79098	79169
62	79239	79309	79379	79448	79518	79588	79657	79726	79796	79865
63	79934	80002	80071	80140	80208	80277	80345	80413	80482	80550
64	80618	80685	80753	80821	80888	80956	81023	81090	81157	81224
65	81291	81358	81424	81491	81557	81624	81690	81756	81822	81888
66	81954	82020	82085	82151	82216	82282	82347	82412	82477	82542
67	82607	82672	82736	82801	82866	82930	82994	83058	83123	83187
68	83250	83314	83378	83442	83505	83569	83632	83695	83758	83821
69	83884	83947	84010	84073	84136	84198	84260	84323	84385	84447
70	84509	84571	84633	84695	84757	84818	84880	84941	85003	85064
71	85125	85187	85248	85309	85369	85430	85491	85551	85612	85672
72	85733	85793	85853	85913	85973	86033	86093	86153	86213	86272
73	86332	86391	86451	86510	86569	86628	86687	86746	86805	86864
74	86923	86981	87040	87098	87157	87215	87273	87332	87390	87448
75	87506	87564	87621	87679	87737	87794	87852	87909	87966	88024
76	88081	88138	88195	88252	88309	88366	88422	88479	88536	88592
77	88649	88705	88761	88818	88874	88930	88986	89042	89098	89153
78	89209	89265	89320	89376	89431	89487	89542	89597	89652	89707
79	89762	89817	89872	89927	89982	90036	90091	90145	90200	90254
80	90309	90363	90417	90471	90525	90579	90633	90687	90741	90794
81	90848	90902	90955	91009	91062	91115	91169	91222	91275	91328
82	91381	91434	91487	91540	91592	91645	91698	91750	91803	91855
83	91907	91960	92012	92064	92116	92168	92220	92272	92324	92376
84	92427	92479	92531	92582	92634	92685	92737	92788	92839	92890
85	92941	92993	93044	93095	93146	93196	93247	93298	93348	93399
86	93449	93500	93550	93601	93651	93701	93751	93802	93852	93902
87	93951	94001	94051	94101	94151	94200	94250	94300	94349	94398
88	94448	94497	94546	94596	94645	94694	94743	94792	94841	94890
89	94939	94987	95036	95085	95133	95182	95230	95279	95327	95376
90	95424	95472	95520	95568	95616	95664	95712	95760	95808	95856
91	95904	95951	95999	96047	96094	96142	96189	96236	96284	96331
92	96378	96426	96473	96520	96567	96614	96661	96708	96754	96801
93	96848	96895	96941	96988	97034	97081	97127	97174	97220	97266
94	97312	97359	97405	97451	97497	97543	97589	97635	97680	97726
95	97772	97818	97863	97909	97954	98000	98045	98091	98136	98181
96	98227	98272	98317	98362	98407	98452	98497	98542	98587	98632
97	98677	98721	98766	98811	98855	98900	98945	98989	99033	99078
98	99122	99166	99211	99255	99299	99343	99387	99431	99475	99519
99	99563	99607	99651	99694	99738	99782	99825	99869	99913	99956

add these logarithms, and then from the table find the number of which this is the logarithm. The position of the decimal point is fixed by the value of the mantissa in the sum of the two logarithms. By adding a whole number to a mantissa and subtracting the same number from the characteristic the mantissa of the final result can always be kept positive.

Examples. — Multiply 376.2 by 0.587:

$$\begin{aligned} \log 376.2 &= \quad 2 + .57541 \\ \log 0.587 &= -1 + .76863 \end{aligned}$$

$$\text{Adding gives } 1 + 1.34404 = 2.34404$$

therefore $376.2 \times 0.587 = 220.8.$

Divide 37.62 by 587:

$$\begin{aligned} \log 37.62 &= 1 + .57541 = \quad 1.57541 \\ \log 587 &= 2 + .76863 = 2 + .76863 \end{aligned}$$

$$\text{Subtracting gives} - 2 + .80678$$

therefore $$\frac{37.62}{587} = 0.06409.$$

NATURAL LOGARITHMS. — (*See also Roots and Powers.*) The base of the so-called natural system of logarithms is the value of the expression $\left(1 + \frac{1}{n}\right)^n$ when n is taken equal to infinity. The numerical value of this expression is found by expanding $\left(1 + \frac{1}{n}\right)^n$ by the binomial theorem (*see Series*), and is equal to $2.718282 +$. This number is usually represented by the symbol ϵ, that is,

$$\epsilon = 2.718282 +, \text{ or } 2.718 \text{ approximately.}$$

Logarithms to this base are readily calculated by means of Taylor's series (*see Series*); also this number ϵ enters in a very simple manner into various mathematical and physical relations (*see Equations, Differential; Transient Electric Phenomena; Hyperbolic Functions; Trigonometric Functions*).

The relation between a logarithm of any number A to any base b and the logarithm of A to any other base a is

$$\log_b A = \frac{\log_a A}{\log_a b}.$$

Hence $$\log_\epsilon A = 2.30259 \log_{10} A.$$

From this last relation the natural logarithm of any number may be found directly from the table of common logarithms.

Example. —

$$\log_\epsilon 376.2 = 2.3026 \times 2.57541 = 5.930.$$

The natural logarithm may also be taken directly from the table of exponential functions (q.v.), remembering that $\log_\epsilon A$ is the number in the margin of the table corresponding to the number equal to A in the columns of the table.

The symbol "ln" is frequently used for the natural logarithm and the symbol "log" without a subscript is usually employed as an abbreviation for the logarithm to the base 10.

[W. A. DEL MAR.]

LUBRICANTS AND LUBRICATION. — (*See also Bearings; Friction.*)

Ordinary lubricants may be classified as follows:

Vegetable Oils. — Commonly employed vegetable oils are linseed, cottonseed, rape and castor. Vegetable oils decompose at comparatively low temperatures. They are used chiefly for compounding with mineral oils.

Animal Fats. — Animal fats ordinarily employed for lubrication are tallow, neat's-foot, lard, sperm, wool grease and fish oil. Like vegetable oils they decompose at comparatively low temperatures and are used chiefly for compounding with mineral oils.

Mineral Oils. — These are all petroleum products, and form the whole or the greater part of most of the lubricants employed.

Solid Lubricants. — Dry graphite, soapstone and mica are sometimes used as lubricants for slow-speed work when the bearing surface is restricted in area and the load to be carried is very large.

"Deflocculated" Graphite. — In 1906 E. G. Acheson discovered a process of producing a fine, pure, unctuous graphite, which when heated with a solution of tannin would remain suspended in water for months. The graphite thus suspended in water, known as "aquedag" has been successfully used as a lubricant (*Trans. A.I.E.E., 1907*). Acheson's "deflocculated" graphite, as the graphite in the finely divided form is called, has also been suspended in oil, the oil emulsion being known as "oildag," making an excellent lubricant.

Greases. — Compounds of oils and fats containing sufficient soap to form a more or less solid mass at ordinary temperatures are called greases. Lime soda or lead soaps are used in these compounds. For very high pressures graphite, soapstone and mica are sometimes added to the grease.

QUALIFICATIONS OF A GOOD LUBRICANT. — The generally accepted conditions of a good lubricant are as follows:

1. "Body" (i.e., viscosity) enough to prevent the surfaces to which it is applied from coming in contact with each other.
2. Freedom from corrosive acid, of either mineral or animal origin.
3. As fluid as possible consistent with sufficient "body."
4. Low coefficient of friction (as determined in a standard bearing).
5. High "flash" and burning points.
6. Freedom from all materials liable to produce oxidation or "gumming."

The examinations to be made to verify the above are both chemical and mechanical, and are usually arranged in the following order:

1. Identification of the oil, whether a simple mineral oil, or animal oil, or a mixture. 2. Density. 3. Viscosity. 4. Flash point. 5. Burning point. 6. Acidity. 7. Coefficient of friction. 8. Cold test.

Test for Fats. — Heat a small quantity of the oil in a small test tube 15 minutes with small pieces of metallic sodium or caustic potash. If fatty oil is present, a soapy mass will form at the top. (*Gebhardt.*)

Test for Tarry Matter. — Dissolve a small quantity of the oil in from 10 to 20 times its bulk of gasoline; tar and other insoluble matter will be precipitated. (*Gebhardt.*)

Specific Gravity. — This is usually made with a hydrometer, graduated according to the Baumé scale. At 60° F.

$$\text{Specific gravity} = \frac{140}{130 + \text{degrees Baumé}}.$$

Viscosity. — Viscosity, or internal friction, is usually determined by observing the time required for a given amount of oil to flow through a standard orifice. By "specific viscosity" is meant the ratio of the time for the oil to run out to that required for an equal volume of water at 60° F. The temperature of the oil should always be observed and stated. Engine oils are usually tested for viscosity at 70° F. and cylinder oils at 212° F. (*Gebhardt.*)

Flash Point. — The flash point is determined by heating a sample of oil in a cup at the rate of 15° F. per minute until a spark will ignite the vapor; the corresponding temperature is the flash point. The flash point as thus determined depends to some extent upon the surface exposed, the size of the spark, the distance between spark and surface of oil and the dimensions of the cup. (*Gebhardt.*)

Burning Point or Fire Test. — By continuing the application of heat and noting the temperature at which the oil itself takes fire and continues to burn, the burning point is determined.

Acidity. — The presence of free acid is determined by shaking up equal quantities of oil and water and testing with litmus paper.

Cold Test. — The cold test is the temperature at which the oil will just flow.

Friction Test. — The coefficient of friction as determined from friction-testing machines gives but little information concerning the action of the oil under the widely different conditions found in practice. (*Gebhardt.*)

Properties of Vegetable and Animal Oils. — The following data are taken from Gebhardt's *Steam Power Plant Engineering.*

Kind of oil	Specific gravity		Flash test, ° F.
	Water as 1.00	Baumé	
Lard	0.9175	23	505
Sperm	0.8815	29	478
Tallow	0.9080	24.5	540
Cottonseed	0.9210	22	518
Linseed	0.9299	19	505
Castor	0.9639	15	...
Palm	0.9046	25	405
Rape-seed	0.9155	23	...

Properties of mineral oils as compounded for ordinary use are given below.

Grease Lubricants. — Tests made on an Olsen lubricant testing machine at Cornell University are reported in *Power*, Nov. 9, 1909. It was found that some of the commercial greases stood much higher pressures than the oils tested, and that the coefficients of friction at moderate loads were often as low as those of the oils. The journal of the testing machine was 3¾ inches diameter, 3⅓ inches long, and the babbitt bearing shoe had a projected area of 5.8 square inches. The speed was 240 revolutions per minute and each test lasted one hour, except when the bearing showed overheating. The following are the coefficients of friction obtained in the tests:

RELATIVE VALUES OF FRICTION COEFFICIENT WITH GREASES AND OILS

Lb. per sq. in.	Mineral grease	Animal grease	Graphite grease	Mineral grease	Engine oil	Engine oil	Grease	Grease
86.2	0.024	0.023	0.04	0.023	0.019	0.015	0.020	0.025
172.4	0.021	0.023	0.05	0.018	0.04	0.022	0.015	0.022
258.6	0.021	0.023	....	0.018	0.06	0.037	0.014	0.020
344.8	0.025	0.025	....	0.019	....	....	0.017	0.020
431.0	0.050	0.035	....	0.028	....	....	0.026	0.019

APPLICATIONS OF VARIOUS TYPES OF LUBRICANTS. — The type of lubricant to use in any case depends upon:

1. The cost due to consumption of lubricant.
2. The saving in annual cost due to lessening of wear of bearings, guides and other rubbing surfaces.
3. The cost of the energy saved (as the result of decreased friction losses) due to the use of the lubricant.

For minimum annual cost the sum of the last two items should equal the first. Estimates of this kind are difficult to make, and the result is that the kind of lubricant used in any specific case is usually determined by experience.

The following table, from a paper in *Power*, December, 1905, p. 750, gives the kind of lubricant ordinarily employed for various purposes, together with their approximate characteristics. The cold test of all these oils, except oil for refrigerating machinery, is given as 30° F. Refrigerating-machinery oil should not solidify above 0° F.

METHODS OF LUBRICATION. — The commonest type of "lubricator" on engines or dynamos is the simple oil cup with sides of glass, so that the level of the oil in the cup can be seen. Any type of lubricator in which the flow of oil can be seen is known as a "sight feed." The flow of oil is regulated by a needle valve in the base of the cup.

Lubrication of Crossheads, Crank Pins, etc. — In applying oil to rubbing surfaces, both of which are in motion, various devices are used by means of which the oil cup can be kept at rest. A stationary oil cup may be used with a "wiper" on the moving member, or a "telescopic," "pendulum" or "centrifugal" oiler may be employed. Sometimes the crank, connecting rod and crossheads are inclosed in a casing the bottom of which is filled with oil so that at each revolution the end of the connecting rod splashes oil over all the parts.

Oil Rings and Chains. — The bearings of small high-speed engines and dynamos are frequently provided with rings or chains running loosely over the journal and dipping into an oil bath in the pedestal below the bearing. The rotation of the journal gives enough motion to the rings to enable them to carry up sufficient oil from the bath to keep the bearing surfaces bathed in oil.

Cylinder Lubrication. — The oil must be forced into steam cylinders against the steam pressure. This is usually accomplished by means of specially constructed cylinder cups, hydrostatic lubricators or force pumps. A brief description of the more common forms of these devices will be found in Gebhardt's *Steam Power Plant Engineering*.

APPLICATIONS AND CHARACTERISTICS OF VARIOUS OILS

Kind of oil and application	Specific gravity, Baumé	Flash test, °F.	Fire test, °F.	Viscosity at 70° F. (Water = 1)
High-pressure cylinder oils: For cylinders using dry steam at from 110 to 210 lb.	25–24.5	600–610	645–660	175–205
General cylinder oil: For cylinders using dry steam at from 75 to 100 lb. Also for air compressor cylinders when the oil is made from steam-refined mineral stock and has a viscosity of 200.	26–25.5	550–585	600–630	180–190
*Wet cylinder oil** For cylinders using moist steam, especially in compound- and triple-expansion engines.	25.8–25.3	560–585	600–630	150–185
Gas-engine cylinder oil†	26.5	320	350	300
Automobile gas-engine oil‡	29.5	430	485	195
Heavy engine and machinery oils: For heavy slides and bearings, shafting and horizontal surfaces.	30.5–29.5	400	440–450	170–195
General engine and machinery oils: For high-speed dynamos and other comparatively heavy machines.	30.8–30	400–420	450–470	175–190
Fine and light machine oils: For fine work, such as printing presses, sewing machines, typewriters, spindles, etc.	32.5–30.2	400	440	110–160
Cutting oils: For cutting tools, screws, etc.	27–23	410–420	475–480	210–175
Refrigerating machine oils	30.2	200	225	165
Wet service and marine oils§	28	430	475	230
Greases: Various kinds, used in special work requiring high pressures and low velocities.	......	....	....	.....

* May contain not over 2 to 6 per cent of refined acidless tallow oil in the high-pressure oils and not over 6 to 12 per cent in the low-pressure oils.

† Neutral mineral oil compounded with soap. The soap will not decompose at high heat, and although not a lubricant serves as a vehicle for carrying some oil.

‡ Owing to lack of body this oil will not deposit carbon on the sparking points.

§ May contain 30 to 40 per cent of pure strained lard oil.

Oil-feed Systems. — In power plants oil is supplied continuously to the bearing surfaces of the various engines and generators by means of an oil-feed system, comprising essentially a supply tank, pump and the necessary piping. Oil filters and purifiers are also used in connection with the oiling system, to eliminate the impurities which collect in it due to dust, wear of bearings, exposure to the heat and to the atmosphere. See article on *Power Stations*.

AMOUNT OF OIL REQUIRED FOR ENGINES. — J. H. Spoor, in *Power*, Jan. 4, 1910, has made a study of a great number of records of the amount of oil used for lubricating cylinders of different engines, and has reduced them to a systematic basis, i.e., the number of pints of oil used in a 10-hour day for different areas of surface lubricated. The surface is determined in square inches by multiplying the circumference of the cylinder by the length of stroke. The results are plotted in a series of curves for different types of engines, and approximate average figures taken from these curves are given below:

PINTS OF CYLINDER OIL IN 10 HOURS

Type of Engine	Square inches lubricated								
	1000	2000	3000	4000	6000	8000	10,000	12,000	18,000
Automatic high speed	2	....	....	....	....	....	....	....	....
Simple slide valve..	0.5	....	....	....	....	....	....	....	....
Compound..........	....	2	....	3.5	4.3	5	5.5	6	6.5
Corliss:									
Average...........	0.9	1.65	2.25	3.75	....	....	....	....	....
Maximum.........	1.2	2.25	....	....	....	....	....	....	....
Minimum.........	....	1.00	....	....	....	....	....	....	....

As shown in the figures under 2000, Corliss, a certain engine may take 2¼ times as much oil as another engine of the same size. The difference may be due to smoothness of cylinder surface, kind and pressure of piston rings, quality of oil, method of introducing the lubricant, etc. Variations in speed of a given type of engine and in steam pressure do not appear to make much difference, but the small automatic high-speed engine takes more oil than any other type. Vertical marine engines are commonly run without any cylinder oil, except that used occasionally to swab the piston rods.

The amount of engine oil required will of course depend upon the number of cups on the engine and the size of the various bearings. For a 1000-h.p. Corliss engine the Vacuum Oil Company state that the amount of engine oil would not exceed twice the amount of cylinder oil required.

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[WM. KENT.]

MACHINE TOOLS, ELECTRICAL OPERATION OF. — (*See also Motors, Industrial Applications of.*) When the work of equipping a machine tool with motor drive is undertaken, there are certain features which should be taken into account and properly analyzed, as the conditions of operation generally vary greatly with the product manufactured. If a tool is intended for a certain specialized kind of work, information on the following points should be given:

1. The exact class of work which the tool is to accomplish.
2. If the power required to remove the metal is not known, then a statement should be made as to the approximate feed and cutting speeds to be taken.
3. Careful analysis should be made of the time required to load and unload the machine, to determine the feasibility of employing auxiliary means other than manual labor for loading the tools.

From this information an approximate determination can be made as to the intermittency of operation of the tool, in order to decide whether an intermittently rated motor or a continuously rated motor will be required. From a knowledge of the physical shape of the work, a determination can be made as to whether an adjustable-speed motor will result in economy of time, if used on this particular class of tool. The tool builder can then decide upon the proper type of controller, and its most desirable location from an operating point of view for the workman.

If a special type of tool is not desired and it is preferable to purchase one with such characteristics that it can be used for general manufacturing, one should determine as nearly as possible the range of material or work for which it will be used in straight manufacturing operations. A knowledge of this will undoubtedly permit of a better motor and tool selection than the simple purchase of a standard stock tool.

TYPE OF MOTOR. — The following table will aid in the choice of the proper motor for machine tool application.

It must be kept in mind that various circumstances, such as size or roughness of work, flywheel capacity, etc., may call for radical departures in choice of motors, this list being compiled to meet average conditions.

Shunt Motors are used in the following cases: when the work is of a fairly steady nature; when considerable range of adjustment of speed is required, as on lathes and boring mills, and on group and line-shaft drives, etc.

Compound-wound Motors are used where there are sudden calls for excessive power of short duration, as on planers, punch presses, etc.

Series Motors should be used where speed regulation is not essential and where excessive starting torque and slow starting speeds are required, as, for instance, in moving carriages of large lathes, in raising and lowering the cross rails of planers and boring mills, and for operating cranes.

When in doubt as to the choice of compound or series motors of small horsepower, the choice might be determined by the simplicity of control in favor of the series motor. Series motors, however, should never be used when the motor can run without load, as the speed would accelerate beyond the point of safety.

Induction Motor. — The alternating-current motor of the squirrel-cage rotor type corresponds to the constant-speed, shunt, direct-current motor, but with a high-resistance rotor it approaches more closely the characteristics of a compound direct-current motor. It is understood that the variable-speed machines, checked in this list under the alternating-current squirrel-cage rotor column, have the necessary mechanical speed changes.

The slip-ring induction motor with external rotor resistance would be used

MOTORS FOR MACHINE TOOLS

Tool	D-C.			A-C. (*See Footnotes*)		
	Shunt	Comp., %	Series	×	#	⊕
Bolt cutter	✓	......	......	×	....	....
Bolt and rivet header	......	20–40	......	×	#	....
Bulldozers	......	20–40	......	×	#	....
Boring machines	✓	......	......	×	....	....
Boring mills	✓	......	......	×	....	....
Raising cross rails on boring mills and planers	......	60	✓	....	#	....
Boring bars	✓	......	......	×	....	....
Bending machines	......	20–40	......	×	#	....
Bending rolls	......	40–80	✓	....	....	⊕
Corrugating rolls	......	20–40	......	×	#	....
Centering machines	✓	......	......	×	....	....
Chucking machines	✓	......	......	×	....	....
Boring, milling and drilling machines	✓	......	......	×	....	....
Drill, radial	✓	......	......	×	....	....
Drill press	✓	......	......	×	....	....
Grinder — tool, etc	✓	......	......	×	....	....
Grinder — castings	......	20	......	×	....	....
Gear cutters	✓	20	......	×	....	....
Hammers — drop	......	20–30	......	....	#	....
Keyseater — milling — broach	✓	......	......	×	....	....
Keyseater — reciprocating	✓	20	......	×	....	....
Lathes	✓	......	......	×	....	✠
Lathe carriages	......	......	✓	....	#	....
Milling machines	✓	......	......	×	....	....
Heavy slab milling	✓	20	......	×	....	....
Pipe cutters	✓	......	......	×	....	....
Punch presses	......	20–40	......	×	#	....
Planers	......	20	......	×	#	....
Planers — rotary	✓	20	......	×	....	....
Saw — small circular	✓	......	......	×	....	....
Saw — cold bar and *I* beam	✓	20	......	×	....	....
Saw — hot	......	20	......	×	....	....
Screw machine	✓	......	......	×	....	....
Shapers	✓	20	......	×	....	....
Shears	......	20–40	......	×	#	....
Slotters	✓	20	......	×	....	....
Swaging	✓	20	......	×	#	....
Tappers	✓	......	......	×	....	....
Tumbling barrels or mills	......	20	......	×	....	....

× Squirrel-cage rotor.
Squirrel-cage rotor — high starting torque.
⊕ Slip-ring induction motor with external rotor resistance.
✠ Might be used for tire lathes as it allows slowing down when cutting hard spots.

for variable speed, but this must not be construed to mean that it corresponds to a direct-current, adjustable-speed motor, as it has the characteristics of a direct-current shunt motor with armature control.

The self-contained, rotor-resistance type would be used for line-shaft drives, and for groups when of sufficient size.

Multi-speed Alternating-current Motors are those giving a number of definite speeds, usually 600 and 1200 or 600, 900, 1200 and 1800 r.p.m., and are made for both constant horse-power and constant torque. These motors would be used where alternating current only was available, or direct current limited; and the speed range of the motor, together with one or two change gears, would give the required speeds.

Shaft Couplings. — In connection with the selection of motors, standard shafts and shaft extensions should be chosen so that spare parts and interchanges may be made with the least cost and time. A number of standard shafts and shaft extensions are shown in the sketches in Fig. 1.

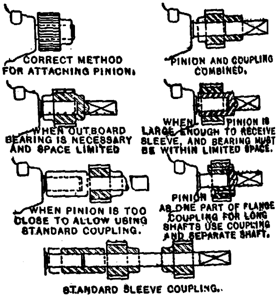

Fig. 1. Standard Shaft Couplings

CONTROL EQUIPMENT. — The choice of control, whether it be for old or new tools, in the majority of cases is fully as important as that of the motor. In selecting the control it is necessary to consider the nature of the work, the accessibility of the controller to the operator, the method of attaching it to the tool and in some cases its relative position to other tools; for instance, an open-type starting rheostat should not be exposed to danger of short-circuit from flying chips.

When installing controllers, accessibility in case of accident should be kept in mind, even though of little importance as far as starting up is concerned. The starting apparatus should be placed where the motor or some of the moving parts can be seen by the operator. On individual motor-driven tools, where the motor is started and stopped many times a day or where the starting conditions are of a severe nature, or where tools are edged along, drum-type controllers with extra heavy starting resistance should be used. For adjustable speed motors, using the drum-type control, the field control should be through fingers making contact on segments of the controller drum and not by sliding contacts on a dial. Motors above 40 or 50 horse-power under these severe conditions are best operated by a master controller which operates contactors for cutting out steps of starting resistance, and if adjustable speed, the field control should be taken care of by fingers making contact on segments of the drum. This class of starting apparatus will stand any quantity of abuse and, by the addition of a simple current limit relay device, becomes practically a fool-proof protection for the motor. There are cases where it might be advantageous to use master controllers and contactors even with smaller motors. The controlling apparatus as well as the motor in the case of individual drives should be attached directly to the tool when possible. This arrangement allows moving the tool by simply disconnecting the leads and connecting them in the new position. In case of portable tools this, of course, is an absolute necessity.

Upon the convenient arrangement of the control depends, to a considerable degree, the output of the tool. The importance of the arrangement from the standpoint of the operator cannot be ignored, since the output of a tool will be materially increased when an operator can start and stop the tool and obtain

at all times maximum cutting speeds by simply turning a handle. The controller must be placed in a safe position and should be accessible for repairs, which very often means that some arrangement is necessary to bring the operating handle within easy access of the operator.

The convenience of control, which bears directly on production, is ignored in the majority of tools where the control is of the greatest importance. A familiar illustration of the convenience of control is the arrangement so commonly seen on lathes, whereby the operating handle travels with the tool carriage and allows the operator at all times a complete control of his tool.

Application of Reversing Motors. — One of the most interesting motor applications of recent date is the use of reversing motors for machine tools. The large increase in production due to this form of drive on planers is now generally appreciated, but the application of the reversing-motor drive in its various forms (which is almost unlimited) is not so well appreciated. It is applicable not only to planers, new and old, but to screw-, worm- and rack-driven slotters, keyseaters, turret lathes, wire- and tube-drawing machinery and to boring mills, when machining projections which are short in comparison with the total travel of the mill or when machining surfaces where projections prevent a complete revolution. Reversing motors are also applicable to that class of reversing machinery which is now reversed through clutches, shifting belts, etc., the cost of maintenance of which is usually high and the efficiency low.

The motors recommended for this service are of the commutating pole type with a speed range usually of from 250 to 1000 r.p.m. in sizes up to and including 100 horse-power planer rating; also a speed range of from 350 to 1200 r.p.m. in sizes up to 35 horse-power planer rating. Other speeds can be obtained when required. These combinations of speeds allow the motor in the majority of cases to be coupled direct to the driving shaft of the machine.

Starting, stopping and reversing are accomplished with sparkless commutation. In order not to brake (dynamically) from high speed in one violent step, means have been provided in the control to accomplish this in three distinct steps, braking down slowly from the high speeds and then quickening the brake action at the lower speeds by cutting out the brake resistance in two steps, thus completing the entire brake action without undue shock, in the shortest possible time. This feature, in addition to the quickness of the brake, will be recognized as a decided advantage in the maintenance of the machine. Incidentally, reversing is accomplished without the delay incident to the use of sluggish relays. After a failure of voltage with the master controller in the running position and upon closing the line circuit breaker, the motor will start up in the regular way, without additional complications in the control. This latter feature is advantageous in the event of the operator failing to return the master switch to the off-position. Cutting and return speeds are entirely independent of each other, so that it is possible to use the slowest cutting speed and the highest return speed, or vice versa, in any combination not exceeding four to one, with thirty-five to seventy cutting speeds and the same number of return speeds, depending on the size of the equipment. Special cases, where overlapping speeds are required or where the entire range of motor speeds is to be used in both directions for cutting, as in plate planers, can be readily provided for when the operating conditions are known.

Example. — In Fig. 2 curves *A*, *B* and *C* are for motor No. 2 and show the ampere input of a 10 horse-power, 1250 r.p.m. motor driving a 36-inch modern type planer through shifting belts. *A* is the return stroke of the planer table at a speed of 68 feet per minute; *C*, the cutting stroke, without cut, at 33.3 feet per minute; *B*, the same cycle but with a cut slightly less than 10 horse-power. The lost time on the cutting stroke, due to the belt slipping, is plainly seen, the

cutting speed falling off from 33.3 feet to 29.4 feet or 13 per cent. Curves *E*, *F* and *G* are made on the same machine when driven by motor No. 1, a 10-horse-

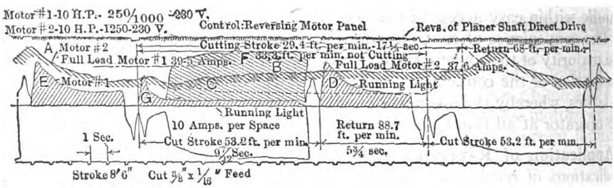

Fig. 2. Tests on 36 in. by 10 ft. Reversible Planer

power, 250 to 1000 r.p.m. reversing motor, direct connected. These curves are superimposed on the above curves for comparative purposes. No attempt was made in this set of curves to duplicate the slow cutting or return speeds of the belt-driven machines as the comparison would have shown power differences only. *E* is the return stroke of the planer table at a speed of 88.7 feet per minute; *G*, the cutting stroke at 53.2 feet per minute; *F*, the same cycle but with a cut of approximately 13 horse-power. The speed drop in this case is motor slow-down only. For comparison the speed of motor No. 1, curve *F*, was chosen as the most economical speed under the conditions which the test was made. The loss in time of the belt drive as compared with the direct-connected reversing-motor drive (the depth of cut and feed being the same in both cases) is 62 per cent.

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[D. B. RUSHMORE, assisted by E. A. LOF.]

MAGNETIC PROPERTIES OF IRON AND OTHER METALS.

—(*See also Electricity and Magnetism, Principles of; Magnetic Testing.*) For the definitions of magnetizing force and flux density see *Electricity and Magnetism, Principles of;* for the relations of the various units in which these quantities are expressed see *Units and Conversion Factors.*

Hysteresis Loop — Residual Magnetism and Coercive Force. — When a magnetic substance which is not magnetized initially is placed in a magnetic field the intensity of which is increased from O to H_m, the flux density produced

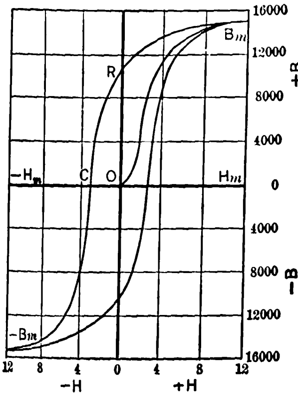

Fig. 1.

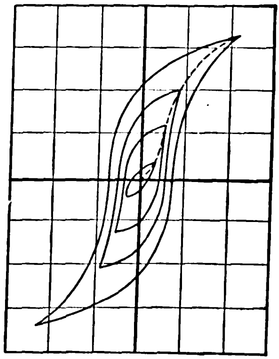

Fig. 2.

in the magnetic substance increases in the manner shown by the curve OB_m in Fig. 1. If the magnetizing force is then decreased to a value H the flux density does not return to the value corresponding to this value of H on the ascending curve, but decreases less rapidly than it increased. This phenomenon is known as "magnetic hysteresis."

When the magnetizing force is reduced to zero the flux density in general has a considerable value (OR in Fig. 1); this value of B is called the "remanent" or "residual" magnetism. To reduce the flux density to zero the magnetizing force must be reversed and increased in the reversed direction to a value OC, called the "coercive force." As the magnetizing force is still further increased in the reversed direction to a value numerically equal to the positive maximum, and then decreased to zero, reversed, and increased again to H_m, the flux density passes through the cycle of values represented by the closed loop. This closed loop is called the "hysteresis loop."

If the iron (or other magnetic substance) is not originally unmagnetized, this hysteresis loop will be shifted above or below the axis of H, but after a number of reversals of the magnetizing force between given positive and negative values, the loop will become practically symmetrical with this axis, particularly if the iron is continually jarred. In the armatures of electrical machines and the cores of transformers, in which the field intensity reverses a large number of times every second, and the iron is continually jarred, the relation between flux density and field intensity after a short interval of time is represented by a symmetrical loop of the form shown in Fig. 1.

The area inclosed by the hysteresis loop depends upon the maximum value of the flux density reached during the cycle, but the general shape remains

about the same. Fig. 2 shows a series of loops corresponding to various values of the maximum flux density. The area of the loop is also different for various kinds of iron or steel. $\frac{1}{4\pi}$ times the area of this loop when B and H are plotted to scale is equal to the ergs of heat developed in the iron per cubic centimeter per cycle. (*See section on Hysteresis Loss, below.*)

Permanent Magnets — Retentiveness. — An examination of this hysteresis loop also makes clear how a bar of steel may be permanently magnetized by placing it in a magnetic field. For, when the bar is removed from the field it retains a flux density approximately equal to the residual magnetism OR. In the case of a cast-iron or steel bar, properly hardened, the bar thus magnetized may be handled with comparative roughness without reducing to any considerable extent the strength of its poles, but in the case of a soft-iron bar even the slightest jar will cause it to lose its magnetism almost entirely. The property possessed by a magnetic substance of retaining its magnetization is called its "retentiveness."

Magnetic Saturation. — The difference between the flux density and magnetizing force, at any point in a magnetized substance, divided by 4π is defined* as the intensity of magnetization J at this point, viz.,

$$J = \frac{B - H}{4\pi},$$

or

$$B = H + 4\pi J.$$

The intensity of magnetization J is proportional to the *excess* of magnetic flux caused by the presence of the magnetic substance. As the magnetizing force in a magnetic substance is increased the flux density B at first increases much more rapidly, for soft iron a thousand or more times, than the magnetizing force H (i.e., the intensity of magnetization established is many times greater than the magnetizing force). However, for larger values of H the intensity of magnetization ultimately becomes constant and any further increase in B cannot be greater than the increase in H. When the intensity of magnetization has reached its saturation value the magnetic substance is said to be "saturated."

Wrought iron, cast iron, cast steel and sheet steel all become practically saturated at magnetizing forces below 100 ampere-turns per inch (50 gilberts per centimeter), but absolute saturation does not occur until the magnetizing force reaches about 5000 gilberts per centimeter, and in the case of some magnetic alloys even higher fields are required for absolute saturation. Hadfield and Hopkinson (*Jour. Elec. Eng., 1911, Vol. 46, p. 237*) give the following, as the results of an extended investigation.

1. The saturation value of J in absolute units for pure iron of density 7.80 is 1680 within 1 per cent. This is slightly lower than the values obtained by Ewing and Low and other experimenters.

2. In an annealed iron-carbon steel in which other elements are present in small proportions the saturation value of J is less than that of pure iron by a percentage equal to 6 times the percentage of carbon.

3. Quenching an iron-carbon alloy from a high temperature reduces the saturation value of J by a large but somewhat uncertain amount.

4. The addition of silicon or aluminium to iron results in a reduction in the saturation value of J which is roughly in proportion to the amount added, as

* The factor 4π arises from the original conception of intensity of magnetization as the magnetic moment per unit volume.

though the addition behaved as an inert diluent. If carbon be present, however, silicon seems to neutralize its action to some extent.

Permeability and Normal B–H Curves. — The magnetic permeability μ of a substance corresponding to any degree of magnetization is usually defined as the quotient of the flux density B by the magnetizing force H, that is

$$\mu = \frac{B}{H}.$$

On account of the hysteresis effect, however, this quotient may have any value within wide limits depending upon how the magnetization is produced. Consequently a more restricted definition of permeability is required in the case of highly magnetic substances like iron and steel. In such cases it is customary to take as the *normal* permeability of the substance corresponding to any given value of the magnetizing force, the quotient

$$\mu = \frac{B_m}{H_m},$$

where B_m is the flux density corresponding to the end of the symmetrical hysteresis loop produced by reversing the magnetizing force a number of times between the values $+H_m$ and $-H_m$. That is, the locus of the ends of the various symmetrical hysteresis loops (the dotted line in Fig. 2) is taken as the *normal* B–H curve of the substance, and the corresponding quotients B/H for any point on this curve is taken as the *normal* permeability corresponding to the value of B and H at this point.

The relation between permeability and magnetizing force or flux density is a complex one. As the flux density increases, μ reaches a maximum at a relatively low flux density (from 3000 to 8000 gausses) and then decreases ultimately to unity when the magnetizing force reaches a value so large that the intensity of magnetization J is negligible in comparison with H. This limiting condition, however, is never reached in practice.

Susceptibility. — The quotient of the intensity of magnetization J divided by the magnetizing force H is called the magnetic susceptibility κ corresponding to this magnetizing force; viz.,

$$\kappa = \frac{J}{H} \text{ whence } \qquad \mu = 1 + 4\pi\kappa.$$

FACTORS AFFECTING THE PERMEABILITY AND HYSTERESIS LOSS. — The normal permeability and the hysteresis loss (the latter is proportional to the area of the hysteresis loop) depend to a very great extent upon the physical structure and chemical constitution of the sample and the heat treatment to which it has been subjected. It has also been recently discovered (*Pender and Jones, Phys. Rev., 1913*) that when sheet steel is annealed in an alternating magnetic field, the permeability is increased in certain cases as much as 50 per cent, but there is no appreciable change in the hysteresis loss. The B–H curves of two samples taken from the same lot of material may even differ considerably. The permeability and hysteresis loss also depend to a slight extent upon whether the sheets are magnetized in the direction of rolling, or transverse thereto, being higher in the latter case.

Temperature and Aging. — Permeability and hysteresis also depend upon the temperature of the sample at the time the observations are taken, though the variation due to ordinary changes of temperature is slight. For very high temperatures, however, all magnetic substances become practically non-magnetic. This temperature corresponds to the major recalescence point, which is about 750° C. for steel of the quality used in armature and transformer

punchings. When steel is kept continuously at a moderately high temperature (100° C.), the hysteresis loop also gradually increases in size, and therefore the energy loss in the magnetic circuits of electric machines due to hysteresis increases with time. This effect is called "ageing." There is practically no ageing of silicon steel.

Chemical Composition of "Electrical" Sheet Steel. — Sheet-steel manufacturers make a special grade of sheet for electrical purposes, which they sell under various trade names. Such steel is always low in carbon content and, except silicon in the so-called silicon steels, all impurities are reduced to small amounts. Within the last decade the use of steel containing about 3 per cent silicon has come into extensive use, particularly for the magnetic circuit of transformers. The permeability of this steel is somewhat lower, in the useful range of flux densities, than that of ordinary electrical steel, but the area of the hysteresis loop, and therefore the hysteresis loss, is from 40 per cent to 60 per cent less; the specific resistance of silicon steel is also about 3 times greater than that of ordinary electrical steel, resulting in a reduction of about 70 per cent in the eddy-current loss (*see curves below*). Aluminium has much the same effect as silicon, but the aluminium alloy is not so easily rolled.

The following are typical chemical analyses of the two kinds of electrical steel, but it should be understood that considerable variations in the proportions of the various constituents occur in practice.

	Ordinary electrical steel *	Silicon steel
	Per cent	Per cent
Silicon	0.01	3.46
Phosphorus	0.08	0.04
Manganese	0.50	0.13
Sulphur	0.03	0.02
Carbon	0.06	0.06

* Parshall & Hobart, Electric Machine Design.

Of a large number of electrical steels analyzed by the Bureau of Standards (*Lloyd and Fisher, Trans. A.I.E.E., 1909, Vol. 28, p. 463*) none showed more than the slightest trace of vanadium.

Annealing of Sheet Steel. — Sheet steel as it comes from the rolling mill may be greatly improved in magnetic properties by proper annealing. The chief requirement seems to be that the steel be brought to a temperature about 100° C. above the major recalescence point, i.e., to a temperature of about 850° C., and then allowed to cool slowly down to a temperature of from 100 to 150° C., when it may then be removed from the annealing furnace and allowed to cool more rapidly. The time required for annealing is from 12 to 36 hours. The annealing is usually done after the punchings have been made, thus eliminating the hardening at the edges produced by the cutting; otherwise this hardening may produce a considerable increase in the hysteresis loss.

TYPICAL *B–H* AND PERMEABILITY CURVES. — In Fig. 3 are given the standard *B–H* curves used by one of the large manufacturing companies, and in Fig. 4 the permeability at various flux densities derived therefrom. It should be understood that these curves represent results obtainable under ordinary commercial conditions (joints not included) on iron and steel of

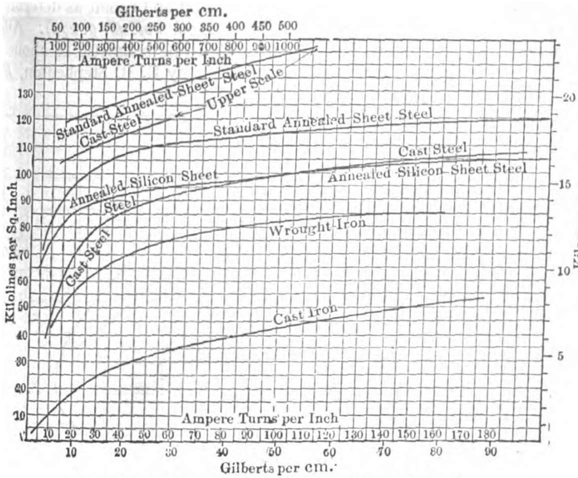

Fig. 3. Normal B-H Curves of Iron and Steel

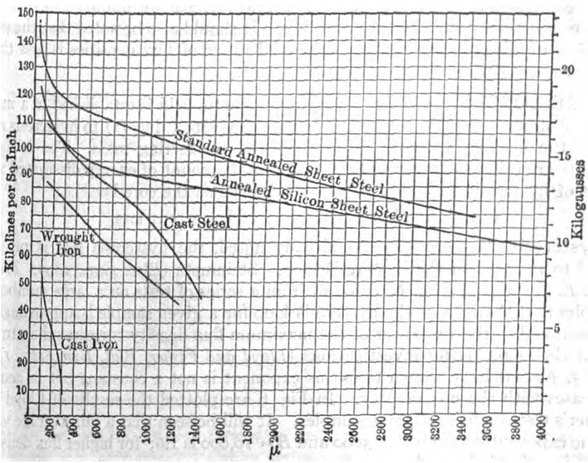

Fig. 4. Permeability Curves of Iron and Steel

the composition found suitable for electrical purposes. Ordinary commercial iron or steel will not, as a rule, have a permeability as high as given by these curves.

In Fig. 5 are given the permeability curves of nickel and cobalt as determined by Fleming, Ashton and Tomlinson (*Phil. Mag.*, *1899, Vol. 48, p. 271*). Alloys of certain non-magnetic metals have been found to be magnetic to about the same extent as nickel; see paper on *Heusler Alloys* by E. B. Stephenson, *Bull. 47* (*1911*), *University of Illinois, Eng. Exp. Stat.*

Maximum Permeability. — As indicated by the curves in Fig 4, the maximum permeability of sheet steel occurs below the range of commercial flux densities. A number of tests by the author showed that this maximum occurs at from 6 to 10 kilolines per square inch, the permeability decreasing rapidly with flux densities less than these values, the complete curve being similar in shape to the curves shown in Fig. 5. The author has obtained a maximum permeability of about 9000 c.g.s. units with ordinary electrical sheet steel, and a maximum of 13,000 c.g.s. units with silicon steel, carefully annealed in small lots in an alternating magnetic field (*Pender and Jones, Phys. Rev., 1913*). These exceptionally high permeabilities, however, are not obtained in practice under commercial conditions of annealing. It should also be noted that the permeability of silicon steel in the commercial range of flux densities is less than that of the ordinary low-carbon electrical steel.

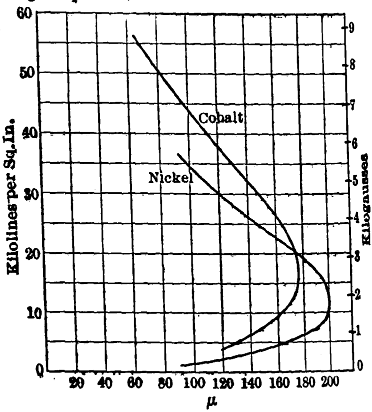

Fig. 5. Permeability Curves of Nickel and Cobalt

CORE-LOSSES. — When a varying magnetic field is established in a magnetic substance a certain amount of heat is developed due (1) to hysteresis and (2) to the electric currents induced in the conducting mass. The induced currents are called eddy currents, and the total amount of energy dissipated as a result of hysteresis and eddy currents is known as the core-loss, i.e.,

$$\text{Core-loss} = \text{hysteresis loss} + \text{eddy-current loss.}$$

Hysteresis Exponent. — As already noted, the hysteresis loss is proportional to the area of the hysteresis loop. Steinmetz (*Elec. Eng., 1890; Trans. A. I. E. E., 1892, Vol. 9, p. 3*) found from a series of tests on a large number of samples that the area of the hysteresis loop for a given sample is approximately proportional to the 1.6 power of the maximum flux density corresponding to the tip of the loop. Later investigations (*Lloyd and Fisher, Bull. Bur. Stds., 1909, Vol. 5, p. 453*) have shown that this exponent is not a constant but in general increases with the flux density. In Fig. 6 are plotted the results of Lloyd and Fisher's tests on five different samples. It will be seen that a fair average value of the exponent between $B = 3000$ and $B = 10{,}000$ is 1.6; for higher flux densities there is a decided trend upward.

Hysteresis Coefficient. — If the magnetic field throughout the iron is *uniform*, and the hysteresis exponent is assumed constant and equal to 1.6, then the power loss can be expressed by the formula

$$P_h = KVfB^{1.6}, \qquad \text{or} \qquad P_h = KWfB^{1.6},$$

where

V = volume of iron,
f = frequency of alternating field, in cycles per second,
B = maximum flux density,
W = weight or mass of the iron.

The value of the constant K depends upon the units in which the other quantities are expressed. If V is in cubic centimeters, B in gausses, f in cycles per second and P_h in ergs per second the formula is usually written

$$P_h = \eta V f B^{1.6}.$$

That is, the loss in ergs per cycle per cubic centimeter is $\eta B^{1.6}$. The constant η is known as Steinmetz's hysteresis coefficient and the quality of iron or steel with respect to hysteresis is frequently expressed in terms of this constant.

The hysteresis loss is also sometimes stated as the watts per pound at 10,000 gausses and 60 cycles. Let w_1 = hysteresis loss in watts per pound at 10,000 gausses and 60 cycles. Then the corresponding value of η is:

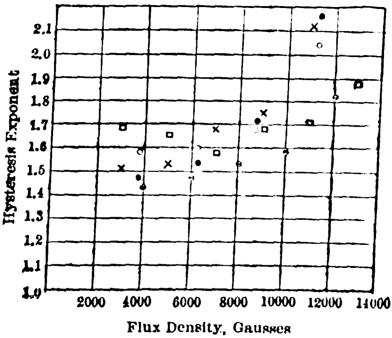

Fig. 6. Variation in Hysteresis Exponent

⊖ Ordinary electrical sheet, unannealed.
● Ordinary electrical sheet, annealed (German).
× 0.7 % silicon, annealed.
□ 3.4 % silicon, annealed.
○ Silicon-steel, annealed (German).

For Specific Gravity = d	Specific Gravity = 7.7	Specific Gravity = 7.5
$\eta = 0.000146\, w_1 d$	$\eta = 0.00113\, w_1$	$\eta = 0.00110\, w_1$

Note that w_1 is the hysteresis loss only, excluding the eddy-current loss, which is discussed below.

The following table gives the value of K in terms of η when the various quantities in the formula for hysteresis loss are in the units stated.

FORMULA FOR CALCULATING HYSTERESIS LOSS

$$P_h = KVfB^{1.6}$$

Power, P_h	Volume, V	Frequency, f	Flux density, B	K =
Ergs per sec.	Cu. cm.	Cycles per sec.	Gausses	η
Watts	Cu. cm.	Cycles per sec.	Kilogausses	$0.00631\,\eta$
Watts	Cu. in.	Cycles per sec.	Kilogausses	$0.1035\,\eta$
Watts	Cu. in.	Cycles per sec.	Kilolines per sq. in.	$0.00525\,\eta$

$$P_h = KWfB^{1.6}$$

Power, P_h	Weight, W	Frequency, f	Flux density, B	$K=$
Ergs per sec.	Grams	Cycles per sec.	Gausses	$\frac{\eta}{d}$
Watts	Kilograms	Cycles per sec.	Kilogausses	$6.31\frac{\eta}{d}$
Watts	Pounds	Cycles per sec.	Kilogausses	$2.86\frac{\eta}{d}$
Watts	Pounds	Cycles per sec.	Kilolines per sq. in.	$0.145\frac{\eta}{d}$

d = specific gravity = 7.7 for ordinary electrical sheets = 7.5 for silicon steel.

Hysteresis Loss for Various Substances. — The hysteresis loss in iron and steel and other magnetic metals depends to so great extent upon their chemical composition, physical structure, heat treatment, etc., that average values have no significance. The following table is intended to give the range in the value of the hysteresis coefficient for iron and steel used in electrical machinery; figures for nickel and cobalt are also included.

VALUES OF HYSTERESIS COEFFICIENT

η = ergs per cycle per cubic centimeter for $B = 1$ gauss

w_1=watts per pound at 60 cycles for $B = 10,000$ gausses

Metal	Values of η		Values of w_1	
	From	To	From	To
Silicon steel, annealed sheets	0.0006	0.0015	0.55	1.36
Ordinary electrical sheets, annealed	0.00095	0.004	0.84	3.5
Soft cast steel	0.003	0.012	2.7	11
Cast iron	0.011	0.016	10	14
Forged steel	0.015	0.025	13	22
Hard cast steel	......	0.028	....	25
Cobalt	0.012	......	11	....
Nickel	0.013	0.040	12	35

The extreme low values of η can seldom be uniformly realized in practice. One of the large manufacturing companies uses for design purposes the values $\eta = 0.00145$ for silicon steel and $\eta = 0.0033$ for ordinary electrical sheets, these values allowing a considerable margin for variations in the quality of the steel. For close design the particular quality of steel to be used should be carefully tested (*see Magnetic Testing*) and the test results used.

In Fig. 7 are plotted hysteresis loss curves for various values of η assuming the $B^{1.6}$ law.

Eddy-Current Loss. — When an alternating magnetic field is established in a conducting material alternating currents are set up in it due to the alter-

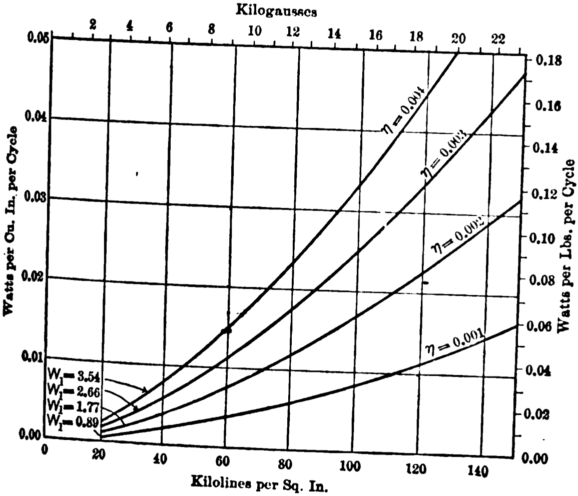

Fig. 7. Hysteresis Loss

nating e.m.f. produced by the alternating field. Consider a sheet of iron, Fig. 8, subjected to a *uniformly* distributed alternating field parallel to its faces. The dots indicate the flux lines perpendicular to the page and the loops the induced currents. Let

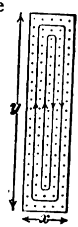

Fig. 8.

B = flux density, in gausses,
x = thickness of plate, in centimeters,
ρ = specific resistance of plate, in abohms per cm-cube,
$\dfrac{dB}{dt}$ = rate of change of flux density with time, gausses per second.

Then the *instantaneous* power loss per cubic centimeter of the sheet, when the thickness x is very small (1 per cent or less) compared with the width of the plate, is

$$p = \frac{x^2}{12\rho}\left(\frac{dB}{dt}\right)^2.$$

The *average* power loss per cubic centimeter for a complete cycle of variation of the flux, in ergs per second, is

$$P_e = \frac{x^2}{12\rho} \times \left(\text{root-mean-square value of } \frac{dB}{dt}\right)^2.$$

It should be noted that this average loss depends upon the form factor (*see Alternating Currents*) of the rate of change of flux density, the latter being proportional to the counter e.m.f. induced by the varying field of the magnetizing coil.

If the induced voltage and therefore the flux density varies according to a sine function of the time, i.e., if $B = B_m \sin(2\pi ft)$, where B_m is the maximum

value of the flux density, f the frequency, and t the time, then the average power loss due to eddy currents per cubic centimeter is

$$P_e = \frac{(\pi x f B_m)^2}{6\rho}.$$

If the voltage wave has a form-factor h, the loss for the *same* B_m is greater than this by the factor $\left(\frac{h}{1.11}\right)^2$; see below.

The corresponding formula for the eddy-current loss per unit volume in a wire or cylinder of radius r, magnetized *parallel* to its axis, to a *uniform* flux density over its cross section is

$$P_e = \frac{(\pi r f B_m)^2}{4\rho}.$$

Eddy-current Coefficient. — In practice the field in the sheets is seldom perfectly uniform, and eddy currents flow from one sheet to the other, unless they are insulated from each other; consequently the eddy-current loss in a core made up of a bunch of sheets is usually greater than that given by the above formula. The eddy-current loss in a laminated core is therefore usually expressed by the formula

$$P_e = K_1 V (xfB)^2,$$

or

$$P_e = K_1 W (xfB)^2,$$

where V and W are the volume and weight respectively of the core and B is the maximum flux density. The coefficient K_1 is determined from actual tests on a suitable sample (*see Magnetic Testing*).

The coefficient K_1 (aside from its dependence upon the units in which the various quantities are measured) depends upon the specific resistance of the iron, the wave shape of the induced voltage, the distribution of flux in the sheets, the degree to which the sheets are insulated from each other, and upon the shape of the magnetic circuit in so far as this affects the distribution of flux. The results of the tests by Lloyd and Fisher also indicate that K_1 depends upon the value of B, or, in other words, that the eddy-current loss does not vary directly as the square of the maximum flux density, all other conditions being the same. At high frequencies, above 100 cycles per second, the loss also increases less rapidly than the square of the frequency, due to the increase in the effective resistance of the sheets as the result of an action similar to the skin effect (q.v.) in a wire carrying a rapidly alternating current.

When the loss P_e is expressed in watts, V in cubic centimeters, f in cycles per second, x in centimeters and B in gausses, the coefficient K_1 may be represented by the symbol ϵ, and the formula for eddy-current loss becomes

$$P_e = \epsilon V (xfB)^2.$$

The eddy-current loss is frequently stated as the watts per pound at 10,000 gausses and 60 cycles for a thickness of sheet corresponding to No. 29 on the sheet-steel gage (= 0.0141 inch = 0.0358 centimeter). Let w_2 = eddy-current loss in watts per pound at 10,000 gausses and 60 cycles for sheets 0.0141 inch thick. Then the corresponding value of ϵ is:

Specific Gravity = d	Specific Gravity = 7.7	Specific Gravity = 7.5
$\epsilon = 0.0000478\, w_2 d$	$\epsilon = 0.000368\, w_2$	$\epsilon = 0.000359\, w_2$

FORMULA FOR CALCULATING EDDY-CURRENT LOSS *

$$P_e = K_1 V (xfB)^2$$

Power, P_e	Volume, V	Thickness of sheets, x	Frequency, f	Flux density, B	$K_1 =$
Ergs per sec.	Cu. cm.	Cm.	Cycles per sec.	Gausses	ϵ
Watts	Cu. cm.	Cm.	Cycles per sec.	Kilogausses	0.1ϵ
Watts	Cu. in.	In.	Cycles per sec.	Kilogausses	10.58ϵ
Watts	Cu. in.	In.	Cycles per sec.	Kilolines per sq. in.	0.254ϵ

$$P_e = K_1 W (xfB)^2$$

Power, P_e	Weight, W	Thickness of sheets, x	Frequency, f	Flux Density, B	$K_1 =$
Ergs per sec.	Grams	Cm.	Cycles per sec.	Gausses	$\frac{\epsilon}{d}$
Watts	Kilograms	Cm.	Cycles per sec.	Kilogausses	$100\frac{\epsilon}{d}$
Watts	Pounds	In.	Cycles per sec.	Kilogausses	$292\frac{\epsilon}{d}$
Watts	Pounds	In.	Cycles per sec.	Kilolines per sq. in.	$7.01\frac{\epsilon}{d}$

d = specific gravity = 7.7 for ordinary electrical sheets = 7.5 for silicon-steel.

Eddy-current Loss in Sheet Steel. — The eddy-current coefficient for various makes of ordinary electrical sheets is subject to much the same variation as the hysteresis coefficient, and the same is true for various makes of silicon steel. The values given in the following table are for uniform space distribution of flux and sinusoidal time variation.

VALUES OF EDDY-CURRENT COEFFICIENT †

ϵ = ergs per sec. per cu. cm. for 1 cycle per sec., thickness of 1 cm., and B = 1 gauss.

W_2 = watts per lb. at 60 cycles, thickness of 0.0141 in., and B = 10,000 gausses.

Kinds of sheets	Values of ϵ			Values of W_2		
	From	To	Average	From	To	Average †
Silicon steel........	0.000043	0.000098	0.000065	0.12	0.27	0.180
Ordinary electrical..	0.00012	0.00025	0.00022	0.34	0.70	0.608

* These formulas assume a sine-wave voltage; if the voltage has a form-factor k, multiply the constant K_1 by $\left(\frac{k}{1.11}\right)^2$.

† From tests by L. T. Robinson, *Trans. A. I. E. E.*, *1911*, *Vol. 30*, *p. 741* and Lloyd and Fisher, *Bull. Bur. Stds.*, *1909*, *Vol. 5*, *p. 453*. The "average" values are those given by Mr. Robinson for "standard" and "alloyed iron" respectively.

Eddy-current-loss curves for ordinary electrical sheets are plotted in Fig. 9, and for silicon steel in Fig. 10, these being based on Robinson's values of w_3 and the formula given above, ***which assumes uniform distribution of flux in the sheets and a sine-wave variation with time.*** The numbers on the curves are the gage numbers of the sheets, viz.,

No. 29 gage = 0.0141 inch thick,
No. 26 gage = 0.0188 inch thick,
No. 24 gage = 0.0250 inch thick,

See *Gages, Sheet Metal.*

RELATION BETWEEN FLUX DENSITY AND IMPRESSED VOLTAGE. — When an iron core is magnetized by a current in a coil surrounding it, the flux established is such that the counter e.m.f. set up in the coil is equal to the impressed e.m.f. less the drop in voltage due to the resistance of the coil. The counter e.m.f. may be measured by connecting a voltmeter to the terminals of a secondary coil wound on the same core, provided the flux leakage between the primary and secondary coils is negligible (*see Magnetic Testing*). The counter e.m.f. per turn will be approximately equal to the impressed e.m.f. per turn provided the resistance of the primary winding is small. Let

A = cross-section of core in square inches,
E = effective value of counter e.m.f. per turn, in volts,
f = frequency in cycles per second,
h = form-factor of counter e.m.f. (h = 1.11 for sine wave),
B = maximum flux density in kilolines per square inch.

Then

$$B = \frac{10^5 E}{4hfA}.$$

Effect of Wave Form on Core-Loss. — Experiments by M. G. Lloyd (*Bull. Bur. Stds., 1908, Vol. 5, p. 381*) show that for a given maximum flux density in the core the hysteresis loss is practically independent of the wave form of the counter induced e.m.f. in the exciting coil, provided the voltage wave does not pass through zero more than twice per cycle. This counter e.m.f. is numerically equal to the impressed e.m.f. if the resistance drop in the exciting coil due to the exciting current is negligible, as is the case in a properly designed transformer. However, the maximum value of the flux density established by a given impressed e.m.f. is inversely proportional to the form factor of this e.m.f. (assuming negligible resistance drop). Put

P_h = hysteresis loss for sine-wave voltage,
P_e = eddy-current loss for sine-wave voltage,
P_h' = hysteresis loss for voltage wave having a form factor h,
P_e' = eddy-current loss for voltage wave having a form factor h,
h = form factor of non-sinusoidal wave.

Then for ***given maximum flux density***

$$P_h' = P_h, \qquad P_e' = \left(\frac{h}{1.11}\right)^2 P_e,$$

that is, the hysteresis loss is independent of the form factor and the eddy-current loss varies as the square of the form factor.

For ***given effective value of voltage wave***

$$P_h' = \left(\frac{1.11}{h}\right)^{1.6} P_h, \qquad P_e' = P_e,$$

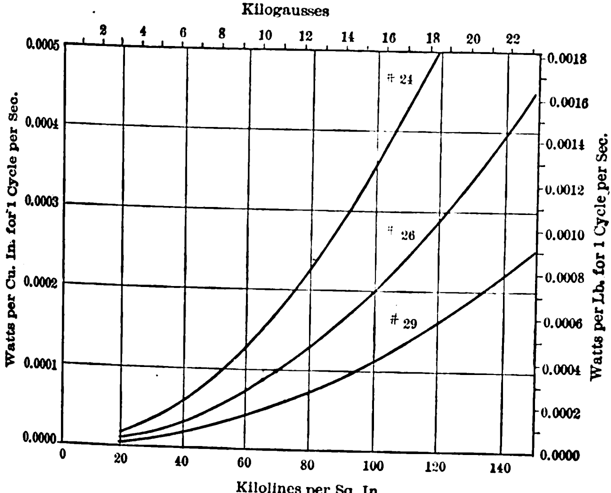

Fig. 9. Eddy-current Loss in Ordinary Electrical Sheet Steel

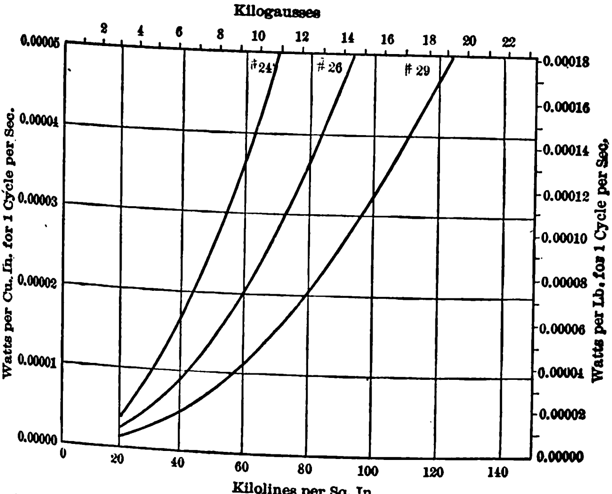

Fig. 10. Eddy-current Loss in Silicon Sheet Steel

that is, the hysteresis loss varies inversely as the 1.6 power of the form factor (approximately only, since the 1.6 "law" is only an approximation to the real facts) and the eddy-current loss is independent of the form factor.

For a given effective value of the voltage wave the total core-loss is therefore less the higher the form factor, i.e., the more peaked the wave.

EXCITING CURRENT ANGLE OF HYSTERETIC ADVANCE.—For a given wave shape of the impressed e.m.f., the wave shape of the exciting current (neglecting eddy currents) is determined by the shape of the hysteresis loop, since the exciting current is proportional to the magnetizing force H. Curve I in Fig. 11 shows the wave shape of the exciting current for a sine wave impressed e.m.f. calculated from hysteresis loop in Fig. 2 for a maximum flux density of 10,000 gausses.

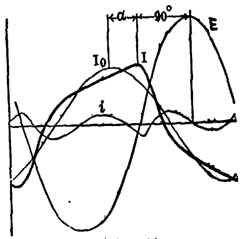

Fig. 11.

The sine curve I_0 is the fundamental of this wave (*see Wave Analysis*), and the curve i is the difference between I and I_0 and consists chiefly of the third harmonic.

The curve E is the sine wave of induced voltage. The flux curve would be a sine curve shifted 90° to the left (ahead of E). The angle α by which the fundamental of the current wave leads the flux wave is called by Steinmetz the "angle of hysteretic advance of phase."

For flux densities below the knee of the B–H curve the effective value of the fundamental current wave I_0 differs but slightly from the effective value of the actual current wave.

In the above discussion the effect of eddy currents is neglected. The effect of these is to increase the exciting current by a component of the same shape as, and in phase with, the impressed e.m.f., thus causing an increase in the effective value of the exciting current and an increase in the angle by which the exciting current leads the flux.

Magnetizing and In-phase Components of Exciting Current.—Let

P_c = total core-loss,
E = back e.m.f. induced in the exciting coil,
I = exciting current,

$$\cos\phi = \frac{P_c}{EI}.$$

Then $I \sin\phi$ is called the magnetizing component of the exciting current and $I\cos\phi$ the in-phase (or energy) component. For magnetizations below the knee of the B–H curve

$$\phi = 90 - \alpha',$$

where α' is the total angle of advance of the exciting current ahead of the flux due to both hysteresis and eddy-current losses.

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[H. PENDER.]

MAGNETIC TESTING. — (*See also Electricity and Magnetism, Principles of; Magnetic Properties of Iron and Other Metals.*) The ordinary magnetic tests of iron and other substances are: (1) The determination of the normal *B-H* curves and hysteresis loop, and (2) The determination of the core-losses, i.e., hysteresis and eddy-current losses. The following is a brief table of contents of this article:

DETERMINATION OF THE NORMAL *B-H* CURVES AND HYSTERESIS LOOP. — Under certain conditions, noted below, the magnetizing force *H* may be calculated in terms of the number of turns per unit length and the current. The flux density *B* may be measured in any one of three ways: (1) By winding on the sample a secondary or "test" coil and connecting in series with it a ballistic galvanometer or equivalent device (e.g., a fluxmeter, q.v.) and noting the deflection of this instrument when the current in the primary or "magnetizing" coil is changed in value or reversed; (2) by measuring the mechanical force required to separate one end of the sample from a yoke which, with the sample, forms a closed magnetic circuit; and (3) by measuring the change in resistance of a spiral of bismuth inserted in an air gap between two parts of the sample or between the sample and a yoke. (4) In addition the permeability of two samples may be compared by a device known as a permeability bridge, in which the detector is a magnetometer or compass. These four methods may be designated respectively as the ballistic, traction, bismuth spiral and bridge methods respectively. The first is best suited for precision measurements, the last three for rapid shop tests where great accuracy is not demanded.

Magnetic Circuit of Testing Apparatus. — The simplest form of circuit is a straight bar, which is placed inside a solenoid or helix, the flux returning through the air. With this type of circuit, however, unless the bar and coil are very long, the demagnetizing action of the magnetic poles formed at the ends and along the sides of the bar render it impossible to calculate with accuracy the resultant magnetizing force in the bar. If the sample is made in the form of a closed iron ring, the mean magnetizing force over the cross-section of the metal may be calculated, but unless the radial thickness of the metal is small compared with the radius of the ring, the magnetizing force will be appreciably greater near the inner edge than near the outer edge, and as the permeability is a function of the magnetizing force, an appreciable error may be introduced. Also, the difficulty of winding the coils on a ring makes its use objectionable.

Bar and Yoke. — As a compromise, the sample is usually made in the form of a rod (or bunch of straight strips, in the case of sheet steel) which is fitted into a massive yoke of low reluctance which completes the magnetic circuit. The magnetizing coil is wound on the sample only. As a first approximation, in case the joints between sample and yoke are well made, the reluctance of the yoke and joints may be neglected, and the magnetizing force per unit length of the sample taken as the total magneto-motive force divided by the length of the sample. Methods have been devised for correcting the effect of the yoke and joints as described below.

Demagnetizing the Sample. — After the sample has been mounted ready for test (inserted in the yoke, if a bar and yoke method is employed) the circuit is thoroughly demagnetized by an alternating magnetizing force, preferably of about 1 period per second, which is gradually reduced from an initial intensity which establishes an induction well beyond the point of maximum induction to be measured to a final value somewhat lower than the lowest induction to be measured.

Ordinary Ballistic Method. — The connections are shown in Fig. 1. A ring sample is here shown, but this may be replaced by a straight sample or by a straight bar in a yoke, Fig. 2. The test coil on the sample, the ballistic galvanometer *G*, the secondary coil of the standard solenoid and a resistance *r* are all connected permanently in series. This

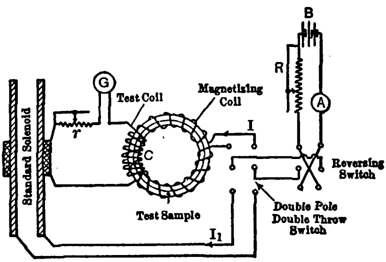

Fig. 1. Connections for Ballistic Method of Measuring Permeability

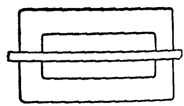

Fig. 2. Bar and Yoke

resistance is adjusted so that the galvanometer shows a suitable deflection when the current in the magnetizing coil of the test sample is reversed. The galvanometer is then calibrated (*see Galvanometers*), keeping this resistance unchanged. A double-pole double-throw switch is provided for connecting the battery circuit at will to the magnetizing coil on the test sample or to the primary coil of the standard solenoid. The sample should be in place when the galvanometer is checked, as the hysteresis and eddy-current losses due to the variable current in the primary during the test have the same effect as an increase in the resistance of the galvanometer circuit. (*See Burrows, C. W., Bull. Bur. Standards, 1909, Vol. 6, p. 31.*)

Precautions. — For a high degree of precision the sample should be placed with its axis perpendicular to the earth's field, the apparatus should be protected from mechanical vibration, and should be sufficiently remote from any strong magnets (such as the permanent magnets in ammeters and voltmeters) in order not to be affected by their magnetic fields.

Calibration of Galvanometer. — Let

N_1 = number of turns in primary of standard solenoid *per centimeter length* (axial),
n_1 = total number of turns in secondary coil of standard solenoid,
n = total number of turns in secondary coil on test sample,
A_1 = mean cross-section, in square centimeters of primary coil of standard solenoid if secondary is on the outside; if the secondary is inside the primary A_1 is the mean cross-section of the secondary,
A = cross-section in square centimeters of the test sample,
a = mean cross-section in square centimeters of the magnetizing coil on test sample if the test coil is on the outside; if the test coil is on the inside a is the mean cross-section of the test coil,
I_1 = current established through primary of solenoid,
D = deflection of galvanometer when this current is reversed,

B = flux density in test sample corresponding to a deflection D when the current through the magnetizing coil on the sample is reversed,

H = magnetizing force in sample corresponding to the flux density B.

The flux in maxwells established by the current I_1 through the primary of the solenoid is then $0.4\pi N_1 I_1 A_1$, and the number of linkages of this flux with the secondary of the standard solenoid, i.e., with the galvanometer circuit, is $0.4\pi n_1 N_1 A_1 I_1$. When the double-throw switch is thrown to connect the battery to the magnetizing coil on the test sample, establishing in this coil a current I and a flux density B, the number of linkages between the flux in the test sample and the galvanometer circuit will be $BnA + Hn\ (a - A)$. If these two linkages are equal, as they will be if the galvanometer shows the same deflection D when I is reversed through the magnetizing coil on the test sample as when I_1 is reversed in the primary of the solenoid, then

$$B = \frac{0.4\pi n_1 N_1 A_1}{nA} I_1 - \frac{a - A}{A} H. \qquad (1)$$

Or, putting

$$K = \frac{0.4\pi n_1 N_1 A_1}{nA},$$

which is a constant for a given solenoid and sample, then

$$B = KI_1 - \frac{a - A}{A} H. \qquad (2)$$

The correction term $\left(\frac{a - A}{A}\right) H$ is usually negligible except for very low values of the flux density.

Hence, by sending various currents I_1 through the primary of the solenoid, reversing these currents and noting the deflection D, a curve may be plotted showing the relation between B ($= KI_1$) and D. Then, when a given current is reversed through the magnetizing coil on the test sample and the deflection D noted, the flux density corresponding to this current may be read directly from the curve. This curve will be practically a straight line unless the damping of the galvanometer is excessive.

The calibration curve holds only for a constant total resistance in the galvanometer circuit (*see Galvanometers*). If it is necessary to change this resistance in order to alter the sensibility of the galvanometer a new calibration curve must be obtained.

Calculation of Magnetizing Force. — Let

N = number of magnetizing turns per centimeter of mean circumference (*see Fig. 1*). If a straight sample with or without a yoke is used, N is taken as the total number of magnetizing turns divided by the free length of the sample in inches.

I = current in magnetizing coil in amperes.

Then the magnetizing force in gilberts per centimeter is

$$H = 0.4\pi NI. \qquad (3)$$

This formula represents the *average* magnetizing force over the section of the ring, if a ring sample is used. If a straight sample is used, the formula is approximate only, due to the reluctance of the return circuit through the air or yoke.

Determination of Normal B–H Curve by Ballistic Method. — The sample is first demagnetized as described above, then the lowest magnetizing force to be used is applied and reversed many times until the iron is brought to

a cyclic magnetic state, that is, until the reversal of the magnetizing force reverses the direction of magnetization without changing its magnitude. The number of reversals required to establish a cyclic condition in soft steel ranges from about 10 at high flux densities to several hundred at low flux densities. In general the harder the steel the fewer the number of reversals required. The galvanometer deflection is then noted, and the corresponding value of B taken from the calibration curve and H calculated from the formula given above. As a check, it is well to carry the iron through the demagnetizing process again, reduce to a cyclic state, and redetermine the point.

Higher points on the curve may be obtained in a similar manner, but it is not necessary to demagnetize the sample between successive points unless it accidentally becomes magnetized above the point being determined.

Determination of Hysteresis Loop by the Ballistic Method.—The sample is first demagnetized and then the maximum magnetizing force (corresponding to the tip of the loop) is applied and a cyclic condition established as described in the preceding paragraph. By suddenly inserting an additional resistance in the magnetizing circuit, e.g., by moving the slider on the resistance R so that less of this resistance is short-circuited, the magnetizing force is reduced to any desired value, and the change in flux density corresponding to this change in magnetizing force is determined by noting the galvanometer deflection. If the calibration curve of the galvanometer has been obtained by the method of reversals, as described above, the change in flux density is twice the value of B as read from the calibration curve, since the actual change in flux corresponding to any ordinate of the calibration curve is twice this ordinate.

The next point on the hysteresis loop is determined in the same manner, first bringing the iron to a cyclic state under the maximum magnetizing force corresponding to the tip of the loop. Data with negative values of the magnetizing force are obtained by reversing the currents in addition to making the adjustments already described. The points on the hysteresis curve may be taken in any order.

In this method of obtaining hysteresis data, the measured quantity is the change in induction when the magnetization is changed at one step from a maximum to any other given point on the hysteresis loop. This method is comparatively free from the irregularities due to the slow creep that occurs when a magnetizing force is applied slowly or changed by small steps. It is also free from irregularities due to variations in the size of step in the "step-by-step" method, in which the magnetizing force is changed from one value to the next lower without restoring it each time to the maximum value. (*See Burrows, Bull. Bur. Standards, 1909, Vol. 6, p. 1.*)

Zero Ballistic Methods for Determining Flux Density.—By using an independent battery circuit to energize the primary of the standard solenoid and establishing the currents I_1 and I at the same time and reversing them simultaneously, it is possible by having the relative directions of these currents right, so to adjust I_1 that for a given value of I the net discharge through the galvanometer is zero, and no deflection occurs. Under this condition B may be calculated directly from equation (1) or (2) above.

Instead of using a standard solenoid with fixed coils, an adjustable mutual inductance, previously calibrated, may be used, and a constant current maintained through its primary, the adjustment being effected by moving the secondary. Let

M = mutual inductance of standard, in henries,
I = current in amperes, in primary of mutual inductance,
n = total number of turns in test coil on sample,

A = cross-section of sample in square centimeters,
B = flux density in sample for balance.

Then

$$B = 10^8 \frac{MI}{nA} = K_1 M,$$

where $K_1 = \frac{10^8 I}{nA}$ = a constant for a given test coil and sample and given current in the primary of the mutual inductance.

The design of a standard mutual inductance suitable for this purpose is described in detail by Burrows, *Bull. Bur. Stds., 1909, Vol. 6, p. 31.*

Burrows' Compensated Double-Yoke Method. — This method, described in detail in Vol. 6 of the Bulletins of the Bureau of Standards (*Reprint No. 117*), is the standard method used by the Bureau of Standards and has also been adopted as the standard method of testing permeability by the American Society for Testing Materials, see *Proc. Am. Soc. Test. Mat., 1912.* The following description of the method is taken from Burrows' paper.

Fig. 3 shows the relative positions of the magnetizing and test coils when double yokes and double rods are used. The lower rod is the one under test.

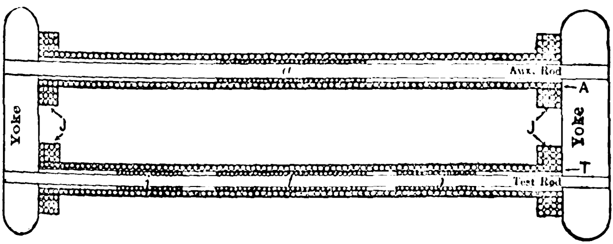

Fig. 3. Double Yoke and Bars

The upper rod is an auxiliary rod of approximately the same magnetic properties as the test specimen. T and A are the two main magnetizing coils, one wound over each rod. Over the four joints are wound four short coils J, each about 1.5 centimeters long. These are connected in series and used as a single coil. t and a are the two test coils surrounding test and auxiliary specimen respectively. j is the end test coil distributed with one-half over each end of the test rod.

Test Samples. — When rods are to be tested these should preferably be of square cross-section, ⅜ by ⅜ inch (0.9525 cm.) and 30 cm. long. With rods of square cross-section it is possible to make a more perfect joint between the rods and yokes. Sheet steel is tested by making up two equal bundles of strips 3 centimeters wide by 50 centimeters in length, having a total weight of 5 kilograms (*Stand. Spec. of Am. Soc. Test. Mat., June, 1912*). Each bundle will be about 2.25 centimeters thick. One bundle takes the place of the test rod, the other takes the place of the auxiliary rod.

Yokes. — The yokes are of soft iron, provided with suitable clamps for holding the sample firmly and making a good magnetic joint. When sheet steel is being tested, the yokes may be made of short strips of the material 3 centimeters wide, the magnetic circuit in this case being a rectangle made up of strips, the joints at the four corners being alternately butt and lap in successive layers.

Magnetizing Coils. — No. 18 A. W. G. double-cotton-covered copper wire is a convenient size. A coil made of ten layers of this wire will stand continu-

ously a current of 1.7 amperes, corresponding to a magnetizing force of $H =$ 171 gilberts per centimeter. For short periods twice this current may be safely employed, giving a magnetizing force of about 350 gilberts per centimeter. The length and diameter of the coils will depend upon the dimensions of the test sample (see above). If the coils are made of round cross-section the hollow core on which they are wound should have a shallow screw thread, 8 threads to the centimeter, cut on the outside. The first layer is wound in this thread, and succeeding layers wound in the same direction between adjacent wires in the previous layer. The various layers are then connected in series. The magnetizing force at the center of such a coil of ten layers, assuming perfect compensation for the rest of the magnetic circuit, is

$$H = 0.4\pi \times 8 \times 10 I = 100.53 I,$$

where I is the current in the coil.

Test Coils.—The test coils are made of fine wire (enameled or silk covered) wound on thin cores of paper, cloth or slotted metal. Coils t and a are placed over the middle portions of the test rod and auxiliary rod, respectively. Over the two ends of the test rod are placed the two halves of a third test coil j. These three test coils have the same number of turns and are spread over a considerable length of rod, so as to prevent any irregularities which may exist in the iron from exerting a preponderating influence. If the test coils are placed inside the magnetizing coils the correction for the flux between the rod and the coil will be small.

Connections for Permeability Test.—Fig. 4 shows diagrammatically the full scheme of electric circuits both primary and secondary. The coils T, J, A, t, j and a are the same as those of the same letters in Fig. 3. The coils M and m are the primary and secondary of the variable mutual inductance (or standard solenoid) used to balance the e.m.f. induced in the test coil.

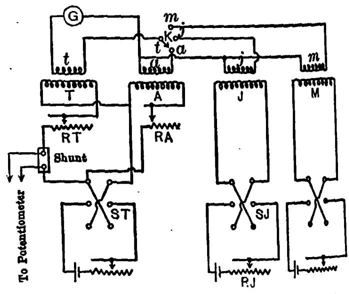

Fig. 4. Connections for Burrows' Method of Testing Permeability

Compensation.—Compensation is secured when the flux across every section of the iron circuit is the same, i.e., when there is no leakage. This condition may be closely realized by adjusting the currents in the three magnetizing coils separately.

The switches ST and SJ are reversed repeatedly, and the resistance RA and RJ adjusted until the three test coils, t, j and a, indicate the same change in flux when the magnetizing currents are simultaneously reversed, i.e., switches ST and SJ reversed simultaneously. With the key K on the point t–a, the equality of flux in the test and auxiliary rods is secured first by adjusting RA until the galvanometer shows no deflection on reversing ST. Then with the key K on the point t–j the flux near the magnetic joints is adjusted to uniformity.

Measurement of Induction.—To measure the induction, K is moved to the point t–m. A reversal of the magnetizing forces produces an impulsive electromotive force acting on the galvanometer, which may be measured as a deflection or may be compensated for by reversing simultaneously a suitable current through a variable mutual inductance, or standard solenoid M.

Measurement of Magnetizing Force. — For accurate work the current should be measured by means of a potentiometer (*see Potentiometers*). If the shunt used for measuring the current has a resistance of 1.0053 ohms and the magnetizing coil T 80 turns per centimeter, then

$$H = 100\ V,$$

where V is the fall of potential across this shunt.

STANDARD INDUCTION TESTS OF THE A.S.T.M. (*adopted June 1, 1912, by the American Society for Testing Materials.*)

The normal magnetic induction is the induction produced by a magnetizing force in a given piece of magnetic material which has been previously demagnetized and then subjected to many reversals of the given magnetizing force.

Both the induction B and the magnetizing force H shall be expressed in terms of the C. G. S. electromagnetic unit (gauss).

Sheet Metal. — The standard normal induction data for sheet material shall consist of the magnetizing forces corresponding to inductions of 2000, 4000, 6000, 8000, 10,000, 12,000, 14,000, 16,000, 18,000, 20,000 gausses, or such as may be obtained without exceeding a magnetizing force of 200 gausses.

The following details are to be observed.

The test material shall consist of 5 kilograms of the strips cut as indicated for the standard core-loss test.

The magnetic circuit shall be a rectangle having the test material for one pair of opposite sides, and the same or different material for the other pair, which may be shorter. The joints at each corner are alternately butt and lap, or may be clamped on the edges.

The magneto-motive force is applied in two sections. The main magnetizing coils shall consist of two equal and uniformly-wound solenoids surrounding the test material. The compensating coils shall consist of four short coils, each having the same number of turns wound closely over the ends of the magnetizing coils.

The test coil surrounds the middle portion of each bundle of test material. Four other test coils of half the number of turns are placed over the test material, approximately midway between the yokes and the center. (This arrangement is similar to Fig. 3 except that two additional test coils corresponding to j are used on the upper bundle.) The two center test coils are joined in series and the four test coils are joined in series. The corresponding ballistic deflections, due to these two test coils, are measures of the magnetic fluxes through the underlying portions of the magnetic circuit. By connecting the two test coils so that the induced electromotive forces oppose each other, and adjusting the current through the compensating magnetizing coils so that there is no resulting ballistic deflection, an approximate uniformity of flux is secured through the greater portion of the test material, and the induction may be measured ballistically in the regular manner. The magnetizing force when the flux is adjusted to uniformity is that calculated from the uniform winding of the main magnetizing solenoids.

The cross-section of the magnetic circuit is determined as in the standard core-loss test.

Rods. — The standard test for rods for use in electromagnets shall consist of the magnetizing forces corresponding to inductions of 2000, 4000, 6000, 8000, 10,000, 12,000, 14,000, 16,000, 18,000, 20,000 gausses, or such as may be obtained without exceeding a magnetizing force of 200 gausses.

The standard test for rods intended for permanent magnets shall consist in the measurement of the magnetizing force, the residual induction and the coercive force corresponding to a maximum induction of 14,000 gausses.

Standard tests shall be made by the Burrows compensated double-yoke method (*described above, and also in Technical Paper No. 117 of the Bureau of Standards*).

Permeability Bridge. — A double-bar and yoke arrangement, such as shown in Fig. 3, may be used without compensating or test coils to obtain a fairly accurate measure of the permeability of a sample, provided the B–H curve of one of the rods is known. A small magnetic needle or compass placed in the air-space between the two rods is used as a detector of magnetic leakage from one rod to the other. The currents in the two magnetizing coils are adjusted until the needle shows no deflection when the currents in the two coils are reversed. The total flux in each rod will then be the same. The magnetizing forces in the two rods are then calculated from the currents in the magnetizing coils (*equation 3 above*) and the flux from the B–H curve of the standard sample.

PERMEAMETERS FOR SHOP TESTS. — A great number of approximate methods have been devised for the rapid testing of permeability in shop work, where a high degree of precision is not required. Some of these are briefly described below.

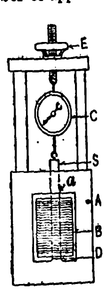

Fig. 5. Thompson Permeameter

Thompson Permeameter (Fig. 5). — In this instrument the flux density is measured in terms of the force required to separate a rod S from a yoke A, when the magnetic circuit formed by the rod and yoke is magnetized by a current in the coil B. The handle E is turned until the sample is pulled away from the yoke and reading on the balance taken at the instant of break.

Let

I = amperes in magnetizing coil,
N = number of turns in magnetizing coil,
l = "equivalent" length of magnetic circuit in inches = distance from a to D plus from 10 to 20 per cent to allow for reluctance of yoke and joints,
P = pull in pounds as read by balance,
A = cross-section of rod S in square inches.

Then the magnetizing force is

$$H = \frac{0.4\pi NI}{2.54\,l},$$

and the flux density is, within the accuracy of measurement by the instrument,

$$B = 1320\sqrt{\frac{P}{A}}.$$

There are several errors in the instrument, however, notably the unavoidable air gap between the sample and the top of the yoke and the contact at D, which makes the instrument unsuitable for the accurate determination of the absolute values of the qualities of iron and steel. It is, however, extremely useful for the comparison of samples where exact absolute values are not required. More nearly absolute values can be obtained by properly calibrating the instrument by using a standard sample the permeability of which at various flux densities has been measured by a more accurate method.

Du Bois Permeameter (Fig. 6). — Another form of traction permeameter is that devised by Du Bois.

The tractive force is measured across a gap in a yoke which completes the magnetic circuit of which the sample or test piece forms part. In the figure P and P_1 are massive soft-iron pole pieces in which the sample S is clamped.

A is a yoke pivoted at *E*. *B* is the magnetizing coil. The resultant moment acting on the balance when a flux is established across the gaps *G* and G_1 is a function of the flux density in the sample (all samples are of the same cross-section). Consequently, the point on the scale at which the weight *W* must be placed for a balance is a measure of the flux density. The scale is calibrated to read directly in flux densities. Due to the reluctance of the yokes and air gaps a correction factor, furnished with the instrument, must be used to obtain the magnetizing force from the ordinary formula.

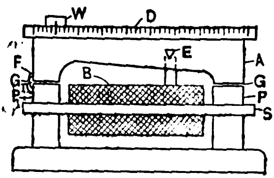

Fig. 6. Du Bois Permeameter

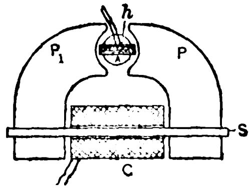

Fig. 7. Koepsel Permeameter

Koepsel Permeameter (Fig. 7). — The magnetic circuit consists of the sample *S* and two heavy soft-iron pole pieces *P* and P_1, with a gap and soft-iron core *A* (similar to the gap and core in the voltmeter). A small coil *h* is suspended in the gap and connected to an auxiliary current supply. The sample is magnetized by current passing through the magnetizing coil *C*. The flux produced in the magnetic circuit causes the coil *h* to deflect an amount proportional to that flux, for a constant strength of auxiliary current. The deflection is indicated by a pointer, attached to the coil *h*, swinging over a scale that is calibrated to read directly in flux densities. The calibration of the instrument does not hold for wide variations in the permeability of the samples since the leakage depends upon this permeability. Also the magnetizing force can be calculated only approximately. These difficulties can be overcome in a measure by placing compensating coils on the yokes *P* and P_1 and sending sufficient current through these to compensate for the leakage.

Esterline Permeameter. — This instrument is similar to the Koepsel apparatus except that the moving coil and core (*h* and *A* in Fig. 7) are replaced by a small direct-current armature. The apparatus then becomes a small separately-excited dynamo. The armature is driven by an auxiliary motor at a constant speed. The voltage across the armature is then directly proportional to the total flux cutting the armature conductors. Leakage is avoided by the use of compensating coils on the pole pieces. The current in these compensating coils is adjusted until a small compass placed near the magnetic circuit of the apparatus shows no deflection when the magnetizing and compensating currents are simultaneously reversed. The apparatus is described in detail in *Proc. Am. Soc. Test. Mat.*, 1906, Vol. 6, p. 320.

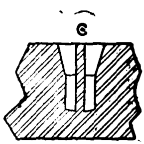

Fig. 8. Drilled Hole for Drysdale Permeameter

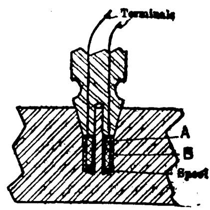

Fig. 9. Drysdale Permeameter

Drysdale Permeameter (Fig. 9). — This device is particularly useful in determining the permeability of large generator frame castings or other large

masses of metal without the preparation of samples. A hole is drilled in the casting at any point desired with a special drill that will leave a hole with a core in the center similar to that shown at *C* in Fig. 8.

A magnetizing coil *A* and a test coil *B* are wound on a soft-iron plug and inserted into this hole as shown. The terminals of both these coils are brought out through small holes in the plug. Measurements are made by the ordinary ballistic method.

Bismuth Spiral. — Bismuth has the property of changing in electrical resistance when put in a magnetic field. The per cent increase in resistance in bismuth when in a magnetic field is nearly proportional to the magnetic density of the field. For measuring permeability, a flat spiral of bismuth wire non-inductively wound is fastened between two plates of mica, the terminals of the coil being brought out through a long insulated handle.

The sample is made in two pieces and held in a yoke such as shown in Fig. 2. The two rods are inserted through the holes in the ends of the yoke and are pushed in until only a small air gap is left between their opposing ends. The bismuth spiral is inserted in this gap. The resistance of the spiral is measured with a Wheatstone's bridge and the flux density determined from a curve furnished with the spiral giving the variation in resistance with varying density. The calculated value of the magnetizing force must be corrected for the reluctance of the yoke and air gap.

CORE-LOSS MEASUREMENTS. — A great number of methods have been suggested for determining the core-loss in a sample of iron or steel when subjected to an alternating field. These methods may be classified as: (1) hysteresis loop method; (2) wattmeter method; (3) mechanical torque methods. Of these the wattmeter method, when a properly constructed magnetic circuit is employed, gives the most reliable results. The hysteresis loop method is tedious and gives no knowledge of the eddy-current loss. The mechanical torque methods necessitate an air gap in the magnetic circuit with a resulting induction in the specimen which is far from uniform; these latter methods, however, are frequently used where high degree of accuracy may be sacrificed to speed in testing, e.g., in comparative shop tests.

Hysteresis Loss from Hysteresis Loop.—The hysteresis loop may be determined by any of the methods described above. The loop is plotted on cross-section paper and integrated. Plot the magnetizing force, in gilberts per centimeter, to a scale of h gilberts per centimeter equal to 1 inch and the flux density to a scale of b gausses equal to 1 inch. Then if A is the area of the loop in square inches, the hysteresis loss, in ergs per cycle per cubic centimeter of iron, is

$$W = \frac{hbA}{4\pi}.$$

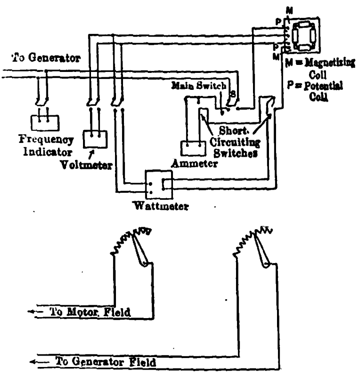

Fig. 10. Connections for Core-loss Tests

Principle of Wattmeter Method of Determining Core-Loss. — Fig. 10 is a complete diagram of connections. The principle of this method is as follows: The sample to be tested is inserted in a magnetizing coil of a known number of

turns, under* which is wound a secondary coil having either the same number of turns or a multiple thereof. The primary coil is connected in series with a source of alternating e.m.f. (which can be adjusted without changing its wave form) and the current coil of a wattmeter. The potential coil of the wattmeter is connected to the secondary coil on the test sample. A voltmeter is also connected across this secondary coil.

Let

N_1 = number of primary turns,
N_2 = number of secondary turns,
A = cross-section of sample in square centimeters,
f = frequency in cycles per second,
k = form factor of secondary voltage,
B = maximum flux density in sample, assumed constant throughout its length,
V = reading of voltmeter,
P = reading of wattmeter,
$R = r + \frac{R_1 R_2}{R_1 + R_2}$, where r is the resistance of the secondary coil, R_1 the resistance of the voltmeter and R_2 the resistance of the potential coil of wattmeter.

The resistance R should be sufficiently large so that the heat developed (V^2/R) in the secondary circuit is small compared to the core-loss, and the resistance r of the secondary coil should also be small compared with R. Then if the instruments are calibrated to read correctly and P and V are read simultaneously, the total core-loss is, assuming r negligible compared with R,

$$P_c = \frac{N_1}{N_2} P - \frac{V^2}{R},$$

and the maximum flux density is

$$B = \frac{10^8 V}{4 kfAN_2}.$$

Test Specimen with Single Coil. — The core-loss test can also be made with a single coil (the magnetizing coil) on the sample with the voltmeter and potential circuit of the wattmeter connected to the terminals of this coil. In this case, however, the voltage read by the voltmeter must be corrected for the resistance drop in the magnetizing coil, and the wattmeter reading for the rI^2 loss in this coil. The two-coil method also has the advantage that the wattmeter reading can be made any multiple of the actual loss by using a proper ratio of N_2/N_1, thus enabling one to measure small losses with greater accuracy.

Control of Impressed E.M.F. — The e.m.f. impressed across the terminals of the magnetizing coil should have a sine wave form for all values of the maximum flux density at which tests are to be made. To secure this condition it is necessary that the e.m.f. of the generator supplying the current have a sine wave form, and that the resistance and reactance of both the generator and the circuit between the generator and the test sample be as small as possible, as the magnetizing current, having a distorted wave form due to hysteresis (see *Magnetic Properties of Iron*), will introduce a voltage drop having a non-sine form, thus distorting the impressed voltage wave.

The impressed voltage should therefore be controlled through the field of a generator having a large capacity compared to the power taken by the test sample, or instead of varying the field a transformer with suitable taps may be used, provided the resistance and leakage reactance of the transformer are small.

* The two windings may also be "sandwiched."

In either case, the generator should give a sine wave at all field excitations within the range used. If this condition cannot be realized the hysteresis and eddy-current losses should be separated by measuring the total loss at two frequencies and the total loss then corrected for form factor (see below).

Form of Specimen for Wattmeter Test. — The specimen may have one of three forms: (1) It may be in the form of straight strips, the flux lines returning through the air; (2) straight strips may be used with a yoke; and (3) the specimen may be arranged to form a closed magnetic circuit in itself.

The first form gives a distribution of flux which is far from uniform. It is possible, however, by determining the flux distribution in the sample by means of an exploring coil to correct the observed losses for this flux variation (*see Robinson, L. T., Trans. A.I.E.E., 1911, Vol. 30, p. 741*).

The second form gives a more uniform flux, but it is necessary to distinguish between the energy supplied to the specimen and that supplied to the yoke. This can only be done satisfactorily by knowing the constants of the yoke, and only then by having the distribution of flux uniform, a condition difficult to secure.

Consequently, for accurate measurements the third form is the most reliable, although for factory use the first or second may prove more convenient where accuracy can be sacrificed for other considerations. The material used should be cut in a form such that only a small part of it is contiguous to a cut edge since all methods of cutting have a hardening effect upon the material bordering upon the cut. This means that the strip, whether straight or in ring form, should not be too narrow. This condition may be dispensed with if all specimens are annealed under definite conditions after cutting to size, and prior to testing.

Two general forms of magnetic circuit are available. The material may be stamped into rings (as in the Esterline apparatus), or the circuit may be built up from straight strips as in the Epstein apparatus. Leakage is most effectually avoided by using rings. With this form of specimen, however, the flux density will not be uniform unless rings of very great diameter are employed, and in the latter case there is a very great waste of material. The non-uniformity of flux existing in rings of small diameter, even when uniformly wound, and the errors resulting therefrom, are discussed by Loyd, M. G., *Bull. Bur. Stds., 1909, Vol. 5, p. 435*. The use of rings is thus restricted to cases where the material is annealed after stamping, and the radial width of the ring should be very small in comparison to its diameter. When rings are employed, the labor of winding each specimen separately with a magnetizing coil may be obviated by the use of the apparatus of Esterline (*see p. 925*) or Möllinger (*E. T. Z., 1901, Vol. 22, p. 379*).

Epstein Method. — This method has been adopted as the standard in Germany and by the American Society for Testing Materials in this country. It is used for commercial testing by both the General Electric and Westinghouse Companies. The method is clearly described in the standard specifications for making core-loss tests given in the following paragraph.

A.S.T.M. Standard Core-loss Tests. (Adopted June 1, 1912 by the *American Society for Testing Materials*.)

The power consumption in electrical sheet steel when subjected to an alternating magnetization is known as the core-loss. The standard core-loss is the total power in watts consumed in each kilogram of material at a temperature of 25° C., when subjected to a harmonically-varying induction having a maximum of 10,000 gausses and a frequency of 60 cycles per second, when measured as specified below. It is represented by the symbol $W_{10/60}$.

The ageing coefficient is the percentage change in the standard core-loss after continued heating at 100° C. for 600 hours.

The standard core-loss shall be measured under the following conditions:

The magnetic circuit consists of 10 kilograms (22 pounds) of the test material, cut with a sharp shear into strips 50 centimeters (19 11/16 inches) long and 3 centimeters (1 3/16 inches) wide, half parallel and half at right angles to the direction of rolling, made up into four equal bundles, two containing material parallel and two containing material at right angles to the direction of rolling, and finally built into the four sides of a square with butt joints and opposite sides consisting of material cut in the same manner. No insulation other than the natural scale of the material (except in the case of scale-free material) shall be used between laminations, but the corner joints shall be separated by tough paper 0.01 centimeter (0.004 inch) thick.*

The magnetizing winding shall consist of four solenoids surrounding the four sides of the magnetic circuit and joined in series. A secondary coil shall be used for energizing the voltmeter and the potential coil of the wattmeter.

These solenoids shall be wound on a form of any non-magnetic non-conducting material of the following dimensions:

Inside cross-section...........................	4 by 4 cm.
Thickness of wall	not over 0.3 cm.
Winding length............................	42 cm.

The primary winding on each solenoid shall consist of 150 turns of copper wire uniformly wound over the 42-centimeter length. The total resistance of the magnetizing winding shall be between 0.3 and 0.5 ohm. The secondary winding of 150 turns of copper wire on each solenoid shall be similarly wound beneath the primary winding. Its resistance shall not exceed 1 ohm.

A voltmeter and the voltage coil of a wattmeter shall be connected in parallel to the terminals of the secondary winding of the apparatus. The current coil of the wattmeter shall be connected in series with the primary winding.

A sine-wave electromotive force shall be applied to the primary winding and adjusted until the voltage of the secondary circuit is given by the equation,

$$E = \frac{4fNkBM}{4lD\,10^8},$$

in which

k = form factor of primary e.m.f. = 1.11 for sine wave,
N = number of secondary turns = 600,
f = number of cycles per second = 60,
B = maximum induction = 10,000,
M = total mass in grams = 10,000,
l = length of strips in centimeters = 50,
D = specific gravity = 7.5 for high-resistance steel
= 7.7 for low-resistance steel,
E = 106.6 volts for high-resistance steel for sine voltage
= 103.8 volts for low-resistance steel for sine voltage.

* The purpose of this paper is to prevent the exposed end of the laminations being forced into the spaces between those in the adjacent side of the other part of the sample and to prevent the formation of eddy currents at the corners which may not be confined to the thickness of the laminations if the paper is not used. The certainty of the measurements is thereby improved by a small but definite amount over the results which are obtained without the paper in the joints. This improvement is of course accomplished at the expense of some increase of leakage flux at the corners and consequently greater departure from absolute uniformity of flux distribution along the length of the sample, also the conditions imposed upon wattmeter are more severe as the general power factor is lower. The flux distribution over a section of the sample is more uniform with the paper. The net result is a definite gain in accuracy.

(L. T. Robinson.)

A specific gravity of 7.5 is assumed for all steels having a resistance of over 2 ohms per metergram, and 7.7 for all steels having a resistance of less than 2 ohms per metergram. These steels are designated as high- and low-resistance steels, respectively.

The wattmeter gives the power consumed in the iron and the secondary circuit. The loss in the secondary circuit is given in terms of the total resistance and voltage. Subtracting this correction term from the total power gives the net power consumed in the steel as hysteresis and eddy-current loss. Dividing this value by ten gives the core-loss in watts per kilogram.

The Procedure. — 1. Cut from the test material a number of strips 3 by 50 centimeters, half parallel and half at right angles to the direction of rolling.

2. Place on the balance a pile of strips weighing 2.5 kilograms. Add a second pile of the same kind, bringing the weight up to 5 kilograms. In each case the weight is taken to the nearest strip. Add in succession two piles of 2.5 kilograms each, of the other kind of strips, bringing the weight up to 7.5 kilograms and 10 kilograms respectively.

3. Secure each bundle by string or tape (not wire) and insert in the apparatus as indicated.

4. Apply the alternating voltage to the primary coil and tap the joints together until the current has a minimum value, as shown by an ammeter in series. Then clamp the corners firmly by some suitable device.

5. Shunt the ammeter and adjust the primary current until the voltmeter indicates the proper value. This adjustment may be made by an auto-transformer by varying the field of the alternator, or by both, but not by the insertion of resistance or inductance in the primary circuit. Simultaneously the frequency must be adjusted to 60 cycles.

6. Read the wattmeter.

7. Calculations. Subtract from the wattmeter reading the instrument losses, which will be constant for any set of instruments and voltage, and divide by 10. The result is the standard core-loss.

Bureau of Standards' Method. — (*Bull. Bur. Stds., 1909, Vol. 5, p. 453; Trans. A.I.E.E., 1909, Vol. 28, p. 439.*) This is a modification of the Epstein method. It differs from the latter in the use of a smaller test specimen, from 1.5 to 2 kilograms (about 4 pounds) of strips 25.4 by 5 centimeters (10 by 2 inches), and in the use of a different form of joint between the four bundles of strips. In other respects it is essentially the same as the Epstein method.

Fig. 11 shows the arrangement of the joint. At the corners of the square, short pieces of test material are bent at right angles and interleaved between the strips of adjacent bundles, as shown in the figure. There are as many of

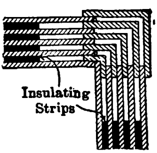

Fig. 11. Detail of Joint Used by N.B.S.

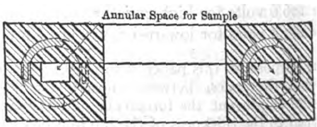

Fig. 12. Esterline Apparatus

these corner pieces as there are test pieces, and they are graduated in length so as to give a uniform lap of about 2 millimeters. A special clamp is tightened over these laps, so as to give a good magnetic joint. The object of these corner pieces is to reduce the leakage and thereby obtain a more nearly uniform flux

throughout the sample. The loss in these corner pieces is small and can be calculated to a sufficient degree of accuracy and allowed for.

Esterline Apparatus For Testing Iron Rings (Fig. 12). — This apparatus is designed for testing rings made up of punchings from sheet steel. It is essentially a solenoid made in the form of a doughnut, but divided into two halves. The sample is inserted by lifting off the top half; the top half is then replaced and the teeth formed by the projecting ends of each upper half-turn fit into little sockets formed at the ends of the corresponding lower half-turn. (*Esterline, J. W., Proc. Am. Soc. Test. Mat., 1906, Vol. 6, p. 320.*)

Ewing Hysteresis Tester. — Strips 5/8 by 3 inches are cut from the sheets to be tested and a bundle of about 7 is used in each test. This sample is rotated at a relatively low speed (to avoid eddy currents) between the poles of a permanent magnet, the magnet being pivoted at the center of rotation and carrying a pointer that deflects over a scale, the deflection depending on the hysteresis loss in the sample. Two standard samples of known hysteresis properties are furnished with the instrument, so that the scale may be calibrated from time to time.

The Blondel Hysteresis Tester. — This apparatus is similar in principle to the Ewing hysteresis tester, but the magnet is revolved instead of the sample. The samples are made in the form of rings which are mounted on a pivoted spindle between the poles of a U-shaped permanent magnet. When the magnet is revolved the sample tends to follow it but is restrained by a spiral spring, and therefore it turns only through an angle such that the torque due to the hysteresis loss is balanced by the opposing torque of the spring.

The Holden-Esterline Core-loss Tester. — This instrument is similar to the Blondel hysteresis tester previously described but an electromagnet is substituted for the permanent magnet and this magnet is driven by a motor at the required frequency. The spring is arranged with a torsion head and pointer that is set to zero when the instrument is not in use.

The angle of torsion necessary to bring the sample back to its zero position when the instrument is operating is read on a scale calibrated directly in watts loss. The speed of rotation being high the eddy-current losses are appreciable, and therefore the instrument reads the combined loss due to hysteresis and eddy currents at the frequency corresponding to the speed of rotation.

SEPARATION OF HYSTERESIS AND EDDY-CURRENT LOSSES. — The total loss is measured by the wattmeter method at two frequencies, say f_1 and f_2, but at the *same effective voltages*. Let

P_1 = total loss at frequency f_1,
P_2 = total loss at frequency f_2,
h_1 = form factor at frequency f_1,
h_2 = form factor at frequency f_2,
E_1 = effective value of induced e.m.f. at frequency f_1,
E_2 = effective value of induced e.m.f. at frequency f_2,
K_h = hysteresis loss at 1 cycle per second and sine wave e.m.f. of 1 volt,
K_e = eddy-current loss at 1 cycle per second and e.m.f. of 1 volt.

Then for a given iron core (*see article of Magnetic Properties of Iron*)

$$P_1 = \left(\frac{1.11\,E_1}{h_1}\right)^{1.6} f_1 K_h + (f_1 E_1)^2 K_e,$$

$$P_2 = \left(\frac{1.11\,E_2}{h_2}\right)^{1.6} f_2 K_h + (f_2 E_2)^2 K_e,$$

whence, by measuring all the other quantities, K_h and K_e may be calculated from these two equations. This method of separation, however, assumes 1.6 as the hysteresis coefficient, which is an approximation.

However, if a sine wave e.m.f. of the same effective value is used at both frequencies, the separation can be made without any other assumption than that the hysteresis loss varies directly as the frequency and the eddy-current loss as the square of the frequency, both of which conditions are in accord with fact when the frequency is less than 100 cycles per second and ordinary thicknesses of sheets are used.

In particular, if f_2 is taken as one-half of f_1 as well as $E_1 = E_2$, both in effective value and wave form, then

$$\text{Eddy-current loss at frequency } f_1 \quad = f_1^2 K_e \; E_1^2 = 2\,(P_1 - 2\,P_2),$$
$$\text{Hysteresis loss at frequency } f_1 \quad = f_1 K_h \; E_1^{1.6} = 4\,P_2 - P_1,$$

and these relations are independent of the law of variation of the hysteresis loss with maximum flux density.

IRON-LOSS VOLTMETER. — As pointed out in the article on *Magnetic Properties of Iron*, the core-loss depends upon the form of the impressed voltage wave. The dotted curves A, B and C in Fig. 13 show the variation in core-loss with form factor when the eddy-current loss is 14 per cent, 20 per cent and 30 per cent of the total loss. The loss corresponding to a sine-wave voltage (form factor 1.11) is taken as 100 per cent.

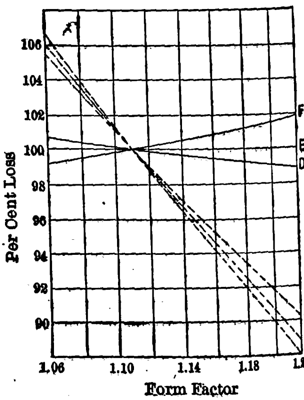

Fig. 13.

In making commercial tests on transformers it is frequently inconvenient to obtain a sine-wave voltage, and it is also equally inconvenient to obtain the form factor of the actual voltage available and the relative value of the eddy-current loss. Yet, for comparative purposes, the core-loss should be referred to a standard sine-wave form. The iron-loss voltmeter, devised by L. W. Chubb (*Trans. A.I.E.E., 1909, Vol. 28, p. 417*) is an instrument designed to read, when connected to a circuit in which the voltage has *any wave form whatever*, the value of the sine-wave voltage which would produce the same total loss as produced by the actual voltage. The meter is calibrated for one frequency only, usually 60 cycles per second. When such an instrument is connected across the transformer and the voltage adjusted until the instrument reads the rated voltage of the transformer, the core-loss read by a wattmeter connected in the usual manner will be the core-loss corresponding to a sine-wave voltage having an effective value equal to the voltage read by the iron-loss voltmeter, irrespective of what the reading of an ordinary voltmeter connected across the line may be.

Chubb's iron-loss voltmeter is essentially an ordinary wattmeter, the current coil of which is connected in series with a winding on a small laminated iron core, these two elements in series being connected directly across the line. The potential circuit of the wattmeter is also connected across the line to the same two terminals as the first or series circuit. The meter therefore has but two terminals. The deflection of the moving element of such a meter, when con-

nected across the supply mains, is proportional to the total power absorbed by the circuits and the iron core. Neglecting the small resistance loss in the series circuit, it is possible, by adjusting the resistance in the potential circuit and the number of turns on the iron ring, to make the equivalent eddy-current loss in the instrument (including the resistance loss in the potential circuit as an eddy-current loss, since it depends on the effective value of the voltage in the same way as the eddy-current loss in the sample) any desired proportion of the total loss. These adjustments are so made that at about 0.6 full-scale deflection the equivalent eddy-current loss on a pure sine-wave voltage at 60 cycles is 20 per cent of the total, this being the average proportion of the hysteresis loss to total loss in commercial 60-cycle transformers.

The instrument is then calibrated to read directly in volts by connecting it in parallel with an ordinary alternating-current voltmeter on a pure sine-wave voltage of the required frequency.

The voltage to be used for a core-loss test on any circuit, irrespective of wave form, is then read by means of this instrument instead of by an ordinary voltmeter. If the transformer under test has the same per cent eddy-current loss as the equivalent eddy-current loss in the iron-loss voltmeter, then the reading of an ordinary wattmeter connected to measure the core loss of the transformer in the ordinary way will be the loss on a sine-wave voltage having an effective value equal to the reading of the iron-loss voltmeter. If the percentage hysteresis loss is different from that of the iron-loss voltmeter, a slight error will be introduced. Curves D and F in Fig. 13 show the error when the eddy-current loss in the transformer is 14 per cent and 30 per cent respectively, instead of 20 per cent.

The voltage may be regulated through a considerable range by the use of a resistance or inductance, thus dispensing with the usual multi-tap transformer or field regulation of generator. The frequency need only approximate the normal values; the final core loss determined at the voltage indicated by the iron-loss voltmeter will be the same as would be obtained on a sine wave of normal frequency and voltage.

Adjustment of Form Factor. — Chubb also gives a method of obtaining a form factor of 1.11 from a wave of any shape. This is shown in Fig. 14. T is the transformer under test, W an ordinary wattmeter, V_1 the iron-loss volt-meter, V_2 an ordinary a-c. voltmeter, R a variable resistance, L a variable inductance, C an aluminum electrolytic condenser having a critical voltage less than that impressed across it. By varying L and R the readings of V_1 and V may be made to agree; under these conditions the form factor of the voltage across the transformer is 1.11. The wave however will not in general be a sine wave.

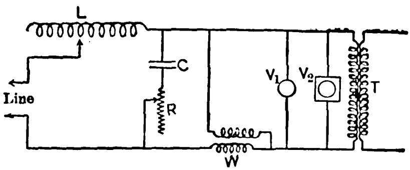

Fig. 14.

BIBLIOGRAPHY. — Gray, A., *Absolute Measurements of Electricity and Magnetism*, London, 1893; Edgecumbe, K., *Industrial Measuring Instruments*, London, 1908; Gerard, Eric, *Measures Electrique*, Paris, 1908; Karapetoff, V., *Experimental Electrical Engineering*, N. Y., 1908. Also the references in the text.

[H. Pender and H. R. Ranken.]

MAXIMA AND MINIMA. — (*See also Derivatives; Series, Mathematical.*) Let y be any function of a variable x, then y will be a maximum or minimum for any value of x which satisfies

$$\frac{dy}{dx} = 0 \qquad (1)$$

provided $\frac{d^2y}{dx^2}$ is not zero. If the second derivative $\frac{d^2y}{dx^2}$ is positive for this value of x, then the corresponding value of y is a minimum; if this second derivative is negative, the corresponding value of y is a maximum.

In case $\frac{d^2y}{dx^2}$ is also zero for the value of x which satisfies (1), the corresponding value of y is not a maximum or minimum unless $\frac{d^3y}{dx^3}$ is also zero and $\frac{d^4y}{dx^4}$ is not zero. When $\frac{d^3y}{dx^3} = 0$, y is a minimum if $\frac{d^4y}{dx^4}$ is positive and a maximum if $\frac{d^4y}{dx^4}$ is negative. In case $\frac{d^4y}{dx^4}$ is also zero, similar relations must hold for the fifth and sixth derivatives, etc.

Example. — Find the maximum and minimum values of

$$y = 2x^3 - 9x^2 + 12x - 3,$$

then

$$\frac{dy}{dx} = 6x^2 - 18x + 12 = 0,$$

$$\frac{d^2y}{dx^2} = 12x - 18,$$

whence y is maximum or minimum for $x^2 - 3x + 2 = 0$, that is, for $x = 2$, or $x = 1$. For $x = 2$, $\frac{d^2y}{dx^2}$ is positive; for $x = 1$, $\frac{d^2y}{dx^2}$ is negative; hence the maximum value of y is 2 and occurs for $x = 1$, while the minimum value of y is 1 and occurs for $x = 2$.

[W. A. Del Mar.]

MECHANICS, PRINCIPLES OF. — (*See also Structures, Simple; Units and Conversion Factors.*) In this article are given the definitions of the more commonly-used mechanical quantities together with a statement of the quantitative laws in accord with which displacements and motion of matter (including deformations) take place. The interrelations of the various units employed for measuring any particular quantity are given in the article on *Units and Conversion Factors*.

DEFINITIONS. — The various terms relating to the displacement and motion of matter and to forces producing these displacements and motions are the following:

Scalar and Vector Quantities. — A mechanical quantity which is not directed in space, e.g., mass, energy, etc., is called a scalar quantity. A mechanical quantity which has a space direction as well as magnitude, e.g., velocity, force, etc., is called a vector quantity. A scalar quantity may be positive or negative, that is, has two "senses," but to specify a vector quantity like force it is necessary to specify not only its magnitude but its direction with respect to one or more fixed lines of reference or axes. Scalar quantities may be treated as ordinary algebraic quantities, and added and subtracted in the usual way. Vector quantities must be added and subtracted *vectorially*, as described in the article on *Vectors*.

Linear Displacement (l). — When a particle P moves from a point A to any other point B, the distance, measured along a straight line, from A to B is called the linear displacement of the point P.* Any unit of length may be used as a unit of displacement; see *Units and Conversion Factors*.

Angular Displacement (θ). — Let a particle P move from a point A to some other point B, and let OX be any arbitrarily-chosen line, or axis. Draw planes through A and OX and through B and OX. Then the angle between the planes AOX and BOX is called the angular displacement of P about OX. * Angular displacement may be measured in degrees, radians or turns; see *Angles* and *Units and Conversion Factors*.

Linear Velocity (v) **and Speed** (s). — The linear velocity or speed of a particle P is the rate of increase with time of the linear displacement of P, i.e.

$$v = \frac{dl}{dt} \qquad \text{or} \qquad s = \frac{dl}{dt}, \tag{1}$$

where dl is the linear displacement in time dt.
Any unit of length per any unit of time may be used as a unit of velocity or speed; see *Units and Conversion Factors*.

Angular Velocity or Speed (ω). — The angular velocity or speed of a particle about a given axis X is the rate of increase with time of its angular displacement about this axis, i.e,

$$\omega = \frac{d\theta}{dt}, \tag{2}$$

where $d\theta$ is the angular displacement in time dt. Any unit of angle per any unit of time may be used as a unit of angular speed; see *Units and Conversion Factors*.

* The positions of A and B (and of OX also in the case of angular displacement) must be referred to some system of coördinates; when the system of coördinates moves, the displacement as above defined is the displacement *relative to* this system of coördinates.

Linear Acceleration (a). — The linear acceleration of a particle P is the of increase with time of the linear velocity of P, i.e.,

$$a = \frac{dv}{dt} = \frac{d^2l}{dt^2}, \tag{3}$$

here dv is the increase of linear velocity in time dt. Any unit of velocity per any unit of time may be used as a unit of acceleration, *e.g.*, centimeters per second per second, miles per hour per hour, etc.; see *Units and Conversion Factors*.

Acceleration Due to Gravity (g). **Gravitational Acceleration Constant** (g_0). — At any given place on the earth a body falling freely in a vacuum has a constant linear acceleration, independent of its size, shape or material. This acceleration is called the acceleration due to gravity (g), and the particular value of this acceleration at 45 degrees latitude and sea level is called the gravitational acceleration constant (g_0). The value of g_0 as adopted by international agreement * is

$$g_0 = 980.665 \text{ cm. per sec. per sec.}$$
$$g_0 = 32.1739 \text{ ft. per sec. per sec.}$$

The value of g for any other location varies but slightly from this value, being at sea level approximately 0.3 per cent greater at the equator, 0.3 per cent less at the poles, and decreasing at the rate of about 0.01 per cent per 1000 feet increase in elevation. See *Landolt-Börnstein's Tables*.

Angular Acceleration (α). — The angular acceleration of a particle P about an axis X is the rate of increase with time of the angular velocity of P about X, i.e.,

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}, \tag{4}$$

where $d\omega$ is the increase in angular velocity in time dt. Any unit of angular velocity per any unit of time may be used as a unit of acceleration, e.g., degrees per second per second, turns per second per second, etc.; see *Units and Conversion Factors*.

Mass or Weight.* — Two bodies are said to have equal masses or weights,† irrespective of their volume, shape or chemical composition, if, when they are suspended simultaneously in a vacuum, one from each end of an equal-armed balance, there is no tipping of the beam of the balance from its original position. This criterion for the equality of two masses holds only in case the bodies and the balance are neither electrically charged nor magnetized, both bodies are supported at the same distance from the earth, and the equilibrium of the balance is not affected by the presence of any other bodies (except the earth) in the vicinity. This is an entirely arbitrary definition, but mass as thus defined is found to be a fundamental property of a body irrespective of its shape, physical state or relation to other bodies. The units of mass or weight and their interrelations are given in the article on *Units and Conversion Factors*.

Density (δ) **and Specific Gravity.** — The density of a uniform substance is defined as the mass of the substance per unit volume. When the substance

* *Troisième Conf. Gen. des Poids et Mes., 1901, p. 66.* See Note 5, p. 3 of this book.

† The term weight is also used for the force exerted by the earth on a portion of matter; in this sense weight is not independent of the relation of the given portion of matter to other bodies, and the term mass is therefore preferable when referring to quantity of matter, since it has not acquired a double meaning. However, engineers almost invariably use the word weight instead of mass for quantity of matter. See the article on *Units and Conversion Factors* for a further discussion of this point.

is not uniform, its density at any point is defined as the mass of an infinitesimally small volume taken about the point divided by this volume; i.e., calling dv the volume and dm the mass of this volume, the density is

$$\delta = \frac{dm}{dv}. \tag{5}$$

In the c.g.s. system the standard unit of density is the gram per cubic centimeter, but any other unit of mass per any unit of volume may be used in either the metric or English system; see *Units and Conversion Factors*. For values of the density of various materials see article on *Weights of Materials*.

The specific gravity of a substance is defined as the ratio of the weight of a given volume of that substance to the weight of an equal volume of water or air. Water is used as the standard of reference for solids and liquids, and air at 0° C. and 760 mm. mercury pressure as the standard of reference for gases. Strictly, the temperature of the water also should be specified, but this is not always done. The variation in the weight of a given volume of water with temperature is slight and for many purposes negligible. When 4° C. is taken as the standard water temperature the specific gravity of a substance is numerically equal to its density in grams per cubic centimeter to within 25 parts in 1,000,000, a difference which is very much less than the degree of precision of ordinary measurements of density.

Surface and Lineal Densities. — Density as defined above is the *volume* density of a body. It is sometimes convenient to use factors which represent the mass or weight of a substance (or other physical quantity) per unit of surface or per unit of length, e.g., the weight per square foot of a plate or the weight per foot of a wire.

Center of Mass, Center of Gravity, or Center of Inertia. — A body of mass M which has any size or shape may be considered as made up of a number of small particles of masses m_1, m_2, m_3, etc., such that $m_1 + m_2 + m_3 + \cdots = M$. These particles may be considered as small as desired, that is, each particle may be considered so small that it occupies but a point in space. Choose any three mutually perpendicular planes, fixed with respect to one another, and represent by x_1, y_1 and z_1 the perpendicular distances of the particle m_1 from these planes respectively, and by x_2, y_2 and z_2 the perpendicular distances of the particle m_2 from these three planes respectively, and so on for the other particles. Then the point whose distances from these three planes are respectively

$$\begin{aligned} X &= \frac{m_1x_1 + m_2x_2 + m_3x_3 + \cdots}{M} \\ Y &= \frac{m_1y_1 + m_2y_2 + m_3y_3 + \cdots}{M} \\ Z &= \frac{m_1z_1 + m_2z_2 + m_3z_3 + \cdots}{M} \end{aligned} \tag{6}$$

is defined as the center of mass, or center of gravity, or center of inertia, of the body. The name center of gravity arises from the fact that the vertical forces acting on all the various particles of a body, due to the pull of the earth, may be considered equivalent to a single force, equal to the sum of these individual forces, applied to the body at this point. The center of mass of a perfectly rigid body is a fixed point with respect to every point in the body; it may, however, lie either within or without the body.

Equations (6) may also be written in terms of the calculus, thus:

$$X = \int \frac{\delta x\, dv}{M}, \quad Y = \int \frac{\delta y\, dv}{M}, \quad Z = \int \frac{\delta z\, dv}{M}, \tag{6a}$$

where dv represents an elementary volume of the body at any point, x, y and z the distances of this point from the three planes of reference, δ the density of the body at this point, and M the total mass of the body. When the density is uniform throughout,

$$X = \int \frac{x\,dv}{V}, \quad Y = \int \frac{y\,dv}{V}, \quad Z = \int \frac{z\,dv}{V},$$

where V is the total volume of the body.

Centroid or Center of Gravity of a Plane Section. — This is defined as the point whose coördinates in the plane of the section are

$$X = \int \frac{x\,dA}{A} \quad \text{and} \quad Y = \int \frac{y\,dA}{A} \tag{6b}$$

referred to any two fixed mutually perpendicular lines in this plane, A being the total area of the section and dA any elementary area of the section. Physically, the center of gravity of a plane section may be defined as the center of gravity of a thin plate having the same shape as that of the section and of uniform thickness and specific gravity. See article on *Structures, Simple,* for the position of center of gravity for various plane sections.

Center of Gravity of Cylinders and Prisms. — The center of gravity of a cylinder or prism with parallel end surfaces and of *any* shaped cross-section, solid or hollow, is the centroid of that cross-section of the cylinder or prism which is halfway between the two ends, provided the cylinder has a uniform density throughout.

Moment of Inertia (I). — Consider any axis of reference X and any particle of mass m at a distance r from this axis; then the product mr^2 is called the moment of inertia of the particle m about the axis X. The moment of inertia of an extended body or system of bodies about any axis is

$$I = m_1r_1^2 + m_2r_2^2 + m_3r_3^2 + \ldots, \tag{7}$$

the summation including products mr^2 for all the particles of the body or system of bodies, r being the distance of the particle from the axis. This may also be written in the notation of the calculus as

$$I = \int r^2\,dm, \tag{7a}$$

where dm represents the mass of any particle and r is the distance from the axis. In case the body has a uniform density δ throughout this may also be written

$$I = \delta \int r^2\,dv, \tag{7b}$$

where dv represents an elementary volume of the body.

Units of Moment of Inertia of Bodies. — The moment of inertia of a body is of the nature of, or has the "dimensions" of, $(\text{length})^2 \times (\text{mass})$. When the mass is expressed in pounds and the distance r in feet then the moment of inertia may be said to be expressed in $(\text{foot})^2$-pounds, and similarly for any other units of mass and length. In applying the formulas below the same set of units must be used throughout. See *Units and Conversion Factors.*

Moment of Inertia of a Plane Section. — The moment of inertia of a plane section about any axis is defined by the relation

$$I = \int r^2\,dA, \tag{7c}$$

where dA represents any elementary area of this section and r the distance of this elementary area from the axis. When the axis is chosen *perpendicular* to the plane of the section *through its center of gravity* the moment of inertia about this axis is called the "polar" moment of inertia. Physically, the moment of inertia of a plane section about any axis may be defined as the moment of inertia about this same axis of a thin plate having the same shape as that of the section and having unit mass per unit of area.

The unit of moment of inertia of a plane surface has the dimensions of length to the fourth power.

Let X and Y be two mutually perpendicular axes in the plane of the section *passing through its center of gravity* and let Z be the axis through the center of gravity perpendicular to this plane and therefore also perpendicular to X and Y; let I_x and I_y be the moments of inertia of the plane section about X and Y respectively, and I_z be the polar moment of inertia of the section. Then

$$I_z = I_x + I_y. \tag{7d}$$

Values of I_x and I_y for various plane figures are given in the article on *Structures, Simple.*

Let X be any axis in the plane of a section, X_0 an axis through the center of gravity of the section parallel to X, x the distance between the two parallel axes X and X_0, I_0 the moment of inertia of the section about X_0, and A the area of the section. Then the moment of inertia of the section about X is

$$I_x = I_0 + x^2A. \tag{7e}$$

Principal Axes and Principal Moments of a Plane Section. — For every plane surface there are two rectangular axes passing through its center of gravity and lying in the surface, such that about one of these axes the moment of inertia is less, and about the other greater, than that about any other axis in the given surface. These two axes are called the *principal axes* and the moments the *principal moments of inertia.* If the plane figure has an axis of symmetry, this axis is one of the principal axes; e.g., a rectangle has two axes of symmetry, one parallel to each side, hence each of these axes is a principal axis. In the case of a circle any axis may be considered a principal axis. The principal axes for various sections are given in the article on *Structures, Simple.*

Moment of Inertia of Cylinders and Prisms. — Let the cylinder or prism be of any cross-section whatever, solid or hollow; let its density be δ, constant throughout; let the length of its own axis between the parallel end surfaces be l; and let I_s be the moment of inertia of the cross-section of the cylinder or prism about any axis X parallel to the length l; then the moment of inertia of the entire cylinder or prism about this axis X is

$$I_c = \delta l I_s. \tag{7f}$$

The density δ must be referred to a volume unit based upon the same unit of length as that in which the dimensions of the cylinder or prism are expressed. For example, if I_s is expressed in centimeters-to-the-fourth-power, δ must be expressed in mass per cubic centimeter, and l must be expressed in centimeters; if δ is in grams per cubic centimeter, then the moment of inertia is in (centimeter)^2-grams.

Radius of Gyration (ρ). — The radius of gyration of a body about any axis is defined by the relation

$$I = M\rho^2 \quad \text{or} \quad \rho = \sqrt{\frac{I}{M}}, \tag{8}$$

where M is the mass of the body and I its moment of inertia about the given axis. When the body has a uniform density throughout the radius of gyration may also be written

$$\rho = \sqrt{\frac{\int r^2 dv}{V}}, \tag{8a}$$

where V is the volume of the body. Radius of gyration is of the same nature as length and is therefore expressed in the same units as length.

Radius of Gyration of Plane Section. — The radius of gyration of a plane section about any axis is defined by the relation

$$\rho = \sqrt{\frac{\int r^2 dA}{A}}, \tag{8b}$$

where A is the total area of the section, dA an elementary area of this section and r the distance of dA from the axis. Values of ρ_x and ρ_y for various sections are given in the article on *Structures, Simple.*

From formula (7d) it follows that if ρ_x and ρ_y represent the radii of gyration of a plane section about two mutually perpendicular axes *passing through the center of gravity* of the section, then the radius of gyration ρ_z of this section about an axis through its center of gravity *perpendicular* to the plane of the section is

$$\rho_z = \sqrt{\rho_x^2 + \rho_y^2}. \tag{8c}$$

For example, for a circular section $\rho_x = \rho_y = \frac{d}{4}$, where d is the diameter of the section. Hence the polar radius of gyration about a perpendicular axis through the center of the circle is $\rho_z = \frac{\sqrt{2}}{4} d$.

From formula (7e) it also follows that if ρ_0 is the radius of gyration about any axis X_0, then the radius of gyration about a *parallel* axis X at a distance x from X_0 is

$$\rho_x = \sqrt{\rho_0^2 + x^2}. \tag{8d}$$

Radius of Gyration of Cylinders and Prisms. — The radius of gyration of a cylinder or prism about any axis X parallel to its own axis is the same as the radius of gyration about the axis X of any plane section of the cylinder or prism, provided the end surfaces are parallel and the density is uniform throughout. This relation holds for a prism or cylinder of any shape of cross-section, whether solid or hollow.

Linear Momentum (mv). — When the *center of mass* of a body of mass m is moving with a linear velocity v with respect to any point P, the product mv is called the linear momentum of the body with respect to this point. Units of linear momentum are of the same nature or dimensions as the units of power; see *Units and Conversion Factors.*

Experience shows that in any system of bodies composed of two bodies or groups of bodies A and B, any change in the linear momentum of the first body or group of bodies A with respect to the center of mass of the system is accompanied by an equal and opposite change in the linear momentum of the second body or group of bodies B with respect to the center of mass of the system, and vice versa. This fact of experience is known as the *Principle of the Conservation of Linear Momentum.*

Angular Momentum ($I\omega$). — When a body has a moment of inertia I about any axis X and is rotating about this axis with an angular velocity ω with respect to any point P, the product $I\omega$ is called the angular momentum of the body about this axis with respect to the given point. Units of angular momentum are of the same nature or dimensions as those of linear momentum and of power; see *Units and Conversion Factors*.

Experience shows that in any system of bodies composed of two bodies or groups of bodies A and B, any change in the angular momentum of the first body or group of bodies A about any axis X is accompanied by an equal and opposite change in the angular momentum of the second body or group of bodies B about this same axis, and vice versa. This fact of experience is known as the *Principle of the Conservation of Angular Momentum*.

Force (f). — In general terms a force is that which produces or tends to produce a change in the state of rest or motion of a body, or "a force is pull or a push." The nature of force is not thoroughly understood, but the effects of a force, e.g., change in motion, the extension or compression of a spring, etc., are readily measured. From the principle of the conservation of linear momentum stated above it follows that any change in the motion of a body or portion of a body A is always accompanied by a change in the motion of some other body or portion of the same or of some other body B. Hence the body A is said to exert a force on the body B, and vice versa.

In a system composed of but two bodies A and B, uninfluenced by the presence of any other bodies, the measure of the force produced on the body A by the body B may be taken as the rate of change with time of the linear momentum of A with respect to the center of mass of the system formed by A and B. The measure of the force produced on B by A is similarly defined. From the principle of the conservation of linear momentum it then follows that the force produced on A by B is equal and opposite to the force produced on B by A, or "action and reaction are equal and opposite."

Stated in a formula the above definition is that the mutual force between any two bodies A and B whose motion is uninfluenced by any other forces is

$$f = m_1a_1 = -m_2a_2, \tag{9}$$

where m_1 and m_2 are the masses of A and B respectively, and a_1 and a_2 are the accelerations of the centers of mass of A and B respectively with respect to the center of mass of the system formed by A and B together.

Force of Gravitation. — For example, consider the case of a mass m at any point above the earth. The earth exerts a force on m and m exerts an equal and opposite force on the earth, and if there are no other forces acting m will move toward the joint center of mass of the earth and the body m, and similarly the earth will move toward this same point. However, as the earth has a mass many times that of any body at or near its surface, the center of mass of the system formed by m and the earth is practically that of the earth itself, and the motion of the earth with respect to this point is inappreciable. Hence the force exerted by the earth on the mass m is, within the limits of error of observation,

$$f = mg, \tag{9a}$$

where g is the acceleration of the center of mass of m with respect to the center of the earth, or with respect to any point fixed with respect to the surface of the earth in the vertical line through the center of mass of m.

Units of Force. — The rational unit of force is that force which will give unit linear acceleration to unit mass; this unit of force is called the "absolute" unit of force. When the mass is expressed in grams and the acceleration in centimeters per second per second, the corresponding absolute unit

of force is called the "dyne;" when the mass is expressed in pounds and the acceleration in feet per second per second, the corresponding absolute unit of force is called the "poundal."

The relation expressed by equation (9a), however, suggests another unit of force which for many purposes is very convenient, and in fact is the unit ordinarily used by engineers. This unit, called the "gravitational" unit of force, is the force exerted by the earth on unit mass; since the value of this force in terms of the absolute unit varies with latitude and elevation, it is also necessary to specify a definite place of measurement of this unit force, or better a *definite* value of the acceleration g, which shall be used in evaluating this force in terms of the absolute unit. The value of g adopted by international agreement is $g = 980.665$ cm. per sec. per sec. $= 32.1739$ ft. per sec. per sec., which (see Note 5 at top of p. 3 of this book) is equal to the acceleration due to gravity at 45 degrees latitude and sea-level; see also above under *Acceleration.* Unfortunately the gravitational units of force are given the same names as the units of mass to which they refer; for example a force of 1 pound is the pull exerted by the earth on a mass of 1 pound at 45 degrees latitude and sea-level. This double use of names often leads to confusion unless one keeps clearly in mind the definitions of unit mass and unit force. See *Units and Conversion Factors.*

When the force acting on a body is expressed in absolute units, the relation between force, mass and linear acceleration is

$$f = ma \quad \text{absolute units.} \tag{9b}$$

When the force is expressed in gravitational units, the relation between force, mass and linear acceleration is

$$f = \frac{m}{g_0} \cdot a \quad \text{gravitational units,} \tag{9c}$$

where g_0 stands for the numerical factor 32.1739 when f is in pounds, m in pounds and a in feet per second per second; similarly when f is expressed in grams, m in grams and a in centimeters per second per second g_0 stands for 980.665.

Measurements of Force. — One seldom has to deal with a simple system of but two bodies, and consequently the above definitions of the measure of a force can seldom be applied to an actual measurement. The value of these definitions is that they fix the unit of force. Forces are measured by balancing the pull of an unknown force against a known force (e.g., gravity or the reacting force of a spring), the principles involved being deduced from the fundamental principle of the conservation of linear and angular momentum when applied to bodies in equilibrium; see the section on *Conditions for Equilibrium* below.

Weight and Force. — The word weight, as already noted, is used to designate both mass and force. A weight of 10 pounds, say, as ordinarily used in reference to "how much" of a substance, means that the piece of matter in question has a mass of 10 pounds. A weight of 10 pounds used in reference to the pull produced by the earth on a piece of matter means that this pull is a force of 10 pounds. From the above definitions it follows that, neglecting the slight variation with latitude and elevation, the *numerical* values of weight used in the two senses are equal, provided mass is expressed in absolute units and force in gravitational units, i.e., a mass of 10 pounds also weighs 10 pounds, but weighs 321.739 *poundals;* see above under *Units of Force.*

Point of Application and Line of Action of a Force. — Any particle or point of a body which is acted upon directly by some external force (e.g.,

a string attached to the body at this point) is said to be the point of application of this external force. A line drawn through this point in the direction of the force is called the line of action of the force. In general, whenever a solid body is acted upon by any number of external forces applied at various points the motion produced is the same as that which could be produced by *not more than two* single external forces. When the points of application of these two resultant forces coincide, then the actual motion produced is the same as that which would be produced by a *single* force acting at that point.

Impulse of a Force. — The impulse of a force is defined as the time integral of the force, i.e., if a force f is applied for a time t, then the impulse is $\int_0^t f\,dt$, account being taken of both the variation of the magnitude and direction of the force during the given time interval. The total change in the linear momentum of a body in any time t is equal to the impulse of the resultant force acting on it during this time. The conception of the impulse of a force is useful in dealing with suddenly applied forces which continue but a short time.

Concurrent and Coplanar Forces. — When the lines of action of all the external forces acting on a body meet in a point these forces are said to form a concurrent system of forces. When all the external forces acting on a body lie in the same plane they are said to form a coplanar system of forces.

Pressure (p). — The perpendicular component of the force per unit area exerted on any surface is called the pressure on that surface. Let df be the perpendicular or normal component of the force acting on an area dA, then the pressure at dA is

$$p = \frac{df}{dA}. \qquad (10)$$

The term pressure is sometimes used as a synonym for force and what is here defined as pressure is called the "intensity of pressure." The rational unit of pressure is force per unit area, e.g., dynes per sq. cm. or pounds per sq. in., but a number of other arbitrary units are employed, such as, one inch of water column, one millimeter of mercury column, one atmosphere, etc. A dyne per square inch is also a barie. A standard atmosphere is the pressure that will support a column of mercury 76 centimeters = 29.9212 inches high at 0° C. at a place where $g = 980.665$ centimeters per second per second; using Thiesen and Scheel's determination of the density of mercury at 0° C. as 13.59545 grams per cubic centimeter (*Zeitsch. f. Instrkde., 1898, Vol. 18, p. 138*) a standard atmosphere is equivalent to 14.6964 pounds per square inch. See *Units and Conversion Factors*.

Absolute Pressure. — The atmosphere exerts a pressure (ranging from about 29 to 30 inches of mercury, depending upon the weather conditions and elevation) upon every surface with which it is in contact. The total pressure on a surface including that applied by any artificial means and that of the atmosphere is called the absolute pressure on the surface.

Torque or Moment of a Force (T). — Consider any axis X and any force acting at a point P at a perpendicular distance r from this axis; let f be the *component* of this force perpendicular to the plane through X and P. Then the product

$$T = rf \qquad (11)$$

is called the moment of the force, or the torque due to this force, about the axis X. The distance r is called the "lever arm" or simply the "arm" of the force about this axis. Both absolute and gravitational units of torque are

used, the units being of the same nature or dimensions as the units of energy (see below), but the two words in the compound names of the energy units are usually reversed. For example the foot-pound is an energy unit but the corresponding unit of torque is called the pound-foot. The conversion factors, however, are identical; see *Units and Conversion Factors*. Torque is also frequently expressed as "torque at unit radius," e.g., a torque of so-many pounds at 1 foot radius is a common expression; a more exact expression for the measure of torque would be "force at unit radius."

Torques about the same axis can be added algebraically, and torques about several axes meeting in a point can be added vectorially.

Couples. — Two equal and opposite parallel forces which are not concurrent (i.e., do not act along the same line) are said to form a couple, and the strength of the couple is defined as the product of either force by the perpendicular distance between their lines of action. The torque produced by a couple about any axis perpendicular to the plane of the two forces is equal to the strength of the couple.

Torque and Angular Acceleration. — The relation between torque, moment of inertia and angular acceleration is the same as that between force, mass and linear acceleration, i.e.,

$$T = I\alpha, \qquad \text{absolute units,} \tag{12}$$

where T is the torque in absolute units, I the moment of inertia and α the angular acceleration, all about the same axis. When the torque is expressed in gravitational units the relation is

$$T = \frac{I}{g_0}\alpha, \qquad \text{gravitational units,} \tag{12a}$$

where g_0 is a number numerically equal to 32.1739 when T is expressed in pound-feet, I in (foot)^2-pounds and α in radians per second per second; similarly for the centimeter-gram-second units $g_0 = 980.665$.

Work and Energy. — When the point at which a force is applied to a body A moves* with respect to the agent producing the force, in such a manner that the force has a component in the direction of the displacement of its application point, the force is said to do work; the body B exerting the force on A is also said to do mechanical work on the body A. As a measure of the mechanical work dW done by the force f, when its point of application moves a distance dl with respect to the body producing the force, is taken the product of the displacement dl by the component of the force f in the direction of this displacement, i.e.,

$$dW = (f \cos\theta)\, dl, \tag{13}$$

where θ is the angle between the direction of the force and the direction of the displacement. For a finite displacement the mechanical work done is

$$W = \int_0^l f\, (\cos\theta)\, dl. \tag{13a}$$

When mechanical work is done on a body a change in the state of motion or in some other condition of the body is always produced. Like changes can also be produced by other means. For example, when two bodies are rubbed together rapidly, mechanical work is done on them by the agent which produces the force which moves one over the other against the opposing force due to friction. As a result, the temperature of the bodies is raised. The

* Either as a result of the motion of the body A as a whole or in consequence of the motion of that part of A to which the force is applied.

temperature of the two bodies may also be raised by placing them near or in contact with a hotter body, without there being any appreciable amount of *mechanical* work done. In general, any change produced in a body or system of bodies by any means whatever, which change can also be produced directly by doing mechanical work on that body or system, or indirectly by doing mechanical work on some other body or system, is said to be due to a transfer of "energy" *to* that body or system; or work * is said to be done on that body or system, irrespective of the means whereby the change is produced.

As a measure of the gain in energy corresponding to any change in a body or system of bodies is taken the amount of mechanical work which would have to be done by a mechanical force to produce this change and no other. For example, water can be heated by stirring it rapidly with a paddle driven by some external force; the total mechanical work done by this force can be expressed in terms of the value of the force, the number of revolutions of the paddle and the diameter of the pulley attached to the paddle, and the mass of the water and the resultant temperature rise can be measured. By making proper corrections for the work done against the various frictional forces other than the opposing force due to the stirring of the water itself and for the loss of energy by radiation, the amount of work required to raise the temperature of a pound of water, say, one degree Fahrenheit can be determined. By this means a very convenient secondary unit of energy can be expressed in terms of the unit of mechanical work, and the secondary unit can be used for expressing the amount of energy involved in various heat effects. See *Heat and Thermal Properties*.

Whenever a change takes place in a body or system of bodies which is the *reverse* of the change which can be produced in it by doing work *on* it, the body or system is said to *lose* energy, or energy is said to be transferred *from* it, or it is said *to do* work. As a measure of the amount of energy lost by the body or system when a given change takes place in it is taken the amount of work which would have to be done on it to restore it to its original condition.

Units of Work and Energy. — The fundamental unit of mechanical work in the c.g.s. system is the work done by a force of 1 dyne when its point of application is displaced (with respect to the agent producing the force) a distance of 1 centimeter in the direction of the force; this unit is called the "erg." The "joule" is equal to 10^7 ergs, by definition. The fundamental unit of mechanical work in the English gravitational system is the foot-pound, which is the work done by a force of 1 pound when its point of application is displaced a distance of 1 foot in the direction of the force; or 1 foot-pound is the work required to raise a mass of 1 pound a distance of 1 foot at a place where the acceleration due to gravity is 980.665 cm. per sec. per sec. or 32.1739 ft. per sec. per sec. Energy is expressed in the same units as mechanical work and in addition various heat units, such as the British thermal unit, the large and small calories, etc., are used; see *Heat and Thermal Properties*. For the relations among the various units see *Units and Conversion Factors*.

Principle of the Conservation of Energy. — Experience indicates that the amount of energy, as above defined, which can be transferred from a body or system of bodies to which no energy is added, is limited; i.e., the energy "possessed by" or "associated with" any body or system of bodies is *finite* in amount. As a rule, only a relatively small portion of the energy

* The word "work" is used by some writers to signify mechanical work only, but the term is a very convenient one to use in referring to the transfer of energy by other means as well, i.e., any change resulting in a transfer of energy to a body may be said to result from the doing of "work" on the body, whether the change is produced by mechanical, electrical or other means.

associated with a body or system can be transferred to another body or system. Experience also justifies the assumption that whenever one body or system of bodies *gains* energy, some other body or system *loses* an *exactly equal amount* of energy. In every instance where this assumption can be tested directly it is found to hold, and every deduction from it has been found to be in accord with experimental fact. Hence this assumption is accepted as a fundamental principle of nature.

Kinetic Energy. — Work is required to set a body in motion, for while its motion is changing it is accelerating and therefore a force must be exerted upon it. From the definitions of acceleration, force and work given above it follows immediately that the work required to change the *linear* velocity of a body from v_0 to v_1 is

$$W_t = \frac{1}{2} m (v_1^2 - v_0^2) \qquad \text{absolute units,}$$

or

$$W_t = \frac{1}{2} \frac{m}{g_0} (v_1^2 - v_0^2) \qquad \text{gravitational units,}$$

where m is the mass of the body and g_0 is the gravitational constant. Similarly, the work required to change the angular velocity of a rigid body about a given axis from ω_0 to ω_1 is

$$W_r = \frac{1}{2} I (\omega_1^2 - \omega_0^2) \qquad \text{absolute units,}$$

or

$$W_r = \frac{1}{2} \frac{I}{g_0} (\omega_1^2 - \omega_0^2) \qquad \text{gravitational units,}$$

where I is the moment of inertia of the body about this axis and g_0 the gravitational constant. The expression

$$\frac{1}{2} mv^2 \qquad \text{or} \qquad \frac{1}{2} \frac{m}{g_0} v^2 \tag{14}$$

is called the "kinetic energy of translation" of the body, and the expression

$$\frac{1}{2} I\omega^2 \qquad \text{or} \qquad \frac{1}{2} \frac{I}{g_0} \omega^2 \tag{14a}$$

is called the "kinetic energy of rotation" of the body, both referred to the point with respect to which the velocities are measured.

Total Kinetic Energy of a System of Bodies. — The total kinetic energy of a moving system of bodies is equal to (1) the kinetic energy of the entire system moving with a velocity equal to the velocity of the center of mass of the system plus (2) the sum of the kinetic energies, rotational and translational, of each constituent body of the system due to the relative motion of this constituent body with respect to the center of mass of the whole system. For example, the total kinetic energy of a railway car is

$$W = \frac{1}{2} Mv^2 + \frac{1}{2} \Sigma I\omega^2 \qquad \text{absolute units,} \tag{14b}$$

$$W = \frac{1}{2} \frac{M}{g_0} v^2 + \frac{1}{2 g_0} \Sigma I\omega^2 \qquad \text{gravitational units,} \tag{14c}$$

where M is the entire mass of the car with full equipment, I the moment of inertia, ω the angular velocity of any rotating part and the summation sign Σ indicates the sum of the products $I\omega^2$ for all the rotating parts. See also *Railways, Energy Requirements for*.

Potential Energy. — The energy possessed by a body in virtue of its position with respect to the earth is usually called potential energy. More generally the term potential energy is used to designate any form of energy other than kinetic energy. In absolute units the increase in the potential energy of a body when it is raised a vertical distance h is mgh; in gravitational units the increase in potential energy is mh.

Power. — By power is meant the time rate of doing work or the time rate of change of energy. Let dW be the work done in time dt, then the power is

$$P = \frac{dW}{dt}. \qquad (15)$$

The power produced by a force f or a torque T can also be expressed as

$$P = fv \qquad \text{or} \qquad P = T\omega, \qquad (15a)$$

where v is the linear velocity of the point of application of the force and ω is the angular velocity of the point of application of the torque. The rational unit of power is the unit of work done or of energy transferred per unit time, such as 1 erg per second, 1 joule per second, 1 foot-pound per second, 1 British thermal unit per second, and the like. 1 joule per second is called the watt, which is also equal to the power corresponding to 1 ampere and a potential difference of 1 volt (*see Units, Practical Electrical*). The horse-power (English and American) is defined* as the power corresponding to 550 foot-pounds per second or 33,000 foot-pounds per minute. The metric horse-power, also called cheval-vapeur, force de cheval, Pferde-kraft, is defined as 75 kilogram-meters per second. The boiler horse-power is defined in the article on *Boilers, Steam*. The interrelations of the various units of power are given in the article on *Units and Conversion Factors*.

CONDITIONS FOR EQUILIBRIUM. — A body or system of bodies is said to be in equilibrium with respect to any external body (e.g., the earth) when (1) there is no change in the motion of the center of mass of the system with respect to this external body and (2) when there is no change in the total angular momentum of the body or system about any axis fixed with respect to this external body. These two conditions require (1) that the resultant of all the external forces† acting on the body or system be zero and (2) that the resultant of all the moments of these external forces, or torques, about any axis acting on the body or system be zero. These two conditions are most conveniently expressed by choosing three mutually perpendicular axes fixed with reference to the body of reference (e.g., the earth) and resolving all the forces into components F_x, F_y and F_z parallel to these three axes and calculating the moments or torques T_x, T_y and T_z of each of these forces about these three axes; then, using the symbol Σ to indicate the algebraic summation of the individual x, y or z components,

$$\left.\begin{array}{ll} \Sigma F_x = 0 & \Sigma T_x = 0 \\ \Sigma F_y = 0 & \Sigma T_y = 0 \\ \Sigma F_z = 0 & \Sigma T_z = 0 \end{array}\right\} \qquad (16)$$

* The Bureau of Standards recommends the adoption of 746 watts as the definition of a horse-power. The older definition given above makes a horse-power 745.701 watts, using the standard value of g_0 given above, namely, 980.665 cm. per sec. per sec. and the legal values of the foot and the pound (*see Units and Conversion Factors*). As this older definition is the one generally employed by all classes of engineers, it seems preferable to the author to retain it. See *Circular No. 34 of the Bureau of Standards* and discussions in the various technical journals during 1912 and 1913.

† i.e., forces any one of which would, if acting alone, produce a motion of the body or system or of some part of it with respect to the reference body.

These two sets of conditions constitute the basis of the entire subject of statics. Certain elementary applications are given in the article on *Structures, Simple.*

Stability. — When a body or system of bodies is in equilibrium and the state is such that when the body or system is displaced slightly it returns of itself to its original condition, the equilibrium is said to be stable; if when displaced slightly the body or system moves farther from its original condition, the equilibrium is said to be unstable; if when displaced slightly it remains in that condition, the equilibrium is said to be stable. The condition for stability may be expressed in a number of ways, viz.: (1) when the potential energy of the system with respect to the body of reference (e.g., the earth) is a minimum the equilibrium is stable; (2) when the forces acting are all gravitational the equilibrium is stable when the center of mass is in the lowest possible position; or (3) when the body or system rests on a number of points, the equilibrium is stable only when the resultant of all the forces acting on the body including its own weight, but excluding the supporting forces, cuts the smallest polygon which can be drawn including all the points of support.

MOTION OF A PARTICLE. — By a particle of matter is meant a portion of matter so small that it may be considered as occupying but a point in space. Let m be the mass of the particle, v its linear velocity at any instant with respect to any arbitrarily chosen set of axes of reference, dv the increase in its velocity in time dt (including both change in direction as well as in magnitude), and let F be the resultant of all the external forces, in absolute units,* acting on the particle. Then the motion of the particle is completely specified by the equation

$$F = m\frac{dv}{dt},$$

where dv is taken to include the change in *direction* as well as the magnitude of the velocity. To take into account the variation of the direction of v as well as the variation in its magnitude, the resultant force F and the velocities may be resolved into components along the three axes, in which case this equation breaks up into the three equations

$$F_x = m\frac{dv_x}{dt}, \qquad F_y = m\frac{dv_y}{dt}, \qquad F_z = m\frac{dv_z}{dt}.$$

Since the velocity along any axis is equal to the time rate of displacement along that axis, $v_x = \frac{dx}{dt}$, $v_y = \frac{dy}{dt}$, and $v_z = \frac{dz}{dt}$, whence these equations may also be written

$$\frac{d^2x}{dt^2} = \frac{F_x}{m}, \qquad \frac{d^2y}{dt^2} = \frac{F_y}{m}, \qquad \frac{d^2z}{dt^2} = \frac{F_z}{m}. \qquad (17)$$

Hence when the forces can be expressed in terms of the coördinates of the point which the particle occupies at each instant, the displacements and velocities can be determined by solving these equations. Note that $\frac{F_x}{m}$, $\frac{F_y}{m}$ and $\frac{F_z}{m}$ are the accelerations a_x, a_y and a_z along the three axes.

Rectilinear Motion. — The simplest case is that of a particle acted upon by a force F which remains constant in direction. One of the axes, say the

* When F is expressed in gravitational units replace m by $\frac{m}{g_0}$, where g_0 is the gravitational constant.

X axis, may be taken parallel to the line of action of the force, in which case equations (17) reduce to the single equation

$$\frac{d^2x}{dt^2} = \frac{F}{m}, \tag{17a}$$

and there is no acceleration along either of the other axes; i.e., the particle moves along the line of action of the force. If the force is also constant in magnitude, $\frac{F}{m} = a$, a constant. In this case the solution of (17a) is (*see Equations*):

$$\left.\begin{aligned} x - x_0 &= \frac{1}{2} a (t - t_0)^2 + v_0 (t - t_0), \\ v - v_0 &= a (t - t_0), \end{aligned}\right\} \tag{17b}$$

where $t - t_0$ represents any interval of time, $x - x_0$ the displacement of the particle during this interval and $v - v_0$ the change in velocity of the particle during this interval. The displacement $x - x_0$ may also be written

$$x - x_0 = \frac{v^2 - v_0^2}{2a}. \tag{17c}$$

Curvilinear Motion; Motion in a Circle. — Consider the special case of a particle moving in a circle of radius r with a constant tangential or peripheral velocity v. Choose the Z axis perpendicular to the plane of the circle. Let θ be the angle (measured counter-clockwise) which a line drawn from the center of the circle to the particle makes at any instant with the X axis, then $(90 - \theta)$ is the angle which this line makes with the Y axis at this instant. At any instant, then,

$$v_x = -v \sin\theta \qquad \text{and} \qquad v_y = v \cos\theta$$

and these components are not constant since θ changes with time. The corresponding accelerations along the two axes are

$$a_x = -v \cos\theta \frac{d\theta}{dt} \qquad \text{and} \qquad a_y = -v \sin\theta \frac{d\theta}{dt}.$$

both of which components are *toward* the center of the circle. But $\frac{d\theta}{dt}$ is the angular velocity of the particle about the Z axis, and this is equal to the peripheral velocity divided by the radius, or $\frac{d\theta}{dt} = \frac{v}{r}$. Whence

$$a_x = -\frac{v^2}{r} \cos\theta \qquad \text{and} \qquad a_y = -\frac{v^2}{r} \sin\theta,$$

and therefore the resultant acceleration has the numerical value

$$a = \sqrt{a_x^2 + a_y^2} = \frac{v^2}{r} \tag{17d}$$

and is *toward* the center of the circle. Hence to cause a particle to rotate in a circle a force equal to $\frac{mv^2}{r}$ must be applied to it in the direction *toward* the center, and the particle in turn pulls away from the center with an equal and opposite force which produces a tension outward along the radius in whatever (e.g., a string) holds the particle to the center. The tendency of the particle to pull outward from the center is called the centrifugal force, and its value

is $\frac{mv^2}{r}$; the equal inward pull required to make the particle move in the circular orbit is called the centripetal force.

Simple Harmonic Motion. — (*See also Wave Analysis.*) By simple harmonic motion is meant a motion such that the acceleration of the moving particle or point at each instant is proportional to, but in the opposite direction from, the displacement. Such motion may be either rectilinear or a rotation about a fixed axis. Rectilinear simple harmonic motion may also be defined, see Fig. 1, as the motion of the projection P on a given diameter YY', of a point Q which moves with a constant angular velocity ω around a circle of radius A. The motion of the bob of a simple pendulum, the position of the end of a piston rod, etc., are examples of simple rectilinear harmonic motion (very nearly).

Referring to the circle diagram in Fig. 1, P_0 represents the projection of the moving point Q at time $t = 0$. The angle made by OQ at any instant with

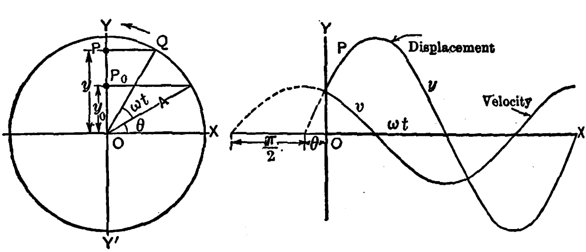

Fig. 1. Harmonic Motion

OX is then $(\omega t + \theta)$. The linear velocity of Q is $A\omega$, and the *component* of this velocity at any instant *along* OY, i.e., the velocity of P, is

$$v = \frac{dy}{dt} = A\omega \cos(\omega t + \theta). \tag{18}$$

Whence the displacement y of P from O is, by integration,

$$y = A \sin(\omega t + \theta), \tag{18a}$$

and the acceleration along OY is, by differentiation,

$$a = \frac{d^2y}{dt^2} = -\omega^2 A \sin(\omega t + \theta), \tag{18b}$$

whence $a = -\omega^2 y$, which agrees with the first definition of harmonic motion.

Period and Frequency. — The period of any kind of an oscillation is defined as the time (usually in seconds) taken for the oscillating point to pass through a complete cycle of values back again to the starting point. The frequency is the number of complete cycles per unit time (usually per second). In the case of a harmonic oscillation the period is

$$T = \frac{2\pi}{\omega} \tag{18c}$$

and the frequency is

$$f = \frac{\omega}{2\pi}. \tag{18d}$$

Amplitude.—The "sine curves" to the right of the circle diagram show the variation with time of the displacement and the velocity (along OY) of the projected point P. Displacement and velocity are plotted along the Y axis and ωt, which is proportional to time, is plotted along the X axis. These curves bear a definite relation to each other and to the "origin" of time, i.e., to the point O. The maximum ordinate of each curve is called the "amplitude" of the curve. The amplitude of the displacement curve is A, the radius of the circle; the amplitude of the velocity curve is ωA.

Phase.—The distance expressed in angular measure, from the origin to the point at which the curve first crosses the X axis in the *rising* direction is called the "phase angle" of the curve. The phase angle is taken positive when it is measured to the *left* from the origin, and negative when measured to the right.* The phase angle of the displacement curve is θ, and the phase angle of the velocity curve in the above case is $\frac{\pi}{2}+\theta$.

Difference in Phase.—The difference in the phase angles of two sine curves of the same frequency is called the "difference in phase" between the two curves; the difference in phase between the displacement curve and the velocity curve in the above case is $\frac{\pi}{2}$ radians or 90 degrees. The curve which has the larger (algebraically) phase angle is said to "lead" the other curve, and the one with smaller (algebraically) phase angle is said to "lag behind" the other one. In the above example the velocity curve leads the displacement curve. Note that the curve to the left is the leading curve.

ROTATION OF A SOLID BODY.—When a solid body moves in such a manner that one straight line through the body remains fixed with respect to any given set of axes of reference, its motion is called simple rotation with respect to these axes. For such motion the external forces must be equivalent to two equal and opposite forces which have a moment only about the axis of rotation. Let T be the value of this moment or torque, I the moment of inertia of the body about this axis and $d\omega$ the increase in the angular velocity about this axis in time dt (the same for each point of the body), then the motion is completely specified by the equation

$$\left.\begin{aligned} &T = I\frac{d\omega}{dt} && \text{(for } T \text{ in absolute units)} \\ \text{or}\quad &T = \frac{I}{g_0}\frac{d\omega}{dt} && \text{(for } T \text{ in gravitational units).} \end{aligned}\right\} \tag{19}$$

For constant torque and therefore constant angular acceleration, i.e., for $\frac{d\omega}{dt}=\alpha=$ constant, the change in angular velocity in the interval of time (t_2-t_1) is

$$\omega_2-\omega_1=\alpha\,(t_2-t_1), \tag{19a}$$

and the angle turned through in the interval (t_2-t_1) is

$$\theta_2-\theta_1=\frac{1}{2}\,(\omega_2+\omega_1)\,(t_2-t_1)=\frac{\omega_2^2-\omega_1^2}{2\,\alpha}. \tag{19b}$$

THREE-DIMENSIONAL MOTION.—The motion of a solid body may in the most general case be expressed in terms of a translation of its center and a rotation of the body around an axis through its center of mass. The motion

* The opposite convention is used by some writers, in which case the equation for the sine curve is $y=A\sin(\omega t-\theta)$.

of the center of mass produced by any number of external forces is the same as would be produced by a single force, equal to the vector sum of the actual forces, acting on a single particle occupying the position of the center of mass in the actual body and having a mass equal to the entire mass of this body. The rotation can be determined by considering the center of mass as fixed and the actual forces then applied. That is, the translation of the center of mass and the rotation about the axis through it can be considered independently of each other; the actual motion is then the resultant of these two types of motion. See *American Civil Engineers' Pocket Book* for a brief treatment of this subject.

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[H. PENDER.]

MENSURATION. — (*See also Angles; Equations; Trigonometry.*) The term mensuration is used in this article to include the relations between the areas and volumes of geometric figures and their linear dimensions.

Triangle. —

$$\text{Area} = \tfrac{1}{2}\,(\text{base}) \times (\text{perpendicular height})$$
$$= \sqrt{s\,(s-a)\,(s-b)\,(s-c)},$$

where a, b and c are the lengths of the three sides respectively, and $s = \tfrac{1}{2}(a+b+c)$.

Trapezoid. —

$$\text{Area} = \left(\frac{a+b}{2}\right) d,$$

where a and b are the lengths of the parallel sides respectively, and d their distance apart.

Parallelogram. —

$$\text{Area} = (\text{base}) \times (\text{perpendicular height}).$$

Parabola. —

$$\text{Area} = \tfrac{2}{3}\,(\text{area of circumscribing triangle}).$$

Cycloid. —

$$\text{Area} = \tfrac{3}{4}\pi \times (\text{altitude})^2,$$

the altitude being the diameter of the rolling circle.

Circle. —

$$\text{Circumference} = 2\pi r = \pi d,$$

$$\text{Area} = \pi r^2 = \frac{\pi}{4} d^2,$$

where r is the radius and d the diameter.

$$\text{Area of segment} = \frac{r^2}{2}\,(\theta - \sin\theta),$$

where θ is the angle in radians (*see Angles*) subtended by the arc of the segment. If n is the height of the segment, measured along the radius perpendicular to the chord,

$$\text{Area of segment} = \pi r^2 M - A\,(r-n),$$

where

$$A = \sqrt{n\,(2r-n)} \quad \text{and} \quad M = \frac{1}{180} \sin^{-1}\left(\frac{A}{r}\right).$$

Ellipse. —

$$\text{Area} = \pi ab,$$

where a and b are the principal semi-axes.

Prism with Parallel Sides and Parallel Ends. —

$$\text{Volume} = (\text{area of end}) \times (\text{Perpendicular Distance between Ends}).$$

Right Circular Cylinder. —

$$\text{Volume} = \frac{\pi}{4} d^2 l,$$

where d is the diameter, and l the length.

$$\text{Total surface of right cylinder} = \pi d\,(l + \tfrac{1}{2} d).$$

Right Circular Cone. —

$$\text{Volume} = \tfrac{1}{3}\,(\text{area of base}) \times (\text{height}),$$
$$= \tfrac{1}{3}\,(\text{volume of circumscribing cylinder}),$$

where r is the radius of base and h the height of the cone.

$$\text{Area of curved surface of a right circular cone} = \pi r\sqrt{h^2 + r^2}.$$

Right Pyramid. —

$$\text{Volume} = \tfrac{1}{3}\,(\text{area of base}) \times (\text{height}),$$
$$\text{Volume of frustum of pyramid} = \tfrac{1}{3}\,(\text{height})\,(A + a + \sqrt{aA}),$$

where A and a are the areas of the ends respectively.

Sphere. —

$$r = \text{radius},$$
$$\text{Area of surface} = 4\pi r^2$$
$$= \tfrac{2}{3}\,(\text{total area of circumscribing cylinder}).$$

Area of the surface of a zone of a sphere = area of zone of the same height as this zone projected on to a cylinder.

$$\text{Volume} = \tfrac{4}{3}\pi r^3$$
$$= \tfrac{2}{3}\,(\text{volume of circumscribing cylinder}).$$

Volume of a frustum of a sphere $= \pi r^2 (k \mp h) - \frac{\pi}{3}(k^3 \mp h^3)$, where k is the distance of its outer face from center and h the distance of its inner face from the center, the negative signs in the brackets to be used if both faces are on the same side of the center and the positive signs if on opposite sides of the center.

Ellipsoid. —

$$\text{Volume} = \tfrac{4}{3}\pi abc,$$

where a, b and c are the three principal semi-axes respectively.

Paraboloid. —

Volume of a paraboloid of revolution equals one half that of the circumscribing cylinder.

[W. A. Del Mar.]

MOTOR-CONVERTERS. — (*See also Converters, Synchronous; Motors, Induction; Standardization Rules.*) A motor-converter is a combination of an induction motor and synchronous converter, with the secondary of the motor and the armature of the converter mounted upon the same shaft and connected together electrically without slip rings. The induction motor receives all the a-c. power, transforms a part of it into mechanical power delivered to the shaft, and also acts as a transformer delivering the rest of the power in electrical form at a lower frequency from its secondary to the armature of the converter. It operates like two induction motors in "concatenation" or "cascade." The object is to obtain the steadiness of a 30-cycle converter on a 60-cycle circuit.

The speed of a motor converter depends upon the supply frequency and varies inversely as the sum of the number of poles in both machines. Thus if a combination of a 6-pole motor and 6-pole converter be operated from a 60-cycle circuit the speed of the armature will be 600 r.p.m. The primary of the induction motor will operate at 60 cycles but the secondary will supply the armature of the converter with 30 cycles. Thus the converter may be built with the good design constants of a 30-cycle machine.

This combination is larger than a converter but smaller than a motor-generator set and its efficiency is lower than that of a converter. It is used somewhat in Europe but not much in the United States.

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[W. I. Slichter.]

MOTOR-GENERATORS. — (*See also Converters, Synchronous; Generators; Motors; Standardization Rules.*) A motor-generator set is a combination of a motor and a generator having separate fields and armatures but mounted on the same shaft with common base and bearings. Combinations of various types of motors and generators are used; some of the more important combinations and their applications are described below.

Direct-current Motor Driving Direct-current Generator. — These sets are used when it is desired to convert low-voltage direct current into high voltage direct current, or vice versa; they are used in preference to a dynamotor (q.v.) when good regulation is desired in the secondary circuit.

Direct-current Motor Driving Alternating-current Generator. — These sets are used for converting direct into alternating current. See also the section on *Inverted Synchronous Converters* in the article on *Converters, Synchronous.*

Induction Motor Driving Direct-current Generator. — The induction motor may be wound for potentials as high as 13,000 volts and the transformation from alternating current at this voltage to direct current may be made without the use of transformers. Since the induction motor has a decreasing speed with increasing load the direct-current generator must be compounded to give good regulation; with proper compounding excellent regulation may be obtained. The induction-motor-generator set is sometimes used in preference to a synchronous converter when the service requires specially good regulation. However, the efficiency of such a set (about 85 per cent at rated load) is less than that of a synchronous converter, even if no transformers are required by the motor-generator set. The motor-generator set also occupies from 50 per cent to 80 per cent more floor space, weighs from 30 per cent to 50 per cent more, and costs from 25 per cent to 50 per cent more than a synchronous converter, in spite of the fact that they are designed to operate at the highest practicable speeds.

Costs and Speeds of Induction-motor-generator Sets. — Very roughly the prices and suitable speeds of induction-motor-generator sets are as follows:

Kw. capacity	R.p.m.	Cost per kw.
200	750	$23
500	500	16
1000	300	12.5

Synchronous Motor Driving Direct-current Generator. — This combination is preferable in many instances to the preceding, because it operates at constant speed and because the field of the synchronous motor may be adjusted to make use of line compounding or to compensate for low power factor in other apparatus on the circuit (*see Motors, Synchronous*). Provision must be made for the direct-current excitation for the synchronous motor. If the direct-current generator is wound for too high a potential a special exciter must be provided. Since there is no load on the set at starting the synchronous motor may be started in the usual manner.

Induction Motor Driving Alternating-current Generator. — An induction motor direct-connected to an alternating-current generator is frequently used to supply alternating current for special purposes at a frequency different from that of the main power station. Thus in power stations such a motor generator

set would be used to transform the 25-cycle power of the main generators to 60-cycle power for lighting circuits. This combination has the disadvantage of decreasing speed, and therefore decreasing frequency, with increasing load. In order that two such sets shall operate in parallel and divide the load properly the resistance of the secondary of both motors must be carefully chosen so that the motors have the same slip at full load. Other types of frequency changers are described below.

Synchronous Motor Driving Synchronous Generator; Frequency Changers.—The constant speed characteristic of a synchronous motor makes it particularly suited as the driving motor of a frequency changer. The generator of the set may be either an ordinary (i.e., synchronous) alternating-current generator or a reversed induction motor; the latter type is described in the next section.

In order that a synchronous-motor-synchronous-generator set may operate properly in parallel with other synchronous apparatus on the two systems to which the motor and generator are respectively connected, the number of poles of both motor and generator and the speed must be carefully chosen. Thus, to change from 25 to 60 cycles per second, the highest speed which allows an exact transformation of frequency is 300 rev. per min., and this requires 10 poles on the 25-cycle machine and 24 poles on the 60-cycle machine. In order to build less expensive sets, it is frequently the custom to use a speed of 750 rev. per min. with a 4-pole machine on the 25-cycle circuit and a 10-pole machine which will give 62.5 cycles for the other circuit. All the other machines on the latter circuit must then also operate at 62.5 cycles. Other combinations of poles and speeds for various combinations of frequency can be readily calculated by means of the usual relation between poles, speed and frequency; see *Generators, Alternating-Current*. Each motor-generator set must be provided with direct current for its field excitation, either from a direct-connected exciter or from a special exciter circuit.

Division of Load on Frequency Changers.—The parallel operation of sets of this character involves certain complicated considerations. In the first place the division of the load depends not only upon the voltage regulation of the two machines constituting a set, but upon the mechanical position of the armature on the shaft. Two machines built apparently the same may not divide the load equally because of inaccuracy in the placing of the keyway, etc. To avoid this trouble it is customary to mount one of the stationary members of one of the machines movable in a cradle so that the angular phase position of this member (usually the armature) with respect to the rotating member (usually the field) may be adjusted. The two sets to be operated in parallel are loaded, and the stationary armature of one machine is rotated in the cradle until the load between the two machines divides properly. This adjustment may also be used for testing, when it is desired to load the two machines one on the other without an external load by adjusting this "phase-angle" so that one machine operates as a generator when the other operates as a motor.

Synchronizing Frequency Changers.—The synchronizing of such machines is difficult, and several very complicated conditions must be satisfied. The theory is quite involved and is discussed at length in a paper by J. B. Taylor on *Parallel Operation*, *Trans.* A.I.E.E., Vol. 25, p. 113. The essential requirement is that it is necessary to synchronize not only the motor with its supply circuit but the generator with the secondary circuit. Thus when the motor has been synchronized properly it may be found that the generator instead of being in phase is 180 degrees out of phase with the secondary circuit. To bring the generator into phase it is necessary to cause it to "slip" one or

more poles; this can be done by reversing the field of the motor one or more times, depending upon the number of poles on the generator and motor. The necessity of more than one reversal is due to the fact that with a large number of poles on both motor and generator there are only a few combinations in which the poles of the motor and the generator of one set match up with the poles of motor and generator of another set.

In practice this difficulty is overcome by the use of a special synchronism indicator (*see Synchronizers*) having two hands appearing on the same dial. One hand shows the phase relation of each member of the set with its proper circuit. It is necessary to synchronize the set when both hands are not only stationary but point to the zero position. If only the motor were synchronized it would be found that both hands were stationary, but the motor hand would point to zero and the generator hand to some other position.

Induction-type Frequency Changers. — When the stator of an induction motor is excited from a supply circuit having a frequency of f_1 cycles per second and the rotor of this motor is driven by another motor in the *opposite* direction to that in which it would rotate due to the currents in its stator winding, the frequency of the current induced in the rotor winding is

$$f_2 = \frac{N_0 + N}{N_0} \cdot f_1,$$

where N_0 = synchronous speed of the motor and N = actual speed at which its rotor is driven. This combination of two motors (the driven motor really acts also as a generator) may therefore be used as a frequency changer.

Neglecting the losses in the driven motor, the electrical input into its stator is

$$\frac{f_1}{f_2} \times (\text{output of set at frequency } f_2)$$

and the mechanical output of the driving motor is

$$\frac{f_2 - f_1}{f_2} \times (\text{output of set at frequency } f_2).$$

While this combination is less expensive than the usual motor-generator set, it has the disadvantage of poor regulation, as every change in the potential of the supply circuit is transmitted to the receiving circuit; it is therefore but seldom used.

SPECIFICATION FOR MOTOR-GENERATORS.* — The following memoranda are intended to assist in writing specifications. See also articles on *Specifications; Motors;* and *Generators.*

General description and use of machine. Motor: see specifications in articles on *Motors.* Generator: see specifications in articles on *Generators.* Whether or not motor and generator are to be on common bed-plate. Exciter for generator field. Limiting over-all dimensions.

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[W. I. Slichter.

* By W. A. Del Mar.

MOTORS, ALTERNATING-CURRENT COMMUTATOR. — (*See also Motors, Direct-current; Motors, Polyphase-induction; Motors, Single-phase Induction; Standardization Rules.*) There are several different types of alternating-current commutator motors designed to operate on single-phase circuits, but they differ chiefly in the electrical connections employed. They may be divided into two general classes, series motors and repulsion motors. While these motors differ in their connections and in slight details in their characteristics, they all have the general characteristics of the d-c. series motor, that is, increasing torque with decreasing speed and a high efficiency over a considerable range of speed. Alternating-current commutator motors with shunt motor characteristics are also used to a limited extent abroad.

GENERAL CHARACTERISTICS. — The torque of any a-c. commutator motor is constant in direction, but pulsating in value, and its average value

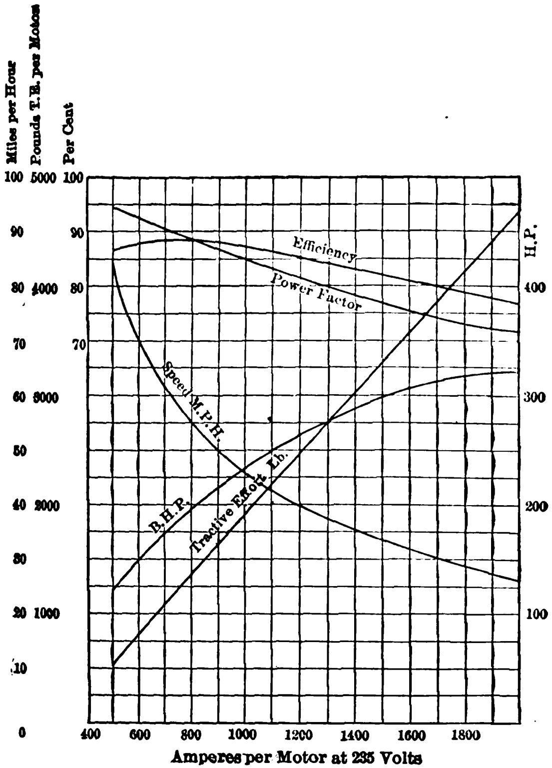

Fig. 1. Characteristic Curves of 1 Motor of the Single-phase Locomotives of the N. Y., N. H. & H. R. R. (St. Ry. Jour., Apr. 1906.)

is proportional to the product of the effective value of the flux and the effective value of the armature current. The direction of the torque may be changed

by changing the direction of the current in the field with respect to the armature, or vice versa. The power factor increases with increase of the speed and therefore decreases with increase of load. The efficiency, while not as good as that of a d-c. motor of the same rating is, however, fairly high. The motors have in addition to the losses common to d-c. motors a core-loss in the field, increased core-loss in the armature, increased commutation loss and increased RI^2 loss in special windings. In Fig. 1 are given the characteristic curves of the a-c. compensated series motors used on the single-phase locomotives of the New York, New Haven & Hartford Railroad.

APPLICATIONS. — The most general application of a-c. commutator motors in large sizes is in railway and hoisting work; see *Railways, Electric*, and *Hoists, Electric*. The same principles of operation are made use of in the devices for starting single-phase induction motors (*see Motors, Single-phase Induction*), the motor being brought up to speed as an a-c. commutator motor and then by a change of connections made to operate as a single-phase induction motor.

DESIGN. — The salient features in the design of the various types of a-c. commutator motors are described briefly below.

Straight A-C. Series Motor. — Since the torque of an ordinary d-c. series motor does not change in direction when the current through both the field and the armature reverses simultaneously, any d-c. series motor will develop a uni-directional torque when connected across a-c. mains. However, when an ordinary d-c. series motor is thus used the power factor of the load taken by it is very low, there is a large eddy-current loss in the field structure, and violent sparking occurs at the commutator. To make a series motor practicable for a-c. service, the field structure must be laminated in order to avoid eddy currents and the field coils must have only a few turns to avoid too great self-inductance, and the consequent low power factor. The greater tendency to spark in the case of the a-c. series motor is due to the *alternating* field flux which interlinks the coils short-circuited by the brushes, thus inducing in these coils a relatively large e.m.f. not present when the motor is operating on a d-c. circuit. This difficulty can be avoided to a certain extent by designing the motor for a small field flux and with but a few turns in series in each armature coil. In general, therefore, single-phase commutator motors are built for low voltages, such as 200, with one turn per coil, multiple-wound armatures and with the armature ampere turns per pole about four times the field ampere turns per pole. The effect of the armature ampere turns can be neutralized by a "compensating" winding described below. The field flux is practically limited to that value which will give 4 volts per turn in the short-circuited armature coil, as this is about the limit that may be commutated with carbon brushes.

Resistance Leads. — As an additional means of preventing sparking, high-resistance leads are frequently used between the commutator segments and the armature coils. These leads are made of a high-resistance metal strip bent back and forth several times, and imbedded in the armature slots along with the armature conductors proper. On account of the dissipation of heat in these leads a lower current density must be used in the armature conductors proper than is used in the case of d-c. motors. The arrangement of the leads is such that the main or useful current passes through two high-resistance leads in multiple as it enters the armature, while the undesirable short-circuited current passes through two high-resistance leads in series. It should be noted, however, that at any instant there is current only in those leads connected to coils which are being short-circuited at this instant.

Compensating Winding. — To obviate the high armature reaction in an a-c. series motor and at the same time to improve the power factor, a "compensating" winding is usually employed. This consists of a distributed winding imbedded in slots in the pole faces and connected usually in series (Fig. 2) with the main field winding and armature in such a manner that the current through it sets up a magnetomotive force which practically neutralizes the effect of the armature ampere turns. When the compensating winding is connected in series with the field and armature, as shown in Fig. 2, the motor is said to be "conductively compensated." An "inductively compensated" motor has this winding short-circuited upon itself and the current in it is induced from the armature by transformer action. Inductive compensation is not operative on d-c. circuits, but is as satisfactory as conductive compensation for a-c. operation.

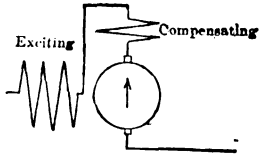

Fig. 2. Compensated Series Motor

Thomson Repulsion Motor. — This motor has a stationary structure or field with a completely distributed winding, which may be wound for any voltage. In this is placed a low-voltage armature designed with all the refinements necessary for single-phase commutator work. The brushes bearing on this commutator are short-circuited upon themselves and are so placed that the line connecting the positive and negative brushes makes an angle α with the neutral axis of the field. The field turns lying within the angle $(90 - \alpha)$ induce a current in the armature winding by transformer action and the field turns lying within the angle α constitute the "exciting" turns and set up the necessary flux to produce the driving force. The arrangement is equivalent to the circuits shown in Fig. 3, although actually there is but a single field winding. This motor then acts exactly like the combination of a transformer and a series motor in one structure. It may be reversed by changing the position of the brushes or by shifting the points of connection of the external circuit to the field or stator winding. This motor operates particularly well near synchronous speed as then it has practically a rotating magnetic field and no excessive commutation difficulties, but at starting and at low speeds the commutation is not as good as that of the compensated series type.

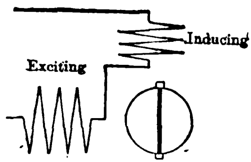

Fig. 3. Thomson Repulsion Motor

Winter-Eichberg Repulsion Motor. — In this motor the short-circuited brushes B_1 are set in the neutral axis of the field due to the stator winding, and an extra set of brushes is set halfway between the main brushes, see Fig 4. Through the latter set of brushes B_2 is sent either the entire main current or a certain portion of it; in the latter case a transformer T connected as shown is required. The main component of the armature current is induced from the stator winding by transformer action; the component entering the rotor through the brushes B_2 serves as the exciting current, setting up the necessary flux to produce the driving force. In a sense, then, the rotor serves both as the armature and the field of the motor, the stator winding acting merely as the primary of a transformer of which the rotor winding is the secondary. By means of a

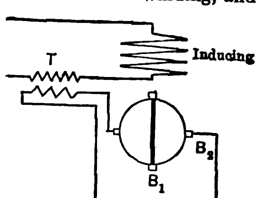

Fig. 4. Winter-Eichberg-Latour Motor

series transformer or compensator with taps the ratio of exciting current to main current may be changed at will and thus the speed and torque of the motor regulated. This is the principal advantage of this type of motor.

Series Repulsion Motor. — Since the repulsion motor has better operating characteristics at a speed near synchronism than the series motor it is better to run with repulsion motor connections. On the other hand, the series motor has better operating characteristics during starting conditions. These facts have lead to the development of the "series repulsion" motor by one of the manufacturing companies. In the control of this motor a compensator with numerous taps is used and the connections so arranged that during starting the armature receives a very large current while the field is excited below the normal value. This gives a considerable torque with little trouble from commutation. As the motor speeds up, the connections are gradually changed until at full speed the motor operates practically as a repulsion motor with the good commutating characteristics of that motor.

DIMENSIONS, WEIGHT AND COSTS. — Due to the low flux densities used and the special windings required an a-c. commutator motor weighs from 50 per cent to 100 per cent more, occupies from 25 per cent to 50 per cent more space, and costs from 50 per cent to 100 per cent more than a 600-volt d-c. motor of the same rating; see *Motors, Direct-current.*

BIBLIOGRAPHY. — Arnold, E., *Wechselstromtechnik*, Vol. V, part 2, Berlin, 1912; Creedy, F., *Single-Phase Commutator Motors*, N. Y., 1913; Fynn, V. A., *The Classification of A. C. Motors*, London; McAllister, A. S., *Alternating Current Motors*, N. Y., 1909; Steinmetz, C. P., *Elements of Electrical Engineering*, N. Y., 1909; *Alternating Current Phenomena*, N. Y., 1908; Crocker & Arendt, *Electric Motors*, N. Y., 1910; Hobart, H. M., *Electric Motors*, London, 1910; Lamme, B. G., *New Haven Motors*, Trans. A.I.E.E., Jan., 1908; Numerous papers, in *Trans. A.I.E.E.*, from1902 to date.

[W. I. SLICHTER.]

MOTORS, DIRECT-CURRENT. — *(See also Alternating Currents; Electricity and Magnetism, Principles of; Generators, Direct-current; Motors, Industrial Applications of; Standardization Rules.)* A motor is a dynamo-electric machine for converting electrical power into mechanical power; that is, it performs the converse function of a generator. Direct-current generators and motors are always interchangeable in function, although a machine which is designed specifically for a motor would probably not make a first-class generator and vice versa. Motors of less than 5 horse-power are usually bipolar; larger machines are multipolar.

The following is a brief outline of the contents of this article:

CLASSIFICATION. — There are four types of direct-current motors, differentiated by their characteristics and the connection of the exciting windings or circuits.

Shunt Motor (Fig. 1). — This motor has only one exciting winding, which is connected across the armature terminals and is thus in parallel or in shunt with the armature. The field winding consists of a large number of turns of fine wire on each pole, and usually the windings on all the poles are connected in series in one circuit. The current in the field depends upon the line voltage and upon the resistance of the field winding. The resistance of the field winding is purposely made high so that the field current will be between 1 per cent and 5 per cent of the full-load current of the motor. The characteristic of the shunt motor is a fairly constant speed for all reasonable values of load.

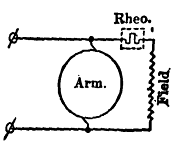

Fig. 1. Shunt Motor

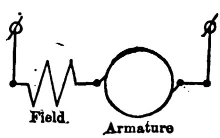

Fig. 2. Series Motor

Series Motor (Fig. 2). — This motor has only one exciting winding, which is connected in series with the armature so that all the current flows through the field as well as the armature. The field winding consists of a few turns of thick wire on each pole and the windings on all poles are connected in series. The current in the field depends upon the load and is thus large with heavy load and small with light load. The resistance of the field winding is purposely made low so that the loss of voltage and power in that circuit will be small. The characteristics of a series motor are a speed varying with every change in load,

high speed at light load and low speed at heavy load. The efficiency is high throughout a wide range of speed. The speed will be dangerously high at no load; thus a series motor must always be connected rigidly to its load. Since the torque is high at low speeds this motor is particularly adapted to work requiring frequent starting.

Compound (or Cumulative) Motor. — This motor has both a series winding and a shunt winding on each pole, wound and connected so that the two windings assist each other in the production of magnetism. It is a combination of a shunt and a series motor designed to give the good starting qualities of the series motor and to avoid the danger of excessive speed at light loads. See also *Motors, Industrial Applications of.*

Differential Motor. — This motor has a shunt and a series winding connected so that they oppose each other in the production of magnetism. The motor therefore has poor starting qualities, increases in speed with increase in load but has no tendency to run at a dangerously high speed. The applications of this motor are very limited.

Inclosed vs. Open Type. — These terms refer to the mechanical housing of the motor. The open type has all its parts freely exposed to the air and is therefore well ventilated. It is intended to be used indoors and in protected places. The inclosed type is intended to be used in exposed locations where there is a liability of dampness or dirt. Special means must be provided to circulate the air inside the machine, but even then an inclosed motor is larger and more expensive than an open motor of the same capacity.

The relative capacities in output of open, semi-inclosed, and totally inclosed motors are shown by the accompanying data on one of a line of typical commercial motors. In general an inclosed motor weighs about 15 per cent more than an open motor of the same capacity in spite of the fact that it is allowed to operate at 15° C. higher temperature by commercial convention.

Type	Output, h.p.	Temp. rise, °C.	Weight in lb. for 700 r.p.m. and given temp. rise
Open	10	40	970
Semi-inclosed	8	40	...
Totally inclosed	5.75	55	1100

METHODS OF RATING. — Motors are rated on the basis of either their continuous or their intermittent capacity.

The continuous rating of a motor is at present (1914) commonly taken as that output in horse-power (or kilowatts) which it will give continuously with a maximum rise in temperature measured by thermometer above the surrounding air at 25° C. not exceeding 40° C. on the field and armature and not exceeding 55° C. on the commutator; see, however, the *Standardization Rules of the A.I.E.E.*

The intermittent rating is at present (1914) commonly taken as that output which the motor will give for one hour (starting at room temperature) with a maximum rise in temperature by thermometer, above the surrounding air at 25° C., not exceeding 90° C. on the commutator and not exceeding 65° C. on any other part; see, however, the article *Standardization Rules of the A.I.E.E.*

VOLTAGE AND CURRENT. — Usual values of voltage for direct-current motors are:

110–125 for small motors on lighting circuits.
220–250 for motors in factories, shops, etc.; on power mains or on the outside mains of a three-wire system.
500–600 for general railway work.
1200 for special railway installations.

The current required for any motor is found by the relation

$$\text{Current} = \frac{\text{Output in h.p.} \times 746}{\text{Efficiency} \times \text{Voltage}}.$$

Usual values for the efficiencies of motors of various sizes are given below.

APPLICATIONS OF MOTORS. — This subject is treated in detail in a separate article on *Motors, Industrial Applications of* (q.v.). The chief applications of continuous-current motors are the following:

Shunt Motor. — Driving shafting, machine tools, blowers, reciprocating pumps; motor generators.

Series Motors. — Railway and all other transportation work; hoists; cranes.

Compound Motor. — Elevators, hoists and machinery that must be started often.

Differential Motor. — Very special applications of small units for peculiar speed conditions.

PRINCIPLES. — The principles upon which a direct-current motor operates are the same as those upon which a direct-current generator operates (*see Generators, Direct-current*). These principles are briefly as follows:

Force Acting on Conductor. — A conductor of length l carrying a current I and placed in a magnetic field having a flux density B is acted upon by a force which is proportional to BlI, which force is in a direction at right angles to the direction of the magnetic flux and at right angles to the length of the conductor.

This in practical form gives the relation

$$T = \frac{p\phi ZI}{852\, m \times 10^8}.$$

T = torque of an armature in pounds at one-foot radius.
p = number of poles.
m = number of armature paths between brushes.
ϕ = flux per pole in armature (lines).
Z = number of active conductors or coil sides on armature.
I = current taken by the armature from line.

This torque is exerted whenever a current flows and is independent of the speed. The core-loss and friction absorb some of the torque so that the torque at the pulley is slightly less than the value given by the formula.

Counter E.M.F. in Conductor. — A conductor of length l moving with a velocity V in a magnetic field of density B has induced in it an electromotive force E proportional to BlV. In practice as soon as the armature starts to move a counter e.m.f. is induced in its conductors which has the value

$$E = \frac{p\phi Zn}{m \times 10^8},$$

where n = revolutions per second.

Thus as soon as the armature moves, this counter e.m.f. tends to stop the flow of current and the impressed e.m.f. must be increased to maintain the flow of current.

The relation between current and counter e.m.f. is given by the equation

$$E_i = E + IR,$$

where E_i = impressed e.m.f.,
E = counter or generated e.m.f.,
R = resistance of armature circuit.

In practice R is made as small as possible so that E and E_i are as nearly equal as possible.

Reversing Rotation.— From a consideration of the equation for the torque it is evident that torque is proportional to the product ϕI, i.e., to the product of the flux in the armature by the current. If the direction of the current through the armature is reversed, that is, if I becomes negative, the product becomes negative and the torque is in the opposite direction. If the direction of the flux is changed (the armature current being unchanged) the direction of torque is reversed. But if both ϕ and I are reversed the torque is not reversed. From this follows the rule for reversing the direction of rotation of any direct-current motor; viz., change the direction of flow of current in either the field or armature winding but not in both.

Speed Control. — From the equation for counter e.m.f. it follows that the speed is proportional to E/ϕ. That is, the speed varies directly as the counter e.m.f. and inversely as the flux. Thus to reduce the speed, decrease the counter e.m.f. by decreasing the e.m.f. impressed on the armature or increase the flux by increasing the field current. To increase the speed perform the converse. To decrease the e.m.f. impressed on the armature a resistance may be inserted between the source of potential and the armature terminals. This is the customary manner of controlling the speed during the starting of motors. (*See also section below on Starting of D-C. Motors.*)

SHUNT MOTOR. — Since the flux in the armature of a shunt motor is practically independent of load, the characteristics of the motor are: approximately constant speed for all reasonable variations of load, torque directly proportional to the armature current irrespective of speed, efficiency high throughout a wide range of load but for only a small range of speed see Fig. 3.

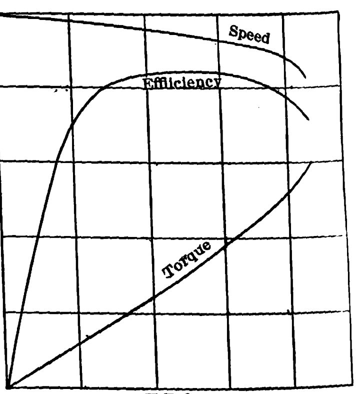

Fig. 3. Speed, Torque and Efficiency Characteristics of a Shunt Motor

Design of Shunt Motor. — The method of design and calculation of shunt motors is the same as for direct-current generators (*see Generators, Direct-current*) except for the minor details noted below.

The armature reaction of a motor is in the opposite direction to that of a generator running in the same direction, and thus the field is distorted in the opposite direction. Hence, if the brushes are to be moved to assist commutation they must be moved in a direction opposite to the direction of rotation of the armature.

The effect of armature reaction is to weaken the field. This causes a tendency to increase the speed and also causes bad commutation.

The stability factor of a motor must be greater than that of a generator because when the motor drops in speed as the load comes on, the field must be weakened to increase the speed. Hence the field is liable to be operated at an excitation less than normal.

Testing of Shunt Motors. — (*See also Standardization Rules of the A.I.E.E.*) The tests on shunt motors may be divided into two classes: (*a*) Commercial, to determine the qualities and serviceability of particular motors; and (*b*) Special, to determine the general characteristics and actions of a type of motor. Commercial tests are the following:

Resistance measurements.
Stray power test, including core-loss and friction.
Input-output test for heating, efficiency and commutation.
Insulation test.

Resistance Measurements are made with the machine cold and later after the heat run. The resistances of the armature winding and field winding are measured, and the brush-contact resistance may be measured but is usually calculated (*see Generators, Direct-current*). For any value of current the resistance losses (RI^2) are calculated from the measured hot resistances.

"Stray Power" Test. — The term "stray power" is applied to the lumped sum of the core-loss and the loss due to friction, bearings and windage. The stray power of a d-c. machine can be determined approximately by impressing normal voltage on the field and letting the machine run as a motor without load, varying the voltage impressed on the armature from about 10 per cent above normal to about 10 per cent below normal. The speed, armature voltage and armature current for each adjustment are observed. Then the stray power for any induced voltage is equal to the armature input less the corresponding RI^2, where R is the armature resistance and I the armature current. The value of stray power for any given load on the motor is then equal to the measured value corresponding to the same counter e.m.f. E, where E is calculated from the impressed voltage E_i by the relation $E = E_i - RI$, I being the armature current for this load and R the armature resistance as before.

If it is desired to determine the stray power more accurately by taking into account the effect of armature reaction, the field current may be adjusted so that the speed on the above run is the same as the load speed. This gives a flux of the same average value as when the machine is under load.

Calculation of Efficiency from Losses. — Let P = total output in kilowatts; R_a = resistance of armature, including brushes; I_a = armature current; I_f = field current; E_i = impressed voltage; S = stray power. Then the per cent efficiency is

$$\epsilon = \frac{100\,P}{E_i(I_a + I_f)} = \frac{100\,P}{P + E_iI_f + I_a^2R_a + S}.$$

Input-output Test with Prony Brake. — The input-output test may be made either by means of a prony or band brake on a pulley, or by using a d-c. generator as a load. If the brake test is made the output is

$$\text{Watts} = \frac{PLN}{7.04},$$

where P = net pull in pounds, L = lever arm or radius in feet, N = revolutions per minute.

Input-output Test with Generator as Load. — With large motors it is desirable to use a generator as a load in making an input-output test. In

this case it is necessary to know the resistance of the generator armature circuit. It is also desirable to have the generator separately excited and to maintain a constant excitation throughout the entire test.

The input of the motor and the output of the generator, together with the speeds of both machines, are observed. A "counter-torque" test must also be made to determine the belt friction loss and the core-loss and friction of the generator. This is performed by making two tests as follows:

(*a*) The motor input is observed when driving the unloaded generator at normal speed first through the regular leather belt and second through a light cotton belt. The difference in input to the motor in the first and second cases gives the belt-friction loss. As this loss is comparatively small, it may frequently be neglected.

(*b*) A regular stray power test (*see above*) is made on the generator when entirely disconnected from the motor. This gives the core-loss and friction of the load machine (generator).

Then for any load during the load run the output of the motor under test is

$$P_1 = P_2 + R_2I^2 + S + F,$$

where P_1 = output in watts of motor, machine 1.
P_2 = output in watts of generator, machine 2.
R_2I^2 = loss in armature winding of generator.
S = stray power of generator for speed and induced voltage at observed load.
F = belt-friction loss.

The ratio of this motor output to the electrical input as observed gives the efficiency of the motor.

Heat Run. — From the input-output test it is also possible to determine the speed regulation, commutation features and heating. The heat run may also be made by "bucking" two machines as described in the article on *Generators, Direct-current.* Small motors will reach a constant temperature in a short time and the heat run need only last 5 or 6 hours for a 100-h.p. motor. A thermometer is usually placed on the machine in a safe and accessible place and read every half hour until it indicates no further rise in temperature.

Insulation Test. — The margin of safety on a 110- or 220-volt motor is usually so great that it is not necessary to make an insulation test. If the motor has been exposed to dampness it may be desirable to make the test after the motor has been thoroughly dried out. The method is indicated in the *Standardization Rules of the A.I.E.E.* (q.v.)

Special Tests. — As a special test there may be obtained a saturation curve of the machine and possibly the distribution of potential around the commutator. These are of particular interest in an adjustable speed motor. In some of these motors with commutating poles there may exist some very high voltages between bars which are not evident except in the bar-to-bar potential test.

SERIES MOTOR. — Since the flux in the series motor is produced by the load current, the flux increases with the current. The torque is proportional to the product ϕI and therefore increases more rapidly than the current. Thus four times full-load torque can be obtained with from two to three times full-load current.

The characteristics of the series motor are: increase of torque faster than increase of current, variation in speed inversely as the load, and high efficiency throughout a wide range of speed as well as load, see Fig. 4.

Design of Series Motors. — In general the method of calculation is the same as for a direct-current generator (*see Generators, Direct-current*). The special considerations are:

A series motor is usually designed to have a large output and low speed at the one-hour rating. At any lesser output the speed will be higher, so the peripheral velocity must be quite moderate at the rated load and speed.

Since the speed is very nearly inversely proportional to the flux the speed curve depends on the shape of the saturation curve, to which very careful attention is paid in designing. By exactly fixing the flux for two extreme values of current the speed for these two values of current is fixed.

The relations between the speed and current of a series motor are shown by the formulas:

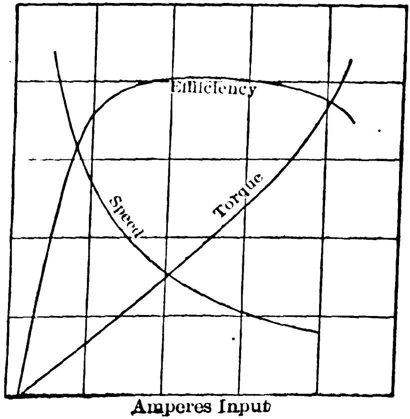

Fig. 4. Speed, Torque and Efficiency Characteristics of a Series Motor

$$E_i = E + IR,$$

$$E = \frac{p\phi Z n}{m\,10^8},$$

$$n = \frac{m(E_i - IR) \times 10^8}{p\phi Z},$$

where

E_i = impressed voltage,
E = counter e.m.f. induced in armature,
R = total resistance of armature and field,
I = current taken by motor,
p = number of poles,
m = number of parallel paths between positive and negative brush sets,
Z = total number of conductors on armature,
n = speed of armature in revolutions per second,
ϕ = total flux per pole in maxwells.

Since the current in the field of a series motor is the same as that in the armature, the ratio of the turns in each is the same as the ratio of ampere-turns or m.m.f's. Thus if the magnetomotive force per pole of the field is to be 1.5 times that of the armature the number of turns will be 1.5 times the number of turns in series in the armature.

Since a series motor is usually an inclosed motor with a one-hour rating its rise in temperature and rating are a direct function of the watts lost and the ability of the mass of the motor to store up this heat energy. In a one-hour run the amount of energy radiated is only about 10 per cent of the amount stored in the mass. For a rise in temperature of 75° C. in one hour there should be about 0.4 pound of material for each watt of loss. This assumes reasonable provision in the construction of the motor for the transfer of the heat from the armature to the field and frame.

Much attention has been directed recently to the ventilation of these motors by drawing air from outside the motor by means of fan blades on the armature and by circulation of the air inside the motor through definite paths. This has considerably increased the weight efficiency of these motors.

In railway motors, which are the most general application of the series motor, commutating poles are very generally used, as this construction makes it possible to obtain a much greater momentary output from a motor of a given size (*see below*).

Testing of Series Motors. — (*See also Standardization Rules of the A.I.E.E.*) To determine properly the speed and torque characteristics of a series motor an "input-output" test must be made, which involves subjecting the motor to actual load and overload conditions. This may be accomplished by running the motor with a prony brake as a load or with a direct-connected generator as a load (*see section above on Testing of Shunt Motors*).

Railway Motor Test. — When two similar motors are available the method used by the manufacturers of railway motors is most desirable. The two motors are direct connected, or geared to each other, and the electrical connections made as in Fig. 5. The test is run through by keeping constant rated voltage on the motor and regulating the load on the motor by changing the load on the generator.

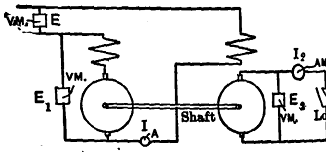

Fig. 5. Railway Motor Load Test

As the two machines are operating under almost exactly the same conditions, their efficiencies are very nearly the same. Thus

$$\text{Efficiency of set} = \frac{E_2I_2}{EI},$$

$$\text{Efficiency of each motor} = \sqrt{\frac{E_2I_2}{EI}}.$$

The speed and torque curves should be made for both directions of rotation of the armature as an incorrect brush setting will give results differing with the direction of rotation. The direction of rotation is changed by reversing the connections of either the field or the armature of the motor.

Commutation is observed during the speed and torque test.

The heat run is made with the same arrangement as the speed-torque test. In making the heat run the motor must start cold or at room temperature. The covers of the inspection openings of railway motors are customarily left open during the heat run.

Losses and Efficiency of Series Motors. — For a more accurate determination of the efficiency and losses the following special tests are made:

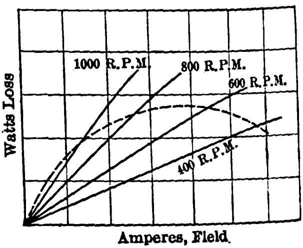

Fig. 6. Core-loss Curves of Series Motor

1. Resistance of armature, brushes and field. These tests are similar to those for a shunt motor, see above.

2. Core-loss test. On account of the variable speed and variable field of a series motor this test consists in repeating the usual core-loss test as described

above for a generator at several different speeds. The field strength is varied step by step throughout the maximum range for each speed. Fig. 6 shows the curves for these different runs and the dotted line connects the points on the different curves that apply to the normal speed curve of the motor.

Insulation Tests. — See *Generators, Direct-current,* and *Standardization Rules of the A.I.E.E.*

STARTING OF DIRECT-CURRENT MOTORS. — A starting box, Fig. 8, or rheostat is always employed in starting direct-current motors in order to reduce the voltage impressed on the motor when it is not running at a high enough speed to generate the proper counter e.m.f.

Let E_i = line voltage,
E = counter e.m.f. (approximately proportional to speed),
I = current,
r = resistance of armature circuit,
R = resistance of starting box or rheostat.

Then

$$I = \frac{E_i - E}{r + R}.$$

At the first instant the motor armature is stationary and $E = 0$; thus $I = \dfrac{E_i}{r + R}$ and the value of R is determined by the desired value of I. As the motor accelerates, E increases. If R remained constant I would decrease to such a small value that there would not be sufficient torque to accelerate the load. The current, and therefore the torque, can be brought back to their original values by changing R to such a value R_1, that

$$I = \frac{E_i - E}{r + R_1}.$$

Fig. 7 shows the sudden rise in current when the resistance is changed and the gradual decrease in current as the speed increases. The number of steps necessary depends upon the ratio of the maximum allowable instantaneous value of the current to the final constant value, upon the value of the armature resistance and upon the inertia of the load.

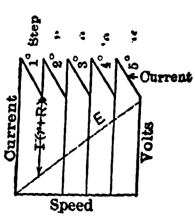

Fig. 7. Motor Current During Starting

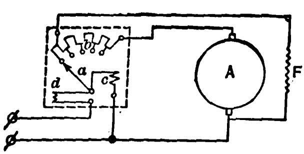

Fig. 8. Starting Box Connections for Shunt Motor

Starting Box.—(*See also article on Rheostats*). A starting box usually contains the following features, as indicated in Fig. 8: (a) a means of opening and closing the circuit supplying all the current to the motor including the field current; (b) a set of resistance steps in series with the motor armature and a means of short-circuiting this resistance step by step; (c) a magnet coil connected across the motor terminals to open the circuit if the impressed voltage fails or falls below a specified value (low-voltage release); (d) a magnet coil carrying the main current to actuate a spring and open the circuit if the current exceeds a

specified value (overload release). The usual connections of a starting box to the line and motor are shown in Fig. 8.

SPEED CONTROL. — There are three methods of varying and controlling the speed of d-c. motors, namely, potential, rheostatic and field systems.

Potential Control or Multi-voltage System. — By means of several generators and several wires various definite voltages are made available, such as 240, 180, 120, etc. By connecting the motor to the 240-volt circuit full speed is obtained; by connecting to the 180-volt circuit ¾ speed is obtained, etc. The shunt field circuit is left connected at all times to a circuit of the proper voltage. A shunt motor with normal field excitation will be stable, that is, it will operate constantly, at the fractional speed. The efficiency will be good at the fractional speeds. A series motor controlled in this manner will be unstable, but for a given torque the speed will be roughly proportional to the voltage.

Rheostatic Control. — A rheostat in series with the armature will reduce the voltage impressed on the armature by an amount proportional to the current, and thereby reduce the speed. The speed is unstable with this arrangement, changing with every change of load, and the efficiency is poor.

Field Control. — By increasing the resistance in series with the field of a shunt motor the speed is increased due to the weakening of the field. If the motor has commutating poles to assure good commutation the speed may be varied in a ratio of 1 to 2, and even 1 to 3 in small sizes. The shunt motor is stable with this method of control and the efficiency is good. In a series motor the field may be shunted by a resistance to increase the speed but the motor is not stable and this practice is not to be recommended.

USE OF COMMUTATING POLES (INTERPOLES) IN VARIABLE-SPEED MOTORS. — In motors intended to be operated over a large variation in speed, obtained by changing the field strength, and in motors which are to be subjected to heavy overloads, it is necessary to use commutating poles in order to obtain good commutation. In a motor without commutating poles the field strength must always be a certain percentage greater than the armature strength to prevent a shifting of the field flux and of the neutral point. Thus,

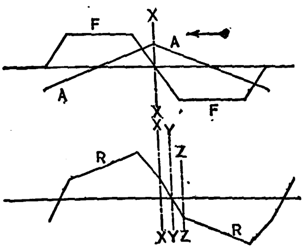

Fig. 9. Flux Distribution without Commutating Poles

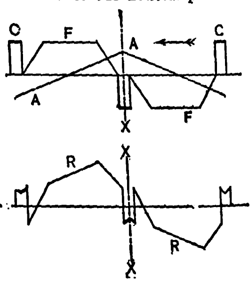

Fig. 10. Flux Distribution with Commutating Poles

if, in Fig. 9, F represents the distribution of field flux when existing alone and A represents a strong armature flux existing alone, then R shows the distribution of the resultant flux when both field and armature are excited.

It will be noticed that the neutral point has been shifted from XX at no load in a direction against rotation to YY at the load considered. The brushes would have to be shifted from XX at no load to a point ZZ beyond YY at load

in order that they shall commutate a coil in a flux which is producing a voltage helpful to commutation.

Resultant Flux with Commutating Poles. — If, however, commutating poles are placed between the main poles and excited with the armature current they will maintain at the geometrical neutral a flux of the direction and value necessary to give good commutation. In Fig. 10, *F* and *A* represent the field and armature flux separately as before and *C* the commutating pole flux that would exist at full load. When at full load these fluxes are combined there exists the resultant flux shown at *R*.

It will be noticed that there remains at the neutral point a small flux of the proper polarity and magnitude to provide an e.m.f. to reverse the current and give good commutation and it is not necessary to move the brushes.

The commutating pole must be of the same polarity as the pole towards which the brush would have to be moved if there were no commutating poles. In fact the principle of commutating poles is nothing more than bringing to the brush a part of the pole instead of moving the brush to the pole. Thus the polarity of the commutating pole is different during motor action from that during generator action. If the windings on the commutating poles are connected in series with the armature the conditions will be correct for either motor or generator action.

INSTALLATION AND ERECTION. — In the installation and erection of a direct-current motor there are certain features which must receive careful attention in order that the machine shall operate properly and not deteriorate with undue rapidity. Although this procedure varies with different motors according to their mechanical construction the following brief memorandum of points to be looked after will be found useful:

1. Base bolted down.
2. Bearings clean and filled with oil.
3. Bearings lined up.
4. Magnet frame bolted to base.
5. Field coils secured in place.
6. Field coils tested for open circuit, wrong connection and polarity.
7. Armature in place.
8. Air gap adjusted by shimming.
9. Measure resistance of armature and field.
10. Measure insulation resistance.
11. Brushes properly fitted and spaced and pressure adjusted to about 1.5 to 2 pounds per brush.
12. Commutator smooth and true.
13. Substantial connections of field circuit.
14. Field adjusted for correct direction of rotation.

The motor must be protected from moisture during shipment and if by accident it becomes damp it must be dried out before it is subjected to a voltage.

OPERATION. — In the operation of a direct-current motor several factors should be considered.

Care. — All motors should be frequently inspected and the following points noted:

1. Bearings filled with proper amount of oil.
2. Brushes securely held in proper position.
3. Brushes fit properly.
4. Commutator smooth: Danger of "high mica" or the insulation between commutator bars projecting above the bars.
5. Air gap true.
6. Commutator not worn in grooves.

Troubles. — In the following paragraphs is given a concise list of the troubles that may be experienced in operating continuous-current motors and their causes as given by Crocker and Wheeler in *Management of Electrical Machinery.*

1. *Sparking at the Commutator.* — Causes: Armature carrying overload. Brushes improperly spaced. Brushes not at proper position. Rough commutator. Poor brush contact. Internal short or open circuit. Field too weak. Unequal strength of poles. Vibration.
2. *Heating of Commutator and Brushes.* — Sparking. Bearing trouble. Bad connections. Brush friction too great.
3. *Heating of Armature.* — Overload. Internal short circuit, moisture or ground. Reversed coil. Excessive eddy currents.
4. *Heating of Field.* — Internal short circuit.
5. *Heating of Bearings.* — Bearings dry or dirty. Shaft out of true. Bearings out of line. Thrust due to belt. Unbalanced magnetic pull.
6. *Noise.* — Armature not balanced. Brushes dry or not set at proper angle. Armature strikes.
7. *Speed Too Low.* — Wrong voltage. Overload. Armature strikes. Bearing too tight.
8. *Speed Too High.* — Wrong voltage. Field too weak.
9. *Motor Stops or Fails to Start.* — Overload, open circuit, wrong connection.

SPECIFICATIONS FOR D-C. MOTORS FOR INDUSTRIAL USE.* — (*See next section for Specifications for Railway Motors.*) The following memoranda are intended to assist in writing specifications. See also article on *Specifications.*

Principal Characteristics and Conditions of Service. — Use to which motor is to be put, kind of load and method of drive. Voltage. Rating, horse-power. Speed.

Style and Description; Details of Construction. — Whether to be open, semi-inclosed or inclosed. Whether to be series, shunt or compound wound; if shunt wound, whether shunt field rheostat is to be supplied; if compound wound, state whether cumulative or differential. Requirements regarding pulley or length of shaft. Whether rails are required. Whether starting rheostat is to be supplied, and if so, its general characteristics.

Performance and Tests. — (*See Standardization Rules of the A.I.E.E.*). Temperature rises upon which ratings are to be based. Details of overload. Efficiency at 25, 50, 75, 100, and 125 per cent load; whether rheostat losses are to be included in calculating efficiencies. Starting torque with full-load current, pound-feet. High-potential tests of insulation. Requirements regarding effect of moisture upon insulation. Regulation; the supply voltage being constant, and the field rheostat fixed, a variation of load from zero to . . . per cent of full-rated load shall cause a variation of speed not greater than . . . per cent. The shunt field rheostat to give speed variation of . . . per cent in steps not greater than . . . per cent and not less than . . . steps, and to carry the current for any speed continuously without undue heating.

SPECIFICATIONS FOR SERIES RAILWAY MOTORS.* — The following memoranda are intended to assist in writing specifications. See also article on *Specifications.*

Principal Characteristics and Conditions of Service. — General statement of the service, giving type of cars, whether current is direct or alternating,

* By W. A. Del Mar.

or both, etc. The motor shall be designed for normal operation at . . . volts and shall operate safety at . . . volts.

Style and Description; Details of Construction. — The frame shall be designed so as to allow the easy removal of armature and field coils. It shall be provided with openings at both ends, and both above and below the shaft, which will enable the inside of the motor to be readily inspected and cleaned. Bearings shall be designed so that lubricant cannot enter the frame, and shall be so located that they may be easily emptied and cleaned. The diameter of the driving axle on which the motor is to be mounted shall be . . . inches. Whether motor is to be of interpole type. Whether natural or forced ventilation.

Brushes and Brush Holder. — The brush holders shall be readily removable through the hand holes. The springs holding the brushes against commutator shall not be relied on to carry current. The brushes shall be staggered or provided with adjustment parallel to the armature shaft so as to prevent the formation of ridges on the commutator.

Clearances.—The minimum distance between motor frame and back of wheel flanges shall be . . . inches, the minimum distance between bottom of motor and top of rail when tires are new shall be . . . inches.

Gears and Gear Case (if any).—Single or double reduction; what gear wheels shall be mounted on; material of wheel and pinion; description of teeth, whether cut or cast, and width of face of wheel or pinion; gear case, material, how suspended, oil-tightness.

Suspension of Motors.—General description and requirements, location of lugs on motor frames.

Data to be Furnished by Bidder. — The armature will be bound with . . . bands. Material and dimensions of the bands. Dimensions of openings in the frame. The brush holders will be adjustable so as to allow . . . inch wear with uniform pressure on the brushes, after the diameter of the commutator has been reduced by . . . inches. The current density in the carbon brushes will not exceed . . . amperes per square inch at normal rated load. The gear ratio will be . . .

Performance and Tests. — (*See also Specifications in article on Locomotives, Electric.*) Either the nominal rating and the continuous ratings at ½, ¾ and full voltage should be specified, or the following data supplied.

Line voltage..
Number of motors per car..............................
Weight of loaded car, exclusive of motors and control equipment.
Diameter of driving wheels............................
Schedule speed..
Distance between stops................................
Duration of stops.....................................
Acceleration miles per hr. per sec....................
Retardation, miles per hr. per sec....................

The engineer should also give a diagram of grades and curves.

Motor Characteristics. — The bidder shall submit diagrams showing speed, tractive effort, efficiency, RI^2 losses, core losses, and any other information bearing on the performance of the motor. Requirements regarding the effect of moisture upon insulation.

Tests. — The motor shall be tested at the Manufacturer's works in the presence of the Engineer's inspector. (In the case of new motor developments it is good practice to make the tests under service conditions; but for standard motors a stand test at the factory is sufficient.) A complete series of tests shall be made upon the first motor manufactured under this specification. These tests shall confirm all the statements made by the bidder in relation to operating characteristics. Should the motor fail to comply with any of these statements, the defects shall be corrected and any changes in construction or design which may be necessary to accomplish this shall be made at the contractor's expense. The first motor shall be submitted to a flashing test to determine the susceptibility of the motor to flash-over on opening the maximum specified line voltage across the motor when running at maximum speed. After the acceptance of the first equipment, any other motors to be supplied under this specification shall be submitted to an approved stand test. The insulation of the armature windings, commutator and field windings, shall be subjected to stated alternating voltages (*see Standardization Rules of the A.I.E.E.*) for a period of one minute.

PERFORMANCE, WEIGHT AND COST. — Usual values of the efficiency and losses, and also values of the weight, speed and cost of shunt and series motors are given in the following tables.

PERFORMANCE OF SHUNT MOTORS

H.P.	Full load efficiency, per cent	Field I^2R, per cent	Friction, per cent	Core-loss, per cent	Armature I^2R, per cent
0.5	70	5	10	5	10
1	79	4	8	4	5
2	82	3.5	7	3.5	4
5	85	3	5	3	4
10	87	2.5	4.5	2	4
20	88	2	4	2	4
25	89	2	3	2	4

WEIGHT, SPEED AND COST OF SHUNT MOTORS

H.P.	Speed, r.p.m.	Weight, pounds	Cost	Speed, r.p.m.	Weight, pounds	Cost
0.5	2200	60	$30	....	....	...
1	2000	100	50	1675	125	$60
2	1650	210	75	1175	250	90
5	1100	650	160	920	600	165
10	850	950	240	700	1100	280
20	680	1650	410	550	1900	470
25	650	1950	500	500	2300	600

PERFORMANCE OF SERIES MOTORS

Commutating Pole Railway Type

H.P.*	Full-load efficiency, per cent	Field I^2R, per cent	Friction, per cent†	Core-loss, per cent	Armature I^2R, per cent
40	79	6.5	5.3	2.2	7.0
50	82	5.0	5.0	2.1	5.9
75	84	4.5	5.0	2.0	4.5
100	85	4.3	5.0	2.0	3.7
150	86.5	3.5	5.0	2.0	3.0

* H.P. for 75° C. rise in one hour.
† Friction includes loss in gearing.

WEIGHT, SPEED AND COST OF SERIES MOTORS

Commutating Pole Railway Type

H.P.*	Speed, r.p.m..	Lb., weight per h.p. †	Cost, dollars per h.p.
40	750	57	11
50	700	53	10.5
75	650	44	8.5
100	625	37	7
150	600	33	6.4

* H.P. for 75° C. rise in one hour.
† Weight includes cast-steel frame and gear pinion and gear case.

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[W. I. SLICHTER.]

MOTORS, INDUSTRIAL APPLICATIONS OF.—(*See also Bearings; Belts and Belting; Blowers and Compressors; Chains and Chain Drive; Conveyors; Couplings, Direct; Cranes; Dredges, Electrically-Operated; Elevators, Electric; Fans; Flywheels for Load Equalization; Gears and Gearing; Hoists, Electric; Machine Tools, Electrical Operation of; Motors, A-C. Commutator, Direct-Current, Polyphase Induction, Single-Phase Induction, Synchronous; Pumps and Pumping Engines; Printing Presses; Ropes and Rope Drive; Shafting; Shovels, Electrical Operation of; Steel Mills, Electric Drive of; Telpherage; Unloaders, Coal and Ore; Valves.*) The application of the electric motor for driving industrial machinery, either individually or in groups, was at first thought to be of value merely in the saving of power through the elimination of the losses due to friction in line shaftings or other forms of mechanical transmissions. A still higher economy has, however, been found to lie in the remarkable effect that it has in increasing the output of the production.

ADVANTAGES OF ELECTRIC DRIVE. — The more important advantages of electric drive are the following:

Location of Machines. — The various machines can be placed in almost any desired position and the use of portable tools is readily made possible, as, for example, when a portable drill and slotter are brought to a heavy casting, the slotter being applied to the outside of the piece at the same time the inside is being drilled.

Head Room. — A clear head room is obtained by the elimination of belting. This gives better illumination and ventilation and permits overhead cranes to be used freely, which is of greatest importance in any factory as it greatly facilitates the handling of the material, resulting in a considerable saving in time and labor, and thus increasing the output.

The constant source of dripping oil from overhead bearings and shafting is eliminated, and the danger which always accompanies the use of belts is overcome.

Centralized Power. — Power can most readily be distributed from a central supply station to the different buildings, and changes or additions to the system can always be made without difficulty.

Reliability. — The electric system offers greater reliability than belt drive. A breakdown is usually confined to a single machine, but with belting and a shafting a breakdown will generally cause a shutdown of a considerable portion of the equipment.

Study of Machine Performance. — Meters of either the recording or indicating type can be installed easily where desired and the performance of every individual machine ascertained. This is a very important point in all industrial undertakings, as it is then possible to maintain all the machinery in the best operating condition. Any excess power taken is at once readily detected and the defect can be promptly corrected. An accurate record can also be kept of the cost of power for the different operations.

GROUP VERSUS INDIVIDUAL DRIVE. — There are two general systems of drive, namely, group and individual, and there is still a diversity of opinion as to the relative advantages of the two. The group drive is to a certain extent an outgrowth of the older system of line shafting. When such a system is to be changed to an electric drive, the most obvious and simple way of making the change is to split the shafting up into such sections as would be most convenient, and drive each of these by means of a comparatively large motor. On the other hand, it may frequently happen that it is necessary to operate only one machine of the entire group for a considerable time, as in overtime work,

and to do this it would then be necessary to keep the motor and the line shafting of the whole group running. Since the efficiency of the motor at this light load would be small, and the friction losses of the entire drive would have to be supplied, it is evident that such a method of operation would result in a waste of power and be most inefficient. Modern installations, therefore, indicate a tendency toward the use of both the group and individual drive.

Influence of Character of Load. — It is generally agreed that all large tools or other machinery should be equipped with individual motors, especially if their service is of an intermittent nature. With the group drive there are two distinct loads, the variable of the machines and the friction of the line shafting and belting. The lower the machine load factor, the greater becomes the percentage of friction load and the more inefficient the group transmission.

Influence of Speed. — Wide ranges of control and the possible variations of speed are reasons which in many cases are sufficient in themselves for the selection of an individual drive. With group drive the methods of speed control for the individual machines are obviously more limited. It is then generally accomplished by shifting of belts on cone pulleys or by change of gears. Both of these methods, however, take a considerably longer time than the simple manipulation of the controller with the individual electric drive.

With individual motor drive it is possible to obtain very fine speed graduations, this benefit, of course, only being derived with a variable speed motor. Another advantage is the fact that it is possible to speed up a machine with a proportional increase in power. This may be necessary whenever a change is made from carbon to high-speed steel.

Influence of Relative Cost. — The increased cost of installation is one of the principal factors that prevent the general installation of individual drives. With this drive the total horse-power rating of the motors installed in the plant will be considerably greater than with group drive, but the maximum power demand of the plant is approximately the same in either case. If power is purchased the price should be based on the actual maximum power demand and not, as sometimes is required, on the total connected horse-power capacity of the motors. This latter method would obviously give a lower basic rate for the group drive, although the higher efficiency of the individual drive would considerably reduce the actual power consumed.

The question of whether or not group or individual drive is to be installed is thus a financial one and each case must be properly analyzed. Individual drive necessarily means a larger investment, but in nearly all cases a much greater percentage income will be realized than if line-shaft drive were employed.

GENERAL CONSIDERATIONS IN THE SELECTION OF MOTORS. — The conditions of capacity and efficiency are both of importance in any motor installation and should therefore be given careful consideration. The installation of a motor having too large a capacity should in general be avoided, unless an increase in the load is to be expected in the near future, because the efficiency of a motor is usually a maximum at its normal rated output, decreasing above and below this point. With alternating-current motors the effect of the power factor must furthermore be considered. This decreases rapidly below normal load, and on account of its bad effect on the regulation of the system it should be kept as high as possible, which can only be done by operating the motors as nearly fully loaded as possible. Ordinarily, however, it is possible to so group the machinery that the motors may be operated near their rated output at all times. Too small a motor is naturally also very undesirable, as it would then in all probability be subject to overloads, which may result in overheating and a burn-out of the motor, causing a shutdown, not only of the motor itself, but also of the machinery which it drives. The op-

erating conditions of the plant may furthermore be such, as for example in steel mills, that the failure of a single motor may necessitate the shutting down of the entire mill.

The selection of suitable motors requires not only complete information on the power required to drive each group of machines or each machine individually but also a thorough knowledge of the motor design and its inherent characteristics to meet the requirements of the load. Some machines will require motors with very heavy starting torque, although running under light load when up to speed, while for others the requirements may be just the opposite. With a variable speed motor the torque-speed characteristics should agree as nearly as possible with the load which the motor is to drive, and the characteristics of adjustable-speed motors as influenced by different systems of control should also be carefully investigated. See also section below on *Data Required to Determine Type and Size of Motor.*

Influence of Motor Efficiency. — A motor of high efficiency is obviously desirable, and it is generally an easy matter to estimate the saving incurred by the installation of a motor of high efficiency as compared to a less efficient one. When a motor is operated for a considerable part of the time on light load, this fact must be given due consideration in the comparison on account of the variation in the motor efficiency for different loads.

Influence of Torque and Speed. — In order to obtain the most satisfactory results from motor drive it is essential that the type as well as the size of the motor be properly adapted to the work contemplated. This is especially important in the case of individual drive, where a wrong selection of the proper motor would be more serious than in group drive. The size of the motor may be ample to operate the machine under normal load but it may not be able to develop a sufficient starting torque, or it may draw a too excessive starting current from the line. For example, to start and accelerate the bridge of a crane requires a motor capable of developing a high starting torque, but after the bridge is accelerated comparatively little power is required to keep it in motion.

The condition of maximum torque must also be given due consideration. A motor driving a heavy punch may, in spite of the flywheel, develop insufficient torque to keep up the speed. As a rule, however, where the motor is large enough for starting and normal operation, but not large enough for the maximum overload required for perhaps only a second or two, the addition of a suitable flywheel will sometimes cut down the maximum torque required. In other cases it may be necessary to install a motor larger than necessary for the average work.

The proper speed regulation is also of importance and a motor must be selected which is best meeting these requirements. The size of the motor is also influenced by the cycle of operation, i.e., whether the load is continuous or intermittent. Careful consideration should be given to this point, and, as previously mentioned, motors will undoubtedly soon be rated to conform to the particular service for which they may be required. See also section below on *Classification of Motors According to Speed.*

Alternating Versus Direct Current. — The choice of alternating or direct current depends largely on local conditions and on the service requirements. With all other conditions the same, the alternating-current system offers many advantages when the distances over which the power must be transmitted are large. A higher transmission voltage may be selected which will diminish the amount of copper needed in the line conductors. The conditions may be such that high-voltage motors can be used, but in other instances step-down transformers may have to be provided and the expense of these as well as other auxiliary apparatus connected therewith must then be considered. It is gener-

ally conceded that the alternating-current system is more reliable than the direct current. This is mostly due to the absence of commutator trouble and to the rugged design of the induction motor.

There is no reason for installing direct-current motors except where a variable speed service is required, and in small plants where such a service is predominating the direct-current system would naturally be the one to install. If the variable-speed feature is only required intermittently, the phase-wound induction motor may be used to advantage. As a rule, a considerable part of industrial machinery will require a constant-speed service, for which the alternating-current motor is admirably adapted, and should direct current be required it can be obtained by installing a motor-generator set.

As a rule it may be said that the alternating-current system should be selected if possible. This would furthermore permit of throwing over to a central-station service, in case it should be found that power could be more economically purchased from the central station than generated on the premises.

CHARACTERISTICS OF MOTORS AFFECTING THEIR APPLICATIONS. — (*See also the separate articles on Motors.*) Motors are divided into two classes — direct-current and alternating-current, according to the system from which they are operated. The direct-current motors are further subdivided into three types, namely, series, shunt and compound motors.

There are also three general types of alternating-current motors, namely, induction, synchronous and commutator motors.

Series Motor. — This motor is used when a powerful starting torque and rapid acceleration are required, without an excessive instantaneous demand of energy. The torque is practically independent of the voltage and at low flux densities varies directly as the square of the current, but as the magnetization approaches saturation it becomes more nearly proportional to the first power of the current. The maximum torque exists at low speed, this being the most valuable feature of the series motor. Dangerously high speeds may be attained by the armature with very light loads, and series motors should for this reason be either geared or direct connected to the load.

Speed Control of Series Motor. — The speed of a series motor on constant potential varies automatically with the load, increasing as the load decreases. The speed may, however, be adjusted if some means of varying the impressed voltage is provided. As the work required of a series motor is very often intermittent in character, the insertion of resistance in the armature circuit to reduce the speed is permissible from an economic standpoint in such cases. In others, such as railway work, where two or four motors are used, reduced voltage is most readily and economically obtained by connecting the motors in series or in series-parallel.

Shunt Motor. — This motor has good starting characteristics and a practically constant speed, varying only slightly with load changes. The speed can, however, be adjusted, either by changing the e.m.f. impressed on the armature, or by changing the field flux.

Speed Adjustment by Armature-voltage Control, i.e., by changing the e.m.f. impressed on the armature, does not change the full-load torque which the motor is capable of exerting, since the rated torque depends only upon field flux and rated armature current. These methods are therefore constant-torque methods and are properly adapted to loads in which the torque remains constant regardless of speed. The method most generally used for varying the impressed e.m.f. with a single-voltage system is by means of inserting resistance in series with the armature. The efficiency with this method is, of course, very low at slow speeds. The speed regulation with varying loads may also be very poor.

There are several systems of controlling the motor speeds by applying different voltages, such as by the use of three-wire generators or two-wire generators with balancer sets or by the Ward Leonard system. This latter system, which is the most practical, consists of a constant-speed motor driving a generator which supplies current to the motor whose speed is to be adjusted. This arrangement is very satisfactory but on account of the expense of providing three full-sized machines instead of one to perform the work, the cost may be prohibitive except with very large motors, such as for hoists, etc.

Speed Adjustment by Shunt-field Control, i.e., by inserting resistance in the shunt-field circuit, is the simplest of all methods of speed variation, but with ordinary shunt motors, the range of speed variation by this means is small. Where a variation of more than from 20 to 30 per cent is desired, a motor of modified design and of a certain increased size is generally required, because the field must be more powerful with respect to the armature than in the case of standard single-speed motors. Variable-speed motors of the field-weakening type are not constant torque, but constant-output motors, i.e., the torque falls proportionally as the speed increases.

A speed variation up to 3 to 1 meets, as a rule, all requirements and such motors can readily be obtained in commercial sizes. Should a greater speed variation be desired, say 4 to 1 or 5 to 1, it is possible to accomplish this by the commutating-pole shunt motor with field control only. A combined field and armature control would, however, be a better method.

Compound Motor. — This motor is provided with both a series and a shunt field. The two fields are usually connected so that they act in the same direction, in which case the motor is called a "cumulative" compound motor. "Differential" compound motors, with the two fields opposing, are sometimes employed for special services. The cumulative, or ordinary, compound motor combines the characteristics of the shunt and series motors, having a speed not extremely variable under load changes, but developing a powerful starting torque and an increasing torque with decreasing load. Motors having a comparatively weak series field are employed extensively in shop practice where the motor may be required to start under heavy load but must maintain an approximately constant speed after starting, or when the load is removed. The heavily compounded motor is used where powerful starting torque and rapid acceleration are necessary, with a speed not varying too widely under load changes, such as for rolling mills, etc.

The speed control employed with compound motors may be any of the various methods explained in connection with the shunt motor. For certain service the control may be entirely rheostatic, the series winding being cut out after the motor has come up to speed.

Induction Motor. — The induction motor is essentially a constant-speed machine, although the speed may be varied either by varying the applied stator frequency or by introducing resistance in the rotor circuit. It is built in two distinct types, namely, the squirrel-cage and the phase-wound.

Squirrel-cage Motor. — The squirrel-cage type is used for constant-speed service with infrequent starting. It has a relatively small starting torque per ampere and draws a large starting current from the line. By increasing the resistance of the rotor, it may however also be built in the smaller sizes for a high starting torque, rapid acceleration and frequent starting, for such applications as sugar and laundry centrifugals, etc., where simplicity of control is desirable. They are also used for operating punches, shears, etc., where a flywheel is provided for storing the energy.

Induction Motor with Wound Rotor. — For service requiring high starting torque combined with moderate starting current a motor with the wound

type of rotor is best adapted. A motor with the resistance mounted inside the rotor should not be used to operate machinery having large inertia or excessive static friction, since full starting current may be required for a long period before the apparatus attains full speed, and, as the capacity of the internal resistance is small, excessive temperatures may result. This type of motor is, as a rule, not built above 200 horse-power due to mechanical difficulties involved in connection with the internal resistance.

A motor with external resistance should be used for moderate and large sizes. The rotor must then be provided with collector rings and brushes. The contact resistance of these as well as the leads and the controller fingers, which are in the circuit all the time, may impair the efficiency and regulation of the motor, especially if the controller and the resistance are located some distance from the motor. The phase-wound induction motor with an external variable rotor resistance is best adapted for a variable-speed service, as the losses necessary to obtain reduced speeds are external to the motor itself.

Multi-speed Induction Motors. — It often happens that the service is such that two or three speeds will be satisfactory for the operation of the machinery and that these speeds must be independent of the load. Under such conditions multi-speed motors can frequently be used. In these motors the different synchronous speeds are produced by changing the number of poles in the magnetic circuit. Each of these speeds is fixed, if no resistance is used in the secondary circuit. With multi-speed motors, as with single-speed motors however, resistance may be used in the secondary circuit for varying the speed.

A change of the number of poles may be produced in any of the following ways:

1. By the use of single magnetic and electric circuits, changing the number of poles by regrouping the coils.
2. By the use of single magnetic circuits and independent electric circuits.
3. By means of separate magnetic and electric circuits, the so-called Cascade connection.

Synchronous Motor. — The speed of a synchronous motor is constant, being fixed by the number of poles and the frequency of the applied voltage. The single-phase type is not self-starting and the polyphase type has in itself a very poor starting torque. They may, however, be made self-starting in the same manner as squirrel-cage induction motors, by the use of an ammortisseur or cage winding, similar in construction to that used for induction motors.

The speed-torque curve of a synchronous motor is similar to that of an induction motor except that the torque values are lower for a given resistance of rotor winding on account of the construction of the machine. The starting winding must be designed with both the load at start and the load at synchronous speed in mind, because too great a slip may cause the motor to shut down when the field is put on. It is, however, seldom that the same motor will be called upon to start a heavy load and at the same time synchronize a heavy load, as the load usually consists principally of either static friction, as in the use of motor-generator sets, line shafting, etc., or it comes up with the speed as in the case of a fan blower or centrifugal pump. The former case would be met by a high-resistance squirrel-cage winding and the latter would require a low resistance.

Single-phase Series Motor. — This type of commutator motor has a very powerful starting torque, high power factor and relatively high efficiency. It is most generally used for traction work, the speed being controlled by varying the applied voltage, which can most readily be done by means of an auto-transformer with a number of taps.

Repulsion Induction Motor. — This type of commutator motor has a limited speed and an increase of torque with decrease in speed. The action of the compensating field insures a power factor approximately unity at full load and closely approaching unity over a wide range in load. In addition it serves to restrict the maximum no-load speed and also permits, where varying speed service is involved, an increase over the synchronous speed.

Starting of Repulsion Motors. — A repulsion motor, if started by directly closing the line switch, will develop about 2½ times full-load torque. The starting current corresponding to full-load starting torque is from 2 to 2¼ times full-load running current. As a general rule, starting boxes are not required up to and including 2 horse-power rating. From 2 to 5 horse-power the use of a rheostat is optional, dependent upon the degree and care to be exercised in maintaining voltage regulation. Starting boxes should, however, preferably be used on sizes above 5 horse-power, especially where light and power circuits are combined.

Reversible Repulsion Motors. — The repulsion motor may be designed for reversible service. This is accomplished by adding an auxiliary reversing winding spaced 90 degrees from the main field winding and connected in series with it. By reversing the relative polarity of the two windings, the direction of rotation is changed in a simpler manner than by mechanical shifting of the brush holder yoke. Instant reversal may be effected from full speed in one direction to full speed in the other, about 200 per cent of normal running torque being developed at moment of speed reversal in either direction.

Variable-speed Repulsion Motors. — In addition to the constant-speed repulsion motor, two other types are also available, one for constant-torque and variable-speed service, the other for adjustable speed independent of torque. In general, variable-speed repulsion motors are not applicable to lathes, boring mills or similar machines where the service requires adjustable speed and constant horse-power at all speeds below and above normal. When a certain amount of variable speed is required at approximately constant torque, such as driving fans, blowers, printing presses, etc., the repulsion motor successfully meets a wide field of application.

POWER RATING OF MOTORS. — (*See also Standardization Rules of the A.I.E.E.*) These rules recommend that with the exception of railway motors, all motor ratings shall be expressed in kilowatts (Kw.) available at the shaft. On account of the hitherto prevailing practice of expressing mechanical output in horsepower, it is, however, also recommended that for machinery of this class the rating may, for the present, be expressed both in kilowatts and in horsepower, as follows: Kw. ——— h.p. ———. The horsepower rating of a motor may for practical purposes be taken as 4/3 of the kilowatt rating.

It is also highly desirable that the motor ratings should closely conform to the actual service requirements, and for this reason the Standardization Rules also recommend the following two kinds of ratings:

1. **Continuous Rating,** when the motor shall be able to operate continuously at its rated output, without exceeding any of the guaranteed limitations.

2. **Short-Time Rating,** when the motor shall be able to operate at its rated output during a limited period, to be specified in each case, without exceeding the guaranteed limitations. Such service includes runs alternating with stoppages of sufficient duration to ensure substantial cooling.

CLASSIFICATION OF MOTORS ACCORDING TO SPEED. — Motors for industrial application may be conveniently classified according to their speed characteristics. The wording of the classification as adopted by the American Association of Electric Motor Manufacturers is as follows:

A. — Constant-speed Motors, in which the speed is either constant or does not vary materially, such as synchronous motors, induction motors with small slip, ordinary direct-current shunt motors, and direct-current compound-wound motors, the no-load speed of which is not more than 20 per cent higher than the full-load speed.

B. — Adjustable-speed Motors. — 1. Shunt-wound motors in which the speed can be varied gradually over a considerable range, but when once adjusted remains practically unaffected by the load, such as motors designed for a considerable range of speed by field variation. 2. Compound-wound motors in which the speed can be varied gradually over a considerable range, as in 1 and, when once adjusted, varies with the load, similar to compound-wound constant-speed motors or varying-speed motors, depending upon the percentage of compounding.

C. — Varying-speed Motors, or motors in which the speed varies with the load, decreasing when the load increases, such as series motors and heavily compounded motors. Examples of heavily compounded motors are those designed for bending roll service and mill service, in which a shunt winding is provided only to limit the light-load operating speed.

D. — Multi-speed Motors (two-speed, three-speed, etc.), which can be operated at any one of several distinct speeds, these speeds being practically independent of the load, such as direct-current motors with two armature windings and induction motors with primary windings capable of being grouped so as to form different numbers of poles.

DATA REQUIRED TO DETERMINE TYPE AND SIZE OF MOTOR. — In selecting a motor for a certain application complete information must be had with regard to the machines to be driven. This is the first step and in some respects one of the most important parts of the problem, as without complete information on the subject it becomes very difficult, and in certain instances utterly impossible, to intelligently select or design a motor and control equipment which will satisfactorily fulfill the conditions to be met in actual operation. In order to facilitate the work of obtaining such information, an outline of the points to be investigated is given below.

1. Description of Machine to be Driven. —

a. Individual or group drive.

b. Photographs, drawings and sketches of machines and connections as complete as advisable, especially such drawings as indicate the size of heavy flywheels or rotating masses whose speed must be varied, and also drawings indicating gearings, transmission devices, etc.

c. Limiting features of product. What are the points in quality and characteristics of product which fix the condition of operation? e.g. limiting speed of tool or mechanism, etc. *Example:* A 26-inch lathe used for finishing requires approximately 3 horse-power to drive. The same lathe for roughing shafting, when equipped with high-speed tool steel and two tools, needs approximately 30 horse-power.

d. Can electric drive approach nearer the conditions wanted?

e. If group drive can be considered for several machines, some of which are only to be operated intermittently, give notes with regard to the latter machines.

f. What arrangements can be made so that intermittently-operated machines need not operate simultaneously, thus giving a smoother load curve and allowing the use of one small motor to drive several intermittent machines?

2. **Cycle of Operation.** —

a. Starting condition. Torque at start of day, and start of each cycle. Is frequent starting and stopping necessary? *Example:* The adjustment of large boring mills affects the controller, etc. Starting torque may be measured approximately if necessary by adjusting a rough beam to shafting. Find distance from axis at which the weight just starts machine. Knowing the weight in pounds the torque may be calculated.

b. Curves of loads, speeds, torque, maximum and average conditions through one complete cycle. Where necessary note power required with machines both loaded and unloaded. From this may be determined losses in shafting and transmission.

c. Time in operation, days per year, per month, per week, hours per day.

d. Reversing conditions, their frequency, their time, full or partial. Some apparatus, e.g., lathes, printing presses (flat-bed type), etc., only require slight backward movement to release tool or to adjust cylinders.

e. Starting. Flywheel or line shafting, friction clutches, etc.

f. Are speeds, torques, accelerations, etc., fixed by conditions of work or of driving machine?

g. Can any parts of cycle be varied with benefit? This should be investigated carefully.

3. **Present Method of Drive.** —

a. Prime mover or source of power.

b. Method of speed variation and speed change or control. Must machine be shut down to change speed?

c. Adaptability to gearing, etc.

d. Speed of machine, size of pulley, size of belt, size of gear or chain.

4. **Mechanical Transmission.** —

a. Rope, belt, chain, gear, direct connection. Direction of pull on belt or gear. Is belt or chain pull on the top or bottom? Is belt tightner advisable?

b. Clutches. Crab clutch, friction or couplings. Rigid, flexible or insulated.

c. Method of speed variation.

d. Brakes, electric. Solenoid or magnetic. Regenerative control, dead load or pumping back.

e. Brakes, mechanical. Band-post brake, disc brake, automatic safety brake, steam, hydraulic, air- or hand-operated.

5. **Conditions of Location.** —

a. Near external source of heat, furnace, etc.

b. Character of dust, conductive, magnetic or wearing, marble or stone, etc.

c. Possibility of fire or explosion to be caused by sparks from motor. Combustible flyings, e.g., cotton mills.

d. Explosive gases. Coal gas, benzine fumes, etc.

e. Presence of injurious gases. Acid fumes or salt air, e.g., SO_2, Cl, etc.

f. Dampness and moisture.

g. Insurance regulations. Obtain and forward copy of State Insurance Rules, or other state or local regulations, e.g., mining laws, municipal laws. Consider also personal danger or liability due to use of high-voltage apparatus.

h. Accessibility for inspection and repairs, oiling, etc.

i. Ventilation.

j. Allowable space for installation, also space for transportation. Down mine shaft, through doorways, etc.

k. Foundations and how attached to same.

l. Sudden temperature variations. This may cause condensation of moisture on windings and insulation.

m. If controlled from distance where will controller or switch be placed? Give approximate length of leads necessary. Must they pass through or under water, in building, on poles, underground?

6. Control. —

a. Intelligence (probable) of operators.

b. Is entire range of operation visible to operator or are special automatic features desirable?

c. Regenerative Control — See item No. 4-*d;* also *Cycle of Operation,* item No. 2.

7. Overload and Safety Devices. —

a. Can electric automatic devices be made to supersede mechanical overload safety devices, such as slipping clutches, braking shafts or crabs?

b. Should these devices be time limit or instantaneous? If former, how long?

c. Are safety devices necessary, other than those to protect overload — e.g., overrun of hoist, stoppage or cessation of load. Must emergency stop be employed, and where placed?

d. Are no-voltage releases required or advisable?

8. Probable Cost of Present Method of Operation. —

a. *Steam Operation.*

1. Coal. Tons per day or month; quality, cost, distance shipped.
2. Feed and other water. City mains or pumped; source, quality.
3. Is steam required for other purposes beside power? e.g., digesters, heating, etc.
4. Can steam be generated from waste gases or other waste products?
5. Distance of transmission, outdoors and indoors.

b. *Air.* Distance of transmission, air pressure, how obtained?

c. *Water Power.* Has electric installation to compete with water power? If so, report separately on this aspect.

9. Strength of Present Equipment. —

a. Strength of line shafting, foundation, transmission or gearing, machine parts.

b. Give estimate of limiting horse-power of items in (*a*), using sketch and dimensions where necessary. The above items should be considered with regard to any change in speed or torque made necessary or desirable, due to the electric drive.

10. Generating Station or Source of Electric Power. — Capacity of station or feeder, frequency, voltage, location, voltage or frequency variations.

BIBLIOGRAPHY. — It is impossible in the limited space available to give references to all the numerous papers in the technical journals dealing with the industrial applications of motors. The reader should consult the files of the *Transactions of the American Institute of Electrical Engineers, American Society of Mechanical Engineers, National Electric Light Association; also the Electrical World, General Electric Review, Electric Journal, Electrical Review, Power, Power and Engineer, Electrical Engineer, Engineering Magazine, Cassiers' Magazine.* A few recent papers are listed below.

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[D. B. RUSHMORE, assisted by E. A. LOF.]

MOTORS, POLYPHASE INDUCTION. — (*See also Electricity and Magnetism, Principles of; Generators; Motors, A-C. Commutator, Direct-current, Single-phase Induction and Synchronous; Motors, Industrial Applications of; Standardization Rules; Transformers.*) An induction motor may be either single, two, or three phase. Single-phase induction motors are treated in another article (*q.v.*). The induction motor is essentially a polyphase transformer with the secondary free to move; the electric energy transferred to the secondary is transformed by this motion directly into mechanical energy.

The following is a brief table of the contents of this article:

DEFINITIONS AND PRINCIPLE OF OPERATION. — Certain terms used in connection with the induction motor can best be defined by a brief statement of the principles underlying its operation.

Primary and Secondary. — By the primary of an induction motor is meant that part which receives energy by direct connection to the source of electric energy; the other member is called the secondary.

Stator and Rotor. — That member of an induction motor which remains stationary, whether it be the primary or secondary, is called the "stator," and the revolving member is called the "rotor." In most machines the primary is the stator. A "squirrel cage" rotor is one in which the conductors are straight bars of copper all connected together at each end of the rotor by copper rings.

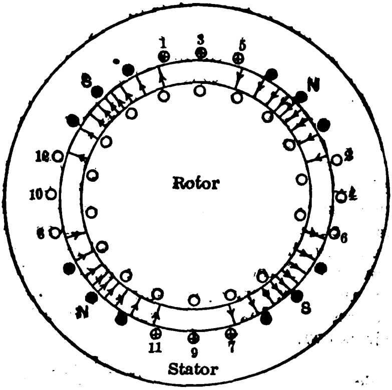

Fig. 1. Elementary Induction Motor

Poles of an Induction Motor. — In Fig. 1 is given a diagram of the primary winding of a two-phase induction motor. The small numbered circles represent the conductors forming the winding of one phase, the small black circles the conductors forming the winding of the second phase. The numbers opposite the circles give the order in which the current in phase 1 passes through the conductors, a cross indicating that the current goes down into the page and the open circles that the current is coming up.

The diagram is drawn to represent that instant at which the current in the second phase is zero. At this instant the distribution of flux in the air gap will be roughly as indicated by the lines with arrows on them; that is, the flux will leave the stator iron in the two regions marked N and enter it in the two regions marked S. Consequently, the current in the winding of phase 1, which consists of 4 bands or groups of conductors, will produce 4 polar regions, or 4 poles, on the stator. As will be shown below, the combined effect of the two phase currents in the two windings is merely to cause a rotation of these polar regions. The "number of poles" is always equal to the number of bands of conductors into which the total winding of each phase is divided. The bands of conductors forming one phase usually overlap the bands of conductors forming the other phase, there being then two or more conductors per slot.

Rotation of Magnetic Flux. — In Fig. 1 is shown the distribution of flux in the gap when the current in phase 2 is zero; the curve marked A in Fig. 2 represents this same state of affairs, the cylindrical surface of the stator here being bent out into a plane, and the ordinates of the curve giving the value of

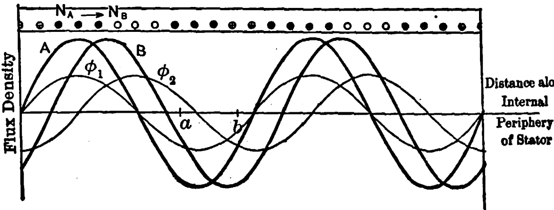

Fig. 2. Rotating Field

the flux density in the gap at each point of this surface. The flux distribution is not a smooth curve as shown, but approximates such a curve.

Next consider the case when the current in phase 1 has decreased to, say, 0.7 of its maximum value and the current in phase 2 has increased to 0.7 of its maximum value (this corresponds to ⅛ of a cycle). Then the flux distribution due to phase 1 remains in the same position as before but is reduced at each point of the gap to 0.7 of its maximum value, i.e., reduces to the curve marked ϕ_1. Similarly, the flux due to the current in phase 2 is similar in shape to the flux due to the current in phase 1, but its position is to the right of the latter by an amount equal to the width of one of the bands of conductors. The distribution of the flux due to the current in phase 2 is then as shown by the curve ϕ_2, the ordinates of which at the instant under consideration are 0.7 of their maximum values.

The resultant flux in the air gap at this instant is then the sum of the curves ϕ_1, and ϕ_2, namely the curve B. That is, the effect of the two fluxes due to the two phases is a resultant flux shifted forward, or moved around the gap, a distance equal to ⅛ the distance between successive north poles, but this resultant flux curve has the same shape and maximum value as before. This is strictly true only if the windings are distributed with absolute uniformity over the internal periphery of the stator. An extension of this analysis will show that the resultant flux remains constant in value at all times but travels around the air gap with a speed of

$$N = \frac{120f}{p} \text{ rev. per min.,}$$

where f is the frequency of the supply and p the number of poles.

This same result holds for a three-phase machine.

Synchronous Speed. — The speed of rotation of the air-gap flux, namely the speed N given by the above formula, is called "synchronous" speed. At light loads the speed of the rotor is very nearly equal to this speed.

Slip. — The slip of an induction motor is the ratio of the difference between the actual speed (N_1) of the rotor and synchronous speed (N) to the synchronous speed (N), i.e.,

$$s = \frac{N - N_1}{N}.$$

The slip may be expressed as a fraction or as a per cent. The slip at standstill is unity; at no load it is very nearly zero. An induction motor driven at a speed higher than its synchronous speed has a negative slip; such is the case in an induction generator.

Electromotive Forces in Secondary. — The electromotive forces in the secondary of a polyphase induction motor are induced by the rotation of the flux produced by the currents in the primary windings, just as the electromotive forces induced in the armature conductors of a generator are induced by the rotation of these conductors in the magnetic field set up by the field winding. In the generator only the conductors move, the field being stationary, but in the induction motor both the field and conductors move. In either case it is the *relative* motion of the field and conductors which determines the e.m.f. induced.

Let v be the linear speed at which the field moves, and let v_1 be the linear speed of the rotor conductors, and B the flux density at any particular conductor C at any instant. Then the e.m.f. induced in this conductor at this instant is $B(v - v_1)l = Blsv$, where s is the slip and l is the length of the conductor (*see also Electricity and Magnetism, Principles of*). The rotor electromotive force is therefore proportional to the slip. As the rotor turns, the conductor C moves slower than the rotating flux, and the state of affairs is just the same as if the flux remained at rest and the conductor moved through the field in the gap at a speed of $v - v_1 = sv$. Hence the electromotive force induced in each rotor conductor is alternating, since the flux which it cuts varies from a positive maximum to a negative maximum, and a consideration of the relative speed of the conductor and the flux will show that the frequency of this induced electromotive force is the frequency in the primary multiplied by the slip.

That is, the secondary electromotive force is proportional to the slip and has a frequency equal to the product of the slip by the frequency in the primary.

Secondary Current and Torque. — The current set up in each rotor conductor by this electromotive force will be practically in phase with this electromotive force, since the rotor conductors have but a small reactance, particularly when the rotor is revolving at a speed near synchronism. Hence the current in any chosen rotor conductor at any instant is proportional to the electromotive force in this conductor at that instant, which in turn is proportional to the flux density at this conductor at this instant. Since the force produced by a magnetic field on a conductor is equal to the product of the flux density by the current by the length of the conductor (*see Electricity and Magnetism, Principles of*), the force acting at any instant on any rotor conductor will then be equal to

$$f = Bli = Bl\left(\frac{Blsv}{r}\right) = \frac{B^2l^2sv}{r},$$

where r is the resistance of the conductor. Since the current and the flux density both change signs at the same time (being in phase) the direction of this force will always be in the same direction and will consequently drive the rotor against whatever opposing force may exist.

Not only is the force on each conductor always in the same direction, but the *total* force acting on *all* the conductors is practically constant for a given value of the slip. Consider any two rotor conductors which are a distance apart equal to ¼th the distance measured along the periphery of the rotor between successive north poles, for example, at a and b in Fig. 2. Then, assuming a sine-wave distribution of flux in the air gap, and calling B_m the maximum flux density, the flux density at a is $B_a = B_m \sin x$ and the flux density at b is $B_b = B_m \cos x$, where x is a function of the distance measured from some fixed point in the air gap. Then the total force on the two conductors at a and b is

$$\frac{l^2 s v B_m^2}{r}(\sin^2 x + \cos^2 x) = \frac{l^2 s v B_m^2}{r}$$

and is therefore constant, since B_m is a constant. Similarly for any other two conductors this same distance apart. Hence the total force on all the conductors is constant. On any practical machine the flux distribution is not an exact sine wave, and there is a slight pulsation in the total force, and therefore in the torque, but this pulsation is extremely small.

Magnetizing Current. — The currents set up in the secondary of an induction motor produce a rotating flux, which travels with the same speed with respect to the primary and in the same direction as the flux set up by the current in the primary, but the direction of this secondary flux at any point in the air gap is opposite to the direction of the primary flux. Hence the resultant flux when there is current in the secondary is equal to the difference of these two fluxes, and this difference remains practically constant irrespective of the secondary currents, just as the resultant flux in a transformer is practically independent of the secondary current. The primary current which would be necessary to produce this *resultant* flux is called the "magnetizing" current, and is very nearly equal to the current in the primary when the motor is running without load, in which case the current in the secondary is extremely small.

METHODS OF RATING. — The Standardization Rules of the A.I.E.E. up to 1914 recommended that the rating of an induction motor should be the load in horse-power which it will deliver continuously at the shaft with a maximum rise in temperature of any part not exceeding 50° C. by thermometer. In commercial practice three variations of this have been developed to suit different conditions, as noted below. See, however, the new ratings recommended in the proposed rules of 1914, given in the article on *Standardization Rules of the A.I.E.E.*

A-Rating. — For cases where there are no excessive overloads and where the load is fairly steady, it is customary to guarantee that the motor will operate continuously at its rated load with a maximum rise in temperature of 40° C., and that subsequently it will deliver a load 25 per cent greater than the rated for two hours with a maximum rise in temperature not exceeding 55° C.

B-Rating. — For cases where there are frequent overloads and for intermittent service, that is, a low load factor, it is customary to guarantee that a motor will deliver its rated load continuously with a rise in temperature not to exceed 35° C., and that it will deliver an overload of 50 per cent for two hours with a maximum rise in temperature not exceeding 55° C.

One-hour Rating. — Certain motors for special intermittent work, as for hoists, elevators, etc., are rated in accordance with the output they will give for one hour.

Starting and Break-down Torque. — In addition to the ability to carry its rated load without excessive heating and with reasonable constants, such as

efficiency, power factor and slip, it is advisable to make sure that the motor is able to start such loads as must be brought up to speed with the motor, as good starting ability in an induction motor involves certain complications and expenses. This subject is treated at length in the section below on *Methods of Starting*. Another important point is that the motor should be able to carry momentary overloads without "breaking down" as it is called, which means gradually decreasing in speed to a standstill when the load is excessive. To be sure of this qualification we must know the maximum output of the motor, which should be at least 50 per cent greater than the rated output.

VOLTAGE. — Motors may be wound for any voltage up to 13,000 but the great majority and all the small motors are wound for voltages of 110, 220 or 440 volts between lines.

FREQUENCY AND SPEED. — Induction motors may be built for any frequency. The higher frequencies are satisfactory in those cases where the load never exceeds normal conditions. Lower frequencies, such as 25, are more favorable where frequent overloads are met with or large starting torques are required.

The speed of the rotor of an induction motor at normal loads approaches within 5 to 10 per cent of the synchronous speed. The synchronous speed is fixed by the frequency of the system and the number of poles of the winding (*see above*) so that for a given frequency of the supply circuit there are only certain speeds available. Thus for 25 cycles we have

1500 for 2 poles;	500 for 6 poles;
750 for 4 poles;	375 for 8 poles, etc.

PHASE CONNECTIONS. — Two-phase or quarter-phase motors are usually wound with independent phase windings. Three-phase motors are connected in Y or Δ, depending upon the convenience of the designing engineer.

In a single-phase and two-phase motor the voltage and current per phase are the same as the voltage between lines and current in line; in a Y-connected three-phase motor the current per phase is equal to the line current, and the voltage per phase is equal to the line voltage divided by $\sqrt{3}$; in a Δ-connected three-phase motor the current per phase is equal to the line current divided by $\sqrt{3}$, and the voltage per phase is equal to the line voltage.

CURRENTS TAKEN BY MOTORS. — Let

P_0 = horse-power output;
I = current in each line;
ϵ = efficiency as a decimal fraction;
$\cos\phi$ = power factor as a decimal fraction;
E = voltage between lines (between one outside wire and the middle wire for three-wire two-phase line).

Then for

$$\text{Two phase: } I = \frac{373\,P_0}{\epsilon E\cos\phi};$$

$$\text{Three phase: } I = \frac{431\,P_0}{\epsilon E\cos\phi}.$$

Usual efficiencies and power factors at full rated load for polyphase motors are as follows:

POWER FACTORS AND EFFICIENCIES

Horse-power	25 cycles		60 cycles	
	Efficiency	Power factor	Efficiency	Power factor
1	0.79	0.78	0.78	0.78
5	0.85	0.88	0.82	0.88
20	0.88	0.91	0.84	0.91
50	0.90	0.92	0.87	0.92
100	0.905	0.925	0.89	0.92
200	0.91	0.925	0.905	0.92

DESIGN. — The methods of calculating two-phase and three-phase motors are practically the same. Most induction motors on single-phase circuits are made with polyphase windings, as the extra winding is necessary in starting.

The factors which must be considered in the design of an induction motor are the same as those considered for a synchronous generator with the addition of the power factor. It is desirable to have a high power factor but a high power factor requires a generous use of material, a small air gap, and a careful arrangement of windings. A high power factor at light load requires a small magnetizing current, and a high power factor at overloads requires a low value of leakage flux.

A small value of magnetizing current is obtained by using a small air gap and a large value of diameter per pole.

A low value of leakage flux is obtained by using a large value of diameter per pole and by subdividing the windings in a large number of slots.

Preliminary Choice of Main Dimensions. — In the discussion below the following symbols are employed:

E = volts per phase;
I = full-load current per phase;
D = diameter of armature, in inches;
L = length of armature, in inches;
p = number of poles;
f = frequency in cycles per second;
g = length of gap, in inches;
B = average flux density in gap, in lines per square inch;
ϕ = flux per pole, in maxwells;
S = turns in series per phase;
N = revolutions per minute;
α = quotient of no-load current divided by full-load current (ranges from 0.45 for a 1-horse-power motor to 0.25 for a 200-horse-power motor);
σ = ampere-conductors per inch of periphery. The values given in the article on *Generators, Alternating-Current*, also apply to the induction motor.
T = ratio of width of tooth at face to slot pitch (T varies from 0.6 to 0.7 for open slots and from 0.9 to 1.0 for overhung slots);
q = ampere-conductors per slot.

Diameter and Length of Armature. — The first problem in the design of an induction motor is the estimation of the proper diameter of armature. There are three methods of determining the proper value of this very important dimension:

(a.) Reference to machines already built, which shows that the diameter varies directly with the number of poles and inversely as the frequency. Thus for customary values of diameter in inches divided by the number of poles:

for 25 cycles, D/p = 5 to 6 inches;
60 cycles, D/p = 2.5 to 3 inches.

(b.) The peripheral speed, being a direct function of the diameter and speed, determines the diameter. As the value of the peripheral speed may vary from 3000 to 10,000 feet per minute, depending on the mechanical construction, this function is not very definite. For lack of more definite figures, 5000 feet per minute may be considered a good average figure for peripheral speed.

(c.) On account of the effect of the magnetizing current on the power factor it is desirable to limit the value of this current to a certain percentage of full-load current (*for values see p. 988*). To accomplish this there is a certain minimum limit to D and L which is expressed approximately in the equation

$$D^3L = \frac{28.7 \times 10^{12} fgk}{\alpha \sigma^2 N^2} \times \text{(Kv-a. rating)}.$$

k = ratio of the actual excitation current to the excitation current for the air gap; $k = 1.2$ for 60 cycles and $k = 1.4$ for 40 cycles.

This gives a relation between D and L. Assuming that for best economy L is equal to the pole pitch $\left(L = \frac{\pi D}{p}\right)$ a value for D is obtained. This formula also indicates the effect on the general design which would result from radical changes in any of the quantities.

The formula given for the preliminary calculation of dimensions of an alternating-current generator (*see article on Generators, Alternating-Current*) may also be used if proper values for T and L be selected.

Air Gap (g) varies in length according to the diameter, usual values being given in the accompanying table.

Diameter, inches	Air gap, inch
0 to 12	0.02 to 0.03
12 to 26	0.03 to 0.04
26 to 50	0.04 to 0.06
50 to 100	0.06 to 0.10
100 up	Diameter/1000

Average Flux Density in Gap (B). — The usual value of the average flux density in the air gap for 25 cycle machines is 30,000 lines per sq. in. and for 60 cycle machines is 25,000 lines per sq. in.

Conductors per Slot (c). — The "effective" number of conductors per slot is the number of conductors *in series* per slot, i.e., if each phase is made up of two windings in parallel, the two conductors of the two parallel windings are counted as one conductor. Hence the effective conductors per slot are

$$c = \frac{2S \times \text{(Number of phases)}}{\text{(Total number of slots)}}.$$

The permissible number of ampere conductors per slot (q) depends upon the permissible current density per square inch of copper, usual values of which are

Size of motor	Voltage	Amperes per sq. in.
Small	Low	3000
Small	High	2000
Large	Low	2000
Large	High	1600
Large	Above 6000	1000

Usual values of q are

Size of motor h.p.	Value of q
Up to 5	Up to 250
5 to 50	250 to 350
50 to 100	350 to 450
100 to 200	450 to 600
Greater	Up to 800

Number and Dimensions of Slots. — The number of slots is

$$\frac{\pi D\sigma}{q}.$$

The conductors should be arranged to give slots having a depth about four times the width, and the width of slot should be about ½ to ⅔ of the pitch of slots at the gap. Machines of small diameter will have slots smaller with respect to the pitch and machines of large diameter will have slots occupying more than ⅔ of the pitch.

The slots of one member at least (usually the rotor) should be overhung, i.e., partly closed at the opening. It is better if both members have partly closed slots, as this reduces the magnetizing current and improves the power factor, but it entails a more expensive method of winding.

The dimensions of the slots are determined by the size of conductors and amount of insulation. The allowance to be made is about as follows:

Type of slot	Voltage	Coil sides per slot	Allowance for insulation	
			Vertical	Horizontal
			in.	in.
Straight	500	2	0.30	0.04
Straight	2200	2	0.45	0.15
Straight	6000	2	0.65	0.25
Overhung	500	4	0.40	0.15
Overhung	2200	4	0.50	0.20

The depth of the slot is found by adding to the total depth of cotton-insulated copper the vertical dimension given in the table.

Turns in Series per Phase (S) may be calculated from the formula

$$S = \frac{\pi D \sigma}{2I \times (\text{number of phases})}$$

or from the formula

$$S = \frac{10^8 E}{4.44 \phi f k},$$

where ϕ is the flux per pole and is given by the formula

$$\phi = \frac{0.7 \pi BDL}{(\text{number of poles})}$$

and k is a constant depending on the wave form and winding distribution (*see next paragraph*). The value of S as obtained from these two formulas must check in the final design.

Flux per pole (ϕ). — The flux per pole is given by the formula

$$\phi = \frac{10^8 E}{4.44 f S k},$$

where k, called the distribution constant, has the values given in the following table, provided the flux distribution in the air gap is sinusoidal.

DISTRIBUTION CONSTANT k

Two phase					Three phase				
Slots per pole	Per cent winding pitch				Slots per pole	Per cent winding pitch			
	100	75	67	50		100	75	67	50
	k	k	k	k		k	k	k	k
2	1.00	....	....	0.71	3	1.00	....	0.87	....
4	0.93	0.85	....	0.66	6	0.97	....	0.84	0.69
6	0.91	....	0.79	0.64	9	0.96	....	0.83	0.68
8	0.905	0.84	....	0.64	12	0.96	0.89	0.83	0.68
12	0.90	0.836	0.78	0.63	18	0.958	0.885	0.83	0.68
Many	0.90	0.833	0.78	0.63	Many	0.958	0.885	0.83	0.68

Choice of Phase Connection. — The decision whether a motor shall be Δ or Y connected depends on such minor details of design as the convenience of arranging the conductors in the slots. For instance, if for 110 volts and Δ connection a desirable flux value and number of conductors would require 7 conductors per slot, a Y connection having 64 volts per phase and 4 conductors per slot could be substituted and would give a practical winding. 64 volts per phase, Y connected, gives 110 volts between lines.

Magnetic Circuit. — A tentative layout of the magnetic circuit is next made in the same manner as described in the article on *Generators, Alternating*.

Current. The values there given for usual values of the flux density also apply to the *maximum* instantaneous flux density in the magnetic circuit of an induction motor. The magnetic circuit must have such dimensions that the exciting current will not be too large, and the slots must be of sufficient size to accommodate conductors of necessary size (*see above*).

Maximum Efficiency and Power Factor. — Since induction motors frequently operate on an intermittent or variable load, it is desirable that the efficiency and power factor be high at fractional loads. It is therefore quite usual to design the motors so that the maximum efficiency comes at ¾ load and maximum power factor at less than full load. This is accomplished with regard to efficiency by making the core-loss and friction small (the core-loss is made small by using low flux densities) and with regard to power factor by making the magnetizing current small by using a small air gap.

PREDETERMINATION OF PERFORMANCE OF AN INDUCTION MOTOR FROM ITS DIMENSIONS. — From the above calculations a preliminary drawing of the motor to scale may be laid out. The next step is to calculate its performance, i.e., predetermine what will be the efficiency, the power factor, and the temperature rise in the various parts. Examples of specific design and tested performance are given below.

Calculation of Exciting Current. — The first step is the calculation of the exciting current. This current is practically constant at all loads, and is equal to the no load current, i.e., the current taken by the motor when it is running light. The exciting current has two components, the magnetizing current, which leads the induced voltage by 90° and a component in phase with the induced voltage, which supplies the core-loss and friction. The magnetizing current is much the larger component, and for preliminary calculations may be taken equal to the exciting current.

Magnetizing Current. — The magnetizing current is calculated by determining the flux density in each part of the magnetic circuit and the ampere turns required for each part. This is most easily done by means of the following tabulation:

MAGNETIC DENSITIES AND M.M.F.'S

(For dimensions refer to Fig. 3.)

Part	Flux per pole	Area	B_{max}	A.T. per inch	Length path	Total A.T.
Stator core.....	$\phi/2$	$h_1 \times l$	$1 \times B_{avg}$	....	$\frac{\pi D_1}{2 \times \text{poles}}$	....
Stator teeth....	ϕ	$r_1 \times l \times \frac{\text{slots}}{\text{poles}}$	$\frac{\pi}{2}$ $1.57 \times B_{avg}$	....	n_1	....
Air gap.........	ϕ	See below	$1.57 \times B_{avg}$	....	g	....
Rotor teeth....	ϕ	$r_2 \times l \times \frac{\text{slots}}{\text{poles}}$	$1.57 \times B_{avg}$	....	n_2	....
Rotor core.....	$\phi/2$	$h_2 \times l$	$1 \times B_{avg}$	....	$\frac{\pi D_2}{2 \times \text{poles}}$	....
					Total.....	....

Stator Core or Yoke. — The flux divides in the core, one-half going each way. The maximum and average* densities are practically the same and equal to ($\phi/2$ divided by the radial depth h_1 times the effective length l). The material is laminated steel of high permeability. The ampere turns per inch length of path are obtained from a magnetization curve of the steel, see *Magnetic Properties of Iron.* The length of path is indeterminate but closely approximates one-half of the pole pitch measured on the circle of diameter D_1. By multiplying the ampere turns per inch obtained from the magnetization curve by the length of path, the ampere turns required to send the flux through this path are obtained.

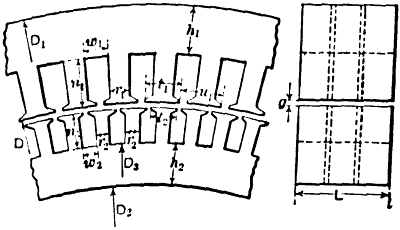

Fig. 3. Dimensions of Magnetic Circuit

Stator teeth. — The average density in the teeth is first obtained. The effective area of one tooth is the area one-third the distance from the face to the root of the tooth as shown at r_1. This gives the average magnetizing force rather than the average density. The total cross section of the path in the teeth is therefore $r_1 \times l \times$ (the number of teeth per pole). Due to the peaked or sinusoidal space distribution of the flux the maximum density in the teeth is 1.57 times the average density. The ampere turns required depend upon the maximum density. From the proper curve determine the ampere turns per inch necessary to establish this density, and by multiplying this quantity by the distance n_1 in inches, the ampere turns for the stator teeth are found.

Air Gap. — The flux is not uniformly distributed in the air gap, especially if the openings of the slots are fairly large. The flux passes through each tooth and spreads out from the iron when it leaves the teeth to cross the gap. The peripheries of the stator and rotor present unequal and dissimilar surfaces passing each other. The lesser of the two is the one that must be considered. The effective area of the air gap is

$$\frac{(t+g)\, l \times (\text{total number of slots})}{0.9\, p}$$

and the maximum density $= \dfrac{1.57 \times \phi}{\text{gap area}}$

(the $t+g$ allows for the spreading of the flux in one direction and $l/0.9$ allows for the spreading in the other direction).

The ampere turns per inch are $0.313\, B_{\max}$. The length of gap being known, the total ampere turns for the gap are found.

Rotor Teeth. — The calculation of the ampere turns for the rotor teeth follows the same method as the calculation for the stator teeth.

Rotor Core. — The same as for the stator core.

The effective value of the magnetizing current is then

$$I_m = \frac{(\text{Total ampere turns}) \times p}{2\sqrt{2}\, S} \text{ for three phases}$$

$$\text{and } I_m = \frac{(\text{Total ampere turns}) \times p}{\sqrt{2}\, S} \text{ for two phases}$$

* By average is here meant the average over the cross section of the maximum instantaneous value of the flux density, which, of course, alternates between fixed positive and negative values.

Resistance per Phase of Primary Winding. — The resistance to direct current, or ohmic resistance, is given by the formula

$$r_1 = \frac{0.0093\, S \times (\text{mean length of turn})}{12{,}000\, an},$$

where

S = turns in series per phase,
a = cross section of one conductor in sq. in.,
n = no. of conductors or circuits in parallel,

and 0.0093 is the resistance at 60° C. of a conductor 1000 feet long and 1 square inch cross section. The mean length of turn is approximately (see Fig. 3)

$$2L + 10\frac{D}{p} \times (\text{pitch as a fraction}).$$

Due to the eddy currents set up in the primary conductors by the total flux and to eddy currents set up in the core by the leakage flux, the "effective" resistance of the primary winding is about 15 per cent greater than the value calculated by the above formula.

Resistance per Phase of Secondary Winding. — In a wound rotor the resistance per phase is found in the same manner as the resistance per phase of the stator winding. The secondary resistance reduced to primary is then equal to $\left(\frac{\text{number prim. turns per phase}}{\text{number sec. turns per phase}}\right)^2 \times$ (actual effective sec. resistance).

The squirrel-cage rotor, in which each slot contains one bar and all bars are short-circuited at each end by a ring presents a more complicated problem. It may be solved as follows:

$$\text{Amp.-cond. per sec. slot} = \frac{\pi D \sigma}{(\text{number of sec. slots})}.$$

σ = amp.-cond. per inch for full-load current.

$$\text{Current density in sec. bars} = \frac{(\text{amp.-cond. per slot})}{(\text{area of one bar})}.$$

Watts lost in bars = 0.775×10^{-6} (vol. of bars) (amp. per sq. in.)^2.

Area of bars per pole = (area of one bar) (number of bars per pole).

$$\text{Current density in rings} = \frac{(\text{current density in bars})\ (\text{area of bars per pole})}{4\ (\text{area of ring})}.$$

Watts lost in rings = 0.775×10^{-6} (vol. of both rings) (density in rings)^2.

$$\text{Resistance of sec. in terms of primary} = \frac{\text{total loss}}{3\ (\text{full load prim. current})^2}.$$

Leakage Reactance. — When the motor is loaded the currents in the secondary set up a counter m.m.f. which causes part of the flux to pass along the air gap instead of into the secondary core. This flux does not interlink both members and is therefore a leakage or useless flux. It is proportional to the load currents and to the permeance of this path. As the greater portion of the path is in the air and is of high reluctance, therefore that part of the path in the iron may be neglected. The flux in both the primary and secondary must be calculated.

Referring to the diagram Fig. 4, the permeance of these two paths are

$$P_1 = \left[\frac{o_1}{3w_1} + \frac{2r_1}{w_1 + q_1} + \frac{p_1}{q_1} + \frac{t_1 - q_2}{6g} + 0.37\frac{l_{s1}}{L}\log_{10}\frac{1.5\, l_{s1}}{V_{s1}}\right]L,$$

$$P_2 = \left[\frac{o_2}{3w_2} + \frac{2r_2}{w_2 + q_2} + \frac{p_2}{q_2} + \frac{t_2 - q_1}{6g} + 0.37\frac{l_{s2}}{L}\log_{10}\frac{1.5\, l_{s2}}{V_{s2}}\right]L$$

where

g = length of gap in inches,
l_e = length of end connection at one end in inches,
V_e = perimeter of end connections of coils in inches.
L = total length of iron in inches.

In these formulas the subscript 1 refers to the primary and subscript 2 to the secondary. The first three terms give the slot reactance, the fourth term gives the "zigzag" or "tooth-tip" reactance, and the last term the reactance of the end connections.

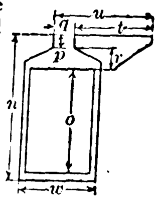

Fig. 4. Slot Dimensions

The primary reactance in ohms per phase is

$$x_1 = 3.2\, P_1 k c_1^2 s_1 \times 2\,\pi f 10^{-8}.$$

The secondary reactance in ohms per phase in terms of the primary turns is

$$x_2 = 3.2\, P_2 k c_2^2 s_2 \times 2\pi f 10^{-8} \left(\frac{c_1 s_1}{c_2 s_2}\right)^2,$$

where

c_1 and c_2 are the conductors in series per slot,
s_1 and s_2 are the slots in series per phase,
f = primary frequency,
k = winding distribution constant (*see above*).

Losses in Induction Motor. — The losses in an induction motor are

Core-loss;
Friction, bearing and windage;
Primary copper loss;
Secondary copper loss.

The first two of these are approximately constant for all loads and the last two vary as the square of the current per phase.

Core-loss. — The distribution of the magnetic flux in an induction motor is quite irregular both in the core and in the teeth. The losses are therefore greater and their calculation more involved than in machines having uniform density in each part. In the primary the frequency of the passage of the secondary teeth introduces pulsations which increase the losses. In the secondary the frequency, being proportional to the slip, is so low that the core-loss is negligible.

In order to avoid too lengthy and complex calculations use is made of empirical constants, by which the easily calculated losses are multiplied to derive the practical loss. The total loss consists of hysteresis and eddy loss in the primary core and primary teeth. The loss in watts per cubic inch for one cycle per second at any magnetic density is found by the curves given in the article on *Magnetic Properties of Iron*.

The losses are then calculated as follows:

Hysteresis Loss:

In primary core $= k_h C_1 f V_c$ watts,
In primary teeth $= k_h C_2 f V_t$ watts.

Eddy Loss:

In primary core $= k_e C_3 f^2 V_c$ watts,
In primary teeth $= k_e C_4 f^2 V_t$ watts,

where V = volume in cu. in., C = respective loss per cu. in. from curves, f = frequency, k_h = empirical constant, 1 to 1.5, k_e = empirical constant, 3 to 4.

The higher values of k_h and k_e are to be used where open slots are used and where the frequency of the passage of the secondary teeth past one primary tooth is high. The core-loss is the sum of all the above losses.

Friction and Windage Loss varies greatly with the style of motor and form of structure, bearings, etc. The loss is made up of bearing friction and wind friction. The latter may be large purposely, as the motor may be designed with fan blades in order to circulate the air for the purpose of keeping the motor cool by ventilation. While each manufacturer has a formula which will calculate the friction loss correctly for machines built according to a particular plan, no general formula can be assumed. The nearest approach to an estimate is obtained from the percentage given in the table below.

Primary and Secondary Copper Losses. — The calculation of the effective resistances of the primary and secondary windings is given above. The primary or secondary copper loss is equal to the product of the number of phases, the effective primary or secondary resistance per phase, and the square of the current per phase. The total copper loss is the sum of the primary and secondary copper losses.

Power Factor. — The power factor ($\cos\phi$) of an induction motor for a given current input (I) may be calculated roughly from the formula

$$\cos\phi = \cos(\alpha + \beta),$$

where $$\alpha = \sin^{-1}\left(\frac{I_m}{I}\right) \text{ and } \beta = \sin^{-1}\left(\frac{XI}{E}\right),$$

and I_m = magnetizing current per phase,
I = total current per phase,
$X = (x_1 + x_2)$ = total reactance per phase, the secondary reactance x_2 being reduced to the primary turns,
E = voltage per phase.

For a more exact formula for power factor and for the usual values of the power factor at rated load, see *below*.

Efficiency. — The efficiency (as a fraction) for a given current input is

$$\epsilon = 1 - \frac{F + C + m(r_1 + r_2)I^2}{mEI\cos\phi},$$

where F = friction loss, in watts; C = total core-loss, in watts; m = number of phases; r_1 = effective primary resistance per phase; r_2 = effective secondary resistance per phase reduced to primary; E = voltage per phase; I = current per phase; $\cos\phi$ = power factor.

The corresponding horse-power output is then

$$P_0 = \frac{\epsilon mEI\cos\phi}{746} \text{ horse-power.}$$

Usual values of the efficiency are given above in section on *Currents taken by Motors*. Usual values of the component losses for 60-cycle motors are given in the following table.

In 25-cycle motors the exciting current is usually greater than in 60-cycle motors and the IX drop less. Thus the power factor will be lower at light loads and higher at overloads than in 60-cycle motors.

Rating, h.p.	$\frac{Im}{I}$	$\frac{XI}{E}$	$\frac{\text{Friction loss}}{\text{Input}}$	$\frac{\text{Core-loss}}{\text{Input}}$	$\frac{r_1 I}{E}$	$\frac{r_2 I}{E}$
1	0.45	0.20	0.06	0.05	0.05	0.05
5	0.35	0.14	0.035	0.04	0.04	0.04
20	0.30	0.13	0.025	0.035	0.035	0.035
50	0.27	0.11	0.015	0.03	0.025	0.03
100	0.26	0.11	0.015	0.03	0.025	0.03
200	0.25	0.11	0.015	0.03	0.025	0.03

The efficiency of 60-cycle motors naturally tends to be higher than of 25-cycle motors. This quality is usually sacrificed by economizing in material and making the 60-cycle motors lighter and cheaper, but of about the same efficiency as the 25-cycle motors.

Slip and Speed. — The slip for any given current is approximately

$$s = \frac{r_2 I}{E},$$

where the symbols are defined in the preceding paragraph.

The synchronous speed is

$$N = \frac{120 f}{p} \text{ rev. per min.},$$

where f is the frequency of the supply and p the number of poles. The actual speed of the motor is then

$$N_1 = (1 - s)\, N.$$

Values of Other Characteristics. — The energy component of the exciting current is

$$I_e = \frac{(\text{Total core-loss}) + (\text{Total friction loss})}{mE},$$

where m is the number of phases and E the volts per phase.

The total exciting current per phase is then

$$I_{00} = \sqrt{I_e^2 + I_m^2},$$

where I_m is the magnetizing current per phase.

The impedance per phase at standstill ("short-circuit" impedance) is

$$Z = \sqrt{(r_1 + r_2)^2 + (x_1 + x_2)^2},$$

where r_1 and r_2 are the effective values of the primary and secondary resistances per phase, the latter being reduced to primary turns and x_1 and x_2 the primary and secondary reactances per phase, the latter being reduced to primary turns.

The current per phase at standstill ("short-circuit" current) is

$$I_s = \frac{E}{Z}.$$

The starting torque in pounds at 1-foot radius is

$$T_s = \frac{7.06\, m r_2 E^2}{N Z^2},$$

where m, r_2, E, and Z are as defined above, and N is the synchronous speed in revolutions per minute.

The slip at maximum output is

$$s_m = \frac{r_2}{r_1 + Z}.$$

The maximum output in watts is

$$P_m = \frac{0.5\, mE^2}{1.3\,(r_1 + r_2) + Z} \text{ watts.}$$

This should be from 1.5 to 3 times the rating of the motor. A 25-cycle motor usually has a greater maximum output or overload capacity than a 60-cycle motor.

Heating. — Since in most motors the primary member is subject to both a core-loss and a copper loss and is stationary, the heating of this member is very important. As the secondary contains only a copper loss and is usually revolving, its rise in temperature is usually much less than that of the primary.

The problem therefore consists of analyzing the flow of heat and drop in thermal potential in the primary as the energy lost in the windings flows to the iron of the core through the insulation in the slots and to the air around the projecting end connections. The core itself is maintained at a temperature above the air by the core-loss. It is therefore necessary to calculate first the rise in temperature of the iron of the core caused by the core-loss and that portion of the copper loss which is conducted to the core through the slot insulation. It is then necessary to calculate the rise in temperature of the copper above the iron and above the air around the end connections.

Since at times in every motor, when the core-loss is great compared with the copper loss, the iron may be hotter than the copper, it is necessary to distinguish between that portion of the power (P_1) which passes between copper and iron (or vice versa), and that portion (P_2) which passes from copper direct to the air. The analysis must take account of the fact that sometimes the heat due to core-loss passes to the copper and is dissipated by the end connections.

The method given here is based on the more exact and elaborate treatment given by Arnold in Vol. V of his treatise *Wechselstromtechnik*, but is simplified by assuming that the copper conductors have such a high heat conductivity that they have the same temperature throughout their length. The result is a value for the average temperature of the copper such as would be found by a resistance measurement. For a method of determining the maximum temperature in any spot the reader is referred to Arnold's treatise.

Temperature Rise of Iron of Core (T_i).—The heat which is dissipated by the core surface consists normally of the core-loss and that part of the primary copper loss occurring in the portion of the winding embedded in the slots. It is

$$H = \text{core-loss} + mI^2r_1\left(\frac{2L}{t}\right),$$

where
m = number of phases,
I = primary current per phase,
r_1 = effective resistance of primary per phase,
L = length of armature core between heads,
t = mean length of a primary turn.

The surface consists of the outer cylindrical surface of the core, the two annular surfaces at the ends, and the surfaces in the air ducts, see Fig. 3. Since the

surface in the air ducts is not as effective as the others, only half the air-duct surface is used.

The area is $A_i = \pi D_1 L + \frac{\pi}{4}(D_1^2 - D^2)(2 + d)$,

where D_1 = outside diameter of stator in inches,
D = inside diameter of stator in inches,
d = number of air ducts.

The rise in temperature of the iron of the core will then be $T_i = \frac{kH}{A_i}$, in degrees Cent., where k ranges from 30 for narrow machines to 50 for long machines.

Temperature Rise of Copper (T_c). — A part of the heat due to the copper loss flows from the copper to the iron through the slot insulation, due to the small difference in temperature between the copper and the iron, $T_c - T_i$, where T_c is the rise in temperature of the copper above the air. The power in watts so dissipated is

$$P_1 = A(T_c - T_i),$$

where

$$A = \frac{U_s L S_1}{d_1 k_1}$$

and U_s = perimeter of slot insulation,
S_1 = total number of slots,
d_1 = thickness of slot insulation,
k_1 = a constant, 200 to 250 (say, 210).

Sometimes T_i is greater than T_c and P_1 becomes negative, which means that some of the energy of the core-loss flows to the windings and is dissipated by the end windings.

Another portion, P_2, of the copper loss flows through the insulation of the end connections to the air. Here there are paths, insulation and air, in series, across which there is a drop in temperature of T_c°C. The flow of energy to the air is affected by the movement of the air, which, in turn, is a function of the peripheral speed V of the rotor. The value of P_2 is

$$P_2 = BT_c,$$

where

$$B = \frac{U_c l_c Z_1}{d_2 k_2 + k_3}$$

and U_c = perimeter of end connections, in inches,
l_c = length of end connections, in inches,
Z_1 = total number of coils,
d_2 = thickness of insulation on same, in inches,
k_2 = a constant, 400 to 500 (say, 430),
k_3 = resistance to flow of heat from insulation to air and is given by the formula,

$$k_3 = \frac{170}{1 + 0.015\,V};$$

where V = peripheral speed of rotor in feet per second.

The total copper loss is $P = P_1 + P_2$, whence

$$T_c = \frac{P + AT_i}{A + B}.$$

This is approximately the average difference in temperature between copper and air as it would be determined by a measurement of resistance.

TESTING OF INDUCTION MOTORS. — (*See also Standardization Rules of A.I.E.E.*) Induction motors may be given either an "input-output" test at load under working conditions, from which the efficiency and power factor may be determined, or the motor may be given a no-load excitation and no-load short-circuit test, from which all the characteristics may be calculated. The latter method requires very little power and is preferable in the case of large motors where it would be expensive to supply power and inconvenient to dissipate the energy.

Excitation Test. — In this test the machine is operated without load at constant frequency with the voltage varied through a range from 30 per cent above rated value down to as low a voltage as will cause the machine to rotate. The current in each phase is noted and by means of two wattmeters the power required is noted. This test is sometimes also made with the machine operating single phase for the purpose of checking the core-loss. The curves are plotted with voltage as abscissas and amperes and watts as ordinates. At very low voltages the core-loss is negligible and therefore the watts input may be taken as friction loss. Around normal voltage the real core-loss may be determined by subtracting from the input the small copper and the friction loss.

Short-circuit Impedance Test. — In this test the starting resistance, if there is any, is short-circuited and the armature is blocked to prevent rotation. A low voltage is applied to the primary until the ammeter in the primary circuit shows a current of from 1 to 1.5 times the full-load value. Two wattmeters are used to read the power input. As the leakage flux, and hence the reactance, varies with the relative position of the rotor and stator teeth, it is customary to take readings with the rotor in several positions. Sometimes the rotor is allowed to revolve very slowly at, say, two to three revolutions per minute. From this test the impedance (Z) of the machine is obtained by dividing the volts per phase by the current per phase; the combined effective resistance (R) of primary and secondary is found by dividing the watts input per phase by the square of the current per phase. This latter is the effective value of the resistance, since the watts input includes all losses due to eddy currents. From these values the total reactance (primary and secondary) of the motor is $X = \sqrt{Z^2 - R^2}$. With the results of the two preceding tests and the measured resistances of the two members, the characteristics of the motor may be calculated either by the Steinmetz Method or the Circle Diagram. (*See below under Performance, Calculation of.*)

Stationary Torque Test (Fig. 5). — This test may be made at the same time the impedance test is made and consists in the measurement of the torque of the motor by means of a brake arm and spring balance. When the current of the machine has been adjusted to a suitable value the brake arm (to which a known weight has been attached to overcome bearing friction) and the spring balance are allowed to move downward through a small arc, during which the spring balance will register a pull (P_1) equal to torque (T) plus the weight (W) minus the friction (F), that is, $P_1 = T + W - F$. The spring balance and brake arm are then raised against the torque through the same small arc and the spring balance will then register a pull (P_2) equal to the torque plus the weight plus the friction, that is, $P_2 = T + W + F$. From these two readings the friction can be eliminated

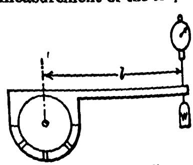

Fig. 5. Stationary Torque Test

and the actual torque of the armature on the shaft is determined. This should be done at two or three positions of the armature. The corrected torque of the machine is found by solving the two equations for T and multiplying T by the lever arm of the brake in feet; this gives the torque in pounds at 1-foot radius.

Load Test. — To make this test on a small machine a prony brake is required, while for a large machine a direct-current generator may be conveniently employed as a load. The induction motor and generator are direct-connected if possible, otherwise belted together. All the constants of the generator are determined, so that its losses under any condition may be calculated. The motor is then allowed to drive the generator, the rated voltage being impressed upon the motor, and the generator is loaded by means of a rheostat or a water-box so that any load may be obtained. The voltage across each phase of the motor, the current in each phase, the total watts input (by two meters), speed, and, if possible, slip (by a slipmeter, *see below*) are all read. Care must be taken that correct voltage and frequency are supplied to the motor. The load on the motor is increased step by step to the maximum output point, which is easily known, since when it is reached a decrease in speed is not accompanied by any increase in output of the motor. The motor is still stable at the maximum output condition, since maximum torque occurs at a lower speed than maximum output.

After the load run the direct-current generator is run as a motor with the same field strength as before, to determine the mechanical losses, first driving the induction motor at the proper speed, and then running alone at the same speed or speeds at which it ran in the load test. Knowing the no-load friction of the induction motor as previously determined in the *Excitation Test*, the increase in friction due to the belt and load is determined and half of it charged against the motor. Let

EI = the output of the direct-current generator in watts,
I^2R = the hot resistance loss in the direct-current armature,
CT = the counter torque or stray power losses in the direct-current generator plus one-half belt loss,
P_0 = watts input to induction motor.

$$\text{Then efficiency of the motor} = \frac{EI + I^2R + CT}{P_0}.$$

From these tests, curves may be plotted for the speed, efficiency, power factor current and torque of the induction motor for any horse-power output.

Use of Stroboscope to Measure Slip. — One type of slipmeter consists of a disk which is attached to the motor shaft and on which alternate sectors of black and white are shown, preferably as many black sectors as there are poles on the motor. If this rotating disk is illuminated by means of an arc light supplied with the frequency impressed on the primary of the motor, the disk will appear to rotate at a speed proportional to the difference between synchronous speed and actual running speed. The number of these revolutions for one minute, will be the slip in turns per minute, which may be translated into a fraction or percentage of synchronous speed.

Slipmeter. — Another type of slipmeter has two disks of insulating material mounted on the same frame, one carrying a wiping finger which makes contact with a button on the other disk. One of the disks is connected to a small synchronous motor which is driven from the same power source as the induction motor under test; the other disk is arranged to be readily attached to the shaft of the induction motor. The circuit of a miniature tungsten lamp is closed when the wiping finger and button are in contact. The number of flashes

of the lamp per minute gives the slip in revolutions of the induction motor behind the synchronous speed.

Heat Runs. — On machines of moderate size heat runs are advisable, but on large size machines heat runs are expensive. It is not always necessary to make a heat run in order to know whether the motor is properly designed, as the losses can be accurately calculated from the excitation and short-circuit tests, and the machine may be run without load but with losses equivalent in value to the losses at full load. The usual heat run consists in operating the machine at a certain output for a period from three to six hours, depending on the size, measuring the resistance before and after the run and taking temperatures by thermometer on the following parts after the run: primary winding, the iron surfaces in the air gap, secondary winding, bearings, frame.

A heat run which will indicate whether there is anything radically wrong with a motor consists in operating it for an hour or two with a voltage 15 per cent above normal, but without load.

Insulation Tests. — High-potential tests are made on the primary in the manner described under *Testing* in the article on *Generators, Alternating-Current*. The value of the high potential for the primary is chosen in accordance with its rated voltage from the *Standardization Rules of the A.I.E.E.* (q.v.). The potential applied to the secondary, however, has no relation to the rated voltage of the machine; as the working potential in the secondary is low, 1000 to 1500 volts is the usual range of testing potential for the secondary.

PERFORMANCE, CALCULATION OF. — The performance of an induction motor at any load may be determined directly from the load tests described above, or the performance at any load may be calculated from the excitation and short-circuit tests by either of the two methods given below. These methods are also applicable to the calculation of performance from the values of the constants calculated from the dimensions of the machine. The first method is given in detail by Steinmetz in his *Elements of Electrical Engineering*, and the Heyland Circle Diagram is given by McAllister in his *Alternating Current Motors*. The former is recommended where accuracy is desired and the latter (graphical) for the student desiring a general understanding of the relations.

Steinmetz Method. — This method is based upon the equivalent circuit of a transformer, as given in the article on *Transformers*. Let

E = impressed voltage per phase,

m = number of phases,

r_1, r_2, x_1, x_2 = resistance and reactance of primary and secondary respectively, per phase and reduced to terms of primary,

s = slip as a decimal fraction (assumed),

$$g = \text{primary no-load conductance} = \frac{\text{core-loss}}{mE^2},$$

$$b = \text{primary no-load susceptance} = \frac{I_m}{E}.$$

Assume a slip s and calculate

$$a_1 = \frac{sr_2}{s^2x_2^2 + r_2^2},$$

$$a_2 = \frac{s^2x_2}{s^2x_2^2 + r_2^2},$$

$$g_1 = g + a_1,$$

$$b_1 = b + a_2,$$

$$c_1 = 1 + r_1g_1 + x_1b_1,$$

$$c_2 = g_1x_1 - b_1r_1,$$

Then

Counter e.m.f. per phase	$e = \dfrac{E}{\sqrt{c_1^2 + c_2^2}}$,
Current per phase	$I = e\sqrt{g_1^2 + b_1^2}$,
Total volt-ampere input to motor	$P' = mEI$,
Watts input to motor	$P = me^2(g_1c_1 - b_1c_2)$,
Watts output of armature	$P_0' = me^2a_1(1 - s)$,
Watts output of pulley	$P_0 = P_0' -$ (friction in watts),
Efficiency	$\epsilon = \dfrac{P_0}{P}$,
Power factor	$\cos\phi = \dfrac{P}{P'}$.

Circle Diagram or Graphical Method (Figs. 6 and 7). — The method of the circle diagram is based on the fact that the electrical reactions in an induction motor (or transformer) may be represented without any great error by the reactions in two parallel inductive circuits as in Fig. 6. I_e and I_m represent the two components of the exciting current, which is assumed constant. R_m and X_m represent the total resistance and reactance (both assumed constant) of the motor and are in series with a variable resistance R_L representing the load.

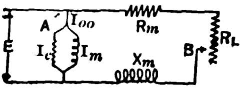

Fig. 6. Equivalent Circuits

The vector relations of the currents are as shown in Fig. 7, where

$OE =$ the impressed voltage per phase,
$OM =$ the current I_{oo} in A,
$MP =$ the variable current in B (not drawn).

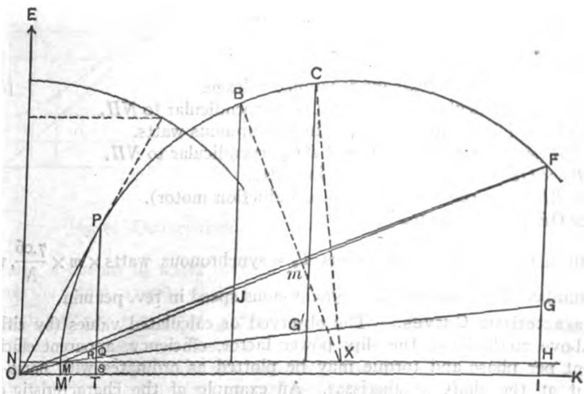

Fig. 7. Circle Diagram

Since the current represented by MP is equal to $\dfrac{E}{\sqrt{(R_m + R_L)^2 + X_m^2}}$, as R_L varies the point P will describe a circle through M, P and F, where $MF = \dfrac{E}{\sqrt{R_m^2 + X_m^2}}$. The total or resultant current will then be OP.

Heyland Circle Diagram (Fig. 7). — The circle diagram upon which the following discussion is based is a modification of Heyland's transformer, or induction motor diagram.

Let
OE = impressed voltage per phase.
OK be at 90° to OE.
OM = exciting current per phase, drawn in phase and magnitude.
OF = short circuit current per phase, drawn in proper phase and magnitude.
IF = energy component of OF.

Join M and F; then MF is the secondary short-circuit current in terms of primary circuit.
Bisect MF at m.
Draw mX perpendicular to MF at middle point, intersecting NH at X.
With X as center and either XM or XF as radius, draw the semicircle MCF. This is the locus of the primary current.
Since $OE \times IF$ = watts input at standstill, draw HG such that $OE \times HG$ = primary I^2R at standstill.
Then $OE \times GF$ = secondary I^2R at standstill.
Draw GM. The vertical distance between GM and NH at any point gives a current which if multiplied by OE gives the power loss in primary.
Choose any point P on circle; then OP = current per phase.
Then $\cos \measuredangle POE = \cos\phi$ = power factor.
MP (not drawn) = secondary current reduced to primary turns.

$PT \times OE$ = power input to primary, in watts.
$TS \times OE$ = no-load loss, in watts.
$QT \times OE$ = total motor loss.
$RT \times OE$ = total primary loss in watts.
$RS \times OE$ = primary copper loss, in watts.
$PR \times OE$ = secondary input, in watts.
$QR \times OE$ = secondary copper loss, in watts.
$QP \times OE$ = motor output, in watts.
$OM \div OP$ = per cent magnetizing current.
$M'T \div OP$ = per cent leakage reactance voltage.

These are all per phase

Draw XC perpendicular to MG and CG' perpendicular to NH.
Then $CG' \times OE$ = maximum torque in synchronous watts.
Draw XB perpendicular to MF and BJ perpendicular to NH.
Then $BJ \times OE$ = maximum output in watts.
$QR \div RP$ = per cent slip (in case of induction motor).
$OP \times OE$ = volt amperes input.

Torque in pounds at 1-foot radius T_s = synchronous watts $\times m \times \frac{7.06}{N}$, where m = number of phases and N = synchronous speed in rev. per min.

Characteristic Curves. — The observed or calculated values (by either of the above methods) of the slip, power factor, efficiency, apparent efficiency, current per phase and torque may be plotted as ordinates with horse-power output at the shaft as abscissas. An example of the characteristic curves thus plotted is shown in Fig. 8. Usual values of the various quantities for different sizes of motors at rated load are given above. The "power factor" shows the relation between the true power input of the machine and the apparent power, called the "volt-amperes." A poor power factor does not involve any greater registration of the watt-hour meter or cost of energy to operate the motor, but it does involve poor regulation of voltage in the system as a whole and larger capacity of wiring, transformers, etc. The "apparent efficiency" is equal to the product of the power factor and efficiency, or is equal

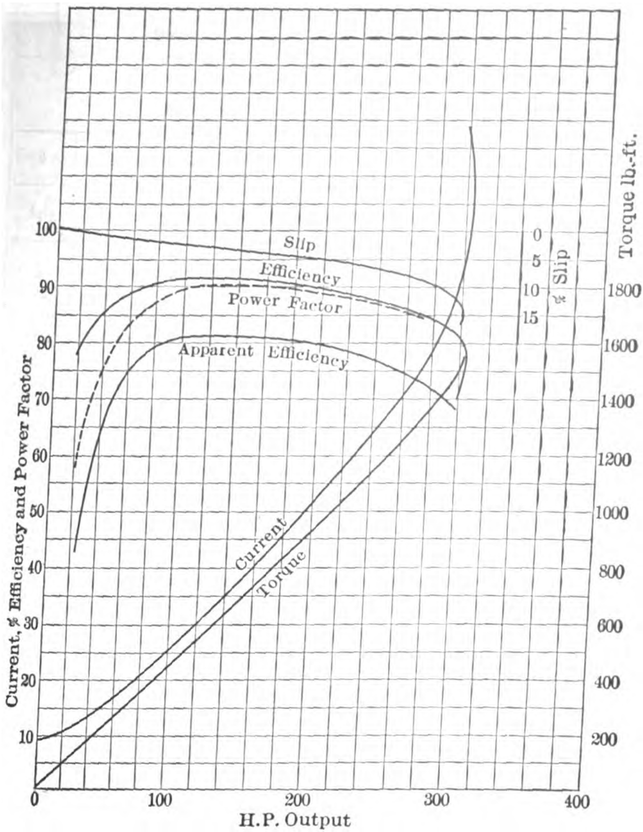

Fig. 8. Characteristic Curves of an Induction Motor

to the ratio of output in watts divided by input in volt-amperes. Its value determines the actual capacity of the lines and transformers supplying the motor.

EXAMPLES OF DESIGN AND PERFORMANCE. — In the accompanying tables are given the essential data for both mechanical and electrical features of six three-phase induction motors. The list of items will be found useful as a guide in collecting data on various machines. Performance data are deduced from tests.

INDUCTION MOTORS

Mechanical Data. (Dimensions in Inches)

Type	1	2	3	4	5	6
	3-ph.	3-ph.	3-ph.	3-ph.	3-ph.	3-ph.
Number of poles	4	4	4	6	6	8
Rating in h. p.	5	15	30	5	10	20
Revs. per min.	750	750	750	1200	1200	900
Prim. volts bet. lines	440	220	220	220	440	220
Prim. connections	Y	Y	Δ	Δ	Δ	Δ
Frequency	25	25	25	60	60	60
Stator						
Outer diam. punchings	17	21	28	17	18.25	26
Inner diam. punchings	12.06	15.07	19.07	12.094	11	19.07
Total length of iron	6	7.5	8	5	3.75	6
No. of ducts	0	0	0	0	0	0
Width of each duct	....	....	....	....	....	....
Total no. slots	60	72	72	54	72	96
Depth of slot	1.25	1.25	1.44	1.56	1.5	1.25
Width of slot	0.32	0.34	0.39	0.50	0.3	0.33
Width of slot at face	0.32	0.34	0.39	0.25	0.3	0.33
Wires per slot	36	16	16	64	30	16
Size of wire	No. 14 B. & S.	No. 10 B. & S.	No. 8 B. & S.	No. 14 B. & S.	No. 15 B. & S.	No. 13 B. W. G
Wires in multiple	1	2	2	2	1	2
Turns in series per ph.	360	96	96	288	360	128
Per cent coil pitch	100	72	67	100	100	75
Rotor						
Outer diam. punchings	12	15	19	12	10.95	19
Inner diam. punchings	8	11	12	8.5	6	14
Total no. slots	37	47	67	72	127	71
Depth of slots	0.56	0.47	0.47	0.94	0.5	0.47
Width of slots	0.56	0.56	0.56	0.34	0.12	0.56
Width of slots at face	0.063	0.063	0.063	0.19	0.02	0.063
Wires per slot	1	1	1	4	1	1
Size of wire or bar	0.5×0.45	0.35×0.5	0.35×0.5	0.34×0.11	0.35×0.09	0.35×0.5
No. in multiple	1	1	1	4	1	1
Cross section each ring, sq. in.	1.3	0.94	2	....	0.198	1.2
Resistance rel. to copper	2	2	2	....	1	2
Air gap on one side	0.03	0.035	0.035	0.047	0.025	0.035

INDUCTION MOTORS

Electrical Data. (All quantities per phase)

Type	1	2	3	4	5	6
Volts per phase, E	254	127	220	220	440	220
Current per ph. at rating	6.6	38	42	8.1	7.3	31.2
Flux per pole, megalines	0.636	1.2	2.07	0.29	0.48	0.645
Magnetizing current I_m	2.1	9	13	2.64	2.4	12.3
Friction, watts	80	175	530	150	295	390
Core-loss, watts	150	390	1240	180	210	760
Primary res. at 60° C., ohms	2.86	0.193	0.13	1.15	2.78	0.213
Second. res. at 60° C., ohms	1.76	0.18	0.17	1.11	3.5	0.21
Short-circuit current	36	212	362	29.4	26	205
$Y =$ (Sh. cir. cur.) $\div I_m$	17	23.6	28	11.2	10.8	16.6
Reactance per ph.	5.52	0.55	0.54	8	5.05	1.10
Eff. at rating	0.83	0.853	0.883	0.83	0.86	0.875
Power factor at rating	0.905	0.91	0.917	0.845	0.90	0.83
$I_m \div I$	0.32	0.24	0.31	0.33	0.33	0.39
$Fr. \div P_0$	0.018	0.013	0.02	0.033	0.034	0.023
Core-loss $\div P_0$	0.033	0.029	0.049	0.04	0.024	0.044
$IR_1 \div E$	0.074	0.058	0.025	0.042	0.046	0.03
$IR_2 \div E$	0.046	0.054	0.032	0.041	0.058	0.03
$IX \div E$	0.143	0.165	0.103	0.29	0.084	0.156
Slip, per cent	0.047	0.042	0.032	0.03	0.038	0.03

METHODS OF STARTING. — In order to start an induction motor of any size without injurious heating either a resistance must be connected in the secondary circuit or the voltage impressed on the primary must be reduced. The two general methods of starting induction motors are known as "Potential Control" and "Rheostatic Control." The same methods are used for speed control (*see below*).

Starting by "Potential Control" Method (Fig. 9). — This method consists of reducing and regulating the voltage impressed on the primary, usually by means of a starting compensator or auto-transformer which provides one or two fractional voltages. In order to make use of this method the secondary must be of higher resistance than with other methods of starting. For this reason and for the reason that there is no need of making any change in the windings, a squirrel-cage rotor winding is customarily used with this method of starting. This winding is made up of one bar per slot, and all bars are connected at both ends to rings. To start the motor the primary is connected to taps on the compensator which give a voltage of from ½ to ⅔ the rated voltage if it is a small motor, and ⅓ to ½ if it is a large motor. A small motor may be brought up to full speed on this voltage but a large motor may require an intermediate step. The connections are shown in Fig. 9. It is customary to adjust the motor resistance and starting voltages to give the relations in the accompanying table.

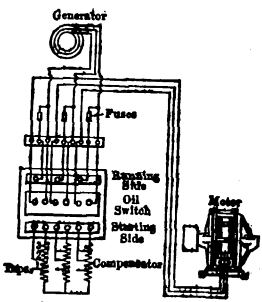

Fig. 9. Potential Control

Voltage on Motor, per cent	Current in line, per cent	Starting torque, per cent
33	75	22
50	175	50
66	300	88
100	700	200

Starting by "Rheostatic Control" Method (Fig. 10). — Better apparent torque efficiency, that is to say, more torque for a given current, is obtained by inserting in the secondary circuit a much greater resistance than can be left permanently in circuit. This is accomplished by having a special starting resistance connected in series with the armature winding and a switch for short-circuiting the resistance either step by step or as a whole, as the motor speeds up. There are two practical methods of doing this:

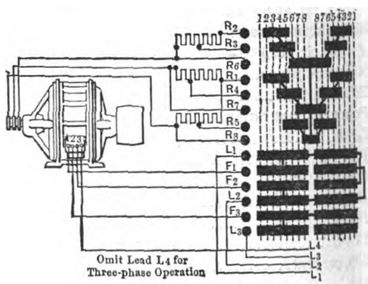

Fig. 10. Rheostatic Control

The first is intended to be used only when the torque required at starting is not very great, in which case the starting resistance may be small and located inside the armature spider. The switch lever is so arranged that the resistance can be short-circuited in steps while the armature is revolving. This obviates the need of collector rings and external connections.

The second method consists in bringing the three terminals of the secondary winding to collector rings. From brushes bearing on these rings conductors lead to external resistances with steps or taps so that the resistance may be short-circuited gradually. This scheme is used where a large starting torque (greater than full load torque) is required. It may be used also for speed control as shown in Fig. 12.

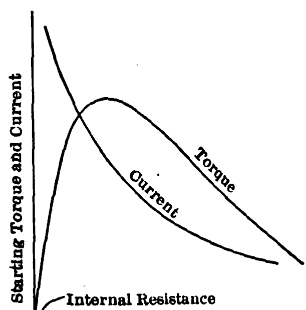

Fig. 11. Starting Resistance, Torque and Current

The proper value of resistance per phase in the secondary is determined by the relation

Torque in pounds at 1 foot =

$$\frac{E^2 r_x}{Z^2} \times \frac{7.06\, m}{\text{r.p.m.}} = \frac{7.06\, m r_x E^2}{N Z^2},$$

where

E = primary voltage per phase,

r_x = total secondary resistance per phase in terms of primary,

$Z = \sqrt{(r_1 + r_x)^2 + X^2}$, where r_1 is the resistance per phase of primary and X the total reactance per phase of both primary and secondary,

m = number of phases,

N = synchronous speed in revolutions per minute.

The relation between starting torque and total resistance of the secondary is shown by the curve in Fig. 11.

SPEED CONTROL OF INDUCTION MOTORS. — (*See also Hoists, Electric.*) The speed of an induction motor may be controlled in five ways:

(a) By varying the potential applied to the primary of motor having a suitable permanent resistance in the secondary.

(b) By varying the resistance in the secondary circuit.

(c) By changing the connections of the primary winding in a manner to change the number of poles.

(d) By varying the frequency of the applied voltage.

(e) By connecting the secondary of one motor to the primary of another, called the "concatenation" method of control.

Potential Control of Speed. — This method is an elaboration of the potential-control method of starting. A suitable resistance or auto-transformer reduces the line voltage to the fractional value desired. In this reduction the energy loss is only about 5 per cent of the amount transformed. The induction motor should have a very large resistance in the secondary, which is preferably of the squirrel-cage type. This resistance gives the motor a speed characteristic such that its full-load speed is some 10 per cent less than that of a normal motor. As the load is increased the speed may fall to about 30 per cent of the no-load value without the motor breaking down or falling out of step, which in the normal motor usually takes place at about 80 per cent of the full-load speed. This motor is not stable in speed as each slight change in the load will cause a change in the speed more or less inversely proportional to the load. The advantages of this method of control are the simplicity of the connections and devices. The disadvantages are the greatly increased heating in the motor itself with the decreased speed. Thus the motor must be larger than if other speed-control methods are used. The table given below shows the efficiencies obtained.

Rheostatic Control of Speed. — With this method (an elaboration of the method of the same name for starting) the secondary or rotor must have a definite winding (which costs more than the squirrel-cage winding used in the preceding method) with slip rings and brushes to lead out the current. The friction and resistance losses due to these brushes decrease the efficiency of the motor a slight amount. The action of this method is based on the principle that in an induction motor the drop in speed for any given torque is proportional to the resistance of the secondary circuit.

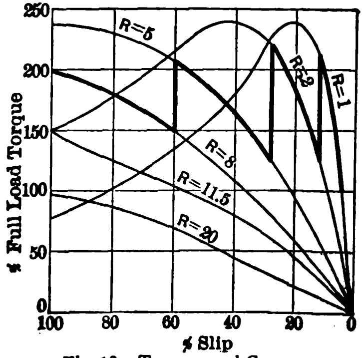

Fig. 12. Torque-speed Curves

Assuming a motor which has at full speed a net resistance of the secondary proper of 1 ohm, the speed-torque curve would be as shown for $R = 1$ in Fig. 12. This motor would have a slip of 5 per cent for full-load torque and of 25 per cent

for maximum torque, a torque at starting of 80 per cent of full-load torque and a very large starting current. If by some means this resistance is doubled the speed-torque curve would be as shown for $R = 2$, which shows a slip of 10 per cent for full-load torque and of 50 per cent for maximum torque. For $R = 5$, $R = 8$, etc., there would be other speed curves. By starting with a resistance of 8 ohms a torque of about 200 per cent of full-load torque would be obtained at starting with about twice full-load current. By allowing the motor to follow this curve until the torque has dropped to the full-load value, the motor would reach 60 per cent of synchronous speed, or 40 per cent slip. Then by reducing the resistance in steps the torque and speed would follow the heavy zigzag line until the motor reached full speed.

With this method of control the torque per ampere remains practically constant as in a shunt motor regulated by resistance in the armature circuit. The efficiency varies directly with the speed as shown in the table below.

The advantages of this method are the higher efficiency and particularly the smaller losses in the motor itself. The losses are in the rheostat. The disadvantages are the necessity of collector rings, brushes, controllers, etc. The motor is not stable at any fractional speed but the speed will change with every change of load.

Change of Poles to Control Speed. — By a proper design of the windings an induction motor may be made to operate with either 4 or 8 poles, 6 or 12 poles, or even 4 or 6 poles or 6 or 8 poles. This is accomplished by a throw-over switch to which taps from the windings are brought. In this arrangement the pitch of the primary winding must be made a compromise between the proper value for the different numbers of poles, and therefore the constants of the motor are not as good as those of a standard motor. It is also necessary to use a squirrel-cage armature, since this is suitable for any number of poles without change of connections. This type of motor operates advantageously only at the two speeds corresponding to the two arrangements of poles and is stable at each of these speeds. If a wider range is desired the potential-control scheme may be combined with it. The speeds and efficiencies with this scheme of control are given in the following table. At half-speed the efficiency is almost double that obtained with the other methods, but the losses in the motor are greater than with the rheostatic control.

COMPARISON OF METHODS OF SPEED CONTROL

(For constant torque equal to torque at full load)

Nominal speed	Potential			Rheostatic			Change of poles		
	Speed	Volts	Eff.	Speed	Volts	Eff.	Speed	Volts	Eff.
No load	1.00	100	..	1.00	100	..	1.00	100	..
Full speed	0.89	100	81	0.96	100	86	0.96	100	86
Three-quarter speed	0.67	66	59	0.72	100	65	....	...	..
Half speed	0.45	57	37	0.48	100	43	0.48	100	74
Quarter speed	0.22	56	17	0.24	100	22	....	...	..

Change of Frequency Method of Control. — The speed of an induction motor at any load varies directly with the frequency of the supply circuit. If two circuits from two alternators of different frequencies are provided the motors may be connected to one circuit for one speed and to the other circuit

for another speed. This method of speed control requires as many separate generators and circuits as the number of speeds desired. It is, therefore, costly and not widely used.

Concatenation Control or Cascade Control. — If two motors have definite windings in both the primary and secondary and are rigidly connected to a common shaft, they may be operated at a fractional speed corresponding to a number of poles equal to the sum of the number of poles on the two motors. With the concatenation control the primary (stator) of motor No. 1 is connected to the supply circuit, the secondary (rotor) of No. 1 is connected to the primary (rotor) of No. 2, and the secondary (stator) of No. 2 is connected to a resistance which is eventually short-circuited. When the two motors have the same number of poles, which is the usual commercial condition, motor No. 1 transforms half the power into mechanical power at the shaft at *half speed* and the remainder into electrical power at half frequency. Motor No. 2 receives this electrical power and transforms it into mechanical power at this same speed. If the number of poles is different, the speed in r.p.m. of the combination is

$$N = \frac{120f}{p_1 + p_2}.$$

These simple relations are exact only on the assumption of no losses and the rotor of No. 2 short-circuited.

The objections to this method of control are that the first motor has to carry the magnetizing current of both motors, thus having a low power factor. The first motor must receive the power for both motors; therefore, it must be the larger of the two and specially designed or there is an inefficient use of material in the second motor. If the two motors are alike (the usual commercial condition) the torque of the two motors in concatenation is not as great as that of the two motors in parallel.

The efficiency at fractional speed is, however, better than with the rheostatic method of control. Only two "free-running" speeds are available with two motors having the same number of poles. This scheme has been adopted frequently for the speed regulation of induction motors used on electric locomotives in foreign countries. In such applications it is frequently the custom to allow one motor to be idle at the higher speed.

INSTALLATION AND ERECTION. — Induction motors are usually built, even in large sizes, as a unit including the bearings, which are usually a part of the end frame of the motor proper. They may be either direct-connected or belted to their load, but the latter method is more general, since each induction motor can only be built for a certain definite speed corresponding to a certain number of poles. The smaller motors need no foundation, and in fact are frequently attached to the wall or to the ceiling, the bearings and end shield being made in such a manner that they may be turned through 90 degrees or 180 degrees so that the oil rings will operate properly under these conditions. In most motors reasonable ventilation, free from dust and dirt, must be available. For certain applications such as cement mills or mines, the motors are built totally inclosed and may then be even submerged in water. In this case, of course, a motor of a given rating is larger and more expensive than one of the open type.

OPERATION. — Small motors are designed to start merely by closing the main switch. With larger motors if the starting switch is at the proper position, potential may be applied to the motor and the starting resistance gradually cut out by moving the switch.

Induction motors are very sensitive to variation in the impressed voltage. A decrease in impressed voltage from 100 to 80 would cause the maximum

output and maximum starting torque to decrease from 100 to 64 and roughly would cause a proportional increase in the heating for a given load.

Unequal voltages in the different phases also cause a decrease in the maximum output and an increase in the heating for any given output. An unbalancing of 25 per cent in voltage would double the heating effect at full load, i.e., would give the same heating as an overload of 50 per cent.

Care in Starting. — Before starting the motor for the first time it is desirable to make sure that the starting device is in operating condition and in the proper position, in order that the motor should not become injuriously heated. Attention should be paid to having the wiring so proportioned as to carry the starting current without an excessive drop in voltage (*see above*).

Faults. — Some of the more common faults occurring in induction motors, together with their signs and remedies, are the following.

Secondary Open-circuited. — The motor will not start and will not take a current greater than the exciting current. The cause is probably due to the starting resistance not being connected in.

One Phase of Secondary Open-circuited. — The motor has a tendency to remain at half synchronous speed although the current is apparently normal. If the armature is blocked it will be found that the current in the three phases will be unbalanced.

One Phase of Primary Open. — The motor will not start and the current will be unbalanced.

One Phase of Primary Reversed. — The currents in the primary will be very much unbalanced when the motor is running and the starting torque will be very slight.

Short-circuited Coil in Primary. — There will be humming when potential is applied to the motor and excessive local heating around the short-circuited coil.

Vibration. — Vibration due to mechanical unbalancing is chiefly noticeable at high speeds and particularly in high-speed machines. If the vibration is due to magnetic unbalancing it is probably caused by inequality in the air gap at different portions of the circumference and with different positions of the armature. This may be detected by measuring the air gap with taper wedges at various points around the circumference, first with the armature in one position and then in several other positions.

SPECIFICATIONS FOR INDUCTION MOTORS.* — The following memoranda are intended to assist in writing specifications. See also article on *Specifications*.

Principal Characteristics and Conditions of Service. — Use to which motor is to be put; kind of load and method of drive. Voltage and number of phases. Rating, horse-power. Frequency and speed.

Style and Description; Details of Construction. — Whether to be open, semi-inclosed or inclosed. Requirements regarding pulley or length of shaft. Whether rails are required. Method of starting; compensator, external resistance or internal resistance; whether motor is to be run at speeds other than full speed. Whether the starting devices are to be supplied.

Performance and Tests. — (*See Standardization Rules of the A.I.E.E.*). Temperature rises upon which ratings are to be based. Details of overload capacity. Efficiency at 25, 50, 75, 100, and 125 per cent load. Starting torque with full-load current, pound-feet. High-potential tests of insulation. Requirements regarding effect of moisture upon insulation.

* W. A. Del Mar.

WEIGHT AND COST. — The weight and cost of an induction motor vary with the type of its armature winding and the character of its mechanical frame and housing, as well as with the speed, frequency and voltage.

The curves in Figs. 13 and 14 give an idea of the weights and costs of a line of 25-cycle and 60-cycle motors respectively, having squirrel-cage rotors (the simplest and least expensive type) and with the simplest form of mechanical frame. They therefore represent minimum values.

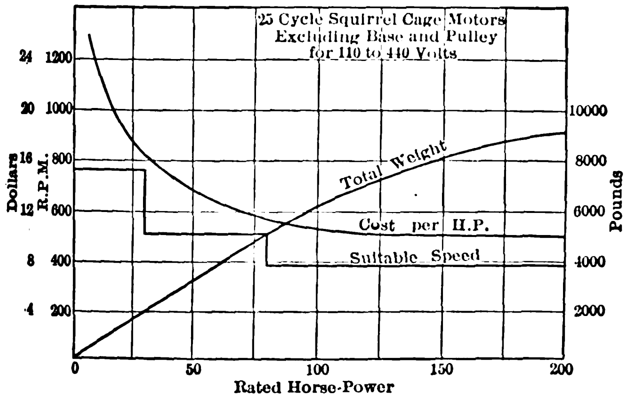

Fig. 13. Weight, Cost and Suitable Speed of 25-cycle Induction Motor

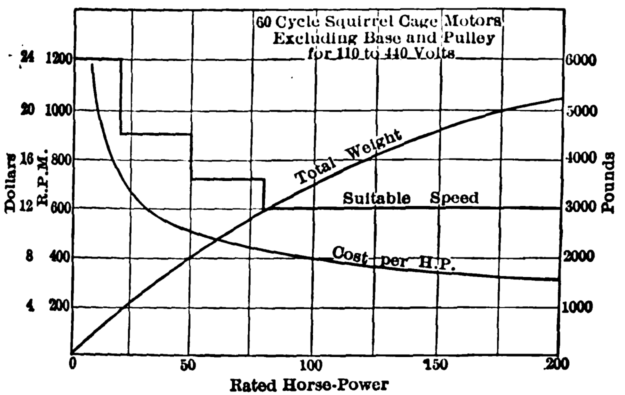

Fig. 14. Weight, Cost and Suitable Speed of 60-cycle Induction Motor

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[W. I. SLICHTER.]

MOTORS, SINGLE-PHASE INDUCTION. — (*See also Motors, Polyphase Induction; Motors, A-C. Commutator*). A two- or three-phase induction motor may be operated as a single-phase machine after it is brought up to speed. Under these conditions it operates at a lower efficiency, lower power factor and with a lower maximum output than it would have as a polyphase motor. The slip for a given output is less in a single-phase motor than in a polyphase motor.

Load and Voltage Rating. — On account of the poorer operating characteristics and particularly on account of the lower maximum output or maximum torque, it is necessary to rate the motor at a lower capacity. When a three-phase motor is operated single-phase with the same voltage between lines, its maximum output will be approximately 40 per cent of the three-phase maximum output.

For best conditions, such as best distribution of losses and ratio of rated to maximum output, it is customary to use a three-phase motor, to reduce the rated output of the motor and to increase the rated terminal voltage in a definite ratio. Thus, if P be the rated output, in watts, of a given motor when operating on a three-phase circuit having a voltage between lines equal to E, then it would be advisable to operate the motor single-phase with a voltage between lines equal to $1.3\ E$, and to assign the motor a rating of from $0.67\ P$ to $0.75\ P$.

This will result in a distribution of losses in the motor quite similar to that which obtains during three-phase operation. The maximum output as a single-phase motor (at the higher voltage) will be about 67 per cent of the maximum output of the three-phase motor. The efficiency and power factor will be reasonable.

In case the voltage of the single-phase supply circuit must be the same, the winding of the motor is changed to give about 75 per cent as many turns in series per phase as for normal three-phase operation.

Exciting Current and Power Factor. — At a given voltage between terminals the volt-amperes input at no load for excitation are practically the same for single-phase and polyphase operation. Thus the no-load current of a single-phase motor is considerably greater than when operating polyphase. The increase in the applied voltage or the decrease in the number of turns makes a still greater increase in the magnetizing current. Thus the power factor of a single-phase motor is very poor at light loads and not very good at rated load.

CALCULATION OF PERFORMANCES. — To predetermine the characteristics of a single-phase motor it is calculated as a three-phase motor for the same voltage between lines as the single-phase circuit. The magnetizing current, core-loss, resistance per phase, and reactance per phase are all calculated as usual. The motor primary may be connected either delta or Y, but for purposes of calculation it is desirable to pro-rate the constants of a Y-connected motor on the assumption that it is delta connected.

To pro-rate the resistance and reactance per phase of primary and secondary, multiply the values for a Y-connection by 3 to obtain the values for the equivalent delta constants. The voltage per phase is presumed to increase in the ratio 1 to 1.73, while the voltage between lines remains the same.

The magnetizing current and the core-loss current are found by dividing the 3-phase values by 1.73.

The single-phase magnetizing current is found by dividing the volt-amperes excitation for three-phase conditions by the single-phase voltage. The core-loss in watts remains practically the same single-phase and the energy component of the single-phase exciting current is equal to core-loss watts divided by rated voltage.

The resistance and reactance of the primary of a single-phase motor are taken the same as the equivalent delta values per phase.

The resistance and reactance of the secondary of a single-phase motor are taken as ⅓ of the three-phase equivalent delta values.

These values are then substituted in the formulæ of the Steinmetz method (*see Motors, Polyphase Induction*) and the characteristics calculated for several assumed values of slip. The only difference in calculation is that the torque in synchronous watts is

$$T = e^2 a_1 (1 - s)$$

and the output of the armature is

$$P = e^2 a_1 (1 - s)^2.$$

It is of course understood that in a single-phase motor the output calculated for one phase is also the total output of the motor.

METHODS OF STARTING. — A single-phase motor has no torque at standstill. It must be started by some device such as a phase-splitting device. It may be started in either direction and as soon as it starts to rotate a slight torque develops which increases with the speed. When this torque has reached a great enough value to overcome the friction and inertia the special starting apparatus may be disconnected and the motor will continue to accelerate to its proper speed.

Starting of Small Motors.— Small motors may be started without auxiliary electric circuits by giving the armature a spin by hand, after which (if there is no load) they will accelerate. Certain small motors are designed with a loose pulley which is clutched at a predetermined speed of the armature by means of a centrifugal governor. These motors are provided with "shading coils" or a small external phase-splitting device, to give them just enough starting torque to overcome their own friction.

Starting of Large Motors. — A phase-splitting device is generally used; either a reactor and resistor or a condenser and resistor may be employed. Commutator devices are also used.

Use of Reactance Coil and Resistor. — The connections employed are shown in Fig. 1. This device consists of a resistance and a reactance connected so as to advance the phase of the e.m.f. impressed on one circuit of the motor and retard the phase of the e.m.f. on another circuit, while the line voltage is impressed on the third winding. This may be accomplished

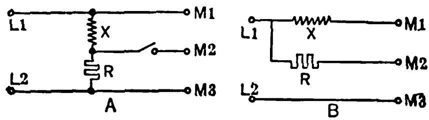

Fig. 1.

either by connecting the resistance and reactance across the line terminals and in multiple with the motor windings as shown in Fig. 1 A, or by connecting the resistance and reactance in series with the respective windings as shown in Fig. 1 B.

In these figures L_1 and L_2 represent line terminals, R is the resistance coil, X the reactance coil and M_1, M_2 and M_3 the three motor terminals. The former method gives a greater starting torque but takes a greater current in proportion to the torque. The latter method is more efficient. The motor must have sufficient resistance in its secondary circuit to give good starting characteristics as a three-phase motor.

The principle of this device is that the voltages between the outside terminals and the middle point are out of phase with each other and form a somewhat

flattened vector triangle similar to that of an unbalanced three-phase system. The ratio of the starting torque obtained with such a device to the normal starting torque with balanced three-phase voltages is the same as the ratio of the altitude of the triangle of vector voltages to the altitude of the equilateral triangle. Thus in Fig. 2 the ratio of altitudes is $30/95 = 0.316$; thus the single-phase starting torque would be 31.6 per cent of the three-phase starting torque.

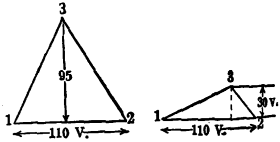

Fig. 2.

Use of Condenser and Resistor. — If a condenser of the proper capacity is substituted for the resistance the starting torque will be increased and the efficiency improved, but this is much more expensive and frequently involves the addition of a transformer across the condenser.

Use of Commutator. — Another method of starting single-phase motors involves the use of a commutator which permits the motor to start as a repulsion motor (which has good starting qualities; see *Motors, A-C. Commutator*), and after the motor has reached a considerable speed the brushes are removed from the commutator and a short-circuited squirrel-cage winding comes into play.

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[W. I. SLICHTER.]

MOTORS, SYNCHRONOUS.—(*See also Alternating Currents; Generators, Alternating Current; Motors, Industrial Applications of; Motors, Polyphase Induction.*) Any alternator will operate as a motor. If two synchronous alternators are connected in parallel to bus-bars supplying a load, and the driving power be removed from one prime mover, the alternator connected to this prime mover will continue to run at the same speed as before, taking power from the other alternator and driving its own prime mover or other apparatus coupled to it; i.e., this alternator acts as motor. The speed of such a motor depends solely upon the speed of the generator or generators supplying electric energy to it; it is therefore said to run in "synchronism" with the source of supply and is called a synchronous motor. The synchronous speed of a synchronous motor having p poles, when supplied with a current of a frequency of f cycles per second, is

$$N = \frac{120f}{p} \qquad \text{rev. per min.}$$

If the load on such a motor increases the speed will not decrease, unless the load reaches such an excessive value that the maximum output or "pull-out torque" is reached, when the motor will drop out of step and come to rest, while the current taken will increase to short-circuit value and the torque decrease to a negligible value.

Differences between an Alternator and Synchronous Motor. — The difference in construction between an alternator and a synchronous motor is that the latter has placed in the face of the field poles, a squirrel-cage winding, which is intended to give good starting torque and to prevent hunting while running. A synchronous motor usually operates better with a higher value of armature reaction than that of a well-designed generator of the same kilowatt rating. This increase in armature reaction is usually obtained in practice by operating the machine as a motor at lower voltage than that for which it would be operated as a generator. Thus a standard 2300-volt generator will operate very satisfactorily as a motor at 2080 volts, and as these are the natural values of the generated and delivered voltages, this characteristic of the synchronous motor fits in very well with customary distribution practice. Thus a standard generator may have a squirrel-cage winding added to its poles and become a good synchronous motor.

Field Excitation. — Synchronous motors always require direct current for field excitation and if a suitable d-c. source is not available an exciter must be provided.

Number of Phases. — Synchronous motors may be single, two or three phase. The single-phase motor is not self-starting and has a considerably lower efficiency than the polyphase motor. It is also more liable to hunt (*see below*) and be unstable, and is therefore far less desirable than a polyphase motor. The two-phase and three-phase motors are very similar in all their characteristics. There is a slight economy in the three-phase over the two-phase motor as there is in the three-phase over the two-phase generator (q.v.).

Terminal Voltage. — Since synchronous motors are usually built with a revolving field and a stationary armature, it is possible to insulate the armature winding for voltages as high as 13,000 and thus obviate the need of transformers in many cases.

Advantages and Disadvantages of Synchronous Motors. — The advantages of synchronous motors as compared to induction motors are: higher efficiency, higher power factor, power factor may be controlled, constant speed,

high voltage, lower cost. The disadvantages are: need of an exciter, will not start under load, possibility of hunting.

Relations of Voltage and Current. — The relations between line voltage and phase voltage are the same as in a-c. generators (q.v.). The current in each line of a three-phase motor is

$$I = \frac{746\,P}{\sqrt{3}\,\epsilon\,E\cos\theta},$$

where

P = horse-power output,
E = voltage between lines,
ϵ = efficiency at load assumed,
$\cos\theta$ = power factor (may be unity).

Usual values for efficiency are about the same as for a-c. generators (q.v.).

APPLICATIONS. — In order to transform from alternating to direct current, or from one kind of alternating current to another differing in frequency, potential or phase relation, motor-generator sets, consisting of a synchronous motor direct connected to one or more generators, are often employed. By this means the potential of the secondary or distribution circuit is independent of the variation in potential of the primary circuit supplying power to the motor. In certain cases it is desired to take power from a 25-cycle circuit and supply power at 60 cycles for lighting purposes. Here a synchronous motor-generator set would be used. Such a set is frequently called a "frequency changer" (*see Motor Generators*). In some applications of electric drive by induction motors one synchronous motor is installed for the purpose of making it take leading current in order to neutralize the lagging current taken by the induction motors. This effect is produced by over-exciting the fields of the synchronous motor. The motor may be used to drive any machinery that does not require much starting torque. Such a motor is called a "rotary phase modifier" or "rotary condenser," see also below.

GENERAL PRINCIPLES. — In any synchronous generator or motor when a current flows in the armature there is a loss of voltage proportional to IZ_0, where I is the current and Z_0 is a hypothetical quantity called the synchronous impedance, which includes the effect of the resistance, the leakage reactance and the demagnetizing effect of the armature current. This quantity is obtained by the synchronous impedance test (*see below*) and is expressed in complex quantities as $Z_0 = r + jx_0$ and in algebra $Z_0^2 = r^2 + x_0^2$ where r is the effective resistance per phase and x_0 is the synchronous reactance per phase. IZ_0 is therefore the drop in voltage per phase in the armature, and this voltage and the current differ in phase by an angle θ, where $\tan\theta = x_0/r$ and $\cos\theta = r/Z_0$. If a synchronous motor is running and generating a counter e.m.f. e and is connected to bus-bars of voltage E, the current flowing in the armature will be proportional to the vector difference between E and e and inversely proportional to Z_0, all taken per phase.

Vector Relations for Motor Action. — In Fig. 1 let E represent the bus-bar or line voltage (per phase) impressed on the terminals of a synchronous motor. Let e represent the counter e.m.f. of the motor and in this case assume $e < E$ and directly opposed to E. Then the difference between E and e will set up a current in the armature of the motor. IZ_0 will be the voltage and the current will lag behind IZ_0 by an angle θ where $\cos\theta = \frac{r}{Z_0}$. Thus the motor takes a current I lagging behind the impressed voltage E.

Fig. 2 shows the relations when $e > E$, then IZ_0 will be reversed in phase as indicated. I will always lag behind IZ_0 by the angle θ and will be found drawn upward. I lags behind e and IZ_0 but I leads the impressed e.m.f. E by an angle $(180° - \theta)$. Thus when the field excitation is increased so that e tends to

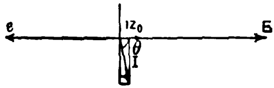

Fig. 1.

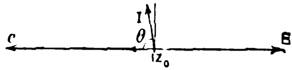

Fig. 2.

become greater than E, the machine takes a current leading with respect to the impressed e.m.f. As in a synchronous generator the field excitation required for a given terminal voltage depends upon the phase relation of the external circuit or the load, so conversely in a synchronous motor the phase relation of the current into the armature at a given terminal voltage depends upon the field excitation and the load.

Fig. 3 shows the relations when e is more than 180 degrees behind E, that is, e is behind the position it had in the preceding examples by an angle α. The vector resultant of E and e will be IZ_0 as shown, leading E. The current I will lag behind IZ_0 by the same angle θ, but is now almost in phase with E and lagging only slightly. Thus power $(EI \cos \phi)$ is being sent into the ma-

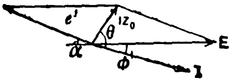

Fig. 3.

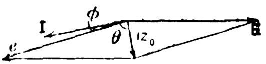

Fig. 4.

chine and it acts as a motor, transforming electrical power into mechanical power. When the machine is running as in Figs. 1 and 2 the power is very small. If, however, a mechanical load is applied, the armature will drop back in position by a slight amount. This causes e to drop back in phase and the machine immediately draws power from the line.

If, as in Fig. 4, power is applied to make the armature move forward and cause e to advance in the opposite direction, the resultant IZ_0 is thrown downward and I, lagging behind IZ_0 by θ, is thrown around almost in phase with e. The machine then becomes a generator, transforming the mechanical power applied into electrical power $eI \cos \phi$.

Synchronous Position. — In the discussion of the action of a synchronous motor it is convenient to employ the term "synchronous position," by which is meant the position which any definite point on the revolving member occupies at the same period of each cycle of time. It is only necessary for the machine to change in synchronous position by a very slight angle α to cause a large energy current to flow. If α should become 90 degrees, theoretically the power would become a maximum, and any increase in α means that θ becomes greater than 45 degrees, the power decreasing and the machine falling out of step. When α becomes 180 degrees a total e.m.f. equal to the sum of e and E is short-circuited by Z_0 and the current is enormous and the power factor low.

Maximum Torque. — If the speed changes for a short instant of time sufficiently to allow a point on the armature to drop back in synchronous position one-half the pitch of one pole (90 electrical degrees), the motor torque will increase from zero to the maximum available in the motor. Thus, for any torque less than the maximum the armature (or revolving field) need only change in speed sufficiently to drop back some distance less than one-half the pitch of a

pole. If the load demands a torque greater than this maximum the armature will drop back more than 90 degrees and will fall out of step and come to rest. In most synchronous motors the maximum torque is about 6 times normal torque.

DESIGN AND CALCULATIONS. — (*See also Generators, Alternating Current.*) The design and calculation of synchronous motors is very much like the design and calculation of alternating-current generators. There is a difference in the proportioning of certain details and there are certain features that are of importance in generators that are not important in motors (i.e., regulation) and the converse is also true.

Armature Reaction. — The armature reaction of a synchronous motor is expressed as $\frac{1.5\sqrt{2}SI}{p}$ for a three-phase machine and $\frac{\sqrt{2}SI}{p}$ for a two-phase machine, where S = turns in series per phase, I = full-load current per phase and p = number of poles. The armature reaction of a motor is usually 30 per cent to 50 per cent greater than that of a generator of the same rating and frequency. The higher value gives greater synchronizing torque per ampere, a better starting torque, and reduces the cross-currents between machines in case of hunting. The armature-reaction ampere turns at full load may be equal to the field ampere turns at no load. Too great an armature reaction is objectionable, because it reduces the energy transfer between two machines and therefore reduces the synchronizing power, that is, the tendency of the machines to hold each other in step.

Excitation. — The excitation of a motor is calculated in the same manner as the excitation of a generator. The magnetic densities are usually a little less. The capacity of the field winding depends upon whether the synchronous motor is to be used as an ordinary motor or to regulate the power factor of a system by over-excitation.

Leakage Reactance. — The leakage reactance may be higher in a motor than in a generator as regulation is not of so great importance. However, too great a reactance will reduce the starting torque of the motor.

Short-circuit Current and Synchronous Impedance. — The short-circuit current of the motor depends upon the leakage reactance and the armature reaction. The short-circuit current and the synchronous impedance may be predetermined from the no-load saturation curve and the calculated leakage reactance per phase.

Let F = excitation in ampere turns per pole for which it is desired to find the short-circuited current,
I = rated current per phase,
x = leakage reactance in ohms per phase,
E = voltage per phase due to F at no load,
S = turns in series per phase,
p = number of poles.

Then the ampere-turns synchronous impedance for full-load current is

$$\frac{2.12\,SI}{p} + \frac{IxF}{E},$$

and the short-circuit current with excitation F is

$$I_0 = \frac{EpF}{2.12\,SE + xpF}.$$

The synchronous impedance at this excitation is $Z_0 = E/I_0$, and the syn-

chronous reactance is $x_0 = \sqrt{Z_0^2 - r^2}$, where r is the resistance of the armature per phase.

Efficiency and Losses. — The losses are predetermined as in a generator. They are: A = friction, B = excitation or field RI^2, C = core-loss, D = armature RI^2; then

$$\text{Efficiency} = \frac{\text{Output}}{\text{Output} + A + B + C + D}.$$

Phase Characteristics or V-Curves. — The phase characteristic is a curve showing the variation in armature current for any given load with varying field excitation. Fig. 5 shows the shape of the curves, which may be determined both by calculation and test. The phase characteristic for any particular load has the general shape of the letter V, and the group of such curves for various loads are frequently called the "V-curves" of the machine. There are two methods of calculating the phase characteristics, the electromotive force method and the magneto-motive force method.

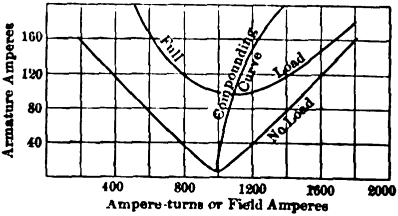

Fig. 5. Phase Characteristics of Synchronous Motor

Electromotive Force Method. —

Let E = line voltage per phase,

e = motor counter e.m.f. per phase corresponding to the excitation to be used,

i = component of current in phase with e,

ei = mechanical output of armature,

i_1 = reactive component of current, positive for leading, negative for lagging,

r = armature resistance per phase,

x_0 = synchronous reactance per phase,

then $E^2 = (e + ri - x_0 i_1)^2 + (x_0 i + r i_1)^2 = e_1^2 + e_2^2$.

E, r and x_0 are constant, e is set by the value of the field current assumed, then i = (watts output)$/e$ and i_1 remains the only unknown quantity. Solving for i_1 the total current $I = \sqrt{i^2 + i_1^2}$ and the power is $e_1 i - e_2 i_1$ (for i_1 lagging). If e is greater than E, then i_1 is leading, and the power factor is $(e_1 i + e_2 i_1)/EI$.

Magneto-motive Force Method. —

Let E = line voltage per phase,

F = field ampere turns per pole for this voltage,

D = any value of field ampere turns per pole,

I = armature current per phase,

i = energy component of current = (power input) ÷ E,

p = number of poles,

ν = field leakage coefficient of machine,

S = turns in series per phase,

i_1 = wattless component of current for excitation D.

Then

$$pF = pD - 2.12\, S i_1 \nu \text{ and}$$

$$i_1 = \frac{pD - pF}{2.12\, S\nu},$$

and

$I = \sqrt{i^2 + i_1^2}$ is the current in armature for power input Ei and for the excitation D assumed. If i_1 is negative ($F > D$) then i_1 is lagging. If $D > F$ then i_1 is positive and leading.

Angular Lag Due to Load (α). — For any load on a synchronous generator there is an angular advance of the generated e.m.f. ahead of the terminal e.m.f., and in a synchronous motor there is an angular lag of the generated e.m.f. behind the terminal e.m.f. This phase displacement is accompanied by a shift in the synchronous position of the armature, which may be calculated and actually measured (*see Tests of Synchronous Motors below*).

To calculate this angle α expressed in electrical degrees (360 degrees per pair of poles), let

E = line or terminal voltage per phase,
e = induced or counter e.m.f. per phase; may be taken from no-load saturation curve for given excitation,
r = resistance of armature per phase,
x_0 = synchronous reactance per phase,
I = current per phase,
ϕ = phase angle between E and I.

Then

$$e^2 = (E - Ir\cos\phi - Ix_0\sin\phi)^2 + (Ir\sin\phi - Ix_0\cos\phi)^2,$$

and

$$\alpha = \sin^{-1}\left[\frac{(Ir\sin\phi - Ix_0\cos\phi)}{e}\right],$$

or roughly

$$\sin\alpha = \frac{I\cos\phi}{\text{short-circuit current}}.$$

The mechanical displacement of the armature for the load $EI\cos\phi$ per phase is $2\,\alpha/p$.

Synchronizing Torque.—The synchronizing power of a machine is a measure of the ability of a machine to keep in step with its supply circuit. It may be expressed in terms of torque per degree of displacement. If P is the kw. output of a motor and α is the angular displacement of the armature in electrical degrees for this load, then

$$\text{Motor torque} = \frac{7050 \times (\text{kw.})}{\text{rev. per min.}},$$

$$\sin\alpha = \frac{(Ir\sin\phi - Ix_0\cos\phi)}{e},$$

and

$$\text{Synchronizing torque} = \frac{7050 \times (\text{kw.})}{\alpha \times (\text{rev. per min.})}.$$

A high resistance between machines reduces the synchronizing torque as it reduces E. A reactance between machines is not as bad as resistance. Increasing the excitation increases e and improves the synchronizing torque.

Hunting; Natural Period. — The rotating part of every synchronous machine acts like a pendulum, tending to swing ahead and behind its normal synchronous position. The mass of the armature (and its flywheel) acts like the mass of the pendulum, and the torque of the machine being proportional to the displacement (α) corresponds to a spring or gravity acting on a pendulum. Such a combination has an "electro-mechanical period" of its own, and if the frequency of this period is in tune with any other pulsating force in the system, such as engine impulses, "hunting" or "surging" may occur.

Boucherot and Kapp have shown that the natural period of any synchronous machine expressed in seconds or fraction of a second is given by the formula:

$$T = 0.308\,N\sqrt{\frac{Wk^2}{fgmEI_0}},$$

where N = r.p.m. of revolving part,
W = total weight of revolving part including any flywheel in pounds,
k = radius of gyration of W in feet,
f = frequency of current, cycles per second,
g = acceleration of gravity, in ft. per sec. per sec. = 32.2,
m = number of phases,
E = terminal voltage per phase,
I_0 = short-circuit current of machine per phase and at excitation used.

The frequency of the natural period expressed in impulses per minute is $f_1 = 60/T$.

The formula shows that the greater the flywheel effect kW^2 (*see Flywheels*) the longer will be the periodic time of a swing. The greater the short-circuit current or excitation the shorter will be the periodic time. The periodic time may be increased by connecting reactance coils in series with the machine between its terminals and the bus-bars.

If T_0 is the periodic time of any other pulsating force in the system, as the strokes of a steam engine, the danger of hunting is greatest when

$$T/T_0 = 1/4,\ 1/3,\ 1/2,\ 1,\ 3,\ 5, \text{ etc.}$$

A tendency to hunt is damped by solid pole pieces, bridges between poles or, best of all, a squirrel-cage winding in the pole face.

Maximum Output. — As the current which would flow during maximum output of a synchronous motor is so great that it would burn up the windings in case this output should last more than a fraction of a second, the value of the maximum output is only of theoretical interest. In practice the maximum output (for a given voltage) is only reached under two conditions: (1) when, due to extraneous causes, the line voltage decreases to a fractional value, the maximum output decreasing as the square of the voltage; (2) when, due to hunting or pulsation, the flow of energy into and out of the machine reaches excessive values momentarily. In one of these swings the power may reach the value of the maximum output or exceed it and the machine shut down. Although the power of the machine drops off gradually after the point of maximum output has been reached, the motor is unstable in this region and is more than likely to shut down when the condition is reached.

Starting Torque. — All synchronous motors must be started either as hysteresis* or induction motors. In the former case the motor requires a high voltage and takes a small current, but as the torque is very slight this method is seldom used in practice. When starting as an induction motor a high armature reaction is desirable and a low leakage reactance. A squirrel-cage winding in the pole face must be provided, and as in an induction motor this squirrel-cage winding must have a cross-section approximating a certain value. If the cross-section is too large, the currents will be excessive and the starting torque not the best. If the cross-section is too small, the currents will be small and the starting torque not the best. While the cross-section of the squirrel-cage winding may be roughly predetermined by treating the ma-

* A piece of iron in a rotating field has a torque produced on it due to the hysteresis and eddy currents set up in it. Such an arrangement may be called a "hysteresis" motor; an open-circuited field of a polyphase synchronous motor therefore forms a hysteresis motor.

chine as an induction motor, this method is not accurate because the construction of the field renders the calculation of the leakage reactance inaccurate. It is thus much better determined empirically.

Rotary Phase Modifiers or Rotary Condensers. — Synchronous motors are sometimes used to improve the power factor and reduce the line current of an installation of a number of induction motors. If a factory has an installation of 100 kw. of induction motors having an average power factor of 71 per cent and taking I amperes, then by installing a synchronous motor of 100 kw. rating, designed to be overexcited, the power available will be doubled and the line current only increased by 41 per cent or to 1.41 I. Such a machine is called a "rotary condenser" and if it is rated at 100 kw. it may give 71 kw. of power and 71 kv-a. to balance the reactive effect of the inductive apparatus. Other relations may be obtained in accordance with the principle of vector combinations.

Let P_1 = true power taken by the induction motors, kw.,
Q_1 = reactive power taken by the induction motors, kv-a.,
$L = \sqrt{P_1^2 + Q_1^2}$ = total kv-a. of induction motors,
P_2 = true power taken by rotary condenser, kw.,
Q_2 = reactive power taken by rotary condenser, kv-a.,
$K = \sqrt{P_2^2 + Q_2^2}$ = total kv-a. of condenser,

then

$$\text{Line kv-a.} = \sqrt{(P_1 + P_2)^2 + (Q_2 - Q_1)^2}.$$

TESTS OF SYNCHRONOUS MOTORS. — Certain tests on synchronous motors are the same as those made on an a-c. generator; the methods of carrying out such tests are described in the article on *Generators, Alternating Current*. The first five of the following tests are of this character:

1. Resistance of armature and field circuits both cold and hot.
2. Saturation curve at no load and under special circumstances at full load.
3. Core-loss.
4. Short-circuit or synchronous impedance.
5. Insulation tests.

6. **Phase Characteristics or V-Curves** at no load, full load and any other specified load. The machine is operated as a motor with the specified load kept constant throughout the run. The voltage and frequency impressed upon the motor are also kept constant. The current in the field is varied from the minimum at which the motor will operate to the maximum (from ¼ normal to 1½ normal) and the variation in current input to armature noted. Readings are taken of load, volts armature, amperes armature and amperes field. A curve is plotted with amperes armature as ordinates, and amperes or ampere turns per pole in field as abscissae. This gives the characteristic V-curves of the synchronous motor (see Fig. 5). The point of minimum current input for each load is very clearly shown. At this point the power factor is unity. At lesser values of field current the armature current is lagging and the power factor poorer. At greater values of field current the armature current is leading. The point of minimum current occurs at a higher value of field current for the greater loads because the field excitation must be increased with the load to overcome the armature reaction due to the load current.

Compounding Curve. — The curve connecting the points of minimum armature current in the group of V-curves is called the compounding curve for unity power factor.

7. Starting Tests. — A low voltage is impressed on the armature and gradually increased until the motor starts. The field circuit is open in two or more places and a high potential voltmeter connected across one section to determine the voltage induced in the field spools by the rotating magnetic flux. The test is repeated for several different initial positions of the revolving part and a record is made of the time required for the machine to reach synchronism. The time at which synchronous speed is reached may be determined by the fact that the induced voltage in the field becomes zero. Readings are taken of volts armature, amperes armature, initial position, maximum volts field, time to reach synchronism. See also section on *Starting*, below.

8. Armature Phase Position. — The phase position of the armature discussed previously may be measured, although the item is only of theoretical interest and not of commercial importance. A synchronous motor is supplied with power from a special alternating-current generator, and on the end of the shaft of each machine is placed a contact-making disk, as shown in Fig. 6.

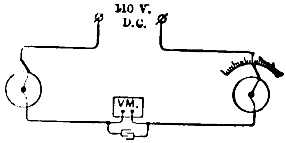

Fig. 6. Contact Method of Measuring Armature Phase Position

A voltmeter is connected in series with a source of direct current, the two disks, and the brushes pressing on the disks. The voltmeter reading is a maximum when the two brushes are in contact with the metal strips at the same time. The brush on the motor may be moved over a graduated scale or arc, so that its position may be varied and the actual angular movement measured. The brush on the generator remains stationary. As the load on the motor is increased it will be found necessary to move its brush in order to keep the voltmeter reading at its maximum value, and the angle β through which the brush has been moved for the change in load gives the phase position of the motor armature with respect to the generator armature. It will be found that the value of β is directly proportional to the load. The difference in phase of the electromotive forces in electrical degrees will be β multiplied by the number of pairs of poles of the motor. If hunting exists, the maximum angle of swing may be determined by moving the motor brush first one way as far as the effect may be noticed, and then in the other direction. (*See Morecroft, Laboratory Manual.*)

SPECIFICATIONS. — Synchronous motors are rated in the same manner as synchronous generators, and the same heating limits and specifications apply. (*See Generators, Alternating Current.*) It is customary to specify the value of the current taken by the motor in starting with no load other than the friction of its own bearings, or its own friction plus that of the machine to which it is connected, in case it is part of a motor-generator set. It is also sometimes mentioned in the specifications that the motor will not hunt providing the total resistance drop between the generator and motor is less than some specified value (10 per cent or 15 per cent).

INSTALLATION AND OPERATION. — The precautions to be taken in installation are the same as for a-c. generators (q.v.). Direct current must be provided for excitation. If a synchronous motor is operated on a polyphase system having unbalanced voltages it will take unequal currents in the different lines and tend to balance the voltages. These unequal currents, however, increase the heating somewhat for a given load.

Starting. — Provision must be made for a reduced voltage for starting the motor, either by means of taps on the transformers or by means of a starting compensator (q.v.). Large motors require two steps in starting, ⅓ and ⅔

of normal voltage. Small motors will start with one step at ½ voltage. The field circuit is opened by the field "break up" switch and a voltage applied to the armature. When the armature current has decreased from its large value at starting to a reasonable value the voltage is increased, step by step. When the motor has reached maximum speed the field is excited and the motor pulls into exact synchronism. The field current is then adjusted until the condition of minimum armature current is found or until the power-factor indicator records unity power factor. The load may then be applied.

If a synchronous motor is to be started often (several times a day) it is desirable to provide a special starting motor which brings the synchronous motor up to a speed a little above synchronism. Synchronizing must then be effected as in a-c. generators. Such a motor would require only about 30 per cent of the full-load current of the motor, and therefore have very little effect on the regulation of the system and not cause disturbance to the lights and other motors on the system. It usually requires less than a minute to bring a motor up to speed.

DIMENSIONS, WEIGHT AND COSTS. — These are approximately the same as for synchronous generators of the same kv-a. rating. (*See Generators, Alternating-Current.*)

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[W. I. Slichter.]

OHMMETERS. — (*See also Bridges for Electrical Measurements; Galvanometers.*) The name ohmmeter is applied to any portable device designed for the direct measurement of electrical resistance. The simplest form of ohmmeter consists of a slide-wire bridge, battery and galvanometer all mounted in a portable case. Another form of ohmmeter utilizes a special form of galvanometer whose deflection, measured on a properly calibrated scale, gives the value of the unknown resistance directly in ohms.

BRIDGE TYPE OF OHMMETER (Fig. 1). — This ohmmeter is based on the Wheatstone-bridge principle, but differs from most Wheatstone bridges in that the rheostat resistance R is kept constant and bridge arms, formed by the wire W, are varied by moving a contact C. Ohmmeters of this type are made in many varied forms. In some a straight slide wire is stretched over a cardboard scale and contact on the wire made with a metallic stylus. In others a slide wire is wound on an insulating cylinder.

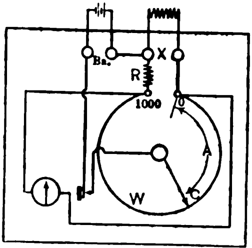

Fig. 1. Slide Wire Ohmmeter

A satisfactory type is that in which the slide wire is mounted on a disk and the disk rotated, the contact being fixed. A high-resistance slide wire is secured by winding a helix of insulated wire on a small mandril and mounting it on the periphery of the disk, removing the insulation from the wire where the contact touches.

The scale may be equally divided into a thousand parts, or may be divided to read directly the ratio of the resistance measured to the resistance of the fixed rheostat resistance R.

When provided with a direct-reading scale, the slide-wire ohmmeter is an exceedingly useful instrument for factory work, where a large number of resistances of various magnitudes are to be measured, and no great accuracy of measurement is required. Where a great number of resistances all having approximately the same value are to be measured, the instrument can be made with a scale covering a small range and good accuracy secured.

It makes a convenient instrument for cable-testing work, especially when provided with telephone and buzzer, as the set may then be used to locate opens by a capacity method. When so equipped it is also possible to measure resistances where earth currents would make ordinary Wheatstone-bridge measurements impossible, as in measuring the ground resistance of lightning rods, etc.

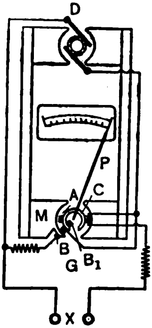

Fig. 2. Evershed Megger

EVERSHED MEGGER (Fig. 2). — This instrument is of the galvanometer type. It is principally useful for the measurement of insulation resistances and other high resistances where extremely high accuracy is not required. The scale is graduated to read directly in megohms, whence the name.

Referring to Fig. 2, D is a small d-c. generator and G a special form of pivoted galvanometer, consisting of a permanent magnet M, a soft iron core C, a current coil A and two pressure coils B and B_1. These three coils are rigidly attached at a fixed angular distance apart to the shaft which carries the pointer P. The generator and galvanometer are mounted in a single box provided with two binding posts for

connecting the unknown resistance X and a handle for turning the armature of the generator by hand.

The coil BB_1, called the pressure coil, is permanently connected across the terminals of D. The current coil A is made to move through an annular gap in such a manner that the field in which it moves is uniform, whereas the pressure coils BB_1 move from a position midway between the poles, where the field is at a minimum, into a stronger and stronger field, the connections being such that the torque due to the current in the current coil is opposed by the torque due to the current in the pressure coil. With no current in the current coil, that is to say, when the resistance to be measured is infinite, current through the pressure coils will cause them to come to rest with their plane at right angles to the magnetic field. When the current through the current coil is increased by putting in lower resistances the current coil drags the moving system round in a clockwise direction; since the pressure coils come into a stronger and stronger field, the resistance to this motion becomes greater and greater. Hence a definite position is assumed by the system for the particular resistance at X.

An increase in voltage would increase the current in both current and pressure coils in the same proportion; consequently the instrument is independent of the voltage of the generator.

To test the insulation of a circuit it is only necessary to set the megger down on a fairly level base, connect the circuit wires to the line and earth terminals, and give the generator handle half a dozen rapid turns, when the index promptly comes to rest and points to the resistance in megohms.

In the cheaper instruments the generator gives a variable voltage depending upon the speed of rotation of the handle. In the higher priced instruments the generator is designed to give a constant voltage for a considerable range in the speed of driving. The voltage at which an insulation test is made is often of great importance (*see Resistance and Conductance*).

COST, RANGE AND ACCURACY. — Bridge type ohmmeters range in price from $35 for ohmmeters having a range of from 0 to 100 ohms and an accuracy of about 2 per cent to $80 for ohmmeters having a range of from 0 to 10,000 ohms and an accuracy of about ½ per cent. Meggers range in price from $187.50 for variable-voltage meggers having a range of from 0 to 10 megohms and an accuracy of about 5 per cent to $375 for constant-pressure meggers having a range of from 10 to 2000 megohms and an accuracy of about 4 per cent.

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[H. Pender and H. R. Ranken.]

OIL, TRANSFORMER. — (*See also Transformers; Circuit Breakers; Insulating Materials, Testing of.*) While mineral, vegetable and animal oils when pure are all good insulators, mineral oils obtained from petroleum products are almost universally adopted for use in transformers and switches. The mineral oil employed for this purpose is commonly called Transil Oil.

Mr. D. B. Rushmore gives the following as the characteristics of oil usually furnished with large, high voltage, water-cooled transformers:

Flash point	130° C.
Burning point	145° C.
Freezing point	−15° C.
Color	white
Spec. gravity at 15.5° C.	0.830
Viscosity (Saybolt 40° C.)	40

Dielectric Strength and Specific Inductive Capacity. — Fig. 1, from Hendrick's paper, gives the puncturing voltage of dry transformer oil for variously-shaped electrodes. These are standard curves suitable for use in design. They are based on the same data as those given by H. W. Tobey (*see Bibliography*). The tests were made under the following conditions:

Curves. — For 4-in. disks; 2-in. balls; 1-in. blunt conical points; needle points; 4-in. disk and needle point. *Material.* — Heavy transformer oil. *Dimensions.* — Specific gravity, 0.868; viscosity at 40° C. = 100 Saybolt. *Composition.* — From Pennsylvania crude. *Treatment.* — Filtered through dried blotting paper. *Method of test.* — Beginning at lowest voltage, each curve is taken up to highest voltage and down again — about 10 to 15 points being taken on each curve. Standard test on oil at beginning and end shows that quality remained nearly constant. *Temperature.* — 20 to 25° C. *Time.* — Instantaneous. *Frequency.* — 60. *Wave.* — Sine. *Number of trials.* — Each point, about five. *Accuracy of curves.* — Plus or minus 10 per cent. *Characteristics.* — (Fig. 1.) Puncture voltage depends very largely on shape of electrodes. Curve 1 — using 4-in. disks. Curve 2 — using 2-in. balls. Curve 3 — using 1-in. blunt conical points. Curve — 4 using needles. Curve 5 — using 4-in. disk and needle.

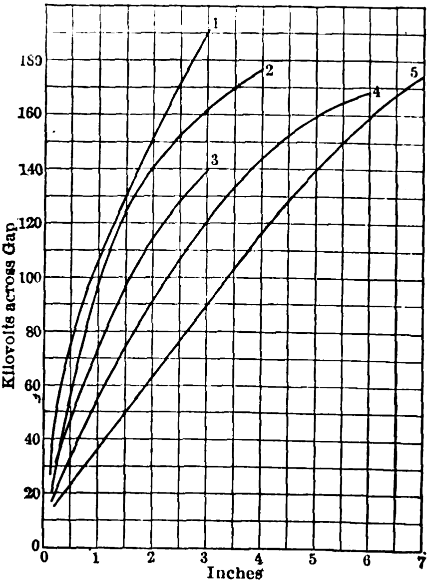

Fig. 1. Puncturing Voltage for Dry Transformer Oil

The specific inductive capacity as determined from these tests was 2.5 between 25 and 100° C.

Effect of Moisture on Dielectric Strength. — The effect of moisture in oil is shown in Fig. 2, which curve is taken from Hendrick's paper (*see Bibliog-*

raphy). The conditions were as follows: *Material.* — Heavy transformer oil. *Dimensions.* — Specific gravity, 0.87; viscosity at 40° C. = 100 Saybolt. *Composition.*—From Pennsylvania crude. *Treatment.* — Oil is first filtered through dry blotting paper, and oil and water then emulsified by mechanical shaker. *Method of test.* — Standard spark gap 0.2 in. between 0.5-in. disks; two separate emulsions; four samples of each; five trials on each sample. *Temperature.* — 20 to 25° C. *Time.* — Instantaneous. *Frequency.* — 75. *Wave.* — Sine. *No. of trials.* — Each point, 40. *Accuracy of curve.* —Plus or minus 5 per cent. *Characteristics.* — Extremely rapid reduction of dielectric strength by minute quantities of water—under 0.01 per cent if thoroughly mixed. *Specific capacity.* — Dry oil = 2.5 at 25 to 100° C. *Notes.* — Practically identical results obtained on light oil whose specific gravity was 0.85 and viscosity at 40° C. was 40 Saybolt.

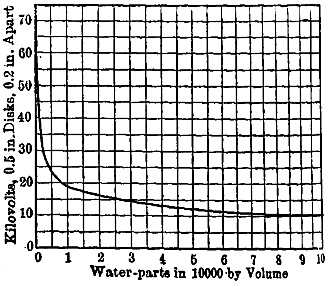

Fig. 2. Effect of Moisture on Puncturing Voltage

Methods of Removing Moisture; Chemical Treatment. — Drying agents, such as calcium chloride, calcium oxide (unslaked lime), calcium carbide and metallic sodium, respectively, are mixed with the oil in the proportion of 6 parts of dehydrating material to 100 parts of oil by weight. To be effective the mixture must stand for three or four days before the drying agent is filtered off. Calcium chloride works faster and more efficiently than any of the other materials mentioned and is most commonly used.

Heat Treatment. — Oil heated to a temperature of 105° C. is rendered quite dry but there is great danger of injuring the oil by overheating. If the pressure on the oil is reduced when heated, the moisture may be evaporated at a lower temperature, thereby reducing the danger of overheating. Dry air blown through moist oil will take up much of the moisture, especially if used in connection with either of the above heat treatments.

Separation by Gravity. — When the oil contains a large amount of water, the water will settle to the bottom if the oil is left undisturbed for a number of days. The dry oil may then be drawn off from the top.

Centrifugal Separation. — If moist oil is placed in a centrifugal separator similar to the De Laval cream separator, most of the water will be forced into the outer casing and the remainder may be extracted by passing the oil through a filter.

Paper-filter Treatment. — When moist oil is forced through a paper filter, the water content will not pass through the filter. Since the process is slow in a gravity filter, high-pressure filters are used in which a pressure pump forces the moist oil through several layers of blotting paper. This type of filter press is especially convenient since the moist oil may be drawn from the bottom of a transformer case, dried in a filter press and fed into the transformer case again at the top. In this method, the oil is not only dried but is also freed from dirt and other foreign matter. (*Bibliography reference, H. W. Tobey.*)

Effect of Temperature on Puncturing Voltage and Insulation Resistance. — The curves in Fig. 3, taken from Tobey's paper, show the effect

of temperature upon the dielectric strength and insulation resistance of transformer oil. Tobey also found that at about −8° C. the dielectric strength increases very rapidly with further decrease of temperature.

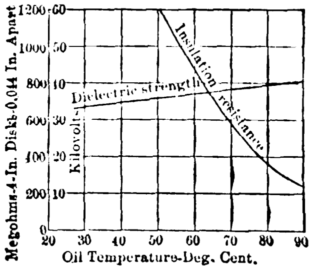

Fig. 3. Effect of Temperature on Puncturing Voltage and Insulation Resistance of Transformer Oil.

TESTING OF TRANSFORMER OIL. — The puncturing voltage, insulation resistance and specific inductive capacity of oil is determined in much the same manner as for other dielectrics; see *Insulating Materials, Testing of.* Full directions for making all the necessary tests on oil are given in a paper by C. E. Skinner, *Elec. Club Jour., Vol. 1, p. 227.*

Test for Moisture. — The presence of moisture in oil can be detected by the hissing sound produced when a very hot nail is dropped into it. A bluish tinge given anhydrous copper sulphate placed in oil also indicates the presence of moisture.

Flash and Fire Tests. — In the "open test" for the flash point of an oil, about 50 cc. of the dry oil are heated in an open vessel. The oil should be heated slowly and uniformly and the temperature noted by a thermometer immersed in the oil. From time to time a small flame is brought on to the surface of the oil and the lowest temperature at which a slight explosion or flash takes place is called the "flash point." In the "closed test" the oil is placed in a closed vessel, the cover of which is perforated with several holes, which may be opened or closed. The oil is heated as in the "open test" and is stirred by a shaft which passes through the center of the cover. To determine the flash point, the holes in the cover are opened simultaneously and a small flame is then directed upon the surface of the oil through one of the openings. The fire test, in which the ignition point of the oil is determined, is carried out in a similar manner as that for the flash point. The ignition point is the lowest temperature at which the oil will continue to burn after a flame has been brought in contact with its surface for a few seconds. (*J. Lewkowitsch, see Bibliography.*)

Evaporation Test. — A small quantity of the oil is dropped upon a watch glass, and, after being carefully weighed, is placed in a constant-temperature oven for a number of hours. The temperature and duration of the test should conform as far as possible with the conditions under which the oil is to be used. After heating as indicated, the oil on the watch glass should again be weighed carefully to note the shrinkage. In general, a test at 50° C. for a period of 24 hours should not show an evaporation of more than 1 per cent. (*Bibliography reference, A. H. Gill.*)

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[R. G. HUDSON.]

OSCILLOGRAPHS. — (*See also Braun Tube; Wave Analysis.*) The oscillograph is essentially a galvanometer of very short period. It is applied in the observation of potentials or currents, as voltmeter or ammeter, mainly when variations are too rapid to be indicated by the more usual instruments; for example in observation of potential waves of generators, potential and current waves in inductive apparatus, short-circuits or switching of transmission lines. As in many cases it is necessary to have two or more curves taken together in their relative phase relation, more than one element is necessary in a practical instrument; it is regularly built as a three-element oscillograph, giving one, two or three curves, as may be required, on the record. One element may be used as a chronograph by connecting it to record a timing wave from an a-c. source of known frequency.

Types of Oscillographs. — The most common form of oscillograph is the moving-coil type, the "coil" being two small thin strips or ribbons arranged very close together, thus forming the two sides of a coil of one turn. A moving iron vane can also be used, but this type has not received extended practical application. The electrostatic oscillograph, which is particularly well adapted for direct observation of high potentials, consists of two insulated strips maintained at a constant potential difference, these strips being caused to vibrate by the varying force of attraction and repulsion due to the charges on two fixed plates connected to the varying high-potential source. The vibrator is similar in construction to the vibrator of the ordinary oscillograph, except that the two strips are connected by a light insulating thread. The strips are connected to the terminals of a storage battery, the middle of which is kept at a potential midway between the potential of the two plates, by being connected between two equal condensers in series across the attracting plates. Still another form of oscillograph is the cathode ray, or Braun, tube; see article on *Braun Tube*. This last type of oscillograph finds its most valuable application in the laboratory study of high-frequency and high-voltage phenomena which are beyond the range of the ordinary oscillograph. The practical application of this type has been found rather difficult.

DESIGN OF MOVING-COIL OSCILLOGRAPH. — The ordinary or moving-coil oscillograph is described in detail below.

Construction of Vibrator. — The vibrator, or moving element (Fig. 1), consists of two strips of flattened wire stretched over bridges, with a very small mirror cemented directly to the strips; the arrangement constitutes a one-turn galvanometer coil of elementary form. The vibrator is placed between the wedge-shaped poles of an electromagnet or of a permanent magnet. It is immersed in a liquid which provides critical damping. The vibrator conductor passes from one terminal post T over the bridges BB_1 in narrow grooves, over a pulley P, back over the bridges in grooves very close to the former, to the other terminal post T_1. The width of the strip is usually 0.005 inch to 0.007 inch. The mirror M is cemented to the strips midway between the bridges, its usual size being 0.060 inch by 0.017 inch. Tension is applied to the strips, for the ordinary vibrator 6 oz. for the two strips, indicated by a small spring balance SB. The vibrator is readily rewired in case of break or burn-out.

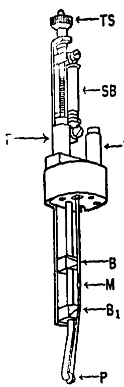

Fig. 1. Oscillograph Vibrator. B,B1 bridges for supporting strip; P, pulley; SB, spring balance indicating tension; TS, tension screw; M, mirror

For efficient operation vibrators are made interchangeable. This is necessary to make expeditious operation possible, and to minimize delay due to accidental destruction of vibrator parts.

Construction of Field Magnets. — The galvanometer field is an electromagnet with wedge-shaped poles. The vibrator is so mounted that the strips lie in the narrow air gap between these poles. A direct current through the winding of the electromagnet produces a very strong field in this air gap. The ampere turns of the electromagnet are sufficient to saturate the pole tips and render the strength of the field practically independent of the voltage applied to the electromagnet windings, at least for ordinary voltage variations. One terminal of the vibrator may be connected electrically to the core of its field magnet, each core being insulated from its field winding for a working pressure of 2,300 volts.

A permanent magnet can be substituted for the electromagnet as the oscillograph field. This construction is practicable for two elements. The permanent magnet oscillograph is somewhat inferior in sensibility, and greatly inferior in insulation between elements, compared with the electromagnetic type of oscillograph.

Optical System. — The light for the oscillograph from a projection arc lamp enters the case at the shutter aperture, and, after being reflected toward the vibrator cell by a total reflecting prism (one for each element), passes through a slit which adjusts the width of the image. It is then reflected by the small vibrator mirror, and passes through a cylindrical lens to the image on screen or film. Very careful adjustment of the optical system is essential to secure records of good photographic intensity.

The arc lamp used as a source of light for the oscillograph is usually a hand-feed lamp with small carbons at right angles, taking 5 to 8 amperes, with *solid positive* carbon in the horizontal position. The light is rendered parallel by a simple projection lens. Any convenient arc lamp, however, may be used with good results. If only an a-c. source is available for the arc, records of fluctuating intensity are obtained, which, however, are usually legible throughout their length. Sunlight source with a heliostat, when circumstances permit, gives photographic records of superior intensity.

The astigmatic optical system of the oscillograph, due to the cylindrical lens, is quite a distinctive one; the geometrical light source for the image in the vertical direction is the adjustable slit, in the horizontal direction it is the vibrator mirror.

Means for Obtaining Photographic and Visual Records. — The photographic record is taken on a moving film or plate. The most practical arrangement is a film, having a length of about 12 inches, on a drum driven at suitable speed by a small motor. A contactor opens the shutter for one revolution of the drum, giving exposure once over the film. The exposure can be adjusted to start at the beginning of the film, or if a record is taken in response to a signal, as of switching or short-circuit, the exposure can be started instantaneously at any part of the film. A long film is desirable in some cases, as in transmission line switching, where the whole disturbance lasting perhaps several seconds is to be taken. By the use of a suitable attachment for long films, records can be taken on films 3 to 5 feet in length.

The motion of the spot of light reflected from the vibrator mirror may be projected as a standing wave on a tracing table, for examination or demonstration. An oscillating mirror, actuated by a cam driven by a small synchronous motor, is given a uniform angular velocity during alternate cycles of the current or voltage observed, and draws out the wave longitudinally. Only alternate waves or cycles are utilized, the intervening waves being cut off by a revolving screen on the motor shaft during the return motion of the mirror. The recur-

rences of the waves are so rapid as to produce an image sensibly continuous to the eye. The synchronous mirror is removed by shift of a simple mechanism to permit photographic record to be taken in the usual manner by revolving film.

Free Period of Vibrator. — The free period of the vibrator is about 1/6,000 second, but some have been constructed to have a free period as high as 1/10,000 second. The higher the free period the less the distortion of the higher harmonics in the current and voltage waves. A free period of 1/6,000 second, is ample to secure practically accurate values of all harmonics having a frequency of less than 1200 cycles per second; with a free period of 1/10,000 second harmonics having a frequency as high as 2000 cycles per second are recorded with practical accuracy.

The free period of a vibrator depends upon the moment of inertia of the vibrating system and upon the tension on the strips; the less the moment of inertia and the greater the tension, the shorter will be the free period. The free period of a given vibrator may therefore be slightly shortened by increasing the tension on the strips, but this in turn decreases the sensibility, i.e., the deflection for unit current through the strips.

To measure the free period of a vibrator it is placed in the cell between the poles of magnet, in the usual way, but with no damping liquid in the cell. It is connected to a d-c. source through an interrupter which makes or breaks the circuit one or more times during the exposure of a film running at as high a speed as possible. The free vibration in decreasing amplitudes is shown on the film, and the period is readily counted, the film speed being known.

Sensibility of Oscillograph. — An oscillograph having a free period of 1/6,000 second requires from 0.1 to 0.2 ampere through the vibrator to give a curve of good amplitude; or about 0.006 ampere is required to give a millimeter deflection from the zero line on the film or tracing table. As ordinarily constructed a deflection of 45 mm. on each side of the zero may be obtained, but a deflection of more than 30 mm. is seldom necessary. The resistance of the vibrator is from 1 to 1.5 ohms; hence to obtain a deflection of 30 mm. about 0.2 volt is required across the vibrator terminals. The sensibility of a vibrator may be much increased by using thinner strips and smaller mirrors, but such arrangements are more delicate and should be used only to meet special requirements.

CONNECTIONS AND ADJUSTMENTS. — For potential, or voltage, curves the oscillograph is connected similarly to a voltmeter; E in Fig. 2. A suitable amplitude of curve is obtained by an external adjustable resistance.

For current curves the oscillograph is connected similarly to a millivoltmeter across a non-inductive shunt; I in Fig. 2. A shunt potential drop of at least 0.1 volt is required, but for convenient adjustment it should be larger, 0.5 volt being a suitable value where practicable. If the current measured is less than 0.2 ampere, no shunt is used, the whole current being taken by the vibrator.

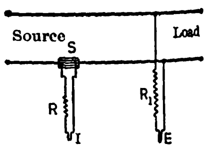

Fig. 2

For waves of flux distribution in generators or motors, the oscillograph is connected to give the voltage of an exploring coil; sometimes in d-c. machines the voltage between commutator bars can be used.

Potential and current transformers are used where the line voltages and currents are too high for practical or convenient direct observation. The oscillograph potential circuit can be connected to the potential transformer along with voltmeter or other instruments, as it is only a fraction of the rated

potential transformer load. For current curves a shunt is connected in the current transformer secondary circuit.

Adjustment of Vibrator and Optical System. — Individual vibrators are provided with movements for vertical and horizontal adjustment of the image. The beam of light is brought vertically to the middle, or axis, of the cylindrical lens; this adjustment is left unchanged, but should be examined occasionally to insure its correctness; this may conveniently be done by viewing the position of the light spot on a thin sheet of paper held against the case where the light comes through to form the image on film or ground-glass screen; the light spot should appear central in the aperture. The horizontal adjustment brings the image to the proper position on the record, and is changed freely according to circumstances. When two curves are to be taken on the record, the images are usually placed for clearness so as not to overlap. If, however, the phase relation is important they are placed with their zero positions near together; it is better not to make them coincident, however, as good superposition is not always secured. After the principal exposure, an auxiliary exposure for zero line is taken with vibrator circuit opened, except in occasional records where the zero line is unnecessary.

Quantitative Measurements with Oscillograph. — The values of the curves may be obtained quantitatively when necessary by reference to a d-c. measurement. For potential curves an observation is taken of a d-c. source of known voltage; for current curves similarly an observation is taken of a measured d-c. current. During these d-c. measurements the resistances in the vibrator circuit should be the same as used during the observations. If a number of observations are taken at different resistances, two or three d-c. calibrations can be made with known resistances, and the calibration for the other resistances computed, assuming the deflection proportional to the total resistance of the vibrator circuit. When potential and current transformers are used, the voltages and currents as directly measured are reduced to terms of line voltages and currents by the transformer ratios. In case of short-circuit currents, where it is not practicable to obtain d-c. currents comparable in amount with those of the observation, a measured d-c. voltage, as from dry cells, can be applied to the oscillograph leads detached from the shunts, the corresponding currents being computed from the shunt resistances.

In many cases, the curve is self-calibrating, a portion of the curve being at a constant d-c. value which can be measured by d-c. instruments, as for instance on generator-field voltage and current curves of short-circuits.

COST OF OSCILLOGRAPH. — An oscillograph complete with a three-element electromagnet galvanometer, optical system, shutter and shutter-operating mechanism, motor and countershaft, photographic and tracing attachments, six film-holders, and the following repair parts: 6 extra suspension strips; 6 vibrator mirrors, 1 box special gold-leaf fuses, 1 bottle mirror cement, 1 bottle damping liquid, costs about $650.

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[L. T. Robinson.]

PAPER, IMPREGNATED. — (*See also Insulating Materials; Wires and Cables, Insulated.*) Wires and cables may be insulated by winding paper ribbon helically around the conductor in successive layers until the desired thickness of insulation is obtained. The paper-covered cable, after being thoroughly dried in a hot vacuum dryer is immersed in resin oil until saturated, and then passed through a lead press, which covers it with a continuous sheathing of lead. The paper serves the double function of affording a conveyance for the oil and a mechanical separator between the conductor and sheath. The oil forms the main insulation.

THE PAPER. — The most commonly used paper for such cables is that made of Manila hemp or musa textilis, a fiber grown in the Philippine Islands for rope making. The original fibers are about 6 millimeters long and have a diameter of 0.024 millimeter but the length is materially decreased in the process of beating described below. The paper made from this fiber is not necessarily the best for cable insulation, but the fact that it has stood the test of twenty years use makes American manufacturers slow to try others. Hemp papers are often called Manilas, and at the present time mixtures of hemp and musa have come into general use, a common proportion being one of hemp and two of musa. Jute is sometimes added, but as it is not permanent and adds nothing to the strength, it should be avoided. German manufacturers are using papers containing long fiber wood-pulp in large quantities, or even made exclusively of such wood-pulp, and claim that they are better than Manila papers (*C. Beaver*).

Manufacture of the Paper. — The process of manufacturing Manila paper is as follows. The fibers are cut up, placed in a boiler with lime or caustic soda and boiled under a pressure of from 30 to 50 pounds for between 5 and 10 hours. They are then emptied out, washed free of alkali and put in a breaker where the fibers are partially disintegrated and washed. Sometimes bleaching is performed at this stage, by adding chloride of lime and again washing. The material is then put into the beater, where it is reduced to the condition necessary for the paper machine. After suitably diluting the material it is passed over sand tables, where gritty matter is deposited and then through strainers, where any coarse particles are retained. It then passes in a continuous flow to the endless wire of the Fourdrinier machine, where the fibers are deposited in the form of a sheet; then to the couch rolls and to the press rolls, where the water is squeezed out. Finally it passes over a series of drying cylinders where it emerges dry and is taken upon reels. (This description is abstracted from a paper by C. Beadle and H. P. Stevens; see *Bibliography*, below.)

Necessary Properties of the Paper. — The paper should have the following qualities.

1. Porosity, in order that it may hold a large amount of oil.
2. Strength and elasticity, in order that it may not break either while being wound or while suffering shrinkage in the dessicator.
3. Freedom from mineral matter, not only to give greater porosity but to avoid electrical weakness.
4. Freedom from alkalies, in order that the impregnating oils shall not be saponified.
5. Freedom from metallic salts, such as chlorides and sulphates, in order to avoid short-circuiting the oil by an electrolyte and also to avoid injuring the conductor.

Strength of the Paper. — The strength of paper is usually stated in terms of the length of a piece the weight of which would be equivalent to its breaking strength. Thus a high-class Manila paper gave the results in the accom-

panying table, the humidity of the air being 65 per cent and the temperature 17° C. (Beadle and Stevens.)

Strength	Elongation
	Per cent
11,450 meters, lengthwise. .	2.8
4,450 meters, crosswise. . . .	7.8
7,950 meters, average.	5.3

The average ratio of longitudinal to transverse strength as determined by C. Beaver from tests on several hundred tons of satisfactory insulating paper is about 2.02. The average elongations were 2.2 per cent for the longitudinal direction and 4.4 per cent for the transverse direction.

OIL FOR IMPREGNATING. — Considerable mystery is maintained by the manufacturers respecting the exact nature of the oils used for impregnating the paper, but the principal ingredient is resin oil, the fluidity of which is reduced to a suitable degree by the addition of resin. One manufacturer brings the resin into chemical combination with the oil.

SPECIFIC RESISTANCE OF IMPREGNATED PAPER. — The specific resistance of impregnated paper for wire and cable insulation depends upon the dryness of the oil and upon the nature and quantity of the substances mixed with it. Increasing the fluidity within certain limits reduces the resistance, increases the flexibility and improves the dielectric strength. The value of K in the formula $M = K \log \frac{D}{d}$ (*see article on Rubber*) varies from less than 1000 to more than 3000, the usual value being near the lower limit.

Temperature Coefficient of Resistance. — The effect of temperature changes upon the resistance of oiled paper is greater than upon rubber and less than upon varnished cloth. The accompanying table is representative of a typical paper insulation, but the properties of different makes vary considerably.

Temperature, °F.	Per cent of resistance at 60°
60	100
65	62.5
70	42.6
75	30
80	23.3
85	18.5
90	15.2

DIELECTRIC STRENGTH. — The dielectric strength of oiled-paper insulation is a very indefinite quantity. It is greatly reduced by the presence of even the minutest trace of moisture and it decreases rapidly with increasing temperature. The gradual ionization of vapor is another cause of decreased dielectric strength. It is therefore not surprising to find different experimenters reporting widely different values of this quantity. Thus, E. Jona says that the average commercial impregnated paper subjected to dielectric stress for an hour with progressively increasing potential, will stand from 8 to 10 kilovolts (effective a-c.) per millimeter and that it is not uncommon to find samples with 20 or 30 per cent greater dielectric strength. E. J. Berg gives 250 to 300 kilovolts per inch, or 10 to 12 kilovolts per millimeter. P. Humann says that high-grade impregnated paper will stand 20 kilovolts per millimeter. The usual testing strength H, in the formula

$$V = \frac{H}{r \log_\epsilon \frac{D}{r}}$$

is about 4 kilovolts per millimeter or 100 kilovolts per inch, but the validity of this formula when applied to compound oils is very doubtful on account

of the heterogeneous nature of the insulation. When impregnated paper is punctured electrically, the paper itself chars, so that even though the oil tends to flow into and repair the gap the short-circuit is maintained (*see Wires and Cables, Insulated*).

SPECIFIC INDUCTIVE CAPACITY. — The specific inductive capacity of plain Manila paper (without oil) is 1.8 (*H. Floy*), while that of Manila paper impregnated with a mixture of resin oil and resin is between 2.4 and 2.6. During operation the constituents of the oil which have higher specific inductive capacities are drawn toward the conductor and tend to remain there permanently, thereby affording a natural grading for the cable.

SPECIFICATIONS. — Specifications for impregnated paper insulation for wires and cables will be found in the article on *Wires and Cables, Insulated.*

BIBLIOGRAPHY. — Beadle, C. and Stevens, H. P. *The Composition and Durability of Cable Papers,* Electrician (London), Vol. 63, 1909; see also the bibliography in the article on *Rubber*.

[W. A. DEL MAR]

PERMUTATIONS AND COMBINATIONS. — (*See also Factorials.*) Each of the *arrangements* which can be made by taking some or all of a number of things is called a permutation.

Each of the *groups* or *selections* which can be made by taking some or all of a number of things is called a combination.

The number of permutations of n things taken r at a time is

$$n(n-1)(n-2)\ \ldots\ (n-r+1).$$

The number of combinations of n things taken r at a time is

$$\frac{n(n-1)(n-2)\ \ldots\ (n-r+1)}{r(r-1)(r-2)\ \ldots\ 3.2.1}.$$

The number of combinations of n things taken r at a time is equal to the number of combinations of n things taken $(n-r)$ at a time.

[W. A. DEL MAR]

PHOTOMETRIC QUANTITIES. — (*See also Illumination, Laws of; Photometry; Vision, Laws of.*) The names and sense of the common photometric units are conventional, though a uniform value of the basic unit of the system, viz., the international candle, was established in 1909 by the national standards laboratories of Great Britain, France and the United States. The standard unit in Germany is the Hefner unit, but the relation that 1 Hefner unit equals 0.9 international candle was officially recognized. The terminology and definitions that follow are in agreement with those recognized by the Geneva Congress in 1896 and the proposals made by the Committee on Nomenclature and Standards of the Illuminating Engineering Society in 1912 (*Trans. Ill. Eng. Soc., Vol. 7, p. 723. See also Standardization Rules of the A.I.E.E.*)

LIGHT FLUX is a measure of the rate of flow of light from a luminous body. It is not identical with the flow of radiant energy but is an evaluation of radiation in terms of the corresponding light sensation. Light flux is proportional to two factors, power radiated and a stimulus coefficient $K\lambda$. The stimulus coefficient varies with the wave-length of radiation as shown in Fig. 1 in the article on *Vision, Laws of*. (*See also Phil. Mag., Vol. 24, p. 853.*)

Lumen. — The lumen, or unit of light flux, denotes the light radiating within one steradian from a source having a uniform luminous intensity of one candle. The steradian, or unit solid angle, is the angular space subtended at the center of a sphere by a portion of its surface equal to its radius squared. An entire sphere includes 12.5664 steradians and a hemisphere 6.2832 steradians. The symbol for luminous flux is F, that for luminous intensity I, and that for steradians ω. The following relations hold:

Flux in any solid angle,	$F = \int I\, d\omega;$
Total flux from source,	$F_s = 12.5664\, I_{ms};$
Flux in lower hemisphere,	$F_{lh} = 6.2832\, I_{mlh};$
Flux in upper hemisphere,	$F_{uh} = 6.2832\, I_{muh};$
Flux in any zone	$F_z = \omega_z\, I_{mz};$

where the I's have the designations given in the following paragraph.

LUMINOUS INTENSITY, commonly termed candle-power, denotes the solid angular density of light flux emitted in the direction considered, or

$$I = \frac{dF}{d\omega}.$$

Although luminous intensity refers in the strict sense to a single direction, mean intensities within certain limits are widely used. The following designations are employed in this article and also the articles on *Illumination, Laws of*, and *Photometry*.

Mean horizontal intensity,	I_h.
Mean spherical intensity,	I_s.
Mean zonal intensity,	I_{mz}.
Mean upper hemispherical intensity,	I_{muh}.
Mean lower hemispherical intensity,	I_{mlh}.

International Candle. — The international candle is the official unit of luminous intensity in France, Great Britain and the United States. The Hefner unit, which is standard in Germany equals 0.9 international candle. For further discussion of photometric standards see *Photometry*.

Distribution Curves. — The luminous intensities at various angles about a light source are commonly represented by polar curves of horizontal and vertical

distribution. Caution should be observed in interpreting such curves. The mean horizontal intensity is equal to the mean polar radius of the curve of horizontal distribution. The mean polar radius and the inclosed area of the curve of mean vertical distribution are entirely lacking in significance. For methods of computing mean intensities from vertical distribution curves, see *Illumination, Laws of*.

Spherical Reduction Factor. — This factor is the ratio of the mean spherical intensity of an illuminant to its mean horizontal intensity.

ILLUMINATION denotes the density of light flux intercepted by a surface or traversing an area in space. The unit of illumination corresponds to unit flux per unit area and is commonly one lumen per square foot, or foot-candle. The metric unit of illumination is the meter-candle, or lumen per square meter, sometimes called the "lux." It has been proposed by Blondel to designate the unit of one lumen per square centimeter by the title "phot." One foot-candle is 10.764 meter-candles and one meter-candle is 0.0929 foot-candles.

For methods of measuring illumination see *Photometry;* for the calculation of illumination, see *Illumination, Laws of;* for values of illumination required for various purposes, see *Vision, Laws of.*

INTRINSIC BRILLIANCY OF COMMON ILLUMINANTS

Illuminant	Candles per sq. in.	Candles per sq. cm.
Crater, carbon arc	84,000	13,000
Magnetite arc	4,000	620
Nernst glower	3,010	470
Incandescent electric lamps:		
Tungsten, 1.25 watts per c-p	1,060	164
Graphitized carbon, 2.5 watts per c-p	750	120
Tantalum, 2 watts per c-p	580	90
Carbon, 3.1 watts per c-p	485	75
Carbon, 3.5 watts per c-p	400	63
Acetylene flame, 1-foot burner	53	8.2
Acetylene flame, 0.25-foot burner	33	5.1
Welsbach mantle	31	4.8
Welsbach mantle, mesh	56	8.7
Mercury arc	14.9	2.3
Kerosene flame	9.0	1.4
Gas flame	2.7	0.4
Frosted tungsten lamp, tip	1.67	0.26
Frosted tungsten lamp, side	6.0	0.93

SURFACE BRIGHTNESS OR INTRINSIC BRILLIANCY is the luminous intensity per unit area of a surface projected on a plane normal to the line of sight. It is measured in candles per square inch or per square centimeter of projected area. Let b denote surface brightness, S the area and θ the angle between the normal to the surface and the line of sight, then

$$b = \frac{dI}{dS \cos \theta}.$$

The specific luminous radiation is closely akin to surface brightness as it denotes the flux emitted per unit area. If the emission agrees with the cosine

law the specific luminous radiation E' and the surface brightness normal to the surface b_0, bear the relation

$$E' = \pi b_0.$$

The brightness of illuminated surfaces is conveniently expressed in lumens emitted per square foot, but this should be clearly distinguished from illumination, or lumens received per square foot.

Mean values of surface brightness or brilliancy of various illuminants are given in the preceding table (*Ives and Luckeish, Elec. W., Vol. 57*).

The presence in the field of view of illuminants of high brilliancy depresses to a marked degree the sensibility and acuity of vision. The upper limit of brightness consistent with best vision has been variously estimated between 4 and 7.5 candles per square inch. (*See Glare in article on Vision, Laws of.*) For method of measurement of surface brightness see Ives and Luckeish, *ref. cit.*

REFLECTION OF LIGHT. — Reflection is regular or diffuse according as the reflector is a polished surface or a matt. Regular reflection is characterized by equal angles of incidence and reflection, the formation of images and the invisibility of the reflecting surface. Diffuse reflection scatters light in all directions, produces no images and renders the reflecting surface luminous. The intensity of light reflected by a perfect matt surface varies in proportion to the cosine of the angle of departure from the normal, and the total flux reflected equals π times the normal intensity. However, some regular reflection always accompanies diffuse reflection and these laws can be applied only approximately. Fair diffusing plates can be prepared from barium sulphate, magnesium oxide or other white materials of extremely fine and even grain. Opal glass with both surfaces carefully depolished is fairly satisfactory for diffuse reflection and transmission of light. Wall coverings of paper, plaster, fabric, paint and kalsomine deviate considerably from the cosine law of diffusion and its corollary and caution must be observed invariably in applying these laws to calculations of illumination. All materials not white, gray or black are selective and vary in their reflecting power with the spectral composition of the light received.

The following approximate values of reflection coefficients have been collected from various sources and are useful in calculations dealing with reflected light.

Reflector	Coefficient	Reflector	Coefficient
Polished silver	0.92	Orange yellow paper	0.34
Silvered mirror	0.80	Light green paper	0.25
White blotting paper	0.80	Light pink paper	0.25
White bond paper	0.75	Light blue paper	0.18
White kalsomine	0.75	Medium blue paper	0.12
Flat white paint	0.66	Dull green paper	0.08
Chrome-yellow paper	0.62	Light red paper	0.10
Cream paper	0.56	Medium red paper	0.08
Flat cream paint	0.53	Medium brown paper	0.08
Flat ivory paint	0.50	Deep red paper	0.06
Light buff paper	0.45		

The above values refer to light having the spectrum of commercial incandescent illuminants. In general flat paints are slightly below papers of the same tone

in reflecting power, whereas glossy paints exceed papers by 0.10 to 0.15 on account of regular reflection.

The most satisfactory device for the measurement of coefficients of diffuse reflection is that described by Nutting, *Trans. Ill. Eng. Soc., Vol. 7, p.* 412.

ABSORPTION OF LIGHT. — Transmitting media possess qualities quite analogous to reflecting surfaces with respect to regular and diffuse transmission and selective absorption depending on color. The absorption of light in the varieties of glassware most used in the lighting art are approximately as follows (*see Trans. Ill. Eng. Soc., Vol. 6, p. 98*);

Kind of glass	Absorption in per cent	Kind of glass	Absorption in per cent
Clear	5–12	Ground	20–30
Light sand blast	10–20	Medium opalescent	25–40
Alabaster	10–20	Heavy opalescent	30–60
Canary	15–20	Flame	30–60
Light blue alabaster	15–25	Signal green	80–90
Heavy blue alabaster	15–30	Ruby	85–90
Ribbed	15–30	Cobalt blue	90–95
Opaline	15–40		

BIBLIOGRAPHY. — Johns Hopkins University *Lectures on Illuminating Engineering*, Vol. 1; Barrows, W. E., *Light, Illumination and Photometry*, N. Y., 1912; Liebenthal, E., *Praktische Photometrie;* Wickenden, W. E., *Illumination and Photometry*, N. Y., 1910; Numerous papers in *Trans. Ill. Eng. Soc., Bull. Bur. Stand.*, and *Elec. World*.

[W. E. WICKENDEN.]

PHOTOMETRY. — (*See also Illumination, Laws of; Vision, Laws of; for definitions of quantities and symbols see Photometric Quantities.*) Photometry is the science of light measurement. Its chief aspects are the determination of (*a*) the intensity of light sources in definite directions, (*b*) mean vertical light distribution, (*c*) the mean intensity and light flux emitted within specified limits of distribution, (*d*) the illumination of surfaces, (*e*) the reflecting properties of surfaces and (*f*) the analysis of the color of light. Its methods are essentially visual and consist in equating the brightness of two diffusing surfaces. As an instrument of physical measurement the eye lacks precision as an estimating device, but serves well in the judgment of equality. Photometric devices are of four general types, according to the method of comparison employed, viz., (1) by equality of brightness of two surfaces visible simultaneously, (2) by equality of contrast between two pairs of surfaces differing slightly in brightness, (3) by the disappearance of flicker when two surfaces are viewed in rapid alternation, and (4) by acuity of vision. The latter process does not admit of precision and is used only for rough comparisons.

Methods of Obtaining Photometric Balance. — The methods of obtaining photometric balance are: (1) varying the distance of one or both light sources from the surfaces compared, according to the law of inverse squares (*see Illumination, Laws of*); (2) varying the angle of incidence of light according to the cosine law (*ibid*); and (3) varying the proportion of light received by means of a sector disk or slit and collimator. In cases (1) and (2), given two diffusing surfaces illuminated to equal brightness b by respective light sources of intensities I_1 and I_2; the reflection or transmission coefficients of the surfaces as K_1 and K_2; the angles of incidence as α_1 and α_2; and the distances from the light sources d_1 and d_2 respectively; then if $\alpha_1 = \alpha_2$ and $K_1 = K_2$ and I_2 is the known intensity of a standard lamp,

$$I_1 = \frac{I_2 d_1^2}{d_2^2}.$$

If α_1 and α_2 or K_1 and K_2 differ slightly, a sensibly correct result is obtained by interchanging the light sources for half the observations and using the mean distances in the inverse square relation.

STANDARD LAMPS. — A primary standard possesses two essential qualities, definiteness and reproducibility from specifications. None of the existing standards of luminous intensity meet both requirements. Of the flame standards, the Hefner and pentane lamps are approximately reproducible and fairly definite with standard atmospheric conditions and fuel. The incandescent electric lamp is capable of definite calibration, but is not strictly reproducible. The present unit of luminous intensity, the international candle, is derived from the mean intensity of a group of incandescent electric lamps maintained by the U. S. Bureau of Standards, in coöperation with similar custodians in France and Great Britain.

Hefner Standard. — The Hefner standard which is the official standard of Germany and which has been extensively used in the United States, is a wick lamp burning amyl acetate of definitely specified chemical and physical properties. Its standard intensity is 0.9 international candle. Expressing the actual intensity as I, corrections for various atmospheric conditions are made as follows:

For variations from the standard flame height of 40 mm.

$$I = 1 + 0.025\,(h - 40) \text{ or } I = 1 - 0.034\,(40 - h)$$

for heights greater or less than 40 mm. respectively, where h is the flame height in millimeters. For barometric variations

$$I = 1 + 0.00011\,(b - 760),$$

where b is the barometric pressure in millimeters. For atmospheric humidity

$$I = 1.049 - 0.0055\,x,$$

where x signifies the liters of water vapor per cubic meter of air at 760 mm. and free from CO_2.

For atmospheric vitiation by CO_2

$$I = 1.012 - 0.0072\,y,$$

where y signifies the liters of CO_2 per cubic meter of dry air.

The objections to the Hefner standard are its low intensity, its reddish color, its flabby flame and its sensitiveness to variation in flame height. The element of uncertainty associated with it is at best not less than 2 per cent.

Pentane Standard. — The pentane standard, as represented by the Vernon-Harcourt type, is essentially an argand burner supplied with pentane-air gas and preheated air. The fuel is formed by passing air over pentane in a saturator box subdivided by baffles. The burner is surmounted at a height of 47 mm. by a cylindrical chimney. An annular chamber surrounding the chimney supplies the interior of the flame with preheated air. The flame is shielded from drafts by a conical, blackened hood having a slit at one side through which the flame is exposed. The chimney is fitted with a mica window showing a gauge line to which the flame height is closely adjusted. Under standard atmospheric conditions, viz., a barometric pressure of 760 mm. and a humidity of 8 liters of water vapor per cubic meter of air the flame should have a horizontal intensity of 10 candles. Experience indicates, however, that it is usually less by 2 to 4 per cent and that it is desirable to calibrate the lamp against a more definite standard. Variations from the above standard atmospheric conditions may be corrected by the equation:

$$I = I_n\,[1 - 0.00567\,(e - 8) + 0.0006\,(b - 760)],$$

where I is the actual intensity, I_n the standard intensity, e the liters of water vapor per cubic meter of air and b the barometric height.

The color and intensity of the flame are convenient for practical purposes, especially for the measurement of the illuminating power of gas. An added advantage in the testing of luminous flames of all sorts resides in the fact that their intensities are affected by atmospheric variations in a manner corresponding quite closely to the changes in the pentane lamp, whereby somewhat troublesome corrections are avoided.

Carcel Lamp. — The Carcel lamp is the recognized standard of luminous intensity in France. It has a central-draft ring burner fitted with a wick of the light-house type burning colza oil. Its standard intensity is 9.61 international candles.

Standard candles are now discredited for all accurate work, though still used extensively in the routine testing of gas.

Incandescent Electric-lamp Standard. — The incandescent electric lamp is superior to all other working standards where corrections for flame luminosity due to atmospheric variations are not required. Carefully selected carbon-filament lamps aged by burning until the hot resistance is constant maintain their candle-power sensibly unchanged for a period of 100 hours or more. With the most precise photometric apparatus such lamps may be standardized

with the mean error of any single determination not exceeding 0.2 per cent. Lamps are standardized for use in a fixed position or in rotation about the principal axis. No difference exists between the precision of the two methods. Lamps are standardized in terms of a definite voltage, current or power consumption, of which the first method is the most common. Lamps standardized for a definite voltage or current require more careful aging and are somewhat less permanent than those standardized for a definite power. Current and voltage values, however, are more readily determined and checked with precision by the use of the potentiometer. It is desirable to have lamps standardized in terms of both voltage and current for the constancy of the lamp can be relied upon as long as both standard conditions exist simultaneously. Carbon filament lamps at 4 watts per candle are most widely used as standards, though the limited experience to date with metal filament lamps shows them to be quite satisfactory. The color value at 4 watts per candle is distinctly more red than illuminants in general use. It is best to keep on hand a group of well-seasoned lamps of all commercial types and to select from these the most appropriate working standard for calibration against the primary carbon standard, so that the latter is used only for checking.

Other Standards have had a very limited use. Among these are the acetylene flame; the Methven screen, which is essentially a sharply-defined portion of a luminous gas flame, calibrated against a primary standard; and the Elliot lamp, which is a kerosene lamp of the student type having a limited portion of its flame exposed by a screen. The utility of these standards is largely in the routine testing of gas.

Relations of Luminous Standards. — The relative intensities of the several standards under standard conditions are given in the following table:

	International candle	Hefner	10-c-p. pentane	Carcel	Bougie decimale	English candle	German candle
International candle	1.00	1.11	0.10	0.104	1.00	0.96	0.95
Hefner	0.90	1.00	0.09	0.0936	0.90	0.864	0.855
10-c-p. pentane	10.00	11.11	1.00	1.04	10.0	9.6	9.5
Carcel	9.61	10.66	0.96	1.00	9.6	9.24	9.19
Bougie decimale	1.00	1.11	0.10	0.104	1.00	0.96	0.95
English candle	1.04	1.154	0.104	0.1	1.04	1.00	0.98
German candle	1.055	1.17	0.105	0.109	1.055	1.02	1.00

SIGHT BOXES. — A photometric sight box consists of two diffusing surfaces and accessories to facilitate the comparison of brightness. The types described following are of greatest utility.

Bunsen Sight Box. — The bunsen screen is a disk of white diffusing paper, a well-defined region of which is made translucent by impregnation with paraffine or other material. The disk is set transversely in a sight box of blackened interior, as shown on the plan in Fig. 1. Light from the sources to be compared enters the apertures A–A, and falls normally on the disk surfaces. Dihedral mirrors M_1 and M_2 enable both sides of the disk to be viewed at the sight tube T. The opaque portion of the disk reflects diffusely, while the translucent region partially reflects and partially transmits the light received. A photometric balance exists when the two sides of the disk appear alike. If both lights are alike in color and the absorption of both regions of the disk is equal, the boundary disk appears and both sides appear uniformly bright. With unequal ab-

sorption, balance exists when equal contrast exists between the opaque and translucent regions on both sides of the disk. The contrast principle is of distinct advantage with slight color differences. The sensitiveness of the screen depends largely on the definition of the boundary of the impregnated portion.

Leeson Disk.—This is a useful modification of the Bunsen type built up by pasting opaque paper disks with accurately matched star-shaped apertures on the two sides of a disk of translucent paper. By a careful selection of the materials the disk may be made to embody either the equality-of-brightness or the equality-of-contrast principle. The paper used should agree closely with the cosine law of diffusion.

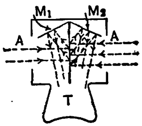

Fig. 1.

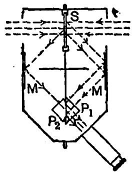

Fig. 2.

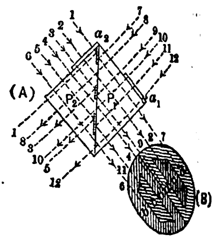

Fig. 3.

Lummer-Brodhun Sight Box.—The plan of this box is shown in Fig. 2. An opaque diffusely-reflecting screen S receives light from the sources to be compared and reflects it along the paths indicated by aid of the mirrors M–M and the prisms P_1–P_2. The prisms present to the eye a composite field in which the brightness of the two sides of S can be conveniently compared. Fig. 2 also shows the arrangement for equality-of-brightness working. The prisms are in optical contact over an elliptical portion of their hypotenusal faces, and the remainder of one is cut away. The central portion of the field is illuminated by direct transmission through the contact area, the outer portion by total reflection from the face of the uncut prism.

For equality-of-contrast working the arrangement of the prisms is as shown in Fig. 3. The hypotenusal face of P_2 is recessed over the area shaded in (B). That of P_1 is plane. Two thin glass absorbing strips a_1 and a_2 are set before the faces of P_1 and P_2 as shown in the plan (A). By tracing the several paths indicated it is seen that the field has the appearance of (B) and that the regions shaded are darkened by the absorption of light in a_1 and a_2. In a state of balance b and d appear equally bright and c and e equally dark in contrast. The degree of contrast created by a_1 and a_2 should be about 3.5 per cent for the best sensitiveness. Accuracy of adjustment and cleanliness of all parts are essential in photometers of the Lummer-Brodhun type.

Comparison of Bunsen, Leeson, and Lummer-Brodhun Boxes.—Bunsen, Leeson and Lummer-Brodhun sight boxes should be so mounted as to permit the complete reversal of the optical system about its axis of symmetry in order that optical asymmetry may be corrected as explained above. The Bunsen and Leeson types are binocular and therefore less fatiguing in a long series of observations than the monocular Lummer-Brodhun type. Furthermore, they are the more readily balanced with slightly flickering light. The Lummer-Brodhun contrast type excels in sensitiveness and general utility with steady lights of equal and slightly dissimilar color.

Flicker Photometer. — The flicker photometer affords the most reliable means of comparing light sources of distinctly unlike color. A field of view alternately illuminated by two such sources displays a flickering appearance which may be due to color dissimilarity or to difference in brightness. Above a moderate rate of alternation the color sensations blend and the disappearance of flicker is a true indication of equal brightness, as conclusively established by Ives, who recommends the following conditions as suited to the best precision: (*a*) a field illumination of 25 meter-candles; (*b*) a photometric field of 2 in. diameter; and (*c*) a background field about 25 in. in diameter surrounding the photometric field and about equal to it in brightness. The latter provision, though not essential to precision, is an aid to comfort.

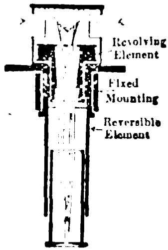

Fig. 4.

Bechstein Flicker Photometer (Fig. 4). — This photometer employs a train of lens and prism oscillating before a fixed diffusing wedge. The field of view consists of a circle and ring, alternately illuminated by the respective sides of the wedge as the lens system revolves. The highest sensitiveness exists at the lowest speed at which the flicker can be made to disappear.

Wild Flicker Photometer (Fig. 5). — The Wild flicker photometer is the simplest of this class. It consists of a disk *D* of white diffusing paper, one-half of which is made translucent by impregnation. This disk is revolved about an axis slightly at one side of the path of light and a mirror *M* reflects one side of the disk to the sight tube *T*. A high degree of precision is claimed by Wild for this device.

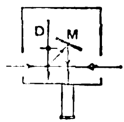

Fig 5.

Color-equalizing Screens. — The simplest method of comparison for lights of dissimilar color is to employ a Lummer-Brodhun photometer in connection with a set of glass screens of graded tints, selecting one which renders the light of the standard lamp equal in color to that of the lamp under measurement. The absorption of each screen for the light of the standard lamp must be determined by means of the flicker photometer, but the inconvenience of using the latter instrument is limited to the process of calibration.

Sector Disk. — The sector disk affords a most convenient means of reducing the intensity of the light received from an illuminant on the photometric screen. It consists of two or more disks with sector apertures revolving on a common axis. By advancing one disk with respect to the other the net aperture may be altered at will. By Talbot's law the intensity of transmitted to incident light equals the ratio of the total angular opening to 360 degrees. The sector disk has the advantage over other absorbing media in that it is adjustable, is not affected by time, and is independent of color.

Photometric Bar. — The devices above described are best suited for use in connection with a photometric bar or bench, which should be level and straight, and preferably greater than 100 inches in length. A plan of the layout is shown in Fig. 6. For convenience the sight box and at least one of the lamps should be mounted on movable carriages. The bench should be provided with a series of screens *D* of dead black material having graded apertures along the photometric axis and with solid screens at the ends. These screens should completely occlude from the sight box all extraneous light, and should protect the eye of the operator from the direct light of the lamps. If these conditions are met the photometric

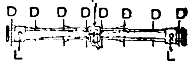

Fig. 6.

room need not be blackened. The sight box should have a dark background, however, and all light in the room should be well diffused. The bar should be provided with a scale of equal divisions and a scale reading directly the ratio of the inverse squares of the distances from the ends of the bar. For use with a standard lamp of definite value in a fixed position a direct reading scale of candle-power is readily obtained from the ratio scale.

Connections for Testing Incandescent Electric Lamps.—In testing incandescent electric lamps the effects of voltage fluctuation must be reduced to a minimum. A storage battery or special generator of very close regulation should be employed if possible. The effect of variations in line voltage is minimized by the method of connection shown in Fig. 7. L_1 and L_2 are two lamps under test. R_1, R_2 and R_3 are rheostats capable of fine adjustment. V is a voltmeter or potentiometer. S_1, S_2 and S_3 are instrument switches. The voltmeter may be made to measure the voltage at L_1, at L_2 or the difference between the two. The former connections are for initial adjustments and checks, the latter for holding the proper relative voltages.

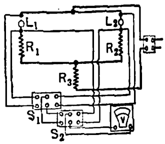

Fig. 7.

Manipulation of Photometric Bar. — Direct comparison between the test lamp and the standard may be made with the two in fixed positions by moving the sight box to a point of balance. In this case half the observations should be made with the sight box reversed. For the substitution method a fixed socket is provided at one end of the scale. The second socket and the sight box are on movable carriages coupled at a fixed distance. A well-seasoned lamp is placed in the movable socket and adjusted to a suitable voltage, which is subsequently held constant. The standard lamp is placed in the fixed socket and a balance secured by setting the movable carriages. The standard lamp is then removed and test lamps substituted in turn, a balance point being observed for each. The intensity of each test lamp equals that of the standard multiplied by the direct ratio of the squares of their distances from the screen at the times of balance. Reversals are unnecessary in the substitution method. The comparison lamp should be checked against the standard at intervals.

MEASUREMENT OF PHOTOMETRIC DISTRIBUTION.—Mean horizontal intensity is measured with the lamp rotating about its vertical axis. Special mountings, driven by motor or hand wheel, are provided for this purpose. Rotators are made universal by provision for the turning of the lamp by definite angular steps about its luminous center in a vertical plane including the photometric axis. The speed of horizontal rotation should be only sufficient to equalize differences in intensity.

Mirror Rotators. — In testing the light distribution of arcs, heavy reflector units, gas lamps, etc., which must remain in an upright position, mirror rotators are generally employed to direct the light from any desired vertical angle toward the photometer. A three-mirror device is shown schematically in Fig. 8. A two-mirror arrangement could also be used. In either case the lamp remains stationary, or is revolved about its vertical axis only, while the mirror system is turned by steps about the photometric axis. Numerous other devices for this purpose are described in standard works on photometry.

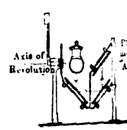

Fig. 8.

INTEGRATING PHOTOMETERS. — An integrating photometer enables the measurement of mean spherical or mean hemispherical intensity, or of the total flux of light from an illuminant, to be made by a single observation.

Ulbricht Integrating Sphere. — As shown schematically in Fig. 9, this apparatus consists of a hollow sphere whose inner surface is coated with a white material giving approximately true diffusion. A window of milk glass is placed in its horizontal axis. With a true spherical diffusing surface the light of an illuminant within the sphere is distributed as two components, viz., direct light, giving an intensity at any point on the surface according to the inverse square and cosine laws, and reflected light, which tends to spread itself uniformly over the entire surface if none is absorbed by obstructions. If then the direct light is screened from the window and it receives only the reflected component its brightness viewed externally is proportional to the total light flux emitted within the globe. The arrangement indicated is well adapted to a substitution method in which the errors due to non-sphericity, imperfect diffusion and to absorption in screens and lamps are minimized as much as possible. The calibrating lamp, whose mean spherical intensity is known, is first lighted and the resulting brightness of the window measured by a photometer. The calibrating lamp is then extinguished and the test lamp lighted, when a similar observation is made. The mean intensity of the test lamp equals that of the standard lamp multiplied by the ratios of the observed brightnesses of the window. A sphere intended for use with arc lamps and other large units should be not less than 5 feet in diameter; for work with incandescent lamps the sphere may be about 2 feet in diameter. The screens should be no larger than necessary to protect the window from direct light and should be coated with the same material as the spherical surface. For convenience the sphere may be divided vertically and at least one-half mounted on wheels. A removable circular section may be provided at the top. A thick paste of barium sulphate and zapon lacquer is largely used for the interior coating. The sphere is well adapted to the photometry of large and asymmetrical light sources. It is not a highly accurate device, but the final error involved need not exceed 5 per cent.

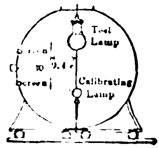

Fig. 9.

PORTABLE PHOTOMETERS. — Weber Photometer. — Portable photometers exist in great variety and have for their purpose the measurement of illumination and of the intensity of light sources in place. The great majority are modifications of the Weber photometer, shown schematically in Fig. 10. This comprises two cylindrical tubes of blackened interior, one fixed and the other attached to it at right angles by means of a sleeve to allow rotation. A Lummer-Brodhun prism device is placed at the junction of their axes, and permits the brightness of two translucent glass plates P_1 and P_2 to be compared at the eyepiece E. P_1 is illuminated by an external source L_1 at a distance d_1. P_2 is movable along the tube by a knurled head and is illuminated by a small standard lamp L_2 mounted in a convenient housing. Assuming both plates to be illuminated by light incident normally at a state of balance, then

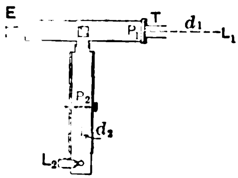

Fig. 10.

$$I_1 = \frac{I_2 T_2}{T_1} \times \frac{d_1^2}{d_2^2},$$

where T_1 and T_2 are the respective transmission coefficients of the two plates. A calibration for intensity measurements is readily obtained by the use of a standard lamp as I_1, and by observing the scale readings d_2 giving a balance with various values of d_1, keeping I_2 constant.

Two methods are available for the measurement of illumination. In the first the terminal tube T is removed and a flush test plate of depolished milk glass

fitted in its place. This test plate is placed in the position where the illumination is to be tested and the instrument balanced as usual.

For the second method of illumination measurement the terminal tube is not removed, but the plate P_1 is omitted. A large, white, diffusely reflecting card is placed in the position where the illumination is to be tested and the tube T pointed in its direction so that the entire cone of light entering T arises from the test plate. So long as this condition is met the inclination and distance of T is not important.

The first method is preferable where it is possible to make the attached test plate coincide with the position of the test. In the second method it is difficult to avoid interference with light which should reach the test plate.

Sharp-Millar Photometer. (Fig. 11.) — This photometer is a modified Weber instrument which is extensively used in America. It comprises an elongated wooden box divided into two compartments, one of which contains a fixed Lummer-Brodhun photometric cube and the other a lamp carriage movable along the box by turning a knurled head H. Observations are made at E at one side of the box. The two compartments are separated by a milk glass window whose brightness is balanced against illumination from an outside source admitted through an elbow tube T. Stray light in the lamp compartment is screened from this window by a series of diaphragms with central apertures. The elbow tube T is fitted on a collar and may be turned to any desired inclination. At the elbow is a circular, reversible plate one side of which is a mirror for measurements of illumination and the other a white diffusing surface for measurements of candle-power. In the former case the end of the tube is fitted with a flush plate of depolished milk glass. The range of the photometer is controlled by a pair of glass absorbing screens mounted in the compartment with the Lummer-Brodhun cube. These plates are respectively of high- and low-absorbing power. They may be turned so that either one may be used to reduce the illumination of either part of the field. But one may be employed at a time. In this way a range from 0.004 foot-candle to 2000 foot-candles may be secured. In the most recent design the need of a voltmeter or ammeter to keep the standard lamp constant is obviated by a small Wheatstone bridge arrangement with a telephone receiver, whereby the lamp may be kept at constant hot resistance. The Sharp-Millar photometer may be used with a detached test plate similar to that described for the Weber photometer.

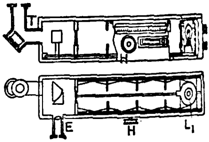

Fig. 11.

COLORIMETRY. — The Ives colorimeter is an instrument for tri-chromatic color analysis. (*See Vision, Laws of.*) It consists essentially of an oblong box, at one end of which are placed four slits, one clear, and the three others equipped respectively with red, green and blue screens. By means of levers the openings of the three colored slits can be altered to read by scales from 0 to 100. By rotating a wheel of lenses the three colors are mixed. The observer views a divided field, one part consisting of the mixture of the three primary colors and the other of the color to be matched as viewed through the clear slit. To make a measurement, the three colored slits are opened until white is matched, and the scales are set to read 100 for each color. Then any color matched by moving the levers can be read off in terms of the per cent of red, green and blue necessary to match white. The precision of the instrument is from 2 to 5 per cent under favorable conditions.

REFLECTING POWER OF SURFACES. — See section on *Reflection of Light* in the article on *Photometric Quantities*.

PRECAUTIONS IN PHOTOMETRIC OBSERVATIONS. — The eyes of the observer should be constantly shaded from bright light to maintain their sensitiveness in a state of dark adaptation. For best photometric sensibility a screen illumination of about 2 foot-candles is desirable. At low intensities the Purkinje effect (*see Vision, Laws of*) may prove disturbing. The precision of photometric settings may often be improved by a process of narrowing down between points equally out of balance. Many good photometricians reject their first observation in a set as untrustworthy. As the best conditions cannot reduce the uncertainty of observations below 0.2 per cent (in many cases it is 3 per cent or more) not more than three figures in the result are significant. A photometer bar should be at least 100 inches long for good results with ordinary illuminants. The distance from a large unit to the screen should not be less than 10 feet if the inverse square law is to be applied.

The voltage of electric lamps, or current, in the case of series lamps, should be measured by the most accurate device obtainable. In life tests of incandescent lamps exact regulation of voltage is of the utmost importance and a sensitive, automatic regulator is most desirable. In measurements of illumination by portable photometers the sources of error to be guarded against are: occlusion of light from the test plate by observer or instrument, uncertainty of standard lamp due to poor electrical regulation, faulty diffusion by the test plate, and low sensibility and Purkinje effect in weak fields. In tests of illuminants in place, the voltage, current or power, or the gas consumption and pressure should be ascertained and recorded if possible. In tests of gas illuminants the volume consumed should be reduced to the corresponding volume at a temperature of 60° F., and a barometric pressure of 30 inches.

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[W. E. WICKENDEN.]

PI (π), VALUE OF. — The letter π is used to represent the ratio of the circumference of a circle to its diameter; it is an incommensurable quantity. Its value is 3.14159265. . . . The value 3.1416 is sufficiently accurate for all ordinary purposes, and for rough calculations the value 22/7 is convenient. The following factors frequently occur:

$\pi = 3.14159$	$\frac{\pi}{4} = 0.785398$	$\frac{1}{2\pi} = 0.159155$
$2\pi = 6.28319$		
$3\pi = 9.42478$	$\frac{\pi}{6} = 0.523599$	$\frac{1}{4\pi} = 0.079578$
$4\pi = 12.56637$		
$\frac{\pi}{2} = 1.57080$	$\frac{4\pi}{3} = 4.18879$	$\pi^2 = 9.86960$
		$\pi^3 = 31.00628$
$\frac{\pi}{3} = 1.04720$	$\frac{1}{\pi} = 0.318310$	$\sqrt[2]{\pi} = 1.77245$
		$\sqrt[3]{\pi} = 1.46459$

PIPES AND PIPING. — (*See also Boilers; Electrolysis of Grounded Structures; Hydraulics, Principles of; Power Stations; Valves.*) In the following table is given a list of the metals ordinarily used for steam, gas and water pipes, together with their average weight per cubic inch, their tensile strength and the expansion per 100 feet for various temperature differences. The expansions are based on data given in Gebhardt's *Steam Power Plant Engineering*.

TENSILE STRENGTH, WEIGHT AND EXPANSION OF PIPE MATERIALS

Material	Lb. per cu. in.	Tensile strength, lb. per sq. in.	Expansion (in. per 100 ft.) Temp. rise above 60° F.				
			100°	200°	300°	400°	500°
Brass, wrought	0.30	50,000	1.15	2.41	3.80	5.38	7.11
Copper, wrought	0.32	30,000	1.08	2.26	3.56	5.05	6.66
Iron, cast	0.26	18,000	0.72	1.50	2.38	3.36	4.44
Iron, wrought	0.28	50,000	0.79	1.65	2.61	3.70	4.89
Lead	0.41	1600 to 2400	....	....	....	....	....
Steel, mild	0.28	65,000	0.79	1.65	2.61	3.70	4.89

Of the above materials mild steel is most widely used for general work. Wrought iron is more expensive but is sometimes preferred. Cast iron is largely used for water service and sanitation, and also to a limited extent where many connections are required, due to the fact that the flanges are cast with the piece. Copper and brass are used only to a limited extent, on account of their high cost. Lead is used to a limited extent for water pipes. Fittings are usually made of brass, cast iron, malleable iron or pressed steel. Wooden stave pipe is also much used in the western part of this country for water supply and power.

DIMENSIONS AND WEIGHT OF COMMERCIAL PIPE. — The size of iron and steel pipes is usually specified in terms of the "nominal" inside diameter. The actual inside diameter is usually greater than the "nominal," the percentage difference being the greatest for small sizes. The thickness of wall and weight per lineal foot of a given size of pipe varies over a considerable range, due to processes of manufacture. Manufacturers specify that "full weight" pipe may have a variation of from 5 per cent above to 5 per cent below nominal or table weight, but "merchant pipe," which is the ordinary pipe carried by jobbers and manufacturers, is almost invariably from 5 to 10 per cent under the nominal weight.

In drawing specifications for pipe, engineers should be careful to state what grade of pipe is desired, whether "merchant," full weight, or extra strong, and in the case of cast-iron pipe the class according to the table on the next page.

Cast-iron Pipe. — The following dimensions and weights are taken from the catalogue of the U. S. Cast-iron Pipe and Foundry Co. (1908). The weights are figured on the basis of a pipe length of 12 feet, and include proportional part of weights of standard sockets.

DIMENSIONS AND WEIGHT OF CAST-IRON PIPE

Nominal inside diam., in.	Class A		Class B		Class C		Class D	
	Thickness, in.	Lb. per ft.	Thickness, in.	Lb. per ft.	Thickness, in.	Lb. per ft.	Thickness, in.	Lb. per ft.
3	0.39	14.5	0.42	16.2	0.45	17.1	0.48	18.0
4	0.42	20.0	0.45	21.7	0.48	23.3	0.52	25.0
6	0.44	30.8	0.48	33.3	0.51	35.8	0.55	38.3
8	0.46	42.9	0.51	47.5	0.56	52.1	0.60	55.8
10	0.50	57.1	0.57	63.8	0.62	70.8	0.68	76.7
12	0.54	72.5	0.62	82.1	0.68	91.7	0.75	100.0
14	0.57	89.6	0.66	102.5	0.74	116.7	0.82	129.2
16	0.60	108.3	0.70	125.0	0.80	143.8	0.89	158.3
18	0.64	129.2	0.75	150.0	0.87	175.0	0.96	191.7
20	0.67	150.0	0.80	175.0	0.92	208.3	1.03	229.2
24	0.76	204.2	0.89	233.3	1.04	279.2	1.16	306.7
30	0.88	291.7	1.03	333.3	1.20	400.0	1.37	450.0
36	0.99	391.7	1.15	454.2	1.36	545.8	1.58	625.0
42	1.10	512.5	1.28	591.7	1.54	716.7	1.78	825.0
48	1.26	666.7	1.42	750.0	1.71	908.3	1.96	1050.0
54	1.35	800.0	1.55	933.3	1.90	1141.7	2.23	1341.7
60	1.39	916.7	1.67	1104.2	2.00	1341.7	2.38	1583.3
72	1.62	1283.4	1.95	1545.8	2.39	1904.2	....	......
84	1.72	1633.4	2.22	2104.2	....	......	....	......

The safe working pressures recommended for the four classes are:

SAFE WORKING PRESSURES, CAST-IRON PIPE

Unit of Pressure	A	B	C	D
Pounds per square inch........	43	86	130	173
Head of water, feet............	100	200	300	400

Welded Pipe. — The first table following is based on Briggs' Standard for sizes up to 10 inches, and upon the National Tube Co's Standard above 10 inches (1910). Weights per foot, up to and including 15 inches internal diameter, are based upon a length of 20 feet including the coupling; weights given for larger sizes are for plain end pipe.

The second table gives the dimensions of "Extra Strong" and "Double Extra Strong" welded tubes (*National Tube Co., 1902*).

DIMENSIONS AND WEIGHT OF WELDED PIPE

Size, nominal internal diam., in.	Diameter in inches		Thickness of metal, in.	Number of threads per in.	Weight of pipe per lin. ft., lb.
	Actual external	Approx. internal			
1/8	0.405	0.269	0.068	27	0.246
1/4	0.540	0.364	0.088	18	0.426
3/8	0.675	0.493	0.091	18	0.570
1/2	0.840	0.622	0.109	14	0.855
3/4	1.050	0.824	0.113	14	1.14
1	1.315	1.049	0.133	11 1/2	1.69
1 1/4	1.660	1.380	0.140	11 1/2	2.29
1 1/2	1.900	1.610	0.145	11 1/2	2.74
2	2.375	2.067	0.154	11 1/2	3.69
2 1/2	2.875	2.469	0.203	8	5.85
3	3.500	3.068	0.216	8	7.66
3 1/2	4.000	3.548	0.226	8	9.24
4	4.500	4.026	0.237	8	10.9
4 1/2	5.000	4.506	0.247	8	12.7
5	5.563	5.047	0.258	8	14.9
6	6.625	6.065	0.280	8	19.2
7	7.625	7.023	0.301	8	23.8
8	8.625	7.981	0.322	8	28.9
9	9.625	8.941	0.342	8	34.3
10	10.750	10.020	0.365	8	41.2
11	11.750	11.000	0.375	8	46.4
12	12.750	12.000	0.375	8	50.9
....	14.000	13.250	0.375	8	56.1
....	15.000	14.250	0.375	8	60.7
....	16.000	15.250	0.375	8	64.9
....	18.000	17.250	0.375	....	70.6
....	20.000	19.250	0.375	....	78.6
....	22.000	21.250	0.375	....	86.6
....	24.000	23.250	0.375	....	94.6

DIMENSIONS OF "EXTRA STRONG" AND "DOUBLE EXTRA STRONG" WELDED TUBES

Nominal diam., in.	Actual outside diam., in.	Thickness, extra strong, in.	Thickness, double extra strong, in.	Actual inside diam., extra strong, in.	Actual inside diam., double extra strong, in.
1/8	0.405	0.100	.....	0.205	.....
1/4	0.54	0.123	.....	0.294	.....
3/8	0.675	0.127	.....	0.421	.....
1/2	0.84	0.149	0.298	0.542	0.244
3/4	1.05	0.157	0.314	0.736	0.422
1	1.315	0.182	0.364	0.951	0.587
1 1/4	1.66	0.194	0.388	1.272	0.884
1 1/2	1.9	0.203	0.406	1.494	1.088
2	2.375	0.221	0.442	1.933	1.491
2 1/2	2.875	0.280	0.560	2.315	1.755
3	3.5	0.304	0.608	2.892	2.284
3 1/2	4.0	0.321	0.642	3.358	2.716
4	4.5	0.341	0.682	3.818	3.136

Riveted Pipes. — Large pipes are frequently made of sheets of boiler steel with riveted joints, with longitudinal, circumferential or spiral seams. The following tables give the necessary data regarding dimensions, rivets, etc. The first table is taken from a catalogue of the Abendroth & Root Mfg. Co.

SHEET IRON AND RIVETS REQUIRED FOR RIVETED PIPES

No. sq. ft. of iron required to make 100 lin. ft. punched and formed sheets when put together			Approx. No. of rivets 1 in. apart required for 100 lin. ft. punched and formed sheets	No. sq. ft. of iron required to make 100 lin. ft. punched and formed sheets when put together			Approx. No. of rivets 1 in. apart required for 100 lin. ft. punched and formed sheets
Diam. in in.	Width of lap in in.	Square feet		Diam. in in.	Width of lap in in.	Square feet	
3	1	90	1600	14	1 1/2	397	2800
4	1	116	1700	15	1 1/2	423	2900
5	1 1/2	150	1800	16	1 1/2	452	3000
6	1 1/2	178	1900	18	1 1/2	506	3200
7	1 1/2	206	2000	20	1 1/2	562	3500
8	1 1/2	234	2200	22	1 1/2	617	3700
9	1 1/2	258	2300	24	1 1/2	670	3900
10	1 1/2	289	2400	26	1 1/2	725	4100
11	1 1/2	314	2500	28	1 1/2	779	4400
12	1 1/2	343	2600	30	1 1/2	836	4600
13	1 1/2	369	2700	36	1 1/2	998	5200

THICKNESS AND WEIGHT PER FOOT OF SHEET IRON

No. of gauge, B.W.G.	Thickness, in.	Weight in lb., black	Weight in lb., galvanized	No. of gauge, B.W.G.	Thickness, in.	Weight in lb., black	Weight in lb., galvanized
26	0.018	0.80	0.91	18	0.049	1.82	2.16
24	0.022	1.00	1.16	16	0.065	2.50	2.67
22	0.028	1.25	1.40	14	0.083	3.12	3.34
20	0.035	1.56	1.67	12	0.109	4.37	4.73

Wooden Stave Pipes are usually built up in place. Staves of redwood, fir, yellow pine, and spruce are used. The staves range from 1¼ to 2½ inches in thickness and from 6 to 8 inches in width; these are held in place by steel bands ranging from ⅜ to ¾ inch in diameter. The interior surfaces of the staves wear smoother by the action of the flowing water and do not become fouled. Stave pipes have been installed in sizes ranging from 18 to 144 inches in diameter, and for heads up to 300 feet.

FORMULAS FOR WEIGHT, CIRCUMFERENCE, SURFACE, CONTAINED VOLUME AND SAFE PRESSURE. — Let

A = external surface per lineal foot in square feet,
C = external circumference in inches,
D = internal diameter in inches,
D_0 = external diameter in inches,
f = factor of safety,
H = safe head in feet of water,
P = safe pressure in pounds per square inch,
S = tensile strength in pounds per square inch,
T = thickness of wall in inches,
V = volume of contents (water, steam or gas) per lineal foot in cubic feet
w = specific weight of metal in pounds per cubic inch,
W = weight of metal per lineal foot in pounds.

Then

$$A = 0.262\,D_0, \qquad C = 3.14\,D_0, \qquad D_0 = D + 2T,$$

$$H = \frac{4.62\,ST}{Df}, \qquad P = \frac{2\,ST}{Df},$$

$$T = \frac{D_0 - D}{2} = \frac{DfP}{2\,S} = \frac{DfH}{4.62\,S},$$

$$V = 0.00545\,D^2, \quad W = 9.42\,w\,(D_0^2 - D^2) = 37.7\,wT\,(D + T).$$

The values of the specific weight w and tensile strength S are given in the table at the beginning of this article. The tensile strength of riveted pipe is about 70 per cent of the tensile strength of the metal.

PIPE FITTINGS. — For dimensions and weights of various pipe fittings see *Kent's Mechanical Engineers' Pocket Book*, pp. 196–207 (8th Edition). See also article in this book on *Valves*.

PIPE COVERINGS. — To prevent loss of heat by radiation, steam and feed-water pipes are usually protected with a cover, 1 inch or more in thickness,

of loose non-conducting material. About 3 B.t.u. per square foot per hour per degree difference in temperature is radiated from a bare pipe under ordinary conditions. By the use of any good commercial covering from 75 to 85 per cent of this loss may be prevented. Pipe covering is usually applied in sections, moulded to fit the pipe, and held in place by bands. The covering for fittings and valves is usually applied in plastic form.

FLOW OF WATER THROUGH PIPES. — The pressure required to force a stream of water through a pipe is usually expressed in terms of the height of a column of water which would produce a static pressure equal to this pressure. A pressure of

1 pound per square inch = 2.31 feet of water column,
1 foot of water column = 0.433 pound per square inch.

Friction and Velocity Head. — The head required to overcome the resistance of a pipe is called the "friction head," and for a given pipe and velocity of flow is proportional to the length of the pipe. The head required to overcome the resistance at the entrance to a pipe is called the "entry head;" in the case of long pipes the entry head is negligible in comparison with the friction head. In addition to these two heads, a certain pressure, and therefore a corresponding head, is required to produce any change in the velocity of flow, as, for example, when water enters a pipe from a reservoir. This velocity head is equal to $\frac{V_1^2 - V_2^2}{64.4}$, when the velocity changes from V_1 to V_2 feet per second. A decrease in velocity gives rise to a *negative* velocity head. In long pipes the velocity head is negligible.

Hydraulic Grade Line. — Imagine a horizontal line drawn over a pipe line from a reservoir, and let this horizontal line be at the same elevation as the surface of the reservoir. From each point of this horizontal line drop perpendiculars equal in length to the loss of head between this point and the reservoir. The locus of the foot of these perpendiculars is called the "hydraulic grade line." In a pipe leading from a reservoir no part of the length of the pipe should be above the hydraulic grade line. If the pipe has vertical curves, valves should be provided at the high points to permit the escape of the air which tends to collect at the top of such curves, otherwise the pipe may become "air-bound," i.e., water will not flow although the supply is higher than the outlet.

Formulas Connecting Velocity, Discharge, and Head. — Various formulas have been proposed to express the relation between velocity, discharge and head. Let

d = diameter of pipe in inches,
D = diameter in feet,
H = loss of head in pipe in feet of water,
L = length of pipe in feet,
p = loss of pressure in pipe in pounds per square inch,
Q = discharge in cubic feet per second,
V = velocity of flow in feet per second.

Unwin gives the following formula:

$$V = 4.012\sqrt{\frac{DH}{fL}},$$

where f is the coefficient of friction. This coefficient depends upon the smoothness and cleanness of the pipe and also upon whether the pipe is straight or

crooked, and upon the velocity. (*See Kent's Mechanical Engineers' Pocket Book.*) Rankine gives the following formula for f for smooth, clean, straight pipe:

$$f = 0.005\left(1 + \frac{1}{12\,D}\right).$$

This formula is approximate only, since it does not take into account the velocity of flow. It is sufficiently accurate, however, for velocities up to 6 feet per second.

Combining these two formulas the following are obtained:

$$V = 16.4\,d\sqrt{\frac{H}{L\,(d+1)}} = 25\,d\sqrt{\frac{p}{L\,(d+1)}},$$

$$V = 57\,D\sqrt{\frac{H}{L\,(D+0.083)}} = 87\,D\sqrt{\frac{p}{L\,(D+0.083)}},$$

$$Q = 0.00545\,d^2V = 0.089\,d^3\sqrt{\frac{H}{L\,(d+1)}} = 0.136\,d^3\sqrt{\frac{p}{L\,(d+1)}},$$

$$Q = 0.785\,D^2V = 44.7\,D^3\sqrt{\frac{H}{L\,(D+0.083)}} = 68\,D^3\sqrt{\frac{p}{L\,(D+0.083)}}.$$

When the pipe is not completely filled with water, the same formulas hold provided d and D are taken as 4 times the hydraulic radius. The "hydraulic radius" is defined as the ratio of the area of the cross section of the water to that portion of the perimeter of the pipe in contact with the water.

These formulas apply approximately to any kind of clean, straight pipe, provided the interior surface is smooth. The formulas involving the head H also apply approximately to any kind of liquid or gas, provided H is taken as the height of a column of the given liquid or gas which will produce a static pressure equal to the fall in pressure in the pipe. The coefficients in the formulas involving the drop of pressure p should then be multiplied by $\sqrt{\frac{62.4}{w}}$, where w = weight in pounds of 1 cubic foot of the given liquid or gas, and 62.4 = weight in pounds of 1 cubic foot of water.

These formulas are for *new, clean, straight pipes*. For cast-iron pipes that have been in service a number of years the loss of head will be larger on account of corrosion and incrustation, and the value of H in the formulas should be multiplied under average conditions by the factors opposite; but they must be used with much discretion, for some waters corrode pipes much more rapidly than others.

10 years	1.3
20 "	1.6
30 "	2.0
50 "	2.6
75 "	3.4

The same figures may be used for wrought-iron pipes which are not subject to a frequent change of water.

From the above formulas for velocity the loss in head due to pipe resistance is

$$H = \frac{LV^2\,(d+1)}{270\,d^2} = \frac{LV^2\,(D+0.083)}{3250\,D^2}.$$

William Cox (*Amer. Mach.*, 1893) gives the following formula for pipes over 6 inches in diameter:

$$H = \frac{L\,(V^2 + 1.25\,V - 0.5)}{300\,d}.$$

The Pelton Water Wheel Co. advocate the following formula for riveted pipe for pipes over 6 inches in diameter:

$$H = \frac{L(V^2 + 1.25 V - 0.5)}{250 d}.$$

Effect of Curves and Valves.—The resistance of curves and valves may be allowed for approximately by taking for L in the above formulas the actual length of the pipe plus a length equal to

$$\frac{kd^2}{d+1} \text{ feet, } \quad \text{or} \quad \frac{KD^2}{D+0.083} \text{ feet,}$$

for each curve or valve, where k and K have the following values:

	45° angle	90° angle	Gate valve	Globe valve	Angle valve
$k=$	0.8	4.2	0.8	8.1	12.4
$K=$	9	49	9	96	148

These coefficients are based on data given by Gebhardt (*Steam Power Plant Engineering*). Variations of 100 per cent or more from the valves given may be expected, depending on the radius of the bends and design of the valves.

Water-Hammer. — From the formula given by Prof. I. P. Church the pressure developed by the instantaneous closing of a valve in a water pipe is

$$P_i = \frac{63.5\, V}{\sqrt{1 + \frac{300{,}000\, d}{MT}}} \text{ pounds per square inch,}$$

where M = modulus of elasticity of the pipe material and the other symbols are as above.

FLOW OF AIR AND GAS THROUGH PIPES. — (*See also preceding section.*) Let

d = internal diameter of pipe in inches,
L = length of pipe in feet,
p = fall of pressure in the pipe in pounds per square inch,
Q = discharge in cubic feet per second,
V = linear velocity in feet per second,
w = weight of 1 cubic foot of air or gas in pounds.

Then from Unwin's formula, using the same coefficients as for the flow of water, *when the fall of pressure is small,*

$$V = 198\, d \sqrt{\frac{p}{wL(d+1)}}. \qquad (1)$$

$$Q = 1.08\, d^3 \sqrt{\frac{p}{wL(d+1)}}. \qquad (2)$$

For air: $\quad w = \dfrac{2.70\, P}{460 + t}$; for any other gas: $\quad w = \dfrac{2.70\, GP}{460 + t}$,

where P = absolute pressure in pounds per square inch and t = temperature in degrees F and G = the specific gravity of the gas referred to air as unity. For ordinary illuminating gas $G = 0.65$.

Various authorities give different values of the numerical coefficients in the formulas for V and Q (*see Kent's Mechanical Engineers' Pocket Book*). The values given above, which are the same as for the flow of water ($w = 62.4$), are sufficiently accurate for rough calculations. The formula for Q may also be written

$$Q = Cd^2\sqrt{\frac{dp}{wL}}, \tag{3}$$

where $C = 1.08\sqrt{\dfrac{d}{d+1}}$, corresponding to the coefficient 1.08 in equation (2).

C is frequently taken as unity for all sizes of pipes.

When the *fall of pressure is large*, the above formulas for Q give approximately the quantity per second at the *average* pressure in the pipe, provided the weight per cubic foot is taken corresponding to this *average* pressure. Or, putting P = the pressure in pounds per square inch at the *outlet* of the pipe, and w = weight per cubic foot at this pressure P, p being the fall in pressure, then the discharge in cubic feet per second at the pressure P is approximately (from equation 2)

$$Q = 1.08\,d^3\sqrt{\frac{p\,(P + 0.5\,p)}{PwL\,(d+1)}}.$$

For d large compared with unity, $\dfrac{d}{d+1}$ may be taken sensibly equal to unity, and this expression may be written

$$Q = 1.08\,d^2\sqrt{\frac{p\,(P + 0.5\,p)\,d}{PwL}}.$$

For $P = 14.7$ pounds per square inch, and $t = 60°$ F., then for air

$$Q_a = 1.02\,d^2\sqrt{\frac{p\,(P + 0.5\,p)\,d}{L}}, \text{ cubic feet per second,}$$

and for illuminating gas of 0.65 specific gravity,

$$Q_g = 1.26\,d^2\sqrt{\frac{p\,(P + 0.5\,p)\,d}{L}} \text{ cubic feet per second.}$$

Instead of the coefficients 1.02 and 1.26 the following values correspond to the coefficients in the formulas used by the authorities named:

	Instead of 1.02	Instead of 1.26
Wm. Cox (*Am. Mach.*, 1902)	0.95	1.18
J. E. Johnson (*Am. Mach.*, 1899)	0.96	1.19
E. A. Rix (*Pac. Coast Gas Assoc.*, 1905)	1.05	1.30

Effect of Bends, Valves, etc. — The Norwalk Iron Works Co. give the following table; radius of elbow and length of pipe are both expressed in terms of the pipe diameter:

Radius of elbow.........	5	3	2	1½	1¼	1	¾	½
Equivalent lengths of straight pipe.........	7.85	8.24	9.03	10.36	12.72	17.51	35.09	121.2

W. L. Saunders (*Compressed Air*, 1902) gives the following figures for the length of pipe in feet equivalent to each of the items listed:

Diam. of pipe, in..	1	1½	2	2½	3	3½	4	5	6	7	8	10
Globe valves.....	2	4	7	10	13	16	20	28	36	44	53	70
Elbows and tees..	2	3	5	7	9	11	13	19	24	30	35	47

FLOW OF STEAM. — The formulas given in the preceding section apply only approximately to the flow of steam. Putting v = volume of 1 pound of steam in cubic feet (*see tables under Steam*), the above formula for flow in cubic feet per second becomes

$$Q = 1.08\, d^3 \sqrt{\frac{vp}{L\,(d+1)}},$$

and the corresponding flow in pounds per second is

$$W = 1.08\, d^3 \sqrt{\frac{p}{vL\,(d+1)}}.$$

G. H. Babcock, in *Steam*, gives the flow in pounds per minute as*

$$W = 87 \sqrt{\frac{wpd^5}{L\left(1+\frac{3.6}{d}\right)}}.$$

The corresponding flow in cubic feet per second may be written

$$Q = 1.45\, d^3 \sqrt{\frac{p}{wL\,(d+3.6)}}.$$

For a given drop in pressure this formula gives a less flow for pipes under 2 inches in diameter and a greater flow for pipes over 2 inches in diameter than is given by the corresponding formula for air or gas.

Effect of Bends, Valves, etc. — According to Briggs the effect of each right angle bend is equivalent to increasing the length 40 diameters, and the effect of each globe valve is equivalent to increasing the length 60 diameters.

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[WM. KENT.]

* w is pounds per cu. ft. at entrance pressure.

POLES FOR OVERHEAD LINES. — (*See also Cross Arms; Distribution Lines; Insulator Pins; Insulators; Transmission Lines.*) The following is a brief table of contents of this article:

Methods of Specifying Pole Dimensions. — A pole for supporting an overhead line is usually specified by its *total* or "nominal" length and by the diameter of its top; e.g., a 40-ft 7-in. top pole. When set, the distance a pole stands above the surface of the ground is less than the nominal length by the amount it sets in the ground. Poles are standard in lengths which are multiples of 5 feet. The ordinary range of length is from 30 to 60 feet.

The top of a pole is sometimes specified by inches circumference instead of by diameter. Poles are standard in diameters which are even multiples of one inch. The ordinary range of top diameter is from 7 to 8 inches except on the Pacific Coast, where from 8 to 10 inches is common.

Taper of Poles. — The taper of various kinds of poles, specified as the difference, measured in inches, between two circumferences 10 feet apart, is given as follows in *Forest Service Bull. No. 84:* Chestnut (Maryland), 3.8 to 4.0; Northern white cedar (Michigan), 5.2; Western yellow pine (California), 4.0; Lodgepole pine (Montana), 3.0; Loblolly pine (Texas), 2.4; Western red cedar (Washington), 3.5. Trees grown upon a high elevation have a greater taper in the trunk than trees grown lower down.

WOODS USED FOR POLES AND CROSS ARMS. — (Based on publication of the Forest Service, see *Bibliography.*) Of the timbers used for poles, chestnut and northern, southern and Idaho cedar easily rank first. Longleaf and shortleaf pine, red cedar, cypress, redwood, locust, catalpa and several of the oaks are used, but in much smaller numbers, and their employment is generally confined to the region of their growth. Still other timbers are used, but in numbers insignificant in comparison with those mentioned above.

For cross arms, longleaf, shortleaf and loblolly pines of the South and Norway pine of the North are most largely used, while the demand for cedar, cypress, spruce and red fir is but little less. Again, as in the case of pole timbers, a third group may be formed of those timbers used in small numbers and very locally.

In 1911, of the 3,418,020 poles used in the United States, 61.5 per cent were cedar, 20.4 per cent were chestnut, the remainder being oak, pine, cypress, etc.

Desirable Timber for Poles. — The several qualities which timber must possess to adapt it to use for poles are stated to be: Durability in contact with the soil, minimum weight, straightness coupled with relatively small size and little taper. The wood must be soft, so that the spikes of a climber may enter readily and at the same time it must have strength to support considerable

weight. These qualities are admirably combined in cedar and in juniper, which commercially is a cedar; no other woods possess so many.

Uncertainties in Names of Timber Trees. — The terms cedar, pine, etc., used in describing poles and cross arms and even the apparently more exact terms, such as white cedar, yellow pine, etc., each cover several kinds of trees and have different meanings in different localities.

There are at least eight pines (of the thirty-five native ones) in the market, some of which so closely resemble each other in their minute structure that they can hardly be told apart; and yet they differ in quality and should be used separately, although they are often mixed or confounded in the trade.

Referring to the use of yellow pine as the material for cross arms the Committee on Overhead Line Construction of the N. E. L. A., says: "Yellow pine is understood to cover what is commonly known as longleaf pine. It is understood that the term is descriptive of quality rather than of botanical species." Forestry Bulletin No. 10 states: "'Yellow pine,' is applied in the trade to all the Southern lumber pines; in the Northeast it is also applied to the pitch pine; in the West it refers mostly to bull pine. 'Yellow longleaf pine,' 'Georgia pine,' chiefly used in advertisement, refers to longleaf pine."

Timbers Ordinarily Used for Poles and Cross Arms. — The principal timber trees from which poles and cross arms are obtained are briefly described below in accordance with the names used in the publications of the Forest Service.

Chestnut. — Chestnut ranks next to the cedars in the quantity of poles used. The reported number purchased in 1909 was 608,000. The sapwood is very narrow, usually from about 1/8 to 3/8 of an inch wide. Chestnut is widely distributed throughout the entire Appalachian mountain region. A small territory embracing parts of Pennsylvania, Maryland, Virginia and West Virginia furnishes nearly all the chestnut poles. Chestnut is not so straight as cedar and is liable to be knotty. It has greater strength, but this advantage is more than counterbalanced by its greater weight, which prohibits long shipments.

Northern White Cedar or Arborvitæ. — This species is very commonly used for poles throughout the central and eastern portion of the United States. The principal source of supply is in the states bordering the Great Lakes. It makes a very desirable pole on account of its durability but is high-priced. The sapwood varies from 1/2 to 1 inch in thickness. A very large portion of northern white cedar poles have unsound butts. Northern white cedar, common in the northern woods of New England, New York and the Lake States, occurs as far south as North Carolina and Tennessee, but only in the mountains where the elevation is sufficiently great to permit northern species to thrive.

On account of its strength, lightness, durability and form it is the most desirable pole timber. Arborvitæ is extremely slow in growth. The sapwood zone is narrow at the butt and gradually widens as the top is approached. The average time it takes to produce a 30-foot arborvitæ pole is about 190 years.

Southern White Cedar. — The number of southern white cedar poles purchased in 1909 was 44,000. The woods known under the general name of "cedar" comprise a number of distinct species which differ in their durability, the white cedar of the southern swamps being somewhat less durable than the cedar of the Lake States. The sapwood, which is usually from 1/2 to 1 inch wide, decays very quickly. Southern white cedar, though sometimes found as far north as southern Maine, is of commercial importance chiefly south of Delaware and New Jersey.

Red Cedar. — A small to medium-sized tree scattered through the forests, or, in the West, sparsely covering extensive areas (cedar brakes). The

red cedar is the most widely distributed conifer of the United States, occurring from the Atlantic to the Pacific and from Florida to Minnesota, but attains a suitable size for lumber only in the Southern and more especially in the Gulf States.

Juniper. — The term Juniper is commonly used by telephone men for southern white cedar; the term also is applied to red cedar. Juniper poles come from Virginia, the Carolinas and other South Atlantic States.

Western Red Cedar. — The light and durable western red cedar is much used for poles on the Pacific coast and throughout the Northwest. Also it competes to a certain extent with northern white cedar in the East, its form and size making it especially desirable for the larger classes of poles. The principal points of production are northern Idaho and western Washington. The relative durability of western red cedar and northern white cedar under similar conditions is not known, and the testimony by pole users on this point is somewhat contradictory.

Cypress. — The cypress is a large deciduous tree, occupying much of the swamp and overflow land along the coast and rivers of the Southern States. Cypress is usually considered a durable wood, and the heartwood is, in fact, one of the most durable of our native species. The sapwood, however, decays quickly and this seriously weakens the pole. The width of the sapwood on pole-size trees is from 3/4 of an inch to 1 1/4 inches. Cypress frequently is too large for use as a pole and has greater value for lumber. Even when its general diameter is small enough the butt will often be so big that it adds too much weight.

Longleaf Pine. — Large tree; forms extensive forests and furnishes the hardest and strongest pine lumber in the market. Coast region from North Carolina to Texas. The longleaf pine is strikingly heavy, hard and resinous, and usually very regular and narrow ringed, showing little sapwood, and differing in this respect from the shortleaf pine and loblolly pine, which usually have wider rings and more sapwood, the latter excelling in that respect.

Shortleaf Pine. — Resembles loblolly pine; often approaches in its wood the Norway pine. The common lumber pine of Missouri and Arkansas, North Carolina to Texas and Missouri.

Loblolly Pine. — Large-sized tree; forms extensive forests; wider-ringed, coarser, lighter, softer, with more sapwood than the longleaf pine, but the two often confounded. This is the common lumber pine from Virginia to South Carolina and is found extensively in Arkansas and Texas, Southern States, Virginia to Texas.

This pine is not durable when used as a pole unless treated with preservatives, but because of its cheapness and ease of impregnation is very desirable if preservative treatment is contemplated. Its distribution, ease of reproduction and rapidity of growth insure a steady and cheap supply. When this timber is used it is necessary to treat the entire pole instead of only the butt, especially in the warmer and more humid localities of the South.

Norway Pine.—Large-sized tree; never forming forests, usually scattered or in small groves, together with white pine; largely sapwood and hence not durable. Minnesota to Michigan; also in New England to Pennsylvania. The Norway pine, which may be confounded with the shortleaf pine can be distinguished by being much lighter and softer. It may also, but more rarely, be confounded with heavier white pine, but for the sharper definition of the annual ring, weight and hardness.

Western Yellow Pine. — Western yellow pine is used for poles to a limited extent in certain parts of the Southwest, where the high cost of more

durable pole timbers makes it necessary to find a cheaper substitute. The life of this timber, untreated, is very short. In the upper part of the San Joaquin Valley of California, where a study of this species was made, untreated pine poles last only two or three years; but since the wood when not exposed to the soil is fairly durable, it is believed that a butt treatment with a good wood preservative will result in a pole that will give good service. A butt-treated pine pole costs considerably less than an untreated cedar pole in this locality.

Lodgepole Pine. — Lodgepole pine is cut to a limited extent for poles. It grows at high altitudes in the Rocky Mountains. It decays quickly in contact with the soil, but is durable when not so exposed. The tree grows tall and straight, with very little taper and makes a well-shaped pole. In certain parts of the West, where there are large bodies of fire-killed lodgepole that remain standing for many years, sound and thoroughly seasoned, conditions for effective treatment are excellent. If given a butt treatment, this dead timber makes a durable pole, and in many localities the cost of the pine pole plus the cost of the treatment is less than that of the Idaho cedar untreated. The sapwood of pole-sized timber may be an inch or an inch and a quarter thick.

DEFECTS IN WOOD USED FOR POLES AND CROSS ARMS. — (*See also Timber.*) The following are the defects in timber which are frequently referred to in specifications for poles and cross arms.

Pith. — The pith of a tree is the central core about which the annual rings are formed. It goes through the tree from top to bottom and branches into the limbs. The pith is quite thick, usually 1/8 to 1/5 inch in Norway pine and in the southern species, though much less so in white pine and is very thin 1/15 to 1/25 inch in cypress, cedar and larch. The pith of the tree is the weakest part on account of the many knots which it invariably and necessarily contains.

Sapwood. — The sapwood of a tree is a zone of wood next to the bark, 1 to 3 or more inches wide and containing 30 to 50 or more annular rings (in coniferous trees). It is of lighter color than the inner, darker part of the log which is the heartwood. Sapwood changes to heartwood as the tree grows.

The width of the sapwood is small for longleaf and white pine and great for loblolly and Norway pines. In old trees of longleaf pine the sapwood forms about 40 per cent of the merchantable log, while in the loblolly and in all young (coniferous) trees the bulk of the wood is sapwood.

Sapwood, being the normal condition of the outer rings of a tree, is not a "defect" in poles, where the whole cross section of the tree (except bark) is used. Being weaker and more liable to decay it is considered a "defect" in pins and cross arms, which are better if made from the heartwood only.

Cup-Shakes. — These are cracks extending circumferentially at one or more places, caused by the separation of the annual rings.

Doatiness. — This is a speckled stain found in beech, American oak and other timber, due to incipient decay. It is produced by imperfect seasoning or by exposure for a long period to a stagnant atmosphere.

Heart-Shakes. — These are splits or clefts occurring in the center of the tree. They are common in nearly every variety of timber and are very serious when they twist in the length, as they interfere with the conversion of the tree into boards or scantlings. They sometimes divide the log in two for a few feet from the end.

Star-Shakes. — When several heart-shakes occur in one tree they are called star-shakes from the appearance produced by their radiation from the center.

Wind-Cracks. — Shakes or splits on the sides of a balk (a log which has been squared off) of timber, caused by shrinkage of the exterior surface, are called wind-cracks.

Dry Rot. — Dry rot is a special form of decay in timber caused by the growth of a fungus which spreads over the surface like a close network of threads, white, yellow or brown, and causes the inside to perish and crumble. Causes which render timber favorable to the growth of this fungus are; large proportion of sapwood; felled at wrong season when full of sap; if cut down in the spring or fall of the year instead of in midwinter or midsummer, when the sap is at rest; stacked for seasoning without sufficient air spaces being left; fixed before thoroughly seasoned; painted or varnished while containing moisture. (Six preceding definitions from *Carpentry and Joinery* by *Paul N. Hasluck.*)

Sound Knot. — A sound knot is one which is solid across its face and which is as hard as the wood surrounding it; it may be either red or black, and is so fixed by growth or position that it will retain its place in the piece.

Loose Knot. — A loose knot is one not firmly held in place by growth or position.

Pith Knot. — A pith knot is a sound knot with a pith hole not more than one-fourth of an inch in diameter at the center.

Encased Knot. — An encased knot is one which is surrounded wholly or in part by bark or pitch. Where the encasement is less than one-eighth of an inch in width on both sides, not exceeding one-half the circumference of the knot, it shall be considered a sound knot.

Rotten Knot. — A rotten knot is one not as hard as the wood it is in.

Pin Knot.— A pin knot is a sound knot not over one-half inch in diameter.

Spike Knot.— A spike knot is one sawn in a lengthwise direction. The mean or average width shall be considered in measuring these knots.

Pitch Pocket. — A pitch pocket is an opening between the grain of the wood containing more or less pitch or bark.

Pitch Streak. — A pitch streak is a well-defined accumulation of pitch at one point in the piece. When not sufficient to develop a well-defined streak, or where the fiber between grains — that is, the coarse-grained fiber, usually termed "spring wood" — is not saturated with pitch, it shall not be considered a defect.

Wane.—Wane is bark, or lack of wood from any cause, on edges of timber.

Shakes. — Shakes are splits in timber which usually cause a separation of the wood between annual rings.

Checks.— Checks are splits in timber, which usually cause a separation of the wood across annual rings. (Last twelve definitions are those used in the timber-test work of the Forest Service in describing defects. *Forest Service Circular 38, Revised.*)

Wind Shake. — A crack or incoherence in timber produced by violent winds while the timber was growing.

Wind. — A turn or bend. A piece of timber is out of wind when it is perfectly straight or flat.

Warped.— Twisted out of shape by seasoning.

Cat-Faces. — Old wounds, partially overgrown, leaving a long, narrow, dead surface exposed.

VOLUME AND WEIGHT OF POLES.— A quick way to find the approximate volume of a pole is to multiply the area of the circle at the center of gravity

by the length of the pole. The formula for the volume, considering a pole as a frustrum of a cone is

$$v = \frac{\pi}{1728}(d_1^2 + d_1d_2 + d_2^2)h,$$

where v = volume in cubic feet; d_1 = diameter at butt in inches; d_2 = diameter at top in inches; h = length of pole in feet.

The following table gives the volume of some standard poles:

Kind of pole	Nominal size		Volume, cubic feet
	Diameter, inches	Length, feet	
Chestnut	7	30	20.0
Southern white cedar	7	30	20.8
Northern white cedar	7	30	17.6
Western red cedar	8	40	27.3
Western yellow pine	8	40	26.0

The weight of a pole may be found by multiplying its volume in cubic feet by its weight per cubic foot. The following table gives the weight per cubic foot.

WEIGHT PER CUBIC FOOT OF POLES

Kind of pole	When cut		When seasoned	
	Weight,* pounds per cubic foot	Moisture, per cent of dry weight	Weight,* pounds per cubic foot	Moisture, per cent of dry weight
Southern white cedar	38.9	88	25.0	21
Chestnut (N. C.)	56.5	101	43.2	54
Chestnut (N. J.)	51.8	85	42.2	50
Chestnut (Pa.)	54.0	92	40.7	45
Chestnut (Md.)	56.4	86	44.9	48
Northern white cedar	34.2	90	22.9	27
Western red cedar	42.4	133	23.5	29
Western yellow pine	66.6	154	30.3	16

* Including contained moisture.

SEASONING.— Poles should be seasoned because it increases their resistance to decay, increases their strength and decreases their weight. The strength of partially seasoned timber, other things being equal, increases as the amount of moisture it contains decreases. Thoroughly seasoned timber of small sizes is sometimes three or even four times as strong as the same timber, when green.

Seasoning of poles reduces their weight, commonly from 16 to 30 per cent, and even more for some species, with a corresponding decrease in the cost of transportation. Thorough seasoning is essential if the poles are to be treated with preservatives. The percentage of moisture in a pole when cut varys with the season when cut as shown in the table at top of p. 1068.

In general, poles cut during the spring and summer lose weight most rapidly. Poles cut during autumn and winter lose weight less rapidly, but more regularly.

Too rapid seasoning may be detrimental to the timber by causing excessive checking. Shrinkage of poles during seasoning is very slight and does not exceed 1 per cent on the circumference.

MOISTURE CONTENT WHEN CUT, PER CENT OF DRY WEIGHT

Kind of pole	Spring	Summer	Autumn	Winter
Southern white cedar (N. C.)	68	77	87	88
Chestnut (N. C.)	97	91	95	101
Chestnut (N. J.)	81	83	81	85
Chestnut (Pa.)	89	92	88	88
Chestnut (Md.)	83	84	85	86
Northern white cedar (Mich.)	77	82	79	90
Western red cedar (Cal.)	...	133	...	...
Western yellow pine (Cal.)	149	147	145	154

The time in months required for poles cut at different periods of the year to season to approximately air-dry weight is as follows:

TIME REQUIRED FOR SEASONING

Kind of pole	Spring	Summer	Autumn	Winter	Moisture content* seasoned
	Months	Months	Months	Months	Per cent
Chestnut (Md.)	5	4	8	7	55
Southern white cedar (N. C.)	3	3	8	5	26
Northern white cedar (Mich.)	12	9	7	6	37
Western red cedar (Cal.)	....	....	....	....	43
Western yellow pine (Cal.)	5	3	9	6	25

* The average amount of moisture remaining in the poles after seasoning as above in per cent of the weight of the dry wood.

ROOFING. — If the top of a pole is lef flat rain water will not run off rapidly and will penetrate the pole by following the grain, causing early decay of the pole top. Poles are accordingly "roofed" by cutting the top to give an inclined surface which is sometimes conical but usually is merely two inclined planes meeting in a horizontal ridge. The angle of the planes with the horizontal is usually 45 degrees. Where a bracket is to be bolted to top of pole a flat strip from ½ to 1 inch in width is sometimes left, instead of a sharp ridge, in order to leave more material in the pole top where the strain from the upper bracket bolt comes. Roofs should be painted to close the grain which is porous, in order to prevent the entrance of water.

PRESERVATIVE TREATMENT. — The forest service estimate that it requires 190 years to grow a 30-foot cedar pole whose average life, when set in the ground in its natural state, does not exceed 15 years. They also estimated in 1907 that 800,000 miles of pole line were then in operation containing 32,000,000 poles and requiring 2,650,000 poles per annum for maintenance. They conclude

that the enormous demand must soon deplete the supply and have made extensive experiments on preservative treatment as a means of conserving the pole supply.

Preservative treatments are used to increase the resistance of poles to decay. The advantages of such treatment are:

1. It increases the life of the pole.
2. It makes possible the use of smaller poles as less allowance need be made for decay.
3. It makes possible the use of species of timber not naturally durable.

The butt of the pole near the ground line is most subject to decay and treatment of the butt alone is usually deemed sufficient. In some of the Southern states the whole pole is subject to decay, in which case the whole pole is treated.

Cause of the Decay of Timber. — Decay of wood is due to low forms of plant life called fungi. The germs of decay are not inherent in the wood. The wood-destroying fungi start from the outside, either from adjacent rotten wood or by spores, which correspond to seeds, being carried by the wind and deposited on the surface. While the fungi from these spores begin at the "outside" of the wood, this surface must be understood to include all holes or cracks which the spores may enter.

Fungi require for their growth and development air, heat, moisture and food. Warmth, preferably between 60° and 100° F., favors decay. Cold retards it and temperatures above 150° F. prevent it. Under water or deep under the surface of the ground where the air is excluded, decay does not take place. Ordinarily wood which is seasoned until it is air-dry does not contain sufficient moisture to support the growth of fungi.

Preservatives. — The best method of checking the growth of fungi is to deprive them of food. This can be done by injecting poisonous substances into the timber. These substances are called preservatives. Of the many antiseptics which have been proposed for the preservation of timber only four have been largely used with success in the United States. These are creosote, zinc chlorid, corrosive sublimate and copper sulphate. Copper sulphate has fallen into almost total disuse. At present creosote and zinc chloride, pure or in mixture, are the only preservatives which are in general use.

Corrosive Sublimate (Bichlorid of mercury). — This is used in the so-called "kyanizing" process. This process consists in steeping the timber in a dilute solution of corrosive sublimate long enough to insure thorough penetration.

Zinc Chloride. — Zinc chloride is an excellent antiseptic; it is obtained by dissolving metallic zinc in hydrochloric acid. This is further diluted by water before it is used for wood preservation. Zinc chloride is much cheaper than creosote, and since it is shipped in the form of a solid the freight charges are considerably less. Zinc chloride is soluble in water, being in fact, injected into the timber in water solution and so when timber treated with it is exposed to moisture the leaching out of the salt is only a question of time. Hence zinc chloride is most commonly used in comparatively dry situations.

Creosote. — Creosote is a by-product of coal tar, which is produced at most plants for the manufacture of illuminating gas and at by-product coke-oven plants. Wood tar, when distilled in a similar manner, gives "wood creosote," which like that derived from coal tar, possesses strong antiseptic properties. There is also on the market a so-called creosote, a by-product of water-gas tar or tar manufactured from kerosene oils, which, for wood preservation, is probably inferior to the true creosote. In general, however, by "creosote" is meant the dead oil of coal tar.

Creosote is not a single chemical compound but a mixture of a number of compounds. Not only does the relative proportion of the several constituents vary, but some may be absent or other compounds, not normally constituents of creosote, may be present. Creosote proper is the fraction of oil passing over between 240° C. and 270° C. during the first distillation of the crude coal tar. In practice, however, many of the creosote oils of commerce contain considerable amounts of materials having boiling points higher than 270° C. and lower than 240° C. Some commercial creosotes are rather thin oils, some are almost entirely solid with naphthalene and some are heavy oils with a large proportion of high-boiling constituents.

An analysis of creosote in well-preserved timbers (*Forest Service Circular 98*), led to the conclusions that light oils, boiling below 205° C., will not remain in timber, but that heavy oils, containing a high percentage of anthracene oil, will remain almost indefinitely and protect the wood from decay and boring animals.

The cost of creosote in carload lots (including transportation) is (1911) about 10 cents per gallon for points east of the Mississippi River and in the vicinity of the Gulf ports west of the Mississippi. West of the Rocky Mountains the cost is about 20 cents per gallon. A gallon of creosote is estimated to be 8½ pounds.

Patented Preservation. — There are many other patented substances known by various names, but most of them have for their base creosote or zinc chloride.

Carbolineum. — Carbolineum, like creosote, is derived from the distillation of coal tar. The compounds included in carbolineum are derived from the coal tar at a higher temperature of distillation and are therefore somewhat different.

Crude Petroleum. — Crude petroleum has been experimented with but there is little definite knowledge of its value as a wood preservative.

Methods of Treatment. — The methods of applying the preservatives to the pole are the brush treatment, open tank treatment and pressure tank treatment.

The brush treatment is applied to a part of the butt at the ground line, the open-tank treatment to the whole butt and the pressure-tank treatment to the whole pole.

The brush treatment is least expensive and gives the least protection and the pressure tank is most expensive and gives the most protection.

For a full description of these methods of treatment and estimates of cost see *U. S. Forest Service Circulars, Nos. 84, 147, etc.*

An estimate in 1911 for 30 ft. 7 in. poles for a two-coat brush treatment of the butts with creosote was from 15 to 20 cents per pole in the East and from 20 to 30 cents per pole in the West. An estimate for the open-tank treatment of the butt was 67 cents per pole with creosote and 65 cents per pole with zinc chloride.

LIFE OF POLES. — Statistics compiled by the National Electric Light Association give the following figures for the average life of *untreated* poles; the figures for *butt-treated* poles are according to estimates by the U. S. Forest Service.

Untreated	Years	Butt-treated	Years
Cedar	13.5	Chestnut	20
Chestnut	12.0	Western cedar	20
Cypress	9.0	Northern white cedar	22
Pine	6.5	Pine, in dry climates	20
Juniper	8.5		

Records of the German Postal and Telegraph Department covering 52 years give an average life of 20.6 years for creosoted pine poles.

SPECIFICATIONS FOR POLES. — In the report of the Committee on Overhead Line Construction of the National Electric Light Association (abbreviated N.E.L.A.) are given very complete specifications for chestnut, eastern white cedar and yellow pine poles. Space does not permit of the incorporation of these specifications here; copies may be had from the Secretary of the National Electric Light Association. The report is also given in the *Proc. N.E.L.A.* for 1911, Vol. II, p. 374.

FORCES ACTING ON A POLE. — A pole is subject to the following forces:

(1) Vertical forces due to weight of pole, wires, sleet, etc., and to downward pull of guys.
(2) Lateral horizontal forces due to wind across line on pole, wire, sleet, etc.
(3) Longitudinal horizontal forces due to unbalanced pull of wires.
(4) Torsional forces due to unbalanced pull of wires.

A pole is strong as regards the vertical forces but weak for horizontal forces and the cross arms are weak for the torsional forces. The theory of good line work is, therefore, first to reduce the horizontal and torsional forces as much as possible by balancing the stresses and second to convert remaining unbalanced horizontal stresses into vertical stresses on the pole by the use of guys.

In practice the lateral horizontal force of the wind is one which cannot ordinarily be provided for by guys. Calculations for strength of poles, when made, are ordinarily limited to the effect of side wind.

Breaking of Pole by Cross Wind. — The principal forces tending to break a pole are wind pressures on pole and conductors when the wind blows transversely. These tend to break it by cross bending.

Let M_1 = moment of the wind on the pole,
M_2 = moment of the wind on the wires,
M = moment of resistance of the pole.

Then the condition that the pole shall not break is that

$$M_1 + M_2 < M.$$

The calculation of M_1, M_2 and M is given below.

Moment of Wind on Pole (M_1). — Moment at ground level due to wind pressure on pole is

$$M_1 = \frac{P_1 H_1^2 (D_1 + 2 D_2)}{72},$$

M_1 = moment at the ground in pound-feet,
P_1 = wind pressure in pounds per sq. ft. of projected area of pole,
H_1 = height of pole in feet,
D_1 = diameter of pole at ground in inches,
D_2 = diameter of pole at top in inches.

The maximum bending moment due to horizontal forces at the top of the pole is ordinarily assumed to be at the ground level; it is really a little below ground level and opposite the center of pressure of the resistance furnished by the ground.

Moment of Wind on Wires (M_2). — Moment at ground level due to wind pressure on the wires is

$$M_2 = \frac{P_2 H_2 n d \ (S_1 + S_2)}{24},$$

M_2 = moment at the ground in lb.-ft.,
P_2 = wind pressure in pounds per sq. ft. of projected area of wires,
H_2 = height of wires above ground in feet,
n = number of wires,
d = diameter of wires (including ice) in inches,
S_1 and S_2 = lengths of adjacent spans in feet.

Where wires are of different diameters or at different levels the formula is to be applied to each size and each level separately and moments summed.

Moment of Resistance (M). — The moment of resistance or strength of a circular pole for cross bending is

$$M = \frac{f\pi D^3}{384} \quad \text{or} \quad = \frac{fD^3}{122}.$$

M = moment of resistance of the section considered in lb.-ft.,
f = fiber stress in pounds per sq. in.,
D = diameter of pole in inches.

The maximum allowable moment M is found by using the maximum allowable value for the fiber stress f.

Fiber Stress (f) **and Actual Tests of Strength.** — The following table gives the value of the fiber stress for various kinds of timber and the actual breaking load from tests of a number of poles.

FIBER STRESS AND BREAKING LOAD

Kind of timber	Test specimen	Fiber stress at elastic limit, pounds per square inch	Fiber stress at rupture,* pounds per square inch	Actual force at rupture, pounds
Arborvitæ (1).........	2 by 2 by 30 in.....	2600	4250	....
Cedar:				
Red, western (3)....	25- ft. pole.........	....	....	1310
Red, western (4)....	25- to 35-ft. poles..	....	....	2215
Red, western (4)....	25- to 35-ft. poles..	....	....	1930
Oregon (4)..........	25- to 35-ft. poles..	....	....	3040
White, Maine (2)....	29- to 31.5-ft poles.	....	3200 to 5600	1235 to 2650
Chestnut, Conn. (2)...	29- to 31.5-ft. poles.	....	4500 to 9780	1540 to 3240
Cypress (1)...........	....	4430	7110	....
Pine:				
Lodgepole (1)........	2 by 2 by 30 in.....	3080	5130	....
Lodgepole (3)........	25-ft. pole..........	....	....	1430
Longleaf (1).........	2 by 2 by 30 in.....	5090	8630	....
Shortleaf (1)........	2 by 2 by 30 in.....	4360	7710	....
Yellow, Cal. (1).....	2 by 2 by 30 in.....	3180	5180	....
Spruce, Engelmann (3)	25-ft. pole.........	....	....	1405

(1) *Forest Service Cir., No. 213;* green, clear pieces.
(2) L. W. Winchester, *Elec. W., March 16, 1911;* top circumference 17 to 24.5 inches, poles set in ground from 4 to 6 ft., force applied 22 ft. to 26 ft. above ground.
(3) *Forest Service Cir., No. 204;* 7 in. top diameter, force applied at top.
(4) *Pac. Tel. & Tel. Co.;* from 6 to 9 in. top diameter, force applied at top.
* Modulus of Rupture.

Weakest Point of a Pole. — A pole is approximately a truncated cone in shape. For a bending force applied at one end such a cone is weakest at the point where the diameter is 3/2 the diameter at the point (near the small end) where the force is applied. A pole with 8-inch diameter at the cross arm is, therefore, weakest where it is 12 inches in diameter and may be expected to break at this point provided this point is above the place where maximum bending occurs. If it is less than 12 inches in diameter at the point of maximum bending then the break may be expected here. This rule must be considered approximate as it neglects the fact that the pole is not homogeneous, i.e., outer annual rings are sapwood and inner are heartwood, and also neglects effect of knots, etc.

ATTACHMENT OF CROSS ARMS TO POLES. — Wooden cross arms are attached to wooden poles:

(1) By gaining the pole, see below.
(2) By one or two lag screws or bolts.
(3) By one or two cross-arm braces.

The forces at the point of attachment which these fastenings must resist are:

(1) A force vertically downward, equal to weight of cross arm, pins, insulators and wire (including sleet).

(2) A horizontal force parallel to axis of arm, equal to pressure of wind blowing across line on wires.

(3) A horizontal force at right angles to axis of arm: (a) toward pole or (b) away from pole and equal to difference in pull of wires on two sides of arm.

(4) A couple in a vertical plane parallel to arm, equal to difference in moments of weight on the two ends of arm.

(5) A couple in a horizontal plane parallel to arm, equal to difference in moments of wire pull on the two ends of arm.

(6) A couple in a vertical plane at right angles to arm, equal to difference in moments of wire pull (caused by pin leverage) in the two directions.

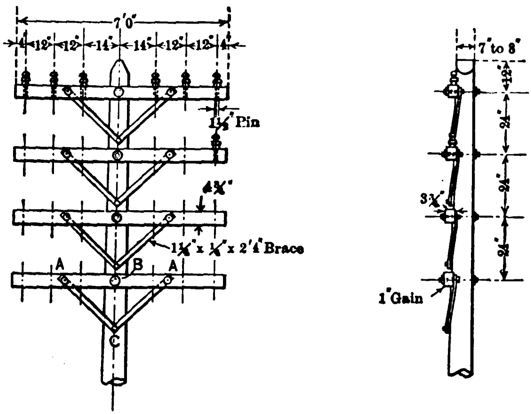

Fig. 1. Pole Top Framing, Single Arm

Framing and Hardware. — Typical framing and hardware for the three principal combinations of arms (single arms, double arms and buck arms on corner poles) are shown in Figs. 1, 2 and 3, respectively. These are standard framings of the Stone & Webster Engineering Corp. The arms used are shown in detail in Fig. 1 of the article on *Cross Arms*. The arms shown are all 6 pin,

but the framing would be the same, except for length of arm for 4-, 8- or 10-pin arms.

Gaining. — A gain is a notch cut in the side of a pole to receive a cross arm. The width (vertical dimension) of the gain should be just large enough for the cross arm. The depth of gain varies from ½ to 1 inch. With gains shallower than ½ inch the cross arm has insufficient support below and the flat bearing surface at the back is inadequate unless the pole is of larger diameter than usual. Deep gains greatly weaken the top of the pole especially when double arms are used. Gains should be painted before arms are attached to prevent moisture entering the wood through the cut surface.

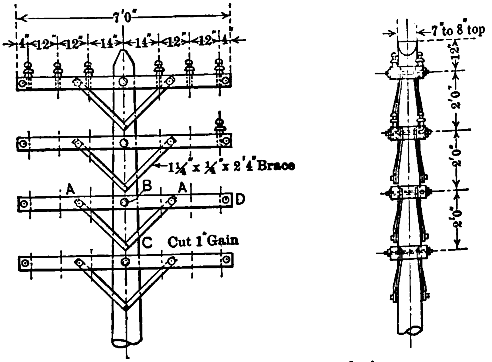

Fig. 2. Pole Top Framing, Double Arms

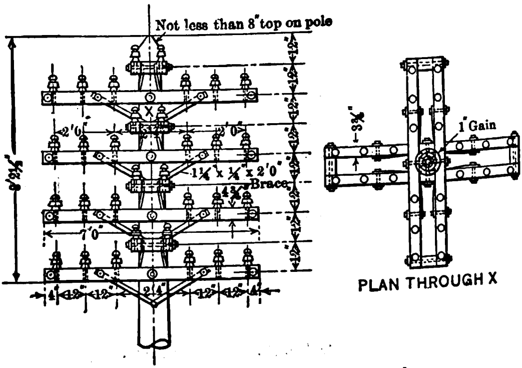

Fig. 3. Pole Top Framing, Corner Poles

Specification for Framing. — Gaining shall be as follows:
Top of pole to center of top gain, 12 inches.
Center to center of gains, 24 inches.

Gains for buck arms shall be located centrally between gains for main arms. Gains shall be cut not over 1 inch deep.

Arms shall be fastened to pole by one through bolt. Back of pole shall be flattened to give true bearing surface for washer under head of bolt. Each arm shall be braced by two braces fastened to back side of arm. Ends of double arms shall be separated by spacing blocks and shall be bolted together.

Specification for Hardware. — All hardware shall be galvanized iron.

Bolts. — Arms shall be fastened to pole with one ⅝-in. machine bolt. Double arms shall be fastened together by one ⅝-in. machine bolt at each end. Above bolts shall have 6-in. thread to allow for variation in thickness of poles. For fastening double arms to poles stud bolts having a nut at each end shall be used. Braces shall be fastened to arms by ⅜-in. machine bolts.

Washers. — Square-cut iron washers shall be used at both ends of bolts fastening arms to poles and at both ends of bolts fastening double arms together. Round-cut washers shall be used with machine bolts for fastening braces to arms.

Nuts. — Square nuts shall be used.

Braces. — Two braces shall be used on each arm. Braces shall be 28 in. by 1¼ in. by ¼ in. except on corner poles, where 24 in. by 1¼ in. by ¼ in. braces shall be used.

Lag Screws. — Braces shall be fastened to pole by ½ in. by 4 in. lag screws.

DEPTH OF SETTING IN GROUND. — The following table gives the depth of setting recommended by the National Electric Light Association.

Total length of pole, feet	Depth of setting, feet		Total length of pole, feet	Depth of setting, feet	
	Straight line	Curves and corners		Straight line	Curves and corners
30	5.0	6.0	60	7.0	7.5
35	5.5	6.0	65	7.5	8.0
40	6.0	6.5	70	7.5	8.0
45	6.5	7.0	75	8.0	8.5
50	6.5	7.0	80	8.0	8.5
55	7.0	7.5			

GUYING OF POLES. — A guy is ordinarily composed of guy wire, clamps, strain insulators, turnbuckles (sometimes) and guy stub or guy anchor.

Guy Wire. — Iron wires are used for guy wires but should always be galvanized (*see Galvanizing*). Solid wire was formerly common but stranded cable (called "strand") is now generally used. Various sizes have been used but the ⅜-inch is probably the best. The N.E.L.A. specification for the two sizes recognized by them is:

Size	Strands	Size of individual wires, B. W. G.	Ultimate breaking strength
Inch			Pounds
¼	7	14	2300
⅜	7	12	5000

Strain Insulators. — Strain insulators are placed in guys to prevent the lower part of the guy wire, which is accessible to the public, becoming charged through leakage or contact with a conductor. Two strain insulators should be used, one located about 5 feet from the pole and the other 8 feet above the ground.

Two types of strain insulator are used. In one the insulation (usually impregnated wood) is in tension, while in the other (usually porcelain) it is in compression. The former has the disadvantage that failure of the insulation causes mechanical failure of the guy. The latter has the disadvantage that the insulator may fail and the guy become charged without the failure being readily apparent. The N.E.L.A. specification favors the latter type and requires a mechanical strength of twice the strength of the guy wire and a wet flash over electrical test of four times the line voltage.

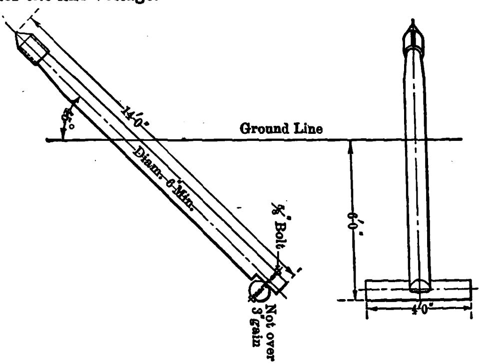

Fig. 4.

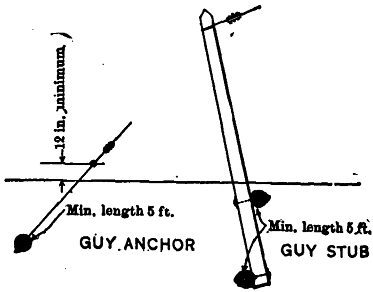

Fig. 5.

Guy Stubs and Guy Anchors. — A common guy stub is shown in Fig. 4. This is made from parts of defective poles and is fastened together with a cross-arm bolt. Fig. 5 shows another common form of guy stub and a common guy anchor.

There are also various forms of patented anchors which screw into the earth or are placed or driven into small holes and then expanded. These are designed to economize labor of installation of standard anchors.

REPAIRING DECAYED POLES. — Where poles have been weakened at the ground line by decay the strength may be restored by cutting off the decayed butt and resetting pole, thus reducing its height by six to eight feet. When reduction of height is not permissible, the pole may be stubbed by setting along side of it a short pole or stub extending a few feet above ground to which the old pole or the undecayed part above ground is bolted or otherwise fastened. This method does not look well and is unsuitable for city distribution lines but has been used for transmission lines. A more recent method is to reinforce the decayed pole by a sleeve of concrete (usually reinforced) extending above and below the decayed portion. (*See Electrical World, April 1, 1909 and Aug. 25, 1913 for Orr patented process.*)

COST OF POLES. — The cost of wooden poles varies between wide limits, depending upon the part of the country in which they are purchased. The following figures are rough approximations and are exclusive of freight.

APPROXIMATE COST OF POLES

Length, feet	Chestnut, Pennsylvania	Cedar, Iowa	Cedar, Minnesota
25	.....	$2.30*	.....
30	$3.00*	3.80*	.....
35	3.50*	6.10*	.....
40	4.25†	7.50*	$8.00†
45	6.00†	9.75*	11.75†
50	8.50†	11.20*	14.50†
55	10.00†	13.80*	.....
60	14.50†	16.70*	.....

* 7-inch top.
† 8-inch top.

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[R. A. PHILIP and CABOT STEVENS.]

POTENTIOMETERS. — (*See also Cells, Standard; Galvanometers; Resistors, Standard.*) A potentiometer is primarily an arrangement of resistances for the accurate comparison of two potential differences by balancing one against the other. In connection with suitable resistance standards (*see Resistors, Standard*) it may also be used for the accurate measurement of electric currents. The accessories for making ordinary d-c. measurements are a standard cell (*see Cells, Standard*) and a galvanometer (q.v.), and suitable keys or switches. For making a-c. measurements certain additional apparatus is required; see below under *Alternating Current Potentiometer*.

USES OF THE POTENTIOMETER. — For the calibration of current, voltage and power-measuring instruments, both d-c. and a-c., the potentiometer is the most accurate and satisfactory instrument available. A technical laboratory relies almost entirely upon the standard Weston (or Clark) cell and a set of standard resistances as its ultimate or primary standards, leaving the testing of the accuracy of these latter to a central standardizing bureau, such as the Bureau of Standards at Washington. It is, of course, convenient to have suitable "precision" ammeters, voltmeters and wattmeters as secondary laboratory standards but, as such secondary standards tend to "lose their calibration," they should be frequently checked against the standard cell and standard resistances by means of a potentiometer.

PRINCIPLE OF THE POTENTIOMETER (Figs. 1 to 3). — The potentiometer in its simplest form consists of a uniform wire stretched over a scale divided into a number of even parts, say 1500. A battery B (Fig. 1) having an e.m.f. of about 2 volts and rheostat R are connected in series with this wire and two contact points A and S are provided, one contact S being movable. G is a galvanometer. At P there may be connected at will a standard cell or any other source of potential difference.

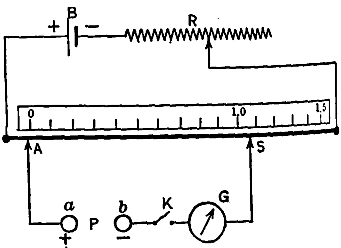

Fig. 1. Simple Potentiometer

At P is first connected a standard cell, say a Weston cell, having an e.m.f. of 1.01830 volts and the contact A is placed at 0 and the contact S at 1.0183 on the scale. By varying the rheostat R it is possible to so adjust the current in the slide wire that the fall of potential between A and S due to the current from the battery B is exactly 1.0183 volts. This can be determined by closing the switch K; if R is properly adjusted there will be no deflection of the galvanometer. Care must be taken that the positive terminals of both B and the standard battery are connected to the same end of the wire.

To measure any other p.d. the standard cell is taken out of circuit and the terminals between which the p.d. is to be measured are connected to a and b respectively. The contact S is then moved until the galvanometer shows a balance. The corresponding position of the contact S, as read on the wire, then gives the value of this p.d. in volts.

Increase of Range by Use of Volt Box or Multiplier. — To measure voltages above 1.5 one must resort to the "volt box" or "multiplier," which is simply a standard high resistance, R_1 (Fig. 2) being provided with taps, so arranged that a definite fraction of the total drop can be measured on the poten-

tiometer. The unknown p.d. is connected at E, Fig. 2, the terminals a and b being the same as the like-lettered terminals in Fig. 1. The potentiometer reading would then be multiplied by 10 or 100 depending on the position of the switch D. The resistance R_1 must be sufficiently large so that the current taken by R_1 does not appreciably affect the value of the p.d. in the circuit being tested.

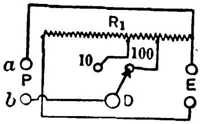

Fig. 2. Volt Box

Increase of Accuracy by Use of Additional Series Resistance. — The accuracy of measurement may be increased by inserting in series with the slide wire a known multiple of the resistance of, say, 1000 divisions of the slide wire. This is equivalent to increasing the length of the slide wire, and therefore, each division on the scale represents a correspondingly smaller fraction of a volt.

Current Measurement. — If it is desired to measure current, the drop across a known low resistance r is measured as shown in Fig. 3. The resistance r is usually so adjusted that its resistance between the potential terminals PP' is an even fraction of an ohm, in which case the potentiometer reads the current directly, with the exception of the proper pointing of the decimal. For instance, using a 0.01 ohm standard the potentiometer reading is to be multiplied by 100.

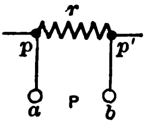

Fig. 3.

Caution. — In using any form of potentiometer a balance with the standard cell in circuit should always be obtained just before and after a balance with the unknown p.d. in circuit. In other words, the potentiometer current should always be checked both before and after a measurement.

TYPICAL DIRECT-CURRENT POTENTIOMETERS. — Potentiometers are made in such varied forms that it would take considerably more space than is available to describe them all. The most generally used direct-current types are the high-resistance "null" type, the low-resistance "null" type and the Brooks deflection type.

The Brooks potentiometer is particularly well adapted to the requirements of a large electrical engineering laboratory, where many instruments must be checked and kept in adjustment, and where it is imperative that the work be done with great speed, combined with a degree of precision ample for engineering work.

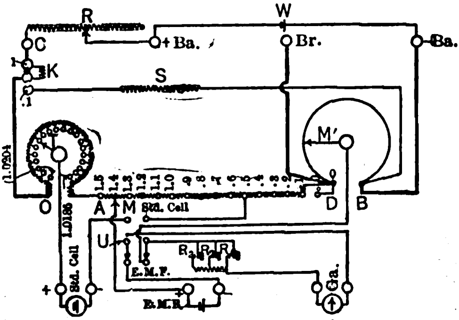

Fig. 4. Leeds & Northrup Potentiometer

Low-resistance Null Type (Fig. 4). — A diagram of the connections of a Leeds and Northrup low-resistance null-type potentiometer is shown in Fig. 4.

Fifteen 5-ohm coils are connected to the studs of a dial switch, the contact M arranged to make contact with any stud. The slide wire DB consists of 11 turns of manganin wire wound on a marble cylinder 6 inches in diameter. The contact M' is mounted on the inside of a light aluminum hood, which serves to protect the slide wire from dust. It moves over the entire length of the slide

wire and carries a scale from which the number of turns and the fraction of a turn can be easily read.

This potentiometer is designed for use with commercial "standard" Weston cells, which differ slightly in e.m.f. Each one, however, is accompanied by a certificate giving its e.m.f.

The switch T should be set at the stud corresponding to this stated e.m.f. In other words, the switch T is used for adjusting the potentiometer to the particular cell which is used. The rheostat R is then adjusted until no current flows through the galvanometer with the double-throw switch U thrown across the contacts marked Std. Cell.

In closing the galvanometer circuit close the keys R_3, R_2, and R_0 in the order stated. R_3 puts the galvanometer in series with a high resistance, R_2 puts the galvanometer in series with a medium resistance and R_0 puts the galvanometer in circuit without series resistance. If the potentiometer is considerably off balance this may thus be detected without causing a violent deflection of the galvanometer, and a closer balance obtained before the other keys are closed.

Range of Low-resistance Null Type. — With the rheostat R adjusted to give a perfect balance the current through the potentiometer is 1/10 ampere, and the drop across each coil and across 1000 divisions of the slide wire is consequently 0.1 volt and across 1 division of the slide wire 0.0001 volt.

Range-lowering Device. — The shunt S serves as a range-shifting device, by which the range may be reduced to 0.1 the normal, one division of the slide wire in this case being equal to 0.00001 volt. This shunt has such a resistance that when the plug opposite K is shifted to the lower hole the drop over AD will be 1/10 its previous value.

High-resistance Null Type (Fig. 5). — A diagram of the connections of a Wolff high-resistance null-type potentiometer is shown in Fig. 5. In this instrument the slide wire is dispensed with altogether. The double-dial switches are so arranged that the total resistance between the terminals B is constant and equal to 15,000 ohms irrespective of the position of the switches. The means whereby this is accomplished is evident upon tracing out the circuits in the diagram. By using this high resistance and keeping it constant, the effect of the contact resistances in the various switches is rendered negligible. The adjustments are made in the same manner as for the simple slide-wire potentiometer, except that instead of moving a sliding contact along a wire, one manipulates the various dial switches. The potentiometer circuit proper and all necessary keys and switches, are provided in this instrument. The accessories necessary for making measurements up to 15 volts are a regulating rheostat of about 5000 ohms total resistance, a standard cell, a suitable galvanometer and two cells of storage batteries or other source of e.m.f. of about 4 volts.

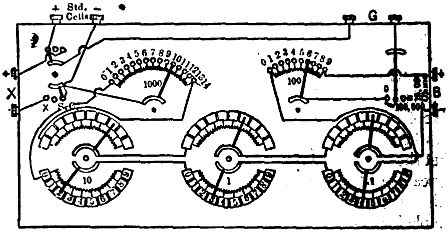

Fig. 5. Wolff's Potentiometer

Range of High-resistance Null Type. — This potentiometer has a range up to 15 volts in steps of 0.0001 volt and up to 1.5 volts in steps of 0.00001 volt.

Brooks Potentiometer (Fig. 6). — In this instrument no attempt is made to obtain an exact balance; the dial switches are set so near to the null point that

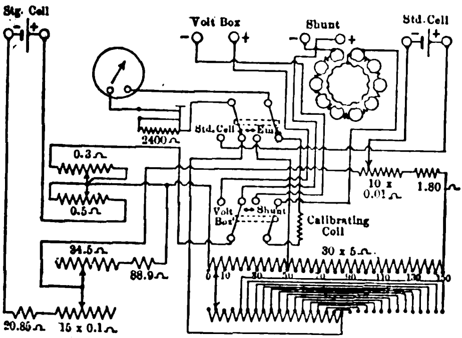

Fig. 6. Connections of Brooks Potentiometer

the galvanometer deflection is small. The galvanometer is so graduated that it gives the amount that must be added to the reading of the slides in order to give the unknown p.d. The potentiometer proper, regulating rheostat, galvanometer and necessary keys and switches are mounted in one box, making the instrument semiportable. The only accessories required to make measurements within the range of the instruments are a standard cell and storage battery.

Range of Brooks Potentiometers. — The instrument is made in two types, one model 3 for general laboratory work and the other model 5 for photometric work. The model 3 potentiometer has a range of from 0 to 1.5 volts (which may of course be extended by the use of a volt box), one step on the hand-operated dial corresponding to 0.05 volt and one division of the galvanometer scale corresponding to 0.001 volt. The model 5 potentiometer is similar to the model 3 but has a range of from 0 to 6 volts, one step on the hand-operated dial corresponding to 0.2 volt and one division of the galvanometer scale corresponding to 0.004 volt. Volt boxes having ranges up to 750 volts are made for use with either of these potentiometers.

The model 3 potentiometer may be used for current measurements up to 400 amperes with shunts that are not necessarily bulky or expensive. The model 5 instrument is not suitable for heavy-current measurements as the shunts require a large drop and are bulky and expensive.

For complete description of these instruments see *Bulletin of the Bureau of Standards, Vol. 8, No. 2.*

ALTERNATING-CURRENT POTENTIOMETER. — The principles involved in the use of the potentiometer for alternating-current measurements are the following: Referring to Fig. 1, a balance is obtained with a battery at B, a standard cell at P and the contact S at the point on the scale corresponding to the e.m.f. of the standard cell, and the current flowing in the potentiometer circuit noted by means of an electrodynamometer or a.c.-d.c. ammeter connected between B and A. The battery B is then replaced by a source of alternating current and the rheostat R adjusted until an alternating current of the same

effective value, as read on the electrodynamometer or ammeter, flows through the potentiometer. The standard cell at P is replaced by the unknown alternating p.d. to be measured and the galvanometer by an alternating-current galvanometer (*see Galvanometers*). To obtain a balance, however, it is now not only necessary that the contact S be moved along until the drop between A and S has the same effective value as the unknown p.d., *but also that this drop and the unknown p.d. have the same frequency and are in the same phase.* Consequently the p.d. at B must be supplied from the same source as the p.d. to be measured, and means must be provided for shifting the phase of one with respect to the other.

The coils of an a-c. potentiometer are wound non-inductively, and consequently the p.d. due to the potentiometer current is in phase with this current. Hence the phase adjustment consists in bringing the potentiometer current into phase with the unknown p.d.

Phase-shifting Device. — One means of shifting the phase of the potentiometer current with respect to the unknown p.d. is to use two small alternators mounted on the same shaft, with the field frame of one adjustable with respect to the other. This, however, can be more conveniently accomplished by using a device similar in construction to a polyphase-induction motor, the primary of which is supplied either from a two- or three-phase circuit, or from a two-phase circuit derived from a single-phase circuit by connecting a resistance and condenser in series, and tapping off the p.d. across the resistance for one phase and the p.d. across the condenser for the other phase. By setting the secondary of this phase shifter at a fixed angle with respect to the primary, a p.d. can be obtained from the secondary at any desired phase with respect to the source of supply. A phase-shifting device based on this principle, and so designed that the change of phase resulting from shifting the position of the secondary does not alter the effective value of the secondary e.m.f., has been recently devised by C. V. Drysdale (*Phil. Mag., 1909, Vol. 17, p. 402*). With this instrument the phase of the potentiometer current and p.d. to be measured may be adjusted to 0.1°.

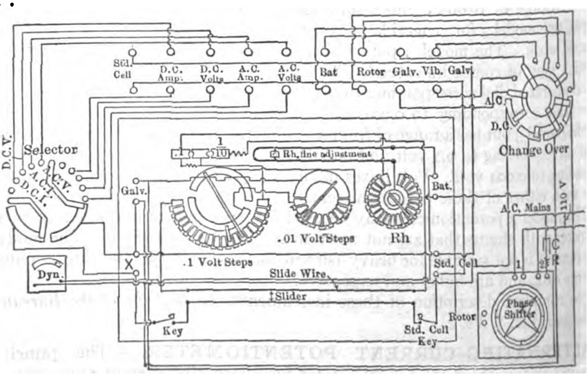

Fig. 7. Drysdale-Tinsley Alternating-current Potentiometer

Drysdale-Tinsley Alternating-current Potentiometer (Fig. 7). — A diagram of connections is shown in the figure. A vibration galvanometer is used as an a-c. detector and is tuned to the impressed frequency. The potentiometer is first balanced with an ordinary battery and standard cell and the reading of the electrodynamometer noted. By means of the change-over switch, alternat-

ing is substituted for direct current and the reading brought to the same point and held there. The phase of the potentiometer current is then roughly adjusted and the unknown p.d. is balanced as nearly as possible by shifting the potentiometer slides. The balance is then improved by shifting the phase of the potentiometer current and by resetting the slides; thus, by a process of double adjustment, the vibration galvanometer is brought to rest.

As the vibration galvanometer is a tuned instrument, *the frequency of the a-c. supply must be kept constant.* Also, the wave shape of the potentiometer current and that of the unknown p.d. must be the same, as the vibration galvanometer shows a *balance for the fundamental frequency only.*

Current and Power-Factor Measurements. — By the use of suitable non-inductive shunts alternating currents may also be measured with this potentiometer. Also, by noting the reading of the phase-shifting device corresponding to a balance first of the given p.d. and then for the given current, the phase angle between the p.d. and current can be obtained by taking the difference of the two readings.

Range of Drysdale-Tinsley Potentiometer. — This potentiometer has a range of from 0 to 1.5 volts in steps of 0.001 volt. With suitable non-inductive volt boxes its range may be extended to 750 volts. It may also be used, in conjunction with suitable shunts, for measuring alternating currents of any value.

PRECISION OF POTENTIOMETER MEASUREMENTS. — The precision obtainable with a potentiometer depends upon the accuracy to which the e.m.f. of the standard cell is known, the accuracy to which the various resistance coils in the potentiometer circuit and shunts are adjusted, and the relative magnitude of the various contact resistances, and the proportion of the total resistance in the potentiometer circuit. For a-c. measurements the accuracy is also dependent upon the inductance and capacity of the coils. The Bureau of Standards calibrate the Weston cell to 1/50 per cent. The various coils of a potentiometer are adjusted to different degrees of accuracy, little attention being paid to the absolute accuracy of coils as long as they bear definite relations to each other.* Instruments are calibrated as potentiometers to give the following accuracies.

PRECISION OF POTENTIOMETER MEASUREMENTS

Kind of potentiometer	Degree of precision, per cent	
	Voltage measurements	Current measurements
Low-resistance null type	1/50	1/20
High-resistance null type	1/50	1/20
Brooks	1/25	1/10

COST OF POTENTIOMETERS. — The following are approximate costs of the potentiometers proper, exclusive of the standard cell, galvanometer, volt box and shunts for current measurements:

Low-resistance null type	$240
High-resistance null type, rheostat not included	250
Brooks deflection type, including galvanometers	350

BIBLIOGRAPHY. — A complete bibliography on potentiometers is given in *Circular No. 21, Bur. Stand.*, 1910; H. B. Brooks, *Bull. Bur. Stand.*, Vol. 8, 1911, p. 395.

[H. PENDER AND H. R. RANKEN.]

POWER-FACTOR INDICATORS AND REACTIVE VOLT-AMPERE INDICATORS. — (*See also Alternating Currents; Generators, Alternating-Current; Wattmeters.*) A power-factor indicator is an instrument designed to give a direct reading, at any instant, of the power factor in a circuit or system of circuits as well as to indicate whether the current is leading or lagging; see *Alternating Currents*. The power factor of a single-phase circuit may be calculated from the readings of an ammeter, voltmeter and wattmeter. The power factor of a balanced three-phase circuit can also be calculated directly from the readings of the two wattmeters used to measure the power (*see Wattmeters*). A direct-reading power-factor indicator, however, is usually to be preferred for station purposes, since it gives the power factor directly and also indicates directly whether the current is leading or lagging.

Under certain conditions it is more convenient to obtain a direct measure of the reactive (wattless) power supplied to a circuit; ordinary wattmeters may be thus used as explained below.

MOVING-COIL TYPE. — Power-factor indicators of this type operate on the same principle as wattmeters of the electrodynamometer type, except that the coils are so arranged and so connected to the circuit that a differential action is produced which depends only on the power factor of the load.

Three-phase, Moving-coil Type. — Fig. 1 is a diagram of the three-phase power-factor indicator built by the G. E. Co. C and C' are current coils in series with one leg of circuit and P_1 and P_2 are potential coils connected as shown through the non-inductive resistances r_1 and r_2. Connections into and out of the moving coils are made by light spirals having very small torque, or through the bearings, so that the final position of the moving system depends solely upon the relative forces between the fixed and moving coils.

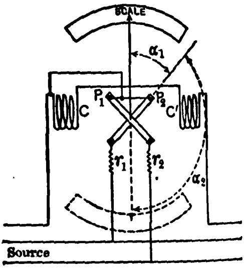

Fig. 1. G. E. Moving-coil Type

If the instrument is connected properly, the current coil acts with the two potential coils like the two elements of polyphase wattmeter (*see Wattmeters*). The torques of the two moving elements are developed in opposite directions. The torque of each element depends on the position of its potential coil. The potential coils are fixed with relation to one another at such an angle that the torque of one element always increases when that of the other decreases, due to the movement. Hence the needle will come to rest where the two torques are equal, and will give a scale reading dependent upon the ratio of the watts supplied to the two elements, i.e., in the case of a balanced load dependent on the ratio $\cos(30 - \theta) \div \cos(30 + \theta)$, where θ is the power-factor angle. Hence, for unity power factor, $(\theta = 0)$, the moving element will take up a position which is symmetrical with respect to the two potential coils, i.e., the pointer will take up a vertical position. For 50 per cent power factor, lagging, $(\theta = 60°)$, the moving element will swing to the right until the axis of the potential coil P_1 coincides with the axis of the current coil. Similarly, for 50 per cent power factor, leading, $(\theta = -60°)$, the moving element will swing to the left until the axis of P_2 coincides with the axis of the current coil. The character and range of the scale can be modified by varying the angle between the two potential coils.

Should the direction of flow of power reverse, the pointer would tend to take up the dotted position in Fig. 1. Some instruments are provided with scales to take advantage of this effect.

Another form of instrument of this type designed for unbalanced systems has three fixed coils, 120° apart, one for each phase. The movable element has three Y-connected non-inductive coils.

Two-phase Moving-coil Type. — Instruments for use on two-phase circuits are similar to the three-phase instrument, shown in Fig. 1, except that for the same scale the relative position of the two coils is different.

Single-phase Moving-coil Type. — Power-factor indicators are used only to a limited extent on single-phase circuits, this being in part due to the difficulty in designing an instrument of this type which will not be affected appreciably by even small changes in frequency. There is also but a limited demand for such instruments.

In construction and principle of operation the single-phase power-factor indicator is similar to the three-phase indicator described above. One of the movable coils is connected in series with a non-inductive resistance, and the other in series with an inductance, and both circuits are connected across the two mains; in this way the necessary phase relations are established in the two potential circuits. Since the current in the inductive circuit varies with the frequency, it is evident that the readings of such an instrument will be influenced by changes of frequency and of wave form. To overcome this difficulty various more or less successful modifications have been made in the simple arrangement described.

MOVING-VANE TYPE.—In place of the movable coils this type of power-factor meter contains a movable soft iron vane, which, in the case of polyphase instruments, is magnetized through a stationary coil carrying a current in phase with the voltage of one phase of the circuit. There is one stationary series coil for each phase, the arrangement being such as to produce a rotating field. The iron vane then takes up a position in which the direction of the flux produced in it by the potential coil when at a maximum is coincident with the direction of resultant flux due to the current coils. In the three-phase instrument three current coils, placed 120° apart, are used; in the two-phase meter two current coils at 90° are used. In the single-phase type the iron vane is magnetized by a stationary coil placed in series with the line, while the rotating field is produced by two potential coils 90° apart, one of which is connected to the line through an inductance, the other being connected to the same line through a non-inductive resistance. For the purpose of damping the deflections, an aluminum disk moving in the field of two permanent magnets is attached to the movable system.

REACTIVE (WATTLESS) VOLT-AMPERE INDICATORS.—Operating conditions on polyphase circuits are sometimes such that the reactive component of the volt-amperes in the circuit does not vary through wide limits of load between light load and full load, whereas the power factor may vary greatly. Under such circumstances it is frequently more convenient to have a direct measure of the reactive volt-amperes rather than the power factor; as, for example, in a railway substation employing synchronous converters (*see Converters, Synchronous*). This may be secured, as noted above, by using wattmeters suitably connected to the circuit. On a balanced three-phase circuit either a single-phase wattmeter, connected as shown in Fig. 2, or a two-element polyphase meter, connected as shown in Fig. 3, may be used; the instrument transformers are omitted when the voltage and current do not exceed the range of the wattmeter. The only difference between the connections shown in Fig. 3 and those for the two-element polyphase wattmeter for measuring power (Fig. 5 in article on *Wattmeters*) is that the two potential circuits are interchanged.

A wattmeter connected as in Fig. 2, reads $EI \sin \theta$, where E is the voltage between wires, I the line current, and θ the power-factor angle; to obtain the total reactive volt-amperes of the load this reading must be multiplied by $\sqrt{3}$. A two-element wattmeter connected as in Fig. 3 reads $2EI \sin \theta$, and to obtain the total reactive volt-amperes of the load this reading must be multiplied by $\sqrt{3}/2$; see article on *Wattmeters*. The instrument in either case may be calibrated to read the volt-amperes directly.

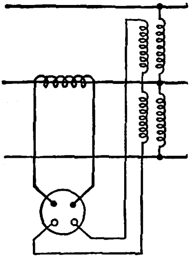

Fig. 2. Connections for Single-phase Reactive Volt-ampere Meter

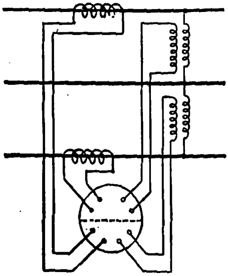

Fig. 3. Connections for Polyphase Reactive Volt-ampere Meter

RANGES OF THE VARIOUS TYPES. — The largest capacity of instrument of the moving-coil type for direct connection to the circuit is 100 amperes and 300 volts for the single-phase and 200 amperes and 600 volts for the two-phase and three-phase types. The range of power factor on standard instruments is from 0.60 leading to 0.60 lagging, although instruments with ranges as low as 0.00 leading to 0.00 lagging have been made.

Instruments of the moving-vane type have a current capacity of 5 amperes. They are made for direct connection to circuits of 110, 220 and 440 volts. For all other capacities and voltages the 5-ampere, 110-volt instrument with suitable current and potential transformers is used. The instruments are made for frequencies of either 25 or 60 cycles per second. The range of power factor on the moving-vane type of meter is unlimited.

Power-factor indicators of either type may be connected to instrument transformers which are used in connection with other meters.

Five amperes and 115 volts are standard for wattmeters used as reactive volt-ampere indicators, with transformers for higher ranges. However, single-phase instruments for direct connection, are furnished up to 200 amperes and 600 volts, and polyphase instruments for direct connection, up to 60 amperes and 600 volts.

COSTS. — The approximate prices of polyphase power-factor indicators of the moving-coil type, for current capacities of from 5 to 100 amperes and for voltages from 100 to 600 volts, range from $45 to $75. The price of the corresponding single-phase instruments, including the phase-splitting reactor, is approximately $10 more. Power-factor indicators of the moving-vane type for voltages from 110 to 440 cost approximately from $30 to $40, exclusive of current or potential transformers.

Single-phase wattmeters calibrated to read reactive volt-amperes cost about $45 each and polyphase wattmeters thus calibrated cost about $65.

BIBLIOGRAPHY. — See *Bibliography* in article on *Synchronizers*.

[L. T. Robinson and O. R. Schurig.]

POWER STATIONS, GAS-ELECTRIC. — (*See also Gas; Gas Engines; Gas Producers; Generators; Power Stations, Steam-Electric; Power Stations, Hydroelectric.*)

Number and Capacity of Units. — As gas engines have little overload capacity a total rating of engines should be installed sufficient to carry the extreme peak load and afford ample emergency reserve. In many cases the kilowatt rating of generators is made but 60 to 65 per cent of the horse-power rating of the engines, in order to provide overload generating capacity. This is not always wise, as the most efficient load of a gas engine is quite near its maximum capacity. The units in very large plants are usually of the horizontal, twin, tandem type and range from 4000 to 6000 h.p. in rating, the latter size being the largest available. Producers are quite flexible in capacity, but heavy forcing is apt to produce clinker and impair the quality of the gas. The total producer capacity usually corresponds with the total engine capacity, unless the load fluctuates greatly and considerable tank capacity for gas storage is available. In most cases not less than three engine and producer units should be provided.

Buildings. — The practice in this field follows quite closely that described in the article on *Power Plants, Steam-Electric* (q.v.). In a few cases producers are set in the open air. Producers and engines are sometimes in separate buildings. In the usual design the engine and producer rooms are parallel and electrical control galleries are provided on the side of the engine room opposite the producer room. The engine room should be spanned by an electrically operated crane with capacity sufficient to handle the heaviest engine part. In some important plants the producer room is also provided with a crane.

Producer Room Lay-out. — The gas producers are usually set in a single row extending the length of the producer room. Each unit comprises one or two gas generators, an evaporator, a scrubbing tower, a gas pump and tar filters grouped so that the path of the gas is toward the engine room. A coal bunker is often provided above the producers, preferably along the outside wall. Down-spouts to the charging bells of the several producers should be somewhat inclined and should have cut-off valves. When desired, automatic weighing hoppers may be attached between the bunker and each down-spout to record the coal consumption of each producer. In some producer rooms a shallow pit is provided along the front of the gas generators so that ashes may be raked into this pit. In other cases ashes are raked out onto the main floor to be carted away. A skeleton gallery of steel connecting the producers at their charging levels is often a convenience.

Engine Room Lay-out. — Horizontal engines are usually set in a single row with piston rods across the engine room. The power end of each unit is on the side of the gas supply and the electrical end on the side nearer the control gallery. In a-c. plants a group of independently driven exciters is placed near the middle of the room. The spaces between units and the clearances at their ends are determined by the room needed for erection, dismantling, making repairs and access during operation.

Piping. — The piping scheme is usually a simple parallel plan. A main gas header is provided and runs the length of the wall between the engine and producer rooms. Branches from this header run to each producer unit to each engine and to the storage tank. Piping may be of relatively light weight and needs no special provision for expansion, but all joints should be secure, as producer gas is of a poisonous nature. The storage tank may properly be at one end of the building. Each producer is usually set beneath a vent pipe or short steel chimney run a few feet above the roof to discharge the smoke and raw gases produced when starting up.

Space Required. — The following table shows the floor space provided per kw. of plant capacity in modern stations.

Plant capacity, kilowatts	Number of engine units	Number of producer units	Square feet per kilowatt	
			Engine room	Producer room
360	3	2	6.0	2.2
700	2	2	4.5	2.9
1,620	3	2	5.5	1.56
3,000	3	2	3.7	0.93
34,000 (a)	17	0	3.0	....

(a) Blast-furnace gas plant.

COST OF GAS POWER PLANTS. — The cost per kilowatt of gas power plants varies with the capacity of the plant, the number of units installed and the source of gas supply. The construction cost of engine plants using natural gas ranges from $65.00 to $100.00 per kw., depending on the size of plant. The construction cost of blast-furnace gas plants, including the necessary gas-cleaning equipment, ranges from $85.00 to $120.00 per kw., and of producer plants from $90.00 to $125.00 per kw. according to capacity. The cost per kw. diminishes very slowly above a plant capacity of 12,000 kw. The division of the construction cost per kw. in a 16,000-kw. producer-gas plant having eight engine units is approximately as follows:

Building and foundations	$ 15.00
Producers	19.50
Engines	40.00
Generators	10.00
Piping and auxiliaries	3.00
Wiring and switchgear	2.50
Service equipment	2.50
Engineering and contingencies	7.50
Total	$100.00 per kw.

Fuel. — Assuming the coal to have 12,000 B.t.u. per lb., about 1⅛ lb. coal are required per kw-hr. generated at full load; 1⅓ lb. per kw-hr. at ¾ load; and 1⅝ lb. per kw-hr. at ½ load. For coal of any other heating value the weight required is approximately inversely proportional to the B.t.u. per lb.

Water. — About 9 gallons of water per kw-hr. are required when the supply water is at 50° F., and about 13 gallons per kw-hr. when at 70° F.

Oil and Supplies. — Cost ranges from 0.06 cent to 0.1 cent per kw-hr.

Labor for Operation. — 1 fireman required for each producer in operation and 1 oiler for each engine in operation. Supervision, switchboard attendance, coal handling, etc., as for a steam-electric station. The total cost of labor for operation is usually about 50 per cent of the fuel cost.

Maintenance. — Total maintenance cost per annum ranges from 1.75 per cent to 2.25 per cent of the first cost of plant, depending upon the load factor since larger plants involve only an increased number of similar units and a proportionately larger building.

Fixed Charges. — These range from 10 per cent to 12 per cent of the first cost of the plant.

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[W. E. Wickenden.]

POWER STATIONS, HYDROELECTRIC. — (*See also Power Stations, Gas-Electric; Power Stations, Steam-Electric; Dams; Generators; Hydraulics, Principles of; Hydrology; Pipes and Piping; Switchgear Equipment for Power Stations; Transformers; Water Wheels; etc.*) This article deals primarily with those features in which a hydroelectric plant differs from a steam electric plant; these features arise chiefly from the nature of the prime movers and from the use of the very high voltages (up to 150,000) at which power from hydroelectric stations is transmitted. The individual constituent items, such as generators, water wheels, transformers, switchgear, etc., are treated in the separate articles, dealing with these subjects. The following is a brief table of contents of this article:

LOCATION OF POWER HOUSE. — The location of the power station of a hydroelectric plant is determined primarily by the head utilized and by the location of the dam (*see Dams*). In the case of low head developments the substructure of the power station is usually a portion of the dam itself or a "wing" offset from it. In choosing the location of both the dam and the position of the power station relative thereto, particular attention should be paid to the liability of ice and débris clogging the intakes. For high head developments, using penstocks, the power station is so located as to give the maximum head consistent with economy of construction and availability of a suitable natural channel for carrying off the discharge. Available sites for storage reservoirs should also be considered.

The location of the development generally has to be such that high-tension transmission is necessary, so that provision must also be made for housing the transforming and high-tension switching apparatus.

DESIGN OF BUILDING. — (*See also Water Wheels and Their Setting.*) The design of power houses differs greatly, depending on the conditions which are to be met. It is affected to a very great extent by natural conditions, such as the location with respect to the stream, the condition of the soil, etc. Low and high head developments require different types of turbines and these may furthermore be of horizontal or vertical construction, necessitating entirely different lay-outs.

A hydroelectric power-house building is generally divided into two longitudinal bays, a front or main bay, containing the turbines and generators, and a rear bay containing the transformers, switching apparatus, etc.; see Figs. 1 and 1a. The two bays are separated either by a wall or by a row of supporting columns. The rear bay is divided into two or more floors and these, in turn, into various rooms or compartments to accommodate the step-up transformers, switches, busbars, lightning arresters, etc.

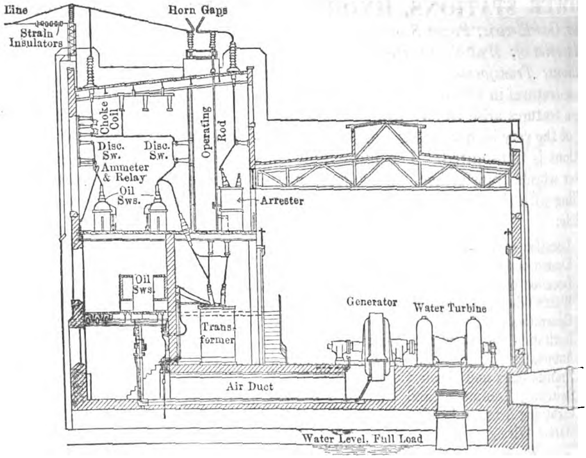

Fig. 1. Cross Section of Power House

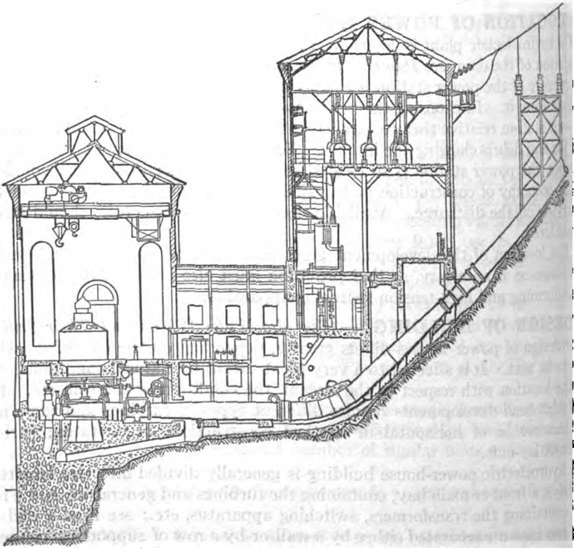

Fig. 1a. Cross Section of Power House; Roof Trusses with Raised Chord

Bearing Power of Soils. — The most important part of the building is the foundation, and careful soundings must be made to ascertain the underlying strata. If bedrock is found within moderate depth, the foundation should be carried down to the same. The safe bearing loads usually allowed for soils in this country are given in the following table:

SAFE BEARING POWER OF SOILS
(*From Baker "Treatise on Masonry Construction"*)

Material	Load in tons per sq. ft.	
	Minimum	Maximum
Rock, hardest kind	200	..
Rock, equal to Ashler masonry	25	30
Brick, equal to Ashler masonry	15	20
Brick of poor quality	4	7
Clay in thick beds always dry	4	6
Clay in thick beds moderately dry	2	4
Clay, soft	1	2
Gravel and coarse sand	8	10
Sand, fine and compact	4	6
Sand, clean and dry	2	4
Alluvial soils and uncertain sand	0.5	1

For foundations it is considered good practice to use somewhat lower values. About one-half is a good working basis for such work, thus allowing a maximum load of about 1000 lb. per sq. ft. for ordinary alluvial soils. Clean sharp sand is considered to be a good bearing soil, and it may only be necessary to cover it with a concrete mat, which requires a minimum amount of concrete. For soft or alluvial soils piling is almost always required. These may be of wood, although in the last few years much use has been made of concrete piles, both plain and reinforced. Such piles are less apt to decay and their bearing power is higher, due to their greater friction. They may also be made of larger diameters than can be obtained with wood piles and a less number is therefore required to support a given load.

Design of Foundations. — The weight on a foundation includes the machines, fittings, the weight of the foundation itself, and, in the case of the turbines, the weight due to the water thrust unless this is balanced. Separate foundations should be provided for the different units so as to isolate any failure as far as possible. Concrete is always used for the foundations.

Machinery foundations should be solid, but buildings may be supported on columns or arches so as to economize the concrete (*see Concrete*). Where there is danger of high water in the tail race, the outside foundation walls should necessarily be made water-tight so as to prevent water from entering the basement. For such cases a sump is generally provided into which the seepage may collect and from which it can be pumped out.

Basements. — A basement should be provided below the generator room when vertical turbines are to be used. That part of the floor on which the turbine discharge casings rest should be reinforced by heavy I-beams, and provision should be made for supporting the penstocks and draft tubes. There must be provided an intermediate basement floor which is generally made of concrete and should be carried on I-beams supported by the concrete piers which also support the generators and the main floor. With horizontal turbines no basement is needed. Ventilating and cable ducts for the generators,

and tunnels for piping, etc., are, however, often installed below the main floor.

Floors. — No combustible material of any kind should, if possible, be used in the construction of a power house. As the sub-structure of the building is generally built of concrete, it is but natural that the floors should also be of concrete. A dark color is preferable so as to render drops of oil inconspicuous. A tile or mosaic floor is smooth, easy to keep clean and has a very handsome appearance if made to conform with the general interior finish of the station.

Walls. — The walls may be either of reinforced concrete construction or of brick with a steel skeleton frame work. Where future extensions are contemplated a false wall is provided on one end of the building. The interior should be kept as light as possible and it is therefore advisable to apply a smooth surface of cement plaster and whitewash or paint the same. For more important stations the walls may be faced with pressed brick and up from the floor to about ten feet with enameled brick so that they may be readily washed and cleaned. Where the extra expense is warranted, the walls may be entirely lined with enameled brick and a wainscoting of contrasting color, preferably olive-green.

Roof. — The roof of the building should always be supported on the steel trusses, carried on the side walls or on steel columns. The slope should not be excessive, two inches per foot being sufficient with gravel covering. This construction requires less material, and is advantageous when the transmission wires are to enter the station through roof entrance bushings, or where the lightning arrester horns are to be installed on the roof.

The roof covering may simply consist of boards covered with roofing paper, tar and gravel. Reinforced concrete is sometimes used in place of boards so as to make an absolutely fireproof construction. Roofs covered with red tile are often used and present a very pleasing appearance. Corrugated iron roofs are objectionable due to the liability of moisture condensing on the inner surface and dripping into the station. They may also cause the station to be extremely hot in the summer unless an insulating lining is provided below the roof trusses to keep out the heat. With tile or metal roofs it is necessary to provide steeper inclines than with gravel roofs so that the water may run off rapidly. Monitors are sometimes provided so as to give additional ventilating facilities.

Roof trusses with a raised chord, shown in Fig. 1a, are in many instances of great advantage in that they provide an increased headroom without unnecessarily raising the walls of the building. This is of special importance in the high-tension part of the station, where ample headroom must be provided for the busses.

Windows. — Good lighting is imperative, and large windows are therefore essential. They should be symmetrically located with regard to the generating units and their design should be such as to harmonize with the building, arched windows being very generally used. Skylights of glass tile placed in the roof will also add considerably to the lighting. The window sashes should preferably be metallic and the glass reinforced with wire netting so as to prevent shattering when broken. Ribbed or non-transparent glass is also desirable, because it keeps out the intense rays of the sun. In order to provide for ventilation, provision should be made so that the windows can be opened, but precaution should be taken so that rain, snow or dust will not blow in on the machinery or apparatus. This is especially important on the switchboard side where the wiring is exposed and it is therefore better practice not to provide any means for opening the windows on that side. For tropical climates the windows which are liable to be opened should be equipped with mosquito screens.

Doors. — The location of the doors is naturally governed by local conditions. One of the openings should be of sufficient size to admit a railroad car, for which

tracks should be provided. Very often these doors are of the rolling type, this design being most economical as regards space.

Traveling Crane. — Provision should always be made for supporting the track for a traveling crane, which should span the generator room and run the full length of the station. The track is generally supported on pilasters in the outside wall and on the steel columns separating the generator and switch rooms. There should be ample headroom allowed so that the various machine parts can be readily removed when repairs are to be made. This is especially important with vertical units where the water-wheel rotor is mounted on the same shaft as the generator field, and in which case it should be possible to lift out the whole revolving element by simply removing the top bracket and bearing of the generator.

Miscellaneous Rooms. — Repair rooms, store rooms, offices, toilets, etc., should be provided. Ample stairway provision is essential so as to permit a ready access to important points, such as between the generator room and the switchboard gallery.

LOCATION OF APPARATUS. — The arrangement of the apparatus should be very carefully considered from the standpoint of simplicity and reliability of operation. The purpose of the station being to give reliable service consideration must also be given to the causes of disturbances and means for minimizing their effects. In anticipating these abnormal or so-called emergency conditions, the failure of every piece of apparatus must be considered as a possibility, and a definite plan worked out for limiting the magnitude and area of such disturbances.

Location of Generators. — The turbo-generator units are located on the main floor and are almost always arranged in a line along the long axis of the station. They should be spaced far enough apart so that ample space for passage is provided between them. Horizontal sets may be installed either at right angles or parallel to the long axis, the latter method being necessary for high heads where impulse wheels are used. The arrangement of the rest of the equipment, such as the transformers, may also be a determining factor in regard to which direction the sets should be installed. If one transformer bank, consisting of single-phase units, is to be installed for each generator, the space occupied by them may be of such a length that it would be more economical to install the turbo-generator sets parallel to the long axis, thus reducing the width of the building.

Location of Turbines. — (*See Water Wheels.*) With horizontal sets the turbines may be located together with the generators in the generator room or in separate wheel chambers built in the dam. The latter practice is only used for very low head developments, where one of the power-house walls forms part of the dam structure. With vertical units the turbines are always located in a basement, the thrust bearing being supported on an intermediate floor below the main floor, unless suspension bearings are used, these being mounted on top of the upper generator-bearing bracket.

Location of Exciters. — The exciters are as a rule installed on the same floor as the main generators and in the center of the station. The advantage of such an arrangement is that the exciters will be located close to the operating switchboard and the amount of copper required for the exciter leads is thus a minimum. The system may readily be sectionalized, one exciter serving the generators located in one-half of the station, and the other the generators on the opposite side. This does not, of course, refer to direct-connected exciters.

Location of Transformers. — Due to their weight, the step-up transformers should preferably be located on the main floor. They are generally installed

in isolated compartments in the rear bay, separated from the generating room by fireproof steel curtains. These compartments should be sufficiently large to allow a good ventilation. A car track is provided on the generator-room floor in front of the transformer compartments whose floors are raised so that the transformers can be run out on the car and moved to some convenient place in the station where repairs can readily be made. For large units it may be necessary to provide a hole in the floor above the repair room so as to enable the transformer core to be lifted out of the tank, or a pit may be provided into which the transformer may be lowered so that sufficient headroom is obtained for lifting out the core. Sometimes the repair room is so situated that the main crane cannot be utilized for dismantling the units. In such a case a chainfall supported from a heavy I-beam in the floor above may be provided.

Transformers Installed Out-of-Doors. — Considerable activity has recently taken place in installing transformers and associated high-tension apparatus outdoors. With the exception of the bushings the transformers for such installations differ comparatively little from the indoor type; the only feature out of the ordinary being the necessity of keeping the moisture from entering the transformer cases under the covers and leads. To prevent this the joints have been made with waterproof gaskets and breathing chambers have been provided.

Special precautions must naturally be taken to protect transformers of the outdoor type both from the extreme heat and from the cold in the winter. The former can readily be obtained by providing sunshades, and in certain instances very good results have been obtained by simply painting the cases white. It is more difficult, however, to provide for the cold winter temperatures, especially with water-cooled transformers. With the transformers in service there seems to be no danger of freezing and if such should be the case some sort of heating grids could readily be provided in the bottom of the tanks. The main difficulty lies in the formation of moisture which takes place when the temperature of the transformer is allowed to fall below that of the surrounding air; this applies also to indoor transformers. Precautions must therefore be taken that this does not happen, and may be accomplished by either reducing the water rate at times of cold weather, or by using the cooling water over and over again. Non-freezing oil may be used in such transformers, but its cost is so high that it is almost prohibitive from a commercial standpoint.

Location of Switchboards and Switchgear. — (*See also Switchboards; Switchgear Equipment for Power Stations.*) The different pieces of apparatus comprising the switching equipment are distributed on the various floors in the switch-section of the station, each story being partitioned to suit the various purposes. The operating room with the control switchboard is generally located on the second floor and in such a position that the operator may have an unobstructed view of the station, and be able to readily communicate with the turbine operators. A balcony, somewhat overhanging the generator room in front of the switchboard, is often provided or the operating room is built with a curved front wall extending out over the generator room.

Location of Oil Switches. — The low-tension oil switches are generally of the enclosed type and, together with the low-tension bus-bars (*see Bus-Bars and Bus-bar Structures*), are located generally in compartments on the main floor back of the transformer compartments. The switches themselves should preferably be set opposite the generator and transformer bank which they control, so as to call for as short a connection as possible and in order that these connections may be of equal length. The high-tension oil switches and bus-bars, and also as a rule the lightning arrester tanks, are installed on the floor above.

Disconnecting Switches. — It is customary to install disconnecting switches on both sides of an oil switch so that they may be entirely disconnected from the circuit when repairs are to be made on them, when the oil tanks are to be refilled, etc. Disconnecting switches may also be used in a number of cases for changing connections, when this is not to be made under load. Such switches should be provided with locking devices, as experience has shown that the magnetic fields caused by short-circuits may cause disconnecting switches to open, which in turn may cause serious disturbances by the arcs set up.

Spacing of Bus-Bars. — (*See also Bus-Bars and Bus-bar Structures.*) The following spacings may be used in laying out bus-bar structures:

Voltage of circuit	Distance in inches		
	Between centers of conductors of opposite polarity	Min. between live parts of opposite polarity	Min. between live parts and ground
3,300	6	2½	2
7,500	9	4	3
15,000	9	5	3½
22,000	12	7¼	6
35,000	18	12	10
45,000	24	16	14
70,000	36	24	21
90,000	48	32	27
110,000	60	38	33

Location of Lightning Arresters. — (*See also Lightning Protectors; Switchgear Equipment for Power Stations.*) The aluminum arrester is now generally used in all high-voltage stations. Both the arrester tanks and the associated horn gaps may be located within the building, or the horn gaps may be placed outside and the tanks inside, or both may be placed outside, provided there is no danger of the electrolyte freezing. Standard equipments of 27,000 volts and below are usually designed as complete units to be installed inside the station, whereas for those above 27,000 volts the horn gaps should preferably be installed outside the station and the tanks inside. Exception to this rule can be made where there is sufficient space in the station over the gaps.

The arrester tanks should naturally be located close to the line entrances. The horn gaps, when installed out-of-doors, may be placed on the roof of the building if roof-entrance bushings are used, or on a separate structure at the side of the building if wall-entrance bushings are used. The location of the arresters should also be such that the path for the discharge from the line conductors to the arresters and ground will be as straight as possible.

Clearance over Horn Gaps. — Wherever horn gaps are mounted inside the building sufficient clearance should be allowed over them. There is no appreciable arc at the gaps, but in abnormal cases where the film has been allowed to get out of order, the arc may be of considerable size. Where there are no busses or inflammable apparatus, the following are the minimum clearances from the tops of horns to be allowed:

Volts	Clearance, feet
Up to 16,100	3
16,101 to 37,900	4
37,901 to 70,000	6

These clearances should be materially increased when there are wires, cables, busses or any inflammable material over the horn gaps.

Above 70,000 volts, the horn gaps should never be placed indoors.

Effect of Climatic Conditions on Arresters Installed Out-of-Doors. — The objection to installing arrester tanks out-of-doors comes from the increased liability of freezing the electrolyte in cold weather and the abnormal film dissolution when exposed to the sun on hot days. The electrolyte may not be injured by freezing, but when frozen the internal resistance of the arrester is considerably increased and hence its discharge rate is materially lowered. Where warm climatic conditions prevail, the arrester should be in as cool a place as possible and protected from the direct rays of the sun. A high initial temperature will reduce the available heat-storage capacity of an arrester and its ability to care for long continuous discharges. A high operating temperature also increases the rate of dissolution of the films which would necessitate more frequent charging. In some cases it may be found advisable to charge two or more times a day. When operating under conditions of high temperature, any failure to periodically charge the arrester increases the liability of damage from a heavy charging current.

WATER WHEELS AND GOVERNORS. — The available types of water wheels and governors and the conditions under which they should be used are treated in detail in the articles on *Water Wheels and Their Settings*, and *Water Wheels, Speed Regulation of.*

GENERATORS AND THEIR CONNECTIONS. — (*See also Generators, Alternating-Current.*) Water-wheel-driven alternators are always of the revolving field type. Machines of this type are generally direct-connected to the water wheel. The horizontal construction has been used mostly, but recently the vertical construction has been used to a great extent, especially in low head developments. The choice is usually determined by hydraulic conditions; see *Water Wheels and Their Settings.*

Use of Power-limiting Reactances. — See article on *Reactance Coils.*

Operation of Generators in Parallel, Synchronizing, etc. — (*See Generators, Alternating-Current; Synchronizers.*)

EXCITATION AND EXCITER SYSTEMS. — It is a good practice to have the combined normal capacity of all the exciters correspond to the excitation required for all the generators, when these are operating at their maximum overload, and at the actual operating power factor. As the exciters are generally designed for a 25 per cent two-hour overload rating, a safe margin in capacity will thus be left for operating auxiliary station apparatus, such as pilot lights, switch and circuit-breaker solenoids, motors, etc. A spare unit to be kept in reserve in case of the break-down of any exciter should generally be provided. This is especially desirable where an uninterrupted service must be secured at any cost and where the exciter units are few in number, as in such a case the shutdown of one exciter would seriously cripple the system.

Amount of Excitation. — The curves in Fig. 2 give approximately the average excitation required for water-wheel-driven alternators of high and slow speeds. It is seen that, as compared to the rating of the generator, the exciter capacity ranges from 0.75 per cent for large high-speed machines to 3 per cent for small slow-speed machines.

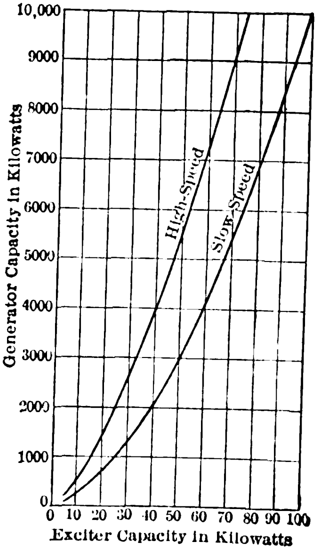

Fig. 2. Excitation Required for Water wheel-driven Alternators

"Time Element" of Exciters. — The "time element" of the exciters should be such that the insertion into its field circuit of an external resistance equal to about three times the resistance of its field circuit, will cause the voltage to drop from 125 to 25 volts in from 4 to 6 seconds. This is particularly important when automatic voltage regulators are used (*see below*), for the exciter voltage must respond quickly to the short-circuiting of the field rheostat by the regulator.

Exciter Voltage. — For large installations a 250-volt system of excitation will generally be found more economical than a 125-volt system. This higher voltage will permit the use of smaller exciter and field switches, leads of reduced size may be used between the exciters and the generator field, and the cross-section of the exciter bus-bars will be reduced. A considerable saving can also generally be accomplished in the exciter itself.

Methods of Driving Exciters. — Although the exciters can be either belt-driven or direct-connected to the machines driving them, the latter practice is almost exclusively used except in the very smallest plants. The direct connection may be either to the main generators, to separate water wheels, or to motors. Sometimes (although rarely) an exciter is connected to both a motor and a turbine, the latter running idle when the motor is carrying the load, and the motor running idle when the turbine is doing the work.

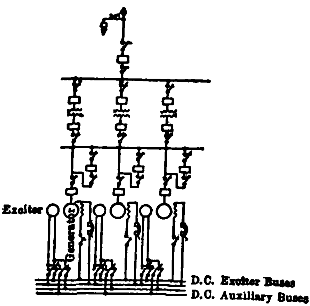

Fig. 3. Exciters Direct Connected to Main Units

Exciters Direct-connected to Main Units. — The practice of installing for each main generator an exciter direct-connected to this unit, Fig. 3, has been used to a considerable extent, especially for small and moderate-size plants. For larger installations this method is, however, now giving place to other systems. When there are not more than three main units in the station, each of the direct-connected exciters should preferably have a capacity sufficient to excite two of the units, so as to provide for reserve in case of damage to one of the exciters. With more than

three units, there seems to be no reason why extra capacity should be provided, and any reserve capacity is then best provided by installing a motor-generator set.

Three-exciter System. — The system which is the most widely used and which offers the greatest reliability, is that in which the excitation is obtained from a common source, consisting of as few exciters as possible, Fig. 4. Three units are then generally provided, of which two are all that are needed for supplying the required excitation, the third unit being held in reserve. Sometimes the two exciters normally in service are driven by water wheels, the reserve unit being motor-driven. From the point of view of economy, however, it is evident that two motor-driven units with a water-wheel driven set as the spare will cost less. In the latter case, however, the exciter trouble with the exciter driven by the prime mover would prevent starting up the system, unless a storage battery were provided, which, however, is usually the case in large stations.

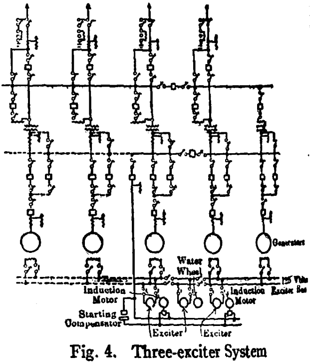

Fig. 4. Three-exciter System

Individual Motor-driven Exciters. — In some of the latest hydro-electric developments an entirely new and quite novel system of excitation is being used, one small motor-driven exciter set being installed for each generator unit, Fig. 5. The exciter has a capacity corresponding to that required by its generator, and its terminals are connected directly to the generator field. The motors of the various exciter sets are fed from one or two low-voltage generators, driven by independent turbines. In addition, means are provided so that if necessary the motors may be connected to the main bus through transformers, two separate sources being thus provided for driving them. This arrangement avoids the objection to motor-driven exciters on the ground that they are liable to fall out of step when a short-circuit occurs on the system.

Electrical Connections of Exciters. — The general practice is to provide one or two sets of common bus-bars to which all the exciters are connected in parallel and from which the fields of the different generators are excited, a rheostat being inserted in each field circuit.

Exciters Direct-connected to Main Units. — The diagram shown in Fig. 3 represents a system where every generator is provided with a direct-connected exciter. There are two sets of bus-bars, one for excitation and the other for auxiliary service. Switches are provided so that the exciters can be connected to either set as desired. The advantage of this system is that one exciter can be connected to the auxiliary bus while the others are operating on the exciter bus. Any fluctuation in the exciter voltage caused by an automatic regulator, for example, will therefore not be felt on the auxiliary or lighting bus, the pressure of which can be kept constant.

Three-exciter System. — The arrangement shown in Fig. 4 is often used for the exciter system when a combination of motor-and water-driven exciters is used. Only one set of exciter bus-bars are shown, although frequently an auxiliary set is also provided as in the previous case. There are e exciters, two of which are driven by induction motors fed from the main

bus-bars, and the third unit, which is held in reserve, is driven by an independent water wheel. Only one starting compensator is needed for the two motors, a common starting and running bus being provided. The system can be sectionalized in two parts if desired, and switches are provided so that the water-wheel-driven exciter can be connected to either side.

Individual Motor-driven Exciters. — Fig. 5 represents the practice when an individual motor-driven exciter is used for each main unit. Each exciter is connected directly to the field of its respective generator. The exciters are not arranged for parallel operation, but are each provided with its own automatic regulator, so that it is possible to compensate for "wattless" or reactive cross-currents between the generators. In the system shown, the induction motors are fed either from two low-voltage alternating-current generators driven by separate water wheels, or alternatively from the main alternating-current bus-bars through stepdown transformers. The auxiliary low-voltage generators are the normal source; the exciter system is thus entirely free from voltage fluctuations or disturbances on the alternating-current system. In another installation of this kind the auxiliary generators are provided for combination drive, one end being connected to a water wheel and the other to an induction motor, which in turn can be connected to the main alternating-current busses through stepdown transformers, unless the voltage will permit of a safe operation of the motors without the transformers. In such an arrangement the exciter sets are not provided with independent group connections to the main bus, as shown in the illustration, as it is considered that a breakdown would most commonly be caused by a clogging up of the turbines, in which case the alternating-current units could be driven by the motors. It would seem, however, that the scheme shown in the diagram is more flexible and reliable.

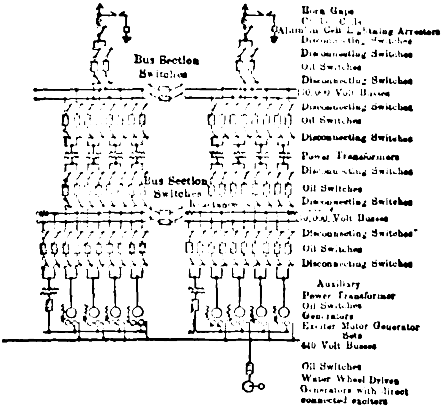

Fig. 5. Individual Motor-driven Exciters

Storage Battery on Exciter Bus. — The use of storage batteries in connection with exciters has been increasing of late. The advantages of such a combination is obvious, as with the failure of the exciters for any reason, the storage battery would automatically keep up the excitation. The storage battery is generally floating on the exciter busses, the pressure of which is kept constant. A separate exciting bus is provided and between this bus and one of the exciter bus-bars a booster is installed which can be operated to either raise or lower the voltage, its field being controlled by an automatic voltage regulator. In case of failure of the exciters the excitation would be furnished by the storage battery, and the booster in connection with the regulator would take care of the voltage regulation.

AUTOMATIC VOLTAGE REGULATION. — Without some form of automatic voltage regulator it is impossible to take care of the heavy swings in the voltage caused by fluctuating power and railway loads. Even in the case

of a purely lighting load it is exceedingly difficult to properly take care of the voltage by hand regulation, especially at peak loads. The present tendency of designing generators for a high internal reactance, in order to reduce destructive short-circuit currents, results furthermore in a rather poor inherent regulation of the generators.

Many different forms of automatic regulators have been devised. Some of them have been designed to operate directly on the alternating-current generator field rheostat by varying the resistance. Such a system has, however, proved to be entirely too sluggish in operation. The most successful type of automatic regulator is the T. A. Regulator in which the regulation is effected entirely in the field circuit of the exciter, by rapidly opening and closing a shunt circuit across the exciter field rheostat. See the article on *Regulators.*

TRANSFORMERS AND THEIR CONNECTIONS. — (*See also article on Transformers.*) The number and size of the transformers depends entirely on the nature of the development and on the conditions to be met. With a moderate voltage development it has in the past been the general practice to install one transformer bank for each generator and having a capacity equal to that of the generator, even if this size was not the most economical. For present modern high-voltage systems where it is undesirable to parallel transmission lines on the high-tension side, or to carry out any high-tension switching, it has become general practice to install the transformers in groups, each having a capacity corresponding to the line, the transformer group and the line thus being considered as a unit. Transmission lines generally have a carrying capacity ranging from 15,000 to 40,000 kw. As the most economical size of high-voltage transformers is from 6000 to 10,000 kw., it is entirely feasible to provide one bank of single-phase transformers for lines up to about 25,000-kw. capacity; but above this capacity it, as a rule, becomes necessary to provide two banks in parallel for each line.

In order to facilitate moving the transformers in or out of their compartments, wheels should be provided in the base or trucks may be installed on which the transformer will rest. The design should also preferably be such that the complete core and coils, with the cover and leads can be lifted from the tank as a unit. Eyebolts are provided for this purpose and also for lifting the entire transformer filled with oil.

Single- versus Three-phase Transformers. — No specific rule can be given regarding the selection of single-phase or three-phase transformers since both designs are equally reliable; local conditions will generally determine which type is preferable. See the article on *Transformers.*

Use of Auto-Transformers. — (*See also Auto-Transformers.*) Auto-transformers are sometimes used in connection with Y-connected generators for obtaining a moderate rise in the voltage. Where such is the case a path must be provided for the flow of the triple-frequency exciting current, which is required for the normal magnetizing of the transformer. With a grounded generator neutral (*see Generators, Alternating-Current, and Grounding of Electric Circuits*), this can be obtained by also grounding the neutral point of the auto-transformers, although it is also highly desirable to connect the two neutrals together as any ground offers more or less of a resistance. If a sure path for the triple-frequency exciting current is not provided, a third harmonic will appear in the no-load e.m.f. from line to neutral and cause an excessive strain in the windings, which under such conditions should in all cases be insulated for a higher voltage than the normal.

Cooling of Transformers. — Transformers for hydroelectric generating stations should obviously be of the water-cooled type. Ordinarily the water

rate to keep a transformer of this type cool is approximately one-half gallon per minute per kw. loss, the temperature of the incoming water being 15° C.

Cooling coils are generally made of extra heavy lap-welded wrought iron pipe with electrically-welded joints. These coils will withstand a hydraulic test of 1000 pounds per square inch. In some cases the quality of water available for cooling purposes may make it necessary to use either brass or copper pipe, in order to avoid corrosion which would prohibit the use of iron pipe.

Drying of Transformers. — It was formerly customary in shipping transformers to pack the cores separately from the tanks. Where the railroad clearances will permit, transformers are now shipped assembled with the oil in the tanks, the cores being securely braced in the tank. In this manner the transformers should arrive at the destination with the insulation and oil practically dry and free from moisture. Where transformers are shipped without the oil in the tanks it is almost invariably necessary to dry them out first. This may be accomplished in several ways, as explained in the article on Transformers (q.v.).

Transformer Oil. — The oil, whether shipped in the transformer case or separately, should always be tested before it is used in service (*see the article on Oil, Transformer*), and should be dried if it punctures at too low a voltage. Oil for transformers of 40,000 volts and over should be dried before using, if it punctures below 35,000 volts. For transformers having voltages less than 40,000 volts, the oil must be dried if it punctures below 25,000 volts. Where oil is dried it may easily be brought to a puncture of 40,000 volts. If a sample contains sediment, it will puncture at a lower voltage than it would without the sediment.

Transformer Connections. — With the three-phase system the transformers are usually connected in delta or Y, and when the Y-connection is used the neutral may be grounded or not. It is a much-disputed question which connection is to be preferred. In general it may, however, be said that in transmission systems where continuity of service is the most important factor, delta-connected transformers (both primary and secondary) are preferable on account of the increased reliability which such a system affords. See articles on *Transformers* and *Grounding of Electric Circuits*.

For high-voltage systems it is, however, now being generally conceded that the Y-connected system with the neutral grounded is preferable, if not almost essential. The fact that any ground will then constitute a short-circuit followed by a shut-down, is outweighed by the limitations of the rise in voltage caused by such grounds. Modern transmission line apparatus must furthermore be designed to withstand the mechanical strains imposed by short-circuits. With a ground on a delta-connected system it is evident that the neutral is shifted from the center of the delta to one corner, and the charging current, which is a function of the voltage from wire to neutral, is therefore increased in proportion, or about 73 per cent. This increased charging current will in turn cause a corresponding increase in the voltage rise which may take place when the lines are cut in circuit at no load. Actual experience has shown that this voltage may reach prohibitive values which, of course, would not be the case with the Y-connected, grounded system.

SWITCHING EQUIPMENT AND ELECTRICAL CONNECTIONS. — Continuity of service is the most essential part in the protection and operation of a large high-tension power system. A maximum degree of reliability can naturally be obtained by providing reserve and duplicate apparatus, the maximum reliability thus being governed by the permissible investment in the apparatus, by the price paid for the power and by the competitive situation.

In small and medium-size power plants the switching equipment may be of the hand-operated type, mounted directly on the back of the switchboard panels

or on a separate framework and operated by hand by means of levers located on the front of the panels. For large modern power houses the switches should, however, always be of the remote control type. The control board is then located so that the operator may obtain the best view of the station, while the switches and bus-bar structure are installed with regard to convenience of wiring and safety.

The switch and bus-bar structure for large stations may be either of the enclosed or open type, but is mostly a combination of the two.

The entire subject of switchgear equipment is discussed in detail in the separate article on *Switchgear Equipment for Power Stations*, which see.

Sectionalized Low-tension Bus. — (*See also Bus-Bars and Bus-bar Structures.*) The generators should preferably be paralleled on a low-tension bus, and where the total capacity is large the bus should be sectionalized, it being the general practice to limit the normal capacity of each section to from 30,000 to 50,000 kv-a. The sections may be connected by means of automatic switches provided with instantaneous overload relays which in case of trouble in one section will immediately disconnect the same from the other sections and thus limit the power which the oil switches will have to rupture to the capacity of one section. In some of the recent systems reactance coils have been inserted between the bus sections, and the sectionalizing switches have been made non-automatic. The reactances generally have such a value that on short-circuit in one section they will limit the total power rush from the two adjacent sections to the short-circuit current of one generator.

In sectionalizing the bus it is desirable to make such provision that sufficient generator capacity to supply the charging current of one transmission line can be entirely separated from the rest and used for testing out the lines. A ring bus will generally insure sufficient flexibility to accomplish this, although for large systems a double bus may be desirable if the extra expense is warranted.

Generator Switches. — The generator switches are usually made non-automatic, as it is of the utmost importance to keep the generators in service, and the possibility of trouble between the generators and busses is rather remote. If, however, automatic protection is desired for the generator switches, the relays (*see Relays*) should be of the definite-time limit type, set very high so as to trip the switches as a last resort, after the automatic switches more remote from the generators have failed to isolate the trouble. Sometimes the generator switches are provided with reverse energy relays which will cut out a damaged generator on the reversal of the power, or the relay may be connected to an alarm bell which will indicate such a condition.

Transmission-line Connections. — (*See also Transmission Lines.*) Double transmission lines are nearly always provided to important load centers, and it is then also desirable to so proportion the line conductors that in case of trouble, one line alone or together with a section of the other can take care of the greater part of the load without causing too poor a regulation. The lines should normally be operated electrically apart from each other, the paralleling and switching being done on the low-tension side of the transformers in both generating and substations. This is in order to avoid high-frequency surges caused by high-tension switching, and also to prevent communication between the lines of disturbances started from any other cause, such as lightning, arcing grounds, etc.

All feeder switches should be provided with instantaneous overload relays (*see Relays*) as it is of course necessary to immediately cut out any damaged feeder and confine the interruption to the smallest area possible.

The diagram of Fig. 6 represents a system in which each transformer bank forms a unit with one line. The high-tension line switches L_9 and L_8 are non-automatic and are only intended for sectionalizing purposes, as are the tie

switches S which should be open under normal operation. The transformer switches T_g and T_s in the generating and substations respectively should be of the automatic type, the former being equipped with inverse time-limit relays and the latter with reverse-energy relays. A short-circuit in one of the lines will cause switches T_g and T_s, belonging to this circuit, to open, thus disconnecting

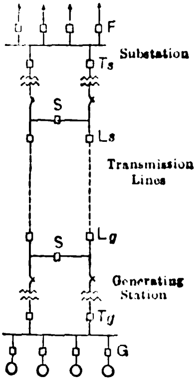

Fig. 6.

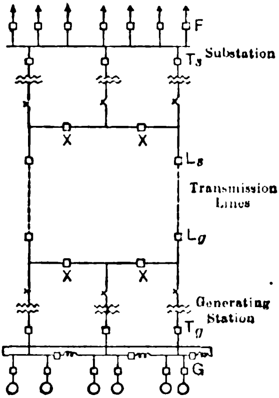

Fig. 7.

it from the system. The load is then shifted over to the other line which remains in service, and may overload the transformers of this line. This will not cause any danger, as the transformers can readily carry up to 100 per cent overload for a few minutes until the operator has had time to open switches L_g and L_s, disconnecting the faulty line from its transformers, and close switches T_g, T_s and S, connecting the transformers in parallel again.

In the system represented in Fig. 7 three transformer banks are provided with only two lines. The connections are similar to those in Fig. 6 except that the high-tension side of the third transformer bank is connected to the outgoing lines through the oil switches X. These should be of the automatic type provided with instantaneous overload relays. A short-circuit in any of the lines will thus cause these switches to trip out with a comparatively small disturbance, after which switches T_g and T_s will open, disconnecting the damaged line.

LIGHTNING ARRESTERS, EQUIPMENT AND CONNECTIONS.— Aluminum cell electrolytic lightning arresters are now used almost entirely for lightning protection of high-voltage transmission systems, see article on *Lightning Protectors*, for description of the various available types of arresters. The arrester, however, is not a universal protector against all kinds of interruptions. For example, while it meets the usual, and most of the unusual, needs in protection against disruptive potentials from lightning, an arrester located in the station cannot, and is not expected to, protect an insulator out on the line from a lightning flash. Neither is it designed to protect against surges of comparatively low voltage.

Arresters for Grounded and Ungrounded Circuits.—It is important to avoid the mistake of choosing an arrester for a thoroughly-grounded neutral when the neutral is only partly grounded; that is to say, grounded through an appreciable resistance. In an arrester for a grounded neutral circuit, each stack of cones normally receives the neutral potential when the arrester discharges;

but if a phase becomes accidentally grounded the line voltage is thrown across each of the other stacks of cones until the circuit breaker opens the circuit. Line voltage is 173 per cent of the neutral or normal operating voltage of the cells and therefore about 150 per cent of the permanent critical voltage of each cell. This means that when a grounded phase occurs this 50 per cent excess dynamic potential is short-circuited through the cells until the circuit breaker opens. The amount of energy to be dissipated in the arrester depends upon the kilowatt capacity of the generator, the internal resistance of the cells and the time required to operate the circuit breakers. It is evident that the greater the amount of resistance in the neutral, the longer will be the time required for the circuit breakers to operate. Therefore, in cases when the earthing resistance in the neutral is great enough to prevent the automatic circuit breakers from opening practically instantaneously, an arrester for a non-grounded neutral system should be installed. It is difficult to determine these factors of ground resistance and time elements in the operation of switches and therefore no mistake can be made by adopting the 4-tank arrester even on grounded Y circuits.

Wiring Connections for Lightning Arresters. — The wiring connections of lightning arresters are an important consideration. The discharge circuit should contain minimum impedance and hence must furnish the shortest and most direct path from line to ground. The most severe disturbances which an arrester is called upon to handle are of high frequencies and it is, therefore, imperative to eliminate all unnecessary inductance. The features favorable for low inductance are short length of conductor, large radius bends and large surface of conductor. For wiring high-voltage arresters the use of copper tubing is therefore recommended. Such copper tubing has the advantage over either copper-strip or solid conductors in that it is easily supported, requires fewer insulators and is, therefore, cheaper to install. From arrester to ground it is sometimes more convenient to use copper strip than tubing. Copper strip, say 1.5 in. by 0.03 in., can be fastened to the station wall leading directly down to ground.

Ground Connections for Lightning Arresters. — In all lightning-arrester installations it is of the utmost importance to make proper ground connections since many lightning-arrester troubles can be traced to bad grounds; see article on *Ground Connections*. As noted in the article referred to, a very satisfactory method of making a ground is to drive a number of one-inch iron pipes six or eight feet into the earth about the station, connecting all these pipes together by means of a copper wire, or preferably, by a thin copper strip. A quantity of salt should be placed around each pipe under the surface of the earth and the ground thoroughly moistened with water. It is advisable to connect these earth pipes to the iron framework of the station, and also to any water mains, metal flumes or trolley rails that are available. For the usual size station the following recommendation is made: place three earth pipes equally spaced near each outside wall, making twelve altogether, and place three extra pipes spaced about six feet apart at a point nearest the arrester. When plates are placed in streams of running water, they should be buried in the mud along the bank in preference to laying them in the stream. Streams with rocky bottoms are to be avoided.

From time to time the resistance of these ground connections should be measured to determine their condition; see article on *Ground Connections*. The resistance of a single pipe ground in good condition has an average value of about 15 ohms. A simple and satisfactory method of keeping account of the condition of the earth connections is to divide the earth pipes into two groups and connect each group to the 110-volt lighting circuit with an ammeter in series. If there is a flow of about 20 amperes the conditions are satisfactory, provided the earth pipes are properly distributed around the station.

Charging of Aluminum Cell Arresters. — This is accomplished as follows:

First. — Operate the charging mechanism so as to bring the charging contact securely against the horn gap and charge for five seconds. The contacts should be so adjusted as to eliminate arcing when gaps are closed. The closing and opening of the charging contact should be performed quickly so as to avoid unnecessary arcing. Note should be made of the size and color of the arc which forms when the contact is broken at the close of the charging period.

Second. — With the horn gaps in normal position reverse the transfer device, thereby interchanging the connections to the ground stack of cones and one of the line stacks.

Third. — The first operation should again be repeated, thus charging the fourth stack of cones, which was originally the ground stack.

When an arrester is first installed and also when one has been off the circuit for several days the initial charging current is sometimes above normal. It is recommended that the cells be charged six or eight times the first day and three times a day during the remainder of the first week. This charging should be performed as just described. After the first week the regular daily charging will usually be found sufficient. It is important that the charging should be done at a time of the day when the line voltage is at a maximum value. The charging period should always be five seconds.

In cases where aluminum lightning arresters are installed in places where the temperature is excessive, it is sometimes advisable to charge the arresters twice a day. This condition will be indicated by an increase in the charging arc and charging current from day to day.

Charging-current Indicator. — The charging-current indicator is a device for measuring the current taken by an alternating-current aluminum arrester during charging; it also indicates the condition of the arrester cells. An arrester in good condition has a charging current of approximately 0.25 ampere on 25-cycle circuits, 0.30 ampere on 40-cycle, and 0.40 ampere on 60-cycle circuits. Should these values be doubled, the arrester must be charged more frequently and the current carefully measured until it comes down to normal. It is only when this additional charging fails to reduce the charging current that an inspection of the cells is necessary.

The essential parts of the charging-current indicator are an ammeter mounted on a specially-constructed switch stick and a set of jacks. These jacks are so connected in the arrester circuit that when the ammeter switch stick is inserted in them and the horn gaps short-circuited, the charging current flows through the meter.

Discharge Recorders. — A knowledge of all discharges is of immense value to operating engineers in studying conditions of abnormal voltage on transmission and cable systems. For this purpose a discharge recorder has been developed, which will register the time and nature of discharges through an arrester. This recorder consists of four spark gaps so arranged that the discharges between lines or between lines and ground pass through the gaps. The spark gaps are assembled with a clock-operated drum in such a manner that a continuous record is obtained, showing all discharges by means of punctures in a moving roll of paper. This paper passes through the gaps at a rate of about one inch per hour which gives an accurate record of the time and duration of each discharge.

Besides being valuable in recording discharges due to abnormal voltages on a system, the discharge recorder is of value in indicating and recording the daily charging of the lightning arresters. With such a recorder it can be told whether the arresters are or are not being properly charged by the station operator; and besides, the puncture gives some indication of the condition of the arrester.

Choke Coils. — Choke coils should always be installed in the power circuit between the lightning arrester and the apparatus to be protected. All choke coils should be very rigidly supported as they are subject to severe mechanical strains when short-circuits occur on the system. See article on *Lightning Protectors*.

Use of Arcing Ground Suppressor. — This apparatus is described in the article on *Ground Detectors and Arcing Ground Suppressors*. One arcing ground suppressor is sufficient for controlling the entire system. When several power stations feed into one transmission system special attention should be given to the best location of the suppressor.

STATION WIRING. — The design and construction of the cabling and wiring system of the station is of equal importance to the rest of the equipment. Experience has shown that in a great number of instances the shut-down of power plants was caused by defective installation of the station wiring. Every cable and wire should have a definite place provided for it in advance, just as much as any other piece of machinery, and wires carrying currents of different voltages should, as far as possible, be kept apart from each other.

Braided vs. Lead-covered Cables. — For generator and low-tension transformer leads, braided or lead-covered cables are used, and may be run in ducts or exposed upon racks in cable-ways or tunnels. Lead-covered cables are necessarily more expensive than braided cables, and their use seems to be justified only in places where protection against water and moisture is required. As the lead sheath is necessarily grounded along its entire length, such cables are more apt to be punctured. As a protection against fire the lead covering is obviously useless and due to its softness it is not very efficient in withstanding mechanical injuries.

Installation of Cables. — (*See also article on Wires and Cables, Insulated.*) In many plants the cables are installed in ducts in the floor, and in such instances the cables should preferably be lead-covered to protect the cables from abrasion when drawn into the ducts. There are, however, several other objections to installing the cables in ducts. With high-voltage single-conductor lead-covered cables, static discharges may take place through the insulation to the lead, which rapidly injures the insulation and a breakdown soon follows. If the cable is not lead-covered a static discharge may take place to the tile duct, this also having a tendency to break down the insulation in time. In multiple-conductor cables this action does not occur, the static activity probably being neutralized. With cables carrying large low-voltage currents the lack of ventilation in the ducts may furthermore cause overheating of the cables.

In a large number of stations the open method of cabling is used. Tunnels or cable subways are then provided in which braided cables are supported in free air upon insulators mounted on racks, these insulators themselves having sufficient insulation to withstand the operating voltage of the cable. Where single-conductor, high-voltage cables are used, they should be separated far enough to prevent static discharges between the cables, and also in order to obtain the best possible ventilation and to minimize trouble in case of short-circuits. The cables should also be rendered fireproof by wrapping them with asbestos tape.

Size of Cables. — Due to the skin effect it is generally considered good practice to limit the size of single-conductor cables to 1,250,000 cir. mils for 25-cycle service and 700,000 cir. mils for 60 cycles. Where the value of current is such that it can be carried safely by one three-conductor cable, this is preferable in every respect to three single-conductor cables. If more than one cable has to be used to carry the current the difficulty of making connections will generally offset the advantages of three-conductor cables, and under such conditions it is usually found to be more advantageous to use single-conductor cables.

High-tension Wiring. — Bare wire or tubing supported on post insulators or hung from suspension insulators is generally used for all high-tension wiring. High-tension station wiring is described in detail in the article on *Switch-gear Equipment for Power Stations.*

OPERATION AND OPERATING RECORDS. — The selection and maintenance of an efficient and reliable operating force is essential. Most modern systems of any size have a method of operation which corresponds to that of a train dispatcher on steam railroads, and where many different plants are attached to the same network, this becomes practically necessary. The directions for operating the different stations and apparatus come from a central source, where the dispatcher has before him a diagram of the whole system and information regarding the capacities of the generators in use and the magnitudes of the loads at the different places of distribution.

Organization of Operating Force. — The organization of the operating force of a hydroelectric generating station is necessarily less complicated than in a steam station. It is determined largely by the location and the arrangement, and there are so many different conditions in such systems that it is impossible to recommend any exact form of organization, as really no two can be quite alike If the station is not too large, it is desired to have the hydraulic superintendent report to the station superintendent, but if the development is of such a magnitude as to require the entire time of a superintendent for each of the departments under consideration, a position is warranted for a man to whom both electric and hydraulic superintendents will report, thus still bringing the responsibility of operation of the two departments under one head.

As a general rule, for the same capacity installed, a plant having horizontal units can get along with a smaller force than one using vertical units. It is a general practice to maintain one man at all times on each of the different levels or floors of the power house, such as the switchboard gallery, the main floor and the basement, where with vertical units the turbines proper as well as the oil pumps and other auxiliaries are located. The man in the basement could in all probability be dispensed with in plants using horizontal units. In addition to these men a chief operator should be provided for each shift, whose duties should carry him to all parts of the building. For a very large station the above force may be entirely inadequate, and for small plants the force may be reduced.

Switching Operations. — The switching operations are determined by the general method of operation. It is desirable to eliminate all high-tension switching under load, due to the fact that such switching may set up surges which may be dangerous to the transformers and other apparatus.

When a line is to be cut into service, the high-tension switches in both the main and substations should be closed first, then the low-tension transformer switch in the generating station should be closed, energizing the transformers and the line, after which the low-tension transformer switch in the substation is closed and the load picked up. In case it becomes necessary to open a high-tension switch in a loaded line, the circuit should if possible first be parallel with another before opening the switch. If, on the other hand, transformers are to be paralleled on both high- and low-tension sides, the low-tension switch should be closed first, assuming that the low-tension bus is energized. Similarly, in cutting out the transformer the low-tension switch should be opened last.

Operating Records. — One of the essential things in connection with the operation of hydroelectric generating stations is the keeping of accurate records. Record sheets should contain only the most important readings, as with complicated forms the attendant generally realizes that a large number of the readings are of no importance and for this reason he becomes very lax in his attention to the readings in general and as a consequence the important ones may suffer

The following description applies to an actual record sheet which has been found to give satisfactory results. The sheet is of the size of ordinary letter paper and is ruled for hourly records of "Water," "Main Units," "Cycles," "Power Factor," "Exciters," "Transformers" and "Floodgates." These items are listed vertically and the sheet is divided into 24 vertical columns, one for each hour. At the top are given the "Forebay" readings and "Tail Race" readings, the difference between which gives the "Effective Head." Immediately below are listed the indicated kilowatts and per cent gate opening of each generator in service, following which are given the "Total Indicated Kilowatts" and "Total % Gate." The total kilowatt-hours during each hour, as read from the watthour meters, is plotted as a block-curve extending across the face of the sheet.

This serves as a better record for the actual station output than the indicated kilowatts. It has been found necessary, however, to follow the indicated kilowatts to serve as a check on the efficiency and condition of the units in general, from time to time, as well as to determine what capacity would be required for short interval peaks. The station voltage is also plotted as a block curve across the face of the sheet.

The exciters form an individual group, and for each exciter the voltage, current and per cent gate opening are recorded.

Transformer records are limited to the temperatures. These are taken hourly, at which time the oil elevation is noted but not recorded. If the transformer is not in service the column in which the temperature is listed is left blank; if in service the temperature is taken and recorded.

Under the item, "Floodgates" the total opening of the floodgates in feet is recorded, rather than each one separately. This record is maintained daily, the flow of the river at each of the stations being followed very closely.

At the bottom of the sheet appear the daily readings of the various generator and feeder watthour meters taken at midnight of each 24 hours. The following items are also recorded at the bottom of the sheet: "Total Generated," or the total output of the station for 24 hours; the "Maximum Hour Time," or the maximum kw-hr. of any particular hour during the day; the "Maximum Kw. Time," or the maximum indicated kilowatts at any particular instant; the "Average Load," obtained by dividing the total kilowatt hours generated by 24; the "Load Factor," obtained by dividing the "Average Load" by the "Max. Kw. Time"; the "Average Flow of the River in Cubic Feet per Second," calculated each day and converted into "Available Capacity of River," which is shown in kw-hr.; the "Available Capacity of Power House," shown in kw-hr., and determined by calculating the capacity of the machines under the average head for 24 hours; the "Kw-hr. Lost," or the difference between what was actually generated by the machines and what could have been secured from the river during the same number of hours.

Any important notes of operation are entered on the back of each day's log sheet. These notes, together with certain records for log sheets, are also entered each day in a log book kept on the operator's desk at all times, for reference purposes. Weather conditions and temperatures are recorded four times daily, at midnight, 6 A.M., noon and 6 P.M. A rain gauge is provided on the roof of the station, from which records of precipitation covering each 24 hours are obtained

CAPITAL COSTS AND APPROXIMATE DIMENSIONS. — (*See also section below on Capital and Annual Costs of Some Typical Plants.*) The cost of a large water-power development is generally very great. The estimates of the amount of power available are always subject to error and many times are greatly exaggerated. On account of unforeseen obstacles in dam construction, it is always possible that the actual cost will exceed the engineering estimates, and

such elements of uncertainty must always be taken into consideration. Moreover, as the large water powers of the country are generally more or less remote from power markets, there is necessitated the construction of expensive high-tension transformer equipments and transmission lines to transmit the power to the point of consumption. This additional cost is often greater than that of the dam and power-house construction. The provision of large storage reservoirs is often necessary in order to meet the irregularities in the flow of streams, while, on the other hand, the most economical utilization of water power often requires the erection of auxiliary steam plants.

The main items entering into the cost of construction of a hydroelectric power plant are: (1) Dam, (2) Water Conductors, (3) Reservoirs, (4) Power house, (5) Land and Water Rights, (6) Transmission Lines. In addition to the cost of the above physical equipment certain overhead and organization expenses must also be included. These may be classified as follows: (7) Engineering and Contingencies, (8) Administration, (9) Organization, (10) Taxes and Insurance, (11) Interest during Construction, (12) Working Capital.

Range of Total Capital Cost per Horse-Power. — Extensive investigations by the Bureau of Corporations show that the cost of a hydroelectric power development, including the construction of dams, the erection of transmission lines and other equipment, ranged from $50 to $375 per horse-power delivered at the substations. These figures represent extremes, as the usual cost will fall between $100, and $200 per horse-power, depending upon physical conditions and the length of the transmission lines.

Approximate Dimensions and Construction Costs. — It is obvious that there are certain minimum costs that can readily be approximated when the rough dimensions of dams, pipe lines, etc., are available, so that rough minimum figures can readily be made. Such rough figures cannot be expected to take into account expensive contingencies that must be anticipated.

Dams. — Given the length and height of dam, the dam structure itself will have a cost of material that can be estimated roughly, and the ordinary cost of placing such material — including tools and forms — can be added, but the extraordinary labor costs due to the construction of expensive foundations and coffer dams and the cost of placing dam material under difficult conditions can, of course, only be estimated by experienced engineers thoroughly familiar with the local conditions.

Expressing the height of dam in yards as h, the approximate sections of dams and approximate costs of dams not used as weirs or spillways are:

SECTIONS AND COST OF DAMS NOT USED AS SPILLWAYS

Type of dam	Batter or slope: Common upstream batter horizontal to vert.	Batter or slope: Common down stream batter	Approx. section in sq. yards	Approx. cost per cubic yard	Approx. cost per lineal yard
1. Earth	2 to 1	3 to 1	$2.5h^2$	$0.50	$1.25 h^2
2. Crib	1½ to 1½	1½ to 1	$1.5h^2$	1.50	2.25 h^2
3. Rock fill	2 to 1	2 to 1	$2.0h^2$	2.00	4.00 h^2
4. Masonry, straight	Vertical	8/10 to 1	$\frac{4}{10}h^2$	12.00	4.80 h^2
5. Masonry, arched	0.15 to 1	0.30 to 0	$\frac{1}{4}h^2$	15.00	3.50 h^2

It should be borne in mind that maintenance and depreciation on Types 1, 2 and 3 are far heavier than on Types 4 and 5.

These figures are intended to cover all costs under ordinary conditions, for the cost per cubic yard is taken sufficiently high; but they cannot be used for extraordinary conditions, producing costs possibly four or five times greater.

Low crib or masonry dams used practically throughout their length as weirs or spillways may easily cost many times these figures.

No figures are given for intakes, owing to the great variety of conditions to be met.

Flumes. — The use of flumes of wood becomes less and less as hydroelectric work becomes more permanent in character. For estimate purposes it is then suggested that the cost of wood-stave or riveted-steel pipe be used instead of using the presumably lower cost of the flume. It is obvious that repairs and depreciation on flumes are heavy, and hence they would seem to have little use in permanent construction.

Low-pressure Pipes. — (*See also Pipes and Piping.*) In hydroelectric work cast-iron pipes are seldom used. Wooden-stave and riveted-steel pipes have been widely used for low pressures. The approximate costs of these, as compiled by A. L. Adams for Chicago, are given in the following tables.

COST PER FOOT OF WOODEN-STAVE PIPE

Including laying but omitting hauling

Diameter in inches	25-foot head	50-foot head	100-foot head	200-foot head
12	$0.42	$0.49	$0.63	$0.85
18	0.69	0.80	1.02	1.46
24	0.79	0.91	1.14	1.61
30	0.96	1.12	1.44	2.06
36	1.19	1.40	1.82	2.65
42	1.40	1.68	2.23	3.33
48	1.55	1.85	2.46	3.67
54	2.23	2.62	3.43	5.02
60	2.85	3.35	4.37	6.40
66	3.21	3.81	5.00	7.38
72	3.65	4.38	5.83	8.73

COST PER FOOT OF RIVETED-STEEL PIPE

Including laying but omitting hauling

Diam. in in.	No. 14	No. 12	No. 10	No. 8	No. 6	1/4 in.	5/16 in.	3/8 in.
12	$0.32	$0.38	$0.44	....	....	....	....	....
18	....	0.57	0.65	$0.78	$0.98	....	....	....
24	....	....	0.85	1.04	1.28	$1.55	$1.99	....
30	....	....	....	1.27	1.59	1.93	2.46	$3.04
36	....	....	....	1.55	1.93	2.30	2.92	3.58
42	....	....	....	1.61	2.18	2.66	3.37	4.12
48	....	....	....	....	2.48	3.03	3.83	4.66
54	....	....	....	....	2.80	3.41	4.29	5.21
60	....	....	....	....	....	3.79	4.75	5.74
66	....	....	....	....	....	4.35	5.21	6.29
72	....	....	....	....	....	4.52	5.66	6.83

High-pressure Piping. — In high heads the high-pressure piping may be either riveted-steel pipe or lap-welded pipe with bolted flanged joints. It is suggested that a percentage amount be allowed for this, as the cost must vary widely for different conditions.

Tunnels. — Tunnels for hydroelectric work are usually lined. Naturally the cost per cubic yard varies, but for ordinary conditions an average unit cost of $15 per cubic yard will cover all expense including timbering and lining. Allowing a velocity of 10 ft. per second (except in the smallest tunnels), approximate dimensions and costs are as follows:

COST AND DIMENSIONS OF TUNNELS

Carrying capacity in cubic feet per second	Velocity in feet per second	Net sectional area, square feet	Dimensions in feet, width by height	Approximate slope, feet per 1000 feet	Approximate cost per linear foot
100	3.6	28	4 by 7	0.46	$16.00
500	10.0	50	7 by 7¼	2.0	28.00
1,000	10.0	100	10 by 10	1.5	56.00
1,500	10.0	150	12 by 12½	1.1	85.00
2,000	10.0	200	14 by 14¼	0.9	115.00
5,000	10.0	500	20 by 25	0.6	280.00
10,000	10.0	1000	30 by 33	0.3	500.00

Canals. — In ordinary earth a velocity too small to produce erosion and yet sufficiently great to prevent undue deposit of silt and other matter, and sufficiently great to prevent growth of weeds, should be used. For preliminary calculations a velocity of 2 ft. per second will give approximate results. For this velocity the following approximate figures will serve:

DATA ON AND COST OF CANALS IN ORDINARY EARTH

Cubic feet per second	Velocity in feet per second, V	Area of wet section, sq. ft.	Water depth, feet	Approximate slope in feet per mile	Approximate cost per running foot	
					Low	High
50	2	25	2.5	4	$0.375	$0.75
100	2	50	3.5	2	0.75	1.50
200	2	100	5	1½	1.50	3.00
300	2	150	6	1	2.25	4.50
400	2	200	7	¾	3.00	6.00
500	2	250	7	¾	3.75	7.50
1000	2	500	10	½	7.50	15.00
1500	2	750	12	½	11.25	22.50
2000	2	1000	12	⅛	15.00	30.00
3000	2	1500	15	¼	22.50	45.00

In rock the velocity in a canal may be much higher, and if the canal be lined 8 ft. per second may be used. For preliminary calculations this velocity will give approximate results. The following approximate figures are on this basis:

DATA ON AND COST OF CANALS IN ROCKS

Cubic feet per second	Velocity in feet per second, V	Area of wet section, sq. ft.	Water depth, feet	Approximate slope in feet per mile	Approximate cost per running foot	
					Low	High
50	8	6.25	2.5	40	$0.32	$1.28
100	8	12.5	3.5	25	0.63	2.50
200	8	25.0	5.0	16	1.25	5.00
300	8	37.5	6.0	12	1.87	7.50
400	8	50.0	7.0	10	2.50	10.00
500	8	62.5	7.0	9	3.25	13.00
1000	8	125.0	10.0	6	6.00	24.00
1500	8	175.0	12.0	4½	8.75	35.00
2000	8	250.0	12.0	3½	12.50	50.00
3000	8	375.0	15.0	3	18.75	75.00

Values are given for lined canals only, since a higher velocity will be allowable without producing too great a lost head and the cost will probably be more favorable.

It must be considered that these figures are wholly approximate since even for a desired useful section the amount of excavation per lineal foot of canal must vary with the character of the route — whether this be flat, rolling or side-hill.

In this connection the following approximate costs based on the annual reports of the United States Reclamation Service are of interest.

APPROXIMATE COST OF EXCAVATION PER CUBIC YARD

Class	Cost per cubic yard		
	Low	High	Fair value
1. Plowable with 4 horses	$0.098	$1.00	$0.18
1a. Plowable with 6 horses	0.1225	2.00	0.30
2. Indurated material	0.29	2.00	0.60
3. Loose Rock	0.35	3.00	0.75
4. Solid Rock	0.60	5.00	2.00
4a. Excavation below plane of saturation	0.20	3.00	1.80
4b. Solid rock under water	....	....	4.50

Tail Race. — Frequently the cost of the tail race is negligible, but for some developments the cost is an appreciable percentage of the total. No data are given for estimating tail race since those given for canals, pipes and tunnels can be used.

Receivers.— No data are given for approximation of the costs of reservoirs, vent-pipes and surge tanks on account of the varied character of these. Generally speaking the percentage of the total hydro-electric cost to be allowed for these is small and a lump sum or a percentage can be allowed for the same.

Hydroelectric Power Houses. — The following figures upon the approximate space and cost of hydroelectric power houses are wholly approximate, and will give only a rough idea of what may be expected under ordinary conditions, without any allowances for high freight, long haulage, unusually expensive labor charges, etc.; and a wide departure from these figures is to be expected. See also the articles on *Water Wheels, Generators, Transformers, Switchgear, etc.*

DIMENSIONS AND COST OF POWER HOUSES

Item	Low head*			Medium head*			High head*		
	Small†	Medium†	Large†	Small†	Medium†	Large†	Small†	Medium†	Large†
Cu. ft. per kw. for hydraulic apparatus	30	20	10	7	6	5	6	5	4
Cu. ft. per kw. for generators, exciters and switchboards (no transformers)	20	18	15	18	13	10	16	12	10
Cu. ft. per kw. for generators, exciters, transformers and switchboards	30	25	20	28	20	15	26	19	15
Total cu. ft. per kw. for hydroelectric power house (without transformers)	50	38	25	25	19	15	22	17	14
Total cu. ft. per kw. for hydroelectric power house (with transformers)	60	45	30	35	26	20	32	24	19
Approx. cost of power-house building not including foundations (without transformers) per kw:									
Low	$5.00	$3.80	$2.50	$2.50	$1.90	$1.50	$2.20	$1.70	$1.40
High	15.00	11.40	7.50	7.50	5.70	4.50	6.60	5.10	4.20
Approx. cost of power-house building, not including foundations (with transformers) per kw.:									
Low	$6.00	$4.50	$3.00	$3.50	$2.60	$2.00	$3.20	$2.40	$1.90
High	18.00	13.50	9.00	10.50	7.80	6.00	9.60	7.20	5.70
Approx. cost per kw. for hydraulic machinery	15.00	12.00	7.00	12.00	10.00	7.00	10.00	8.00	5.00
Approx. cost per kw. for exciters, generators, switchboards and cables (without transformers)	24.00	15.00	10.00	20.00	12.00	9.50	15.00	9.50	7.50
Approx. cost per kw. for exciters, generators, switchboards, transformers and cables	32.00	22.00	16.00	27.00	17.00	14.00	22.00	14.00	12.00
Approx. cost per kw. for complete power-house and foundations (without transformers)	54.00	38.40	24.50	39.50	27.70	21.00	31.60	22.60	16.70
Approx. cost per kw. for complete power-house and foundations (with transformers)	65.00	47.50	32.00	49.50	34.80	27.00	41.60	29.20	22.70

* Low head = 50 to 200 ft.
Medium head = 200 to 600 ft.
High head = 600 and above.

† Small capacity = 200 to 1000 kw.
Medium capacity = 1000 to 5000 kw.
Large capacity = 5000 kw. and above

It is assumed that the large capacity stations with transformers will be for a line voltage of from 60,000 to 110,000, the medium from 40,000 to 60,000 and the small from 10,000 to 40,000.

Overhead and Organization Expenses. — In addition to the expenditures for the actual construction of the physical plant allowance must be made for the following items to cover overhead and organization expenses.

Engineering and Contingencies. — This item should cover all the cost of engineering, drafting and supervision of construction and of all the items properly chargeable to construction engineering; 5 per cent of the construction cost is a conservative estimate for this item. An equal amount (5%) is also generally allowed to cover contingencies, errors, etc.

Administration. — All items which go to make up the cost of administration for construction, general office expense, etc., come under this head. An additional charge of 5 per cent of the construction cost should also be ample to cover these expenses.

Organization. — This item covers the cost of organization and promotion, such as legal expenses, allowance for brokerage connected with the disposal of the securities, discount on the same, etc., 5 to 10 per cent of the construction cost is usually allowed for this item.

Taxes and Insurance. — Taxes must be provided for until the operation is begun, and usually for some time thereafter, until the income is available for such expenses. Similarly with insurance, including fire, casualty, etc. One per cent of the construction cost is generally allowed to cover these items.

Interest During Construction. — Allowance must also be made for the accrued interest on the idle capital during the construction period.

Working Capital. — A certain amount of money should be provided for working capital, the amount depending on the nature of the business transactions. About 1 per cent of the construction may be allowed to cover this item.

OPERATING COST AND FIXED CHARGES. — (*See also section below on Capital and Annual Costs of Some Typical Plants.*) The cost of hydroelectric power can be considered as made up of two parts: the operating expenses and the fixed charges. The former consist of: (1) Labor; (2) Administration; (3) Oil, Waste, etc.; (4) Maintenance and Repairs; and the fixed charges of: (5) Interest; (6) Depreciation; (7) Taxes and Insurance.

It is difficult to give any general figures as to the cost of generating hydroelectric power, on account of the widely varying cost of such developments. The load factor also has a very great bearing on the cost, much more so than with steam plants because the items making up the power cost are affected very little by a change in the load.

The annual operating cost per kilowatt of generator capacity in general (excluding transmission costs) ranges between the following values:

Labor	\$1.00 to \$2.00
Administration	0.25 to 0.50
Oil, Waste, etc.	0.25 to 0.50
Maintenance & Repairs	1.00 to 2.00
Total operation	\$2.50 to \$5.00

Or, considering the annual cost items as percentage of the capital cost, the following figures are representative for a large station for which the capital cost is \$125 per kilowatt, transmission costs excluded:

	Per cent
Labor	1.00
Administration	0.25
Oil, Waste, etc.	0.25
Maintenance & Repairs	1.00
Taxes & Insurance	1.00
Depreciation	3.50
Total	7.00

The cost of power per kilowatt is thus 7 per cent of \$125 or \$8.75. Power at the bus bars of such a station could then be sold for \$16, say, per kilowatt-year, leaving \$7.25, or approximately 6 per cent for interest on the investment.

Primary and Secondary Power. — Hydroelectric power is generally sold as primary and secondary power. The former must be supplied continuously and is usually marketed by the horse-power-year or kilowatt-year. The secondary power is only available during certain months of the year, and the price obtained for it is necessarily considerably less than for the primary power. To maintain the primary power at the most economical value a steam auxiliary station is usually required; this is frequently obtained by operating the hydroelectric plant in conjunction with an existing steam plant.

CAPITAL AND ANNUAL COSTS OF SOME TYPICAL PLANTS. — Below are given data on some typical plants, ranging in size from 750 to 16,000 horse-power.

Minidoka Project. — The following data illustrate the development and power cost of the Minidoka project of the U. S. Reclamation Service:

COST OF GENERATING STATION

Capacity of station, 7000 kw.
Transmission voltage, 33,000
Hydraulic head, 46 feet

Item	Total cost	Cost per kw.
Building	\$82,000	\$11.70
Hydraulic machinery	73,000	10.40
Electric machinery	83,000	11.80
Freight and hauling	26,200	3.75
Erection	55,500	7.90
Tailrace	60,000	8.50
Roads and telephone lines	7,300	1.10
Camp and permanent quarters	23,200	3.30
Engineering and incidentals	11,100	1.55
Administration charges, etc	15,000	2.10
Total	\$436,300	\$62.30

ANNUAL COST OF OPERATION

Item	Expense per year
Operation:	
Labor	\$5,700
Supplies	950
Repairs:	
Labor	900
Supplies and material	300
Superintendence, clerical, camp, etc	1,700
General expense and administration	450
Operating expense	\$10,000

A depreciation of 5 per cent ($21,800) has also been charged to this development. No taxes or interest is charged, the undertaking being done by the Government. Assuming 7 per cent for interest and taxes the total operating expenses would amount to $62,000. A total of about 15 million kw-hr. were delivered during the year, thus corresponding to a cost of $0.0041 per kw-hr.

Costs Reported by Ontario Commission. — The following table gives the estimated cost of development and yearly operating expenses of various plants from reports of the Ontario Hydro-Electric Power Commission.*

Location of development	Available head in feet	Developed power in horse-power	Estimated capital cost	Cost per horse-power	Annual operating expenses, including administration	Annual maintenance and repairs	Depreciation	Interest at 4 per cent	Total annual charges
			$	$	$	$	$	$	$
Healey's Falls, Lower Trent River	60	8,000	675,000	84.38	16,875	13,500	13,500	27,000	70,875
Middle Falls, Lower Trent River	30	5,200	475,000	91.37	11,875	9,500	9,500	19,000	49,875
Maitland River (*a*)	80	1,600	325,000	203.12	5,665	2,754	2,755	13,000	24,174
Saugeen River	40	1,333	250,000	187.53	4,840	3,247	3,247	9,984	21,318
Severn River (Big chute) (*b*)	52	4,000	350,000	87.50	17,433	8,571	8,571	14,000	48,575
South River	85	750	115,000	153.33	4,100	2,620	2,620	4,534	13,874
St. Lawrence River, Iroquois, Ont.	12	1,200	179,000	149.16	6,864	5,119	5,118	7,151	24,252
Mississippi River, High Falls "A" (*c*)	78	2,400	195,000	81.25	9,391	3,840	3,841	7,777	24,849
Mississippi River, High Falls "B"	78	1,100	123,000	111.82	6,390	2,491	2,491	4,908	16,280
Dog Lake, Kaministiqula River	310	13,675	832,000	61.00	13,760	16,427	15,927	32,278	79,392
	310	6,840	619,700	91.00	11,296	10,632	10,132	24,787	56,847
Cameron Rapids	39	16,350	815,000	50.00	16,375	17,327	16,727	32,561	82,990
	39	8,250	600,000	73.00	14,390	11,478	10,978	24,008	60,854
Slate Falls	40	3,686	357,600	97.00	6,000	6,634	6,334	14,303	33,271
	40	1,843	260,000	141.00	6,000	3,868	3,669	10,400	23,937

(*a*) Expensive dam.
(*b*) Inexpensive construction of canal and headworks.
(*c*) Includes storage development.

* Capital costs cover hydraulic development (such as dams, headworks, pipe lines), power house, hydraulic and electrical equipment, with one spare unit, and step-up transformer station with electrical equipment. It does not include cost of vested rights and land damages or transmission lines.

BIBLIOGRAPHY. — Below are given references to some of the more important hydroelectric stations. A great amount of valuable data on the control and transmission of energy from hydroelectric stations is also given in the report of the Engineering Data Committee of the A.I.E.E., entitled *Engineering Data Relating to High Tension Transmission Systems* presented at the annual convention of the A.I.E.E., June 25, 1914.

Company	State	Voltage	Journal	Year
Appalachian Power Co.	Virginia	88,000	Elec. World	1912
Arizona Power Co.	Arizona	45,000	Elec. World	1910
Central Colorado Power Co.	Colorado	100,000	Elec. World	1910, '11, '12
Central Georgia Power Co.	Georgia	66,000	Elec. World	1911, '13
			South. Elec.	1911
Connecticut River Power Co.	Vermont	66,000	Pow. & Eng.	1909
			G .E. Rev.	1911
			Power	1911
East Creek Elec. Lt. & Pow. Co.	New York	60,000	Elec. World	1912
			G. E. Rev.	1912
Eastern Michigan Power Co.	Michigan	140,000	Elec. World	1912
			Eng. News	1912
Georgia Power Co.	Georgia	110,000	South Elec.	1912
			G. E. Rev.	1914
Great Falls Water Power, & Town Site Co.	Montana	102,000	G. E. Rev.	1911
			Elec. World	1912
Great Northern Power Co.	Minnesota	60,000	Pow. & Eng.	1908
			Elec. World	1906
Great Western Power Co.	California	100,000	J. El. P. & G.*	1910, 1912
			Elec. World	1909
North Carolina Electrical Power Co.	No. Carolina	66,000	Elec. World	1912
Mississippi River Pow. Co.	Iowa	110,000	Eng. Rec.	1911, 1912
			Elec. World	1912
			G. E. Rev.	1914
Mohawk Hydroelectric Co.	New York	22,000	Elec. World	1911
Ontario Power Co.	Ontario	110,000	Elec. World	1912
Pacific Gas & Electric Co.	California	60,000	Elec. World	1912
			J. El. P. & G.*	1910
			Power	1912
Pacific Light & Pow. Corp.	California	150,000	Eng. Rec.	1912
		& 60,000	J. El. P. & G.*	1912
			Elec. World	1912
Pacific Power & Light Co.	Oregon	66,000	Elec. World	1912
Pennsylvania Water & Power Co.	Pennsylvania	70,000	Elec. World	1912
Portland Railway, Light & Power Co.	Oregon	57,000	Elec. World	1908, '12
			J. El. P. & G.*	1913

* Journal of Electricity, Power and Gas.

Company	State	Voltage	Journal	Year
Puget Sound Power Co...	Washington	55,000	Elec. World	1912
			Eng. News	1912
			Eng. Rec.	1912
			J. El. P. & G.*	1912
Rio de Janeiro Tramway, Lt. & Pow. Co..........	Brazil	88,000	Elec. World	1909
San Joaquin Light & Power Co...............	California	69,500	J. El. P. & G.*	1908, '12
			Power	1912
Shawinigan Water & Power Co...............	Quebec	100,000	Elec. World	1912
Sierra San Francisco Power Co...............	California	104,000	J. El. P. & G.*	1909, '12
Southern California Edison Co..................	California	33,000	El. Rev.	1911
			Elec. World	1907
			Power	1911
Southern Power Co.......	N. C. & S. C.	100,000	Elec. World	1910, '11
			Elec. Jour.	1911
			G. E. Rev.	1909, '10
			Pow. & Eng.	1909
			Eng. Rec.	1909
U. S. Reclamation Service	Arizona	45,000	Elec. World	1910, '11
Washington Water Power Co.....................	Washington	60,000	Elec. World	1908, '12
			Eng. Rec.	1912

* Journal of Electricity, Power and Gas.

In addition to the above the following articles on the subjects listed contain valuable data: **Exciters and Excitation:** *Elec. World*, 1907, Vol. 49, p. 880; 1912, Vol. 59, p. 1247; *Trans. A.I.E.E.*, 1912, Vol. 31, p. 1841; *G. E. Rev.*, 1912, Vol. 15, p. 626; 1914, Vol. 17, p. 567. **Voltage Regulation:** *G. E. Rev.*, 1912, Vol. 15, p. 468, p. 530; 1912, Vol. 15, p. 44, p. 626; *Trans. A.I.E.E.*, 1912, Vol. 31, p. 1841; *Elec. Jour.*, 1911, Vol. 8, p. 943; 1912, Vol. 9, p. 609; *Elec. World*, 1912, Vol. 60, p. 996. **Station Wiring:** *Elec. Jour.*, 1904, Vol. 1, p. 123; 1906, Vol. 3, p. 412; 1907, Vol. 4, p. 43; *G. E. Rev.*, 1913, Vol. 16, p. 361. **Operation:** *G. E. Rev.*, 1913, Vol. 16, p. 355. **Ventilation of Station:** *G. E. Rev.*, 1914, Vol. 17, p. 572. See also *Bibliography* in articles on the component apparatus.

[D. B. Rushmore, assisted by E. A. Lof.]

POWER STATIONS, STEAM-ELECTRIC. — (*See also Power Stations, Gas-electric; Power Stations, Hydroelectric; Boilers; Condensers; Generators; Steam Engines; Steam Turbines; etc.*) The following is a brief table of contents of this article:

LOCATION. — The selection of a site for a power plant depends on the following factors:

(1) The cost and availability of land, adequate in area and suitable in form for present and future needs.

(2) Provision for the economical handling of fuel and ashes. The site should afford navigable water frontage if possible and railroad connection in every case unless the delivery of coal by water is assured at all seasons. In comparing sites the cost of dredging channels and of track construction, including right of way, should be carefully considered.

(3) The nature of the water supply. Ample water supply from natural sources suited to all the needs of the plant is highly desirable. The life, efficiency and cost of maintenance of boilers and condensers depend greatly on the quality of the water supply. Water analyses should be made in connection with preliminary surveys.

(4) The bearing power of the sub-soil for foundations, the probability of costly difficulties in construction, the elevation of ground water, the normal, maximum and minimum stages of adjacent bodies of water.

(5) The general character of the surroundings and the existence of restrictive ordinances relating to smoke, noise, vibration and the movement of coal cars.

(6) Proximity to the load center of gravity, if power is to be distributed at low voltages.

FOUNDATIONS. — (*See also Concrete; Concrete, Reinforced.*) The design of foundations depends on the bearing power of the soil, the concentration of loads and the necessity of suppressing vibration. For table of the bearing power of various soils see the article on *Power Stations, Hydroelectric*.

The taking of borings is usually essential to the proper design of foundations, especially where the site is on alluvial soil near a water frontage. Concentrated loads may reach a maximum of 15 tons per square foot. Foundation footings should rest on rock whenever practicable. Soils of inadequate bearing power are reinforced by driving piles of wood or concrete at points of concentrated load or under the entire foundation, as the soil may require. Rafts of reinforced concrete resting on a soil stratum or on piling are often used to distribute loads, to prevent the flowage of alluvial soil and to reduce the transmission of vibration. The foundation structures proper are usually of reinforced concrete. The following table of safe loads on foundations is given in Snell's *Power House Design*.

Foundations for machinery are usually made separate from those of the walls to reduce the transmission of vibration. Boiler-room equipment is usually carried on the steel framing of the superstructure, but main generating units have separate foundation piers of concrete. Foundations for reciprocating engines have extended bases, and in extreme cases cushions of felt, sand or rubber composition are provided to suppress vibration. Anchor bolts for machinery are

TABLE I. — SAFE LOADS ON FOUNDATIONS

Loads in tons per square foot

Good concrete	4
Steel rails in concrete	8
Concrete piles	12
Ordinary bricks in cement mortar	5
Hard bricks in cement mortar	8
Blue bricks in cement mortar	12

accurately located in concrete work by means of templates. Bolts may be cast in the foundations, or holes may be provided, together with side holes to permit the adjustment of the bolts. When the concrete has set, the machine is lifted into place, aligned and leveled and the interstices run in with cement grout.

The datum line of a power plant should be fixed by consideration of water levels. Tidal limits and flood water stages fix the levels of condenser intake and outlet tunnels, of engine room floors and of furnace grates.

SUPERSTRUCTURE. — A power house should be fireproof in every respect, clean, well-lighted, well-ventilated and well-drained. The skeleton structure is usually of steel with wall panels of brick or of reinforced concrete. All structural members should be computed with a factor of safety of not less than 3 (*see Structures, Simple*). The most heavily-loaded members are the pillars carrying overhead bunkers and those supporting an engine-room crane. The roof should be of fire-proof material, truss-supported and with sufficient pitch to insure good drainage. The roof may properly have glazed monitors to assist in lighting and ventilation. Windows in the generating room and switch houses should be designed to exclude rain when partly open.

Interior walls are usually finished in brick set with close joints and neatly pointed. Glazed brick or tile are appropriately used for walls in generating rooms. The upper portion of such walls should be of light color. The basement floors are usually of smooth cement. Generating room floors should be of tile, brick or other material not tending to form dust. The boiler room floors are usually hard-burned brick or special concrete. The battery rooms should have floors of brick or acid-resisting asphalt. Stairways are preferably of iron and should be provided with non-slip treads.

Recent practice has evolved a standard type of building division, providing a boiler room and a generating room side by side and separated by a solid wall. All electrical control apparatus is placed at the side of the generating room opposite the boiler room on galleries outside the crane span or in a separate section of building. This general ground plan has a number of important advantages. The framing may be proportioned to the loads in the different sections, the boiler plant and generating plant may be extended with equal facility, all wiring is isolated from steam piping, dirt and smoke are excluded from the generating room and accidents may be isolated in the section in which they arise.

BOILER-ROOM LAYOUT. — (*See also Boilers; Chimneys; Draft, Mechanical; Fuel; Smoke Prevention; Steam; Stokers, Mechanical; Feed-water Heaters and Purifiers.*)

Capacity and Number of Boilers. — The rating of boilers is purely nominal and their evaporative capacity is limited largely by the rate at which fuel can be economically burned in their furnaces. A well-designed unit is capable of giving from 75 to 100 pounds of equivalent evaporation per hour per rated horsepower, though forcing to this extent involves some sacrifice of efficiency. The

boiler plant should have sufficient steaming capacity to operate all steam machinery at its maximum output during the period of peak load, plus reserve capacity to insure against boiler shut-downs. The most economical boiler capacity depends largely on the form of the load curve. It is economical to force boilers to very high outputs during short and severe peaks, due to the reduction of investment and of fuel required for banking fires. With a very even load curve lacking extended banking periods it is most economical to install capacity sufficient to carry the average load at the highest efficiency. In modern public service stations it is customary to draw on boilers up to 200 or 250 per cent of their nominal ratings during peak loads. In a large number of modern stations ranging in capacity from 400 to 10,000 kw. the average boiler installation was found to be 0.4 boiler horse-power per kilowatt of generating capacity. In very large stations of recent design this ratio is from 0.25 to 0.3 boiler horse-power per kilowatt.

The simplest arrangement possible is to group with each generating unit one or two boilers, but this scheme lacks operating flexibility. In large steam plants the boilers are all operated in parallel on a common steam header, though provision is often made to isolate groups of boilers in emergencies. With this arrangement the boiler plant may properly be subdivided into the number of units affording the greatest economy and convenience. Large boilers are usually more efficient than small. The unit costs of boilers, stokers, piping, flues, air ducts and building are apt to be less for large boilers than for small. The crippling of a large boiler withdraws from service a larger portion of the total capacity and correspondingly larger reserve equipment may be needed. There is no evidence to show that small boilers afford greater safety than large. In general a boiler plant should comprise not less than 4 units if continuous operation is anticipated.

Grouping of Boilers. — Two common boiler room plans exist. In one the boilers are ranged in a single or double row facing a firing aisle which runs the length of the plant. In the other there are several lateral firing aisles, each serving a double row of boilers. The former plan is appropriate when the aggregate length of firing aisle does not exceed the length of the generating room. The latter plan lends itself well to the unit or group scheme of connection to generators. When the greatest economy of ground space is necessary, the boiler plant is double-decked, but this plan complicates the handling of fuel and ashes, reduces the natural light and ventilation and requires a building of very heavy framing. A basement space is provided below the boiler room. This space contains the ash hoppers, air ducts for forced draft and ash disposal equipment. Boiler feed pumps, blowers, hotwells, feed water heaters and flues are often placed in this space. The head room of the basement should be not less than 10 feet.

Boiler Spacing and Clearances. — With few exceptions boilers are set in batteries of two, with a space of 5 feet or more between batteries. A space not less than 5 feet wide should be left behind the settings for repair work, access to blow-off valves and minor piping. If the main piping or flues occupy this space it should be at least 8 feet wide. Each firing aisle should afford ample space to withdraw boiler tubes, drums and furnace structures for replacement or repairs. The width is usually between 18 and 25 feet and varies with the type of boiler and furnace. Clear head room above the boilers should be ample to install and repair the main piping and valves and to remove and replace boiler drums. An allowance of from 10 to 12 feet is usually ample. The following table gives minimum allowances of floor space for water-tube boilers of the most widely-used types, set in batteries of two:

TABLE II. — MINIMUM BOILER ROOM SPACE ALLOWANCES FOR WATER-TUBE BOILERS

400-hp. units in batteries of two...........	1.25 sq. ft. per hp.
500-hp. units in batteries of two...........	1.20 sq. ft. per hp.
600-hp. units in batteries of two...........	1.05 sq. ft. per hp.

In power plants of modern design the actual boiler room area per boiler horse power ranges from 0.9 to 2.00 square feet, the average being very nearly 1.4 square feet.

Location of Flues. — Boiler flues may be placed on the floor at the rear of the settings, in the basement space beneath the rear of the boilers, or may be carried overhead. Flues may be of brickwork or of steel plates reinforced with angle-iron stiffeners. In the most compact designs steel flues are used over the rear of the settings. Brick flues should be lined with firebrick. Steel flues are built up from 1/8-inch plates when indoors and of 3/16-inch plates outdoors. Flues should be as short, air-tight and straight as possible and should preferably have an upward gradient toward the chimney. It is good practice to allow from 4.5 to 5.5 square feet of flue section for each 1000 pounds of coal burned per hour in the boilers served. Branch flues should be equipped with swivel dampers to permit boilers to be shut down independently.

Location of Economizers. — Economizers are most frequently set on a steel gallery above the rear of the boiler setting. Less frequently they are set on the boiler room floor behind the boiler settings or on a floor above the boiler room. Each economizer unit is provided with a by-pass flue below or behind the economizer setting. A soot chamber is provided beneath the tubes and should be from 2 to 2.5 feet in depth. Access must be allowed along the front of the economizer to permit the opening of the cleaning holes in the bottom branch pipes. Clear space of 10 feet or more is required above the setting to permit the withdrawal and replacement of the tubes. Economizers are usually set in brickwork and subdivided into sections, each associated with a battery of boilers. A large central economizer can often be installed at lower cost, but affords less flexibility in operation.

Chimneys and Mechanical Draft Appliances. — *(See also Chimneys; Brick and Brick Masonry; Draft, Mechanical.)* Brick chimneys are usually carried down to foundations independent of the building and outside of its walls. In exceptional cases where greatest space economy is necessary, brick chimneys are carried by steel columns integral with the framing of the boiler house. Steel stacks are usually supported on cast-iron base plates carried by the structural frame work of the boiler house. Fans for forced draft are commonly set at centrally located points on the boiler room floor and distribute air to the various furnaces through ducts of sheet steel beneath the main floor. Induced draft fans are usually installed in duplicate and are located at the bases of the chimneys. Short steel chimneys are generally employed with induced draft systems.

Coal and Ash Handling Equipment. — *(See also Conveyors; Cranes; Fuel; Hoists, Electric; and Unloaders.)*

Coal Storage, External. — Continuity of fuel supply is a vital necessity to power stations. Insurance against interruptions of delivery is commonly made by use of internal bunkers and external coal storage yards. The amount of fuel to be kept in reserve is a local problem and depends on the certainty of delivery, the fluctuation of the market, the rate of deterioration of coal in storage and the danger of spontaneous combustion. Coal exposed to the weather loses heat value at a rate which depends on its content of volatile fuel and which may

exceed 1 per cent per month. Coal stored in deep piles is liable to spontaneous ignition, especially if it contains much sulphur. Both difficulties may be obviated by storing coal in basins under water. Bituminous coal should not be piled deeper than 35 feet unless submerged. As a precautionary measure iron pipes may be sunk into coal piles at intervals and the temperature read periodically by a suspended thermometer. A coal yard is commonly spanned by a gantry crane carrying an automatic grab bucket for distributing and reclaiming.

Coal Storage, Internal. — Internal storage is provided in overhead bunkers supported by the boiler house framing above the firing aisles. Bunkers are of two general types, suspended steel tanks hung from the framing, and hopper-shaped structures of reinforced concrete incorporated into the building proper. Suspended bunkers are usually concrete-lined and have a limited storage capacity, the practicable limit being about 10 tons per linear foot. Built-in bunkers are best adapted to large storage capacities up to 40 tons per linear foot. Bunkers should be divided by transverse bulkheads to increase their strength and assist in isolating trouble from spontaneous combustion. Hopper bottoms should have a slope of 45° or more to make them self-clearing. Cut-off gates should be provided at point of attachment to down-spouts. Automatic weighing and recording hoppers may be installed between bunkers and down-spouts to good advantage as their records assist in keeping check on boiler performances. Down-spouts should be not less than 12 inches in diameter and should be slightly inclined to lessen the tendency of the coal to pack. The firing aisle is sometimes equipped with an electrically-operated traveling hopper which may draw coal from any desired bunker section and distribute it to the several stokers.

The bunker capacity desirable in a boiler plant depends on the extent of the external storage and the facilities for fuel handling. Bunker capacity sufficient for from 4 days' to 7 days' supply is generally adequate. Very large bunkers are costly and are apt to lead to trouble from spontaneous ignition. In computing bunker capacity it is customary to allow 40 cubic feet per ton of coal.

Coal Handling. — The handling of coal which is delivered by rail and is to be delivered directly to overhead bunkers is most readily accomplished by the following plan. The loaded car is run over a track hopper into which it dumps from beneath. The hopper delivers to a crusher which reduces the coal to a uniform size suitable for the use of the stokers. The coal is delivered by the crusher to an elevator which may consist of some type of skip hoist, inclined belt or endless chain of buckets. If a skip hoist or belt is used the coal is dumped into a receiving hopper after its ascent and is finally distributed to the bunker by a horizontal belt, flight or bucket conveyor. A chain of pivoted buckets may be used as both elevator and distributor.

When the boiler room is arranged on the unit plan, i.e., boilers in rows facing transverse firing aisles, each aisle should have its bunker system, track hopper, crusher, elevator and conveyor. Such a conveyor may properly be of the pivoted-bucket type and may serve to elevate and distribute coal or to collect ashes from the ash hoppers beneath the boilers and deliver them to an ash bunker built out over the railroad track.

Reliability is a most important factor in all coal-handling systems and is promoted by making the system mechanically simple and rugged, by installing duplicate sets of equipment and by the sectionalizing of bunkers and conveyor outfits into independent units.

Coal delivered by water is usually handled by a clam-shell or grab-bucket unloader which dumps it into a receiving hopper. After being crushed the coal is conveyed to the storage yard or bunkers by equipment similar to that just described.

Ash Handling. — If ashes are handled by a conveyor when wet or hot the corrosive action on the buckets may make maintenance costly. When the ashes are not handled by a conveyor system it is customary to provide a track running beneath the ash hoppers on which small cars may be run to haul ashes to the dump.

Feed-water Systems. — (*See also Pipes and Piping; Feed-water Heaters and Purifiers; Pumps and Pumping Engines; and Valves.*) Condensing plants usually draw feed water from the hot-wells of the condenser system. When surface condensers are used a small amount of make-up water must be added from outside sources. The water discharged from jet condensers may be used for boiler feeding if of suitable quality. Non-condensing plants operating in connection with steam heating systems usually draw feed water from the return system, with added make-up water as required. Open heaters of all types are placed on the suction side of feed pumps. Closed heaters are placed on the delivery side. Meters are usually placed in the delivery pipe and should be in duplicate if continuous indication is important. Otherwise they should be by-passed, as are all heaters and economizers, to provide for cleaning and repairs during operation.

Water is supplied to boilers through a feed main which is run along the front or rear of the boilers, often in the basement space below. Double and ring mains are occasionally used though the gain in reliability is doubtful. An auxiliary injector main running direct to boilers from the source of cold water is sometimes installed as a reserve. Iron pipe is generally employed with cold water and brass pipe for water above 200° F., or water which has a pitting tendency. Screwed joints are generally used with pipe diameters less than 2 inches. Larger pipes are fitted with screwed flanges.

Feed pumps should be installed in duplicate on each feed main or cross-overs provided between pumps. There is wide divergence in the location of feed pumps. In most plants of the unit type each feed pump is associated with the auxiliaries of a generating unit and is cared for by the turbine operator or oiler. In other cases feed pumps are grouped in a central position on the main floor of the boiler room and are cared for by a water tender. In other cases the feed pumps are placed in a separate basement pump room.

A relief valve should be placed in the pump delivery to prevent strains from excess pressure. The size of feed pipe is usually such as to allow a maximum velocity of from 300 feet to 400 feet per minute. At least two valves, a regulating valve and a check valve, should be placed in each boiler branch.

Condenser Water System. — (*See also Condensers, Steam; Cooling Systems for Power Stations.*) Cooling water for condensers is obtained from an adjacent body of water whenever possible and in other cases from a cooling pond or the basin beneath a cooling tower. In the former case large concrete intake and discharge tunnels are usually provided. These run beneath the generating room in alignment with the intake and discharge pipes of the condensers. The cross-section of these tunnels should be ample to keep the flow of water down to 5 or 6 feet per second. Intakes should have generous openings fitted with trash racks and should if possible be placed at a considerable distance upstream from the discharge outlet. In some cases it is necessary to build a baffle wall in the stream between the two tunnels. A shut-off gate is usually provided in each tunnel to facilitate cleaning and repairs. The tunnels should be of sufficient depth to insure an adequate supply of water at the lowest stage of tide or stream flow. The circulating system of surface condensers is fully enclosed and the work done by the circulating pump is merely that necessary to overcome the fluid friction in the system.

GENERATING ROOM LAY-OUT. — (*See also Condensers, Steam; Cranes; Feed-water Heaters and Purifiers; Generators, Alternating Current; Generators, Direct Current; Lubricants and Lubrication; Pipes and Piping; Pumps and Pumping Engines; Steam Engines; Steam Turbines; and Valves.*)

Capacity and Number of Units. — A generating plant should have sufficient capacity to serve its peak load with any one generating unit shut down. The reserve capacity needed above normal requirements may be provided most economically by selecting types of equipment capable of giving large overloads in emergencies, by maintaining in service condition obsolete machinery which is physically sound but uneconomical, by installing a large reserve storage battery, or by tying in parallel several power plants so that a moderate reserve may be shared in common. More reserve capacity is needed when a plant contains few large units than with many of small size. A smaller number of units than four is disadvantageous, due to the relatively large reduction of capacity by the disabling of one. A larger number than eight units has no inherent advantages. Large units are generally more economical than small units if kept well loaded. If the plant has a normal daily period of very light load it may be economical to employ one small unit well suited to this load.

Types of Equipment. — Steam-turbine generators are now almost exclusively for alternating-current generation in units exceeding 500 kw. The turbine has little or no advantage over the engine in smaller sizes and in non-condensing plants the engine is often superior. On account of its high speed the turbine is poorly adapted to the direct driving of d-c. generators. Large d-c. generators are becoming obsolete in steam plants, for it is usually more economical to generate alternating current in turbine units and convert it to direct current either locally or in distant substations. A self-contained unit comprising boiler, engine and condenser, known as the *locomobile*, has been largely used in very small European plants and is now being introduced in America.

Arrangement of Generating Rooms. — The modern standard power house has a long and narrow generating room placed between a boiler house and a switch house or series of electrical control galleries. The generating room is spanned by an electrically-operated crane. The generating units are usually ranged along this room in a single or double row. Each unit comprises a prime mover, electric generator, condenser and the associated pumps and their motive power. In most instances a basement space or series of open pits is provided below the main floor in which the condensers, pumps and most of the piping are located.

Arrangement of Turbines. — In many plants using vertical-shaft turbines the basement has been omitted and the auxiliaries grouped about the base of the turbine on the main floor. In such cases the condenser is either incorporated into the base of the turbine or is placed on the floor immediately beside it. This grouping of equipment on a single floor has several operating advantages. All apparatus has good light. A single operator can give efficient attention to a large group of equipment, and machinery so placed usually receives closer attention than it would in a pit or basement. All pieces can be readily handled by the crane without interference with other apparatus. The ample head room is advantageous in making repairs. The basement plan is especially economical of floor space, but makes the auxiliaries relatively less accessible. Placing condensing equipment in open pits beside the engine or turbine piers makes it accessible for inspection and for handling by the crane.

In determining the arrangement of a generating room it is essential to provide each element with ample space for all needed attention during operation or repairs. Sufficient clear floor space about a unit is usually provided to permit it to be dismantled without removing the parts to a distance. Clear trucking

space is also desirable. The separate pieces of machinery should be so arranged that the crane can be conveniently used in assembling or dismantling any piece without interference with others. Clear overhead space below the crane hook should be sufficient to permit any heavy part to be lifted clear of its setting and carried away. The crane girder and trolley must clear all roof trusses and lighting fixtures. Clearance must be allowed at the generator ends of horizontal turbine units to withdraw the revolving field structures.

Arrangement of Condensers. — In setting surface condensers clear working space must be allowed at both ends to remove the heads and at one end clear space must be allowed to permit the withdrawal and replacement of tubes. Jet condensers should be set in such a manner that the head can be conveniently opened for repairs. Condensers in general should be set below the level of the associated prime mover and as close as possible to its exhaust port. The condenser connection should provide natural drainage for condensed steam. It should have as few joints as practicable to avoid occasion for air leaks. It is usually desirable to provide a copper expansion section in this connection as joints in a rigid pipe are difficult to keep air-tight with varying temperatures. The arrangement of a central condenser serving a group of prime movers is uncommon in electric power plants. Barometric jet condensers are often placed outside of the wall of the building on account of the long tail pipe required.

Floor Space in Generating Rooms. — The floor space provided per kilowatt varies greatly with the size, type and arrangement of equipment. The following data from modern steam-turbine stations are illustrative of the range of best practice.

TABLE III. — SPACE COVERED BY STEAM-TURBINE POWER STATIONS

Station No.	Capacity, kilowatts	Boiler room, square feet per kilowatt	Turbine room square feet per kilowatt	Total, square feet per kilowatt
1	3,000	0.71	0.70	1.41
2	8,000	0.74	0.69	1.43
3	11,000	1.11	0.70	1.81
4	13,500	1.13	0.50	1.63
5	16,000	0.90	0.37	1.27
6	24,000	0.92	0.40	1.32
7	30,000	0.44	0.40	0.84
8	32,000	0.60	0.30	0.90
9	100,000	0.48	0.17	0.65

Oiling Systems. — (*See also Lubricants and Lubrication.*) Steam power plants afford three classes of lubrication problems, viz., wearing surfaces exposed to high temperature steam, as in cylinders, valve chests and stuffing boxes; atmospheric surfaces of open guides and journals; and enclosed surfaces, chiefly journals. Steam surfaces are lubricated with cylinder oils of mineral origin supplied in atomized form by forcing the oil in small quantities into the steam supply pipe. Oil may be supplied by a local force-feed pump or hydrostatic lubricator or may be supplied from a central tank and pump supplying a group of cylinders. Bearings, guides and other exposed surfaces are lubricated by oil or grease supplied by hand or from adjustable feed cups. Enclosed surfaces are usually lubricated by splashing or by flooding with oil from a central source of supply. The flooding system is most efficient. Oil from a central tank is

forced by pump or gravity to the various working parts whence it returns through a drip system to collecting pans and filters. After purification a small amount of make-up oil is added and the reclaimed supply restored to the oiling system.

PIPING SYSTEMS. — (*See also Pipes and Piping; and Valves.*) The various piping systems in a power plant are subdivisible into the following groups: (*a*) High-pressure steam piping between boilers, main units and auxiliary engines; (*b*) exhaust piping to condensers; (*c*) exhaust piping to feed-water heaters; (*d*) atmospheric-exhaust piping; (*e*) feed-water piping; (*f*) cooling-water piping; (*g*) pipe-drainage system and (*h*) oil piping. To facilitate the identification of pipes of various classes it is desirable to paint each a distinctive color.

High-pressure Steam Piping. — The chief considerations in laying out high-pressure piping are (*a*) to produce a reliable system without complexity; (*b*) to make all joints permanently steam tight; (*c*) to take up all expansion strains; (*d*) to maintain proper steam pressure at all points of delivery; (*e*) to reduce to an economic minimum the loss of heat by radiation and (*f*) to drain from the entire system all water of condensation. In many older systems of piping elaborate ring and multiple headers, with numerous by-passes, cross-overs and duplicate-connection branches were employed in the endeavor to promote reliability by making possible the isolation of any fault. Reliability is sought in modern systems by very simple connections with skillful design and the best possible construction.

Unit and Parallel Systems of High-pressure Piping. — The ***unit system*** and the ***parallel system*** of connection with their various modifications are now most extensively used. The unit system of piping connects a separate group of boilers to each prime mover and its auxiliaries. The parallel system provides a large steam header running the length of the plant into which all boiler branches deliver and from which all prime movers are supplied. Unit systems are usually provided with cross-overs between unit steam and water headers to permit the parallel operation of different sections in emergencies. Parallel systems are often provided with sectionalizing valves to permit the isolation of any section in case of accident. Diagrammatic sketches of unit and parallel grouping are shown in Figs. 1 and 2.

Size of Piping. — Pipe sizes for high-pressure work are generally determined by the maximum allowable steam velocity. It has been found satisfactory and economical to allow a maximum velocity of 6000 feet per minute with saturated steam and from 9000 to 12,000 feet per minute with superheated steam. The flow to piston engines is intermittent and the pipe size should be proportioned according to the velocity during admission unless a receiver is installed. The steam header of a parallel system serves as a reservoir to equalize pressures and prevent vibrations in the piping. Its cross section is properly proportioned according to the maximum cross flow of steam with any boiler section inoperative.

Joints. — Joints in high-pressure piping are usually of the screwed type for diameters under 3 inches and of the flanged type for larger sizes. Flanged joints may be made between ground faces or by aid of gaskets, the former type being preferable for high-pressure steam. An excellent joint is made by drawing up with loose collar flanges the turned-over ends of pipe sections with faces ground true, often called a Van Stone joint. Such joints are more expensive than rigid flanged joints but the possible swing of the pipe about its axis is a great advantage in the final aligning and connecting up of a system with many branches.

Provisions for Expansion. — Provision for the expansion of pipe without straining joints and fittings is imperative. Between two points which must be rigidly fastened expansion is taken up in expansion loops and bends. The

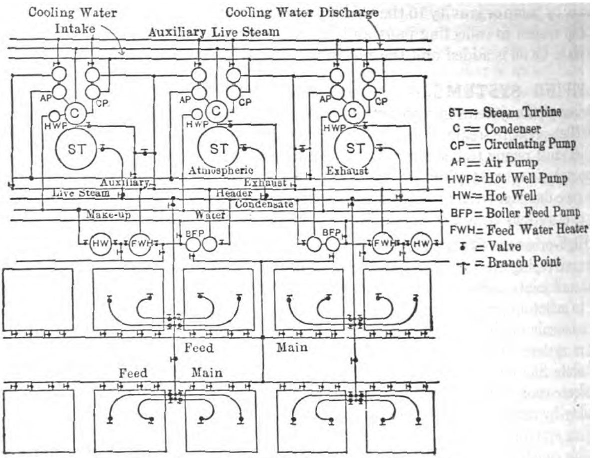

Fig. 1. Parallel Method of Connection

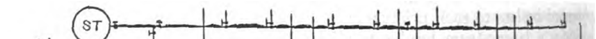

Fig. 2. Unit Method of Connection

radius of such bends should in all cases be not less than five times the pipe's diameter and at each end of the curve there should be a length of straight pipe not less than twice the diameter. Welded flanges are recommended for the attachment of bends. When bends cannot be used recourse must be had to swivel joints or slip joints, which should be avoided if possible. When screwed joints are employed expansion is commonly provided for by the use of sections having single, double or triple swing about a screwed connection. Pipes of considerable length must be anchored at more than one point to prevent vibration. Between each anchor expansion bends are required and the pipe must be support-

ed by hangers, roller brackets or pedestals allowing longitudinal motion. Anchorage is usually made at points of attachment to branches, and fittings with anchor bases are employed for the purpose.

Lagging. — Live-steam pipes, boiler-feed pipes, steam drums, receivers and separators should be covered with heat-insulating material to reduce the radiation losses to a minimum. The loss from bare pipe is approximately 3 B.t.u. per square foot per hour for each degree difference of temperature. Good commercial coverings, such as magnesia, felt, asbestos, mineral wool, etc., will prevent from 75 to 90 per cent of this loss if properly applied.

Drainage. — Saturated steam in passing through pipes undergoes a small amount of condensation due to friction and heat radiation. The presence of water in steam piping is a source of danger, for a water slug, if picked up by the moving column of steam, may be driven with tremendous force against any opposing surface, such as a valve, sharp bend or cylinder, with destructive results. Pipes should be slightly inclined so that water drains away from the prime movers and every point where water may collect must be provided with a drip connection. These drip connections are run through traps to the hot-well so that the hot water may be returned to the boilers. In other systems the drip water is returned directly to the boiler. Separators are often installed at the inlets to reciprocating engines and exhaust steam turbines taking wet or saturated steam, to drain the moisture from the entering steam. Drip connections should always be made to these separators. Bleeder connections to live steam lines are provided in order that the water condensed in warming up the pipe when steam is turned on may be drawn off.

Valves. —Valves for steam pipes are of two general types, gate valves and globe valves. Either type may be outside screw or inside screw, according as the screw of the spindle is outside or inside of the casting. Outside screw types are preferred for high-pressure work as the position of the spindle is then an index showing whether this valve is open or closed.

Check valves are required in boiler connections to prevent steam flowing into the boiler when cold or when its pressure is below that of the header to which it is connected. Emergency valves are often provided to cut off the steam under abnormal conditions, such as the bursting of a pipe or the racing of an engine. These often take the form of a weighted valve which closes itself when a trip is released. Electric motors and hydraulic pistons are sometimes connected to ordinary valves to provide for remote control during emergencies.

Blow-off valves of boilers are subjected to very severe service and are made exceptionally rugged. Such valves must close without leaks, open readily and furnish a free path for the ejection of scale and sediment. The wearing parts of such valves should be readily renewable. Best practice requires the use of two blow-off valves or a valve and a cock. The steam, water and sediment are usually blown through a tank partially filled with water before being exhausted to the air.

A few rules relative to the installation of high-pressure valves and piping may be noted. All valves of a diameter above 6 inches should be by-passed to facilitate opening under pressure. Valve stems are often placed horizontally to lessen the tendency to form water pockets. Angle valves should be selected whenever convenient because of the greater room in them. Branches from mains to boilers should have at least two valves, an ordinary stop valve next to the main and an automatic check valve near the boiler. Valves are best placed at the highest points in the pipe to simplify the drip system. When the flow of steam is intermittent heavy valves should not be placed far to one side of a line joining the points where the pipe is supported as they may cause vibration.

Branches from a main to a prime mover should have a stop valve at the highest point near the main. A receiver-separator from three to four times the volume of the high-pressure cylinder should be placed as close to the throttle of a piston engine as possible to equalize steam flow and drain moisture from the steam. When superheated steam is used cast-steel fittings and valves are preferred to cast iron. Branch pipes are almost invariably connected to the top of the main to prevent water from passing into the branches. When superheated steam is supplied to the main units and saturated steam to the auxiliaries a separate steam main for the latter is employed.

Exhaust Steam Piping. — The size of the exhaust pipe of a prime mover is determined by the permissible back pressure. High vacuum requires ports and exhaust piping of large diameter and the length of pipe to the condenser as short as possible. An atmospheric-relief valve which is normally closed by air pressure against a spring or weighted lever but which opens automatically when the vacuum fails should be installed in the condenser pipe. Atmospheric-exhaust pipe is usually spiral riveted as no precaution against leaks is required. This usually terminates in an enlarged exhaust head which contains baffles to drain from the steam condensed water and oil before it discharges to the air. A common exhaust main for a number of units is often employed. When exhaust steam is used for heating purposes the system is supplied through a back-pressure valve which automatically opens the atmospheric exhaust if the back pressure exceeds that required to operate the heating system and closes when normal back pressure is restored.

GENERATOR AND CONTROL EQUIPMENT. — (*See also Batteries, Storage, Applications of; Bus-Bars and Bus-bar Structures; Circuit Breakers; Generators, Alternating Current; Generators, Direct Current; Reactance Coils; Regulators; Relays; Switches; Switchboards; Switchgear Equipment for Power Stations; Transformers; Transformers, Instrument; Wires and Cables.*)

Direct-current Generators. — Three-wire lighting service may be provided (*a*) by the connection of generators in sets of two in series, (*b*) by the use of three-wire generators with external or internal compensator coils or (*c*) by the use of voltage balancer sets associated with standard two-wire generators. Railway generators are operated at or near 600 or 1200 volts and are grounded at one pole, usually the negative. Series fields and equalizer connections may be on either the positive or the negative side. Equalizer switches are often carried by pedestals near the generator terminals to save wiring to the switchboard. Satisfactory voltage regulation for railway systems is usually provided by the use of compound generators. The close voltage regulation required by lighting systems is best provided by the use of a regulator of the Tirrill type. (*See Regulators.*)

Alternating-current Generators. — Synchronous 3-phase generators are used most extensively. Induction generators are often advantageous in connection with exhaust-steam turbines. Synchronous turbo-alternators are now designed with high internal reactance to prevent excessive transient currents immediately after the creation of a short-circuit. If the internal reactance is inadequate they may be connected to the bus-bars through reactance coils. (*See Reactance Coils.*) In some cases the generators are operated at half the bus-bar voltage and are connected through raising auto-transformers wound with large reactance.

Grounding the Neutral. — Three-phase alternators are usually Y-connected and provision is frequently made for the grounding of one generator at the neutral point to prevent dangerous potential rise should a wire become accidentally grounded. More than one grounded neutral in a group of genera-

ors in parallel is undesirable, due to the possibility of a third-harmonic circulating current passing through the neutral connection.

Air Ducts for Forced Ventilation. — Modern turbo-alternators are designed for forced ventilation. A system of ducts should be provided, either overhead or beneath the floor of the generating room from which each generator may draw a supply of clean outside air. This air is forced through the ventilating spaces of the stator and rotor by the fan action of the rotor. Discharge ducts are also provided in case it is undesirable to discharge the heated air to the generating room. It is desirable to equip such ducts with dampers so that the air may be discharged indoors or outdoors as desired. In many cases where the air supply is dusty it is desirable to install air-conditioning equipment in the intake of the system. See *General Electric Review*, Vol. 16, p. 627 for a full discussion of turbo-generator ventilation.

Excitation. — Fig. 3 shows the exciter capacity required by modern alternators. In small plants each alternator is often provided with its individual exciter driven by the main shaft. Large stations are provided with central

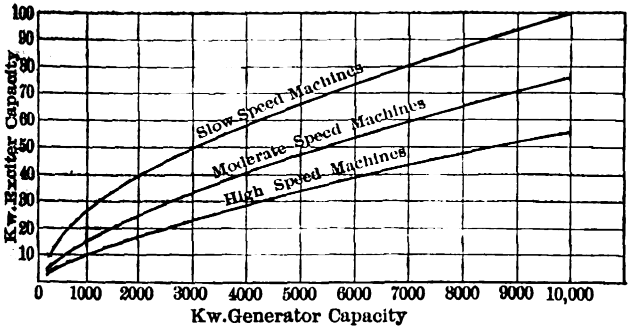

Fig. 3. Exciter Capacity Required by A-C. Generators

systems of excitation comprising not less than two, and generally more, direct-current generators, driven by independent motive power. At least one exciter in every plant should be steam-driven. Three-phase induction motors are quite generally used for the electric driving of exciters. The exciter system is frequently reinforced by a floating storage battery to insure the supply of exciting current in every emergency. (*See Batteries, Storage, Applications of.*) All alternator fields are supplied through adjustable rheostats from a set of excitation bus-bars, except that in a few very large plants the excitation system is sectionalized for the sake of reliability. Exciters are commonly rated at 125 or 250 volts, and are usually compound-wound, with magnetic circuits normally in a state of low saturation. The best position for exciters in large stations is generally near the center of the generating floor.

Voltage Control. — In both alternating- and direct-current stations very sensitive regulation of the bus-bar voltage may be obtained by the use of appropriate types of Tirrill regulators. (*See Regulators.*) When the load variations of different feeders are quite unlike it is often desirable to provide each feeder or group of parallel feeders with independent feeder regulators, which may be hand-operated or may be automatically controlled by voltage relays.

Transformers. — The arrangement of the transformers depends to some extent upon the type used. When it is not convenient or possible to allow above

each transformer clear head-room for crane handling, the transformer cells may be arranged to open at one side along a track and have raised floors at the level of a flat car, so that the assembled units may be slid into or out of place.

Air-blast Transformers. — These are largely used for step-up service in power stations in connection with feeders operating at 20,000 volts or less. Railway transmission lines and feeders operated at more than 20,000 volts are usually supplied through oil-insulated, water-cooled transformers. Air-blast transformers are usually placed above a common pressure pit whose air-supply is drawn from out-doors by electrically operated fans. Each transformer may be equipped with adjustable dampers to regulate the air supply.

Oil-insulated Transformers. — In some cases oil-insulated transformers are isolated in fire-proof cells built of concrete or masonry. The fire-risk is usually not sufficient to warrant isolation for each unit, but it is often desirable to enclose each 3-phase bank. Large oil-insulated transformers should be connected to an oil-drainage system to facilitate the withdrawal and replacement of oil.

Switching Equipment and Wiring. — For descriptions and standard arrangements see the following articles: *Bus-Bars and Bus-bar Structures; Circuit Breakers; Regulators; Relays; Switches; Switchboards; Switchgear for Power Stations; Synchroscopes; Transformers, Instrument;* and the articles on the various kinds of meters.

Wiring. — In direct-current stations the power conductors are usually rubber-insulated copper cables run from generators to switch-board and thence to outside circuits in ducts of tile or fiber. In some cases the duct system is dispensed with and the conductors are carried on open wallracks spaced by insulating knobs. In small alternating-current stations the wiring to and from the switchboard is usually in a duct system, but the switchboard and bus-bar wiring is open and is supported by porcelain insulators on walls and on a light frame work of pipes or angle-irons.

In plants of 12,000 kilowatts and up, operating at voltages of 20,000 or less, it is customary to isolate all conductors of unlike polarity as fully as possible in fireproof cells and barriers. The essential features of such a system of isolation are as follows: — (1) Each horizontal run of conductor is drawn into its individual duct of tile or fibre set in a concrete floor; (2) each bus-bar is mounted in a separate horizontal cell of concrete or masonry; (3) each instrument transformer and disconnecting switch is in a separate cell; (4) each pole of a bottom-connected oil switch or each complete top-connected switch is in its separate cell; and (5) vertical fire-proof barriers are placed between all vertical runs of conductor. Isolation of this nature is seldom practiced in systems above 30,000 volts, but open overhead wiring with generous spacing is employed.

In an enclosed system the power connections are appropriately formed by single-conductor copper cables, insulated with rubber or with cambric tape, and covered with a fire-resisting sheath. Lead sheathing is seldom employed unless the wiring is exposed to dampness. The cross section of such conductors is determined by the safe limits of temperature rise.

COST OF STEAM POWER STATIONS. — Power station costs are subject to wide variations with the type and elaborateness of equipment, construction difficulties and rated capacity.

Capital Costs. — Illustrative data showing the range of unit costs of the chief divisions of plant equipment are given by O. S. Lyford, Jr., and R. W. Stoval of the Westinghouse, Church, Kerr Co., in the Electric Journal, 1912, Vol. 9, p. 322, as follows:

TABLE IV. — COSTS OF STEAM-ELECTRIC POWER STATIONS

Capacity, 2000 to 20,000 kw., based on maximum continuous capacity of generators at 50° C. rise

Item	Dollars per kw.	
	High	Low
Preparing Site: Dismantling and removing structures, making construction roads, tracks, etc.	0.25	0.00
Yard Work: Intake and discharge flumes for condensing water, railway siding, grading, fencing, sidewalks	2.50	1.00
Foundations, including foundations for building, stacks and machinery, together with excavation, piling, waterproofing, etc.	6.00	1.00
Boiler Room Equipment, including boilers, stokers, flues, stacks, feed pumps, feed-water heater, economizers, mechanical draft, and all piping and pipe covering except for condenser water	24.00	12.00
Turbine Room Equipment, including steam turbines and generators, condensers with condenser auxiliaries and water piping, oiling system, etc.	22.00	12.00
Electrical Switching Equipment, including exciters, masonry switch structure with all switchboards, switches, instruments, etc., and all wiring except for building lighting	5.00	2.00
Service Equipment, such as cranes, lighting, heating, plumbing, fire protection, compressed air, furniture, permanent tools, coal and ash-handling machinery, etc.	5.00	2.50
Building, including frame, walls, floors, roofs, windows and doors, coal bunkers, but exclusive of foundations, heating, plumbing and lighting	12.00	4.00
Starting Up. — Labor, fuel and supplies for getting plant ready to carry useful load	1.00	0.50
General Charges, such as engineering, purchasing, supervision, clerical work, construction plant and supplies, watchmen, cleaning up	6.00	3.00
Total Cost, except land and interest during construction	83.75	38.00

The same authorities give the costs of foundations at from \$1.25 to \$4.00 per square foot of building area, depending on the nature of the sub-soil. Table III shows that the ground area of buildings ranges from 0.8 to 2.0 square feet per kw. Building costs range from 8 to 12 cents per cubic foot of space, over-all. Power plant buildings range from 50 to 100 cubic feet per kw.

The writer has correlated miscellaneous data on power plant costs in the form of curves, see Fig. 4. The curves are shown in an additive sense, e.g., for a 12,000 kilowatt plant, boiler room equipment and piping cost \$17.50 per kw., generating plant \$37.00 − \$17.50 = \$19.50 per kw., etc., the total cost except land and interest during construction being \$66.00 per kw.

Operating Cost and Fixed Charges. — The cost of producing electrical energy comprises two groups of items, viz., fixed charges on the investment in the power plant to cover interest, depreciation, taxes and insurance, and the

TABLE V.—COSTS OF BOILER ROOM EQUIPMENT.

Item	Dollars per boiler horse-power	
	High	Low
Boilers, except settings	$11.00	$8.00
Superheaters	3.00	0.00
Stokers	5.50	3.00
Masonry settings of boilers	3.50	2.00
Flues	1.50	0.75
Stacks	4.00	2.00
Economizers	4.00	0.00
Mechanical draft	3.00	0.00
Feed pumps	1.50	0.50
Feed heaters	1.00	0.40
All piping and pipe covering	10.00	6.00
Coal chutes and ash hoppers	1.25	0.00
Miscellaneous items	1.00	0.50
Totals	$50.25	$23.15

cost of operating the plant. Fixed charges on the investment range from 10 to 14 per cent of the investment per annum and are practically independent of the load factor of the station. The amount assignable to each kilowatt-hour is therefore inversely proportional to the load factor and equals, in cents,

$$F = \frac{100\,RI}{8760\,K},$$

where I is the investment per kw. of capacity, in dollars, R is the rate of fixed charges expressed as a decimal fraction and K the annual station load factor expressed as a decimal fraction.

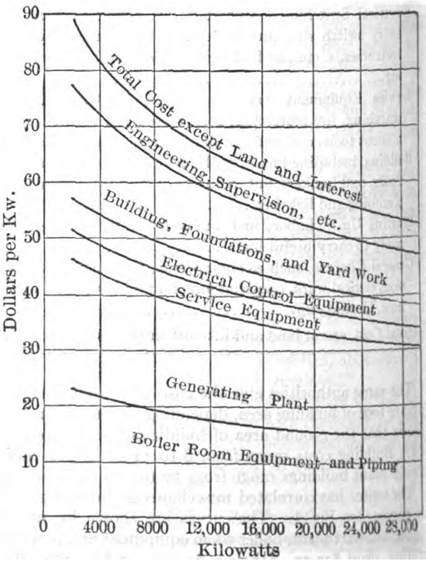

Fig. 4. Approximate Division of Construction Cost of Steam-electric Power Plants. (In additive sense, see text.)

Operating costs include supervision, labor, fuel, other materials consumed and current repairs. The cost of supervision per kw-hr. varies almost inversely

with the load factor. Labor cost per kw-hr. in boiler plants vary in almost direct proportion to the weight of coal burned per kw-hr. Labor cost per kw-hr. in generating rooms varies greatly with the size of the station and the number of units operated, since the number of men required depends more on the number of units of apparatus to be cared for than on the ratings of these units. Large plants are therefore at a large advantage in the item of labor cost. The cost of fuel per kw-hr. varies directly with the price of heat units in the coal burned and inversely with the over-all efficiency of the station. In modern steam turbine stations the cost of supplies other than fuel averages about 20 per cent of the total labor cost. The cost of current repairs in such stations is from $0.75 to $1.50 per annum per kw. of generating capacity. A high load factor is distinctly favorable to good physical efficiency, due to the reduction of stand-by and light-load losses, and to economy in the use of labor. H. G. Stott has pointed out that the production cost per kw-hr. varies inversely with the 4th root of the annual load factor. (*See Proc. A.I.E.E., Vol. 32, p. 1127.*)

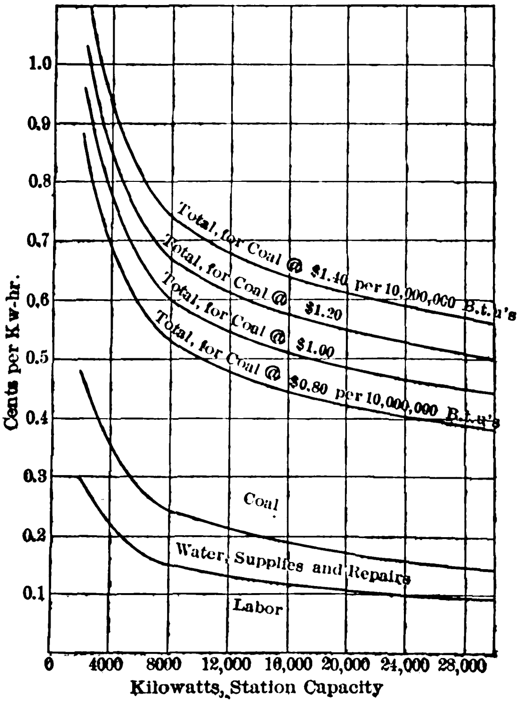

Fig. 5. Approximate Division of Operating Expenses of Steam-electric Power Plants. (In additive sense, see text.)

The writer has correlated the records of the operating costs of a considerable number of railway and public service station having load factors between 25 and 33 per cent, and from these data has prepared the curves in Fig. 5. (For the B.t.u. equivalent of a ton of coal see article on *Fuel*.) These curves are shown in an additive sense, in the same manner as the curves in Fig. 4.

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[W. E. Wickenden.]

PRINTING PRESSES, ELECTRICAL OPERATION OF.—(*See also Motors, Industrial Applications of.*) Owing to the great variety of work performed, printing machinery as a rule requires a certain degree of speed variation in order that with a given equipment and operating force, the maximum of high-grade production may be turned out. All necessary speed variations are most readily, economically and satisfactorily obtained with electric drive and control. Each press can then at all times be under instant control, so that the entire time of the operator may be devoted to the work in hand.

Individual drive is particularly applicable to printing establishments, as in this class of service the ratio is small between the power required to drive the machines running idle and when performing actual productive work. With group drive the ratio between average and connected load is high, due to the large and constant character of the friction losses. The economy in power consumption is, therefore, in favor of separately-driven units, which may be shut down when not operating productively.

Printing presses may be divided into three principal classes, viz., job, flat-bed and web presses.

JOB PRESSES. — The job press is the smallest type and requires from ¼ to 1 horse-power, depending upon the size. The motor is generally mounted on or near the press and the drive may involve either a short belt with idler or occasionally some form of friction device. With these presses a speed variation of 60 per cent is often required, which, as a rule, is accomplished by armature control, shunt-wound motors being used with direct-current installations and single-phase repulsion motors with alternating-current.

FLAT-BED PRESSES are considerably larger than job presses and consist of a reciprocating bed containing the type form and a main cylinder on which the impressions are made. The presses require from 2 to 10 horse-power to drive them according to work and size. In the majority of plants a 50 to 60 per cent speed variation is desired, and as these presses require a considerable torque at starting it is usual to provide compound-wound direct-current motors or phase-wound polyphase induction motors. The series winding of the compound-wound motors is generally weaker than for standard motors, 10 per cent being an average value. The speed control is accomplished by field control for direct-current motors and secondary control for induction motors. Most presses of this type allow of mounting the motor under the bed toward the front of the machine and a belt drive with an idler is almost always used.

WEB PRESSES vary greatly in size and come under the head of rotary presses. Different makers have different classifications for the various sizes, such as the number of decks or webs, as 3-deck, 5-deck, 2-web, etc., or according to the number of groups of which they consist, as quadruple, sextuple, octuple, etc. A modern high-speed sextuple press, for example, has three paper rolls and six plate cylinders, each cylinder generally being four plates wide. The plate cylinders revolve at a maximum speed of 300 r.p.m. A wide range of operating speeds must therefore be provided on the controller as the press will often be called upon to operate at speeds as low as one-half of the above.

All of these presses require a slow threading speed, usually about 10 r.p.m. This speed must be steady as the operators have to thread the paper from roll to roll around the cylinder up over the carriers to the folder, and should the press turn by jerks it might mean the loss of a finger, hand or arm. The control must also provide for "jogging" or "inching along" when making the press ready for service, that is turning the cylinders through a small fraction of a revolution so as to bring them to the desired position for putting on the plates. Brakes must

also be provided so that in case of emergency the press may be brought immediately to rest.

Single-motor Equipment. — Motor equipments for web presses are of either the single or two-motor type. The former is used with small and medium-size direct-current units but is not practicable above 30 or 35 horse-power. The principal objection to single-motor equipments is the difficulty in maintaining a constant slow speed, due to the wide variation in torque during any given revolution of the press cylinders. The waste of power in the series and shunt resistances is also a disadvantage, particularly for the larger sizes. Compound-wound motors are generally used with the single-motor equipments so as to provide for the comparatively-heavy starting torque. Speed regulation down to 10 per cent is readily accomplished by connecting a resistance in parallel with the armature circuit which, in combination with the series resistance, gives a fixed slow speed.

Two-motor Equipment. — The two-motor equipment consists, as the name implies, of two motors, one of small size for driving the press at slow speed through a worm-gear reduction, and a large motor to drive the press at the full-producing speed through ordinary direct gearing. The reduction gearing for the small motor is furthermore provided with an automatic ratchet and pawl clutch, which mechanically disconnects the small motor when the press is being accelerated by the large motor.

Either direct- or alternating-current motors may be used with two-motor equipments. When direct-current is used the small motor should be compound wound to insure sufficient starting torque. It is also a common practice to make the large motor compound wound, so as to accelerate the press more easily. With alternating-current the smaller motor may be of the squirrel-cage type but the larger must be of the phase-wound slip-ring type, as a smooth speed variation from the 10 per cent threading speed to the full running speed is most essential.

The capacity of the motors depends on the size and make of the press, and the accompanying table is only intended to give an approximate idea of the power required:

Type of press	Horse-power	
	Small motor	Large motor
Quadruple	5	35
Sextuple	7	50
Double quadruple	Two 5½	Two 35
Double sextuple	Two 7½	Two 50
Octuple	10	70

Control of Two-motor Drive. — Full automatic control in connection with two-motor drive has been almost universally adopted on larger presses. The essentials of this control are the complete automatic control of the press speed from any number of push-button stations located at different points about the press. From any of these stations it is possible to start the press, increase or decrease the speed or stop it. Five buttons are generally provided with each station marked "Fast," "Slow," "Stop," "Safe" and "Run."

Pressure on the "Fast" or "Slow" button causes the press to speed up or slow down until the button is released, when the press will continue to run at the

speed it has then attained. The manipulation of the "Stop" button will immediately stop the press, through the operation of a dynamic brake on direct-current equipments and a solenoid brake on alternating-current equipments. Pressure on the "Safe" button at any station opens the control circuit and renders the equipment inoperative until the "Run" button at that particular station is closed, releasing the "Fast" button. A "Jog" button is frequently also provided, particularly on a-c. equipments, by means of which the cylinders can be inched along to the desired position when plating is done.

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[D. B. RUSHMORE, assisted by E. A. LOF.]

PROGRESSION. — (*See also Series, Mathematical.*) There are two kinds of progression, arithmetical and geometrical.

ARITHMETICAL PROGRESSION. — Quantities are said to be in arithmetical progression when they increase or decrease by a common difference.

Let a = first term of the progression,
d = the amount by which any term is greater than the next preceding, i.e., the common difference,
n = the number of terms in the progression.

Then the successive terms of the progression are

$$a,\ a + d,\ a + 2d,\ a + 3d, \text{ etc., to } n \text{ terms.}$$

The last term is

$$a + (n - 1)\, d.$$

The sum of the n terms is

$$\frac{n}{2}[2a + (n - 1)\, d].$$

GEOMETRICAL PROGRESSION. — Quantities are said to be in geometrical progression when they increase or decrease by a common ratio. Let

a = the first term of the progression,
r = the ratio of any term to the next preceding term,
n = the number of terms in the progression.

Then the successive terms of the progression are

$$a,\ ar,\ ar^2,\ ar^3, \text{ etc., to } n \text{ terms.}$$

The last term is

$$ar^{n-1}.$$

The sum of the n terms is

$$\frac{a(1 - r^n)}{1 - r}.$$

[W. A. DEL MAR.]

PUMPS AND PUMPING ENGINES. — (*See also Boilers; Blowers and Compressors; Condensers; Fans; Power Stations; Steam Engines; Steam Turbines.*) Pumps may be classified according to (1) their mode of action as piston, plunger, centrifugal, rotary, jet and direct pressure; (2) their motive power, as engine-driven, turbine-driven, motor-driven, and power-driven (by belt or gearing); (3) the number of cylinders (or their equivalent), as simplex, duplex and triplex; (4) the number of stages or pump elements in series, as single-stage and multi-stage; (5) the mode of connection, as direct-acting, crank-driven and geared. Large steam-driven pumps are usually called pumping engines.

Piston and Plunger Pumps are the most common in use. In the single-acting type, water is taken in on one stroke and discharged on the return stroke. In the double-acting pump water is taken in and discharged on both strokes. In the direct-acting piston or plunger pump the steam piston and the water piston or plunger are both secured to the same piston rod and the steam is used non-expansively. Their steam consumption is consequently high, unless special compensating devices, substitutes for a flywheel, are used, as in the Worthington high-duty and the d'Auria pumping engines. Large piston pumps are usually provided with a crank and flywheel, so that the steam may be used expansively.

Centrifugal Pumps consist essentially of a rotating impeller which draws water in at its center and a stationary casing which guides the water to the discharge outlet. Centrifugal pumps having stationary guide-vanes inside of the casing are called "turbine" pumps. Centrifugal pumps are especially suited for low heads and large volumes, but are also built multi-stage for high heads. They are not as efficient as high-grade pumping engines, but are considerably cheaper.

Rotary Pumps. — Pumps with two parallel geared shafts carrying vanes or impellers which mesh with each other, and other forms of positive-driven apparatus, in which the water is pushed at a moderate velocity, instead of being rotated at a high velocity as in centrifugal pumps, are known as rotary pumps. They have an advantage over reciprocating pumps in being valveless, and over centrifugal pumps in working under widely-varying heads. They are usually not economical, but when carefully designed with the impellers of the correct cycloidal shape, like those used in positive rotary blowers, they give a moderately high efficiency.

Injectors. — The injector is a form of steam pump commonly used for feeding boilers. If a cylindrical tube 1 or 2 inches in diameter is reduced to one-half its diameter, or thereabouts, at one portion of its length, by gradual reduction and enlargement, and a smaller tube or nozzle inserted inside of it, so that the end of the smaller tube approaches the reduced section of the larger tube, then if the larger tube be connected to a supply of water. and steam at considerable pressure be introduced through the smaller tube or nozzle, the water will be drawn into the larger tube and ejected from its outer end with such force as to cause it to overcome the pressure in the boiler supplying the steam, and thus to feed water into the boiler. The apparent paradox of the injector is explained by the fact that the violent rush of steam into the water gives the latter a high velocity and the momentum thereby induced cannot be overcome in a limited space without the exertion of a force greater than that of the steam in the boiler. As a boiler feeder the injector has a remarkably high efficiency, for so much of the heat energy of the steam as is not converted into mechanical work is carried into the boiler as heat in the water, but as a pump for ordinary purposes its efficiency is very low, since less than 1 per cent of the thermal

energy of the steam is converted into work. The usefulness of the injector as a boiler feeder is limited by the fact that it will not handle hot water.

Direct-pressure Pumps. — These are of two types, the pulsometer and the air lift. In the pulsometer the water is raised by suction into the pump chamber by the condensation of steam within it, and is then forced into the delivery pipe by the pressure of a new quantity of steam on the surface of the water. Two chambers are used, which work alternately, one raising while the other is discharging. The air-lift pump consists of a vertical water pipe with its lower end submerged in a well and a smaller pipe delivering air into it at the bottom. The rising column in the pipe consists of air mingled with water, the air being in bubbles of various sizes, and therefore lighter than a column of water of the same height; consequently the water in the pipe is raised above the level of the surrounding water. The pulsometer is used for pumping out pumps, drains, etc., and the air lift for pumping from wells.

PERFORMANCE. — The performance of a pump is usually expressed in terms of steam consumption of the steam cylinder or engine driving it, the indicated horse power of the cylinder or engine driving it (or the brake horse power of the motor in case of a motor-driven pump), the mechanical efficiency of the pump, and the slip.

Useful or Water Horse Power. — Let p = difference in pressure in pounds per square inch between inlet and outlet of pump = pressure in pounds per square inch indicated by gauge on force main $\pm$ pressure in pounds per square inch indicated by gauge on suction main ($+$ if this reads below atmospheric pressure, $-$ if above) $+$ pressure in pounds per square inch corresponding to the distance between the two gauges; H = head in ft. corresponding to p; Q = actual discharge* in cubic feet per minute; W = pounds per minute discharged; w = weight of the fluid per cubic foot. For water at 62° F., w = 62.36 pounds and is about 0.1 per cent greater at 40° F., and 4 per cent less at 212° F. Then the useful or water horse power is

$$P_w = \frac{144\,pQ}{33{,}000} = \frac{pQ}{229} = \frac{pW}{229\,w} = \frac{HW}{33{,}000} = \frac{HwQ}{33{,}000}.$$

For water at 62° F., and approximately at any temperature between 32° and 212° F.,

$$P_w = \frac{pW}{14{,}300} = \frac{HQ}{529}.$$

Slip. — By the slip of a pump is meant the ratio of the difference between the piston displacement and the water delivered to the piston displacement.

Mechanical Efficiency. — By mechanical efficiency is usually meant the ratio of the useful or water horse power to the indicated horse power of the steam cylinder driving it, or to the brake horse power of the motor in the case of a motor-driven pump. Sometimes the useful or water horse power is figured on the basis of the amount of water corresponding to the piston displacement; the actual water delivered is then equal to the piston displacement multiplied by (1 − slip). The power required to deliver water at a given rate in the first case is equal to the useful or water horse power divided by the mechanical efficiency; in the second case the power required is equal to the useful or water

* Q is sometimes taken as equal to the piston displacement, which is greater than the actual water delivered, due to the slip of the water past the piston and valves.

horse power (reckoned on the basis of piston displacement) divided by the product of the mechanical efficiency and (1 − slip). (*See table below.*)

Duty. — The performance of a steam-driven pump is also expressed as the number of foot-pounds of useful work done by the pump (144 pQ × time in minutes) per million B.t.u. in the steam supplied to it above the temperature of the boiler feed water. (If the boiler takes water from several sources at different temperatures, the B.t.u. added by the boiler from each source must be reckoned separately.) This ratio is called the "duty" of the pump. Duty is also sometimes expressed as the useful work in foot-pounds done by the pump per 1000 pounds of dry steam supplied to it.

The efficiency of a pump falls off rapidly with use, due to wear of the moving parts, unless it is kept in first-class condition. The figures in the following table are for average full-load conditions at rated speed in ordinary practice:

EFFICIENCY AND DUTY OF PUMPS.

Type of Pump	Pounds steam per hour per h.p. of Useful Work	Mechanical efficiency	Per cent slip	Duty, million ft. lb. per million B.t.u.
Piston pumps:				
Small duplex, direct-acting	100 to 200	40 to 60	5 to 20	10 to 20
Compound, direct-acting, non-condensing	40 to 80	70 to 90	2 to 20	25 to 50
Ditto with "high duty" attachment	20 to 33	60 to 80	2 to 20	60 to 100
Single cylinder, flywheel, non-condensing	30 to 50	70 to 90	2 to 20	40 to 60
Multi-cylinder, fly wheel, non-condensing	25 to 40	70 to 90	2 to 20	50 to 80
Multi-cylinder, fly wheel, condensing	13 to 20	70 to 95	2 to 20	80 to 160
Direct-connected, motor-driven	....	50 to 80	2 to 20	60 to 80
Geared pumps	....	50 to 90	2 to 20	30 to 90
Centrifugal pumps:				
Single-stage for low heads	22 to 66	40 to 70	....	30 to 90
Multi-stage for high heads	25 to 50	40 to 60	....	40 to 80
Rotary pumps	26 to 50	50 to 80	5 to 20	40 to 75
Pulsometer	100 to 400	....	....	5 to 20
Air lift	....	10 to 40	....	10 to 25

Performance of Injectors. — Kneass, *Theory of the Injector*, states that the pounds of water delivered (w) per pound of steam supplied is equal, very approximately, to the ratio of the B.t.u. per pound in the steam supplied, reckoned from the temperature (t) of the discharge water to the difference between the temperature (t) of the discharge water, and the temperature (t_0) of the suction water. That is, letting r = heat of evaporation, h = heat of liquid, and x = quality of the steam, then

$$w = (rx + h - t + 32) \div (t - t_0).$$

ELECTRIC DRIVE.* — (*See also Motors, Industrial Applications of.*) Since the reciprocating pump is essentially a low-speed machine, limited to about fifty

* By D. B. Rushmore.

revolutions per minute or thereabouts, it requires a speed reduction, as by gearing, for connection to the driving motor. The centrifugal pump, on the other hand, is suitable for direct connection to motors operating at speeds of up to 3500 r.p.m.

Motors for Reciprocating Pumps. — In starting large reciprocating pumps the water may be delivered through a by-pass until the motor is up to speed, when the by-pass is gradually closed and the water delivered into the system. The load at starting, therefore, only consists of the friction losses, and usually does not exceed 25 per cent of the full-load torque. Small- and medium-size pumps may, however, be required to start under full load.

When direct-current motors are used, the compound-wound type is generally selected for single-acting pumps on account of the rather pulsating load, but for double and triplex pumps having steadier load characteristics the shunt-wound type is used to advantage. Both squirrel-cage and phase-wound induction motors are suitable, the latter, as a rule, being selected where it is desirable to reduce the starting current to a minimum or where a somewhat variable speed is required. Synchronous motors may and are frequently used for driving large pumps. By-pass valves must then, however, be provided for reducing the torque at starting as previously mentioned.

Motors for Centrifugal Pumps. — On account of the peculiar characteristics of centrifugal pumps special care is required in the selection of the motor drive. With a reciprocating pump operating at constant speed an increase of the resistance increases the pressure and therefore the load on the motor; but with a centrifugal pump an increase of the resistance reduces the load. The volume of water delivered by a reciprocating pump is not affected by the reduction of the head, but the required power is reduced. A reduction of the head with a centrifugal pump, however, increases the volume of water, and as the efficiency at the same time goes down rapidly, the load increases. It is, therefore, of importance to know what this overload, caused by a reduction of the head, amounts to and the duration of this overload; and the capacity of the motor should as a rule be governed by the low and not the high head conditions.

The condition of starting must also be given careful consideration in selecting the motor. In starting a centrifugal pump the discharge valve may be entirely closed until the motor comes up to speed, so that the motor may start as nearly light as possible. At rest the torque required is small, usually from 15 to 25 per cent of full-load torque, and this drops from 5 to 6 per cent as soon as the machine starts turning over. The pump casing is full of water, however, and as the machine comes up to speed this water is churned around in the casing, causing the motor to load up as it approaches full speed, when with pumps of the usual design it takes from 40 to 50 per cent of full-load torque to drive it even though pumping no water. Shunt-wound direct-current motors and either squirrel-cage or phase-wound induction motors are well adapted for this type of pump and will readily meet the above conditions. A synchronous motor may lead to difficulties unless proper precautions are taken in designing the starting winding and auxiliary starting equipment.

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[Wm. Kent.]

PYROMETERS. — (*See also Temperature and Thermometers.*) A pyrometer is any device for measuring high temperatures. By high temperature as here used is meant a temperature beyond the range of the ordinary mercury thermometer, say 350° C. and up. A great number of pyrometric methods have been proposed, the more important of which will be found treated in detail in Burgess and Le Chatelier's *Measurement of High Temperatures*, New York, 1912. The following brief treatment is adapted from this work.

CLASSIFICATION OF PYROMETERS. — The various types of pyrometers may be classified as follows. Some of these are more fully treated in the following sections.

Gas Pyrometer (Pouillet, Becquerel, Sainte-Claire-Deville, Barus, Chappuis, Holborn, Callendar, Day). — Utilizes the measurement of change in pressure of a gaseous mass kept at constant volume. Its great volume and its fragility render it unsuitable for ordinary measurements; it serves only to give the definition of temperature and should only be used to standardize other pyrometers.

Calorimetric Pyrometer (Regnault, Violle, Le Chatelier, Siemens). — Utilizes the total heat of metals, platinum in the laboratory and nickel in industrial works. It was formerly used for intermittent researches in industrial establishments because its employment demands almost no apprenticeship and because the cost of installation is not great. The cost of a metal ball to withstand over 1000° C. prohibits its use above this temperature in industrial works. Other and more convenient types of pyrometers have almost entirely superseded the calorimetric pyrometer.

Total Radiation Pyrometer (Rosetti, Langley, Boys, Féry, Thwing). — Utilizes the total heat radiated by warm bodies. Its indications are influenced by the variable emissive power of the different substances. Convenient for the evaluation of very high temperatures which no thermometric substance can withstand (electric arc, sun, very hot furnaces), or when it is not convenient to approach the body whose temperature is wanted. Can be made self-registering.

Optical Pyrometer (Becquerel, Le Chatelier, Wanner, Holborn-Kurlbaum, Morse). — Utilizes either the photometric measurement of radiation of a given wave length of a definite portion of the visible spectrum, or the disappearance of a bright filament against an incandescent background. Its indications, as in the preceding case, but to a much less degree, are influenced by variations in emissive power. The intervention of the eye aids greatly the observations, but diminishes notably their precision. This method is mainly employed in industrial works for the determination of the temperatures of bodies difficult of access — for example, of bodies in movement (casting of a metal, the hot metal passing to the rolling mill). Can be used to estimate the highest temperatures and is the best method for use above 1700° C. in laboratory and industrial works.

Electric Resistance Pyrometer (Siemens, Callendar, Waidner and Burgess). — Utilizes the variations of electric resistance of metals (platinum) with the temperature. This method permits of very precise measurements to 1000° C., but requires the employment of fragile apparatus. It merits the preference for very precise investigations in laboratories. As a secondary instrument for the reproduction of a uniform temperature scale throughout the range in which the platinum resistance thermometer can be used, to 1000° C. except in very heavy wire, it is unsurpassed in precision and sensibility. It is also now constructed in convenient form for industrial use.

Thermoelectric Pyrometer (Becquerel, Barus, Le Chatelier). — Utilizes the measurement of electromotive forces developed by the difference in temperature of two similar thermoelectric junctions opposed one to the other. In employing for this measurement a Deprez-d'Arsonval galvanometer with movable coil or a millivoltmeter one has an apparatus easy to handle and of a precision amply sufficient for industrial and many scientific uses. With a potentiometer an instrument is obtained of the highest precision, available for use to 1600° C., or even to 1750° C. with proper precautions. This pyrometer was used for a good many years in scientific laboratories, before it spread into general industrial use, where it also renders most valuable service.

Contraction Pyrometer (Wedgwood). — Utilizes the permanent contraction which clayey materials undergo when submitted to temperatures more or less high. It is employed today only in a few pottery works.

Fusible Cones (Seger). — Utilize the unequal fusibility of earthenware blocks of varied composition. Give only discontinuous indications. Such blocks studied by Seger are spaced so as to have fusing points distant about 20° C. In general use in pottery works and in some similar industries.

Other Pyrometers (Hobson, Uhling-Steinbart, Job, Fournier). — There are a number of other pyrometers which have been found suitable in special cases or which for one reason or another have been found convenient in some particular line of work. Among these are the various industrial instruments based on the relative expansion of metals or of a metal and graphite used in air blasts and metal baths and pyrometers based on the flow or on the pressure of air or vapor.

TOTAL-RADIATION PYROMETERS. — These instruments utilize the radiant heat of *all* wave lengths given off by the body whose temperature is to be measured or by an auxiliary body at the same temperature. By means of a focusing device the radiant heat from a small portion of the hot surface is caused to fall upon a suitable detector. Various devices have been used as detectors, but in modern radiation pyrometers the thermocouple in conjunction with a galvanometer, millivoltmeter or potentiometer is almost universally employed. Instead of the thermocouple a spiral bi-metallic spring, with a pointer attached, is used in Féry's *spiral* pyrometer.

Total-radiation pyrometers can be made self-registering by simply substituting for the indicating galvanometer a suitable recording instrument.

Conditions of Use. — To obtain accurate results with a radiation pyrometer it should be sighted upon the bottom of a closed-end tube inserted into the furnace or bath. The radiating properties of the bottom of such a tube approach very closely those of a "black body" (*see Heat and Thermal Properties*); i.e., the total energy radiated upon the receiving device is proportional to the difference in the fourth powers of the absolute temperature of the hot body and that of the receiving device. Since the latter temperature is usually low compared with the temperature of the hot body, the radiation is practically proportional to the fourth power of the absolute temperature of the hot body. Hence, if the relation between the absolute temperature T_0 and the deflection D_0 of the galvanometer attached to the thermocouple is known at one temperature, and the deflection is known to be proportional to the amount of heat falling on the thermocouple, then the absolute temperature T corresponding to any other deflection D is

$$T = T_0 \sqrt[4]{\frac{D}{D_0}}.$$

It should be noted that this law does not apply unless the instrument is focused upon a "black body." If the instrument is focused upon objects in the

open air its readings, if calibrated by the above law, will be too low, due to the selective radiating properties of all materials. However, the instrument may be calibrated by comparison with a standard pyrometer, e.g., a thermoelectric pyrometer, to give true surface temperatures when sighted upon any particular kind of surface, but this calibration will not hold for other surfaces.

Féry Radiation Pyrometer. — In this apparatus a concave mirror (gold on glass) is used to focus the rays upon the thermocouple. The gold mirror may be considerably tarnished without seriously influencing the readings; and if the aperture of the furnace sighted upon is of sufficient size and the telescope in focus, the temperature readings are practically independent of the distance. The instrument takes its final reading very promptly with only slight creep.

Foster has also transformed the Féry telescope into a "fixed-focus pyrometer" by putting the thermocouple and the aperture at the *conjugate foci* of the gold mirror. In a similar instrument recently issued by the Brown Pyrometer Company, the sighting of the instrument is facilitated by the use of a finder such as used with photographic cameras. The Féry pyrometer of constant-focus type has been coupled directly to a long closed-end tube by Whipple, so that the closed end may be plunged directly into the hot region or melted metal.

Thwing Radiation Pyrometer. — In Thwing's apparatus the reflecting mirror is replaced by a conical cone which by multiple reflection concentrates the radiation at its apex on one or more thermocouples in series with a portable galvanometer.

OPTICAL PYROMETERS. — These instruments utilize only the visible portion of the spectrum, and their indications depend on the comparison, by the eye, of the equality of brightness of two images, one of the object whose temperature is sought and the other a standard light source. Such instruments are therefore essentially the same as photometers (*see article on Photometers*). As a rule the comparison is made with approximately monochromatic light, the images being viewed through a colored glass, usually red.

Temperature and Intensity of Illumination. — Wien's Law. — Wien has shown that the intensity of the light given out by a "black body" (*see Heat and Thermal Properties*) may be expressed by the formula

$$I = A\epsilon^{-\frac{B}{T}},$$

where A and B are constants for a *given wave length*, ϵ is the base of the natural system of logarithms and T the *absolute temperature*. In general, however, the energy of a given wave length radiated by a body in the open air is less than that of a "black body," i.e., its emissive power is less than unity, and the emissive power varies with the temperature. The above law, however, is usually assumed to apply to such bodies, and although the temperature as thus obtained may differ from the true (gas thermometer) temperature by from 50 to 100° C., a *consistent* scale of temperature is obtained for any *particular* substance observed.

"Black Body" Temperature. — The temperatures indicated by a radiation pyrometer that has been calibrated against a black body, or on the assumption of the laws of "black body" radiation, are known as black-body temperatures. Thus, were a piece of iron and a piece of porcelain both at 1200°, the optical pyrometer, which used the red light emitted by these bodies, would give, as the temperature of these bodies, 1140° and 1100° C. respectively. This means that iron and porcelain at 1200° C. emit red light of the same intensity as is emitted by a black body at 1140° and 1100° C. respectively.

Féry Absorption Pyrometer (Fig. 1). — The optical system of this pyrometer is shown in the figure. pp' are a pair of absorbing-glass wedges, G is a

mirror with only a narrow central strip silvered over, L is the standard light source, focused on the mirror G by the lens l. The resultant field, when observing a small crucible, is then as shown at ab. r is a red glass in the eye-piece. The instrument has a fixed angular aperture, so that no correction has to be made for focusing or for varying distance from furnace. The range of the instrument may be extended by the use of auxiliary absorbing glasses A, A'. The instrument is movable about a horizontal axis, which is a convenience in sighting.

A setting is made by adjusting the thickness of the absorbing glasses pp' by moving the wedges together by means of a micrometer screw. Let x = setting of micrometer screw, T = absolute temperature, then, assuming Wien's law to apply

$$T = \frac{a}{x+b},$$

where a and b are two constants, for a given set of wedges pp', and can be obtained by observing two known temperatures.

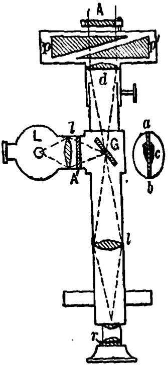

Fig. 1. Féry Absorption Pyrometer

Le Chatelier's Optical Pyrometer. — The Féry pyrometer is a modification of an earlier form devised by Le Chatelier. The latter used an iris diaphragm instead of the wedges, the setting of the instrument being accomplished by changing the aperture. With this arrangement greater sensibility can be obtained, but with the decided disadvantage that the calibration will then hold only for a fixed distance of the instrument from the object viewed.

Wanner Pyrometer (Fig. 2). — In this instrument a Nicol prism is used to vary the relative intensities of the light received from the standard lamp and from the object viewed, the angle through which the prism must be turned to give equal illumination being a measure of the relative intensity of the light emitted from the two sources. The principle is the same as that of a König spectrophotometer (*see Photometers*).

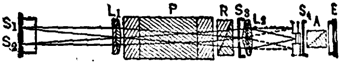

Fig. 2. Wanner Pyrometer

The slit S_1 is illuminated by light from the comparison source, a small 4-volt electric lamp not shown in the figure reaching S_1 after diffuse reflection from a right-angled prism placed before S_1. Light from the object whose temperature is sought enters the slit S_2. If the analyzer is at an angle of 45 degrees with the plane of polarization of each beam, and if the illumination of S_1 and S_2 is of the same brightness, the eye will see a single red circular field of uniform brightness. If one slit receives more light than the other, one-half of the field will brighten, and the two may be brought to equality again by turning the analyzer carrying a graduated scale, which may be calibrated in terms of temperature. Let ϕ = angle through which prism is turned to establish equal illumination, T = absolute temperature of furnace or bath; then, assuming Wien's law to apply,

$$T = \frac{a}{\log(\tan\phi) - b},$$

where a and b are constants determined by calibration. Only two known tem-

peratures need be observed to obtain these constants, but it is safer to plot a calibration curve from several known temperatures.

In the latest form of this instrument the details of its mechanical construction have been improved, and it has been made direct-reading by providing a second scale on the instrument graduated in temperatures, corresponding, of course, to a definite normal point and for a source approximating a black body.

Incandescent Lamp Pyrometers. — In this type of instrument the current through the filament of an incandescent lamp is adjusted until a portion of the filament is of the same color and brightness as the object. When this occurs this part of the filament becomes invisible against the bright background, and the current then becomes a measure of the temperature as given either by a thermocouple or in terms of the intensity of illumination. This principle appears to have been first used by Morse and independently developed by Holborn and Kurlbaum. An absolute match of both color and brightness cannot be made unless monochromatic light is used or unless the lamp filament and viewed object radiate similarly.

Holborn-Kurlbaum Form (Fig. 3). — A small 4-volt electric incandescent lamp L with a horseshoe filament is mounted in the focal plane of the objective and of the eye-piece of a telescope provided with suitable stops D, D, D, and a focusing screw S for the objective. The lamp circuit is completed through a two-cell storage battery B, a rheostat, and a milliammeter.

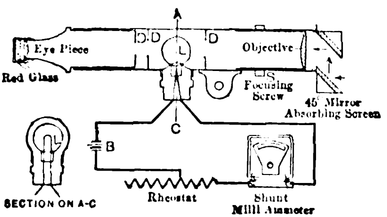

Fig. 3. Holborn-Kurlbaum Pyrometer

The determination of a temperature consists in focusing the instrument upon the incandescent object, thus bringing its image into the plane AC, and adjusting the current by means of the rheostat until the tip of the lamp filament disappears against the bright background, when a previous calibration of current, in terms of temperature for the particular lamp used, gives the temperature by reading the milliammeter. Above 800° C. one or more red absorbing glasses are used.

For the calibration of the instrument it is necessary to find empirically the relation between the current through the lamp and the temperatures for a number of temperatures, and then interpolate either analytically, or more conveniently, graphically. The calibration will be an independent one for each lamp used. If the lamp is not aged its indications may change by as much as 25° C. with time, but after twenty hours' heating at 1800° C. it will undergo no appreciable further changes over a period of many months if not heated above 1500° C.

By the substitution of tungsten for carbon filaments even greater permanence may be had, but the selective radiation of the metallic filament may then be a source of error or inconvenience in certain cases.

Morse and Henning Pyrometers. — The Morse pyrometer is based on the same principle as the Holborn-Kurlbaum, but the construction is somewhat different. The Henning pyrometer is essentially a combination of the Holborn-Kurlbaum instrument with a spectrometer for utilizing monochromatic light.

RESISTANCE PYROMETERS. — These instruments are based upon the variation of electrical resistance with temperature, and are suitable for measuring temperatures throughout the range from the lowest obtainable temperatures to about 1200° C. Platinum wire is usually employed as the resistor, though nickel may be used for temperatures up to 400° C.

Platinum Scale of Temperatures. — The platinum scale of temperatures is defined by the relation

$$p_t = \frac{R_t - R_0}{R_{100} - R_0} \times 100,$$

where R_0 and R_{100} are the resistances of the platinum wire at the temperature of ice and boiling water (760 millimeters mercury pressure) respectively, and R_t is the resistance at any other temperature t. For this scale to agree with the gas thermometer scale would require that the temperature coefficient of resistance of platinum (on the gas scale) be a constant equal to

$$c = \frac{R_{100} - R_0}{100\, R_0}.$$

As a matter of fact the temperature coefficient is not constant but the relation between temperature (on the gas scale) and resistance is

$$R_t = R_0\,(1 + at - bt^2)$$

where a and b are constants. Consequently the platinum temperature and the gas temperature differ by an amount

$$t - p_t = \delta\left(\frac{t}{100} - 1\right)\frac{t}{100},$$

where

$$\delta = \frac{10{,}000\, b}{a - 100\, b}.$$

The fundamental constants of a platinum thermometer are then R_0, c, and δ. These can be determined by measuring the resistance at three known temperatures, *viz.*, melting point of ice, boiling point of water and boiling point of sulphur (444.7° C. on the gas scale). For the purest platinum $\delta = 1.50$ and $c = 0.0039$. The value of δ increases with the amount of impurity.

CORRECTIONS TO t FOR SMALL CHANGES IN δ

Centigrade scale				Centigrade scale			
	Δt for $\Delta\delta = 0.01$		Δt for $\Delta\delta = 0.01$		Δt for $\Delta\delta = 0.01$		Δt for $\Delta\delta = 0.01$
50	−0.002	300	+0.060	550	+0.247	800	+0.560
100	.000	350	.087	600	.300	850	.637
150	+ .008	400	.120	650	.357	900	.720
200	.020	450	.157	700	.420	950	.807
250	.037	500	.200	750	.487	1000	.900

Computations of t from p_t are made by the table on p. 1149, as if the thermometer had $\delta = 1.50$. The above corrections (Δt) are then applied to the computed values of t for the value of δ proper to the thermometer.

Example. Let $p_t = 470.00$, whence $t = 500.00°$ C. by table on p. 1149. If $\delta = 1.52$, the corrected value of t is 500.40° C.

VALUES OF TEMPERATURE CENTIGRADE (t) IN TERMS OF PLATINUM TEMPERATURES (p_t) FOR THERMOMETERS WITH $\delta = 1.500$

p_t	t	Difference for 1° p_t	p_t	t	Difference for 1° p_t	p_t	t	Difference for 1° p_t	p_t	t	Difference for 1° p_t
0	0.000	0.985	250	255.99	1.066	500	534.89	1.170	750	844.26	1.313
10	9.867	0.988	260	266.67	1.070	510	546.62	1.175	760	857.42	1.319
20	19.762	0.991	270	277.38	1.073	520	558.40	1.180	770	870.65	1.326
30	29.687	0.994	280	288.13	1.077	530	570.22	1.185	780	883.95	1.333
40	39.641	0.997	290	298.92	1.081	540	582.10	1.190	790	897.32	1.340
50	49.625	1.000	300	309.75	1.084	550	594.03	1.195	800	910.76	1.347
60	59.639	1.003	310	320.61	1.088	560	606.00	1.200	810	924.28	1.355
70	69.683	1.006	320	331.51	1.092	570	618.03	1.205	820	937.87	1.363
80	79.758	1.009	330	342.46	1.096	580	630.11	1.210	830	951.54	1.370
90	89.863	1.012	340	353.44	1.100	590	642.24	1.216	840	965.28	1.378
100	100.00	1.015	350	364.46	1.104	600	654.43	1.222	850	979.10	1.386
110	110.17	1.018	360	375.52	1.108	610	666.67	1.227	860	993.01	1.394
120	120.37	1.021	370	386.62	1.112	620	678.97	1.232	870	1007.00	1.403
130	130.60	1.024	380	397.76	1.116	630	691.32	1.238	880	1021.07	1.411
140	140.86	1.027	390	408.95	1.120	640	703.73	1.244	890	1035.23	1.420
150	151.16	1.031	400	420.18	1.125	650	716.20	1.250	900	1049.47	1.428
160	161.49	1.034	410	431.45	1.129	660	728.73	1.256	910	1063.80	1.437
170	171.85	1.038	420	442.77	1.134	670	741.32	1.261	920	1078.21	1.445
180	182.25	1.041	430	454.13	1.138	680	753.97	1.267	930	1092.71	1.455
190	192.68	1.044	440	465.53	1.142	690	766.67	1.274	940	1107.31	1.464
200	203.14	1.048	450	476.97	1.146	700	779.44	1.280	950	1122.00	1.474
210	213.64	1.052	460	488.46	1.151	710	792.27	1.286	960	1136.79	1.484
220	224.18	1.055	470	500.00	1.156	720	805.17	1.293	970	1151.69	1.494
230	234.75	1.058	480	511.58	1.160	730	818.13	1.299	980	1166.68	1.503
240	245.35	1.062	490	523.21	1.165	740	831.16	1.306	990	1181.76	1.513
250	255.99	1.066	500	534.89	1.170	750	844.26	1.313	1000	1196.95	1.524

For a full discussion of the platinum temperature scale and its relation to the gas-thermometer scale, see Burgess and Le Chatelier, *Measurement of High Temperatures.* The brief discussion given above is adapted from this treatise, from which also the tables are taken.

Forms of Resistance Thermometers and Pyrometers. — A complete outfit consists of the thermometer proper or "bulb," the indicating or recording instrument and two or three dry cells for operating the same. Where the indicating or recording instrument is to be in circuit continuously, it is best to operate it from a direct-current lighting circuit or storage-battery system.

With the proper switching device a number of bulbs at widely-separated points may be used with the same indicator or recorder.

Thermometer Proper or Bulb. — Since practically every problem in temperature measurement presents different conditions there can be no general type or form of resistance thermometers. The following descriptions are of some of the thermometers now on the market that have proved satisfactory for certain classes of work.

Resistance thermometers for use below 125° C. are of relatively simple construction, for in this case silk-insulated nickel wire may be used. Such a thermometer is shown in section in **Fig. 4.** This particular thermometer has an over-all length *D* of 10¼ inches, a length of winding *C* of 4½ inches and a diameter *A* of ⅜ inch. The same type of thermometer is also made with a number of different lengths from 3¾ inches over-all to 12¼ inches over-all, and one form, designed to be particularly quick acting, is encased in a thin steel tube ¼ inch outside diameter. This type of thermometer is largely used for the measurement of the temperature of air, solids, concrete, water, and grains and fruit in transit. It may also be used as a wet-bulb thermometer.

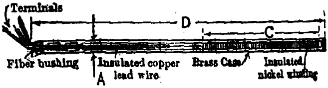

Fig. 4. Resistance Thermometer for Use up to 125° C.

The thermometer shown in Fig. 5 is a platinum resistance bulb for measurement of the highest precision. The winding is of pure platinum wound on a mica cross; the protecting tube is of royal Berlin porcelain. For shop use it is well to protect the porcelain tube with a seamless nickel case. Such a case will maintain its mechanical strength after long exposures to high temperatures. Thermometers of this type may be used up to 1000° C. This type of thermometer is also provided with a special fireproof head and with a steel protecting case in addition to the nickel case. This is necessary where the thermometer is to be used in connection with case hardening.

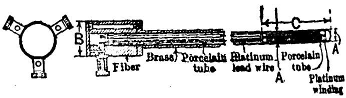

Fig. 5. Thermometer for Use up to 100° C.

The Leeds and Northrup Company make a bulb designed especially for measuring the temperature of generators, transformers, etc. The resistance winding of this bulb is about 3 inches long and is mounted on a flexible stem 30 inches or more in length, the width over-all is ½ inch and the thickness 1/25 inch. This resistance coil is wound non-inductively and is therefore unaffected by neighboring magnetic fields.

Indicating Devices. — The application of the various bridge, potentiometer, differential galvanometer and ohmmeter methods (q.v.) to the measurement of the resistance of the platinum coil and the methods of compensating for the resistance of the leads are described in detail in Burgess and Le Chatelier's *Measurement of High Temperatures*. Two types of indicators are employed: (1) the balance type which requires the adjustment of a rheostat to produce zero deflection of the indicating instrument, the temperature then being read in terms of the balancing resistance, current or p.d., and (2) the direct-reading type in which the deflection of the instrument, read on a properly calibrated scale, gives the temperature directly.

THERMOELECTRIC PYROMETERS. — These instruments consist of a thermocouple and some device for measuring the e.m.f. generated due to the difference in temperature between the hot and cold junctions of the thermocouple, for which the relation between e.m.f. and temperature difference is known. The thermocouple consists of two parallel wires of different materials joined together at one end but insulated from each other throughout their length. The two ends which are joined form the hot junction and the two open ends are usually referred to as the cold junction, although they are not connected directly to each other. The two cold ends are, however, connected electrically through the device used for measuring the e.m.f. generated, thus forming a closed circuit.

Thermocouple Circuits. — The net e.m.f. tending to establish a current in such a circuit as described above is the algebraic sum of all the contact e.m.f.'s (*see Electricity and Magnetism, Principles of*) in the circuit. Wherever there is a change in the chemical nature or physical properties of the conductors forming such a circuit there exists such a contact e.m.f. These various contact e.m.f.'s are functions of the temperature of the transition points, but if the entire circuit is at the same temperature throughout, the net e.m.f. is zero (provided the circuit is formed solely of *metallic* conductors). If, however, *any* transition point is at a higher temperature than another, the net e.m.f. will in general not be zero. Consequently, when a thermocouple is used to measure the difference in temperature between the hot and cold junctions of the couple itself, care must be taken to eliminate the e.m.f.'s due to the temperature differences between the various other transition points in the circuit.

Transition points occur (1) at the junction of any two dissimilar metals; (2) wherever there is a change in the homogeneity of the structure of any of the metals in the circuit, and (3) wherever there is a change in the temperature of the metal, although it may be perfectly homogeneous. To avoid errors due to the e.m.f.'s at the transition points other than the hot and cold junctions of the couple itself, it is necessary to use for the latter metals which give a relatively *high* e.m.f. for a given temperature difference between the two junctions and which are also *homogeneous* throughout, and to so dispose the rest of the circuit that the e.m.f.'s produced therein as the result of temperature differences neutralize one another.

Metals for Thermocouples. — For high-precision thermoelectric pyrometers platinum against platinum-iridium or platinum-rhodium alloys (containing 10 per cent iridium or rhodium) are usually employed. For industrial purposes cheaper metals and alloys are used, such as nickel, copper, iron and alloys of these metals; such couples are usually referred to as *base* metal couples, as contrasted with the *noble* metal couples formed of platinum, iridium, etc.

Thermocouples for Industrial Use. — For industrial use cheap and robust couples are required. Such couples may be made either entirely of the

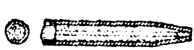

Fig. 6. Base Metal Couple

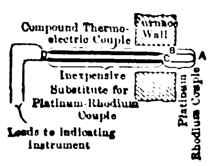

Fig. 7. Compound Couple

base metals or a base-metal couple may be used in series with a short length of a platinum-alloy couple, the latter only being exposed to the highest temperature. In Fig. 6 is shown a convenient form of a simple base-metal couple, and in Fig. 7 one form of compound couple.

Simple base-metal couples can be made of low resistance and their e.m.f. may therefore be measured with a fair degree of accuracy by low-resistance millivoltmeters of robust construction. Base-metal couples made of alloys or metals possessing critical regions, accompanied by the absorption or liberation of heat, are liable to give discordant results on reheating. These effects are particularly marked in couples of considerable size, and are enhanced by varying the depth of immersion of the couple in the test bath or furnace.

Although a very considerable number of base-metal thermocouples has been put upon the market in recent years, there appears to be very little certain

knowledge available as to the exact composition, thermoelectric properties, and behavior of most of them, some of which are quite complex alloys, as, for example, of Ni, Cr, Al and Cu.

Insulation and Protection of Thermocouples. — The two leads should be insulated from one another throughout their length. Use may be made in the laboratory of glass tubes or pipe stems, or of thread of pure asbestos wound about the two wires, by crossing it each time between the two so as to make a double knot in the form of an 8, each of the wires passing through one of the loops of the 8. This is a convenient method of insulation for laboratory use, although ordinary asbestos is likely to contain impurities which will damage the couple. With this arrangement it is impossible to go above 1200° or 1300° C., at which temperature asbestos melts. A more satisfactory insulation, however, is had by means of thin tubes of hard porcelain standing 1500° C. or of quartz tubes standing about 1200° C. Quartz, however, gradually crystallizes and crumbles above 1200° C., and in the presence of a volatile reducing agent, as graphite or hydrogen, volatile silicides are formed above 1200° C., which will destroy platinum.

For industrial installations use may be made of small fire-clay cylinders of 100 millimeters in length and 10 millimeters in diameter, pierced in the direction of the axis by two holes of 1 millimeter diameter, through which pass the wires, or hard porcelain tubes may be used. One or another of the other forms of insulator is added in sufficient numbers. They are placed, according to the case, in an iron tube or in a porcelain tube. The porcelain tube should be employed in fixed installations in which the temperatures may exceed 800° C.

E.M.F.'s of Thermocouples. — By the e.m.f. of a thermocouple is meant the e.m.f. developed in a circuit consisting solely of the metals forming the thermocouple itself; i.e., the e.m.f.'s in the external circuit are not considered. If E_{sa} is the e.m.f. of a couple made of a metal a against any standard metal s, and E_{sb} is the e.m.f. of a couple made of a metal b against this same standard metal s, the e.m.f.'s in each case being for the same temperature of the cold junction (standard temperature is 0° C.) and for the same difference in temperature between the hot and cold junctions, then the e.m.f. of a couple made of a and b is, for these same temperatures

$$E_{ba} = E_{bs} + E_{sa} = E_{bs} - E_{as}.$$

(In this notation *rises* of potential are considered to be in the direction of the subscripts, that is, E_{ba} is the rise of potential from b to a). Consequently the insertion of a metal wire of any material in the circuit of a thermocouple does not change the net e.m.f., *provided this wire is of uniform structure and temperature throughout.* Consequently the hot junction may be soldered with any other metal which will not melt at the temperature to which the junction is exposed.

Electromotive Force Formulas. — No simple relation exists between the e.m.f. (E) of a thermocouple and the temperature difference (t) between the hot and cold junctions. A number of approximate formulas, however, has been suggested. The following, due to Holman, is one of the simplest.

$$E = mt^n,$$

where m and n are constants, the cold junction being kept at 0° C. For convenience in plotting and calculation this may be written

$$\log E = n \log t + \log m.$$

Holman's formula when applied to couples made of platinum and its alloys of iridium and rhodium gives results accurate to about 2° C. for t between

200° C. and 1200° C. Typical values of n and log m are given below (Cambridge Scientific Instrument Company).

	n	log m
Pt against (Pt 90%, Ir 10%)............	1.102	0.895
Pt against (Pt 90%, Rh 10%)..........	1.189	0.526

The values of these constants, however, differ with different makes of couples, and in any particular case should be obtained from the maker as determined by calibration.

Another formula, due to Holborn and Day, is the following,

$$E = -a + bt + ct^2,$$

where a, b and c are constants. Between 300° C. and 1100° C. this formula gives results accurate to 1° C. when applied to couples made of platinum and the various platinum alloys.

Ideal wire (*see Wires, Resistance*) and copper form a very satisfactory couple of relatively large e.m.f. suitable for a range of temperature between 0° and 400° C. With the cold junction at 0° C. the temperature of the hot junction and the e.m.f. are related as follows:

Temp. Hot Junct., °C.........	50	100	150	200	250	300	350
E.m.f., microvolts...	2.3	4.65	7.05	9.45	11.90	14.43	17.08

Thermoelectric Power (H) of Thermocouples. — The thermoelectric power of a thermocouple is defined as the increase in the e.m.f. developed per degree increase in the difference in temperature between the hot and cold junctions. This quantity, which is equal to $b + 2ct$ in Holborn and Day's formula, depends in general upon the temperature (t) of the hot junction; the following values are close approximations within the range stated.

THERMOELECTRIC POWERS OF THERMOCOUPLES
Microvolts per °C. increase in temperature, t in °C.

Thermocouple	Thermoelectric power	Temperature range, ° C.	Authority
Pt and 90% Pt, 10% Rh.............	$4.3+0.0088\,t$	0–1300	Le Chatelier
Pt and 90% Pt, 10% Ir..............	$11.3+0.0104\,t$	0–1000	Le Chatelier
Pt and Ni..........................	$7.8+0.01325\,t$	300–1300	Burgess
Cu and Ni..........................	$24.4+0.016\,t$	0–235	Pécheux
Cu and Constantan *................	$42.3+0.058\,t$	0–320	Pécheux
Pt and Fe (forged).................	$2.5+0.0210\,t$	700–1000	Le Chatelier

* 60% Cu, 40% Ni.

Measurement of E.M.F. of Thermocouples. — Two methods may be used to measure the electromotive force of a couple: the potentiometer method and the galvanometric method (*see Potentiometers; Galvanometers*). The first is the more accurate and is usually made use of in laboratories. The second method is simpler, but possesses the inconvenience of giving only indirectly the

measure of the electromotive force by means of a measurement of current strength.

There are sources of error, however, inherent in the galvanometric method, such as effects of lead resistance and temperature coefficients of leads and galvanometer, which are difficult if not impossible of complete elimination even with the best apparatus available. The potentiometer method, on the other hand, may be made, in so far as the measurements of e.m.f. are concerned, as exact as may be desired, or so that the only outstanding uncertainties are inherent in the thermocouple itself. These uncertainties, such as inhomogeneity, conduction along the wires, variable zero and actual change of e.m.f., are sometimes overlooked, giving rise to illusory accuracy.

The application of these two methods to the measurement of the e.m.f.'s of thermocouples is described in detail in Burgess and Le Chatelier's *Measurement of High Temperatures*.

Cold-junction Correction. — When the galvanometer method is used, it is often not convenient to keep the junctions of the couple to the lead wires of the galvanometer at a definite temperature, although the galvanometer itself may be so far removed from the furnace that its temperature changes are slight. Except in the roughest kind of work, allowance has to be made for the cold-junction temperatures, which may be measured by an auxiliary thermometer.

Calling t_0 the cold-junction temperature for which the instrument reads correctly, t the observed temperature of the cold junction, the correction to apply to the observed temperature readings of the galvanometer, otherwise supposed to read correctly for a given thermocouple, usually lies between $\frac{1}{3}(t - t_0)$ and $(t - t_0)$, depending on the type of couple and the temperatures of both hot and cold junctions. This question has been treated in detail for several types of couple by C. Offenhaus and E. H. Fischer (*Electrochem. and Met. Ind.*, *1908, Vol. 6, p. 362*).

Elimination of Cold-junction Correction. — Bristol has also devised an automatic compensator for cold-end temperatures, shown in Fig. 8, consisting of a small glass bulb and capillary tube partially filled with mercury, into which a short loop of fine platinum wire dips. This is inserted in the thermoelectric circuit close to the cold junction. Changes in temperature cause the mercury to expand or contract, cutting in or out resistance in the circuit. This acts in opposition to the change in e.m.f. with temperature at the cold end, so that a balance may be established if the parts are properly designed.

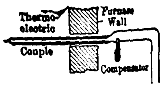

Fig. 8. Bristol's Compensator

In the Thwing instruments the elimination of the temperature variations of the cold ends of the couple, where they can be brought close to the galvanometer, is affected by a device consisting of a compound strip of two metals having unequal coefficients of expansion, so attached to the spring controlling the pointer that the reading of the galvanometer when no current is flowing is the temperature of the surroundings.

CALIBRATION OF PYROMETERS. — The methods employed are described in detail in Burgess and Le Chatelier's *High Temperature Measurements*. In general, the reading of the pyrometer is observed when the fire end is at the melting or boiling point of some solid or liquid. The melting and boiling points which possess the greatest reliability are given in the article on *Heat and Thermal Properties*.

COST, RANGE AND PRECISION OF TYPICAL PYROMETERS. — These items are indicated *approximately* in the table below. The precision stated is on the assumption of proper calibration and careful use.

Type	Range, ° C.	Precision ° C.	Cost, dollars
Nickel ball and calorimeter	100 to 1000	....	40
Féry radiation pyrometer	800 and up	(a)	450
Thwing radiation pyrometer	800 and up	(a)	180
Féry absorption pyrometer	900 and up	(a)	330
Wanner pyrometer	900 and up	(a)	200
Holborn-Kurlbaum pyrometer	900 and up	(a)	royalty basis
Resistance thermometer, " bulb " only	Up to 125	0.3	10
Resistance thermometer, " bulb " only	−200 to 1000	(b)	40
Balance indicator	(c)	(b)	75
Direct-reading indicator	(c)	(b)	45
Thermoelectric pyrometer			
Platinum-iridium couple (24 in. long)	(c)	(b)	30
Base-metal couple	(c)	(d)	10
Seger cones, each	600 to 2000	20–40	0.02
Iron-graphite expansion pyrometer	100 to 800	(b)	15 to 30

(a) Depends on upper limit, (b) depends on range and conditions of use, (c) depends on conditions under which it is used, (d) with millivoltmeter.

BIBLIOGRAPHY. — Burgess and Le Chatelier's *Measurement of High Temperatures*, N. Y., 1912. In this treatise will also be found a complete bibliography of the entire subject of pyrometry.

[H. PENDER AND H. R. RANKEN.]

RAILS, TRACK AND THIRD. — (*See also Bonds, Railway Track; Railways, Electric, Traction Systems for; Third-rail Systems; Trolley Systems.*) For city streets the track rail of the girder type, as standardized by the American Society of Civil Engineers, has been adopted as standard. Both the 7-inch and the 9-inch sizes are used, but the latter is better adapted to street paving. For interurban and trunk-line work the A.S.C.E. standard* "T" rail is used in sizes ranging from 40 to 100 pounds to the yard. Rails are generally used in 30 ft. lengths, but for city work where low resistance in the return circuit is essential (*see Electrolysis of Grounded Structures*) 60 ft. lengths are used. The longer rail also gives a better roadbed. A new type of rail, known as the Romapac rail, has recently been introduced, in which the base and head are rolled separately the head being crimped on to the base when the rail is installed, with the joints in the head staggered with the joints in the base; see *Bonds, Railway Track.*

RESISTANCE AND CHEMICAL COMPOSITION. — The chemical composition of steel rails with respect to the impurities or elements other than iron (chiefly carbon, manganese, phosphorus, sulphur and silicon) varies over a considerable range, depending upon the process of manufacture. According to J. A. Capp, the specific resistance of an ordinary† steel rail may be taken as a rough indication of the total impurities present. Fig. 1 shows the results of tests on a number of samples of different makes, ranging in total impurities

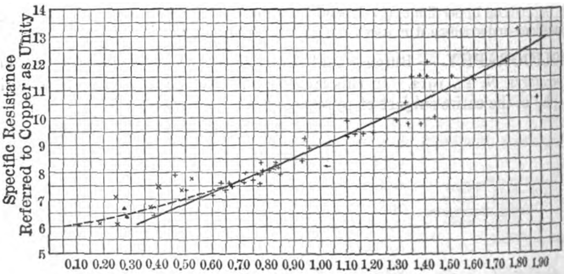

Fig. 1. (From Trans. A.S.M.E.)

from 0.1 per cent to 1.9 per cent, the specific resistance referred to copper as unity ranging from 6.1 to 13.3. The greater hardness caused by the presence of impurities is an advantage in the case of track rails which offsets the disadvantage of low conductivity, but it is usually economical to employ for the third or contact rail a rail of fairly high conductivity.

The compositions given in the tables on the following page are recommended by the authorities quoted.

* A modification of the present standard is now (1913) under consideration. The New York Central and Pennsylvania Railroads have their own standards.

† This does not apply to special forms of rails submitted in the process of rolling to extra heavy pressures.

PER CENT IMPURITIES AND SPECIFIC RESISTANCE OF STEEL RAILS

Item	Track rails			Third rails		
	Am. El. Ry. Assoc.			J. A. Capp		A. H. Armstrong
	Lower limit	Recommended	Upper limit	Recommended	Upper limit	Recommended
Carbon	0.60	0.68	0.75	0.15	0.20	0.12
Manganese	0.60	0.80	0.80	0.30	0.40	0.40
Phosphorus	....	....	0.04	0.06	0.06	0.10
Sulphur	....	....	....	0.06	0.06	....
Silicon	....	....	0.20	0.05	0.05	0.05
Approx. spec. res. referred to copper as unity		12.5		8.0		

PER CENT IMPURITIES AND SPECIFIC RESISTANCE OF SOME TYPICAL THIRD RAILS

Railroad	Per cent of impurities					Spec. res. referred to copper as unity
	Carbon	Manganese	Sulphur	Phosphorus	Silicon	
Long Island	0.080	0.022	0.029	0.074	0.074	7.55
District & Met. Ry., London	0.05	0.19	0.06	0.05	0.03	6.4
Manhattan Ry.	0.098	0.485	0.158	0.085	0.022	8.98
I. R. T. Subway, N. Y.	0.161	0.561	0.055	0.091	trace	8.56
New York Central	0.10	0.40	<0.08	<0.10	<0.05	7.85
Detroit River Tunnel	0.10	0.40	<0.08	<0.10	<0.05	7.85

Allowance for Wear in Resistance Calculations. — Resistance calculations should be made for rails worn down to the weight at which they will be scrapped. It is usual to scrap rails when they have lost from 10 per cent to 20 per cent of the original weight, depending upon the importance of the line. The resistances in the above table should therefore be increased from 10 per cent to 20 per cent.

RESISTANCE OF T-RAILS, A.S.C.E. STANDARD SECTION (1913)

(Full Cross section)

Weight, lb. per yard	Cross section, sq. in.	Area, millions of circular mils	Spec. res. 12.5 times that of copper		Spec. res. 8 times that of copper *	
			Ohms per 1000 ft.	Ohms per mile	Ohms per 1000 ft.	Ohms per mile
40	3.90	4.95	0.0261	0.138	0.0167	0.0882
45	4.40	5.60	0.0231	0.122	0.0148	0.0782
50	4.90	6.23	0.0208	0.110	0.0133	0.0702
55	5.40	6.86	0.0189	0.0996	0.0121	0.0637
60	5.90	7.50	0.0173	0.0911	0.0110	0.0583
65	6.40	8.14	0.0159	0.0840	0.0102	0.0538
70	6.90	8.77	0.0148	0.0779	0.00944	0.0499
75	7.437	9.45	0.0138	0.0729	0.00884	0.0467
80	7.80	9.9	0.0131	0.0689	0.00835	0.0441
85	8.34	10.5	0.0122	0.0645	0.00781	0.0413
90	8.83	11.2	0.0115	0.0609	0.00738	0.0390
95	9.30	11.8	0.0109	0.0570	0.00701	0.0370
100	9.82	12.5	0.0104	0.0547	0.00664	0.0350

* To find the resistance of rails of any specific resistance x referred to copper as unity multiply these resistances by x and divide by 8.

Resistance of Romapac Rails. — The conductivity of a rail is affected not only by the chemical composition but also by the treatment in manufacture, especially with regard to the pressure to which it is subjected in the rolling mills. The conductivity of the flange of a rail is greater than that of the head owing to the more direct pressure to which the flange is submitted in the rolling mill. In the Romapac rail, which has a separately rolled head crimped on to the base after the latter is installed, both the head and base are subjected to unusually direct pressure in the rolling mills. Romapac rails tested at Chicago contained about 1.27 per cent of impurities and if rolled in the ordinary way would have a resistivity of about 10.5 times that of copper. They were found to have a resistivity of about 6.5 times that of copper.

A-C. Resistance and Reactance. — See articles on *Trolley Systems, Overhead*; *Signaling, Railway*.

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[W. A. DEL MAR.]

RAILWAYS, ELECTRIC, TRACTION SYSTEMS FOR. — (*See also Automobiles, Electric; Bonds, Railway Track; Car Barns and Inspection Sheds; Cars, Electric; Conduits and Conduit Lines; Control Systems for Railway Motors; Depreciation; Electrolysis; Locomotives, Electric; Locomotives, Steam; Motors; Power Stations; Rails, Track and Third; Rectifiers; Signaling, Railway; Specifications; Substations, Railway; Third-rail Systems; Railways, Energy Requirements for; Railways, Location and Permanent Way for; Transmission Lines; Trolley Systems; Wires and Cables.*) The component parts of a railway system are the following: Generating station, usually three-phase alternating current; high-tension transmission line; substations with transformers and synchronous converters; low-tension distribution system, trolley and feeders; motive power or motor equipments; bonding and feeders for return circuit.

HISTORICAL DEVELOPMENT. — The history of electric traction is given briefly by a few salient epoch-making events.

1835. T. Davenport of Vermont built and exhibited a small model electric vehicle operating on a circular track.

1842. Davidson, Edinburgh, Scotland built a seven-ton car operated by primary batteries and electro-magnets.

1851. G. G. Page operated a small locomotive with primary batteries on the Baltimore and Ohio Railroad at Washington.

1877. S. D. Field in San Francisco operated a small car driven from a stationary remote generator.

1882. J. R. Finney first used the over-running trolley at Allegheny, Pa.

1883. Leo Daft equipped a full-sized passenger train running from Saratoga to Mt. McGregor, together with the over-running-trolley construction.

1886. Bently and Knight introduced the first conduit system in Cleveland, O.

1887–8. Several small installations.

1888. Frank Sprague equipped the Richmond Street Railway and demonstrated the practicability of the system. Bently and Knight equipped a road at Allegheny City. Thomson-Houston system installed at Washington, D. C. Van Depoele brought out the under-running trolley. The directors of the Boston West End System decided to adopt electric traction.

1896. Baltimore and Ohio tunnel at Baltimore electrified; the first electrification of a steam road.

For references to the original papers describing these early installations, see Burch, *Electric Traction for Railway Trains*, N. Y., 1911.

APPLICATIONS OF ELECTRIC TRACTION. — There are certain well defined fields in which electric traction is superior to other methods, the most important of which are the following:

1. Where the frequency of stops is so great as to require a high rate of acceleration in order to make a good schedule speed. If the service requires trains of several cars, any desired number of these cars can be made motor cars and thus a sufficient weight on the driving wheels can be economically secured to give the required adhesion and tractive effort.

2. Where local conditions prohibit the nuisance of the smoke, exhaust gases and noise of steam locomotives, as in cities and tunnels.

3. In heavy trunk-line service where the density of traffic is so great that a high load factor can be obtained with respect to both the power house and distribution system. The *operating* cost of an electric train is always less than the operating cost of a steam train but the *fixed* charges (interest on investment and depreciation) of the electric system are high, for the first cost for the electric equipment, viz., locomotives, motor-car equipment, distribution

system and power house, is much greater than the first cost of steam locomotives, and their accessory equipment. If there are sufficient trains in a system so that the pro rata share of the fixed charges for each train is less than the difference between the operating cost of an equivalent steam equipment and the operating cost of the total electric equipment then electric traction is advantageous.

SYSTEMS OF ELECTRIC TRACTION. — Three different types of motors are in use for electric traction, the direct-current motor, the single-phase commutator motor, and the three-phase induction motor; see articles on *Motors*. The trolley or third-rail voltages (i.e., volts between trolley and track rails or between third rail and track rails) and the motor voltages employed are as follows:

Direct-current Systems. — Trolley or third-rail voltages of 600, 1200 and 2400 volts are in use. The motors for the 600-volt system are designed for full trolley voltage. For the 1200-volt system both 1200-volt and 600-volt motors are used, in the latter case each pair of 600-volt motors being permanently connected electrically. For the 2400-volt system 1200-volt motors are used, each pair of motors being permanently connected in series electrically.

Single-phase Systems. — Trolley voltages (third rails are not used) of from 3000 to 11,000 volts and frequencies of 15 and 25 cycles per second are in use. The trolley voltage is stepped down to from 200 to 500 volts, for which voltage the motors are designed.

Three-phase Systems. — Two trolley wires for each track are required, the two wires and the track forming the necessary three conductors for the three-phase distribution. The usual frequency is 25 cycles per second. The voltages used between the two trolleys and between each trolley and track range from 6000 to 11,000 volts. Transformers are employed to step this three-phase voltage down to from 400 to 600 volts between motor terminals.

Comparison of the Various Systems. — For ordinary street railway service the 600-volt d-c. system is almost universally employed, but for interurban and trunk-line service there is a great difference of opinion as to which of the various systems is the most economical when all the factors are taken into account. The factors which must be considered in comparing the three systems in any particular case are the following:

1. For a given weight and length of trolley or third rail the per cent power loss for a given amount of power transmitted varies inversely as the square of the trolley or third-rail voltage.

2. The higher the trolley or third-rail voltage the fewer are the number of substations required for the same efficiency of distribution and weight of conductor.

3. The higher the trolley or third-rail voltage the more costly is the insulation and supporting structure, and also the greater is the cost of maintenance of the distribution system.

4. Both the first cost and the annual expense of the substations are less for the a-c. systems than for the d-c. systems, since for the former static transformers only are required whereas for the latter rotary converters must be used.

5. The relatively low power factor of a-c. motors (80 to 90 per cent) as well as the relatively low power factor of the line (due to the reactance of the a-c. trolley wire and track return) gives rise to a greater power loss in the a-c. distribution system for the same power delivered than in the case of the d-c. system, and this great loss and lower power factor makes necessary the employment of generating apparatus of greater kv-a. capacity.

6. The 600-volt d-c. motor, for the same horse-power rating and speed, costs less, weighs less and occupies less space than either type of a-c. motor. The high-voltage d-c. motors cost more, weigh more and occupy more space than the 600-volt type.

7. With the a-c. motors transformers are required on the locomotive, which adds to the cost and weight of the locomotive equipment.

8. The 600-volt d-c. motor costs less to maintain and is liable to fewer operating troubles than any of the other motors.

9. With the commutating type of a-c. motor the power lost in the control equipment is practically negligible, since the "potential" type of control can be used. For both the d-c. motor and the induction motor a resistance control is necessary, with consequent loss in power (*see Control Systems for Railway Motors*).

10. The induction motor is inherently a constant-speed machine, and consequently the power input varies directly as the opposing resistance. The d-c. motor and the a-c. commutator motor are inherently variable-speed machines, and the power input varies approximately as the square root of the opposing resistance, the speed at the same time falling off.

11. The three-phase induction motor, when kept connected electrically to the source of power, automatically operates as a generator when the train is going down grade at a speed greater than the synchronous speed of the motor, the motor thus returning power to the line and at the same time acting as a brake preventing any considerable increase in speed. "Regeneration," as this action is called, can also be obtained with the other types of motors but only at increased expense for the additional control equipment required.

Examples of the Use of the Various Systems. — In the following tables are listed (1) all the steam railroad electrifications in this country in 1913; (2) the interurban roads in this country using the a-c. system and (3) those using the high-voltage (over 750 volts) d-c. system; and (4) some of the more important steam railroad electrifications in Europe.

ANALYSIS OF AN ELECTRIC RAILWAY PROJECT. — In the analysis of a particular problem the following general line of procedure is followed:

1. Determine the number and capacity of cars to supply the service desired.
2. Determine the power and energy required to propel these cars at the schedule speed desired; see *Railways, Energy Requirements for.*
3. Select the motors to correspond to the power determined in 2; see *Railways, Energy Requirements for.*
4. Lay out distribution of cars, by train diagrams if necessary; see *Railways, Energy Requirements for.*
5. Calculate the capacity of the low-potential distribution system and of the substations; see *Substations, Railway; Railways, Energy Requirements for; Third-rail Systems; Trolley Systems.*
6. Determine the capacity of the generating stations and transmission system; see *Power Stations; Railways, Energy Requirements for; Transmission Lines.*
7. Estimate the first cost of the system; see articles on the various items involved.
8 Estimate the cost of operation; see articles on the various items involved.
9. Estimate the earning power of the system.

A. In the case of a new road the earning power must exceed the sum of the operating cost and fixed charges by an amount sufficient to pay dividends.

B. In the case of the electrification of a steam road it must be possible to show either: (1) that the result of electrification has been to reduce the operating charges by an amount more than sufficient to pay the fixed charges on the electrical apparatus, or, (2) that the result of electrification will increase the capacity of the road or attract sufficient new business so that the increased earning power will more than balance the increased fixed charges.

TABLE I. — ELECTRIFIED * STEAM ROADS IN THE UNITED STATES (1913)

Name of road	Year electrified	Miles of single track	Main line, tunnel, terminal	Trolley or 3rd rail	Voltage trolley A. C. or D. C.	Motor cars or locomotives
Nantasket Beach	1895	14	M. L.	T.	700 D-C.	M. C.
Baltimore & Ohio	1896	6.6	Tun.	3d R.	650 D-C.	L.
Chicago & Oak Park	1896	22	M. L.	3d R.	600 D-C.	M. C.
Hartford-New Britain	1896	11	M. L.	T.	500 D-C.	M. C.
Chicago South Side	1897	47	M. L.	3d R.	600 D-C.	M. C.
Hoboken Manufacturers	1897	4.5	Term.	T.	500 D-C.	L.
Buffalo & Lockport	1898	21	M. L.	3d R.	600 D-C.	L. & M. C.
Stamford-New Canaan †	1898	10	M. L.	T.	600 D-C.	M. C.
Providence, W. & Bristol	1899	33	M. L.	T.	625 D-C.	M. C.
Manhattan El.	1902	119	M. L.	3d R.	600 D-C.	M. C.
Northwestern Pacific North Shore	1903	34	M. L.	3d R.	600 D-C.	M. C.
Tafts-Central Vil., Conn.	1904	19.5	M. L.	T.	600 D-C.	M. C.
Hartford-Melrose, N. H.	1904	30	M. L.	T.	600 D-C.	M. C.
West Shore	1905	106	M. L.	3d R.	620 D-C.	M. C.
N. Y. Central	1906	234	Term.	3d R.	660 D-C.	L. & M. C.
Middletown-Meriden	1906	8.5	M. L.	T.	600 D-C.	M. C.
West Jersey & Sea Shore	1906	160	M. L.	3d R.	600 D-C.	M. C.
Denver Interurban	1907	51	M. L.	T.	11,000 A-C.	M. C.
Baltimore & Annapolis ‡	1907	26	M. L.	T.	6,600 A-C.	M. C.
Middletown-Berlin	1907	9.5	M. L.	T.	600 D-C.	M. C.
Erie R. R., Rochester	1907	40	M. L.	T.	11,000 A-C.	M. C.
Grand Trunk	1908	12	Tun.	T.	3,300 A-C.	L.
Visalia	1908	40	M. L.	T.	3,300 A-C.	L. & M. C.
N. Y., N. H. & H.	1908	276	Term.	T.	11,000 A-C.	L.
Denver & Intermountain	1909	17	M. L.	T.	500 D-C.	M. C.
Great Northern	1909	6	Tun.	T.	6,600 A-C.	L.
Mich. Central, Detroit	1910	19	Tun.	3d R.	600 D-C.	L.
Peninsular R. R., Cal.	1910	97	M. L.	T.	600 D-C.	L. & M. C.
Salt Lake & Ogden	1910	50	M. L.	T.	700 D-C.	L. & M. C.
Penn. — L. I. R. R.	1910	309	Term.	3d R.	675 D-C.	L. & M. C.
Southern Pacific, Oakland	1911	96	M. L.	T.	1,200 D-C.	M. C.
Boston & Maine (Hoosac Tunnel)	1911	25	Tun.	T.	11,000 A-C.	L.
Butte, Anaconda & P.	1912	114	M. L.	T.	2,400 D-C.	L.

* In whole or in part.
† Since 1908 single phase a-c., 11,000 volts.
‡ Changed to d-c. in 1913.

TABLE II.—ELECTRIC INTERURBAN ROADS IN THE UNITED STATES (1913)

Using the Single-phase A-C. System

Name of road	Year opened	Miles of single track	Voltage at trolley	Motors, number and H.P.
Indianapolis & Cincinnati	1904	112	3,300	4-100
Westmoreland County	1905	7	1,300	4- 50
Vallejo & Napa Valley	1905	34	3,300	4-100
Warren and Jamestown	1905	26	3,300	4- 50
Sea Cliff, L. I.	1905	6	2,200	2- 50
Toledo & Chicago	1906	43	3,300	4- 75
Spokane & Inland Empire	1906	162	6,600	4-100
Ft. Wayne & Springfield	1907	22	6,600	4- 75
Pittsburg & Butler	1907	39	6,600	4-100
Richmond & Chesapeake	1907	16	6,600	4-125
Windsor Essex & Lake S.	1907	40	6,600	2-100
Anderson Traction	1907	20	3,300	4- 75
Hanover & York	1908	21	6,600	4- 75
Chicago, L. S. & S. Bend	1908	90	6,600	4-125
Stamford-New Canaan	1908	8	11,000	4-125
Shawinigan Railway	1908	1	6,600	4-150
Rock Island Southern	1910	52	11,000	4-100
N. Y., Westchester & Boston	1911	63	11,000	4-150

TABLE III.—ELECTRIC INTERURBAN ROADS IN THE UNITED STATES (1913)

Using the High-voltage D-C. System

Name of road	Date	Miles of single track	Voltage at trolley	Motors, number and H.P.
Indianapolis & Louisville	1907	42	1200	4- 75
Central California Traction	1908	69	1200	4- 75
Pittsburg, Harmony & Butler	1908	77	1200	4- 75
Aroostook Valley	1910	15	1200	4- 50
Milwaukee Electric Railway	1910	68	1200	4-125
Oakland-Antioch	1910	35	1200	4- 75
Shore Line Electric	1910	52	1200	4- 75
Southern Cambria	1910	24	1200	4- 75
Washington-Baltimore	1910	89	1200	4- 75
Piedmont Traction	1911	125	1500	4- 90
Ft. Dodge & Southern	1911	145	1200	4-125
Southern Pacific	1911	96	1200	4-125
Davenport & Muscatine	1912	30	1200	4- 50
Nashville-Gallatin	1912	23	1200	4- 75
Oregon Electric	1912	140	1200	2- 75
Kansas City & St. Joseph	1912	70	1500	4-...

TABLE IV.—SOME OF THE MORE IMPORTANT STEAM RAILROAD ELECTRIFICATIONS IN EUROPE (1913)

Name of road	Country	Miles of single track	Main line, tunnel, terminal	Trolley or 3rd rail	Trolley voltage	Locomotives or motor cars
Northeastern Railway	England	82	Term.	Both	600 D-C.	Both
London, Brighton & S. C.	England	62	M. L.	T.	6,600 A-C.	M. C.
Metropolitan District	England	50	Tun.	3d R.	600 D-C.	M. C.
Metropolitan	England	60	Tun.	3d R.	600 D-C.	Both
Swedish State	Sweden	100	M. L.	T.	15,000 A-C.	L.
Paris-Orleans	France	46	Term.	3d R.	600 D-C.	Both
Midi Railway	France	75	M. L.	T.	12,000 A-C.	Both
Prussian State:						
Hamburg	Germany	17	M. L.	T.	6,600 A-C.	M. C.
Magdeburg	Germany	23	M. L.	T.	10,000 A-C.	L.
Bavarian State	Germany	14	M. L.	T.	5,500 A-C.	Both
Baden State, Wiesenthal	Germany	34	M. L.	T.	10,000 A-C.	Both
St. Polten-Mariazell	Austria	68	M. L.	T.	6,500 A-C.	L.
Swiss Federal:						
Burgdorf-Thun	Switzerland	26	M. L.	T.	750 A-C.	Both
Simplon Tunnel	Switzerland	26	Tun.	T.	3,000 A-C.	L.
Bernese Alps	Switzerland	55	M. L. & Tun.	T.	15,000 A-C.	Both
Rhatische Bahn	Switzerland	48	......	......	....	......
Italian State:						
Giovi, Genoa	Italy	26	M. L.	T.	3,000 A-C.	L.
Milan-Porto Ceresio	Italy	81	M. L.	3d R.	660 D-C.	Both
Valletina	Italy	70	M. L.	T.	3,000 A-C.	L.

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[W. I. SLICHTER.]

RAILWAYS, ENERGY REQUIREMENTS AND MOTOR EQUIPMENT FOR. — (*See also Locomotives, Electric; Motors; Power Stations; Railways, Electric, Traction Systems for; Railways, Location and Permanent Way for; Substations, Railway.*) From a consideration of the forces acting on a moving train it is possible to determine the motor capacity and energy required to operate it when the profile and contour of the road, the time table and the characteristics of the available motors are known. The various items are treated in the following sequence:

UNITS AND ABBREVIATIONS. — Throughout this article the various quantities employed will be expressed in the following units unless specifically stated otherwise: distances in feet, weights in tons of 2000 pounds, forces in pounds, speeds in miles per hour (abbreviated mph.), accelerations in miles per hour per second (abbreviated mphps.), mechanical power in horse power, energy in watt-hours.

FORCES ACTING ON A TRAIN. — The forces tending to accelerate a train are the tractive effort developed by the motors and the component of the weight along the track on down grades. The forces which retard the motion of the train are the various frictional forces and the component of the weight along the track on up-grades; also in braking, the frictional force due to the brakes. All the various frictional forces, except the braking resistance, such as track friction, journal friction, air friction, etc., which oppose the motion of a train on a straight track are usually considered together and are referred to as the "train resistance." The extra friction due to track curvature is usually considered as an equivalent up-grade.

Tractive Effort and Draw-bar Pull. — The tractive effort of a motor is the force exerted by the motor at the rim of the driving wheel to which the motor is geared. The tractive effort of a locomotive is the force exerted by the locomotive at the rim of the drivers. The draw-bar pull of a locomotive is the force transmitted through the draw-bar of the locomotive,* and is less than its tractive effort by an amount equal to the resistance due to the rolling friction of the locomotive wheels on the track and the air resistance of the locomotive.

Direct-current motors deliver a uniform tractive effort for a given current; the tractive effort of alternating-current motors pulsates to some extent with the alternations of the current; the tractive effort of a steam locomotive varies

* In the case of a steam locomotive, the draw-bar pull usually refers to the force transmitted through the coupling between the tender and train; i.e., the tender is considered as a part of the locomotive.

from 28 to 50 per cent above and below its average value during each revolution of the drivers.

Train Resistance. — The total train resistance includes the following:

Rolling friction, which depends on the stiffness of the rail and the rigidity of the roadbed, and upon the speed and the weight of the train.

Journal friction, which depends on the speed, weight of the train, temperature and condition of lubrication.

Effect of oscillation and concussion, which depends upon the square of the speed, the weight of the train, and upon the condition of the track and roadbed.

Air resistance, which depends upon the square (approximately only) of the speed, the end area and length of train (number of cars), upon the shape of the end surface, and upon the kind of platforms or vestibules, but is independent of the weight of the train.

In view of the numerous factors which affect the train resistance, it is practically impossible to devise a formula for the resistance per ton of train weight which will be applicable to all types of cars and track construction. Numerous formulas have been suggested, most of which are of the form

$$r = A + Bv + Cv^2,$$

where r = the resistance in pounds per ton, v = speed in miles per hour and A, B and C are constants determined by experiment. The values of these constants naturally depend upon the type of cars, nature of roadbed, number of cars in the train, etc.; consequently the values of these constants as obtained from tests on various types of equipment and track construction differ considerably.

Burch (*Electric Traction for Railway Trains, New York, 1911*) gives the following values for these constants under normal conditions of track and weather. In winter the total train resistance may be as much as 60 per cent greater than the values calculated from the above formula, when these values of the constants are used. An electric locomotive is to be treated as a single car, its weight averaged with the weight of the cars. In the case of a steam locomotive and tender, the tender is to be treated as a car, but the locomotive resistance is to be added as a separate item (*see below*).

Value of *A*. — This constant depends chiefly upon the average total weight of car and load. Let ω = this average total weight *per car* in tons; then for

ω =	15	20	25–30	35	40–45	50	70
A =	6.0	5.5	5.0	4.5	4.0	3.5	3.0

Value of *B*. — This constant depends primarily upon the nature of the track and roadbed and also to some extent upon the weight and type of the car. Burch gives the following values:

Passenger cars on excellent track..................	0.06–0.11
Passenger cars on ordinary track..................	0.10–0.15
Freight cars on ordinary track....................	0.05–0.06

The heavier the car the higher the value of this coefficient.

Value of *C*. — This constant depends primarily upon the air resistance at the head and rear ends of the train and along the sides of the train. It may be expressed approximately by the formula

$$C = \frac{Ka\,(0.9 + 0.1\,N)}{Nw},$$

where a = the cross section of car in square feet, N = number of cars, w = average weight of car and load in tons, and K = 0.0010 for parabolic ends,

0.0020 for wedge-shaped ends, 0.0028 for vestibule cars, 0.0030 for open platforms, 0.0033 for freight cars and 0.0040 for flat fronts.

Tests made on three-car trains in the Boston-Cambridge subway gave a value of approximately 0.0050 for the constant K, the higher value being due to the extra air resistance in the tunnel. *(See Elec. Ry. Jour., 1912, Vol. 40, p. 280.)*

Burch's Table of Train Resistance. — Using the above values of the constants Burch has calculated the following table for train resistance.

TRAIN RESISTANCE FOR ELECTRIC TRAINS

N = number of cars (including electric locomotive, if any);
w = average weight of car loaded, in tons (= total weight of train divided by N);
r = train resistance in pounds per ton;
v = speed in miles per hour;
a = cross section of car in square feet;
A and B constants in the formula; K taken as 0.0030 throughout.

$$r = A + Bv + \frac{Ka\,(0.9 + 0.1\,N)\,v^2}{Nw}. \qquad (1)$$

TABLE I. — VALUES OF TRAIN RESISTANCE

	N	w	A	B	a	Speed in miles per hour 10	20	30	40	50	60
Passenger Trains	1	15	6.0	0.11	100	9.1	16.2	27.3	42.4	61.5	84.6
	1	20	5.5	0.12	100	8.2	13.9	22.6	34.3	49.0	66.7
	1	25	5.0	0.13	100	7.5	12.4	19.7	29.4	41.5	....
	1	35	4.5	0.13	100	6.7	10.5	16.1	23.4	32.4	....
	1	45	4.0	0.13	110	6.0	9.6	14.5	21.2	28.8	....
	2	15	6.0	0.11	100	8.2	12.6	19.2	28.0	39.0	52.2
	2	20	5.5	0.12	100	7.5	11.2	16.5	23.5	32.1	42.4
	2	25	5.0	0.13	100	7.0	10.2	14.8	20.8	28.0	36.5
	2	35	4.5	0.13	100	6.3	9.0	12.6	17.2	22.8	29.3
	2	45	4.0	0.13	110	5.7	8.2	11.5	15.6	20.5	26.3
	3	15	6.0	0.11	100	7.9	11.4	16.5	23.2	31.5	41.4
	3	20	5.5	0.12	100	7.3	10.3	14.5	19.9	26.5	34.3
	3	25	5.0	0.13	100	6.8	9.5	13.2	17.9	23.7	30.1
	3	30	4.5	0.13	100	6.2	8.7	12.0	16.1	21.0	26.7
	3	35	4.5	0.13	100	6.1	8.5	11.4	15.2	19.6	24.6
	3	45	4.0	0.13	110	5.6	7.8	10.5	13.9	17.8	22.4
	4	25	5.0	0.13	100	6.7	9.2	12.4	16.4	21.3	26.8
	4	30	4.5	0.13	100	6.1	8.4	11.3	14.9	19.1	24.0
	4	35	4.5	0.13	100	6.1	8.2	10.9	14.1	18.0	22.3
	4	45	4.0	0.13	110	5.5	7.6	10.0	13.0	16.5	20.4
	6	25	5.0	0.13	100	6.6	8.8	11.6	15.0	19.0	23.6
	6	35	4.5	0.13	100	6.0	8.0	10.3	13.1	16.4	20.0
	6	45	4.0	0.13	110	5.5	7.3	9.5	12.1	15.1	18.4
	8	35	4.5	0.13	100	6.0	7.8	10.0	12.5	15.5	....
	8	45	4.0	0.13	110	5.4	7.2	9.2	11.7	14.4	....
	12	45	4.0	0.13	110	5.4	7.0	9.0	11.2	13.7	16.4

TABLE I.—VALUES OF TRAIN RESISTANCE—*Continued*

	N	w	A	B	a	Speed in miles per hour					
						10	20	30	40	50	60
Freight Trains	10	30	5.0	0.06	110	5.8	7.0	8.7	10.7	....	....
	20	30	5.0	0.06	110	5.7	6.8	8.2	9.9	....	....
	30	40	4.0	0.06	110	4.7	5.6	6.7	8.1	....	....
	50	40	4.0	0.06	110	4.7	5.6	6.7	8.0	....	....
	40	50	3.5	0.06	110	4.2	5.0	6.0	7.2	....	....

Other Formulas for Train Resistance.—Of the numerous other formulas which have been proposed for train resistance the two most commonly employed are Armstrong's formula

$$r = \frac{50}{\sqrt{Nw}} + 0.03\,v + \frac{0.002\,a\,(0.9 + 0.1\,N)\,v^2}{Nw}$$

and Mailloux's formula

$$r = 3.5 + 0.15\,v + \frac{(0.25 + 0.02\,N)\,v^2}{Nw}.$$

The symbols have the same meaning as above. Armstrong's formula gives considerably lower values than those given by Burch, except for heavy trains; Mailloux's formula gives lower values than those given by Burch except for heavy trains at low speeds, but the difference in most cases is not very great. For other formulas proposed from time to time see Burch, *Electric Traction for Railway Trains*. See also an excellent paper on the resistance of freight trains by E. C. Schmidt in Trans. A.S.M.E., May, 1910.

Resistance of Steam Locomotives.—The American Railway Engineering Association recommend the formula for the resistance of a steam locomotive

$$r_1 = 18.7 + \frac{80\,X}{w_1},$$

where r_1 is the resistance (between cylinder and rim of drivers) per ton weight on the drivers, X is the number of driving axles and w_1 the total weight in tons on all the drivers. Tests by the American Locomotive Company showed that the resistance per ton weight on the drivers is 22.2 pounds.

Train Resistance at Starting.—The formulas given above are not applicable to speeds below about 10 mph. The New York Central tests on electric trains show that the train resistance decreases with decrease in speed to 10 mph., but that as the speed still further decreases the resistance per ton increases. The resistance at starting may be from 6 to 18 pounds per ton, depending upon the condition of the bearings, track, etc., and upon the duration of the stop preceding the starting. These figures also apply to freight trains. Tests on the Rock Island system showed that in the case of a train which had stood overnight in cold weather (i.e., which had become "frozen up"), the starting resistance was 30 pounds per ton. The slack in the car couplings, however, renders it unnecessary for a locomotive to exert sufficient effort to start all the cars at once.

Grades and Curvature.—(*See Railways, Location and Permanent Way for.*) An actual up-grade of G per cent produces a retarding force of 20 G pounds per

ton, and a down-grade of G per cent produces an accelerating force of 20 G pounds per ton. A curve always gives rise to a retarding force which ranges from 0.5 to 1 pound per ton per degree of curvature. Using the higher figure each degree of curvature may be taken equivalent to an up-grade of 0.05 per cent. Note that for angles of curvature up to 12 degrees the angle in degrees may be taken equal to $5730 \div R$ where R is the radius of curvature in feet.

ACCELERATION AND BRAKING. — The permissible rate of acceleration depends upon a number of factors.

1. The rating of the motors; the larger the motors the higher the tractive effort they can develop and therefore the greater the acceleration.

2. The weight on the driving wheels of the car or locomotive; the maximum tractive effort that a motor car can exert without slipping the wheels is from 15 to 20 per cent of the weight on the drivers (*see below under Adhesion Coefficient*).

3. The comfort of the passengers; the higher the acceleration rate the more difficult is it for a passenger to maintain his equilibrium. This also depends to some extent upon the uniformity of the acceleration.

4. To make a given schedule speed with the least amount of energy the acceleration rate should be as high as possible. Very high rates of acceleration, however, are not in general justified on this score, as the increase in the size of the motors required may more than offset the saving of energy.

The following rates of acceleration represent common practice:

TABLE II. — ACCELERATION RATES

Service	Miles per hour per second
Steam locomotive, freight service	0.1 to 0.2
Steam locomotive, passenger service	0.2 to 0.5
Electric locomotive, passenger service	0.3 to c.6
Electric motor cars, interurban service	0.8 to 1.3
Electric motor cars, city service	1.5 to 2.0
Electric motor cars, rapid transit service	1.5 to 2.0
Highest practical rate	2.0 to 2.5

Braking. — The maximum retardation in braking is limited by the comfort of the passengers, and injury to equipment, a retardation of 1.5 miles per hour per second being the usual practical limit for electric or steam passenger trains, although 2.5 miles per hour per second is sometimes attained. For freight trains the braking retardation is from 0.7 to 0.8 miles per hour per second. The higher the rate of braking the less the energy consumption for a given schedule speed.

"Acceleration Constant." — The tractive effort required to give to one ton (2000 pounds) a linear acceleration of one mphps. is 91.2 pounds. To accelerate a train of W tons requires a tractive effort of 91.2 aW pounds to produce a linear acceleration of a mphps., but on account of the accompanying angular acceleration of the rotating parts an additional force is required. This additional force is proportional to the linear acceleration a and also depends upon the radius of gyration (*see Mechanics, Principles of*) of all rotating parts, and upon the gear ratio of the motors (i.e., ratio of number of teeth in gear to number of teeth in pinion). The effect of the moment of inertia may be looked upon either as increasing the effective weight W or as increasing the acceleration

constant, the acceleration constant being defined as the quotient of total accelerating force divided by the product of weight and linear acceleration.

The increase in effective weight (in tons) due to any rotating axle or wheel is $\frac{M}{2000}\left(\frac{K}{r}\right)^2$, where M is the weight in pounds of the part in question, K its radius of gyration (*see Inertia, Moment of*) and r its actual radius, both in feet. Each motor armature adds to the effective weight $\frac{M}{2000}\left(\frac{\rho K}{r}\right)^2$ tons, where M is the weight of the armature in pounds, K the radius of gyration of the armature in feet, r the radius in feet of the wheel to which it is geared and ρ is the gear ratio. The total additional weight W_r is the sum of the above items for all the rotating parts. The total force in pounds required to produce the acceleration of a mphps. is then CaW, where W is the actual weight of the train in tons and $C = 91.2\left(1 + \frac{W_r}{W}\right)$. This quantity C is the corrected acceleration constant, and this corrected value should be used in all calculations.

The acceleration constant is raised by the flywheel effect discussed above by about 5 per cent (i.e., $W_r/W = 0.05$) for heavy cars and locomotives, and between 5 per cent and 10 per cent for light low-speed cars, 8 per cent being an average figure. However, C is usually taken as 100, corresponding to an increase in effective weight of about 10 per cent. A given linear acceleration of a mphps. then requires an accelerating force of $100a$ pounds per ton.

TRACTIVE EFFORT REQUIRED. — Let

F = tractive effort, in pounds per ton, exerted by motors,
G = per cent actual grade (+ for up-grade),
g = degrees of curvature,
r = train resistance, in pounds per ton,
a = acceleration in mphps (− for retardation).

Then the tractive effort required per ton of total train weight is

$$F = 100a + r + 20G + g. \qquad (2)$$

Example. — Given a train of three 45-ton cars moving with a speed of 20 mph. and accelerating at a rate of 1.5 mphps. up a 1 per cent grade on a straight track; what is the total tractive effort required?

Answer: $(100 \times 1.5 + 7.8 + 20 \times 1)\, 3 \times 45 = 24{,}000$ pounds.

GEAR RATIO AND SPEED. — By gear ratio is meant the ratio of the number of teeth in the gear on the wheel axle to the number of teeth in the pinion on the motor shaft. A gear ratio greater than 6:1 is seldom used for railway motors. For a given torque developed by the driving motor, the tractive effort at the wheel rim and the linear speed for a given current depend upon the gear ratio and wheel diameter. Let D = the diameter of the wheel in inches, K = the gear ratio, F = the tractive effort for a given current input; then the tractive effort F_1 for this same current input but for a wheel diameter D_1 and gear ratio K_1 is

$$F_1 = \frac{DK_1}{D_1K}F.$$

If V is the speed corresponding to the tractive effort F for a given motor voltage, then the speed V_1 corresponding to the tractive effort F_1 for the same motor voltage and current is

$$V_1 = \frac{D_1K}{DK_1}V.$$

If the gear ratio is low, the maximum speed will be high and the rate of acceleration low; if the gear ratio is high, the maximum speed will be low and the rate of acceleration high.

For a given motor equipment, train weight, schedule, and profile the energy consumption and temperature rise of the motors depend upon the gear ratio selected, since this in turn determines the amount of coasting; see section below on *Importance of Coasting.* The proper gear ratio can be found only by trial calculation, plotting speed-time and distance-time curves from motor curves based upon different gear ratios, and calculating the energy consumption and temperature rise in the motors as described below.

MAXIMUM POSSIBLE TRACTIVE EFFORT — ADHESION COEFFICIENT. — The adhesion or "tractive" coefficient is the quotient (expressed usually as per cent) of the tractive effort in pounds which will slip the drivers, divided by the weight in pounds on the drivers. Burch gives the values in the following table. The maximum possible tractive effort is the product of the adhesion coefficient (as a decimal fraction) by the weight (in pounds) on the drivers.

TABLE III. — ADHESION COEFFICIENTS

Condition of track	Without sand	With sand
Most favorable condition	35	40
Clean, dry rail	28	30
Thoroughly wet rail	18	24
Greasy moist rail	15	25
Sleet-covered rail	15	20
Dry-snow-covered rail	11	15

Maximum Grade Train can Ascend. — Let

W = total weight of train in tons,
W_d = total weight on all drivers in tons,
p = adhesion coefficient in per cent,
r = train resistance in pounds per ton of total weight,
G = per cent grade,
g = degree of curvature,
a = acceleration in miles per hour per second.

Then the maximum tractive effort which the drivers can exert is $20\,pW_d$ pounds, and therefore $(r + 20\,G + g + 100\,a)\,W$ must be less than $20\,pW_d$, or the maximum per cent grade which the train can ascend is*

$$G = \frac{pW_d}{W} - \frac{(r + g + 100\,a)}{20}. \tag{3}$$

This grade is greater the less the acceleration, or the greater the retardation. The greater the speed before the train strikes the grade, the greater may the retardation be without bringing the train to rest on the grade, and therefore the steeper the grade it may ascend.

* To be exact W_d should be multiplied by $\sqrt{1 - (G/100)^2}$, but except for very heavy grades this correction is negligible.

Example. — Assume no acceleration or retardation and no curvature, a train resistance of 8 pounds per ton, an adhesive coefficient of 15 per cent, and 25 per cent of total weight of train on drivers. Then the maximum grade the train can ascend is $G = 15 \times 0.25 - 8/20 = 3.35$ per cent.

The highest permissible grade is when all the weight is on the drivers, e.g., single cars or trains of motor cars with all axles equipped with motors. On steam freight roads the maximum grade seldom exceeds 2 per cent, and is usually considerably less, except in very mountainous country.

Weight of Locomotive. — The weight of locomotive required to accelerate a train weighing W tons at the rate of a miles per hour per second up a grade of G per cent on a g degree curve against a frictional resistance of r pounds per ton, when the q per cent of the weight is on the drivers and the coefficient of adhesion is p per cent, is given by the following formula:

$$\text{Weight of locomotive} = \frac{5W}{pq}(100\,a + r + 20\,G + g).$$

Example: What weight of locomotive is required to accelerate a 400-ton train at the rate of 0.5 mile per hour per second up a 0.1% grade against a frictional resistance of 8 lb. per ton, when 80% of the weight is on the drivers and the coefficient of adhesion is 20%?

$$\text{Weight of locomotive} = \frac{5 \times 400}{20 \times 80}(50 + 8 + 2) = 75 \text{ tons.}$$

POWER REQUIRED AT GIVEN SPEED. — Let

r = train resistance in pounds per ton of total train weight,
G = per cent grade,
g = degree of curvature,
a = acceleration in mphps.,
v = speed in mph.,
W = total weight of train in tons.

Then the power required *at the rims of the drivers* is $1.99\,v\,(r + 20\,G + g + 100\,a)$ watts per ton, or $p_0 = 2.67 \times 10^{-3}\,vW\,(r + 20\,G + g + 100\,a)$ horse power, total. (4)

The *power input* p_i to the car or locomotive is equal to the power at the rims of the drivers divided by the over-all efficiency ϵ of the controller, motors and gears, i.e.,

$$p_i = \frac{1.99\,Wv\,(r + 20\,G + g + 100\,a)}{1000\,\epsilon} \quad \text{kilowatts} \qquad (5)$$

Efficiency of Motors. — The over-all efficiency of the motors and gears when the motor is operating at full line voltage does not vary considerably for loads ranging from 50 to 150 per cent of rated load. The maximum efficiency is usually at about rated load, and has the values given in Table IV. At 50 and 150 per cent load, the efficiency may be from 3 to 10 per cent less, depending upon the design and the type of motor. The variation in efficiency with load is usually greater with alternating-current series than with direct-current motors.

Average Over-all Efficiency During Controller Period. — The over-all efficiency of the motors and controller depends upon the type of control employed and upon the resistance inserted or the connections made by the controller. Most modern controllers for interurban cars are provided with a current-limiting device which limits the current to a given value, usually the value of the current corresponding to the 1-hour rating of the motor. Under these conditions the *average* over-all efficiency of the motors and controller over the whole of the *controller period* (i.e., from time of starting until full

TABLE IV. — MAXIMUM OVER-ALL EFFICIENCY OF MOTORS AND GEARS AT RATED VOLTAGE.

Horse power, 1-hour rating	Kind of motor	Max. eff., per cent
30–100	D.C. geared	83–88
100–250	D.C. geared	88–89
250–500	D.C. gearless	91–93
50–200	A.C. series geared	70–80*
200–500	3-phase induction geared	85–89

* Including step-down transformers.

line voltage is established across each motor or until all resistance is cut out) may be expressed as follows, on the assumption (which is very nearly exact) that the speed of the motor varies directly as the voltage impressed across its terminals:

Let ϵ = the over-all efficiency of the motors and gears at end of controller period. Then for straight resistance control with direct-current or induction motors the average efficiency during the controller period is $\epsilon/2$; for series-parallel control with 2 direct-current motors the average efficiency during the controller period is $2\epsilon/3$; with 4 direct-current motors first all in series, then 2 in series, then 2 series sets in parallel, and then all 4 motors in parallel the average efficiency during the controller period is $8\epsilon/11$; for alternating-current series motors with voltage control the average efficiency during the controller period (including losses in step-down transformers) is about 0.90ϵ. Hence for a motor having an 85 per cent efficiency at the 1-hour rating and the starting current limited to the corresponding current rating, the average over-all efficiency of the controller, motors and gear during the controller period is

For straight resistance control	43 per cent
For series-parallel control, direct-current motors	57 per cent
For series-series-parallel control, direct-current motors	62 per cent

Example. — Given a train of three 45-ton cars each car equipped with two 200-horse-power direct-current motors. What is the input to each motor if the speed is 30 mph. and the train is accelerating at 1.0 mphps. up a 2-per-cent grade, the track being straight, and full line voltage being across each motor?

Answer: From Table I the train resistance is $r = 10.5$, and from equation (4) the total power output is

$$p_0 = 2.67 \times 10^{-3} \times 30 \times 135\,(10.5 + 20 \times 2 + 100 \times 1)$$
$$= 1627 \text{ horse-power,}$$

or $1627/6 = 271$ horse power per motor. Assuming an efficiency of 87 per cent, the input to each motor is $271 \times 0.746/0.87 = 232$ kilowatts, and the total input to the train $6 \times 232 = 1392$ kilowatts.

If the train accelerates at this same rate from rest to a speed of 30 mph. at the end of the controller period (series multiple control) the grade being 2 per cent throughout, the average output of the motor would be the output corresponding to half speed or 15 mph., or 136 horse power per motor, requiring an average input of $136 \times 0.746/0.57 = 178$ kilowatts, and the average input to the train during the controller period would be $6 \times 178 = 1068$ kilowatts.

SPEED-TIME AND DISTANCE-TIME CURVES. — To determine the energy required to propel a car or train a given distance over a given track in a

given time requires the consideration of a number of factors which can best be taken into account by the construction of various kinds of time curves. Such curves may be constructed with practically any degree of accuracy desired when the profile of the track, the weight of the train, the various resistances, schedule speed, time of stops, etc., are accurately known. Such data are, however, seldom known with any great precision, and consequently elaborate methods of plotting and calculation are seldom justified. Below will be given (1) some results of actual tests, (2) a rough but simple method of approximating the energy requirements, (3) a step-by-step method, which, though tedious in application, is susceptible of any degree of accuracy desired, provided the given data are accurately known, and (4) an analytical method by which the effect of changes in the time of coasting, rates of acceleration and braking, etc., may be predetermined.

The following terminology will be employed:

Speed-time Curve. — A curve showing the speed (in mph.) plotted as ordinates against elapsed time (in seconds) as abscissas; e.g., the curve *ABCEF* in Fig. 1. A speed-time curve may be conveniently divided into four parts, namely:

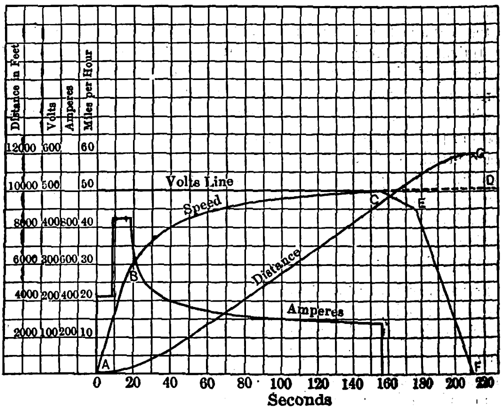

Fig. 1.

Controller Period. — The period from starting until full line voltage is established across each motor; i.e., the portion *AB* of the curve in Fig. 1.

Motor-Curve Period. — The period during which the motor is operating on full line voltage; i.e., the portion *BC* in Fig. 1. The relation between speed and tractive effort during this portion of the run is fixed by the motor characteristics; specifically by the speed-torque curve of the motor and the gear ratio.

Coasting Period. — The period during which the car or train is coasting; i.e., the portion *CE* in Fig. 1.

Braking Period. — The period during which the brakes are applied; i.e., the portion *EF* in Fig. 1.

Distance-time Curve. — A curve showing the distance covered (in feet) plotted as ordinates against elapsed time (in seconds) as abscissas; e.g., the curve *AG* in Fig. 1.

Current-time Curve. — A curve showing the line current (in amperes) plotted as ordinates against elapsed time (in seconds) as abscissas; e.g., the curve marked "Amperes" in Fig. 1.

Voltage-time and Power-time Curves. — Curves showing respectively the voltage per motor (or the line voltage) and the power input to the motor (or train) plotted as ordinates against elapsed time (in seconds) as abscissas.

Average Speed and Schedule Speed. — The average speed V is the total distance run L' (in miles) divided by the time (in hours) the train is actually running. The schedule speed S is the total distance run (in miles) divided by the total time (in hours) of the run from one end of the road to the other, including time of all stops at intermediate stations. If the *total* time of all the *intermediate* stops is T_s' seconds, then

$$V = \frac{S}{1 - \dfrac{T_s' S}{3600\, L'}}, \tag{6}$$

where V and S are in miles per hour and L' is the *total* length of route in *miles*.

Duration and Frequency of Station Stops. — The duration of each stop for surface cars ranges from 5 to 10 seconds, for elevated and subway trains from 10 to 30 seconds, for interurban trains from 10 to 40 seconds. The stops per mile are the reciprocal of the average distance in miles between stops.

Average Equivalent Grade (G). — Grades may be taken into consideration by calculating the sum H_1 of all the rises on upgrades and the sum H_2 of all the drops on down grades, and taking for the average "equivalent" upgrade in per cent

$$G = \frac{100\,(H_1 - 0.5\,H_2)}{L}, \tag{7}$$

where H_1 and H_2 are in feet and L is the total length of the route in feet. On a round trip $H_1 = H_2$, and the "equivalent" grade in per cent is

$$G = \frac{50\,H_1}{L}.$$

This method of dealing with grades is equivalent to assuming that half the kinetic energy stored in the train on down grades is utilized in taking the train up the following upgrade. The amount of energy thus rendered available of course will depend upon the amount of braking necessary on down grades to prevent excessive speeds and also upon the location of the stops with respect to the grades. The figure ½ is taken as an approximate average; this figure may be varied as seems reasonable in view of the actual profile.

Average Angle of Curvature (g). — Curves may be taken into account by finding the average curvature, i.e., finding for each curve the product of the degree of curvature by the length of the curve, adding all these products and dividing by the length of the route.

ENERGY CONSUMPTION FROM TESTS. — The table on p. 1176 gives the energy consumption, as found by tests, for a number of typical services. This table will be found useful as a rough check on any calculations made for a specific service. Methods of making such calculations are given in subsequent paragraphs.

TABLE V.—WATT-HOURS PER TON-MILE AT CARS, FROM ACTUAL TESTS

	N.Y.N.H.& H.R.R.		N.Y.C.&H.R.R.		N.Y.N.H.& H.R.R.		N.Y.C.&H.R.R.		N.Y. N.H. &H. R.R.	Interborough Subway, N.Y. Express	Interborough Subway, N.Y. Local	Northern Texas Trac. Co.		Urban trolley lines.
Service	P	P	P	P	P	P	P	P	F	P	P	P	P	P
A. C. or D. C.	A. C.	A. C.	D. C.	D. C.	A. C.	A. C.	D. C.	D. C.	A. C.	D. C.	D. C.	D. C.	D. C.	D. C.
Locomotive or cars	L.	L.	L.	L.	L.	L.	L.	L.	L.	C.	C.	C.	C.	C.
Train weight, tons	316	285	266	250	477	493	432	434	1438	310.6	310.6	25.4	25.4 4	4–12
Length of run, miles	20.6	20.6	23.9	23.9	20.6	20.6	17.1	17.1	16.8	12.9	6.6	28.4	28.4	.....
Average grade, per cent	+0.053	−0.053	+0.129	−0.129	+0.053	−0.053	+0.129	−0.129	−0.52	0	0	0	0	.....
Schedule speed, mph.	22.1	22.1	23.2	22.8	44.7	49.0	44.1	43.9	35.9	20.2	12.4	28.6	29.6	.....
Stops per mile	0.63	0.68	0.84	0.75	0.048	0.048	0.059	0.059	0.06	0.62	1.76	0.32	0.27	2–10
Watt-hours per ton-mile	85.4	74.2	81.6	59.4	35.0	30.0	29.2	17.6	25.9	58.2	78.6	75.5	72.5	100 to 160
References	1	1	2	2	1	1	2	2	1	3	3	4	4	

NOTE.—While single-phase and direct-current service are given above in adjacent columns, the operating conditions are not identical and these figures are not intended to bring the two systems into comparison.

Abbreviations.—P = passenger; F = freight; L = locomotive; C = cars;

References.—
1 = W. S. Murray, *Trans. A.I.E.E. 1911, Vol. 30, p. 1497.*
2 = E. B. Katte, *Trans. A.I.E.E., 1911, Vol. 30, p. 1497.*
3 = L. B. Stillwell, *Elec. Ry. Jour. 1908, Vol. 32, p. 6.*
4 = E. R. Roberts and I. H. Sherwood, *Trans. A.I.E.E. 1903, Vol. 20, p. 1080.*

APPROXIMATE METHOD OF CALCULATING ENERGY CONSUMPTION.—The following method is based upon simple kinetic principles, and, if certain characteristics of the run are known, gives the actual energy output at the wheel rims. This fact makes the method useful, not only for rough calculations, but also to check calculations made by the step-by-step method.

When the method is applied to checking purposes, the column of Table VI. headed "Actual energy output" should be used, and the input calculated from the known efficiencies. When applied to rough calculations, the column headed "Approximate electrical energy input" should be used. In the latter case the maximum speed and length of run with power on are not known, but it is possible to assume certain values, based upon experience, which will give a rough approximation to the energy required. Let

V = average running speed in miles per hour,
V_m = maximum speed in miles per hour,
L = length of run in miles,
L_p = distance traveled, with power on, in miles,
$n = 1/L$ = number of stops per mile including one terminus,
r = average train resistance, in lb. per ton (Say that corresponding to a speed from 10 to 20 per cent greater than the average speed),
G = average equivalent grade, in per cent,
g = average curvature in degrees,
$K = \frac{V_m}{V}$ = ratio of maximum to average speed; see Table VII,
$Q = \frac{L}{L_p}$ = ratio of length of run to distance traveled with power on; see Table VII.

TABLE VI.—OUTPUT AT WHEEL RIM AND INPUT TO CAR IN WATT-HOURS PER TON-MILE

Energy for	Actual energy output at wheel rims of cars	Approx. electrical energy input to cars
Acceleration	$\frac{V_m^2}{36.2L}$	$\frac{K^2nV^2}{25}$
Train resistance	$\frac{1.99\,rL_p}{L}$	$\frac{2.9\,r}{Q}$
Grades	$\frac{39.8\,GL_p}{L}$	$\frac{57\,G}{Q}$
Curves	$\frac{1.99\,gL_p}{L}$	$\frac{2.9\,g}{Q}$
Total	Sum	Sum

NOTE.—$25 = 36.2\,\epsilon$, $57 = 39.8 + \epsilon$, and $2.9 = 1.99 \div \epsilon$, where ϵ is the efficiency, taken as 0.7. The formula for energy due to curves assumes each degree of curvature to be equivalent to a train resistance of one pound per ton, which is probably high.

TABLE VII.—VALUES OF K AND Q

Stops per mile n	K		Q
	Locomotive Passenger Trains	Single cars, multiple-unit trains and freight trains	All trains
0	1.00	1.00	1.00
0.1	1.18	1.10	1.11
0.2	1.35	1.18	1.24
0.3	1.48	1.25	1.38
0.4	1.60	1.31	1.52
0.5	1.68	1.36	1.67
0.6	1.75	1.40	1.78
0.7	1.82	1.44	1.89
0.8	1.86	1.47	1.99
0.9	1.90	1.50	2.07
1.0	1.93	1.52	2.15
1.2	1.93	1.56	2.24
1.4	1.93	1.59	2.34
1.6	1.94	1.62	2.44
1.8	1.94	1.65	2.52
2.0	1.95	1.68	2.58
2.5	1.95	1.75	2.71
3.0	1.96	1.80	2.81
3.5	1.96	1.85	2.87
4.0	1.97	1.90	2.91
4.5	1.97	1.94	2.95
5.0	1.98	1.97	3.00
over 5.0	2.00	2.00	3.00

(The above method was worked out by D. C. Woodbury and W. A. Del Mar, the table being based upon a large number of actual and calculated runs.)

Example.—A multiple-unit train has a speed (excluding stops) of 25 miles per hour and makes 0.8 stops per mile. It ascends an average grade of 0.143 per cent. What will be its energy consumption in watt-hours per ton-mile?

From Table VII we find for $n = 0.8$, that $K = 1.47$ and $Q = 1.99$. Then using the formulas in Table VI, the results in the table on the next page are obtained, assuming 6.5 pounds per ton for friction.

Efficiency of Run.—The formulas given above enable one to judge the effect upon the energy consumption, of altering any of the principal physical elements upon which the run is based.

From the formulas for kinetic energy it is obvious that a low value of K means a low energy consumption. A low value of K, however, means a "square" speed-time curve; i.e, for low energy consumption the controller, acceleration and braking periods should be as short as possible, or in other words, the rate of acceleration and braking should be as great as practicable.

The quantitative effect of changing any of these variables may be estimated by the analytical method given further on.

Energy for	Watt-hours per ton-mile	
Acceleration..............	$\frac{1.47^2 \times 0.9 \times 25^2}{25} =$	48.5
Train resistance...........	$\frac{2.9 \times 6.5}{1.99} =$	9.5
Grades....................	$\frac{57 \times 0.143}{1.99} =$	4.1
Total......................		62.1

This example worked out by the step-by-step method gave 60.5 watt-hours per ton-mile.

STEP-BY-STEP METHOD OF PLOTTING SPEED-TIME CURVES. — There is no way of exactly predetermining a speed-time curve except by a number of successive trials. That is to say, the time the current is kept on, the time of coasting and the time of braking must each be guessed and it is usually necessary to make a number of trials, by varying the proportion of motor run, coasting and braking, before the given distance is traversed in the desired time.

If the characteristics of the train and its equipment are expressed numerically, the principles of mechanics enable such trial runs to be plotted on paper and the proper proportion of motor run, coasting and braking, selected to make the train travel the desired distance in the given time. For a given motor equipment on a given route it is possible to plot by a step-by-step method these speed-time curves, and then from these curves and the characteristic curves of the motors the various characteristics, such as energy consumption, root-mean-square current, etc., may be determined. The accuracy of this method depends solely upon the accuracy with which the assumed data are known. The necessary data are:

Profile and alignment of road.
Characteristic curves of the motors.
W = total weight of train in tons,
T = the time of run in seconds between successive stops,
I_0 = the permissible starting current, or
a = the acceleration in miles per hour per second during the controller period,
b = the braking rate in miles per hour per second,
r = the train resistance in pounds per ton at any speed.

Determination of Acceleration and Retardation Rates. — From the motor characteristics, which are usually given in the form indicated in Fig. 2, determine the tractive effort and speed corresponding to the permissible starting current. This is usually taken as the current corresponding to the 1-hour rating; in the case of the motor whose characteristics are shown in Fig. 2, this current is 315 amperes, the speed 19.6 mph. and the tractive effort 4300 pounds. If each car is equipped with two motors, the weight corresponding to each motor is half the weight of the car. If the cars weigh 44 tons each, then the tractive effort per ton developed by the motors at this speed is $4300/22 = 195$ pounds per ton. At the point where the motors are changed from series to parallel the speed

will be approximately one-half the speed at rated voltage or 9.8 mph. If the average line voltage is less than the rated voltage, these speeds should be reduced in proportion to the ratio of the actual to the rated voltage. In the case selected the average line voltage is 571 volts; hence the speeds corresponding to the parallel and series points are 18.6 and 9.3 mph. respectively. These speeds and the corresponding tractive efforts are entered into Table VIII.

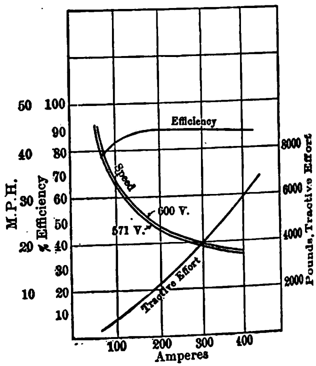

Fig. 2.

From the motor characteristics determine the tractive effort (f) in pounds per ton and the corresponding speeds (v), corrected for line voltage, up to the maximum permissible speed, and enter these in Table VIII. Also enter into this table the train resistance corresponding to the various speeds. In the table given, the train resistance is calculated from Burch's formula for three 44-ton cars, the cross section of each car being 120 square feet, and the constant K is taken equal to 0.0050 (subway service). The formula is then

$$r = 4.0 + 0.13\,v + 0.0055\,v^2,$$

where v is the speed in miles per hour.

The available tractive effort in pounds per ton for acceleration on any grade is then $f - r - 20\,G'$, and the acceleration rate is (assuming an acceleration constant of 100)

$$a = \frac{f - r - 20\,G'}{100},$$

where G' is the "equivalent" grade (including curvature, see above), in per cent. The value of a is calculated for grades of 0, 1, 2, 3, etc., per cent, both up and down, including the largest up and down grade. These values are entered into Table VIII. The maximum speeds are the speeds for which the acceleration becomes zero. These can be obtained by plotting the accelerations against speed, and finding the speed at which the curves cross the speed axis.

Similar calculations should also be made for the retardation on up-grades when the train strikes such a grade, with power on, at a higher speed than the free running speed on the grade, and also for the coasting period when no power is on. When power is on, the friction of the gears and motors is allowed for in the motor tractive effort curve, the tractive effort curve being the gross tractive effort less these losses. The friction of the gears and motors may be taken as approximately 5 per cent of the rated tractive effort, which, in the special case under consideration is $0.05 \times 195 = 10$ pounds per ton. The total train resistance in coasting is then the normal train resistance plus the resistance of gears and motors.

In the Tables a + acceleration signifies an actual increase in velocity, a − acceleration a retardation, or decrease in velocity.

The braking rate is assumed constant, usually $b = 1.5$ mphps.

TABLE VIII. — ACCELERATION RATES

	Speed, mph.	Motor tractive effort, pounds per ton	Train resistance, pounds per ton	Acceleration rates, mphps., = a						
				Per cent equivalent grade given in first line						
	v	f	r	+3	+2	+1	0	−1	−2	−3
Accelerating, power on	9.3	195	6	1.29	1.49	1.69	1.89	2.09	2.29	2.49
	18.6	195	8	1.27	1.47	1.67	1.87	2.07	2.27	2.47
	20	159	9	0.90	1.10	1.30	1.50	1.70	1.90	2.10
	22	112	10	0.42	0.62	0.82	1.02	1.22	1.42	1.62
	25	74	11	0.03	0.23	0.43	0.63	0.82	1.03	1.23
	30	45	13	......	......	0.12	0.32	0.52	0.72	0.92
	35	27	15	......	......	......	0.12	0.32	0.52	0.72
	40	18	18	......	......	......	0.00	0.20	0.40	0.60
Retarding power on	40	18	18	−0.60	−0.40	−0.20	......	......	......	......
	35	27	15	−0.48	−0.28	−0.08	......	......	......	......
	30	45	13	−0.28	−0.08	......	......	......	......	......
Coasting, no power	50	0	34	−0.94	−0.74	−0.54	−0.34	−0.14	+0.06	+0.26
	45	0	31	−0.91	−0.71	−0.51	−0.31	−0.11	+0.09	+0.29
	40	0	28	−0.88	−0.68	−0.48	−0.28	−0.08	+0.12	+0.32
	35	0	25	−0.85	−0.65	−0.45	−0.25	−0.05	+0.15	+0.35
	30	0	23	−0.83	−0.63	−0.43	−0.23	−0.03	+0.17	+0.37
	25	0	21	−0.81	−0.61	−0.41	−0.21	−0.01	+0.19	+0.39
	20	0	19	−0.79	−0.59	−0.39	−0.19	+0.01	+0.21	+0.41

NOTE. — The train resistance is assumed to be 6 lb. per ton from zero speed to 9.3 miles per hour.

Construction of Acceleration and Retardation Time Curves. — The next step is to construct a set of acceleration speed-time and distance-time curves, a set of coasting speed-time and distance-time curves, and a braking speed-time and distance-time curve. The construction of these curves is based on the following relations:

Let

v_1 = the speed in miles per hour at time t_1,

v_2 = the speed in mph. at time t_2,

$v = \dfrac{v_1 + v_2}{2}$ = average speed during the interval $t_2 - t_1$,

a_1 = the acceleration in miles per hour per second at time t_1,

a_2 = the acceleration in miles per hour per second at time t_2,

$a = \dfrac{a_1 + a_2}{2}$ = average acceleration during the interval $t_2 - t_1$; (for the first step take a speed corresponding to half the speed at end of controller period),

x_1 = the distance in feet from the starting point at time t_1,

x_2 = the distance in feet from the starting point at time t_2.

Then, for a small change in speed,

$$t_2 - t_1 = \frac{v_2 - v_1}{a} \quad \text{seconds} \tag{8}$$

and the distance covered in this interval is

$$x_2 - x_1 = 1.466\, v\, (t_2 - t_1) \qquad \text{feet} \qquad (9)$$

or, when the speed is plotted against time,

$$x_2 - x_1 = 1.466 \times (\text{Area of speed-time curve between } t_2 \text{ and } t_1). \qquad (10)$$

From equation (8), using the values of a given in Table VIII, the time at which any speed is reached may be calculated. The results of such calculations for the special case under consideration are given in Table IX, and are plotted in Figs. 3, 4 and 5. The distance-time curves are found by planimetering the speed-time curves and multiplying by 1.466 (*see equation 9*), and are also plotted in Figs. 3, 4 and 5.

TABLE IX.—DATA FOR ACCELERATING, COASTING AND RETARDING SPEED TIME CURVES

	Speed v	Total time in seconds to accelerate from rest						
		Per cent equivalent grade						
		+3	+2	+1	0	−1	−2	−3
Accelerating, power on	0.0	0	0	0	0	0	0	0
	9.3	7	6	6	5	4	4	4
	18.6	14	12	11	10	9	8	7
	20	17	13	12	11	10	9	8
	22	20	16	14	12	11	10	9
	25	33	23	19	16	14	12	11
	30	Max. speed =25.3	Max. speed =27.5	37	26	21	18	16
	35	..	..	Max. speed =32	49	33	26	22
	40	..	..	..	132	52	37	29
		Total time in seconds to retard from 40 mph.						
Retarding, power on	40	0	0	0	..	..	..	..
	35	9	15	36	..	..	..	..
	30	22	43	Min. speed =37	..	..	..	..
	....	Min. speed =25.3	Min. speed =27.5	..	..	..	..	..
		Time in seconds required for speed to decrease 5 mph. to speed given in first column.					To increase 5 mph. from from given speed	
Coasting, power off	45	5	7	10	15	40	67	18
	40	6	7	10	17	53	48	16
	35	6	8	11	19	77	37	15
	30	6	8	11	21	125	31	14
	25	6	8	12	23	250	28	13
	20	6	8	13	25	..	25	13

The speed during the braking period t seconds *before* the train stops is $v = bt$ and the distance to travel to come to rest is $x = \frac{1}{2} bt^2$.

In the special case of a braking rate of 1.5 mphps.,

$$v = 1.5\,t \text{ miles per hour,}$$

and the distance to travel to come to rest is

$$x = \frac{1.466 \times 1.5\,t^2}{2}.$$

The corresponding curves are given in Fig. 6.

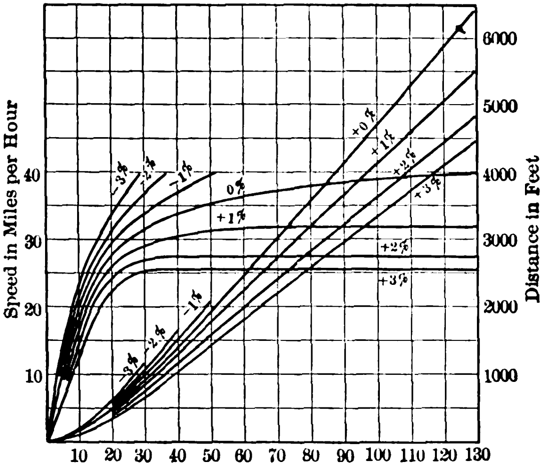

Fig. 3. Accelerating

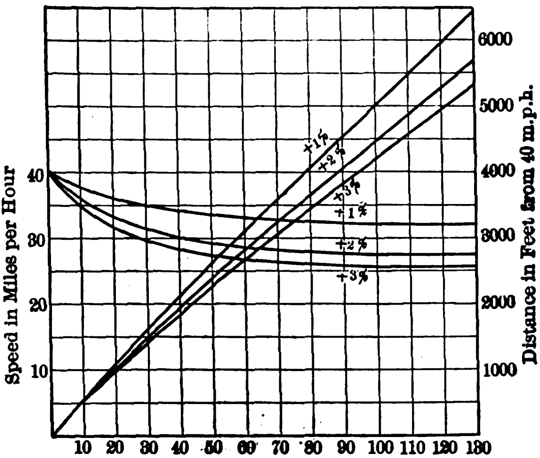

Fig. 4. Retarding, Power On

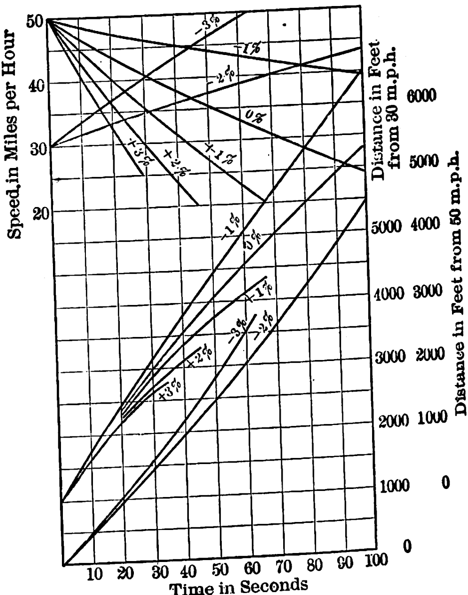

Fig. 5. Retarding, Power Off

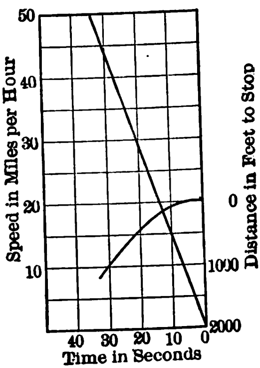

Fig. 6. Braking

Time Curves for Given Profile and Alignment. — With these four sets of curves the speed-time curve for any given run with this particular equipment may be rapidly constructed. For intermediate grades interpolation may be readily made. In Fig. 7 is given a profile and alignment between two stops. The first step is to make up a table like Table X, dividing the route into sections such that the "equivalent" grade (= actual grade plus, say, 0.05 per cent for each degree of curvature, assuming a resistance of 1 pound per ton per degree of curvature) is the same throughout each section.

TABLE X. — "EQUIVALENT" GRADES

Stop	Distance between stops, feet	Length of section in feet	Per cent grade = G	Radius of curvature in feet	Degree of curvature = g	"Equivalent" grade $G' = G + 0.05\,g$.
A	....	505	......	....	...	......
	....	145	......	1130	5.0	+0.25
	....	908	+0.70	1130	5.0	+0.95
	....	77	+0.50	1130	5.0	+0.75
	....	633	+0.50	....	...	+0.50
	....	273	+3.00	....	...	+3.00
	....	273	+3.00	800	7.2	+3.36
	....	247	......	800	7.2	+0.36
	....	94	......	....	...	......
	....	308	−2.95	....	...	−2.95
	....	800	+3.00	....	...	+3.00
	....	1862	−3.00	....	...	−3.00
	....	192	−3.00	5000	1.2	−2.94
	....	234	+0.12	5000	1.2	+0.18
	....	178	+0.12	....	...	+0.12
	....	69	+0.12	5000	1.2	+0.18
	....	358	......	5000	1.2	+0.06
B	7321	165	......	....	...	......

For a complete round trip over the entire route the time curves must be plotted for the entire route in both directions. The run in one direction between two stations only will be considered in the numerical calculations given below.

Next lay off on a piece of tracing cloth, see upper part of Fig. 7, a distance equal to the time of run between the two stations (156 seconds in the example), to the same scale as Figs. 3 to 6. The braking speed-time curve can be laid off directly at the far end of the run by placing Fig. 6 under the tracing cloth and tracing the curve. Similarly, by placing Figs. 3 and 4 under the tracing cloth and tracing for the proper distance the curve corresponding to the proper grade, an acceleration curve can be built up until this curve intersects the braking curve. If the total distance, as read off from the corresponding distance-time curve, is greater than the actual distance, it will be necessary to introduce a coasting period of proper duration to make the total distance as read off from the distance-time curves equal to the actual distance. This can be done by placing Fig. 5 under the tracing cloth, and by cut and try finding the proper amount of coasting.

In case there are curves or crossovers, requiring a reduction in speed at certain points, these reductions should be allowed for in plotting the speed-time

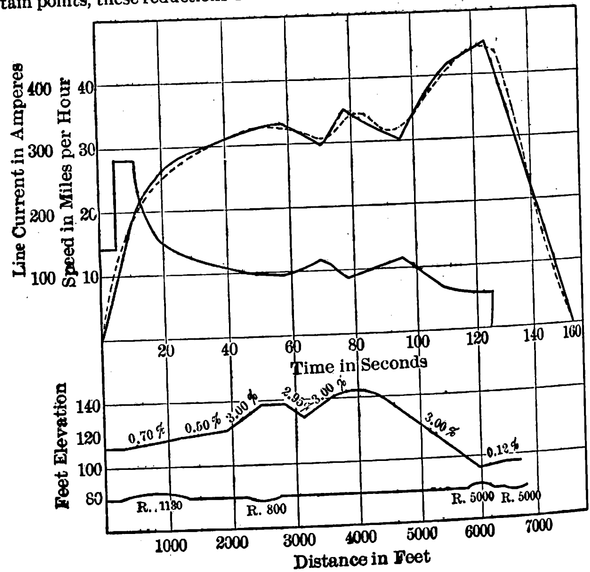

Fig. 7.

curve. A rough rule for the permissible speeds on properly constructed curves is that

$$\text{Speed on curve} = \sqrt{\text{Radius of Curve}},$$

where the radius is in feet and the speed in miles per hour.

The dotted curve in Fig. 7 was obtained from test.

Current-time Curve. — The motor current for any speed may be taken directly from the speed-time curve and motor characteristics. From the motor currents the line current is readily found by multiplying by *half* the number of motors during the *series* portion of the controller period and by the total number of motors during the rest of the time power is on. Current-time curves (line current) for the various grades may also be drawn once for all in the same manner as the speed-time curves in Fig. 3. The curve with the square shoulder in Fig. 7 is the current-time curve for the example considered.

Watt-hours per Ton-Mile. — During the first half of the controller period the input per motor is equal to the product of the current per motor by approximately one-half the line voltage (series-parallel control assumed); during the rest of the time that power is on, the input per motor is equal to the product of the motor current by the line voltage. Hence, calling A the area of the current-time curve from the start until power is shut off, and I_0 the starting current, E the average line voltage, T_1 the seconds duration of the controller period, L the length of the run in miles, M the number of motors and W the weight of the train in tons, then to a fair approximation,

$$\text{Watt-hours per ton-mile} = \frac{EM(A - 0.25 I_0 T_1)}{3600\, WL}.$$

A more accurate, but tedious method, is to plot a power-time curve by multiplying the total current per train at successive intervals of time by the average line voltage during this interval, and then integrating this curve to find the total watt-seconds. This area, in watt-seconds, divided by $(3600\ WL)$ will give the watt-hours per ton-mile. In making such a calculation note that during the series part of the controller period the current per train is equal to the current per motor multiplied by *half* the number of motors.

Root-mean-square Current per Motor. — This may be found by squaring the ordinates of the current-time curve (current per *motor*), plotted as described above, and dividing by the time of run including stops and taking the square root of the quotient. Or, the current-time curve may be plotted in polar co-ordinates, taking time in seconds as the angle in degrees and current in amperes as the radius vector. See Fig. 8. Calling B the area of this curve, the unit of area being the square whose side has a length corresponding to 1 ampere, then the root-mean-square current is

$$I_e = 10.7\sqrt{\frac{B}{T+T_s}},$$

where T is the time that the train is moving and T_s the standing time, both in seconds.

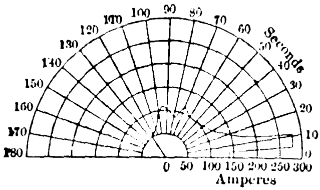

Fig. 8.

Average Motor Voltage. — For a simple run, such as shown in Fig. 1, average motor voltage during the controller period is equal to approximately 55 per cent of the line voltage, assuming 10 per cent of line voltage across the motors at the instant of starting. Let

E = average line voltage,
T_1 = time of controller period, from speed-time curve,
T_2 = time motor is running on full line voltage,
T = total time that the train is moving,
T_s = total standing time.

Then average motor voltage for the entire run is

$$E_m = \frac{0.55\,ET_1 + ET_2}{T+T_s}.$$

For a complex run, where the controller is shut off and put on again during the run, a voltage-time curve may be plotted and the average ordinate obtained by integration.

MOTOR CAPACITY. — A railway motor is usually rated in terms of the output in kilowatts or horse-power which it will give when run for 1 hour at rated voltage with a temperature rise above the surrounding air not exceeding 75° C. in any part of the motor, other than the commutator (*see Standardization Rules*).

The size of motor required for any particular service (i.e. for a given route, schedule, weight of car, line voltage and per cent coasting) depends upon two factors, (1) the motor must be of such a size that the maximum current required will not produce harmful sparking at the brushes or dangerous mechanical stresses in any part of the motor, and (2) the temperature must not rise to a value which will cause the insulation to deteriorate.

Size of Motor Limited by Commutation and Mechanical Stresses — **The maximum current is usually that required at starting, and since the start-**

ing current remains practically constant up to the point where full line voltage is impressed across the motor, the corresponding maximum horse-power output of the motor can be calculated directly from equation (4), when v is taken as the speed at the end of the controller period and W as the weight of the train per motor, i.e., W is taken equal to the total weight of train divided by the number of motors. A safe rule for non-interpole motors in single-car or multiple-unit service is to limit the starting current to a value equal to the rated current. For interpole motors in like service the starting current may safely be 25 to 50 per cent in excess of the rated current.

In locomotive work a heavier starting current is sometimes demanded, and due to the low acceleration rate during the starting period the motor must carry this current for a longer interval than in the case of single-car or multiple-unit service. In selecting a motor for such service, information should be obtained from the manufacturer as to the maximum current which the motor can safely carry during a limited period, say for 5 minutes. This maximum current may be limited by sparking at the commutator, by mechanical stresses, or by local heating of the windings. See also section below on *Size of Motor Limited by Short-time Heating*.

TABLE XI.—RATED AND CONTINUOUS CAPACITY OF WESTINGHOUSE RAILWAY MOTORS

	Type, Westinghouse	Rated H.P.	Rated voltage	Rated amperes	Continuous current capacity		Weight in pounds
					At 300 volts	At 400 volts	
Non-interpole	92A	35	500	65	30	28	2265
	101B2	40	500	75	35	33	2780
	93A2	60	500	105	50	46	3440
	112B	75	500	135	60	55	3490
	121	85	500	150	80	75	4254
	119	125	550	196	95	85	4682
	114	160	550	245	120	110	5300
	113	195	550	300	150 at 350 v.	...	6550
Interpole	323A	32	500	58	28	26	1890
	307	50	600	73	37	35	2850
	306	50	500	87	44	40	2661
	310	60	500	107	50	46	3510
	305	60	500	107	50	46	3550
	304	75	500	130	60	55	3550
	303	100	550	158	70	65	3950
	302	125	550	195	95	85	4685
	301	160	550	246	120	110	5510
	300	200	550	310	150	130	6475

Size of Motor Limited by Heating.—The heat developed in a railway motor is carried partly by conduction through the several parts and partly by

convection through the air to the motor frame, whence it is distributed to the outside air. As the temperature of the several parts is thus dependent not only upon their own internal losses, but also upon the temperature of the neighboring parts, it becomes necessary to determine the actual value and distribution of losses in a railway motor for a given service in order to determine with precision what the temperature rise will be; or, *vice versa*, to determine what size of motor will be required to avoid too great a temperature rise.

For ordinary electric railway calculations, however, in view of the other uncertain elements which enter, it is usually sufficiently accurate to assume that the *relative* temperatures in the different parts of the motor are independent of the *relative* values of the copper and core losses. The copper loss, i.e., the rate at which heat is developed in the windings, is proportional to the square of the current, and the core loss, i.e., the rate at which heat is developed in the core, due to hysteresis and eddy currents, may be taken as proportional to the first power of the voltage across the motor terminals (this latter relation being approximate only). When the motor reaches a constant temperature under a constant load, the temperature of any part will then be proportional (approximately) to the total power (kilowatts) lost in the windings and core. Similarly, under a fluctuating load continuing over a long period during which there are no excessively long breaks or excessive overloads, the temperature becomes fairly constant and the rise is proportional to the average power (kilowatts) lost during this period. There will be times at which the temperature rise will exceed this average and times at which it will be less, but on account of the heat storage capacity (or thermal capacity) of the materials of which the motor is made the fluctuations in temperature will be very much less than the fluctuations in the load.

Size of Motor Limited by Average Temperature Rise. — The manufacturers supply information as to the current which any motor will carry continuously (on stand test) without overheating, at various voltages from one-half to full voltage; see Table XI. From this information, making the assumptions noted in the preceding paragraph, it is possible to determine the approximate temperature of the motor for any given run or series of runs. The process is to calculate the root-mean-square current per motor and the average motor voltage for the particular service contemplated, using the methods given above in the paragraphs headed *Root-Mean-Square Currrent per Motor* and *Average Motor Voltage.* Call these values of the r.m.s. service current and average motor voltage I_e and E_m respectively. Let I_c be the continuous-current capacity at a given voltage E_c, as given by the manufacturer (see Table XI), and let T_c be temperature rise corresponding to this continuous rating. (Motors having ordinary fibrous insulation are rated on the basis of a 75° C. temperature rise on stand test, which corresponds to about 65° C. rise in actual service, due to better ventilation; see *Standardization Rules of the A.I.E.E.;* hence for ordinary motors T_c is 65° C.). Let J_c be the corresponding core loss and K_c the corresponding copper loss and $L_c = J_c + K_c$ be the corresponding total electrical losses. Then the total electrical losses corresponding to the average load are

$$L_a = \frac{E_m}{E_c} J_c + \left(\frac{I_m}{I_c}\right)^2 K_c,$$

and the average temperature attained by the motor in service will be approximately

$$T_a = \frac{L_a}{L_c} \cdot T_c.$$

For safe operation the average temperature rise T_a should never exceed the value T_c, which for motors with ordinary fibrous insulation is 65° C.

Approximate Values of J_c and K_c. — When the core loss and copper loss are not given separately, a rough estimate of J_c and K_c may be made by assuming that at rated load (one-hour rating and line voltage) the core loss is, say, ¼th of the total electrical losses. The total electrical losses L_r in kw. at rated load may be found from the characteristic curves of the motor by using the formula

$$L_r = (1.05 - \epsilon)P,$$

where P is the one-hour rating in kilowatts, ϵ the efficiency of the motor with gears, and the 0.05 takes into account the frictional losses in the motor and the gears. Let I_r be the rated current and E_r the rated voltage; then

$$J_c = \frac{L_r}{4}\frac{E_c}{E_r} \qquad \text{and} \qquad K_c = \frac{3\,L_r}{4}\left(\frac{I_c}{I_r}\right)^2.$$

Size of Motor Limited by Short-time Temperature Rise. — When the service is such that the motor must take a heavy current for a comparatively long interval (e.g., a long starting period or a heavy grade for a considerable distance) followed by a like period of light load or no load, the average temperature for the run, as calculated above, may be within the required limits, but the short-time temperature rise may be excessive. This short-time temperature rise depends upon the heat-storage capacity of the motor, i.e., upon the *energy* loss (number of *kilowatt-hours* of heat developed in it) required to raise its temperature one degree, say, assuming no radiation of heat from its surface. The one-hour rating of a motor is an indirect measure of this heat-storage or thermal capacity.

The temperature-time curve during the first hour's application of a load is practically a straight line whose slope is proportional to the load. The rise in temperature of the motor due to a short-time load may then be assumed to be proportional to the *energy* (kilowatt-hour) input during this time, and the factor of proportionality may be obtained from the one-hour rating as follows: Let T_r = the temperature rise at the end of one hour due to a load equal to the one-hour rating of the motor (rated current and rated voltage), L_r = the total electrical losses in kilowatts corresponding to the rated load, L_a = the total electrical losses in kilowatts corresponding to the average load in service (L_r and L_a may be estimated by the method given in the preceding section), L_p = the total electrical losses in kilowatts corresponding to any given short-time or peak load, t_p = the number of minutes' duration of this peak load. Then during this interval t_p the rise of temperature above the average value T_a is

$$T_p - T_a = \frac{t_p\,T_r}{60\,L_r} \cdot (L_p - L_a).$$

T_p as calculated from this formula gives approximately the maximum temperature rise during the run. For safe operation this maximum temperature rise T_p should not exceed the safe limit stated by the manufacturer. For motors with ordinary fibrous insulation T_p should not exceed 75° C.

Final Choice of Motor. — No motor should be employed for a given service which does not meet the above requirements regarding the maximum current and heating limits. A larger motor than that fixed by these requirements may prove the cheaper in the long run, if by using such a motor the energy consumption can be materially reduced by increasing the amount of coasting during the run. In any event the motor should be of sufficient capacity to permit of a reasonable amount of coasting under normal conditions, so that there will be a sufficient margin in which to make up for lost time, due to unexpected slow-downs or extra stops.

ANALYTICAL METHOD OF PREDETERMINING ENERGY AND MOTOR EQUIPMENT. — The following method was suggested by that developed by Cary T. Hutchinson, in two papers in the *A.I.E.E. Trans., Vol. 19, p. 129, 1902,* and *Vol. 22, p. 657, 1903.* In this method a speed-time curve similar in shape to the curve *ABCEF* in Fig. 1 is assumed. That is, the acceleration during the controller period, the train resistance and the braking retardation are all assumed constant, but a "motor-curve" period (*BC* in Fig. 1) is also taken into account, this latter constituting the essential difference between this method and the "straight-line" speed-time curve method frequently employed for approximate calculations. The introduction of this motor-curve period in the calculations enables one to approximate much more closely actual working conditions, and the results are much more accurate. In addition this method enables one to predetermine, without choosing any particular equipment, the effect of rate of acceleration, rate of braking, per cent of coasting, etc.

TABLE XII. — AVERAGE MOTOR CHARACTERISTICS

y = ratio of any given speed to speed at rated input	f = ratio of tractive effort at this speed to rated tractive effort	p = ratio of power input at this speed to rated input
0.80	2.06	1.82
0.85	1.67	1.54
0.90	1.38	1 30
0.95	1.19	1.14
1.00	1.00	1.00
1.05	0.83	0.87
1.10	0.71	0.78
1.15	0.61	0.70
1.20	0.53	0.625
1.25	0.46	0.57
1.30	0.41	0.525
1.35	0.365	0.48
1.40	0.32	0.45
1.45	0.295	0.425
1.50	0.27	0.395
1.55	0.23	0.375
1.60	0.215	0.35
1.65	0.20	0.33
1.70	0.18	0.315
1.75	0.17	0.30
1.80	0.16	0.29
1.85	0.14	0.28
1.90	0.13	0.265
1.95	0.125	0.26
2.00	0.12	0.25

Average Motor Characteristics. — To take into account the motor-curve portion of the speed-time curve it is necessary to consider the speed,

tractive effort and current input characteristics of the motors. However, instead of using the motor characteristics for any specific motor, the average characteristic curves of direct-current motors* given by Mr. Hutchinson are employed, see Table XII. These average characteristics were calculated by plotting the characteristic curves for the various sizes of direct-current motors manufactured by the General Electric and Westinghouse Companies, expressing the various quantities (current, speed, and tractive effort) as fractions of their values at rated load. Such curves are found to lie very close together, which justifies the use of a single set of curves representing the averages for the various motors. Mr. Hutchinson's curves were calculated from the characteristics of non-interpole motors, but have been found to check also quite closely with the curves for interpole motors. It is also found that these curves have practically the same shape irrespective of what point, between 75% and 125% of rated load, on the curves is taken as unity.

Method of Calculation. — The following symbols are employed:

n = number of stops per mile = total number of stops including one terminus divided by the distance between termini in miles,

V = average running speed in miles per hour,

$T = \dfrac{3600}{nV}$ = average running time between stops in seconds,

a = acceleration in miles per hour per second,†

b = braking rate in miles per hour per second,

r = train resistance in pounds per ton corresponding to a speed from 10 to 20% greater than the average speed V,

G = "equivalent" grade in per cent (*see above*),

g = average curvature, in degrees (*see above*),

$c = \dfrac{r + 20G + g}{100}$ = average "effective" coasting retardation in miles per hour per second,

ϵ = over-all efficiency (expressed as a fraction) of motors and gears at rated load; ϵ is about 0.85 for direct-current motors and about 0.75 for alternating-current motors, the latter figure including step-down transformer losses,

s = ratio of total standing time (including stops and lay overs) to total time that the train is moving,

W = total weight of train in tons,

M = total number of motors for the entire train,

E = average line voltage.

* The method is also applicable to alternating-current motors, but average characteristic curves for such motors are not available. They may, however, be readily constructed, or the characteristic curves given for direct-current motors may be used as an approximation.

† When the starting tractive effort is given, then a is to be calculated from the formula $a = (F - r) \div 100$, where F is the tractive effort in pounds per ton and r the train resistance corresponding to a speed from 10 to 20% greater than V. The actual starting acceleration will be greater than this, due to the lower train resistance at low speeds. Vice versa, if the starting acceleration is given, then the calculated tractive effort (and therefore the starting current and horse-power) at end of the controller period will produce a starting acceleration in excess of the assumed value. As far as the energy consumption and heating are concerned, however, these differences in the actual and assumed accelerations will be balanced by the higher train resistance at the higher speeds.

Calculate

$$\beta = \frac{a}{c},$$

$$A = \frac{V}{T} = \frac{nV^2}{3600},$$

$$m = 2A\left(\frac{1}{c} - \frac{1}{b}\right),$$

$$q = \frac{a}{A}(1 + m).$$

Select a value of the ratio

$$x = \frac{\text{time power is applied}}{\text{time of controller period}}.$$

A run of specified distance can be made in a given time with various values of this ratio x, since this ratio depends upon the proportion of the time that the train coasts; i.e., the greater the coasting time the smaller the value of x, see Fig. 1. The less the value of this ratio, however, the greater will be the starting current required, and therefore the larger the motor capacity in order to avoid sparking. By carrying through the calculations for several values of x one can determine the relation between energy consumption, time of coasting and starting current, and, by plotting, find the minimum value of the energy consumption corresponding to the data assumed, see Fig. 9. In typical rapid-transit service (1 or more stops per mile) minimum energy consumption usually corresponds to a value of x between 2 and 5, i.e., for the controller period lasting from 50 to 20 per cent of the total time power is on.

From Tables A to D, pp. 1196 and 1197, find the values of y, λ, u and i* corresponding to the selected value of x and the ratio β. Using these values calculate

$$J = \frac{x + \beta y}{q}, \quad \text{and} \quad H = \frac{\lambda + \beta y^2}{2q}.$$

Then calculate the quantities listed in the following table.

---

* The analytical expressions for x, λ, u and i in terms of y, f and p (see column headings of Table XII for definitions) are

$$x = 1 + \int_1^y \frac{\beta\, dy}{(1 + \beta) f - 1}$$

$$\lambda = 1 + 2\int_1^x y\, dx$$

$$u = \frac{1 + \beta}{18.1\beta}\left[0.75 + \int_1^x p\, dx\right]$$

$$i = 199\left(1 + \frac{1}{\beta}\right)\sqrt{1 + \int_1^x p^2\, dx}.$$

In the expression for u the constant 0.75 is for series-parallel control. For straight resistance control change 0.75 to 1.00 and for series-series-parallel control change 0.75 to 0.69. For a-c. commutator motors change 0.75 to 0.56. Tables A to D were made up by calculating the value of the above expressions graphically, using for y, f and p the values given in Table XII. Similar tables may readily he calculated for a-c. commutator motors, or for any other type of motor whose "percentage" characteristics differ from Table XII.

Speed at end of controller period in miles per hour....	$V_1 =$	$\dfrac{V}{J+\sqrt{m(H-J^2)}}$
Maximum speed in miles per hour................	$V_m =$	yV_1
Watt-hours per ton-mile...........................	$U =$	$\dfrac{unV_1^2}{\epsilon}$
Ratio of total running time to time on controller.....	$X =$	$\dfrac{aV}{AV_1}$
Ratio of coasting time to running time.............	$C =$	$\dfrac{b(X-x)-ay}{(b-c)X}$
Horsepower output at end of controller period......	$P_0 =$	$\dfrac{(a+c)V_1W}{3.74M}$
Starting current per motor *......................	$I_0 =$	$\dfrac{199(a+c)V_1W}{ME\epsilon}$
Root-mean-square current per motor (including stops) *..................................	$I_e =$	$\dfrac{aiV_1W}{ME\epsilon\sqrt{X(1+s)}}$
Average voltage across motor terminals (including stops)..	$E_m =$	$\dfrac{(x-0.45)E}{X(1+s)}$

* For alternating-current series motors insert the power factor (as a decimal) in the denominator of the formula.

Example. — Consider the following example:

Number of stops per mile.....................	$n = 0.938$
Average running speed........................	$V = 28.7$ mph.
Acceleration rate.............................	$a = 1.84$ mphps.
Braking rate..................................	$b = 1.60$ mphps.
Train resistance at 34.4 mph..................	$r = 15$ lb. per ton.
Average equivalent grade......................	$G = 0.22$ per cent
Average curvature.............................	$g = 1.2$ per cent
Average effective coasting retardation...........	$c = 0.21$
Efficiency of motors and gears at rated load.....	$\epsilon = 0.88$
Ratio of standing to running time (20 sec. stops).	$s = 0.15$
Total weight of train in tons..................	$W = 132$ tons
Total number of motors........................	$M = 6$
Average line voltage..........................	$E = 571$

Then

$$\beta = \frac{1.84}{0.21} = 8.76,$$

$$A = \frac{0.938\,(28.7)^2}{3600} = 0.214,$$

$$m = 2 \times 0.214\left(\frac{1}{0.21} - \frac{1}{1.60}\right) = 1.77,$$

$$q = \frac{1.84\,(1 + 1.77)}{0.214} = 23.8.$$

Then (*see above for meaning of symbols*)

$x =$		1	2	4	6	10
$y =$	....	1.00	1.43	1.69	1.83	1.96
$\lambda =$	....	1.00	3.52	9.88	17.0	32.1
$u =$	....	0.0462	0.0822	0.126	0.163	0.228
$i =$	....	222	260	281	301	321
$J =$	$\frac{x + 8.76y}{23.8}$	0.410	0.608	0.788	0.925	1.143
$H =$	$\frac{\lambda + 8.76y^2}{47.6}$	0.205	0.452	0.735	0.973	1.380
$V_1 =$	$\frac{28.7}{J + \sqrt{1.77(H - J^2)}}$	43.1	28.8	23.1	20.9	19.2
$U =$	$1.07 u V_1^2$	92	73	72	76	90
$X =$	$\frac{247}{V_1}$	5.73	8.57	10.7	11.8	12.9
$C =$	$1.15\left(1 - \frac{x}{X}\right) - \frac{1.32y}{X}$	0.72	0.67	0.51	0.36	0.06
$P_0 =$	$12.07 V_1$	520	348	279	253	232
$I_0 =$	$17.8 V_1$	770	512	411	371	342
$I_e =$	$\frac{0.076 i V_1}{\sqrt{X}}$	304	195	151	139	131
$E_m =$	$\frac{497(x - 0.45)}{X}$	47	90	165	233	367

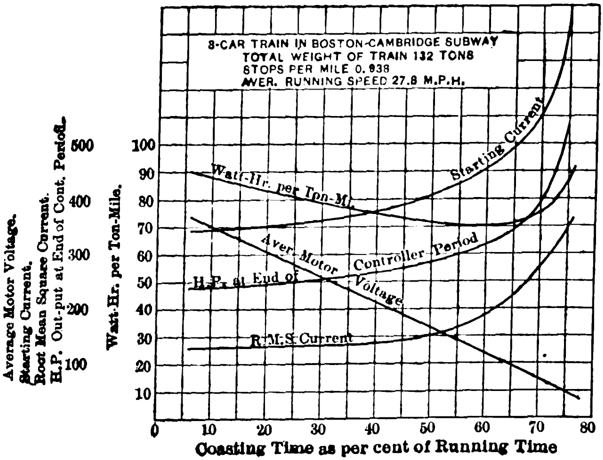

Fig. 9.

These results are plotted in Fig. 9. An actual test on this road was made, showing an average coasting time of 10 per cent. The table on p. 1198 gives the

TABLE A. — VALUES OF y IN TERMS OF x

x	Numbers in first line give values of $\beta = \frac{a}{c}$											
	1	2	3	4	5	6	7	8	9	10	15	20
1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.5	1.17	1.22	1.24	1.26	1.27	1.28	1.28	1.28	1.29	1.29	1.29	1.29
2	1.20	1.30	1.35	1.38	1.40	1.42	1.42	1.43	1.43	1.43	1.45	1.45
2.5	1.22	1.34	1.41	1.45	1.47	1.49	1.51	1.51	1.52	1.53	1.55	1.56
3	1.23	1.37	1.44	1.49	1.53	1.56	1.58	1.59	1.60	1.61	1.63	1.64
4	1.24	1.37	1.48	1.55	1.60	1.63	1.66	1.68	1.70	1.72	1.76	1.79
5	1.24	1.38	1.50	1.58	1.65	1.70	1.73	1.76	1.79	1.81	1.86	1.89
6	1.24	1.38	1.51	1.60	1.67	1.73	1.77	1.81	1.84	1.87	1.94	1.98
7	1.24	1.38	1.52	1.61	1.68	1.75	1.81	1.85	1.88	1.92	2.00	2.05
8	1.24	1.39	1.52	1.62	1.70	1.77	1.83	1.88	1.91	1.95	2.05	2.11
9	1.24	1.39	1.53	1.63	1.71	1.78	1.84	1.90	1.94	1.98	2.10	2.16
10	1.24	1.39	1.53	1.63	1.72	1.80	1.86	1.92	1.97	2.00	2.13	2.20
12	1.24	1.39	1.53	1.64	1.73	1.81	1.88	1.95	2.00	2.05	2.20	2.28
14	1.24	1.39	1.53	1.64	1.74	1.83	1.90	1.97	2.02	2.07	2.24	2.35
16	1.24	1.39	1.53	1.64	1.75	1.84	1.92	1.98	2.05	2.10	2.27	2.40
18	1.24	1.39	1.53	1.65	1.75	1.85	1.93	2.00	2.07	2.12	2.31	2.44
20	1.24	1.39	1.53	1.65	1.76	1.86	1.94	2.02	2.09	2.15	2.34	2.48

Example. — For $x = 2$ and $\beta = 3$, $y = 1.35$.

TABLE B. — VALUES OF λ IN TERMS OF x

x	Numbers in first line give values of $\beta = \frac{a}{c}$											
	1	2	3	4	5	6	7	8	9	10	15	20
1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.5	2.12	2.14	2.15	2.16	2.16	2.17	2.17	2.17	2.17	2.17	2.17	2.17
2	3.30	3.40	3.46	3.48	3.50	3.50	3.52	3.52	3.52	3.52	3.52	3.52
2.5	4.50	4.74	4.84	4.90	4.92	4.94	4.96	4.98	5.00	5.00	5.02	5.04
3	5.70	6.10	6.28	6.38	6.44	6.50	6.52	6.54	6.58	6.58	6.61	6.65
4	8.16	8.84	9.30	9.44	9.60	9.70	9.80	9.83	9.90	9.90	10.0	10.1
5	10.6	11.6	12.3	12.6	12.8	13.0	13.1	13.2	13.3	13.4	13.6	13.6
6	13.0	14.4	15.4	15.7	16.1	16.4	16.6	16.8	17.0	17.2	17.4	17.7
7	15.4	17.1	18.1	18.9	19.4	19.9	20.3	20.5	20.6	20.9	21.3	21.7
8	18.0	19.8	21.2	22.2	23.0	23.7	24.1	24.5	24.8	25.0	25.5	25.9
9	20.4	22.6	24.2	25.4	26.2	27.0	27.7	28.0	28.3	28.8	29.7	30.0
10	23.0	25.6	27.4	28.6	29.6	30.5	31.2	31.9	32.2	32.7	33.8	34.3
12	27.8	31.0	33.6	35.2	36.8	38.0	39.0	39.7	40.2	40.8	42.3	43.4
14	32.6	36.6	39.4	41.8	43.8	45.8	46.5	47.3	48.1	48.9	51.0	53.0
16	37.4	42.2	45.6	48.2	50.6	52.7	54.1	55.8	56.8	57.7	60.3	62.2
18	42.2	47.6	51.6	54.8	57.4	60.0	62.0	63.3	64.8	66.0	69.6	72.0
20	47.2	53.4	58.0	61.6	64.8	67.2	69.4	71.2	73.0	74.2	79.0	82.0

TABLE C.—VALUES OF u IN TERMS OF x

x	Numbers in first line give values of $\beta = \frac{a}{c}$											
	1	2	3	4	5	6	7	8	9	10	15	20
1	0.082	0.0620	0.0550	0.0515	0.0495	0.0481	0.0472	0.0466	0.0460	0.0456	0.0445	0.0440
1.5	0.125	0.0922	0.0813	0.0760	0.0729	0.0709	0.0695	0.0685	0.0676	0.0670	0.0642	0.0627
2	0.161	0.115	0.100	0.0940	0.0895	0.0863	0.0843	0.0830	0.0819	0.0810	0.0780	0.0769
2.5	0.195	0.134	0.118	0.109	0.104	0.101	0.0980	0.0960	0.0948	0.0938	0.0902	0.0888
3	0.230	0.155	0.133	0.123	0.117	0.113	0.110	0.107	0.106	0.104	0.100	0.0975
4	0.298	0.193	0.162	0.149	0.140	0.135	0.131	0.128	0.126	0.124	0.118	0.115
5	0.362	0.232	0.194	0.175	0.163	0.156	0.150	0.146	0.143	0.141	0.135	0.133
6	0.431	0.270	0.222	0.199	0.185	0.176	0.170	0.165	0.162	0.159	0.151	0.147
7	0.500	0.310	0.252	0.223	0.207	0.196	0.188	0.182	0.179	0.176	0.166	0.161
8	0.565	0.347	0.279	0.247	0.227	0.215	0.207	0.200	0.195	0.191	0.179	0.174
9	0.634	0.386	0.309	0.270	0.249	0.233	0.223	0.216	0.210	0.206	0.192	0.188
10	0.705	0.425	0.338	0.293	0.270	0.252	0.241	0.232	0.227	0.221	0.204	0.199
12	0.833	0.500	0.390	0.339	0.307	0.287	0.273	0.263	0.255	0.249	0.230	0.220
14	0.965	0.580	0.450	0.384	0.349	0.324	0.309	0.296	0.286	0.278	0.251	0.239
16	1.11	0.653	0.504	0.430	0.388	0.360	0.340	0.327	0.314	0.305	0.273	0.259
18	1.24	0.738	0.560	0.475	0.427	0.395	0.373	0.356	0.342	0.331	0.293	0.278
20	1.38	0.809	0.620	0.521	0.468	0.430	0.406	0.387	0.371	0.359	0.311	0.290

TABLE D.—VALUES OF i IN TERMS OF x

x	Numbers in first line give values of $\beta = \frac{a}{c}$											
	1	2	3	4	5	6	7	8	9	10	15	20
1	398	300	267	249	239	232	228	224	221	220	213	209
1.5	452	334	297	278	267	260	253	250	248	245	238	233
2	488	355	312	292	280	272	267	262	259	257	249	243
2.5	518	369	322	301	289	280	274	270	267	263	257	250
3	546	383	337	311	298	289	281	277	273	270	261	254
4	599	411	352	327	310	300	292	287	282	279	270	263
5	647	432	369	338	321	309	301	295	290	288	277	270
6	688	455	382	350	331	318	310	303	299	295	283	275
7	730	475	397	360	340	326	317	310	303	300	289	280
8	770	497	409	369	347	331	321	315	310	305	292	284
9	806	513	421	379	355	340	329	321	315	310	297	288
10	845	531	431	387	361	344	333	327	320	315	301	291
12	911	565	454	403	377	358	345	337	329	323	307	295
14	975	600	479	422	391	370	356	347	338	331	313	300
16	1030	630	500	440	404	380	365	353	344	339	318	303
18	1085	656	518	451	414	390	372	360	350	343	320	307
20	1140	681	531	462	423	399	380	368	357	350	326	309

actually observed values of the various quantities and the corresponding values taken from the calculated curves in Fig. 9 for a coasting time of 10 per cent.

	Observed	Calculated
Watt-hours per ton-mile	91	88
Horse-power output at end of controller period	249	241
Starting current per motor	370	345
Root-mean-square current per motor	132	131
Average motor voltage	337	350

From Table XI the proper size of motor would be the interpole motor No. 300, which is rated as 200 horse-power at 550 volts. The motor actually used was a Westinghouse motor, designated as 300 – *B*, and rated at 225 horse-power at 600 volts. The temperature rise from test, after 13 hours in service, was

Armature	65° C.
Commutator	75° C.
Series field	46° C.
Interpole field	52° C.
Frame	37° C.

Importance of Coasting and Selection of Gear Ratio.—A study of Fig. 9 shows that a much less energy consumption would be required for a coasting period of 50 per cent instead of the actual coasting period of 10 per cent, namely 72 watt-hours instead of 88 watt-hours. The starting current required in order to obtain this higher coasting time is 406 amperes instead of 345 amperes, or an increase in the starting current of 18 per cent. If no change in the size of motor were made, this would require approximately the same percentage increase in the gear ratio. The root-mean-square current would increase from 131 to 150, but the average motor voltage would drop from 350 to 170. The motor could therefore probably operate at this higher gear ratio without seriously overheating, but it would be safer to use a larger size motor, particularly as the starting current of 406 amperes is also close to the safe commutating limit.

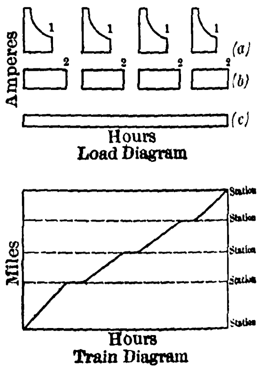

Fig. 10.

TRAIN AND LOAD DIAGRAMS.—The current-time curve for a train making a number of stops may be represented as shown by (*a*) in Fig. 10. On a railway line where there are several trains, the total current may be obtained by placing the current curve for each train at its proper place in the time scale, and adding the ordinates of the curves. Such a process is very tedious and unnecessary where there is a large number of trains. In such cases the high and low parts of the curves become staggered with respect to one another more or less according to the laws of chance, so that each current curve may be replaced by a rectangle of the same area but with a base extending over the entire running time as shown by (*b*) in Fig. 10.

When this is done the kilowatts and amperes per train are derived from the watt-hours per ton-mile by the following formulas:

$$\text{kilowatts} = \frac{WV}{1000} \times (\text{watt-hours per ton-mile}),$$

$$\text{amperes} = \frac{WV}{E} \times (\text{watt-hours per ton-mile}),$$

where

W = weight of train in tons,

V = average running speed (excluding stops) in miles per hour,

E = line voltage.

The time when the current is cut off is indicated by 1, and that when the train stops, by 2, in Fig. 10.

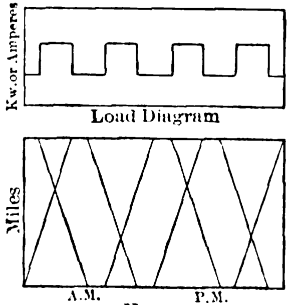

Fig. 11.

Another approximation, which is even more often used, is to replace the series of rectangles shown at b, by a single rectangle as shown at c in Fig. 10. The area of this rectangle will be equal to the sum of the areas of the smaller rectangles or current curves. Using this approximation, kilowatts or amperes may be obtained from the above formulas, taking, however, V to be the schedule speed (i.e., speed including stops). The procedure is to plot a train diagram showing when each train comes on and off the line, neglecting intermediate stops, as shown for a simple case in Fig. 11. Each time a train comes on or off, the corresponding kilowatts or amperes are added to, or subtracted from, the load diagram.

Power Required for Car Heating and Lighting. — In addition to the energy required for propelling the cars, a very appreciable amount is also required in the winter, for heating them, and a small amount at night for lighting. In making up a load diagram this energy should be included.

The average power for car heating varies, of course, with the climate and time of year. The following figures represent usual requirements in the northern parts of the United States:

TABLE XIII. — HEATING AND LIGHTING OF CARS

Length of car, feet	Average* kw. for lighting	Average† kilowatts for heating	
		Average conditions	Severe conditions
14–20	0.25	3.5	4
20–28	0.35	4.5	5.5
28–34	0.55	5.5	7.5
34–40	0.70	7.5	10.5

* During the hours lights are on, using Tungsten lamps.
† During the time car is in service.

Substation and Power-station Loads. — The load diagram obtained as described above gives the total load at the trains. To obtain the load at a sub-

station, the kilowatts or amperes must be increased by a suitable amount to allow for the losses in the distribution system. The load diagram of the power station should allow for all transmission and distribution losses between the power house and substation (*see Power Stations; Substations; Third-rail Systems; Trolley Systems*), and also for all auxiliary power, such as that required for station lighting, shop machinery, etc.

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[W. A. Del Mar and H. Pender.]

RAILWAYS, LOCATION AND PERMANENT WAY FOR.— (*See also Railways, Energy Requirements and Motor Capacity for; Railways, Electric, Traction Systems for; Third-rail Systems; Trolley Systems.*) There are four general steps in the determination of the location of a railway, (1) the reconnoissance, which is a personal examination by the locating engineer of the country through which the railway is to run, (2) a preliminary survey and investigation more particularly of the topographic features of the proposed road, (3) office study of the data obtained in (1) and (2), and (4) the field location and the preparation of final location plans.

RECONNOISSANCE. — First a careful examination of the country should be made, with a view not only of obtaining a line which will be reasonably straight and as free from grades as possible, but also with a view of selecting a line or lines which will earn a suitable revenue. An electric railway reconnoissance calls for a much more detailed study of the country than a steam railroad location and a very much more detailed study of the amount of population, its growth, and the likelihood of the population making use of the proposed line. The adapting of an electric line to take in the various intermediate centers of population is practically a necessity if the income of the line is to be sufficient to take care of the investment. The engineer should therefore realize fully that the commercial and financial matters relating to the road are more important than the engineering problems, and should always bear in mind that the road is constructed for the purpose of selling transportation.

Electric railway locations are divided in general into two classes: those which are operated on the public highways, and those which are operated upon their own private rights of way, where they are free to make the fastest practicable schedules. Speed and comfort must be considered. Both of them are, as a rule, better obtained on private locations. It is essential in this reconnoissance for the engineer to make a careful estimate of the probable gross income of the road, as well as to choose two or three lines which will later on be investigated by preliminary surveys.

PRELIMINARY SURVEY. — After the favorable lines have been determined by the locating engineer in his reconnoissance, a party is sent into the field to make a preliminary survey. This is usually a linear traverse run by transit and tape. After this a level party determines the profile of this line, this party being followed by a topographical party which sketches in the 5-foot contours, the locations of brooks, highway crossings, the number and kinds of industries located within say a mile or so of the line, the location of existing recreation points and picnic grounds, as well as the possible location and development of those which do not now exist. The engineer running the preliminary line should keep in mind that the railroad is built for the convenience of the public and that any inconvenient detail of the road will have a material effect upon its gross income.

The topographical party should note the character and property value of buildings or other structures which the line may affect, the character of soil (whether rock or gravel), the location of possible borrow pits, etc. In some locations the topographical features should extend a considerable distance from the transit line; in other locations, where, for example, the line is passing through a narrow valley, the topographical features need not be taken very far from the transit line. While the line should be as straight as possible, some curves are necessary. In locating curves consideration should be taken of the fact that electric traction permits of very rapid acceleration and retardation, and therefore sharper curves are permissible than in steam railroad practice because of the shorter time required to pass around it, even at the

slower speed rendered necessary because of its greater curvature. The time lost in passing around the curve as well as the permissible speed are factors which should be taken into consideration. Roughly speaking, a safe rule is that the speed on curves in miles per hour can be equal to the square root of the radius in feet.

OFFICE STUDY. — On the preliminary map an accurate paper location is drawn and an estimate is made of the cost of the road and of the probable gross earnings, as well as of the fixed charges and the cost of operating and maintaining. This study is one of great importance and if the reconnoissance and preliminary survey are not thoroughly accomplished, errors in judgment may very easily creep into this part of the work. The office study shows the final alinement and the proposed grades of the road.

FIELD LOCATION. — After the completion of the office study the line is finally located in the field. Field location requires running out the line by straight lines and curves, using easement curves at the beginning and end of all changes in direction; the preparation of property maps showing the location required for the railway and all parcels of property affected by actual takings for slope easements; as well as studies of grade crossing eliminations where they are necessary. At this time soundings or borings are taken for bridges or other structures. Cross sections are made of the line, from which careful estimates are made. These cross sections are used in computing the final payment for the work of construction.

Locations in Existing Highways. — Such locations usually have the great advantage of easy grades which frequently cannot be so cheaply obtained upon private land. Locations in highways have the disadvantage of requiring much slower speed. Where such locations are made it is necessary to survey the highways as laid out by the proper officials, and not infrequently it is necessary to consider the widening of such highways by takings on either side. The electric road, for example, may be a 2-track road laid out in the center of the roadway, in which case the road would possibly occupy nearly the full width of the existing highway. In this case the electric road would be required to purchase additional property on either side and probably to build roadways for other vehicles in lieu of the portion which the electric line occupies. When this is done high speeds can sometimes be maintained. Where the electric road occupies one side of an existing highway, the speed often has to be reduced to a relatively low rate. This affects the amount of traffic which the road will attract and thereby reduces the gross earnings.

DEFINITIONS OF TERMS USED ON RAILWAY MAPS AND PROFILES. — Some of the more common terms are defined below.

Curves. — A curve is generally composed of successive arcs of circles joining two straight lines or tangents. When these arcs are of varying radii decreasing with the distance from the tangents the curve is said to be compounded or spiralled. To insure smooth riding the curve should begin with a large radius and gradually grow sharper until the circular part is reached. At the same time the outer rail should be gradually raised (see below) until it reaches its maximum elevation. A curve so built is called an easement curve. Curves, if properly eased, may be run at full speed. On high-speed lines curves sharper than 10 degrees should be avoided.

The following terms are in general use:

The point of curvature (P. C.) is the beginning of the curve, and the point of tangency (P. T.) is the end of the curve, going in the direction of the surveyor's stationing on the line.

The point of intersection (P. I.) is the point where the two tangents through P. C. and P. T. intersect.

The tangent distance is the distance between the P. C. or P. T. and P. I.

The degree of curvature is the angle subtended at the center of a curve by a chord 100 ft. long. Up to a curvature of 12 degrees the radius of curvature may be found to a close approximation by dividing 5730 by the degree of curvature. The exact relation between the degree of curvature C and the radius R is

$$R = \frac{50}{\sin \frac{C}{2}}.$$

The middle ordinate is the perpendicular distance from the center of a chord to the curve.

Elevation of Outer Rail on Curves.—The outer rail on curves is raised above the inner an amount depending upon the velocity of cars and the degree of the curve. This elevation must be gradually attained. The amount of elevation required in order to make the weight of the car just balance the centrifugal force on the curve is

$$E = \frac{DV^2}{32.2\,R} \text{ feet,}$$

where D is the distance between center lines of the two rails (not the track gage), V the speed of car in feet per second, and R the radius of curvature in feet. For standard track gage, viz., 4 feet 8½ inches from inside edge of the *head* of one rail to the inside head of the other rail, and taking $R = 5730/C$, where C is the curvature in degrees, the above expression for the elevation becomes

$$E = 0.000325\,V^2C \text{ inches.}$$

Grades. — A railroad grade is expressed in per cent, this per cent being the number of feet vertical rise in a horizontal distance of 100 feet; i. e., a 4 per cent grade means a rise of 4 feet in a horizontal distance of 100 feet. Calling L the distance in feet along the track and H the vertical rise in feet in this distance, the per cent grade may also be expressed as

$$G = \frac{100\,H}{\sqrt{L^2 - H^2}},$$

which for a grade less than 10 per cent is equal to $100\,H/L$ with an error of less than 1 part in 100.

Grades should be as small as financially practicable. It may be cheaper to operate over a grade than to pay interest on the sum needed to reduce it. Some steam road grades are as steep as 4 per cent, but such grades are extremely costly to operate. Two per cent is about the limit for steam roads and most roads try to keep grades down to about 0.5 per cent. On electric lines operating single trolley cars grades as steep as 10 per cent exist, but no grade over 6 per cent ought to be used unless it is absolutely unavoidable.

Virtual and Momentum Grades. — A driving force or tractive effort is required to accelerate a train, and a driving force or tractive effort is required to cause a train to ascend a grade at constant speed; hence an increase of the speed of the train may be considered, as far as the motive power is concerned, as equivalent to an up-grade. *Vice-versa,* a decrease of speed may be considered as equivalent to a down-grade. The energy required to give the train a speed of v miles per hour is the same as the energy required to raise it a height of $0.0334\,v^2$ feet. Hence, if to the actual elevation of the profile at each point is added a height of $0.0334\,v^2$ feet, where v is the velocity of the train at this

point, and a line is drawn showing the sum of the actual elevation and this velocity head at each point, this line may be looked upon as the "virtual profile" of the road, and the slope of this virtual profile, at any point, i.e., the change in its elevation per hundred feet, is called the "virtual grade" at this point.

The virtual grade at any point on an actual up-grade will be less than the actual grade, if the speed of the train is decreasing as it passes this point; the grade in this case is sometimes called a "momentum grade." That is, if the train has a high speed when it strikes an up-grade, and the operating requirements of the division will permit of a slowing down of the train as it ascends the grade, then the effective or virtual grade will be less than the actual grade. Under such circumstances a given locomotive can pull a given train up a short steep grade on which it could neither start this train nor pull it at constant velocity. A momentum grade must be comparatively short.

Ruling Grade.—The limiting or ruling grade on a division is that grade which limits the weight of the train which can be hauled over this division by the regular motive-power unit. Ordinarily, it is the maximum grade on the division, but if *momentum* grades are relied upon, the ruling grade might be the maximum virtual grade. Reliance upon momentum grades is not always considered good practice, since a heavy train which might get over this grade if it strikes the grade with sufficient momentum, cannot start on this grade if it should have to stop on the grade due to some emergency. When steep grades occur at only one or two places in a division it is sometimes economical to use helper engines on these steep grades, thus making possible the hauling of heavier trains, the weight of train then usually being limited by maximum grade on the remainder of the division.

Switches, Frogs, Cross-overs, Etc.—A switch is an arrangement of rails, which permits a car or train to pass from one track to another. A cross-over consists of a pair of switches connected by a short piece of track in such a way as to allow a car or train to pass or cross over from one track to another parallel one. A turn-out or siding is a short length of track parallel to the main track and connected thereto by a switch at each end; when used as a means of passing cars or trains it is usually called a turn-out, and when used primarily for storing cars or loading it is called a siding.

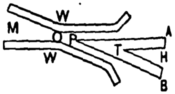

Fig. 1.

A frog, Fig. 1, is that part of the switch where the two inner rails cross each other, and is so designed as to allow the flange of the wheel to pass over without "riding up" on the rail. A guard rail is set close to the switch rail opposite the frog. The lead of a frog is the distance of a frog from the frog point P, see Fig. 1, to the point of the switch. The point of the switch is the tip end of the movable rail of the switch which is planed down to fit closely against the main rail. The number of a frog is equal to the quotient of the distance from P to the line AB divided by the length of the line AB.

Turn-outs.—Turn-outs are short sidings used on single-track roads to permit cars running in opposite directions to pass each other. The proper location of turn-outs is governed to a certain extent by topography. They can be readily located by drawing up a train diagram; see *Railways, Energy Requirements and Motor Capacity for*. To keep the number of turn-outs at a minimum the time intervals between cars (headway) should all be multiples of the shortest time interval. For accurate results the diagram should be carefully drawn to a scale large enough so that locations can be measured with an error not to exceed 200 feet for distance and one minute for time.

CONSTRUCTION OF PERMANENT WAY.—In the preparation of the roadbed for ballast, ties and track, it should be noted that rails settle more or less and allowance must be made for this when grading is in progress. The surface of the roadbed should be flat, from 14 to 20 feet wide for a single-track road, and from 9 to 10 feet wider for a double-track road, with a ditch at each side for drainage. On this the ballast of broken stone, gravel or cinders is laid varying from 6 to 12 inches thick under the ties. The ballast should ultimately be brought up level with the tops of the ties.

Ballast. — Broken stone is best but most expensive in first cost and maintenance. It is clean and dustless. Gravel is cheaper and very satisfactory from a maintenance standpoint. It is apt to be very dusty. Cinders are fairly satisfactory but are very dirty.

Ties.—Ties are of the following materials in the order named: Oaks, southern pine, Douglas fir, cedar, chestnut, cypress, western pine, tamarack, hemlock, redwood, lodge-pole pine and white pine; see article on *Timber*. The average cost is from 80 to 30 cents each. Treated ties cost 95 cents. With the rapidly increasing cost of ties, preservative treatment is becoming important. Creosoting is best. Zinc chloride treatment is satisfactory in arid regions only. The average life of an untreated tie is 5 years for hemlock, 7 to 12 years for white oak, and 15 years for cedar ties. Treated ties may last 30 years, but must be protected from mechanical injury by rails and spikes. For this purpose tie-plates and screw spikes should be used. Ties are spaced about 2 feet between centers.

Rails. — See article on *Rails, Track and Third.*

Grade Crossings. — On high-speed lines grade crossings should be avoided wherever practicable. It costs less to avoid them in the beginning than to eliminate them afterwards. To eliminate an existing grade crossing costs from $40,000 up unless the topography is unusually favorable.

COST OF PERMANENT WAY. — When tracks are laid in paved streets the railway usually has to pay for paving 18 inches of the street outside the rails. The total cost of such track, excluding overhead construction, and using 8- or 9-inch girder rails is about $5 per foot of single track. When the track is constructed with a light T-rail, as may be the case when laid on a private right of way or at the side of a street where there is no paving, the cost may be as low as $1 per foot, exclusive of ballast, bonding, and overhead construction. Ballast will cost from $1000 for cheap gravel to $3000 for thin rock, per mile of track. Thick rock ballasting as used on steam railroads may go as high as $5000 per mile.

Maintenance Costs. — Maintenance of way costs will depend largely on the character of the original construction and also upon the character and amount of traffic. On typical Massachusetts street and interurban railways maintenance of way expenses are reported as from $200 to over $500 per mile of track. $200 per mile is not sufficient to maintain the road and track in an adequate manner, but from $350 to $380 is probably a fair figure.

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[L. E. MOORE.]

REACTANCE COILS OR REACTORS.—(*See also Alternating Currents; Inductance and Inductive Reactance.*) Coils designed particularly for the purpose of introducing reactance (*see Inductance and Inductive Reactance*) into a circuit, when connected therein, are called "reactance coils" or "reactors."

APPLICATIONS. — Reactance coils are used wherever (1) it is desired to produce a voltage drop in an a-c. circuit without producing a proportionate loss in power (*see Controllers*); (2) to introduce inductance into a circuit where a compounding effect is to be obtained by passing a leading current through the coils (*see Converters, Synchronous*); (3) in large power stations to limit the short-circuit current; and (4) as "choke coils" in connection with lightning arresters (*see Lightning Protectors*).

RATING OF REACTANCE COILS. — It has been common practice to rate a reactance coil in terms of the kv-a. absorbed by it when carrying such a current (of given frequency) as will produce a temperature rise of 40° C. above a 25° C. room temperature; see, however, the recent recommendations regarding rating in the article on *Standardization Rules*. Usually the resistance is negligible compared with the reactance, and under these conditions the kv-a. rating $= xI^2$.

Per cent Reactance. — A reactance coil having a reactance of x ohms is said to have a "per cent reactance" equal to $(100\,x\,I) \div V$, where I is the normal or full-load current through it and V the *total* normal voltage across the circuit in which it is placed; i.e., the percentage reactance is the percentage ratio of the reactance drop at normal current to the total voltage impressed on the circuit in which it is inserted. When the coil is to be inserted in one branch of a Y, the voltage V is the volts to neutral.

In the case of a three-phase generator, three-phase transformer or bank of transformers, or set of three single-phase reactance coils, the Y reactance, x, (*see Alternating Currents*) is related to the per cent reactance, p, by the formula

$$x = \frac{10\,pE^2}{(kv\text{-}a.)}, \tag{1}$$

where E is the *kilovolts* between phases (i.e., between the terminals of the Y) and $(kv\text{-}a.)$ stands for the total kilovolt-ampere rating of the three phases.

DESIGN OF IRON-CORE REACTANCES. — When a high inductance is required the coils are usually wound on an iron core, which usually contains an air gap, for the inductance of a coil on a completely closed iron core is by no means a constant, due to the variation of the permeability of the iron with the magnetizing current, and a straight-line relation between voltage and current does not hold. When a relatively low inductance is required the iron core may be omitted entirely. Reactance coils for use in connection with synchronous converters have an iron core.

In iron-core reactances the air gap is usually of such a length that practically all of the reluctance of the circuit is in the air gap. Hence in calculating the reactance of the coil the reluctance of the iron portion of the path may be neglected without materially affecting the results, provided normal magnetic densities exist in the iron (60,000 to 80,000 lines per square inch, maximum values). As the gap is made adjustable such errors as result from this approximation are allowed for by adjusting the gap afterwards.

The inductance L in henries and the reactance x in ohms are given approximately (neglecting effect of iron) by the equations

$$L = \frac{3.2\,S^2 A}{10^8\,l} \quad \text{and} \quad x = 2\pi f L, \tag{2}$$

where S = number of turns in coil; A = cross section of path in gap, in square inches; l = length of gap, in inches; f = frequency in cycles per second. This must, however, be checked to see that the magnetic density B is not too great at the maximum current to be used, thus, $B = (3.2 \times \sqrt{2}\, SI) \div l$, where I = the effective value of current; B = maximum value of flux density = 60,000 to 80,000 lines per sq. in.

The losses in such a coil are the core-loss in the iron and the RI^2 in the copper. See article on *Magnetic Properties of Materials* for the method of calculating the core-loss. The procedure in proportioning the various parts is similar to that employed in the design of transformers (q.v.).

POWER-LIMITING REACTANCES.* — (*See also Power Stations.*) In order to avoid the prohibitive expense of high-voltage insulation, power-limiting reactances are always designed to be installed in the low-tension circuits. From the standpoint of economy, this requirement prohibits the use of a magnetic core. When power-limiting reactances are installed at a distance greater than one-half their diameter from any iron or steel structure, no appreciable eddy current or hysteresis losses will be produced in such structures.

The inductance and reactance of the coils can be calculated from the formula for a short solenoid given in the article on *Inductance and Inductive Reactance.*

It is desirable to reduce the size of reactance coils as much as possible, and they are therefore now usually designed for a temperature rise of 70° C. As to the dielectric test, the A.I.E.E. rules recommend 2¼ times the line voltage plus 2000 for one minute, from conductor to ground.

Location of Reactance Coils. — As noted above the reactance coils may be inserted in the leads from the generator, between sections of the low-tension bus-bars, or in the low-tension leads of the transformers. Which one of these locations or combinations is preferable depends on a number of conditions. Modern water-wheel-driven generators are now designed for a very high inherent reactance and external power-limiting reactances are usually inserted between the bus-bar sections or in the low-tension transformer leads.

Reactances in Generator Leads. — With reactances in the generator leads the current flowing in the armature winding is limited, and this method therefore affords an excellent protection for the generator itself. An objection to generator reactances is the fact that a short-circuit on or near the bus-bars will cause a voltage drop on all the feeder circuits connected thereto.

Reactances between Bus-bar Sections. — With reactance inserted between the bus-bar sections the trouble is confined to the particular section on which short-circuit takes place. Bus-section reactances afford no protection to the generators connected to the bus to which the faulty line is connected, but they give added protection to the generators on the other sections.

Reactances in Low-tension Leads of Transformers. — Reactances in the low-tension leads of the transformer banks are of considerable value for protecting against short-circuits in the lines, where they, of course, mostly take place. They are, however, not of value if the short-circuit should occur on the low-tension bus or in the generators or in their leads. There is also a constant loss of power in the reactance coils when they are inserted in the transformer leads, as is also the case when they are installed in the generator leads. For large systems this may reach a considerable value and must not be ignored when the selection of reactances is made.

Calculation of Short-circuit Kilovolt-amperes. — The short-circuit current for any arrangement of reactances and for a short at any point can be found

* By D. B. Rushmore.

with sufficient accuracy by neglecting the resistances of the apparatus and connections and calculating the resultant reactance by applying successively the formulas for two reactances in series or in parallel, as the case may be. Note that the resultant reactance X_s of two reactances, x_1 and x_2 in series, and the resultant reactance X_p of two reactances, x_1 and x_2 in parallel, are respectively

$$X_s = x_1 + x_2,\ X_p = \frac{x_1 x_2}{x_1 + x_2} \qquad (3)$$

provided the resistances of the circuits are negligible. The reactances, x_1, x_2, etc., of the various circuits can be calculated from the percentage reactance by formula (1). Let X_r be the resultant reactance of all the circuits feeding into the short circuit, and let E be the kilovolt rating of the generators, then

$$\text{Total short-circuit kv-a.} = \frac{1000\,E^2}{X_r} \qquad (4)$$

For example, consider the case of four, 12,000 kv-a., three-phase, 11,000-volt generators feeding into a bus which is divided into two sections, A and B, with two generators feeding into each section, and one transmission line from each section, each line fed through a bank of transformers. A set of reactance coils, having 6 per cent reactance, connects the two bus sections. The inherent reactance of each transformer bank is 8 per cent, and the inherent reactance of each generator is 20 per cent. The reactance of each generator is then 2.0 ohms; of each transformer bank 0.4 ohm, and of each set of reactance coils 0.6 ohm. Let a short circuit occur between the three wires of the line connected to the bus section B. We then have $2.0/2 = 1.00$ ohm in series with 0.6 ohm, or a total of 1.6 ohms, which total is in parallel with the other two generators, i.e., in parallel with 1.0 ohm, giving a resultant reactance up to the transformer bank of 0.6 ohm; this is in series with the transformer bank, giving a final resultant reactance, $X_r = 0.6 + 0.4 = 1.0$ ohm. Hence the total kilovolt-amperes, from formula (4), is 121,000, or about 10 times the rating of each generator.

DIMENSIONS, WEIGHT AND COST.—Iron-core reactances with air gap occupy from 0.30 to 0.50 cubic feet per rated kv-a., weigh from 20 to 50 pounds per rated kv-a. and cost from $2.00 to $4.00 per rated kv-a., the first figure in each case applying to air-blast reactances and the latter figure to oil-cooled reactances. Power-limiting reactances occupy from 0.3 to 0.6 cubic feet per rated kv-a., weigh from 20 to 30 lbs. per rated kv-a. and cost from $4 to $12 per rated kv-a. The first figure in each case applies to reactances having a rating of approximately 500 kv-a. and the latter for ratings of approximately 50 kva-a. These figures apply to 60-cycle reactances and should be increased from 10 to 15 per cent for 25-cycle reactances.

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[W. I. Slichter.]

RECTIFIERS. — (*See also Arc, Electric; Converters; Motor-Generators.*) The term rectifier is applied to any stationary apparatus or rotating commutator for transforming alternating into direct current, or vice versa. Three types of rectifiers are in commercial use, viz., the mercury-arc rectifier, the electrolytic rectifier and the rotating commutator driven by a synchronous motor, but only the first type is at present of much importance commercially.

MERCURY-ARC OR MERCURY-VAPOR RECTIFIER. — The operation of this rectifier depends upon the fact that a tube containing mercury vapor under a low pressure and having one electrode of mercury, and the other of some other conductor, offers a very high resistance to a current tending to flow through the tube from the mercury to the other electrode, but has a very low resistance to a current flowing in the opposite direction, provided the current is once started by forming an arc in the tube, e.g., by tilting the tube so that the mercury touches for an instant the other electrode. See article on *Arc, Electric*.

Application. — Mercury-arc rectifiers find their chief application as a means of charging storage batteries and supplying a certain series type of d-c. arc lamp (*see Lamps, Electric*) from an a-c. supply. In the latter case the rectifier receives its current from a constant-current transformer and delivers a direct current of constant value. Mercury-arc rectifiers have also been tried out experimentally on electric cars and locomotives, power being supplied to the car or locomotive from a high-voltage a-c. trolley, stepped down to a lower voltage by transformers and converted into low-voltage direct current by means of the rectifiers. Although this scheme has not yet proved satisfactory commercially, it seems to be particularly promising when used in connection with high voltage (1200-volt) railway motors. The difficulty lies chiefly in producing a tube of rugged and lasting qualities.

Connections for Single-phase Operation. — The connections employed in practice are shown in Fig. 1. The complete equipment consists of a source of alternating current *HG*, the rectifier tube *AA'*, two reactances *E* and *F* and the load represented as a storage battery *J*. The rectifier tube is an exhausted glass vessel in which are two graphite electrodes (anodes *AA'*) and one mercury cathode *B*. Each anode is connected to a separate side of the a-c. supply, and also through one-half of the main reactance to the negative side of the load. The cathode *B* is connected to the positive side. There is also a small starting electrode *C* connected to one side of the a-c. circuit through a resistance and used for starting the arc. When the rectifier tube is rocked so as to form and break a bridge of mercury between the cathode *B* and starting anode *C* a small arc is formed. This produces mercury vapor in the tube and the arc immediately jumps to one or the other of the main anodes and alternates on these during regular operation.

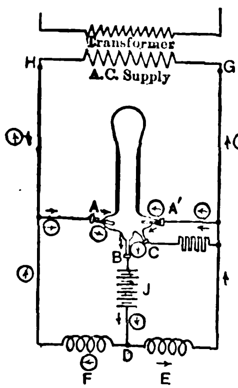

Fig. 1. Mercury-vapor Rectifier

Mode of Operation. — To analyze the operation, assume an instant when the terminal *G* is positive and *H* negative. The positive current will then flow from anode *A'* to cathode *B*, through the load *J* to *D* and through reactance *F* back to *H*. The current cannot jump from *A'* to *A* on account of the high

counter electromotive force of the arc. The small arrows surrounded by circles show the path of the current during this half cycle. During the next half cycle the terminal H is positive and the current flows to A through the tube to B, through the load J, reactance E and back to G. The small arrows show the path of the current during this half cycle. Hence during a whole cycle the cathode B is continuously negative but first one anode is active and then the other. If the voltage and current should become zero coincidentally the arc would become extinguished and operation cease. Hence the reactances E and F are introduced. At the end of the first half cycle described, when the line voltage drops to zero, the inductance of F maintains the current and a local circuit is formed through A, B and F, which maintains the arc until the voltage at G has risen to a value which will maintain the arc.

The rectifier thus makes use of both half waves, or the entire alternating current, and the result is a uniform pulsating uni-directional current. On account of the reactance in the circuit, this current in the load never falls to zero and, in fact, with sufficient reactance, may be made very nearly constant. But this extreme is not always desirable, as it distorts the current wave in the a-c. supply circuit.

Arc Rectifiers on Polyphase Circuits. — Rectifiers may be arranged on two-phase and three-phase circuits, and in fact there is an advantage in the arrangement of the reactances in these cases.

Efficiency of Mercury-arc Rectifier. — The losses in the rectifier correspond to a constant counter e.m.f. of about 14 or 15 volts, thus the efficiency of the tube is constant at all loads and at high voltages is very high; the higher the voltage the higher the efficiency. In fact, for high voltages the losses in the transformer and reactance coils form the major part of the total losses. On account of the small losses in the tube itself, there is very little heat to be dissipated, and rectifier tubes of large capacity are very small in bulk.

Kilowatt Capacity of Mercury-arc Rectifier. — Rectifiers have been built for voltages up to 6000. The tubes may be of glass or steel. The glass tubes have a capacity of about 40 amperes and the steel tubes from 200 to 300 amperes. Several rectifiers may be operated in parallel for large currents. The power factor of the combination of tube and controlling devices is about 90 per cent. Rectifiers have been built of a capacity sufficient to operate motor cars and locomotives.

Dimensions, Weights and Costs of Mercury-arc Rectifier Outfits. — A standard outfit consisting of rectifier tube, transformer, reactance, switchboard panel, switches and instruments to supply a load of 30 amperes at 110 volts d.c. would occupy a floor space 16×19 in., have a height of about 64 in., and weigh complete about 600 pounds. The first cost of the outfit would be between $200 and $250 and the cost of renewing the tube about $20. The net efficiency would be about 78 per cent. Such an equipment is frequently used for charging the batteries of electric vehicles from a-c. supply circuits.

ELECTROLYTIC RECTIFIERS. — (*See also Condensers; Lightning Protectors.*) The operation of this type of rectifier is in a general way the same as that of the mercury-arc rectifier, being based on the fact that a certain electrolytic cell having electrodes of different metals (e.g., aluminum and steel electrodes in a solution of ammonium phosphate) have the property of allowing a current to pass in only one direction. By suitably combining two of these cells both the half waves of an alternating current may be rectified. Such rectifiers have a low efficiency and due to the large losses in them heat very rapidly. They are therefore applicable only to the rectification of small currents. Examples are the electrolytic rectifier of Nodon described at the International Electric Congress of 1904.

MECHANICAL RECTIFIERS. — These usually consist of a commutator driven by a synchronous motor. Each commutator has as many live segments as there are poles on the synchronous motor. Alternate segments are connected to the source of alternating currents, and brushes properly spaced and bearing on the commutator collect the direct current. The objections to these devices are that if there is much inductance in the d-c. circuit (e.g., a motor) there will be a great deal of sparking, or if the load has a constant counter e.m.f. (e.g., a storage battery) there will be sparking. By alternating live and dead segments, this trouble can be obviated, provided the current to be rectified is not too large and the proportions of the live and dead segments are properly chosen. Commutator rectifiers of this type capable of rectifying currents up to 50 amperes at 200 volts have been constructed. The principal losses are in the driving motor; the efficiency is poor at light loads.

Many attempts have been made to produce a satisfactory rectifier by various schemes for interchanging the connections to the a-c. circuit at the end of every half wave, but none of them have as yet proved satisfactory for rectifying currents of any considerable magnitude. The difficulty lies chiefly in providing a means of preventing the formation of a spark when the circuit is interrupted or commutated.

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[W. I. Slichter.]

REGULATORS. — (*See also Batteries, Storage, Applications of; Controllers; Rheostats; Starters, Motor; Switchgear Equipment for Power Stations.*) Devices for adjusting and controlling the voltage in an electric circuit are called regulators. They may be divided into two classes, depending on whether they operate directly on the circuit or indirectly by means of the excitation of the generator.

FEEDER OR POTENTIAL REGULATORS, as the name implies, operate to raise or lower the feeder voltage. Where a feeder circuit is operated from a constant potential bus and it is desirable to compensate for the drop in the feeder, there is usually installed a regulator which adjusts the voltage impressed upon the feeder so as to maintain proper voltage at the point of distribution.

With d-c. circuits the only way to raise the voltage of the feeder above that of the bus is to connect a booster or a storage battery (q.v.) in series. The feeder voltage may be lowered by connecting in resistance. Owing to the inefficiency of resistance control and the complication of providing boosters or batteries, feeder control of d-c. circuits is used only in special cases, such as for the distribution system of a direct-current railway.

Feeder control for a-c. circuits is accomplished by transformer action. Feeder regulation for such circuits can be made in various ways, the most common being the "induction" type and the "step" type.

Induction Regulators, also called "induction potential regulators," are used on either single-phase or three-phase circuits and are arranged for hand operation, motor operation with distant control, or for full automatic operation by means of relays. An induction regulator is a special type of transformer, built like an induction motor (q.v.) with a coil-wound secondary. The primary is permanently connected across the feeder circuit, and the secondary which is connected in series with the feeder is normally stationary, but is movable at will for the purpose of adjusting the voltage. In comparison with the step-type regulator (*see below*) the induction regulator possesses the advantage of being operated without short-circuiting any transformer coils. It has the disadvantages of a large magnetic leakage and a high value of exciting current.

Single-phase Induction Regulator. — This regulator has a secondary induced voltage whose value depends on the relative angular position of the primary and secondary coils, but which is always in time phase with the primary voltage. This induced voltage is a maximum when the axes of the coils coincide and it is zero when the coils are at right angles to one another. The resultant feeder voltage is equal to the arithmetical sum (or difference) of the primary and secondary voltages.

Polyphase Induction Regulator. — This regulator has a rotating flux of constant value, set up by the primary current. Thus the secondary induced voltage is also constant in value but differs in phase from the normal line voltage depending on the angular relation between the primary and secondary windings. The resultant delivered voltage is the vector sum of the primary and secondary voltages, and its value varies with the position of the movable member.

Step-type Regulators, also called "contact-voltage regulators," consist essentially of a stationary transformer provided with a large number of secondary taps for cutting in and out sections of the transformer windings. The taps are connected with a dial or drum so that any pair of taps can be connected to the feeder circuit according to the voltage required.

The moving arm on the dial-type regulator is usually arranged so that in passing from the position of maximum boost the number of secondary turns in series with the circuit is reduced in equal steps until the turns are all cut out. Further rotation in the same direction throws over the reversing switch and

then cuts in the same secondary turns in opposition to the main voltage until the position of maximum bucking is reached, when a stop prevents any further rotation in that direction. A similar stop prevents overtravel in the position of the maximum boost.

Automatic Operation of Feeder Regulators with either single- or three-phase circuits is obtained by the action of a voltage relay, which may or may not have a compensating device. This relay acts in conjunction with the motor on the regulator so that, as the load comes on or as the bus-voltage drops, the motor will turn the regulator in such a direction as to increase the voltage. By means of a compensator which can be set for a certain resistance and a certain inductive drop, the voltage at the point of distribution can be maintained constant independent of the amount and power factor of the load, provided the total drop is within the range of the regulator.

FIELD REGULATORS. — In order to maintain practically-constant voltage on a-c. and d-c. generators, or to have these machines compound automatically to take care of feeder drop, field regulators of either non-automatic or automatic types are employed. The former are hand-controlled field rheostats (*see Rheostats*). The latter are made in various forms, of which the Tirrill regulator is the best known in America.

Tirrill Regulator. — This regulator depends for its operation upon the rapid opening and closing of a circuit that shunts the field rheostat, and thus changes the resistance in the field circuit of the generator to be regulated. For d-c. service the regulator usually works upon the main generator field and for a-c. service upon the field of the exciter. In both cases the rheostat is so adjusted that when in circuit it tends to lower the voltage considerably below normal, and when the rheostat is short-circuited the generator voltage rises. The regulator automatically closes the shunt circuit when the voltage drops to a predetermined value and opens the shunt circuit when the voltage rises above that value.

The main features of the Tirrill regulator and those which have been so conducive to its successful operation are: (1) the method of control by shunting the rheostat and (2) the fact that with the total range of regulation from no load to full load the maximum travel of the only moving parts, the vibrating contacts, is only 1/32 inch. The vibrations are so rapid that the time factor is reduced to a minimum and there are no retarding effects due to dash pots or other damping devices.

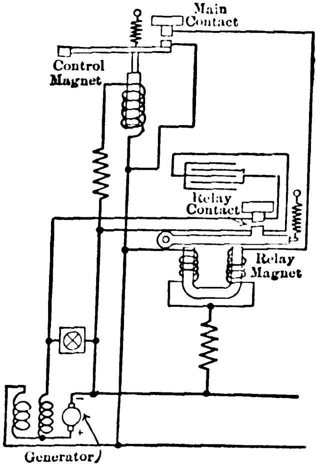

Fig. 1. Connections for Direct-current Tirrill Regulator

Tirrill Regulator for Direct-current Generator (Fig. 1). — This consists essentially of a main control magnet whose winding is connected across the generator terminals, and a differentially-wound relay magnet. When the effect of the potential winding increases, due to a rise in the generator voltage, the contact of the main control magnet is opened and in turn one winding of the relay magnet is deënergized. Thus the relay contact is opened and the short-circuit removed from the generator rheostat. When the generator volt-

age drops, the main contact is closed and the differentially-wound relay magnet acts to short-circuit the field rheostat. The relay contacts are shunted by a condenser to reduce sparking.

In case the generator is to overcompound for line drop a current winding is added to the main control magnet and is connected across a shunt in one of the local mains. The action of this current winding opposes the action of the potential winding in the control magnet end, and thus makes the generator overcompound for line drop. Regulators of this type can be adjusted for a line drop up to 15 per cent.

Where the generators are shunt wound a separate regulator is required for each machine which is operating at any time. Where several small compound machines are operating in parallel and all of the regulating is done by one machine, the others trailing after, provision is made for connecting any machine to the regulator. For field currents in excess of about 3 amperes at 250 volts, it is found that one relay contact is not sufficient, and for such cases regulators are built with as many as 10 relays, all being operated by a single main control magnet. For still larger d-c. generators whose field current could not be handled by a multiple-contact regulator, it is necessary to supply a separate exciter or exciters and the regulator then controls the field circuit of the exciter as described below for a-c. generators.

Tirrill Regulator for Alternating-current Generator (Fig. 2).—This works on the exciter field. The main contacts with this type of regulator are acted on by two sets of control magnets, one connected across the exciter bus

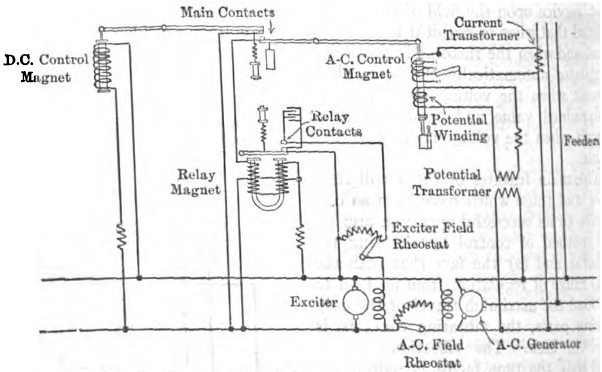

Fig. 2. Connections for Alternating-current Tirrill Regulator

and tending to move the main contacts further apart as the exciter voltage rises, and the other acted upon by a-c. potential and current coils. Suitable springs and counterweights allow the proper adjustment to be made. When the main contact closes it energizes the relay magnet, thus closing the relay contact, short-circuiting the exciter rheostat and raising the exciter voltage and consequently the generator voltage. The use of the exciter voltage as one of the main control circuits prevents the generator voltage "overshooting," for as the exciter voltage rises to bring up the a-c. voltage the d-c. control tends to keep the main contacts apart and so reduce the voltage again.

The compensating current winding of the a-c. solenoid is provided with a dial switch to give any amount of compensation required for the feeder circuit in which the current transformer is located. Where it is desired to compensate

for both resistance and inductive drop under varying power factors a special compensator is provided. A modification of the regulator to take care of the large exciters has a number of relay contacts all operated at the same time from the one set of control contacts, the various relays being shunted by condensers to reduce the sparking. A single regulator may serve a number of alternators if they are operated in parallel and if all use the same exciter. If two or more exciters are operated in multiple, a single regulator will suffice; if not operated in multiple, a separate regulator must be installed for each exciter.

REGULATORS, STORAGE BATTERY. — See articles on *Batteries, Storage*.

COSTS. — Tirrill regulators for d-c. service are built for pressures up to 550 volts and for controlling a total excitation current up to 30 amperes at 250 volts. The regulators for use with exciters are built for machines with a range of 2 to 1 from no-load to full-load excitation current. For a-c. service Tirrill regulators are built for controlling a total excitation current of 96 amperes with a range of 2 to 1 from no-load to full-load excitation current. The cost of both d-c. and a-c. types varies considerably according to the character of the installation, but for rough preliminary estimates may be taken as ranging from $230 for a total excitation current of 2.5 amperes at 250 volts to $910 for a total excitation current of 55 amperes at 250 volts.

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[S. Q. HAYES.]

RELAYS. — (*See also Circuit Breakers; Telegraphy; Telephony; Regulators; Signaling, Railway.*) A relay is a device which opens or closes a local circuit under predetermined electrical conditions of the main circuit. There are three general classes of relays as determined by their respective functions of (1) protecting the main circuit, (2) regulating the current in the main circuit, or (3) signaling the condition of the main circuit.

PROTECTIVE RELAYS are auxiliary devices supplied for use with circuit breakers on systems that require protection more selective and flexible than that afforded by the usual control features of automatic circuit breakers (q.v.). The closing of a local circuit through the action of the relay causes in turn the tripping of the circuit breaker.

Types of Protective Relays. — The types which are most commonly employed are given in the following table, together with the apparatus with which each type is applied and the protection furnished by the operation of the relay.

Relays are built to furnish protection against overvoltage, no voltage, overload, no load, reverse load and reverse phase. They may operate either directly or in connection with other relays, and may be made instantaneous or provided with a time limit either of definite duration (time-limit relay) or inversely proportional to the extent of overload (inverse-time-limit relay).

TYPES OF RELAYS

Type of Relay	Application	Operations	Approximate cost per relay*
D-C. overvoltage	Storage batteries d-c. apparatus	Prevents overcharging Prevents damage from excess voltage	$13.00
D-C. reverse current	Storage batteries rotary converters	Prevents discharging into charging source Protects against reversal of flow of energy	43.00
D-C. low-voltage release	D-C. motors	Operates if voltage falls below given value	13.00
D-C. underload	Storage batteries	Disconnects on completion of charge	13.00
A-C. overload	Feeders, motors, rotary converters, transformers	Protects against overload	14.50
A-C. overload and reverse load	Feeders, generators	Protects against overload and reversal of flow of energy	32.50
A-C. low-voltage	Induction motors	Protects against fall in line voltage	14.50
A-C. reverse phase	Synchronous apparatus	Protects against reversal of phase progression	32.50

* Costs do not include shunts for d-c. relays or current and potential transformers for a-c. relays, as the cost of these varies greatly with current and voltage.

Definite Time-limit Relays are used with circuits where the service must be maintained at all hazards no matter how great is the overload, provided it does not last more than a definite period of time, say from two to eight seconds. The allowable length of this time depends on the ability of the system to withstand such conditions; also on the length of time required for various feeder breakers to trip out and thus relieve the system protected by the breaker with definite time limit.

Inverse Time-limit Relay gives a selective action, whereby the time of operation varies inversely with the load. With this type of relay the faulty line carrying the heavier load usually will have its breaker tripped out before the other breakers are affected.

Example of Use of Relays. — The various types of relays that may be used to advantage in a system are shown diagrammatically in Fig. 1, where

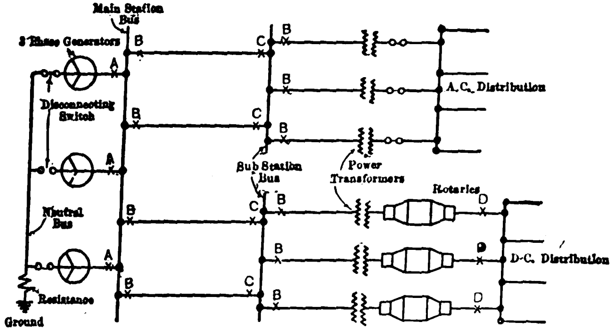

Fig. 1. Diagram Illustrating Use of Various Types of Relays

three a-c. generators operate in parallel with their neutral points grounded through a resistance and feed a common bus supplying current to power transformers, rotaries, etc., for a-c. and d-c. distribution. Relays for the operation of circuit breakers are inserted as follows:

At A — a-c. overload and reverse load relays.
B — a-c. overload time-limit relays.
C — a-c. reverse load, time-limit relays.
D — d-c. reverse current, inverse time-limit relays.

Principle of Operation of a protective relay depends somewhat on the duty to be performed. The actuating mechanism usually involves such a motive device as a solenoid and core, or a rotating motor. The mechanisms described below are those most frequently used for the respective types of service.

D-C. Overvoltage Relay. — One design consists of a permanent magnet which forms a base and two iron cores mounted parallel to the base and attached to it at one end with an eccentrically-pivoted armature carrying a suitable contact. With no current flowing the two cores exert an equal pull (due to the magnetic field of the permanent magnet) on the pivoted armature, which, due to its eccentric mounting, is pulled up against one core. When voltage is impressed on the coils wound on the iron cores the magnetism in one pole is strengthened and in the other weakened. At the predetermined overvoltage the position of the armature is reversed, notwithstanding its eccentric mounting, and the contact is closed, tripping the breaker.

D-C. Reverse-current Relays for instantaneous operation are similar in construction to overvoltage relays (*see preceding paragraph*). The solenoids

however, are operated from a shunt in such a way that with the current in the normal direction the contact is held open, but when the current reverses in direction the contact is closed, thus tripping the breaker.

Reverse-current Time-limit Relays are built on the principle of a permanent-magnet d-c. ammeter operated from a shunt. In normal operation the armature of the relay tends to turn in one direction but is restrained by a stop; in the case of current reversal the armature turns in the opposite direction, its rate of movement being proportional to the strength of the current. The angle through which the armature has to turn to close the contacts is adjustable by moving the stationary contact.

A-C. Overload Relays are usually built in single-phase units and for either instantaneous, definite time-limit, or inverse time-limit operation. The relay consists essentially of a solenoid and core. In the instantaneous relay the core lifts immediately upon occurrence of overload and closes or opens the circuit-breaker contacts when the current in the coil reaches a certain value. With the inverse time-limit relay the movement of the core is retarded by a bellows which has an adjustable valve and which is mounted above the coil. With the definite time-limit relay the same kind of bellows and valve is used but the solenoid does not work directly on the bellows. When the overload occurs the core rises instantly, compressing a spring which in turn acts on the bellows. If the core, due to continued overload, keeps the spring in compression the required time, the air will be forced out of the bellows and the tripping circuit operated. Relays of this type are usually operated from current transformers, but are sometimes mounted on a high-tension insulator and connected in the high-voltage circuit.

A-C. Reverse-load Relays are usually made of the wattmeter type with moving contacts adjustable to trip the breaker at any predetermined load in the reverse direction. They are made so as to have practically the same torque at low power factor as at high, and to be operative when the voltage drops almost to zero.

Relay Switch. — Where, as frequently happens, the tripping contacts of a relay will not carry enough current to trip the circuit breaker, a relay switch is employed. In this case the relay merely energizes the operating coil of the relay switch whose contacts can be made suitable for any reasonable amount of current.

Rating of Protective Relays for overload service is given usually in terms of maximum current in the main circuit (secondary of transformer with a-c.). For reverse-load relays the rating is expressed as the percentage of reverse load on which they will operate. For relays which are not instantaneous in their action a time rating in seconds is usually given.

REGULATING RELAYS. — See article on *Regulators*.

SIGNALING RELAYS. — See articles on *Telegraphy; Telephony; Signaling, Railway*.

COSTS. — See table above.

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[S. Q. HAYES.]

RESISTANCE AND CONDUCTANCE, ELECTRIC. — (*See also Alternating Currents; Bridges for Electrical Measurements; Electricity and Magnetism, Principles of; Resistors, Standard; Skin Effect; Wires and Cables; Wires, Resistor.*) The general definition of the resistance R' of a substance between any two equipotential surfaces intersecting the path of a current I is

$$R' = \frac{P_h}{I^2}, \tag{1}$$

where P_h is the power dissipated as heat between the two equipotential surfaces and I is the *effective* value of the *total* current from one surface to the other. In the case of a varying current this dissipation of heat may occur in four different ways, viz., (1) as heat due to the conduction current through the substance, (2) as heat due to dielectric hysteresis accompanying the displacement current through the substance (when the latter is an insulator), (3) as heat due to magnetic hysteresis accompanying the varying magnetic flux produced by the current and (4) as heat due to eddy currents induced in neighboring conductors.

In the case of a continuous (non-varying) current the last three effects do not occur, and the heat is that due to the conduction current only. The resistance offered by a substance to a continuous current is called the "true," "ohmic," "continuous-current" or "direct-current" resistance, as distinguished from the "effective" resistance offered by the substance to a varying current. The effective resistance, even when there are no losses due to dielectric or magnetic hysteresis, is in general greater than the ohmic resistance, due to the skin effect; see *Skin Effect*.

Similarly, the general definition of conductance is

$$G' = \frac{P_h}{V^2}, \tag{2}$$

where P_h has the same meaning as above and V is the *effective* value of the potential difference between the two equipotential surfaces. When the voltage is non-varying the conductance is called the "true," "ohmic," "continuous-current" or "direct-current" conductance as distinguished from the "effective" conductance to a varying current.

In the case of a *continuous*, or constant current I, the above expressions for resistance and conductance reduce to

$$R = \frac{V}{I}, \qquad \text{and} \qquad G = \frac{I}{V}, \tag{3}$$

provided there is no source of e.m.f. between these two surfaces. The ohmic resistance and conductance, given by equations (3), are reciprocals of each other, but this is not true for the effective resistance and conductance.

RESISTIVITY AND CONDUCTIVITY. — When the current stream lines from one equipotential surface to the other are straight and parallel and uniformly distributed, and l is the distance between the two surfaces and A the area of each, the ohmic resistance and conductance of the prism or cylinder of the substance thus formed are respectively

$$R = \rho\frac{l}{A}, \text{ and } G = \gamma\frac{A}{l}, \tag{4}$$

where ρ and γ (which are reciprocals) are called the resistivity* and conductivity* of the substance respectively. For a given material at constant temperature throughout ρ and γ are constants.

* Also called specific resistance and specific conductance.

Units of Resistivity and Conductivity. — From (4) it is evident that ρ is the resistance of a cube of the substance of unit length on each edge when the current stream lines are uniformly distributed and parallel to four of the edges of the cube; similarly, γ is the conductivity of such a unit cube. Resistivity is therefore frequently expressed as ohms or microhms per centimeter cube or per inch cube, and conductivity as mhos or mega-mhos per centimeter or inch cube. Note that a resistivity of x microhms per centimeter (or inch) cube is equivalent to a conductivity of $\frac{1}{x}$ mega-mhos per centimeter (or inch) cube.

Ohms per Mil-foot. — It is frequently convenient in dealing with wires to express lengths in feet and cross sections in circular mils. The corresponding value of the resistivity ρ is then expressed in ohms per mil foot.

Ohms per Meter-gram. — The cross section of a bar or wire of uniform cross-section is equal to the volume of the bar divided by its length, and the volume of the bar is in turn equal to its mass divided by the density of the material of which it is made. Hence, using the same symbols as above, and in addition calling m the mass of the conductor and δ its specific gravity,

$$R = \rho\delta \frac{l^2}{m}. \tag{5}$$

For a given material at a given temperature $\rho\delta$ is also constant, hence

$$R = k\frac{l^2}{m}, \tag{5a}$$

where k is a constant (equal to $\rho\delta$) for a given material at a given temperature; this factor k is then called the specific resistance of the conductor in "ohms per meter-gram" when l is in meters and m in grams.

Pounds per Mile-ohm. — In telephone and telegraph practice the specific resistance of a conductor is also expressed as the weight w, in pounds, of a wire one mile long having a resistance of one ohm. This weight w is sometimes called the "mile-ohm equivalent" of the wire.

"Annealed Copper Standard." — Matthiessen's Standard. — In 1862 Matthiessen published the results of a number of determinations of the resistivity of copper. Recent determinations of the conductivity of a number of samples of commercial annealed copper wire at the Bureau of Standards (*Bull. Bur. Stand., 1911, Vol. 7, p. 103*) gave a mean value of the resistivity very close to Matthiessen's value. A conductivity corresponding to 0.15328 ohm per meter-gram at 20° C. has been adopted (1912) by the Bureau and by the American Institute of Electrical Engineers (1914)* as the "standard" of conductivity, the name "Annealed Copper Standard" being suggested for it. (*See Standardization Rules of the A.I.E.E.*) Matthiessen's Standard, formerly used by the American Institute of Electrical Engineers as the basis for their wire tables corresponded to 0.141729 ohm per meter-gram at 0° C.

To reduce the value of the Annealed Copper Standard to resistivity and conductivity in volume units the value of 8.89 has been adopted as the density of copper in grams per cubic centimeter at 20° C. which corresponds to 8.90 grams per cubic centimeter at 0°C.

Per cent Conductivity. — The Bureau of Standards recommends that whenever the conductivity of a sample is expressed as a percentage, the measured resistivity or conductivity be corrected to reduce it to the value it would have

* Also adopted by the national electrical engineering societies in Germany and France.

at 20° C.; see section on *Temperature Coefficient of Resistance*, below. A conductivity of P per cent is equivalent to

$15.328 \div P$ ohms per meter-gram at 20° C.
$172.41 \div P$ microhms per centimeter cube at 20° C.
$67.87 \div P$ microhms per inch cube at 20° C.
$1037.1 \div P$ ohms per mil-foot at 20° C.

For example, a conductivity of 60 per cent is equivalent to a resistance of 17.285 ohms per mil-foot.

CALCULATION OF THE OHMIC RESISTANCE OF A CONDUCTOR. — Values of the resistivity of various materials are given in the table below. Experiment shows that the resistance of a given length of wire of uniform cross section is independent of the shape into which the wire is bent, provided the diameter of the wire is small compared to the radius of curvature of the curve into which it may be bent. This condition is almost always realized in practice, and consequently formulas (4) and (5a) are in general directly applicable to the calculation of the resistance of a wire whether the wire be straight or curved or wound into a coil of any shape. These formulas are also applicable to the calculation of the resistance of a rod or bar, provided the rod or bar is not bent into a sharp curve, and the distance between its points of connection to the circuit is large compared to the linear dimensions of its cross section. Care should be taken, however, to express all quantities in the proper units; for example, the resistance in *ohms* of a wire which has a specific resistance of 1.6 microhms per centimeter cube, a length of 1000 feet and a cross section of ¼ square inch, is

$$R = 1.6 \times 10^{-6} \frac{1000 \times 12 \times 2.54}{0.25 \times (2.54)^2} = 0.0302 \text{ ohm}.$$

Resistance Formulas When the Stream Lines are Not Parallel and Uniformly Distributed. — When the stream lines of the current are not all of the same length, as, for example, when a current is established in a heavy short bar bent into a sharp curve, the resistance of the bar can be calculated only when the distribution of these stream lines is known. Again, when the stream lines of the current are not parallel, e.g., the leakage current through the insulation of a cable, these formulas are not applicable. However, when the formula for the capacity between any two conductors is known (*see Capacity and Charging Current*), the resistance of the insulation, if uniform throughout, may be found by multiplying the *reciprocal* of the capacity, viz., $\frac{1}{C}$, by $\frac{K\rho}{4\pi}$, where ρ is the resistivity of the insulation and K the specific inductive capacity in the capacity formula. See articles on *Rheostats* and *Wires and Cables, Insulated.*

TEMPERATURE COEFFICIENT OF ELECTRIC RESISTANCE. — The resistance temperature coefficient β_t of a substance *at* any temperature t is defined as the rate of change of the resistance at this temperature, viz. $\left(\frac{dR}{dt}\right)_t$, divided by the resistance R_t at this temperature, i.e.,

$$\beta_t = \frac{1}{R_t}\left(\frac{dR}{dt}\right)_t. \tag{6}$$

The *mean* temperature coefficient α_t between any two temperatures t and t_1 *referred to* the temperature t is defined as the *average* change in the resistance in this interval per degree change of temperature, viz.,

$$\alpha_t = \frac{R_{t_1} - R_t}{R_t\,(t_1 - t)}. \tag{7}$$

General Expression for Change of Resistance with Temperature.—In general, the relation between the resistance R_t of a given mass of a substance at any temperature t may be expressed in terms of its resistance R_0 at zero degrees as follows:

$$R_t = R_0 (1 + at + bt^2 + \ldots), \tag{8}$$

where a, b, etc., are constant coefficients.

Linear Relation Between Resistance and Temperature.—When all the coefficients except the first one, a, are of negligible magnitude, i.e., when the relation between the resistance and temperature is a *linear* one, then, from the above definitions, the *mean* temperature coefficient (α_t) referred to a given temperature t is equal to the temperature coefficient (β_t) *at* that temperature, and the zero degree temperature coefficient α_0 ($= \beta_0 = a$) and the t' degree temperature coefficient $\alpha_{t'}$ are related as follows:

$$\alpha_{t'} = \frac{\alpha_0}{1 + \alpha_0 t'}, \tag{9}$$

and

$$R_t = R_0 (1 + \alpha_0 t) = R_{t'} [1 + \alpha_{t'} (t - t')], \tag{10}$$

where R_0 is the resistance at zero degrees, $R_{t'}$ the resistance at t' degrees and R_t the resistance at t degrees.

For most metals the simple linear relation expressed by equation (10) represents the experimental facts within practical limits of accuracy and for ordinary temperature ranges; values of α_0 are given in the table below. For dielectrics, however, the relation between the resistance and temperature is by no means linear, and several terms in such an expression as equation (8) are needed to represent the facts; see *Insulating Materials, Properties of*, particularly the subheadings *Cambric, Paper and Rubber.*

Calculation of Change in Resistance With Temperature.—Equation (10) may be conveniently expressed in the form

$$\frac{R_{t'}}{R_t} = \frac{T_0 + t'}{T_0 + t}, \tag{11}$$

where T_0 is written for the reciprocal of α_0, viz., $T_0 = \dfrac{1}{\alpha_0}$. (11a)

Equation (11) will be found very convenient for calculating the change of resistance with temperature and the change of temperature corresponding to two measured resistances, particularly when a slide rule is used.

T_0 is approximately equal to the number expressing the absolute zero on the mercury thermometer scale (except for magnetic metals) and is sometimes called the "inferred absolute zero" for the particular metal in question.

MEASUREMENT OF RESISTANCE, RESISTIVITY AND TEMPERATURE COEFFICIENT.—The simplest method of measuring a resistance in ordinary engineering work is to send a direct current through the conductor and measure this current by means of an ammeter (q.v.) and measure the potential difference across it by means of a voltmeter (q.v.). For measuring very low resistances, however, and for high-precision measurements, a bridge method should be used; see *Bridges for Electrical Measurements.*

VALUES OF RESISTIVITY AND TEMPERATURE COEFFICIENT.—The resistivity and temperature coefficient of the more common metals and alloys are given in the table below.

RESISTIVITY AND TEMPERATURE COEFFICIENT OF RESISTANCE*

1 microhm per centimeter cube $= \frac{1}{2.5400}$ microhms per inch cube
= 6.0153 ohms per mil-foot
= 0.01 δ ohms per meter-gram, where δ is the specific gravity.
= 57.08 δ pounds per mile-ohm.

ρ microhms per cm. cube at 20° C. $= \frac{172.4}{\rho}$ per cent conductivity at 20° C.

The zero degree Fahrenheit temperature coefficient is equal to

$$\frac{5\alpha_0}{9 - 160\alpha_0}.$$

Substance (*Numbers refer to authorities, top of p. 1225*)	Remarks	Microhms per centimeter cube at 0°C. ρ_0	Mean temperature coefficient referred to 0° C. Temperature range °C. From	To	α_0
Advance (3)	Copper-nickel	48.8	......	......	0.000018
Aluminum (1)	Pure	2.62	0	100	0.00423
Aluminum (7)	Wire, 61% cond.	2.607	......	......	0.00423
Alum. bronze (1)	97 Cu+3 Al	8.85	15	......	0.000897
Argentan (1)	61.6 Cu+15.8 Ni+22.6 Zn	28.5	0	160	0.000387
Brass (1)	90.9 Cu+9.1 Zn	3.64	0	100	0.00204
Brass (1)	65.8 Cu+34.2 Zn	6.29	0	100	0.00158
Bronze (1)	88 Cu+12 Sn+0.94 P	17.8	19	92	−0.00050
Calido (8)	Ni+Cr+Fe	100.0	......	......	0.00034
Carbon† (1)	Graphite	400 1150	25	387	−0.0006 −0.0012
Carbon (1)	Incand. lamp	4000	25	335	−0.0003
Climax (3)	Nickel-steel	87.1	......	......	0.00055
Constantan (1)	60 Cu+40 Ni	49.0	0	100	0.0000±
Copper (7)	**Annealed Standard**	**1.589**	**0**	**100**	**0.00427**
Copper (1)	Electrolytic	1.56	0	100	0.00428
Copper (1)	Hard-drawn	1.60	0	100	0.00408
Copper-iron (2)	0.4% Fe	4.08	0	100	0.00155
Excello (4)	......	91.4	......	......	0.00016
Ferro-nickel (3)	......	27.1	......	......	0.00216
German silver (3)	18% Ni with Cu and Zn	33.1	......	......	0.00031
Gold (1)	99.9% Au	2.20	18	100	0.00368

* See *Wires and Cables, Bare*, for the resistance of various sizes of wires; *Wires, Resistance*, for additional data on resistance wires; and *Insulating Materials, Properties of*, for the resistivity of dielectrics. See also the articles on *Aluminum*, *Copper*, and *Rails*.
† See also p. 565.

RESISTIVITY AND TEMPERATURE COEFFICIENT OF RESISTANCE — *Continued*

Substance (*Numbers refer to authorities, top of p. 1225*)	Remarks	Microhms per centimeter cube at °C. ρ_0	Mean temperature coefficient referred to 0° C. Temperature range °C. From	To	α_0
Ideal (8)	Cu+Ni	49.0	......	......	0.0000±
Ia Ia soft (4)	Copper-nickel	47.1	......	......	0.000005
Ia Ia hard (4)	Copper-nickel	50.2	......	......	−0.000011
Iron (1)	Very pure	8.85	0	100	0.00625
Iron (1)	soft steel	11.8	10	35	0.00423
	hard steel	45.6	10	35	0.00161
Iron, cast (2)	soft	74.4	......	......	....
Iron, cast (2)	hard	97.8	......	......	....
Krupp metal (5)	Nickel steel	85.0	......	......	0.00070
Lead (1)	Pure	19.8	0	100	0.00411
Lead-bismuth (1)	42.3 Pb+57.7 Bi	63.3	......	......	....
Manganese-copper (1)	70 Cu+30 Mn	100	0	100	0.00004
Manganin (3)	Cu+Mn+Ni	41.4	......	......	0.000011
		73.8	......	......	0.000039
Mercury (1)	....	94.07	0	100	0.0008649†
Molybdenum (6)	Hard drawn	4.9	0	170	0.0050
Molybdenum (6)	Annealed	4.2	0	170	0.0050
Monel metal (3)	Copper nickel	40.8	......	......	0.00206
Nichrome (3)	....	98.7	......	......	0.00045
Nichrome II (3)	....	109.2	......	......	0.00016
Nickel (1)	Electrolytic	6.93	0	100	0.00618
Nickel (3)	Commercial wire	9.9	......	......	0.0039
Nickel steel (1)	4.35% Ni	29.4	......	......	....
Phosphor-bronze (1)	....	7.75	......	......	....
Platinum (1)	Drawn	11.0	0	100	0.00367
Platinum-iridium (1)	80 Pt+20 Ir	31.6	−100	100	0.002±
Platinum-rhodium (1)	90 Pt+10 Rh	21.1	15	......	0.00143
Rheotan (1)	....	44.6	0	......	0.00041
Rose's metal (1)	48.9 Bi+23.5 Sn+27.6 Pb	64.5	0	94.3	0.0023
Silver (1)	Electrolytic	1.47	0	100	0.00400
Steel (*see Iron*)*					
Superior (4)	Nickel-steel	87.1	......	......	0.00081
Tantalum (1)	Pure	14.6	0	100	0.0033
Therlo (3)	Cu+Mn+Al	46.7	......	......	0.0000056
Tin (1)	....	10.5	18	100	0.00465
Tungsten (6)	Hard drawn	5.42	0	170	0.0051
Tungsten (6)	Annealed	4.37			
Wood's metal (1)	55.7 Bi+13.7 Sn+13.7 Pb+16.2 Cd	51.8	0	69.8	0.0023
Yankee silver (3)	....	33	......	......	0.000154
Zinc (1)	Pure	5.38	18	100	0.00402

* See also *Rails, Track and Third*. † To be used in equation (8) with $b = 0.00000112$.

Sources of Data in Preceding Table. — (1) Landolt and Börnstein's Physical-Chemical Tables, 1912 Edition. (2) Smithsonian Physical Tables. (3) Driver Harris Wire Co. (4) Herman Boker and Co. (5) Thomas Prosser and Son. (6) Dr. Frink, Trans. Am. Electro-chem. Soc.. 1910, Vol. 17. (7) *Copper Wire Tables*, Circ. No. 31, Bureau of Standards, 1914. (8) Electrical Alloy Co.

Resistivity and Temperature Coefficient of Some Common Solutions. — (*See also Electrochemistry, Principles of.*) The table on p. 1226 is based on data given by Kohlrausch and Holborn (*Leitvermögen der Elektrolyte, Leipsig,* 1898). The temperature coefficient given is that corresponding to an increase of temperature from 18° C. to 19° C., and is a negative quantity, i.e., for an increase of temperature the resistance decreases. The resistivity temperature coefficient of aqueous solutions diminishes rapidly with increase of temperature, i.e., the higher the temperature the less is the decrease in resistance for each degree increase in temperature.

Resistivity and Temperature Coefficient of Ordinary Water. — The resistivity of ordinary tap or river water ranges from 1200 to 12,000 ohms per centimeter cube, ordinarily being between the limits 2000 and 5000 ohms per centimeter cube. The change of the resistance of such water with temperature between the limits 0° C. and 100° C. may be represented to a fair degree of approximation by the formula

$$R_t = \frac{40\, R_{20}}{20 + t},$$

where R_{20} is the resistance at 20° C. and t is any other temperature between 0° C. and 100° C. (*From Tests by Applequest and McKenny, Mass. Inst. of Tech.*, 1912.)

RESISTIVITY OF SOLUTIONS AND THEIR TEMPERATURE COEFFICIENT OF RESISTANCE

$$1 \text{ ohm per centimeter cube} = \frac{1}{2.540} \text{ ohm per inch cube.}$$

By per cent solution is meant the weight of the dissolved salt or acid expressed as a percentage of the weight of the solution.

For a dilution less than 5% the resistivity is approximately inversely as the per cent of dissolved salt or acid, i.e., a 2 per cent solution of common salt has a resistivity of approximately $14.9 \times 5/2 = 37$ ohms per centimeter cube. As noted above the resistivity of ordinary tap water ranges from 1200 to 12,000 ohms per centimeter cube; a solution made from such water can not of course have a resistivity greater than that of the water.

Substance	Per cent solution	Ohms per centimeter cube at 18° C.	18° C. temperature coefficient
HNO_3	5	3.90	−0.015
	10	2.18	−0.014
	20	1.41	−0.014
	30	1.28	−0.014
	40	1.37	−0.015
	50	1.59	−0.016
	60	1.96	−0.016
HCl	5	2.53	−0.016
	10	1.59	−0.016
	20	1.31	−0.015
	30	1.51	−0.015
	40	1.94	−0.015
H_2SO_4	5	4.80	−0.012
	10	2.55	−0.013
	20	1.53	−0.014
	30	1.35	−0.016
	40	1.47	−0.018
	50	1.85	−0.019
	60	2.68	−0.021
	70	4.64	−0.026
KOH	5	5.84	−0.019
	10	3.19	−0.019
	20	2.01	−0.020
	30	1.85	−0.022
	40	2.23	−0.027
$ZnSO_4$	5	52.3	−0.022
	10	31.2	−0.022
	20	21.4	−0.024
	30	22.5	−0.027
$CuSO_4$	5	53.0	−0.021
	10	31.3	−0.022
	15	23.7	−0.023
Na_2SO_4	5	24.4	−0.024
	10	14.6	−0.025
	15	11.3	−0.026
Na_2CO_3	5	22.2	−0.025
	10	14.2	−0.027
	15	12.0	−0.029
$NaCl$	5	14.9	−0.022
	10	8.25	−0.021
	15	6.09	−0.021
	20	5.10	−0.022
	25	4.68	−0.023
NH_4Cl	5	10.9	−0.020
	10	5.63	−0.019
	15	3.86	−0.017
	20	2.97	−0.016
	25	2.48	−0.015

BIBLIOGRAPHY. — See references in text.

[H. PENDER and R. G. HUDSON.]

RESISTORS, STANDARD, AND RESISTANCE BOXES.— (*See also Bridges for Electrical Measurements; Resistance and Conductance; Wires, Resistance; Rheostats.*) The primary standard of resistance is a mercury column of certain specified dimensions (*see below*). Secondary or commercial standards are made in two forms, viz.: (1) resistance standards each consisting of a single coil, carefully calibrated, mounted in a metal case, and (2) the ordinary resistance box, which contains a group of coils of known resistance.

The ultimate or primary standard of resistance, when measured in the international units, is the resistance of a column of mercury at 0° C. having a length of 106.300 centimeters and having a mass of 14.4521 grams (*see Units, Practical Electrical*).

STANDARD MERCURY RESISTANCE. — For the practical realization of such a standard, the International Conference on Electrical Units held in London in 1908 adopted the following specifications:

The glass tubes used for mercury standards of resistance must be made of a glass such that the dimensions may remain as constant as possible. The tubes must be well annealed and straight. The bore must be as nearly as possible uniform and circular, and the area of cross-section of the bore must be approximately one square millimeter. The mercury must have a resistance of approximately one ohm.

Each of the tubes must be accurately calibrated. The correction to be applied, to allow for the area of the cross-section of the bore not being exactly the same at all parts of the tube, must not exceed 5 parts in 10,000.

The mercury filling the tube must be considered as bounded by plane surfaces placed in contact with the ends of the tube.

The length of the axis of the tube, the mass of mercury the tube contains and the electrical resistance of the mercury are to be determined at a temperature as near to 0° C. as possible. The measurements are to be corrected to 0° C.

For the purpose of the electrical measurements, end vessels carrying connections for the current and potential terminals are to be fitted on to the tube. These end vessels are to be spherical in shape (of a diameter of approximately four centimeters) and should have cylindrical pieces attached to make connections with the tubes. The outside edge of each end of the tube is to be coincident with the inner surface of the corresponding spherical end vessel. The leads which make contact with the mercury are to be of thin platinum wire fused into glass. The point of entry of the current lead and the end of the tube are to be at opposite ends of a diameter of the bulb; the potential lead is to be midway between these two points. All the leads must be so thin that no error in the resistance is introduced through conduction of heat to the mercury. The filling of the tube with mercury for the purpose of the resistance measurements must be carried out under the same conditions as the filling for the determination of the mass.

The resistance which has to be added to the resistance of the tube to allow for the effect of the end vessels is to be calculated by the formula

$$A = \frac{0.80}{1063\pi}\left(\frac{1}{r_1} + \frac{1}{r_2}\right) \text{ohm},$$

where r_1 and r_2 are the radii in millimeters of the end sections of the bore of the tube.

The mean of the calculated resistances of at least five tubes shall be taken to determine the value of the unit of resistance.

For the purpose of the comparison of resistances with a mercury tube, the measurements shall be made with at least three separate fillings of the tube.

STANDARD SINGLE-COIL RESISTANCES. — The mercury standard resistance is cumbersome to work with and must be kept at constant zero temperature. Secondary standards, made of wire having a low temperature coefficient, are therefore universally employed in ordinary testing laboratories. The unit is provided with suitable heavy copper terminals so arranged that it may be hung from mercury cups and dipped into an oil bath. The object of the oil bath is to keep the temperature constant. Very low-resistance units, designed to carry large currents, are kept cool by means of water circulating in a coil of pipe within the case itself. Low-resistance units are also provided with "potential terminals," see Fig. 1, the stated resistance being the resistance between these terminals.

Reichsanstalt and N.B.S. Types. — There are at present two recognized types of resistance standards, the Reichsanstalt type and the National Bureau of Standards (or N. B. S.) type. Fig. 1 shows a 0.1-ohm standard of the first type, and Fig. 2 a 1-ohm standard of the N. B. S. type.

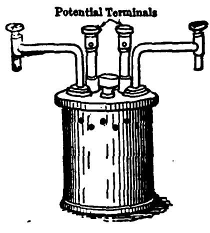

Fig. 1. Reichsanstalt Type Resistance Standard

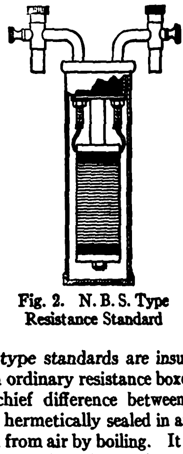
Fig. 2. N. B. S. Type Resistance Standard

The Reichsanstalt type and the N. B. S. type standards are insulated and baked in much the same manner as the coils in ordinary resistance boxes, though of course more carefully calibrated. The chief difference between the two types of standards is that the N. B. S. type is hermetically sealed in an oil-filled brass case, the oil having previously been freed from air by boiling. It is claimed that the N. B. S. units hold their calibration better than the Reichsanstalt units, the latter being subject to slight variations of resistance in climates where wide changes of humidity occur. (*See Bulletin Bureau of Standards, Vol. 5, p. 413.*)

Precision and Current-carrying Capacity of Standard Resistances. — The makers of standard units usually guarantee their accuracy as given below. The current-carrying capacity of the units is also given.

REICHSANSTALT TYPE, STANDARD RESISTANCE

Size of unit	Max. amp.	Error not greater than	Size of unit	Max. amp.	Error not greater than
0.0001	100	1/25 %	10	.3	1/100 %
0.001	30	1/25	100	.1	1/100
0.01	10	1/50	1,000	.03	1/100
0.1	3	1/50	10,000	.01	1/100
1	1	1/100	....	....	....

The National Bureau of Standards will calibrate any of these units and guarantee their accuracy of calibration to the above degree of precision, the charge being $4 to $6 per unit according to the value of the resistance.

RESISTANCE BOXES. — A brief description of the construction and arrangement of the coils in ordinary resistance boxes is given below.

Construction of Coils. — The coils are always wound non-inductively, that is, there are as many turns carrying the current in the right-handed direction as in the left-handed direction; this construction is illustrated in Fig. 3. Manganin wire (*see Wires, Resistor*) is usually employed. This wire is double-silk or enamel insulated, and is wound on wood or metal spools. Metal spools do not change in shape as wooden spools are liable to do, and since they more readily conduct the heat away, they may be safely used with larger currents.

The wound spools are then dipped in shellac and baked from 10 to 15 hours at a temperature of 140° C. This baking removes the tension from the wire due to the winding and the resistance is rendered constant, whereas coils not treated in this manner will change their resistance to some extent long after being wound. The resistance coils are adjusted to the desired values by varying the length of the wire, and copper terminals are silver soldered to the ends of the resistance wire. The coils are then ready for soldering in place in resistance boxes.

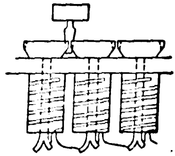

Fig. 3. Resistance Coil Construction

Arrangement of Coils. — The principal arrangements employed at the present time in the construction of resistance boxes are the 1, 2, 3, 4 plan, the 1, 2, 2, 5 plan and the decade plan.

The 1, 2, 3, 4 and the 1, 2, 2, 5 Plans. — Resistance boxes built on the 1, 2, 3, 4 plan have coils of the following resistances: 1, 2, 3, 4, 10, 20, 30, 40, 100, 200, 300, 400, 1000, 2000, 3000, 4000 ohms. These coils are all in series and each block on the top of the box is connected to a junction between successive coils. Inserting a plug therefore short-circuits a coil, see Fig. 3. In the 1, 2, 2, 5 plan the construction is similar, but the coils of any group have resistances in the ratio 1 : 2 : 2 : 5. Any resistance from 1 to 10,000 ohms can be obtained by either of these plans. On account of the large number of plugs to be manipulated, the large number of plug contacts and the necessity of making a mental summation of the values unplugged, these arrangements are being superseded in modern resistance boxes by the decade plan described below.

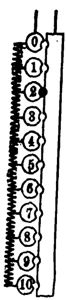

Fig. 4. 10-coil Decade

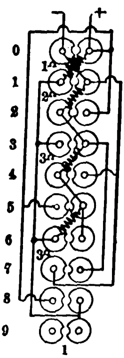

Fig. 5. 4-coil Decade

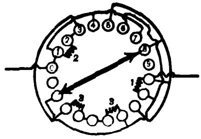

Fig. 6. 4-coil Dial Decade

Decade Plan. — The name decade arises from the use of 10 plug holes (but only one plug) for each group of coils, each hole corresponding to a given number of units, tens, hundreds, etc.

There may be either 10, 9 or 4 coils per set. The 10-coil arrangement is shown in Fig. 4, a 4-coil plug decade arrangement in Fig. 5, and a 4-coil dial decade arrangement in Fig. 6. The 9-coil decade differs from the 10-coil decade only in the omission of the tenth coil.

The only gain in having the tenth coil is to make it possible to check the total resistance of the 10 coils of one decade against any one coil of the next higher decade. The 4-coil arrangement possesses all the advantages of the 9-coil decade, and has the added feature of fewer coils to get out of adjustment.

Precision of Resistance Boxes. — The precision to which the various coils in a resistance box are adjusted depends on the design and use to which the box is to be put. In the cheaper boxes the actual resistance of any coil may differ from the stated resistance by as much as 1/5 per cent, whereas higher grades of boxes can be had having an accuracy of 1/50 per cent.

Precautions to be Taken. Care of Resistance Boxes. — The directions under these same headings given in the article on *Bridges for Electrical Measurements* also apply to the use and care of resistance boxes.

COSTS. — An ordinary 10,000-ohm resistance box (with coils for obtaining all resistances from 1 to 10,000 ohms) with the coils accurate to 1/5 per cent costs about $30. A high-grade 10,000-ohm resistance box with coils accurate to 1/25 per cent costs about $90. The costs of Reichsanstalt standard resistances are approximately as follows:

0.0001 ohm	$40	1 to 1000 ohms	$20
0.001 ohm	30	10,000 ohms	25
0.01 and 0.1 ohm	30		

BIBLIOGRAPHY. — Same as for *Bridges for Electrical Measurements*, q. v.

[H. PENDER AND H. R. RANKEN.]

RHEOSTATS AND RESISTORS. — (*See also Controllers; Control Systems for Railway Motors; Motors; Regulators; Resistors, Standard; Starters, Motor.*) A rheostat is a resistance device, usually adjustable, placed in a circuit for the purpose of regulating the current in that circuit. If used in connection with a machine, it may serve for regulating the speed, input, output, voltage or power factor. The name resistor is also used synonomously with rheostat, but is sometimes limited to mean a non-adjustable resistance device. This article treats primarily of the design of the resistance units, or resistors, used in the various controlling and starting devices employed commercially and in the laboratory; for their application, connections, etc., see the articles listed at the head of this article.

General Principles of Design. — Rheostats used for starting motors or for other intermittent work are usually designed on the assumption that no heat is radiated by them during the period (usually from 15 to 30 seconds) that they are carrying current. On this assumption the maximum allowable current I which a resistor can carry for t seconds without exceeding a temperature rise of T degrees is proportional to the square root of the ratio of T to t, the factor of proportionality depending upon the specific resistance and size of the current conductor, and the specific heats, weights and volumes of the materials heated, including the insulating material in which the resistors are embedded; see *Heat and Thermal Properties*. However, since there are so many variables entering into this factor K, the usual method employed in designing a rheostat is to determine experimentally the relations between current, temperature rise and time for a series of resistance units and to plot these relations in a set of curves. See also below under *Commercial Forms of Rheostats.*

The maximum temperature rise in the case of rheostats designed to carry a continuous load will occur when the heat radiated equals the heat generated. The proper resistance units for any service are determined from capacity curves for a variety of units subject to various conditions. Capacity curves are obtained by plotting as ordinates degrees of final rise of temperature against watts dissipated as abscissas. See also below under *Commercial Forms of Rheostats.* Natural draught is usually depended on for ventilation. For very large capacities forced ventilation is occasionally applied, if the space available is limited, but little is saved on the initial investment, and the maintenance expense is increased.

COMMERCIAL RHEOSTATS. — The term "commercial rheostat" is commonly meant to include besides the resistance elements the complete switching and control mechanism which serves to vary the resistance in the circuit for any particular kind of regulation.

Forms of Resistance Elements for Commercial Rheostats. — The more common forms of resistance elements are described below.

Standard "Unit" Type of Resistance Element. — Units of this type are made in either a flat or cylindrical form. One form of flat unit consists of a moulded flat core of vitreous material on which the resistance wire is wound; the surface is then coated with a special cement and baked. Thus the resistance material is protected from injury and made proof against moisture.

Standard cylindrical units usually have a core of asbestos tubing, or sometimes of metal tube coated with a suitable insulating material, such as enamel. A wire of low temperature coefficient is used; see *Wires, Resistance.* The tube with winding is covered with a suitable insulating compound, and porcelain bushings or metal rings (the latter for the clip type) are placed on the ends. The units are then thoroughly baked and mounted rigidly on a frame or in a suitable case. The coating protects the resistance material from mechanical injury,

forms a good conductor of heat, and in case of a burn-out prevents appreciable arcing and unwinding of the wire. As a further protection, a sheet metal covering is sometimes placed around the cement.

The sizes and capacities of cylindrical units cover the following ranges approximately: power capacity, from 30 to 350 watts; current capacity, from 0.06 to 40 amperes; resistance, from 0.1 to 10,000 ohms; length, from 4 to 22 inches; diameter, from 1 to 2.5 inches. The watt capacity for these resistance units varies from approximately 1.5 to 3.5 watts per square inch of surface for continuous service, when assembled in frames affording good ventilation. This type of rheostat is rarely employed when currents of more than 50 amperes are to be handled.

Plate Type of Resistance Element. — In this type the resistance coils are attached to a circular base of insulating material. The coils are either covered with an insulating, heat-conducting cement and baked, or are enclosed in a ventilated iron case. Suitable contacts and a switch arm are provided. Currents higher than 60 amperes are rarely handled by this type of rheostat. The common sizes of plates range from 9 to 15 inches in diameter.

Grid Type of Resistance Element. — This type of rheostat, Fig. 1, consists of cast-iron grids assembled on horizontal rods bolted to pressed steel end-plates, and is the type generally used for large currents. The rods are covered with a mica insulating sleeve. If mounted on a frame affording good ventilation, the continuous capacity of the standard grids is approximately 700 amperes per square inch of cross section for a maximum rise of 240° C.

Switching and Control Mechanism for Rheostats. — (*See also Starters, Motor.*) With those rheostats which contain distinct resistance elements, or units, the terminals of the elements are brought out to metal contacts usually arranged in a circle on an insulating, fireproof plate. The contact of the switch arm for small currents up to 25 amperes per contact arm consists of an ordinary straight finger contact brush; for currents up to 350 amperes, solid sliding plungers with evenly-faced surfaces held by springs against the contact segments are used in the switch-arms. For higher currents the laminated brush has been found more satisfactory because of the more uniform contact obtained. Most designs are based on the rule that for maximum current, 20° C. rise of temperature above surrounding air should not be exceeded for contacts and face parts. When hand-operated rheostats cannot be mounted on the panel, chain and sprocket drive is usually employed. Gear control is also occasionally used. Large rheostats are frequently operated by solenoids or motors. Motor-operated rheostats are usually employed for currents in excess of 350 amperes.

Special Forms of Commercial Rheostats. — For any particular class of service, the nature of the regulation desired, and the kind of circuit to which the rheostat is to be connected will determine the arrangement, size and number of the resistance steps necessary as well as the proper type of switching mechanism and control. See also *Motors, Industrial Application of.*

Field Rheostats. — The number of steps for a field rheostat depends on the closeness with which adjustments of field current are to be made and on the range desired. For ordinary conditions a rheostat resistance equal to the resistance of the generator field is satisfactory. Machines which are regulated by automatic voltage regulators frequently require a rheostat resistance of from 2 to 4 times that of the generator field. Field rheostats for currents up to 60 amperes are usually in form of the plate type or unit type. For higher capacities the grid type is employed. Field rheostats commonly have from 30 to 70 divisions of resistance. Double this number may be obtained in case of two or more plates, if the plates have the levers staggered. Field rheostats are either hand-controlled or solenoid- or motor-operated.

Field Discharge Resistors. — These resistances are placed across the field of motor or generator whenever the main-line switch is opened, for the purpose of providing a gradual dissipation of the electromagnetic energy stored in the field, thus limiting the arc on opening and reducing the back electromotive force due to the inductance of the field circuit. They are commonly designed for 15-second duty. In general, a resistance equivalent to that of the field is recommended. They are frequently included with the field rheostats.

Starting Rheostats. — See *Starters, Motor.*

LABORATORY RHEOSTATS AND RESISTORS. — (*See also Resistors, Standard.*) On account of the extreme variety of the requirements for rheostats in a laboratory, there are but few standard forms of rheostats for laboratory use on the market. Therefore, most laboratory rheostats are of special design.

A good form of laboratory rheostat for moderately-large currents is one consisting of a thin metal tube, on which enameled resistance wire is wound. The sliders should carry two or more laminated brushes, below which the enamel is scraped off. For fixed resistors the slider is omitted. The continuous capacity of this form of resistor of diameter from 1 inch to 2 inches ranges from 4 to 6 watts per square inch of surface, if ventilation is good.

Grooved porcelain tubes or rectangular slate slabs wound with bare resistance wire will carry on an average from 2 to 4 watts per square inch continuously.

Resistance wire (*see Wires, Resistance*) wound on open wooden frames is frequently used for laboratory rheostats.

Data on Galvanized Iron Wire. — Iron wire, although it has a much lower resistance than the various forms of resistance wires on the market, is much cheaper and is therefore sometimes used. The following data will be found useful in designing iron wire rheostats.

DATA ON GALVANIZED IRON WIRE

B. & S. gage	Circular mils	Maximum* allowable current	Feet required for 110 volts
20	1,022	2.5	594
19	1,288	2.9	626
18	1,624	3.5	673
17	2,048	4.2	710
16	2,583	5.0	750
15	3,257	6.0	790
14	4,107	7.1	840
13	5,178	8.5	886
12	6,530	10.1	941
11	8,234	12.0	990
10	10,380	14.3	1054
9	13,090	17.1	1103
8	16,510	20.3	1354

* In air with free radiation.

Carbon Rheostats. — This type of rheostat consists of a series of carbon plates stacked in a suitable frame and provided with an adjusting screw such that the pressure between the plates may be varied. Alternate plates of two different sizes are sometimes used in large rheostats to secure greater radiating

surface. These rheostats are very useful when continuous variation of resistance over relatively small ranges is desired; they are also very durable and cheap.

Data for the Design of Carbon Rheostats. — The following data is taken from an article by C. R. Moore in the *Elec. Rev. and West. Elec.*, 1912, Vol. 60, p. 672. Carbon plates, either circular or square, varying in thickness from 1/8 to 1/4 inch are satisfactory, the sizes from 1/8 to 3/16 inch being preferable because of the more uniform contact obtained between the plates. Since practically all of the resistance is in the contacts between successive plates the plate surfaces must be *ground very smooth*. The current-carrying capacity for such rheostats varies from 5 to 12 amperes per square inch with an average of 7 amperes per square inch. For temperature rises ranging from 40 to 85° C. the radiating coefficient is from 0.002 to 0.005 watt per square inch of radiating surface per degree centigrade rise. For pressures ranging from 7.5 to 180 lb. per sq. in., the variation in resistance per square inch per contact based on average results is shown in the following table:

RESISTANCE OF CARBON-PLATE RHEOSTATS

Pressure, lb. per sq. inch	Resistance, ohms per sq. in. per contact	Pressure, lb. per sq. inch	Resistance, ohms per sq. in. per contact	Pressure, lb. per sq. inch	Resistance, ohms per sq. in. per contact
7.5	0.0230	35	0.0063	90	0.0029
10.0	0.0185	40	0.0057	100	0.0027
12.5	0.0145	45	0.0052	120	0.0024
15.0	0.0125	50	0.0047	140	0.0021
20.0	0.0100	60	0.0041	160	0.0018
25.0	0.0082	70	0.0036	180	0.0015
30.0	0.0071	80	0.0032		

Example for Design of Carbon Rheostats. — Given a circuit of 5 ohms resistance connected to constant potential mains of 110 volts, the current is to be adjusted between the limit of 17 and 21.5 amperes by means of a series rheostat. Hence the corresponding voltage drops through the rheostat are 25 and 2.5 volts respectively, which calls for a resistance ranging from 1.47 ohms to 0.116 ohm. For a current-carrying capacity of 7 amperes per square inch, the plates must have an area of 21.5 ÷ 7 = 3 sq. in. approximately; square plates 1.75 by 1.75 in. may therefore be used. A convenient thickness for plates of this size is 1/8 in. With 8 lb. per sq. in. as the minimum pressure, the resistance per plate is 0.022 ÷ 3 = 0.0073 ohm (from table and area of plates as computed above). Hence the number of plates for 1.47 ohms resistance is 1.47 ÷ 0.0073 = 200; thus the length of the rheostat is approximately 200 × 0.125 ≒ 25 in. The minimum resistance of 0.116 ohm requires 0.116 ÷ 200 = 0.00058 ohm per plate, which is equivalent to 3 × 0.00058 = 0.00174 ohm per square inch per contact. Hence, from the table the maximum pressure must be 165 lb. per sq. in., which is equivalent to a total pressure of 3 × 165 = 495 lb. acting on the frame supporting the carbon blocks.

Radiating surface must be provided for a maximum of 25 × 17 = 425 watts. With a radiating constant of 0.004 watts per square inch per degree centigrade rise of temperature, a maximum rise of temperature of 75° C. will require a radiation of 0.004 × 75 = 0.30 watts per square inch. Hence a radiating surface of 425 ÷ 0.30 = 1420 sq. in. must be provided. Since the 200 plates represent a radiating surface (edges only) of only 200 × 4 × 0.125 × 1.75 = 175 sq.

in., every other plate must be of a larger size. This may be secured by using 100 small plates 1¾ by 1¾ by ⅛ inch, having a total radiating surface (edges only) of 87.5 sq. in., and 100 plates 3 by 3 by ⅛ inch, having a total edge surface of $3 \times 0.125 \times 4 \times 100 = 150$ sq. in.; the total area of their projecting sides represents $(3 \times 3 - 3) \times 2 \times 100 = 1200$ sq. in. Thus the total radiating surface is $87.5 + 150 + 1200 = 1437.5$ sq. in.

SUBMERGED WIRE RHEOSTATS. — Steady loads for temporary work may be obtained from galvanized iron resistors placed in a river or tailrace or in a barrel continuously fed with supply water. The proper diameter and length of wire for any set of requirements may be calculated from the following formulas:

$$d = KI^{2/3}, \qquad l = \frac{d^2E}{112\,I},$$

in which d = diameter of wire in mils, I = the current in amperes, E = the impressed voltage, l = length of wire in feet, 112 = average value of ohms per mil-foot of galvanized iron wire. Values of K for barrels or tanks range from 3.25 to 2.75, for tailraces or rivers of moderate flow, 2.75 to 2.25, for conditions of rapid flow, 2.25 to 2.0. Wooden sticks with fairly-sharp edges will make reliable supports for the wire. For more careful work the wires may be held by porcelain cleats screwed to wooden frames. The heating of these wires in water is so great that there must be no obstructions to a free circulation of the cooling water, otherwise failure may occur due to the formation of gaseous envelopes surrounding the wire. The water used must be clean to prevent rapid destruction by electrolysis. When tanks or barrels are used, it is preferable to raise the container above the ground (on wooden blocks about two feet high) so as to avoid seriously grounding the circuit through the stream of cooling water.

DATA FOR SUBMERGED RHEOSTATS OF GALVANIZED IRON WIRE

From *Rheostats for Dynamo Load Tests*, Am. Elec., 1903, Vol. 15, p. 512.

Size of wire, B. & S. Gage	Safe carrying capacity, amperes	Minimum length in feet for safe carrying capacity at different voltages			Feet per ohm, *hot*
		110 volts	220 volts	500 volts	
4	584	66	131	298	348.0
5	489	62	124	282	276.0
6	412	59	117	266	219.0
7	347	55	110	250	173.5
8	293	52	103	235	137.5
9	245	47	94	214	109.1
10	205	45	90	205	86.5
11	173	42	84	191	68.6
12	145	40	80	182	54.3
13	122	38	76	173	43.2
14	103	36	72	164	34.2
15	88	34	68	155	27.2
16	71	32	64	145	21.5
17	60	30	60	136	17.1
18	50	29	58	132	13.5
19	42	27	54	123	10.4
20	36	25	50	114	8.5

WATER RHEOSTATS. — A water rheostat serves as a simple, cheap and satisfactory means of dissipating large amounts of energy. It is extensively used in the commercial testing of apparatus and for other purposes where an adjustable, high-power-capacity resistance is required. Permanent installations are sometimes made in hydroelectric stations to improve hydraulic or electric regulation and to furnish an artificial load.

Resistivity of Water and Common Solutions.—The resistivity of ordinary water at 20° C. (= 68° F.), taken from streams, wells or reservoirs, usually ranges from about 800 to 2000 ohms per inch cube (2000 to 5000 ohms per centimeter cube) though extremes may be found as low as 500 and as high as 5000 ohms per inch cube (1200 and 12,000 ohms per centimeter cube respectively). In water rheostats designed for less than 1000 volts it is usually desirable to increase the conductivity of the water by adding a salt (common salt, copper sulphate or other cheap salt) or an acid (usually sulphuric). For example, the resistivity of a 5 per cent solution of common salt at 18° C. is 14.9 ohms per inch cube (38 ohms per centimeter cube). The resistivity of various solutions of different strengths is given in the article on *Resistance and Conductance.*

As noted in the article just referred to, the resistance of ordinary water and of salt and acid solutions diminishes rapidly with increase of temperature. To a close approximation the change in the resistance of ordinary clean water between 0° C. and 100° C. may be calculated from the formulas

$$R_t = \frac{40\,R_{20}}{20 + t}, \qquad \text{or} \qquad R_{t'} = \frac{72\,R_{68}}{4 + t'}.$$

where R_{20} is the resistance at 20° C. = R_{68}, the resistance at 68° F., and t is any other temperature centigrade and t' any other temperature Fahrenheit.

Before designing a large high-voltage rheostat it is desirable to determine both the resistivity and temperature coefficient of a sample of the water which will be used. The sample should be tested in a miniature, low-voltage rheostat. If the large rheostat is to be operated on alternating current, the test with the miniature rheostat should also be made with alternating current, in order to avoid the effects of polarization. Determinations for a direct-current rheostat should be made with direct current, using a miniature rheostat of the same materials as are to be used in the large rheostat, and operated at the same current density and voltage. In designing a rheostat for permanent use data should also be obtained on the variations in the resistivity of the water throughout the year, for these variations are very marked in some cases.

Low-voltage Water Rheostats. — A simple form of water rheostat for direct-current or single-phase low-voltage work can be made of an oil barrel and two iron plates. One plate is placed on the bottom of the barrel. An insulated copper wire is connected to this plate and brought up along the side of the barrel and out through the top, or it may be attached to the underside of the barrel by means of a bolt passing through the plate and the bottom of the barrel. The other terminal is attached to a movable plate which is held by a window cord strung over two pulleys, the plate being held in position by a suitable counterweight at the free end of the cord. The barrel is filled with water, to which a relatively strong solution of common salt ($NaCl$), sal ammoniac (NH_4Cl) or washing soda (Na_2CO_3) is added in sufficient amount to give the required conductivity. It is essential that the substance should be dissolved before it is added to the water barrel and that the solution should be poured in very carefully, because a very small amount of a salt solution will add considerably to the conductivity of the water. A single-phase load of 100 amperes at 100 volts may be carried continuously in a 40-gallon barrel without causing the water to boil. The plates should have a surface (one side only) of at least 1 square inch per ampere.

For low-voltage three-phase work a barrel with three pipes, suitably mounted and arranged to be raised or lowered by means of a cord and pulleys, is sometimes used. The pipes should have an immersed surface (outside surface only) of at least 1 square inch per ampere.

A serious objection to the use of a water rheostat containing a salt or acid is that the resistance for a given setting of the electrodes does not remain constant, due to the evaporation of the water, rendering the solution more concentrated. The increase in the temperature of the solution also causes very appreciable changes in resistance. When direct current is used the change in the concentration of the solution at the two electrodes (polarization) also causes a variation in the resistance with time.

High-voltage Water Rheostats. — For voltages above 1000 the conductivity of the tap or river water available is usually ample, without having recourse to the use of salts or acids. The use of ordinary water also makes it possible to keep a continuous flow of water through the rheostat and thus maintain a practically constant temperature. The discussion of such high-voltage rheostats given below is adapted from an excellent paper on this subject by E. A. Ekern in the *Stone and Webster Public Service Journal* for Nov., 1911.

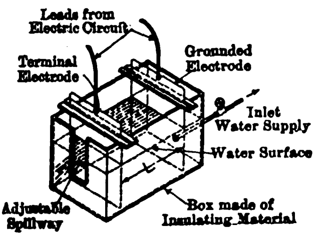

Fig. 1. Open-column Type

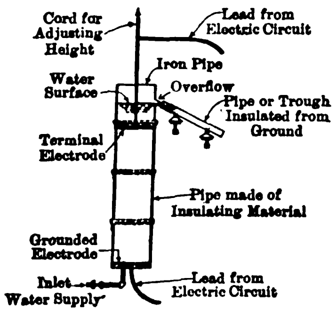

Fig. 2. Closed-column Type

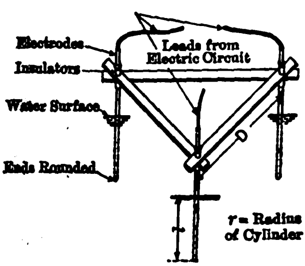

Fig. 3. Open Type, Three-phase Rheostat

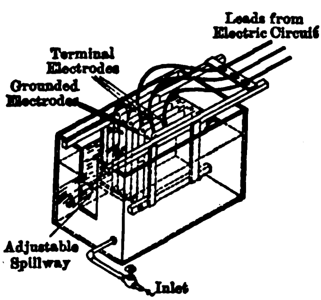

Fig. 4. Mixed Type, Three-phase

Types of High-voltage Water Rheostats. — Fig. 1 illustrates a simple form of such a rheostat, and will be referred to as the "open-column" type, and Fig. 2 illustrates another simple form which will be referred to as the "closed-column" type. The illustrations are for single-phase rheostats; three-phase rheostats are made by combining three single-phase elements with the neutral point grounded. Fig. 3 illustrates a three-phase type suitable for immersion

in an open body of water; this type will be referred to as the "open type." The type shown in Fig. 4, which is a combination of the column type and open type, will be referred to as the "mixed type;" the illustration shows a three-phase rheostat with three plates for terminal electrodes sandwiched between four neutral electrodes. The plates should preferably have rounded edges.

Calculation of Resistance. — The resistance of the column type of rheostat (Figs. 1 and 2) is computed by the formula

$$R = \rho \frac{L}{A}, \tag{1}$$

where ρ = the resistivity of the water in ohms per inch cube (as noted above ρ is usually between 800 and 2000 ohms per inch cube), L = the length of the column in inches, and A = the area of the cross section of the column in sq. in.

For the open type (Fig. 3) the resistance between any cylindrical electrode and the neutral is given by the formula

$$R = \frac{\rho}{2.73\, l} \log_{10}\left(\frac{2\, D}{d}\right), \tag{2}$$

where ρ = the resistivity of the water in ohms per inch cube, l = the wetted length of cylinder in inches, D = distance between cylinders in inches (center to center), and d = external diameter of each cylinder in inches. The equation is sufficiently accurate for all practical purposes when $2\, D \div d$ is greater than 80 and may be used with but small error when this ratio is as low as 40. (*See also the article on Resistance and Conductance.*)

The resistance to neutral of the mixed type (Fig. 4) is computed by the formula

$$R = \frac{R_1 R_2}{R_1 + R_2}, \tag{3}$$

where

$$R_1 = \rho \frac{S}{2\, A} \qquad \text{and} \qquad R_2 = \frac{\rho}{1.37\, \lambda} \log_{10}\left(\frac{4\, S}{t}\right), \tag{4}$$

R_1 is the resistance between the face of any one terminal plate and the two neutral plates between which it is placed and R_2 is the resistance between the wetted edge (assumed rounded) of the terminal plate and the two plates between which it is placed. The letters in the expressions for R_1 and R_2 have the following meanings: ρ = resistivity of the water in ohms per inch cube, S = the distance between adjacent plates (terminal plate to neutral plate) in inches. A = area in square inches (one side only) of terminal plate, t = thickness of each terminal plate in inches, λ = total length in inches of wetted edge (perimeter) of one terminal plate. The expression for R_2 is a rough approximation, but as R_2 is small compared with R_1, the error in the resultant resistance R due to an error in R_2 is relatively small.

Allowable Current Density. — A current density at the electrodes in excess of 4 amperes per square inch is accompanied by a "spitting" or arcing and there is danger of short circuit. This limiting current density applies to clean water, but as there is always a likelihood of foreign materials getting into the water, it is not advisable to use a current density between flat plates in excess of 1 ampere per square inch, but at the edges of the plates or where there is a free and large circulation of water a current density as high as 3 amperes per square inch may be safely used.

Minimum Distance between Electrodes. — Calling σ the maximum allowable current density and v the voltage to neutral, then in terms of the notation defined above, the minimum spacing of the electrode is;

For column type (Figs. 1 and 2) $$L = \frac{v}{\rho \sigma}. \tag{5}$$

For open type (Fig. 3) $$D = \frac{d}{2} \log^{-1}\left(\frac{0.869\, v}{\rho d \sigma}\right). \tag{6}$$

In the mixed type the limiting current density is usually at the edge of the plates, and the minimum spacing in this case is given by the relation:

For mixed type (Fig. 4) $$S = \frac{t}{4} \log^{-1}\left(\frac{0.869\, v}{\rho t \sigma}\right). \tag{7}$$

The logarithms are all common logarithms (i.e., to the base 10).

Minimum Size of Electrodes. — Using the same symbols as above, and in addition calling I the current per phase (terminal to neutral), the minimum area of each plate in the column type (Figs. 1 and 2) is $A = I \div \sigma$ square inches, and in the open type (Fig. 3) the minimum diameter of the electrodes is $d = I \div (\pi l \sigma)$ inches. In the mixed type the total area of the terminal plate (both sides and the edges) is approximately $(\rho S) \div R$, where R is the total required resistance. The area of the flat surface of the plate (one side only) may be taken from 10 to 15 per cent less than half of this total surface, to allow for the edges. The actual division of current between the edges and the flat surfaces will be inversely as the resistances R_1 and R_2 given by equations (4).

Example of Calculation of Dimensions. — The more complicated case of a mixed type of rheostat is chosen, to illustrate the use of both types of formulas. Power to be dissipated 3000 kw. at 2300 volts, three-phase. Then voltage to neutral is $V = 1328$ volts, current per phase $I = 753$ amperes, corresponding to a resistance of 1.762 ohms. Assume the plates to be ¼ inch thick, then $t = 0.25$. Allowing a maximum current density of 3 amperes per sq. in. at the edges, and taking $\rho = 800$, the minimum spacing is then, from equation (7), $S = 5.27$ inches.
The total surface (both sides and edges) of each terminal plate, assuming $\rho = 1200$, to allow for an increase in the assumed resistivity of the water, will be $(1200 \times 5.27) \div 1.762 = 3580$ sq. in., and the area of the plate should then be say 15 per cent less than $3580 \div 2$, or 1500 sq. in. approximately. A plate 60 by 25 inches will be a suitable size. The two resistances R_1 and R_2 are then from equation (4), $R_1 = 2.1$ ohms and $R_2 = 11.6$ ohms, and from equation (3) $R = 1.775$, which practically agrees with the required value of 1.762 ohms.

Supply of Water. — Due to the change in the resistivity of water with increase in temperature it is preferable to supply the rheostat with sufficient water to prevent much of an increase of temperature. At a hydroelectric station it is usually possible to use an open-type rheostat (Fig. 3) placed in a running stream or in a large body of water. When the water supply is limited it is sometimes necessary to allow the water to heat or even boil in extreme cases. Even in the case of limited water supply, however, a small steady overflow should be allowed, in order to prevent an accumulation in the rheostat of the salts contained in the water.

For an increase of $T°$ F. in temperature the water required to carry off P kilowatts, assuming no radiation or evaporation, is

$$\frac{0.91\, P}{T} \text{ cu. ft. per min.} = \frac{6.8\, P}{T} \text{ gallons per min.} \tag{8}$$

If only G gallons of water per minute are available and the water is allowed to boil, then calling T the difference between the boiling point and the temperature of the supply water in degrees Fahrenheit, the water evaporated when P kilowatts are supplied to the rheostat would be

$$G_e = \frac{P}{143} - \frac{GT}{970} \text{ gallons per min.} \tag{9}$$

Example. — To dissipate 3000 kw. with a 10° F. rise in the temperature of the water would require (6.8 × 3000) ÷ 10 = 2040 gallons per min. If only 60 gallons per minute are available at 62° F., $G_e = 3000 \div 143 - 60 \times (212 - 62) \div 970 = 11.7$ gallons per min. will be evaporated, leaving 48.3 gallons per minute for the overflow.

Notes on Design of Water Rheostats. — The column type of rheostat may be used for any capacity and voltage. For voltages up to 23,000, the mixed type is usually preferable for large loads. For higher voltages, the open type may then be employed to advantage. Common iron pipe or steel plates are satisfactory materials for electrodes. Rheostats designed for permanent use should have galvanized electrodes. For voltages of 2300 volts and less, well-oiled and shellacked wooden containing vessels for the water have been found satisfactory. Open-or mixed-types of rheostats may be put in any tank or body of water where the clearance to sides or bottom is greater than one-half the spacing of electrodes. Column type rheostats up to 44,000 volts having columns built of vitrified glazed tile cemented together with Portland cement shellacked over have been operated successfully. Open-type rheostats made of iron pipes insulated and held securely above the water surface are very satisfactory. Mixed-type rheostats having plates separated by common porcelain line insulators and all clamped into a wooden frame have been used very successfully. Plates or electrodes should always be so arranged and located as to obtain free circulation of water. Where insulating material may be partially or intermittently in and out of the water, material may be deposited from the water and sufficient leakage surface must be provided. Innumerable arrangements and contrivances for the operation of rheostats may be designed but the important considerations are sufficient insulation to protect the operator, and positive movements to obtain the required degree of adjustment. Water rheostats are practically non-inductive. A slightly-leading current is obtained due to electrostatic capacity, but it is often more than offset by the inductive reactance of the conductors.

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[O. R. Schurig.]

ROOTS AND POWERS. — The symbol a^n where n is a positive whole number means a multiplied by itself n times; n is called the exponent of a. The conception of an exponent is also extended to include negative and fractional exponents as follows: Let m be a positive whole number and let $n = -m$; then the expression $a^n = a^{-m}$ is defined as equivalent to $\frac{1}{a^m}$; that is, a^{-m} is taken as equivalent to the reciprocal of a^m. Again, let m be a whole number and let $n = \frac{1}{m}$; then the expression a^n is defined as equivalent to $a^{\frac{1}{m}} = \sqrt[m]{a}$; that is, $a^{\frac{1}{m}}$ is taken to represent the m root of a. In general, then, the properties of exponents are the following, where m and n may be positive or negative whole numbers or fractions:

$$a^{-m} = \frac{1}{a^m} \qquad (a^m)^n = a^{mn}$$

$$a^{\frac{1}{m}} = \sqrt[m]{a} \qquad \sqrt[n]{a^m} = a^{\frac{m}{n}}$$

$$a^m a^n = a^{m+n} \qquad a^0 = 1$$

$$\frac{a^m}{a^n} = a^{m-n}$$

The calculation of roots and powers can be conveniently carried out by means of logarithms (q. v.); for the highest accuracy, a seven or nine place table should be used. Note that

$$\log a^n = n \log a$$

$$\log \sqrt[n]{a} = \frac{1}{n} \log a$$

[W. A. Del Mar.]

ROPES AND ROPE DRIVE. — (*See also Wires and Cables, Bare.*) The following terms relating to ropes and cordage are commonly employed:

Yarn. — Natural fibers twisted together.
Thread. — Two or more *small yarns* twisted together.
String. — Same as thread except of little larger yarns.
Strand. — Two or more *large yarns* twisted together.
Cord. — Several *threads* twisted together.
Rope. — Several *strands* twisted together.
Hawser. — A rope of three *strands*.
Shroud-laid rope. — A rope of four strands.
Cable. — Three *hawsers* twisted together.

In a strand the yarns are laid up left-handed; in a rope the strands are laid up right-handed; in a cable the hawsers are laid up left-handed.

WEIGHT AND STRENGTH OF ROPES. — The following tables give the weight per foot and breaking strength of hemp and steel ropes. Cotton ropes have approximately the same weight per foot as hemp ropes, but their breaking strength is only 60 per cent of that of hemp ropes. Iron ropes have the same weight per foot as steel ropes, but only 50 per cent of the breaking strength of cast-steel ropes.

APPROXIMATE BREAKING STRENGTH OF STEEL-WIRE ROPES

(*Trenton Iron Company*)

6 strands of 19 wires each					6 strands of 7 wires each				
Diam. rope, inches	Lb. per ft.	Approximate breaking stress, lb.			Diam. rope, inches	Lb. per ft.	Approximate breaking stress, lb.		
		Cast steel	Extra strong steel	Plow steel			Cast steel	Extra strong steel	Plow steel
2¼	8.00	312,000	364,000	416,000	¼	0.10	4,800	5,400	......
2	6.30	248,000	288,000	330,000	1½	3.55	136,000	158,000	182,000
1¾	4.85	192,000	224,000	256,000	1⅜	3.00	116,000	136,000	156,000
1⅝	4.15	168,000	194,000	222,000	1¼	2.45	96,000	112,000	128,000
1½	3.55	144,000	168,000	192,000	1⅛	2.00	80,000	92,000	106,000
1⅜	3.00	124,000	144,000	164,000	1	1.58	64,000	74,000	84,000
1¼	2.45	100,000	116,000	134,000	⅞	1.20	48,000	56,000	64,000
1⅛	2.00	84,000	98,000	112,000	¾	0.89	37,200	42,000	48,000
1	1.58	68,000	78,000	88,000	11/16	0.75	31,600	36,800	42,000
⅞	1.20	52,000	60,000	68,000	⅝	0.62	26,400	30,200	34,000
¾	0.89	38,800	44,000	50,000	9/16	0.50	21,200	24,600	28,000
⅝	0.62	27,200	31,600	36,000	½	0.39	16,800	19,400	22,000
9/16	0.50	22,000	25,400	29,000	7/16	0.30	13,200	15,000	17,100
½	0.39	17,600	20,200	22,800	⅜	0.22	9,600	11,160	12,700
7/16	0.30	13,600	15,600	17,700	5/16	0.15	6,800	7,760	......
⅜	0.22	10,000	11,500	13,100	9/32	0.125	5,600	6,440	......
5/16	0.15	6,800	8,100	......	....	......	......	....	...

MANILA ROPE

(*From Blue Book of Amer. Mfg. Co., N. Y.*)

Diameter of rope, inches	Approx. pounds per foot	Breaking strength, pounds	Diameter of rope, inches	Approx. pounds per foot	Breaking strength, pounds
½	0.085	1,750	1⅜	0.65	13,200
⅝	0.13	2,730	1½	0.77	15,700
¾	0.20	3,950	1⅝	0.90	18,500
⅞	0.26	5,400	1¾	1.04	21,400
1	0.34	7,000	2	1.36	28,000
1⅛	0.43	8,900	2¼	1.73	35,400
1¼	0.53	10,900	2½	2.13	43,700

Factor of Safety. — For ordinary purposes the maximum safe stress in wire ropes should be about one-third the ultimate, and for shafts and elevators about one-fourth the ultimate. In estimating the stress due to the load for shafts and elevators, allowance should be made for the additional stress due to acceleration in starting. For short inclined planes not used for passengers a factor of safety as low as 2½ is sometimes used, and for derricks in which large sheaves cannot be used and long life of the rope is not expected, the factor of safety may be as low as 2.

For power transmission by hemp ropes a factor of safety of 36 (referred to the nominal strength of the rope) is usually employed (*C. W. Hunt, Trans. A.S.M.E., Vol. 12, p. 230*).

Splicing and Knots. — See *Kent's Mechanical Engineers' Pocket-Book.*

ROPE DRIVING. — There are two methods of putting ropes on the pulleys, the multiple-rope and the single-rope systems. In the multiple-rope system several endless ropes are employed, and therefore there are as many splices as there are single ropes. In the single-rope system one endless rope is used, making several turns around the pulleys or sheaves, and therefore with but a single splice. In the first case the individual ropes are spliced in place, being made very taut at first, and less so as the rope lengthens, stretching until it slips, when it is respliced. In the second method tension pulley must be used to give the necessary adhesion and also to take up the wear.

Power Transmitted by a Rope. — The formulas given in the article on *Belts and Belting* are directly applicable to rope transmission, but may be put in a more convenient form as follows. Let

V = velocity of rope, in feet per minute,
d = diameter of rope, in inches,
m = weight, in pounds, of 1 foot of rope 1 inch in diameter,
T_1 = actual tension in driving side of rope, in pounds per circular inch (i.e., the tension in pounds in a rope 1 inch in diameter),
f = coefficient of friction between rope and pulley,
n = number of half-turns the rope makes around pulley,
$C = 1 - \epsilon^{-nf\pi}$, where the value of $\epsilon^{-nf\pi}$ is taken from the table of ϵ^{-x} in the article on *Exponential Functions*, putting $x = nf\pi$.

Then horse-power transmitted by each taut rope from one pulley to the other is

$$P = \frac{CT_1d^2V}{33,000}\left(1 - \frac{mV^2}{116,000\,T_1}\right). \tag{1}$$

Power Transmitted by Hemp Rope. — A formula developed by C. W. Hunt (*Trans. A.S.M.E., Vol. 12, p. 230*) is frequently employed. This formula may be written

$$P = \frac{d^2V}{248}\left[1 - \left(\frac{V}{8500}\right)^2\right], \qquad (2)$$

which is the same as formula (1) when $n = 1$, $f = 0.35$, $C = 0.67$, $m = 0.32$ and $T_1 = 200$. The following table is calculated from Hunt's formula. For temporary work a rope of given diameter may be used with safety to transmit twice the horse power given in the table.

HORSE POWER TRANSMITTED BY HEMP ROPE AT VARIOUS SPEEDS

Computed from formula (2) given above

Diam. of ropes, inches	Speed of the rope in feet per minute									
	1500	2000	2500	3000	3500	4000	4500	5000	6000	7000
½	1.45	1.9	2.3	2.7	3.0	3.2	3.4	3.4	3.1	2.2
⅝	2.3	3.2	3.6	4.2	4.6	5.0	5.3	5.3	4.9	3.4
¾	3.3	4.3	5.2	5.8	6.7	7.2	7.7	7.7	7.1	4.9
⅞	4.5	5.9	7.0	8.2	9.1	9.8	10.8	10.8	9.3	6.9
1	5.8	7.7	9.2	10.7	11.9	12.8	13.6	13.7	12.5	8.8
1¼	9.2	12.1	14.3	16.8	18.6	20.0	21.2	21.4	19.5	13.8
1½	13.1	17.4	20.7	23.1	26.8	28.8	30.6	30.8	28.2	19.8
1¾	18.0	23.7	28.2	32.8	36.4	39.2	41.5	41.8	37.4	27.6
2	23.2	30.8	36.8	42.8	47.6	51.2	54.4	54.8	50.0	35.2

Power Transmitted by Iron and Steel Ropes. — Wm. Hewitt of the Trenton Iron Company gives the following formula for the power transmitted by a steel rope, excluding the losses due to journal friction of the sheaves,

$$P = Bd^2v, \qquad (3)$$

where v is the velocity of the rope in *feet per second* and B has the following values.

B for steel rope on	Number of half turns around sheave = n					
	1	2	3	4	5	6
Iron	5.61	8.81	10.62	11.65	12.16	12.56
Wood	6.70	9.93	11.51	12.26	12.66	12.83
Rubber or leather	9.29	11.95	12.70	12.91	12.97	13.00

For iron rope take for B one-half the figures given in the above table.

Hewitt's formula (3) is equivalent to (1) when the centrifugal force of the rope is neglected and T_1 and f are as follows:

Kind of rope	Working tension T_1 in lbs. per circ. inch		Coeff. of friction $= f$
	Iron	Steel	
Iron	3600	7200	0.16
Wood	3600	7200	0.23
Rubber or leather	3600	7200	0.40

It should be noted that the coefficient of friction decreases rapidly with moisture or grease on the rope and sheaves, and consequently to transmit a given amount of power the tension must then be increased.

In the following table is given the horse-power that may be transmitted by a steel rope making a single half turn ($n = 1$) on wood-filled sheaves. This table agrees approximately with formula (3). The transmission of more than 250 h.p. by a single steel rope making a single half turn on filled sheaves is impracticable, as the filling would be rapidly cut out due to the increased tension (i.e., total tension $= T_1d^2$) and high velocities required. If the rope makes several half turns around the sheave, however, a greater amount of power can be transmitted at a given speed and tension than when only a single half turn is used.

HORSE-POWER TRANSMITTED BY A STEEL ROPE ON WOOD-FILLED SHEAVES

Diameter of rope, in.	Velocity of rope in feet per second									
	10	20	30	40	50	60	70	80	90	100
1/4	4	8	13	17	21	25	28	32	37	40
5/16	7	13	20	26	33	40	44	51	57	62
3/8	10	19	28	38	47	56	64	73	80	89
7/16	13	26	38	51	63	75	88	99	109	121
1/2	17	34	51	67	83	99	115	130	144	159
9/16	22	43	65	86	106	128	147	167	184	203
5/8	27	53	79	104	130	155	179	203	225	247
11/16	32	63	95	126	157	186	217	245	...	...
3/4	38	76	103	150	186	223	...	...	...	...
7/8	52	104	156	206	...	...	...	...	...	...
1	68	135	202	...	...	...	...	...	...	...

The horse-power that may be transmitted by iron ropes is one-half of the above.

Power Lost Due to Friction of Sheaves and Shafts. — In the above formula no allowance is made for the friction of the sheaves and shafts. Wm. Hewitt of the Trenton Iron Company gives the following expression for the horse-power lost in friction

$$6 \times 10^{-6} (W + G_1 + G_2) v,$$

where $W =$ total weight of rope, $G_1 =$ total weight of terminal sheaths and shafts, $G_2 =$ total weight of intermediate sheaves and shafts, all in pounds, and $v =$ velocity of rope in feet per second.

Diameters of Sheaves. — The following table gives the minimum diameter of sheaves recommended by Hunt for hemp ropes and by Hewitt for steel and iron ropes. The larger the diameter of the sheaves the greater the life of the rope.

MINIMUM DIAMETER OF SHEAVES

Diameter of rope, in.	Hemp	Steel			Iron		
		7-wire	12-wire	19-wire	7-wire	12-wire	19-wire
¼	..	20	15	12	40	30	24
5/16	..	25	19	15	50	38	30
⅜	..	30	22	18	60	45	36
7/16	..	35	26	21	70	53	42
½	20	40	30	24	80	60	48
9/16	..	45	33	27	90	68	54
⅝	24	50	37	30	100	75	60
11/16	..	55	41	32	110	83	66
¾	30	60	44	35	120	90	72
⅞	36	70	52	41	140	105	84
1	42	80	59	47	160	120	96
1¼	54	..	..	..	...	...	..
1½	60	..	..	..	...	...	..
1¾	72	..	..	..	...	...	..
2	84	..	..	..	...	...	..

Tension Measured by Sag. — (*See also article on Transmission Lines.*) Let

w = weight of rope per foot, in pounds,
L = distance between centers of sheaves in feet,
S = sag in feet midway between sheaves.

Then the tension at the sheaves, corresponding to the sag S, is approximately

$$T = \frac{wL^2}{8S} + wS \qquad \text{pounds}$$

for either the driving or the slack rope.

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[Wm. Kent.]

RUBBER. — (*See also Gutta-Percha; Insulating Materials; Wires and Cables, Insulated.*) Rubber is derived from the milky secretion or latex of certain tropical trees, creepers and shrubs found chiefly in America, Africa, Ceylon and Malacca. When these plants are tapped, a thick milky-looking fluid or latex exudes from them. This latex is composed of very minute oil-like refractive globules, varying in size, which are in a state of rapid Brownian movement in a clear transparent liquid, called the serum. Besides these *caoutchouc* globules, or rubber-gum proper, the serum contains resins, protein, enzymes and various organic and inorganic compounds. Rubber or India rubber is the dried-up or coagulated latex. In Brazil coagulation is effected principally by dry heat or smoking. A wooden paddle is dipped in the latex and held over a smoky fire until the latex has coagulated. This process is repeated until the caoutchouc layers have become sufficiently thick, when the lump of raw rubber is cut off, dried for several days and despatched usually as "fine Para biscuits" to a trading center. It is believed that the excellence of Para rubber is partly due to this mode of preparation, the smoke having a preservative effect and the coagulation in concentric layers adding greatly to the "nerve" of the rubber. Para entrefine, Negro Heads and Sernamby are usually prepared from fine Para rubber which adheres to the tree during tapping or to the vessels containing the latex.

IMPURITIES IN COMMERCIAL RUBBER. — Commercial rubber contains, in addition to the caoutchouc, a number of foreign substances, such as sand, bark, etc., which can be removed by mechanical washing, followed by drying.

Acetone Extract. — In addition to the pure rubber gum, washed rubber contains resins and proteins which are soluble in acetone and are therefore often known as "acetone extract."

The accompanying table shows the loss in washing and the percentage of acetone extract in the best brands of raw rubber.

IMPURITIES IN COMMERCIAL RUBBER

(*Abstracted from a very comprehensive table in "Lectures on India Rubber," Edited by D. Spence, London, 1909.*)

Brazilian Rubber

Trade name	Geographical origin	Mean loss on washing, per cent	Acetone extract in washed dry rubber, per cent
Para, fine Island, soft cure	Brazil, the islands of the lower Amazon and its delta, and also other parts of the State of Para	17–20	1.9–2.1
Para entrefine, Islands entrefine		18–25	Varies
Negro Heads, or Islands Coarse Sernamby		35–40	2–6
Fine Para, upriver, hard cure	Amazon district. Also the districts drained by its large tributaries	15–20	1.9–2.9
Upriver entrefine, hard entrefine		18–25	Varies
Upriver coarse or Manaos Scrappy Negroheads		18–25	1.5–1.8
Cameta Negroheads	Southwestern Para	37–42	1.2–2.2

IMPURITIES IN COMMERCIAL RUBBER — *Continued*

Brazilian Rubber — *Continued*

Trade name	Geographical origin	Mean loss on washing, per cent	Acetone extract in washed dry rubber, per cent
Caucho Balls	Amazon district and its lower tributaries	25-35	3.6-4
Caucho Slabs and Strips		35-42	4-8
Ceará Scraps	Ceará, Piauhy and Rio del Note	29-	2.1
Maniçoba			
Matto-Grosso, Virgin sheets, white Para	Matto-Grosso	15-30	2.5-3.5
Matto-Grosso, Negroheads	Matto-Grosso	25-35	2.5-6
Miscellaneous South and Central American Rubber			
Bolivian, fine medium	Bolivia	15	1.6
Virgin, coarse, entrefine	"		
Uncut Bolivian	"		
Mollendo	South Bolivia, Peru	15-25	1.9-3
Peruvian, fine, medium and scrappy	Peru	15-22	1.9-3
Peruvian Balls (also Caucho)	"	20-35	3.6-4
Orinoco, also Angostura	Venezuela	18-22	1.9-2.9
Eastern Rubber			
Plantation Rubber, as fine biscuits, sheets, fine crepe, scraps and block	Malacca and Ceylon	2-7	2-4
African Rubber			
Upper Congo, black	Congo, Angola	10	3.1
Loando Niggers, red fine and fine black.	" "	10-20	3
Kassai, fine	" "	7-10	3.3

MANUFACTURE OF RUBBER INSULATION. — The washed, dried rubber is passed between heavy rollers and flattened into thin sheets. It is then cut into small pieces and again passed through the rollers with a large proportion of fine powder consisting usually of inert mineral substances, wax, hydrocarbons and sulphur. The mixture is thus masticated until all its constituents are thoroughly mixed and a smooth homogeneous paste obtained. This process is known as compounding.

Adulterants Used in Compounding. — Experience has shown that 60 or 70 per cent of mineral adulterant, or even a greater proportion of rubber substitute, may be added to rubber gum, before the essential qualities of the rubber

cease to predominate. The majority of commercial 30 per cent insulating compounds have compositions which fall within the following limits.

Ingredient	Per cent
Rubber	30–32
Whiting	0–30
Zinc oxide	28–67
Litharge	1–12
Ozokerite or paraffin	2–4

In addition to the above adulterants, from 2% to 4% of sulphur is added to the compound, the greater part of which combines with the rubber in the vulcanizing process (see below).

Barium sulphate, sublimed white lead, lead carbonate, lamp-black, talc, magnesium carbonate, red lead, barium carbonate and other substances are also used in small quantities. Talc is often objected to, as making the compound porous, and lampblack, as rendering analysis difficult.

Applying the Compound to Wires. — The rubber compound is applied to the wire by "tubing" machines, or is applied in strips, and the wire thus covered with the compound is coiled up ready for vulcanizing. See article on *Wires and Cables, Insulated.*

Vulcanizing. — If exposed for a long time to air and sunlight, rubber looses its elasticity and finally oxidizes completely into resinous matter soluble in acetone. By vulcanization, however, rubber is rendered more or less immune from deterioration by weathering. Vulcanization is the chemical union of rubber gum with sulphur or sulphur chloride. It takes place at a temperature of from 248° to 302° F.

The coils of wire, covered with the compound as above described, are placed in a suitable chamber to which steam at the proper temperature is admitted. The time required for vulcanization depends upon the thickness of the insulation, the nature of the compound, the temperature and pressure of the steam, etc., ranging from 2 to 8 hours.

Sulphur Required for Vulcanization. — The amount of sulphur required to produce vulcanization varies with the brand of rubber and the nature of the adulterants with which it is mixed. The ratio of the weight of combined sulphur to the weight of caoutchouc, which is insoluble in acetone, is called the vulcanization coefficient. The highest grades of 30 per cent Para insulation usually have a coefficient between 5 per cent and 10 per cent.

The vulcanization of some brands of rubber cannot be accomplished without either an excess of sulphur or the presence of some mineral accelerator, such as red lead. Such rubber is to be avoided where permanency is an important consideration. It does not follow from this that red lead is a detrimental ingredient.

SPECIFIC RESISTANCE. — The specific resistance of 30 per cent Para rubber compounds is extremely variable, all values between 150 millions and 4000 millions of megohms per inch cube at 60° F., after electrification for one minute, being encountered. Cable manufacturers usually specify the specific resistance in terms of the constant K in the following logarithmic equation (*see Wires and Cables, Insulated*).

$$M = K \log_{10} \frac{D}{d},$$

where

M = insulation resistance of a cable, megohm-miles,
d = diameter of cylindrical conductor,
D = diameter of cable over its insulation,
K = 5.8 × (millions of megohms per inch cube at 60° F.).

Hence K varies from 870 to 23,000, but its usual commercial value is between 3000 and 8000.

The specific resistance is some indication of the proportion of rubber gum, in the sense that other conditions being equal, it increases with the amount of gum. It also depends upon the dryness, it being often possible to double the megohms by drying the compound in a dessicator. It is even more affected by the amount of mineral wax present, this substance closing the pores and itself possessing a very high resistance. Furthermore, strip-laid compound has greater specific resistance than the seamless.

Temperature Coefficient of Resistance. — The specific resistance of rubber compound decreases rapidly with increase of temperature, this variation being somewhat less, the greater the proportion of rubber gum in the compound. At any temperature T the rate of change of the resistance per degree of temperature rise is approximately proportional to the resistance R at this temperature, i.e.,

$$\frac{dR}{dT} = -CR,$$

where C is a constant, which for high-grade 30 per cent Para compound ranges from 0.02 to 0.03, the usual value being 0.025. The minus sign before the C indicates that the resistance actually decreases with rise of temperature.

Calling R_{60} the resistance at 60° F., and integrating this equation gives for the resistance at any other temperature T the relation

$$R_T = R_{60}\,\epsilon^{(60-T)C}.$$

See article on *Exponential Functions* for values of ϵ^x.

Change of Resistance with Time of Electrification. — The apparent specific resistance of rubber decreases with the time of electrification, a steady state being reached after a period of electrification, depending upon the temperature, this period being less the higher the temperature. Above 120° F. the steady state is reached almost immediately; at ordinary testing temperatures, such as 80° F. to 100° F., the steady state is not reached for an hour or more.

DIELECTRIC STRENGTH. — The disruptive strength of rubber insulation is generally given as between 350 and 450 kilovolts per inch or about 14 to 18 kilovolts per millimeter, effective a–c. values (*E. Jona, Proc. Inst. Elect. Eng., St. Louis, 1907, Vol. 2, p. 550; H. Osborne, Trans. A.I.E.E., 1910, Vol. 29, p. 1553, etc.*). The usual stresses in commercial testing do not exceed one-half of the above amounts. A series of tests with alternating voltage on high-grade commercial wires in 3-foot lengths gave an average dielectric strength of about 600 kilovolts per inch, effective value, with progressively-increasing testing potential.

The disruptive strength, however, is not a constant quantity; it depends upon the time of application of the testing potential, amount of moisture present and upon the temperature.

The relation between dielectric strength and time of electrification is given approximately by the following table:

SPECIFIC INDUCTIVE CAPACITY. — The specific inductive capacity of pure rubber is about 2.3 (*Floy*), but the vulcanized compounds used for insulation have specific capacities ranging between 3 and 4. The specific capacities of several compounds of stated composition are given by E. Jona (*St. Louis, 1904*), but they cannot be considered as representative of American practice.

Time of electrification, minutes	Relative dielectric strength
1	130
3	110
5	100
10	90
15	85
30	80

SPECIFICATIONS. — Specifications for rubber insulation for wires and cables will be found under *Wires and Cables, Insulated*.

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[W. A. Del Mar.]

SECHOMETER. — (*See also Bridges for Electrical Measurements.*) The sechometer is an instrument used in connection with a Wheatstone bridge for changing the direct current from a battery to an alternating current, and commutating the portion of this current that flows in the galvanometer circuit to a direct current. The sechometer may be used in the measurement and comparison of self-and mutual inductances and in the measurement of the resistance of electrolytes.

Fig. 1 shows the relative position of the commutators and brushes of a sechometer when connected in a bridge circuit. *A* is the commutator to which the galvanometer is connected and *B* the one to which the battery is connected. They are on the same shaft and turn together.

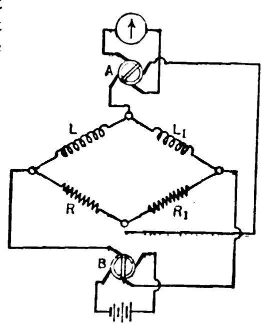

Fig. 1. Connections for Sechometer

The sechometer is not an altogether satisfactory instrument, due to the variation in the various contact resistances when used for any length of time. Whenever possible some other method of accomplishing the desired results should be employed, e.g., the use of alternating current with some sort of a-c. detector, such as a telephone receiver, electrodynamometer or a-c. galvanometer.

[H. Pender and H. R. Ranken.]

SELENIUM. — Selenium, an element of rare occurrence in nature, is found in sulphur and as a selenide in combination with various metals. Vitreous selenium, i.e., selenium which has been fused, is dark brown in color and has a specific gravity of 4.28. Its melting and boiling points are 217° C. and 700° C. respectively. Vitreous selenium is a very poor conductor of electricity, its resistance being about 60,000 ohms per centimeter-cube at room temperature. When vitreous selenium is annealed it assumes a crystalline form sometimes called metallic selenium. As a result of the annealing process, the electrical resistance is considerably reduced (the amount depending upon the thoroughness of the annealing) and becomes a function of the intensity of illumination, a property which is possessed to a lesser degree by tellurium and carbon.

Light-sensitive selenium is said to be light-positive or light-negative depending upon whether its resistance decreases or increases respectively when carried from dark to light. Sensitive selenium is usually light-positive. The resistance of light-negative selenium is usually less than that of light-positive selenium.

SELENIUM LIGHT-CELLS. — A selenium cell or unit is made by connecting several narrow strips of light-sensitive selenium in parallel between the edges of two brass plates. The higher the resistance the higher the sensibility of such a cell, that is, the greater the ratio of its conductivity in the light to its conductivity in the dark. The conductivity of a cell made of high-resistance selenium may show a change in conductivity of 20,000 per cent (200 times) when taken from direct sunlight to a dark room. Cells made of low-resistance selenium (light-negative) show a much smaller change in conductivity, seldom over 50 per cent for a change from direct sunlight to dark.

Effect of Intensity of Illumination. — The resistance of the cell when exposed to light depends upon the time of exposure and the intensity of the illumination. If a strong light is suddenly thrown upon a high-resistance cell placed in a dark room, the resistance of the cell decreases to its minimum value in a fraction of a second and then begins slowly to increase again. If the intensity of illumination is not so great or the cell has a comparatively low resistance the time taken to reach a minimum resistance may be several minutes. The majority of experimenters on this subject agree that the change of resistance varies as the square root of the intensity of illumination.

Effect of Wave Length of Light, Temperature, etc. — It is found that the greenish-yellow rays are the most effective. The sensibility of most cells decreases as the temperature increases. In certain selenium cells a large potential difference impressed across the terminals of the cell produces a variation of resistance in the cell similar to that caused by an exposure to light.

Uses of Selenium Cells. — The light-sensitive property of selenium has been utilized in the photophone and in connection with the electrical transmission of pictures. In the photophone a beam of light reflected from a mirror attached to the vibrating disk of a telephone transmitter is made to play across a selenium cell connected in series with a battery and a telephone receiver. In this manner, speech may be transmitted through space by means of light waves.

In the transmission of pictures a large number of selenium cells are usually made to reproduce the light and shade of the picture. The light reflected from each point on the picture is then represented at the distant point by a corresponding intensity of current, which may reproduce again electrochemically the intensity of the reflected light at each point.

BIBLIOGRAPHY. — Brown, F. C., *Phys. Rev.*, 1911, Vol. 33, pp. 1 and 403; Crum, *Phys. Rev.*, 1911, Vol. 33, p. 538; Ries, *Phys. Zeit.*, 1908, Vol. 9, p. 9.

[R. G. Hudson.]

SEPARATORS, STEAM. — (*See also Boilers.*) Ordinary forms of steam boilers, without superheaters, generally deliver steam containing not more than 1.5 per cent of moisture. When the water level rises too high, however, or when the water contains substances which cause foaming, the percentage of moisture in the steam may be much higher. Condensation in long lines of steam pipes also increases the moisture, and for this reason it is customary to place in the pipe line near the engines one or more steam separators. These usually operate by suddenly changing the direction of the flowing steam, so that the particles of water are by their momentum carried out of the flowing current and projected against a baffle or bend in the pipe or separator chamber, where they are collected and removed through a trap. Separators are also frequently installed in the exhaust steam piping to eliminate the cylinder oil contained in the exhaust steam.

BIBLIOGRAPHY. — Gebhardt, G. F., *Steam Power Plant Engineering*, N. Y., 1909; *Catalogues of Manufacturers.*

[WM. KENT.]

SERIES, MATHEMATICAL. — (*See also Equations, Differential; Wave Analysis.*) In calculating the numerical values of a function for given numerical values of the variable, it is frequently convenient to express the function as a sum of a series of terms each of which is a simple algebraic function of the variable. The simplest form of such a series is the

Binomial Theorem. —

$$(x+a)^n = x^n + nax^{n-1} + \frac{n(n-1)}{2!}a^2x^{n-2} + \frac{n(n-1)(n-2)}{3!}a^3x^{n-3} + \ldots a^n$$

Where $2!$ means 1×2, $3!$ means $1 \times 2 \times 3$, etc.

A particular case of this series is when $x = 1$, $a = \frac{1}{n}$ and n is taken infinitely large. The expression $\left(1+\frac{1}{n}\right)^n$ is not unity, but has the value

$$e = 1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \cdots$$

This is the base of the natural system of logarithms and is numerically equal to 2.718282 +.

Taylor's Series. — Let $f(x)$ be any function of x, and for brevity write

$$f'(x) \text{ for } \frac{df(x)}{dx}, \quad f''(x) \text{ for } \frac{d^2f(x)}{dx^2}, \text{ etc.}$$

Then
$$f(x+h) = f(x) + hf'(x) + \frac{h^2}{2!}f''(x) + \frac{h^3}{3!}f'''(x) +, \text{ etc.}$$

This expression is known as Taylor's series. Such a series is a useful expression for $f(x+h)$ only when the terms of the higher order are negligibly small, in which case the series is said to be convergent.

Maclaurin's Series is the special case of Taylor's Series when the x in the latter is taken equal to o and the h in the latter is put equal to x. In this case

$$f(x) = f(o) + xf'(o) + \frac{x^2}{2!}f''(o) + \frac{x^3}{3!}f'''(o) +, \text{ etc.}$$

Where by $f(o)$ is meant the value of $f(x)$ for $x = o$, by $f'(o)$ is meant the value of $f'(x)$ when $x = o$, etc.

The following are useful expressions derived from the above relations:

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \text{etc.}$$

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \text{etc.}$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \text{etc.}$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \text{etc.}$$

$$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \text{etc.}$$

$$\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \text{etc.}$$

[W. A. Del Mar.]

SHAFTING. — (*See also Bearings; Belts and Belting.*) Ordinary shafting for mill work is usually made of solid steel rods, although hollow shafting is sometimes used.

Diameter of Shaft. — The proper size of shafting to use to transmit a given amount of power may be expressed by the formula

$$d = \sqrt[3]{\frac{CP}{N}},$$

where d is the diameter in inches, P the horse-power transmitted, N the speed in revolutions per minute, and C a constant depending upon the desired factor of safety, the number and location of pulleys or gears, the weight of pulleys or gears and belting, etc. Various authorities assign different values to C, ranging from 40 to 125, the lower value for short shafts used simply for transmitting power, a higher value for line shafts (carrying several pulleys), and the highest value for head shafts carrying the main driving pulley.

Distance Between Bearings. — The distance between bearings may be expressed by the formula

$$L = \sqrt[3]{Bd^2},$$

where L is in feet, d in inches and B is a constant dependent upon the elasticity of the shaft, the load on the shaft and the allowable deflection. The latter is usually taken as 1/100 of an inch per foot of length. The Pencoyd Iron Works take $B = 720$ for bare shafts, and $B = 140$ for shafts carrying pulleys.

Speed of Shafting. — The following represents average practice.

Machine shops	120–240 revolutions per minute.
Wood-working	250–300 revolutions per minute.
Cotton and woolen mills	300–400 revolutions per minute.

BIBLIOGRAPHY. — See Kent's *Mechanical Engineers' Pocket-Book* and the various works on *Machine Design* listed in the Bibliography at the end of article on *Bearings*.

[Wm. Kent.]

SHOCK, ELECTRIC. — The physiological effects of a shock by lightning or by artificially-produced electricity may be classified into two groups: the major effects, cessation of respiration or heart action and the minor effects, fractures and internal injuries due to falls, and burns due to contact with electric arcs.

Removal of Victim from Circuit. — If a switch or circuit breaker in the circuit is close at hand, open the circuit at once or cut away the conductors with a wooden-handled hatchet. If the circuit cannot be opened immediately, break the contact of the victim with the conductor by moving the conductor or the victim, using any available dry non-conductor such as a board, rope or clothing. *Do not touch the victim with the bare hands under any circumstances unless insulated thoroughly from the ground.*

First Aid for Slight Shock. — If the victim is seen to be breathing, administer heart stimulants such as alcohol, ether or ammonia either by the mouth or hyperdermically, produce external warmth by rubbing the body or by application of hot substance, loosen constricting clothing about the neck and chest and present aromatic spirits to the nostrils to excite consciousness. Give plenty of fresh air.

Artificial Respiration. — If the victim does not breathe, send for the nearest doctor, remove any foreign body (tobacco, gum, false teeth, etc.) from his mouth and throat and begin artificial respiration at once, even if the victim appears to be dead. Proceed as follows:*

(*a*) Lay the subject on his belly, with arms extended as straightforward as possible and with face to one side, so that nose and mouth are free for breathing; see Fig. 1. Let an assistant draw forward the subject's tongue.

Fig. 1.

(*b*) Kneel, straddling the subject's thighs and facing his head: rest the palms of your hands on the loins (on the muscles of the small of the back), with fingers spread over the lowest ribs, as in Fig. 1.

(*c*) With arms held straight swing forward slowly so that the weight of your body is gradually but not violently brought to bear upon the subject, see Fig. 2. This act should take from two to three seconds. Immediately swing backward so as to remove the pressure, thus returning to the position shown in Fig. 1.

* Report of Commission on Resuscitation from Electric Shock issued by National Electric Light Association.

(*d*) Repeat deliberately twelve to fifteen times a minute the swinging forward and back — a complete respiration in four or five seconds.

(*e*) As soon as this artificial respiration has been started, and while it is being

Fig. 2.

continued, an assistant should loosen any tight clothing about the subject's neck, chest or waist.

(*f*) *Continue the artificial respiration, if necessary at least an hour, without interruption,* until natural breathing is restored, or until a physician arrives. If natural breathing stops after being restored, use artificial respiration again.

(*g*) Do not give any liquid by mouth until the subject is fully conscious.

Other Methods of Inducing Respiration are: (1) withdraw the tongue and allow it to recede periodically, the whole manipulation occurring about fifteen times in a minute; (2) dash cold water upon the face; (3) rub spine with ice; (4) massage chest in region of heart; and (5) administer oxygen gas (obtained at drug stores) by placing cone over mouth and nose.

After Treatment. — After the victim breathes again, treatment may be administered as outlined above in the case of a slight shock. If any bones have been fractured or if the victim appears to have received internal injuries, do not move the victim any more than is necessary and prepare for removal to the nearest hospital.

Treatment of Burns and Blisters. — If the victim has received burns, raw or blistered surfaces should be protected from the air. Blisters should not be opened. Cut around any clothing that sticks and saturate the adhering cloth or cotton dressing with picric acid (0.5 per cent) or a solution of baking soda (one teaspoonful to a pint of water). Wounds may be coated with a paste of flour and water or may be protected with machine, transformer, linseed or olive oil or vaseline. The dressing should be covered with cotton, gauze, lint, clean waste or clean handkerchiefs held tightly in place by bandages. Oil should not be applied to dry charred burns, a light, dry bandage being preferable.

Indication of Death. — Efforts to revive the victim should not cease until death is indicated by the following appearances:* "complete cessation of breathing and heart action, eyelids half-closed and pupils dilated, jaws clenched, tongue appearing between teeth with frothy mucus about mouth and nostrils, fingers semi-contracted with increasing coldness and pallor of surface."

[R. G. Hudson.]

* J. A. Austin, M.D., *Manual of First Aid.*

SHOVELS, ELECTRICAL OPERATION OF. — (*See also Cranes; Motors, Industrial Applications of.*) The operating cycle of an electrically-operated shovel is about 20 seconds, the component times being: hoist 8 to 10 seconds, thrust 10 seconds and swing 10 to 12 seconds, the thrust being in operation at the same time as the hoist and swing are operating. The motors to meet these requirements must have a sufficiently low armature inertia to permit of rapid acceleration under small power, and are therefore generally of the crane or mill-type construction.

Hoist Motors. — In the case of the hoist, considerable advantage may be gained in this respect by using two motors of one-half the capacity each instead of one motor of the full capacity, as the power required for accelerating is much less. For example: Assume a shovel that requires a 225-horse-power, 514 r.p.m. motor for the hoisting operation. Such a motor requires 144 horse-power seconds for bringing up to full speed, whereas if two 115-horse-power, 600 r.p.m. motors are substituted in its place, each motor requiring 63.5 horse-power seconds or both 127 horse-power seconds to bring up to speed, there would be a saving of about 12 per cent in the power required to accelerate the motor alone. On the other hand, such an arrangement may require the use of bevel gears and additional shafting.

Swing Motor. — This motor, although not subject to the severe overloads and shocks encountered on the hoist motors, is subject to frequent reversals, and, as rapid acceleration is required, a motor of similar design as for the hoist should be used.

Thrust Motor. — This motor differs somewhat in its operation in that it is practically stalled during the digging operation, although it may revolve or overhaul according to conditions, and is operated at full speed only after the hoist operation is completed. Its duty is to keep the dipper against the bank, and it must therefore stand still and exert torque most of the time. For this reason its design should be very rugged, and the motor should be able to develop a heavy torque for short intervals of time while standing still, or rotating very slowly.

Location of Motors. — The hoist and swinging motors are as a rule located on the car and are geared to the drums through suitable reducing gears, while the thrust motor as a rule is mounted directly on the boom and communicates its motion to the bucket staff through reducing gears connected to a pinion engaging a rack on the staff.

Type and Size of Motors. — The motor equipments may be either of the direct- or alternating-current type. The direct-current series motor has ideal characteristics for this class of service, but it requires the use of a motor-generator set when only an alternating-current supply is available. With the ordinary phase-wound induction motor it is not possible to obtain the maximum torque at starting and it may therefore be necessary to use a motor slightly larger than what would have been the case with direct current to obtain the same result. It may be possible that alternating-current commutator motors may find a useful field in connection with this service, although the objection to the single-phase type would be the heavy unbalancing effect on the system, especially at the beginning of each operating cycle, i.e., when the hoist and thrust motors are being started.

The approximate size of motors required for the various operations of a number of different size shovels may be obtained from the following table:

Control Equipment. — The magnetic contactor control is preferable for all the motors on the shovel. (*See Controllers.*) The hoist and thrust motors are then provided with a notching-back arrangement which will automatically cut

Weight of shovel in tons	Size of dipper in cubic yards	Average cycles of operation per minute	Horse power of motors		
			Hoist	Thrust	Swing
135	6	2	150	75	75
120	5	2	150	50	50
105	4	2	150	50	50
95	3¼	2¼	100	50	50
80	2½	2½	100	30	30
65	2	2¾	60	30	30
45	1½	3	60	20	20
25	¾	3½	40	20	20

resistance into the circuit when the current exceeds a certain limiting value due to the motors becoming stalled by digging too deep or striking rocks or other obstructions, thus protecting the motors from damage. Sometimes the hoist and swing motors are provided with automatic magnet control while the thrust motor is hand controlled, an ordinary drum controller being provided for this motor.

Brakes for the different shovel motions are generally of the air-brake type, a small air compressor being provided on the shovel for this purpose.

BIBLIOGRAPHY. — *Electrically-operated Shovels*, Eng. Rec., 1911, Vol. 63, p. 58; Patterson, W. H., *Electrically-operated Shovels*, Elec. Jour., 1910, Vol. 7, p. 853; *Cost Record of an Electric Power Shovel*, El. Ry. Jour., 1909, Vol. 33, p. 15[illegible]

[D. B. Rushmore, assisted by E. A. Lof.]

SHUNTS. — (*See also Ammeters; Galvanometers; Voltmeters; Wattmeters.*) When heavy currents are to be measured it is impracticable in many cases to pass the entire current through the coils of the instrument proper. In such cases it is necessary therefore to pass the bulk of the current through a parallel circuit or shunt and measure only a known fraction of the total current.

CONSTRUCTION. — For laboratory work standard resistance units (*see Resistors, Standard*) are commonly employed as shunts when a high degree of precision is required. Special forms of resistance boxes, usually called shunt boxes, are also used. For switchboard work shunts made of sheets of manganin or other high-resistance metal of low temperature coefficient (*see Wires, Resistance*) are usually employed. Several sheets of metal are used in order to provide a large radiating surface. These sheets are brazed into heavy copper terminals for clamping to the bus-bars or other heavy conductors; heavy terminals are necessary to insure a uniform distribution of current in the sheets. The contact surfaces must be kept bright and the bolts tight.

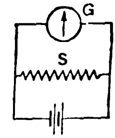

Fig. 1. Galvanometer and Shunt

THEORY OF SHUNTS. — Fig. 1 shows diagrammatically a simple shunt such as used with an ammeter or galvanometer. The galvanometer (or ammeter) current is

$$I_G = \frac{S}{G+S} \cdot I_B,$$

where G and S are the resistances of galvanometer and shunt respectively and I_B is the total current from the battery or other source.

For example, if the galvanometer has a resistance of 9900 ohms and the shunt a resistance of 100 ohms, the galvanometer current will be 1/100 of the battery current. Shunt boxes made on this principle are usually provided with resistance coils giving a ratio of currents of 1/10, 1/100 and 1/1000 when used with a galvanometer having a given resistance. The corresponding readings of the ammeter or galvanometer, multiplied by 10, 100, 1000, or 10,000 will then give the current in the battery circuit.

"Universal" Galvanometer Shunts. — Simple shunts such as above described give even ratios of currents only when used with a galvanometer of a certain definite resistance. A set of coils arranged as shown in Fig. 2, when used as described below, will give even ratios when used with a galvanometer of any resistance. A shunt of this kind is known as a universal shunt. The Ayrton and Mather shunt is arranged on this principle.

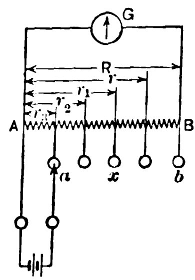

Fig. 2. Diagram of Ayrton and Mather Universal Shunt

A study of the circuits will show that if the galvanometer is calibrated (*see Galvanometers*) with the terminal a at b, that is, with all the resistance R in parallel with the galvanometer, the same calibration may be used for a connected to any other terminal x, provided the current values from the calibration curve are multiplied by the ratio R/r_x, where r_x is the resistance in the box between that terminal and A. The various resistances are usually arranged so that these ratios or "multiplying powers" of 10, 100, 1000, and 10,000 may be obtained. The galvanometer should have a relatively low resistance compared with the total resistance of the shunt.

Effect of Temperature Changes. — The resistivity temperature coefficient of a shunt made of manganin is very much lower than the temperature coefficient of the copper circuit of the ammeter or galvanometer. Consequently changes in temperature will produce a change in the relative resistances of the shunt and ammeter or galvanometer, thus changing the multiplying power. For accurate measurements a correction should therefore be made in the nominal multiplying power of the shunt. With a universal shunt and a relatively low-resistance galvanometer the effect of temperature changes is less marked, since the resistances r_x and R are both in the same box and are usually made of manganin or other metal having a low temperature coefficient.

COSTS. — A 4-coil laboratory shunt box for use with an ordinary galvanometer and giving multiplying ratios of 10, 100, 1000 and 10,000 costs about $20. Switchboard shunts cost from $3 to $90 depending upon their current-carrying capacity, the lower figure applying to a 10-ampere shunt and the higher figure to a 10,000-ampere shunt.

BIBLIOGRAPHY. — Drysdale, C. O., *Phil. Mag.*, Vol. 16, p. 136; Patterson and Rayner, *Proc. Ins. Elec. Eng.*, Vol. 42; Edgcumbe, K., *Industrial Measuring Instruments*, London, 1908; Argmagnot, H., *Instruments et Methods de Mesures Electriques Industrielles*, Paris, 1902.

[H. Pender and H. R. Rankin.]

SIGNALING, RAILWAY. — Railway signaling is the art of conveying information to the person or persons in immediate charge of the movement of a train. The means for conveying the information are various, including simple movements of the hands or arms and more or less complicated combinations of positions of semaphore arms by day, or colored lights by night.

CARDINAL PRINCIPLE OF RAILWAY SIGNALING. — The cardinal principle of design of all signal circuits, apparatus and systems, is that any failure of any part, such as breaks, open or short circuits, grounds, etc., if having any effect on the signal whatever, shall result in a stop indication.

CLASSIFICATION OF SIGNALING SYSTEMS. — The two main divisions of railway signaling are block signaling and interlocking signaling. The former has to do with keeping trains which are running on the same track properly spaced. The latter has to do with the handling of trains over tracks which intersect at points of crossing or divergence, and has for its object the prevention of conflicting movements, the proper routing of trains, and the insurance that the movable parts of the track are in their right positions before the signals governing movement over them can be made to give a proceed indication.

Block signals may be divided into two main classes: non-automatic and automatic. In the United States there are 71,469 miles of track protected by the former as against 36,943 miles of track protected by the latter. Of the non-automatic blocks, those on 70,493 miles of track are operated by telegraph or telephone.*

NON-AUTOMATIC BLOCK SIGNALING. — Systems in this class are of two kinds, non-controlled and controlled.

Non-controlled Manually Operated System. — The manually-operated signals used in connection with the telephone or telegraph blocks are located at passenger stations, junctions or other convenient points where operators are available. They are put in the proceed position to permit a train to enter the next block provided information has been received by the operator from the next station in the direction of traffic that the preceding train has passed out of the block. They are placed in the stop position on the passage of the train.

The operators have blanks on which they enter the designation numbers of each train they admit to a block, together with the time of entrance. On the same sheet is checked off the departure of the train from the block as advised by the operator at the leaving end of the block. The train despatcher, located at some central point, is also kept informed of all train movements, so that in case the schedule is deranged for any cause he can give the necessary orders to expedite traffic, such as changing orders for passing points and giving superior rights to certain trains.

Defects of Non-controlled Manually Operated System. — The defects of the system are obvious, as misunderstandings may arise between operators, trains may be checked off by mistake as having left a block when such is not the case, or trains may part and the fact not be noted by the block operator because of his failing to note the absence of the tail lights carried by the rear car of a train when the first division passes his station.

Controlled Manually Operated System. — These defects are partially overcome by the use of controlled manual block signals. In such systems electric locks are applied to the levers operating the manual signals. The locks are included in circuits running between block stations, and so arranged that when an operator wishes to place a signal in the proceed position he has

* From tables compiled by the Interstate Commerce Commission, January 1, 1914.

first to ask (by bell signal or otherwise) for an unlock from the next station in the direction of traffic, and coöperate with the operator at that station in the proper manipulation of the circuits to get his unlock. A further check on the operators is sometimes obtained by the use of track circuits at the stations to effect a certain degree of control of the apparatus locking the signal levers by the passage of the trains themselves. These systems, however, are all inferior, in the degree of protection given, to the automatic blocks with signals controlled by continuous track circuits, which are described below.

Staff System. — Another form of controlled manual block signal is the staff system in which the possession of a small metal cylinder, or "staff," gives the engineer permission to run through a block. These staffs are normally in one or the other of a pair of instruments called staff instruments, one of a pair being at each end of a block. Only one staff can be taken from a pair of instruments at a time because of their locking features, controlled by circuits between the instruments, requiring the coöperation of two people, one at each instrument, to abstract a staff. Until this staff is replaced in one or the other instrument, no other staff can be withdrawn. Following movements with more than one train in the block can be accomplished by using a divisible staff, made in several sections which must be screwed together in a specified order to permit the insertion of the staff in an instrument when the last train has brought the remainder of the staff through the block.

There is also an automatic staff system in which the coöperation of the second person is not required.

There are about 508 miles of track equipped with the staff system in the United States.*

AUTOMATIC BLOCK SIGNALING. — This subject will be treated under three main headings, in the order stated: Track Circuit, Signals and Their Mechanisms, and Location of Block Signals.

TRACK CIRCUIT FOR AUTOMATIC BLOCK SIGNALS. — The standard means of control of automatic block signals is the track circuit. It is the safest means known because the control is continuous and reliable.

Steam-road Track Circuit (Fig. 1). — The figure shows the elements of a steam-road track circuit. They are a source of electrical energy, means for limiting the flow of current from the source, a section of the rails of a track

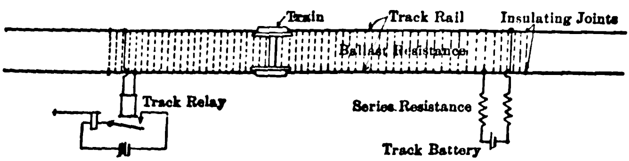

Fig. 1. Elements of a Steam-road Track Circuit

insulated by special rail joints from adjoining rails, the leakage path constituted by the ties and ballast, and an electromagnetic type of relay. The contacts of the relay open and close the circuit which effects the operation of the signal. When there is no train on the track circuit, the contacts of the relay are closed due to the energizing of the relay by current flowing over the rails, which act as conductors from the source of electrical energy. The signal is made to indicate the absence of a train on the track circuit under these conditions. The

* From tables compiled by the Interstate Commerce Commission, January 1, 1913.

presence of a train on the track circuit, or any other cause depriving the relay of energy, causes the relay to open its contacts, which results in the signal taking the stop position. The use of means for limiting the flow of current is to prevent exceeding the capacity of the source when a train shunts the rails, and at the same time to cause such drop of potential across the rails as will insure the opening of the relay.

Electric-road Track Circuit. — The track circuit for a road using electric propulsion, if a direct-current track relay is used, must, in order to avoid false clear signals, have the voltage point at which the relay closes its contacts higher than any voltage which may exist across its terminals, with a train on the track circuit, due to the flow of the propulsion current through the rails. Standard practice, to avoid the possibility of such false closing of the relay contacts, is the use of an alternating-current relay which is not operative to close its contacts under the influence of the propulsion current.

Single-rail Track Circuit (Fig. 2). — Fig. 2 shows an arrangement for d-c. propulsion roads where only one rail is used as the main return for the

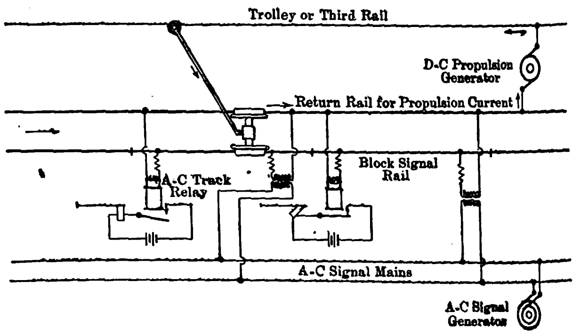

Fig. 2. A-C. Track Circuit, Single-rail System, D-C. Propulsion

propulsion current, the other rail being given up to the signaling current. The resistances used between the rails and the transformer, and between the rails and the relay, limit the flow of propulsion current through those pieces of apparatus. The inductive shunt across the relay serves the same purpose in forming a by-pass for propulsion current. The inductive shunt and the transformer supplying the track circuit have cores with open magnetic circuits to minimize the magnetizing effects of such direct current as may flow through their windings. An arrangement like the above is limited in practical use to those situations where the drop in voltage due to the propulsion current, along the length of continuous rail opposite any section of block rail, is under 50 volts. It finds its greatest application in subway and terminal or interlocking work. Where the number of tracks and shortness of track circuits bring the maximum voltage across the terminals of the relay, due to propulsion current, down to about 5 volts, a relay may be used with resistance enough in its winding to render unnecessary the use of the inductive shunt and resistance between it and the rails.

With a signaling current of higher frequency than the propulsion current and a selective relay, the single-rail track circuit is applicable to a-c. propulsion roads.

Double-rail Track Circuit (Fig. 3). — Where the conductivity of the return propulsion circuit is not sufficient to permit giving up one rail to the signaling current, the arrangement shown in Fig. 3 is used. Both rails are sectioned by insulating joints and the return propulsion current is carried around the insulating joints by means of impedance bonds (i.e., coils having a high inductance and low resistance) which are joined electrically at the middle points of their windings. The flow of the propulsion current is opposed

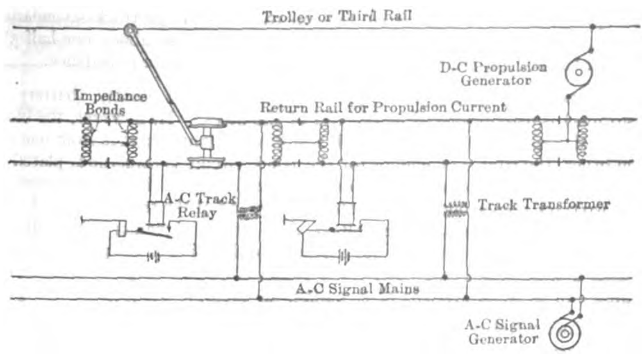

Fig. 3. A-C. Track Circuits, Double-rail System, D-C. Propulsion

only by the ohmic resistance of the bonds and their connections. The full impedance is, however, offered to the flow of the alternating signaling current from rail to rail, and it is, therefore, possible to maintain a difference in potential across the rails sufficient to operate the track relay.

Where the propulsion current is alternating, a higher frequency is used for the track circuit and the relay made correspondingly selective. 25-cycle current is ordinarily used for the signals on direct-current propulsion roads and 60-cycle current on alternating-current propulsion roads.

Source of Energy for Track Circuit. — The source of electrical energy for supplying the track circuit is generally in steam-road practice a gravity battery, in which case no external current-limiting means are necessary. Generally two or three cells are used in multiple, but where the leakage between rails is high, larger numbers are used in series-multiple combinations. Primary batteries of the caustic-potash type are coming into use. The storage battery is used to a limited extent and is especially advantageous on two- or four- track systems where the number of track circuits and amount of apparatus per unit distance is relatively large.

Use of Alternating Current. — Alternating current is rapidly coming into use for signaling on steam roads. Where alternating current is used for track circuits, it is almost universally used to operate the signals and supply the lamps for the night indications. 25- or 60-cycle current is used, the former generally resulting in slightly lower power consumption and being generally favored where the current has to be generated for the signaling system alone and no commercial emergency sources are available. Where such commercial emergency sources *are* available, as is generally the case in thickly-settled districts, the frequency of such sources being ordinarily 60 cycles, that frequency is often chosen for the signaling system.

Transformers for Track Circuits. — The source of alternating current for each block is either a low-voltage (5 to 20 volts) secondary of a transformer

whose primary is connected to the transmission line, or a similar secondary of a small special air-cooled transformer with a 110-volt primary. In the latter case the special transformer may be housed with other signal apparatus and its primary supplied from the 110-volt wires brought from the commercial transformer, whose primary is connected to the transmission line. The same 110-volt wires may be used also to supply the motors, line relays and "hold clear" devices, and carbon lamps used for lighting the signals. If Tungsten lamps are used, they may be supplied from suitable taps on the 110-volt primary of the special transformer or from taps on the low-voltage track secondaries.

Track circuits are generally supplied with current at about one-half the voltage of the source, due to the drop over the intermediate impedance.

Location of Energy Supply.—The source of energy (battery or transformer) is usually placed at the end of the block. If, however, the block is long and impedance bonds are used, resulting in large leakage from one rail to the other, the source of energy for the track circuit is sometimes placed at the center, and the control wires for the signal carried through the contacts of the two track relays, one being located at each end of the track circuit. Under these conditions, because of the voltage drop of the signaling current in the rails, there is only a limited distance extending either side of the point of supply in which a train will shunt both track relays simultaneously. This is taken advantage of in some systems of signaling, an example being given in Fig. 7.

D-C. Track Relay.—The direct-current track relay most commonly used on steam roads has two or four contacts closed when energized, and has a resistance of four ohms. The working voltage across its terminals is generally 0.4 or 0.5 volt under good weather conditions, but may be as low as 0.35 volt during wet weather.

A-C. Track Relay.—There are two types of a-c. track relays called, respectively, "single-energizing circuit type" and "two-energizing circuit type," according to whether they take all of the energy to operate them from the track, or part from the track and part from a local source. These types are built on either one or both of two distinct principles: (1) on the principle of the electrodynamometer (q.v.) and (2) on the principle of the induction motor; (see *Motors, Polyphase Induction* and *Watthour Meters*).

The induction-motor type has the advantage over the electrodynamometer type, in that, its angular motion being unrestricted, a given pressure at the contact points can be produced with a less expenditure of energy. There is a limitation to this, however, due to the friction of the reduction gearing, which must not be so great that it cannot be overcome by the counterweight provided to open the contacts when the relay is deënergized.

With all types of railway signal relays, the addition of "back" contacts to be closed when the relay is deënergized, means the expenditure of more energy to satisfactorily close the "front" contacts, because of the extra counterweighting necessary to develop the necessary pressure on the back contacts.

Single-energizing-circuit Type of Track Relay.—Relays of this type are made on the induction motor principle in order to make them selective between the signal and propulsion currents. The following forms of construction are typical.

(1) Segment of sheet aluminum set in motion by the shifting magnetic field due to the "shaded" pole pieces of a laminated "C"-shaped core which carries the winding. This form is very largely used on short track circuits on d-c. propulsion roads.

(2) Double segment of sheet aluminum acted upon differentially on either side of its axis by the two sets of "shaded" poles of a double "C"-shaped

core. The core is so designed that one or the other set of poles is stronger, depending on the respective amounts of propulsion or signaling current traversing the winding. This form is used on a-c. propulsion roads, being made selective between 25 and 60 cycles, the latter frequency being used for the signaling current.

(3) Centrifuge driven by split-phase induction motor. This form is used on a-c. propulsion roads. It is selective by virtue of the centrifuge being designed not to close the contacts when driven at a speed corresponding to the lower frequency of the propulsion current.

Typical power figures at the working points are as follows:

Form	Frequency	Volt-amperes	Volts*	Power factor	Number of contacts closed when energized
(1)	25	5.0	3.0	0.55	4
....	60	8.5	5.0	0.55	4
(2)	60	20.0	3.1	0.55	2
(3)	60	6.0	2.3	0.70	10

* Track windings of relays.

Two-energizing-circuit Type of Track Relay. — The following forms of construction are typical: (1) Electrodynamometer; used on d-c. propulsion roads and to some extent on steam roads. (2) Induction motor with gearing; the rotor is a cylindrical shell of copper with a stationary laminated core; used on d-c. propulsion roads and very largely on steam roads. (3) Centrifuge driven by induction motor; used on a-c. propulsion roads; see also preceding section.

The local windings of all two-energizing-circuit relays are generally supplied at from 10 to 110 volts The objection to the higher voltage is the small size of wire and consequently greater liability to damage by lightning.

The following typical power figures are for "two-position" relays, i.e., relays with one energized and one deënergized position. Relays which are caused to assume two energized positions by reversals of the current in one of the windings are termed "three-position" relays, and generally require twice as much power in one of the windings as two-position relays.

Form	Frequency	Track			Local		Number of contacts closed when energized
		Volt-amp.	Power factor	Volts	Volt-amp.	Power factor	
(1)	25	0.70	0.90	0.35	36	0.70	4
......	60	1.60	0.80	1.60	33	0.34	4
(2)	25	0.038	0.65	0.15	2.4	0.70	10
......	60	0.024	0.65	0.15	11.0	0.70	10
(3)	60	1.56	0.60	1.3	11.0	1.0	10

The adjustment of the phase relations of the currents in two-energizing circuit relays has to be carefully cared-for in the choice of the impedance used between transformers and rails, and between local windings and the source of energy supply.

Single- Versus Two-circuit Types. — As a track circuit is, from an engineering point of view, nothing but a very inefficient transmission circuit,* track circuits above a certain length can be worked most economically with a two-energizing-circuit type of relay, as the amount of energy transmitted over the rails can be reduced to a certain point by increasing the amount of energy supplied economically to the second winding from a local source. For track circuits below a certain length, however, the energy supplied to the track circuit for the single-energizing-circuit type of relay is less than the total supplied to the track circuit and the local winding of the two-energizing-circuit type.

The above consideration, taken in conjunction with the fact that a relay with one energizing circuit usually is cheaper than one with two energizing circuits, and does not require the installation of a source of energy supply for a local winding, generally points to the choice of the single-energizing-circuit type for track circuits less than 1500 feet in length. On the other hand, as the single energizing circuit type of relay may be falsely energized by current from an adjacent track circuit, due to broken down insulating joints, it does not give the same degree of protection against this failure as the two energizing circuit type, with the source of current supply to adjacent track circuits of opposite instantaneous polarity.

Impedance Bonds. — Impedance bonds are wound with strap copper varying from 57,000 cir. mils for 22,000-volt, 25-cycle propulsion to 220,000 cir. mils for ordinary d-c. 600-volt interurban work, and 800,000 cir. mils for heavy 600-volt d-c. traction work. The corresponding resistances across rails are respectively about 0.014, 0.0014 and 0.00045 ohm. The respective continuous propulsion current per rail ratings are 50, 500 and 1500 amperes. Typical impedance values from rail to rail are respectively 10 ohms at 2 volts and 16 ohms at 9.2 volts, 60 cycles; and 0.3 ohm at 1.5 volts, and 0.18 ohm at 1.5 volts, 25 cycles.

Though the resistance of the connection around an insulating joint formed by an impedance bond may be relatively high as compared with the resistance of the ordinary rail joint bond, the small number of impedance bonds in a system compared to the number of rail joint bonds results in but a very slight percentage increase in the total resistance of the return propulsion circuits.

The impedance bonds for direct-current propulsion roads are provided with an air gap in the magnetic circuit to limit the change in impedance due to unequal amounts of propulsion current flowing in the two halves of the winding. Good practice is to provide for satisfactory operation with a maximum difference equal to 20 per cent of twice the rated capacity per rail, e.g., a 1500 ampere per rail bond should operate satisfactorily with a difference of 600 amperes between the currents in the two rails.

Impedance of Track Circuits. — The table on the following page gives the impedance per 1000 feet of track under various conditions of bonding in common practice and for the values of current commonly used for the energization

* The inefficiency of the track circuit as a transmission line is due to the drop through the current limiting means between the source of energy and the rails (and in the single-rail system between the relay and the rails), the high reactive drop in the rails, the distributed leakage of ballast and ties, and, in the two-rail system, the concentrated leakage through the impedance bonds. The voltage at which the track winding of a relay is designed to operate is selected first, to obtain safe and reliable operation, and second, to obtain as high a degree of efficiency of the track circuit as is permitted by the characteristics of the track circuit. This generally results, with relays taking a comparatively large amount of energy from the track, in higher voltages for track windings of relays used on steam roads than for those used on electric roads, though the total energy put into the track circuit is less in the former than in the latter case.

IMPEDANCE OF BONDED RAILS TO SIGNAL CURRENTS

Ohms per 1000 feet of track

Weight of rail, lb. per yard	Bonding*	27.5-ft. rails				30-ft. rails				33-ft. rails			
		25~		60~		25~		60~		25~		60~	
		z	P.F.	z	P.F.	z	P.F.	z	P.F.	z	P.F.	z	P.F.
100	To capacity	0.10	0.40	0.25	0.40	0.10	0.40	0.25	0.40	0.10	0.40	0.25	0.40
	2 No. 6 copper	0.13	0.72	0.28	0.56	0.13	0.70	0.28	0.56	0.13	0.69	0.27	0.54
	1 No. 8 iron 1 No. 6 copper	0.17	0.83	0.30	0.65	0.16	0.82	0.30	0.63	0.15	0.79	0.29	0.62
	2 No. 6 c.c. — 40%	0.19	0.87	0.32	0.69	0.19	0.86	0.32	0.69	0.17	0.84	0.31	0.68
	2 No. 6 c.c. — 30%	0.25	0.91	0.36	0.75	0.22	0.91	0.35	0.74	0.20	0.88	0.34	0.73
	2 No. 8 iron	0.40	0.97	0.50	0.88	0.36	0.96	0.47	0.87	0.34	0.96	0.44	0.85
90	To capacity	0.10	0.43	0.26	0.43	0.10	0.43	0.26	0.43	0.10	0.43	0.26	0.43
	2 No. 6 copper	0.14	0.73	0.29	0.58	0.13	0.72	0.28	0.58	0.13	0.70	0.27	0.54
	1 No. 8 iron 1 No. 6 copper	0.17	0.83	0.31	0.67	0.16	0.82	0.31	0.64	0.16	0.80	0.29	0.62
	2 No. 6 c.c. — 40%	0.19	0.87	0.33	0.71	0.19	0.87	0.33	0.70	0.17	0.84	0.31	0.68
	2 No. 6 c.c. — 30%	0.23	0.91	0.36	0.76	0.26	0.91	0.36	0.76	0.20	0.89	0.34	0.73
	2 No. 8 iron	0.40	0.97	0.51	0.89	0.37	0.97	0.48	0.88	0.35	0.96	0.45	0.86
85	To capacity	0.10	0.46	0.26	0.46	0.10	0.46	0.26	0.46	0.10	0.46	0.26	0.46
	2 No. 6 copper	0.14	0.74	0.29	0.60	0.13	0.73	0.29	0.59	0.13	0.71	0.28	0.58
	1 No. 8 iron 1 No. 6 copper	0.17	0.84	0.32	0.68	0.17	0.83	0.31	0.67	0.16	0.81	0.30	0.65
	2 No. 6 c.c. — 40%	0.19	0.88	0.33	0.72	0.19	0.87	0.33	0.69	0.18	0.85	0.32	0.70
	2 No. 6 c.c. — 30%	0.23	0.91	0.37	0.77	0.23	0.91	0.36	0.77	0.21	0.89	0.35	0.76
	2 No. 8 iron	0.41	0.97	0.52	0.89	0.37	0.97	0.49	0.88	0.35	0.96	0.46	0.84
80	To capacity	0.11	0.48	0.26	0.48	0.10	0.48	0.26	0.48	0.11	0.48	0.26	0.48
	2 No. 6 copper	0.14	0.75	0.29	0.62	0.14	0.73	0.29	0.60	0.13	0.72	0.29	0.60
	1 No. 8 iron 1 No. 6 copper	0.17	0.84	0.32	0.69	0.17	0.84	0.31	0.68	0.16	0.82	0.31	0.67
	2 No. 6 c.c. — 40%	0.20	0.88	0.34	0.73	0.20	0.88	0.34	0.73	0.18	0.85	0.33	0.71
	2 No. 6 c.c. — 30%	0.23	0.91	0.38	0.78	0.23	0.91	0.37	0.78	0.21	0.89	0.36	0.76
	2 No. 8 iron	0.41	0.97	0.53	0.89	0.37	0.97	0.49	0.88	0.35	0.96	0.47	0.87
70	To capacity	0.11	0.52	0.27	0.52	0.11	0.52	0.27	0.52	0.11	0.52	0.27	0.52
	2 No. 6 copper	0.15	0.77	0.30	0.65	0.14	0.76	0.30	0.65	0.14	0.75	0.30	0.64
	1 No. 8 iron 1 No. 6 copper	0.18	0.86	0.33	0.72	0.17	0.85	0.33	0.71	0.17	0.82	0.32	0.70
	2 No. 6 c.c. — 40%	0.20	0.89	0.36	0.75	0.20	0.89	0.35	0.75	0.18	0.86	0.34	0.74
	2 No. 6 c.c. — 30%	0.24	0.92	0.39	0.80	0.24	0.92	0.38	0.81	0.22	0.90	0.37	0.78
	2 No. 8 iron	0.42	0.97	0.54	0.90	0.38	0.97	0.51	0.89	0.36	0.96	0.48	0.87

* c.c. = copper clad.

RESISTANCE OF BONDS TO SIGNAL CURRENTS

Ohms per 1000 feet of track

Bond wires per joint	27.5-ft. rails	30-ft. rails	33-ft. rails	
2 No. 6 copper	0.057	0.052	0.048	Bond wires 48 in. long. No allowance is made for conductance by the splice bars.
1 No. 6 copper and 1 No. 8 iron	0.098	0.089	0.082	
2 No. 6 copper clad — 40%	0.124	0.112	0.103	
2 No. 6 copper clad — 30%	0.166	0.150	0.138	
2 No. 8 iron	0.348	0.315	0.291	

of the track elements of relays. Where propulsion current is flowing in the rails, the power factor corresponding to the value of the propulsion current should be used to determine the most adverse conditions of drop of potential of signaling current between the point of supply to the track circuit and the relay.

Resistance of Leakage Path. — The resistance of the leakage path between rails in ohms per thousand feet of track varies with the nature of the ballast, the condition of the ties and the weather conditions. Two ohms per thousand feet is a low wet weather value for track with gravel ballast.

In connection with the calculations involving the values of rail impedance given in the table on p. 1267 the following values for resistance of ballast and ties between rails may be used. They are given for ballast clear of the rails.

	Ohms per 1000 feet of track
Wet gravel	3
Dry gravel	6
Wet broken stone	6
Dry broken stone	16

Transmission-Line Voltages. — Transmission-line (between generator and signal transformers) voltages in standard practice vary from 1100 to 4400 volts inclusive, though in some special cases of subway and elevated road work, or where the mains are carried on existing telegraph poles, they may be lower.

SIGNALS AND THEIR MECHANISMS. — Signal indications for block and interlocking work are given in standard railroad practice by semaphores. The semaphore consists of a wood, or enameled steel, blade or arm, fastened at its inner end to a "spectacle." The spectacle is a casting, or combination casting and metal stamping, embodying the hub for the shaft on which the semaphore rotates, and the colored glass roundels which change position before a lamp fitted with a clear lens, to give the night indications corresponding with the positions of the semaphore. The lamp is provided with a long-time oil burner (usually seven days) or incandescent bulbs.

Weight and Torque of Semaphores. — The weight of the semaphore is distributed to give a maximum torque (tending to return the semaphore to the stop position) varying with different types of operating mechanisms from about 30 to 100 pound-feet. As the torque curve is necessarily a sine curve, a spectacle moving through 90° is generally designed to have maximum torque at about 55°.

Spectacles operated manually, which can be pulled to the stop position, may be lighter than those which, in returning to the stop position, have to return parts of the mechanism which cause them to assume the proceed position.

In automatic signals gravity is depended on almost universally to return the signal to stop.

Semaphore Positions and Colors of Lights. — The three fundamental signal indications are:

1. Stop,* 2. Proceed with caution, 3. Proceed. The standard method of giving these indications is in the upper right-hand quadrant as indicated in Fig. 4, the respective positions of the semaphore arm being respectively at 0°, 45°, and 90°. The corresponding colors of the night lights differ with different

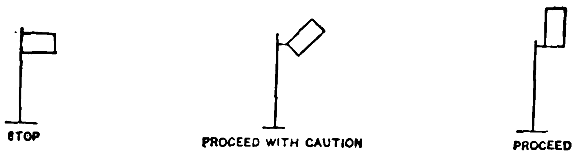

Fig. 4. Standard Semaphore Positions

roads; a common practice is to use red for stop, green for proceed with caution and white for proceed, but on account of the danger of a white light being falsely displayed because of the breakage of the colored roundels it is considered best practice to use red, yellow and green, for the respective indications enumerated above.

It is also good (but less up to date) practice to give these indications in the lower right-hand quadrant, the horizontal position of the semaphore arm always indicating stop. Where the view to the right of the signal mast is obscured, as by the poles along a trolley road, or where clearances are small, due to high walls close to the right-of-way, as through a city, the semaphore is sometimes operated in the upper left-hand quadrant.

Two-position semaphores, indicating stop and proceed only, are also largely used, the proceed indication generally being given at from 60° to 90° depending on the individual road.

Power Mechanisms. — Power signals are operated by "bottom" or "top post" mechanisms. Electricity, compressed air and gas are used as motive power. Of the track mileage protected by automatic semaphores approximately 90 per cent has electrically-operated mechanisms. Compressed gas and air for automatic blocks are limited in use to very few roads, though these include some of the most important ones. The gas is supplied in tanks. The air is distributed in pipe mains.

Electrically-operated Bottom-post Mechanism. — Electric bottom-post mechanisms generally consist of a motor which, by gearing and levers, transmits the movement necessary to clear the semaphore through a vertical rod extending inside the mast between the mechanism and a crank arm on the semaphore shaft. The best types have interposed in the movement transmission system, as near the connection to the vertical rod as possible, an electric latching device called a *slot*. The operation of the semaphore to, and maintenance in, a proceed position depends on this latch being energized, the motor cutting out automatically after performing its work. The location of the *slot* in the movement system close to the vertical rod reduces to a minimum the number of parts moved by the return of the semaphore to the stop position, under the influence of gravity, and, consequently, reduces the possibility of

* An automatic signal in the "stop" position may be passed "according to rule" after bringing train to stop. An interlocking signal in the "stop" position may be passed only after bringing train to stop and receiving authority to pass the stop signal from an authorized person.

the signal sticking in the proceed position due to the development of any friction.

The control of the circuit supplying the motor and slot is effected by the contacts of the track relay.

Electrically-operated Top-post Mechanism. — In top-post mechanisms the motor acts directly on the semaphore shaft through a train of gears. The semaphore in its return to the stop position has to drive the gear train and usually the motor. The ratio of gearing in top-post mechanisms is made comparatively low (about 120 : 1) to diminish friction when going to the stop position, and the motors are consequently very slow speed. The latching of the semaphore in the clear position is accomplished by electromechanical means or by magnetic induction. The latter method obviates the danger of the signal sticking in the proceed position due to the adhesion of contacting surfaces in the latching device.

Power Required for Electrically-operated Signal Mechanisms. — D-c. motors and latching devices are generally supplied with current from local primary or storage batteries at from 8 to 10 volts. With a maximum semaphore torque of between 30 to 50 pound-feet and clearing 90° in about 10 seconds, the motors will consume between 2 and 2½ amperes. The latching device consumes from 0.008 amperes to 0.02 amperes, depending on the voltage and winding resistance.

Under the same conditions as given for d-c. motors, a-c. commutator motors will consume slightly less than 1 ampere at 110 volts at about 0.8 power factor, and induction motors will consume between 2.8 and 3.5 amperes at the same voltage but at about 0.5 power factor.

The electric latching device consumes approximately from 5 to 10 watts in the a-c. mechanisms, the power factor varying widely with the design.

Pneumatically-operated Signal Mechanisms. — The pneumatic mechanism is the simplest of those used for operating signals, consisting of merely a 3-inch by 4-inch cylinder with a metal-packed brass piston, a magnet valve for controlling the admission of air and suitable mechanical connections to the semaphore shaft. The electromagnet valve controlling the admission of air is used as the "slot," as its deënergization shuts off the air and exhausts the cylinder to atmosphere. The pneumatic mechanisms are readily adaptable for bottom- or top-post operation. The air pressure carried varies from 40 to 100 pounds. Ninety pounds is a common pressure.

The advantages of the pneumatic mechanisms are their simplicity, reliability and over-load capacity.

Gas-operated Signal Mechanisms. — The gas-operated mechanisms are similar in principle to those operated by air, but are complicated by the apparatus incidental to the tank supply being at a very much greater pressure than that at which the gas is used in the signal cylinder.

Day-light Signals. — In places where space or view is constricted, powerful light signals are sometimes used to give the day, as well as night, indications in place of the semaphore. Notable examples of this are the New York terminal of the Pennsylvania Railroad, and some of the most modern signal installations on electric interurban roads.

These light signals are generally hooded to make them more visible in sunlight. They are provided with concentrating lenses, and sometimes with specially-constructed reflectors arranged so that light entering from the exterior through the lens will not be reflected out again and give a false indication when the lamp behind the lens is not lighted.

The candle power behind the lens varies from 10 to 120, depending on the

type of lamp and the efficiency of the lenses and reflectors used. The indication given is sufficiently powerful, even in bright sunlight, to permit of the satisfactory operation of high-speed interurban electric cars.

LOCATION OF BLOCK SIGNALS. — Automatic block-signal systems have to take care of such varied fundamental conditions as are found, on the one hand, on single-track roads where the distances between passing sidings may be as great as four miles or more, and the traffic relatively infrequent and, on the other hand, on roads of four tracks, or more, where the traffic is dense, the headway short, and trains of both high- and low-speeds have at times to be handled over parts of the same tracks.

Home and Distant Signals. — The signal at the entrance to a block indicating the presence or absence of a train in that block is called the "home signal." In order that a train may be given warning in time to stop at the entrance to an occupied block, a second signal, called the "distant signal," indicating the position of the home signal at the entrance to the block, is located at a point sufficiently distant to enable the runner to act on the indication and bring his train to a standstill with a reasonable factor of safety before passing the stop signal. This distance will vary with the conditions of grade, curvature of track, and character of train equipment and may be from 3500 to 4200 feet or more.

In three-position signaling the semaphore at the beginning of a block is used to give the home signal for that block and the distant signal for the next block ahead. In two-position signaling the position of the home signal is repeated by a semaphore of distinctive shape, displaying a distinctively-colored light at night. This distant signal may, where the blocks are short, be located on the same mast with the preceding home signal. If the blocks are long and the rules of the road require a train to be immediately brought under control when advance information is received that the home signal is at stop, it is best to avoid loss of time by placing the signal giving the advance information on a separate mast at the proper distance from the home signal. This applies also to three-position signaling.

With distant signals it is, therefore, evident that in order to have following trains run continually under clear signals, they must be separated by a distance equal, at least, to the distance run by the preceding train during the time occupied by the clearing of the home and distant signals plus the length of the intervening block plus the distance between the home and distant signals.

The relations stated above give the basis for the theoretical location of automatic block signals to safely obtain maximum traffic capacity. In practice one

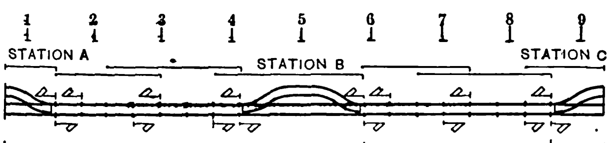

Fig. 5. Arrangement of Single-track Signaling in Both Directions

must consider character of traffic, congested conditions at approaches to terminals, busy passenger stations and junction points, and local conditions such as view and opportunity for suitable foundations, in their effect on signal location.

Arrangement of Signals on Single-track Road. — Fig. 5 shows one arrangement of single-track signaling in both directions used by a large western

system. The horizontal lines terminating at one end opposite signals and at the other end opposite insulating joints (the latter indicated by the short marks across the lines representing the rails) show the extent of track governing each signal by virtue of the track circuit.

Even where the distance between stations (or sidings) is less than a mile, it is important that intermediate home signals be placed between stations (or sidings) in order to prevent a train on the main track at a station from causing possible detention to an incoming train. It will be noticed by reference to the figure that eight signals are used to protect traffic, six between stations, and two within station limits. This number of signals is necessary to adequately protect all traffic and at the same time to insure proper flexibility, since, with this arrangement, trains can move up to the station limits under full protection from both directions, even though the main line may be occupied within these limits. This is because a train in a station affects only the entrance signals, and consequently does not affect approaching traffic on either side of the station, neither does a train on one side of a station influence traffic on the other side.

Fig. 6 shows the circuits for the system shown in Fig. 5.

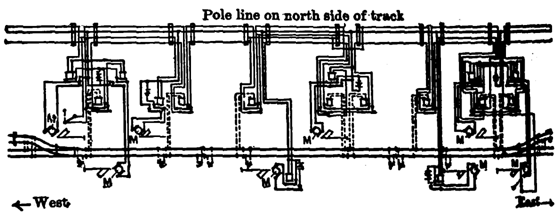

Fig. 6. Circuits for System Shown in Fig. 5

Special Arrangement for Single-track Interurban Road. — Fig. 7 shows a system of signaling which is particularly applicable to interurban single-track signaling where special conditions sometimes render it extremely desirable to

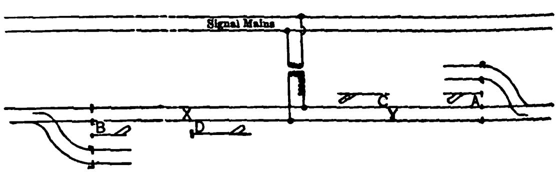

Fig. 7. System for Single-track Interurban

permit more than one car, moving in the same direction, between sidings, and yet give both head-on and rear-end protection.

Because of the impedance of the rails and the characteristics of the voltage supply to the track circuit, track relay at signal "A" is deënergized only by a train between "A" and "X." Similarly, track relay at signal "B" is deënergized only by a train between "B" and "Y." Signals "A" and "B" are connected so that both will go to "stop" on the entrance of a car at either end of the block, but for following cars the signal at the entering end will clear up after

any preceding car has gone half the distance between sidings plus half the distance $X-Y$. $X-Y$ is generally about 3000 feet where distance $A-B$ is about 2 miles. If, by any misunderstanding, cars going in opposite directions should enter a stretch of track between sidings, they would be stopped by signals "C" and "D," located sufficiently far apart to allow for a proper factor of safety in stopping.

Trolley Wheel Operated Signals. — Block signals controlled from a contactor operated by the trolley wheel do not meet the requirements of high-speed signaling as to reliability, and are of value principally in preventing the loss of time due to two cars moving in opposite directions entering the same stretch of track, where conditions of vision and speed are such that there is small chance of harm resulting from a collision.

Arrangement of Signals on Double-track Road. — Fig. 8 shows a typical a-c. automatic block-signaling system with circuits for a two-track road. It will be noted that the *control* of the third position of the signals is obtained by the use of *line wires* running between the signals and supplied with current by circuit controllers operated by the signals.

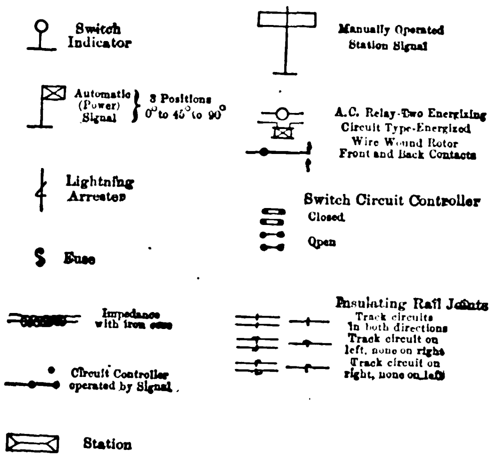

Fig. 8a. Key to Fig. 8

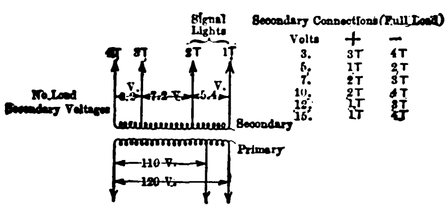

Fig. 8b. Details of Transformer Shown in Fig. 8

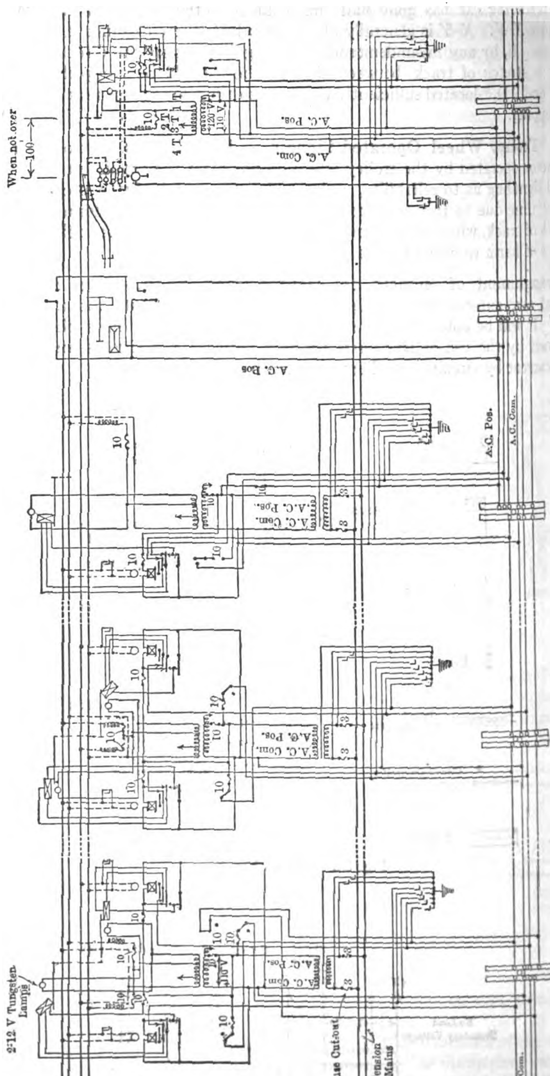

Fig. 8. A-C. Automatic Block System for Double-track Steam Road

The use of two-energizing-circuit track relays as *polarized* relays permits *dispensing* with *control line wires*, in obtaining the control of the third position of three-position signals. The polarized relays close contacts in two positions according to direction of current in track winding and open them when deënergized, as by the presence of a train on the track circuit. The polarity of the current supplied to the track circuit in the rear of any signal depends on the position of that signal and the circuit controllers attached to it. This method is known as *polarized wireless control* and has its counterpart in direct-current signaling.

Maximum Capacity Arrangement. — Fig. 9 shows the latest development of signaling to give maximum capacity where high- and low-speed trains are run on close headway. The main idea is to make possible proper *speed control*. It will be noted that the high-speed trains receive sufficient warning to stop at the proper point in spite of the spacing of signals being made close to get maximum capacity with low-speed trains, which may run for short distances on the high-speed tracks.

Beginning with the first signal behind the occupied block at the left, the indications are respectively "Stop, then proceed according to rule," "Proceed, prepared to stop at next signal," "Proceed, prepared to pass next signal at medium speed," "Proceed."

INTERLOCKINGS AT CROSSINGS, JUNCTIONS AND TERMINALS. — All interlockings are designed with the view of insuring that signals governing traffic over any movable track must be in the stop position before the track parts can be shifted, that the track parts cannot be moved under a train, that they must be locked in the proper position before the signals governing over them can be placed in the proceed position, and that conflicting signals cannot be given.

The apparatus at an "interlocking" consists of the signals and movable

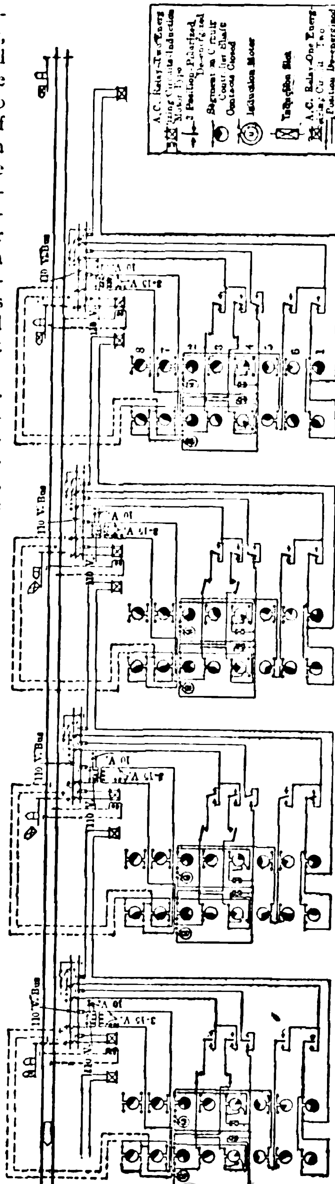

Fig. 9. Circuits of Arrangement to give Maximum Track Capacity

track parts with their operating mechanisms and intermediate apparatus, the central-controlling interlocking machine (sometimes called an "interlocker") located in a signal tower or station, and auxiliary apparatus. Three types* of interlockings are used: (1) mechanical interlocking, in which movement is transmitted to the signals and track parts by means of wire and pipe connections to the levers in the interlocking machine; (2) electric interlocking in which the signals and track parts are moved by electric motors controlled through the interlocking machine; and (3), electro-pneumatic interlocking in which the signals and track parts are moved by compressed air controlled electrically through the interlocking machine. The last two types are called power interlockings as contrasted with the mechanical interlockings.

Almost all large terminals in the United States use electro-pneumatic interlockings. It is most rapid in operation and its simplicity of construction and maintenance, its overload capacity and low voltage give great reliability. In large and busy installations these points are generally considered to overbalance the excess of cost of compressing the air over that of charging the batteries in electric interlockings.

Signal Apparatus.—Interlocking signals differ from automatic signals in external appearance only in such details as are necessary to enable the runner to differentiate them. The differences are generally confined to shapes and markings of arms, locations on masts, etc., and call attention to modifications in the significance of some of the aspects of interlocking signals as distinguished from automatics, e.g., see section above on *Semaphore Positions*. In mechanical interlockings signals are operated by wire or pipe connections to the mechanical levers in the interlocking machine. Sometimes a low voltage electric operating mechanism is used, though the switches remain mechanically operated. In such cases the signals are controlled through contacts operated directly from the mechanical lever or from contacts on a small auxiliary lever properly interlocked with the other levers of the machine.

In power interlockings the signal-operating mechanisms are essentially the same as those used for automatic block signals and are described above in the section on *Power Mechanisms*. Their control and operating circuits replace the mechanically operated pipe and wire connections of the mechanical interlocking signals.

Track Apparatus.—The track apparatus comprises switches, movable point frogs, detector bars, derails, bolt locks, drawbridge locks, etc., with their operating mechanisms.

The detector bar is a piece of steel 3/8 inch or 1/2 inch thick, 2 1/4 inches wide and varying in length up to 55 feet. It is supported beside the head of the rail in the vicinity of a movable part of the track, so that it is capable of vertical motion above the head of the rail when no car is present over it.

On account of the fact that with power interlocking and a wide head of rail a mechanical detector bar is likely to be forced up outside the wheels of a car over a switch, mechanical bars are being supplanted by short track circuits, called detector track circuits, which are utilized to lock the switch-control lever in the interlocking machine against being thrown under a train while a train is over a movable track part.

In all types of interlocking the movable part of the track is locked in position by connecting to it a horizontal bar, called the lock rod, which moves with the track part at right angles to the track. The lock rod is provided with holes

* Combinations of (1) and (2) are rapidly coming into use, and are of great advantage when the tower space is restricted and a change of track layout calls for an increase of capacity over that of the existing "mechanical" machine. The combination ...alled "Electro-mechanical."

or notches into which a bar, moving in guides parallel to the track, may be moved when the switch, or other moving part, is in its correct position. This latter bar, which engages the lock rod, receives its motion from the connection which operates the detector bar. The sequence of movements at the switch is: (1) the lifting of the detector bars; (2) the withdrawal of the bar engaging the lock rod (the latter operation not being capable of completion unless it has been possible to raise the detector bar to the full extent of its travel); (3) the movement of the switch, or other track part; (4) the lowering of the detector bar; (5) the entrance of the locking bar into the hole or notch in the lock rod, provided the moving part has made its full travel and is in its correct position.

Intermediate Apparatus. — The movements enumerated in the above paragraph are generally effected in mechanical interlockings by pipe connections from track parts and their locking devices to corresponding levers in the mechanical interlocking machine. Bell cranks are used in the pipe connections to change the direction of motion. Sometimes all the movements enumerated are effected in proper order by a device located beside the track, called a "switch and lock movement," which may be operated by one pipe connection and machine lever. In power interlockings switch and lock movements are almost always used, power apparatus at the track and electric circuits replacing the mechanically-operated pipe connections.

Power Apparatus for Electric Interlockings. — The power supply for electric interlockings is derived from storage batteries, used in duplicate sets to provide for charging, or in single sets charged while in service or floated on the charging circuit. The nominal voltage is 110, but exceeds this figure if the batteries are charged in operation, or floated, or if, as is sometimes the case, more than 55 cells are used. A 120-ampere-hour cell is a common size for isolated plants. At such plants the charging is generally done two or three times a week by a gas-driven generator. A switch motor operating a single switch without mechanical detector bars in two seconds takes an average current of about 3½ amperes. With about 200 feet of detector bar the corresponding current is about 4½ amperes. A motor signal operating in the same time takes about 2½ amperes to clear and one-tenth ampere to hold. Solenoid dwarf signals, which are quite commonly used, operate almost instantaneously and have a resistance of about 12 ohms and require 0.3 ampere to hold, resistance being cut in when the signal clears.

Power Apparatus for Electro-pneumatic Interlocking. — Each signal cylinder in electro-pneumatic interlocking is provided with an electromagnetic pin valve fastened directly to the cylinder. The piston is metal-packed, single-acting and self-sealing at the end of its stroke under pressure. The cylinder bore is 3 inches and the stroke 4 inches except where two cylinders act jointly to obtain three positions of the semaphore. Admission of air is obtained by the energization of the magnet valve, and exhaust by the deënergization, the piston returning under the influence of the counterweight of the semaphore.

The switch cylinders are double acting, and vary from four inches to seven and one-half inches in diameter, depending on the character of the load, and have a stroke varying from a length equal to the throw of the switch up to 10 inches. The smaller cylinders are used to act directly on the switch points without bars, the larger ones to operate (through the medium of a switch and lock movement) the heavier movable-point frogs in connection with one end of a double slip with their complement of bars. The stroke of the larger cylinders is generally 8 inches. Air is admitted to the cylinder by a slide valve. The latter is shifted by two small single-acting pistons, the admission of air to which is controlled by electromagnetic pin valves similar to those used on the signal cylin-

ders. The slide valve is locked in its extreme positions by a pin actuated by a small piston controlled by a third electromagnet valve whose energization must precede the operation of either of the pistons which shift the slide valve.

The control of the various magnet valves is effected by contacts operated by the interlocking machine levers. Storage battery to give about fourteen volts is generally used and the individual currents may vary from 1/40 to 1/80 of an ampere. From 75 to 100 pounds air pressure is generally carried.

Interlocking Machine for Mechanical Interlocking. — The interlocking machine comprises the levers, or controller handles, corresponding to the apparatus governed, and the mechanical, or mechanical and electrical, interlocking devices between the levers themselves, and between the levers and the apparatus governed by them.

The mechanical locking between levers is effected in the "locking bed." This is an iron plate, with two sets of parallel grooves. One set of grooves contains cold-rolled steel bars of rectangular cross section connected to and moved by the levers. The other set of grooves, at right angles to the first, contains shorter bars with ends shaped to engage with projections or depressions on the bars connected to the levers. The cross bars engage with the lever bars in such a manner that a lever cannot be moved unless the lever bar affecting the other end of the cross bar is in a definite position.

In a mechanical interlocking the operation of the detector bar and locking bar operating in conjunction with a movable track part, is generally effected by a separate lever in the interlocking machine, called the lock lever. This lever is interlocked with the proper signal levers so that the latter must have been operated to put the corresponding signals to stop before the lock lever can be operated to raise the detector bar and unlock the switch. The switch lever is, in turn, interlocked with the lock lever so that if the lock lever cannot make its full stroke, due, for example, to a train being over the detector bars, the switch lever cannot be thrown. If, after the switch lever is thrown, the lock lever cannot be thrown fully back to its original position, due, for example, to the switch not having made full travel, and the locking bar not being able to properly engage the lock rod, the interlocking between the lock and signal levers prevents the latter from being operated to put the signals in the proceed position. In mechanical interlocking plants rods connected to the signals are also often made to engage with the lock rod of the switch.

Further protection is often obtained in mechanical machines by attaching to moving parts connected to the levers circuit controllers and electromagnetic locks. The electric locks act by gravity when deënergized to engage with a moving part connected to the lever. The circuit which energizes them may be carried through circuit controllers attached to the signal or track mechanisms, or through the circuit controllers on other levers.

Interlocking Machine for Power Interlockings. — In power interlockings the levers are small as they have to operate only the mechanical locking in the machine, and contacts for controlling the various circuits. The strain which can be brought on the mechanical locking is thereby reduced, and it is made correspondingly smaller than in mechanical interlocking machines. In these types of machines where the levers give a rotary motion to the circuit controllers it is possible, by the use of the two extreme positions of the levers, to control signals governing over the same route in opposing directions by the same lever. Only circuits for signals in one direction are closed in an extreme position and the indication referred to below being received before the levers can be returned to the middle position, where it releases the mechanical locking, insures against signals being placed in the proceed position if opposing signals controlled by the same lever have not returned to stop.

Depending on the type a power interlocking machine may occupy less than one-quarter of the space taken by a mechanical machine, and require but one-quarter of the operators. Space is also saved on the ground, as the pipe connections and foundations are replaced by wires.

In power interlockings the completion of the stroke of a movable-track part and its proper locking is "indicated" on the machine by the energization of an electromagnetic lock which allows the lever to complete its full stroke, thereby releasing the proper mechanical locking in the machine. The circuit for the lock is carried through contacts on the moving mechanism on the ground. The return of a signal to the stop position is indicated in a similar manner.

Control of Current for Electric Interlockings. — The supply of current to the signals, switches, etc., may be effected directly by the controller in the interlocking machine, or by simple or polarized secondary controllers located at the various signals and switches, the windings of the secondary controllers being in turn supplied in whole or in part with current from the machine's lever contacts. The use of the secondary controllers permits the wires between them and the interlocking machine to be small, the power for operating the signals, switches, etc., being supplied through contacts operated by the secondary controller from mains running through the plant.

On account of the small amount of current used to operate the secondary controllers, compared to the amount used by the apparatus they supply, the contacts in the interlocking machine may be as small and compact as in an electro-pneumatic interlocking machine, and therefore offer the same excellent opportunity for intercontrol of circuits in the machine itself. This is particularly important, for instance, where it is desired to effect *route locking* (see below), where it may be necessary to have certain signals and switches enter into a great variety of combinations.

Indication Current for Power Interlockings. — Among the various methods of providing the current for "indication" may be mentioned: (1) the use of the same current as is used for control and operation, called "battery indication;" (2) the use of polarized apparatus in connection with the preceding, or in connection with current of a different character from the operating current, to reduce the chances of false indications due to crosses, grounds, etc.; (3) utilization of the momentum of the operating electric motor to generate the indication current after the motor has completed the movement of the apparatus; (4) the operation of the electric motor as a motor generator after it has completed the movement of the apparatus, the generated current being distinctive in character, viz., alternating or of greater voltage than the operating current and of such direction as to selectively operate polarized magnets.

Auxiliary Apparatus. — Various relays, indicators, annunciaters, emergency time lever releases and special circuits are used in connection with or in addition to the essential interlocking apparatus to expedite the handling of traffic and to meet special conditions.

Approach Locking. — Where electric locking is effective while a train is approaching a signal which has been set for it to proceed, to prevent manipulation of levers or devices which would endanger that train, it is termed *approach locking.*

Route Locking. — When electric locking is so arranged that it takes effect when a train passes a signal to prevent manipulation of levers which would endanger the train while it is within the limits of the route entered, it is termed *route locking.*

Sectional Route Locking. — To minimize the limitation of trackage capacity which results from route locking, especially when the route lies across the through tracks, *sectional route locking* is used. This is route locking so

arranged that a train in clearing each section of a route releases the locking affecting that section. The detector track circuits are used to accomplish this type of locking. When *route locking* is not used it is necessary, in order to safely obtain full trackage capacity, to space the signals so close that no considerable amount of track is locked by the clearing of any one signal. Where the track layout and traffic conditions are such that short switching moves predominate over through routing this latter method is superior in giving greater capacity and flexibility.

AUTOMATIC STOPS. — Automatic stops have been in use for a number of years in connection with elevated and subway roads. They have been operated by direct mechanical connection to the signals or by separate mechanisms working in conjunction with the signals, and have consisted of arms located on the roadbed so that when elevated they were in position to engage with and operate a connection to the brake system on the car. It is evident that as a train may travel a considerable distance after the brakes are applied, and as the engineman or runner is dependent on the signals for the knowledge necessary to the proper control of his train, the relation of signal and stop locations must be such that in case the signals are properly observed the brakes will not be automatically applied, while if the contrary is the case, the space intervening between the train causing the display of the danger signal and the automatic stop protecting it must be sufficient to allow a following train being brought to rest without a collision.

This may be accomplished in two ways: (1) by using an overlap (i.e., extending the control of a signal to a point sufficiently beyond the next succeeding signal), which increases the spacing of trains and diminishes the capacity of the road; (2) by giving the runner an advance indication of the position of the signal at which it may be necessary to make a stop, so that if necessary he may bring his train under control in time to stop at the proper point. In the latter case the automatic stop is located to bring the train to rest at the stop signal, in case the speed has not been properly reduced. Up to the present (1913) automatic stop installations have been almost universally on the overlap system.

On account of clearances and atmospheric conditions, the automatic stop problem for steam roads is very difficult of solution. The problem is further complicated by the fact that there are involved the different conditions affecting freight and passenger traffic, a differentiation between the absolute stop indication of an interlocking signal and the permissive stop indication of an automatic block signal, and a differentiation between the speeds permissable over different routes, as indicated by an interlocking signal, with two or more arms, located at a point of divergence.

The above considerations indicate that the satisfactory solution of the automatic-stop problem for the majority of roads must follow the trend of modern signaling, and be based on speed control.

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[L. F. Howard.]

SKIN EFFECT. — (*See also Electricity and Magnetism, Principles of; Resistance and Conductance, Electric.*) A conductor of finite cross-section may be looked upon as made up of separate filaments, just as a beam may be looked upon as made up of separate fibers. The inductances of the various filaments which make up such a conductor are different, due to the fact that the exterior filaments are linked by fewer flux lines than the interior filaments; see *Electricity and Magnetism, Principles of,* and *Inductance and Inductive Reactance.* Consequently, when the same potential gradient (varying with time, however) is established through all the filaments of such a conductor, by connecting it to some external source of alternating or varying e.m.f., the self-induced back e.m.f. in the interior filaments will be greater than in those filaments nearer the surface, and therefore the resistance drop in the interior filaments must be less than in the surface filaments. This can be brought about only by the current distributing itself over the cross-section of the conductor in such a manner that the current density in the interior of the wire will be less than at the surface, i.e., the current is forced toward the surface filaments or "skin" of the wire; hence the term "skin-effect" for this phenomenon.

Factors upon which the Skin Effect Depends. — The self-induced e.m.f. depends not only upon the amount of flux set up but also upon the rapidity of its variation; hence the skin effect becomes more pronounced the greater the frequency of the impressed e.m.f. It is also greater the larger the cross section of the conductor, the greater the conductivity of the conductor and the greater its magnetic permeability. It also depends slightly upon temperature since the conductivity changes with temperature.

Change in Resistance and Inductance due to Skin Effect. — As a consequence of the skin-effect the effective resistance of a conductor to alternating currents is greater than to direct currents, but the *internal* inductance *decreases* with the frequency; the external inductance is not altered; see *Inductance and Inductive Reactance.* Whereas, however, the internal inductance with increasing frequency approaches a limiting value, the resistance increases indefinitely as the frequency approaches an infinite value. The change of resistance is always relatively much larger than the change in the total inductance.

The effects just described are, for the most part, negligible at low frequencies, except in the case of heavy conductors and in coils wound with stout wire in several layers. In the latter case, however, the diminution of the inductance, due to the irregular distribution of the current, is masked, to a greater or less degree, by the effect of the capacity between the windings of the coil, which gives rise to an *increase* of the inductance with the frequency. For the same reason the resistance is increased more than it would be by the eddy currents alone.

Unfortunately, the rigorous or approximate solution of the problem at high frequencies for the various cases which arise in practice is in many instances very difficult, if not impossible.

SKIN EFFECT IN STRAIGHT ROUND WIRES. — An accurate solution for the case of straight *solid** wires of circular cross-section has been given in a number of different forms by various scientists. A summary of the formulas is given in a paper by Rosa and Grover, *Bull. Bur. Stds., 1912, Vol. 8, p. 172.* The calculations are most conveniently made by the use of the

* Tests at the Massachusetts Institute of Technology in 1913 indicate that the formulas for solid wires also apply to round *stranded* wires of the *same cross section of metal,* not the same over-all diameter.

tables given in Rosa and Grover's paper, which are given in a condensed form below. Let

f = frequency in cycles per second,
μ = permeability of the wire, *assumed constant,*
R = direct-current resistance, in ohms, of 1000 feet of the wire; see tables in the article on *Wires and Cables, Bare.*
L = direct-current inductance, in millihenries per 1000 feet, of a *non-magnetic* wire of the same *cross section* as that of the given wire, and at the given spacing between wires, taken from the tables in the article on *Inductance and Inductive Reactance.*

Calculate the quantity

$$x = 0.02768\sqrt{\frac{\mu f}{R}}, \tag{1}$$

and take from the following table the corresponding values of K_1 and K_2. Then the alternating-current resistance at the frequency f is

$$R' = K_1 R \qquad \text{ohms per 1000 ft.} \tag{2}$$

and the alternating-current inductance of the given wire at the frequency f is

$$L' = L + 0.01524\,(\mu K_2 - 1) \qquad \text{millihenries per 1000 ft.} \tag{3}$$

For x greater than 7 the following relations hold to within less than 1 per cent, the error being less the greater the value of x:

$$R' = \left(\frac{x}{2.828} + 0.25\right)R, \tag{4}$$

$$L' = L + 0.01524\left(\frac{2.828\,\mu}{x} - 1\right). \tag{5}$$

Example.—Take the case of a 1,000,000-circular-mil copper cable (assumed equivalent to a solid wire of the same cross section; see footnote on preceding page), frequency 60 cycles, and return wire 10 ft. away. Then at 25° C., R = 0.0108 ohm per 1000 ft. of wire, L = 0.3494 millihenry per 1000 ft., for direct current. The value of x is

$$x = 0.02768\sqrt{\frac{1 \times 60}{0.0108}} = 2.06.$$

From the table below K_1 = 1.088 and K_2 = 0.957, and the alternating-current resistance at 25° C. is therefore $R' = 1.088 \times 0.0108 = 0.0118$ ohm per 1000 ft. and the alternating-current inductance is $L' = 0.3494 + 0.01524\,(1 \times 0.957 - 1) = 0.3487$ millihenry per 1000 ft.

SKIN-EFFECT FACTORS FOR SOLID ROUND WIRES

x	K_1	K_2	x	K_1	K_2	x	K_1	K_2
0.0	1.00000	1.00000	3.8	1.60314	0.71729	10.5	3.97477	0.26832
0.1	1.00000	1.00000	3.9	1.64051	0.70165	11.0	4.15100	0.25622
0.2	1.00001	1.00000	4.0	1.67787	0.68632	11.5	4.32727	0.24516
0.3	1.00004	0.99998	4.1	1.71516	0.67135	12.0	4.50358	0.23501
0.4	1.00013	0.99993	4.2	1.75233	0.65677	12.5	4.67993	0.22567
0.5	1.00032	0.99984	4.3	1.78933	0.64262	13.0	4.85631	0.21703
0.6	1.00067	0.99966	4.4	1.82614	0.62890	13.5	5.03272	0.20903
0.7	1.00124	0.99937	4.5	1.86275	0.61563	14.0	5.20915	0.20160
0.8	1.00212	0.99894	4.6	1.89914	0.60281	14.5	5.38560	0.19468
0.9	1.00340	0.99830	4.7	1.93533	0.59044	15.0	5.56208	0.18822
1.0	1.00519	0.99741	4.8	1.97131	0.57852	16.0	5.91509	0.17649
1.1	1.00758	0.99621	4.9	2.00710	0.56703	17.0	6.26817	0.16614
1.2	1.01071	0.99465	5.0	2.04272	0.55597	18.0	6.62129	0.15694
1.3	1.01470	0.99266	5.2	2.11353	0.53506	19.0	6.97446	0.14870
1.4	1.01969	0.99017	5.4	2.18389	0.51566	20.0	7.32767	0.14128
1.5	1.02582	0.98711	5.6	2.25393	0.49764	21.0	7.68091	0.13456
1.6	1.03323	0.98342	5.8	2.32380	0.48086	22.0	8.03418	0.12846
1.7	1.04205	0.97904	6.0	2.39359	0.46521	23.0	8.38748	0.12288
1.8	1.05240	0.97390	6.2	2.46338	0.45056	24.0	8.74079	0.11777
1.9	1.06440	0.96795	6.4	2.53321	0.43682	25.0	9.09412	0.11307
2.0	1.07816	0.96113	6.6	2.60313	0.42389	26.0	9.44748	0.10872
2.1	1.09375	0.95343	6.8	2.67312	0.41171	28.0	10.15422	0.10096
2.2	1.11126	0.94482	7.0	2.74319	0.40021	30.0	10.86101	0.09424
2.3	1.13069	0.93527	7.2	2.81334	0.38933	32.0	11.56785	0.08835
2.4	1.15207	0.92482	7.4	2.88355	0.37902	34.0	12.27471	0.08316
2.5	1.17538	0.91347	7.6	2.95380	0.36923	36.0	12.98160	0.07854
2.6	1.20056	0.90126	7.8	3.02411	0.35992	38.0	13.68852	0.07441
2.7	1.22753	0.88825	8.0	3.09445	0.35107	40.0	14.39545	0.07069
2.8	1.25620	0.87451	8.2	3.16480	0.34263	42.0	15.10240	0.06733
2.9	1.28644	0.86012	8.4	3.23518	0.33460	44.0	15.80936	0.06427
3.0	1.31809	0.84517	8.6	3.30557	0.32692	46.0	16.51634	0.06148
3.1	1.35102	0.82975	8.8	3.37597	0.31958	48.0	17.22333	0.05892
3.2	1.38504	0.81397	9.0	3.44638	0.31257	50.0	17.93032	0.05656
3.3	1.41999	0.79794	9.2	3.51680	0.30585	60.0	21.46541	0.04713
3.4	1.45570	0.78175	9.4	3.58723	0.29941	70.0	25.00063	0.04040
3.5	1.49202	0.76550	9.6	3.65766	0.29324	80.0	28.53593	0.03535
3.6	1.52879	0.74929	9.8	3.72812	0.28731	90.0	32.07127	0.03142
3.7	1.56587	0.73320	10.0	3.79857	0.28162	100.0	35.60666	0.02828

SKIN EFFECT IN THIN STRIPS AND TUBES. — The following formulas are exact for a flat strip of infinite width and at an infinite distance from any other conductor carrying a current; they also apply with a close degree of approximation to a strip which has a width of 10 or more times its thickness or to a tube which has a circumference 10 or more times its thickness, provided no other conductor carrying a current is closer than a distance of 10 times the thickness of the strip or tube. In the case of several strips in parallel and close to one another, the ratio of the a-c. to the d-c. resistance (and the same

also applies to the a-c. and d-c. inductance) is something between the ratio which would hold for each strip separately and the ratio which would hold for a single strip having a thickness equal to the total thickness of all the strips. The closer the strips are together the nearer is the true ratio to that which would hold for a strip having the total thickness of all the strips.

Let t = thickness of strip, or *twice* the thickness of the wall of a tube, in centimeters,
w = width of strip or *half* the mean circumference of tube, in centimeters,
l = length of strip or tube in centimeters,
ρ = specific resistance of conductor, in microhms per centimeter cube, at the given temperature,
μ = magnetic permeability of conductor in absolute units,
f = frequency in cycles per second,

$$x = 0.1987\, t \sqrt{\frac{\mu f}{\rho}}.$$

Then d-c. resistance is

$$R = \frac{10^{-6}\rho l}{wt} \text{ ohms.}$$

The d-c. internal * inductance is

$$L_i = 1.047 \times 10^{-6}\, l \frac{\mu t}{w} \text{ millihenries.}$$

The ratio of the a-c. to the d-c. resistance is

$$\frac{R'}{R} = \frac{x}{2}\left(\frac{\sinh x + \sin (57.3\, x)^0}{\cosh x - \cos (57.3\, x)^0}\right)$$

and the ratio of the a-c. to the d-c. internal inductance is

$$\frac{L_i'}{L_i} = \frac{3}{x}\left(\frac{\sinh x - \sin (57.3\, x)^0}{\cosh x - \cos (57.3\, x)^0}\right)$$

For x less than unity these ratios are unity to within 0.6% and 0.2% respectively, i.e., the a-c. resistance is practically equal to the d-c. resistance and the a-c. internal inductance is practically equal to the d-c. internal inductance. For x greater than 6 the following formulas are accurate to within 0.5%:

$$\frac{R'}{R} = \frac{x}{2} \quad \text{and} \quad \frac{L_i'}{L_i} = \frac{3}{x}.$$

To a very rough degree of approximation the skin effect in a conductor of any shaped cross section may be approximated by using the above formulas for a strip, taking for the effective width w one-half the perimeter of the section (in centimeters) and for its effective thickness twice the area of the section (in square centimeters) divided by its perimeter. In general, then, for the same area the skin effect will be less the greater the perimeter of the section. If the section approaches more nearly that of a solid circle than that of an elongated rectangle, the formulas for a solid round wire will give more accurate results.

SKIN EFFECT IN IRON AND STEEL CONDUCTORS.—The skin effect in conductors having a variable permeability is not susceptible of mathematical calculation. For tests on iron and steel conductors of square section, rails, etc., see report of the International Railway Test Commission, St. Louis,

* The external inductance, which is uninfluenced by the skin effect, depends upon the distance away of the return conductor; see *Inductance and Inductive Reactance.*

1903; as the permeability of iron and steel ranges between such wide limits, the data given in this report should be used with caution. See also the article in this book on *Trolley Systems, Overhead; Signalling, Railway.*

Skin Effect in Copper-clad Steel Wires. — The data given below are from tests made in the Electrical Engineering Research Laboratory of the Massachusetts Institute of Technology, on samples furnished by the Duplex Metals Co.

Size of wire, B. & S. gage, or diameter in inches	**Solid** 12	**Solid** 8	**Solid** 4	**Stranded** 3/16″	**Stranded** 1/4″	**Stranded** 1/2″
Per cent conductivity, manufacturer's rating..........	30	30	40	30	30	30
Total metal cross section in circular mils...............	6530	16,510	41,740	28,750	45,710	183,750
Ratio of A-C. to D-C. resistance at 20° C.:						
At 25 cycles per sec......	1.00	1.00	1.00	1.00	1.00	1.01
At 60 cycles per sec......	1.00	1.00	1.00	1.00	1.00	1.06
At 500 cycles per sec......	1.02	1.07	1.11	1.14	1.15	1.59
At 1000 cycles per sec......	1.05	1.14	1.16	1.31	1.31	2.05
At 5000 cycles per sec......	1.24	1.29	1.27	2.01	2.22	3.16
Increase in inductance,† ΔL, millihenries per 1000 ft.:						
At 25 cycles per sec......	0.18*	0.100	0.031	0.119	0.089	0.07*
At 60 cycles per sec......	0.18*	0.099	0.030	0.117	0.088	0.07*
At 500 cycles per sec......	0.15*	0.068	0.007	0.086	0.059	0.023
At 1000 cycles per sec......	0.085	0.038	−0.002	0.057	0.038	0.007
At 5000 cycles per sec......	0.011	−0.002	−0.009	0.010	0.003	−0.014

* Approximate.

† Calling L the inductance of a solid round, non-magnetic wire of the same cross section and on the given spacing, taken from the tables in the article on *Inductance and Inductive Reactance*, the alternating-current inductance of the copper-clad wire on this spacing, is

$$L' = L + \Delta L \qquad \text{millihenries per 1000 ft.}$$

BIBLIOGRAPHY. — A complete bibliography on the formulas for skin effect is given in the paper by Rosa and Grover, *Bull. Bur. Stand.*, 1912, Vol. 8, p. 172; see also Steinmetz, C. P., *Transient Electrical Phenomena and Oscillations*, N. Y., 1909.

[H. Pender.]

SMOKE PREVENTION. — The direct cause of smoke from boiler furnaces is that the gases distilled from the coal are not completely burned in the furnace before coming in contact with the surface of the boiler, which chills them below the temperature of ignition.

Smoke may be prevented from forming if each particle of gas, as it is made by distillation from coal, is immediately mixed thoroughly with hot air. Even if smoke is formed by the absence of conditions for preventing it, it may afterwards be burned if it is thoroughly mixed with air at a sufficiently-high temperature. It is easy to burn smoke when it is made in small quantities, but when made in great volumes it is difficult to get the hot air mixed with it unless special apparatus is used. In boiler firing the formation of smoke must be prevented, as the conditions do not usually permit of its being burned.

Essential Conditions. — The essential conditions for preventing smoke in boiler fires may be enumerated as follows:

1. The gases must be distilled from the coal at a uniform rate.
2. The gases, when distilled, must be brought into intimate mixture with sufficient hot air to burn them completely.
3. The mixing should be done in a fire-brick chamber.
4. The gases should not be allowed to touch the comparatively-cold surfaces of the boiler until they are completely burned. This means that the gases shall have sufficient space and time in which to burn before they are allowed to come in contact with the boiler surface.

Every one of these four conditions is violated in the ordinary method of burning coal under a steam boiler. (1) The coal is fired intermittently and often in large quantities at a time, and the distillation proceeds at so rapid a rate that enough air cannot be introduced into the furnace to burn the gas. (2) The piling of fresh coal on the grate in itself chokes the air supply. (3) The roof of the furnace, or tubes of the boiler, is a cold shell instead of a fire-brick arch, as it should be, and the furnace is not of a sufficient size to allow the gases time and space in which to be thoroughly mixed with the air supply.

Methods of Prevention. — In order to obtain the conditions for preventing smoke it is necessary: (1) That the coal be delivered into the furnace in small quantities at a time. (2) That the draft be sufficient to carry enough air into the furnace to burn the gases as fast as they are distilled. (3) That the air itself be thoroughly heated either by passing through a bed of white-hot coke or by passing through channels in hot brickwork, or by contact with hot fire-brick surfaces. (4) That the gas and the air be brought into the most complete and intimate mixture, so that each particle of carbon in the gas meets before it escapes from the furnace its necessary supply of air. (5) That the flame produced by the burning shall be completely extinguished by the burning of every particle of the carbon into invisible carbon dioxide.

If a white flame touches the surface of a boiler, it is apt to deposit soot and to produce smoke. A white flame itself is the visible evidence of incomplete combustion.

Anthracite Coal. — The first remedy for smoke is to obtain anthracite coal. If this is not commercially practicable, then obtain, if possible, coal with the smallest amount of volatile matter. Coal of from 15 to 25 per cent of volatile matter makes much less smoke than coals containing higher percentage. Provide a proper furnace for burning coal. Any furnace is a proper furnace which secures the conditions named in the preceding paragraphs. Next, compel the firemen to follow instructions concerning the method of firing.

Bituminous Coal. — It is impossible with coal containing over 30 per cent of volatile matter and with a water-tube boiler, with tubes set close to the grate

and vertical gas passages, as in an anthracite setting, to prevent smoke even by the most skillful firing. This style of setting for a water-tube boiler should be absolutely condemned. A Dutch oven setting, or a longitudinal setting with fire-brick baffle walls, is highly recommended as a smoke-preventing furnace but with such a furnace it is necessary to use considerable skill in firing.

Steam Jets and Mechanical Stokers. — Mechanical mixing of the gases and the air by steam jets is sometimes successful in preventing smoke, but it is not a universal preventive, especially when the coal is very high in volatile matter, when the firing is done unskillfully, or when the boiler is being driven beyond its normal capacity. It is essential to have sufficient draft to burn the coal properly and this draft may be obtained either from a chimney or a fan. There is no especial merit in forced draft, except that it enables a larger quantity of coal to be burned and the boiler to be driven harder in case of emergency, and usually the harder the boiler is driven, the more difficult it is to suppress smoke.

Down-draft furnaces and mechanical stokers (q.v.) of many different kinds are successfully used for smoke prevention, and when properly designed and installed and handled skillfully, and usually at a rate not beyond that for which they are designed, prevent all smoke. If these appliances are found giving smoke, it is always due either to overdriving or to unskillful handling. It is necessary, however, that the design of these stokers be suited to the quality of the coal and the quantity to be burned, and great care should be taken to provide a sufficient size of furnace with a fire-brick roof and means of introducing air to make them completely successful.

BIBLIOGRAPHY. — Gebhardt, G. F., *Steam Power Plant Engineering* N. Y., 1909; Kent, Wm., *Steam Boiler Economy*, N. Y., 1914; Kent's *Mechanical Engineers' Pocket-Book*, N. Y., 1913.

[WM. KENT.]

SPARK GAP FOR MEASURING HIGH VOLTAGES.—One of the simplest methods of obtaining a measure of the value (maximum value in case of an alternating voltage) of a high voltage is to determine the length of air gap between two electrodes across which the given voltage will just cause a spark to pass. The voltage (maximum value) required to break down such a gap depends upon: 1. the shape and size of the electrodes; 2. the presence of other conductors in the vicinity of the gap; 3. the time of application of the voltage; and 4. upon the temperature and pressure of the air. The dependence of the break-down voltage upon the shape and size of the electrode is due not only to the effect of these factors upon the potential gradient in the gap but also to the fact that the maximum potential gradient at which air breaks down is dependent upon the distribution of the electrostatic field in the gap (*see article on Corona*) and is greater for very short gaps (0.5 cm. or less) than for long gaps.

For the same *maximum* value of the voltage it has been found that the striking distance is independent of the wave shape and frequency, and is the same for direct as for alternating voltages.

Time of Application of Voltage.—Air (and other gases) will stand for a short period of time (i.e., a few seconds) a much larger potential than it will stand for an indefinitely long period of time; this phenomenon is known as dilatation. After a voltage has been applied to a gap for about one minute, the apparent dielectric strength becomes sensibly constant. When measurements must be made with great rapidity, dilatation may be prevented by illuminating the gap by an arc-lamp. This procedure reduces the value of the sparking voltage, but only to a slight extent. (The theory of the action is that the ultra-violet light from the arc-lamp ionizes the air in the gap, thereby enabling the spark to pass at once.)

NEEDLE SPARK GAP.—Steinmetz in 1898 (*Trans. A.I.E.E., Vol. 15, p. 281*) made a careful determination of the voltages required to produce a spark between spherical, cylindrical and pointed (needles) electrodes. Steinmetz concluded that the sparking voltage between needle points (a sine wave of potential being used) was especially constant for any given set of conditions and that after the curve connecting spark potential and distance between needle points had been carefully ascertained, the needle spark gap offered a very valuable means for measuring high voltages. Results obtained under like conditions have checked very well for voltages up to about 100,000. Above this voltage, however, it has been found difficult to duplicate conditions nearly enough for different observers to obtain results which are in agreement.

A.I.E.E. Needle Spark-gap and Spark-over Voltages.—See paragraphs 272 and 274 of the *Standardization Rules of the A.I.E.E.* (q.v.)

Effect of Pressure and Temperature on Needle-gap Voltage.—According to H. J. Ryan (*Trans. A.I.E.E., 1904, Vol. 23, p. 101*), the break-down voltage for a needle gap is proportional to the barometric pressure. The A.I.E.E. table is for normal atmospheric pressure and a correction should therefore be applied when the atmospheric pressure differs appreciably from 29.92 inches (76 cm.). No data on the effect of change of temperature are available; for ordinary ranges of temperature the effect is probably small.

Fisher's Spark Gap.—Fisher (*Trans. Int. Elec. Cong., 1904, Vol. 2, p. 20[illegible]*) has suggested the use of a needle gap with concave disks of about 10-inch diameter placed back of the points, these disks reducing the brush discharge and rendering the readings more consistent. The calibration for such a gap is different from that for two opposing needles; see Fisher's paper. Fisher also found

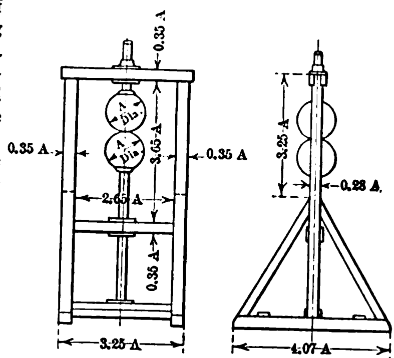
that the degree of sharpness of the needles had a decided effect upon the breakdown voltage, and that the sharper the points the more consistent the results.

SPHERE SPARK GAP. — Farnsworth and Fortescue (*Proc. A.I.E.E., Feb., 1913*) have suggested the use of a spark gap between two spheres. The following is adapted from their paper.

Besides giving inconsistent results the needle-point spark gap is cumbersome and requires a great deal of space. If constructed according to the A.I.E.E. rules, the space required for a spark gap to measure 300,000 volts would be approximately 20 feet by 12 feet by 12 feet as compared with a space of 4 feet by 5 feet by 8 feet required for a sphere spark gap having a range from 50,000 to 412,500 volts. In addition to this advantage the sphere spark gap breaks down with a sharply-defined spark discharge without previous formation of corona, provided the distance of separation is less than the diameter of either sphere. For greater distances of separation the corona forms first and the sparking voltage for a given length of gap becomes more or less variable.

Another advantage of the sphere spark gap is that the terminals do not have to be renewed after each discharge, as is the case with the needle gap.

Construction of Sphere Spark Gap. — Fig. 1 shows the arrangement recommended by Farnsworth and Fortescue and the relative dimensions of the supporting structure in terms of the sphere diameter, A. Being constructed vertically they use a very small floor space as compared with equivalent horizontal needle-point gaps. The top sphere is stationary but slightly adjustable in height so as just to make contact with the lower sphere when it is set for zero separation. The lower sphere is mounted on a piece of brass tubing which carries a threaded bushing on its lower end. This bushing works on a carefully-threaded rod having a pitch of two per centimeter. The bushing being graduated to fiftieths on its circumference, separation may be measured to the nearest 1/100 cm. directly, thus providing a micrometer adjustment. Being made of large parts the whole arrangement is mechanically strong and the spheres are kept in constant alignment. Being mounted on large wheels the spark-gap sets are very portable and may also be picked up by a crane without risk of damage.



Note:- A variation of 1 cm. in thickness and width of wooden parts is permissible.

Fig. 1. Relative Dimensions of Sphere Spark Gap

A.I.E.E. Sphere Spark-gap and Spark-over Voltages. — See paragraphs 272 and 275 of the *Standardization Rules of the A.I.E.E.* (1914 edition).

BIBLIOGRAPHY. — In addition to the references in the text the following papers contain useful data on spark potentials: Kowalski and Rappel, *A-C. Spark Potentials*, Phil. Mag., 1909, Series 6, Vol. 18, p. 699; Russel, *Dielectric Strength of Insulating Materials*, Electrician, 1907, Vol. 60, p. 160; Phil. Mag., 1906, Series 6, Vol. 11, p. 237; Proc. Phys. Soc. Lond., 1906, Vol. 20, p. 49; Warburg, *Spark Potentials*, Ann. der Phys., 1901, Series 4, Vol. 5, p. 811; Pashen, *Spark Potentials*, Ann. der Phys., 1889, N. F., Vol. 39, p. 69. See also bibliography in article on *Corona*.

[W. S. Gorton.]

SPECIFICATIONS AND CONTRACTS. — (*See also Standardization Rules and Standard Specifications; also under name of apparatus.*) Webster defines a specification as a written statement containing a minute description or enumeration of particulars, and a contract as a formal writing which contains the agreement of parties, with the terms and conditions, and which serves as a proof of the obligation. A "preliminary" specification is a description or enumeration of a purchaser's requirements when calling for bids. A "contract" specification is a description or enumeration of labor and material to be supplied under a contract of which the specification is an integral part. A "manufacturer's" specification is a description or enumeration of secondary details peculiar to the work of the individual manufacturer. This also is often made an integral part of a contract.

POLICY TO BE FOLLOWED. — When writing a specification the engineer should bear the following points in mind:

1. A preliminary specification is a commercial instrument designed to describe labor, material or results desired by a purchaser, with the object of enabling competitive bidders to estimate with equal facility upon the amount of a contract.
2. As a rule, contractors are as honest as the struggle for commercial existence permits them to be, and every unnecessary or unfair clause in a specification has its part in limiting competition and lowering the standard of honesty among contractors. A similar remark applies to requirements of which the purchaser cannot ascertain or enforce the fulfillment.
3. The standards of the national engineering societies should be followed unless local conditions prohibit them, and manufacturer's standards should be followed if low bids are to be expected.
4. No specification should contain a blanket clause covering the furnishing of unnamed contingencies unless all bidders are to have an equal opportunity to ascertain what such contingencies are likely to be. If such a clause is made very broad, it is unlikely that the courts would hold it valid.
5. Before drawing the specifications, determine to what extent the contractor is to be made responsible for the final results.
6. Avoid specifying proprietary articles or material as far as possible in order not to restrict competition.
7. Quality is an important factor in cost, hence unless the very best materials and workmanship are required, in spite of the higher cost, it is necessary to adopt some fair commercial standard.
8. There are many minor items in specifications which cannot be minutely described without making the specifications unduly long. In such cases it is good practice to specify that these items shall be made "to the reasonable satisfaction of the inspector," the word "reasonable" saving the contractor from arbitrary and unjustifiable actions of the inspector by enabling him to refer the reasonableness of such actions to the arbitrament of the courts.

FORM OF SPECIFICATIONS. — All specifications should be written in clear, concise language, free from ambiguity, and should be in convenient form for reference purposes.

It is desirable to number the paragraphs or clauses of all specifications to facilitate reference, and every item of the work should be alloted a separate clause.

There are two general classes of specifications. In one class results only are specified and all requirements should be rigorously exact. In the other class details of construction are specified and requirements can be stated in approximate terms only. Wherever practicable the former class of specification should be used. A combination of the two classes is also commonly adopted, but i

such cases an agreement as to results does not bind the contractor if the methods of arriving at these results are also specified. (*See J. C. Wait, Eng. News, June 8, 1905*). In either case considerable study should be given to securing a happy medium between brevity and elaboration of details. The degree of detail should be governed very largely by the magnitude and importance of the work.

The following is suggested as a suitable plan to follow in drawing up specifications for materials. It is the result of a study of this subject by a special committee of the American Society for Testing Materials.

Whatever form is adopted should be closely adhered to in writing all the specifications for a given job.

1. The specification shall be divided into eight "Parts."

2. The titles of these Parts shall be in large upper-case type centered over the text and preceded by a Roman numeral.

3. The titles of these Parts shall be as follows unless conditions necessitate some variation. In case a Part is omitted, the Roman numerals shall be continued in an unbroken sequence.

I. Manufacture.
II. Chemical Properties and Tests.
III. Physical Properties and Tests: Mechanical, Electrical, Magnetic, Thermal, etc.
IV. Standard Sizes, Dimensions, Weights, Gauges, etc.
V. Workmanship and Finish.
VI. Packing, Marking and Shipping.
VII. Inspection and Rejection.
VIII. Definition of Terms. (If a specification contains numerous terms that admit of ambiguity, they shall be defined under this sub-title.)

4. Each "Part" of a specification shall be divided into "Clauses" or "Sections," which shall be numbered continuously throughout the specification in Arabic numerals. Every Section or Clause of printed specifications shall have a marginal heading in bold-face type, briefly indicative of its content. If specifications are on one side of the paper, the clause titles shall be in the left-hand margin; if printed on both sides, as in a book, the clause titles shall be in the outside margins.

5. Each Section or Clause may be subdivided into paragraphs distinguished by lower-case italics in parenthesis.

6. Directly after the title of the specification insert sections of an introductory, descriptive or general character. No sub-title shall precede this matter, which shall precede Part I.

7. Desired values, rather than permissible limits, shall be given, followed by a statement with respect to permissible variations.

8. In so far as practicable, specified values shall be expressed in tabular form.

9. The Style sheets of the A.I.E.E., A.S.T.M., etc., should be followed in all matters pertaining to typography, standard terms, abbreviations, spelling, etc.

The words "shall" and "will" are used in the following sense by the American Society for Testing Materials. Use "shall" wherever the specifications are to be made binding on parties of the first or second part. Use "will" wherever the specifications are intended to express a declaration of purpose not mandatory upon the parties of the first or second part. Many engineers use "shall" to express a command binding on the Contractor and "will" to express a declaration of purpose binding the Purchaser.

Specifications for machinery, apparatus or construction work may be written in similar form using different Part headings. In the case of apparatus and machinery, the following sequence of Parts has been found very practical by the author of this article.

The object of the specification and general conditions to be met having been stated, the following parts follow:

I. Summarized Description of principal Characteristics and Conditions of Service.
II. Style and Description of Apparatus; Details of Construction.
III. Dimensions, Weights, Drawings and Schedules.
IV. Work to be done by other Contractors.
V. Performance and Tests.
  (a) Performance which may be checked by mere observation.
  (b) Factory Tests.
  (c) Performance and Tests after Erection.
VI. Workmanship and Finish.
VII. Packings, Marking, Shipping and Delivery.
VIII. Inspection and Rejection.
IX. Guarantees.
X. Conditional Payments. (The details of conditional payments depending upon results of inspection and tests.)
XI. Definition of Terms. (If a specification contains numerous terms that admit of ambiguity, they shall be defined under this sub-title.)

It will be observed that the sequence of topics in the above schedule approximately follows the order of the life history of the machine. The form may be extended upon this principle to cover any kind of work.

POINTS TO BE COVERED IN SPECIFICATIONS. — A specification should often cover the following points.

Items 1 to 3 refer to preliminary specifications only. Items 4 and 5 are frequently included in the contract proper.

1. These specifications are intended to furnish such information to the Bidders as will enable them to prepare detail plans upon which to give prices. Should any Bidder consider the requirements of these specifications prohibitive to the free exercise of his skill, any suggestions made by him will be duly considered.

2. Before a contract is awarded, final specifications will be prepared by the Company.

3. In comparing proposals due consideration will be given by the Company to availability, reliability, simplicity, cost of maintenance and quickness of delivery.

4. The Engineer agrees to consider all drawings submitted by Bidders as confidential, and that he will not show any such drawings to other manufacturers, whether they are tendering under this specification or not.

5. Definition of words describing the parties to the contract, such as "Contractor," "Purchaser," "Company," "Engineer."

6. Person or persons concerned and their respective powers.

7. Where work is to be done or material delivered.

8. The point where the work of two contracts meet should be carefully designated in both contracts, not merely in a general way, but specifically to the minutest detail.

9. The Contractor shall be responsible for the correctness of all drawings even after they are approved by the Engineer.

10. Materials ordered or work commenced prior to the approval of the drawings will be at the Contractor's risk.

11. No approved drawing may be changed without the approval of the Engineer.

12. The Contractor shall inspect the work of other contractors whose work affects his and shall notify the Engineer of anything which injuriously affects

his work. The Contractor shall give these other contractors the privilege of inspecting his work in so far as it affects the acceptance of their work.

13. Contractor shall not drill or in any way impair the strength of buildings or structures except with the written consent of the Engineer.

14. All work shall conform to the requirements of the "National Board of Fire Underwriters," and to all Government regulations.

15. The Contractor shall obtain all necessary permits from the City or County.

16. All like parts shall be interchangeable as far as practicable.

17. The Contractor shall install and maintain in his offices and at the site of the work such telephones as may be required by the Engineer, and shall place them at the disposal of the Engineer or his representative for any purpose relating to the execution of the contract.

18. The Standardization Rules of the A.I.E.E. and other similar documents should be followed wherever possible.

19. The provisions of the preliminary specification and those subsequently agreed upon between Company and Bidder should be incorporated as part of the final or contract specification.

POINTS TO BE COVERED IN CONTRACT. — In addition to items 5 and 6 above, the Engineer should make sure that the following points are included in the contract. The actual preparation of the contract, at least where the amount of money involved is large, should be left to a competent lawyer.

1. Description and value of bond and indemnity to be delivered by Contractor to Company.
2. Statement of fire insurance to be secured by Contractor for Company.
3. Protection of Company by Contractor against losses resulting from letters patent.
4. The work to be performed in such a way as not to interfere with safety and continuity of service.
5. Responsibility for damage to work from fire, floods, storm, earthquake, or any other cause whatsoever.
6. Contiguous work by other contractors shall not be delayed.
7. Statement of liquidated damages to be paid in the event of work not being completed on the date agreed.
8. The Contractor shall not transfer the contract without permission.
9. Time of commencement of work, rate of progress and date of completion.
10. The character of the methods and appliances to be used and grade of workmen to be employed.
11. Method of payment, time of settlement and basis thereof.
12. Arbitration and settlement of disputes.
13. Extras and claims therefor.

(The subject of Engineering contracts is more fully covered in the books by J. B. Johnson and J. I. Tucker, cited in the Bibliography.)

SPECIFICATIONS FOR MACHINES. — The following clauses are often useful in specifications for machinery. As presented here, they are necessarily somewhat special, and should not be adopted without modification unless after due consideration they are found applicable:

1. The Contractor shall furnish the services of experienced erecting engineers, who shall be constantly in charge of the work at the sites of erection, together with all necessary machinists, electricians, riggers and laborers to properly and quickly unload, erect, adjust, operate and test the machine.
2. The Company will supply crane service at a stated price per hour.
3. The Contractor will be held responsible for all operations connected with the handling of the machine on the premises of the Purchaser, and the Con-

tractor is to defray, make good or repair any loss, damage or cost occasioned by, or in consequence of, the careless or negligent handling of such machine or its parts.

4. Another Contractor will construct suitable foundations in accordance with detail plans to be prepared by the Engineer; data and dimensions required to prepare said plans to be furnished by the Contractor. The foundations will be constructed to the entire satisfaction of the Contractor, so far as they affect the final acceptance by the Engineer of the work furnished by said Contractor. The intention of this clause is to bar any claim by the Contractor that the foundations have not been constructed in such a manner as to enable him to fulfill all the provisions of the specification and the contract.

5. After the machine is erected and ready for regular service, the Contractor shall superintend the operation of it for a stated period. Attendance for this purpose will be furnished by the Company. The Contractor shall give full instructions regarding the adjustment, care, operation and maintenance of the machine to the representatives of the Company.

6. All castings shall be filled and carefully rubbed until practically smooth and of a uniform surface, then painted and rubbed and again painted before shipment. After erection all painted parts of the machine shall receive two finishing coats of hard drying paint of a color to be approved by the company, numbered as directed and finally given two coats of hard drying varnish. All finished steel or iron work shall be brightly polished and carefully slushed with white lead and tallow before shipment. After erection all finished work shall be cleaned and polished and the machine left in perfect condition.

7. The Contractor shall furnish a complete set of case-hardened wrenches mounted on a polished oak board, and all special tools or implements necessary for adjusting and handling the machine. He shall also furnish all oil and grease cups, oil gauges or other devices required to complete the oiling system.

Specifications for dynamo-electric machinery may also advantageously contain clauses on the following subjects:

8. Maximum permissible dimensions.
9. Leveling of foundations.
10. Who supplies and erects field-resistance boxes, dial plates, chains and panels.
11. Maximum reduction of commutator depth permissible by turning.
12. Amount of brush adjustment to allow for brush wear when commutator has been turned down to minimum size.
13. Staggering of brushes to prevent ridges on commutator.
14. Where terminal boards are to be located.
15. Tinning of connections.
16. Smoothness of armature slots.
17. Rigidity of overhanging armature windings.
18. Strength of armature-binding wire.
19. Means for turning over rotating part for inspection and repair.

BIBLIOGRAPHY. — American Society for Testing Materials, *Rules Governing the Form but not the Substance of Specifications*, Report of Committee E-5, 1912; Butler, H. L., *Specifications for Engineering Work*, Engineering and Contracting, 1909, Vol. 31, p. 98; Cochran, J., *The Requirements and Theory of the Advertisement or Notice to Bidder on Contracts for Public Works*, Eng. N., 1911, Vol. 66, p. 306; Fowle, F., *Engineering Specifications*, Trans. A.I.E.E., Vol. 30, 1911; Fowler, Chas. E., *Law and Business of Engineering and Contracting*, N. Y., 1909; Johnson, J. B., *Engineering Contracts and Specifications*, N. Y. 1903; Tucker, J. I., *Contracts in Engineering*, N. Y., 1910; Wait, John C., *Engineering and Architectural Jurisprudence*, N. Y., 1898: *Law of Operations Preliminary to Construction*, N. Y., 1910: *Law of Contracts*, N. Y., 1901.

[W. A. Del Mar.]

STANDARDIZATION RULES OF THE A.I.E.E. — (*See also Standardization Rules and Standard Specifications.*) The first set of Standardization Rules issued by the American Institute of Electrical Engineers was the "Report of the Committee on Standardization" presented and accepted by the Institute June 26, 1899, and published in the *Trans. A.I.E.E.*, Vol. 16, p. 255. A revision of these rules was presented and adopted June 20, 1902, and these revised rules were published in the *Trans. A.I.E.E.*, Vol. 19, p. 1075. These rules were again revised and adopted by the Institute June 21, 1907, and appear in the *Trans. A.I.E.E.*, Vol. 26, p. 1795. Another revision of the Standardization Rules was approved by the Board of Directors on June 27, 1911, and appears in the *Trans. A.I.E.E.*, Vol. 30, p. 2535.

Since 1911 the Standards Committee of the Institute has been working on a complete revision of the rules, particularly with reference to methods of rating of electrical machinery. At a special meeting of the Board of Directors of the Institute held July 10, 1914, at which a draft of these new rules was presented, the following resolution was passed:

RESOLVED, that the rules reported by the Standards Committee be and hereby are adopted subject to editorial revision by the committee for the purpose of correcting errors and clarifying the real intent of the rules, the same to take effect December 1, 1914.

Below is reprinted the draft of the new Standardization Rules as presented at this meeting of the Board of Directors. The resolution passed by the Board would indicate that the rules which are to go into effect on December 1, 1914, will not differ materially from the draft given below.

Copies of the 1911 Edition of the Rules may be obtained from the Headquarters of the Institute, 33 W. 39th St., New York City, for 10 cents per copy in paper covers or for 25 cents per copy in cloth covers. The new rules will also undoubtedly be on sale in pamphlet form.

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DEFINITIONS

NOTE. — The following definitions are intended to be practically descriptive, and not scientifically rigid. The definitions of currents given below apply also, in most cases, to electromotive force, potential difference, magnetic flux, etc.

1. **A Direct Current** is a unidirectional current. As ordinarily used, the term designates a practically non-pulsating current.

2. **A Pulsating Current** is a current which pulsates regularly in magnitude. As ordinarily employed, the term refers to unidirectional current.

3. **A Continuous Current** is a practically non-pulsating direct current.

4. **An Alternating Current** is a current which alternates regularly in direction. Unless distinctly otherwise specified, the term "alternating current" refers to a periodic current with successive half waves of the same shape and area.

5. **An Oscillating Current** is a periodic current whose frequency is determined by the constants of the circuit or circuits.

6. **Cycle.** — One complete set of positive and negative values of an alternating current.

7. **Electrical Degree.** — The 360th part of a cycle.

8. **Period.** — The time required for the current to pass through one cycle.

9. **Frequency.** — The number of cycles or periods per second. The product of 2π by the frequency is called the *angular velocity* of the current.

10. **Root-Mean-Square or Effective Value.** — The square root of the mean of the squares of the instantaneous values for one complete cycle. It is usually abbreviated r.m.s. Unless otherwise specified, the numerical value of an alternating current refers to its r.m.s. value. The r.m.s. value of a sinusoidal wave is equal to its maximum value divided by $\sqrt{2}$. The word "virtual" is sometimes used in place of r.m.s., particularly in Great Britain.

11. **Wave-Form or Wave-Shape.** — The shape of the curve obtained when the instantaneous values of an alternating current are plotted against time in rectangular coördinates. The distance along the time axis corresponding to one complete cycle of values is taken as 2π radians, or 360°. Two alternating quantities are said to have the same wave-form when their ordinates of corresponding phase (see §13) bear a constant ratio to each other. The wave-shape, as thus understood, is therefore independent of the frequency of the current and of the scale to which the curve is represented.

12. **Simple Alternating or Sinusoidal Current.** — One whose wave-shape is sinusoidal.

Alternating-current calculations are commonly based upon the assumption of sinusoidal currents and voltages.

13. **Phase.** — The distance, usually in angular measure, of the base of any ordinate of an alternating wave from any chosen point on the time axis, is called the phase of this ordinate with respect to this point. In the case of a sinusoidal alternating quantity, the phase at any instant may be represented by the corresponding position of a line or *vector* revolving about a point with such an angular velocity ($\omega = 2\pi f$) that its projection at each instant upon a convenient reference line is proportional to the value of the quantity at that instant.

14. **Non-Sinusoidal Quantities** are quantities that cannot be represented by vectors of constant length in a plane, and the following definitions of phase, active component, reactive component, etc., are not in general applicable. Certain "equivalent" values, as defined below, may, however, be used in many instances, for the purpose of approximate representation and calculation.

15. **Crest-Factor or Peak-Factor** is the ratio of the crest or maximum value to the r.m.s. value. The crest factor of a sine-wave is $\sqrt{2}$.

16. **Form Factor** is the ratio of the r.m.s. to the algebraic mean ordinate taken over a half-cycle beginning with the zero value. If the wave passes through zero more than twice during a single cycle, that zero shall be taken which gives the largest algebraic mean for the succeeding half-cycle. The form factor of a sine-wave is 1.11.

17. **Distortion-Factor** of a wave is the ratio of the r.m.s. value of the first derivative of the wave with respect to time, to the r.m.s. value of the first derivative of the equivalent sine wave.

18. **Equivalent Sine Wave.** — A sine wave which has the same frequency and same r.m.s. value as the actual wave.

*19. **Phase Difference: Lead and Lag.**—When corresponding cyclic values of two sinusoidal alternating quantities of the same frequency occur at different instants, the two quantities are said to differ in phase by the angle between their nearest corresponding values, e.g., the phase angle between their nearest ascending zeros or positive maxima. That quantity whose maximum value occurs first in time is said to lead the other, and the latter is said to lag behind the former.

*20. **Counter-Clockwise Convention.**—It is recommended that in any vector diagram, the leading vector be drawn counter-clockwise with respect to the lagging vector,† as in the accompanying diagram, where *OI* represents the vector of a current in a simple alternating-current circuit lagging behind the vector *OE* of impressed e.mf.

*21. **The Active or In-Phase Component** of the current in a circuit is that component which is in phase with the voltage across the circuit; similarly the active component of the voltage across a circuit is that component which is in phase with the current. The use of the term *energy component* for this quantity is disapproved.

*22. **The Reactive or Quadrature Component** of the current in a circuit is that component which is in quadrature with the voltage across the circuit; similarly the reactive component of the voltage across the circuit is that component which is in quadrature with the current. The use of the term *wattless component* for this quantity is disapproved.

*23. **Reactive Factor** is the sine of the angular phase difference between voltage and current, or the ratio of the reactive current or voltage to the total current or voltage.

*24. **Reactive Volt-Amperes.** — The product of the reactive component of the voltage by the total current, or of the reactive component of the current by the total voltage.

*25. **Non-Inductive Load and Inductive Load.** — A *non-inductive* load is a load in which the current is in phase with the voltage across the load. An *inductive* load is a load in which the current lags behind the voltage across the load. A *condensive* or *anti-inductive* load is one in which the current leads the voltage across the load.

26. **Power in an Alternating-Current Circuit** is the average value of the products of the coincident instantaneous values of the current and voltage for a complete cycle, as determined by a wattmeter.

* Note. — Definitions 19, 20, 21, 22, 23, 24, 25 refer strictly only to cases where the voltage and current are both sinusoidal (see §12).

† See Publication 12 of the International Electrotechnical Commission (Report of Turin Meeting, Sept. 1911, p. 78).

27. **Volt-Amperes or Apparent Power.** — The product of the r.m.s. value of the voltage across a circuit by the r.m.s. value of the current in the circuit. This is ordinarily expressed in kv-a.

28. **Power Factor** is the ratio of the power (cyclic average as defined in §26) to the volt-amperes. In the case of sinusoidal current and voltage, the power factor is equal to the cosine of their difference in phase.

29. **Equivalent Phase Difference.** — When the current and e.m.f. in a given circuit are non-sinusoidal, it is customary, for purposes of calculation, to take as the "equivalent" phase difference the angle whose cosine is the power factor (see §28) of the circuit. There are cases, however, where this equivalent phase difference is misleading, since the presence of harmonics in the voltage wave, current wave, or in both, may reduce the power factor without producing a corresponding displacement of the two wave forms with respect to each other; e.g., the case of an a-c. arc. In such cases the components of the equivalent sine waves, the equivalent reactive factor and the equivalent reactive volt-amperes may have no physical significance.

30. **Single-Phase.** — A term characterizing a circuit energized by a single alternating e.m.f. Such a circuit is usually supplied through two wires. The currents in these two wires, counted positively outwards from the source, differ in phase by 180° or a half-cycle.

31. **Three-Phase.** — A term characterizing the combination of three circuits energized by alternating e.m.f.'s which differ in phase by one-third of a cycle; i.e., 120°.

32. **Quarter-Phase, also called Two-Phase.** — A term characterizing the combination of two circuits energized by alternating e.m.f.'s which differ in phase by a quarter of a cycle; i.e., 90°.

33. **Six-Phase.** — A term characterizing the combination of six circuits energized by alternating e.m.f.'s which differ in phase by one-sixth of a cycle; i.e., 60°.

34. **Polyphase** is the general term applied to any system of more than a single phase. This term is ordinarily applied to symmetrical systems.

35. **Per Cent Drop.** — In electrical machinery the ratio of the internal resistance drop to the terminal voltage is called the "*per cent resistance drop.*"

36. Similarly the ratio of the internal reactance drop to the terminal voltage is called the "*per cent reactance drop.*"

37. Similarly the ratio of the internal impedance drop to the terminal voltage is called the "*per cent impedance drop.*"

Unless otherwise specified, these per cent drops shall be referred to rated load and rated power factor.

38. In the case of transformers, the per cent drop will be the primary drop (reduced to secondary turns) plus the secondary drop, in per cent of secondary terminal voltage.

39. In the case of induction motors, it is advantageous to express the drops in per cent of the internally-induced e.m.f.

40. **The Load Factor** of a machine, plant or system is the ratio of the average power to the maximum power during a certain period of time. The average power is taken over a certain period of time, such as a day, a month, or a year, and the maximum is taken over a short interval of the maximum load within that period.

In each case, the interval of maximum load and the period over which the average is taken should be definitely specified, such as a "half-hour monthly"

load factor. The proper interval and period are usually dependent upon local conditions and upon the purpose for which the load factor is to be used.

41. **Plant Factor** is the ratio of the average load to the rated capacity of the power plant.

42. **The Demand** of an installation or system is the load which it puts on the source of supply, as measured at the receiving terminals. The demand may be as specified, contracted for, or used. It may be expressed either in kilowatts, kilovolt-amperes, amperes or other suitable units.

43. **Maximum Demand** of an installation or system is its greatest demand, as measured not instantaneously but over a suitable and specified interval, such as a "five-minute maximum demand."

44. **Demand Factor** is the ratio of the maximum demand of any system or part of a system to the total connected load of the system, or of the part of the system, under consideration.

45. **Diversity Factor** is the ratio of the sum of the maximum power demands of the subdivisions of any system or parts of a system to the maximum demand of the whole system or of the part of the system under consideration, measured at the point of supply.

46. **Connected Load.** — The combined continuous rating of all the receiving apparatus on consumers' premises connected to the system or part of the system under consideration.

47. **The Saturation Factor** of a machine is the ratio of a small percentage increase in field excitation to the corresponding percentage increase in voltage thereby produced. Unless otherwise specified, the saturation factor of a machine refers to the excitation existing at normal rated speed and voltage. It is determined from measurements of saturation made on open circuit at rated speed.

48. **The Percentage of Saturation** of a machine at any excitation may be found from its saturation curve of generated voltage as ordinates, against excitation as abscissas, by drawing a tangent to the curve at the ordinate corresponding to the assigned excitation, and extending the tangent to intercept the axis of ordinates drawn through the origin. The ratio of the intercept on this axis to the ordinate at the assigned excitation, when expressed in percentage, is the percentage of saturation and is independent of the scales selected for excitation and voltage. This ratio, as a fraction, is equal to the reciprocal of the saturation-factor at the same excitation, deducted from unity, or if f be the saturation factor and p the percentage of saturation.

$$p = 100\left(1 - \frac{1}{f}\right)$$

49. **Magnetic Degree.** —The 360th part of the angle subtended, at the axis of a machine, by a pair of its field poles. One mechanical degree is thus equal to as many magnetic degrees as there are pairs of poles in the machine.

50. **The Variation in Prime Movers** which do not give an absolutely uniform rate of rotation or speed, as in reciprocating steam engines, is the maximum angular displacement in position of the revolving member expressed in degrees, from the position it would occupy with uniform rotation, and with one revolution taken as 360°.

51. **The Variation in Alternators** or alternating-current circuits in general, is the maximum angular displacement, expressed in electrical degrees (one cycle = 360°), of corresponding ordinates of the voltage wave and of a wave of absolutely constant frequency equal to the average frequency of the alternator or circuit in question, and may be due to the variation of the prime mover.

52. **Relations of Variations in Prime Mover and Alternator.**—If p is the number of pairs of poles, the variation of an alternator is p times the variation of its prime mover, if direct-connected, and pn times the variation of the prime mover if rigidly connected thereto in such a manner that the angular speed of the alternator is n times that of the prime mover.

53. **The Pulsation in Prime Movers,** or in the alternator connected thereto, is the ratio of the difference between the maximum and minimum velocities in an engine-cycle to the average velocity.

54. **Capacity.** — The two different senses in which this word is used sometimes lead to ambiguity. It is therefore recommended that whenever such ambiguity is likely to arise, the descriptive term *power capacity* or *current capacity* be used, when referring to the power or current which a device can safely carry, and that the term "*Capacitance*" be used when referring to the electrostatic capacity of a device.

55. **A Resistor** is a device, commonly known as a resistance, used for the operation, protection or control of a circuit or circuits.

56. **A Reactor** is a coil, winding or conductor commonly known as a reactance coil or choke coil, possessing inductance, the reactance of which is used for the operation, protection or control of a circuit or circuits.

57. **The Efficiency** of an electrical machine or apparatus is the ratio of its useful output to its total input.

SYMBOLS AND ABBREVIATIONS

58. *The list recommended is given in the article on Abbreviations and Symbols, p. 1.*

59. E_m, I_m and P_m should be used for maximum cyclic values, e, i and p for instantaneous values, E and I for r.m.s. values (see §10) and P for the average value or active power. These distinctions are not necessary in dealing with continuous-current circuits. In print, vector quantities should be represented by bold-face capitals.

CLASSIFICATION OF MACHINERY

60. The machinery under consideration in these rules may be classified in various ways, these various classifications overlapping or interlocking in considerable degree. Briefly, they are Direct-Current or Alternating-Current, Rotating or Stationary. Under Rotating Apparatus there are two principal classifications: *First,* according to the function of the machines; Motors, Generators, Boosters, Motor-Generators, Dynamotors, Double-current Generators, Converters and Phase Modifiers; *Second,* according to the type of construction or principle of operation; Commutating, Synchronous, Induction, Unipolar, Rectifying. Obviously some of these groups could be rationally included in either classification, e.g., Motor-Generators and Rectifying Machines.

In the following, the self-evident definitions are for the most part omitted.

FUNCTIONAL CLASSIFICATION OF ROTATING MACHINES

61. **A Generator** is a machine which transforms mechanical power into electrical power.

62. **A Motor** transforms electrical power into mechanical power.

63. **A Booster** is a generator inserted in series in a circuit to change its voltage. It may be driven by an electric motor (in which case it is termed a motor-booster) or otherwise.

64. **A Motor Generator** is a transforming device consisting of a motor mechanically coupled to one or more generators.

65. **A Dynamotor** is a transforming device combining both motor and generator action in one magnetic field, either with two armatures, or with one armature having two separate windings and independent commutators.

66. **A Direct-current Compensator** or **Balancer** comprises two or more similar direct-current machines (usually with shunt or compound excitation) directly coupled to each other and connected in series across the outer conductors of a multiple-wire system of distribution, for the purpose of maintaining potentials of the intermediate wires of the system, which are connected to the junction points between the machines.

67. **A Double-current Generator** supplies both direct and alternating currents from the same armature-winding.

68. **A Converter** is a machine employing mechanical rotation in changing electrical energy from one form into another. A converter may belong to either of several types, as follows:

69. **A Direct-current Converter** converts from a direct current to a direct current, usually with a change of voltage. Such a machine may be either a motor generator or a dynamotor.

70. **A Synchronous Converter** (also called a Rotary Converter) converts from an alternating to a direct current, or vice-versa. It is a synchronous machine with a single closed-coil armature.

71. **A Cascade Converter,** also called a **Motor Converter,** is a combination of an induction motor with a synchronous converter, the secondary circuit of the former feeding directly into the armature of the latter; i.e., it is a synchronous converter concatenated with an induction motor.

72. **A Frequency Converter** converts the power of an alternating-current system from one frequency to another, with or without a change in the number of phases, or in the voltage.

73. **A Rotary Phase-Converter** converts from an alternating-current system of one or more phases to an alternating-current system of a different number of phases, but of the same frequency.

74. **A Phase-Modifier,** also called *a Phase-Advancer*, is a machine which supplies reactive volt-amperes to the machine; e.g., induction motor, or to the system to which it is connected. Phase modifiers may be either synchronous or asynchronous.

75. **A Synchronous Phase-Modifier,** sometimes called a Synchronous Condenser, is a synchronous motor, running either idle or with load, the field excitation of which may be varied so as to modify the power-factor of the system, or through such modification to influence the load voltage. The function of a Synchronous Phase-Modifier is to supply reactive volt-amperes to the system with which it is connected.

CONSTRUCTIONAL CLASSIFICATION OF ROTATING MACHINES

Commutating Machines:

76. **Direct-current Commutating Machines** comprise a magnetic field of constant polarity, an armature and a multi-segmental commutator connected therewith. These include: Direct-current Generators; Direct-current Motors; Direct-current Boosters; Direct-current Motor-Generators and Dynamotors; Direct-current Compensators or Balancers; and Arc Machines.

77. **Alternating-current Commutating Machines*** comprise a magnetic field of alternating polarity, an armature and multi-segmental commutator connected therewith.

78. **Synchronous Commutating Machines** include synchronous converters, cascade-converters and double-current generators.

79. **Synchronous Machines** comprise a constant magnetic field and an armature receiving or delivering alternating currents in synchronism with the motion of the machine; i.e., having a frequency strictly proportional to the speed of the machine. They may be subdivided as follows:

80. **An Alternator** is a synchronous alternating-current generator, either single phase or polyphase.

81. **A Polyphase Alternator** is a polyphase synchronous alternating-current generator.

82. **An Inductor Alternator is a Synchronous Alternator** in which both field and armature windings are stationary and in which masses of iron or inductors, by moving past the coils, alter the magnetic flux through them. It may be either single phase or polyphase.

83. **A Synchronous Motor** is a machine structurally identical with a synchronous alternator, but operated as a motor.

84. **Induction Machines** include apparatus wherein the primary and secondary windings rotate with respect to each other; i.e., induction motors, induction generators, certain types of frequency converters and certain types of rotary phase-converters.

85. **An Induction Motor** is an alternating-current motor, either single phase or polyphase, comprising independent primary and secondary windings, one of which, usually the secondary, is on the rotating member. The secondary winding receives power from the primary by electromagnetic induction.

86. **An Induction Generator** is a machine structurally identical with an induction motor, but driven above synchronous speed as an alternating-current generator.

87. **Unipolar or Acyclic Machines** are direct-current machines, in which the voltage generated in the active conductors maintains the same direction with respect to those conductors.

SPEED CLASSIFICATION OF MOTORS

88. Motors may, for convenience, be classified with reference to their speed characteristics as follows:

89. *a.* **Constant-speed Motors,** in which the speed is either constant or does not materially vary; such as synchronous motors, induction motors with small slip and ordinary direct-current shunt motors.

90. *b.* **Multi-speed Motors** (two-speed, three-speed, etc.), which can be operated at any one of several distinct speeds, these speeds being practically independent of the load, such as motors with two armature windings, or induction motors with controllers for changing the number of poles.

* Definitions of a-c. commutator-motors have not yet been agreed upon. The differences of opinion are fundamental and relate to the whole system to be employed in naming the numerous types. One example of this difference is in connection with the definition of the term "Repulsion-Motor," some desiring to extend its use to cover all a-c. commutator motors with short-circuited brushes, and others to substitute more systematic names for the various species of short-circuited brush motors.

91. *c.* **Adjustable-speed Motors,** in which the speed can be varied gradually over a considerable range, but when once adjusted remains practically unaffected by the load, such as shunt motors designed for a considerable range of field variation.

92. *d.* **Varying-speed Motors,** or motors in which the speed varies with the load, ordinarily decreasing when the load increases; such as series motors, compound-wound motors and series-shunt motors.

CLASSIFICATION OF ROTATING MACHINES RELATIVE TO THE DEGREE OF ENCLOSURE OR PROTECTION

93. The following types are recognized:

(1) Open	(7) Self-ventilated
(2) Protected	(8) Drip-proof
(3) Semi-enclosed	(9) Moisture resisting
(4) Enclosed	(10) Submersible
(5) Externally ventilated	(11) Flame-proof
(6) Water-cooled	(12) Flame-proof slip-ring enclosure

94. No. 1. **An "open" machine** is of either the pedestal-bearing or end-bracket type where there is no restriction to ventilation, other than that necessitated by good mechanical construction.

95. No. 2. **A "protected" machine** is one in which the armature, field coils and other live parts are protected mechanically from accidental or careless contact, while free ventilation is not materially obstructed.

96. No. 3. **A "semi-enclosed" machine** is one in which the ventilating openings in the frame are protected with wire screen, expanded metal, or other suitable perforated covers, having apertures not exceeding ¼ of a square inch (1.6 sq. cm.) in area.

97. No. 4. **An "enclosed" machine** is so completely enclosed by integral or auxiliary covers as to prevent a circulation of air between the inside and outside of its case, but not sufficiently tight to be termed air-tight.

98. No. 5. **An "externally ventilated" machine** has its ventilating air supplied by an independent fan or blower external to the machine.

99. No. 6. **A "water-cooled" machine** is one which mainly depends on water circulation for the removal of its heat.

100. No. 7. **A "self-ventilated" machine** differs from an externally ventilated machine only in having its ventilating air circulated by a fan, blower, or centrifugal device integral with the machine.

If the heated air expelled from the machine is conveyed away through a second pipe attached to the machine, this should be so stated.

101. No. 8. **A "drip-proof" machine** is one provided with ventilating openings, so protected as to exclude falling moisture or dirt.

102. No. 9. **A moisture-resisting machine** is one in which all parts are treated with moisture-resisting material. Such a machine shall be capable of operating continuously or intermittently in a very humid atmosphere, such as in mines, evaporating rooms, etc.

103. No. 10. **A "submersible" machine** is a waterproof machine capable of withstanding complete submersion for four hours without injury.

104. No. 11. **A "flame-proof" machine** is a machine in which the enclosing case can withstand, without injury, any explosion of gas that may occur within it, and will not transmit the flame to any inflammable gas outside it.

105. No. 12. **Flame-proof Slip-ring Enclosure.** — An induction motor in which the slip rings and brushes alone are included within a flame-proof case should not be described as a flame-proof machine, but as a machine "with flame-proof slip-ring enclosure."

STATIONARY INDUCTION APPARATUS

107.* **Stationary Induction Apparatus** changes electric energy to electric energy through the medium of magnetic energy without mechanical motion. It comprises several forms, distinguished as follows:

108. **Transformers,** in which the primary and secondary windings are ordinarily insulated one from another.

109. **High-voltage, Low-voltage, Primary, Secondary.** — The terms "high-voltage" and "low-voltage" are used to distinguish the winding having the greater from that having the lesser number of turns. The terms "primary" and "secondary" serve to distinguish the windings in regard to energy flow, the primary being that which receives the energy from the supply circuit, and the secondary that which receives the energy by induction from the primary.

110. **The Rated Current of a Constant-potential Transformer** is that secondary current which, multiplied by the rated-load secondary voltage, gives the kv-a. rated output. That is, a transformer of given kv-a. rating must be capable of delivering the rated output at rated secondary voltage, while the primary impressed voltage is increased to whatever value is necessary to give rated secondary voltage.

The rated primary voltage of a constant-potential transformer is the rated secondary voltage multiplied by the turn ratio.

111. **The Voltage Ratio** of a transformer is the ratio of the r.m.s. primary terminal voltage to the r.m.s. secondary terminal voltage under specified conditions of load.

112. **The Current Ratio** of a current-transformer is the ratio of r.m.s. primary current to r.m.s. secondary current under specified conditions of load.

113. **The Ratio of a Transformer,** unless otherwise specified, shall be the ratio of the number of turns in the high-voltage winding to that in the low-voltage winding; i.e., the "turn-ratio."

114. **The Marked Ratio** of an instrument transformer is the ratio which the apparatus is designed to possess under average conditions of use. When a precise ratio is required, it is necessary to specify the voltage, frequency, load and power factor of the load.

115. **Auto-transformers** have a part of their turns common to both primary and secondary circuits.

116. **Voltage Regulators** have turns in shunt and turns in series with the circuit, so arranged that the voltage ratio of the transformation or the phase relation between the circuit-voltages is variable at will. They are of the following three classes:

117. **Contact Voltage Regulators,** in which the number of turns in one or both of the coils is adjustable.

118. **Induction Voltage Regulators,** in which the relative positions of the primary and secondary coils are adjustable.

119. **Magneto Voltage Regulators,** in which the direction of the magnetic flux with respect to the coils is adjustable.

* There is no §106 in the original, [Editor].

120. **Reactors** or **Reactance-Coils,** also called **Choke Coils;** a form of stationary induction apparatus used to supply reactance or to produce phase displacement.

INSTRUMENTS

121. **An Ammeter** is a measuring instrument, indicating in amperes.

122. **A Voltmeter** is a measuring instrument, indicating in volts.

123. **A Wattmeter** is an instrument for measuring electrical power, indicating in watts.

124. **Recording Ammeters, Voltmeters, Wattmeters, etc.,** are instruments which record graphically upon a time-chart the values of the quantities they measure.

125. **A Watt-hour Meter** is an instrument for registering watt-hours. This term is to be preferred to the term "integrating wattmeter."

126. **A Line-drop Voltmeter Compensator** is a device in connection with a voltmeter, which causes the latter to indicate the voltage at some distant point of the circuit.

127. **A Synchroscope, sometimes called Synchronoscope,** is a device which, in addition to indicating synchronism, shows whether the machine to be synchronized is fast or slow.

STANDARDS FOR ELECTRICAL MACHINERY

128. **Notes.** — The expression "machinery" is here employed in a general sense in order to obviate the constant repetition of the words "machinery or induction apparatus."

129. All temperatures are to be understood as centigrade.

130. The expression "capacity" is to be understood as indicating "capability" except where specifically qualified, as, for instance, in the case of allusions to electrostatic capacity, i.e., capacitance.

131. Wherever special rules are given for any particular type of machinery or apparatus (such as railway motors, railway substation machinery, switches, etc.), these special rules shall be followed, notwithstanding any apparent conflict with the provisions of the more general sections. In the absence of special rules on any particular point, the general rules on this point shall be followed.

132. **Objects of Standardization.** — To ensure satisfactory results, electrical machinery should be specified to conform to the Institute Standardization Rules in order that it shall comply, in operation, with approved limitations in the following respects so far as they are applicable.

Operating temperature	Efficiency
Mechanical strength	Power factor
Commutation	Wave shape
Insulation strength	Regulation

133. **Capacity of an Electrical Machine.** — So far as relates to the purposes of these Standardization Rules, the Institute defines the Capacity of an Electrical Machine as the load or task of which it is capable for a specified time (or continuously), without exceeding in any respect the limitations herein set forth.

Except where otherwise specified, the capacity of an electrical machine shall be expressed in terms of its *output*. For exceptions see §140 and 418.

134. **Rating of an Electrical Machine.** — Capacity should be distinguished from Rating. The Rating of a machine is the output marked on the

Rating Plate, and shall be based on, but shall not exceed, the maximum* load which can be taken from the machine under prescribed conditions of test. This is also called the rated output.

135. **A.I.E.E. and I.E.C. Ratings.** — When the prescribed conditions of test are those of the A.I.E.E. Standardization Rules, the rating of the machine is the Institute Rating. When the prescribed conditions of the test are those of the I.E.C. Rules, the rating of the machine is the I.E.C. rating. A machine so rated in either case shall bear a distinctive sign upon its rating plate.

136. **Standard Temperature and Barometric Pressure for Institute Rating.** — The Institute Rating of a machine shall be its capacity when operating with a cooling medium of the ambient temperature of reference (40° for air or 25° for water, see §§153 and 157) and with barometric conditions within the range given in §156. See §168.

UNITS IN WHICH RATING SHALL BE EXPRESSED

137. **Direct-Current Generators.** — In the case of direct-current generators, the rating shall be expressed in kilowatts (kw.) available at the terminals.

138. **Alternators and Transformers.** — In the case of alternators and transformers, the rating shall be expressed in kilovolt-amperes (kv-a.) available at the terminals, at a specified power factor. The corresponding kilowatts shall also preferably be stated.

139. **Motors.** — In the case of motors, the rating shall be expressed in kilowatts† (kw.) available at the shaft. (An exception to this rule is made in the case of Railway motors, which for some purposes are also rated by their kilowatts *input*, see §418.)

140. **Auxiliary machinery,** such as regulators, phase controllers, resistors, reactors, balancer sets, stationary and synchronous condensers, etc., shall have their ratings expressed in terms of the functions which they perform. It is essential to specify also the voltage of the circuits on which the machinery may appropriately be used.

KINDS OF RATING

141. There are two kinds of rating: namely, (1) rating for continuous service, i.e., "continuous rating;" (2) rating for discontinuous service, i.e., "short-time rating."

142. **Continuous Rating.** — A machine rated for continuous service shall be able to operate continuously at its rated output, without exceeding any of the limitations referred to in §132.

* The term "maximum load" does not refer to loads applied solely for mechanical, commutation, or similar tests.

† Since the input of machinery of this class is measured in electrical units and since the output has a definite relation to the input, it is logical and desirable to measure the delivered power in the same units as are employed for the received power. Therefore, the output of motors should be expressed in kilowatts instead of in horse power. However, on account of the hitherto prevailing practice of expressing mechanical output in horse power, it is recommended that for machinery of this class the rating shall, for the present, be expressed both in kilowatts and in horse power, as follows:

kw.———— h.p.————

The horse-power rating of a motor may, for practical purposes, be taken as 4/3 of the kilowatt rating.

In order to lay stress upon the preferred future basis, it is desirable that on Rating Plates, the Rating in kilowatts shall be shown in larger and more prominent characters than the rating in horse power.

143. Short-time Rating. — A machine rated for short-time service (i.e. service including runs alternating with stoppage of sufficient duration to ensure substantial cooling) shall be able to operate at its rated output during a limited period, to be specified in each case, without exceeding any of the limitations referred to in §132. Such a rating is a short-time rating.

144. Nominal Ratings. — For railway motors and railway substation machinery, certain nominal ratings are employed. See §§391 and 415.

145. Duty-cycle Operation. — Many machines are operated on a cycle of duty which repeats itself with more or less regularity. For purposes of rating, either a continuous or a short time "equivalent load" may be selected which shall simulate as nearly as possible the thermal conditions of the actual duty cycle.

146. Standard Durations of Equivalent Tests shall be for machines operating under specified duty-cycles:

5 minutes	60 minutes
10 minutes	120 minutes
30 minutes	and continuous

Of these the first five are short-time ratings selected as being thermally equivalent to the specified duty cycle.

When, for example, a short time rating of 10 minutes' duration, is adopted, and the thermally equivalent load is 25 kw. for that period, then such a machine shall be stated to have a 10-minute rating of 25 kw.

147. In every case the equivalent short-time test shall commence only when the windings and other parts of the machine are *within 5° C. of the ambient temperature* at the time of starting the test.

148. In the absence of any specification as to the kind of rating, the continuous rating shall be understood.*

Machines marked in accordance with §135 shall be understood to have a continuous rating unless otherwise marked in accordance with §146.

HEATING AND TEMPERATURE

149. Temperature Limitations of the Capacity of Electrical Machinery. — The capacity, so far as relates to temperature, is usually limited by the maximum temperature at which the materials in the machine, especially those employed for insulation, may be operated for long periods without deterioration. When the safe limits are exceeded, deterioration is rapid. The insulating material becomes permanently damaged by excessive temperature, the damage increasing with the length of time that the excessive temperature is maintained, and with the amount of excess temperature, until finally the insulation breaks down.

150. The result of operating at temperatures in excess of the safe limit is to shorten the life of the insulating material. This shortening of life is, in certain special cases, warranted, when necessary for obtaining some other desirable result, as, for example, in some instances of railway motors, in providing greater power within a limited space. See §419. Exceptions may also be noted in the cases of contactors, arc-lamp magnet windings, etc., designed and constructed for operation at relatively-high temperatures.

* An exception is made in the case of machines for railway service, where in the absence of any specification as to the kind of rating, the "nominal rating" as defined in §§391 and 415 shall be understood.

151. There does not appear to be any advantage in operating at lower temperatures than the safe limits, so far as the life of the insulation is concerned. Insulation may break down from various causes, and generally, when these breakdowns occur, it is not due to the temperature at which the insulation has been operated, provided the safe limits have not been exceeded.

152. **The Ambient Temperature** is the temperature of the fluid or fluids which, coming into contact with the heated parts of a machine, carries off its heat convectively.

The cooling fluid may either be led to the machine through ducts, or merely surround the machine freely. In the former case the ambient temperature is to be measured at the intake of the machine. In the latter case see §163.

153. **Ambient Temperature of Reference for Air.** — The standard ambient temperature of reference, when the cooling medium is air, shall be 40° C.

154. The permissible rises in temperature given in column 2 of the table in §188 have been calculated on the basis of the standard ambient temperature of reference, by subtracting 40° from the highest temperatures permissible, which are given in column 1 of the same table.

155. A machine may be tested at any convenient ambient temperature, but whatever be the value of this ambient temperature, the permissible rises of temperature must not exceed those given in column 2 of the table in §188.

156. **Altitude.** — Increased altitude has the effect of increasing the temperature rise of some types of machinery. In the absence of information in regard to the height above sea level at which the machine is intended to work in ordinary service, this height is assumed not to exceed 1000 meters (3300 feet). For machinery operating at an altitude of 1000 meters or less, a test at any altitude less than 1000 meters is satisfactory, and no correction shall be applied to the observed temperatures. Machines intended for operation at higher altitudes shall be regarded as special. When a machine is rated for service at altitudes above 1000 meters (3300 ft.) the normal permissible temperature rise, until more nearly accurate information is available, shall be reduced by 1 per cent for each 100 meters (330 ft.) by which the altitude exceeds 1000 meters. Water-cooled oil transformers are exempt from this reduction.

157. **Ambient Temperature of Reference for Water-cooled Machinery.** — For water-cooled machinery the standard temperature of reference for incoming cooling water shall be 25° C., measured at the intake of the machine.

158. **In Testing Water-cooled Transformers,** it is important, especially for the smaller sizes, to maintain the temperature of the ingoing water within 5° C. of the surrounding air. Where this is impracticable, the reference ambient temperature shall be taken as that indicated by the resistance of the windings, when the disconnected transformer is being supplied with the normal amount of cooling water and the temperature of the windings has become constant.

159. **Machinery Cooled by Air Led to the Machine from a Distance through Ventilating Ducts.** — In this case the temperature of the ingoing air shall be measured at the intake of the machine. The ambient temperature shall be determined in the manner specified in §158 for water-cooled transformers.

160. **Rotating Machines.** — In the case of rotating machines, the above method becomes inapplicable, and recourse must be had to a weighted mean between the temperatures of the circulating air and of the surrounding air. If the necessary thermal data are known, this weighted mean can be calculated; but it shall be permitted to employ a conventional weighted mean, by giving a weight of four to the circulating air and of one to the surrounding room air, provided that these two temperatures during the test do not differ by more than 10 °C.

161. **Machines Cooled by Other Means.** — For machines cooled by other means, special rules are necessary.

162. **Outdoor Machinery Exposed to Sun's Rays.** — Outdoor machinery not protected from the sun's rays at times of heavy load must receive special consideration as regards ambient temperature.

163. **Measurement of the Ambient Temperature During Tests of Machinery.** — The ambient temperature is to be measured by means of several thermometers placed at different points around and half way up the machine at a distance of 1 to 2 meters (3 to 6 feet), and protected from drafts, and abnormal heat radiation, preferably as in §165.

164. The value to be adopted for the ambient temperature during a test is the mean of the readings of the thermometers (placed as above) taken at equal intervals of time during the last quarter of the duration of the test.

165. **Errors due to the Time Lag.** — In order to avoid errors due to the time lag between the temperature of large machines and the variations in the ambient air, all reasonable precautions must be taken to reduce these variations and the errors arising therefrom. Thus, the thermometer for determining the ambient temperature shall be immersed in a suitable liquid, such as oil, in a suitable heavy metal cup. This can be made to respond to various rates of change, by proportioning the amount of oil to the metal in the containing cup. A convenient form for such an oil-cup consists of a massive metal cylinder with a hole drilled partly through it. This hole is filled with oil and the thermometer is placed therein with its bulb well immersed. The larger the machine under test, the larger should be the metal cylinder employed as an oil-cup in the determination of the ambient temperature. The smallest size of oil cup employed in any case shall consist of a metal cylinder 25 mm. in diameter and 50 mm. high (1 in. in diameter and not less than 2 in. high).

166. **In Testing Transformers** and sometimes other machines it will often be desirable to avoid errors due to time lag in temperature changes by employing an idle unit of the same size and subjected to the same conditions of cooling as the unit under test, for obtaining the ambient temperature as described in § 158 and §159.

167. **Where Machines are Partly Below the Floor Line** in pits, the temperature of the rotor shall be referred to a weighted mean of the pit and room temperatures, the weight of each being based on the relative proportions of the machine in and above the pit. Parts of the stator constantly in the pit shall be referred to the ambient temperature in the pit.

168. **Corrections for the Deviation of the Ambient Temperature, at the Time of Test, from the Reference Value of 40° C.** In view of numerous experiments which have shown that the effect on the temperature rise of the precise value of the ambient temperature at the time of test, is small, obscure and of doubtful direction, no correction shall be made for ambient temperature deviations from the standard value of 40° C. It is, however, desirable that tests should be conducted at ambient temperatures not lower than 25° C. Exception to this rule is made in the case of air-blast transformers, in which, if the ingoing air temperature during the test differs from 40° C., correction on account of difference in resistance and difference in convection shall be made by changing the "observable" temperature rise of the windings by 0.5 per cent for each degree centigrade. Thus with a room temperature of 30° C. the "observable" rise of temperature shall be increased by 5 per cent, and with a room temperature of 15° C. the "observable" rise of temperature shall be increased by 12.5 per cent.

169. Duration of Heat Run. — For practical purposes the duration of a test of a machine for continuous service shall be prolonged until the difference between the temperature of the machine and the ambient temperature is practically constant. Temperature measurements, when possible, shall be taken during operation, as well as when the machine is stopped. The highest figures thus obtained shall be adopted. In order to abridge the long heating period, in the case of large machines, reasonable overloads of current during the preliminary period are suggested for them.

OPERATING TEMPERATURES

170. The actual temperatures attained in the different parts of a machine, and not the rises in temperature, affect the life of the insulation of the machine. (See §§149 to 151.)

171. The temperatures in the different parts of a machine which it is desired to ascertain are the maximum temperatures reached in those parts.

172. As it is usually impossible to determine the maximum temperature attained in insulated windings, it is convenient to apply a correction to the measured temperature, to approximate the difference between the actual maximum temperature and the measured temperature by the method used. This correction or margin of security is provided to cover the errors due to fallibility in the location of the measuring devices, as well as inherent inaccuracies in measurement and methods.

TEMPERATURE MEASUREMENTS

173. In determining the temperature of different parts of a machine, three methods will be considered. One or other of these methods, as set forth below, will usually be appropriate for commercial measurements on any particular type of machine.

174. **Method No. 1. Thermometer Method.** — This method consists in the determination of the temperature by mercury or alcohol thermometers, by resistance thermometers, or by thermocouples, any of these instruments being applied to the hottest accessible part of the *completed* machine, as distinguished from the thermocouples or resistance coils imbedded in the machine, as described under Method No. 3.

175. When Method No. 1 is used, the hottest-spot temperature shall be estimated by adding a hottest-spot correction of 15° C. to the highest temperature observed.

176. *Exception.* — In cases where the thermometer is applied directly to the surface of a bare winding, such as an edgewise strip conductor, or a copper casting, a hottest-spot correction of 5° C. instead of 15° C. shall be made, in order to allow for the unlikelihood of locating the thermometer at the hottest spot.

177. **Method No. 2. Resistance Method.** — This method consists in the measurement of the temperature of windings by their increase in resistance, corrected* to the instant of shut-down when necessary. In the application of

* Whenever a sufficient time has elapsed between the instant of shut-down and the time of the final temperature measurement to permit the temperature to fall, suitable corrections shall be applied so as to obtain as nearly as practicable the temperature at the instant of shut-down. This can sometimes be approximately effected by plotting a curve, with temperature readings as ordinates and time as abscissas, and extrapolating back to the instant of shut-down. In other instances, acceptable correction factors can be applied.

In cases where successive measurements show *increasing* temperatures after shut-down, the highest value shall be taken.

this method careful thermometer measurements must also be made whenever practicable without disassembling the machine,* in order to increase the probability of revealing the highest observable temperature. Whichever method yields the higher temperature, that temperature shall be taken as the "highest observable" temperature and a hottest-spot correction of 10° C. added thereto.

178. In the case of resistance measurements, the temperature coefficient of copper shall be deduced from the formula $1/(234.5 + t)$. Thus, at an initial temperature $t = 40°$ C. the temperature coefficient or increase in resistance per degree centigrade rise is $1/(274.5) = 0.00364$. The following table, deduced from the formula, is given for convenience of reference.

Temperature of the winding, in degrees C. at which the initial resistance is measured	Increase in resistance of copper per ohm per degree C.
0	0.00427
5	0.00418
10	0.00409
15	0.00401
20	0.00393
25	0.00385
30	0.00378
35	0.00371
40	0.00364

179. In Field Coils of Low Resistance, where the joints and connections form a considerable part of the total resistance, the measurement of temperature by the resistance method shall not be used.

180. The Temperature of the Windings of Transformers is always to be ascertained by Method 2. In the case of air-blast transformers, it is especially important to place thermometers near the air outlet.

182.† Method No. 3. Imbedded Temperature-detector Method.—Thermocouples or resistance coils, located as nearly as possible at the estimated hottest spot. This method is only to be used with coils placed in slots.

183. By Building into the Machine suitably placed thermocouples or resistance coils, a temperature not much less than that of the hottest spot will be disclosed. When these devices are adopted for such temperature determinations, a liberal number shall be employed, and all reasonable efforts consistent with safety shall be made to locate them at the various places where the highest temperatures are likely to occur.

184. Temperature-Detectors should be placed in at least two sets of locations. One of these should be between coil and core, and one between the top and bottom coils, where two coils per slot are used. Where only one coil per slot is used, one set of detectors shall be placed between coil and core, and one set between coil and wedge.

185. Method No. 3 should be applied to all stators of machines with wide cores (50 cm. — 20 in. — and over) and to all machines of 5000 volts and over, if of over 500 kv-a., regardless of core width.

* As one of the few instances in which the thermometer check cannot be applied in Method No. 2, the rotor of a turbo-alternator may be cited.

† There is no § 181 in the original. [Editor].

186. Correction Factor for this Method.—On two-layer machines with couples between coils, and between coil and slot, add 5° to the highest reading. In single-layer machines with couples between coil and core and between coil and wedge, add to the highest reading 10° C. plus 1° C. per 1000 volts above 5000 volts of terminal pressure.

TEMPERATURE LIMITS

187. The following table gives the limits for the hottest-spot temperatures of insulations. The permissible limits are indicated in column 1 of the table. The limits of temperature rise permitted under rated-load conditions are given in column 2, and are found by subtracting 40° C. from the figures in column 1. Whatever be the ambient temperature at the time of the test, the rise of temperature observed must never exceed the limits in column 2 of the table. The highest temperatures attained in any machine corresponding to the output for which it is rated must not exceed the values indicated in column 1 of the table and clauses following.

188. Table of Hottest-spot Temperatures and of Corresponding Permissible Temperature Rises.

Class	Description of insulation	Column 1 Highest permissible temperatures for hottest spot	Column 2 Highest permissible temperature rise of hottest spot above 40° for the purpose of fixing the Institute rating
		° C.	° C.
A1	Cotton, silk, paper and other fibrous materials, not so treated as to increase the thermal limit.	95	55
A2	Similar to A1, but treated or impregnated and including enameled wire.	105	65
B	Mica, asbestos or other material capable of resisting high temperatures, in which any Class A material or binder, if used, is for structural purposes only, and may be destroyed without impairing the insulating or mechanical qualities.	125	85

189. **Note.**—The Institute recognizes the ability of manufacturers to employ Class B insulation successfully at maximum temperatures of 150° C. and even higher. However, as sufficient data covering experience over a period of years at such temperatures are at present unavailable, the Institute adopts 125° C. as a conservative limit for this class of insulation, and any increase above this figure should be the subject of special guarantee by the manufacturer.

190. Class C.—For fireproof and refractory materials, such as pure mica, porcelain, etc., no limit is specified.

191. When a lower-temperature class material is comprised in a completed product to such an extent, or in such ways, that its subjection to the temperature limits allowed for the higher-temperature class material, with which it is associated, would affect the integrity of the insulation either mechanically or electrically, the permissible temperature shall be fixed at such a value as shall afford ample assurance that no part of the lower-temperature class material shall be subjected to temperatures higher than those approved by the Institute and set forth above. See also §150.

192. **Table Summarizing the Temperature Conditions under the Three Preceding Methods of Measurement for Insulations of Classes A_1, A_2 and B.** (See next page.)

SPECIAL CASES OF TEMPERATURE LIMITS

193. **Temperature of Oil.** — The oil in which apparatus is immersed shall in no part be subjected to an observable temperature in excess of 90° C.

194. **Water-cooled Transformers.** — In these the hottest-spot temperature shall not exceed 85° C.

195. **Railway Motor Temperature Limits,** see §419.

196. **Squirrel-cage and Amortisseur Windings.** — In many cases the insulation of such windings is largely for the purpose of making the conductors fit tightly in their slots, and the slightest effective insulation is ample. In other cases there is practically no insulating material on the windings. Consequently, the temperature rise may be of any value such as will not occasion mechanical injury to the machine.

197. **Collector Rings.** — The temperature of collector rings shall not be permitted to exceed the "hottest-spot" values set forth in §188 for the insulations employed either in the collector rings themselves, or in adjacent insulations whose temperatures would be affected by the heat from the collector rings. The temperature of the rings shall in no case exceed 130° C.

198. **Commutators.** — For commutators so constructed that no difficulties from expansion can occur, the following temperature limits are prescribed:

Current per Brush Arm	Maximum Permissible Temp.
200 amperes or less	130° C.
200 to 900 amperes	130° C. less 5° for each 100 amperes increase above 200.
900 amperes and over	95° C.

In no case shall the observable temperature be permitted to exceed the values given in §188 for the insulation employed, either in the commutator or in any insulation whose temperature would be affected by the heat of the commutator.

199. **Cores.** — The temperature of the iron core in contact with the windings must not exceed the limits of temperature and temperature rise permitted for the windings themselves.

200. **Other Parts** (such as brush-holders, brushes, bearings, pole-tips, cores, etc.). All parts of electrical machinery other than those whose temperature affects the temperature of the insulating material, may be operated at such temperatures as shall not be injurious in any respect. But no part of continuous-duty machinery subject to handling in operation, such as brush-rigging, shall have a temperature in excess of 100° C. for more than a very brief time.

Class	Permissible Hottest-spot Temp.	Method I Thermometer only			Method II Resistance (With thermometer check when practicable)			Method III Imbedded thermocouples or resistance coils								
								Double-layer windings for all voltages			Single-layer windings 5000 volts or less			Single-layer windings above 5000 volts		
		Hottest-spot Correction	Limiting Observable Temp.	Limiting Temp. Rise above 40°	Hottest-spot Correction	Limiting Observable Temp.	Limiting Temp. Rise above 40°	Hottest-spot Correction	Limiting Observable Temp.	Limiting Temp. Rise above 40°	Hottest-spot Correction	Limiting Observable Temp.	Limiting Temp. Rise above 40°	Hottest-spot Correction	Limiting Observable Temp.	Limiting Temp. Rise above 40°
A_1	95°	15	80	40	10	85	45	5	90	50	10	85	45	10+(E-5)*	85−(E-5)	45−(E-5)
A_2	105°	15	90	50	10	95	55	5	100	60	10	95	55	10+(E-5)	95−(E-5)	55−(E-5)
B	125°	15	110	70	10	115	75	5	120	80	10	115	75	10+(E-5)	115−(E-5)	75−(E-5)

* In this formula *E* represents the rated pressure between terminals in kilovolts. Thus for a three-phase machine of 11 kilovolts between terminals the hottest-spot correction to be added to the maximum observable temperature will be 16°C.

ADDITIONAL REQUIREMENTS

201. **Short-circuit Stresses.** — The Institute recognizes the self-destructibility, both mechanical and thermal, of certain sizes and types of machines when subjected to severe short-circuits, and recommends that ample protection be provided in such cases, external to the machine if necessary.

202. **Over-Speeds.** — All types of rotating machines shall be so constructed that they will safely withstand an over-speed of 25 per cent, except in the case of steam turbines, which, when equipped with emergency governors, shall be constructed to withstand 20 per cent over-speed.

In the case of series motors, it is impracticable to specify percentage values for the guaranteed over-speed on account of the varying service conditions.

Water-wheel generators shall be constructed for the maximum runaway speed which can be attained by the combined unit.

203. **Momentary Loads.** — Machines shall be required to carry momentary loads of 150 per cent rated load, and commutating machinery shall commutate successfully under this condition. Successful commutation is such that neither brushes nor commutator are injured by the test.

Machines for Duty-cycle Operation shall be rated according to their equivalent load, either on the short-time or continuous basis, but intended for operation with widely fluctuating loads, shall commutate successfully under their specified operating conditions. See §139.

204. **Stalling Torque of Motors.** — Motors for continuous service shall, except when otherwise specified, be required to develop a running torque at least 175 per cent of that corresponding to the running torque at their rated load without stalling.

Obviously, duty-cycle machines must carry their peak loads without stalling.

WAVE FORM

205. **The Sine Wave** shall be considered as standard except where deviation therefrom is inherent in the operation of the machine.

206. **The Deviation of Wave Form** from the sinusoidal is determined by superposing upon the actual wave (as determined by oscillograph), the equivalent sine wave of equal length, in such a manner as to give the least difference, and then dividing the maximum difference between corresponding ordinates by the maximum value of the equivalent sine wave. A maximum deviation of the wave from sinusoidal shape not exceeding 10 per cent is permissible, except when otherwise specified.

EFFICIENCY AND LOSSES

207. **Machine Efficiency** is the ratio of the power delivered by the machinery to the power received by it.

208. **Plant Efficiency** is the ratio of the energy delivered from the plant to the energy received by it in the same period of time,* that period of time to be suitably chosen.

209. **Conventional Efficiency** of machinery is the ratio of the output to the sum of the output and the losses; or of the input minus the losses to the input; when, in either case, conventional values are assigned to one or more of these losses. The need for assigning conventional values to certain losses arises from the fact that some of the losses in electrical machinery are practicably

* An exception should be noted in the case of the efficiency of storage batteries.

indeterminable, and must, in many cases, either be approximated by an approved method of test, or else values recommended by the Institute and designated "conventional" values shall be employed for them in arriving at the "conventional efficiency." Efficiencies based upon conventional losses shall be specifically stated to be conventional efficiencies.

210. **Efficiency Determination.** — Input and output determination of efficiency may be made directly, measuring the output by brake, or equivalent, where applicable. Within the limits of practical application the circulating power method, sometimes described as the Hopkinson or "loading back" method, may be used; in machines where none of these methods are practicable the conventional efficiency should be used, especially in the case of large machines of high efficiency.

211. Values for the indeterminate losses may also be obtained by brake or other accurate test, and used in estimating actual efficiencies of similar machines, by the separate-loss method.

212. **Normal Conditions.** — The efficiency shall correspond to, or be corrected to, the normal conditions herein set forth, which shall be regarded as standard. These conditions include voltage, current, power-factor, frequency, wave shape, speed, temperature or such of them as may apply in each particular case.

213. **Measurement of Efficiency.** — Electric power shall be measured at the terminals of the apparatus. In polyphase machines sufficient measurements shall be made on all phases to avoid errors of unbalance.

214. **Point at Which Mechanical Power Shall be Measured.** — Mechanical power delivered by machines shall be measured at the pulley, gearing, or coupling, on the rotor shaft, thus excluding the loss of power in the belt or gear friction. See, however, §415.

215. **The Efficiency of Alternating-current Apparatus** shall be measured when the current is in phase with the terminal voltage, unless otherwise specified, or unless a definite phase difference is inherent in the apparatus, as in induction machinery.

216. **Efficiency of Alternating-current Apparatus in regard to Wave Shape.** — In determining the efficiency of alternating-current apparatus the sine wave is to be considered as standard, unless a different wave form is inherent in the operation of the apparatus. See §205.

217. **Temperature of Reference for Efficiency Determinations.** — The efficiency, at all loads, of all apparatus, shall be determined at, or corrected to, a reference temperature of 75° C.

218. **The Losses in Constant-potential Machinery,** either of the stationary type, or of the constant-speed rotary type, are of two classes; namely, those which remain substantially constant at all loads, and those which vary with the load. The former include iron losses, windage and friction, also I^2R losses in any shunt windings. The latter include I^2R losses in series windings. The constant losses may be determined by measuring the power required to operate the machine at no load, deducting any series I^2R losses. The variable loss at any load may be computed from the measured resistance of the series windings and the given load current.

219. **Stray Load-Losses.** — The above simple method of determining the losses and hence the efficiency is only approximate, since the losses which are assumed to be constant do actually vary to some extent with the load, and also because the actual loss in the copper windings is sometimes appreciably greater than the calculated I^2R loss. The difference between the approximate losses

as above determined and the actual losses is termed the "stray load-losses."* These latter are due to distortions in electric or magnetic fluxes from their no-load distributions or values, brought about by the load current. They are usually only approximately measurable or may be indeterminable.

220. Table of Losses. — Losses in apparatus may be classified as follows:

Accurately measurable or determinable	Approximately measurable or determinable	Indeterminable
a. No-load core losses including eddy-current losses in conductors at no-load	*c.* Brush friction loss	*h.* Iron loss due to flux distortion
b. Load I^2R in windings No load I^2R in windings	*d.* Brush-contact	*i.* Eddy-current losses in conductors due to transverse fluxes occasioned by the load currents
	e. Losses due to windage and to bearing friction	*k.* Eddy-current losses in conductors due to tooth saturation resulting from distortion of the main flux.
	f. Extra copper loss in transformer windings, due to stray fluxes caused by load currents	*l.* Tooth-frequency losses due to flux distortion under load
	g. Dielectric losses	*m.* Short-circuit loss of commutation

221. Evaluation of Losses. — The larger individual losses are either accurately or approximately determinable, but certain of the indeterminable losses reach values in various kinds of machinery which require that they should be taken into account.

Methods of measuring, approximating or allowing for these various losses are given below.

LOSSES TO BE TAKEN INTO ACCOUNT IN VARIOUS TYPES OF MACHINES

222. Continuous-current Commutating Motors and Generators.

No-load core-losses (Acc. Meas. or Deter.).

I^2R loss in windings (Acc. Meas. or Deter.).

Brush contact I^2R loss (Approx. Meas. or Deter.).

Unless otherwise specified, use the Institute Standard of 1 volt for contact drop per brush; — i.e., 2 volts for total brush drop — for either carbon or graphite brushes. See §§232 and **429.**

Friction of bearings and windage (Approx. Meas. or Deter.).

Rheostat losses, when present (Acc. Meas. or Deter.).

Brush friction (Approx. Meas. or Deter.).

All indeterminable load losses (including stray load iron losses) which may be important, which vary with the design, and for which no satisfactory method

* In the Table of §220 stray load-losses include *f, h, i, k, l* and *m*, but do not include increased core losses due to increased excitation for compensating internal drop under load.

of determination has been found, shall be included as zero per cent in estimating conventional efficiency.

223. Synchronous Motors and Generators.

No-load core-losses. (Acc. Meas. or Deter.)

I^2R loss in windings. (Acc. Meas. or Deter.) based upon rated kw. and power factor.

Stray load-losses. (Indeterminable). In approximating these losses, the method described in §236 shall be employed.

Friction of bearings and windage. (Approx. Meas. or Deter.) (Brush friction and brush-contact loss is negligible.)

Rheostat losses, when present (Acc. Meas. or Deter.) corresponding to rated kw. and power factor.

224. Induction Machines.

No-load core-losses. (Acc. Meas. or Deter.)

I^2R losses in windings. (Acc. Meas. or Deter.)

Stray load-losses. (Indeterminable.) In approximating these losses the method described in §240 shall be employed.

Brush friction when collector rings are present. (Approx. Meas. or Deter.)

Brush contact loss. (Approximately Meas. or Deter.) Unless otherwise specified, use the Institute Standard of 1 volt for contact drop per brush; — for either carbon or graphite brushes. See §232.

Friction of bearings and windage. (Approx. Meas. or Deter.)

225. Commutating A-C. Machines.

No-load core-losses. (Acc. Meas. or Deter.)

I^2R losses in windings. (Acc. Meas. or Deter.)

Brush friction. (Approx. Meas. or Deter.)

Brush contact loss. (Approx. Meas. or Deter.) Unless otherwise specified, use the Institute Standard of 1 volt for contact drop per brush; — for either carbon or graphite brushes. See §§232 and 429.

Friction of bearings and windage. (Approx. Meas. or Deter.)

Short-circuit loss of commutation. (Indeterminable.) Iron loss due to flux distortion. (Indeterminable.) Eddy-current losses due to fluxes varying with load and saturation. (Indeterminable.)	The Institute is not at this time prepared to make recommendations for approximating these losses.

226. Synchronous Converters.

No-load core-losses. (Acc. Meas. or Deter.)

I^2R losses in windings. (Approx. Meas. or Deter.) based on rated kw. and power factor.

Brush friction. (Approx. Meas. or Deter.)

Brush contact loss. (Approx. Meas. or Deter.) Unless otherwise specified, use the Institute Standard of 1 volt for contact drop per brush; — for either carbon or graphite brushes. See §232.

Short-circuit loss of commutation. (Indeterminable.) Iron loss due to flux distortion when present. (Indeterminate.) Eddy-current losses due to fluxes varying with load and saturation. (Indeterminable.)	These losses, while usually of low magnitude, are erratic, and the Institute is not at this time prepared to make recommendations for approximating them.

Friction of bearings and windage. (Approx. Meas. or Deter.)

For the booster type of synchronous converter, where the booster forms an integral part of the unit, its losses shall be included in the total converter losses in estimating the efficiency.

227. Transformers. — *No-load Losses.* These include the core-loss and the I^2R loss due to the exciting current (Acc. Meas. or Deter.) and the dielectric hysteresis loss in the insulation (Approx. Meas. or Deter.). At low power factors the total losses due to treating the exciting current in this way are likely to be too low.

Load-Losses.—These include I^2R loss in windings, and eddy-current losses in windings and core due to fluxes varying with load. (Approx. Meas. or Deter.) See §240 for the method of approximating these losses.

DETERMINATION OR APPROXIMATION OF LOSSES IN ROTATING MACHINERY

228. Bearing Friction and Windage may be determined as follows: Drive the machine from an independent motor, the output of which shall be suitably determined. The machine under test shall have its brushes removed and shall not be excited. This output represents the bearing friction and windage of the machine under test.

In the case of engine-type generators, one-half the output of the driving motor shall be charged against the generator for windage. The remainder, considered as bearing friction, shall be debited to the prime mover.

229. Brush Friction of Commutator and Collector Rings. — Follow the test of §228 taking an additional reading with the brushes in contact with the commutator or collector rings. The difference between the output obtained in the test in §228, and this output shall be taken as the brush friction. Note. — The surface of commutator and brushes should already be smooth and glazed from running when this test is made.

230. No-load Core-Loss. — Follow the test in §229 with an additional reading having the machine excited. The difference between the output value of §229 and the output value of this reading shall be taken as the no-load core-loss. This no-load core-loss shall be taken with the machine excited, so as to produce rated terminal voltage.

231. No-load Core-Loss at the Internal Voltage Corresponding to Rated Load. — This shall be taken as in §230, except that the machine shall be excited so as to produce at the terminals the voltage corresponding to the calculated internal voltage for the load and power factor under consideration. For synchronous machines, since no generally accepted method has been determined for obtaining the stator reactance, the internal voltage shall be determined by adding resistance drop to the terminal voltage.

232. Brush Contact Loss depends largely upon the material of which the brush is composed. As indicating the range of variation the following table will be of interest:

Grade of brush	Volts drop across one brush contact. (Average of positive and negative brushes)
Hard carbon	1.1
Soft carbon	0.9
Graphite	0.5 to 0.8
Metal-graphite types	0.15 to 0.5*

* The former for largest proportion of metal.

One volt drop per brush shall be considered as the Institute Standard drop corresponding to the I^2R brush-contact loss, for carbon and graphite brushes. Metal-graphite brushes, shall be considered as special. See §429.

233. **Field-rheostat Losses** which are normally present shall be included in the generator losses where there is a field rheostat in series with the field magnets of the generator, even when the machine is separately excited.

234. **Ventilating Blower.**—When a blower is supplied as part of the machine set, the power required to drive it shall be charged against the machine set; but not against the machine.

235. **Losses in Other Auxiliary Apparatus.** — Auxiliary apparatus, such as a separate exciter for a generator or motor, shall have its losses charged against the plant of which the generator and exciters are a part, and not against the generator.

236. **Stray Load-Losses in Synchronous Generators and Motors.** — These include iron losses and eddy-current losses in the copper due to fluxes varying with load and due to saturation.

Stray load-losses are to be determined by operating the machine on short-circuit and at rated-load current. This, after deducting the windage and friction and I^2R loss, gives the stray load-loss result thus obtained for polyphase generators and motors. These losses in single-phase machines are large; but the Institute is not yet prepared to give a method for measuring them.

237. **Stray Load-Losses in Induction Machines.** — These include eddy-current losses in the stator copper, and other eddy-current losses due to fluxes varying with the load. In wire-wound machines these are usually negligible.

With rotor removed and for a given stator current, measure the input through the stator at different frequencies. Plot a curve of loss against frequency. At low frequencies, the loss becomes constant, indicating the I^2R value. The difference between this I^2R value and the total loss at normal frequency shall be taken as the stray load-loss. This method is not accurate with induction motors in which the slots are entirely closed. In such machines these losses may be greater.

238. **Induction Motor Rotor I^2R Loss.** — This should be determined from the slip whenever the latter is accurately determinable, using the following equation:

$$\text{Rotor } I^2R \text{ loss} = \frac{\text{Output} \times \text{slip}}{1 - \text{slip}}.$$

In large slip-ring motors, in which the slip cannot be directly measured by loading, the rotor I^2R loss shall be determined by direct resistance measurement; the rotor full-load current to be calculated by the following equation:

$$\text{Current per ring} = \frac{\text{watts output.}}{\text{Rotor voltage at stand-still} \times \sqrt{3} \times K}.$$

This equation applies to three-phase rotors. For rotors wound for two phase use 2 instead of the $\sqrt{3}$. K may be taken as 0.95 for motors of 150 kw. or larger. The factor K usually decreases as the size of motor is reduced, but no specific value can be stated for smaller sizes.

DETERMINATION OR APPROXIMATION OF LOSSES IN TRANSFORMERS

239. **No-load Losses.** — These shall be measured with open secondary circuit at the rated frequency, and with an applied primary voltage giving the

rated secondary voltage plus the IR drop which occurs in the secondary under rated-load conditions. These no-load losses include core losses, consisting of hysteresis and eddy-current losses in the core, as well as dielectric loss in insulation due to electrostatic flux, which latter loss increases rapidly with temperature, and the test should therefore preferably be made at the reference temperature of 75° C.

240. **Stray Load-Losses.** — These shall be measured by applying a primary voltage sufficient to produce rated-load current in the primary and secondary windings, the latter being short-circuited. The stray load losses will then be equal to the input decreased by the measured I^2R losses in both windings. It is ordinarily immaterial whether the high-voltage or low-voltage winding is used as the primary winding in this test.

241. **Volt-ampere Ratio of Transformers.** — The volt-ampere ratio, which should not be confused with real efficiency, is the ratio of the volt-ampere output to the volt-ampere input of a transformer, at any given power factor.

242. **Methods of Loading Transformers for Temperature Tests.** — Wherever practicable, transformers should be tested under conditions that will give losses approximating as nearly as possible to those obtained under normal or specified load conditions, maintained for such a time as is necessary for the temperature to reach a steady value. The maximum temperature rises measured during this test should be considered as the observable temperature rises for the given load.

An approved method of making these tests is the "*loading back*" method. The principal variations of this method are:

243. (a) **With Duplicate Single-phase Transformers.** — Duplicate single-phase transformers may be tested in banks of two, with both primary and secondary windings connected in parallel. Normal magnetizing voltage should then be applied and the required current circulated from an auxiliary source. One transformer can be held under normal voltage and current conditions while the other may be operating under slightly abnormal conditions.

244. (b) **With One Three-phase Transformer.** — One three-phase transformer may be tested in a manner similar to (a), provided the primary and secondary windings are each connected in delta for the test. Normal three-phase magnetizing voltage should be applied and the required current circulated from an auxiliary single-phase source.

245. (c) **With Three Single-phase Transformers.** — Duplicate single-phase transformers may be tested in banks of three, in a manner similar to (b) by connecting both primary and secondary windings in delta and applying normal three-phase magnetizing voltage and circulating the required current from an auxiliary single-phase source.

246. **Note.** — Among other methods that have a limited application and can be used only under special conditions may be mentioned:

(1) Applying dead load by means of some form of rheostat.

(2) Running alternately for certain short intervals of time on open circuit and then on short-circuit, alternating in this way until the transformer reaches steady temperature.

In this test the voltage for the open-circuit interval and the current for the short-circuit interval shall be such as to give the same integrated core-loss, and the same integrated copper loss, as in normal operation.

DIELECTRIC TESTS OF MACHINERY

247. **Basis for Determining Test Voltages.** — The test voltage which shall be applied to determine the suitability of insulation for commercial operation is dependent upon the kind and size of the machinery, and its normal operating

voltage, upon the nature of the service in which it is to be used, and upon the severity of the mechanical and electrical stresses to which it may be subjected. The voltages and other conditions of test which are recommended have been determined as reasonable and proper for the great majority of cases and are proposed for general adoption, except when specific reasons make a modification desirable.

248. **Condition of Machinery to be Tested.**—Commercial tests shall, in general, be made with the completely assembled machinery and not with individual parts. The machinery shall be in good condition, and high-voltage tests, unless otherwise specified, shall be applied before the machine is put into commercial service, and shall not be applied when the insulation resistance is low owing to dirt or moisture. High-voltage tests shall be made at the temperature assumed under normal operation. High-voltage tests to determine whether specifications are fulfilled are admissible on new machines only. Unless otherwise agreed upon, high-voltage tests of a machine shall be understood as being made at the factory.

249. **Points of Application of Voltage.**—The test voltage shall be successively applied between each electric circuit and all other electrical circuits and metal parts grounded.

250. **Interconnected Polyphase Windings** are considered as one circuit. All windings of a machine, except that under test, shall be connected to ground.

251. **Frequency, Wave Form and Test Voltage.**—The frequency of the testing circuit shall not be less than the rated frequency of the apparatus tested. A sine-wave form is recommended. See §205. The test shall be made with alternating voltage having a crest value equal to the $\sqrt{2}$ times the specified test voltage.

252. **Duration of Application of Test Voltage.**—The testing voltage for all classes of apparatus shall be applied continuously for a period of 60 seconds.

253. **Apparatus for Use on Single-phase, 3-Phase-Delta or 3-Phase-Star Circuits.**—Apparatus, such as transformers, which may be used either on single-phase circuits, or in star or delta connection on three-phase circuits, shall have the maximum voltage of the circuits on which they may be used indicated on the rating plate and the test shall be based on such maximum voltage.

VALUES OF TEST VOLTAGES

254. **The Standard Test for all Classes of Apparatus, Except as Otherwise Specified, Shall be Twice the Normal Voltage of the Circuit to Which the Apparatus is Connected, Plus 1000 Volts.**

255. *Exception.*—**Alternating-current Apparatus connected to Permanently Grounded Single-phase Circuits, for use on Permanently Grounded Circuits of more than 3000 Volts** shall be tested with 2.73 times the voltage of the circuit to ground + 1000 volts.

256. *Exception.*—**Distributing Transformers.** Transformers for primary pressures from 550 to 5000 volts, the secondaries of which are directly connected to consumers' circuits and commonly known as distributing transformers, shall be tested with 10,000 volts from primary to core and secondary combined. The secondary windings shall be tested with twice their normal voltage plus 1000 volts.

257. *Exception.*—**Auto-Starter Transformers or Starting Compensators,** used for starting purposes, shall be tested with the same voltage as the test voltage of the apparatus to which they are connected.

258. *Exception.*—**Household Devices.** Apparatus taking not over 660* watts and intended solely for operation on supply circuits not exceeding 250 volts shall be tested with 900 volts.

259. *Exception.*—**Apparatus for Use on Circuits of 25 Volts or Lower,** such as bell-ringing apparatus,† electrical apparatus used in automobiles, apparatus used on low-voltage battery circuits, etc., shall be tested with 500 volts.

260. *Exception.*—**Field Windings of Alternating-current Generators** shall be tested with 10 times the exciter voltage, but in no case with less than 1500 volts.

261. *Exception.*—**Field Windings of Synchronous Machines,** including motors and converters requiring to be started from alternating-current circuits, shall be tested with 5000 volts, when the field is wound for 125 volts, and with 8000 volts when the field is wound for 250 volts or more. In no case shall the test voltage be lower than that given in §260.

262. *Exception.*—**Phase-wound Rotors of Induction Motors Required to Reverse in Service.** In order to allow for the extra voltage caused by the increased frequency at the instant of reversal, this test shall be four times stand-still voltage plus 1000.

263. *Exception.*—**Switches and Circuit Control Apparatus** above 600 volts, shall be tested with 2¼ times rated voltage plus 2000. See §§367 to 387.

264. *Exception.*—**Assembled Apparatus.** Where a number of pieces of apparatus are assembled together and tested as an electrical unit, they shall be tested with 15 per cent lower voltage than the lowest required on any of the individual pieces of apparatus.

265. **Testing Transformers by Induced Voltage.**—Under certain conditions it is permissible to test transformers by inducing the required voltage in their windings, in place of using a separate testing transformer. By required voltage, is meant a voltage such that the line end of the windings shall receive a test to ground equal to that required by the general rules.

266. **Transformers with Graded Insulation** shall be so marked. They shall be tested by inducing the required test-voltage in the transformer and connecting the successive line leads to ground.

MEASUREMENT OF VOLTAGE IN DIELECTRIC TESTS OF MACHINERY

267. **Use of Voltmeters and Spark-Gaps in Insulation Tests.**—When making insulation tests on electrical machinery, every precaution must be taken against the occurrence of any spark-gap discharges in the circuits from which the machinery is being tested. A non-inductive resistance of about one ohm per volt shall be inserted in series with one terminal of a spark gap. If the test is made with one electrode grounded, this resistance shall be inserted directly in series with the non-grounded electrode. If neither terminal is grounded, one-half shall be inserted directly in series with each electrode. In any case this resistance shall be as near the measuring gap as possible and not in series with the tested apparatus. This resistance will damp high-frequency oscillations at the time of breakdown and limit the current which will flow. A water tube is the most reliable resistance. Carbon resistors should not be used because their resistance becomes very low at high voltages.

* This rule does not include bell-ringing transformers of ratio 125 to 6 volts. See Code.
† The present Code power limit for a single outlet.

268. For Machinery of Low Capacitance.—When the machinery under test does not require sufficient charging current to distort the high-voltage wave shape, or change the ratio of transformation, the spark gap should be set for the required test voltage and the testing apparatus adjusted to give a voltage at which this spark gap just breaks down. This adjustment should be made with the apparatus under test disconnected. The apparatus should then be connected, and with the spark gap about 20 per cent longer, the testing apparatus is again adjusted to give the voltage of the former breakdown, which is the assumed voltage of test. This voltage is to be maintained for the required interval.

269. For Machinery of High Capacitance.—When the charging current of the machinery under test may appreciably distort the voltage wave, or change the effective ratio of the testing transformer referred to, the first adjustment of voltage with the gap set for the test voltage should be made with the apparatus under test connected to the circuit and in parallel with the spark gap.

When making arc-over tests of large insulators, leads, etc., partial arc-over of the tested apparatus may produce oscillations which will cause the measuring gap to discharge prematurely. The measured voltage will then appear too high. In such tests the "equivalent" ratio of the testing transformer should be measured by gap to within 20 per cent of the arc-over voltage of the tested apparatus with the tested apparatus in circuit. The measuring gap should then be greatly lengthened out and the voltage increased until the tested apparatus arcs over. This arc-over voltage should then be determined by multiplying the voltmeter reading by the equivalent ratio found above. Direct measurement of the spark-over voltage over one gap by another gap should always be avoided.

270. Measurements with Voltmeter.—In measuring the voltage with a voltmeter, the instrument should preferably derive its voltage from the high-tension circuit, either directly as with an electrostatic voltmeter, or through an auxiliary *ratio transformer*. It is permissible to measure the voltage at other places, such as the transformer primary, provided corrections can be made for the variations in ratio caused by the charging current of the machinery under test, or provided there is no material variation of this ratio. In any case, when the apparatus to be tested is insufficiently large in relation to the testing apparatus to cause wave distortion, the voltage must be checked by spark gap, as in §275.

A crest-voltage voltmeter has advantages over an r.m.s. voltage voltmeter in determining the maximum cyclic value of the testing e.m.f., since it eliminates the necessity for determining the crest factor of the e.m.f. wave.

271. Measurements with Spark Gaps.—If proper precautions are observed, spark gaps may be used to advantage in checking the calibration of voltmeters when set up for the purposes of high-pressure tests of the insulation of machinery.

272. Ranges of Voltages.—For such calibrating purposes:

The *Needle Spark-Gap* should preferably be used for voltages from 10 kv. to 50 kv. because of the larger air gaps involved.

A *Sphere Spark-Gap* should be used above 50 kv.

273. The Needle Spark-Gap.—The needle spark gap shall consist of new sewing needles supported axially at the ends of linear conductors which are at least twice the length of the gap. There must be a clear space around the gap for a radius of at least twice the gap length.

274. Sparking Distance.—The sparking distances in air between needle points for various root-mean-square sinusoidal voltages in mm. are as follows:

NEEDLE-POINT SPARK-OVER VOLTAGES WITH NO. OO SEWING NEEDLES

(At 25° C. and 760 mm. barometer)

R.M.S. kilovolts	Millimeters	R.M.S. kilovolts	Millimeters
10	11.9	40	62
15	18.4	45	75
20	25.4	50	90
25	33	60	118
30	41	70	149
35	51	80	180

The above values refer to a relative humidity of 80 per cent. Variations from this humidity may involve appreciable variations in the sparking distance.

275. **The Sphere Spark-Gap.**—The standard sphere spark-gap shall consist of two suitably mounted metal spheres.

No extraneous body or external part of the circuit shall be near the gap within twice the diameter of the spheres.

The shanks should not be greater in diameter than one-fifth the sphere diameter. Metal collars, etc., through which the shanks extend, should be as small as practicable and should not during any measurement come closer to the sphere than the maximum gap length used in that measurement.

The sphere diameter should not vary more than 0.1 per cent and the curvature, measured by a spherometer, should not vary more than 1 per cent from that of a true sphere of the required diameter.

All gaps are affected by barometric pressure and temperature, and if tests are not made at 25° C. and 760 millimeters barometer, appropriate corrections must be applied. The Institute is not at present prepared to make recommendations as to the amount of such correction.

The sparking distances between different spheres for various r.m.s. sinusoidal voltages will be assumed to be as follows:

276. SPHERE GAP SPARK-OVER VOLTAGES

(At 25° C. and 760 mm. barometer)

Kilo-volts	Sparking distance in millimeters					
	125-mm. spheres		250-mm. spheres		500-mm. spheres	
	One sphere grounded	Both spheres insulated	One sphere grounded	Both spheres insulated	One sphere grounded	Both spheres insulated
40	19.1	19.1				
50	24.4	24.4				
60	30	30	29	29		
70	36	36	35	35		
80	42	42	41	41	41	41
90	49	49	46	45	46	45
100	56	55	52	51	52	51
120		71	64	63	63	62
140		88	78	77	74	73
160		110	92	90	85	83
180			109	106	97	95
200			128	123	108	106
220			150	141	120	117
240			177	160	133	130
260					148	144
280					163	158
300					177	171
320					194	187
340					214	204
360					234	221

The sphere gap is more sensitive than the needle gap to momentary rises of voltage, and the voltage required to spark over the gap should be obtained by slowly closing the gap under constant voltage, or by slowly raising the voltage with a fixed setting of the gap.

INSULATION RESISTANCE OF MACHINERY

277. **The Insulation Resistance of a Machine** at its operating temperature shall be not less than that given by the following formula:
Insulation resistance in megohms =

$$\frac{\text{voltage at terminals}}{\text{rated capacity in kv-a.} + 1000}.$$

The formula only applies to dry apparatus. Such high values are not attainable in oil-immersed apparatus.

Insulation resistance tests shall, if possible, be made at a d-c. pressure of 500 volts. Since the insulation resistance varies with the pressure, it is necessary that, if a pressure other than 500 volts is to be employed in any case, this other pressure shall be clearly specified.

The order of magnitude of the values obtained by this rule is shown in the following table:

Rated voltage of machine	Megohms		
	100 kv-a.	1000 kv-a.	10,000 kv-a.
100	0.091	0.05	——
1,000	0.91	0.50	0.091
10,000	9.1	5.0	0.91
100,000	——	50	9.1

278. It should be noted that the insulation resistance of machinery is of doubtful significance by comparison with the dielectric strength. The insulation resistance is subject to wide variation with temperature, humidity and cleanliness of the parts. When the insulation resistance falls below that corresponding to the above rule, it can in most cases of good design, and where no defect exists, be brought up to the required standard by cleaning and drying out the machine. The insulation resistance test may therefore afford a useful indication as to whether the machine is in suitable condition for the application of the dielectric test.

REGULATION

DEFINITIONS

279. **Regulation.**—The regulation of a machine in regard to some characteristic quantity (such as terminal voltage or speed) is the change in that quantity occurring between any two loads. Unless otherwise specified, the two loads considered shall be zero load and rated load and at the temperature attained under normal operation. The regulation may be expressed by stating the numerical values of the quantity at the two loads, or it may be expressed by the "percentage regulation" which is the percentage ratio of the change in the quantity occurring between the two loads to the value of the quantity at either one or the other load, taken as the normal value. It is assumed that all parts of the machine affecting the regulation maintain constant temperature between the two loads, and where the influence of temperature is of consequence, a reference temperature of 75° C. shall be considered as standard. If change of temperature should occur during the tests the results shall be corrected to the reference temperature of 75° C.

The normal value may be either the no-load value, as the no-load speed of induction motors; or it may be the rated-load value as in the voltage of a-c. generators.

It is usual to state the regulation of d-c. generators by giving the numerical values of the voltage at no load and rated load, and in some cases it is advisable to state regulation at intermediate loads.

280. **The Regulation of D-C. Generators** refers to changes in voltage corresponding to gradual changes in load and does not relate to the comparatively large momentary fluctuations in voltage that frequently accompany instantaneous changes in load.

In determining the regulation of a compound-wound d-c. generator, two tests shall be made, one bringing the voltage down and the other bringing the voltage up between no load and rated load. These may differ somewhat, owing to residual magnetism. The mean of the two results shall be used.

281. In Constant-potential A-C. Generators, the regulation is the rise in voltage (when the specified load at specified power factor is thrown off) expressed in per cent of normal rated-load voltage.

282. In Constant-current Machines, the regulation is the ratio of the maximum difference of current from the rated-load value (occurring in the range from rated load to short-circuit, or minimum limit of operation), to the rated-load current.

283. In Constant-speed Direct-current Motors, and induction motors, the regulation is the ratio of the difference between full-load and no-load speeds to the no-load speed.

284. In Constant-potential Transformers, the regulation is the difference between the no-load and rated-load values of the secondary terminal voltage at the specified power factor (with constant primary impressed terminal voltage) expressed in per cent of rated-load secondary voltage, the primary voltage being adjusted to such a value that the apparatus delivers rated output at rated secondary voltage.

285. In Converters, Dynamotors, Motor-generators and Frequency Converters, the regulation is the change in the terminal voltage of the output side between the two specified loads. This may be expressed by giving the numerical values or as the percentage ratio.

286. In Transmission Lines, Feeders, etc., the regulation is the change in the voltage at the receiving end between rated non-inductive load and no load, with constant impressed voltage upon the sending end. The percentage regulation is the percentage change in voltage to the normal rated voltage at the receiving end.

287. In Steam Engines, Steam Turbines and Internal Combustion Engines, the percentage speed regulation is usually expressed as the percentage ratio of the maximum variation of speed to the rated-load speed in passing slowly from rated load to no load (with constant conditions at the supply).

288. Fluctuation. — If the test is made by passing suddenly from rated load to no load, the immediate percentage speed regulation so derived shall be termed the fluctuation.

289. In a Hydraulic Turbine, or other water motor, the percentage speed regulation is expressed as the percentage ratio of the maximum variation in speed in passing slowly from rated load to no load (at constant head of water), to the rated-load speed.

290. In a Generator Unit consisting of a generator combined with a prime mover, the speed or voltage regulation should be determined at constant conditions of the prime mover, i.e., constant steam-pressure, head, etc. It includes the inherent speed variations of the prime mover. For this reason the regulation of a generator unit is to be distinguished from the regulation of either the prime mover, or of the generator combined with it, when taken separately.

CONDITIONS FOR TESTS OF REGULATION

291. Speed and Frequency. — The regulation of generators is to be determined at constant speed, and of alternating-current apparatus at constant frequency.

292. Power Factor. — In apparatus generating, transforming or transmitting alternating currents, the power factor of the load to which the regulation refers should be specified. Unless otherwise specified, it shall be understood as referring to non-inductive load, that is to a load in which the current is in phase with the e.m.f. at the output side of the apparatus.

293. **Wave Form.** — In the regulation of alternating-current machinery receiving electric power, a sine wave of voltage is assumed, except where expressly specified otherwise. See §205.

294. **Excitation.** — In commutating machines, rectifying machines and synchronous machines, such as direct-current generators and motors, as well as in alternating-current generators, the regulation is to be determined under the following conditions, so as to maintain the field adjustment constant at that which gives rated-load voltage at rated-load current.

(1) In the case of separately excited field magnets, constant excitation.

(2) In the case of shunt machines, constant resistance in the shunt-field circuit.

(3) In the case of series or compound machines, constant resistance shunting the series-field windings.

295. **Tests and Computation of Regulation of A-C. Generators.** — Any one of the three following methods may be used. They are given in the order of preference.

Method (a). — The regulation can be measured directly by loading the generator at the specified load and power factor, then reducing the load to zero, and measuring the terminal voltage, with speed and excitation adjusted to the same values as before the change. This method is not generally applicable for shop tests, particularly on large generators, and it becomes necessary to determine the regulation from such other tests as can be readily made.

296. **Method (b).** — This consists in computing the regulation from experimental data of the open-circuit saturation curve and the zero-power factor saturation curve. The latter curve, or one approximating very closely to it, can be obtained by running the generator with over-excited field on a load of idle-running under-excited synchronous motors. The power factor under these conditions is very low and the load saturation curve approximates very closely the zero power factor saturation curve. From this curve and the open circuit curve, points for the load saturation curve for any power factor can be obtained by means of vector diagrams.

To apply method (b), it is necessary to obtain from test, the open-circuit saturation curve OA, Fig. 1, and the saturation curve BC at zero power factor and rated-load current. At any given excitation Oc, the voltage that would be induced on open circuit is ac, the terminal voltage at zero power factor is bc, and the apparent internal drop is ab. The terminal voltage dc at any other power factor can then be found by drawing an e.m.f. diagram * as in Fig. 2, where ϕ is an angle such that $\cos\phi$ is the power factor of the load, be the resistance drop (IR) in the stator winding, ba the total internal drop and ac the total induced voltage, ba and ac being laid off to correspond with the values obtained from Fig. 1. The terminal voltage at power factor $\cos\phi$ is then cb

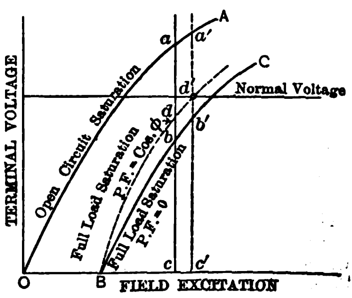


Fig. 1

* Method b, for deducing the load saturation curve, at any assigned power factor, from no-load and zero-power-factor saturation curves obtained by test, must be regarded as empirical. Its value depends upon the fact that experience has demonstrated the reasonable correctness of the results obtained by it.

of Fig. 2 which, laid off in Fig. 1, gives point d. By finding a number of such points, the curve Bdd' for power factor $\cos\phi$ is obtained and the regulation at this power factor (expressed in per cent) is $\frac{100 \times a'd'}{d'c'}$, since $a'd'$ is the rise in voltage when the load at power factor $\cos\phi$ is thrown off at normal voltage $c'd'$.

Generally, the ohmic drop can be neglected, as it has very little influence on the regulation, except in very low-speed machines, where the armature resistance is relatively high, or in some cases where regulation at unity power factor is being estimated; for low power factors, its effect is negligible in practically all cases. If resistance is neglected, the simpler e.m.f. diagram, Fig. 3, may be used to obtain points on the load saturation curve for the power factor under consideration.

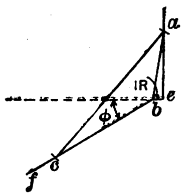


Fig. 2

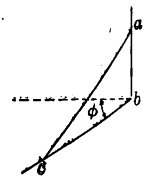


Fig. 3

297. Method (c).—Where it is not possible to obtain by test a zero-power-factor saturation curve as in (b), this curve can be estimated closely from open-circuit and short-circuit curves, by reference to tests at zero power factor on other machines of similar magnetic circuit. Having obtained the estimated zero-power-factor curve, the load saturation for any other power factor is obtained as in (b).

Thus Method (c) is the same as (b), except that the zero-power-factor curve must be estimated. This may be done as follows: In Fig. 4 OA is the open-circuit saturation curve and OE the short-circuit line as shown by test. The zero-power factor curve corresponding to any given current BF will start from point B and for machines designed with low saturation and low reactance will follow parallel to OA, as shown by the dotted curve BD which is OA shifted parallel to itself by the distance OB. In high-speed machines, or in others having low reactance and a low degree of saturation in the magnetic circuit, the zero-power-factor curve will lie quite close to BD, particularly in those parts that are used for determining the regulation. This is the case with many turbo-generators or high-speed water-wheel generators. In many cases, however, the zero-power-factor curve will deviate from BD, as shown by BC, and the deviation will be most pronounced in machines of high reactance, high saturation and large magnetic leakage. The position of the actual curve BC with relation to BD can be approximated with sufficient exactness by investigating the corresponding relations as obtained by test at zero power factor on machines of similar characteristics and magnetic circuit. Or curve BC can be calculated by methods based on the results of tests at zero power factor. After BC has been obtained, the saturation curve and regulation for any other power factor can be derived as in Method (b).

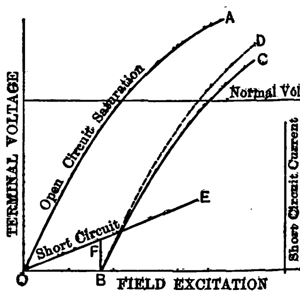


Fig. 4

298. **Tests and Computation of Regulation for Constant-potential Transformers.**—The regulation can be determined by loading the transformer and measuring the change in voltage with change in load at the specified power factor. This method is not generally applicable for shop tests, particularly on large transformers.

The regulation for any specified load and power factor can be computed from the percentage resistance drop and percentage reactance drop in the windings as follows:

299. To compute the regulation of a constant-potential transformer, it is necessary to obtain the equivalent resistance R and impedance drop E_z. The equivalent resistance R of primary and secondary combined is found by multiplying the secondary resistance by the square of the ratio of turns and adding it to the primary resistance. The impedance voltage E_z is found by short-circuiting the secondary winding and measuring the volts necessary to send rated-load current through the primary.

300. The reactance drop is then

$$IX = \sqrt{E_z^2 - \left(\frac{P}{I}\right)^2}$$

where P = impedance watts as measured in the short-circuit test.

Let

E = rated primary voltage,
IR = resistance drop in volts,
IX = reactance drop in volts,

$$q_r = 100\,\frac{IR}{E} = \text{per cent resistance drop,}$$

$$q_x = 100\,\frac{IX}{E} = \text{per cent reactance drop.}$$

301. Then

(1) For unity power factor,

$$\text{Per cent regulation} = q_r + \frac{q_x^2}{200}.$$

302. (2) For inductive loads of power factor m and reactive factor n,

$$\text{Per cent regulation} = mq_r + nq_x + \frac{(mq_x - nq_r)^2}{200}.$$

TRANSFORMER CONNECTIONS, SINGLE-PHASE TRANSFORMER

303. **Marking of Leads.**—The leads of single-phase transformers shall be distinguished from each other by marking the high-voltage leads with the letters A and B, and the low-voltage leads with the letters X and Y. They shall be so marked that the potential difference between A and B shall have the same direction at any instant as the potential difference between X and Y.

In accordance with the above rule, the terminals of single-phase transformers shall be marked as follows:

304. (1) High- and low-voltage windings in phase:

$$\begin{array}{c} A \text{———} B \\ X \text{———} Y \end{array}$$

305. (2) High- and low-voltage windings 180 deg. apart in phase:

$$\begin{matrix} A \text{———} B \\ Y \text{———} X \end{matrix}$$

306. To operate transformers thus marked in parallel, it is only necessary to connect similarly marked terminals together (provided that the reactances and resistances of the transformers are such as to permit of parallel operation).

307. Single-phase Transformers with More than Two Windings.—Transformers possessing three or more windings (each being provided with separate outgoing leads) shall have the leads connected to two of their windings, lettered in accordance with the preceding paragraph. The remaining leads shall be distinguished from the others by a subscript. For example, transformers possessing four secondary leads connected to two distinct similar windings for multiple-series operation, shall be lettered as follows:

$$\begin{matrix} A \text{———} B \\ \left\{\begin{matrix} X \text{———} Y \\ X_1 \text{———} Y_1 \end{matrix}\right\} \end{matrix}$$

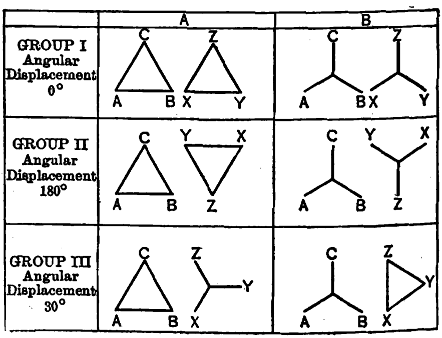
This indicates that the low-voltage winding consists of two disconnected parts, one part having terminals XY and the other part having terminals X_1Y_1. For multiple connection, X and X_1 are connected together and Y and Y_1 are connected together. For series connection, Y is connected to X_1.

308. Neutral Lead.—An outgoing 50 per cent (neutral) tap lead should be lettered N.

309. Internal Connections.—The manufacturer shall furnish a complete diagrammatic sketch of internal connections, and all taps and terminals of the transformer shall be marked to correspond with numbers or letters in the sketch.

THREE-PHASE TRANSFORMERS

310. Three-phase transformers ordinarily have three or four leads for high-voltage, and three or four leads for low-voltage windings. To distinguish the various leads from each other, and also to distinguish between the various phase relations obtainable, the three high-voltage leads should be lettered ABC and the three low-voltage leads XYZ. In addition, it should be distinctly stated in which of the three groups given in the accompanying diagram the transformer belongs.



The rules given above for single-phase transformers in regard to the neutral tap, and also in regard to internal connections, are applicable to three-phase transformers.

311. Angular Displacement.—The angular displacement between high- and low-voltage windings is the angle in the accompanying diagram, the lines passing from a neutral point through A and X respectively. Thus, in Group I,

the angular displacement is zero degrees. In Group 2, the angular displacement is 180°, and in Group 3 the angular displacement is 30°.

312. **Parallel Operation of Three-phase Transformers.** — Three-phase transformers, lettered in accordance with the above rules, will operate correctly in parallel, if their percentage resistance drops are equal, and their percentage reactance drops, at their rated loads, are equal. It is furthermore necessary that the angular displacements between high-voltage and low-voltage windings shall be equal, i.e., that the transformers shall belong to the same group in the accompanying diagram. It is then only necessary to connect together similarly marked leads.

INFORMATION TO BE GIVEN ON THE RATING PLATE OF A MACHINE*

313. (a) It is recommended that the rating plate of machines which comply with the Institute rules shall carry a distinctive special sign, such as "A.I.E.E. Rating."

(b) The absence of any statement to the contrary on the rating plate of a machine implies that it is intended for continuous service and for the standard ambient temperature of reference. See §§153 and 157.

(c) The rating plate of a machine intended to work under various kinds of rating must carry the necessary information in regard to those kinds of ratings.

(d) The rating plate, in addition to the name of the manufacturer and the serial number, must give the following information. See also §§391 and 415.

314. **Generator, Direct-current.**
Shunt, series, or compound.
Output, in kw., with statement as to the kind of rating.
Terminal pressure, in volts.
Current, in amperes.
Speed, in revolutions per minute.

315. **Motor, Direct-current.**
Shunt, series or compound.
Output, in kw., with statement as to the kind of rating.
Terminal pressure, in volts.
Current, approximate, in amperes.
Speed, in revolutions per minute.

316. **Transformer.**
Frequency, in cycles per second.
Number of phases.
Output at the secondary terminals in kv-a., with statement as to the kind of rating.
Primary pressure, in volts.
Secondary pressure, in volts. See §110.
Lead markings and diagram of internal connections as stated in §§303 to 312.

317. **Alternator.**
Frequency, in cycles per second.
Number of poles.
Number of phases.
Output, in kv-a., with statement as to the kind of rating and power factor corresponding to rated output.
Pressure between terminals, in volts, corresponding to the rated output.

*Information, for which space on the rating plate cannot be provided, shall be furnished on a subsidiary rating certificate.

Current in amperes.
Speed in revolutions per minute.
Excitation pressure, in volts.
Maximum exciting current, in amperes, required to maintain rated voltage under rated load.

318. **Synchronous Motor.**
Frequency, in cycles per second.
Number of poles.
Number of phases.
Mechanical output, in kw., with statement as to the kind of rating.
Pressure between terminals, in volts, corresponding to the rated output.
Current in amperes.
If the motor is intended to work with a power factor different from unity the necessary information shall be given.
Speed, in revolutions per minute.
Excitation pressure, in volts.
Maximum exciting current, in amperes, required to maintain rated power factor under rated load.

319. **Synchronous Converter.**
Frequency in cycles per second.
Number of poles.
Shunt or compound.
Number of phases.
Output at commutator in kilowatts, with statement as to kind of rating. In case of a converter for railroad service, both nominal and continuous ratings.
A-c. terminal pressure in volts.
D-c. terminal pressure in volts.
Current from commutator in amperes.
Speed in revolutions per minute.

320. **Induction Motor.**
Frequency, in cycles per second.
Number of poles.
Number of phases.
Mechanical output, in kw., with statement as to the kind of rating.
Pressure between terminals, in volts.
Current, in amperes.
Speed, in revolutions per minute, at rated output.
Secondary pressure (initial) when starting.

STANDARDS FOR WIRES AND CABLES

321-337. **Terminology.**—Same as in *Circular No. 37 of the Bureau of Standards. Given in full in the article on Wires and Cables, Insulated.*

SPECIFICATION OF SIZES OF CONDUCTORS

338. The sizes of solid wires shall be stated by their diameter in mils, the American Wire Gage (Brown and Sharpe) sizes being taken as standard. The sizes of stranded conductors shall be stated by their cross-sectional area in circular mils. For brevity, in cases where the most careful specification is not required, the sizes of solid wires may be stated by the gage number in the American Wire Gage, and the sizes of stranded conductors smaller than 250,000 circular mils (i.e., No. 0000 A.W.G. or smaller) may likewise be

stated by means of the gage number in the American Wire Gage of a solid wire having the same cross-sectional area. Furthermore, an exception is made in the case of "Flexible Stranded Conductors," for which see §341 below. In stating large cross-sections, it is sometimes convenient to use a circular inch (507 sq. mm.) instead of 1,000,000 circular mils.

STRANDING

339. The Standard Concentric Stranding Table printed in Circulars 31 and 37 of the Bureau of Standards, is adopted.

STANDARDIZATION OF CONCENTRIC STRANDING

Range of sizes		Number of wires. Standard concentric-lay cables
Circular mils	Sq. mm.	
2,000,000 to 1,600,000	1015 to 810	127
1,500,000 to 1,100,000	760 to 560	91
1,000,000 to 550,000	507 to 280	61
500,000 to 250,000	253 to 127	37
No. 0000 to No. 1 A.W.G.	107 to 42	19
No. 2 and smaller	33	7

340. **Sectional Area of Cables.**—The cross-sectional area of a cable shall be considered to be the sum of the cross-sectional areas of its component wires, when laid out straight and measured perpendicular to their axes.

341. **Flexible Stranding.**—Conductors of special flexibility should ordinarily be made with wires of regular A.W.G. sizes, the number of wires and size being given. The approximate gage number or approximate circular mils of such flexible stranded conductors may be stated.

342. **Correction for Lay.**—The resistance and mass of a stranded conductor are greater than in a solid conductor of the same cross-sectional area, depending on the lay (i.e., the pitch of the twist of the wires). Two per cent shall be taken as the standard increment of resistance and of mass. In cases where the lay is definitely known, the increment should be calculated and not assumed.

The direction of lay is the lateral direction in which the strands of a cable run over the top of the cable as they recede from an observer looking along the axis of the cable.

CONDUCTIVITY OF COPPER

343. The following I.E.C. rules are adopted:*

The following shall be taken as normal values for standard annealed copper:

(1) At a temperature of 20° C., the resistance of a wire of standard annealed copper one meter in length and of a uniform section of 1 square millimeter is 1/58 ohm = 0.017241... ohm.

(2) At a temperature of 20° C. the density of standard annealed copper is 8.89 grams per cubic centimeter.

* See I. E. C. Publication No. 28 "International Standard of Resistance for Copper," March, 1914.

(3) At a temperature of 20° C., the "constant mass" temperature coefficient of resistance of standard annealed copper, measured between two potential points rigidly fixed to the wire, is 0.00393 = 1/254.45...per degree centigrade.

(4) As a consequence, it follows from (1) and (2) that, at a temperature of 20° C. the resistance of a wire of standard annealed copper of uniform section, one meter in length and weighing one gram, is $(1/58) \times 8.89 = 0.15328\ldots$ ohm.*†

344. Copper Wire Tables.—The copper-wire tables published by the Bureau of Standards in Circular No. 31 are adopted. These Tables are based upon the I.E.C. rules stated in §343.

HEATING AND TEMPERATURE OF CABLES

345. Maximum Safe Limiting Temperatures.—The maximum safe limiting temperature in degrees Cent. at the surface of the conductor in a cable shall be:

For impregnated paper insulation	$(85 - E)$,
For varnished cambric	$(75 - E)$,
For rubber insulation	$(60 - 0.25\,E)$,

where E represents the r.m.s. operating e.m.f. in kilovolts between conductors.

Thus, at a working pressure of 3.3. kv., the maximum safe limiting temperature at the surface of the conductor or conductors in a cable would be:

For impregnated paper	81.7° C,
For varnished cambric	71.7° C,
For rubber	59.2° C.

ELECTRICAL TESTS OF CABLES

346. Lengths Tested.—Electrical tests of insulation on wires and cables shall be made on the entire length to be shipped.

347. Immersion in Water.—Electrical tests of insulated conductors not enclosed in a lead sheath shall be made while immersed in water after an immersion of twelve (12) hours, if insulated with rubber compound, or if insulated with varnished cambric. It is not necessary to immerse in water insulated conductors enclosed in a lead sheath.

In multiple-conductor cables, without waterproof over-all jacket of insulation, no immersion test should be made on finished cables, but only on the individual conductors before assembling.

348. Dielectric-strength Tests. Object of Tests.—Dielectric tests are intended to detect weak spots in the insulation and to determine whether the dielectric strength of the insulation is sufficient to enable it to withstand the voltage it is likely to be subjected to, in service, with a suitable factor of assurance.

The initially-applied voltage must not be greater than the working voltage, and the rate of increase shall not be over 100 per cent in 10 seconds.

* Paragraphs (1) and (4) of §343 define what are sometimes called "volume resistivity," and "mass-resistivity" respectively. This may be expressed in other units as follows:—volume resistivity = 1.7241 microhm-cm. (or microhms in a cm. cube) at 20° C. = 0.67879 microhm-inch at 20° C., and mass resistivity = 875.20 ohms (mile, pound) at 20° C.

† For detailed specifications of commercial copper, see the "Standard Specifications" of the American Society for Testing Materials.

349. **Factor of Assurance.** — The factor of assurance of wire or cable insulation shall be the ratio of the voltage at which it is tested to that at which it is used.

350. **Test Voltage.** — The dielectric strength of wire and cable insulation shall be tested at the factory by applying an alternating test voltage between the conductor and sheath or water.

351. **The Magnitude and Duration of the Test Voltage** should depend upon the dielectric strength and thickness of the insulation, the length and diameter of the wire or cable, and the assurance factor required, the latter in turn depending upon the importance of the service in which the wire or cable is employed.

The following test voltages shall apply unless the departure is considered necessary in view of the above circumstances. Rubber-covered wires or cable for voltages up to 7 kv. shall be tested in accordance with the National Electric Code. Standardization for higher voltages for rubber insulated cables is not considered possible at the present time.

Varnish cambric and impregnated paper insulated wires or cables shall be tested at the place of manufacture for five (5) minutes in accordance with the table given below.

Different engineers specify different thickness of insulation for the same working voltages. Therefore, at the present time the test kv. corresponding to working kv. given in the table below are based on the *minimum* thickness of insulation specified by engineers and operating companies.

RECOMMENDED TEST KILOVOLTS CORRESPONDING TO OPERATING KILOVOLTS

Operating kv.	Test kv.	Operating kv.	Test kv.
0.5	2.5*	5	14
0.5	3	10	25
1	4	15	35
2	6.5	20	44
3	9	25	53
4	11.5		

* For minimum insulation thickness of 1/16 inch.

352. **The Frequency of the Test Voltage** shall not exceed 100 cycles per second, and should approximate as closely as possible to a sine wave. The source of energy should be of ample capacity.

353. **Where Ultimate Break-down Tests** are required, these shall be made on samples not more than 6 meters (20 ft.) long. The maximum allowable temperature at which the test is made for the particular type of insulation and the particular working pressure, shall not be greater than the temperature limits given in §345.

354. **Multiple-conductor Cables.** — Each conductor of a multiple-conductor cable shall be tested against the other conductors connected together with the sheath or water.

INSULATION RESISTANCE

355. **Definition.** — The insulation resistance of an insulated conductor is the electrical resistance offered by its insulation, to an impressed voltage tending to produce a leakage of current through the same.

356. Insulation resistance shall be expressed in megohms for a specified length (as for a kilometer, or a mile, or one thousand feet), and shall be corrected to a temperature of 15.5° C. using a temperature coefficient determined experimentally for the insulation under consideration.

357. Linear insulation resistance, or the insulation resistance of unit length, shall be expressed in terms of the megohm-kilometer, or the megohm-mile, or the megohm-thousand-feet.

358. Megohms Constant.—The megohms constant of an insulated conductor shall be the factor "K" in the equation

$$R = K \log_{10} \frac{D}{d}$$

where R = the insulation resistance, in megohms, for a specified unit length, D = outside diameter of insulation, d = diameter of conductor. Unless otherwise stated, K will be assumed to correspond to the mile unit of length.

359. Test.—The apparent insulation resistance should be measured after the dielectric-strength test, measuring the leakage current after a one-minute electrification, with a continuous e.m.f. of from 100 to 500 volts, the conductor being maintained positive to the sheath or water.

360. Multiple-conductor Cables.—The insulation resistance of each conductor of a multiple-conductor cable shall be the insulation resistance measured from such conductor to all the other conductors in multiple with the sheath or water.

CAPACITANCE OR ELECTROSTATIC CAPACITY

361. Capacitance is ordinarily expressed in microfarads. Linear capacitance, or capacitance per unit length, shall be expressed in microfarads per unit length (kilometer, or mile, or one thousand feet) and shall be corrected to a temperature of 15.5° C.

362. Microfarads Constant.—The microfarads constant of an insulated conductor shall be the factor "K" in the equation.

$$C = \frac{K}{\log_{10} \frac{D}{d}},$$

where C = the capacitance in microfarads per unit length, D = the outside diameter of insulation, d = the diameter of conductor. Unless otherwise stated, K will be assumed to refer to the mile unit of length.

363. Measurement of Capacitance.

For Low-voltage Cable.—The capacitance shall be measured by comparison with a standard condenser. For long lengths of high-voltage cables, where it is necessary to know the true capacitance, the measurement should be made at a frequency approximating the frequency of operation.

364. Paired Cables.—The capacitance shall be measured between conductors of pairs, the other wires being connected to the sheath or ground.

365. Electric Light and Power Cables.—The capacitance of low-voltage cables is generally of but little importance. The capacitance of high-voltage cables should be measured between the conductors and also between each conductor and the other conductors connected to the lead sheath or ground.

366. **Multiple-conductor Cables** (not paired). The capacitance of each conductor of a multiple conductor cable shall be the capacitance measured from such conductor to all of the other conductors in multiple with the sheath or the ground.

RATING AND TESTING OF SWITCHES AND OTHER CIRCUIT-CONTROL APPARATUS

SWITCHES

367. The following rules apply to **Switches** of above 600 volts. (For 600 volts and below, see Code.*)

Definition. — A device for making, breaking or changing connections in an electric circuit.

368. **Rating.** — (a) By amperes to be carried with not more than 30° C. rise on contacts and current-carrying parts. (b) By normal voltage of circuit on which it may be used.

369. **Performance and Tests.**

(a) **Heating Test** with rated current applied continuously until temperature is constant; ambient temperature 40° C.

(b) **Dielectric Test** at 2¼ times rated voltage plus 2000. See §263.

CIRCUIT BREAKERS

370. **Definition.** — A device designed to open a current-carrying circuit without injury to itself. A circuit breaker † may be:

(a) An automatic circuit breaker, which is designed to trip automatically under any predetermined condition of the circuit, such as an underload or overload of current or voltage.

(b) A manually-tripped circuit breaker, which is designed to be tripped by hand.

Both types of operation may be combined in one and the same device.

371. **Rating.** — (a) By normal current-carrying capacity. (b) By normal voltage. (c) By amperes which it can interrupt at normal voltage of the circuit.

372. **Performance and Tests.** — The heating test shall be made with normal current; in oil circuit breakers, same oil must be used for heating tests as for rupturing tests. Rise on contacts not to exceed 30° C. Rise on tripping solenoids and accessory parts not to exceed 50° C. Ambient temperature of reference, 40° C.

373. **Dielectric Test.** Same as §369.

374. **Rupturing Test** must be made with the current specified under §371 (c), and at normal voltage.

NOTE. — Although circuit breakers should be considered as devices alone, no account being taken, in the rating, of the system on which they are to be used; yet in applying circuit breakers to any given service, it may be necessary to take into account the system on which they are to be used, with all its characteristics.

* By the term "Code" is meant "National Electrical Code" as recommended by the National Fire Protection Association.

† These rules refer only to circuit breakers of above 550 volts For 550 volts and below, see Code.

Allowances must be made for the reactance, resistance, etc., of the circuit to be controlled, as these have a direct bearing on the maximum current flow.

In some systems it has been found that the pressure rises so high during switching that higher insulation tests than that specified in §369 should be given.

FUSES

(For circuits up to and including 600 volts, see Code.)

NOTE. — Complete standardization of these fuses above 600 volts according to the method of the Code is not advisable at this time, but is expected to be accomplished by an eventual extension of the Code. Until such extension is made the following definitions and ratings may be followed.

375. **Definition.** — An element designed to melt or dissipate at a predetermined current value and intended to protect against abnormal conditions of current.

NOTE. — The terminals, tubes, etc., which go with the fuse proper are included in the definition.

376. **Rating.** — Fuses shall be rated at the maximum current which they are required to carry continuously and at the normal voltage of the circuit on which they are designed to be used.

Fuses may be divided into two classes:

(1) Those designed to protect the circuit and apparatus both against short-circuit and against definite amounts of overload (e.g. fuses of the code which open on 25 per cent overload).

(2) Those designed to protect the system only against short-circuits; (e.g. expulsion fuses which blow at several times the current which they are designed to carry continuously). The line separating these two classes is not definitely fixed.

377. **Temperature.** — Coils or windings (such as accompany fuses of the magnetic blow-out type) should not exceed the limits set for machine coils having the same character of insulation. (See §§188 and 191.) The highest temperature for the fuse proper should not exceed the safe limit for the material employed (e.g. the temperature of the fiber tube of an enclosed fuse should not exceed the safe limit for this material, but an open-link metal fuse may be run at any temperature which will not injure the fuse material; except that no application of the above rule shall contravene the Code).

378. **Test.** — For fuses intended for use on circuits of small capacity, or in protected positions on systems of large capacity, see Code. For large power fuses intended for service similar to that required of circuit breakers, see §§370 to 374, or the Code as far as the latter applies.

LIGHTNING ARRESTERS

379. **Definition.** — A device for protecting circuits and apparatus against lightning or other abnormal potential rises of short duration.

380. **Rating.** — Arresters shall be rated by the voltage of circuit on which they are to be used.

Lightning arresters may be divided into two classes:

(a) Those intended to discharge for a very short time;

(b) Those intended to discharge for a period of several minutes.

381. **Performance and Tests.** Dielectric test same as §369.

The resistance of the arrester at double potential and also at normal potential, determined by observing the discharge currents through the arrester.

(c) In the case of any arrester using a gap, a test shall be made of the spark potential on either direct-current or 60-cycle a-c. excitation.

(d) The equivalent sphere gap under disruptive discharge shall also be measured, using a considerable quantity of electricity.

(e) The endurance of the arrester shall be tested to continuous surges.

PROTECTIVE REACTANCES

382. **Definition.** — A reactance for protecting circuits by limiting the current flow and localizing the disturbance under short-circuit conditions.

383. **Rating.**

(a) In kilovolt-amperes absorbed by normal current.

(b) By the normal current, frequency and line (delta) voltage for which the reactance is designed.

(c) By the current which the device is required to stand under short-circuit conditions.

384. **Performance and Tests.**

The Heat Test should be made with normal current and frequency applied until the temperature is constant. The temperature should not exceed the safe limits for the materials employed. See §§188 and 191.

385. **Dielectric Test.** — 2¼ times line voltage plus 2000, for one minute, from conductor to ground.

NOTE. — The reactance shall be so designed as to be capable of withstanding, without mechanical injury, rated current at normal frequency, suddenly applied.

RESISTOR OR RHEOSTAT

386. **Definition.** — Any device commonly known as a resistance used for operation or control. See Code.

INSTRUMENT TRANSFORMERS

387. **Definition.** — A transformer for use with measuring instruments, in which the conditions in the primary circuit as to current and pressure are represented with high numerical accuracy in the secondary circuit.

Under this heading and for more general use.

(a) A current transformer is a transformer designed for series connection in its primary circuit with the ratio of transformation appearing as a ratio of currents.

(b) A potential (voltage) transformer is a transformer designed for shunt or parallel connection in its primary circuit, with the ratio of transformation appearing as a ratio of potentials (voltages).

For further definitions relative to instrument transformers see §§111-114.

STANDARDS FOR ELECTRIC RAILWAYS

DEFINITIONS

388. **Transmission System.** — When the current generated for an electric railway is changed in kind or voltage, between the generator and the cars or locomotives, that portion of the conductor system carrying current of a kind

or voltage substantially different from that received by the cars or locomotives, constitutes the *transmission system.**

389. **Distribution System.**—That portion of the conductor system of an electric railway which carries current of the kind and voltage received by the cars or locomotives, constitutes the *distribution system.**

390. **Substation.**—A substation is a group of apparatus or machinery which receives current from a transmission system, changes its kind or voltage, and delivers it to a distribution system.

RATING OF SUBSTATION MACHINERY

391. **Nominal Rating of Substation Machinery.**—The nominal rating of a substation machine shall be the maximum output at 100 per cent power factor, which, having produced a constant temperature, may be increased 50 per cent for two hours without exceeding the standard ultimate temperature rise.

Substation machines shall be capable of carrying a load of twice their nominal rated load, for a period of five minutes, without disqualifying them for continued service. They shall also be capable of carrying a load of three times the nominal rated load for one minute. These overloads shall be applied after a continuous run at nominal rated load.

392. **Continuous Rating.**—The continuous rating of a substation machine shall be that load, at 100 per cent power factor, which it will carry continuously with a temperature rise not exceeding that set forth in §188, and fulfilling the other requirements set forth in these rules and summarized in §132.

CONDUCTOR AND RAIL SYSTEM

393. **Contact Conductors.**—That part of the distribution system other than the traffic rails, which is in immediate electrical contact with the circuits of the cars or locomotives, constitutes the contact conductors.

394. **Contact Rail.**—A rigid contact conductor.

395. **Overhead Contact Rail.**—A contact rail above the elevation of the maximum equipment line.†

396. **Third Rail.**—A contact conductor placed at either side of the track, the contact surface of which is a few inches above the level of the top of the track rails.

397. **Center Contact Rail.**—A contact conductor placed between the track rails, having its contact surface above the ground level.

398. **Underground Contact Rail.**—A contact conductor placed beneath the ground level.

399. **Gage of Third Rail.**—The distance, measured parallel to the plane of running rails, between the gage line of the nearer track rail and the inside gage line of the *contact surface* of the third rail.

400. **Elevation of Third Rail.**—The elevation of the contact-surface of the third rail, with respect to the plane of the tops of running rails.

* These definitions are identical in sense, although not in words, with those of the Interstate Commerce Commission, as given in their Classification of Accounts for Electric Railways.

† The contour which embraces cross sections of all rolling stock under all normal operating conditions.

401. **Standard Gage of Third Rails.**—The gage of third rails shall be not less than 26 inches (66 cm.) and not more than 27 inches (68.6 cm.).

402. **Standard Elevation of Third Rails.**—The elevation of third rails shall be not less than 2¾ inches (68.9 mm.) and not more than 3½ inches (89 mm.).

403. **Third-rail Protection.**—A guard for the purpose of preventing accidental contact with the third rail.

404. **Trolley Wire.**—A flexible contact conductor, customarily supported above the cars.

405. **Messenger Wire or Cable.**—A wire or cable running along with and supporting other wires, cables or contact conductors.

A primary messenger is directly attached to the supporting system. A secondary messenger is intermediate between a primary messenger and the wires, cables or contact conductors.

406. **Classes of Construction.**—Overhead trolley construction will be classed as *Direct Suspension* and *Messenger or Catenary Suspension*.

407. **Direct Suspension.**—All forms of overhead trolley construction in which the trolley wires are attached, by insulating devices, directly to the main supporting system.

408. **Messenger or Catenary Suspension.**—All forms of overhead trolley construction in which the trolley wires are attached, by suitable devices, to one or more messenger cables, which in turn may be carried either in *Simple Catenary*, i.e., by primary messengers, or in *Compound Catenary*, i.e., by secondary messengers.

409. **Supporting Systems** shall be classed as follows:

410. **Simple Cross-span Systems.**—Those systems having at each support a single flexible span across the track or tracks.

411. **Messenger Cross-span Systems.**—Those systems having at each support two or more flexible spans across the track or tracks, the upper span carrying part or all of the vertical load of the lower span.

412. **Bracket Systems.**—Those systems having at each support an arm or similar rigid member supported at only one side of the track or tracks.

413. **Bridge Systems.**—Those systems having at each support a rigid member supported at both sides of the track or tracks.

414. **Standard Height of Trolley Wire on Street and Interurban Railways.**—It is recommended that supporting structures shall be of such height that the lowest point of the trolley wire shall be at a height of 18 feet (5.5 m.) above the top of rail under conditions of maximum sag, unless local conditions prevent. On trackage operating electric and steam road equipment and at crossings over steam roads, it is recommended that the trolley wire shall be not less than 21 feet (6.4 m.) above the top of rail, under conditions of maximum sag.

RAILWAY MOTORS

RATING

415. **Nominal Rating.**—The nominal rating of a railway motor shall be the mechanical output at the car or locomotive axle, measured in kilowatts, which causes a rise of temperature above the surrounding air, by thermometer, not exceeding 90° C. at the commutator, and 75° C. at any other normally accessible part after one hour's continuous run at its rated voltage (and frequency

in the case of an alternating-current motor) on a stand with the motor covers arranged to secure maximum ventilation without external blower. The rise in temperature, as measured by resistance, shall not exceed 100° C.*

416. The statement of the nominal rating shall also include the corresponding voltage and armature speed.

417. **Continuous Rating.** — The continuous ratings of a railway motor shall be the inputs in amperes at which it may be operated continuously at ½, ¾ and full voltage respectively, without exceeding the specified temperature rises (see §420), when operated on stand test with motor covers and cooling system, if any, arranged as in service. Inasmuch as the same motor may be operated under different conditions as regards ventilation, it will be necessary in each case to define the system of ventilation which is used. In case motors are cooled by external blowers, the volume of air on which the rating is based shall be given.

418. **Maximum Input.**—Railway motors shall be capable of carrying twice the current corresponding to their nominal rating for a period of five minutes, without flash-over or mechanical injury. They shall also be capable of carrying a load of three times their nominal rating for one minute under the same conditions. These overloads shall be applied when the motor is at the temperature which it would acquire when operating at its continuous rating.

TEMPERATURE LIMITATIONS

419. **The Allowable Temperature** in any part of a motor will be governed by the kind of material with which that part is insulated. In view of space limitations, and the cost of carrying dead weight on cars, it is considered good practice to operate railway motors for short periods at higher temperatures than would be advisable in stationary motors. The following temperatures are permissible:

TEMPERATURES

Class of material†	Maximum observable temperature of windings			
	Short periods		Continuous	
	By therm.	By resist.	By therm.	By resist.
*A*₁	100	125	85	110
B	115	145	100	130

* This definition differs from that in the 1911 edition of the Rules, principally by the substitution of a kilowatt rating for the horse-power rating and the omission of a reference to a room temperature of 25° C. The horse-power rating of a railway motor may, for practical purposes, be taken as 4/3 of the kilowatt rating. On account of the hitherto prevailing practice of expressing mechanical output in horse power, it is recommended that, for the present, the capacity be expressed both in kilowatts and in horse power, a double rating, namely,

kw.———————approx. equiv. h.p.———————

In order to lay stress upon the preferred future basis, it is desirable that on rating plates, the rating in kilowatts shall be shown in larger and more prominent characters than the capacity in horse power.

† See §188.

420. With a view to not exceeding the above temperature limitations the continuous ratings shall be based upon the *temperature rises* tabulated in the accompanying table:

TEMPERATURE RISES ON STAND TEST *

Class of material†	Temperature rises of windings	
	By thermometer	By resistance
A2	65	85
B	80	105

* The temperature rise in service may be very different from that on stand test. See §440 for relation between stand test and service temperatures, as affected by ventilation.

† See §188.

421. Field-control Motors.—The nominal and continuous ratings of field-control motors shall relate to their performance with the operating field which gives the maximum motor rating. Each section of the field windings shall be adequate to perform the service required of it, without exceeding the specified temperature rises.

CHARACTERISTIC CURVES

422. The Characteristic Curves of railway motors shall be plotted with the current as abscissas and the tractive effort, speed and efficiency as ordinates.

423. Characteristic Curves of Direct-current Motors shall be based upon full voltage, which shall be taken as 600 volts, or a multiple thereof.

424. In the Case of Field-control Motors, characteristic curves shall be given for all operating field connections.

EFFICIENCY AND LOSSES

425. The Efficiency of railway motors shall be deduced from a determination of the losses enumerated in §§426, 427, 429 and 430. (See also §§436 and 437.)

426. The Copper Loss shall be determined from resistance measurements corrected to 75° C.

427. The No-load Core-loss, Brush Friction, Bearing Friction and Windage shall be determined as a total under the following conditions:

In making the test, the kind of brushes and the brush pressure shall be the same as in commercial service. With the field separately excited, such a voltage shall be applied to the armature terminals as will give the same speed for any given field current as is obtained with the field current when operating at normal voltage under load. The sum of the losses above-mentioned, is equal to the product of the counter-electromotive force and the armature current. Under load, these losses shall be taken as follows [in accompanying table].

Per cent of nominal load	Loss as per cent of no-load losses
100 and over	130
75	120
50	115
25	110
15	100

428. In case it is desired to separate the core-loss from the other losses above described, this may be accomplished by measuring the power required to drive the motor at any given speed without gears, by running it as a series motor on low voltage and deducting this loss from the sum of the no-load losses at corresponding speed. (See §437.)

The core-losses at other loads shall be assumed as follows: At full continuous rated input 1.2 times no-load core-loss. At half continuous rated input 1.1 times no-load core-loss. The multipliers for other loads shall be in the same proportion.

429. **The Brush Contact Resistance Loss** to be used in determining the efficiency may be obtained by assuming that the sum of the drops at the contact surfaces of the positive and negative brushes is three volts.

430. The loss in single-reduction gearing and axle bearings varies with type, mechanical finish, age and lubrication. The following [accompanying] values, based on accumulated tests, shall be used in the comparison of motors:

LOSSES IN AXLE BEARINGS AND SINGLE-REDUCTION GEARINGS

Per cent of nominal rating	Losses as per cent of input
100 or over	3.0
75	3.5
50	4.5
25	6.0

ELECTRIC LOCOMOTIVES

431. **Rating.**—Locomotives shall be rated in terms of the adhesive weight, nominal one-hour tractive effort, continuous tractive effort and corresponding speeds.

432. **Adhesive Weight.**—The adhesive weight expressed in pounds shall be the sum of the weights on the drivers and of the drivers themselves.

433. **Nominal Tractive Effort.**—The nominal tractive effort, expressed in pounds, shall be that exerted when the motors are operating at their nominal (one-hour) rating.

434. **Continuous Tractive Effort.**—The continuous tractive effort, expressed in pounds, shall be that exerted when the motors are operating at their full-voltage continuous rating, as indicated in §417.

In the case of locomotives operating on intermittent service, the continuous tractive effort may be given for ½ or ¾ voltage, but in such cases the voltage shall be clearly specified.

435. **Speed.**—The rated speed, expressed in miles per hour, shall be that at which the continuous tractive effort is exerted.

APPENDIX I: RAILWAY MOTORS

436. In comparing projected motors and in case it is not possible or desirable to make tests to determine mechanical losses, the following values of these losses, determined from accumulated experience, will be found useful. They include axle-bearing losses, gear losses, armature-bearing losses, brush-friction losses and windage.

Per cent of nominal rating	Losses as per cent of input
150	5.0
125	5.0
100	5.0
75	5.0
60	5.3
50	6.5
40	8.8
30	13.3
25	17.0

437. The core-loss of railway motors is sometimes determined by separately exciting the field and driving the armature of the motor to be tested, by a separate motor having known losses and noting the differences in losses between

driving the motor light at various speeds and driving it with various field excitations.

The core-losses at other loads shall be assumed as follows:
At full continuous rated input 1.2 times no-load core-loss;
At half continuous rated input 1.1 times no-load core-loss.
The multiplier for other loads shall be in the same proportion.

438. **Selection of Motor for Specified Service.** — The following information relative to the service to be performed is required in order that an appropriate motor may be selected.

(a) Weight of total number of cars in train (in tons of 2000 lb.) exclusive of electrical equipment and load.
(b) Average weight of load and durations of same, and maximum weight of load and durations of same.
(c) Number of motor cars or locomotives in train, and number of trail cars in train.
(d) Diameter of driving wheels.
(e) Weight on driving wheels, exclusive of electrical equipment.
(f) Number of motors per motor car.
(g) Voltage at train with power on the motors — average, maximum and minimum.
(h) Rate of acceleration in m.p.h. per second.
(i) Rate of braking (retardation in m.p.h. per second).
(j) Speed limitations, if any (including slowdowns).
(k) Distances between stations.
(l) Duration of station stops.
(m) Schedule speed including station stops in m.p.h.
(n) Train resistance in pounds per ton of 2000 pounds at stated speeds.
(o) Moment of inertia of revolving parts, exclusive of electrical equipment.
(p) Profile and alignment of track.
(q) Distance coasted as a per cent of the distance between station stops.
(r) Time of layover at end of run, if any.

439. **Stand Test Method of Comparing Motor Capacity with Service Requirements.**—When it is not convenient to test motors under actual specific service conditions, recourse may be had to the following method of determining temperature rise.

440. The essential motor losses affecting temperatures in service are those in the motor windings, core and commutator. The mean service conditions may be expressed as a close approximation, in terms of that continuous current and core-loss which will produce the same losses and distribution of losses as the average in service.

A stand test with the current and voltage which will give losses equal to those in service will determine whether the motor has sufficient capacity to meet the service requirements. In service the temperature of an enclosed motor (§97), well exposed to the draught of air incident to a moving car or locomotive, will be from 75 to 90 per cent (depending upon the character of the service) of the temperature rise obtained on a stand test with the motor completely enclosed and with the same losses. With a ventilated motor (§§98 and 100), the temperature rise in service will be 90 to 100 per cent of the temperature rise obtained on a stand test with the same losses.

441. In making a stand test to determine the temperature rise in a specific service, it is essential in the case of a self-ventilated motor (§100), to run the armature at a speed which corresponds to the schedule speed in service.

In order to obtain this speed it may be necessary, while maintaining the same total armature losses, to change somewhat the ratio between the I^2R and core-loss components.

442. **Calculation for Comparing Motor Capacity with Service Requirements.**— The heating of a motor should be determined, wherever possible, by testing it in service, or with an equivalent duty cycle. When the service or equivalent duty cycle tests are not practicable, the ratings of the motor may be utilized as follows to determine its temperature rise.

443. The motor losses which affect the heating of the windings are, as stated above, those in the windings and in the core. The former are proportional to the square of the current. The latter vary with the voltage and current, according to curves which can be supplied by the manufacturers. The procedure is therefore as follows:

444. (a) Plot a time-current curve and a time-voltage curve for the duty cycle which the motor is to perform, and calculate from these the root mean-square current and the equivalent voltage which with r.m.s. current will produce the average core-loss.

445. (b) If the calculated r.m.s. service current exceeds the continuous rating, when run with average service core-loss and speed, the motor is not sufficiently powerful for the duty cycle contemplated.

446. (c) If the calculated r.m.s. service current does not exceed the continuous rating, when run with average service core-loss and speed, the motor is ordinarily suitable for the service. In some cases, however, it may not have sufficient thermal capacity to avoid excessive temperature rises during the periods of heavy load. In such cases a further calculation is required, the first step of which is to calculate the temperature rise due to the r.m.s. service current, and equivalent voltage.

Let t = temperature rise
$p_0 = I^2R$ loss, kw.
p_c = core-loss, kw.
with r.m.s. service current, and equivalent service voltage.

T = temperature rise
$P_0 = I^2R$ loss, kw.
P_c = core-loss, kw.
with continuous load current corresponding to the equivalent service voltage.

Then

$$t = T\,\frac{p_0 + p_c}{P_0 + P_c}\text{, approximately.}$$

447. (d) The thermal capacity of a motor is approximately measured by a coefficient equal to the ratio of the electrical loss in kw. at its nominal (one-hour) capacity, to the corresponding maximum observable temperature rise.

448. (e) Consider any period of peak load and determine the electrical losses in kilowatt-hours during that period from the *electrical* efficiency curve. Find the excess of the above losses over the losses with r.m.s. service current and equivalent voltage. The excess loss divided by the coefficient of thermal capacity will equal the extra temperature rise due to the peak load. This temperature rise added to that due to the r.m.s. service current, and equivalent voltage, gives the total temperature rise. If the total temperature rise in any such period exceeds the safe limit, the motor is not sufficiently powerful for the service.

449. (f) If the temperature reached due to the peak loads does not exceed the safe limit, the motor may yet be unsuitable for the service, as the peak loads may cause excessive sparking and dangerous mechanical stresses. It is, therefore, necessary to compare the peak loads with the short-period overload capacity. If the peaks are also within the capacity of the motor, it may be considered suitable for the given duty cycle.

APPENDIX II: ILLUMINATION AND PHOTOMETRY*

460. **Luminous Flux** is radiant power evaluated according to its capacity to produce the sensation of light.

461. **The Luminous Intensity** of a point source of light is the solid angular density of the luminous flux emitted by the source in the direction considered; or it is the flux per unit solid angle from that source.

462. **Candle.** — The unit of luminous intensity, maintained by the National Laboratories of France, Great Britain and the United States. This unit, which is used also by many other countries, is frequently referred to as the international candle. The Hefner unit is 0.90 of the international candle.

463. **Candle-Power.** — Luminous intensity expressed in candles.

464. **Lumen.** — The unit of luminous flux, equal to the flux emitted in a unit solid angle (steradian) by a point source of one candle-power.

465. **Illumination** on a surface is the luminous flux-density over that surface, or the flux per unit of intercepting area.

Defining equation: Let E be the illumination and S the area of the intercepting surface. Then

$$E = \frac{dF}{dS} \qquad \text{or, when uniform,} \qquad E = \frac{F}{S}.$$

466. **Lux** is a unit of illumination equal to one lumen per square meter. The C. G. S. unit of illumination is one lumen per square centimeter. For this unit Blondel has proposed the name "Phot." One millilumen per square centimeter (milliphot) is a practical derivative of the C. G. S. system. One foot-candle is one lumen per square foot and is equal to 1.0764 milliphots. The foot-candle is the commonly employed unit of illumination in English-speaking countries.

467. **Exposure.** — The product of an illumination by the time. Blondel has proposed the name "phot-second" for the unit of exposure in the C. G. S. system.

468. **Brightness, *b*, of an Element of a Luminous Surface from a Given Position** is the luminous intensity per unit area of the surface projected on a plane perpendicular to the line of sight, and including only a surface of dimensions negligibly small in comparison with the distance to the observer. It is measured in candles per square centimeter of the projected area.

* Sections 460 to 496, on Illumination and Photometry have been taken from the Publications of the Illuminating Engineering Society, after conference with its Committee on Nomenclature and Standards. (See reports of that Committee.)

Defining equation: Let θ be the angle between the normal to the surface and the line of sight, and dI the luminous intensity of the element. Then

$$b = \frac{dI}{dS \cos \theta}.$$

469. **Normal Brightness, b_0, of an Element of a Surface** (sometimes called **Specific Luminous Intensity**) is the luminous intensity of the element taken normally to the surface of the element, and is expressed in candles per square centimeter.

In practice, the brightness b of a luminous surface, or element thereof, is observed, and not the normal brightness b_0. For surfaces for which the cosine law of emission holds, the quantities b and b_0 are equal.

Defining equation:

$$b_0 = \frac{dI}{dS} \qquad \text{or, when uniform,} \qquad b_0 = \frac{I}{S}.$$

470. **Specific Luminous Radiation.** — The luminous flux density emitted by a surface, or the flux emitted per unit of emissive area. It is expressed in lumens per square centimeter.

Defining equation: Let E' be the specific luminous radiation. Then, for surfaces obeying Lambert's cosine law of emission

$$E' = \pi b_0.$$

471. **Coefficient of Reflection.** — The ratio of the total luminous flux reflected by a surface to the total luminous flux incident upon it. It is a simple numeric. The reflection from a surface may be regular, diffuse or mixed. In perfect regular reflection, all of the flux is reflected from the surface at an angle of reflection equal to the angle of incidence. In perfect diffuse reflection the flux is reflected from the surface in all directions in accordance with Lambert's cosine law. In most practical cases, there is a super-position of regular and diffuse reflection.

472. **Coefficient of Regular Reflection** is the ratio of the luminous flux reflected regularly to the total incident flux.

473. **Coefficient of Diffuse Reflection** is the ratio of the luminous flux reflected diffusely to the total incident flux.

Defining equation: Let m be the coefficient of reflection (regular or diffuse). Then, for any given portion of the surface,

$$m = \frac{E'}{E}.$$

474. **Primary Luminous Standard.** — A recognized standard luminous source reproducible from specifications.

475. **Representative Luminous Standard.** — A standard of luminous intensity adopted as the authoritative custodian of the accepted value of the unit.

476. **Reference Standard.** — A standard calibrated in terms of the unit from either a primary or representative standard and used for the calibration of working standards.

477. **Working Standard.** — Any standardized luminous source for daily use in photometry.

478. **Comparison Lamp.** — A lamp of constant but not necessarily known candle-power against which a working standard and test lamps are successively compared in a photometer.

479. **Test Lamp,** in a photometer, a lamp to be tested.

480. **Performance Curve.** — A curve representing the behavior of a lamp in any particular (candle-power, consumption, etc.) at different periods during its life.

481. **Characteristic Curve.** — A curve expressing a relation between two variable properties of a luminous source, as candle power and volts, candle-power and rate of fuel consumption, etc.

482. **Mean Horizontal Candle-Power** of a lamp. — The average candle-power in the horizontal plane passing through the luminous center of the lamp.

It is here assumed that the lamp (or other light source) is mounted in the usual manner, or, as in the case of an incandescent lamp, with its axis of symmetry vertical.

483. **Mean Spherical Candle-Power** of a lamp. — The average candle-power of a lamp in all directions in space. It is equal to the total luminous flux of the lamp in lumens divided by 4π.

484. **Mean Hemispherical Candle-Power of a lamp** (upper or lower). — The average candle-power of a lamp in the hemisphere considered. It is equal to the total luminous flux emitted by the lamp in that hemisphere divided by 2π.

485. **Mean Zonal Candle-Power** of a lamp. — The average candle-power of a lamp over a given zone. It is equal to the total luminous flux emitted by the lamp in that zone divided by the solid angle of the zone.

486. **The Spherical Reduction-Factor** of a lamp

$$= \frac{\text{mean spherical candle-power}}{\text{mean horizontal candle-power}}.$$

487. **The Spherical Reduction-Factor** should only be used when properly determined for the particular type and characteristics of each lamp. The spherical reduction-factor permits of substantially-accurate comparisons being made between the total lumens, or mean spherical candle-powers of different types of incandescent lamps, and may be used in the absence of proper facilities for direct measurement of the total lumens or mean spherical candle-power.

488. **The Specific Output of Electric Lamps** is properly stated in terms of lumens per watt at lamp terminals. The use of the term efficiency in this connection should be discouraged.

When auxiliary devices are employed in circuit with a lamp, the specific output should be referred to lamp terminals, unless otherwise specified.

489. **The Specific Consumption** of an electric lamp is its watts consumption per lumen. "Watts per candle" is a term used commercially in connection with incandescent lamps, and denotes watts per mean horizontal candle-power.

490. **Photometric Tests** in which the results are stated in candle-power should be made at such a distance from the source of light that the latter may be regarded as practically a point. Where tests are made in the measurement of lamps with reflectors, the results should always be given as "apparent candle-power" at the distance employed, which distance should always be specifically stated.

491. **Basis for Comparison.** — Either the total flux of light in lumens, or the mean spherical candle-power, should always be used as the basis for com-

paring various luminous sources with each other, unless there is a clear understanding or statement to the contrary.

492. Incandescent Lamps, Rating. — It is customary to rate incandescent lamps on the basis of their mean horizontal candle-power; but in comparing incandescent lamps in which the relative distribution of luminous intensity differs, the comparison should be based on their total flux of light measured in lumens, or on their mean spherical candle-power.

493. Life Tests. — Similar filaments may be assumed to operate at the same temperature, only when their lumens per watt consumed are the same. Life tests are comparable only when conducted under similar conditions as to filament temperatures.

494. In Comparing Different Luminous Sources not only should their candle-power be compared, but also their relative form, brightness, distribution of illumination and character of light.

495. Symbols.

Photometric magnitude	Name of unit	Symbols
1. Luminous flux	Lumen	F, Ψ
2. Luminous intensity	Candle	I, Γ
3. Illumination	Phot., foot-candle, lux	E, β
4. Exposure	Phot-second	Et
5. Brightness	Apparent candles per sq. cm. Apparent candles per sq. in.	b
6. Normal brightness	Candles per sq. cm. Candles per sq. in.	b_0
7. Specific luminous radiation	Lumens per sq. cm. Lumens per sq. in.	$E'\beta'$
8. Coefficient of reflection		m

496. In view of the fact that the symbols heretofore proposed conflict in some cases with symbols adopted for electric units by the International Electrotechnical Commission, it is proposed that where the possibility of any confusion exists in the use of electric and photometric symbols, an alternative system of symbols for photometrical quantities should be employed. These should be derived exclusively from the Greek alphabet, for instance:

Luminous intensity .... Γ
Luminous flux .... Ψ
Illumination .... β

STANDARDS IN TELEPHONY AND TELEGRAPHY

TENTATIVE DEFINITIONS *

501. After careful consideration it does not seem that the time is yet ripe for a formal standardization of terms and definitions used in telephony and telegraphy. Many of the terms commonly employed are used in more than a single way, and conversely, many pieces of apparatus and many constants which are essentially identical from a physical standpoint have been and are known by more than one designation.

* Comments or suggestions should be forwarded to the Chairman of the Sub-Committee on Telephone and Telegraph Standards.

502. **Damping of a Circuit.** — The damping, at a given point, in a circuit from which the source of energy has been withdrawn, is the progressive diminution in the effective value of electromotive force and current at that point resulting from the withdrawal of electrical energy.

503. **Damping Constant.** —The damping constant of a circuit is a measure of the ratio of the dissipative to the reactive component of its admittance or impedance.

Applied to the admittance of a condenser or other simple circuit having capacity reactance, the damping constant for a harmonic electromotive force of given frequency is the ratio of the conductance of the condenser or simple circuit at that frequency to twice the capacity of the condenser at the same frequency.

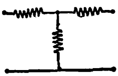
Applied to the reactance of a coil or other simple circuit having inductive reactance, the damping constant for a harmonic current of given frequency is the ratio of the resistance of the coil or circuit at that frequency to twice the inductance at the same frequency.

504. **Equivalent Circuit.** — An equivalent circuit is a simple network of series and shunt impedances, which, at a given frequency, is the approximate electrical equivalent of a complex network at the same frequency and under steady state conditions.

NOTE. — As ordinarily considered, the simple networks, as defined, are the electrical equivalents of complex networks only with respect to definite pairs of terminals, and only as to sending-end impedances, and total attenuation. A further requirement is that the only connections between the pairs of terminals are those through the net-work itself.

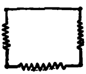
505. **"T" Equivalent Circuit.** — A "T" equivalent circuit is a triple star or "Y" connection of three impedances externally equivalent to a complex network.

Symbol:



506. **"U" Equivalent Circuit.** — A "U" equivalent circuit is a delta connection of three impedances externally equivalent to a complex network. It is also called a "π" equivalent circuit.

Symbol:



IMPEDANCE

507. **Mutual Impedance.** — The mutual impedance, for alternating currents, between a pair of terminals and a second pair of terminals of a network, under any given condition, is the negative vector ratio of the electromotive force produced between either pair of terminals on open circuit to the current flowing between the other pair of terminals.

508. **Self Impedance.** — The self impedance between a pair of terminals of a network, under any given condition, is the vector ratio of the electromotive force applied across the terminals to the current produced between them.

LINE CHARACTERISTICS

509. **Characteristic Impedance.** — Characteristic impedance of a line is the ratio of the applied electromotive force to the resulting steady-state current upon a line of infinite length and uniform structure, or of periodic recurrent structure.

NOTE. — In telephone practice the terms (1) line impedance, (2) surge impedance, (3) iterative impedance, (4) sending-end impedance, (5) initial sending-end impedance, (6) final sending-end impedance, (7) natural impedance and (8) free impedance, have apparently been more or less indefinitely and indiscriminately used as synonyms with what is here defined as "characteristic impedance."

510. **Sending-End Impedance.** — The sending-end impedance of a line is the vector ratio of the applied electromotive force to the resulting steady-state current at the point where the electromotive force is applied.

NOTE. — See note under "Characteristic Impedance." In case the line is of infinite length of uniform structure or of periodic recurrent structure, the sending-end impedance and the characteristic impedance are the same.

511. **Propagation Constant.** — The propagation constant per unit length of a uniform line, or per section of a line of periodic recurrent structure, is the natural logarithm of the vector ratio of the steady-state currents at various points separated by unit length in a uniform line of infinite length, or at successive corresponding points in a line of recurrent structure of infinite length. The ratio is determined by dividing the value of the current at the point nearer the transmitting end by the value of the current at the point more remote.

512. **Attenuation Constant.** — The attenuation constant is the real part of the propagation constant.

513. **Wave-length Constant.** — The wave-length constant is the imaginary part of the propagation constant.

LINE CIRCUITS

514. **Ground-return Circuit.** — A ground-return circuit is a circuit consisting of one or more metallic conductors in parallel, with the circuit completed through the earth.

515. **Metallic Circuit.** — A metallic circuit is a circuit of which the earth forms no part.

516. **Two-wire Circuit.** — A two-wire circuit is a metallic circuit formed by two paralleling conductors insulated from each other.

517. **Superposed Circuit.** — A superposed circuit is an additional circuit obtained from a circuit normally required for another service, and in such a manner that the two services can be given simultaneously without mutual interference.

518. **Phantom Circuit.** — A phantom circuit is a superposed circuit, each side of which consists of the two conductors of a two-wire circuit in parallel.

519. **Side Circuit.** — A side circuit is a two-wire circuit forming one side of a phantom circuit.

520. **Non-phantomed Circuit.** — A non-phantomed circuit is a two-wire circuit which is not arranged for use as the side of a phantom circuit.

521. **Simplexed Circuit.** — A simplexed circuit is a two-wire telephone circuit, arranged for the super-position of a single ground-return signalling circuit operating over the wires in parallel.

NOTE. — In view of the use of the term "Simplex Operation" in telegraph practice, it is felt that the designation "Simplexed Circuit" as applied to the arrangement described is not a happy one.

522. **Composited Circuit.** — A composited circuit is a two-wire telephone circuit, arranged for the super-position on each of its component metallic conductors, of a single independent ground-return signalling circuit.

523. **Quadded or Phantomed Cable.** — A quadded (or phantomed) cable is a cable adapted for the use of phantom circuits.

NOTE. — The type of cable here defined has frequently been designated as "Duplex Cable" — a term which is objectionable, both on account of its lack of description and its widely different use in telegraph practice.

LOADING

524. **Loaded Line.** — A loaded line is one in which the normal inductance of the circuit has been altered for the purpose of increasing its transmission efficiency for one or more frequencies.

525. **Series-loaded Line.** — A series-loaded line is one in which the normal inductance has been altered by inductance serially applied.

526. **Shunt-loaded Line.** — A shunt-loaded line is one in which the normal inductance of the circuit has been altered by inductance applied in shunt across the circuit.

527. **Continuous Loading.** — A continuous loading is a series loading in which the added inductance is uniformly distributed along the conductors.

528. **Coil Loading.** — A coil loading is one in which the normal inductance is altered by the insertion of lumped inductance in the circuit at intervals. This lumped inductance may be applied either in series or in shunt.

NOTE. — As commonly understood, coil loading is a series loading, in which the lumped inductance is applied at uniformly-spaced recurring intervals.

529. **Microphone.** — A contact device designed to have its electrical resistance directly and materially altered by slight differences in mechanical pressure.

530. **Relay.** — A relay is a device by means of which contacts in one circuit are operated under the control of electrical energy in the same or other circuits.

531. **Resonance.** — Resonance of a harmonic alternating current of given frequency, in a simple series circuit, containing resistance, inductance and capacity, is the condition in which the positive reactance of the inductance is numerically equal to the negative reactance of the capacity. Under these conditions the current flow in the circuit with a given electromotive force is a maximum.

532. **Retardation Coil.** — A retardation coil is a reactor (reactance coil) used in a circuit for the purpose of selectively reacting on currents which vary at different rates.

NOTE. — In telephone and telegraph usage the terms "impedance coil," "inductance coil," "choke coil" and "reactance coil" are sometimes used in place of the term "retardation coil."

533. **Skin Effect.** — Skin effect is the phenomenon of the non-uniform distribution of current throughout the cross section of a linear conductor, occasioned by variations in the intensity of the magnetic field due to the current in the conductor.

534. **Telephone Receiver.** — A telephone receiver is an electrically operated device designed to produce sound waves or vibrations which correspond in form to the electromagnetic waves or vibrations actuating it.

535. **Telephone Transmitter.** — A telephone transmitter is a sound-wave or vibration-operated device designed to produce electromagnetic waves or vibrations which correspond in form to the sound waves or vibrations actuating it.

TRANSFORMERS

536. The Coefficient of Coupling of a Transformer. — The coefficient of coupling of a transformer at a given frequency is the vector ratio of the mutual impedance between the primary and secondary of the transformer, to the square root of the product of the self-impedance of the primary and of the secondary.

537. Repeating Coil. — A term used in telephone practice meaning the same as transformer, and ordinarily a transformer of unity ratio.

RADIO*

538. Acoustic Resonance Device. — One which utilizes in its operation mechanical or other resonance to the audio frequency of the received impulses.

539. Antenna. — A system of conductors designed for radiating or absorbing the energy of electromagnetic waves.

540. Atmospheric Absorption. — That portion of the total loss of radiated energy due to atmospheric conductivity.

541. Audio Frequencies. — The normally audible frequencies lying between 20 and about 20,000 cycles per second. (See also Radio Frequencies.)

542. Capacity Coupler. — An apparatus which electrostatically joins portions of two circuits, and thereby permits the transfer of electrical energy between these circuits through the action of electric forces.

543. Coefficient of Coupling. — See §536 above.

544. Conductive Coupler. — An apparatus which magnetically and electrically joins two circuits having a common conductive portion (also known as a Direct Coupler).

545. Counterpoise. — A system of electrical conductors forming one plate of a condenser, the other plate of which is the ground. For alternating current it may be used to replace a direct connection to ground.

546. Damping of a Circuit. — See §502.

547. Damping Factor of a Simple Circuit. — The ratio of the effective resistance of that circuit to twice the effective inductance at any frequency. (The reciprocal of a time.) This term applies only to circuits capable of carrying free alternating currents. (See §503 above.)

548. Detector. — That portion of the receiving apparatus which, connected to a circuit carrying currents of radio-frequency, and in conjunction with a self-contained or separate indicator, translates the radio-frequency energy into a form suitable for operation of the indicator. This translation may be effected either by the conversion of the radio-frequency energy, or by means of the control of local energy by the energy received.

549. Electromagnetic Wave. — A progressive disturbance characterized by the existence on the wave front of electric and magnetic forces acting in directions which are perpendicular to each other and to the direction of propagation of the wave.

550. Forced Alternating Current. — One produced in any circuit by the application of an alternating electromotive force.†

* Sections 538 to 567 have been inserted after conference with the Standards Committee of the Institute of Radio Engineers.

† In power applications termed simply an alternating current.

551. **Free Alternating Current.**—That produced by an isolated electrical displacement in a circuit having capacity, inductance and *less* than the critical resistance.*

552. **Critical Resistance of an Oscillating-current Circuit.**—Twice the square root of the ratio of the inductance of that circuit to the capacity of that circuit both expressed in practical units. This term applies only to circuits capable of carrying free alternating currents.

553. **Group Frequency.**—The number of distinguishable alternating-current groups occurring per second in an electrical circuit.

NOTE 1.—The group referred to above is, in general, mainly a free alternating current which is substantially damped to extinction before the beginning of the following group or train.

NOTE 2.—The acoustic pitch of the note in the receiving station is, in general, determined by the group frequency at the transmitting station.

NOTE 3.—The term "Group Frequency" replaces the term "Spark Frequency."

554. **Inductive Coupler.**—An apparatus which magnetically joins portions of two electric circuits.

555. **Linear Decrement of a Circuit Containing a Resistance Element Equivalent to a Spark.**—The difference of successive current amplitudes in the same direction divided by the larger of these amplitudes. (In circuits containing such an element, not the ratio of successive current amplitudes, but their difference is constant, and characteristic of the damping.)

556. **Logarithmic Decrement.**—Logarithmic decrement of a circuit containing inductance, capacity and constant resistance is one-half the ratio of the electrical energy withdrawn from that circuit during a cycle to the total energy present in that circuit at the beginning of the cycle. It also equals the natural logarithm of the ratio of successive current amplitudes in the *same* direction.

NOTE.—Logarithmic decrements are standard for a complete period or cycle.

557. **Radio Frequencies.**—Those above 20,000 cycles per second. (See also Audio Frequencies.)

NOTE.—It is not implied that radiation cannot be secured at lower frequencies and the distinction from audio frequencies is merely one of definition.

558. **Resonance to an Alternating Current.**—See §531 above.

559. **A Resonance Curve** gives the relation between circuit power, current or voltage at various frequencies of excitation as a function of those frequencies.

560. **A Wave-length Resonance Curve** is one wherein the abscissas are ratios of specified wave lengths to the resonant wave length, and the ordinates are ratios of the energy (or square of the current at corresponding specified wave lengths to the energy (or square of the current) at the resonant wave length. The scale of ordinates and abscissas shall be equal.

561. **A Frequency Resonance Curve.**—One wherein the abscissas are ratios of specified frequencies to the resonant frequency, and the ordinates are ratios of the energy (or square of the current at corresponding specified fre-

* In power applications termed simply an oscillating current. See §5

quencies to the energy (or square of the current) at the resonant frequency. The scales of ordinates and abscissas shall be equal.

562. **A Standard Resonance Curve,** unless otherwise specified, is assumed to be a wave-length resonance curve.

563. **Selecting.** — The process of adjusting an element driven by a plurality of simultaneous impulses until the ratio of desired response to undesired response is a maximum.

564. **Sustained Radiation** consists of electromagnetic waves of constant amplitude (such as are emitted from an antenna in which flows a forced alternating current).

565. **Tuning.** — The process of securing the maximum indications by adjusting the time period of a driven element. (In transmitter or receiver.)

566. **Wave-length Meter.** — A radio-frequency measuring instrument calibrated to read wave lengths.

567. **Rating.** — 1. All radio transmitting sets shall be rated in actual power output measured in the antenna.

NOTE. — The group or audio frequency of the note of the station should be stated as well (except for sustained wave sets, where that characteristic should be mentioned).

2. The over-all efficiency of a radio transmitting station shall be the ratio of the actual power output as measured in the antenna to the power input supplied to the first piece of electrical machinery which is definitely a part of the radio equipment.

STANDARDIZATION RULES AND STANDARD SPECIFICATIONS. — (*See also Standardization Rules of the A.I.E.E.*) During the last decade many of the engineering societies have adopted rules and specifications covering materials, apparatus and methods of procedure. The *Standardization Rules of the American Institute of Electrical Engineers* are given in full in the preceding article. A brief reference to the standardization rules and standard specifications of some of the other engineering societies, with directions for procuring copies of these rules and specifications, are given in this article. A statement of the nature of the publications of the International Electrotechnical Commission and of the Bureau of Standards at Washington is also given.

AMERICAN ELECTRIC RAILWAY ENGINEERING ASSOCIATION. — (*Secretary, E. B. Burritt, 29 West 39th St., New York City.*) This Association issues an *Engineering Manual*, which contains all standards set by the Association, all practices recommended by it, and those practices which, while they have not been formally adopted either as standards or recommended practice, have been discussed and put forward by the Association's various committees. The *Manual* is in loose leaf form and consists of printed matter, drawings and illustrations. The *Manual* is revised each year by the Committee on Standards. The price of the *Manual*, including all sections in a binder, is $4.00 to non-members.

AMERICAN RAILWAY ENGINEERING ASSOCIATION. — (*Secretary, L. H. Fricht, 962, Monadnock Block, Chicago, Ill.*) This Association issues a *Manual* containing the various committee reports which have been formally adopted by the Association. The following items are covered: Roadway; Ballast; Ties; Rail; Track; Buildings; Wooden Bridges and Trestles; Masonry; Signs; Fences and Crossings; Signals and Interlockings; Records and Accounts; Rules and Organization; Water Service; Yards and Terminals; Iron and Steel Structures; Economics of Railway Location; Wood Preservation; Electricity. The cost of the *Manual* is $3.00 to non-members.

AMERICAN SOCIETY OF MECHANICAL ENGINEERS. — (*Secretary, Calvin W. Rice, 29 West 39th St., New York City.*) The Power Test Committee of this Society has under preparation at the present time (1914) a set of *Rules for Conducting Performance Tests of Power Plant Apparatus*, embracing the following subjects: Boilers; * Reciprocating Steam Engines; * Steam Turbines; Complete Steam Power Plants; Pumping Machinery; * Compressors, Blowers and Fans; Locomotives; * Gas Producers; Gas and Oil Engines;* Complete Gas Power Plants; Waterwheels. A preliminary draft of these "Codes" appears in the *Journal* of the Society for November, 1912.

AMERICAN SOCIETY FOR TESTING MATERIALS. — (*Secretary, Edgar Marburg, University of Pennsylvania, Phila., Pa.*) This Society issues every September a *Year Book* which contains all the standard specifications of the Society in their latest revised form. These specifications at present (1914) cover Steel Rails, Structural Steel, Various Steel Objects, Steel Castings, Wrought Iron, Pig Iron, Iron Castings, Locomotive Material, Magnetic Tests of Iron and Steel (*see Magnetic Testing*), Various Kinds of Copper Wire and Wire Bars (*see Wires and Cables, Bare*), Spelter, Manganese Bronze Ingots, Cement, Lime, Clay Products, Preservative Coatings, Road Materials, Timber, Methods of Testing Strength of Materials, and Methods for Metallographic Tests. The price of the *Year Book* is $5.00 to non-members.

* Earlier "Codes" of the Society dealing with these subjects were issued as follows: Boilers, 1899; Steam Engines, 1902; Pumping Engines, 1891; Locomotives, 1894; Gas Engines, 1902.

ELECTRIC POWER CLUB. — (*Secretary, C. W. Roth, 1410 West Adams St., Chicago, Ill.*) This Club is an association of corporations, firms and individuals engaged in the manufacture of electric motors and generators. Its standardization rules, which are printed in a pamphlet entitled *The Electric Power Club*, are confined to sizes of shaft, pulley, etc., speeds, nomenclature, ratings for different kinds of service, and other matters that are industrial in contradistinction to those that are technical or scientific. The pamphlet referred to may be obtained free of charge from the Secretary of the Club.

ILLUMINATING ENGINEERING SOCIETY. — (*Secretary, J. D. Israel, 29 West 39th St., New York City.*) The Committee on Nomenclature and Standards of this Society has drawn up a report of *Proposed Definitions* (*see Trans. Ill. Eng. Soc., Dec., 1912, Vol. 4*). The definitions in this report are substantially the same as those incorporated in the *Standardization Rules of the A.I.E.E.* (1914 edition.)

INSTITUTE OF RADIO ENGINEERS. — (*Secretary, E. J. Simon, 81 New St., New York City.*) See the reports of the Standards Committee of this Institute. Sections 538 to 567 of the *Standardization Rules of the A.I.E.E.* (q.v.) were incorporated in those rules after conference with this Committee.

NATIONAL ELECTRIC LIGHT ASSOCIATION. — (*Secretary, T. C. Martin, 33 West 39th St., New York City.*) In the reports of the annual meetings of this Association are given the recommendations of the various committees dealing with the following subjects: Railway Rates, Transportation, Accounting, Prime Movers, Overhead Line Construction, Lamps, Electrical Apparatus, Meters, Terminology, Grounding Secondaries, Underground Construction, Electrical Measurements and Values, and Street Lighting.

This Association issues, jointly with the **Association of Edison Illuminating Companies** (*Secretary, Geo. C. Holberton, Pacific Gas and Electric Co., San Francisco, Cal.*), a pamphlet entitled *Code for Electricity Meters*. This *Code* deals primarily with the use of watthour meters in connection with the sale of electric energy in both small and large amounts. The following list of chapter headings gives a general idea of the scope of the *Code:* Definitions, Standards and Measuring Instruments, Metering, Specifications for Acceptance of Types of Meters, Specifications for Acceptance of Auxiliary Apparatus, Installation Methods, Methods of Tests, System Tests, Maintenance Methods. The price of the pamphlet is 50 cents.

The N.E.L.A. also issues the *Electrical Meterman's Handbook*, written and compiled by the Committee on Meters. This book is intended primarily for practical meter men, as contrasted with the *Code for Electricity Meters*, which latter is primarily a guide to metering specifications for central station managements and for state and civic commissions. The *Meterman's Handbook* deals with the subject of metering in great detail, containing over 1000 pages. It may be had from the Secretary of the Association for $2.00.

NATIONAL FIRE PROTECTION ASSOCIATION. — This Association issues the *National Electric Code* which contains the rules and requirements of the National Board of Fire Underwriters for electric wiring and apparatus. The *Code* is revised every two years, the next revision being in 1915. A *List of Electrical Fittings* is also issued by the National Board of Fire Underwriters. This is a list of fittings which have been examined and are suitable for the use intended; this list is revised semi-annually. The *Code* and *List of Electrical Fittings* may be obtained free of charge from the National Board of Fire Underwriters, 135 William St., New York City.

OTHER ENGINEERING SOCIETIES.—The reports of the committees of the following societies also contain matter relative to standards of various kinds:

American Electrochemical Society, *Secretary*, J. W. Richards, Lehigh University, South Bethlehem, Pa.

Association of Railway Electrical Engineers, *Secretary*, J. Andreucetti, Room 411, C. & N. W. Terminal Building, Chicago, Ill.

Association of Iron and Steel Electrical Engineers, *Secretary*, W. T. Snyder, McKeesport, Pa.

Electric Vehicle Association of America, *Secretary*, A. J. Marshall, 29 West 39th St., New York City.

Railway Signal Engineers' Association, *Secretary*, C. C. Rosenberg, Times Building, Bethlehem, Pa.

Society of Automobile Engineers, *General Manager*, Coker F. Clarkson, 1790 Broadway, New York City. This society issues a *Handbook* containing the standards recommended for use in the construction of automobiles and automobile accessories.

Among the foreign electrical societies which have issued standards of various kinds may be noted the following:

British Electrical and Allied Manufacturers' Association, King's House, Kingsway, London, W. C., England.

Incorporated Municipal Electrical Association, *Secretary*, C. McArthur Butler, London, England.

Institution of Electrical Engineers, *Secretary*, P. F. Rowell, Victoria Embankment, London, W. C., England.

Verband Deutscher Electrotechniker, *Secretary*, H. Dettmar, Königgratze, Strasse, 106, Berlin, S. W. 11.

INTERNATIONAL ELECTROTECHNICAL COMMISSION. — (*General Secretary's Office, 28 Victoria St., Westminster, London, S. W., England.*) This Commission, which was organized in London in 1906, is made up of National Committees whose membership as a rule is confined to members of the national engineering societies in the several countries. National Committees at present exist in about 25 countries. The United States National Committee, (*Secretary, Dr. A. E. Kennelly, Harvard University, Cambridge, Mass.*) is appointed by the President of the American Institute of Electrical Engineers, and keeps in close touch with the A.I.E.E. Standards Committee. The expenses of the Central Office in London are met by contributions from the various National Committees. In addition to the National Committees there are a number of Special Committees, appointed by the President of the Commission. There are at present Special Committees on Prime Movers, Rating, Nomenclature, and Symbols and Units.

The Commission meets in full session about every two years. The next meeting will be in San Francisco in 1915. These meetings are for the purpose of discussing and acting upon the reports and recommendations of the Special Committees.

The Central Office has, from time to time, published bulletins giving reports of the meetings of the different committees and of the Commission. With the exception of the list of *International Symbols* (*see Symbols and Abbreviations*), no definite standards have yet been published by the Commission. The bulletin containing the list of *International Symbols* may be had of the General Secretary for 2/1 *d*.

BUREAU OF STANDARDS. — The Bureau of Standards at Washington is a branch of the U. S. Department of Commerce. In addition to making tests and comparisons of measuring apparatus, the Bureau carries on numerous

researches related to the establishment and maintenance of the various standards and units of measurement, the development of measuring instruments and methods of measurement, the determination of physical constants and the properties of matter. The results of these investigations are published in pamphlet form, issued in two separate series: (1) *Scientific Papers* and (2) *Technological Papers*. In addition, the Bureau issues from time to time *Circulars* giving useful technical data, standard specifications for apparatus, description of the nature of the standard tests carried out at the Bureau, etc.

A complete list of the publications of the Bureau will be furnished by the Bureau upon request and free of charge; this list is contained in *Circular No. 24* which is brought up to date from time to time. *Circular No. 24* also contains a brief summary of each of the *Scientific* and *Technological Papers*, any one or more of which will also be furnished by the Bureau upon request and free of charge.

The *Bulletin of the Bureau of Standards* is a serial publication in which the *Scientific* and *Technological Papers* are first published, the individual pamphlets referred to above being reprints from the *Bulletin*. Each number of the *Bulletin* contains about 150 pages, the separate numbers being issued as material accumulates, at intervals of about three months. Four numbers constitute a volume. Nine volumes had been issued at the beginning of 1914. The complete *Bulletin* is furnished free to educational and scientific institutions. Individuals may obtain separate numbers in paper covers (current or back numbers) at 25 cents per copy, complete volumes bound in cloth at $1.50 per volume, and may subscribe in advance at the rate of $1.00 per volume to receive the four separate numbers as issued. Orders and payments for the *Bulletin* should be addressed to the Superintendent of Documents, Washington, D. C., and *not* to the Bureau of Standards.

[H. PENDER.]

STARTERS, MOTOR. — (*See also Controllers; Motors, various types of Alternating-Current; Motors, Direct-Current; Rheostats.*) An auxiliary resistance which is used with a motor, either d-c. or a-c., during acceleration is called a starter, or a starting rheostat. A compensator, or induction starter, consists of an auto-transformer and a switch, by the operation of which a reduced voltage is supplied to the terminals of an induction motor for starting.

STARTERS FOR DIRECT-CURRENT MOTORS. — The starters for use with constant speed d-c. motors consist usually of a resistance to be inserted in the armature circuit, this resistance being gradually cut out by the movement of a contact arm over a face plate as the motor comes up to speed. Such rheostats connect the motor field in circuit at the first step and are provided with various safeguards such as low-voltage release, overload release, etc.

Resistance Steps for D-C. Starters. — The resistance is designed to limit the starting current to about 1½ times full load current with enough steps to insure smooth starting. Motors of 10 h.p. or less usually are brought up to speed in about 15 seconds and larger motors in about 30 seconds. Resistors should carry full load current for two minutes, starting cold, with a temperature rise not exceeding 250° C.

Face-plate Type of Starters with Low-voltage Release (Fig. 1). — Fig. 1 shows the connections of a typical d-c. starter used for motors up to 10 h.p. at 110 volts. This starter is provided with a low-voltage release and with an arcing tip at the first contact. The low-voltage release consists of a spring coiled around the pivot of the rheostat arm for returning it to the off-position and an electromagnet for retaining the arm in the on-position as long as the line voltage continues above a predetermined minimum value. The arcing tip at the first contact provides a spring break and thus prevents arcing when the contact arm leaves the first contact in going to the off-position.

For motors larger than 10 h.p. a brush-contact switch is used to short-circuit the starting box when the contact arm is in the running position. A magnetic blow-out may also be used in place of the arcing tip.

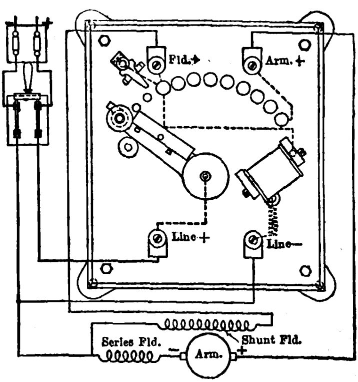


Fig. 1. Starting Box with Low-voltage Release

Overload Release. — This is often furnished, especially with motors of large capacity. This device consists essentially of a solenoid which is connected in series with the motor armature and which, in case of a predetermined overload, opens the circuit of the low-voltage coil. This action releases the contact arm and thus stops the motor.

Mounting of Starting Boxes. — These starting rheostats are frequently mounted on a small panel, together with the line switches and fuses or circuit breaker required for the complete protection of the motor circuit.

Multiple-switch Starters (Fig. 2). — With the larger d-c. motors where the starting conditions are severe the face-plate type of starter is not found satisfactory, so recourse is had to multiple-switch starters. These consist essen-

tially of a number of suitable switches mounted on a panel and a separate resistance, usually of cast-iron grids. The switches are mechanically interlocked so that it is necessary to close them in a given sequence.

The type of starter shown diagrammatically in Fig. 2 is used with 110-volt motors up to 300 h.p. and with 220- and 500-volt motors up to 600 h.p. The first switch on the right of the diagram is provided with magnetic blow-out coils and acts as a circuit breaker. It also has voltage release device. Protection against inadvertently leaving part of the switches open when starting is provided by means of an auxiliary push-button switch that must be held closed until the last switch on the left is closed, thus energizing the coil of the low-voltage release.

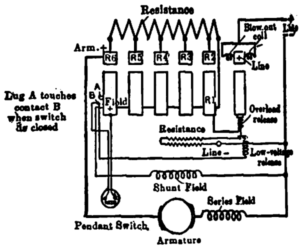


Fig. 2. Connections of Multiple-switch Starter

Starters with Automatically-operated Contactors. — In starting a motor the energy required to overcome the inertia of the motor and the apparatus it is driving must be admitted gradually, which is accomplished by introducing resistance into the armature circuit. If the time taken in starting is too long the resistance may be injured by overheating; if the time is too short the motor may be damaged or the supply circuits seriously disturbed. For these reasons a motor starter that will automatically take care of the proper rate of acceleration presents many advantages. Such a starter can be devised by means of contactor switches and suitable relays, which are operated either by the variation of the voltage drop across a resistance as the motor speeds up, or by the decrease in current which permits a coil to drop its core.

Contactor Starter with Series-relay Control (Fig. 3). — Acceleration by series-relay control is satisfactory where the voltage does not vary more than 12½ per cent either way from a constant value. Fig. 3 shows the connections of a compound-wound motor with a three-switch magnet-switch controller arranged for this type of control of speed acceleration. The relay switch opens when the motor current exceeds a safe value and no further reduction of the starting resistance can be made until the current decreases and allows the relay switch to close.

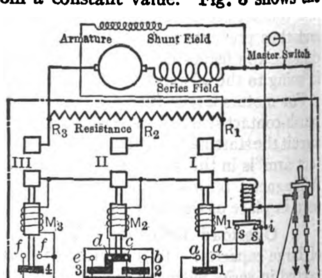


Fig. 3. Diagram of Contractor Starter with Series-relay Control

When the main switch is closed, the closing of the master switch connects coil M_1 across the circuit, thus energizing its core, closing main contacts I and bridging gap a–a by interlocking contact I. The motor starts with all resistance R_3R_1 in series. The high starting current causes the relay switch to open gaps s–s in the control circuit. As soon as the starting current has fallen to a limit predetermined by the relay adjustment the relay core drops, and gap s–s is closed. The second magnet switch then operates, simultaneously closing contacts II, bridging gap b–c by contact 2, and opening gap c–d and closing gap d–e by contact 3.

Contacts II short-circuit resistance R_2R_1, causing an increase in the motor current, and the relay switch again opens gap s–s. But the opening of gap c–d and the closing of gap d–e has, meanwhile, connected coil M_2 across the circuit independently of gap s–s; so that while the relay switch can delay the closing of the magnet switch it has no control over one that is closed. As gaps a–a and b–c are bridged by contacts 1 and 2 respectively, the circuit through coil M_3 will be closed as soon as the motor current has decreased enough to allow the relay switch to drop back and close gap s–s. As soon as the coil m_3 is energized, contacts III are closed, and gap f–f is bridged by contact 4. Contacts III connect the motor directly across the circuit and the bridge across gap f–f removes coil M_3 from any further control by the relay switch.

Contactor Starter with Resistance-drop Control (Fig. 4). — Acceleration by voltage control is applicable where the line voltage does not vary more than 5 per cent either way from a constant value. The arrangement required for this method is given in Fig. 4, which shows the connections of a compound-wound motor with a three-switch magnet-switch controller arranged for speed acceleration by voltage control; that is, the successive operation of the switches depends upon the voltage drop in the starting resistance.

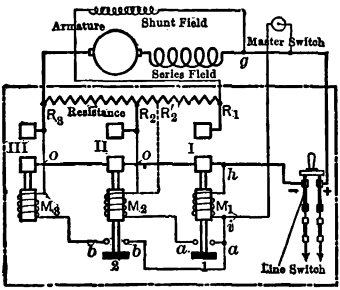


Fig. 4. Diagram of Contactor Starter with Resistance-drop Control

The operation of the controller is started by closing the line switch and the master switch, which connects coil M_1 across the circuit. The core of M_1 rises, main contacts I are closed, and interlocking contacts a–a are bridged by contact I. Main contacts I connect the shunt field directly across the main circuit and close the circuit through the series field, armature and starting resistance R_3R_1. The coils M are wound for full line voltage, except that for higher than 220 volts, 220-volt coils with resistance in series are used. It is evident that the promptness with which the magnet switches act depends largely on the voltage applied to the coil terminals. For example, 180 volts will cause a 220-volt switch to close much more slowly than will the full 220 volts.

When contact I bridges contacts a–a the operating circuit through coil M_2 and resistance R_2R_1 is closed. This coil is then subjected to full line voltage less the drop in resistance R_2R_1. While the starting current is high this voltage drop is considerable, and the closing of the second magnet switch is thereby delayed until the motor current has fallen to a strength that can be approximately predetermined. The operation of the second magnet switch closes contacts II and bridges contacts b–b by contact 2. Contacts II short-circuit resistance R_2R_1 and the bridge across b–b closes the circuit through coil M_3 and resistance R_3R_2. When contacts II close, the starting current increases momentarily and on account of the voltage drop in resistance R_3R_2 the closing of the third magnet switch is delayed in the same manner as described in connection with contacts II.

The lag in the operation of the accelerating magnet switches can be adjusted by changing the connection point of the coils M_2 and M_3 to the resistance. For example, if coil M_2 was connected to the point R_2', as shown by the dotted line, there would be less resistance in series with the coil and consequently a higher voltage would be applied to the coil terminals. The nearer the connection points of the two terminals of coils M_2 and M_3 are to the negative end of the

resistance R_1, the less will be the voltage drop affecting the operation of the second and third magnet switches. If both of these coil terminals were connected to R_1, or to the negative side of the circuit at points *o–o*, coil M_2 would receive the full voltage as soon as contacts *a–a* are bridged and coil M_3 as soon as contacts *b–b* are bridged. The three magnet switches would then close in quick succession, the delay in the operation of the second and third switches being only that caused by their own time element. If no other delaying element is introduced the acceleration will be very rapid, the starting current high and the tax on the motors severe. Acceleration by time element of the switches alone is useful where very quick starting is required.

Contactors for Starters with Acceleration Control. — (*See also Magnets and Solenoids.*) The success of starters with acceleration controls, such as described in the two preceding sections, depends largely on the contactors or contactor switches. Contactors are switches or circuit breakers which are held in the closed position by some auxiliary power, such as a solenoid or compressed air. Contactors of the design shown in Fig. 5 are built in sizes of 100, 250, 350, 500 and 1000 amperes. They consist essentially of a contact that is closed by the action of a solenoid which raises its plunger vertically when the coil is energized and allows it to drop back by gravity, assisted by springs, when the coil is deënergized. The main contacts are above the solenoid and are protected by magnetic blow-out coils so placed on each side of the main contact that the arc is forced quickly to the front and blown out. (*See also Control Systems for Railway Motors.*)

These contactors are ordinarily used in connection with a master switch or controller, and protective relay switches of various kinds to insure the performance of various functions, such as the automatic cutting in and out of resistance in the secondary of an induction motor to maintain constant input to a flywheel set, or any similar features that may be desired.

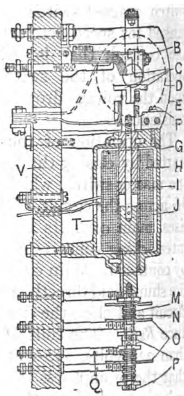


Fig. 5. Section through Contactor. B—Graphite Arcing Contact; C—Copper Contacts; D—Blowout Coil; E—Arc Shield; F—Arc Shield Fastener; G—Magnet Cap; H—Magnet Coil; I—Stationary Core; J—Plunger; M—Insulating Tube; N—Interlock Spring; O—Interlock Disks; P—Insulating Bushings; Q—Interlock Fingers; T—Magnet Frame; V—Panel

STARTERS FOR INDUCTION MOTORS may be divided into two classes: (1) those used with motors having a squirrel cage or short-circuited secondary, and (2) those for motors having a wound secondary. In the former case the starting is done by impressing on the primary a voltage sufficient to induce in the short-circuited secondary the current required to develop the proper starting torque, and then transferring the primary connections to full voltage. With induction motors having phase-wound secondaries the method of starting is to connect the primary circuit directly to the line, with the secondary winding short-circuited through a resistance which is cut out in one or more steps.

With squirrel-cage motors up to about 7½ h.p. it is usually feasible to connect the primary immediately to the full line voltage without drawing an abnormal current from the line. With larger squirrel-cage motors this is not feasible, and consequently there have been developed various means of reducing the impressed voltage supplied to the motor.

Starting of Polyphase Motors with Change of Line Connection. — With two-phase motors operated from a two-phase, four-wire system which is

fed from mesh-wound or interconnected generators or transformers, the motor can be thrown on the side circuits, where 71 per cent voltage is available for starting, and can then be thrown over to full voltage for running. With either two-phase or three-phase motors where 50 per cent of line voltage is sufficient for starting and which have two parallel windings per phase, these windings may be connected in series for starting and parallel for running. With three-phase motors that will develop sufficient starting torque on 58 per cent of line voltage and which have their windings suitably designed, these windings may be connected in star for starting and delta for running. All of these methods give only one starting voltage and they usually require a type of winding that is not conducive to the best design of the motor, but they are advantageous in certain cases because they do not require any additional apparatus other than a double-throw switch.

Starting Compensators. — Under normal conditions the most satisfactory means of obtaining the reduced voltage for starting induction motors with squirrel-cage secondaries is by the use of auto-transformers or compensators (q.v.). The auto-transformers supplied for starting induction motors are provided with taps permitting the choice of any one of several voltages. The auto-transformers are designed for starting service only and are not intended to be left permanently in circuit.

Connections of Starting Compensator (Fig. 6). — The connections of the starting compensator for a two-phase motor are shown diagrammatically in Fig. 6, the switching mechanism being omitted for the sake of simplicity. Two auto-transformers are used with a two-phase motor; with a three-phase

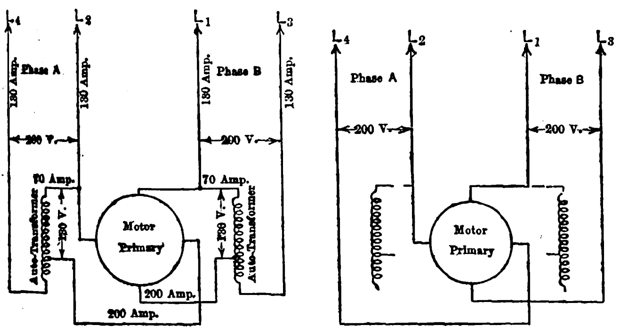


Fig. 6. Connections of Starting Compensator

motor there would be used either three Y-connected or two V-connected auto-transformers. In the starting position the voltage at the motor primary which is connected to the auto-transformer is cut down by the auto-transformers from 200 to 130 in this particular case.

The auto-transformers are provided with taps giving 50, 65 and 80 per cent of line voltage for starting, though it is usually found that the 65 per cent tap gives the proper conditions for average service. With the switch in the running position the auto-transformers are disconnected from the circuit and the motor connected to the full line voltage.

Switch Mechanism. — The switch consists of two sets of butt contacts, copper rods abutting against brass rods, one set of contacts being closed for starting and one for running. When the switch is closed in either direction

coiled springs acting on each pair of rods are compressed so that excellent electrical contact is assured until more than ¼ inch has worn off the end of each rod. The handle of the switch has three positions and locks automatically in the off and running positions but has to be held in the starting position. In starting a motor the handle is first moved very quickly to the starting position and held there until the motor gets up to speed; the handle is then moved quickly past the off position to the running position where it becomes locked. A mechanical device prevents throwing the handle to the switch from the off position to the running position without moving first to the starting position.

Automatic Protection must be secured by means of fuses or circuit breakers in the running leads. If desired, heavier fuses can be installed in the starting leads to give protection against excessive starting currents. A modification of the starting switch is frequently supplied that embodies automatic overload features in the running position, so that no outside protective devices are required. With this type of switch the overload release device consists of two solenoids with plungers, each solenoid being connected in series with a phase of the motor circuit. When the current exceeds a predetermined amount the plunger rises and trips a catch holding the switch in the running position, allowing the switch to open by gravity and disconnect the motor. An oil-filled dash-pot on each solenoid plunger gives an inverse time element feature. The switch contacts trip independently of the handle so that the switch cannot be held closed on an overload.

Resistance Starters for Induction Motors. — With motors having a wound secondary it is customary to connect the primary to full line voltage at starting and to short-circuit the secondary through a resistance. As the motor speeds up, this secondary resistance is cut out in one or more steps until at full speed the secondary is short-circuited. By properly designing the resistors for continuous service instead of for intermittant service, this type of control can be used for speed regulation as well as for starting purposes (*see Controllers*).

With constant-speed induction motors up to 200 h.p. output it is sometimes possible to mount the starting resistance upon the rotor spider, and to control it by butt contact switches operated by a rod which passes through the hollow rotor shaft. By moving this rod out or in by means of a knob at its outer end the resistance may be connected into the secondary circuit for starting or be disconnected therefrom after the motor has come up to speed. With large motors the terminals of the secondary windings are connected by means of slip rings and a drum controller to the resistance, which in this case consists of three resistors mounted separately from the motor. Each resistor has one terminal connected with one of the three phases of the motor secondary. The other terminals of the resistors are connected in star and grounded to the frame of the controller. The drum of the controller short-circuits the various sections of the resistors in steps.

Resistance Steps for Induction Motor Starters. — Wound secondary motors at standstill act as transformers and the resistance in the secondary circuit has to absorb practically the entire input to the motor; as the motor speeds up, the secondary voltage drops off and resistance is cut out of circuit at such a rate that the motor draws from the line about 50 per cent more than full load primary current. Grid resistors for starting service will rise about 250° F. in two minutes. Grid resistors for speed regulation are designed for the same rise on continuous service.

Starters for "Self-starting" Synchronous Motors. — Where self-starting synchronous motors are used they are provided with a squirrel-cage winding on the rotor in addition to the usual field poles and field coils. Owing to this squirrel-cage winding they are started up as induction motors and controlled by

the same type of starting devices. Suitable auxiliary apparatus, such as field rheostats and field switches, must be provided.

CALCULATION OF STEPS FOR RESISTANCE STARTERS.* — (*See also Rheostats.*) Resistance starters usually have from four steps for small motors to seven or ten for large sizes. The number of steps used is commonly somewhat larger than the computed values as a safeguard against excessive starting currents due to improper handling by inexperienced operators. Starters for motors starting under heavy loads are frequently designed with two or three extra steps by which it is permitted to switch the loaded machine on the line with a low current and to raise the current step by step. On the other hand, in stations where experienced operators are available, large motors and synchronous converters are often started on three or four steps.

Calculation for Shunt-motor Starters. — The design is usually based on a fixed maximum current on each step of the starter, as well as on a fixed minimum current for each step. The minimum current must not be taken as less than the load current required for the maximum load with which the motor is to be started.

Let

I_m = the maximum current on each step of the starting rheostat,
I_0 = the lowest current for which the steps are to be designed,
E = the fixed line voltage,
$C = I_m \div I_0$.

Let $K - 1$ be the number of steps required in the rheostat, and let $R_1, R_2, \ldots R_k$ be the total resistances in the circuit, including the resistance of the motor, for the successive positions of the contact arm. K is the number of working contacts over which the contact arm moves, the first contact being when all the resistance of the rheostat is in the circuit and the Kth contact corresponding to all the resistance of the rheostat cut out; whence $R_k = r =$ the motor resistance and $R_1 = E \div I_m$.

It can be shown that the successive resistances form a geometric series with the ratio C, whence

$$K = 1 + \frac{\log\left(\dfrac{E}{rI_m}\right)}{\log C}, \qquad \text{and} \qquad R_1 = \frac{E}{I_m}, \qquad R_2 = \frac{R_1}{C}, \qquad R_3 = \frac{R_2}{C}, \qquad \text{etc.}$$

The value of C must be so chosen that K is a whole number. For most motors which are likely to start under load, the value of C is taken about 1.5. For machines which are started with no load, a larger value for C is usually taken, the limiting value being determined by the ratio of maximum allowable current at starting to current at no load. The maximum allowable current is commonly limited to 1.5 times full-load current. Occasionally twice full-load current is taken, but the lower value is preferable, particularly if both power and lighting services are connected to the same feeders.

Design of Series Motor Starters. — An algebraic method for the computation of the various steps for series-motor starters is rather complicated because the field flux varies with the line current. A graphic method based on equal fluctuations of current on each step of the starting rheostat is as follows.

Let E = the fixed line voltage, I_m the maximum allowable current during starting, and I_0 = the minimum allowable current during starting.

Construct the rectangle AI_mOB as in Fig. 7, in which (AI_m) is the maximum allowable current during starting, and $(OR_1) = E \div I_m$ is the entire resistance

* By O. R. Schurig.

of the circuit when the contact arm of the starter is on the first operating contact. Lay off (AI_0) equal to the minimum current during starting. Determine from the magnetization curve of the machine, at some definite speed the armature voltage E_0 for the current I_0, and the armature voltage E_m for the current I_m at this same speed (*see Motors, Direct-current*). Lay off (OB_0) equal to $(OB) \times \frac{E_0}{E_m}$, and draw the two straight lines (I_mB_0) and (I_0B). Lay off (Or) equal to the resistance of the motor between terminals, and draw (rs) parallel to (OB), and find the point D where (rs) intersects (I_mB_0). Then draw the zig-zag line $I_0v_1Cv_3Fv_5$ . . . If this line does not meet the point D, then the ratio $I_0 \div I_m$ and the corresponding ratio $E_0 \div E_m$ and the distances (AI_0) and (OB_0) must be altered until the new zig-zag line meets the new point D. Then extend the horizontal line Cv_1, Fv_3, . . . until they cut the vertical axis, at R_2, R_3, . . . The resistances of the successive steps (resistances *between* successive contacts) are then equal to the distances (R_1R_2), (R_2R_3), . . .

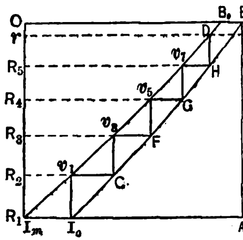


Fig. 7. Graphic Method of Design of Series Motor Starters

For machines working on a straight line magnetization curve, the steps for the starter would be equal, as the line (I_0B) would be parallel to the line (I_mB_0).

Graphic Method for Shunt-motor Starters.—The above method may also be applied to the determination of the steps of a shunt-motor starter, in which case the point B_0 coincides with B.

COST OF MOTOR STARTERS.—The following figures are approximate only and are intended merely to give a rough idea of the cost of some of the common types of starters.

COST OF STARTERS FOR 550-VOLT MOTORS*

Horse-power of motor	1	5	25	100	250
Face-plate type (Fig. 1)	$3.75	$6	$18	$63	....
Multiple-switch type (Fig. 2)	....	....	....	112	$148
Series-relay type (Fig. 3)	....	....	91	115	180
Resistance-drop type (Fig. 4)	....	....	76	110	165
Compensators (Fig. 6)	....	....	59	118	200
Resistance starters for induction motors	....	....	....	221	253
Compensators for "self-starting" synchronous motors	....	....	59	118	200

* Starters for 110- or for 220-volt motors cost about the same as for 550-volt motors.

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[S. Q. HAYES.]

STEAM. — (*See also Boilers; Calorimeters, Steam; Pipes and Piping; Steam Engines; Steam Turbines; etc.*) Steam at a given pressure and at a temperature such that any decrease in temperature, the pressure being kept constant, will cause a condensation of water, is said to be "saturated." When the temperature is such that a decrease of temperature, the pressure being kept constant, can take place without the formation of water, the steam is said to be "superheated," and the number of degrees that the temperature can be lowered before the formation of water takes place is called the "degrees of superheat." Saturated steam may contain fine particles of water in the form of spray or mist, in which case the steam is said to be "wet," while if there is no such moisture present the steam is said to be "dry." (When the steam produced by a boiler is wet the steam is also said to be "primed," and the amount of moisture is called the amount of "priming.") Superheated steam is always dry. The weight of the actual (dry) steam in a pound of wet steam (steam and moisture) is called the "quality of the steam." The temperature of saturated steam at a given pressure is the same whether the steam be dry or wet, and is the same as the boiling point of water at that pressure. The temperature of saturated steam, or the boiling point, depends solely upon the pressure of the steam.

TOTAL HEAT OF STEAM. — The amount of heat required to change one pound of water at 32° F. into steam at any pressure, p, is called the "total heat" of the saturated steam at this pressure. The number of B.t.u. required to raise the temperature of one pound of water from 32° to the temperature of the saturated steam at the given pressure is called the "heat of the liquid." The difference between the total heat and the heat of the liquid is called the "heat of evaporation."

In the case of superheated steam, heat is also required to raise the temperature of the steam from the temperature corresponding to saturation to the temperature of the superheated steam. The total number of B.t.u. required to change one pound of water at 32° F. into superheated steam is given in the second table below.

ENTROPY OF STEAM. — (*See also Thermodynamics, Principles of.*) In dealing with steam the change in entropy resulting from adding to one pound of the water at 32° F. an amount of heat necessary to raise its temperature to the boiling point is called the "entropy of the water," the change in entropy during evaporation, i.e., the heat of evaporation divided by the absolute temperature of the boiling point, is called the "entropy of evaporation," and the entropy of the water plus the entropy of evaporation is called the "entropy of the steam." The entropy of the water is approximately the quotient of the heat added to 1 lb. from 32° to the boiling point divided by the average of these two temperatures above absolute zero.

STEAM TABLES. — Tables giving the values of the various properties of steam for the range of temperatures and pressures met with in practice have recently been recomputed by Marks and Davis, using the latest experimental results as the basis for their calculations. The following tables are condensed from those given in their *Steam Tables and Diagrams* (N.Y., 1909).

Saturated Steam. — Using the symbols at top of the table:

Gage pressure in lb. per square in.	$= p - 14.7$
Vacuum, inches of mercury,	$= 29.92 - 2.036\,p$
Entropy of evaporation	$= N - n$
Pounds per cubic foot	$= \dfrac{1}{v}$.

SATURATED STEAM

Inches mercury	Abs. Press., lb. per sq. in.	Temp., °F.	Spec. Vol., cu. ft. per lb.	Heat of Liq., B.t.u.	Heat of Evap., B.t.u.	Total Heat, B.t.u.	Entropy of Liq.	Total Entropy
	p	t	v	h	L	H	n	N
29.72	0.1	35.0	2935	3.02	1071.7	1074.7	0.0062	2.1727
29.51	0.2	53.1	1524	21.18	1061.6	1082.8	0.0423	2.1127
29.31	0.3	64.5	1042	32.57	1055.2	1087.8	0.0640	2.0776
29.11	0.4	72.9	794	40.95	1050.6	1091.6	0.0800	2.0531
28.90	0.5	79.7	643	47.74	1046.9	1094.6	0.0926	2.0339
28.70	0.6	85.3	540.7	53.32	1043.8	1097.1	0.1029	2.0183
28.49	0.7	90.2	466.8	58.20	1041.1	1099.3	0.1118	2.0053
28.29	0.8	94.5	411.5	62.49	1038.7	1101.2	0.1195	1.9942
28.09	0.9	98.3	367.9	66.28	1036.6	1102.9	0.1265	1.9843
27.88	1	101.8	333.0	69.8	1034.6	1104.4	0.1327	1.9754
25.85	2	126.2	173.5	94.0	1021.0	1115.0	0.1749	1.9180
23.81	3	141.5	118.5	109.4	1012.3	1121.6	0.2008	1.8848
21.78	4	153.0	90.5	120.9	1005.7	1126.5	0.2198	1.8614
19.74	5	162.3	73.3	130.1	1000.3	1130.5	0.2348	1.8432
17.70	6	170.1	61.9	137.9	995.8	1133.7	0.2471	1.8285
15.67	7	176.9	53.6	144.7	991.8	1136.5	0.2579	1.8161
13.63	8	182.9	47.27	150.8	988.2	1139.0	0.2673	1.8053
11.60	9	188.3	42.36	156.2	985.0	1141.1	0.2756	1.7958
9.56	10	193.2	38.4	161.1	982.0	1143.1	0.2832	1.7874
7.52	11	197.8	35.1	165.7	979.2	1144.9	0.2902	1.7797
5.49	12	202.0	32.4	169.9	976.6	1146.5	0.2967	1.7727
3.45	13	205.9	30.0	173.8	974.2	1148.0	0.3025	1.7664
1.42	14	209.6	28.0	177.5	971.9	1149.4	0.3081	1.7604
0.00	14.7	212.0	26.8	180.0	970.4	1150.4	0.3118	1.7565
Pounds gage								
0.3	15	213.0	26.3	181.0	969.7	1150.7	0.3133	1.7549
5.3	20	228.0	20.1	196.1	960.0	1156.2	0.3355	1.7320
10.3	25	240.1	16.3	208.4	952.0	1160.4	0.3532	1.7136
15.3	30	250.3	13.7	218.8	945.1	1163.9	0.3680	1.6991
20.3	35	259.3	11.9	227.9	938.9	1166.8	0.3808	1.6868
25.3	40	267.3	10.5	236.1	933.3	1169.4	0.3920	1.6761
30.3	45	274.5	9.39	243.4	928.2	1171.6	0.4021	1.6665
35.3	50	281.0	8.51	250.1	923.5	1173.6	0.4113	1.6581
40.3	55	287.1	7.78	256.3	919.0	1175.4	0.4196	1.6505
45.3	60	292.7	7.17	262.1	914.9	1177.0	0.4272	1.6432
50.3	65	298.0	6.65	267.5	911.0	1178.5	0.4344	1.6368
55.3	70	302.9	6.20	272.6	907.2	1179.8	0.4411	1.6307
60.3	75	307.6	5.81	277.4	903.7	1181.1	0.4474	1.6252
65.3	80	312.0	5.47	282.0	900.3	1182.3	0.4535	1.6200
70.3	85	316.3	5.16	286.3	897.1	1183.4	0.4590	1.6151

SATURATED STEAM (*Continued*)

Pounds gage	Abs. Press., lb. per sq. in. p	Temp., °F. t	Spec. Vol., cu. ft. per lb. v	Heat of Liq., B.t.u. h	Heat of Evap., B.t.u. L	Total Heat, B.t.u. H	Entropy of Liq. n	Total Entropy N
75.3	90	320.3	4.89	290.5	893.9	1184.4	0.4644	1.6105
80.3	95	324.1	4.65	294.5	890.9	1185.4	0.4694	1.6061
85.3	100	327.8	4.43	298.3	888.0	1186.3	0.4743	1.6020
90.3	105	331.4	4.23	302.0	885.2	1187.2	0.4789	1.5980
95.3	110	334.8	4.05	305.5	882.5	1188.0	0.4834	1.5942
100.3	115	338.1	3.88	309.0	879.8	1188.8	0.4877	1.5907
105.3	120	341.3	3.73	312.3	877.2	1189.6	0.4919	1.5873
110.3	125	344.4	3.58	315.5	874.7	1190.3	0.4959	1.5839
115.3	130	347.4	3.45	318.6	872.3	1191.0	0.4998	1.5807
120.3	135	350.3	3.33	321.7	869.9	1191.6	0.5035	1.5777
125.3	140	353.1	3.22	324.6	867.6	1192.2	0.5072	1.5747
130.3	145	355.8	3.11	327.4	865.4	1192.8	0.5107	1.5719
135.3	150	358.5	3.01	330.2	863.2	1193.4	0.5142	1.5692
140.3	155	361.0	2.92	332.9	861.0	1194.0	0.5175	1.5664
145.3	160	363.6	2.83	335.6	858.8	1194.5	0.5208	1.5639
150.3	165	366.0	2.75	338.2	856.8	1195.0	0.5239	1.5615
155.3	170	368.5	2.68	340.7	854.7	1195.4	0.5269	1.5590
160.3	175	370.8	2.60	343.2	852.7	1195.9	0.5299	1.5567
165.3	180	373.1	2.53	345.6	850.8	1196.4	0.5328	1.5543
170.3	185	375.4	2.47	348.0	848.8	1196.8	0.5356	1.5520
175.3	190	377.6	2.41	350.4	846.9	1197.3	0.5384	1.5498
180.3	195	379.8	2.35	352.7	845.0	1197.7	0.5410	1.5476
185.3	200	381.9	2.29	354.9	843.2	1198.1	0.5437	1.5456
190.3	205	384.0	2.24	357.1	841.4	1198.5	0.5463	1.5436
195.3	210	386.0	2.19	359.2	839.6	1198.8	0.5488	1.5416
200.3	215	388.0	2.14	361.4	837.9	1199.2	0.5513	1.5398
205.3	220	389.9	2.09	363.4	836.2	1199.6	0.5538	1.5379
210.3	225	391.9	2.05	365.5	834.4	1199.9	0.5562	1.5361
215.3	230	393.8	2.00	367.5	832.8	1200.2	0.5586	1.5344
220.3	235	395.6	1.96	369.4	831.1	1200.6	0.5610	1.5327
225.3	240	397.4	1.92	371.4	829.5	1200.9	0.5633	1.5309
230.3	245	399.3	1.89	373.3	827.9	1201.2	0.5655	1.5293
235.3	250	401.1	1.85	375.2	826.3	1201.5	0.5676	1.5276
245.3	260	404.5	1.78	378.9	823.1	1202.1	0.5719	1.5244
255.3	270	407.9	1.72	382.5	820.1	1202.6	0.5760	1.5214
265.3	280	411.2	1.66	386.0	817.1	1203.1	0.5800	1.5185
285.3	300	417.5	1.55	392.7	811.3	1204.1	0.5878	1.5129
385.3	400	444.8	1.17	422	786	1208	0.621	1.489
485.3	500	467.3	0.93	448	762	1210	0.648	1.470
585.3	600	486.6	0.76	469	741	1210	0.670	1.453

SUPERHEATED STEAM

v=specific volume in cubic feet per pound, **h**=total heat, from water at 32° F. in B.t.u. per pound, **n**=total entropy, from water at 32°.

Abs. pres., lb. per sq. in.	Temp. sat. steam, °F.		Degrees of superheat, Fahrenheit 20	50	100	150	200	250	300	400
20	228.0	v	20.73	21.69	23.25	24.80	26.33	27.85	29.37	32.39
		h	1165.7	1179.9	1203.5	1227.1	1250.6	1274.1	1297.6	1344.8
		n	1.7456	1.7652	1.7961	1.8251	1.8524	1.8781	1.9026	1.9479
40	267.3	v	10.83	11.33	12.13	12.93	13.70	14.48	15.25	16.78
		h	1179.3	1194.0	1218.4	1242.4	1266.4	1290.3	1314.1	1361.6
		n	1.6895	1.7089	1.7392	1.7674	1.7940	1.8189	1.8427	1.8867
60	292.7	v	7.40	7.75	8.30	8.84	9.36	9.89	10.41	11.43
		h	1187.3	1202.6	1227.6	1252.1	1276.4	1300.4	1324.3	1372.2
		n	1.6568	1.6761	1.7062	1.7342	1.7603	1.7849	1.8081	1.8511
80	312.0	v	5.65	5.92	6.34	6.75	7.17	7.56	7.95	8.72
		h	1193.0	1208.8	1234.3	1259.0	1283.6	1307.8	1331.9	1379.8
		n	1.6338	1.6532	1.6833	1.7110	1.7368	1.7612	1.7840	1.8265
100	327.8	v	4.58	4.79	5.14	5.47	5.80	6.12	6.44	7.07
		h	1197.5	1213.8	1239.7	1264.7	1289.4	1313.6	1337.8	1385.9
		n	1.6160	1.6358	1.6658	1.6933	1.7188	1.7428	1.7656	1.8079
120	341.3	v	3.85	4.04	4.33	4.62	4.89	5.17	5.44	5.96
		h	1201.1	1217.9	1244.1	1269.3	1294.1	1318.4	1342.7	1391.0
		n	1.6016	1.6216	1.6517	1.6789	1.7041	1.7280	1.7505	1.7924
140	353.1	v	3.32	3.49	3.75	4.00	4.24	4.48	4.71	5.16
		h	1204.3	1221.4	1248.0	1273.3	1298.2	1322.6	1346.9	1395.4
		n	1.5894	1.6096	1.6395	1.6666	1.6916	1.7152	1.7376	1.7792
160	363.6	v	2.93	3.07	3.30	3.53	3.74	3.95	4.15	4.56
		h	1207.0	1224.5	1251.3	1276.8	1301.7	1326.2	1350.6	1399.3
		n	1.5789	1.5993	1.6292	1.6561	1.6810	1.7043	1.7266	1.7680
180	373.1	v	2.62	2.75	2.96	3.16	3.35	3.54	3.72	4.09
		h	1209.4	1227.2	1254.3	1279.9	1304.8	1329.5	1353.9	1402.7
		n	1.5697	1.5904	1.6201	1.6468	1.6716	1.6948	1.7169	1.7581
200	381.9	v	2.37	2.49	2.68	2.86	3.04	3.21	3.38	3.71
		h	1211.6	1229.8	1257.1	1282.6	1307.7	1332.4	1357.0	1405.9
		n	1.5614	1.5823	1.6120	1.6385	1.6632	1.6862	1.7082	1.7493
220	389.9	v	2.16	2.28	2.45	2.62	2.78	2.94	3.10	3.40
		h	1213.6	1232.2	1259.6	1285.2	1310.3	1335.1	1359.8	1408.8
		n	1.5541	1.5753	1.6049	1.6312	1.6558	1.6787	1.7005	1.7415
240	397.4	v	1.99	2.09	2.26	2.42	2.57	2.71	2.85	3.13
		h	1215.4	1234.3	1261.9	1287.6	1312.8	1337.6	1362.3	1411.5
		n	1.5476	1.5690	1.5985	1.6246	1.6492	1.6720	1.6937	1.7344
260	404.5	v	1.84	1.94	2.10	2.24	2.39	2.52	2.65	2.91
		h	1217.1	1236.4	1264.1	1289.9	1315.1	1340.0	1364.7	1414.0
		n	1.5416	1.5631	1.5926	1.6186	1.6430	1.6658	1.6874	1.7280
280	411.2	v	1.72	1.81	1.95	2.09	2.22	2.35	2.48	2.72
		h	1218.7	1238.4	1266.2	1291.9	1317.2	1342.2	1367.0	1416.4
		n	1.5362	1.5580	1.5873	1.6133	1.6375	1.6603	1.6818	1.7223

FLOW OF STEAM THROUGH A NOZZLE. — The rate of flow of steam through a nozzle increases as the difference in the pressures on the two sides of the nozzle increases, until the absolute pressure p_0 of the atmosphere into which the nozzle discharges reaches 58 per cent of the initial absolute pressure p of the steam. For greater differences in pressure, i.e., for $\frac{p_0}{p} < 0.58$, the rate of flow through a given nozzle depends only upon the initial absolute pressure of the steam. For $\frac{p_0}{p} < 0.58$ the percentage change in the volume of the steam as it passes through the nozzle also remains constant, the ratio of initial volume to expanded volume being 1.624. The following formulas have been given by the authorities stated for the flow of steam through a nozzle when $\frac{p_0}{p} < 0.58$. The notation is P = initial absolute pressure in pounds per square inch; A = smallest cross section of nozzle in square inches; W = flow in pounds per minute; x = quality of steam $= \frac{100 - y}{100}$ where y is the per cent of moisture; D = superheat in degrees F.

	Dry saturated	Moist	Superheated
Napier	$W = 0.857AP$	$\frac{AP^{0.97}}{x}$	$AP^{0.97}(1 + 0.00065\,D)$
Grashoff	$W = AP^{0.97}$		

FLOW OF STEAM THROUGH PIPES, VALVES, AND BENDS. — (*See Pipes and Piping.*)

BIBLIOGRAPHY. — Marks and Davis, *Steam Tables and Diagrams*, N. Y., 1910; *Peabody's Steam Tables*, N. Y., 1909; Ennis, W. D., *Applied Thermodynamics*, N.Y., 1910. See also bibliographies in the articles on *Boilers, Steam;* and *Steam Engines.*

[WM. KENT.]

STEAM ENGINES. — (*See also Condensers; Power Stations; Steam; Steam Turbines.*) A steam engine is a machine in which the energy of heat is converted into mechanical energy by means of the pressure of steam upon one or more parts of the machine. This article treats of the reciprocating engine only; steam turbines (q.v.) are treated in a separate article.

CLASSIFICATION. — The principal classifications of reciprocating engines are the following:

Single- and Double-acting Engines. — If the steam acts on only one side of the piston it is a single-acting engine; if on both sides, a double-acting.

Throttling and Automatic Engines. — A throttling engine is one in which the speed of the engine is governed by a throttle valve in the steam pipe, used to vary the pressure of the steam admitted to the engine. An automatic cut-off engine is one in which steam is always admitted at full pressure, but is cut off by a valve or valves at different points in the stroke according to the load, the point at which the cut-off takes place being automatically controlled by a governor.

Throttling as a means of governing is seldom employed in other than small engines. It affords a simple and reliable means of speed control for pumps, etc. Automatic regulation of cut-off is capable of closer speed regulation and permits higher ratios of expansion.

Condensing and Noncondensing Engines. — A condensing engine is one in which the steam is exhausted into a condenser, by means of which its pressure is reduced usually to within 1 to 3 pounds per square inch above a perfect vacuum. A non-condensing engine is one whose exhaust steam is discharged at or above atmospheric pressure. Any engine can be operated condensing or noncondensing, but certain minor modifications are usually made in the design of an engine when it is to be operated normally condensing.

Binary-vapor Engine. — This is a very special form of condensing engine. Sulphur dioxide, instead of water, is used as a cooling medium in a surface condenser. The sulphur dioxide in condensing the exhaust steam is itself vaporized and this vapor, under a pressure of about 175 pounds per square inch, is used expansively in a secondary reciprocating engine. The exhausted sulphur dioxide is discharged into a surface condenser in which it is liquefied by cooling water, much the same as in refrigerating practice, and used over and over again. This type of engine has never come into extensive use.

High- and Low-speed Engines. — This classification refers to rotative (fly wheel) speed only; engines having a rotative speed of 150 revolutions per minute or less are usually classified as low-speed engines; when the fly-wheel speed is greater than 150 revolutions per minute the engine is called a high-speed engine. A high-speed engine may have a lower *piston speed* than a low-speed engine; the relation between piston speed and rotative speed depends solely on the length of stroke.

Compound and Multiple-expansion Engines. — A compound engine is one in which the steam is partially expanded in a smaller cylinder and then carried to one or more larger cylinders in which it is further expanded. If the cylinders are in line with each other, using a common piston rod, it is called a tandem compound; if the smaller and larger, or high-pressure and low-pressure, cylinders are side by side it is called a cross-compound. The tandem type is simpler, lighter, cheaper and more compact, and serves well where exact balancing and uniform crank effort are not essential. The cylinders may be vertical or horizontal. Vertical engines take up less floor space, but are more costly than horizontal engines.

An engine in which the total expansion of the steam is divided into three stages, high-, intermediate- and low-pressure, is a triple-expansion engine; if into four stages, a quadruple-expansion engine. Multiple-expansion engines (other than compound) are seldom used for driving electric generators.

Classification According to Valve Gear. — Engines are also classified according to their valve gear (*see Valve Gear, below*), as Corliss engines, slide-valve engines, piston-valve engines and poppet-valve engines.

DESIGN AND CONSTRUCTION. — For a more complete treatment of the design and construction of steam engines see Kent's *Mechanical Engineers' Pocket-Book*. A brief discussion of valves and governors is given below.

Valves and Valve Gear. — Three types of valves are used to control the admission and exhaust of the steam to and from the cylinder, namely the slide valve, Corliss valve and poppet valve.

Slide Valves. — Fig. 1 shows a cylinder with a plain slide valve of the ordinary type. The valve rests in a V-shaped groove in the bottom of the steam chest, and is held up against its seat by pressure of the steam. The valve spindle passes through a channel in the back of the valve, as shown by the transverse section of the cylinder and steam chest, which allows the valve to press against its seat without springing the valve spindle.

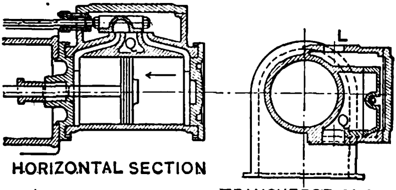


Fig. 1. Plain Slide Valve

In the drawing, the valve is shown moved over to the left so as to allow steam from the steam chest to pass into the head end of the cylinder and force the piston toward the left. Steam from the crank end of the cylinder can flow through a cavity in the valve to the exhaust space Q. Steam enters by the

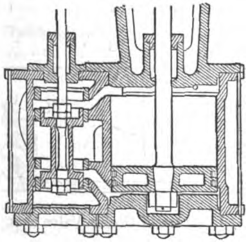


Fig. 2. Piston Valve

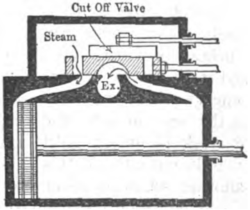


Fig. 3. Riding Cut-off Slide Valve

opening L (transverse section) and escapes through the exhaust space Q. As the piston moves toward the end of the forward stroke, the valve is moved by the eccentric to the right, and it first shuts off the supply of steam from the head end and the exhaust from the crank end of the cylinder, and then opens the supply of steam for the crank end and the exhaust for the head end just before the return stroke is begun.

Other types of slide valves are the piston valve and the riding cut-off valve. In the former, Fig. 2, is used a piston filling an auxiliary cylinder which takes

the place of the steam chest used with the common type. The riding cut-off slide valve, Fig. 3, is essentially a double slide valve. The riding cut-off controls the point of cut-off only, the points of admission, release and compression being controlled by the main valve. (*See Fig. 8 below.*)

Any adjustment of the simple slide valve changes all four events of the stroke, namely admission, cut-off, release and compression. With the riding cut-off slide valve the cut-off can be varied independent of the other events.

Corliss Valve. — To make all four events of the stroke independently adjustable, four separately controlled valves are necessary. One of the first successful four-valve engines was invented by George Corliss in 1848. Various modifications of the original Corliss valve gear have since been made, but the same general principle of operation is embodied in practically all modern four-valve engines.

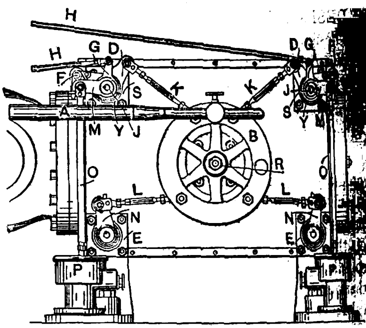


Fig. 4. Corliss Valve Gear

Fig. 4 shows the general external appearance of a simple Corliss valve gear and Fig. 5 is a simplified diagram showing the principle of operation.

"The steam valves work in the chambers *SS*, and the exhaust valves in the chambers *EE*. The double-armed levers *DD* work loosely on the hubs of the steam bonnets; they are connected to the wrist plate *B* by the rods *KK*; the levers *MM* are keyed to the valve stems *JJ*, and are also connected by the rods *OO* to the dashpots *PP*. The double-armed levers *D* carry at their outer ends steam hooks *FF*, these being provided with hardened-steel catch plates which engage with arms *MM*, making the arm *M* and the hook *F* work in unison until steam is to be cut off. At this point another set of levers or cams *GG*, connected by the cam rods *HH* to the governor, come into play, causing the catch plates on the hooks *F* to release the arms *MM*, the outer ends of which are then pulled downwards by the dashpot plunger, causing the steam valves to rotate on their axis and thus cut off steam. The exhaust valve arms *N* are connected to the wrist plate by the rods *LL*; and it is seen that all the valves receive their motion from the wrist plate *B*; the latter receives its motion from the hook rod *A*. This rod is attached to a rocker arm; to this arm the eccentric rod is also attached.

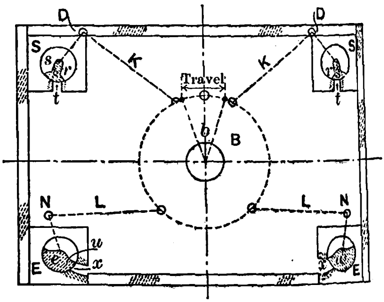


Fig. 5. Diagram of Corliss Valve Gear

"In order to obtain a greater range of cut-off in Corliss engines, a separate steam and exhaust eccentric is used. With two eccentrics the admission and

exhaust valves can be adjusted independently, and steam may be cut off anywhere, nearly to the end of the stroke." (*From Types of Modern Engines and Their Valve Setting, by M. C. Myers, Boston, 1910.*)

Poppet Valves. — This type of valve for engine cylinders is largely used in Europe, but only to a limited extent in this country. Fig. 6 illustrates one type of poppet valve and valve gear, known as the Sulzer valve gear.

Comparison of Different Types of Valves. — Multi-valve engines are more expensive than single-valve types, but give better economy, due to the independent regulation of cut-off and other events of the stroke, and to the reduced requirements of clearance and port space. Rotary valves are more difficult to make and keep steam-tight when used with superheated steam than the poppet and ordinary types. The erosive action and the severe temperature strains caused by superheat add much to the difficulty of maintaining a good valve fit, but the difficulties are minimized in the best designs.

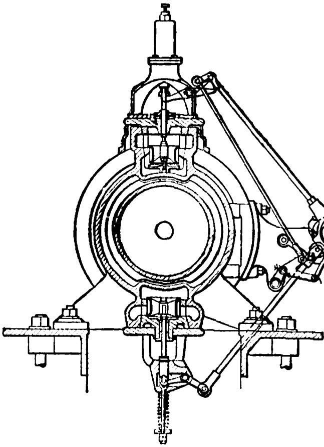


Fig. 6. Sulzer Valve Gear

Governors. — Two types of governors are used, viz., the pendulum or flyball and the flywheel types.

Pendulum Governor.—The construction of the pendulum governor is too well known to need description. This type of governor can be made to control the quantity of steam admitted to the cylinder either by opening or closing a throttle valve, or by varying the point of cut-off. In the latter case a suitable link motion (*see below*) must be provided.

Flywheel or Shaft Governor. — This type of governor is now largely used, especially for high-speed automatic engines.

The Rites governor, Fig. 7, one of the commonest forms of flywheel governors, is of the single-weight or inertia type. The center of gravity of the weight is located approximately at *G*. The governor is shown in position for latest cut-off; the rotation is in the direction of the arrow. Any increase in speed tends to make the center of gravity of the weight seek a position further away from the center of the shaft and causes the weight to swing on its pivot in a direction opposite to that of the wheel; the eccentric is attached to the weight, and this movement brings the center of the eccentric nearer the center of the shaft, and increases its angular advance, thus effecting an earlier cut-off.

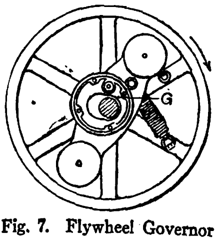


Fig. 7. Flywheel Governor

This and other types of shaft governors are described in detail in a small book on *Shaft Governors* (*Hill Pub. Co., 1908*).

Link Motion. — Link motions, of which the Stephenson link is the most commonly used, are designed for two purposes: first, for reversing the motion of the engine, and second, for varying the point of cut-off by varying the travel

of the valve. The Stephenson link motion is a combination of two eccentrics, called forward and back eccentrics, with a link connecting the extremities of the eccentric rods, so that by varying the position of the link the valve rod may be put in direct connection with either eccentric or may be given a movement controlled in part by one and in part by the other eccentric. When the link is moved by the reversing lever into a position such that the block to which the valve rod is attached is at either end of the link, the valve receives its maximum travel, and when the link is in mid-gear the travel is the least and cut-off takes place early in the stroke.

DEFINITIONS PERTAINING TO RATING AND PERFORMANCE OF STEAM ENGINES. — The following terms are commonly employed.

Indicated Horse-power. — The indicated horse-power P of an engine is found by the formula

$$P = \frac{pLAN}{33,000},$$

in which p is the mean effective pressure, in pounds per square inch, as found by an indicator (*see below*), L the length of the stroke in feet, A the area of the piston in square inches (corrected for area of the piston rod), N the number of single strokes per minute, equal to the number of revolutions of a single-acting, or twice the number of revolutions of a double-acting engine.

Brake Horse-power. — The brake horse-power of an engine is the power delivered by its shaft as determined by a dynamometer or Prony brake. It is equal to the indicated horse-power minus the power absorbed in friction of the engine. In well-designed engines working under normal loads the friction is usually from 8 to 12 per cent of the indicated power; the total power absorbed in friction is nearly a constant at all loads, so that its percentage increases to 100 as the load decreases to zero, when the whole of the indicated horse-power is absorbed in overcoming friction.

Rated Horse-power. — When an engine is commercially rated at a certain horse-power it is understood that it will deliver that power when run under those conditions for which it is designed, such as speed, steam pressure, back pressure, etc., and cutting off at that fraction of the stroke which will give its best economy. Its rated overload capacity is the power it will deliver with the same speed, steam pressure and back pressure, and cutting off at the latest point which the design of the valve motion will permit.

Indicator Diagram. — An indicator diagram is a diagram (see Fig. 8), showing the steam pressure in the engine cylinder at each point of the stroke. Such a diagram may be actually obtained by means of a steam-engine indicator, which is an instrument which causes a pencil to record on paper the pressure in the cylinder at every point of the stroke. The diagram drawn by the pencil shows whether the valves are properly adjusted, and it is also used in figuring the power developed in the cylinder, and approximately the steam consumption. A diagram of a noncondensing engine in which the steam is cut off at about one-quarter of the stroke is shown in Fig. 8.

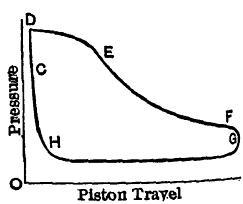


Fig. 8. Indicator Diagram of Simple Engine

The lines and points have the following significance.

Point of Admission, C, is the point at which the steam valve opens.

Admission Line, *CD,* shows the rise of pressure due to the admission of steam to the cylinder by opening the steam valve.

Steam Line, *DE,* is drawn when the steam valve is open and steam is being admitted tc the cylinder.

Point of Cut-off, *E,* is the point where the admission of steam is stopped by the closing of the valve. It is often difficult to determine the exact point at which the cut-off takes place. It is usually located where the outline of the diagram changes its curvature from convex to concave.

Expansion Curve, *EF,* shows the fall in pressure as the steam in the cylinder expands doing work.

Point of Release, *F,* shows when the exhaust valve opens.

Exhaust Line, *FG,* represents the change in pressure that takes place when the exhaust valve opens.

Back-pressure Line, *GH,* shows the pressure against which the piston acts during its return stroke.

Point of Exhaust Closure, *H,* is the point where the exhaust valve closes. It cannot be located definitely, as the change in pressure is at first due to the gradual closing of the valve.

Compression Curve, *HC,* shows the rise in pressure due to the compression of the steam remaining in the cylinder after the exhaust valve has closed.

Initial Pressure is the pressure acting on the piston at the beginning of the stroke.

Terminal Pressure is the pressure above the line of perfect vacuum that would exist at the end of the stroke if the steam had not been released earlier. It is found by continuing the expansion curve to the end of the diagram.

Other Definitions. — In addition to the terms defined above, the following are commonly employed:

Throttle Pressure is the pressure in the steam pipe at the entrance to the throttle valve.

Mean Effective Pressure is that equivalent constant pressure which will do the same amount of work on the piston per stroke as is done by the varying pressure shown by the indicator card. This may be calculated by dividing the area of the card by the length and multiplying by the scale of the spring used in the indicator.

Clearance. — The portion of the cylinder volume, including the steam ports, not swept through by the piston but which is nevertheless filled with steam when admission occurs is called the clearance volume. It ranges from 1 per cent of the piston displacement in very large engines to 10 per cent or more in small high-speed engines.

Ratio of Expansion is the ratio of the piston displacement (in low-pressure cylinder in case of a multiple-expansion engine) to the volume of the steam admitted through the throttle valve at each stroke (or half stroke in case of a double-acting engine), the volume being that corresponding to the pressure on the engine side of the throttle valve.

Wire Drawing, as applied to steam, is the reducing of its pressure, due to its flowing through restricted pipes and passages.

EFFICIENCY OF A STEAM ENGINE. — By the thermal efficiency of a steam engine is meant the quotient of the B.t.u. per hour equivalent to the

indicated horse-power divided by the total number of B.t.u. in the steam supplied per hour; or, putting

W = pounds of steam supplied per hour per indicated horse-power,

x = quality of the steam supplied (= lb. of dry steam per lb. of wet steam; see article on *Steam*),

L = heat of evaporation per pound of steam at throttle pressure, in B.t.u. per pound,

h = heat of the liquid at throttle pressure, B.t.u. per pound,

h_e = heat of the liquid of feed water, B.t.u. per pound, taken at the temperature corresponding to the pressure in the exhaust pipe near the engine. (*See A.S.M.E. code, 1913.*)

Then the thermal efficiency for saturated steam is

$$E_t = \frac{2546.5}{W\,(xL + h - h_e)}.$$

If the steam is superheated, let

H = total heat of superheated steam at throttle pressure and degree of superheat, in B.t.u. per pound.

Then the thermal efficiency is

$$E_t = \frac{2546.5}{W\,(H - h_e)}.$$

The above formulas are also applicable to the calculation of the over-all efficiency, including friction losses, if W is taken as the pounds of steam supplied per hour per brake horse-power.

Efficiency of Rankine Cycle. — The maximum possible thermal efficiency is that of an engine performing the ideal Carnot's cycle (*see Thermodynamics, Principles of*), but this cycle is not very closely simulated by ordinary engines, even when the losses are neglected. Instead the efficiency of the "Rankine cycle" is employed as a standard of reference. This is an ideal cycle which assumes that the work done by the engine is equal to the "maximum work" (*see Thermodynamics*) corresponding to an adiabatic expansion from throttle pressure to exhaust pressure and an isothermal condensation of the exhaust steam and a compression of the condensed water to throttle pressure.

Rankine Efficiency for Dry Saturated Steam. — Let

H_1 = total heat, B.t.u. per pound, of dry saturated steam at throttle pressure,

H_2 = total heat, B.t.u. per pound, of dry saturated steam at exhaust pressure,

h_2 = heat of the liquid, B.t.u. per pound, at exhaust temperature,

T_2 = temperature of exhaust steam, in °F.,

N_1 = total entropy of 1 pound of dry saturated steam at throttle pressure,

N_2 = total entropy of 1 pound of dry saturated steam at exhaust pressure.

Then the efficiency is

$$E_r = \frac{H_1 - H_2 + T_2\,(N_2 - N_1)}{H_1 - h_2}.$$

Rankine Efficiency for Wet Steam. — In addition to the above symbols let

H_w = total heat, B.t.u. per pound, of wet steam at throttle pressure,

N_w = total entropy of 1 pound of wet steam at throttle pressure,

T_1 = temperature of steam at throttle pressure, in °F.

Then the efficiency is

$$E_r = \frac{H_1 - H_2 + T_2\,(N_2 - N_1) - (N_1 - N_w)\,(T_1 - T_2)}{H_w - h_2}.$$

Rankine Efficiency for Superheated Steam. — In addition to the above symbols let

H_s = total heat, B.t.u. per pound, of superheated steam at throttle pressure,
N_s = total entropy of 1 pound of superheated steam at the given degree of superheat at throttle,
t_s = degree of superheat at throttle, in °F.

Then the efficiency is

$$E_r = \frac{H_1 - H_2 + T_2 (N_2 - N_1) + (T_1 + 0.5\, t_s - T_2)(N_s - N_1)}{H_s - h_2}.$$

Efficiency Ratio Referred to the Rankine Cycle. — This is the ratio of the actual efficiency of the engine (referred to the indicated horse-power) to the efficiency of the Rankine cycle (using dry steam) working between the same limits of pressure, i.e.,

$$\text{Eff. Ratio} = \frac{E_t}{E_r}.$$

STEAM ENGINE EFFICIENCIES—SATURATED STEAM.
(*From Gebhardt's Steam Power Plant Engineering.*)

Gage pressure	Noncondensing				Condensing — 1 lb. absolute pressure			
	Carnot cycle	Rankine cycle	Best actual (1907)	Eff. ratio %	Carnot cycle	Rankine cycle	Best actual	Eff. ratio %
25	7.5	7.3	5.5	76	22.6	21.0	11.6	55
50	11.2	10.7	8.5	80	25.7	23.5	13.5	60
75	13.7	13.0	10.4	80	27.8	25.3	15.9	61
100	15.7	14.8	12.0	81	29.5	26.7	20.2	76
125	17.3	16.3	13.5	83	30.8	27.8	20.3	74
150	18.7	17.5	14.3	82	32.0	28.8	21.6	75
175	19.8	18.5	14.8	80	32.9	29.6	21.9	74
200	20.8	19.3	15.2	79	33.7	30.2	22.6	75
225	21.6	19.9	15.5	78	34.5	30.6	22.6	74
250	22.4	20.5	....	..	35.1	31.0	....	..
275	23.0	21.0	....	..	35.6	31.3	....	..
300	23.6	21.4	....	..	36.0	31.5	....	..

The actual thermal efficiencies of multiple expansion engines using superheated steam range from about 19 to 23 per cent.

Engine Losses. — The actual efficiencies of steam engines are necessarily much lower than the efficiency of the Rankine cycle due to the thermal and mechanical losses. These losses consist chiefly of (1) cylinder condensation, (2) steam leakage, (3) incomplete expansion and compression on return stroke, (4) friction, (5) clearance loss, (6) radiation and (7) the admission of wet steam.

Cylinder Condensation is due to the chilling of steam by the cooler cylinder walls during admission and early expansion, thus reducing the active heat during expansion. Condensation increases with the range of expansion per cylinder and with the duration of the cycle. The condensation loss is

augmented by the formation of a water film on the cylinder walls. It is successfully reduced by expanding the steam in several stages, thus reducing the range of temperature in each cylinder and rendering the heat carried to the exhaust in that cylinder available in succeeding cylinders. Other remedies for condensation are: increasing the rotative speed; steam jackets about the cylinders; and reheaters to dry the steam between cylinders. The most positive and effective remedy is superheat sufficient to keep the steam dry at least during admission.

Steam Leakage past valves, pistons and packing increases with the pressure difference. It is an important element of loss in all piston engines and tends to increase with the wear of moving parts. Leakage is especially serious in cylinders employing a wide range of pressures. Flow of steam through orifices, in pounds, increases directly as the pressure (Napier's rule), and through pipes as $\sqrt{\text{density}}$. (*p. 845* Kent's *Mechanical Engineers' Pocket-Book.*)

Incomplete Expansion and Compression. — As a rule the release occurs before the piston reaches the end of the stroke and the point of closure comes before the piston reaches the end of the back stroke. There is a consequent loss of expansion on the forward stroke. The piston also has to do work in compressing the steam remaining in the cylinder on the reverse stroke. These two features are, as a rule, necessary; the first to reduce cylinder condensation, and the second for its cushioning effect. The first always, and the second usually, results in a net loss.

Friction varies with the type and condition of the engine, is greater in compound engines than in simple engines, and is nearly independent of the load, increasing but slightly as the load increases. It ranges from 4 to 20 per cent of the rated output.

Losses Due to Radiation, Clearance and Moisture. — With well-lagged cylinders radiation losses are small. Some clearance is necessary in every engine, but the amount is much greater for high speeds than for low speeds. The loss due to it is trifling in slow-speed engines of long stroke. Moisture dilutes the steam admitted but has little effect on the consumption of *dry* steam; the consumption of the fluid (steam and moisture) is of course increased in proportion to the percentage of moisture present.

ECONOMY OF STEAM ENGINES.* — The performance of a steam engine is frequently expressed in terms of the number of pounds of steam per hour required per indicated horse-power, per brake horse-power or per kilowatt (if used to drive an electric generator). The number of pounds of steam per unit of output is called the "economy" of the engine or of the combined engine and generator unit.

Such data may be very misleading unless comparisons are based on identical conditions of steam pressure, superheat, vacuum, etc. A true measure of economy applicable to all conditions may be expressed in net heat units consumed per unit of output, as B.t.u. per kilowatt-hour. In such determinations of economy all heat returned from the exhaust to the boiler should be credited to the engine or turbine.

The commonest means of gaining good engine economy are: raising the initial pressure, compounding, condensing and superheating.

Effect of Steam Pressure on Economy and Capacity. — There can be no universal rule connecting initial pressure and working economy. Compound

* Adapted from lecture notes of Prof. W. E. Wickenden.

engines are better adapted to high pressures than simple engines. Experience indicates that the following gauge pressures are desirable:

Simple, condensing engine without steam jackets	60 lb.
Simple, condensing engine with steam jackets........	80 lb.
Simple, non-condensing engine without steam jackets .	100 lb.
Compound, non-condensing engines	175 lb.
Compound, condensing engines	150 lb.
Triple-expansion, condensing engines	175 to 200 lb.
Quadruple-expansion condensing engines	200 lb.

An important advantage from the use of high pressures is the greater capacity which can be developed from a given engine. The ratio of pressure and capacity is nearly a direct one. High pressures tend to improve economy but not to the extent theoretically available, due to the counteracting effects of condensation and steam leakage with increasing pressure ranges. High steam pressures, however, usually add to the expense of piping and its maintenance. From an examination of a large number of engine performances Stevens and Hobart give the general relation of pressure and economy as shown in Fig. 9.

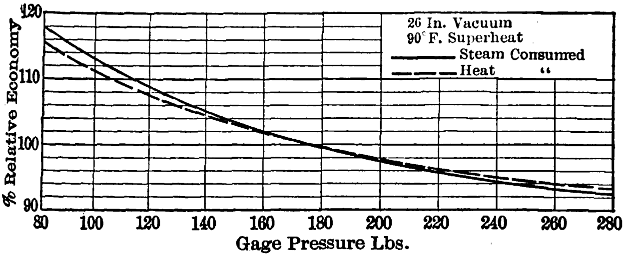


Fig. 9. Effect of Steam Pressure on Economy of Compound Piston Engines

Effect of Multiple Expansion on Economy. — The loss due to cylinder condensation increases with an increase of the difference of temperature of the steam during admission and exhaust, and therefore tends to become greater as the initial pressure is higher. By dividing the total range of expansion into two or more stages, the range of temperature in any one cylinder is decreased, and this lessens the total loss due to cylinder condensation.

Triple and quadruple expansion afford substantial economies with constant load as in pumping and marine service, but have little advantage with the variable loads of electric service to compensate for their great weight, complexity, bulk and cost. Compound engines with high cylinder ratios permit more complete expansion, but have less overload capacity than engines of smaller ratios. For electric service more than 18 expansions is seldom profitable. A cylinder ratio of 1 : 2.5 is well adapted to a pressure of 100 pounds; 1 : 3 to 125 pounds; 1 : 3.5 to 150 pounds and 1 : 4 to 175 pounds or above. Compound engines are relatively uneconomical at light loads, but have a greater economical range of overloads than simple engines. Compound engines, with a moderate cylinder ratio, usually give their best economy at ¼ cut-off in the high-pressure cylinder and will safely carry an overload of 50 per cent.

Effect of Condensing on Economy. — Condensing serves to increase both the capacity and economy of engines. The theoretical gain from condensing to various back pressures is indicated in Fig. 10. Actual engines can avail themselves of but a part of this gain, the proportion falling as the vacuum is increased due to the added condensation and leakage. Expansion to very

low pressures requires cylinders of excessive volume. Condensing equipment, especially that for high vacuum, adds much to the first cost of a plant and hence

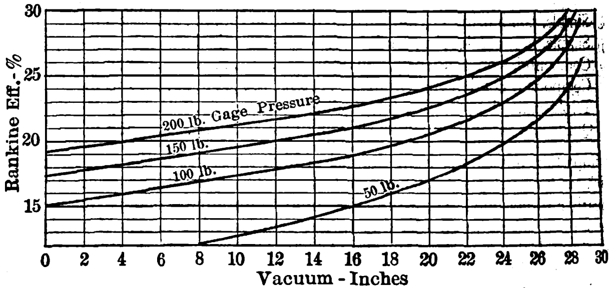


Fig. 10. Effect of Vacuum on Theoretical Efficiency

to its burden of fixed charges. (*See article on Condensers.*) Condensers require from one to three pumps and the thermal gain is reduced by the amount of their heat consumption. The temperature of the exhaust steam is lowered as the vacuum increases and so reduces the heat which can be reclaimed in the feed water. It is of great importance that the net gain rather than the apparent gain be considered in determining the economics of various vacua. Reduced steam consumption lessens the necessary investment in steam-generating equipment and piping, and this tends to somewhat counterbalance the investment in condensing apparatus. (*See article on Power Stations.*)

Effect of Superheating on Economy. — Superheating the steam is more effective than using steam jackets and reheaters as a preventive of cylinder condensation under the usual central-station conditions. With proper valve structures there is a decided net saving in heat consumption as the superheat is increased within reasonable limits, but this procedure usually adds to the

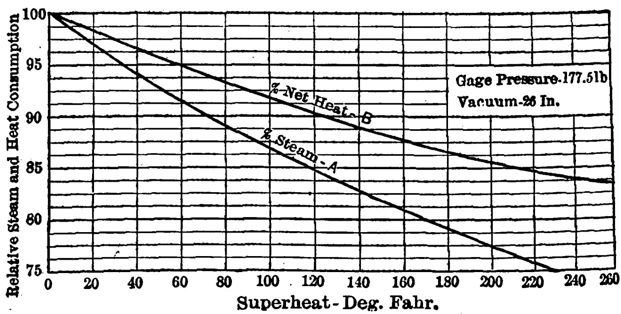


Fig. 11. Effect of Superheat on Piston-engine Economy

cost of piping, fittings and their upkeep. Stevens and Hobart report the mean relationship between superheat and economy in compound and multiple expansion engines, as shown in Fig. 11. It is especially important that economic comparison be based on the heat unit consumption as in Curve B. Additional advantages from superheating are the reduction of condensation, radiation losses and pressure drop in steam pipes.

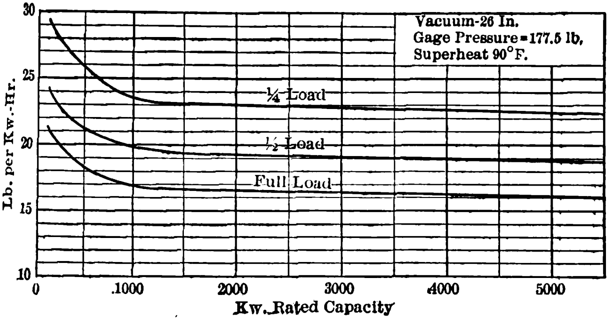


Fig. 12. Steam Consumption of Representative Compound Engine

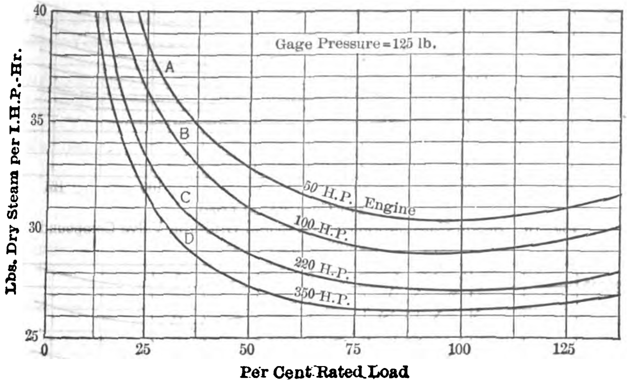


Fig. 13. Steam Consumption of Good Piston Valve Simple High-speed, Non-condensing Engines

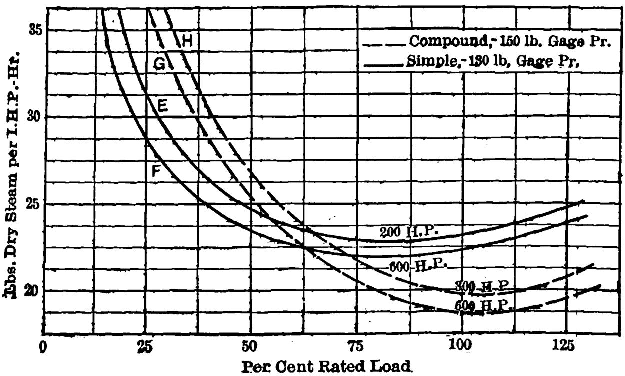
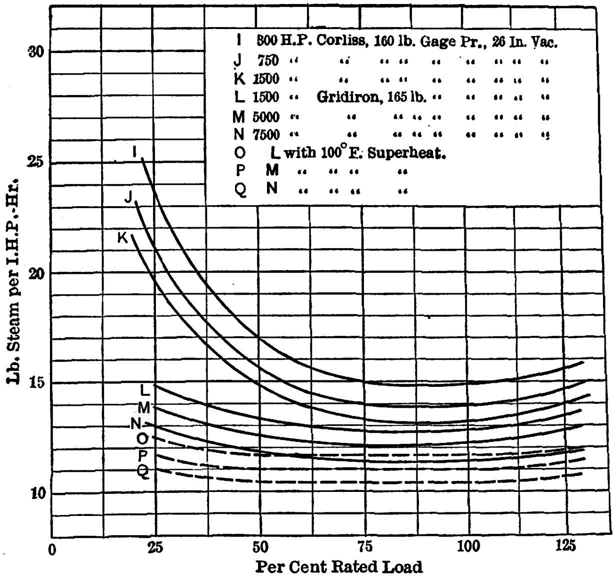
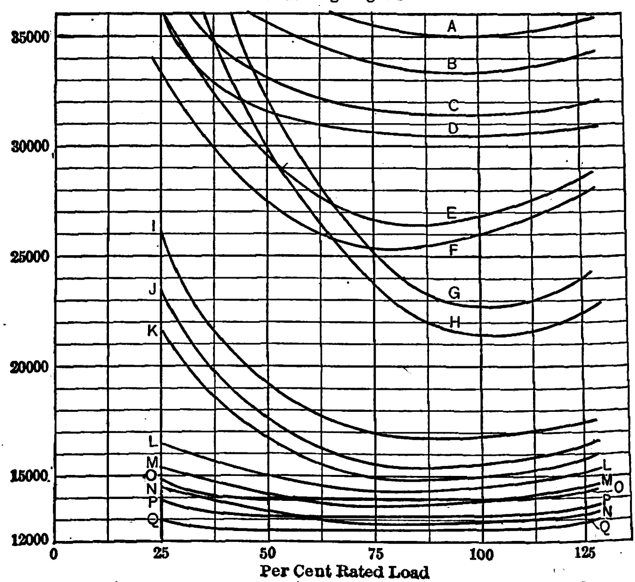


Fig. 14. Steam Consumption of Good 4-Valve Non-condensing Engines



Per Cent Rated Load

Fig. 15. Steam Consumption 4 Corliss and Grid-iron Valve Compound Condensing Engines



Per Cent Rated Load

Fig. 16. Engine Performance in B.t.u. per Indicated Horse-power-hour

Variation of Economy with Load. — Other factors affecting the economy of engines are the size of the unit, the speed, the mode of governing and the uniformity of load. Stevens and Hobart give the relations between engine sizes, loads and economy shown in Fig. 12 as averages for compound piston engines. Simple engines as a rule show smaller variation in economy with partial loads than compound engines. Typical economy curves for simple and compound engines with saturated steam and compound engines with superheated steam are shown in Figs. 13 to 16. The curves in Fig. 16 correspond to like lettered curves in the other figures.

Economy of the Binary-vapor Engine. — With this type of engine there has been obtained an economy of only 8.36 pounds of steam per indicated horse-power-hour, corresponding to a heat consumption of 158.3 B.t.u. per minute. This was the best recorded performance in steam-engine practice at the time of the report (1907) and has not been noticeably exceeded since (1913).

DIMENSIONS AND WEIGHT OF ENGINES depend so largely upon the type, speed, etc., that it is impossible to give representative figures in the space available. The following table from the *Elec. World*, (*Sept.*, *1902*), gives the dimensions of some very large engines in New York City.

DIMENSIONS OF SOME LARGE RECIPROCATING ENGINES

Name of station	Metropolitan	Manhattan	Kingsbridge	Rapid Transit	Edison
Type of engine	Vert. cross-comp.	Double 2 hor. 2 vert. cyls.	Vert. cross-comp.	Double, 2 hor. 2 vert. cyls.	3-cyl. vert.
Rated horse-power	4500	8000	4500	8900	5200
Cylinders, all 60-in. stroke, in.	46, 86	44, 88	46, 86	42, 86	43½, 2-75½
Piston rods, diam., in.	9, 10	8	9, 10	8, 10	9
Crank pins, in.	14×14	18×18	14×14	20×18	22 & 16×14
Wrist pins, in.	14×14	12×12	14×14	12×12	14×14
Shaft length	27 ft. 4 in.	25 ft. 3 in.	27 ft.	25 ft. 3 in.	35 ft.
max. diam.	37 in.	37 in.	39 in.	37 in.	29⅜ in.
bearings, in.	34×60	34×60	34×60	34×60	26×60

The shafts are hollow, with a 16-inch hole, except the Edison which have a 10-inch hole. The speed of all the engines is 75 revolutions per minute, or 750 feet per minute. The crank pins of the Manhattan and Rapid Transit engines are each attached to two connecting rods, side by side, horizontal and vertical, each rod having a bearing 9 inches long on the pin. The crank pins of the Edison engine are 16 inches in diameter for the side cranks, and 22 inches for the center crank.

TESTING OF STEAM ENGINES. — The actual work done by an engine may be tested by a brake or friction dynamometer, or in the case of engines driving electric generators by measuring the electric energy delivered, correction being made for the efficiency of the generator. The work done in the cylinder is obtained by taking numerous indicator diagrams and calculating

from them the mean effective pressure and the horse-power. The steam consumption is tested by condensing all the steam discharged by the engine and weighing the water of condensation. When a condenser is not available the feed water delivered to the boilers may be weighed instead, precautions being taken to insure that all of the steam made by the boiler is used by the engine. Full directions for making steam-engine tests will be found in the "Code of 1902" in *Trans. A.S.M.E.*, of that year, and reprinted by the Society in a pamphlet of 78 pages. For a condensed abstract of the Code see Kent's *Mechanical Engineers' Pocket-Book, p. 988.* (A revised Code will probably be printed in 1914.)

Plant Tests. — In the introduction to the report above referred to the Committee says:

The heat consumption of a steam-engine plant is ascertained by measuring the quantity of steam consumed by the plant, calculating the total heat of the entire quantity, and crediting this total with that portion of the heat rejected by the plant which is utilized and returned to the boiler. The term "engine plant" as here used should include the entire equipment of the steam plant which is concerned in the production of the power, embracing the main cylinder or cylinders; the jackets and reheaters; the air, circulating, and boiler-feed pumps, if steam driven; and any other steam-driven mechanism or auxiliaries necessary to the working of the engine. It is obligatory to thus charge the engine with the steam used by necessary auxiliaries in determining the plant economy, for the reason that it is itself finally benefited, or should be so benefited, by the heat which they return, it being generally agreed that exhaust steam from such auxiliaries should be passed through a feed-water heater, and the heat thereby carried back to the boiler and saved.

SPECIFICATIONS. — (*See also article on Specifications.*) In obtaining bids for steam engines the prospective purchaser or his engineer usually furnishes only general specifications, stating the kind of work to be done by the engine, such as driving a cotton mill, or an electric power plant, and the following requirements: (1) Indicated horse-power required, (*a*) Maximum, (*b*) Average; (2) Steam pressure available at engine; (3) Maximum absolute back pressure; (4) Number of revolutions; (5) Required steam consumption.

The bidders then furnish with their bids complete detailed specifications with guarantees of the engines they propose to supply, with drawings, when these are required. The bidders are usually prepared to contract for the installation, erection and starting of the engine, especially if it is of a large size.

FOUNDATIONS FOR STEAM ENGINES. — Engine foundations may be built of brick or concrete; the latter is more commonly employed in modern practice. The concrete is made with proportions of one barrel of Portland cement, three of sand, and five of crushed stone.

If the foundation is to be of concrete a wooden box is built in the ground, the inside of the box being the shape of the foundation. Over the top of this box are a number of pieces of joist, from which the anchor bolts or holding-down bolts are hung. Most engine builders furnish drawings giving location of foundation bolts and size of foundation required. The nuts at the top of these bolts are blocked up to the height they are to be when the engine is in place.

The heads of the bolts are at the bottom. A cast-iron plate, eight inches by eight inches by one inch, with a square recess for the bolt head, is at the bottom of each rod. Around each bolt a piece of four-inch galvanized-iron gutter pipe is placed, and the top end of the pipe stuffed with waste to keep the mason from dropping cement into the space between the pipe and the bolt. The concrete is now dumped in and tamped down. After it has set, the wooden joists holding the bolts may be taken away.

The engine is next put over the foundation and supported on iron bars one-half inch square or perhaps larger, these bars being placed near the bolts. By means of wedges, etc., the engine is leveled; the main shaft is placed parallel with the shafting which it is to drive. The bolts are next tightened fairly tight. As it is impossible to core the holes in the engine bed exactly as called for on the drawing, the leeway given by the space between the bolts and the iron pipe surrounding it takes care of variations of from an inch to an inch and a half.

A wall of putty, sand, or cement about one inch high is built about one inch from the engine bed. A thin, neat Portland cement, mixed to the consistency of a thick cream, is poured under the engine bed and serves to fill the space around the bolts and to give the bottom of the bed a perfect bearing over the entire foundation. Before the cement becomes hard, the edge may be trimmed up nicely by means of a trowel; the bolts are then tightened again.

Sometimes type metal instead of cement is poured under the engine bed.

OPERATION. — Full directions for setting the valves of a steam engine will be found in a little book by M. C. Myers, entitled *Types of Modern Engines and Their Valve Setting* (Boston, 1910). Other useful data on the operation and performance of steam engines will be found in a book published by the Crosby Steam Gauge and Valve Co., entitled *Steam Engine Indicator*. See also the more elaborate treatises listed in the bibliography below.

Speed Control. — The speed of an engine can always be controlled by changing the opening of the throttle valve, but this method is usually wasteful. A more efficient method is to change the point of cut-off. This can be done while the engine is running by using a suitable link motion (*see above*), or by changing the setting of the governor spring if the governor is of the "automatic" type. On engines driving alternators in parallel this is usually accomplished by having an electric motor arranged to act directly on the governor spring so that the tension of the spring may be controlled from the switch-board.

Equalizing Variable Load by Storing Heat in Hot Water. — There is no satisfactory method for equalizing the load on the engines and boilers in electric-light stations. Storage batteries have been used, but they are expensive in first cost, repairs and attention. Mr. Halpin, of London, proposes to store heat during the day in specially constructed reservoirs. As the water in the boilers is raised to 250 pounds pressure, it is conducted to cylindrical reservoirs resembling English horizontal boilers, and stored there for use when wanted. In this way a comparatively small boiler-plant can be used for heating the water to 250 pounds pressure all through the twenty-four hours of the day, and the stored water may be drawn on at any time, according to the magnitude of the demand. The steam engines are to be worked by the steam generated by the release of pressure from this water, and the valves are to be arranged in such a way that the steam shall work at 130 pounds pressure. A reservoir 8 feet in diameter and 30 feet long, containing 84,000 pounds of heated water at 250 pounds pressure, would supply 5150 pounds of steam at 130 pounds pressure. At a steam consumption of 18 pounds per horse-power-hour, such a reservoir would supply 286 effective horse-power-hours.

COST OF STEAM ENGINES. — The price of steam engines varies not only with their size and weight, but also with their style, design and workmanship. When builders are asked to submit bids for the same rated power of engine, with the same steam pressure and number of revolutions and of the same general type, such as cross-compound condensing, it is not uncommon to receive prices of which the highest is double that of the lowest, the range being say from $10 to $20 per horse-power for an engine of 500 horse-power. For specially

designed engines requiring new patterns to be made the price may be considerably above $20. The price also varies with the condition of the market, discounts from regular prices of as much as 20 per cent being sometimes quoted in periods of dull business. The catalogues of builders usually give the weights of standard sizes of engines and a rough approximation to the probable price may be made by taking it at from 5.5 to 10 cents per pound, the lower price being for engines with heavy bed-plates and of the more simple forms.

From a number of tables and sets of curves kindly furnished by Mr. Jay M. Whitham, Consulting Engineer, Philadelphia, containing the actual bids he has received for different sizes and styles of engines, the following approximate expressions have been deduced, where P is the rated horse-power of the engine.

Type of Engine	Cost in dollars
Corliss engines:	
Single cylinder, noncondensing	$500+9P$
Single cylinder, condensing	$500+11P$
Compound, noncondensing	$1200+10P$
Compound, condensing	$1200+12P$
Compound slide-valve engine:	
Portable (locomotive) boiler, noncondensing engine and stack	$300+15P$

The cost of condensing engines is exclusive of condenser.

Professor Wickenden gives the following values for engines used to drive electric generators, these values being based on the actual costs of a large number of installations.

Type of Engine	Cost in dollars
Simple high-speed engines and settings	$350+9P$
Compound high-speed engines and settings	$1000+16P$
Simple low-speed engines and settings	$1400+11.5P$
Compound low-speed engines and settings	$2500+14.5P$

Cost of Operation. — See articles on *Power Stations* and *Depreciation*.

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[Wm. Kent.]

STEAM TURBINES. — (*See also Condensers, Steam; Power Stations; Steam; Steam Engines.*) Steam turbines may be classified as impulse, reaction, and composite; single-stage and multi-stage; nozzle expansion and blade expansion; vertical and horizontal. A set of nozzles or stationary guide blades and the rotating vanes through which the steam passes immediately upon leaving the nozzles or stationary blades constitute a single "stage." If there are several sets of stationary nozzles or guide blades and rotating vanes through which the steam passes successively, the turbine is called a "multi-stage" turbine.

APPLICATIONS OF STEAM TURBINES. — The steam turbine, when designed to use steam economically, is characterized by extremely high speed of rotation. The special field of application of the steam turbine is therefore the driving of electric generators, centrifugal blowers and pumps, and other machinery in which a high speed of the motor shaft is desirable. When lower speeds are desired in the motor shaft a reduction gear of some kind is placed between it and the turbine shaft. See also articles on *Generators; Power Stations; Pumps.*

RATING OF STEAM TURBINES. — Turbine ratings are usually based on maximum sustained load. Momentary overload capacity is very large and moderate overloads of considerable duration can be carried but may require the admission of high-pressure steam to low-pressure stages by means of a secondary valve. Small turbines for driving pumps, blowers, etc., are rated in horse-power. Turbines used to drive electric generators are usually rated in connection with the generator, the combined unit or turbo-generator being rated in kilowatts. Turbo-alternators of 20,000 kilowatts capacity each are in use (1913), and still larger units are considered practicable.

DESIGN AND CONSTRUCTION OF STEAM TURBINES. — Below are noted briefly some of the chief features in the design and construction of steam turbines. For further details see the treatises by Stevens and Hobart, Moyer, Stodola, Foster, Thomas, etc.

Impulse Turbines. — An impulse steam turbine of the simplest form is a wheel similar to a water wheel, which is moved by a jet of steam impinging at high velocity on its blades. By expansion of the steam its pressure energy is converted into kinetic energy, and the wheel is propelled by the impact of the steam moving at high velocity.

De Laval Turbine. — The distinguishing features of this turbine are the diverging nozzles, in which the steam expands down to the atmospheric pressure in non-condensing, and to the vacuum pressure in condensing wheels; a single forged-steel disk carrying the blades on its periphery; a slender, flexible shaft on which the wheel is mounted and which rotates about its center of gravity; and a set of reducing gears, usually 10 to 1 reduction, to change the very high speed of the turbine to a moderate speed for driving machinery.

The number and size of nozzles vary with the size of the turbine. The nozzles are provided with valves, so that for light loads some of them may be closed, and a relatively high efficiency is obtained at light loads. The taper of the nozzles differs for condensing and noncondensing turbines. Some turbines are provided with two sets of nozzles, one for condensing and the other for non-condensing operation.

Zolley or Rateau Turbine. — The Zolley or Rateau turbines are developments of the De Laval and consist of a number of De Laval elements in series, each succeeding element utilizing the exhaust steam from the preceding. The steam is partly expanded in the first row of nozzles, strikes the first row of

buckets and leaves them with practically zero velocity. It is then further expanded through the second row of nozzles, strikes a second row of moving buckets and again leaves them with zero velocity. This process is repeated until the steam is completely expanded.

Curtis Turbine, made by the General Electric Company, is an impulse wheel of several stages. Steam is expanded in nozzles and enters a set of three or more blades, at least one of which is stationary. The blades are all non-expanding, and the pressure is practically the same on both sides of any row of blades. In smaller sizes of turbines, only one set of stationary and movable blades is used, but in large sizes there are from two to five sets, each forming a pressure stage, separated by diaphragms containing additional sets of nozzles. The smaller sizes and the more recent (1914) larger sizes have horizontal shafts. Earlier large sizes have vertical shafts supported on a step bearing supplied with oil or water under a pressure sufficient to support the whole weight of the shaft and its attached rotating disks.

Reaction and Composite Turbines.— In a simple reaction turbine steam, expanding through a set of blades or nozzles capable of rotating about a fixed axis, produces a reacting force and drives the rotating part in the opposite direction to the motion of the steam. Hero's engine is the prototype of the reaction turbine. The term "reaction" turbine, however, as ordinarily used, refers to a turbine in which the reaction principle is combined with the impulse principle, i.e., to turbines in which jets of steam striking blades or buckets inserted in the rim of a wheel give it a forward impulse, and escaping from it in a reverse direction react upon it.

Parsons Turbine. — This is a reaction turbine in which there are a large number of rows of blades, mounted on a rotor or revolving drum. Between each pair of rows there is a row of stationary blades attached to the casing, which take the place of nozzles. A set of stationary blades and the following set of moving blades constitute what is known as a stage. The steam expands and loses pressure in both sets. The speed of rotation, the peripheral speed of the blades and the velocity of the steam through the blades are very much lower than in the De Laval turbine. The rotor, or drum, on which the moving blades are carried, is usually made in three sections of different diameters, the smallest at the high-pressure end, where steam is admitted, and the largest at the exhaust end. In each section the radial length of the blades and also their width increase from one end to the other, to correspond with the increased volume of steam. The Parsons turbine is built in the United States by the Westinghouse Machine Co. and by the Allis-Chalmers Co.

The Westinghouse Double-flow Turbine. — For sizes above 5000 kw. a turbine is built in which the impulse and reaction types are combined. It has a set of non-expanding nozzles, an impulse wheel with two velocity stages (that is, two wheels with a set of stationary non-expanding blades between), one intermediate section and two low-pressure sections with Parsons blading. After steam has passed through the impulse wheel and the intermediate section, it is divided into two parts, one going to the right- and the other to the left-hand low-pressure section. There is an exhaust pipe at each end. In this turbine, the end thrust, which has to be balanced in reaction turbines of the usual type, is almost entirely avoided. Other advantages are the reduction in size and weight, due to higher permissible speed; blades and casing are not exposed to high temperatures; reduction of size of exhaust pipes and of length of shaft; avoidance of large balance pistons.

Comparison of Impulse and Reaction Turbines. — Moyer gives the following comparison of impulse and reaction turbines:

Impulse	*Reaction*
1. Few stages.	1. Many stages.
2. Expansion in nozzles.	2. No nozzles.
3. Large drop in pressure in a stage.	3. Small drop in pressure in a stage.
4. Initial steam velocities 1000 to 4000 feet per second.	4. All steam velocities low, 300 to 600 feet per second.
5. Blade velocities 400 to 1200 feet per second.	5. Blade velocities 150 to 400 feet per second.
6. Best efficiency when the blade velocity is nearly half the initial velocity of steam.	6. Best efficiency when the blade velocity is nearly equal to the highest velocity of the steam.

Experience has not made manifest any marked and consistent difference in economy and reliability between the impulse and reaction types.

Vertical Versus Horizontal Turbines. —There is also no marked difference in economy and reliability of the vertical and horizontal types. The former is more economical of floor space, especially when the base type of condenser is employed, but requires more head room, as sufficient overhead space must be allowed to permit the lifting clear, and removal of, the shaft from the casing.

Reduction Gear for Steam Turbines. — Double spiral reduction gears, usually of a ratio of 1 to 10, are used with the De Laval turbine to obtain a velocity of rotation suitable for dynamos, centrifugal pumps, etc. G. W. Melville and J. H. McAlpine have designed a similar gear, with the pinion carried in a floating frame supported at a single point between the bearings to equalize the strain on the gear teeth, for reducing the speed of large horizontal turbines to suitable speeds for marine propellers. A 6000-horse-power gear with reduction from 1500 to 300 revolutions per minute has been tested, giving an efficiency of 98.5 per cent (*Eng'g, Sept. 17; Eng. News, Oct. 21 and Dec. 30, 1909*).

TURBINES VERSUS RECIPROCATING ENGINES.* — The inherent advantages of the turbine are: (1) The perfect continuity and uniformity of effort. This permits the working parts to be made light and gives the uniform angular velocity which is desired for the parallel running of alternators. (2) The high velocity of rotation and of steam flow. This tends to great compactness, permits light weights, provides for the full expansion to high vacuum without enormous volume of expansive space, cheapens the cost of turbine and alternator, and is favorable to the efficiency of the latter. The effect of items (1) and (2) on weights, sizes and costs may be best appreciated by comparing turbine and gas-engine sets of equal maximum capacity. (3) The reduction to a minimum of condensation, radiation and leakage. (4) The absence of all lubrication from the steam space. Difficulties of lubrication with high superheat are thus obviated and the condensed steam is free from oil. (5) The maintenance of good economy over a wide range of fractional loads. (6) The great momentary overload capacity obtained from the high inertia of the rapidly rotating parts. To match this advantage in slow-speed reciprocating engines requires very heavy working parts and the addition of an expensive flywheel. The reciprocating engine can be built to equal the turbine in economy over a wide range of loads but the refinements necessary to this end are costly. For equal heat consumption the costs of large engine-alternator units are from 35 to 50 per cent greater than those of turbine sets.

Low-pressure Turbines compounded with high-pressure reciprocating engines are coming into extensive use where it is necessary to increase the capacity or improve the efficiency of existing engine plants. By combining the

* Adapted from lecture notes by Prof. W. E. Wickenden.

superior efficiency of the engine in the pressure range above the atmosphere with that of the turbine below the atmospheric range a resultant superior to the efficiency of either single type may be secured. Standard piston engines are able to sustain full-rated load when run non-condensing and often will carry from 25 to 50 per cent above rated load without danger or excessive wear. Under such conditions the water rate per kilowatt-hour is high but all the heat rejected in the exhaust is available to a low-pressure turbine; hence the net economy of the combined engine and turbine may be considerably superior to that of the engine when run condensing. By proportioning the turbine to efficiently utilize the exhaust steam and by connecting to it a high-vacuum condenser, the initial capacity of the unit may be doubled or even trebled, and if the engine is in good condition, the resultant efficiency may be superior to that obtainable from a new complete-expansion turbine of equal capacity.

Structurally, the low-pressure turbine is similar to the full-pressure type with the high-pressure stages omitted. It is, therefore, somewhat shorter but the steam spaces are necessarily greater than in a full-pressure type of equal rating. It is highly desirable that an efficient separator be installed between the engine and the turbine as the presence of moisture in the steam adds greatly to the friction against the turbine blades. When the exhaust is to be returned to the boiler it is also necessary to extract the oil from the steam as it leaves the engine.

EFFICIENCY AND LOSSES. — The maximum theoretical efficiency of a steam turbine is the efficiency of the Rankine ideal engine between the temperatures of admission and exhaust (*see Steam Engines*).

The several losses which tend to reduce the efficiency of turbines below the theoretical maximum are: (1) residual velocity, or the kinetic energy due to the velocity of the steam escaping from the turbine; (2) friction and imperfect expansion in the nozzles; (3) windage, or friction due to rotation of the wheel in steam; (4) friction of the steam traveling through the blades; (5) shocks, impacts, eddies, etc., due to imperfect shape or roughness of blades; (6) leakage around the ends of the blades or through clearance spaces; (7) shaft friction; (8) radiation. The sum of all these losses amounts to about 25 per cent of the available energy in the largest and best designs and to 50 per cent or more in small sizes or poor designs.

Condensation, leakage and radiation are of small magnitude. Windage and friction are especially marked in the high-pressure stages and the turbine is most efficient in its low-pressure stages, in direct contrast to the piston engine. The mechanical friction is relatively small, due to the small loads on wearing surfaces and the effectiveness with which they may be lubricated. Steam friction, windage and condensation are much increased by moisture in the steam. Superheated steam is particularly advantageous in turbines because of its low heat conductivity, dryness and high fluidity.

STEAM ECONOMY OF TURBINES.* — Like all heat engines the efficiency of the turbine depends on the range of expansion and the reduction of losses. Increasing the expansion range by raising the admission pressure affords a relatively small gain in efficiency. The chief advantage resulting from high initial pressure is in the decreased size of the turbine of a given capacity. The relation of initial pressure to the steam and heat consumption per unit of output varies somewhat with the individual turbine, but the general relation with fixed conditions of vacuum and superheat is shown in Fig. 1.

The advantage gained by increasing the range of expansion at the lower end greatly exceeds that gained by an equal pressure increment at the upper end of

* Adapted from the lecture notes of Prof. W. E. Wickenden.

the cycle. In the utilization of high vacua the turbine greatly surpasses the engine. Fig. 1 shows the relation of vacuum to turbine efficiency for typical units. This relation depends to a marked degree on the design of the particular turbine and is therefore not identical in all units of a given general type.

Effect of Vacuum on Economy. — The economical limit of vacuum in turbines is much higher than in engines. The attainment of very high vacua involves a very heavy investment in condensers and auxiliaries and a considerable consumption of heat for their operation. The ultimate effect on plant economy is therefore decidedly less than the percentage gain indicated in Fig. 1. Factors unfavorable to high vacuum are a poor load factor for the unit; costly, warm or corrosive cooling water; the necessity of artificially cooling condensing water when the supply is limited; and very cheap fuel.

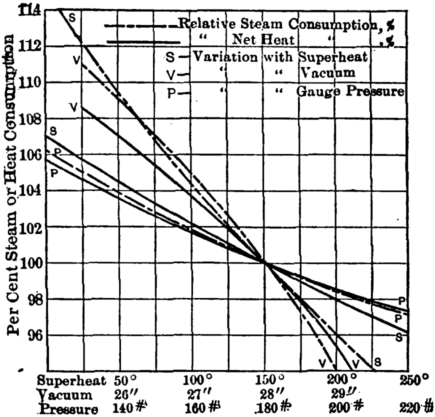

Fig. 1. Relations of Turbine Efficiency to Steam Conditions and Vacuum

The question of the most economical vacuum must be determined by carefully weighing against each other the annual charges on the condenser system, interest, depreciation, attendance, repairs, and the net cost of water for its operation included, and, on the other hand, the annual saving in total fuel consumed plus the saving on boiler-plant fixed charges and operation. Such

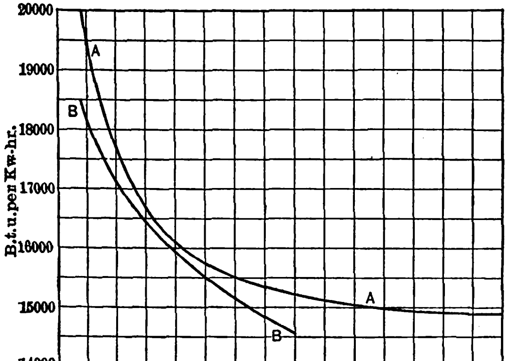

Fig. 2. Average Full-load Net Heat Consumption of Turbines

A. American types, mean values from guarantees and tests
B. Results of 50 European Tests

an analysis takes due account of the heat consumed in driving the auxiliaries. Where an abundant supply of cold condensing water is available, experience indicates that the highest vacuum attainable is a good investment in connection with large units operated at load factors of 60 per cent, or above. With ordinary central-station conditions vacua of from 27 to 28 inches are most advantageous.

Effect of Superheat on Economy. — Superheat of almost any practicable degree improves turbine efficiency, though the net saving is much less than that indicated by the water rate, due to the extra heat content of superheated steam of high temperature. On account of the influence of superheat on the first cost and maintenance expense of piping, valves and fittings, also because of the difficulties in providing for expansion of turbine parts at very high temperatures, the tendency of American practice is to exceed a steam temperature of 500° F. only in very large units, though the tendency in Europe is toward a higher limit. Fig. 1 shows the effect of superheat on steam consumption and net heat consumption of steam turbines, the relation being practically uniform for all types.

Effect of Size and Type on Economy. — The efficiency of the turbine, like that of the engine, improves as the size of the unit increases, see Fig. 2. Large units are relatively cheaper than small, and afford better economy in piping, condensing equipment, space, attendance and maintenance.

A turbo-generator is rated as a single unit. The momentary and sustained overload capacity of the turbine proper is very large, but the generator is much restricted by its ability to dissipate the heat developed in its parts. The usual ratings are based either upon maximum sustained capacity (*M* in Fig. 3) or upon normal sustained capacity with an allowable overload of 25 per cent for a two-hour period (*N* in Fig. 3).

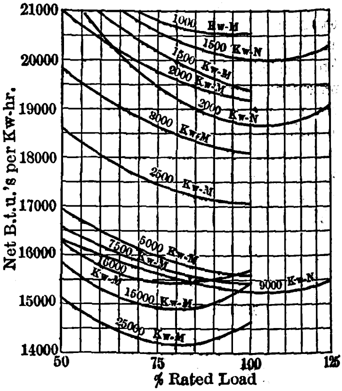

Fig. 3. Net Heat Consumption of Modern Turbines

M. Maximum Rating. N. "Normal" Rating

Experience has not made manifest any marked or consistent differences in economy and reliability between the several commercial types. The vertical type is not well suited to the highest speeds, but is especially economical of floor space in connection with the base type of condenser. It requires greater head-room than the horizontal type. Fig. 4 shows the steam consumption of a number of small non-condensing turbines (*see paper by G. A. Orrok, Trans. A.S.M.E., 1909*). All curves are test results.

ECONOMY OF COMBINED TURBINE AND ENGINE UNIT. — The following is reported by H. G. Stott and R. J. S. Pigott in *Jour. A.S.M.E.*, Mar., 1910. The steam engine is one of the 7500-kilowatt Manhattan type engines at the 59th St. Station of the Rapid Transit Co., New York, with two 42-inch horizontal high pressure and two 86-inch vertical low pressure cylinders, and the turbine, also 7500 kilowatt, is of the vertical three-stage impulse type. The principal results are summarized as follows: An increase of 100 per cent in the maximum capacity and 146 per cent in the economical capacity of the plant; a saving of about 85 per cent of the condensed steam for return to the

boilers [it was previously wasted]; an average improvement in economy of 13 per cent over the best high-pressure turbine results, and of 2.5 per cent (between

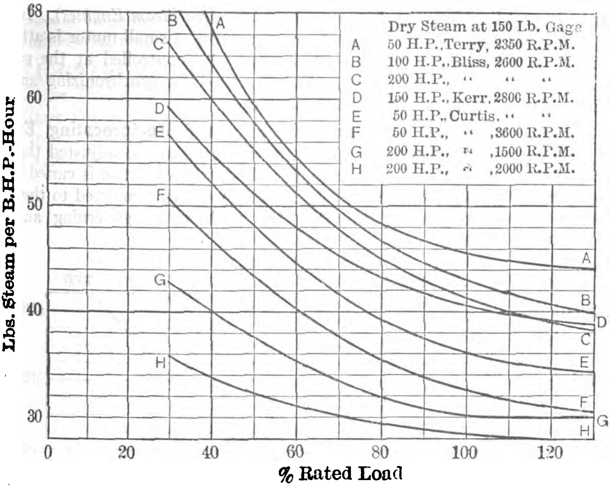

Fig. 4. Steam Consumption of Small Non-condensing Steam Turbines

7500 and 15,000 kilowatts) over the results obtained by the engine alone; an average thermal efficiency between 6500 and 15,500 kilowatts of 20.6 per cent. [This efficiency is not quite equal to that reached by triple-expansion pumping engines.]

TESTING OF STEAM TURBINES. — Turbines are tested in the same way as reciprocating steam engines, measuring the power by a Prony brake or by an electric generator whose efficiency at different loads is known, and the steam consumption by weighing the water discharged from the condenser.

SPECIFICATIONS. — See *Specifications* in article on *Steam Engines.*

OPERATION.* — Several important features are noted below. For additional information see article on *Power Stations.*

Oil-pressure System for Step Bearing of Vertical Turbine. — When running the end of the shaft is lifted slightly from its seat and supported by a forced circulation of water or oil under a pressure of from 500 to 1200 pounds per square inch. This pressure is produced by a special step-bearing pump which may be of the steam-driven, direct-acting type or of the motor-driven, triplex type, and is reinforced by a heavily weighted plunger accumulator. When a double pump equipment is installed the second may serve as a reserve and, if of the motor-driven type, may readily be arranged to automatically take up the load if the accumulator falls to a certain point, due to the failure or inadequacy of the primary pump. The horizontal-shaft type is subject to an end thrust, which must be neutralized by dividing the turbine into two sections through which the steam flows in opposite directions or by the use of pressure rings opposed to the thrust on the turbine.

* Adapted from lecture notes by Prof. W. E. Wickenden.

Speed Control. — Turbines having nozzle expansion are most efficiently governed by varying the number of active nozzles. Reaction turbines are governed by the intermittent throttling of steam at the admission valve. The governor is usually of the shaft or flywheel type (*see Steam Engines*). In both types, when used for driving alternators in parallel, a small motor is attached to the governor spring so that the tension may be controlled at the switchboard. This affords a convenient control of speed for synchronizing and adjusting the loads taken by the several machines.

Speed Control of Combined Turbine and Reciprocating Engine Unit. — In central-station service the two elements are so adjusted that the turbine takes all the steam from the engine, the load on the two is varied simultaneously and the two are electrically coupled by being connected to the same bus bars. This arrangement simplifies the problem of governing and the

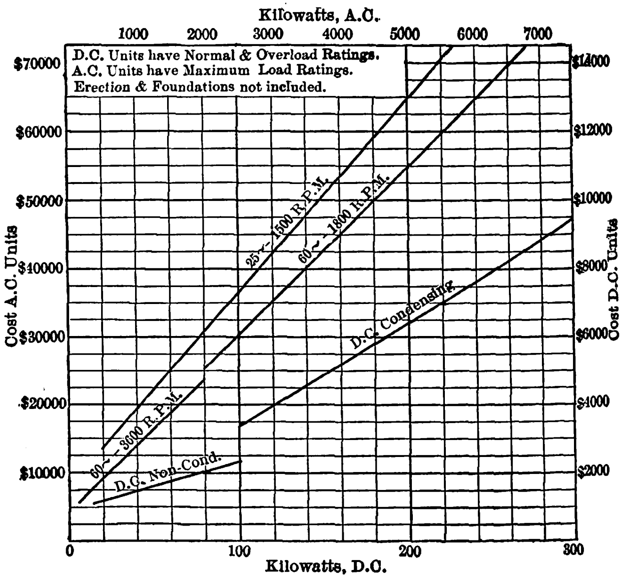

Fig. 5. Approximate Cost of Turbo-generator Sets

division of load. It has not been found advantageous to place a governor on the turbine under such conditions. The turbine operates under variable admission pressure, depending on the load. The turbine should be equipped with a speed-limiting relay, which opens the atmospheric exhaust from the engine should the speed of the turbine become unsafe.

COST OF STEAM TURBINES. — Small turbines for driving pumps, blowers, etc., cost from $20 to $40 per horse-power.

In Fig. 5 are given the total costs of various turbo-generator sets in 1912. These costs include both the turbine and generator but do not include condensers and auxiliaries. The curves were kindly supplied by Prof. W. E. Wickenden and represent the average of the actual costs of a number of units of various

capacities. Impulse and reaction turbines cost approximately the same. Turbine costs are subject to considerable variation, the tendency being a decided decrease in the cost per kilowatt from year to year. The costs given in the curves should therefore be checked against actual quotations, even when used in preliminary estimates.

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[WM. KENT.]

STEEL. — (*See also Iron, Pig and Cast; Castings, Iron and Steel; Rails, Track and Third.*) Carbon steel (the ordinary commercial steel) may be defined as an alloy of iron and carbon produced by any one of the methods described in the following article. It differs from wrought iron in not being slag bearing and from cast iron in being malleable at certain temperatures.

Slag is the substance which collects on the surface of molten iron or steel in the blast furnace, converter or open-hearth furnace. Its constitution varies with the process but it includes oxides of the various impurities found in the iron such as silicon, aluminum, phosphorus, calcium and magnesium. It may also include some oxide of iron. The slag from blast furnaces is often very rich in calcium owing to the amount of limestone added to the charge as a flux. The manufacture of Portland cement from blast-furnace slag is an important industry. The process of puddling used in making wrought iron involves the mixing of some of the slag with the iron, thus making it an integral part of the finished product as explained in the article on *Iron, Wrought.*

METALLURGY. — The various processes by which steel is at present manufactured are described in the following paragraphs:

Bessemer Process. — This process consists of the burning out of most of the impurities and carbon contained in molten iron by the aid of a blast of unheated air, and subsequent recarburization of the mass by the addition of spiegel-eisen or ferro-manganese. The molten material is placed in a pear-shaped vessel called a converter and the blast is introduced through openings in the bottom. After the process has continued long enough to convert the iron into steel, the molten material is either poured into iron molds forming ingots which are later rolled or hammered into commercial shapes, or else cast into *steel castings.*

Two processes are in use, the acid and the basic; in the former the converter lining is of refractory acid materials composed principally of silica, so that the phosphorus and sulphur in the charge are unreduced; in the latter, the converter is lined with burnt dolomite, and lime is added to the charge to reduce the phosphorus and sulphur. The heat necessary for the process is obtained from the combustion of silicon in the acid process and of phosphorus in the basic process. The acid process alone is in use in the United States owing to the prevalence of low phosphorus and sulphur ores. Steel of fair quality is produced by this process at low cost and with great rapidity, but Bessemer steel is gradually being replaced for important structures by the more reliable open-hearth steel.

Open-hearth Process. — This process consists of the reduction of the carbon and other impurities contained in a mixture of pig iron, scrap iron or scrap steel, and iron ore by exposure for seven or eight hours to an intense heat obtained from a mixture of producer gas and air, the process being stopped when the bath is found to have the right proportions as determined by physical and chemical analyses. The steel is then withdrawn from the furnace and either poured into ingots or cast into usable forms. Steel of excellent quality for structural purposes is thus produced in large quantities, a single furnace often having a capacity of 50 tons.

Duplex Process. — This process consists of a combination of the Bessemer and open hearth; pig iron is first blown into the converter to remove the silicon and a portion of the carbon; it is then transferred to the open hearth, where it is dephosphorized and the carbon reduced to the proper percentage. The process permits the use of a poorer grade of material than the Bessemer, and gives a higher grade of finished product, while the speed of production is superior to that of the open hearth.

Crucible Process. — This process consists of melting down, in covered clay or graphite crucibles, wrought iron (sometimes diluted with steel scraps) with charcoal, ferro-manganese and various so-called physics, such as salt, oxide of manganese, etc. The crucibles hold from 50 to 100 lb. each. Crucible steel is high in carbon and low in sulphur and phosphorus. It is superior to Bessemer and open-hearth steel and is used for tool making.

Cementation Process. — This process consists of converting wrought iron into steel by piling iron bars and charcoal together, bringing to a yellow heat and keeping in this condition for a period extending from ten days to two weeks. The steel thus obtained is known as blister steel. If the blister steel is cut, piled, heated and forged, *shear* steel is produced; if the operation is repeated on shear steel, *double shear* steel is obtained. Blister steel is frequently melted in the crucible to free from slag; the resulting product is of the highest quality and is used in making the finest grades of cutting tools, such as razors. Case hardening is a special form of cementation by which the outer surfaces of articles forged from wrought iron or steel are hardened. Harveyizing of armor plate is also a cementation process.

MECHANICAL TREATMENT. — (*See also Steel Mills, Electric Drive of.*) Steel made by any of the preceding processes may be put into usable form either by casting, or by forging while hot under the hammer, or usually by rolling while hot between heavy grooved rollers, the latter process being used for rods, plates, rails, and structural shapes such as angles, channels and I-beams. The rolling process is usually divided into two parts: (*a*) the reduction of the ingot into *blooms* or *billets* by the *blooming mill;* and (*b*) the reduction of the blooms or billets into commercial shapes by the *shaping mill.*

The blooming mill consists of a pair of horizontal grooved rollers, one above the other, between which the ingot is passed back and forth, the rolls being gradually forced closer together. These rolls are actuated by a reversing engine. This process reduces the ingot from a block of steel perhaps 16 inches square by 6 feet in length to a bar of small cross-section, usually rectangular and of considerable length. This is then cut into sections several feet in length which are called blooms if the cross-section contains more than 36 square inches and billets if less. In England this mill is called a cogging mill.

The shaping mill is similar to the blooming mill, but there are usually three non-reversing rolls in a vertical plane, the bloom passing forward between the lower and middle rolls, and returning between the middle and upper roll.

HEAT TREATMENT; CRITICAL TEMPERATURE. — When iron or steel cools from a high temperature, say 1000° C., the rate of cooling is not uniform, but is marked by certain periods of retardation accompanied by spontaneous evolution of heat, the opposite phenomena occurring when the temperature is increased. There are in general three of these critical points which are designated by metallurgists by the letters A_1, A_2 and A_3, A_1 being at the lowest temperature. Owing to *lag* the changes occur at slightly lower temperatures on cooling than on heating, and to distinguish between them, the cooling points are designated Ar_1, Ar_2 and Ar_3, and the heating points Ac_1, Ac_2 and Ac_3. These points vary with the percentage of carbon, and for certain grades of steel the three cooling points may be merged into one and also the three heating points. With favorable conditions when the temperature Ar_1 is reached an increase in temperature of the metal may be detected by its change of color. This point is called the point of recalescence. At this temperature also a softening of the metal occurs. An interesting feature of the point A_2 is that at a higher temperature steel is *non-magnetic,* but in passing through Ar_2 it becomes suddenly *magnetic.* *See Magnetic Properties of Iron and Other Metals.*

The approximate values of these critical points in degrees Centigrade as given by Sauveur are shown in the following table:

CRITICAL TEMPERATURES IN DEGREES CENTIGRADE

Grade of steel	Low carbon	Medium-high carbon	High carbon
Carbon content, per cent	0.1	0.45	Above 0.85
Ar_1	675	650 to 700	675
Ar_2	750	725	675
Ar_3	850	725	675
Ac_1	700	...	...
Ac_2	750	...	...
Ac_3	875	...	...

ANNEALING. — Steel with a coarse structure due to rapid cooling may be improved in grain by heating to a temperature varying with the carbon content and cooling slowly. The accompanying table gives the annealing temperatures which are commonly specified for this purpose.

Range of carbon content, per cent	Range of annealing temperature	
	° C.	° F.
Less than 0.12	875 to 925	1607 to 1697
0.12 to 0.29	840 to 870	1544 to 1598
0.30 to 0.49	815 to 840	1499 to 1544
0.50 to 1.00	790 to 815	1454 to 1499

The object should be kept at the annealing temperature long enough to insure a uniform temperature throughout and then cooled. The higher the carbon, or the greater the ductility and softness required, the slower should be the cooling. It is usually sufficient to remove the object from the furnace and to allow it to cool in air protected from rain, snow and drafts. Steel containing more than 0.50 per cent carbon should cool more slowly until the color dies out before removing from the furnace; it may then be removed and cooled in air. To remove the effects of cold working the object should be heated to about 775° C. and cooled with a slowness depending upon the thickness.

For detailed specifications see *Year Book of Amer. Soc. for Test. Mat.*

CARBON STEEL. — The alloy of carbon and iron of which ordinary carbon steel is composed is divided into the five following groups by Sauveur:

Grade of steel	Hardness	Carbon content, per cent
Very low carbon	Very soft	Not over 0.10
Low carbon	Soft	Not over 0.25
Medium-high carbon	Half hard	0.26 to 0.60
High carbon	Hard	Over 0.60
Very high carbon	Hard, extra hard	Over 1.25

The properties of these different groups when made under normal conditions depend upon the relative amounts of the following constituents:

Ferrite, or iron free from carbon, this being the same as the ferrite in wrought iron (q.v.).

Pearlite, consisting of a mixture of iron carbide and ferrite.

Cementite, or iron carbide, with the formula Fe_3C, combined with a varying amount of carbide of manganese.

Austenite and Martensite. — Steel that is suddenly cooled from a high temperature contains also certain other compounds of iron and carbon, the two of most importance being known as *austenite* and *martensite.* The former is generally considered as a solid solution of carbon, or of the carbide Fe_3C, in iron in the allotropic condition in which the latter exists at a high temperature (above 875° C.). It is not a constituent of constant composition, and is seldom found in ordinary steels since it is rapidly transformed in cooling into an aggregate of ferrite and cementite. To obtain it highly carburized steel may be suddenly cooled from a temperature of 1000° C. or more by a bath such as ice cold water. It may also be obtained by slow cooling if the steel contains a considerable portion of manganese or nickel. Martensite is generally believed to be an early stage in the transformation of austenite.

Phosphorus and Sulphur. — The common injurious impurities in carbon steel are phosphorus and sulphur, both of which cause brittleness and should be allowed only in small quantities. Specifications (*see section below on this topic*) frequently forbid more than 0.05 per cent of sulphur and 0.04 per cent of phosphorus. Other impurities commonly occuring in small quantities in steel are silicon and manganese, which in small quantities help to reduce blow holes by their deoxidizing action. These elements while often present in higher percentages than the carbon have comparatively little influence upon the strength of ordinary carbon steel.

Composition of Low-carbon Steel. — Low-carbon steel consists chiefly of a mass of ferrite with pearlite occuring at the junction of the grains of ferrite, the percentage of the former decreasing in medium and hard steels until in steel having 0.8 per cent to 0.9 per cent carbon, it disappears entirely, the steel then consisting exclusively of pearlite. In still higher carbon steels some of the pearlite is replaced by free cementite, the substance which gives steel its hardening property.

Strength and Weight of Carbon Steel. — The tensile strength of steel varies with its composition. For ordinary carbon structural steel the working values given in the following table may be adopted with safety:

Nature of stress	Working strength, pounds per square inch
Tension on net section, and extreme fiber stress in bending	16,000
Compression in columns*	$16{,}000 - \frac{70\,l}{r}$, with a maximum of 14,000
Shear on net section of plate-girder webs and on machine-driven shop rivets	12,000
Bending on extreme fiber of pins	24,000
Bearing on pins and shop-driven rivets	24,000
Bearing on hand-driven rivets	18,000
Shear on hand-driven rivets	9,000
Modulus of elasticity	28,000,000 lb. per sq. in.

* In the expression for compression in columns, $\frac{l}{r}$ = maximum value of ratio of the unsupported length of column to radius of gyration, both values being expressed in inches. This ratio should be restricted by the form of the column so that it will not exceed 100 for main members and 120 for lateral and other secondary members.

Steel weighs approximately 490 lb. per cu. ft., or 2 per cent more than wrought iron.

Specifications for Structural Steel and Steel Castings.—The American Society for Testing Materials has adopted standard specifications for steel to be used in various classes of structures. (*See A.S.T.M. Yearbook.*) Among the items covered are the chemical and physical properties of the material, the specimens and methods of tensile and bend tests and the permissible variation in weight and gauge. The following tables give the requirements as to chemical composition and tensile properties as given in the Society's specifications for structural steel for bridges and ordinary steel castings as adopted Aug. 25, 1913.

CHEMICAL COMPOSITION

Elements considered	Structural steel	Rivet steel	Steel castings
Phosphorus, max., per cent;			
Acid	0.06	0.04	0.06
Basic	0.04	0.04	0.06
Sulphur, max., per cent	0.05	0.04	0.05

TENSILE PROPERTIES

Properties considered	Structural steel	Rivet steel	Steel castings
Tensile strength, lb. per sq. in.	55,000 to 65,000	48,000 to 58,000	60,000 minimum
Elongation in 8 in., min., per cent	$\frac{1{,}500{,}000}{\text{Tensile strength}}$	$\frac{1{,}500{,}000}{\text{Tensile strength}}$	....
Elongation in 2 in., min., per cent	22	....	20

SPECIAL OR ALLOY STEELS. — The more usual forms of special steels and their uses, composition and general characteristics are as follows:

Nickel Steel. — Steel containing a considerable percentage of nickel, but generally not more than $3\frac{1}{2}$ per cent, is called nickel steel. This steel has a greater strength and a higher elastic limit than carbon steel, and practically the same modulus of elasticity; it is more ductile in proportion to its strength and somewhat harder than carbon steel of like properties, it is more expensive because of the cost of the nickel, and the additional cost of working. Nickel steel has been used considerably in recent years for bridges of large size, for steamer shafts, automobile parts, etc.

Invar Steel. — This is a nickel steel containing about 36 per cent of nickel. Its coefficient of expansion is practically zero. It is used in making steel tapes for base line measurements in geodetic surveying.

Manganese Steel. — The steel commonly known as manganese steel contains from 12 per cent to 13 per cent of manganese and generally from 1.25 per cent to 2.00 per cent of carbon for castings; for forgings or rolled steel, less carbon is used. It is austenitic in composition and is very hard without brittleness, making it well fitted for railroad frogs, railroad rails or curves and burglar-proof safes. It is adapted principally to parts that can be cast, since working it with tools is practically impossible and forging is difficult. It is not used for structural work as its elastic limit is so low as to offset the advantage of its great strength. Like nickel, the manganese interferes with the transformation of austenite to pearlite during cooling. It is often called "Hadfield" steel from the name of its inventor.

Chrome Steels. — Steels containing from 1 per cent to 2 per cent of chromium are very hard and have a high elastic limit. They are well adapted for armor-piercing projectiles, steel balls, files and automobile gears. Krupp armor plate is said to contain 3.25 per cent nickel, 1.5 per cent chromium and 0.25 per cent carbon.

Vanadium Steels. — The addition to steel of a small amount of vanadium, usually less than 0.3 per cent, adds greatly to the elastic limit, ductility and resilience. Vanadium is especially advantageous in increasing strength, toughness and temper of nickel and chromium steels. This steel is particularly useful for springs, axles and automobile parts.

Silicon Steels. — The only silicon steels in use contain less than 5 per cent of silicon. The alloy recommended by Hadfield, its patentor, contains 2.75 per cent silicon with the smallest possible amounts of carbon, manganese and other impurities. Such steel has a greater magnetic permeability than the purest iron, and also a high electrical resistance. Its magnetic hysteresis is low. It is a valuable material for use in electromagnetic and in electrical generating machinery. See *Magnetic Properties of Iron.*

Tungsten Steels. — These steels have the property of becoming martensitic upon heating to a high temperature and air cooling and are said to be "self-hardening." Such steels are used for springs, magnets and with manganese added for self-hardening tools. See also *High-speed Steels and Magnetic Steels, below.*

Self-hardening Steels. — Steel which is hard without tempering and cannot be annealed by known processes is called self-hardening steel. All such steels are non-magnetic. Any steel which is in an austenitic condition at atmospheric temperature is self-hardening. (*See Mushet Steel, below.*)

Ternary and Quartenary Steels. — These are steels consisting of three or four essential components, viz., iron, carbon and one or two alloys. Some of the more important ternary steels are nickel steel, manganese steel, chrome

steel, tungsten steel and silicon steel. Among quartenary steels are nickel-chromium, nickel-manganese, nickel-vanadium, tungsten-chromium, etc. The effect of these various alloys upon carbon steels is very striking and their possibilities are very great.

Chrome-nickel Steel. — The chrome-nickel steels in use are pearlitic and contain moderate amounts of carbon, nickel and chromium. Such steel has a high elastic limit combined with high ductility and has greater hardness, resilience and wearing power than carbon steels. It is used in automobile construction and for the manufacture of armor plate.

Chrome-tungsten Steel. — Tungsten and chromium have the property of producing a finely martensitic structure in steel when heated to a high temperature and cooled in air. The martensite in this case is so stable that the steel may be heated to a red heat without losing hardness. Such steels are known as "high speed steels" (q.v.).

High-speed Steels. — These steels have a finely martensitic structure which is stable up to red heat. They may contain from 0.25 per cent to 1.00 per cent of carbon, from 5 per cent to 25 per cent tungsten, 2 per cent to 10 per cent of chromium and seldom over 0.40 per cent of manganese. Tungsten may be replaced wholly or in part by molybdenum. Steels of this character are of great importance in machine work as tools made from them can be driven without injury until the cutting edge is red hot.

Mushet Steel. — This steel contains both tungsten and manganese. It is a self-hardening steel and has been used for many years for tools which make very heavy or deep cuts such as is required for armor plates. The speed allowable is a little greater than for carbon steel, but it lasts a long time without grinding.

Magnetic Steels. — Steels containing 4 per cent to 5 per cent of tungsten and from 0.5 per cent to 0.7 per cent of carbon if heated to red heat and quenched in water will retain their magnetism better than carbon steel. The addition of chromium increases the permanency but decreases the magnetic force.

PRICE OF STRUCTURAL STEEL, F.O.B. PITTSBURGH. — The following prices show market quotations in April, 1913 (*from Iron Age, April 10, 1913*). I-beams, 3 to 15 in.; channels 3 to 15 in.; angles 3 to 6 in. on one or both legs ¼ in. thick and over; and zees, 3 in. and over, 1.45 to 1.70 cents per lb.; *extras* on other shapes and sizes are as follows:

I-beams over 15 in.	0.10	cents per lb.
H-beams over 18 in.	0.10	” ” ”
Angles over 6 in. on one or both legs	0.10	” ” ”
Angles, 3 in. on one or both legs, less than ¼ in. as per steel bar card, Sept. 1, 1909	0.70	” ” ”
Tees, structural sizes (except elevator, hand rail, car truck and conductor rail)	0.05	” ” ”
Angles, channels and tees, under 3 in. wide as per steel bar card, Sept. 1, 1909	0.20 to 0.80	” ” ”
Deck beams and bulb angles	0.30	” ” ”
Hand rail tees	0.75	” ” ”
Cutting to lengths, under 3 ft. to 2 ft. inclusive	0.25	” ” ”
Cutting to lengths, under 2 ft. to 1 ft. inclusive	0.50	” ” ”
Cutting to lengths under 1 ft.	1.55	” ” ”

No charge for cutting to lengths 3 ft. and over.

FREIGHT RATES ON FINISHED IRON AND STEEL PRODUCTS. — (*From Iron Age, April 11, 1913.*) Freight rates in April, 1913, from Pitts-

burgh in carloads, per 100 lb. were: New York, 16 cents; Philadelphia, 15 cents; Boston, 18 cents; Buffalo, 11 cents; Cleveland, 10 cents; Cincinnati, 15 cents; Indianapolis, 17 cents; Chicago, 18 cents; St. Louis, 22½ cents; Kansas City, 42½ cents; Omaha, 42½ cents; St. Paul, 32 cents; Denver, 84½ cents; New Orleans, 30 cents; Birmingham, Ala., 45 cents; Pacific coast, 80 cents on plates, structural shapes and sheets No. 11 and heavier; 85 cents on sheets Nos. 12 to 16; 95 cents on sheets No 16 and lighter; 65 cents on wrought pipe and boiler tubes.

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[C. M. Spofford.]

STEEL MILLS, ELECTRIC DRIVE OF.—(*See also Flywheels for Load Equalization; Motors, Industrial Applications of.*) The application of electric motors to steel mill service may be divided in two general classes; 1. that dealing with the application of motor drive to the main rolls, that is to those rolls in which the ingot or billet is reduced in section, and 2. that dealing with the problems of electrifying the numerous other auxiliary machines and devices, such as tables, screw-downs, charging machines, etc.

MAIN-ROLL APPLICATION.—The term "mill" is sometimes used to designate a single stand or group of stands, and sometimes to include the main rolls and all auxiliaries involved in the production of a given class of materials. The stands are generally classified according to the arrangement of rolls and method of operation, that is two-high or three-high, the two-high being either reversing or non-reversing, see Fig. 1.

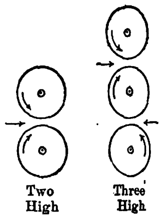

Fig. 1. Diagram of Two-high and Three-high Rolls

Reversing and Non-reversing Mills.—The great majority of rolling mills in this country are of the non-reversing type where the rolls run continuously in one direction. Flywheels are used to equalize the input to the motor. The ideal combination would be to use a motor of sufficient capacity to carry the average load and a flywheel which would take care of all the peaks. Such a wheel would be too large and a compromise between the motor and flywheel must generally be made. As opposed to the heavy flywheel effect required by the two-high or three-high non-reversing mill, the minimum flywheel effect consistent with the necessary speed and torque is desirable for reversing mills. In order to avoid the reflection of peak loads of these motors back upon the system, it is customary to interpose a flywheel motor-generator set with one or more generators between the power station and the mill. In order to minimize the flywheel effect of the reversing motor, two or more direct-current motors are often mounted on the same shaft and their armatures connected in series. This arrangement renders possible the use of smaller armature diameters with consequent reduction of acceleration losses and also permits the use of a high-voltage generator on the flywheel set. The mill motor is separately excited and perfect speed control is obtained by varying the excitation of the generator on the flywheel set. Direct-current generators can be designed for 2400 volts to give excellent results when operating two 1200-volt motors in series.

Power Required.—Rolling mill loads are irregular in the extreme due to the intermittent character of the process. For any particular ingot the motor load consists of periods of heavy duty increasing in length with the length of the metal in the pass, interspersed with periods of friction load only. With a given mill the load varies widely with the difference in the section rolled, differences in temperatures of metal, personal equation of the operator, etc. The practical determination of the power required to roll steel is a matter of elaborate and extensive tests under widely varying conditions. For methods and results see papers in *Bibliography*. The information for predetermining the sizes of motor and flywheel for an installation must cover the following points:

Type of mill; rail, plate, etc.	Time between passes.
Diameter and speed of rolls.	Elongation in each pass and total.
Weight of ingot.	Initial length.
Initial and final section.	Average and maximum rate of rolling.
Number of passes.	Temperature of metal.

Load Curves.—From these data and a thorough understanding of the many variable factors encountered in steel-mill practice, it is possible to lay out load curves similar to the ones shown in Figs. 2, 3 and 4. From such curves it is then possible to determine the proper size of the motor and the flywheel.

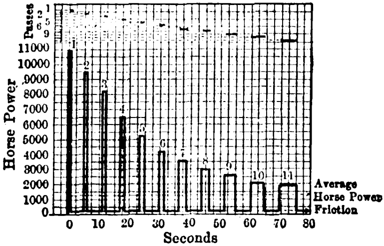

Fig. 2.

The typical curve shown in Fig. 2 represents a load curve calculated for a two-high single-stand reversing blooming mill in which a 3000-pound ingot is reduced from initial to final section in eleven consecutive passes occurring at 5-second intervals. This curve is characteristic of blooming mills, roughing mills, etc., which in general give the lowest load factor. The work done in horse-power seconds is indicated by the area under the curve. Fig. 3 is representative of a large and mixed class with medium load factor, such as small merchant mills, bar mills, sheet mills, etc. Fig. 4 shows a load factor so high as to be scarcely approximated except in merchant mills with a large number of stands working at full capacity.

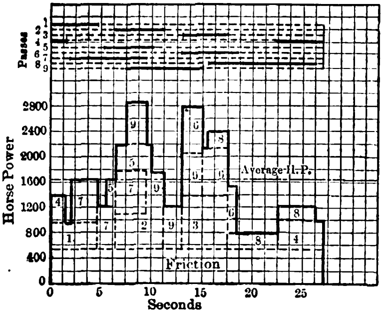

Fig. 3.

In determining the most suitable size of motor for the work indicated by the load curve full consideration must be given to the fact that excessive loads may be encountered at times due to the low temperatures of the metal in the pass, too heavy draft or other causes, and ample margin allowed. Assuming for the moment that by properly combining simultaneous passes on the several stands and the application of a suitable flywheel the load curve has been flattened out as much as appears possible, the question of continuity of service and duration of peaks must be considered to determine whether the limiting feature will be maximum torque or heating. With rolling-mill motors the limiting feature of design is usually torque rather than heating. The motor rating is determined by a consideration of the root-mean-square value of the current for a single cycle, and the frequency and duration of successive cycles.

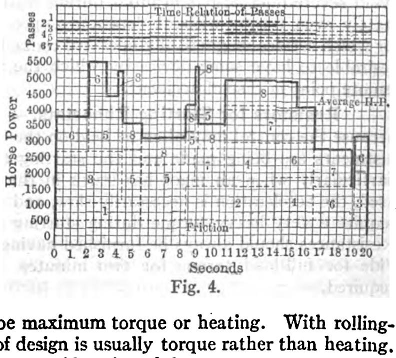

Fig. 4.

Use of Induction Motors. — The choice of alternating- or direct-current motors for main rolls must be determined by a careful consideration of several factors, chief of which are the capacity of individual units, constant, variable or adjustable speeds, reversing or non-reversing, transmission distance, existing power-station equipment, etc.

It is in general conceded that for large units and constant speed, constant-load service induction motors with phase-wound rotors possess many advantages over direct-current motors. This is due to the ruggedness of construction and simplicity of operation of the induction motor, and to the facility with which alternating current may be stepped up to any desired voltage, thus rendering the problem of economic power distribution relatively simple.

For the comparatively few main roll drives where it is essential that close regulation be maintained for a large number of speeds, each constant under varying loads, a direct-current motor with shunt characteristics has until recently been practically necessary. It is now possible to obtain these characteristics by using standard induction motors with phase-wound rotors and speed-regulating sets. The method of avoiding the dissipation of energy as heat in the control rheostat consists in replacing the non-inductive external resistance by a compensated commutator motor which may be mounted either on the shaft with the main motor, or more commonly as one element of a two-unit motor-generator set. In the latter case the second element is a squirrel-cage induction motor, driven slightly above synchronism by the commutator motor and operating as an induction generator, thus returning to the line energy proportional to the slip of the main motor less the losses in the regulating set itself.

Control Equipment for Induction Motors. — The control for large steel mill motors is important because of the size of the apparatus involved and the extreme importance of continuous operation. Most motors for main-roll drive are started up at the beginning of a shift, immediately brought up to full speed, and left running until the end of the shift. Provision must be made for running a short time at reduced speed when desired, for making adjustments to the mill, and for reversing, in order to back out ingots which have stuck in the mill. If a flywheel is used, there should also be provision for quick stopping, so that if a spindle is broken, the motor can be brought to rest quickly and a new spindle substituted without delay. Current-limiting relays are desirable in order to avoid severe fluctuations in input during starting and to maintain an approximately even torque without depending upon the intelligence of the operator, for which service magnetically-operated contactors are necessary. It is always desirable to have some device by which the motor can be shut down from a remote point in case of accident.

Resistors for Starting Rheostats. — The resistance should be liberal, because the control may often be held on the first point for some time when adjustments are being made and the starting torque may sometimes be heavy, particularly when an ingot has stuck in the rolls. Also, where a flywheel is used, the acceleration is necessarily slow, and some mills, particularly cold rolls, require a very heavy torque during starting and until they have warmed up. Resistances should always be furnished having at least enough capacity to provide for full-load torque for two minutes. In special cases more will be required.

Contactors and Oil Switches. — Where 440- or 550-volt alternating current is used, contactors can be employed for both primary and secondary, and remote control is an easy matter. 2200- or 6600-volt motors require oil switches in the primary, which cannot be operated readily by means of alternating current. They can be operated with direct-current, but this complicates the system and makes the operation of the alternating-current motor dependent

upon the direct-current supply. As remote control is not essential in the large majority of cases, hand-operated oil switches can be used.

Simple-control System. — Fig. 5 shows a typical diagram of connections for a simple-control system. The incoming line passes through discon-

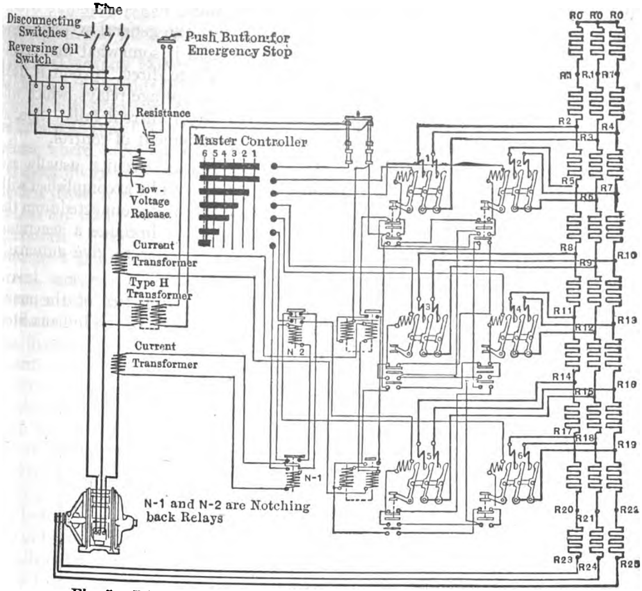

Fig. 5. Diagram of Mill-control System with Induction Motor

necting switches by means of which the motor and all the control equipment can be cut off when repairs or adjustments are to be made. The hand-operated oil switch is made double-throw to provide for forward and reverse operation. A push button or control switch located in the mill can be used to trip the switch by short-circuiting the no-voltage release for emergency stopping. The slip rings are connected permanently to a three-phase resistance. Contactors (double-pole, where sizes permit) are used for short-circuiting successive portions of the resistance as the motor accelerates. The operating current for the contactors passes through a master controller so that the motor can be held with all resistance in circuit or on any one of several intermediate points between that and complete short-circuit. Current-limit relays in series with the primary line govern the rate at which the contactors close.

No-voltage release is obtained automatically on the secondary contactors, because they are operated by current received from the primary line. If the primary oil switch is opened for any cause, the operating current for the contactors is interrupted, so that they open. The acceleration is always governed by the current-limit relays independent of the position of the master controller. For ordinary starting and for stopping the master controller can be left on the full running position and the oil switch simply closed and opened.

It is standard practice to use 50 per cent more ohms in the secondary resistance than the value required to give full-load torque at standstill. This value limits

the current on reversal at full speed to 133 per cent of full-load current, so that full-speed reversal for quick stopping causes no more shocks on the motor or line than will be experienced in ordinary operation. If a flywheel is used, a small amount of resistance (sufficient to give 5 or 10 per cent slip at full load) is left permanently connected in the secondary.

Where motors having two or more speeds are used, more switching equipment is necessary to provide for speed changing, but the general system is the same. Where remote control is necessary, the system is somewhat more complicated, but the same general principles apply. Where direct current is used for operating the control, it is necessary to provide no-voltage relays, which will shut down the motor in case either alternating or direct current fails. Special cases, such as concatenated sets, require special arrangement of control.

Direct-current Motors are used only on low-voltage circuits, usually 250 volts, and can be handled by contactors. Quick stops can be accomplished with d-c. motors by dynamic braking, by which the armature is disconnected from the line and connected across a resistance so that the motor becomes a generator. Compound motors should be used with flywheels in order to give automatic reduction in speed as the load increases.

Characteristics of Gary Equipment. — The characteristics of the main-roll motor equipment for the rail, billet, and sheet-bar mills of the Indiana Steel Company, Gary, Indiana, are given in the following table:

CHARACTERISTICS OF MAIN ROLL MOTORS

Rail mill

No. of motors*	H.P.	R.p.m.	Flywheel effect†	Weight complete, pounds
2	2000	214	4,312,000	392,000
1	6000	83	11,600,000	749,000
1	2000	68	8,950,000	578,000
1	6000	88	11,600,000	749,000
1	6000	75	14,100,000	783,000
Billet mill				
2	2000	214	4,312,000	392,000
3	6000	83	11,600,000	624,000
Sheet-bar Mill				
1	6000	83	11,600,000	624,000

* All motors 25 cycles, 6600 volts.
† Weight of flywheel in pounds times square of radius of gyration in feet.

AUXILIARY MOTOR APPLICATIONS.—The application of electric motors for auxiliary steel-mill machinery has been used for a long time. The nature of this service is unusually severe and has led to the development of the mill-type motor for both d-c. and a-c. service. These motors are designed to withstand heavy overloads and abnormally rapid acceleration, and their construction is such that various parts are interchangeable, which greatly facilitates repairs and minimizes the delays incident to the same.

All A-C. versus Mixed System.—Whether a-c. or d-c. motors should be used for driving the auxiliary machinery in a steel mill is a problem which has been very seriously discussed, and has led to the use of two recognized systems. These are known as the all a-c. system, where no direct current is used, and the mixed system, where direct current is used for the small motors.

A great many factors must be considered in comparing the two systems. In the first place, it is to be assumed that power will primarily be alternating-current, as the transmission distances ordinarily preclude the use of direct-current generators. It would therefore seem to be simplest and most efficient to step down to a suitable voltage through static transformers and use alternating-current motors. The mixed system involves additional expense for motor-generator sets and entails considerable power loss due to the low efficiency of conversion. On the other hand, direct-current motors are lower in first cost than induction motors and a higher power factor is maintained on the entire system where they are used. In the mixed system an increase in power factor is effected by eliminating the lagging current of the induction motors, and, in addition, the motor-generator sets can be equipped with synchronous motors which will take a leading current from the line and offset part of the lagging current on the rest of the system. The increase in power factor enables a reduction to be made in the size and cost of transformers and generators, and also increases their efficiency due to the lower currents which they are required to handle and to the decreased excitation required by the generators.

In order to make as nearly as possible a general determination of these factors, an exhaustive study of the problem was made by Messrs. Shover and Cheney and the results presented in a paper before the Association of Iron and Steel Electrical Engineers. (*See also General Electric Review, Aug., 1912.*) The conclusions reached were as follows:

1. The all a-c. system costs slightly more than the mixed system.
    a. First cost higher for 22,000 volt transmission than for 6600.
    b. First cost higher for gas engines than for turbines. From this it appears that the higher the first cost of power supply the less favorable is the use of the all a-c. system.
2. The lower the power factor, the greater is the cost of the all a-c. system for both percentages of auxiliary load.
3. The less the percentage of auxiliary load the less the cost of the all a-c. system for both power factors.
4. The annual costs of the all a-c. system considered are lower than those of the mixed system.
5. The actual operating costs, i.e., excluding interest, depreciation, taxes and insurance, of the all a-c. system are considerably less than those of the mixed system.
6. The cost of maintenance of the mixed system is based on an estimate and not on actual records. Should this item be entirely neglected, the results in nine out of sixteen cases would show an excess of annual costs for the all a-c. system, but the amount is so small that accurate calculations for any individual case would be necessary to determine the relative advantages.

7. When the saving in output due to the reduced delays in the all a-c. system is taken into consideration, the saving in annual costs will be largely increased; and even should the difference in motor maintenance be neglected there would still be a considerable saving in annual costs for the all a-c. system.

In conclusion, then, for a rolling mill properly motored, where the percentage of power required for auxiliary apparatus (exclusive of pumps, etc.) is 25 per cent or less of the total power delivered to that mill, and where the power factor of the entire mill including both main and auxiliary apparatus is 70 per cent or over, the authors feel amply justified in saying that the all a-c. system will show a saving in annual cost, to say nothing of its greater simplicity and more satisfactory operation.

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[D. B. RUSHMORE, assisted by E. A. LOF.]

STOKERS, MECHANICAL. — (*See also Boilers.*) Mechanical stokers serve a twofold purpose: they make possible a more uniform and a more complete combustion of the fuel and thereby prevent the formation of smoke, and they also reduce the amount of labor required in handling and firing the fuel. To insure the prevention of smoke a fire-brick arch should also be used over the front part of the furnace.

There are five classes of stokers on the market. (1) The "chain-grate stoker," made by Babcock and Wilcox and others, is a continuous belt of grate bars driven over sprockets at the front and rear of the furnace. Coal is fed continuously upon the grate from the hoppers located above the boiler, is carried by the grate the entire length of the furnace at such a rate that it is completely burned by the time it reaches the back of the furnace. (2) The "front-feed step grate" (the Roney and the Wilkinson stokers) consists of a series of stepped grate bars, slightly inclined from the horizontal, and a dumping grate at the bottom which receives and discharges the ashes. The bars are given a slow rocking or sawing motion by means of a small engine or motor. (3) The "side-feed step grate" (the Murphy stoker) is similar to the front-feed step grate, but is arranged so that the fuel is fed into the furnace from the two sides. (4) The "underfeed stoker" (the Jones and Taylor stokers) pushes fresh coal into the fire from beneath. The volatile matter of the freshly fired fuel then has to pass through a body of ignited coke and is thus heated to the ignition point before coming into contact with the supply of hot air immediately above, where it is rapidly burned. (5) In the "sprinkler type" of stoker the fuel in finely divided form is sprinkled uniformly over the entire grate. In the Little Giant stoker this is accomplished by discharging the coal into a cast-iron chute which extends over the front part of the grate, from which it is blown into the furnace by a steam jet.

COSTS. — The following costs are taken from Gebhardt's *Steam Power Plant Engineering*. Approximate costs of stokers suitable for a Babcock and Wilcox boiler of 350 horse-power capacity; 45 square feet of grate surface; height of chimney above grate, 175 feet; coal burned, Illinois screenings; cost of installation not included:

Chain grate and appurtenances	$1500
Roney stoker	1300
Murphy stoker and furnace	1350
Jones underfeed stoker	1400

The maintenance charge of a chain-grate stoker should not exceed $5 per boiler per year, with proper care, and the stoker should have a life of 15 years. If the stokers are neglected or abused the maintenance charge may be as high as $100 per year. Stokers are often condemned by their owners as inefficient and inferior to hand firing, because no particular attention has been paid to them beyond filling the hopper with coal. They should be operated in strict accordance with the principles of their design.

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[Wm. Kent.]

STRENGTH AND ELASTICITY. — (*See also Buildings, Allowable Unit Stresses in; Mechanics, Principles of; Structures, Simple.*) When opposing external forces are applied to a body the latter is in general deformed more or less, and, in the case of solids, when the applied forces are increased sufficiently the body ruptures.

STRESSES. — A stress is the internal resisting force set up within a body opposing the external forces which tend to deform it. When the external forces cause a stretching of the body the stress is called a tension; when the external forces compress the body the stress is called a compression; when the external forces cause a relative slipping of the particles in two contiguous parallel planes in the body the stress is called a shear.

Unit Stress. — By unit stress is meant the stress per unit area; in the case of tension or compression the area is taken perpendicular to the line of action of the forces producing the stress; in the case of shear the area is taken parallel to the forces producing the stress. Unless otherwise stated the stress is assumed uniformly distributed over the area upon which it acts.

Ultimate Stress or Ultimate Strength. — The ultimate stress which a body will stand, or the ultimate strength of the body, is the greatest stress which can be produced in the body without rupturing it. Ultimate strength for tension and compression are usually quite different; the former is usually called tensile strength and the latter compressional strength. The ultimate strength of materials is usually specified in terms of the stress per unit area; see *Units and Conversion Factors.*

DEFORMATIONS OR STRAINS. — The deformation accompanying any stress is usually called a strain. The strain corresponding to a tension is called an elongation, the strain corresponding to a compression is a shortening, the strain corresponding to a shear is called a detrusion. An elongation is usually specified as the ratio (or percentage) of the increase in length to the original length of the specimen. For example, if a rod 10 feet long is stretched by an applied force to a length of 10.2 feet, the elongation is $(10.2-10)/10 = 0.02$ or 2 per cent. The term elongation is frequently used to designate specifically the *maximum* elongation just before rupture.

Hooke's Law. — Experiment shows that when the unit stress in a body does not exceed a certain value it bears a constant ratio to the resulting unit strain (ratio of change in size to original size). This fact is known as Hooke's Law. In the case of a tension the change in size is usually taken as the change in length, no attention being paid to the decrease in cross-section, which is usually negligible, except just before rupture.

ELASTICITY. — A body deformed under stress will return to its original shape when the stress is removed, provided it has not been strained beyond the point at which the proportionality between stress and strain ceases. This ability to return to its original form after deformation is called elasticity.

Elastic Limit. — The stress per unit area corresponding to the point at which the proportionality between stress and strain ceases is called the elastic limit of the material.

Permanent Set. — When a body is stressed beyond its elastic limit, the deformation per unit increase in stress becomes greater than it was before this point was reached, and the material takes a permanent "set," i.e., when the stress is removed the body does not return to exactly its original form. As a measure of the "set" is taken the ratio of the permanent change in size (after the stress is removed) to the original size.

Modulus of Elasticity. — Let ΔF denote the change in the total tension producing a change Δl in a rod of length l and cross section A; then the quotient of the increase in the unit stress by the increase in the elongation per unit length, is called the modulus of elasticity (Young's modulus) of the material, and may be designated by the symbol M, i.e.,

$$M = \frac{\Delta F}{\Delta l} \cdot \frac{l}{A}.$$

This modulus has the dimensions of pounds per square inch.

VALUES OF TENSILE STRENGTH AND MODULUS OF ELASTICITY.— Values of these properties given by different authorities are extremely variable, since the measured values depend largely upon the chemical composition, heat treatment, age, size and shape of the test specimen.

Alloys, Miscellaneous.— See articles on *Alloys* and *Wires, Resistance.*

Aluminum. — See article on *Aluminum.*

Belting. — Tensile strength, in pounds per square inch: Cotton, 4500 to 8900; Single leather, 3200 to 5900; Double leather, 2200 to 5400. See also article on *Belts and Belting.*

Brass Wire. — Tensile strength ranges from about 50,000 to 150,000 pounds per square inch; modulus of elasticity is about 14×10^6 pounds per square inch.

Brick. — See article on *Bricks and Brick Masonry.*

Bronze Wire. — The tensile strength of phosphor-bronze wire ranges from about 44,000 to 140,000 pounds per square inch; of silicon-bronze wire, from 95,000 to 115,000 pounds per square inch.

Cement. — See article on *Cement.*

Concrete. — See article on *Concrete.*

Copper. — See articles on *Copper* and *Wires and Cables, Bare.*

Earth. — See *Soils,* below.

Glass.—Tensile strength of common green or of flint glass ranges from about 2500 to 5000 pounds per square inch. Modulus of elasticity ranges from about 8.5×10^6 to 11.5×10^6.

The crushing or compressive strength of glass ranges from about 13,000 to 40,000 pounds per square inch.

Granite. — Crushing or compressive strength ranges from about 9700 to 34,000 pounds per square inch.

Iron. — See articles on *Iron, Pig and Cast,* and *Iron, Wrought.*

Lead. — Tensile strength ranges from about 2600 to 3300 pounds per square inch.

Limestone. — Crushing or compressive strength ranges from about 6000 to 25,000 pounds per square inch.

Marble. — Crushing or compressive strength ranges from about 7500 to 21,000 pounds per square inch.

Nickel. — Tensile strength of cast nickel ranges from about 40,000 to 85,000 pounds per square inch, and tensile strength of annealed nickel ranges from about 70,000 to 95,000 pounds per square inch. Modulus of elasticity ranges from about 24×10^6 to 27×10^6 pounds per square inch.

Rope. — See *Ropes and Rope Drive.*

Soils. — See articles on *Power Stations, Hydroelectric,* and *Buildings, Allowable Unit Stresses in.*

Steel. — Tensile strength in pounds per square inch:

Bessemer:	56,000 to 74,000,	Open-hearth:	50,000 to 69,000,
Cast:	67,000 to 106,000,	Wire:	50,000 to 450,000.

Modulus of elasticity of various kinds of steels usually ranges between 27×10^6 and 35×10^6 pounds per square inch. See also articles on *Steel* and *Wires and Cables, Bare.*

Timber. — See articles on *Timber* and *Poles for Overhead Lines.*

Tin. — Tensile strength ranges from about 4000 to 5000 pounds per square inch; modulus of elasticity from about 2.5×10^6 to 6×10^6 pounds per square inch.

Zinc. — Tensile strength ranges from about 7000 to 30,000 pounds per square inch; modulus of elasticity is about 12×10^6 pounds per square inch.

Wood. — See *Timber,* above.

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[H. PENDER AND R. G. HUDSON.]

STRUCTURES, SIMPLE. — (*See also Building Laws; Cement; Concrete; Iron; Mechanics, Principles of; Steel; Timber; Towers for Transmission Lines.*) A structure in the sense used by engineers is a member or combination of members constructed to hold forces in equilibrium. The common structures are beams, girders, columns and trusses.

A brief outline of the contents of this article is as follows:

DEFINITIONS AND FUNDAMENTAL RELATIONS. — The more common terms used in the treatment of structures are defined below.

Center of Gravity; Moment of Inertia; Radius of Gyration; Stress and Strain. — See *Mechanics, Principles of.*

Neutral Axis. — The neutral axis of the cross-section of a stressed bar is the locus of those points in the plane of the cross-section at which the direct fiber stress equals zero. If the bar is of homogeneous material and subjected to flexure (*see Bending Moment, below*) only, the neutral axis passes through the center of gravity of the cross-section, and if the loads causing flexure are applied in the plane of one of the principal axes, the neutral axis coincides with the other principal axis. This is the condition commonly occurring in wooden and steel beams. Table I which follows gives the location of the neutral axis for such cases only. If the cross-section is subjected to combined flexure and direct stress, the neutral axis will not pass through its center of gravity, but will deviate from it by an amount depending upon the ratio of bending moment to direct stress, reaching infinity when the stress is direct tension or compression only, as in the case of a tie rod or column centrally loaded.

If the beam is of unhomogeneous material the position of the neutral axis must be determined by calculation. The reinforced concrete beam is the only common example of such a case. For formulas for this case, see *Concrete, Reinforced.*

Section Modulus. — If c is the normal distance from the neutral axis of a given plane surface to the most remote portion of its perimeter, and I the moment of inertia of the surface about its neutral axis the expression I/c is called the section modulus. For symmetrical beams exposed to pure flexure only, c = one-half the depth of the beam. Tabular values of this quantity for plane surfaces of varying shapes are given in Table I and for the ordinary structural steel beams in Tables III to VI, which follow.

Moment of Resistance. — If s is the allowable unit stress per square inch upon the extreme fiber of a beam or girder, the expression $\frac{sI}{c}$ is called the moment of resistance since its value equals the maximum bending moment which the member may carry without causing a stress greater than the allowable unit stress.

Statical Moment. — The statical moment of a plane surface about a given axis lying in the plane is the summation of the products of each elementary area,

dA, of which the surface is composed, times the distance of its center of gravity from the given axis, distances above the axis being considered positive, and below negative. It may be expressed mathematically as follows: (See Fig. 1.)

Let Q = statical moment of a given area about an axis Y-Y lying in the plane of the area,

dA = an elementary area,

y = distance of center of gravity of the area dA from the axis,

then $Q = \int y dA$.

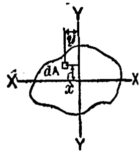


Fig. 1.

The statical moment of an area about an axis of symmetry evidently equals zero since the positive lever arm of any elementary area will be balanced by a corresponding negative lever arm.

Equations of Equilibrium. — (*See also Mechanics, Principles of.*) The following laws apply to a structure lying in a plane and acted upon by forces lying in the same plane:

1st. The algebraic sum of the components of all the forces acting parallel to any axis in the plane of the forces must equal zero.

2nd. The algebraic sum of the moments of all the forces about any axis at right angles to the plane of the forces must equal zero.

If we resolve the applied forces into components parallel to two rectangular axes, OX and OY, and let ΣX and ΣY equal the algebraic sum of the forces parallel respectively to OX and OY, and ΣM the algebraic sum of their moment about any axis, normal to the plane of the forces, these two laws will be fully comprehended by the three equations:

$$\Sigma X = 0, \ \Sigma Y = 0, \ \Sigma M = 0.$$

If OX and OY are respectively horizontal and vertical these equations take the generally used form:

$$\Sigma H = 0, \ \Sigma V = 0, \ \Sigma M = 0.$$

Unless a structure satisfies all these conditions it cannot be in equilibrium.

Reactions. — The forces which the abutments or piers exert upon structures, such as beams, girders or trusses, are called the reactions. Each reaction may, in general, have three unknown factors, viz., direction, magnitude and point of application. In order to simplify computations it is customary for girders or trusses to eliminate three of these unknowns by the method of construction; e.g., in end-supported trusses and long girders, the ends are usually supported on pins of comparatively small diameter thereby fixing the points of application of both reactions. Moreover, the pin at one end is in turn supported on a set of rollers thereby making that reaction normal, or nearly so, to its supporting surface, thus fixing the direction of one of the reactions. These refinements are not generally applied to short girders the ends of which are supported on steel plates or castings. Even in such cases, however, the point of application is fixed within comparatively small limits, and the base plate at one end is usually planed thus making the reaction at that end approximately normal to the supporting surface. For ordinary beams no special provision is made to fix any of the reaction conditions but each reaction is assumed to be normal to the supporting surface and to act at its center.

When the unknown reaction factors are thus reduced to three, their values may be determined by the three equations of equilibrium previously given, this being the usual procedure in the case of beams on two supports as illustrated later. For beams supported at more than two points, the three-moment equation (*see below*) may be applied.

Three Moment Equation. — This is an equation connecting the moments at three adjoining points of support of a continuous structure. It is strictly applicable only to structures having a constant moment of inertia and on level supports but is applied approximately to structures the moment of inertia of which is not constant.

The formula for concentrated loads is as follows:

Let M_c = moment at any support,
M_a = moment at adjoining support on left,
M_b = moment at adjoining support on right,
L_1 = length of span to left of support at which M_c acts,
L_2 = length of span to right of support at which M_c acts,
P_1 = any load in span L_1 and k_1L_1 its distance from left support,
P_2 = any load in span L_2 and k_2L_2 its distance from support at left of that span.

Then

$$M_aL_1 + 2\,M_c(L_1 + L_2) + M_bL_2 = \Sigma P_1L_1^2(k_1^3 - k_1) + \Sigma P_2L_2^2\,(3\,k_2^2 - k_2^3 - 2\,k_2).$$

For uniform load of w_1 lb. per ft. over span L_1 and w_2 lb. per ft. over span L_2 the preceding formula becomes

$$M_aL_1 + 2\,M_c(L_1 + L_2) + M_bL_2 = -\frac{1}{4}w_1L_1^3 - \frac{1}{4}w_2L_2^3.$$

The application of these formulas to successive series of spans enables the moments at all supports to be computed and from these the reactions and shears may also be determined, and the structure completely solved; see section below on *Girders, Design of.*

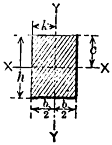
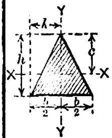
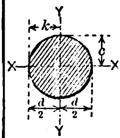
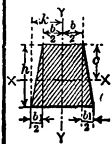
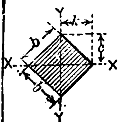
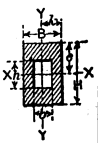
Shear. — The shearing force or shear at any section of a body is the force which tends to produce slipping along the given section. Shearing failure may be due either to transverse fracture or to slipping of the fibers on each other. Of the ordinary structural materials wood is the only one of fibrous character and shearing failure in this material frequently occurs on planes parallel to the fibers.

Bending Moment; Flexure. — The bending moment at any section of a body due to a set of coplanar forces is the resultant moment about an axis passing through the center of gravity of the section of all the forces on either side of the section, it being understood that the section and the axis are perpendicular to the plane of the forces. Fractures due to excessive bending moment occur through longitudinal failure of the fibers either by tension or crushing. The bending which results from the application of a bending moment is termed flexure.

Dead and Live Loads. — The loads acting upon a given structure may be divided into two distinct types: viz., quiescent loads which are known as *dead loads* and moving or intermittent loads which are known as *live loads.* The first class includes all loads which are fixed in magnitude and position such as the weight of the structure itself and such superimposed loads as the floor of a building or bridge; the second class includes such loads as crowds of people, merchandise, snow, wind and vehicles of all sorts. The live load through its rapidity and irregularity of application is more injurious in its effect than the dead load, hence in a structure subjected to live loads, either the value of the live load should be increased as explained below under *Allowances for Impact,* or the allowable unit stress in the material should be reduced. The former method seems to be more logical and is commonly adopted at the present time by leading structural engineers.

(Text continued on p. 1428.)

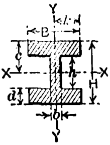
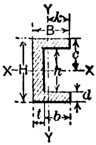
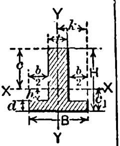
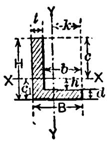
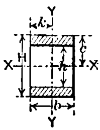
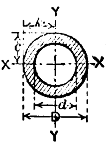
TABLE I.—PROPERTIES OF SECTIO

Section. Axes $X-X$ and $Y-Y$ pass through center of gravity	Distance from $X-X$ to extreme edge of section c	Moment of inertia with respect to axis $X-X$ I_x	Section modulus with respect to axis $X-X$ $\frac{I_x}{c}$	Radius of gyration with respect to axis $X-X$ ρ_x
	$\frac{h}{2}$	$\frac{1}{12}bh^3$	$\frac{1}{6}bh^2$	$\frac{h}{\sqrt{12}}=0.289h$
	$\frac{2}{3}h$	$\frac{1}{36}bh^3$	$\frac{1}{24}bh^2$	$\frac{h}{\sqrt{18}}=0.236h$
	$\frac{d}{2}$	$\frac{\pi d^4}{64}=0.0491d^4$	$\frac{\pi d^3}{32}=0.0982d^3$	$\frac{d}{4}$
	$\frac{h}{3}\left(\frac{3b+2b_1}{2b+b_1}\right)$	$\frac{h^3}{36}\left(\frac{6b^2+6bb_1+b_1^2}{2b+b_1}\right)$	$\frac{h^2}{12}\left(\frac{6b^2+6bb_1+b_1^2}{3b+2b_1}\right)$	$\frac{h}{2b+b_1}\sqrt{\frac{6b^2+6bb_1+b_1^2}{18}}$
	$\frac{b}{\sqrt{2}}=0.707b$	$\frac{b^4}{12}$	$\frac{\sqrt{2}}{12}b^3=0.118b^3$	$\frac{b}{\sqrt{12}}=0.289b$
	$\frac{H}{2}$	$\frac{BH^3-bh^3}{12}$	$\frac{BH^3-bh^3}{6H}$	$\sqrt{\frac{BH^3-bh^3}{12(BH-bh)}}$

PLANE SECTIONS

Distance from Y–Y to extreme edge of section	Moment of inertia with respect to axis Y–Y	Section modulus with respect to axis Y–Y	Radius of gyration with respect to axis Y–Y
k	I_y	$\frac{I_y}{k}$	ρ_y
$\frac{b}{2}$	$\frac{1}{12}hb^3$	$\frac{1}{6}hb^2$	$\frac{b}{\sqrt{12}}=0.289b$
$\frac{b}{2}$	$\frac{1}{48}hb^3$	$\frac{1}{24}hb^2$	$\frac{b}{\sqrt{24}}=0.204b$
$\frac{d}{2}$	$\frac{\pi d^4}{64}=0.0491d^4$	$\frac{\pi d^3}{32}=0.0982d^3$	$\frac{d}{4}$
$\frac{b+b_1}{2}$	$\frac{hb^3}{12}+\frac{hb_1^3}{48}+\frac{bb_1h(3b+2b_1)}{24}$	$\frac{\frac{hb^3}{6}+\frac{hb_1^3}{24}+\frac{bb_1h(3b+2b_1)}{12}}{b+b_1}$	$\frac{1}{12}\sqrt{\frac{24b^3+6b_1^3+12bb_1(3b+2b_1)}{2b+b_1}}$
$\frac{b}{\sqrt{2}}=0.707b$	$\frac{b^4}{12}$	$\frac{\sqrt{2}}{12}b^3=0.118b^3$	$\frac{b}{\sqrt{12}}=0.289b$
$\frac{B}{2}$	$\frac{HB^3-hb^3}{12}$	$\frac{HB^3-hb^3}{6B}$	$\sqrt{\frac{HB^3-hb^3}{12(BH-bh)}}$

TABLE I.—PROPERTIES OF

Section. Axes $X-X$ and $Y-Y$ pass through center of gravity	Distance from $X-X$ to extreme edge of section c	Moment of inertia with respect to axis $X-X$ I_x	Section modulus with respect to axis $X-X$ $\frac{I_x}{c}$	Radius of gyration with respect to axis $X-X$ ρ_x
	$\frac{H}{2}$	$\frac{BH^3-h^3(B-b)}{12}$	$\frac{BH^3-h^3(B-b)}{6H}$	$\sqrt{\frac{BH^3-h^3(B-b)}{12[BH-h(B-b)]}}$
	$\frac{H}{2}$	$\frac{BH^3-h^3b}{12}$	$\frac{BH^3-h^3b}{6H}$	$\sqrt{\frac{BH^3-h^3b}{12(BH-hb)}}$
	$H-\frac{1}{2}\left(\frac{tH^2+bd^2}{tH+bd}\right)$	$\frac{1}{3}(Bc_1^3-bh^3+tc^3)$	$\frac{1}{3}\frac{Bc_1^3-bh^3+tc^3}{H-\frac{1}{2}\left(\frac{tH^2+bd^2}{tH+bd}\right)}$	$\sqrt{\frac{Bc_1^3-bh^3+tc^3}{3[BH-b(c+h)]}}$
	$H-\frac{1}{2}\left(\frac{tH^2+bd^2}{tH+bd}\right)$	$\frac{1}{3}(Bc_1^3-bh^3+tc^3)$	$\frac{1}{3}\left(\frac{Bc_1^3-bh^3+tc^3}{H-\frac{1}{2}\frac{tH^2+bd^2}{tH+bd}}\right)$	$\sqrt{\frac{Bc_1^3-bh^3+tc^3}{3[BH-b(c+h)]}}$
	$\frac{H}{2}$	$\frac{b}{12}(H^3-h^3)$	$\frac{b}{6}\frac{H^3-h^3}{H}$	$\sqrt{\frac{H^2+Hh+h^2}{12}}$
	$\frac{D}{2}$	$\frac{\pi}{64}(D^4-d^4)$	$\frac{\pi}{32}\frac{D^4-d^4}{D}$	$\frac{1}{4}\sqrt{D^2+d^2}$

PLANE SECTIONS — (*Continued*)

Distance from Y–Y to extreme edge of section	Moment of inertia with respect to axis Y–Y	Section modulus with respect to axis Y–Y	Radius of gyration with respect to axis Y–Y
k	I_y	$\frac{I_y}{k}$	ρ_y
$\frac{B}{2}$	$\frac{2dB^3+hb^3}{12}$	$\frac{2dB^3+hb^3}{6B}$	$\sqrt{\frac{2dB^3+hb^3}{12(2dB+hb)}}$
$\frac{HB^2-hb^2}{2(BH-hb)}$	$\frac{Ht^3+2db^3}{3}-(Ht+2db)(b-k)^2$	$\frac{Ht^3+2db^3}{3k}-\frac{(Ht+2db)(b-k)^2}{k}$	$\sqrt{\frac{Ht^3+2db^3}{3(Ht+2db)}-(b-k)^2}$
$\frac{B}{2}$	$\frac{(H-d)t^3+dB^3}{12}$	$\frac{(H-d)t^3+dB^3}{6B}$	$\sqrt{\frac{(H-d)t^3+dB^3}{12(H-d)t+12dB}}$
$\frac{HB^2-(H-d)b^2}{2(Ht+bd)}$	$\frac{Ht^3+db^3}{3}-(Ht+db)(b-k)^2$	$\frac{Ht^3+db^3}{3k}-\frac{(Ht+db)(b-k)^2}{k}$	$\sqrt{\frac{Ht^3+db^3}{3(Ht+db)}-(b-k)^2}$
$\frac{b}{2}$	$\frac{(H-h)b^3}{12}$	$\frac{(H-h)b^2}{6}$	$\frac{b}{\sqrt{12}}=0.289b$
$\frac{D}{2}$	$\frac{\pi}{64}(D^4-d^4)$	$\frac{\pi}{32}\frac{(D^4-d^4)}{D}$	$\frac{1}{4}\sqrt{D^2+d^2}$

Impact is the name given to the dynamic force which is added to the forces due to live and dead loads to obtain the true forces acting upon structures; see section below on *Allowances for Impact.*

Beam or Girder. — A beam is a bar of wood, metal, concrete or other stress-resisting substance supported at certain definite points along its length and loaded transversely to its longitudinal axis, which is usually but not always, placed in a horizontal position. The name girder is commonly applied to large beams built up of structural members; for example, a plate girder is a steel or iron structure made up of plates and angles, or other structural shapes, riveted together.

Floor Beam; Panel.—Frequently girders and trusses receive all their live load and much of their dead load through other members called floor beams. Fig. 2 illustrates such construction. The distance between adjacent floor beams is the panel length.

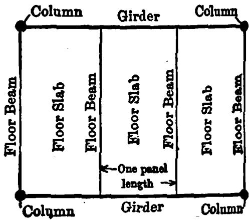


Fig. 2.

Column. — A column is a member intended primarily to resist direct compression, although it may also be subjected to bending stresses due either to transverse loads or to eccentric application of the direct loads.

Truss. — A truss is a structure consisting of separate bars constructed to carry either direct tension or direct compression. These bars are connected at their ends and occasionally at intermediate points, the points of connection being called joints. The connections are sometimes made by riveting the bars directly together and sometimes by riveting them to a common steel plate, the truss in either case being called a *riveted truss.* The connections may also be made by fastening together with a large steel pin all the members meeting at a joint; such a truss is called a *pin truss.* The outer forces should be applied at the joints only, since the members are not intended to carry bending. This is accomplished by the use of floor beams in a bridge and purlins in a roof.

PROPERTIES OF PLANE SECTIONS. — In Table I are given the location of the center of gravity, and values of the moment of inertia, section modulus and radii of gyration of the more common plane sections.

TABLES OF STRUCTURAL SHAPES.—Tables II to VI give the dimensions and properties of the commonly used shapes of structural steel; namely, sheared plates, angles with either equal or unequal legs, I-beams, and channels. For information upon other metallic structural material see the publications of the various steel manufacturing companies.

TABLE II.—SHEARED PLATES*

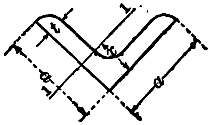
Width in inches	Thickness in inches																		
	3/16	1/4	5/16	3/8	7/16	1/2	9/16	5/8	11/16	3/4	13/16	7/8	15/16	1	1 1/8	1 1/4	1 1/2	1 3/4	2
	Maximum length in inches																		
24– 29	400	525	575	600	600	600	600	600	575	575	550	550	525	525	500	450	425	375	350
30– 35	375	525	550	600	600	625	625	600	575	575	550	500	475	475	450	450	400	375	350
36– 41	375	475	525	550	550	575	575	575	575	550	525	500	475	475	450	425	400	375	350
42– 47	400	525	550	575	600	600	600	575	575	575	525	500	500	500	475	425	400	375	350
48– 53	400	525	575	600	600	600	600	600	575	575	550	550	525	525	500	450	400	375	350
54– 59	400	525	550	600	600	625	625	600	575	575	550	550	525	525	500	450	400	375	350
60– 65	375	525	550	600	600	625	625	600	575	575	550	550	525	525	475	425	400	350	325
66– 71	350	475	500	575	575	600	600	600	575	575	550	550	525	525	475	425	375	350	325
72– 77	325	425	450	525	550	575	575	575	575	575	550	525	500	500	475	425	375	350	300
78– 83	...	400	425	475	500	525	525	525	525	525	500	475	450	450	425	375	325	300	275
84– 89	...	375	400	425	450	475	475	475	475	475	450	450	425	425	375	350	300	275	250
90– 95	...	325	350	375	400	425	425	425	425	425	400	400	375	375	350	325	280	260	250
96–101	...	300	325	350	375	400	400	400	400	400	375	375	350	325	300	275	260	250	225
102–107	...	275	300	325	350	375	375	375	375	375	350	350	325	300	275	250	240	220	220
108–113	...	250	275	300	325	350	350	350	350	350	325	325	300	275	250	250	225	200	175
114–119	...	175	200	225	250	275	275	275	275	300	275	275	250	250	225	200	175	160	150
120–125	...	...	175	200	225	250	250	250	250	275	250	250	225	225	200	200	175	160	150
126	...	...	...	...	...	...	...	175	175	200	200	200	175	175	160	160	150	144	144
Diam. of Head, in.	72	115	117	124	124	127	127	127	127	127	127	127	127	127	127	127	127	127	127

Minimum diameter of heads = 30 inches.

From *Cambria Steel*, George E. Thackray, Engineer.

* Edges trimmed by shearing. Narrower plates are rolled to dimensions and can be obtained in practically any width; even inches should be used.

TABLE III.—STANDARD ANGLES WITH EQUAL LEGS



Dimensions	Thickness	Weight per foot	Area of section	Distance of center of gravity from back of leg	Moment of inertia axis 1-1	Section modulus axis 1-1 $= \frac{I}{a-x}$
$a \times a$	t		A	x	I	S
Inches	Inch	Pounds	Sq. in.	Inches	Inches⁴	Inches³
1½×1½	⅛	1.23	0.36	0.42	0.08	0.072
"	3/16	1.80	0.53	0.44	0.11	0.104
"	¼	2.34	0.69	0.47	0.14	0.134
"	5/16	2.86	0.84	0.49	0.16	0.162
"	⅜	3.35	0.98	0.51	0.19	0.188
"	7/16	3.82	1.12	0.53	0.21	0.214
2 ×2	3/16	2.44	0.72	0.57	0.27	0.19
"	¼	3.19	0.94	0.59	0.35	0.25
"	5/16	3.92	1.15	0.61	0.42	0.30
"	⅜	4.7	1.36	0.64	0.48	0.35
"	7/16	5.3	1.56	0.66	0.54	0.40
"	½	6.0	1.75	0.68	0.59	0.45
2½×2½	3/16	3.07	0.90	0.69	0.55	0.30
"	¼	4.1	1.19	0.72	0.70	0.39
"	5/16	5.0	1.47	0.74	0.85	0.48
"	⅜	5.9	1.73	0.76	0.98	0.57
"	7/16	6.8	2.00	0.78	1.11	0.65
"	½	7.7	2.25	0.81	1.23	0.72
"	9/16	8.5	2.50	0.83	1.34	0.80
3 ×3	¼	4.9	1.44	0.84	1.24	0.58
"	5/16	6.1	1.78	0.87	1.51	0.71
"	⅜	7.2	2.11	0.89	1.76	0.83
"	7/16	8.3	2.43	0.91	1.99	0.95
"	½	9.4	2.75	0.93	2.22	1.07
"	9/16	10.4	3.06	0.95	2.43	1.19
"	⅝	11.5	3.36	0.98	2.62	1.30
"	11/16	12.5	3.65	1.00	2.81	1.40
3½×3½	5/16	7.2	2.09	0.99	2.45	0.98
"	⅜	8.5	2.48	1.01	2.87	1.15
"	7/16	9.8	2.87	1.04	3.26	1.32
"	½	11.1	3.25	1.06	3.64	1.49
"	9/16	12.4	3.62	1.08	3.99	1.65
"	⅝	13.6	3.98	1.10	4.33	1.81
"	11/16	14.8	4.34	1.12	4.65	1.96

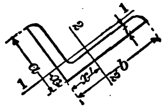
From *Cambria Steel*, George E. Thackray, Engineer.

TABLE III.—STANDARD ANGLES WITH EQUAL LEGS—(*Continued*)

Dimensions	Thickness	Weight per foot	Area of section	Distance of center of gravity from back of leg	Moment of inertia axis 1–1	Section modulus axis 1–1 $=\frac{I}{a-x}$
$a \times a$	t		A	x	I	S
Inches	Inch	Pounds	Sq. in.	Inches	Inches⁴	Inches³
3½×3½	¾	16.0	4.69	1.15	4.96	2.11
"	13/16	17.1	5.03	1.17	5.25	2.25
"	⅞	18.3	5.36	1.19	5.53	2.39
4 ×4	5/16	8.2	2.40	1.12	3.71	1.29
"	⅜	9.8	2.86	1.14	4.36	1.52
"	7/16	11.3	3.31	1.16	4.97	1.75
"	½	12.8	3.75	1.18	5.56	1.97
"	9/16	14.3	4.18	1.21	6.12	2.19
"	⅝	15.7	4.61	1.23	6.66	2.40
"	11/16	17.1	5.03	1.25	7.17	2.61
"	¾	18.5	5.44	1.27	7.66	2.81
"	13/16	19.9	5.84	1.29	8.14	3.01
"	⅞	21.2	6.23	1.31	8.59	3.20
6 ×6	⅜	14.9	4.36	1.64	15.39	3.53
"	7/16	17.2	5.06	1.66	17.68	4.07
"	½	19.6	5.75	1.68	19.91	4.61
"	9/16	21.9	6.43	1.71	22.07	5.14
"	⅝	24.2	7.11	1.73	24.16	5.66
"	11/16	26.5	7.78	1.75	26.19	6.17
"	¾	28.7	8.44	1.78	28.15	6.66
"	13/16	31.0	9.09	1.80	30.06	7.15
"	⅞	33.1	9.73	1.82	31.92	7.63
"	15/16	35.3	10.37	1.84	33.72	8.11
"	1	37.4	11.00	1.86	35.46	8.57
8 ×8	½	26.4	7.75	2.19	48.65	8.37
"	9/16	29.6	8.68	2.21	54.09	9.34
"	⅝	32.7	9.61	2.23	59.43	10.30
"	11/16	35.8	10.53	2.25	64.64	11.25
"	¾	38.9	11.44	2.28	69.74	12.18
"	13/16	42.0	12.34	2.30	74.72	13.11
"	⅞	45.0	13.23	2.32	79.58	14.02
"	15/16	48.1	14.12	2.34	84.34	14.91
"	1	51.0	15.00	2.37	88.98	15.80
"	1 1/16	54.0	15.87	2.39	93.53	16.67
"	1⅛	56.9	16.73	2.41	97.97	17.53

From *Cambria Steel*, George E. Thackray, Engineer.

TABLE IV. — STANDARD ANGLES WITH UNEQUAL LEGS



Dimensions	Thickness	Weight per foot	Area of section	Distance of center of gravity from back of longer leg	Moment of inertia axis 1–1	Distance of center of gravity from back of shorter leg	Moment of inertia axis 2–2
$b \times a$	t		A	x	I	x'	I'
Inches	Inch	Pounds	Sq. in.	Inch	Inches⁴	Inch	Inches⁴
2½×2	3/16	2.75	0.81	0.51	0.29	0.76	0.51
"	1/4	3.62	1.06	0.54	0.37	0.79	0.65
"	5/16	4.5	1.31	0.56	0.45	0.81	0.79
"	3/8	5.3	1.55	0.58	0.51	0.83	0.91
"	7/16	6.1	1.78	0.60	0.58	0.85	1.03
"	1/2	6.8	2.00	0.63	0.64	0.88	1.14
"	9/16	7.6	2.22	0.65	0.69	0.90	1.24
3 ×2½	1/4	4.5	1.31	0.66	0.74	0.91	1.17
"	5/16	5.6	1.62	0.68	0.90	0.93	1.42
"	3/8	6.6	1.92	0.71	1.04	0.96	1.66
"	7/16	7.6	2.22	0.73	1.18	0.98	1.88
"	1/2	8.5	2.50	0.75	1.30	1.00	2.08
"	9/16	9.5	2.78	0.77	1.42	1.02	2.28
"	5/8	10.4	3.05	0.79	1.53	1.04	2.46
3½×2½	1/4	4.9	1.44	0.61	0.78	1.11	1.80
"	5/16	6.1	1.78	0.64	0.94	1.14	2.19
"	3/8	7.2	2.11	0.66	1.09	1.16	2.56
"	7/16	8.3	2.43	0.68	1.23	1.18	2.91
"	1/2	9.4	2.75	0.70	1.36	1.20	3.24
"	9/16	10.4	3.06	0.73	1.49	1.23	3.55
"	5/8	11.5	3.36	0.75	1.61	1.25	3.85
"	11/16	12.5	3.65	0.77	1.72	1.27	4.13
"	3/4	13.4	3.94	0.79	1.83	1.29	4.40
3½×3	5/16	6.6	1.93	0.81	1.58	1.06	2.33
"	3/8	7.9	2.30	0.83	1.85	1.08	2.72
"	7/16	9.1	2.65	0.85	2.09	1.10	3.10
"	1/2	10.2	3.00	0.88	2.33	1.13	3.45
"	9/16	11.4	3.34	0.90	2.55	1.15	3.79
"	5/8	12.5	3.67	0.92	2.76	1.17	4.11
"	11/16	13.6	4.00	0.94	2.96	1.19	4.41
"	3/4	14.7	4.31	0.96	3.15	1.21	4.70
"	13/16	15.8	4.62	0.98	3.33	1.23	4.98
"	7/8	16.8	4.92	1.00	3.50	1.25	5.24

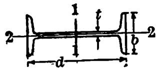
From *Cambria Steel*, George E. Thackray, Engineer.

TABLE IV. — STANDARD ANGLES WITH UNEQUAL LEGS — (*Continued*)

Dimensions	Thickness	Weight per foot	Area of section	Distance of center of gravity from back of longer leg	Moment of inertia axis 1–1	Distance of center of gravity from back of shorter leg	Moment of inertia axis 2–2
$b \times a$	t		A	x	I	x'	I'
Inches	Inch	Pounds	Sq. in.	Inches	Inches[4]	Inch	Inches[4]
4 ×3	5/16	7.2	2.09	0.76	1.65	1.26	3.38
"	3/8	8.5	2.48	0.78	1.92	1.28	3.96
"	7/16	9.8	2.87	0.80	2.18	1.30	4.52
"	1/2	11.1	3.25	0.83	2.42	1.33	5.05
"	9/16	12.4	3.62	0.85	2.66	1.35	5.55
"	5/8	13.6	3.98	0.87	2.87	1.37	6.03
"	11/16	14.8	4.34	0.89	3.08	1.39	6.49
"	3/4	16.0	4.69	0.92	3.28	1.42	6.93
5 ×3	5/16	8.2	2.40	0.68	1.75	1.68	6.26
"	3/8	9.8	2.86	0.70	2.04	1.70	7.37
"	7/16	11.3	3.31	0.73	2.32	1.73	8.43
"	1/2	12.8	3.75	0.75	2.58	1.75	9.45
"	9/16	14.3	4.18	0.77	2.83	1.77	10.43
"	5/8	15.7	4.61	0.80	3.06	1.80	11.37
"	11/16	17.1	5.03	0.82	3.29	1.82	12.28
"	3/4	18.5	5.44	0.84	3.51	1.84	13.15
5 ×3½	5/16	8.7	2.56	0.84	2.72	1.59	6.60
"	3/8	10.4	3.05	0.86	3.18	1.61	7.78
"	7/16	12.0	3.53	0.88	3.63	1.63	8.90
"	1/2	13.6	4.00	0.91	4.05	1.66	9.99
"	9/16	15.2	4.47	0.93	4.45	1.68	11.03
"	5/8	16.8	4.92	0.95	4.83	1.70	12.03
"	11/16	18.3	5.37	0.97	5.20	1.72	12.99
"	3/4	19.8	5.81	1.00	5.55	1.75	13.92
"	13/16	21.3	6.25	1.02	5.89	1.77	14.81
"	7/8	22.7	6.67	1.04	6.21	1.79	15.67
6 ×3½	3/8	11.7	3.42	0.79	3.34	2.04	12.86
"	1/2	15.3	4.50	0.83	4.25	2.08	16.59
"	5/8	18.9	5.55	0.88	5.08	2.13	20.08
"	3/4	22.4	6.56	0.93	5.84	2.18	23.34
"	7/8	25.7	7.55	0.97	6.55	2.22	26.39
6 ×4	3/8	12.3	3.61	0.94	4.90	1.94	13.47
"	1/2	16.2	4.75	0.99	6.27	1.99	17.40
"	5/8	20.0	5.86	1.03	7.52	2.03	21.07
"	3/4	23.6	6.94	1.08	8.68	2.08	24.51
"	7/8	27.2	7.98	1.12	9.75	2.12	27.73

From *Cambria Steel*, George E. Thackray, Engineer.

TABLE V. — STANDARD I-BEAMS



Depth of beam	Weight per foot	Area of section	Thickness of web	Width of flange	Moment of inertia axis 1–1	Section modulus axis 1–1	Radius of gyration axis 1–1	Moment of inertia axis 2–2	Radius of gyration axis 2–2	Coefficient of strength*
d		*A*	*t*	*b*	*I*	*S*	*r*	*I'*	*r'*	
In.	Lb.	Sq.in.	In.	In.	In.^{4}	In.^{3}	In.	In.^{4}	In.	
3	5.50	1.63	0.17	2.33	2.5	1.7	1.23	0.46	0.53	17,650
3	6.50	1.91	0.26	2.42	2.7	1.8	1.19	0.53	0.52	19,140
3	7.50	2.21	0.36	2.52	2.9	1.9	1.15	0.60	0.52	20,710
4	7.50	2.21	0.19	2.66	6.0	3.0	1.64	0.77	0.59	31,810
4	8.50	2.50	0.26	2.73	6.4	3.2	1.59	0.85	0.58	33,890
4	9.50	2.79	0.34	2.81	6.7	3.4	1.54	0.93	0.58	35,980
4	10.50	3.09	0.41	2.88	7.1	3.6	1.52	1.01	0.57	38,070
5	9.75	2.87	0.21	3.00	12.1	4.8	2.05	1.23	0.65	51,590
5	12.25	3.60	0.36	3.15	13.6	5.4	1.94	1.45	0.63	58,100
5	14.75	4.34	0.50	3.29	15.1	6.1	1.87	1.70	0.63	64,630
6	12.25	3.61	0.23	3.33	21.8	7.3	2.46	1.85	0.72	77,460
6	14.75	4.34	0.35	3.45	24.0	8.0	2.35	2.09	0.69	85,270
6	17.25	5.07	0.47	3.57	26.2	8.7	2.27	2.36	0.68	93,110
7	15.00	4.42	0.25	3.66	36.2	10.4	2.86	2.67	0.78	110,410
7	17.50	5.15	0.35	3.76	39.2	11.2	2.76	2.94	0.76	119,400
7	20.00	5.88	0.46	3.87	42.2	12.1	2.68	3.24	0.74	128,560
8	18.00	5.33	0.27	4.00	56.9	14.2	3.27	3.78	0.84	151,660
8	20.25	5.96	0.35	4.08	60.2	15.0	3.18	4.04	0.82	160,510
8	22.75	6.69	0.44	4.17	64.1	16.0	3.10	4.36	0.81	170,970
8	25.25	7.43	0.53	4.26	68.0	17.0	3.03	4.71	0.80	181,430
9	21.00	6.31	0.29	4.33	84.9	18.9	3.67	5.16	0.90	201,300
9	25.00	7.35	0.41	4.45	91.9	20.4	3.54	5.65	0.88	217,930
9	30.00	8.82	0.57	4.61	101.9	22.6	3.40	6.42	0.85	241,460
9	35.00	10.29	0.73	4.77	111.8	24.8	3.30	7.31	0.84	264,990

From *Cambria Steel*, George E. Thackray, Engineer.

* Divide by span in feet to determine total allowable dead load in pounds, including weight of beam, uniformly distributed over length of beam. Table is based upon a fiber stress of 16,000 lb. per sq. in. For allowable live and dead load reduce load as determined from table by amount of impact. Compression flange should be supported laterally at intervals not greater than 20 times its width. If this cannot be done reduce coefficients to one-half of above value where unsupported length divided by width = 70, and proportionally for intermediate values between 20 and 70.

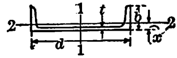
TABLE V. — STANDARD I-BEAMS — (*Continued*)

Depth of beam	Weight per foot	Area of section	Thickness of web	Width of flange	Moment of inertia axis 1-1	Section modulus axis 1-1	Radius of gyration axis 1-1	Moment of inertia axis 2-2	Radius of gyration axis 2-2	Coefficient of strength*
d		*A*	*t*	*b*	*I*	*S*	*r*	*I'*	*r'*	
In.	Lb.	Sq.in.	In.	In.	In.⁴	In.³	In.	In.⁴	In.	
10	25.00	7.37	0.31	4.66	122.1	24.4	4.07	6.89	0.97	260,470
10	30.00	8.82	0.45	4.80	134.2	26.8	3.90	7.65	0.93	286,250
10	35.00	10.29	0.60	4.95	146.4	29.3	3.77	8.52	0.91	312,390
10	40.00	11.76	0.75	5.10	158.7	31.7	3.67	9.50	0.90	338,530
12	31.50	9.26	0.35	5.00	215.8	36.0	4.83	9.50	1.01	383,670
12	35.00	10.29	0.44	5.09	228.3	38.0	4.71	10.07	0.99	405,800
12	40.00	11.76	0.56	5.21	245.9	41.0	4.57	10.95	0.96	437,170
15	42.00	12.48	0.41	5.50	441.8	58.9	5.95	14.62	1.08	628,270
15	45.00	13.24	0.46	5.55	455.8	60.8	5.87	15.09	1.07	648,310
15	50.00	14.71	0.56	5.65	483.4	64.5	5.73	16.04	1.04	687,530
15	55.00	16.18	0.66	5.75	511.0	68.1	5.62	17.06	1.03	726,740
15	60.00	17.65	0.75	5.84	538.6	71.8	5.52	18.17	1.01	765,960
18	55.0	15.93	0.46	6.00	795.6	88.4	7.07	21.19	1.15	942,880
18	60.0	17.65	0.56	6.10	841.8	93.5	6.91	22.38	1.13	997,680
18	65.0	19.12	0.64	6.18	881.5	97.9	6.79	23.47	1.11	1,044,740
18	70.0	20.59	0.72	6.26	921.2	102.4	6.69	24.62	1.09	1,091,800
20	65.0	19.08	0.50	6.25	1169.5	117.0	7.83	27.86	1.21	1,247,490
20	70.0	20.59	0.58	6.33	1219.8	122.0	7.70	29.04	1.19	1,301,110
20	75.0	22.06	0.65	6.40	1268.8	126.9	7.58	30.25	1.17	1,353,400
24	80.0	23.32	0.50	7.00	2087.2	173.9	9.46	42.86	1.36	1,855,310
24	85.0	25.00	0.57	7.07	2167.8	180.7	9.31	44.35	1.33	1,926,950
24	90.0	26.47	0.63	7.13	2238.4	186.5	9.20	45.70	1.31	1,989,700
24	95.0	27.94	0.69	7.19	2309.0	192.4	9.09	47.10	1.30	2,052,440
24	100.0	29.41	0.75	7.25	2379.6	198.3	8.99	48.55	1.28	2,115,190

From *Cambria Steel*, George E. Thackray, Engineer.

* Divide by span in feet to determine total allowable dead load in pounds, including weight of beam, uniformly distributed over length of beam. Table is based upon a fiber stress of 16,000 lb. per sq. in. For allowable live and dead load reduce load as determined from table by amount of impact. Compression flange should be supported laterally at intervals not greater than 20 times its width. If this cannot be done reduce coefficients to one-half of above value where unsupported length divided by width = 70, and proportionally for intermediate values between 20 and 70.

TABLE VI. — STANDARD CHANNELS



Depth of Channel	Weight per foot	Area of section	Thickness of web	Width of flange	Moment of inertia axis 1–1	Section modulus axis 1–1	Radius of gyration axis 1–1	Moment of inertia axis 2–2	Section modulus axis 2–2	Radius of gyration axis 2–2	Distance of center of gravity from outside of web
d		*A*	*t*	*b*	*I*	*S*	*r*	*I'*	*S'*	*r'*	*x*
In.	Lb.	Sq.in.	In.	In.	In.⁴	In.³	In.	In.⁴	In.³	In.	In.
5	6.50	1.95	0.19	1.75	7.4	3.0	1.95	0.48	0.38	0.50	0.49
5	9.00	2.65	0.33	1.89	8.9	3.5	1.83	0.64	0.45	0.49	0.48
5	11.50	3.38	0.48	2.04	10.4	4.2	1.75	0.82	0.54	0.49	0.51
6	8.00	2.38	0.20	1.92	13.0	4.3	2.34	0.70	0.50	0.54	0.52
6	10.50	3.09	0.32	2.04	15.1	5.0	2.21	0.88	0.57	0.53	0.50
6	13.00	3.82	0.44	2.16	17.3	5.8	2.13	1.07	0.65	0.53	0.52
6	15.50	4.56	0.56	2.28	19.5	6.5	2.07	1.28	0.74	0.53	0.55
7	9.75	2.85	0.21	2.09	21.1	6.0	2.72	0.98	0.63	0.59	0.55
7	12.25	3.60	0.32	2.20	24.2	6.9	2.59	1.19	0.71	0.57	0.53
7	14.75	4.34	0.42	2.30	27.2	7.8	2.50	1.40	0.79	0.57	0.53
7	17.25	5.07	0.53	2.41	30.2	8.6	2.44	1.62	0.87	0.56	0.55
7	19.75	5.81	0.63	2.51	33.2	9.5	2.39	1.85	0.96	0.56	0.58
8	11.25	3.35	0.22	2.26	32.3	8.1	3.10	1.33	0.79	0.63	0.58
8	13.75	4.04	0.31	2.35	36.0	9.0	2.98	1.55	0.87	0.62	0.56
8	16.25	4.78	0.40	2.44	39.9	10.0	2.89	1.78	0.95	0.61	0.56
8	18.75	5.51	0.49	2.53	43.8	11.0	2.82	2.01	1.02	0.60	0.57
8	21.25	6.25	0.58	2.62	47.8	11.9	2.76	2.25	1.11	0.60	0.59
9	13.25	3.89	0.23	2.43	47.3	10.5	3.49	1.77	0.97	0.67	0.61
9	15.00	4.41	0.29	2.49	50.9	11.3	3.40	1.95	1.03	0.66	0.59
9	20.00	5.88	0.45	2.65	60.8	13.5	3.21	2.45	1.19	0.65	0.58
9	25.00	7.35	0.61	2.81	70.7	15.7	3.10	2.98	1.36	0.64	0.62
10	15.00	4.46	0.24	2.60	66.9	13.4	3.87	2.30	1.17	0.72	0.64
10	20.00	5.88	0.38	2.74	78.7	15.7	3.66	2.85	1.34	0.70	0.61
10	25.00	7.35	0.53	2.89	91.0	18.2	3.52	3.40	1.50	0.68	0.62
10	30.00	8.82	0.68	3.04	103.2	20.6	3.42	3.99	1.67	0.67	0.65
12	20.50	6.03	0.28	2.94	128.1	21.4	4.61	3.91	1.75	0.81	0.70
12	25.00	7.35	0.39	3.05	144.0	24.0	4.43	4.53	1.91	0.78	0.68
12	30.00	8.82	0.51	3.17	161.6	26.9	4.28	5.21	2.09	0.77	0.68
12	40.00	11.76	0.76	3.42	196.9	32.8	4.09	6.63	2.46	0.75	0.72
15	33.00	9.90	0.40	3.40	312.6	41.7	5.62	8.23	3.16	0.91	0.79
15	35.00	10.29	0.43	3.43	319.9	42.7	5.57	8.48	3.22	0.91	0.79
15	40.00	11.76	0.52	3.52	347.5	46.3	5.44	9.39	3.43	0.89	0.78
15	45.00	13.24	0.62	3.62	375.1	50.0	5.32	10.29	3.63	0.88	0.79
15	50.00	14.71	0.72	3.72	402.7	53.7	5.23	11.22	3.85	0.87	0.80

From *Cambria Steel*, George E. Thackray, Engineer.

ALLOWANCES FOR IMPACT. — The force applied to a structure by a moving load such as a locomotive or electric car is a function of its weight and method of application. The value of the former can usually be determined with a reasonable degree of accuracy; the effect of the latter cannot in general be so determined since it depends upon such uncertain factors as rapidity of application, irregularity of track, improper counterbalancing of locomotive driving wheels, swaying action of crowds of people and similar causes. The allowance for impact (so-called) ranges from 0 to 100 per cent of the live load and is determined by empirical rules as indicated in the formulas below. An exception to this rule is made in the case of timber structures where the effect of impact is commonly allowed for by using a low unit stress. (*See Timber.*)

Impact on Buildings. — No allowance is generally made for impact on buildings except with special loadings such as moving machinery, swinging cranes, etc., for which the designer should use his judgment. The building laws of the larger cities fix arbitrary loads and unit stresses by which the designer must be governed; see article on *Buildings, Allowable Unit Stresses in.*

Impact on Highway Bridges Carrying Electric Railways. — The following extract from the *Specifications for Bridges Carrying Electric Railways* adopted by the Massachusetts Railroad Commission may be safely used for such cases.

"The total maximum stress in any piece shall be computed by adding together the dead and live stresses, the live loads being placed in the most unfavorable position, together with a percentage of the live stress to allow for impact and vibration. This added percentage shall be as follows:

For floor beams and stringers	25 per cent
For floor beam hangers	40 per cent
For all counters	40 per cent

For other members in trusses, and for main girders:

When the "loaded length" is 20 feet or less	25 per cent
When the "loaded length" is 200 feet or more	10 per cent

and proportionally for intermediate lengths."

Impact on Steam Railroad Bridges. — The following rule of the American Railway Engineering and Maintenance of Way Association is commonly used in the United States:

"The dynamic increment of the live load shall be added to the maximum computed live-load strains* and shall be determined by the formula $I = S\dfrac{300}{L + 300}$,

where I = impact or dynamic increment to be added to live-load strains,
S = computed maximum live-load strain,
L = loaded length of track in feet producing the maximum strain in the member.

For bridges carrying more than one track, the *aggregate length of all tracks* producing the strain shall be used. Impact shall not be added to strains produced by longitudinal, centrifugal and lateral or wind forces."

CALCULATION OF REACTIONS. — The reactions upon a truss, girder or beam lying in a plane and acted upon by forces lying in the same plane may be determined by the application of the equations of equilibrium (*see preceding section*) provided there are but two reactions which together have three unknown components; such a structure is said to be statically determined. If more than three unknown reaction components exist the reactions cannot be computed by statics, and the structure, although stable, is statically undetermined; if less than

* Strains as here used means stresses in the more modern meaning of the words.

three exist the structure is unstable. If the structure is supported at more than two points it cannot be made statically determined except by the insertion in the structure of special devices, such as hinged joints. The tower shown in Fig. 3 is statically determined with respect to the outer forces since the points of application of the reactions at a and b are fixed in position, and the reaction at b is also fixed in direction. If neither of the reactions were fixed in direction the structure would be indeterminate; if both of the reactions were fixed in direction the tower would be unstable, e.g., if both ends were on rollers and neither end bolted to the masonry.

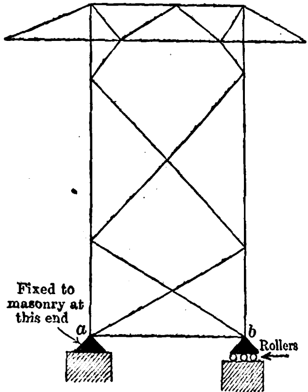


Fig. 3.

Such towers when used for transmission lines are seldom built with rollers or even planed plates, and frequently the bottoms of the columns are imbedded in a concrete base, the latter condition making it impossible to accurately compute the reactions. Similar towers for railroad and highway bridges are usually made determinate.

Method of Computation. — The following example illustrates the analytical method of computing reactions on statically determinate structures. The method may be applied equally well to uniformly distributed loads provided the resultants of such loads are used in place of concentrated loads.

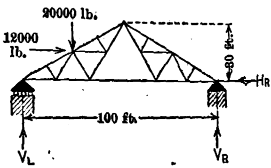


Fig. 4.

To Compute the Reactions on Truss Shown in Fig. 4. — Let V_L = left reaction, and V_R and H_R = the components of the right reaction, the direction of which is unknown. V_L will be vertical since it is supported on rollers bearing on a horizontal surface. V_R and V_H may be assumed to act in either direction. A negative sign in the final result indicates that the force acts in the opposite direction to that assumed. The determination of the reactions by the application of the equations of equilibrium (*see above*) may be carried out as follows:

1st. Apply $\Sigma M = 0$ about right end. This gives $100\,V_L + 12{,}000 \times 15 - 20{,}000 \times 75 = 0$. Hence $V_L = 13{,}200$ lb.

2nd. Apply $\Sigma H = 0$. This gives $H_R - 12{,}000 = 0$. Hence $H_R = +\,12{,}000$ lb.

3rd. Apply $\Sigma V = 0$. This gives $V_R + 13{,}200 - 20{,}000 = 0$. Hence $V_R = +\,6800$ lb.

In these results a positive sign shows that the reaction acts as indicated in Fig. 4.

Reactions on Continuous Girders. — See sections which follow on *Continuous Girders, Beams, Slabs* and *Trusses.*

CALCULATION OF SHEAR. — The nomenclature used in the discussion of shear is as follows:

V = the total external shear on any section in pounds,
P = magnitude of a single concentrated load in pounds,

w = a uniform load in pounds per foot,
L = span of beam in feet (distance center to center of supports),
p = length of a panel in feet,
x = distance in feet from a given section to one of the points of support,
n = total number of equal panels into which a girder is divided,
z = number of panels between a given panel and the more remote abutment.

Method of Computation. — The magnitude of the shear at any section of a body due to a set of coplanar forces may be readily computed in the following manner: Resolve each force into two components, parallel and perpendicular respectively to the given section; the algebraic sum of the components parallel to the section of all the forces on either side of the section equals the shear. The shear is generally considered positive when the resultant force is *upward* on the *left* of the section.

Curve of Shear. — A curve of shear is a line the ordinate to which at any point equals the shear on the given body at the section where the ordinate is measured. Fig. 5 shows typical curves of this sort.

Maximum Shear with Single Load or Uniform Load. — In a simple end-supported beam, girder or truss a concentrated load causes maximum shear at a given section when placed an infinitesimal distance from the section on the side toward the more distant point of support; the magnitude of this shear equals that of the nearer reaction. A concentrated load causes a maximum shear on the beam when placed an infinitesimal distance to one side of either point of support; this shear equals the magnitude of the load itself. A uniformly distributed live load causes maximum shear at a given section when placed over the entire distance from the section to the more distant point of support; its value equals that of the nearer reaction and is given by the equation $V = wx^2/2L$. Its maximum value equals $wL/2$ at either end of the beam when the latter is fully loaded.

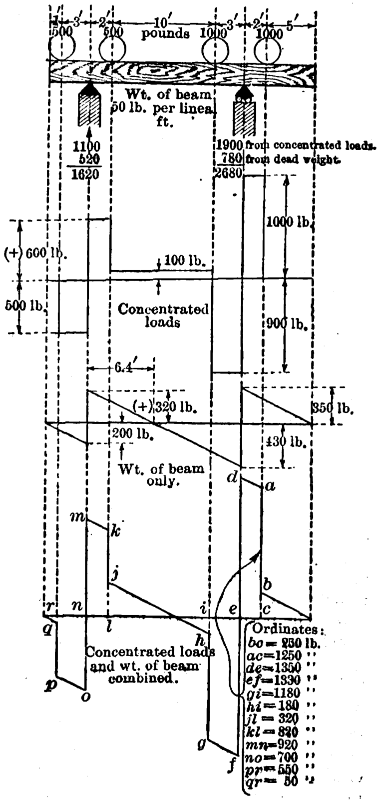


Fig. 5. Curves of Shear. Positive results are shown above the axis. Curve shown by full line

End-supported Girders or Trusses with Loads Applied by Floor-Beams. — A concentrated load causes maximum shear in any panel when

placed at the end of the panel nearer the more remote reaction. The value for equal panels is given by the equation $V = Pz/n$. The maximum shear occurs in the end panel and its value for equal panels is $V = P\ (n - 1)/n$.

Maximum positive shear in any panel due to uniform load occurs when all panel points to the right are assumed to be loaded with full panel loads, and all panel points to the left are unloaded; its value equals the left reaction. This method is approximate but is on the safe side and is commonly used.

The maximum positive shear occurs in the left panel and for girders with equal panels is given by the formula:

$$V = \frac{wp}{2}(n - 1).$$

The same rules are applicable for the maximum negative shear by interchanging left and right in the discussion.

Computation of Maximum Shear for System of Concentrated Loads. — The shear at any section of a beam, girder or truss, due to a system of concentrated loads such as wheel loads of a moving crane, electric car or electric locomotive, equals the algebraic sum of either reaction and the loads lying between that reaction and the given section. In case the girder is divided into panels and the shear in a given panel is to be computed, only that portion of the load acting in the panel under consideration which is carried by the floor-beam at the end of the panel nearer the selected reaction should be deducted from the reaction. To determine the position in which a system of concentrated loads should lie to give maximum shear it is often necessary to proceed by trial. The use of the following simple rules may be helpful.

(*a*) The maximum shear at a given section of a simple beam always occurs with one of the loads at an infinitesimal distance to one side of the section. This load should usually, but not always, be one of the heavier loads of the system; e.g. one of the driving wheels of a locomotive. The proper load may be determined by starting with the first load just to the left of the section and moving the loads to the left until $\sum \frac{Pa}{L} \eqsim P'$. In this expression $P =$ any load which may be on the span during the process of moving the loads, and a is the distance which it is moved. $P' =$ the load which passes to the left of the section as the loads are moved from one position to another.

(*b*) The maximum shear in a given panel of a girder always occurs with one of the loads directly over one of the adjoining floor-beams. This load should usually, but not always, be one of the heavier loads.

(*c*) The position of the loads for maximum positive shear in an intermediate panel may be determined as follows. Place the first load of the system at the right end of the panel and move the system to the left until $\sum \frac{Pa}{L} \eqsim \sum \frac{P_1 a_1}{p}$. Apply the same criterion to the second load and following loads until the position giving maximum shear is reached. In the above expression, P and a are as before, $P_1 =$ any load which may be at any time in the panel under consideration during the process of moving the loads, and $a_1 =$ the distance which P_1 may move in the panel.

If no load comes on or goes off the span and if no load passes out of the panel, $a = a_1$ and we may write

$$\sum \frac{P}{L} \eqsim \sum \frac{P_1}{p}.$$

It follows that for this case the first load should lie at the panel point unless the average load per foot on the entire span is greater than the first load divided by a panel length, in which case the second load should be tried at the panel point and so on until the position for maximum shear is determined.

(*d*) The maximum shear in the end panel of a girder equals the maximum moment at the first panel point divided by the length of the panel. (*See section on Bending Moment, below.*)

CALCULATION OF BENDING MOMENT. — The nomenclature used in the discussion of bending moment is as follows:

M = external bending moment at any section in foot pounds,
P = magnitude of a single concentrated load in pounds,
w = a uniform load in pounds per foot,
L = span of beam in feet (distance center to center of support),
p = length of a panel in feet,
x = distance in feet from a given section to one of the supports,
n = total number of equal panels into which a given girder is divided.

To determine the magnitude of the bending moment at a given section it is necessary to obtain the algebraic sum of the products of every force by its distance from the neutral axis of the section. A mistake commonly made is the failure to consider all the forces, particularly horizontal forces. The moment is considered positive if it is clockwise on the *left* of the section; it follows that the moment is also positive if it is counter-clockwise on the *right* of the section.

Curve of Moments. — A curve of moments is a line whose ordinate at any point equals the moment on the given body at the section where the ordinate is measured. Fig. 6 shows a typical set of moment curves.

Maximum Moment with Single Load or Uniform Load. — For simple end-supported beams, girders and trusses a single concentrated load causes maximum moment at a given section when placed at the section. The following formula gives its value:

$$M = \frac{P(L-x)}{L}x.$$

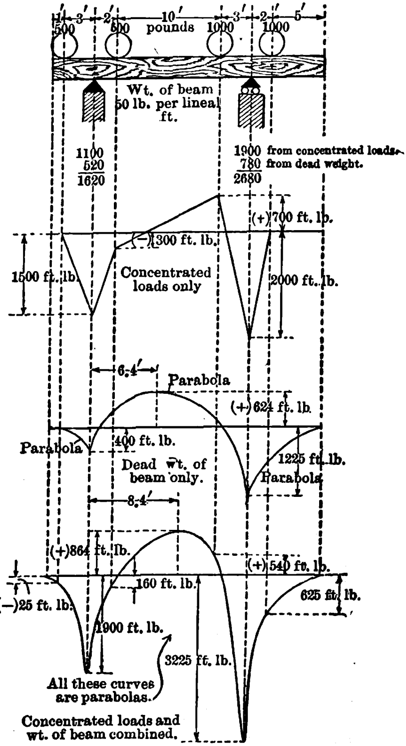


Fig. 6. Curves of Moments

The maximum possible moment occurs when $x = L/2$ and has the value $M = PL/4$.

For simple end-supported structures a uniform load causes maximum moment at any section when it covers the entire beam. Its value at any section is

$$M = \frac{wx}{2}(L-x).$$

The maximum possible moment occurs at the center of the span and equals $wL^2/8$.

End-supported Girders or Trusses with Loads Applied by Floor-Beams. — Moments at panel points are exactly the same as moments at corresponding points on simple girders. It is seldom necessary to consider moments between panel points for concentrated loads. For a uniform load the moment between floor-beams varies uniformly; i.e., the moment due to that portion of the load applied through the floor-beams.

The maximum moment due to a concentrated load occurs at the panel point nearest the center with the load at that point. Its value is given by the following equations:

For even number of equal panels, $M = \frac{Ppn}{4}$.

For odd number of equal panels, $M = \frac{Pp}{4n}(n^2 - 1)$.

The maximum moment due to a uniform load occurs at the panel point nearest the center with load over entire span. Its value is:

For even number of equal panels, $M = wL^2/8$.

For odd number of equal panels, $M = \frac{1}{8}\left[wL^2\left(1 - \frac{1}{n^2}\right)\right]$.

Computation of Moment for System of Concentrated Loads. — The moment at any section due to a system of concentrated loads, such as wheel loads of a moving crane, electric car or electric locomotive equals the algebraic sum of the moments of either reaction and the loads lying between it and the given section, the lever arm for each load being its distance from the section. In case the girder is divided into panels and the section under consideration is at some intermediate point in a panel only that portion of the load applied in this panel which is transmitted to the girder by the floor-beams between it and the reaction used should be considered.

Determination of Maximum Moment at a Given Section. — The maximum moment at any section of a simple beam or at any panel point of a girder loaded through floor-beams always occurs with some load at the section (usually one of the heavier loads). This load should be one which when located just to the right of the section makes the average load per foot of all the loads on the span to the right of the section greater than the corresponding average load per foot on the left of the section, and which reverses this condition when placed just to the left of the section. Such loads may be selected by trial; if more than one satisfies this criterion the largest moment must be determined by actual computation.

Determination of Position of Maximum Moment on a Beam. — Previous articles deal with maximum moment at a given section. The maximum moment on a beam caused by a system of concentrated loads usually occurs near the center but not at it. The following method may be used to determine the location of the section at which the maximum moment occurs:

(*a*) Select, by inspection, one of the heavier loads which would probably be located at the center to give the maximum moment at the center.

(*b*) Determine by inspection the loads which would be on the span when this load is near the center and compute the position of their resultant.

(*c*) Place the loads on the beam in a position such that the center of the span lies half way between this resultant and the assumed load. If the loads on the span in this position correspond to those assumed under *b*, the moment at the load should be computed; otherwise, another trial may be made. If more

than one set of loads corresponds to this condition the moment at the load should be computed for both sets of loads.

(*d*) Apply the same method to any other load which would possibly give maximum moment at the center.

Since the maximum moment at any section always occurs with some load at the section this method if applied to enough loads will give the absolute maximum moment. It is often unnecessary to try more than one load, and seldom more than two. It should be noted that with two equal loads such a distance apart that with one at the center the other will not be on the span, but that with the center of the span midway between the resultant and one of the loads both will be on the span, the maximum moment may possibly occur with either condition, both of which should be investigated.

BEAMS; FORMULAS AND COSTS. — The effect of the external forces upon any cross-section of a simple beam is to cause bending moment and shear, the former usually being the predominant factor in determining the size of the beam. If the beam is subjected to axial forces as well as transverse forces, it becomes a combination of beam and column, or beam and tie, and can no longer be considered as a simple beam. If a beam has an end which

Fig. 7.

projects beyond its adjoining point of support, it is called a *cantilever beam*, the over-hanging end forming the cantilever portion. A beam supported at more than two points of support is a *continuous beam*. A beam having one or both ends rigidly fixed either by being built into a wall, or by being fastened to another member is called a *fixed ended beam*. Fig. 7 represents all of these types.

Beam Formulas for Ordinary Cases. — As a preliminary step in the design of beams, it is necessary to determine the maximum value of the external bending moments, shears and reactions on the beam by the methods given in the sections above which deal with these subjects inclusive. With these items determined, the following formulas of mechanics (*see Mechanics of Materials, Merriman, 10th Edition, Arts. 41 and 108*) may be employed for the design of beams of ***symmetrical section*** and ***homogeneous material*** loaded with ***transverse loads*** applied in the ***plane*** of one of the ***principal axes***. For the treatment of beams of unhomogeneous material (reinforced concrete) see article on *Concrete*.

Let M = maximum external bending moment on beam in inch-pounds,
I = moment of inertia in (inches)^4 about the neutral axis lying perpendicular to the plane of the external loads. (This axis in a symmetrical beam lies at mid-height.)
c = distance in inches from neutral axis to extreme fiber, equals one-half the depth for symmetrical beams,
s = allowable value of fiber stress (working value),
Q = statical moment about the neutral axis of that portion of the cross-section lying above the neutral axis,
v = allowable intensity of longitudinal shear in lb. per sq. inch,
V = maximum external shear on beam in pounds,
b = thickness of beam at neutral axis.

Then,
$$M = \frac{sI}{c} \qquad \frac{I}{c} = \frac{M}{s},$$

and
$$v = \frac{VQ}{bI} \qquad \frac{Q}{bI} = \frac{v}{V},$$

Steel Beams. — Such beams are made in the shape of the letter I in order to obtain maximum strength for a given amount of material. They are rolled to desired shape and size while hot. For dimensions, allowable loads and properties of the standard sizes made in the United States see Table V above. In addition to the I-beams listed in the table of standard I-beams which are manufactured by all the leading structural steel makers, the Bethlehem Steel Co. manufacture certain special beams of the same general shape but either of greater depth or with wider flange, and the Carnegie Steel Co. also manufacture certain special types of beams. These beams may often be used to great advantage.

Cost of Steel Beams. — The cost of steel beams in place depends upon the cost of transportation, punching, riveting and other necessary shop work and the cost of erection and painting. The base price at the mills is quoted each week in the *Iron Age;* the figures which follow came from the issue of June 19, 1913, and are more representative of average prices than those at the date (August, 1914) of publication of this book.

I-beams under 15 in., 1.45¢ to 1.50¢ per lb.
I-beams over 15 in., 1.55¢ to 1.60¢ per lb.
H-beams over 18 in., 1.55¢ to 1.60¢ per lb.
For cutting to length, under 3 ft. to 2 ft. inclusive, 0.25¢ per lb.
For cutting to length, under 2 ft. to 1 ft. inclusive, 0.50¢ per lb.
For cutting to length, under 1 ft., 1.55¢ per lb.
No charge for cutting to lengths 3 ft. and over.

For total cost in place an estimate of 3 cents per pound is ordinarily safe.

PLATE GIRDERS; FORMULAS AND COSTS. — A typical plate girder with the various parts is shown in Fig. 8. Plate girders are used for openings

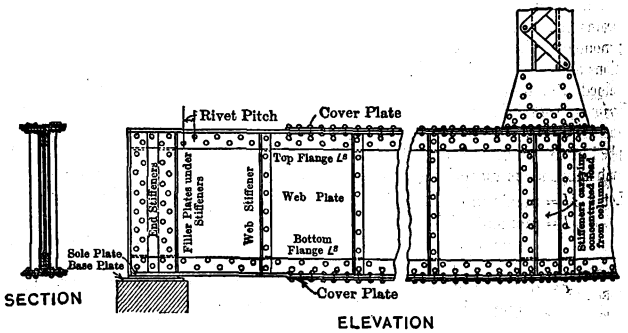


Fig. 8. Typical Plate Girder

where either the load to be carried is too heavy or the clear span too great to be spanned with safety by rolled beams. The maximum length and depth are limited by the possibility of transportation. Probably the largest girder yet made was designed for the Boston & Albany Railroad for use at Worcester, Mass. It is 122 feet 6 inches long, 10 feet 11½ inches deep, and weighs 170 tons. The ordinary and economical depth of plate girders is from ⅙ to 1/12 the span; ⅛ the span is a common value. The maximum depth is usually restricted to 10 feet 6 inches or less.

Essential Points in Plate-Girder Design are as follows:

a. The determination of area of web required to carry the maximum external shear.

b. The determination of area of flange required to resist the maximum external bending moment.

c. The determination of the flange rivet spacing necessary to transmit the flange stress from web to flange.

d. The determination of spacing and size of stiffeners required to prevent the web plate from buckling sidewise, and of size of stiffeners at all points of application of concentrated loads together with number of rivets required therein.

e. The determination of length of flange cover plates.

f. The design of splices for web, angles and cover plates.

g. The design of bearing plates to transmit end reactions into masonry.

It is impossible to give in the limited space available here the necessary formulas and data for the design of plate girders. For further information see Spofford's *Theory of Structures*, N. Y., 1911.

Design of Box Girders. — Box girders resemble plate girders, but have two or more webs as indicated in Fig. 9. They are used where conditions require a shallow girder of great strength. The rules applicable to plate girders apply in general to box girders, but owing to their shallow depth it is usually advisable to determine their strength against bending by the general beam formula $M = \frac{sI}{c}$, the moment of inertia being computed with due allowance for rivet holes (*see Spofford's Theory of Structures, N. Y., 1911, Art. 62 and 63*). The shear in a two-web girder may be considered as equally divided between the two webs, and in a three-web girder as carried half by the center web and half by the two side webs combined. The flange rivets, stiffeners, etc., should be computed on the same basis.

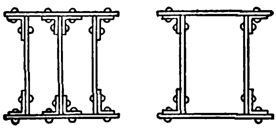


Fig. 9.

Cost of Girders. — The cost of girders is usually computed on a cent-per-pound basis. Price of girders erected in place and painted usually ranges in the Eastern states from 2½ to 4 cents per pound, depending upon the state of the market, location, amount and complexity of the work, rigidity of specifications and inspection, cost of maintaining traffic (in railroad work), etc. The price of 3½ cents per pound may be considered a fair average allowance for approximate estimates. In the West this figure should be materially increased to cover the additional freight rate from Pittsburgh or Chicago.

For basic prices of material see below under *Price of Structural Material*. For freight rates to various points from Pittsburgh, see below under *Freight Rates*.

COLUMNS; FORMULAS AND COSTS. — Steel columns may be made of single rolled sections such as I-beams or channels, or of compound sections consisting of single sections riveted together. A typical steel column is shown in Fig. 10. Single sections of steel columns should be limited in length for convenience in erection and economy in shipment to 60 feet over all or even less. Lengths up to 120 feet may, however, be shipped in one piece if necessary.

Cast-iron columns are made by pouring liquid iron into molds. They may be of various shapes, but are always of comparatively short lengths seldom exceeding 15 or 20 feet. Common shapes are shown in Fig. 11. To obtain freedom from initial stresses due to unequal rates of cooling and from stresses due to

eccentricity with centrally applied loads, the shells of hollow cast-iron columns should be of uniform thickness throughout. This result may best be secured by pouring the column in an upright position; if cast in a horizontal position the core must be carefully restrained against flotation.

Timber columns should consist of single sticks and may be either circular or rectangular in cross-section. Two lengths are sometimes connected by means of cast-iron caps which also furnish seats for beams.

Concrete columns are made by pouring concrete into wooden or metal molds; they may be made of any desired shape or length. Such columns are generally reinforced by longitudinal steel rods extending from end to end of the column and held in place at intervals by transverse steel rings. The transverse reinforcement may be made to add materially to the strength of the column by placing the rings at frequent intervals or by making it in the form of a spiral extending from end to end of the column, the column then being called a hooped column. The strengthening effect of the transverse reinforcement is due to its influence in preventing the bursting tendency of the concrete when subjected to compression.

Splices. — Steel columns are commonly spliced by bringing the abutting ends into close contact after first planing them at right angles to the axis. Splice plates are used on webs and flanges to hold the sections in line. If the column carries no bending moment the size of splice plates and number of rivets may be determined by the designer's judgment; if the column carries flexure as well as direct stress the splice must be sufficient to transmit the flexure.

Changes of cross-sections in steel columns may be accomplished by changing the thickness of the material or by adding additional material. An advantageous location of a splice is at a section where the cross-section area is to be changed. If the column carries only direct compression and is loaded at intervals throughout its length such a section should be near the point of application of one of the loads and on that side of it in which the smaller stress occurs, e.g., in a high building column the section should be changed just *above* one of the floors. For columns carrying flexure as well as direct stress the splices should be located where the bending moment is small, as for example at a point of contraflexure. Cast-iron columns should

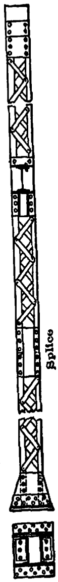


Fig. 10.

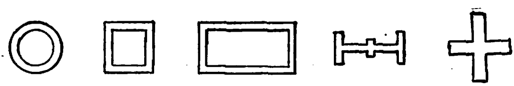


Fig. 11. Cross Sections of Typical Cast-iron Columns

not be used for positions where a considerable amount of flexure may occur or where they may be subjected to severe shock or vibration. Their use in construction is practically confined to buildings of moderate height. They may be spliced by bolting the flanges. In interior building columns the load is sometimes transferred from one section to another by means of a pintle extending between the beams which are supported on the cap of the lower column. Such a joint depends for its lateral rigidity upon the friction between the beams and column cap and can evidently transmit no bending moment.

Ratio of Unsupported Length of Column to Radius of Gyration. — The controlling factor in column formulas is the value of the ratio between the unsupported length of the column and the radius of gyration of its cross-section. If the column is of constant cross-section and restrained against lateral deflection at

both ends only, then l equals the total length of the column and r its least radius of gyration. If the column is held against lateral deflection in every direction at one or several intermediate points, the value of $\frac{l}{r}$ equals the largest value existing between any two adjoining lateral supports. If the column is held at an intermediate point in one direction only, the value of $\frac{l}{r}$ equals the maximum obtainable by using for l the length of either section, or the total length of the column, and for r in each case the corresponding least radius of gyration referred to any axis about which the column is free to bend. If the column is of variable cross-section the designer must use his judgment in determining the value of r to be used, but generally the value should be that at the middle of the unsupported length of the particular portion of the column that is under consideration.

Condition of Ends. — It was formerly assumed that the strength of columns used in practice depended very largely upon the condition of the ends. At the present time, however, no difference is made between round-ended, pin-ended and square-ended columns as used in ordinary practice. For columns of length such that Euler's formula should be applied, a distinction should be made between round-ended and square-ended columns (*see following section*). Columns with one free end should receive special consideration.

Recommended Formulas for Steel Columns.—

For Columns of Ordinary Length:

$$\frac{P}{A} = 16{,}000 - 70\frac{l}{r}, \tag{1}$$

l/r not to exceed 120 for principal members,
l/r not to exceed 150 for secondary members,
P/A not to exceed 14,000 lbs.

For meaning of symbols, see list following equation (4).

For Long Columns having values of l/r greater than allowed in formula (1), the column may fail by collapsing rather than crushing; in this case the following formulas should be used:

$$\text{Fixed Ends} \quad \frac{P}{A} = \frac{4\pi^2 E}{c}\left(\frac{r}{l}\right)^2, \tag{2}$$

$$\text{Round Ends} \quad \frac{P}{A} = \frac{\pi^2 E}{c}\left(\frac{r}{l}\right)^2, \tag{3}$$

$$\text{One Free End} \quad \frac{P}{A} = \frac{\pi^2 E}{4c}\left(\frac{r}{l}\right)^2, \tag{4}$$

where P = total allowable centrally applied load on column in pounds including proper allowance for impact,
A = minimum area of cross-section in square inches,
$\frac{l}{r}$ = maximum ratio of length to radius of gyration for any laterally unsupported section of column,
c = suitable factor of safety, usually 5 to 6,
E = modulus of elasticity in pounds per square inch.

Recommended Formulas for Cast-iron Columns. —

$$\frac{P}{A} = 6100 - 32\frac{l}{d}, \qquad \frac{l}{d} \text{ not to exceed } 40, \qquad (5)$$

where P = total allowable centrally applied load on column in pounds including proper allowance for impact,
A = minimum area of cross-section in square inches,
d = diameter of circular column or shorter side of rectangular column in inches.

Recommended Formulas for Timber Columns. —

For longleaf yellow pine, $$\frac{P}{A} = 1300\left(1 - \frac{l}{60\,d}\right), \qquad (6)$$

For shortleaf pine and spruce, $$\frac{P}{A} = 1100\left(1 - \frac{l}{60\,d}\right), \qquad (6a)$$

where P = total allowable centrally applied load on column in pounds (disregarding impact),
A = area of cross-section in square inches,
$\frac{l}{d}$ = unsupported length divided by least radius of gyration.

The values in (6) and (6a) are for railroad bridges. For highway bridges increase these values by 25 per cent. For buildings protected from the weather and reasonably free from impact, increase these values by 50 per cent. For other timbers, multiply value for longleaf yellow pine by ratio between working compressive stress of timber in question and of longleaf yellow pine. (*See also section on Unit Stresses in article on Timber.*)

Steel Column Details. — Columns composed of separate parts should be so constructed that the various parts will resist buckling as a unit; otherwise, the strength of the column will be no greater than that of the individual pieces combined. As an illustration of this, the column shown in Fig. 10 may be considered. If the two channels are not connected, the strength of this column will be only double that of the two separate channels, and the radius of gyration will have the low value corresponding to a single channel. Similarly a wooden column composed of thin planks bolted loosely together has little if any greater strength than the same number of separate planks standing alone. To fasten together the ribs of a steel column, plates or diaphragms may be used throughout its length, the plates forming a part of the main or stress bearing cross-section, or plates and lattice bars may be used as shown in Fig. 10. The width of the lattice bars is usually taken as the minimum value allowable for the rivets used. For columns consisting of two channels, the minimum sizes of lattice bars given in the accompanying table may be used. In no case should single lattice bars have a thickness less than 1/40 the distance between rivets connecting them to the channels; but for double lattice bars riveted at their intersection the thickness may be 1/60 of the distance.

Size of channel, in.	Lattice bar section, in.
8 and 9	2¼ by 5/16
10	2¼ by 5/16
12	2¼ by 5/16
15	2½ by 5/16

For columns of unusual size, the lattice bars should be carefully designed. (*See Spofford's Theory of Structures, N. Y., 1911, Art. 145.*)

Rivet Pitch in Built Steel Columns. — For columns carrying direct stress only, the rivets should not be farther apart than 6 inches or 16 times the thinnest plate connected and at each end or at each point of application of concentrated loads they should not be farther apart than four diameters of the

rivet. See also sections below on *Design of Tension Members* and *Design of Riveted Joints.*

Eccentrically Loaded Columns. — If the resultant force acting at any cross-section of a column is not applied at its center of gravity the column is said to be eccentrically loaded. The effect of an eccentric load is to subject the column to a combination of direct stress and flexure, and thereby to increase the maximum fiber stress over what it would otherwise be. A similar condition arises if the resultant force on the cross-section is the resultant of an axial force acting at the center of gravity of the section and a couple causing flexure at the section. The maximum fiber stress at such a section may be determined by the following closely approximate formula for the usual case where the resultant force acts in the plane of one of the principal axes and the neutral axis for flexure coincides with the other principal axis.

$$s = \frac{P}{A} + \frac{My}{I - \frac{Pl^2}{10E}}, \tag{7}$$

where s = maximum fiber stress at any section in pounds per square inch,
P = resultant axial force on section in pounds,
l = unsupported length of column in inches,
A = area of cross-section in square inches,
M = bending moment on section in inch-pounds,
I = moment of inertia of cross-section about neutral axis in (inches)^4,
E = modulus of elasticity in pounds = 29,000,000 for steel,
y = distance in inches from neutral axis to most remote fiber. (Neutral axis as used in this formula is the neutral axis for flexure.)

Flexure in a Building Column Carrying an Eccentric Load. — The case of a building column supporting a track for a traveling crane is a common one. The curve of moments for such a case is shown by the dashed line in Fig. 12 in which P is the applied load acting on the track at a distance x from the center of the column. The maximum bending moment always occurs at the load and depends upon the height of application of the latter. Its maximum value is Px and occurs when the load is at the top of the column.

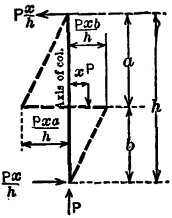


Fig. 12.

Proportions of Columns. —The following specifications should be closely followed:

In compression members the metal shall be concentrated as much as possible in webs and flanges. The thickness of each web shall be not less than one-thirtieth of the distance between its connections to the flanges. Cover plates shall have a thickness not less than one-fortieth of the distance between rivet lines.

The open sides of compression members shall be provided with lattice bars and shall have tie plates as near each end as practicable. Tie plates shall be provided at intermediate points where the lattice is interrupted. In main members the end tie plates shall have a length not less than the distance between the lines of rivets connecting them to the flanges, and intermediate ones not less than one-half this distance. Their thickness shall not be less than one-fiftieth of the same distance.

Abutting joints in compression members when faced for bearing shall be spliced on four sides sufficiently to hold the connecting members accurately in place. All other joints in riveted work, whether in tension or compression, shall be fully spliced.

Where splice plates are not in direct contact with the parts which they con-

nect, rivets shall be used on each side of the joint in excess of the number theoretically required to the extent of one-third of the number for each intervening plate.

Column Caps, Bases and Brackets. — The proper application of the applied loads to the column, and the distribution of the column load over the footing, is a matter of great importance. The three following rules should be applied in designing column caps:

a. Provide sufficient bearing area so that the crushing strength of the column, or of the loading beam, girder or column will not be exceeded.

b. Arrange connections to eliminate eccentric application of the load, or if this is impossible to reduce the eccentricity to the smallest possible amount.

c. Arrange connections so that the stress-bearing portions of the column shall be directly under the stress-transmitting portion of the loading member, e.g., if a column receives load from a girder, the outstanding legs of the end-stiffeners on the girder should be located directly over the webs of the columns.

Typical column bases are shown in Fig. 13. The same general rules should be observed in the design of column bases as in the design of column caps. The advantage of a separate base is that it can be set up on the foundation and leveled to receive the column with less difficulty than would be the case with the finished column. Such bases should always be planed on top and the base of the column should also be planed. Steel columns to which cap and base plates are shop riveted should be planed at each end before these plates are riveted on, and the cap and base plates are also generally planed on top and bottom respectively.

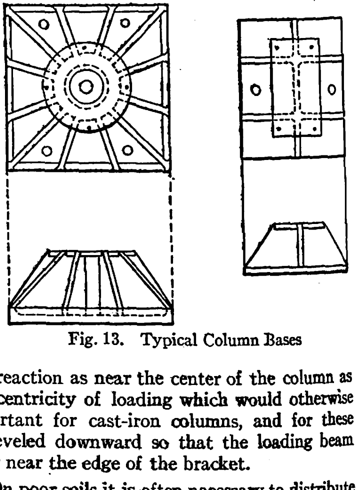


Fig. 13. Typical Column Bases

It is important that brackets for connections of girders to columns should always be designed so as to throw the resultant beam reaction as near the center of the column as possible in order to reduce the eccentricity of loading which would otherwise occur. This is particularly important for cast-iron columns, and for these the brackets should be slightly beveled downward so that the loading beam or girder cannot be supported at or near the edge of the bracket.

I-Beam Column Footings. — On poor soils it is often necessary to distribute heavy column loads over an area so large as to require a special footing which may be economically constructed of steel beams, in concrete. The beams should not be closer together than 3 inches to permit concrete to be easily placed between and around them. The method of design of an I-beam footing for the ordinary case, that of a column carrying direct load only, is as follows: (1) Determine the number of square feet required in the footing; (2) assume the number of tiers of I-beams; (3) assume the entire load above any given layer of beams (weight of over-lying footing courses and column load) to be distributed equally over all the beams in that layer and assume that each beam is uniformly loaded; (4) determine the maximum moment on the beam, which will occur at the center and equal approximately

$$M = \frac{W}{8n}(l - l_1) \tag{8}$$

where W = load on layer of beams under consideration, n = number of beams, h = portion of beam loaded from above, in feet, and l = length of beam loaded from below, in feet; (5) determine in the usual manner the size of beam required to carry this bending moment; (6) test the web thicknesses of the beams selected to see if they are sufficient to transmit the applied load without buckling, applying formula (1).

Pile Footings. — If the ground is soft, piles may be necessary to provide sufficient bearing area. For the allowable bearing value for these see Baker's *A Treatise on Masonry Construction*, N. Y., 1909.

Cost of Columns and Footings. — Steel columns cost about the same as steel girders, values for which are given in the section on *Cost of Girders*, above. In determining the weight of columns it should be remembered that the details will add materially to the weight of the stress-bearing section, especially in the case of latticed columns, where the excess may amount to as much as 50 per cent. For the cost of footings, the I-beams may be computed on a pound price basis from the weekly quotations given in *The Iron Age* with a reasonable allowance for transportation and delivery, or more accurate figures can be obtained from a local dealer. For cost of cast-iron columns see section on *Cost* in article on *Iron, Pig and Cast*. For the cost of concrete and excavation, see articles on these respective subjects.

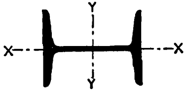
Tables for Steel Columns. — The allowable load for a number of simple steel columns and the important properties of the columns are given in Tables VII to X. To determine the allowable load for built-up columns not given in the table the radius of gyration may be interpolated for columns having the same general dimensions and the unit stress computed by substitution in the column formula. The areas and other properties of the simple structural shapes, such as beams, angle irons, and channels, are given in Tables II to VI above.

These tables are all based on the formula

$$\frac{P}{A} = 16{,}000 - 70\frac{l}{\rho}$$

with a maximum value of 14,000 and are calculated for Cambria Steel Co.'s sections. The load is assumed to be applied at the center of gravity of the column and to act parallel to its axis.

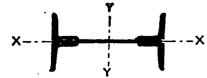
TABLE VII.—STEEL I-BEAM COLUMNS



ρ_x = radius of gyration about axis X-X
ρ_y = " " " " " Y-Y

Depth of beam, in.	Width of flange, in.	Weight per foot, lb.	Thickness of web, in.	Area of column, sq. in.	Least radius of gyration ρ_x, in.	ρ_y, in.	Max. allowable load P in thousands of pounds for various values of $l \div \rho$. To determine strength of column use maximum possible value of $l \div \rho$. If column is unsupported laterally use for ρ the value ρ_x.											
							$\frac{l}{\rho}=10$	20	30	40	50	60	70	80	90	100	110	120
5	3.00	9.75	0.21	2.87	0.65	2.05	40.2	40.2	39.9	37.9	35.9	33.9	31.9	29.8	27.8	25.8	23.8	21.8
6	3.33	12.25	0.23	3.61	0.72	2.46	50.5	50.5	50.2	47.7	45.1	42.6	40.1	37.5	35.0	32.5	30.0	27.4
7	3.66	15.00	0.25	4.42	0.78	2.86	61.9	61.9	61.4	58.3	55.3	52.2	49.1	46.0	42.9	39.8	36.7	33.6
8	4.00	18.00	0.27	5.33	0.84	3.27	74.6	74.6	74.1	70.4	66.6	62.9	59.2	55.4	51.7	48.0	44.2	40.5
9	4.33	21.00	0.29	6.31	0.90	3.67	88.3	88.3	87.7	83.3	78.9	74.5	70.0	65.6	61.2	56.8	52.4	48.0
10	4.66	25.00	0.31	7.37	0.97	4.07	103.2	103.2	102.4	97.3	92.1	87.0	81.8	76.6	71.5	66.3	61.2	56.0
10	5.10	40.00	0.75	11.76	0.90	3.67	164.6	164.6	163.5	155.2	147.0	138.8	130.5	122.3	114.1	105.8	97.6	89.4
12	5.00	31.50	0.35	9.26	1.01	4.83	129.6	129.6	128.7	122.2	115.8	109.3	102.8	96.3	89.8	83.3	76.9	70.4
12	5.21	40.00	0.56	11.76	0.96	4.57	164.6	164.6	163.5	155.2	147.0	138.8	130.5	122.3	114.1	105.8	97.6	89.4
15	5.50	42.00	0.41	12.48	1.08	5.95	174.7	174.7	173.5	164.7	156.0	147.3	138.5	129.8	121.1	112.3	103.6	94.8
15	5.84	60.00	0.75	17.65	1.01	5.52	247.1	247.1	245.3	233.0	220.6	208.3	195.9	183.6	171.2	158.8	146.5	134.1
18	6.00	55.00	0.46	15.93	1.15	7.07	223.0	223.0	221.4	210.3	199.1	188.0	176.8	165.7	154.5	143.4	132.2	121.1
18	6.26	70.00	0.72	20.59	1.09	6.69	288.3	288.3	286.2	271.8	257.4	243.0	228.5	214.1	199.7	185.3	170.9	156.5
20	6.25	65.00	0.50	19.08	1.21	7.83	267.1	267.1	265.2	251.9	238.5	225.1	211.8	198.4	185.1	171.7	158.4	145.0
20	6.40	75.00	0.65	22.06	1.17	7.58	308.8	308.8	306.6	291.2	275.8	260.3	244.9	229.4	214.0	198.5	183.1	167.7
24	7.00	80.00	0.50	23.32	1.36	9.46	326.5	326.5	324.2	307.8	291.5	275.2	258.9	242.5	226.2	209.9	193.6	177.2
24	7.25	100.00	0.75	29.41	1.28	8.99	411.7	411.7	408.8	388.2	367.6	347.0	326.5	305.9	285.3	264.7	244.1	223.5

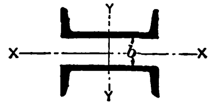
TABLE VIII. — STEEL PLATE AND ANGLE COLUMNS



ρ_x = radius of gyration about axis X-X
ρ_y = " " " " " Y-Y

Depth base to base of angles, in.	Size of web, in.	Size of angle, in.	Width of flange, in.	Area of column, sq. in.	Weight per foot of column exclusive of details, lb.	Least radius of gyration ρ_x, in.	ρ_y, in.	Max. allowable load P in thousands of pounds for various values of $l \div \rho$. To determine strength of column use maximum possible value of $l \div \rho$. If column is unsupported laterally use for ρ the value ρ_x.											
								$\frac{l}{\rho}$ = 10	20	30	40	50	60	70	80	90	100	110	120
6½	6 by ¼	3 by 2½ by ¼	6¼	6.79	23.1	1.24	2.41	95.1	95.1	94.4	89.6	84.9	80.1	75.4	70.6	65.9	61.1	56.4	51.6
6½	6 by ⅝	3 by 2½ by ⅝	6⅝	15.94	54.4	1.43	2.29	223.2	223.2	221.6	210.4	199.3	188.1	176.9	165.8	154.6	143.5	132.3	121.1
7½	7 by ¼	3½ by 2½ by ¼	7¼	7.50	25.6	1.46	2.88	105.0	105.0	104.3	99.0	93.8	88.5	83.3	78.0	72.8	67.5	62.3	57.0
7½	7 by ⅝	3½ by 2½ by ⅝	7⅝	17.82	60.9	1.65	2.76	249.5	249.5	247.7	235.2	222.8	210.3	197.8	185.3	172.8	160.4	147.9	135.4
8½	8 by ¼	3 by 2½ by ¼	6¼	7.29	24.8	1.19	3.25	102.1	102.1	101.3	96.2	91.1	86.0	80.9	75.8	70.7	65.6	60.5	55.4
8½	8 by ⅝	4 by 3 by ⅝	8⅝	20.96	71.4	1.82	3.14	293.4	293.4	291.3	276.7	262.0	247.4	232.7	218.0	203.3	188.6	173.9	159.2
10½	10 by ¼	3 by 2½ by ¼	6¼	7.79	26.5	1.16	4.07	109.1	109.1	108.3	102.8	97.4	91.9	86.5	81.0	75.6	70.1	64.7	59.2
10½	10 by ⅝	4 by 3 by ⅝	8⅝	22.21	75.7	1.77	3.98	310.9	310.9	308.7	293.2	277.6	262.1	246.6	231.0	215.4	199.9	184.3	168.8
12½	12 by ¼	3 by 2½ by ¼	6¼	8.29	28.2	1.12	4.87	116.1	116.1	115.2	109.4	103.6	97.8	92.0	86.2	80.4	74.6	68.8	63.0
12½	12 by ⅝	6 by 3½ by ⅝	12⅝	29.69	101.1	2.68	4.93	415.7	415.7	412.7	391.9	371.1	350.4	329.6	308.8	288.0	267.2	246.4	225.6
14½	14 by 7/16	7 by 3½ by 7/16	14 7/16	23.76	80.8	3.05	5.92	332.6	332.6	330.2	313.6	297.0	280.4	263.8	247.1	230.5	213.8	197.2	180.6
14½	14 by ⅝	7 by 3½ by ⅝	14⅝	33.48	113.7	3.13	5.85	468.7	468.7	465.4	442.0	418.5	395.1	371.6	348.2	324.7	301.3	277.9	254.4
16½	16 by 5/16	5 by 3½ by 5/16	10 5/16	15.23	51.8	1.94	6.59	213.2	213.2	211.7	201.0	190.4	179.7	169.1	158.4	147.7	137.1	126.4	115.7
16½	16 by ⅝	5 by 3½ by ⅝	10⅝	29.73	101.2	2.08	6.48	416.2	416.2	413.3	392.5	371.6	350.8	330.0	309.2	288.4	267.6	246.8	225.9
16½	16 by 7/16	7 by 3½ by 7/16	14 7/16	24.65	83.8	3.00	6.75	345.1	345.1	342.7	325.4	308.1	290.9	273.6	256.4	239.1	221.8	204.6	187.3
16½	16 by ⅝	7 by 3½ by ⅝	14⅝	34.72	118.0	3.08	6.69	486.1	486.1	482.6	458.3	434.0	409.7	385.4	361.1	336.8	312.5	288.2	263.9

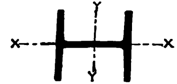
TABLE IX.—STEEL LATTICED CHANNEL COLUMNS



ρ_y = radius of gyration about axis Y-Y
b = minimum allowable distance apart of channels

Depth of channel, in.	Weight per foot of each channel, lb.	Width of flange, in.	Thickness of web, in.	Area of section, sq. in.	Weight per foot exclusive of details, lb.	Radius of gyration ρ_y, in.	b in.	Max. allowable load P in thousands of pounds for various values of $l \div \rho$. To determine strength of column use maximum possible value of $l \div \rho$. If column is unsupported laterally use for ρ the value ρ_y. $\frac{l}{\rho}=10$	20	30	40	50	60	70	80	90	100	110	120
6	8.0	1.92	0.20	4.76	16.00	2.34	3.52	66.6	66.6	66.2	62.8	59.5	56.2	52.8	49.5	46.2	42.8	39.5	36.2
6	15.5	2.28	0.56	9.12	31.00	2.07	2.90	127.7	127.7	126.8	120.4	114.0	107.6	101.2	94.8	88.5	82.1	75.7	69.3
7	9.75	2.09	0.21	5.70	19.50	2.72	4.22	79.8	79.8	79.2	75.2	71.3	67.3	63.3	59.3	55.3	51.3	47.3	43.3
7	19.75	2.51	0.63	11.62	39.50	2.39	3.48	162.7	162.7	161.5	153.4	145.3	137.1	129.0	120.8	112.7	104.6	96.4	88.3
8	11.25	2.26	0.22	6.70	22.50	3.10	4.90	93.8	93.8	93.1	88.4	83.8	79.1	74.4	69.7	65.0	60.3	55.6	50.9
8	21.25	2.62	0.58	12.50	42.50	2.76	4.20	175.0	175.0	173.8	165.0	156.3	147.5	138.8	130.0	121.3	112.5	103.8	95.0
9	13.25	2.43	0.23	7.78	26.50	3.49	5.64	108.9	108.9	108.1	102.7	97.3	91.8	86.4	80.9	75.5	70.0	64.6	59.1
9	25.0	2.81	0.61	14.70	50.00	3.10	4.82	205.8	205.8	204.3	194.0	183.8	173.5	163.2	152.9	142.6	132.3	122.0	111.7
10	15.0	2.60	0.24	8.92	30.00	3.87	6.32	124.9	124.9	124.0	117.7	111.5	105.3	99.0	92.8	86.5	80.3	74.0	67.8
10	25.0	2.89	0.53	14.70	50.00	3.52	5.66	205.8	205.8	204.3	194.0	183.8	173.5	163.2	152.9	142.6	132.3	122.0	111.7
10	35.0	3.18	0.82	20.58	70.00	3.35	5.18	288.1	288.1	286.1	271.7	257.3	242.8	228.4	214.0	199.6	185.2	170.8	156.4
12	20.5	2.94	0.28	12.06	41.00	4.61	7.68	168.8	168.8	167.6	159.2	150.8	142.3	133.9	125.4	117.0	108.5	100.1	91.7
12	30.0	3.17	0.51	17.64	60.00	4.28	7.06	247.0	247.0	245.2	232.9	220.5	208.2	195.8	183.5	171.1	158.8	146.4	134.1
12	40.0	3.42	0.76	23.52	80.00	4.09	6.60	329.3	329.3	326.9	310.5	294.0	277.5	261.1	244.6	228.1	211.7	195.2	178.8
15	33.0	3.40	0.40	19.80	66.00	5.62	9.52	277.2	277.2	275.2	261.4	247.5	233.6	219.8	205.9	192.1	178.2	164.3	150.5
15	40.0	3.52	0.52	23.52	80.00	5.44	9.16	329.3	329.3	326.9	310.5	294.0	277.5	261.1	244.6	228.1	211.7	195.2	178.8
15	55.0	3.82	0.82	32.36	110.00	5.16	8.54	453.1	453.1	449.8	427.2	404.5	381.9	359.2	336.5	313.9	291.2	268.6	245.9

TABLE X. — STEEL H-COLUMNS



ρ_x = radius of gyration about axis X-X
ρ_y = " " " " " Y-Y

Depth of column, in.	Width of flange, in.	Weight per foot, lb.	Thickness of web, in.	Area of section sq. in.	Least radius of gyration, ρ_x in.	ρ_y, in.	Maximum allowable load P in thousands of pounds for various values of $l \div \rho$. To determine strength of column use maximum possible value of $l \div \rho$. If column is unsupported laterally use for ρ the value ρ_x.											
							$\frac{l}{\rho}$ = 10	20	30	40	50	60	70	80	90	100	110	120
8	8.00	34.5	0.31	10.17	2.01	3.46	142.4	142.4	141.4	134.2	127.1	120.0	112.9	105.8	98.6	91.5	84.4	77.3
8½	8.16	53.0	0.47	15.53	2.07	3.57	217.4	217.4	215.9	205.0	194.1	183.3	172.4	161.5	150.6	139.8	128.9	118.0
9	8.32	71.5	0.63	21.05	2.12	3.68	294.7	294.7	292.6	277.9	263.1	248.4	233.7	218.9	204.2	189.4	174.7	160.0
9½	8.47	90.5	0.78	26.64	2.17	3.80	373.0	373.0	370.3	351.7	333.0	314.4	295.7	277.1	258.4	239.8	221.1	202.5
10	10.00	54.0	0.39	15.91	2.51	4.32	222.7	222.7	221.1	210.0	198.9	187.7	176.6	165.5	154.3	143.2	132.1	120.9
10½	10.16	77.0	0.55	22.59	2.57	4.43	316.3	316.3	314.0	298.2	282.4	266.6	250.7	234.9	219.1	203.3	187.5	171.7
11	10.31	99.5	0.70	29.32	2.62	4.55	410.5	410.5	407.6	387.0	366.5	346.0	325.5	304.9	284.4	263.9	243.4	222.8
11½	10.47	123.5	0.86	36.32	2.67	4.67	508.5	508.5	504.8	479.4	454.0	428.6	403.2	377.7	352.3	326.9	301.4	276.0
12	12.00	78.0	0.47	22.94	3.01	5.18	321.2	321.2	318.9	302.8	286.8	270.7	254.6	238.6	222.5	206.5	190.4	174.3
12½	12.16	105.0	0.63	30.94	3.07	5.30	433.2	433.2	430.1	408.4	386.8	365.1	343.4	321.8	300.1	278.4	256.8	235.1
13	12.31	132.5	0.78	38.97	3.13	5.41	545.6	545.6	541.7	514.4	487.1	459.9	432.6	405.3	378.0	350.7	323.5	296.2
13½	12.47	161.0	0.94	47.28	3.18	5.53	661.9	661.9	657.2	624.1	591.0	557.9	524.8	491.7	458.6	425.5	392.4	359.3
14	14.00	99.0	0.51	29.06	3.50	6.07	406.8	406.8	403.9	383.6	363.3	342.9	322.6	302.2	281.9	261.5	241.2	220.9
14½	14.16	130.5	0.67	38.38	3.56	6.18	537.3	537.3	533.5	506.6	479.8	452.9	426.0	399.2	372.3	345.4	318.6	291.7
15	14.31	162.0	0.82	47.71	3.62	6.30	667.9	667.9	663.2	629.8	596.4	563.0	529.6	496.2	462.8	429.4	396.0	362.6
15½	14.47	195.0	0.98	57.35	3.67	6.41	802.9	802.9	797.2	757.0	716.9	676.7	636.6	596.4	556.3	516.2	476.0	435.9
16	14.62	227.5	1.13	66.98	3.73	6.53	937.7	937.7	931.0	884.1	837.2	790.4	743.5	696.6	649.7	602.8	555.9	509.0
16½	14.78	261.5	1.29	76.93	3.77	6.65	1077.0	1077.0	1069.3	1015.5	961.6	907.8	853.9	800.1	746.2	692.4	638.5	584.7

TRUSSES, DESIGN OF. — All trusses may be divided into two general classes based upon the methods necessary to determine the bar stresses; if these stresses can be determined by the application of the three equations of equilibrium (*see paragraph on Equations of Equilibrium, above*), they are statically determined; otherwise they are statically undetermined. It should be noted that a truss may be statically undetermined with respect to the outer forces, i.e., the reactions cannot be determined by statics; or it may be statically undetermined with respect to the inner forces, i.e., the bar stresses cannot be determined by statics.

Trusses that are statically indeterminate through having superfluous bars or reactions are frequently constructed because of their supposed rigidity and economy, but it is doubtful if such trusses are desirable.

Statically determined trusses may be distinguished from statically undetermined trusses by comparing the number of unknown reaction components plus

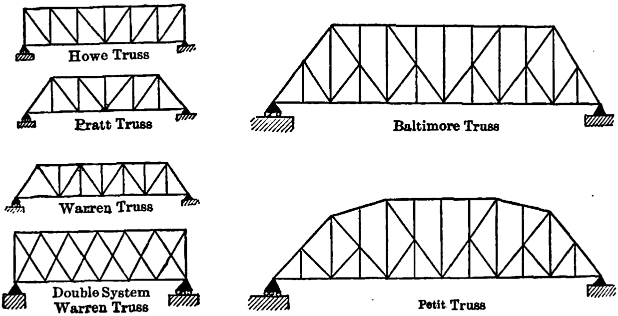


Fig. 14. Bridge Trusses

the number of bars with the number of joints. In general it may be stated that any planar truss supported at two points will be statically determined provided the number of bars equals $(2n-3)$, where n is the number of joints. If the number of unknown bars and reaction components combined exceed twice the number of joints the truss is statically indeterminate but may usually be computed by methods based upon its elasticity. If the number of unknowns is less than twice the number of joints the truss is unstable and may collapse; such structures should never be constructed.

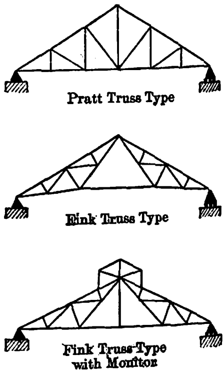


Fig. 15. Roof Trusses

The preceding test should not be used alone to determine whether a truss is or is not stable. The framework should also be so put together as to make it consist of a series of triangles.

Common Types. — All the trusses shown in Figs. 14 and 15, except the double-system Warren truss and the roof truss with monitor, are statically determined; the former may be made statically determined by omitting one of the bars, say a diagonal; the latter by omitting one properly chosen bar.

Theory of Trusses. — The common theory of trusses is based upon the following assumption:

That the members are connected at the intersections of their center of gravity lines by frictionless pins, and that in consequence the stresses in the various members are all direct stresses. The assumption of frictionless pins is approximate particularly in the case of riveted trusses; however, the results obtained by this method are sufficiently accurate for trusses of ordinary proportions and are in common use. Great care should be used in design to insure that center of gravity lines meet at the joints, otherwise the connection will be eccentric and the truss exposed to bending as well as direct stress.

The methods of computing stresses in ordinary trusses involve only the correct application of the three equations of statics (*see section on Equations of Equilibrium, above*). Three methods are in common use: viz., method of joints, method of moments, and method of shear. All of these are methods of sections and are applied by passing a section through the bar under consideration, cutting the truss apart and applying the equations of equilibrium to the outer forces on one of these sections and the stress in the bar under consideration. The first two of these methods are described below.

Method of Joints. — The method of joints is the most general of the three methods of computing the stresses in trusses; it may be applied either analytically or graphically.

Analytical Method. — The mode of procedure for this method is as follows: (1) Computation of reactions. (2) Selection of a joint at which only two bars meet. (3) Application of the two equations of equilibrium, $\Sigma H = O$ and $\Sigma V = O$, to the outer forces and unknown bar stresses acting at this point. The bar stresses should be assumed as tension, that is, as acting away from the joint; a negative result in any case would indicate compression. (4) Consideration of any other joint at which only two unknown bars meet and determination of the stresses in these bars in the same manner, treating known bar stresses as outer forces.

Graphical Method. — This method consists of drawing polygons of forces for each joint in succession. It may be employed readily by draftsmen and others who are not familiar with truss theories. It is also self-checking and reasonably rapid. It is less accurate than the analytical method, but if the diagrams are carefully constructed it is accurate enough for ordinary structures.

The mode of procedure is: (1) A sketch of the structure is drawn to any suitable scale and on it are shown all the outer forces including reactions. (2) All the forces and bars are designated by letters so located that each force and each bar will lie between two letters and only two, as illustrated by Fig. 16. (3) A polygon of outer forces is drawn. This should be drawn to a scale of sufficient size to give the desired accuracy. The forces should be plotted in the order in which they are reached by going around the figure in a clockwise direction, and should be lettered at the ends by the letters in the order obtained by this clockwise rotation. This polygon should close if the reactions have been determined

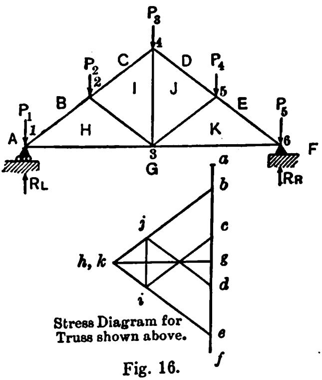


Fig. 16.

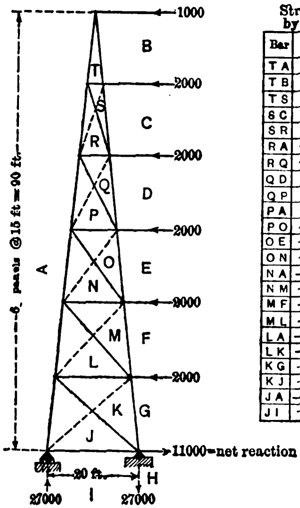
correctly. (4) A triangle of forces for each joint is drawn, beginning at any joint where an outer force and two bars only meet, and proceeding thence, joint by joint, selecting the joints in such an order that at no joint will there be more than two undetermined forces to consider. The sides of these triangles representing the outer forces are the sides of the force polygon. The sides representing bar stresses should be lettered at the ends by the letters obtained by going around the joints in a clockwise direction. The diagram thus drawn should form a closed figure. (5) The magnitude and character of the bar stresses are determined from the diagram. The magnitude of the stress in any member equals the length of the line of the diagram parallel to the bar in question measured to the scale of the force polygon; its character is determined by the order in which the letters are reached in going about any joint in a clockwise direction. For example, to determine the character of the stress in bar *CI* of Fig. 16, note that *ci* in the stress diagram acts downward to the left, as determined by the order in which the letters are reached in going around joint 2; hence the stress in *CI* also acts downward to the left, or toward the joint, since the bar is above the joint, and is therefore compression. A similar result is obtained by considering joint 4. For this joint clockwise reading gives the designation of the bar as *IC*, and *ic* in the stress diagram acts upward to the right, that is toward joint 4, since the bar is below this joint.

Solutions by Graphical Method. — Fig. 16 shows a small truss drawn to scale and with all the outer forces represented in direction and point of application. The force polygon is *abcdefga;* this is a straight line, since all the forces are vertical. In it $ab = P_1$, $bc = P_2$, etc. The reactions R_L and R_R are represented by *ga* and *fg*. The triangle of forces for joint 1 is *gabhg* (*gb* is the resultant of R_L and P_1). Reading around joint 1 in a clockwise direction shows that the magnitude and direction of the stress in bar *BH* is determined by the length and direction of *bh* in the force polygon; as this acts downward to the left, it acts toward joint 1 and hence is compression. In the same manner the stress in *HG* is found to be tension. The stresses in the other bars may be determined similarly.

Fig. 17 shows a simple tower with wind forces acting on one side only. Ordinarily the wind forces on a tower would be equally divided between the two sides but they are all taken on one side here in order to more clearly illustrate the method. The only difference in the result would occur in the stresses in the horizontal members. In this case the diagonals in all panels are assumed to be tension members, only one of which in any one panel is in action under the forces shown. Inspection shows that in each case the bar shown dotted should be considered as out of action. The stress diagram and the scaled values of stresses in the bars are also shown in the figure.

Method of Moments. — This method of finding truss stresses is based upon the application of the equation $\Sigma M = 0$. It is very useful for determining stresses in individual bars of many trusses, but is not so general as the method of joints and is frequently inapplicable to some bars even in the simplest trusses. Like the method of joints, it is also a method of sections, the truss being considered as divided into two portions and the equilibrium of one of these portions under the influence of the outer forces and the stress in the cut bar being considered. The method can be used to determine the stress in a given bar when all the other bars cut by the section, or their prolongations, meet at a point not on the line of action of the given bar. This point should be selected as the origin of moments.

Design of Tension Members. — The design of tension members involves little more than the selection of bars with sufficient net area to carry the total stress without exceeding the allowable unit stress. Steel or iron tension



Stresses by scale

Bar	Stress pounds
T A	− 4500
T B	+ 4500
T S	− 2000
S C	+ 4500
S R	+ 4750
R A	− 9000
R Q	− 3000
Q D	+ 9000
Q P	+ 5000
P A	−13500
P O	− 4000
O E	+13500
O N	+ 5700
N A	−18000
N M	− 5000
M F	+18000
M L	+ 6500
L A	− 22500
L K	− 6200
K G	+22500
K J	+ 7200
J A	−27000
J I	+ 3000

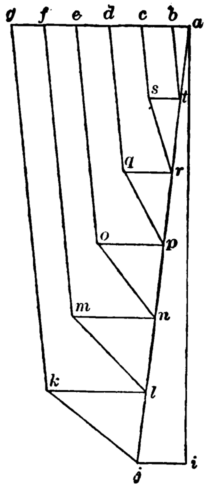


Fig. 17.

members may be divided into two general types: viz., solid bars rectangular or circular in cross-section, and built-up members composed of structural shapes riveted together. Solid bars are used generally in pin trusses for diagonals and bottom chord members, and in Howe trusses for verticals. Built-up members are generally employed for tension members in riveted trusses and for the end hangers in pin-trusses.

Solid Tension Pieces. — The eye bar shown in Fig. 18 is commonly used for the solid bar type of tension members. Such bars are made by most of the large steel manufacturers and are fully described in their handbooks. The heads of these bars are designed so that the bar if tested to destruction will fail in the body rather than in the head, and the engineer should specify that full-sized tests should give this result and not attempt to proportion the heads. A good rule to observe in selecting bars is to keep the thickness between

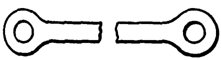


Fig. 18.

Fig. 19.

one-sixth and one-third the width. Eye-bars are generally used in pairs, since an odd number of bars would give a poor arrangement on the pin. For counters adjustable eye-bars, such as those shown in Fig. 19, may be used, the two parts being connected by a turnbuckle or sleeve nut; iron rods with loops formed by welding may be used if the stresses are small. In proportioning adjustable members allowance must be made for the decrease in section due to the screw threads. It is usually advisable to upset the screw end, so as to give sufficient area at the root of the thread to make the bar as strong there as elsewhere. For short rods, however, the labor cost involved in this process may be greater than the saving of material would warrant.

Riveted Tension Bars may be made of various sections such as channels, plates and angles, etc. Although these members do not need latticing or tie-plates to keep the separate parts from buckling, as in the case of columns, some connection between them should be used to make the different parts act together. The design of these details must be left to the judgment of the engineer.

Design of Riveted Joints.— A riveted connection may fail in one of the following ways: (*a*) by the shearing of the rivets, (*b*) by the crushing of the rivets or of one of the pieces upon which they bear, (*c*) by the tearing of the rivets through one of the connected pieces.

Shearing Strength of Rivet. — Under (*a*) it should be noted that the allowable shearing value of the rivet may be found by multiplying its cross-section area by the allowable shearing stress per square inch, and that the area of a ⅞-inch rivet is 0.60 square inch, and of a ¾-inch rivet 0.44 square inch. In designing rivets to resist shear the plane upon which the maximum shear occurs must always be determined. If the maximum shear be equally distributed over two planes the rivet is said to be in *double shear.*

Bearing Strength of Rivet. — The permissible bearing, or crushing strength of a rivet against a given plate is determined by multiplying the allowable bearing strength per square inch by the diameter of the rivet and the thickness of the plate in question.

Distance of Rivet from Edge of Member. — To prevent the failure noted under (*c*) the following empirical rule may be used: rivets may not be spaced closer than three times the diameter, and the distance of a rivet from the edge or end of a piece may not be less than 1¼ inches for a ⅞-inch rivet if the edge in question be rolled or planed, or 1½ inches if it be sheared, though where possible this distance should be at least twice the diameter of the rivet. For other sizes of rivets proportional allowances should be made. These values refer to the center of the rivet in each case.

Riveted Joints Carrying Torsion. — For cases where torsion as well as direct stress has to be transferred by rivets, as is sometimes the case where brackets carrying cranes are riveted to the sides of columns or where girders are riveted to columns, the conditions shown by Figs. 20 and 21 may occur. For the case shown in Fig. 20 the rivets carry torsion only, and each rivet may be assumed to offer a resistance to torsion proportional to its distance from the center of gravity of the group. The resistance to torsion of such a group is

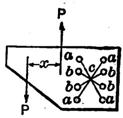


Fig. 20.

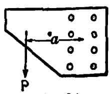


Fig. 21.

$$R = \frac{rI}{d},$$

where r = the allowable working value of the most stressed rivet in pounds,
I = summation of the squares of the distances in inches from the center of gravity of the group of rivets to each rivet,
d = distance in inches from center of gravity of group of rivets to the most stressed rivet,
R = resistance to torsion of the group of rivets in pounds.

In applying the formula to the case shown in Fig. 20 $d = ac$ and the stress in the rivets at a is r. For the case shown in Fig. 21 this method must be modified to allow for the effect of the vertical load. To make this correction it is only necessary to determine the allowable resistance to torsion consistent with the rivet also carrying its share of the vertical load.

CONTINUOUS GIRDERS, BEAMS, SLABS AND TRUSSES. — These are structures which are continuous over several points of support. They are statically indeterminate (*see section on Trusses, above*) with respect to the outer forces and hence the moments and shears should be determined by the "three moment equation" (*see above*). Continuous girders are commonly used in reinforced concrete construction, in which case the continuity is allowed for by the following simple rules recommended by the Joint Committee on Concrete and Reinforced Concrete. (*See article on Concrete.*)

(*a*) "That for floor slabs the bending moments at center and at support be taken as $\frac{wl^2}{12}$ for both dead and live loads, where w represents the load per linear foot and l the span length.

(*b*) "That for beams the bending moment at center and at supports for interior spans be taken as $\frac{wl^2}{12}$, and for end spans as $\frac{wl^2}{10}$ at center and at adjoining support, for both dead and live loads.

(*c*) "In the case of beams and slabs continuous for two spans only, the bending moment at the central support should be taken as $\frac{wl^2}{8}$ and near the middle of the span as $\frac{wl^2}{10}$.

(*d*) "At the ends of continuous beams the amount of negative moment which will be developed will depend on the condition of restraint or fixedness, and this will depend on the form of construction used. There will usually be some restraint, and there is likely to be considerable. Provision should be made for the negative bending moment, but, as its amount will depend on the form of construction, the coefficient cannot be specified here, and must be left to the judgment of the designer."

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[C. M. Spofford.]

SUBSTATIONS, LIGHTING. — (*See also Batteries, Storage, Applications of; Bus-bars and Bus-bar Structures; Circuit Breakers; Converters, Synchronous; Distribution Lines; Distribution Systems; Lighting Plants; Motor-Generators; Regulators; Relays; Substations, Railway; Switchboards; Switchgear Equipment for Power Stations; Transformers.*) Three types of substations are employed in lighting systems viz.:

Substations for Voltage Conversion and Regulation on A-C. Systems. — These substations receive power from high-tension transmission lines or primary feeders designed for economical cross section of conductor rather than for small voltage drop. The standard equipment of such substations includes bus-bars and oil circuit breakers for high-tension lines, step-down transformers, bus-bars, oil switches and voltage regulators for delivery lines, and often in addition constant-current transformers serving local series street-lighting circuits. The construction of bus-bar and switch structures is similar to that in power stations for the control of circuits of equal power. Transformers are often of the air-blast type, and are usually grouped in banks of three single-phase units. The constant-current transformers supplying d-c. arc circuits are associated with mercury-arc rectifiers. The equipment is usually arranged to make the current path through the substation as short and direct as possible. Each outgoing line has its individual equipment of voltage regulators in the best practice.

Substations for the Conversion of Frequency. — Such substations are usually combined with the conversion of voltage. These substations receive power from 3-phase, 25-cycle primary feeders, convert it to 60 cycles and transform its voltage for the supply of local 60-cycle lighting feeders. The type of frequency-changer most used is a pair of synchronous machines of the proper frequency ratio running on a common shaft. When space is restricted the vertical-shaft type is generally used. By proper regulation of the field excitation of the motor element it is possible to control the power factor and terminal voltage of the primary feeders to the most advantageous values.

Substations for the Conversion of Alternating Current to Direct Current. — The alternating current is received from high-tension, 3-phase, primary feeders, and is converted to low-voltage direct current for the supply of local d-c. lighting circuits. Substations of this type are largely used in connection with the 3-wire lighting networks in large cities. The standard equipment comprises bus-bars and oil switches for incoming lines, either (1) synchronous converters associated with lowering transformers and voltage regulating accessories, or (2) motor-generators, and direct-current switchboard equipment. In very many cases the above equipment is supplemented by a large floating storage battery to insure the continuity of service in case of breakdowns. A converter must be associated with step-down transformers while the motor element of the motor-generator may be wound for voltages up to 20,000. The converter equipment is somewhat more efficient and in most cases slightly less expensive. The motor-generator equipment affords the greater range of power-factor compensation.

For methods of voltage control see *Converters, Synchronous.*

In converter substations the step-down transformers are usually in banks of three single-phase units. Transformer banks supply converters individually and are not connected in parallel on the low-tension side. Converters of 300 kw. and higher are usually wound 6-phase. The neutral point of a well balanced 3-wire system may be derived from the neutral of the transformer bank. For storage battery arrangements see *Batteries, Storage, Applications of.*

BIBLIOGRAPHY. — See articles on *Power Stations.*

[W. E. Wickenden.]

SUBSTATIONS, RAILWAY. — (*See also Converters, Synchronous; Railways, Energy Requirements for; Switchboards; Switchgear Equipment; Transformers.*) Substations are used for electric railways when the length of the road is so great that the whole road cannot be supplied with one power station at the voltage required by the motors without either an excessive drop in voltage or a prohibitive amount of copper or both. Practically all electric railways have developed to such an extent that substations are required for successful operation.

ECONOMIC CONSIDERATIONS. — The location, capacity and number of substations is a matter for careful study and calculation, involving not only the cost of the copper required for distribution and the cost of the substations, but also the distribution of traffic and special local conditions. The fundamental economics of the subject are expressed by the general theorem that the cost of operating the substations plus the cost of interest and fixed charges on investment in substations and line copper shall be a minimum. This is explained by the fact that if to any given arrangement an additional substation be added with the proper rearrangement of the spacing, the amount of copper used in the distributing system may be considerably decreased on account of the lesser distance between substations. If the saving in interest on the value of the copper is greater than the cost of operation of this new substation plus the interest and fixed charges on the first cost of the substation, the change is warranted.

Allowable Voltage Drop. — The distance between substations depends directly upon the allowable loss in voltage, the amount of copper in the trolley and inversely as the load. The allowable loss in voltage is a matter of the special conditions of each road. However, roads may be divided into two classes and the allowable drop in voltage for average and maximum load conditions specified, as in the accompanying table.

Service	Average load	Maximum load
	Per cent	Per cent
City	8	15
Interurban	12	30

The two conditions which determine the allowable drop in voltage are the effect on the lights and the effect on the control circuit of the car, each of which factors requires a lesser drop in voltage than the actual operating characteristics of the motors.

Distance between Substations. — For the service usually found in practice the following tabulation shows the average or customary distance between substations.

Where the length of track receives its power from only one direction the allowable distance for a given drop in voltage is one-quarter the distance allowable between substations.

Capacity Required. — The capacity of the substation depends upon the traffic and the size of the cars, since if there are many cars operating it is probable that only a few will be starting simultaneously and it is during starting that the cars demand the maximum amount of power. It is, therefore, necessary to determine the number of cars which will be located on the section of

Volts	Type	Miles between substations	
		Single-track road	Double-track road
600	Direct-current, Trolley	10	10
600	Direct-current, 3d Rail	13	19
3,300	Single-phase, Trolley	17	23
1,200	Direct-current, Trolley	19	25
1,200	Direct-current, 3d Rail	38	43
6,600	Single-phase, Trolley	45	50
11,000	Single-phase, Trolley	..	70

track supplied by one substation and the frequency of starting. In interurban service the number of cars on a given section is given by the formula:

$$X = \frac{2 \times (\text{length of section}) \times (\text{number of cars per hour})}{\text{schedule speed in miles per hour}}.$$

The average load on the substation is equal to the average power demand of each car times the number of cars on the section divided by the efficiency of the distribution system. In a complex system the local conditions must be studied to determine how many of these cars are liable to be starting at once. In other than city roads a load factor of from 0.3 to 0.5 may be used to determine the maximum load on a substation, and in single-track interurban roads it is customary to assume one car starting and one car running. See article on *Railways, Energy Requirements for.*

TYPES OF SUBSTATIONS. — In practice there are two types of substations: 1. for transforming from alternating to direct current, and 2. for transforming from high-voltage alternating current to low-voltage alternating current.

In standard direct-current railways it is customary to generate power in the power stations at about 2200-volt three-phase alternating current, step up by means of transformers to 33,000 volts 3-phase, transmit over any distance to the substations at this voltage, transform from 33,000 volts to approximately 400 volts 3 phase and then to convert to direct current at 600 volts in which form the power is used by the motors of the cars. This is the standard a-c.-d-c. substation. In certain single-phase a-c. railways it is found necessary to transmit the power over long distances at a higher voltage than can be collected from the trolley, in which case the a-c.-a-c. transformer substations are installed to convert the alternating current at the high transmission voltage to alternating current at a voltage convenient for the trolley and collector devices.

SUBSTATIONS FOR TRANSFORMING FROM ALTERNATING TO DIRECT CURRENT. — In these stations it is almost universally the custom to employ synchronous converters to convert from a-c. to d-c., rather than motor-generator sets, as the converters have a higher efficiency and may be compounded to give the desired regulation. Thus the converter makes possible a saving in first cost, space and energy as compared to the motor-generator set. A standard a-c.-d-c. railway substation usually contains the

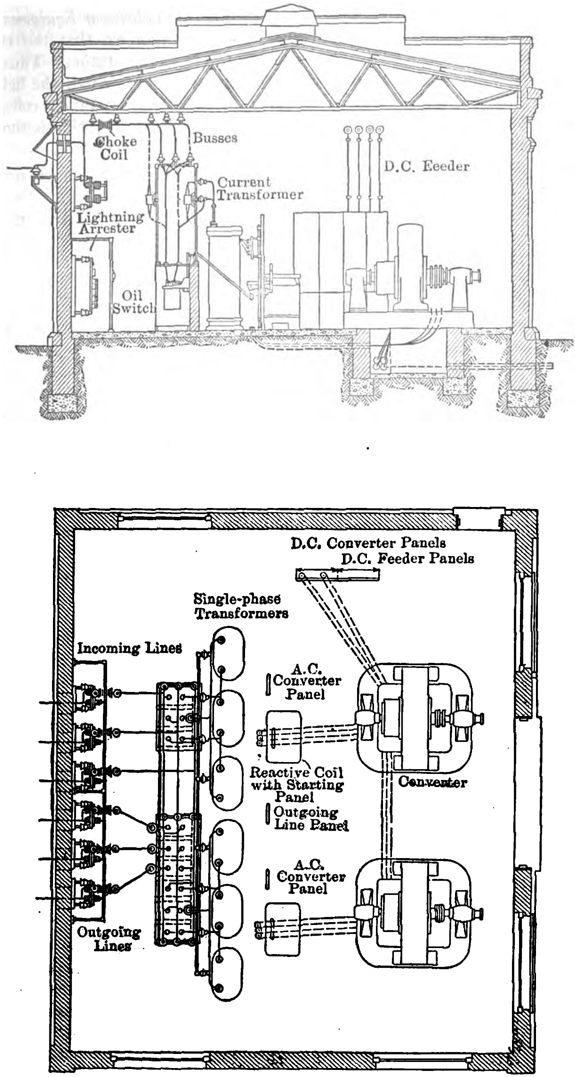

Fig. 1. Plan and Elevation of Typical Railway A-C.-D-C. Substation

following pieces of apparatus: converters, transformers, reactances, blowers, cables, switchboard.

Arrangement of Apparatus.—(*See also article on Switchgear Equipment.*) It is customary to arrange the apparatus in a substation so that the current travels in as nearly as possible a straight line across the station. Thus, with the incoming line on one side there are in the order mentioned: the lightning arresters, oil switches, transformers, a-c. switchboard, reactance coils, converters, d-c. line panel. A typical arrangement for a small station is shown in Fig. 1; see also Fig. 16 in article on *Switchgear Equipment*.

Floor Space Required.—For substations with all apparatus on one level the floor space required is about 0.20 square feet per kilowatt. In cities where real estate is expensive, it may be desirable to put the oil switches and lightning arresters in a gallery on the upper level and the transformers on a floor above, but this involves more expense for attendance.

Standard Voltages and Frequencies.—The following three-phase voltages have been standardized for railway work: 11,000 volts with transformers delta connected on the high-tension side; 33,000 volts with transformers *Y* connected; 66,000 volts with transformers *Y* connected. A frequency of 25 cycles per second is employed in almost all railway installations, as converters are more reliable at this frequency. Certain stations which are obliged to take power from previously existing power stations use a frequency of 60 cycles.

Converters.—(*See article on Converters, Synchronous, for data regarding converters.*) Compound-wound converters are used where the load is variable in order to maintain a constant voltage at the direct-current terminals of the converter irrespective of load. The converter by means of its series winding is made to take a leading current which, in passing through the reactance of the line, the transformers or through an additional reactance introduced for the purpose, tends to neutralize the line drop. (*See Transmission Lines.*) In city service where a large number of cars are operating on one section, the load is fairly constant and shunt-wound converters are generally used.

Both 3-phase and 6-phase converters are universally used, the 3-phase for the smaller capacities and the 6-phase converters for the larger capacities. In a 3-phase converter for a direct e.m.f. voltage of 600 at no load, the transformers should supply the converter with 370 volts between rings or between lines. In the 6-phase converter it is customary to use the diametrical connection, in which each transformer secondary supplies 430 volts to the converter, giving 600 volts at the commutator. Each converter for railway work is customarily supplied with a "speed-limiting" device on one end of the shaft and an "end-play" device on the other end. (*See Converters, Synchronous.*)

Methods of Starting Converters.—(*See also below under Operation.*) There are several methods of starting rotary converters, as is explained under *Converters, Synchronous*. Starting from the alternating-current end as an induction motor is most desirable for railway work, as it avoids the necessity of synchronizing and requires less time. The ability to start a machine quickly and get it on the line in the shortest possible time is very important in railway work, and is an advantage inherent in this method of starting. Three-phase converters are started, by means of suitable starting switches, from one-half voltage taps on the transformer secondaries and take approximately full-load current from the line. Six-phase converters are started from $\frac{1}{3}$ voltage taps and take $\frac{3}{4}$ full-load line current. Since 60-cycle converters usually take a greater starting current than 25-cycle converters and are usually operated on a system supplying power for other purposes, where voltage disturbances are objectionable, special means of starting 60-cycle converters are frequently employed.

Transformers. — (*See article on Transformers.*) The transformers used in railway substations may be either of the oil-cooled, air-blast or water-cooled type. The oil-cooled type is used where the units are small in capacity, i.e., less than 400 kilowatts, and where the expense of the complications for air blast or water cooling are not warranted. Air-blast transformers are used in all except the smallest sizes, and for voltages up to and including 33,000; the objections to their use are the necessity of providing a pit, air ducts and blower to supply the ventilation. Water-cooled transformers (which are oil-insulated transformers with water circulating in a special coil submerged in the oil) are built in sizes from 500 kilowatts upwards and for all voltages. Their use depends upon the availability of water for cooling purposes. The usual aggregate capacity of transformers for the various sizes of synchronous converters is about the same as the converter capacity.

Transformer Connections. — The transformers may be either of the single-phase or three-phase type. For small or moderate installations the single-phase type is preferable on account of the economy of maintaining only one single-phase transformer as a spare for a whole station. In railway work it is customary to connect the secondary of the transformers in delta for three-phase, because of the possibility of operating at reduced output on open delta in case of failure of one transformer. The primary windings of the transformer are usually connected *Y* with grounded neutral. It is common practice to provide transformers for railway work with four 2½ per cent taps on the high-potential winding, in order to use similar transformers in all substations and yet make allowances for the difference in the line drop between the power station and the various substations. Either ⅓- or ½-voltage taps are provided on the low-potential side for starting the converters.

Blowers for Air-blast Transformers (*see Blowers and Compressors*) are usually driven by 25-cycle 3-phase induction motors receiving power from the low-tension side of the transformers. The amount of air required per minute per kilowatt rating of each transformer ranges from 3 cubic feet in the large sizes to 5 cubic feet in the small sizes. This air is supplied at a pressure of from ⅜ to 1 ounce per square inch. The blowers must be capable of supplying this amount of air with an allowance of about 10 per cent for leakage in the air-blast chamber. It is customary to provide two blower sets each capable of supplying air for all the transformers in the station and maintaining one as a reserve unit. In a very large station 3 blower sets are sometimes provided, two of which are together capable of supplying the service and the third is a reserve. A rough idea of the size of the motor necessary to drive a blower for a given purpose may be obtained from the following formula:

$$\text{Horse-power} = \frac{(\text{cu. ft. air per minute}) \times (\text{pressure in ounces})}{1200}.$$

Air Duct for Air-blast Transformers. — The pit or trench over which air-blast transformers are placed must be made water-tight and well drained, as any moisture accumulating is liable to be carried into the transformers where it will injure the insulation.

Voltage Regulation. — Use of Reactances. — The voltage at the d-c. bus-bars is kept constant irrespective of load by means of line compounding, which consists in adjusting the shunt-field excitation of each converter so that the converter takes lagging current at no load, operates at unity power factor at about ¾ load and takes leading current at all loads greater than ¾ load. To accomplish this compounding, it is necessary that there should be a certain amount of reactance between the power-station bus-bars and the converters. (*See Converters, Synchronous.*) There is seldom enough reactance in the trans-

mission line for this purpose, so that additional reactance is inserted either by the use of special transformers having considerable leakage reactance, or by means of reactance coils. The usual reactance coil has a capacity in kv-a. equal to 15 per cent of the kilowatt rating of the converter, i.e. with full-load current passing through the reactance the voltage measured across its terminals would be 15 per cent of the voltage to neutral on the a-c. side of the converter.

Reactance coils are either oil cooled or air blast depending upon the transformers used and are usually built with the three circuits in one unit, with the starting switches for the converters mounted upon the frame. For 6-phase converters a 3-phase reactance coil is used, but since the current per wire is one-half that of the current of a 3-phase converter of the same rating, the reactance per line must be twice as great for the 6-phase converter.

Switchboards. — *(See also article on Switchboards.)* The following sections of switchboards are standard for converter substations.

1. Incoming a-c. line panel.
2. Outgoing a-c. line panel.
3. High-tension a-c. converter panel.
4. D-c. converter panel.
5. D-c. feeder panel.
6. Equalizer and negative panel (on the converter).

Where a substation is tapped off a transmission line at an intermediate point it is good practice to bring the transmission line into the substation, interpose control switches, and then carry the circuit out of the substation on to the next substation. For this reason, in all but terminal substations, it is customary to provide both an incoming and an outgoing a-c. line panel. In connection with the a-c. panel of the switchboard there are a line switch, lightning arrester, choke coil, current transformers and main oil switch, by means of which potential may be removed from all transformers. Between the transformers and the converter are the starting switches, reactance coil and possibly measuring devices.

Direct-current Panels. — Single-pole switchboard panels are used, the positive main bus-bar being the only one on the board. The negative terminals of the converters are connected with switches to the negative or ground return bus-bar, which is frequently located beneath the converter. The series field is connected on the negative side, and the equalizer, series-field-shunt and field break-up switches are frequently placed on the machine itself. The equipment of a standard d-c. converter panel comprises one of each of the following:

Carbon break circuit breaker with overload and low-voltage release, the latter interconnected to the speed-limit device.
Illuminated dial ammeter with shunt.
Field rheostat.
Two-point receptacle.
Single-pole main switch.
Single-pole double-throw station lighting switch.
Watt-hour meter.

Storage Battery. — In certain installations, where reliability of service is of utmost importance, a storage battery is installed, operating in parallel with the converters, and preferably regulated by a series booster. Such a storage battery will not only take care of the load for a considerable period of time in case either the transmission line or converters are out of action, but it will also improve the load factor of the system and thereby the line regulation. In

other cases, where the service does not warrant the outlay for such a storage-battery system, a small storage battery is sometimes installed to take care of the lighting of the substation in case the converters should shut down accidentally. See articles on *Batteries, Storage*.

Ventilation. — As the transformers, reactance coils and converters all dissipate a considerable amount of energy in the form of heat, a substation must be well ventilated in order that the temperature of the air surrounding the apparatus is not so great as to cause an injurious temperature in the apparatus itself. This is particularly important where oil-cooled transformers are used.

Crane. — Where ground space is limited it is good economy to provide a crane in order that the various pieces of apparatus may be lifted over each other when they are taken apart for repairs, as otherwise considerable space must be left to move them about to and from the entrance. See article on *Cranes*.

Operation of Converter Substations. — The only apparatus in a converter substation that requires any great amount of attention are the synchronous converters. As noted above these are usually started from the a-c. supply.

Instructions for Starting a Synchronous Converter from A-C. Supply. —

1. Open all switches except main negative (on machine).
2. Close a-c. line switch feeding busses.
3. Close H.T. transformer switch.
4. Close starting switch on low-voltage taps.
5. When converter reaches synchronism as shown by low frequency of swings of d-c. voltmeter, close equalizer switch.
6. Close switch between series field and the shunt to series field.
7. Correct polarity if necessary by throwing shunt-field break-up switch to reverse position, leaving it closed only momentarily, then throw to running position.
8. Connect a-c. terminals of converter to full voltage of transformers by throwing starting switch to running position.
9. Close d-c. circuit breaker.
10. Adjust field rheostat.
11. Close main d-c. switch.
12. Adjust for correct division of load, power factor and voltage.

Load Factor and Efficiency. — The load factor (q.v.) of a converter substation is usually low, from 30 per cent to 50 per cent, i.e., the load on the station is relatively light except during the morning and afternoon rush-hours, 7 to 9 A.M., and 5 to 7 P.M respectively (*see Train Dynamics*). The all-day efficiency of the station itself, or the ratio of the kilowatt-hours output to the kilowatt-hours input, is less than the efficiency at maximum or rated load; but the over-all efficiency between the a-c. generators in the power house and the cars is about constant throughout the day, since the efficiency of a transmission line increases with decrease of load, thus offsetting the low light-load efficiencies of the transformers and converters. In general practice in interurban 600-volt railways the maximum and all-day efficiencies of the various apparatus are approximately as given in the following table:

Apparatus	Full-load eff., per cent	All-day, eff., per cent
Step-up transformers	98	97
High-tension line	95	98
Step-down transformers	97	94
Converters	90	88
Low-tension distribution	85	88
Over-all, a-c. generator to motors	69	69

First Cost of Converter Substations. — Synchronous converters for railway purposes cost from $9 to $11 per kw., the transformers from $5 to $10 per kw., and the switchgear equipment from $3 to $11 per kw. of substation rating. The total first cost of substations is subdivided approximately as given in the accompanying table.

Item	Per cent of total
Electrical apparatus	67
Building and fixtures	17
Wiring and erection	15
Miscellaneous	1

In an approximate way the cost of substations including building, erection and installation, for 33,000-volt transmission, 25-cycle converters, 600 volts on trolley is as follows:

Station capacity, kw.	No. Converters	Capacity of each converter, kw.	Approximate cost of station
300	1	300	$15,000
600	2	300	25,000
800	2	400	31,000
1000	2	500	36,000

Cost of Operation of Converter Substations. — Labor or wages of the attendants is the principal item in the cost of operation of a substation. Supplies and maintenance form a very small percentage of the cost. In general only one man per shift is necessary, as the principal duties are to start up the machines in the morning and occasionally during the day if they drop out of step due to a short-circuit. Frequently the duties of the attendant can be combined with that of ticket seller or express agent. Where the duty is only attending to the substation, $1800 to $2000 per year may be taken as the cost of operation of a substation, allowing for the two shifts a day.

PORTABLE SUBSTATIONS. — Circumstances frequently occur under which the traffic on a certain branch of a railway is very heavy for only a few

weeks in the year, and possibly for only two or three days in the year. Moreover traffic may be exceptionally heavy on one branch for one short period and on another branch at another period. In such cases as these it would be very expensive to provide on each branch substations having a capacity to meet the heavy demand. To meet such conditions "portable" substations are used. These consist of what are practically large steel furniture cars, in which are installed one synchronous converter and the necessary transformers and control devices. Side tracks are provided at points on the branch lines and when needed this portable substation is hauled to the place desired and its high-tension terminals are connected to the high-tension transmission line, provision for which must be made in building the line, and its direct-current terminals are connected to the trolley and the rail respectively. Such a substation may be of great convenience and value for interurban roads where summer parks, circuses and athletic games occasionally render traffic conditions difficult. The car is limited in its size by the standard clearance outlines of the roads over which it must pass. It usually weighs, with all its equipment, about 75 tons. Apparatus having a capacity of 500 kw. may be installed in such a car and the primary voltage may be as high as 33,000 volts (*see article on Switchgear Equipment*).

HIGH-VOLTAGE DIRECT-CURRENT SUBSTATIONS. — Several electrical railways at present are operated at 1200, 1800 and even 2400 volts d-c. between trolley and ground. For this purpose power is transformed from alternating current by means of either motor-generator sets or synchronous converters. At first the motor-generator set was considered more fitted to the purpose as the design of the commutator is not as restricted in a generator as in a converter and the interpole made it possible to operate at a high potential between brushes. Subsequently, synchronous converters with interpoles and operating at 25 cycles were developed, and are now in successful operation. At first two 600-volt machines were connected in series to give 1200 volts. Later, machines were developed which would give 1200 volts on each machine and these became available for 2400 volts by connecting two in series. A number of installations of this type are in successful operation at present.

ALTERNATING-CURRENT SINGLE-PHASE SUBSTATIONS. — For a single-phase railway operating at a high voltage on the trolley (11,000 usually) it is not necessary to place substations as near together as in the case of d-c. systems. However, even with 11,000 volts on the trolley, there comes a limit to the length of road which may be supplied from one feeding point. In this case it becomes necessary to step up the voltage at the power station to 33,000 or some higher voltage and step down again to the trolley voltage at distant substations. A substation for this purpose need contain only transformers and control devices. It is much simpler and less expensive than the converter substation. It has even been a matter of discussion as to whether such a substation would require continual attendance to maintain proper operation. This is, however, a matter which must be decided by local conditions. If oil-cooled transformers are used there is no moving apparatus in the station, and the only need of an attendant is in case of accident or to reset a circuit breaker in case it goes out as a result of an overload.

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[W. I. SLICHTER.]

SWITCHBOARDS. — (*See also Bus-Bars; Circuit Breakers; Meters; Switches; Switchgear Equipment for Power Stations.*) The term switchboard formerly comprised all apparatus in that portion of an installation devoted to collecting, measuring and distributing the electric energy. In small plants all of the switchgear is mounted on a single structure which is then called the switchboard. In large modern plants the control and measuring pieces of apparatus are frequently mounted on one structure, and the busbars and switches on another. In such cases the term switchboard is confined to the structure, whether panel, pedestal, or desk type, which supports the meters, controlling handles and similar devices. The following article discusses the arrangements of apparatus on representative switchboards selected to illustrate the general trend of switchboard design. For a description of the respective structural arrangements required by direct control and distant control, see the article on *Switchgear Equipment in Power Stations*.

CONSTRUCTION OF PANEL SWITCHBOARDS. — The earliest so-called "panel" boards were made of wooden panels. The various switches, instruments, etc., each on its own base, were attached to the wooden panel, with the wiring on either the front or the rear. The next step in advance was the elimination of the wooden panel and framework. Each piece of apparatus was mounted on a marble slab, which was arranged for placing in an angle-iron framework, and switchboards were made by combining the necessary ammeter, voltmeter, switch and rheostat slabs to make the panels for the different generators, feeders, etc. This form of construction was entirely fireproof but various disadvantages ultimately lead to its being superseded by the modern design of panel switchboards with the apparatus grouped on panels made of one or more comparatively large pieces of marble, or slate.

The design of modern panel switchboards has undergone the standardizing process that has been applied to all electrical apparatus. By careful study of requirements the vast majority of plants of moderate size can be equipped with control switchboards that are made up by assembling standard panels.

Framework of Panel Boards. — In general the framework of the standard switchboard may be divided into two types. The cheaper and smaller boards are mounted on a framework of gas pipe, and usually comprise panels about 4 feet high with a space between them and the floor. The larger and more expensive boards have a total height of about 7 feet 6 inches, extend down to the floor, and are provided with an angle iron or pipe frame.

Gas-pipe Frames. — Although the gas-pipe construction is considerably lighter than the angle-iron construction it has been found amply secure for these smaller switchboards and in fact some manufacturers use gas-pipe construction for most of their larger switchboard installations. Where the number of panels does not exceed four or five the complete board can sometimes be shipped with the panels attached to the framework and most of the small wiring, etc., undisturbed, but if the boards are large ones the panels and frame are shipped separately.

Angle-iron Frames. — For larger and more expensive panels a framework of angle-iron construction is frequently used. Each panel of a total height of 90 inches is provided with 2 by 3 by ¼ inch or some similar section angle irons with the narrow web bolted to the panel. These angle irons extend from the bottom of the panel to within ½ inch of the top. The vertical angles on adjacent panels are bolted together through the 3-inch web. They are provided with corner angles for bolting at the bottom to a 6 by 2-inch channel iron forming the base of the frame and at the top to two ½ by 1⅜-in. iron strips. The channel iron and the top irons are made continuous the entire length of the

board, if this is not over 16 feet. Each panel is shipped bolted to the two angle irons that form its individual frame, which obviates any necessity of disconnecting the wiring between the various slabs making up the panel. This practice of shipping the framework with the panels reduces to a large extent the breakage due to rough handling and facilitates the erection of the board at its destination.

Switchboard Panels. — After trying various materials practically all switchboard builders have adopted either slate or marble, although in a few instances soapstone, brick or steel has been used. The marble used in switchboards is usually of the grade known as "Blue Vermont," although occasionally "White Italian," or "Pink Tennessee" is used.

Comparison of Slate and Marble. — Where switchboard panels are to be given a black finish the choice of slate or marble is largely a question of cost and insulation. Slate is considerably cheaper and somewhat stronger than marble and where the voltage of live metal parts mounted on the panels does not exceed 750 volts it answers just as well. This makes it suitable for all boards except those having ground-detector receptacles, fuse blocks or similar apparatus mounted on the material of the panel and connected to a circuit of 1100 volts or more.

Finish Used on Panels. — The marble is sometimes polished on the front face and bevels, and occasionally on the edges and back. The present standard finish for marble is a dull black marine finish applied to honed panels. Ordinary slate, owing to its irregular color and marking, is usually given an enamel or marine finish and natural black slate is given an oil finish.

Dimensions of Two-piece Panels. — The total height of standard switchboard panels is 90 inches from the channel iron. The division resulting in a 25-inch lower slab as furnished by one company is due to the fact that these particular dimensions were best adapted to the line of switches, circuit breakers, meters, etc., which were in use at the time the standard railway switchboard panels were first brought out. The lower 25-inch slab was used for rheostat face plates having the contacts and contact mechanism on the front of the panel. In order to correspond with the old d-c. panels the a-c. panels were brought out having a lower slab 25 inches high and a main slab 65 inches high. The similar design of another company provides panels with slabs 62 inches and 28 inches high which dimensions were suited to their apparatus at the time of standardization.

Dimensions of Three-piece Panels. — When the present standard brush-type, carbon-break circuit breaker was designed it was found advisable to mount this breaker at the top of the panel in order to take advantage of the tendency of an arc to rise. As all these standard breakers up to 3000 amperes capacity required a vertical space of less than 20 inches, it was soon decided to divide the main upper 65-inch panel into two slabs, one portion being 20 inches high to contain the circuit breaker and the other portion 45 inches high to contain the meters, switches, etc. For this reason the standard d-c. panels of one maker, both for railway and for light and power work, are divided into three slabs, the upper 20 inches high, the middle 45 inches and the lower 25 inches. Nearly all switchboard builders are now following this practice of putting heavy circuit breakers on separate slabs.

TYPICAL DIRECT-CURRENT SWITCHBOARDS. — A few standard equipments are described in the following paragraphs.

Small Isolated Plant Switchboards (Fig. 1). — Where there is only a single d-c. generator of small capacity with one or two feeder circuits and there is little likelihood of additional equipment being needed, a board such as shown in Fig. 1 can be used. This panel, 36 inches high by 16 inches wide, is provided with a

polarized ammeter and voltmeter 5 inches in diameter, single-pole carbon breaker, 2-pole main switch, field rheostat in the generator circuit, and switches with fuses in the feeder circuits. This type of board is limited to 10 kw. at 125 volts and 20 kw at 250 volts.

2-Wire Switchboards (Fig. 2). — For the control of small capacity, d-c. generators operating in parallel on a 2-wire, d-c. circuit the connections are made

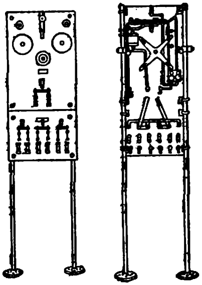

Fig. 1. Direct Current Isolated Plant Switchboard

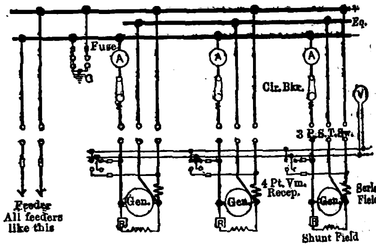

Fig. 2. Connections of 2-wire Switchboard

as indicated in Fig. 2. Each generator is provided with a 3-pole, single-throw switch, a single-pole circuit breaker, an ammeter, and rheostat and a 4-point voltmeter receptacle used for connecting the voltmeter to any machine. Each feeder is provided with a 2-pole switch with inclosed fuses, and a lamp ground detector is connected across the bus-bars.

3-Wire Switchboards. — Fig. 3 shows the diagram of connections for a typical d-c., 3-wire installation and indicates the method of deriving the neutral connection from the middle point of auto-transformers; the use of series fields

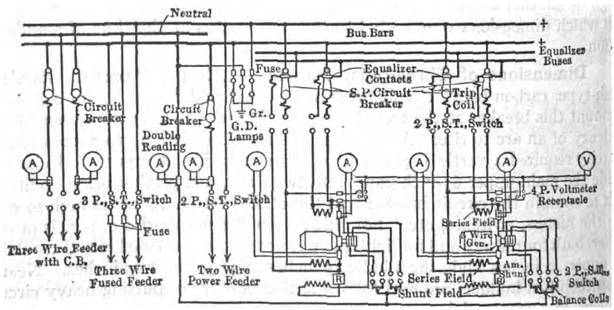

Fig. 3. Connections of 3-wire Switchboard

in the positive and negative circuits; the placing of the ammeter shunts on the terminal boards of the machines so as to be connected inside the series fields; and the 4-pole circuit breakers or the equivalent two single-pole breakers with equalizer contacts to open the positive, positive equalizer, negative and negative equalizer circuits.

Double-bus Railway Switchboards. — For direct-current railway or tramway work at about 600 volts it was for a long time the standard practice to bring both the positive and the negative leads from the generator to the switchboard and to locate both the positive and negative busbars on the switchboards, such boards being called "double-bus" boards. With these boards it was customary to use compound-wound d-c. generators with the series fields in the armature circuit which connected to the overhead trolley. The equalizer connection was usually made through a switch at the machine and the equalizer connection ran from one machine to the other without going to the panel board.

"Single-bus" Railway Switchboards (Fig. 4). — Modifications in this class of switchboards came from placing the series fields in the grounded circuit instead of in the trolley circuit and running only one polarity, usually the positive, to the switchboard. The equalizer switch and the negative switch, if the latter is used, are mounted at the machine. Fig. 4 is a typical diagram of a single-bus switchboard with only one polarity on the board. The advantages of this single-bus arrangement are greater security against short circuits, narrowing of the generator panels with consequent reduction in length of switchboard, reduction in amount of cable required, and the use of bare copper strap or cables between machines for the negative and equalizer busses.

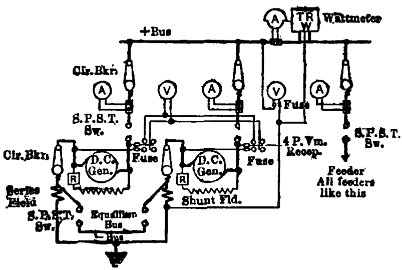

Fig. 4. Connections of Railway Switchboard

Rotary-converter Switchboards. — For rotary converters or motor-generator sets the panel switchboard, if such is used, takes care of both the d-c. and a-c. circuits, which usually are controlled by separate panels. The d-c. panels are practically the same as those used for d-c. generators. The a-c. panels usually correspond with a-c. feeder panels and control the high-tension side of the step-down transformers used with the rotary, where the rotary is made self-starting from the a-c. end. With such machines, in addition to the panels for the high-tension side of the transformers, it is customary to furnish a small slab mounted near the step-down transformers which is used for connecting the rotary to low-voltage taps for starting and then to the full-voltage taps for running. In many rotary-converter installations a combination of direct control for the d-c. circuits and distant control for the a-c. circuits works out to advantage.

Mercury Rectifier Switchboards. — Another combination of a-c. and d-c. equipment met with in switchboard practice is that furnished for mercury rectifiers used in conjunction with constant-current d-c. arc lights fed from a constant-potential a-c. system. The apparatus supplied usually comprises a two-pole oil switch with fuses for the primary circuit of the regulator, plug switches in the secondary circuit, a mechanism for tilting the bulb of the mercury rectifier, and a high-voltage d-c. ammeter placed under a glass cover. The switchboards furnished with low-tension rectifiers are of similar arrangement, except that they are fitted with low-tension meters and switches, and have a single-pole starting switch which automatically applies a rheostat load for starting.

Other Arc-light Switchboards. — For use with d-c. series arc generators, switchboards are frequently supplied with a multiplicity of plug switches to

permit any generator or feeder circuit to be connected in series with any other circuit or circuits. (*See Switches.*)

Exciter Panels, although d-c., are almost invariably combined with and form part of a-c. switchboards. The d-c. sections of the board are naturally considered with the a-c. sections and made to correspond with them in general design. Exciter panels are either single or double, depending on whether they control one or two machines. Usually no fuse equipment or other automatic protection is provided with exciter panels, as the sudden opening of the field circuit due to the blowing of an exciter fuse or the tripping of an exciter breaker is apt to injure the insulation of the generator and cause far greater damage to the plant than the overloading or short circuiting of an exciter. Where there are several large exciters operating in parallel with each other or a large storage battery, it is occasionally considered good practice to furnish reverse-current circuit breakers in the exciter circuits.

TYPICAL ALTERNATING-CURRENT SWITCHBOARDS. — Some standard equipments are described and illustrated in the following paragraphs.

Small Low-voltage A-C. Boards (Fig. 5), made for circuits of not over 600 amperes capacity at 500 volts, are usually built of panels 48 inches high on pipe frames. Each a-c. generator may have its own exciter as shown in Fig. 5, or the exciters may be operated in parallel and controlled from exciter panels. As shown on this diagram the d-c. circuits are taken care of by an exciter rheostat, a generator rheostat and a 2-pole field switch with discharge resistance. Each a-c. generator is provided with a 3-pole single-throw knife switch, three ammeters, an 8-point voltmeter receptacle, and a synchronizing receptacle and plug. Each feeder has a 3-pole switch and an ammeter.

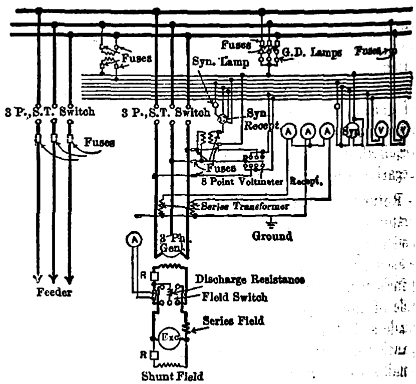

Fig. 5. Connections of Low-voltage Alternating-current Switchboard. (Only one Generator Shown)

A lamp ground detector and a voltmeter are connected in such a manner that the voltage can be read across any phase of any machine. In this diagram lamps are used for synchronizing.

Small High-voltage A-C. Boards. — A similar arrangement of panels may be used for small-capacity high-voltage switchboards, using oil switches in place of knife switches in the generator and feeder circuits, and operating the voltmeter and lamp ground detector from potential transformers.

Large Low-voltage A-C. Boards. — Fig. 6 shows some typical panels for low-voltage a-c. circuits where the cost of equipment is kept to a minimum by simplifying the equipment of instruments, omitting the sub-panels (the lower sections of slabs), and using ornamental cast-iron legs to cover the portion of the framework which projects below the main panel. The panels shown are selected as representative of this type of construction, not as an actual switchboard. The bracket instruments comprise the synchroscope and two volt-

meters, one for the bus and one for the machines. The first panel at the left-hand end is a 1000-ampere 2-phase generator panel with double-throw

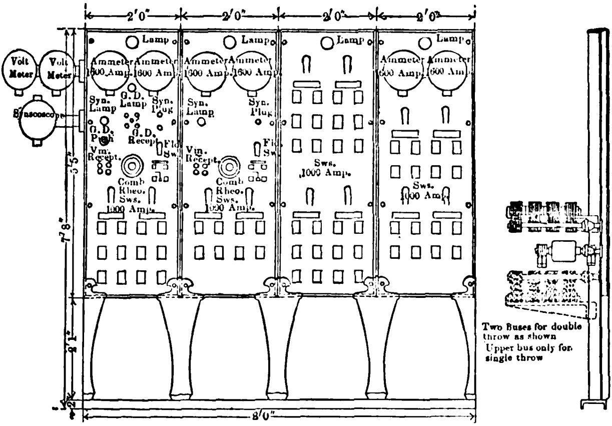

Fig. 6. Low-voltage Alternating-current Switchboard Panels

main switches for connecting to either of two sets of busbars. The panel is also provided with illuminating lamp, two ammeters, 6-point voltmeter receptacles, synchronizing lamp receptacles, plug ground detector receptacle, lamp, push-button rheostat mounting, and field switch. The second panel is a corresponding single-throw generator panel, and the two remaining panels are double-throw and single-throw feeder panels. The side view shows the arrangement of the bus-bar supports, switch studs and connections.

For more important installations sub-panels are furnished. The generator equipment then includes field ammeters and indicating wattmeters, and a synchroscope is provided for paralleling the generators.

Large High-voltage A-C. Boards (Fig. 7) are almost invariably provided with oil switches (or oil circuit breakers), mounted either on the switchboard or apart from it. All a-c. instruments are operated from current and potential transformers. Fig. 7 gives the diagram of a typical switchboard consisting of one exciter panel, two generator panels, and one feeder panel. The connections are shown as though viewed from the back of the board and the leads from the current and potential transformers are bunched into cables.

Station Voltmeter and Synchronizer. — On this switchboard there are 2 voltmeters and 1 synchroscope mounted on swinging brackets at the end. The inner voltmeter is operated through cable 4B from potential transformers connected to the bus. These transformers are used also for the potential circuits of the generator wattmeters and for synchronizing. The other voltmeter is connected through suitable receptacles to the potential transformers of any machine. The synchroscope and synchronizing lamps are operated by inserting the suitable plug in the synchronizing receptacle. When the plug is so inserted, it connects the three inner segments by means of one ring. At the same time the bus transformer is connected to the bottom terminals of the synchroscope and the generator transformer to the top terminals. A lamp at the synchroscope and one on the particular generator panel are connected in parallel with the synchroscope and flash up and down in step with it.

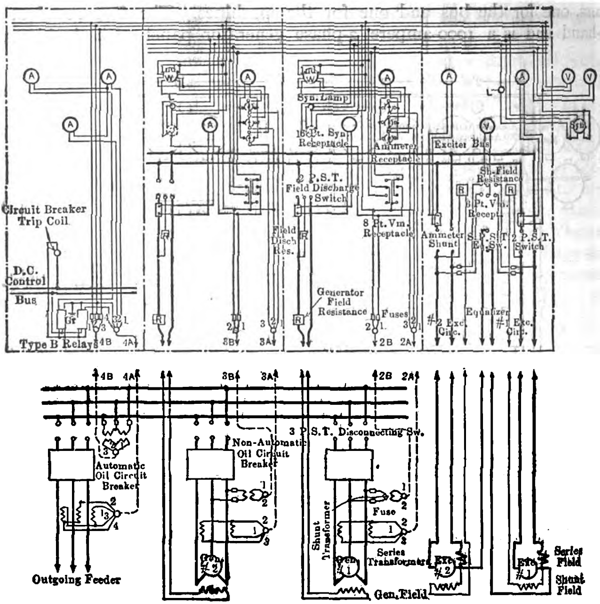

Fig. 7. Connections of High-voltage Alternating-current Switchboard

Exciter Panel. — The first panel (on the right looking at the rear) controls 2 exciters and is provided with 2 2-pole main switches, 1 single-pole equalizer switch, 2 ammeters operated from shunts, 2 rheostats, and 1 exciter voltmeter connecting to either exciter by means of a 6-point receptacle and a 4-point plug.

Generator Panels are each equipped with a 3-pole non-automatic oil circuit breaker with disconnecting switches, 2 current transformers, 1 potential transformer, 1 2-pole field switch with discharge resistance, 1 field ammeter operated from a shunt, 1 polyphase indicating wattmeter, 1 a-c. ammeter with three receptacles for reading the current in any phase, 1 8-point voltmeter receptacle, 1 6-point synchronizing receptacle, and 1 synchronizing lamp.

Feeder Panel. — This is equipped with 1 3-pole automatic oil circuit breaker, 3 disconnecting switches, 3 current transformers, 1 pair of single-phase over-load relays, and 3 ammeters. This system is a simple one with 1 set of exciter bus-bars and 1 set of a-c. bars with a moderate number of instruments, but even this gives some idea of the complexity met with in the detail diagram of a large and complicated a-c. switchboard.

Large Distant-control A-C. Switchboards. — Where the amount of power to be controlled is large or the voltage is above 3300, it is customary to mount the oil circuit breakers apart from the board and to operate them either mechanically or electrically. On these boards the usual equipment of instruments is mounted at the top of the panel, the switch-control handles are in the middle and the time-limit relays at the bottom. Targets and signal lamps to

indicate the condition of the feeder are furnished with electrically operated boards.

SWITCHBOARDS OF PEDESTAL AND POST TYPE. — Where the number of generators is comparatively small in comparison with the number of feeder circuits, it is frequently of advantage to use control pedestals and instrument posts for the generator circuits and to take care of the feeder circuits by means of a panel switchboard. The instrument posts and control pedestals can readily be added with additional machines, without disturbing the symmetry of the arrangement.

Control Room of Ontario Power Co. — An example of this general arrangement is the control room of the distributing station of the Ontario Power Company at Niagara Falls, Ontario (*El. Rev. & W. El., 1911, Vol. 59, p. 1036*), for the control of seven 8770-kv-a., 12,000-volt, 3-phase generators, with banks of three 3000-kv-a. transformers stepping up to 60,000 volts. In this room there are the following switchboards: 1 panel board to control the 60,000-volt feeder circuits running to Rochester, Syracuse, etc., 1 control pedestal and 1 instrument post for each of the 12,000-volt generators; and 2 smaller pedestals for the control of the exciter circuits.

Each of the seven control pedestals is equipped with push-button control for the generator field rheostats and is provided with a white signal lamp which lights up when the field circuit is closed. There are 2 electrically operated oil circuit breakers in each main generator circuit, one being placed in the power house and the other in the distributing station. The circuits from the generator, after passing through these two breakers, are connected by a breaker to one 12,000-volt bus-bar in the distributing station, or can pass through another breaker to a common connection, where it branches and passes either through a breaker to a second 12,000-volt bus-bar or through a breaker to the low-tension side of the step-up transformers. Each circuit breaker is controlled from the generator pedestal. Suitable synchronizing lamps and receptacles are also placed on these pedestals. The pedestals are 5 feet high and each occupies a floor space of approximately 24 inches by 14 inches. Each post is provided with a single-phase synchroscope, a frequency meter, a three-phase power factor indicator, and transformer and generator ammeters. The total height of this post is 9 feet and the width occupied by the meters is 2 feet 7⅝ inches.

CONTROL DESKS (Figs. 8 and 9). — Where it is desired to have a very compact arrangement and to control the generators and feeders from the same switchboard, the control desk has many advantages; particularly where a group system of circuits is used and it is desirable to have a miniature bus-bar to show the general scheme of connections and the arrangement of circuits in use. The desk has mounted on it the field switches, field rheostat handles, and small switches for operating electrically the various controllers, circuit breakers, etc. The field switches and rheostats may also be operated electrically.

Location of Instruments with Control Desks. — The instruments can be mounted either on panels forming the back of the control desk (Fig. 8), or on an instrument frame back of and usually higher than the top of the control desk (Fig. 9), or on instrument posts. The grouping of the instruments is made so far as possible to correspond with the grouping of the control devices. As it is possible for the station operator to become confused in determining the instruments belonging to a certain generator or feeder, whose controlling devices are on the desk, card holders or name plates are placed both on the desk and on the panels. Where instrument panels form the back of the control desk, the instruments as a rule are arranged to correspond in location with the controlling devices for the same circuits. When an independent instrument frame is used,

this is usually arranged at such a height that the station operator standing at the control desk can look over the top of the desk and under the bottom of the instrument frame, out into the station, and readily observe the operation of the machine which he expects to control.

Control Desk with Instrument Panels. — Fig. 8 shows the plan view, front elevation, and end view of a control desk having instrument panels which

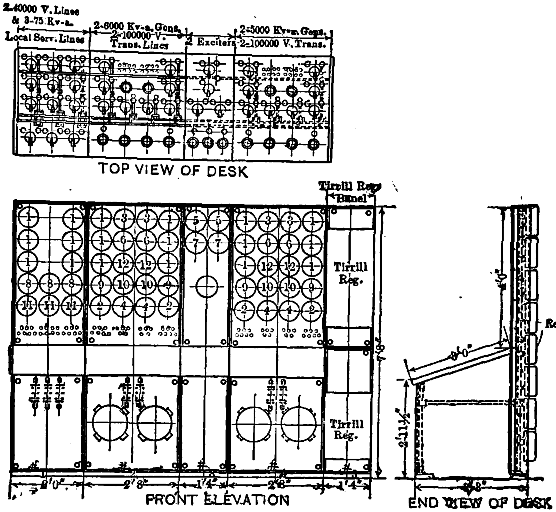

Fig. 8. Control Desk with Instrument Panels

come down to the top of the horizontal section of the desk. This desk when fully equipped will take care of two 300-kw., 250-volt, waterwheel-driven exciters; four 6000-kv-a., 4000-volt, 3-phase generators; four banks each of three 1750-kv-a., single-phase transformers stepping up to 100,000 volts; three 1750-kv-a., 3-phase transformers stepping up to 40,000 volts; four 100,000-volt transmission lines and two 40,000-volt transmission lines. The miniature bus on the face of the desk shows the various connections made by the breakers. In this plant the exciter circuits and the field circuits of the generators are controlled by electrically operated switches, as are also the main circuits of the generators, transformers and transmission lines.

Control Desk with Separate Instrument Frame. — Fig. 9 shows the plan view, elevation and side view of a control desk provided with 7-inch diameter round pattern meters located on an instrument frame back of and above the desk. The height of the instrument frame is 7 feet 8 inches, which corresponds to the height of standard 90-inch switchboard panels mounted on the usual 2-inch channel iron base. This control desk with a total length of 12 feet, when fully equipped, is to take care of six 10,000-kv-a., 3-phase, 6600-volt generators, six banks each of three 3333 kv-a. step-up transformers, and four 110,000-volt transmission lines.

As shown by the miniature bus system placed on the top of the control desk, each of the six sections intended for a generator and its bank of transformers is provided with a generator ammeter, transformer ammeter, generator indicating

wattmeter, power-factor meter, voltmeter, and field ammeter, as well as four circuit-breaker controllers. Each generator, with the low-tension side of its

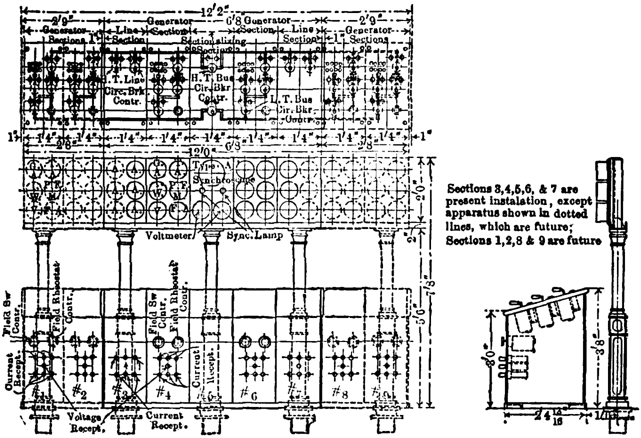

Fig. 9. Control Desk with Separate Frame for Instruments

bank of step-up transformers, is provided with three breakers, one in the main generator circuit connecting the generator to a generator bus, one for connecting this generator bus to a main bus, and the third for connecting the generator bus to the low-tension side of the step-up transformers. With this arrangement under normal conditions each generator will supply current to its own bank of step-up transformers, but in emergency any generator or any transformer can be connected to the low-tension bus, which is sectionalized in the middle by means of an electrically operated breaker. On the high-tension side each bank of transformers connects through a single breaker to a high-tension bus that is sectionalized in the middle by means of an electrically operated breaker. Two high-tension lines are fed from each half of this sectioned bar.

Dimensions of Panels. — The smaller panels intended for use with gas-pipe framework are made in single slabs and as a rule have a height of 48 inches and a width of either 22 inches or 32 inches, although some of the panels are smaller. As few of the smaller d-c. panels have to be made wider than 22 inches or 24 inches and as 1¼ inches is ample thickness for a 24 by 48-inch slab, the thickness of 1¼ inches has been adopted as a standard for most of the small d-c. panels. In order to secure sufficient mechanical strength the 32-inch panels are made 1½ inches thick. As these wide panels are usually required for alternating-current generator panels this thickness of 1½ inches has been usually adopted as standard for a-c. panels. For heavier panels 2-inch marble and slate have been adopted as standards for mechanical reasons. This thickness is required for the heavy switches, circuit breakers, etc., often furnished on these switchboards. When a board has both a-c. and d-c. panels the thickness of all panels is made the same.

Beveled Edges. — The edges of all panels are usually beveled to improve their appearance and to insure against chipping, as it is almost impossible to handle square-edged marble or slate slabs without injury. The front edges are ordinarily beveled with a 45-degree bevel of either ⅜ inch or ½ inch, measured

in the plane of the panel. It has also been found advisable to use a small bevel of 1/16 inch or 1/8 inch on the back edges to prevent chipping.

SPECIFICATIONS AND TESTS. — See article on *Switchgear Equipment for Power Stations.*

COSTS. — The cost of any particular type of switchboard is arrived at by summing up the costs of the various instruments, switches, etc., and adding to this the cost of the framework, slabs and wiring. The following figures are approximate only.

Gas pipe for framework....................	$0.07 per linear foot
2 by 3 by 1/4-inch angle iron................	0.11 per linear foot
6 by 2-inch channel iron..................	0.20 per linear foot
1/2 by 1 3/8-inch strips....................	0.10 per linear foot

Slabs only, without drilling	Marble	Slate
48 by 24 by 1 1/4-inch panel..........................	$5.12	$2.42
48 by 32 by 1 1/2-inch panel..........................	8.99	4.15
90 by 24 by 2-inch panel..........................	15.60	7.00

The cost of drilling, wiring and erection ranges from 5 to 25 per cent of the total cost of the board erected complete. See also article on *Switchgear Equipment for Power Stations.*

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[S. Q. HAYES.]

SWITCHES. — (*See also Circuit Breakers; Switchboards; Switchgear Equipment for Power Stations; Wiring of Buildings.*) A switch is a device for mechanically opening an electric circuit. Various types of knife, drum, plug and oil switches are used; some representative types are described below.

Fundamental Requirements are (1) a switch must, when closed, carry the rated current without excessive drop, usually from 5 to 15 millivolts, or excessive heating, usually 28° C. temperature rise; (2) it must take care of overloads met in practice; (3) it must be designed to prevent or render harmless any arcs that are formed when being opened; (4) it must, when open, insulate all live parts for the maximum potential of the circuit.

KNIFE SWITCHES.— The rules of the National Board of Fire Underwriters relative to knife switches advise for pure copper blades a current density at contact surfaces of not over 75 amperes per square inch. The recommended minimum spacings between points of opposite polarity for various currents and voltages are given in the accompanying table.

Full-load current, amperes	Inches between points of opposite polarity		
	125 v. d-c.	250 v. d-c. 500 v. a-c.	600 v. d-c.
100	1¼	2¼	4½
200–300	2¼	2½	4½
400–600	2¾	2¾	4½
800–1000	3	3	4½

Throw and Poles of Switches. — A switch closing a circuit only when thrown in one position is called a single-throw switch; a switch closing a circuit when thrown in either of two positions (e.g., up or down) is called a double-throw switch. A switch closing only one side of a circuit (one blade) is called a single-pole switch; one closing both sides (two blades) is called a double-pole switch. Evident abbreviations are used, such as S.P.S.T. for single-pole single-throw; D.P.D.T. for double-pole double-throw.

Single- and Multi-blade Knife Switches. — Up to 1000 amperes in capacity knife switches are usually made with single blades. For larger capacity two or more blades per pole are supplied in order to secure sufficient contact surface without making the blades and jaws of abnormal width.

Field Switches (Fig. 1). — A modification of the standard knife switch (q.v.) as shown in Fig. 2 is furnished for use in connection with field circuits. These field switches, whether single-pole or double-pole, single-throw or double-throw, are provided with an auxiliary contact and extra jaw which are used for connecting a resistance across the field terminals before the field is disconnected from the source of supply. The field discharges through this resistance without any inductive kick such as will occur on the sudden opening of a highly inductive circuit without such a device.

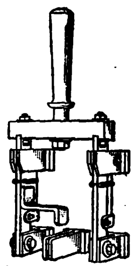

Fig. 1. Quick-break Field Switches

Auxiliary Breaks and Quick-break Attachments are furnished with knife switches in many cases so as to make it impossible to draw a dangerous arc

by opening the switch slowly. Fig. 2 shows a typical quick-break knife switch. The main-switch blade here carries auxiliary blades, which are attached to the main blades by a hinge and spring. When the switch is opened, the auxiliary blades are held in the jaws by friction until they are suddenly jerked out by the spring tension. A similar attachment is used on the field switch shown in Fig. 1.

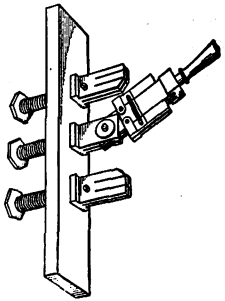

Fig. 2. Quick-break, Single-pole, Double-throw Switch

Starting Switches. — A modification of the standard knife switch is the multi-point starting switch used occasionally for starting d-c. motors, or rotaries or motor-generator sets from the d-c. end. These multi-point starting switches consist usually of single-pole, single-throw switches with several break jaws connected to various steps of the starting resistance.

Disconnecting Switches. — In all high-tension circuits it is customary to install knife-type disconnecting switches for isolating feeders, oil circuit breakers, etc., or for making various connections that do not have to be opened under load. In American practice the knife switches for 3300 volts or less are usually mounted directly on a base of soapstone, marble or similar material; for higher voltages insulators of various kinds are used to support the switch jaws. Up to 33,000 volts these disconnecting switches are made for either front connection, rear connection, or both, but for higher voltages they are almost invariably made for front connection only.

Disconnecting Switches Mounted on Pillar-type Insulators. — Fig. 3 shows a disconnecting switch designed for 13,000 volts. A similar construction is used for voltages up to 110,000. The insulators are of the pillar type and are given a dry test of three times normal voltage. On the larger sizes of this type of switch a trussed blade is furnished to secure rigid construction, and safety catches are supplied to prevent the switches jarring open. The caps holding the jaw blades are clamped to a wall or other flat structure after they are removed from the wooden templates on which they are shipped. On the 110,000-volt switch of one make the center line of the blade is 36¼ inches from the base and the jaws are 44¹⁄₁₆ inches on centers.

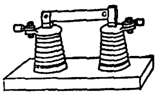

Fig. 3. 13,000-volt Disconnecting Switch

Disconnecting Switches Mounted on Suspension Insulators. — For voltages above 110,000 a type of disconnecting switch using the suspension type of insulators is often found more satisfactory. The main feature of this type of switch consists of a series of suspension-link insulators with a funnel-shaped contact at the lower end. This funnel-shaped contact can be dropped over a pin contact attached to the terminal of an oil circuit breaker or transformer. A flexible lead connects from the funnel-shaped movable contact to the busbars or other source of power.

BRUSH SWITCHES have laminated copper brushes which press against solid copper blocks when closed. They are occasionally employed instead of the usual knife switches (q.v.) to handle large currents at low voltage, since large knife switches with their large rubbing contact surfaces are sometimes difficult to manipulate. By using a toggle or similar device to increase the pressure between the laminated brush and the solid contact blocks such excellent contact is obtained that 200 or 300 amperes per square inch of contact can be used and a small potential drop and a low temperature rise still be obtained.

Electrically-operated Brush Switches. — Brush contact switches are particularly well adapted for electrical operation. As brush switches are normally held closed by a toggle, latch or similar device and as they tend to come open on the release of the toggle or latch, when electrically operated they are practically non-automatic circuit breakers (q.v.). Electrically-operated brush switches are frequently used in field circuits and in the equalizer connections of large machines where it is desired to locate the equalizer switches at the generators and to control the switches from the switchboard.

DRUM SWITCHES are often used with electrically-operated devices such as rheostats, field switches and circuit breakers. In the type shown in Fig. 4 the operating handle and direction dial are mounted on the front of the switchboard, while the contacts are back of the board and are made part of a drum-type controller. This construction removes even the d-c. operating voltage from the front of the board and readily adapts itself for use on 550-volt control circuits. This

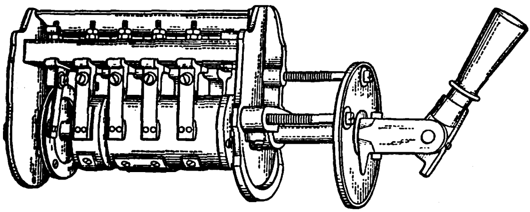

Fig. 4. Drum-type Control Switch

particular control is intended for use with oil circuit breakers. The handle is so designed that after turning to the trip position it can be lifted to stand at right angles to the plane of the dial plate when the circuit is to be put out of service for any length of time. In this position all the indicator lamps are disconnected. For use with this controller an electromechanical indicator or lamp indicator with colored prisms is supplied.

PLUG SWITCHES are commonly used for interrupting small currents at low voltages. They are particularly applicable for such uses as reading the voltage across various phases or circuits. By placing suitable receptacles on generator and feeder panels and using only one voltmeter plug switch, one instrument in a suitable location can be used for reading the voltage across any phase of any circuit without the possibility of trouble from an attempt to connect two or more circuits to the voltmeter at the same time. Other types of receptacles and plugs are used in connection with current transformers for connecting instruments into any phase of any circuit. By using suitable current and potential receptacles it is feasible to connect in testing meters for calibrating the switchboard instruments without removing them from the board.

Synchronizer Plug Switches may be fitted with auxiliary contacts so that it is impossible to close an electrically-operated breaker in a generator or similar circuit without putting the synchronizing plug in the receptacle used for synchronizing that particular circuit. The tripping circuit, however, is kept independent of the synchronizing receptacle so that the breaker may be tripped out independently of the position of the plug.

Plug Switches for Arc Service are built in capacities of 10 amperes or more suitable for use on circuits up to 10,000 volts. Each switch comprises a tube of fiber or similar material, with socket contacts at each end, and a plug consisting of a metallic rod or tube with an insulating handle. The fiber tubes are

usually mounted on the rear of the switchboard. The plug when inserted through the hole in the panel connects the contacts at each end of the tube.

Pull- or Push-Button Switches of twin type are supplied for the operation of oil circuit breakers. The switch operated by one button closes the breaker, and the switch operated by the other button opens it. All the contacts and wiring are mounted on the back of the panel, so that the operator cannot touch a live circuit. By using pull buttons in place of push buttons there is little likelihood of the attendant operating the device unintentionally when cleaning or working about the switchboard. Red and green indicating lamps with prismatic lenses are used for signals. A little red and green target located between the button shows the last movement that has been made so that if the target shows one color and the indicating lamps another it is known that the breaker has been tripped automatically.

Push-Button Switches with Signal Devices are sometimes used for transformer-type ground detectors, engine-room signals and similar equipment. These are frequently arranged as the equivalent of double-throw switches normally maintained in one position by a spring to make one set of connections, and making other connections when pushed in by hand or some mechanism.

Button and Snap Switches of various designs are used for interior lighting circuits (*see Wiring of Buildings*).

OIL SWITCH. — (*See Circuit Breakers.*) There is usually no distinction drawn between the terms "oil switch" and "oil circuit breaker," although in some cases the term "oil switch" is applied to a device with jaw or similar contacts that tend to remain closed, and the term "oil circuit breaker" is applied to a device with butt, brush, cone wedge or similar contacts that tend to come open and must be held closed by a toggle, latch or similar mechanism.

END-CELL SWITCHES. — (*See Batteries, Storage.*)

COSTS. — Switches vary so much in design that it is impossible to give a comprehensive table of costs in a limited space. The following table gives the approximate range of prices of a few typical switches.

Type	Voltage	Amperes	Cost
S.P.S.T.	110 to 600	100 to 3000	$1.65 to $72.10
S.P.D.T.	110 to 600	100 to 3000	2.87 to 103.50
D.P.S.T.	110 to 600	100 to 3000	2.67 to 144.20
D.P.D.T.	110 to 600	100 to 3000	4.47 to 207.00
S.P.D.T. quick break	110 to 600	100 to 3000	5.35 to 108.00
Disconnecting	13,000	300	9.00
Disconnecting	30,000	300	15.00
Disconnecting	60,000	300	30.00
Disconnecting	110,000	300	60.00
Brush switches	110 to 600	2000 to 10000	95.00 to 350.00
Simple plug switch	110	10	1.00
Simple snap switch	110	10	0.20

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[S. Q. Hayes.]

SWITCHGEAR EQUIPMENT FOR POWER STATIONS.—(*See also Bus-bars; Circuit Breakers; Fuses; Lightning Protectors; Power Stations; Substations; Switches; Switchboards.*) This article is a summary of modern practice in regard to the arrangement of the control and switching apparatus in generating stations, transforming stations and converting stations. The special building arrangements to suit the switching apparatus are also discussed.

In the earliest plants the switchgear, which then comprised only knife switches, plugs, fuses and lamps, was scattered around the station in a more or less haphazard way, or possibly assembled on one of the walls, so that practically no space was allotted to it that could possibly be used for any other purpose. The next step with increasing amount of auxiliary apparatus involved the placing of the switchgear on a panel switchboard near the wall where very little room was taken up by it. As stations grew still larger, proportionately more space had to be allotted to the switchgear, until in the modern high-voltage, large-capacity plant, one portion of the building, or in some cases a special building, is assigned to the switchgear and designed especially for its proper housing.

As regards switchgear equipment power stations may be classified as follows:

The switching requirements of the several classes are treated in the sections following.

DIRECT-CONTROL SWITCHGEAR IN GENERATING STATIONS.—Direct-control panel switchboards are usually installed for a-c. or d-c. lighting, power and railway service of low voltage and moderate size. The switchboards for such plants are of the panel type and may be located on the station floor, on a platform or in a gallery. All the switching appliances are mounted directly on the panels of the switchboard. With such equipment it is fairly simple to locate the switchboard in such a manner as to reduce to a minimum the amounts of connecting cables and similar material that depend on the relative position of the switchboards and generators.

The amount of space required and the amount of cables, bus-bars and wiring needed in a direct-current railway generating station can be reduced to a minimum by arranging to have only one polarity, usually the positive, on the switchboard, and to place the negative and equalizer busses in the basement or in a conduit near the machine, locating equalizer and negative switches at the machine.

DISTANT-CONTROL SWITCHGEAR IN A-C. GENERATING STATIONS DISTRIBUTING AT GENERATOR VOLTAGE.—In stations that distribute current at the generator voltage there are three usual locations for the bus-bars, oil circuit breakers and control apparatus, depending principally on the amount of space needed for this portion of the installation. These locations are: 1. at the end of the building; 2. at the side of the building, and 3. in a separate switch-house.

Location of Switchgear. — The end of the building is a favorite location for the switchgear when the number of feeders is such that this location provides sufficient space for the breakers and the bus-bars, making due allowance for probable future additions. With this arrangement it is customary in large plants to provide a number of galleries for the switching equipment. The switchboard is usually placed on one of the upper galleries so that the station attendant can readily watch the operation of the machines which he is controlling.

Where the end of the building does not provide sufficient space the switching equipment is frequently located along one of the side walls, usually the side remote from the boiler room in a steam station, or from the incoming penstock in a hydraulic station. The switching equipment when arranged in one or more galleries along the side of the building can easily be extended, as the lengthening of the building provides for the switchgear proportionately with the space available for the generating equipment. With this arrangement it is usually customary to locate the generator breakers directly opposite the individual machines and to run the bus-bars the length of the station; the length of the generator leads will then be reduced to a minimum and it is sometimes possible to use bare conductors for these leads. The switchboard itself, if electrical operation is provided, may be located either on one of the side galleries or at the end of the building in such a position that the switchboard attendant can readily watch the operation of the machines which he is controlling.

An extension of this scheme of utilizing the side walls is to provide a separate switch-house and to control all of the apparatus electrically from a switchboard in the main building or from a switchboard in the switch-house as preferred.

Structures for Bus-bars and Circuit Breakers. — The arrangement of the supporting structures for bus-bars and circuit breakers is influenced chiefly by the voltage and capacity of the circuits. For circuits of moderate capacity and 3000 volts or less, mechanical control is usually employed, though electrical control is sometimes used; for circuits of large capacity or of higher voltage than 3000, electrical control is practically universal.

Structures for Mechanical Control. — Fig. 1 shows a typical arrangement of distant mechanical control applied to circuit breakers mounted on the wal or on a metal framework. The view shows a section through a 2300-volt, 3-phase generator circuit. Each circuit is provided with a 3-pole oil circuit breaker which can be connected to the three-phase bus-bars supported on the wall brackets. Sprocket-operated face plates with suitable resistors are located in the basement to allow for the regulation of the voltage on the generators. The current and voltage transformers for the instruments are located back of the board on a suitable framework or on the wall.

Structures for Electrical Control of Moderate Capacity. — Fig. 2 shows a section and the front and rear elevations of a part of the structure for use with the breakers and bus-bars for the control of four 500-kw., 2300-volt, 3-phase generators, nine 3-phase feeders and six single-phase feeders. Though the capacity and voltage of these machines would have permitted the use of hand-operated oil circuit breakers, other considerations caused the adoption of electrical operation for the breakers and of inclosed bus-bars. The relative locations of the circuit breakers, disconnecting switches, series transformers, shunt transformers, bus-bars, etc., are indicated in the cut. The bus-bars of laminated copper strap are supported on petticoat insulators, and porcelain floor tubes are used for insulating the leads where they pass through the back wall of the structure. The breakers here used are self-contained and all three poles are in the same compartment. The disconnecting switches are mounted directly on soapstone bases.

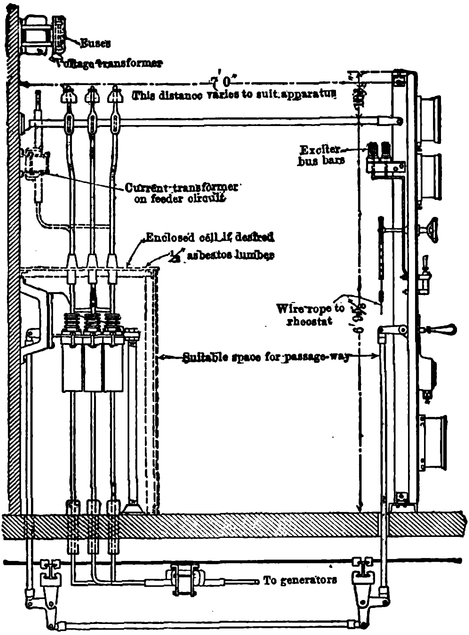


Fig. 1. Structure for Distant Control of 2300-volt, 3-phase Circuit

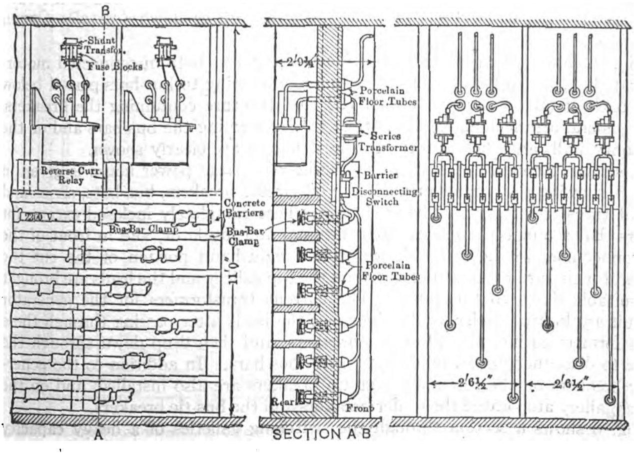


Fig. 2. Structure for Electrical Control of 2300-volt, 3-phase Circuit

The front, side and rear views of a structure for 3-phase, 6600-volt solenoid-operated oil switches of moderate breaking capacity are shown in Fig. 3. The fireproof masonry compartments and bus-bars, connections, etc., are separated

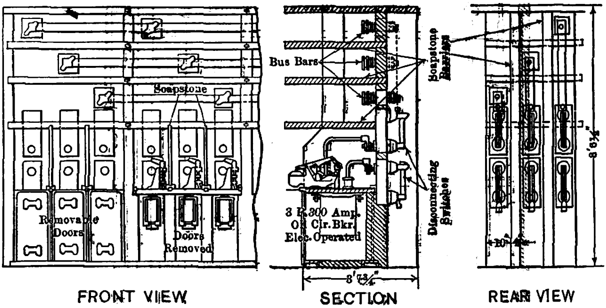


Fig. 3. Structure for Electrical Control of 6600-volt, Solenoid-operated Oil Circuit Breaker

by shelves, walls, septums, etc., in such a manner that no two conductors of opposite polarity are in the same compartment. The bus-bars of laminated copper strap are supported on the insulators that act as bushings for the leads through the walls. The disconnecting switches, partly front connected and partly rear connected, are mounted on porcelain pillars placed on soapstone bases and arranged for completely isolating the breaker.

Structures for Electrical Control of Large Capacity (Figs. 4 to 6).—For larger capacity plants distributing at the generator voltage there are in use two principal types of oil circuit breakers. With motor operation the leads are usually brought out at the bottom of the oil tanks, and with solenoid operation usually at the top. In the latter case the leads are carried through the back wall so that the connections may be run either up or down (*see also Circuit Breakers*).

Fig. 4 shows the typical ways of arranging the bottom-connected motor-operated 13,200-volt oil circuit breaker for connecting to bus-bars placed below (A), back of (B) or independent of (C) the structure containing the breakers. The general arrangement of the copper tubing forming the bus-bars and of the masonry walls and shelves forming the structure are clearly shown.

A section through the switching galleries of a large power house is given in Fig. 5, showing the arrangement of the oil circuit breakers, bus-bars, series and shunt transformers, etc. The busbars are here completely inclosed except for doors that are placed opposite each terminal and insulator, and in front of the disconnecting switches. As shown in the right-hand portion of the cut the generator circuit breakers are located on the top gallery and the leads are brought in suitable ducts to this point. The current transformers for the generator circuit are located under a false floor and the leads after passing through these transformers go into the oil circuit breakers, and then drop down through the floor to disconnecting switches and to the bus-bars. In addition to the generator breakers on the top gallery, group breakers are also installed, and on the lower gallery are located the feeder breakers and the bus-tie breakers.

Fig. 6 shows a section through the switching galleries of a heavy capacity 12,300-volt, two-phase generating station. This station controls the necessary switching equipments for the control of eight 8000-kv-a. turbo-generators and

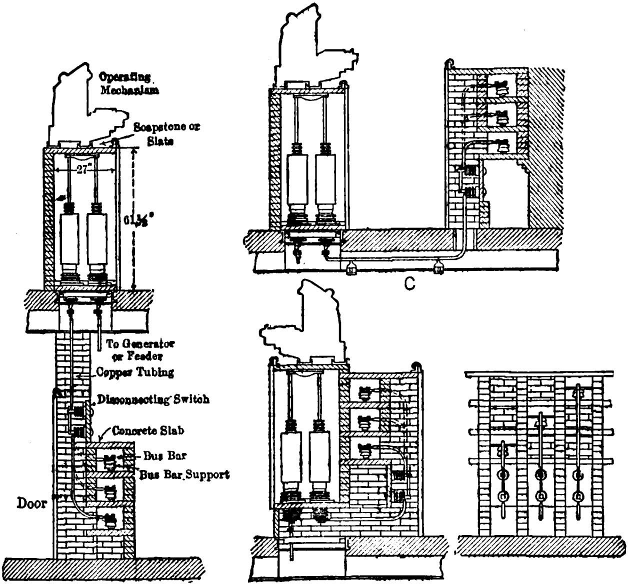


Fig. 4. Structure for Bottom-connected 13,200-volt Circuit Breakers

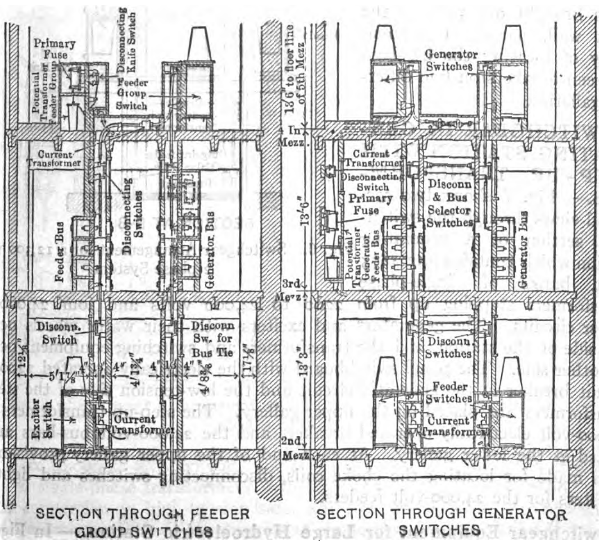
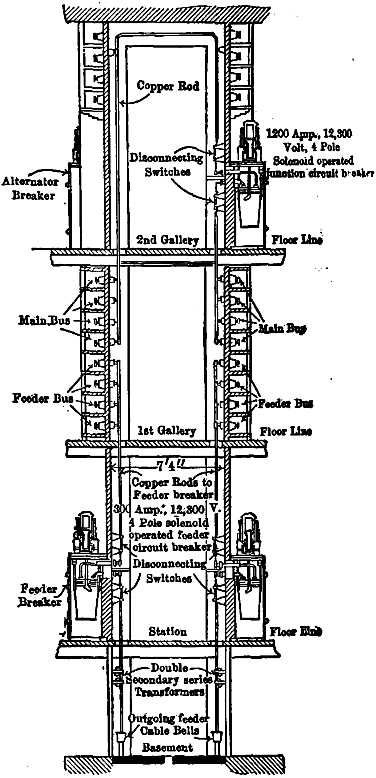


Fig. 5. Section through Switching

forty feeders with a large number of local service circuits. The connections are so made that each generator feeds through its own circuit breaker on to a generator bus, which can be connected through a second breaker to the main bus or through either of two other breakers to two sets of group busses, each group bus supplying current to five feeder circuits. The generator bus-bars are located directly above the breakers in the upper gallery. The main bus-bars are the top sets on the middle gallery and the feeder group bus-bars are the lower sets on the middle gallery. The bus structures are arranged back to back in such a manner that the main bus-bars form one continuous ring, sectioned by means of knife switches and circuit breakers, while the group bus-bars can be connected to form a second ring.

The designs shown in Figs. 5 and 6 show one of the advantages resulting from the use of top-connected breakers with the leads brought out through the back wall, namely the possibility of locating the bus-bars between breakers on two different galleries.

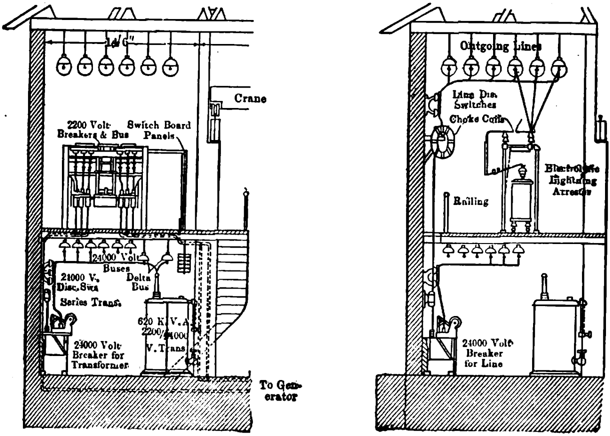


SECTION AT B-B

Fig. 6. Switchgear Arrangement for a 12,300 volt, 2-phase System

SWITCHGEAR IN GENERATING STATION WITH STEP-UP TRANSFORMERS. — Fig. 7 shows two sectional views through the switchgear section of a generating station which contains four 1875 kv-a. banks of single-phase transformers stepping up from 2200 to 24,000 volts and four 24,000-volt feeder circuits. The generators and exciters with their water wheels occupy one side of the station, and the transformers and switching equipment occupy the other side. The panel switchboard with the electrically operated 2200-volt circuit breaker in the generator circuit and the low-tension side of the step-up transformers are placed on the upper gallery. The step-up transformers with 24,000-volt electrically-operated breakers and the 24,000-volt bus-bars are all placed in the lower gallery. At each end of the upper gallery provision has been made for locating the choke coils, disconnecting switches and lightning arresters for the 24,000-volt feeders.

Switchgear Equipment for Large Hydroelectric Station. — In Fig. 8 is given a sectional view through a generating station with two 500-kw., 250-volt



Generator Connections | Line Connections

Fig. 7. Switchgear Arrangements for a 24,000-volt Station

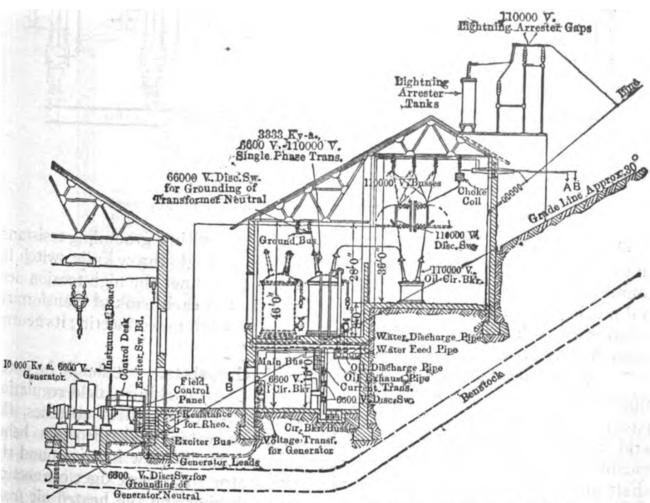


Fig. 8. Switchgear Arrangement for a 60,000-kv-a., 110,000-volt Hydro-electric Station

exciters; six 10,000-kv-a., 6600-volt, 3-phase generators; six banks each of three 3333-kv-a. single-phase transformers with delta connection on low-tension side and star connection on high-tension side; and four 20,000-kv-a., 110,000-volt, 3-phase transmission lines. The diagram of connection for this station is shown in Fig. 9.

Bus Connections. — Each generator as shown in Fig. 9 is normally used with its own bank of step-up transformers. Three circuit breakers, however, are provided with each transformer and generator group so that, if desired, any generator or any bank of transformers may be connected to the 6600-volt

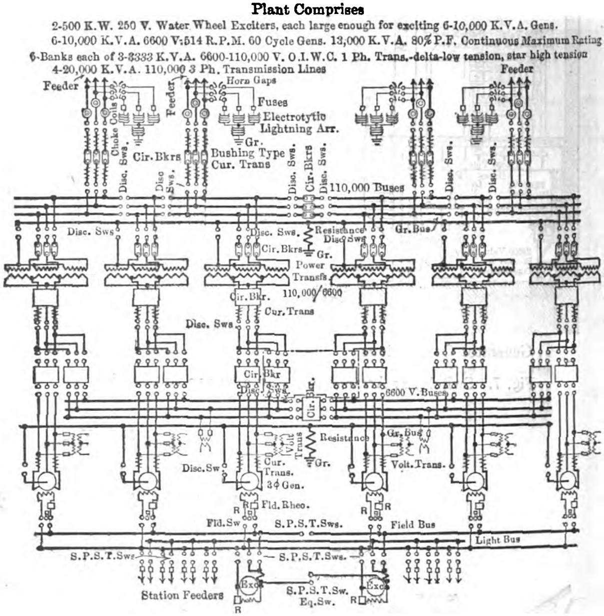


Fig. 9. Diagram of Connections for Plants

bus-bars. The low-tension neutral bus is provided with a grounding resistance and each generator is furnished with a single-pole, single-throw knife switch for connecting it to this grounding bus. In a similar manner the high-tension neutral bus is provided with a grounding resistance, and each bank of transformers is furnished with a single-pole, single-throw knife switch for connecting its neutral point to this ground bus.

Ventilation of Generators and Field Rheostats. — It should be noted that it is intended to locate directly opposite each machine a field-regulating panel containing electrically operated field rheostats and field switches, the grid resistances used with these electrically operated field rheostats being located just outside the building. Each generator will draw in air around the shaft and discharge it at the bottom of the stator to a short duct connecting with the tail race. With this arrangement of discharging the heated air from the generators into the tail race and locating the field rheostat resistors of the generators outside the building, the question of securing proper ventilation is greatly simplified. The generator building is separated from the transformer and switch house by about 20 feet in order to secure better lighting and better ventilation for both buildings.

Switching Gallery. — The control desk, instrument board, and exciter switchboard are placed on a gallery in such a manner that the station attendant

standing at the desk can readily observe the operation of the generator which he is controlling. The switching gallery is on a level with the basement floor of the transformer and switch house so that the switchboard attendant can readily pass into the basement of the switch house.

Switch House. — The basement of the switch house contains electrically operated oil circuit breakers, bus-bars, disconnecting switches, series and potential transformers for the 6600-volt generator and transformer circuits, as well as the oil and water piping for the various transformers. The 6600-volt circuit breakers and bus-bars are inclosed in masonry compartments, but the 110,000-volt circuit breakers and bus-bars are open, owing to the great difficulty and expense of installing masonry compartments for 110,000 circuits and the doubtful benefit to be obtained by such a course. On the main floor the transformers are arranged in one row and are mounted on cast-iron bases provided with wheels in such a manner that any transformer can be rolled out onto a low truck and then pulled outside the building, after the various water pipes, oil pipes, low-tension and high-tension connections have been opened.

ARRANGEMENT OF SWITCHGEAR FOR INDOOR STEP-DOWN TRANSFORMER STATIONS. — Fig. 10 shows the section of a transformer station for the control of two 44,000-volt, 3-phase incoming lines,

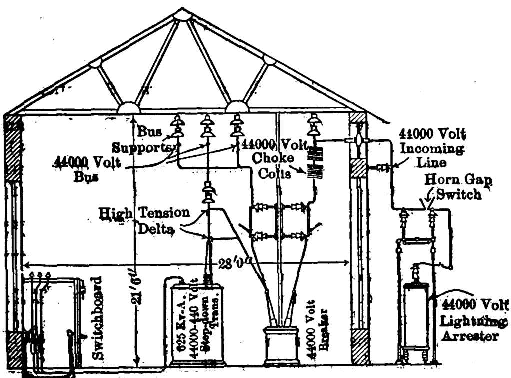


Fig. 10. 44,000-volt Step-down Transformer Substation

four banks each of three 625-kv-a. single-phase step-down transformers, and a number of 6600-volt feeder circuits. The 44,000-volt lightning arresters with their horn-gap disconnecting switches are placed out of doors, and the circuit breakers, transformers, disconnecting switches, and busses are located indoors as shown. The incoming leads pass through porcelain bushings in the walls to the choke coils and through disconnecting switches to the 44,000-volt breakers. From these breakers the current passes through other disconnecting switches to the 44,000-volt bus-bar that is hung from the ceiling by means of suspension insulators. From this bus the current passes through other disconnecting switches to the circuit breakers, and thence to the high-tension side of the step-down transformers.

As an example of a station designed for still higher voltages, Fig. 11 shows the arrangement of a plant for two 140,000-volt, 3-phase incoming lines, three banks of step-down transformers and the necessary switching equipment. In this

station the electrolytic lightning arresters are located on the roof of the station, and the disconnecting switches, circuit breakers, transformers, etc., are inside the station.

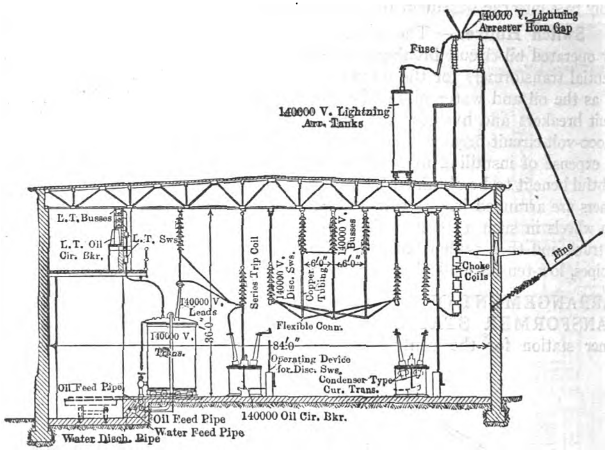


Fig. 11. 140,000-volt Step-down Transformer Substation

Comparison of Top- and Bottom-connected Breakers and Closed and Open Wiring (Fig. 12). — (*See also Bus-bars; Circuit Breakers.*) In order to illustrate the difference in the design of the station made necessary by the use of bottom-connected breakers and inclosed bus-bars for the high-tension circuits, Figs. 12A, 12B and 12C show three different designs of switching equipment for the control of 66,000-volt, 3-phase step-down transformers (each 10,000 kv-a.) supplying current to the two sets of 13,200-volt bus-bars.

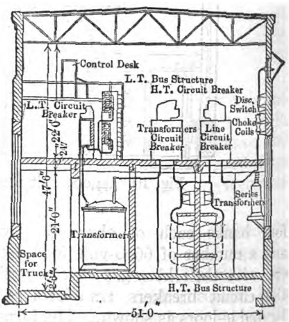


Fig. 12A. Bottom-connected Breakers, Inclosed H.T. Bus-bars, 66,000-volt Substation

Fig. 12A shows the general arrangement of the circuit breakers, bus-bars, connections, etc., using bottom-connected breakers of standard design and arranging to locate the 66,000-volt bus-bars with their disconnecting switches on the lower floor. In order to provide sufficient headroom for lifting the coils and iron out of a transformer case, it is necessary to slide the transformer into the passageway and run it along to the central portion of the building where the floor has been raised under the control desk in such a manner as to provide the necessary headroom. With this arrangement the total height of the building from the floor line to the roof girders is 47 feet 6 inches.

Fig. 12B shows the arrangement necessary if it is desired to use top-connected breakers and still inclose the 66,000-volt bus-bars. With this arrangement the 66,000-volt bus-bars, as well as the 13,200-volt circuit breakers with their

bus-bars, and the control desk, are placed on the upper floor, while the 66,000-volt breakers themselves with their disconnecting switches are located on the main floor near the transformers. With this arrangement any transformer can have its coils or iron removed as soon as it is slid into the passageway. The

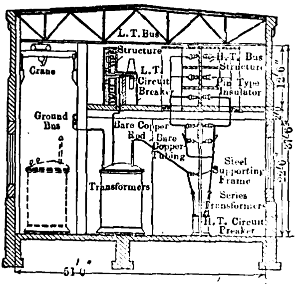


Fig. 12B. Top-connected Breakers, Inclosed H.T. Bus-bars, 66,000-volt Substation

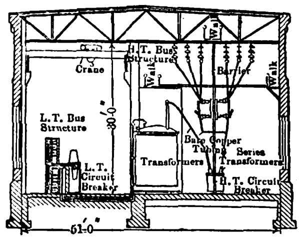


Fig. 12C. Top-connected Breakers, Open H.T. Bus-bars, 66,000-volt Substation

building arranged in this manner requires a height of 37 feet 6 inches from the floor line to the roof girders and requires a second floor the same as shown in Fig. 12A.

Fig. 12C shows the arrangement of this same station with top-connected breakers and open bus-bars and wiring for the 66,000-volt circuits. With this arrangement there is no necessity of a second floor. The height of the building is greatly reduced as the distance from the floor line to the bottom of the roof girders is only 30 feet.

ARRANGEMENT OF SWITCHGEAR FOR OUTDOOR TRANSFORMER STATIONS. — The use of outdoor transformers, switchgear and protective devices should be considered when designing high-voltage installations where it is essential to keep the first cost of the installation down to a minimum.

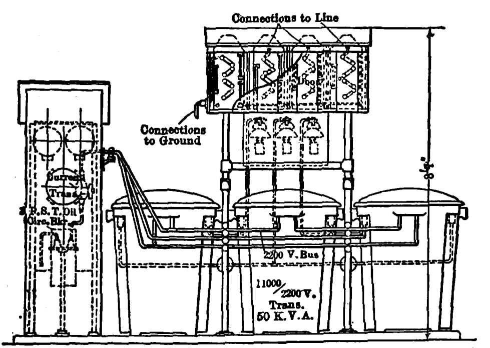


Fig. 13. Switchgear Arrangement for 110,000-volt, 150-kv-a. Outdoor Transformer Station

11,000-volt Station. — In Fig. 13 is shown an installation of three 50-kv-a., 11,000-volt, single-phase transformers with outdoor type expulsion fuse

switches and non-arcing lightning arresters. These arresters are placed in a wooden box with a swinging door. The low-tension switchboard is placed in a similar wooden box with a glass door so that the readings of the ammeter, voltmeter and watt-hour meter can be taken without opening the switch box.

33,000-volt Station. — A 33,000-volt installation with three single-phase self-cooling transformers is shown in Fig. 14. The layout also includes a three-pole, high-capacity, steel-tank, oil circuit breaker, a three-phase electrolytic lightning arrester and a small shed (not shown) containing the low-voltage switchboard.

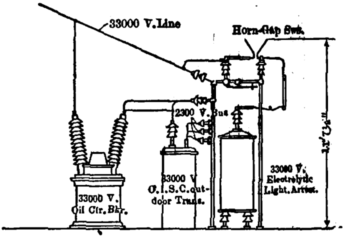


Fig. 14. Switchgear Arrangement for 33,000-volt, Outdoor Transformer Station

When installing these breakers all joints are painted with a heavy gum to safeguard against the entrance of moisture, and the breakers are painted regularly the same as any steel work exposed to the weather. Samples of oil are taken from the bottom and the top of the circuit-breaker tanks at regular intervals and if any evidence of an excessive amount of carbon in the oil is found the oil is immediately changed.

110,000-volt Station. — Fig. 15 shows a 110,000-volt installation with two incoming lines, two out-going lines, four banks each of three 3333-kv-a., oil-insulated, water-cooled transformers stepping down to 11,000 volts, with

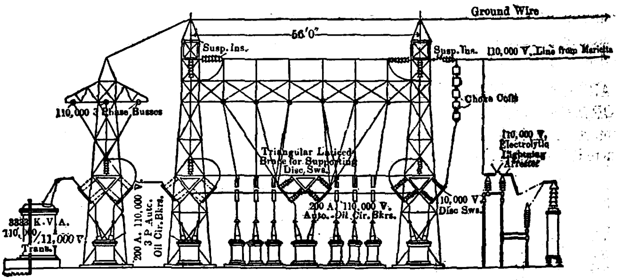


Fig. 15. Switchgear Arrangement for 110,000-volt, Outdoor Transformer Station

provision for two future 110,000-volt lines and additional banks of step-down transformers. The high-tension busses and the disconnecting switches are mounted on steel structures supported on transmission-line towers. The oil circuit breakers and transformers are located on the ground below the disconnecting switches. The transformers are mounted on wheels so that they can be run on a transfer truck and then hauled along a track until they are in the proper position to be pulled into the station for inspection and repair.

Owing to the number and the capacity of the low-tension circuit breakers needed for properly distributing the 11,000-volt current in an installation of this capacity, it is necessary to provide a small building (not shown) to contain the low-tension oil circuit breakers and the low-tension bus-bars. For this service, the circuit breakers and bus-bars have to be installed in masonry compartments owing to the enormous current available on the busses in case of short-circuit. The building also contains the small local-service transformers

with motor-generator sets, storage battery for the electrical operation of the main circuit breakers, and the switchboard on which the controlling devices for the various circuits are mounted.

Advantages and Disadvantages of Outdoor Stations. — The foregoing illustrations of outdoor stations indicate the tendency of design for such installations. The advantages of outdoor apparatus such as described are: (1) the cheapening of the installation due to saving in building, and (2) the reduction in life and fire hazard, owing to the fact that in an outdoor installation the apparatus can be well scattered without materially increasing the expense of the installation. The disadvantages are: (1) the absence of protection from the weather when inspecting, overhauling, or making repairs, and (2) the danger of trespassers to themselves and to the apparatus. The outdoor *apparatus* is somewhat more expensive than the corresponding indoor apparatus, but the difference in cost will be usually more than offset by the saving in *building* investment.

SWITCHGEAR EQUIPMENT FOR CONVERTING STATIONS IN BUILDINGS. — The proper grouping of the apparatus in a rotary-converter station depends on the voltage of the a-c. circuit, size of rotaries, type of transformers, and similar features. The building varies accordingly, provided the shape and size of the available lot is such as not to hamper the design of the station.

The sectional view of a rotary-converter substation containing 1000-kw., 6-phase rotaries with air-blast transformers fed from 13,200-volt, underground circuits is shown in Fig. 16. The incoming leads from the cable ducts pass

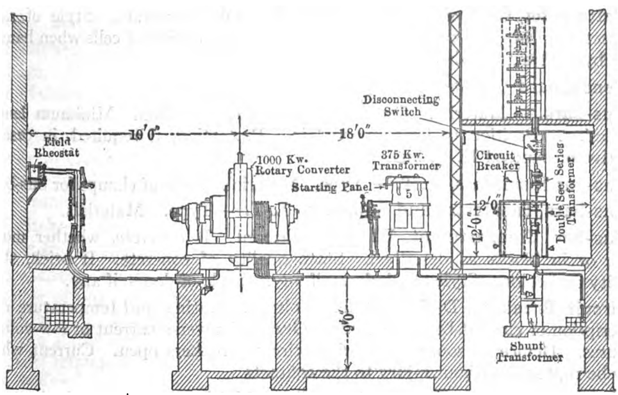


Fig. 16. Arrangement of Switchgear Equipment in Rotary Converter Station

through an oil breaker and disconnecting switches to the bus-bars that are located on a gallery. Provision is made for an additional set of bus-bars and an additional set of disconnecting switches to be installed at a later date so that any breaker may be connected to either of the two sets of busses.

The circuits from these bus-bars pass back through other disconnecting switches and breakers to the high-tension terminals located at the bottom of the air-blast transformers. The low-tension leads from the transformers go to a starting panel provided with double-throw switches that permit low voltages to be impressed on the rotary for the purpose of starting and full voltage for running. The rotaries are provided with series fields on the negative side.

The negative and equalizer switches are placed on a pedestal at the machine, and the negative and equalizer busses run on a bracket in the basement. The positive leads run to the panel board near the left-hand wall and the positive bus is located on the back of this board. The railway feeders are run out through underground ducts. All of the high-tension a-c. circuits are provided with electrically operated breakers controlled from the main switchboards. It will be noted that the entire design of this station hinges on the proper arrangement of the switching equipment.

SWITCHGEAR EQUIPMENT FOR PORTABLE CONVERTING STATIONS. — Many interurban electric railways have portable substations located in freight cars and arranged for ready transportation to whatever point requires their temporary service. A typical installation of this kind consists of a rotary converter; oil-insulated, self-cooling transformers; and the necessary panel switchboard, high-tension oil circuit breaker, and lightning-protective devices. The apparatus may be so arranged in the car that the operator is convenient to the handle of the high-tension oil breaker, the panel switchboard, and the commutator end of the rotary converter.

SPECIFICATIONS FOR SWITCHGEAR EQUIPMENT.* — The great variety of conditions to be met and the numerous ways of meeting each condition in switchboard work make it impossible to write specifications in the standard form suitable for general use. This specification is therefore in the form of a series of memoranda to assist in writing a more specific one. *(See also general article on Specifications.)* The items are arranged in alphabetical order.

Barriers. — Purpose. Material. Method of support. Dimensions.

Battery for Control. — Number of cells. Discharge rate. Style of rack. Protection of exposed copper from acid fumes. Condition of cells when handed over by contractor (whether charged or not).

Benchboard *(see Switchboard)*.

Bus-bars. — Material. Conductivity. Style of joints. Minimum length of section. Position to be supported in. Protection, if required, in case of positive or exposed H. T. bus.

Bus-bar Insulators, H. T. — Type. Material. Style of clamps for bus-bars.

Bus-bars, Insulators and Supports, L. T. — Type. Materials.

Bus-bar Compartment, H. T. — Material. If concrete, whether monolithic or in blocks as described. State quality of concrete. If brick, state quality and color. Style of windows, if any. Style of doors, if any.

Circuit Breakers, D-C. — Style. Voltage. Rating and temperature rise. Description of current-break features. Overload reverse-current or low-voltage features. Device to sound gong when circuit breakers open. Current which may be ruptured without injury to the contacts.

Conduits in Floors and Walls. — Size. Finish. Shall be laid when contractor is notified that floor is ready. Junction boxes, if any, shall have interiors of stated finish and shall be provided with a neat metal cover flush with the floor. Style of pipe joint to be such that any single length may be removed without disturbing others.

Crane Service Connection. — Apparatus required and location. Wiring.

False Floors. — Material of slabs. Description of slabs. Description of piers for supporting slabs. Style of fastening between slabs and piers.

Gongs. — Type and location. To what apparatus they are to be connected.

Ground Connections. — General description. Dimensions.

* By W. A. Del Mar.

High-potential Tests. — After installation, the apparatus and wiring shall be subjected to the potential tests specified in the following table, the tests being performed in accordance with the Standardization Rules of the American Institute of Electrical Engineers. (Give table of test voltages for different classes of apparatus and wiring, preferably from Section of the *Standardization Rules*, q.v.)

Instruments. — Type. Finish. Rating and description of scale. Accuracy at various parts of the scale to be within stated limits of accuracy. Location and style of terminals. Style, rating and temperature rises of shunts. Magnetic shielding.

Insulation (*see Insulating Materials*).

Lightning Arrester Equipment. — Type and purpose. Accessibility of all parts. Removability of all parts which may be injured by a stroke of lightning. Switches for disconnecting arresters from line. Description of ground plates and ground connections. Choke coils. Barriers.

Name Plates. — Description. Dimensions.

Relays. — Style. Location. Function.

Rotary-converter Starting Accessories. Method. Description of apparatus pertaining thereto.

Station Shunt. — Style. Location. Whether in positive or negative bus. Rating and temperature rise.

Switchboards. — Purpose. Material and color. General dimensions. Bench or upright. Number of sections per panel. Style of support. Sills, quality and finish (should not be painted if they are to be set in concrete). Size of bevel. Barriers: material, size and finish. Illumination.

Switches, H. T., Electrically Operated. — Style. Number of phases. Voltage. Rating and temperature rise. Accessibility of all parts. Description of concrete or brick compartment. Description of barriers between phases. Style of doors. Structure in base or elsewhere for disconnection switches. Voltage limits between which control apparatus shall operate successfully. Device to indicate positively at the controlling board whether the oil switch is open or closed. Device to sound gong when oil switch opens. Amount of oil to be supplied.

Switches, Oil, Hand-operated. — Style. Number of phases. Voltage. Rating and temperature rise. Location.

Switches, Knife or Toggle, Hand-operated. — Style and material. Number of poles and throws. Voltage. Rating and temperature rise. Quick break features, if required. Current density in metal and at contacts shall not exceed a stated value. With or without base, and style of base, if any. Front or rear terminals. Style of terminals. Long-handled hooks for h.t. knife switches.

Switches and Circuit Breakers, L.T., Electrically Operated. — Circuit breakers (*see also Circuit Breakers, D.C.*). Switches (*see also Switches, H.T.*). Device to indicate positively at control board whether switch is open or closed. Voltage limits between which control apparatus will operate sucessfully.

Transformers for Instruments. — Style. Ratio. Rating and temperature rises. Maximum permissible percentage error in ratio.

Wiring, Power Circuits. — Use. Location. Shall be clamped so that wires will not be dislodged should any joint or terminal become loose. Description of protection where cables pass through floors or walls. Tagging with number of circuit. Conductivity. Size or carrying capacity. Conductivity of clamps or terminals. Style of insulation.

TOTAL COST OF SWITCHGEAR EQUIPMENT

These costs should be used with caution; *see paragraph on Costs in text, p. 1503.*

No.	Type of station	Breaker equipment	Volts	Number of Gen.	Number of Feed.	Number of Trans.	Number of Lines	Price
1	Generating	Hand operated, moderate capacity	2,300	1	..	..	..	$450
2	Generating	Sol. operated, small capacity	2,300	1	..	..	..	525
3	Generating	Sol. operated, moderate capacity	6,600	1	..	..	..	825
4	Generating	Motor operated, large capacity	13,200	1	..	..	..	1,100
5	Generating	Solenoid operated, large capacity	13,200	1	4	..	..	12,000
6	Generating	Solenoid operated, large capacity	12,300	1	5	..	..	15,000
7	Gen. and transformer	Solenoid operated, moderate capacity	24,000	4	..	4	4	18,000
8	Gen. and transformer	Solenoid operated, large capacity	110,000	6	..	6	4	85,000
9	Gen. and transformer	Solenoid operated, large capacity	110,000	6	4	4	2	130,000
10	Transformer, indoor	Solenoid operated, moderate capacity	44,000	..	..	4	2	14,000
11	Transformer, indoor	Solenoid operated, large capacity	140,000	..	4	2	1	27,000
12	Transformer, indoor	Solenoid operated, large capacity	110,000	..	10	4	2	95,000
13	Transformer, outdoor	Expulsion fuses	11,000	..	1	1	1	325
14	Transformer, outdoor	Hand operated	33,000	..	1	1	1	2,200
15	Transformer, outdoor	Elec. operated, large capacity	110,000	..	6	4	4	60,000
16	Rotary, indoor	Elec. operated, large capacity	13,200	2	4	2	2	7,500
17	Rotary, portable	Hand operated, small capacity	16,500	1	1	1	1	1,150

Wiring for Control and Instruments. — Use. Location. Accordance with city ordinances and rules of "National Board of Fire Underwriters." Ends of each conductor shall be tagged with numbered tags of stated material and design fastened with brass wire. Wires which it is inadvisable to keep together shall be in different pipes as directed by the Engineer. Conductivity. Size. Style of insulation.

COSTS. — It is impossible to give reasonably accurate figures upon the cost per kv-a. of switch-gear equipment because of the large number of independent factors entering each case, such as type of station, voltages, number of generators, feeders, transformers and lines, single-throw or double-throw switching arrangements, etc. The figures in the preceding table may be used as a basis for approximate preliminary estimates, the prices including all switches, circuit breakers, bus-bars, structures, cables, wiring, lightning arresters, panel boards, control desks, instrument transformers and including erection and installation with all main wiring between apparatus and switching equipment.

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[S. Q. Hayes.]

SYNCHRONIZERS AND SYNCHROSCOPES. — (*See also Alternating Currents; Converters, Synchronous; Generators, Alternating-Current; Power Stations; Substations; Switchgear Equipment for Power Stations.*) The two principal functions of any synchronizing device are to indicate (1) when the two circuits to which it is connected are operating at the same frequency, and (2) when the voltages of the two circuits are in phase with each other. The voltages of the two circuits must also be equal in effective value; this is usually determined by means of voltmeters connected to the two circuits. The process of synchronizing is simplified if the synchronizer indicates whether the machine to be synchronized is operating at a frequency lower or higher than that of the system to which it is to be connected.

The name "synchroscope" is usually applied to any device which, in addition to indicating synchronism, shows whether the machine to be synchronized is fast or slow (*see Standardization Rules of the A.I.E.E.*). The name "synchronizer" is used in a broader sense for any kind of synchronizing device.

Synchronizing Lamps. — The simplest form of synchronizer consists of two incandescent lamps shunted around the main switch which connects the machine to the bus-bars; see Fig. 9A, p. 648. Equality in frequency, voltage and phase between the bus-bars and the machine to be synchronized is then indicated by continued darkness of the lamps. For three-phase circuits a similar arrangement may be used, if the lamps are connected between corresponding phases of the two machines which are to be brought into synchronism with each other. If the machines are not in synchronism the frequency of pulsations of brightness of the lamps will be proportional to the difference of the frequencies of the two machines. The lamps do not show whether the machine is running too fast or too slow, nor is the point of exact synchronism very definitely indicated, as it requires an appreciable potential to cause the lamps to glow. The latter difficulty may be avoided in the case of single-phase machines by employing the connections shown in Fig. 9B, p. 648; when the lamps are thus connected they indicate synchronism when at maximum brightness.

Use of Transformers with Synchronizing Lamps. — Transformers are frequently used in connection with the lamp. The arrangement with two transformers is to have the primary of one connected across the terminals of the machine, and the primary of the other connected across the bus. The secondary windings are connected in series with the lamp. The secondary connections may be arranged so as to have either darkness or maximum brightness at the instant of zero phase displacement between the bus-bar voltage and the voltage of the machine. Instead of using two separate transformers a single transformer with two primary windings and one secondary winding may be used; such a transformer is called a "synchronizing transformer." The primary windings are usually connected to give full voltage across the lamps at synchronism.

Siemens and Halske Three-phase Synchronizing Lamps. — Three similar lamps L_1, L_2 and L_3 are connected between the three terminals A_1, A_2 and A_3 of the machine and the bus-bars B_1, B_2 and B_3. Lamp L_1 is connected between corresponding phases, that is, between A_1 and B_1. Lamp L_2 is connected between A_2 and B_3 and lamp L_3 is connected between A_3 and B_2. Synchronism will be reached when L_1 remains dark; L_2 and L_3 will then be equally bright. If the machine falls out of step, the lamps will successively grow bright and dark in rotation, no two lamps being in darkness or at maximum brightness at the same time, i.e., the light will appear to travel from one lamp to another at a definite speed and in a definite direction. The speed of rotation of the light is proportional to the difference of frequency between the voltage of the bus-bars

and that of the machine. The direction in which the light appears to travel indicates whether the frequency of the machine is greater or less than that of the bus-bars. For high-tension circuits, potential transformers are required.

Dial Synchroscopes. — Dial synchroscopes are similar in construction to power-factor indicators (q.v.), but are connected between the two circuits which are to be brought into synchronism. They are designed to indicate by a movable pointer whether the machine to be synchronized is running too fast or too slow, or whether it is in synchronism with the circuit to which it is to be connected.

Moving Coil Type. — A fixed coil is connected through a resistor across the bus; in polyphase circuits across one phase. The movable coil is connected through a condenser to the terminals of the machine to be synchronized, and is normally held in a position at right angles to the fixed coil by means of a spring. There will be no torque acting on the movable coil when the machine and bus voltages are either exactly in phase or 180° out of phase with respect to each other, because in either case the currents in the fixed and movable coils are in quadrature. When the two circuits are not in synchronism, the pointer attached to the movable element will swing back and forth over the scale. In order to distinguish the point of synchronism from that of 180° phase displacement, a translucent glass scale is placed in front of the pointer, the scale being illuminated by a lamp connected to the low-tension side of a "synchronizing transformer" which causes the lamp to be lighted when coincidence of phase between machine and bus occurs. Hence the pointer is visible only during every other swing, i.e., it will appear to rotate in one direction; the direction of rotation indicates whether the machine is fast or slow.

This instrument is applicable to single-phase as well as polyphase circuits, as it is usually unnecessary to indicate synchronism of more than one phase on any machine. The moving-coil synchroscope is ordinarily used with transformers, stepping the line voltage down to 110 volts.

Rotating Iron-vane Type. — This instrument is similar to the moving-vane type of power-factor meter (q.v.). The movable iron vane is magnetized by a stationary coil connected across the machine to be synchronized. A rotating field is produced by passing current from the bus-bars through a split-phase winding and two fixed coils placed nearly 90° apart. The pointer indicates at any instant the phase displacement between the voltages of bus-bars and machine. The speed and direction of its rotation depends on the difference in frequency of the two circuits. When exact synchronism is reached the pointer will remain at rest in a vertical position. The instrument is usually connected through potential transformers; on polyphase circuits only one phase is used.

Lincoln Synchroscope. — This instrument has a rotating iron core carrying two coils placed nearly 90° apart; one coil is connected through a resistance, and the other through an inductance, both being supplied from the machine to be synchronized. A bipolar, laminated iron field is magnetized by means of a coil connected to the bus-bars. The operation of this instrument is similar to that of the moving-coil type of single-phase power-factor meter. A pointer attached to the moving system indicates at every instant the difference of phase angle, as well as the approximate difference of frequency, between the voltage of the bus-bars and that of the machine to be synchronized. The current has to be led into the armature coils through slip rings as the armature is free to revolve. These instruments are furnished for direct connection to circuits of 110 or 220 volts; other circuits require transformers.

Automatic Synchronizers. — These are designed for the purpose of entirely eliminating the judgment of the switchboard operator as to the selection of the proper instant for closing the switch connecting the incoming machine to the bus. Automatic synchronizers are particularly desirable when remote-control

oil switches with appreciable time elements are used; see *Circuit Breakers.* The voltage and speed of the incoming machine must be regulated by the attendant. The principal parts of the automatic synchronizer are two solenoids and a movable contact-making sector which is operated by the joint action of the cores of the two solenoids. There are two equal windings A_1 and A_2 on solenoid A and two windings B_1 and B_2 on solenoid B. The windings A_1 and B_1 are connected in series, and A_2 and B_2 are connected in series. The first circuit is connected across the bus and the second across the machine through suitable potential transformers. Connections are such that when the two potentials are of equal value and have the same frequency and phase, the pull on one core will be zero and that on the other will be a maximum. The cores of the solenoids are connected at the top through a centrally-pivoted lever to which the contact-making sector is connected.

At exact synchronism the pull on the core of A is zero and that on the core of B is a maximum; at 180° phase displacement the pull on the core of A is a maximum and that on the core of B is zero; when the voltages are in quadrature the two cores receive equal pulls. The sector will oscillate back and forth when there is a difference of frequency, the oscillations being similar to the pulsations of the synchronizing lamp. When B is fully drawn in, i.e., for equality and zero phase difference of the two voltages, the sector closes the contacts which close the energizing circuit of the relay switch. The latter closes the circuit for operating the main oil switch. The relay switch is usually necessary, as the current required to operate the main oil switch is greater than the current-carrying capacity of the two synchronizer contacts. Provision is also made for introducing a time element such as to prevent coupling the machine to the bus when the voltages are only temporarily in phase. Adjustments may be performed for varying this time element in accordance with the time element of the main oil switch, and for varying the maximum allowable voltage difference and frequency difference at the instant of connecting the machine to the bus.

Complete equipment for automatic synchronizing includes (1) the automatic synchronizer; (2) a relay switch, mentioned above; (3) one control switch per generator, by which the main switch may be opened but not closed; (4) one potential transformer per generator; (5) one potential transformer for the bus-bars. The potential transformers used in connection with the station meters may be used with the automatic synchronizer. It is preferable to have a set of synchronizing lamps in addition to the above equipment, as the automatic synchronizer gives no direct indication of the frequency and phase of the incoming machine. Automatic synchronizers can be used only in connection with electrically-operated switches or circuit breakers, the auxiliary circuits being supplied from an exciter bus or some other independent source.

COSTS. — A moving-coil synchroscope costs approximately $55; a 9-inch moving-vane synchroscope costs approximately $45; a Lincoln synchroscope for voltages of from 110 to 220 costs from $45 to $55; transformers and accessories are extra in all cases. An automatic synchronizer exclusive of auxiliary equipment costs approximately $125; the auxiliary equipment comprising one relay switch, one control switch, and two potential transformers for 6600-volt circuits at 60 cycles per second costs approximately $85, exclusive of the electrically-operated switch. Only one synchroscope or synchronizer is required in a station, although a spare instrument is usually installed.

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[O. R. SCHURIG.]

TELEGRAPH INSTRUMENTS AND APPARATUS. — (*See also Telegraph Lines; Telegraph Systems; Wireless Telegraphy.*) The more important instruments and apparatus used in the various systems of telegraphy are described below.

KEYS. — A telegraph key is essentially a switch for opening and closing a circuit. Various types of keys are in use.

Standard Morse Key (Fig. 1). — The knob end of the lever is normally held raised so as to open the key by an adjustable coiled spring. The auxiliary switch arm constitutes the "circuit closer" by which the circuit of the line is kept closed, through the key, while the key is not in use.

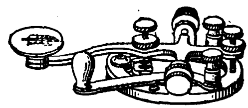


Fig. 1. Standard Morse Key

Double-acting Key. — While the key of the general type illustrated in Fig. 1 is still standard, several later forms, having a sidewise rather than an up and down motion of the lever, have come into some vogue. One type of these, known as the double-acting key, has its operating lever arranged to swing in a horizontal arc between two stationary contact points. The key lever normally stands clear of both points, leaving the circuit open. A movement either to the right or left will close the circuit.

Vibroplex. — Another type of key is represented by such instruments as the Vibroplex, which are more properly semi-automatic transmitters. The motion of the key lever in this is similar to that of the double-acting key just described, the lever normally standing in the middle or open position. When the key is pushed to the left, it closes the circuit in the ordinary way to make a dash. When pushed to the right, a vibratory reed is set in motion, which opens and closes the circuit rapidly to form a succession of dots, the number of dots being controlled by the length of time the key is held in this position. By this means the operator is relieved of the muscular effort of rapidly vibrating the key to form dots.

Operator's Paralysis. — The objects of the double-acting key and of the Vibroplex and other similar transmitters are, in the main, twofold: To relieve or prevent the malady known as "operator's paralysis" or "loss of grip," which sometimes follows long continued use of the ordinary key; and to increase the speed of transmission.

AUTOMATIC TRANSMITTERS. — (*See also Telegraph Systems.*) The transmitters in automatic systems usually employ a paper tape, perforated in accordance with the code to be sent. This tape, so prepared, is caused, by means of revolving rollers, to pass rapidly through the transmitter, the perforations serving as they pass to permit electrical contacts to be made which send the proper impulses to the line.

SOUNDERS (Fig. 2). — A standard form of sounder is shown in Fig. 2. The winding of the electromagnet is connected directly between the binding posts shown on the base of the instruments. The armature is attached to a lever mounted on trunnions in a fixed support. The adjustable stops provided for the lever permit a slight up and down movement of its free end, the downward movement being caused by the pull of the electromagnet, and the upward one by the force of an adjustable retractile spring. The rugged construction of the armature lever and its co-operating parts is required to produce the necessary loudness and quality of sound.

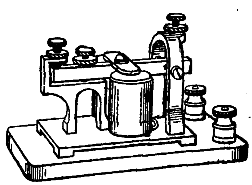


Fig. 2. Morse Sounder

This, with the heavy retractile spring necessary for the prompt return movement of the lever, makes the instrument require more energy for its operation than can usually be delivered to it directly over the line wire. On this account the sounder magnet is usually placed in a local circuit, which is in turn controlled by the more sensitive relay, the magnet of which is placed directly in the line.

When used in local circuits sounders usually have a magnet resistance of about 4 ohms, and require about ¼ ampere for their proper operation. Where the sounder magnet is placed directly in the line, it is termed a main line sounder, and differs in no respect from the local sounder except that its magnet is wound to about 20 ohms.

RELAYS. — Two kinds of telegraph relays are employed; the ordinary or Morse relay, and the polarized relay. The armature of a Morse relay always moves in the same direction irrespective of the direction of the current impulse, while the armature of a polarized relay moves in one direction for a positive impulse, and in the opposite direction for a negative impulse.

Morse Relay (Fig. 3). — The Morse relay has the same essential parts as the sounder, with the addition of a pair of contacts closed by the armature lever when in its attracted position. The armature lever is usually mounted vertically, and is made light in construction so as to be capable of the necessary rapid to and fro movements with a minimum expenditure of energy. The retractile spring is light and delicately adjustable. An extra pair of binding posts is added to form the terminals of the local circuit, these binding posts being wired to the armature and the magnet frame, respectively, so that the circuit between them will be closed upon the attraction of the armature. The back contact of the relay is non-conducting, so as to avoid closing the circuit when the armature is released. A typical form of relay, known as the "pony relay," is shown in Fig. 3.

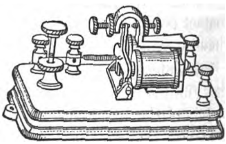


Fig. 3. Morse Relay

Polarized Relay. — A polarized relay is similar to a Morse relay except that the end of the armature which plays between the soft-iron pole pieces is permanently magnetized. For example, if this end of the armature is a north magnetic pole, then, obviously, the armature will be moved in one direction or the other, according as the magnetizing force of the coil tends to set up a south pole in one or the other of the poles. The polarity of the poles, of course, changes with the direction of the current in the coil.

RECORDERS OR REGISTERS. — (*See also Telegraph Systems.*) Recorders or registers are used only where for some reason it is desired automatically to record a message. Simple forms of comparatively slow speed registers are used in connection with district telegraph messenger and fire alarm telegraph service. More elaborate high speed recorders are used in connection with automatic and printing systems. A special form of recorder, known as a siphon recorder, is used on submarine lines.

Slow Speed Registers. — The paper upon which the message is to be recorded is a narrow strip of tape, carried on a reel. A clock-work motor is provided for moving the tape at uniform rate under the recording pen or stylus. The receiving magnet of the register is placed in a local circuit controlled by a relay in the line, and this magnet, when attracted, serves either to move the pen or stylus against the tape or the tape against the pen or stylus, in either case resulting in a mark upon the tape. Some registers record by merely embossing the tape; others by marking thereon with a pen or pencil; and still others by actually punching holes in the tape.

High Speed Recorders. — In very high speed automatic and printing systems receiving devices have been developed giving a record in ink on ordinary paper, or a record on a sensitized paper, or in ordinary print. For a description of the more important printing recorders see below under *Telegraph Systems*.

Chemical Recorders.—For very high speed work with non-printing telegraphs the chemical recorder has been generally found the most satisfactory. In this the tape is impregnated with a solution of some unstable chemical compound which will easily be decomposed upon the passage through it of an electric current, and which, either by changes within itself or by combination with the material of the pen through which the current passes, will cause a definite discoloration of the tape upon the passage of the current.

A common sensitizing solution is formed of 5 parts of prussiate of potash, 150 parts of ammonic nitrate, and 10 parts of water. Under electric decomposition, this solution sets free an acid which attacks the iron of the pen, leaving a blue mark on the paper.

Siphon Recorder. — The high electrostatic capacity of long submarine cables makes necessary some modification of the methods ordinarily employed in overland work. This high capacity results in the variations in the signaling current being less sharply defined than on land lines. The receiving apparatus for long cables must, therefore, be responsive to minute variations in current to such an extent as to preclude the use of the responsive devices employed on land lines.

The siphon recorder is merely a special form of recording galvanometer. The galvanometer proper is of the D'Arsonval or swinging coil type. The motion of the coil is imparted to a very light glass tube bent to form a siphon and suspended with its long end opposite the receiving tape and its short end dipping into a small tank of ink. The siphon moves across the width of the tape, the contact with the tape being agitated so as to be nearly frictionless. Displacements in this record line to the left of an imaginary zero line represent dots, those to the right, dashes. The Continental code is universally employed when a siphon recorder is used.

TELEGRAPH REPEATERS. — When, for any reason, it is desired to transmit messages from one line to another line without manual retransmission, the telegraph repeater is used. By its use lines of excessive length may be avoided, and a consequent increase of speed may be attained. Again, the repeater is often useful in automatically transmitting messages from a through circuit to a side circuit.

Non-Automatic or "Button" Repeater (Figs. 4 and 5). — A fundamental conception of the repeater may be gained from Fig. 4, where a relay magnet

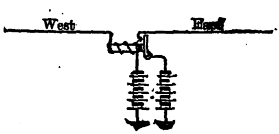


Fig. 4.

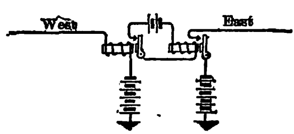


Fig. 5.

placed in one line causes an armature to act as a key to make and break the other line. This conception may be carried a step farther by considering Fig. 5, where a relay magnet in the West line receives impulses from the distant West station, and controls a local circuit containing a battery and a second relay magnet, which latter causes its armature to act as a key in transmitting signals

to the East line. From these two figures it is obvious that such a repeating scheme would transmit in one direction only, providing no facilities for transmitting from East to West.

The "button repeater" of early telegraph history consisted practically of two such sets of relays as are shown in Fig. 5, these sets being arranged for transmission in opposite directions. Only one set of relays could be kept connected to the line, for when both are connected in, the impulses sent out from the repeating station on, say, the West line react on the line relay of that line at the repeating station in such way as to act through its local circuit to send impulses back on the East line. This difficulty was overcome in the button repeater by providing a button switch, operated by an attendant at the repeater station, this switch being thrown manually in one direction or the other, according to the direction of transmission desired.

Automatic Repeaters. — The necessity of an attendant for the purpose of working the switch of a repeater has long since been overcome by the production of the so-called *automatic repeaters*, of which there are many forms. In these, the arrangement is such as to automatically prevent the transmitter of the line *that is being repeated into from reacting to make and break the line over* which the signals are initially sent.

The Milliken Repeater (Fig. 6).— This is one of the most common of the automatic repeaters for the simple Morse system. Its action may be

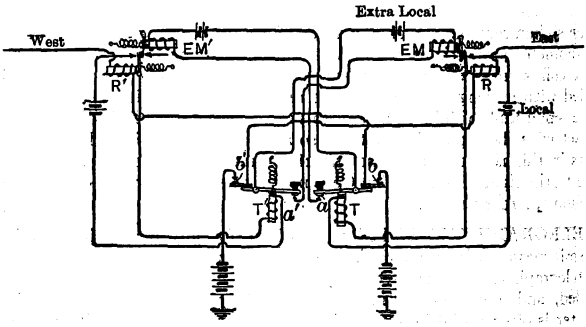


Fig. 6. Milliken Repeater

understood by reference to Fig. 6. If the line East is sending, the relay *R* will respond to the make and break impulses of the key at the distant station on that line, this line being closed to ground and battery at the repeater station through the contacts *b′* on transmitter *T′*, which is held closed by an action that will be described. The operation of the relay *R* of the line East will, by its local circuit, cause the corresponding operation of the transmitter *T*, and this, by the make and break at the tongue *b*, will open and close the line West in accordance with the originally transmitted impulses. The armature of the relay *R* will be free to work, because as long as the magnet of transmitter *T′* remains energized it will hold the circuit of the extra magnet *EM* closed at the point *a′*, and thus will hold the armature of that magnet out of the way of the armature of the relay *R*. The extra magnet *EM′*, associated with the relay *R′* of the line West, prevents this relay from breaking the West line in response to the repeated signals, for the following reason. When the line West is closed by the transmitter *T*, the current through the relay *R′* holds its armature attracted. When

the line West is opened, however, by the transmitter T, the circuit of the extra magnet EM' is also opened by this transmitter, and the retractile spring of the extra magnet prevents the armature of the relay R' from falling back. In this way the line relay of the West line cannot react on its own transmitter to cause makes and breaks in the transmitting line. When the operator at the distant station on the West line breaks, the relay R' is then permitted to open and thus cause the transmitter T' to open the line East.

Duplex and Quadruplex Repeaters.—Automatic repeaters are more simple for duplex and quadruplex systems than those already described for "single" wire working. The reason for this is that in the duplex or quadruplex the transmission from a given sending operator is always to a corresponding receiving operator, and — as between these operators — it is never in the reverse direction. Hence, a one-way repeater only is required for any one of the several simultaneous transmissions. It therefore suffices to place the transmitter at the repeating station under the control of the neutral or polar relays of the duplex or quadruplex, the repeating operation for each set being in one direction only.

CURRENT SUPPLY.—Either a battery or a dynamo may be used to supply the current required for telegraph transmission.

Gravity Battery.—The gravity battery was once almost universally used for supplying current in telegraph systems. It is still largely used on unimportant lines and in small offices where it is not economical to employ machine generators or secondary batteries.

Storage Battery.—The storage battery, on account of its extremely low internal resistance, is capable of handling any desired number of lines in parallel, and this led to the use of the so-called "universal battery system," in which a single battery, with one pole grounded, serves the line wires which are tapped off from the other pole. Suitable resistances were included in the lines of lower resistance, if desired, to compensate for the variation in resistance among the lines. One battery, thus used, would not suffice for an office, however, since with the various systems of duplex and quadruplex working it is necessary to send currents of alternate polarity to line, thus making necessary the use of another battery with its opposite pole grounded. Usually also a reserve was provided for each of these batteries, so that one could be used while another was being charged.

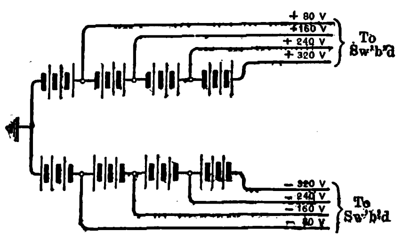


Fig. 7. Storage Battery for Current Supply

In Fig. 7 is shown an arrangement of storage battery supply that provides for any gradation of voltage of either polarity in an obvious manner. By using a duplicate of this arrangement, one set may be charged while the other is being used.

The Dynamo.—The dynamo has generally supplanted both the primary and secondary batteries in all offices large enough to warrant its use. In Fig. 8 a dynamo arrangement employed by the Western Union Telegraph Company is shown. In this five machines constitute a set, their armatures being connected in series, taps being made as indicated to give the required voltages. All of these machines are separately excited except the fifth, which

latter is a shunt machine and furnishes current not only for line supply but also for exciting the fields of all the other machines. Each field circuit is provided with a separate rheostat *R* for varying the field strength. Another set like this is employed to furnish currents of the opposite polarity, and still another set of five machines is employed as a reserve, this being so arranged that

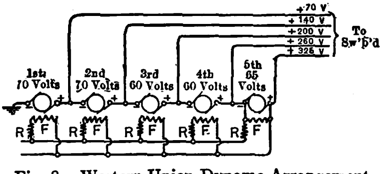


Fig. 8. Western Union Dynamo Arrangement

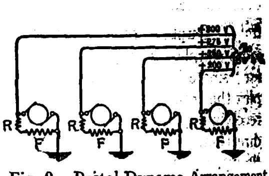


Fig. 9. Postal Dynamo Arrangement

it may be made to generate either positive or negative according to which of the regular sets it is to replace. The dynamo arrangement employed by the Postal Telegraph-Cable Company is shown in Fig. 9.

TELEGRAPH SWITCHBOARDS (Fig. 10).—Telegraph switchboards were the precursors of the vastly more complicated telephone switchboards. The telegraph switchboards which have been most commonly used in America are of two types—the "strap and disc" and the "cross-bar."

A simple "strap and disc switchboard" with its connections is shown in Fig. 10. The lines terminate in the vertical straps and the instrument circuits, each including key and relay, terminate in the horizontal rows of discs, the discs in each row being connected together, as indicated by the dotted line. This figure also shows a typical method of line and instrument protection.

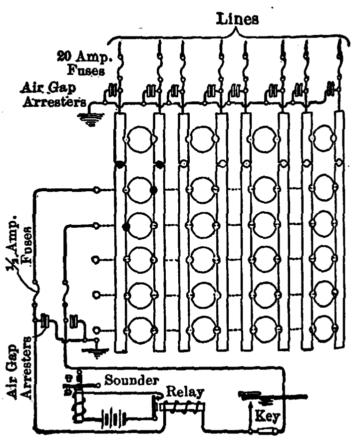


Fig. 10. Strap and Disc Switchboard

COSTS.—The following prices were current in 1913: standard Morse key, $2.20; double-acting key, $10.00; Vibroplex, $12.00; standard Morse sounder, $3.00; standard Morse relay, $7.00; polarized relay, $20.00; ink recorder, $35.00; punching recorder, $75.00. See also *Batteries, Generators,* etc.

BIBLIOGRAPHY.—See Bibliography in article on *Telegraph Systems.*

[S. G. McMeen.]

TELEGRAPH LINES. — (*See also Poles and Cross-Arms; Telephone Lines; Transmission Lines; Wires and Cables.*) The conductor over which transmission is effected is commonly referred to as the line. These conductors may exist as open wires, or they may be disposed in aerial, underground and submarine cables.

OPEN-WIRE LINES. — Most of the open-wire telegraph lines are of hard-drawn copper or of galvanized iron. Recently a wire known as copper-clad steel, consisting of a core of steel wire with an outer shell of copper welded thereto, has appeared as a serious competitor. For the properties of these wires see *Wires and Cables, Bare*.

Hard-drawn copper wire is vastly superior to iron in point of conductivity and in its ability to withstand the action of the elements. Galvanized-iron wire is advantageous in point of first cost, it has a somewhat greater strength, but has a life ranging from 4 to 20 years, according to atmospheric conditions. Copper-clad steel has a conductivity superior to that of iron, but inferior to that of copper. It is stronger and somewhat cheaper than hard-drawn copper, and should possess the same ability as copper to indefinitely withstand the action of the elements.

The mile-ohm values (i.e., the weight in pounds of a wire one mile long having a resistance of one ohm) of the various grades of wire used for telegraphic purposes are as follows: E.B.B. iron, 4700 pounds; B.B. iron, 5500 pounds; steel, 6500 pounds; hard-drawn copper, 895 pounds; copper-clad steel (having conductivity 40 per cent of solid copper), 2010 pounds.

No. 9 B. & S. gauge hard-drawn copper wire, weighing 209 pounds per mile, is more used than any other size of hard-drawn copper wire for telegraphic purposes; and No. 8 B. W. G. galvanized-iron wire, B.B. grade, weighing 378 pounds per mile, is the principal standard in iron wire.

AERIAL CABLES. — The paper-insulated, lead-covered cable of the type employed in telephony (*see Telephone Lines*) but with single wires instead of twisted pairs, is rapidly supplanting the old form of rubber-insulated cable formerly used in telegraphy. Sometimes the cores are saturated with insulating compound, and sometimes they are left "dry," according to whether the consideration for greater safety outweighs the desirability for lower capacity. For aerial work such cables are supported from messenger wires, either by means of clips or by a wrapping of marline around both the cable and the messenger, this form of support being applied by means of the well-known "Spinning Jenny."

UNDERGROUND CABLES. — (*See also Wires and Cables, Insulated.*) Underground telegraph cables are of the same type as those used for aerial. The practice of placing them in ducts extending from manhole to manhole, so as to facilitate their withdrawal, has largely replaced the older practice of burying the cable in such manner as to necessitate its being dug up for replacement or repairs.

SUBMARINE CABLES. — For comparatively short lengths, in crossing rivers or bays where the current is not swift and the bottom is of such a nature as to naturally embed the cable, ordinary multiple-conductor, lead-covered cables, with either paper or rubber insulation, provided with a heavy braiding, are often employed. Where paper insulation is used, it is customary in telegraphic work to thoroughly impregnate the core so as to minimize the bad effect of moisture; but this, of course, increases the electrostatic capacity of the conductors.

Where great tensile stresses or abrasion are likely to occur, the core, after being provided with its lead sheath, is served with a thick cushoning layer of tarred jute, and then with an armor of closely wrapped galvanized-steel wires, over which an additional protecting layer of tarred jute is sometimes placed.

Long Submarine Cables. — For long submarine cables, such as those across the Atlantic Ocean, a single conductor is employed, this being in the form of stranded copper, weighing from 70 to 650 pounds per nautical mile. This conductor is then encased in several layers of gutta-percha. The insulated core, so formed, is protected with a thick layer of jute, outside of which the armoring of galvanized-steel wires is placed. After being sheathed, the whole cable is sometimes covered with a thick layer of tarred tape. The shore ends of cables, where there is greater liability to mechanical injury, are armored with a layer of very heavy galvanized-steel wires.

LIMITING TRANSMISSION DISTANCE. — The length of the line over which a signal can be transmitted depends upon the strength of the current impulses sent out, the sensitiveness of the receiving instruments, the resistance and capacity of the line and upon the number of stations connected to the line.

Open Wire Lines. — The following table, compiled by Mr. F. F. Fowle, shows the result of his calculations as to the limiting lengths for through working (no intermediate stations) of simplex, duplex and quadruplex Morse transmission with standard apparatus over open wire lines with ground return having different resistances per mile, and also the maximum line distances for way circuits equipped with 35-ohm line relays.

Resistance per mile in ohms	Limiting length of line for Through Working, miles			Limiting length of Way Circuits, miles		
	Duplex	Simplex	Quadruplex	A*	B*	C*
2	783	597	531	597	391	313
3	658	510	442	510	363	299
4	580	450	386	450	341	286
6	485	376	313	376	305	265
8	425	331	268	331	285	248
10	384	299	236	299	260	234
15	318	248	186	248	225	208
20	278	217	156	217	201	188
25	250	195	135	195	183	174
30	229	179	120	179	170	162
40	200	156	98.7	156	150	144
50	180	140	84.3	140	...	...

* *A*, stations at ends of line only; *B*, stations every 10 miles; *C*, stations every 3 miles.

CONSTRUCTION OF LINES. — (*See Telephone Lines.*)

BIBLIOGRAPHY. — See Bibliography in article on *Telegraph Systems*.

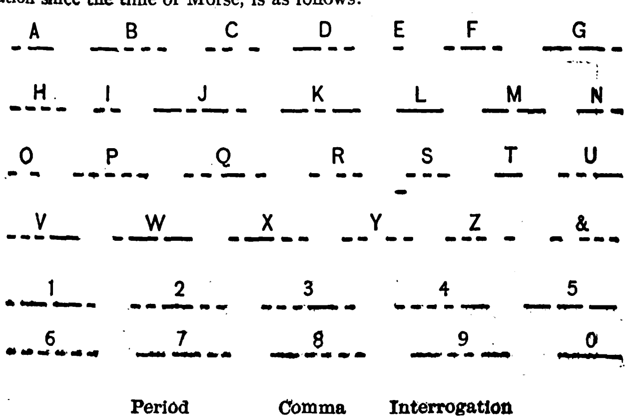
[S. G. McMEEN.]

TELEGRAPH SYSTEMS. — (*See also Telegraph Instruments and Apparatus; Telegraph Lines.*) The following is a brief table of contents of this article:

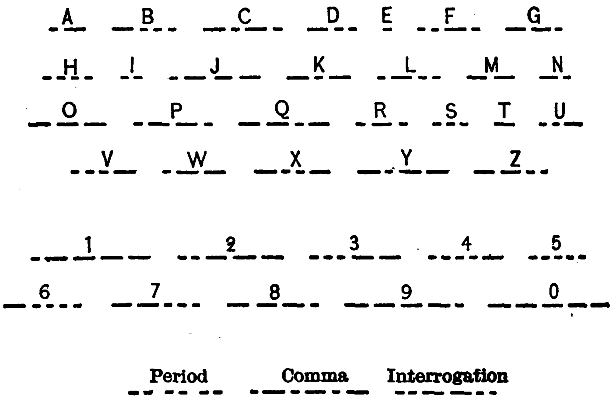
With slight modification the original system of electric telegraphy devised by Morse is still largely used, the great bulk of telegraph work still being done by it. In its simplest form it consists of a circuit, connecting the points between which intelligence is to be transmitted, a battery or other generator in the circuit for supplying the current, a key at each station for facilitating the opening and closing of the circuit by hand, and a sounder for giving response to the opening and closing of the circuit by audible clicking.

TELEGRAPH CODES. — Two different codes are used in telegraphy, the Morse code and the Continental code.

Morse Code. — The Morse code is an arbitrary system of interpreting the various letters, numeral digits and punctuation marks employed in writing, this interpretation being intelligible either audibly or visually. This code is formed of dots, dashes and spaces. The dot is made by a quick depression of the key followed by its immediate release; the dash by a depression of the key with a delayed release; and the space by permitting a short interval to elapse between the completion of two dots. Thus, the letter *a* is represented by a dot and a dash, and it is formed by depressing the key, immediately releasing it, depressing the key again and holding it a slight interval before release. Similarly, the letter *i* is represented by two dots, and it is formed by depressing and releasing the key twice in rapid succession. The letter *o* is an example of a so-called space letter, and is represented by dot-space-dot. It differs from the letter *i* only in that an interval of time is allowed to elapse between the two dots. The Morse code, as it has been employed practically without modification since the time of Morse, is as follows:



Continental Code. — The so-called Continental Code differs from the Morse code in that it is composed entirely of dots and dashes, employing no spaces. It is almost universally used for submarine lines, and for land lines in nearly all countries except the United States. This Continental or universal code is as follows:



SIMPLEX OR MORSE SYSTEM. — There are two types of circuits used, the so-called "open-circuit" system and the "closed-circuit" system. In both cases but one line wire is employed, the earth forming the return circuit.

Closed-circuit System (Fig. 1 and 2). — This system is mainly used in the United States. In its simplest form the keys and the sounder magnets are placed in series in the line with a source of current, as shown in Fig. 1. A

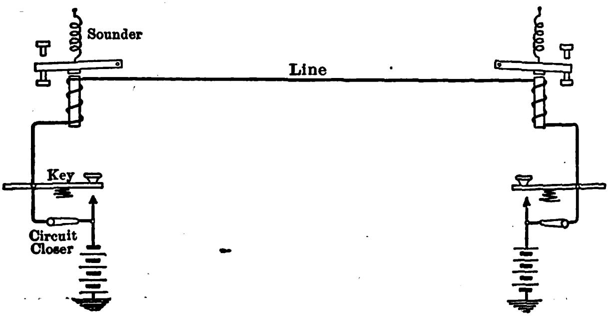


Fig. 1. Simple Telegraph Circuit

switch or "circuit closer" is provided with each key, the function of which is to keep the line circuit closed around the key contacts when the key is not in use. When an operator begins to send he opens this circuit closer, thus making his key effective in controlling the line circuit. When the line is idle, the circuit is closed at all stations, and all sounder levers are held down by the magnetizing effects of the current flowing in the line.

In Fig. 1 but two stations are shown, one at each end of the line. Obviously intermediate or way stations could be added by merely placing the necessary keys and sounders in the line circuit at the desired points.

A more common practice is to employ a line relay at each station as the device directly responsive to the line currents, using this line relay to control a local circuit containing a local battery and a sounder. The reason for the choice of this slightly more complicated system for commercial practice is that the sounder, in order to make the necessary amount of noise, is required to be of a rather sturdy nature and is not as readily responsive to the comparatively small line currents as is the relay, which, having no other function than to be responsive,

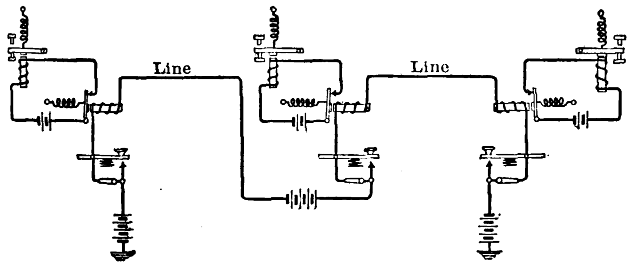


Fig. 2. Closed Circuit System with Way Stations

is readily made of the required sensitiveness. A line equipped for three such stations is shown in Fig. 2. While the constant flow of current in the closed-circuit system results in the use of a greater amount of current than would otherwise be necessary, it has the advantage of keeping the line under test at all times and of not requiring a line battery at each of the stations, as is the case with the open-circuit system.

Open-circuit System (Fig. 3). — This is largely used abroad. A battery is required at each station. Each key has a front and back contact so arranged

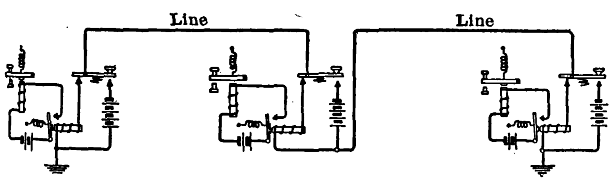


Fig. 3. Open Circuit System with Way Stations

as to hold the relay in the line circuit when the key is raised, and to cut the relay out of the line circuit and substitute the battery for it when the key is depressed. While, therefore, the line circuit is normally closed from one end to the other, the relays are not energized, since there is no current in the line. As a result, both the line and the local batteries stand in normally open circuits, and there is no waste of current. Besides requiring a line battery, sufficient for the operation of the entire line, at each station, this system does not hold the line and its instruments under automatic test conditions, which is automatically done by the constant flow of current in the closed-circuit system. A three-station open-circuit system is shown in Fig. 3.

Sound Reading.—In the first use of Morse telegraphy the received characters were recorded on a moving tape and subsequently translated. It was found in practice, however, that the operators soon learned to read what was coming over the wire by the clicking of the instruments, and, although this method of sound reading was condemned at first as being dangerous, it has survived to such an extent that the major portion of the telegraph work of the world is to-day received in this way.

Speed of Handling Messages by Morse System.—The speed of handling messages by Morse was limited by the speed of the receiving operator in writing down the message, as long as this was done in ordinary handwriting. The use of the typewriter for transcribing has, however, considerably increased the possible speed, and the limitation where the typewriter is used is that of the sending operator to "write" the Morse code. These speed limitations exist where the line conditions permit. A poorly working line may serve to reduce the possible speed far below the limitations prescribed by the ability of the operators. While speeds of over fifty words a minute have been attained by experts during short periods of time, and while operators often do send and receive thirty-five and forty words a minute in actual commercial work, the average under working conditions is far below this, and is probably under twenty words a minute.

DUPLEX SYSTEMS.—Duplex telegraphy involves the simultaneous sending of two messages in opposite directions over a single line. Two different methods are employed for making the receiving apparatus unresponsive to the home key. These two methods are termed the "differential" and "bridge" methods, respectively. There are also two different methods of sending impulses over the line, one of which depends on variations in current strength and the other on changes in current direction.

Differential Method (Fig. 4).—The differential method is readily understood from a consideration of Fig. 4, which shows a line relay having two windings in opposite directions. Current from the battery is supplied through one of these windings to the line, and through the other of these windings to the artificial line represented by a resistance at the left. The resistance and other characteristics of the artificial line are made approximately equal to those of the real line, so that impulses of current sent by means of the key from the battery to the line will produce no effect on the relay, because an equal current passes also through the artificial line. Obviously, however, there is no such differential action for incoming currents, and, therefore, the relay is responsive to currents from the distant station.

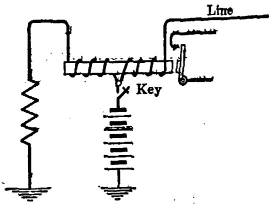


Fig. 4. Principle of Differential Method

The differential method is ordinarily employed in land telegraphy.

Bridge Method (Fig. 5).—The bridge method of rendering the home relay unresponsive to the home key and responsive to the distant key is illustrated in principle in Fig. 5. In this the resistances shown are so proportioned with respect to the resistance of the line that the line relay will receive no current upon closure of the key, for the

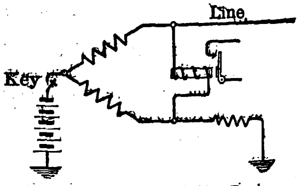


Fig. 5. Principle of Bridge Duplex

same reason that the galvanometer in a Wheatstone bridge does not respond when the bridge is balanced.

The bridge method finds its chief use in overland and in submarine telegraphy.

Stearns Duplex for Land Lines (Fig. 6). — The Stearns duplex for land lines operates on the increase and decrease in current strength, without changing the direction of current flow; and for the reason that it employs the differential method of making the home relay unresponsive to the home key, it is commonly referred to as the Stearns differential duplex system.

A simplified diagram of the Stearns differential duplex is shown in Fig. 6. It is to be noted that the batteries at opposite ends of the line have their opposite poles grounded, so that whether one or the other, or both, of the batteries are

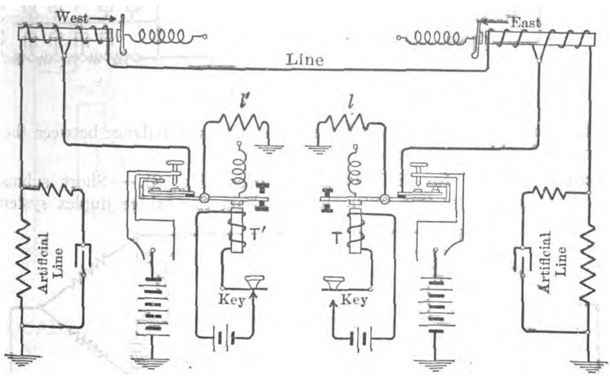


Fig. 6. Stearns Differential Duplex

in the circuit the current flows in the same direction. The key controls a local circuit of an electromagnetic transmitter T or T'. The tongue of each transmitter normally engages the contact on the under side of the bend in the lever; when the latter is attracted by its magnet upon the closure of the key, the tongue makes contact with the adjustable screw on the standard, and this prevents the tongue from moving farther upward, so that the making of this contact is immediately followed by the breaking of the normally closed contact of the tongue. Because these transmitters make one contact before breaking the other, they are called "continuity-preserving transmitters." When the key is open, the corresponding transmitter keeps its end of the line connected to ground through the resistance l, but upon the closure of the key the transmitter causes the substitution of the battery at that end of the line for this resistance. This resistance l sometimes is called the "spark coil," since its use tends to prevent sparking at the transmitter contacts, and its function is to compensate for the internal resistance of the line battery.

The artificial line shown at each station is composed of resistances and condensers arranged for ready adjustment, so as to balance not only for the resistance of the line, but for its capacity also.

Stearns Duplex for Submarine Cables. — Fig. 7 shows the Stearns duplex arrangement of terminal apparatus for cable working. This is a bridge method. The normal position of the keys grounds the cable conductor, thus discharging it. Alternately depressing the two keys gives the cable alternate negative and positive charges, thus sending dots and dashes. By moving the

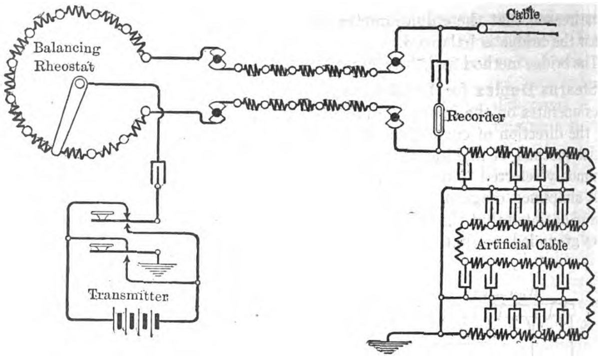


Fig. 7. Stearns Cable Duplex

arm of the balancing rheostat, delicate adjustments of balance between the real and artificial cables are secured.

Jacobs' Duplex for Submarine Cables (Fig. 8). — Short submarine cables frequently have two conductors, making the bridge duplex system of

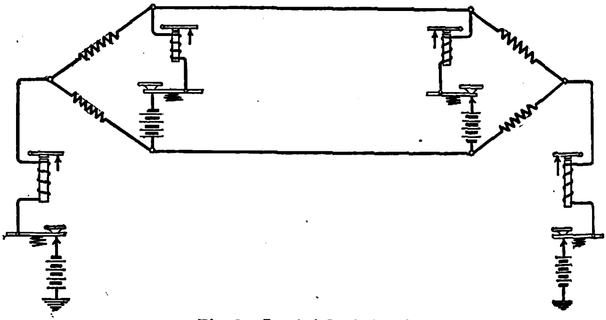


Fig. 8. Jacobs' Cable Duplex

Fig. 8 available. This is similar to simplex working in combined telephony and telegraphy. This method is found preferable to using each conductor separately for the two transmissions, since it avoids inductive interference between the two wires.

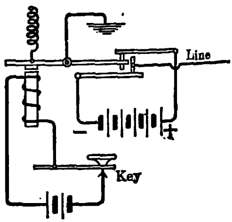


Fig. 9. Pole-changing Transmitter—Polar Duplex

Polar Duplex (Fig. 9 and 10). — The polar duplex depends for the principal feature of its operation on the fact that a polarized relay will move its armature oppositely for opposite directions of current. The key used in sending operates to close and open a local circuit which controls a pole-changing transmitter. The action of this is made clear in Fig. 9. With the key closed, as shown in that figure, the negative side of battery will be connected to line and the positive side to ground.

With the key open, the reverse will be true. The circuit of the polar duplex is shown in Fig. 10.

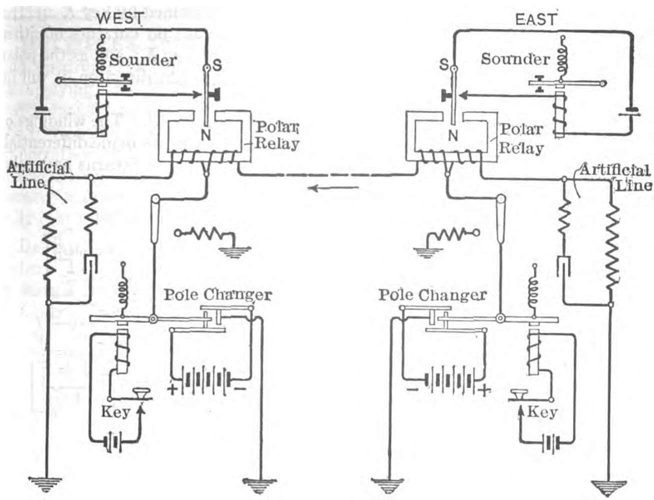


Fig. 10. Polar Duplex

QUADRUPLEX SYSTEMS. — In quadruplex systems four messages, two in each direction, may be transmitted simultaneously over a single line wire.

Edison Quadruplex (Fig. 11 and 12). — The Edison quadruplex depends for its principle of operation on a combination of the two duplex systems, the Stearns and the polar just described. At each station two relays are provided, one, the neutral relay, responsive to currents in either direction, and the other, the polar relay, responsive only to currents in one direction. At each

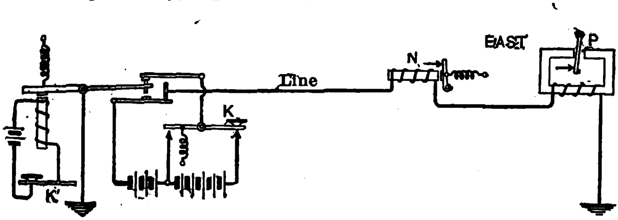


Fig. 11. Principle of Edison Quadruplex

station also two key-controlled transmitters are provided, one of which operates, as in the polar duplex, to change the direction of current in the line and the other, as in the Stearns duplex, to increase and diminish the strength of current flow.

The principle of transmitting in one direction by means of the Edison quadruplex is indicated in Fig. 11, where N is the neutral relay and P the polar relay at the East station. K and K' are transmitting keys at the West station, key

K operating to alternately weaken and strengthen the line current, and *K'* operating as a pole changer to reverse the line current. As shown, the line will always have some current flow, the amount being determined by key *K* and the direction by key *K'*. As the neutral relay will operate on currents of either direction, if strong enough, it will be responsive only to key *K*; and as the polar relay will operate on either current strength, if in the right direction, it will be responsive only to key *K'*.

The circuits of the Edison quadruplex are shown in Fig. 12. The windings of the neutral relays *N* and of the polar relays *P* are in each case made differential, for the same reasons as pointed out in connection with the Stearns and polar

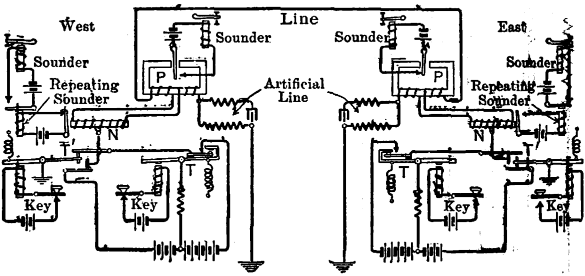


Fig. 12. Edison Quadruplex

duplex systems. At each station of the "quad" system one winding of each relay is in the line, and the other in the artificial line. As will be seen from Fig. 12, the two transmitters at each station modify the main battery current, as pointed out in connection with Fig. 11, the continuity-preserving transmitter *T* controlling the current strength and the pole-changing transmitter *T'* controlling its direction.

Key Systems for Quadruplex Working. — In the quadruplex so far described, the current was supplied by batteries, and the strength-varying transmitter operated to accomplish its purpose by alternately cutting a portion of the line battery out of and into the circuit. Where dynamos furnish the current, a different scheme is employed. One system, known as the Field key system, is illustrated in principle in Fig. 13. Two dynamos are employed, having opposite poles grounded. The pole changer merely chooses between the two live poles for securing its reversals in polarity. The continuity-preserving transmitter varies the current strength in the line by changing the resistance relations as shown. Thus, when the key is closed, the 1200-ohm resistance is short-circuited and the 900-ohm "leak" resistance to ground is cut off. The

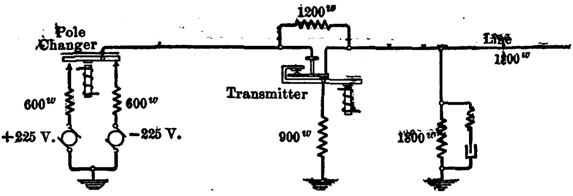


Fig. 13. Field Key System

current under these circumstances flows to line at full strength. When the key is opened, the 1200-ohm resistance is introduced in the line and the 900-ohm leak is made effective. The resistances shown are those for an 1800-ohm line.

MULTIPLEX SYSTEMS. — The term "multiplex" should include duplex and quadruplex systems, but practice has been otherwise, and has resulted in applying the term "multiplex" principally to a system wherein two synchronously moving switches, one at each end of the line, serve to connect the corresponding sets of telegraph instruments momentarily and successively in operative relation with the line. While each set of instruments, therefore, is connected with the line for only a small fraction of the total time, the successive connections recur so rapidly as to afford, to all intents and purposes, a practically continuous use of the line.

Delany Synchronous Multiplex System (Fig. 14). — The principle of the Delany synchronous multiplex system is shown in diagram in Fig. 14. Six sets of instruments, each consisting of a pole-changing key and a polar relay, are shown at each end of the line. The two switch arms of the synchronous rotary

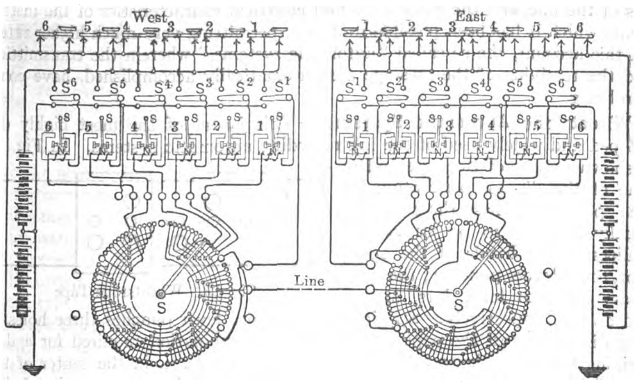


Fig. 14. Delany Synchronous Multiplex

switches S are made to revolve in unison, so that the correspondingly numbered sets of instruments at each end will be simultaneously connected with the line. As the switch arms make about 180 revolutions per minute, and as each set of instruments is connected with the line twelve times at each revolution, it follows that each set of instruments is connected with the line thirty-six times per second. This is a much shorter interval than that required to make the shortest dot, and, therefore, any closure of a key which an operator may make will necessarily cover a time during which at least one connection will be made with the line. In the particular embodiment of the Delany system shown polarized relays and pole-changing keys are used, the advantage of this being that the relay armature will remain in the position to which it was last brought by a current impulse, in spite of the rapid interruptions caused by the synchronous switch. A small switch lever, S^1, S^2, etc., is associated with each key and relay and is moved by the operator to one or the other of its positions, according to whether he is sending or receiving.

The synchronizing mechanism for the switch movements is ingenious and interesting. The two arms are each driven by step-by-step motors, the driving impulses being regulated by means of tuning forks, as accurately tuned to each

other as possible. Obviously, complete synchronism could not be obtained by this means alone, and, therefore, the two driving devices are each brought into association with the line by the rotary switch arms six times during each revolution, the arrangement being such that if one switch arm arrives on one of the dead or synchronizing contacts slightly in advance of the other, it will apply a corrective measure to its motor device, which will tend to slow it down. This corrective measure is in the form of an electromagnet, in the field of which the tuning fork vibrates, and the energization of this magnet exerts a retarding influence on the rate of vibration, and, therefore, on the rate of rotation of the synchronous switch.

This system has been used to some extent commercially with as high as six simultaneous transmissions on comparatively short lines, and four or fewer on long lines. Obviously, the time constant of the line enters as a limiting factor to a greater extent than in the ordinary systems of Morse telegraphy.

AUTOMATIC SYSTEMS. — The limitations in speed of transmission over an ordinary manually operated telegraph line are mainly those due to the ability of the operators to transmit and receive, the electrical characteristics of the line, and the mechanical and electrical characteristics of the instruments which it has been possible to devise, permitting very much higher rates. On this account, the so-called "automatic systems," wherein the transmitting and the receiving of the message are automatically accomplished, have come into being.

Wheatstone Automatic System. — This is one of the most highly developed and widely used of the ink recording automatic systems. In Fig. 15 is shown a piece of the tape and a plan of the die for perforating it. The perforator has three levers usually adapted to be struck by mallets in the hands of the operator, these levers punching the necessary holes to form dots, spaces and dashes, respectively. Striking the left-hand lever punches three holes, 1, 2, 3, directly across the paper, this being the combination required for a dot. Striking the center lever punches a single small hole, 4, in the center of the tape, thus making a space, while the third or right-hand lever punches four holes, arranged as 5, 6, 7 and 8, thus setting up a combination for a dash. The function of the small holes along the center line of the tape is to effect the proper driving of the tape, these being engaged by the points of a star wheel in connection with the feeding mechanism. As this tape is driven through the transmitter, one row of larger holes determines the intervals at which the positive pole of the battery shall be put to line, and the other row of large holes the intervals during which the negative pole is connected. As the two members which engage these holes are arranged so that one is slightly in advance of the other, it follows that the holes 1 and 3 will cause first a negative impulse, immediately followed by a positive impulse, this being the combination for a dot. The two holes, 5 and 8, will likewise cause a negative, followed by a positive, impulse, but the two will be separated by an interval of time, this being the combination for a dash. The Wheatstone is thus a so-called "double-current" system, depending for its operation on the actual reversal of the line current.

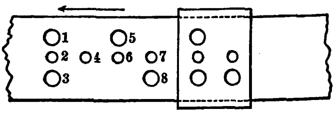


Fig. 15. Wheatstone Tape

Wheatstone Receiver. — The receiver of the Wheatstone system is in the form of a polarized relay, the armature of which will stay in either of its extreme positions even though the current which moved it there ceases. The inking is done by a very small ink wheel which is constantly rotated in close proximity to the moving paper tape. The shaft on which this ink wheel revolves

is so mounted as to be deflected by the movements of the relay armature. The ink wheel receives its ink on the back stroke from another wheel which revolves in a tank of ink. On the forward stroke the ink wheel is brought close enough to the moving paper tape to make a mark, although there is no actual contact between the wheel and the tape.

Chemical Systems. — These systems, once employed in commercial work to a considerable extent, have been in large measure superseded either by the ink recording systems or by ordinary Morse. They present possibilities for future development, however, which give them more than an historic interest.

Anderson Chemical Automatic System (Fig. 16). — The salient points of the Anderson chemical automatic system are shown in Fig. 16. The

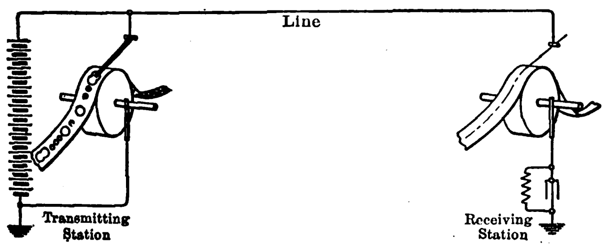


Fig. 16. Principle of Chemical Automatic System

passage of a hole in the transmitting tape under the contact pen serves to short-circuit the transmitting battery and thus cause a cessation of current in the line. The condenser at the receiving end, previously charged by the flow of current from the battery, then discharges into the line, this being intended to cause an actual reversal of current flow. This is an interesting example of so-called "double-current" working with a single battery. At the receiver the chemically saturated tape passes under a contact pen, so that the current is compelled to flow through the paper. Its flow decomposes the solution and marks the paper, as already described. The reverse flow of current from the condenser tends to prevent the occurrence of "tailings."

Speed of Anderson System. — This system, like the Wheatstone, records in dots and dashes, and, according to Maver, it has been found capable of transmitting 600 words per minute over a 1000-mile line having a resistance of 2500 ohms, under all sorts of weather conditions. On a shorter line, 360 miles in length, having a resistance of about 700 ohms, the astonishing rate of 3000 words per minute has been accomplished.

WRITING TELEGRAPHS OR AUTOGRAPHIC SYSTEMS. — Writing or autographic telegraphs are to be distinguished from printing telegraphs in that they write the received messages in script, while the printing systems actually print them as from type.

In these systems the message to be transmitted is written down by the sender as with a pen or stylus, while at the receiving end a recording pen simultaneously executes the same movements and thus writes the message in the same or very similar handwriting. Nearly all autographic systems depend on resolving the movements of the transmitting pen or stylus into two component straight-line movements at right angles to each other, electromagnetically reproducing these component rectilinear motions at the receiving station, and recombining them there into the resultant movement, which, therefore, traces a path closely approximating that originally made by the sending stylus.

Robertson System.—Fig. 17 is a schematic diagram of this system and clearly illustrates the principle of autographic systems. Two line wires are employed, each including a battery and a variable-resistance element

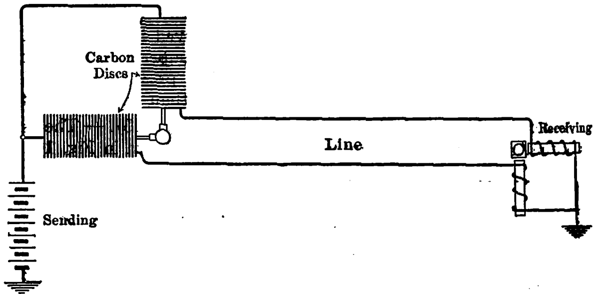


Fig. 17. Principle of Robertson System

at the transmitting station, and an electromagnet at the receiving station. The variable-resistance elements are composed in this case of piles of carbon discs arranged at right angles to each other, these being so related to the transmitting stylus that its movements in writing will cause, due to variations in pressure on the carbon discs, corresponding variations in resistance in the paths through the discs. The receiving pen is actuated by the armature common to the two magnets at the receiving end. As the respective pulls of these two magnets are at right angles and vary as the strength of the currents flowing, the two components of the transmitting stylus are reproduced and these are recombined to produce the corresponding movements of the receiving pen.

Telautograph.—This is another autographic system, due largely to Elisha Gray. It employs two wires, and like the system just described resolves the complex motion of the transmitting pencil into two rectilinear components at right angles, reproduces these at the distant end, and recombines them to obtain the facsimile writing. The transmitting stylus operates two bell-crank arms carrying rollers which operate over rheostat contacts to cause the variations in resistance and current strength. At the receiving end the pen is attached to the arms of two bell cranks which are operated on by solenoids, which lie in uniform magnetic fields and which, under the influence of the varying currents passing through them, cause the required movements of the bell cranks.

Pollak-Virag System.—In this the message is prepared by perforating a tape, the passage of which through the transmitter results in currents being sent over the two line wires in proper sequence, direction and strength to cause the receiving instrument to record the message in a style very similar to longhand writing. Two circuits are obtained from the two wires; one the metallic circuit and the other the two wires in parallel with earth return. At the receiving end two instruments closely resembling telephone receivers are affected by the currents received over the two circuits. The diaphragms of these receivers control the movements, on horizontal and vertical axes, respectively, of a small mirror. A ray of light reflected from this mirror is caused to move upon a rapidly moving strip of light-sensitized paper, and the co-ordinated movements on the two axes of the mirror under the influence of the currents received result in writing. The photographic paper is automatically passed

through the necessary developing and fixing baths, the time required in this process being only about twelve seconds.

PRINTING TELEGRAPHS. — These are systems in which the received messages are automatically recorded in print. Most printing telegraph systems depend upon synchronism being maintained between the two wheels at the transmitting and receiving stations; although in some systems, such as the Murray, for instance, no such requirement exists.

Ticker Systems. — In the ticker systems, which are widely employed for transmitting stock-market quotations, the type wheels are moved synchronously a step at a time, either by causing the same set of line impulses to affect each, or by causing the wheel at the sending station, in its revolution, to transmit impulses which turn the distant wheel. The wheels must not only revolve at the same rate, but a given letter on each must come opposite a given fixed point simultaneously. By stopping the wheels when the desired letter is brought opposite the paper, an impulse of different character is sent which causes the imprint to be made. Some ticker systems operate over one line wire, and the impulses which cause the rotation of the type wheel occur so rapidly as not to affect the printing magnet, but upon their cessation the printing magnet included in the same circuit is brought into play. Many of the ticker systems operate over two line wires and employ two type wheels, one carrying letters and the other figures. Means are provided for shifting either the wheels or the tape, so as to print from either as desired.

The Hughes System. — This is largely employed in Europe, and is a synchronous type-wheel system in which the type wheels revolve continuously rather than by step-by-step movements. Electric motors furnish the driving power. The wheels are adjusted to revolve at as nearly the same rates as possible and suitable corrective means are brought into play over the line wire to maintain complete synchronism. The transmitting apparatus resembles the keyboard of a piano, a key being assigned to each character. In connection with this keyboard there is a rotating contact device revolving in synchronism with the type wheels, and the arrangement is such that the depression of any key will cause a contact to be made at the exact instant when the letter corresponding to that key is in printing position at both instruments.

Speed of Hughes System. — The Hughes system is capable of operating with great accuracy at a speed in the neighborhood of from thirty to forty words per minute.

Phelps System. — The Phelps system is a modification of the Hughes and operates on the same general principles. It has superseded the Hughes system in the United States.

The Buckingham System. — One difficulty with the ordinary step-by-step system is that the great number of impulses required per letter necessarily slows down the speed. The Buckingham printing system is designed to operate with a maximum of six current impulses per letter, the necessary choice between letters being effected by various combinations of short and long current impulses in either direction, with short and long intervening spaces. The Buckingham system resembles the Wheatstone, in that the spaces are made by negative currents and the dots and dashes by positive. As in the Wheatstone system, also, the message is prepared by perforating the tape in such manner as automatically to send the required combinations of dots, dashes and spaces, but a different code is used, in which each letter and other character is composed of three positive impulses, dashes or dots, or both, variously combined with long or short negative impulses, corresponding to spaces. By a most ingenious mechanism at the receiving end these positive and negative currents, instead

of being received in dots and dashes on a moving tape, are made to effect proper selection of the various printed characters, and to print them in desired order in page form. The type wheels are four in number, all mounted side by side on the same shaft and rotating together. Each type wheel carries eight letters or characters, making thirty-two in all. By means of levers, operated by electromagnets, the type wheels are, under the action of the code impulses, given any desired lateral or rotary motion to bring the required type face into printing position. The space between the letters is utilized to give the printing impulse.

Tape-punching Machine for Buckingham System.—The tape-punching machine of the Buckingham system employs a standard typewriter keyboard, one depression of the key effecting the tape punching for a complete character. The maximum speed possible is about 80 words per minute, as against 20 to 40 words per minute in the mallet method of punching Wheatstone tape.

Speed of Buckingham System.—This system has a possible average speed of transmission of about 100 words per minute, and is reported to have worked on a duplex circuit between New York and Chicago with an average speed of 80 words per minute in each direction.

The Barclay System.—This has been employed extensively by the Western Union Telegraph Company, and resembles in many respects the Buckingham system. It uses the same code alphabet and the same general method of transmitting, but the impulses received result in the operation of a group of magnets which in turn affect the operating levers of a Blickenderfer typewriter, which does the actual printing. The printing is done on ordinary message blanks.

Speed of Barclay System.—The rate of transmission possible is approximately 100 words per minute in each direction when operated as a duplex.

The Rowland System.—This is one of the most highly developed of the printing systems, and was for a time used to considerable extent by the Postal Telegraph Cable Company between New York and Chicago, and on other circuits. Essentially the system is a one-way quadruplex, but as it is capable of being worked duplex it becomes in effect an "octoplex," transmitting four simultaneous messages in each direction over one line wire.

The line current employed is alternating, generated by a dynamo at one end, which runs a synchronous motor at the other, thus effecting the required synchronism. The necessary current combinations to effect the selection and printing of a given letter are brought about by reversing certain of the half waves in each group of line current waves. The simultaneous transmission of four messages in one direction is provided for by an adaptation of the multiplex system, a distributing switch giving each set of four transmitting and receiving instruments successive and simultaneous control of the line. Since each letter is represented by a specific combination of positive and negative currents, it is possible by means of these combinations to effect the closing of the proper local circuits at the receiving end to cause the paper to be pressed against the type wheel at the time when the letter required is opposite it.

Speed of Rowland System.—Each transmitter and receiver has a speed of about 30 words per minute, making a total transmission of 240 words per minute when the octoplex method of working is employed.

The Murray Automatic System.—This is a non-synchronous printing system. Messages are first prepared by perforating a paper tape. This tape is then fed into the transmitting device and permits currents to flow over the

line in such a manner as to cause the receiving device, which in itself is a tape-perforating machine, to perforate another tape there. This receiving tape is a replica of the transmitting tape, and when passed through an automatic typewriter the perforations are translated into Roman type letters, the message being in page form.

Speed of Murray System. — This system is capable of transmission speeds upward of 200 words per minute, but speeds of from 100 to 125 words per minute have been found to be the best for practical working.

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[S. G. McMeen.]

TELEPHONE INSTRUMENTS AND CIRCUITS. — (*See also Standardization Rules of the A.I.E.E.; Telephone Lines; Telephone Traffic; Wireless Telephony.*)

The chief subjects treated in this article are:

The telephone transmits speech and other sounds by this cycle of action: sound waves in air vibrate a diaphragm; this vibration produces fluctuating current in a line joining the transmitter to a distant receiver; the fluctuating current produces vibration in the diaphragm of the receiver; these vibrations produce sound waves in air.

Bell's original telephone utilized one device as both transmitter and receiver. It survives in its original form as a receiver, but seldom is used as a transmitter, more powerful devices having supplanted it.

RECEIVERS. — The original telephone consisted of a magnet, a coil of fine wire and a diaphragm, arranged as in Fig. 1. *d* is a diaphragm, *m* is a magnet,

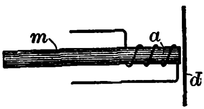


Fig. 1. Principle of Permanent Magnet Receiver

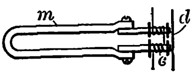


Fig. 2. Principle of Bipolar Receiver

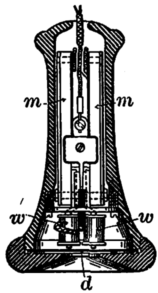


Fig. 3. Section of Bipolar Receiver

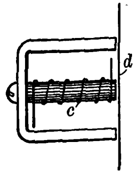


Fig. 4. Principle of Direct Current Receiver

and *a* is a coil. The magnet was first made of soft iron, getting its magnetism from direct currents in the line; a permanently magnetized bar of hard steel with soft iron poles (Figs. 2 and 3) soon was substituted for this soft iron core.

Bipolar Receiver. — The bipolar receiver is the present standard form, as in Fig. 2, in which *d*, *m* and *c* represent, as before, the diaphragm, magnet and coil. Both poles of the permanent magnet are presented to the diaphragm, being extended by soft iron pole pieces, each carrying a bobbin wound with fine, silk-covered or enameled wire. The wires of the two bobbins are connected in series. When used to transmit speech, such an arrangement is called a magneto transmitter.

The magnet, winding and diaphragm of a receiver are assembled in a case of hard rubber or an imitation thereof, adapted at one end to fit the ear and at the other to hang on a hook switch. Fig. 3 is a section of a typical assembled receiver. *m, m* are permanent magnets, arranged to produce opposite poles at the ends of the soft iron cores carrying the windings *w, w*; *d* is the diaphragm. In good types of receivers the entire mechanical arrangement of parts is supported from the ear end of the shell and not from the other end. This precaution is necessary in order to prevent the adjustment of the pole pieces to the diaphragm being upset by the difference in expansion of the shell and magnet.

"**Direct-current Receiver** (Fig. 4).—For use in lines carrying direct current during conversation, as in common-battery systems, the iron-core type of receiver recently has been revived. They are extremely simple and cost little to make. It is not yet established that they are as efficient as permanent-magnet receivers, but the difference seems to be slight.

The Monarch Telephone Manufacturing Company uses a receiver which has two pairs of windings, as in Fig. 5. One pair is of large impedance, serving to magnetize the cores, while the other, of smaller impedance, serves to actuate the diaphragm. Direct currents pass through both pairs of windings, their magnetizing effects being additive, but the direct current through the coils farther from the diaphragm is the larger. Voice currents pass through both pairs of windings and their effects are additive, but the voice currents through the windings nearer to the diaphragm are the larger.

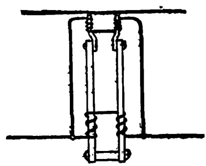


Fig. 5. Monarch Receiver

Head Receiver.—A head receiver is a compact one carried by a band fitting the operator's head, leaving her hands free. The compactness is gained by making the magnets small and placing them parallel with the diaphragm.

TRANSMITTERS (Figs. 6 and 7).—Present standard carbon transmitters are many times more powerful than the best magneto transmitters. Carbon transmitters and their circuits take many forms, but in all the function is to produce current fluctuation as a consequence of diaphragm vibration caused by sound.

The elements of a transmitter are a diaphragm, two polished solid carbon electrodes, and a charge of polished carbon granules. Vibration of the diaphragm moves one electrode to compress and release the granules between the electrodes. Fig. 6 illustrates the elements; *d* is the diaphragm, *m* the movable and *f* the fixed electrode. Fig. 7 is a section of a typical granular carbon transmitter.

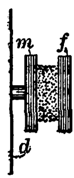


Fig. 6. Principle of Transmitter

All carbon transmitters operate by varying the resistance of some material proportionally to movement of the diaphragm. Receivers, on the other hand, are caused to operate by a varying magnetic flux. The reason for the excellent action of carbon in varying its contact resistance under pressure is not known.

RINGERS OR BELLS (Fig. 8).—The present standard telephone signal is a two-gong bell, actuated by alternating current. Its principle is that of the polarized telegraph relay. Its construction, as in Fig. 8, is of two coils, a permanent magnet *N*, two gongs and an armature *S*.

Magneto Generator (Fig. 9).—The bells of telephones in all exchanges are rung by alternating current sent out from the central office power plant. In magneto systems the subscriber calls the central office by operating a magneto generator, which is a part of his telephone, and with this generator

the subscriber also rings other bells on the same line. The magneto generator is a standard part of subscribers' equipment for rural and other party lines, being the best way for a subscriber to do signaling between various telephones on one line. The armature has many turns of fine wire (Nos. 32 to 36 B. & S.

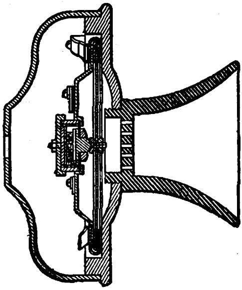


Fig. 7. Granular Carbon Transmitter

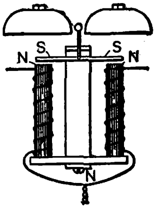


Fig. 8. Alternating Current Ringer

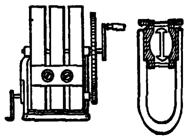


Fig. 9. Magneto Generator

gage), of a resistance of from 300 to 1000 ohms. A frequency of 20 cycles and an e.m.f. of from 60 to 80 volts can be produced by turning the crank briskly by hand.

BATTERIES. — (*See articles on Batteries.*) The transmitter of a telephone set requires direct current and produces from it the alternating (or otherwise fluctuating) voice currents, which pass over the line to the distant receiver. A central storage battery furnishes the direct current over the line in "common battery systems," or "central energy systems." A primary battery furnishes the direct current in systems not having a central storage battery for the purpose.

Dry Cells. — The primary battery usually is of the dry cell type and is associated directly with the telephone set. Such an arrangement is called a local battery set. The cells have an e.m.f. of about 1.5 volts, and can give a current on short circuit as high as 15 amperes. Dry cells, designed particularly for local battery telephone service, have higher internal resistance than those intended for gas engine ignition and other heavy current intermittent work. Longer life, with only slightly less efficiency, is obtained by increasing the internal resistance. Dry cells may be used for but a few minutes at a time.

Gravity Cells. — Other types of primary cells are used to energize transmitters. Gravity cells are suitable for operators' telephones in small exchanges, and can furnish current steadily with no periods of rest. Gravity cells must not stand long on open circuit. They give about 1 volt per cell. Three or four in series are suitable for a transmitter.

Fuller Cells are suitable for heavy service (many long conversations per day), and also can stand on open circuit without harm. They approach nearest the storage battery in e.m.f. (about 2 volts) and output. They are sloppy and costly to maintain and are used less widely now than heretofore.

Storage Batteries give good results in local battery sets; they require charging by direct current from outside themselves, however, and suffer from the lack of frequent charges. They are suitable for but few conditions of local battery use for those reasons. The most widely used method of charging storage batteries is by motor-generator sets, the motor being adapted to the source of energy and the generator being shunt wound.

POLE CHANGERS. — Ringing current in central offices is usually produced by alternating current generators or by pole changers. The latter are vibrating contact-makers having a natural frequency corresponding to the frequency of the current desired. They are operated by direct current from a storage battery or other source. The Warner pole changer, Fig. 10, is widely used for ringing ordinary telephone bells at a frequency of from 16 to 20 cycles per second. Pole changers of multiple frequency are also used to ring harmonic bells on party lines.

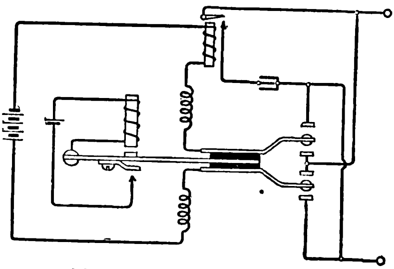


Fig. 10. Warner Pole Changer

IMPEDANCE COIL. — An impedance coil is a single coil of wire wound on an iron core.

INDUCTION COIL. — A step-up transformer used in a telephone circuit is called an induction coil in telephone parlance. Fig. 11 shows the use of such a transformer in a local battery set. The battery and the carbon button are connected in series with its primary winding; the line and the receiver are connected in series with its secondary winding. The function of the induction coil is to convert the local variations of direct current of low e.m.f. into alternating currents of higher e.m.f. in order that it may pass over the line with less loss.

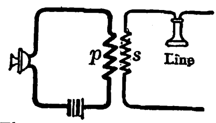


Fig. 11. Induction Coil in Local Battery Set

Local battery induction coils have a large ratio between the two windings. The primary winding is of low resistance and of fewer turns than the secondary winding. Single silk insulated wire is used for both windings; practice varies as to wire sizes and turns; a representative example has 350 turns of No. 20 B. & S. gage wire in the primary and 2400 turns of No. 30 B. & S. gage wire in the secondary. Other examples use from 200 to 700 turns in the primary, 700 to 5500 turns in the secondary, No. 20 to No. 28 B. & S. gage wire in the primary and No. 26 to No. 36 B. & S. gage wire in the secondary.

The induction coils in common battery sets have different functions from those in local battery sets. Figs. 12 and 13 show two forms of common battery telephones using them. In most cases one function of the induction coil is to in-

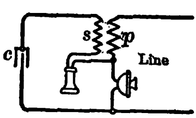


Fig. 12.

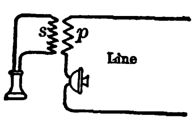


Fig. 13.

sulate the receiver from the direct current of the line, so that its permanent magnetism never shall be opposed by direct current flowing in its coil. In Fig. 12 the induction coil has the further function that, co-operating with the condenser *c* it assists the transmitter to convert the direct current of the line into an alternating voice current. The condenser alternately receives and gives up charges, responsive to the state of the varying resistance of the transmitter.

This particular transforming function is lacking in all circuits having the general type of Fig. 13.

REPEATING COIL. — A repeating coil is any transformer used in a telephone circuit. A one-to-one ratio repeating coil is frequently connected in such a manner that when equal currents flow through each winding in the same (or opposite, depending upon the point of reference) direction, the magnetic fields of the two currents *neutralize* each other, and, therefore, practically no impedance is offered to the flow of such currents, but each winding by itself has a high impedance. As the result of such an arrangement the repeating coil offers a high impedance to any current tending to flow through one winding which is not accompanied by an equal current in the proper direction in the other winding. One application of a repeating coil is shown in Fig. 29. One of the windings of a transformer or induction coil can also be used as a repeating coil as illustrated in Fig. 32.

HOOK SWITCH. — The time during which a subscriber's telephone is idle much exceeds the time in which it is in use. *When idle, it must be ready to receive or to make a call.* When in use, its transmitter must be able to receive direct currents and its receiver to receive voice currents. The hook switch makes these changes *automatically.*

TELEPHONE REPEATERS. —A telephone repeater is similar in principle to a telegraph relay. A repeater to be successful must be designed so that a weak current passing through the "primary circuit" of the repeater will cause a local source of energy located at the repeater to set up a current wave in the "secondary" of the repeater of practically the same form as in the primary, but of increased amplitude; in addition the secondary wave thus set up must not distort the primary wave. To be commercially successful a repeater must be so designed that it can be shunted or bridged across the line and be used for transmitting in both directions. The simplest form of repeater is a receiver which has its diaphragm connected mechanically to the movable electrode of a transmitter. Repeaters are used to a limited extent, but unless special care is used in locating the instrument at the "electrical center" of the line and in keeping it carefully adjusted, the secondary wave set up is reflected back into the primary or transmitter circuit, is again amplified by the repeater, and so on, which cumulative action may be such as to produce a "howling" noise and render the repeater useless. Pupin, the inventor of the loading coil, has suggested the use of an induction motor driven above synchronism as a repeater, but up to the present such a device has not been perfected.

PROTECTIVE DEVICES. — Telephone apparatus and lines are protected from damage due to abnormal currents by means of fuses supplemented by two important co-operating elements. The latter are heat coils or "sneak current arresters," and air-gap arresters. Heat coils operate by short circuiting a pair of contacts when small currents flow for long times or when large currents flow for short times. Air-gap arresters operate when potentials of 350 volts or more exist at their terminals. The operation of either a heat coil or an air-gap arrester short circuits the line, grounds the wires in trouble, and blows the fuse. On account of the high resistance of the telephone instruments, an abnormal current in the line may not be large enough to blow a fuse unless there is a dead short circuit. The heat coil insures such a short circuit whenever the amount or duration of the current exceeds a safe value.

The relation of the three elements to each other and to lines and apparatus is shown in Fig. 14. The fuses should be located at the point of junction between wires which are "exposed" and wires which are "not exposed" to possible contact with a source of dangerous current. All aerial wires and aerial cables are

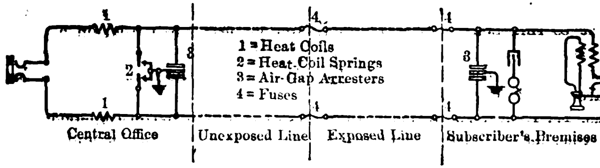


Fig. 14. Arrangement of Protective Devices

exposed. Underground and indoor wires are to be considered unexposed, if the indoor wires are run in accordance with good rules, such as those of the National Electrical Code.

TELEPHONE SETS. — The several elements necessary at a subscriber's station are associated to form series magneto sets, bridging magneto sets and common battery sets.

Series Magneto Sets (Fig. 15) are suitable for use on single party lines, and are widely so used in exchange practice, and, unfortunately, to some extent on lines having more than one telephone (party lines). Fig. 15 is the circuit of a series set. When the receiver is on the hook, the bell is connected between the terminals of the set, the battery and receiving circuits both are open, and if the generator is not turned its armature winding is shunted. When the generator is turned in calling, the shunt is broken automatically.

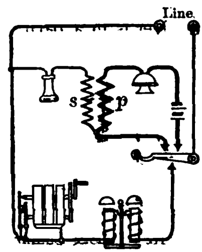


Fig. 15. Series Local Battery Magneto Set

In series sets the voice currents must pass in series *through* all but one of the ringers of a line. The impedances of the ringers, therefore, must be as *low* as possible, yet high enough to respond to ringing current of reasonable e.m.f. 80 to 100 ohms is the usual resistance for both coils of a series ringer. The cores usually are shorter than in bridging ringers.

Bridging Magneto Sets (Fig. 16) are better than series magneto sets for party lines. The ringer is bridged across the line permanently. The generator is bridged across the line when the crank is turned. The receiver and transmitter circuits are closed when the hook is up. Three of the four circuits of the set are open when it is not in use, as in Fig. 16.

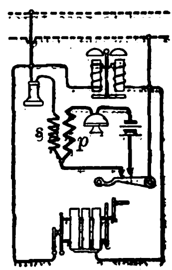


Fig. 16. Bridging Magneto Set

In bridging sets the voice currents must pass *by* all of the ringers of a line. The impedance of the ringers, therefore, must be as *high* as possible, yet low enough to respond to ringing current of reasonable e.m.f. 1000 to 1800 ohms is a usual resistance for the two coils of a bridging ringer. The cores are usually longer than in series ringers.

Common Battery Sets (Figs. 17–19) have no magneto or local battery and can, therefore, be used only on lines leading to a central office. A storage battery at the central office, common to all the lines, supplies current to them for two purposes, for talking and for signaling the operator; hence the name "common battery." The absences of the magneto generator and the local battery simplify construction

and operation of the instrument. Common battery telephones cannot send out current to ring bells, as can magneto telephones; when used on a party line they can, therefore, call each other only indirectly, by asking the central office operator to send the necessary ringing current over the line. Common battery sets are nearly always bridging sets.

Figs. 17, 18 and 19 are typical circuits of common battery telephones. Fig. 17 is that used by the Associated Bell Telephone Companies and in Western Electric Company's apparatus throughout the world. As the condenser is "opaque" to direct current, but "translucent" to alternating current in a degree dependent upon the frequency, no direct current from the central office normally will flow through the set when the hook switch is down, but alternating current from the central office can actuate the ringer. When the switch-hook is up, direct current from the central office flows through the transmitter *t* and the primary *p* of the induction coil.

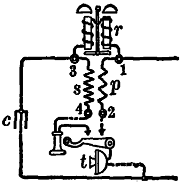


Fig. 17. Bell Common Battery Desk Set

In the circuit of Fig. 18, used by the Kellogg Switchboard & Supply Company, the ringer, in series with a condenser C_2, normally is bridged across the line. When the switch-hook is up, the transmitter is in series with the line and can vary its current. The receiver is in a path "opaque" to direct current, but "translucent" to voice current, because of the condenser C_1. The receiver path, including the condenser

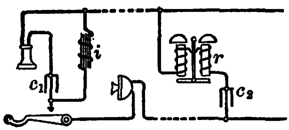


Fig. 18. Kellogg Common Battery Set

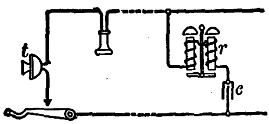


Fig. 19. Desk Set with Direct Current Receiver

C_1 is shunted by a coil *i* of low resistance but of high inductance, which is opaque to voice currents but translucent to direct currents. No direct currents can pass through the receiver.

The circuit of Fig. 19 is one in which all of the direct current intentionally is allowed to pass through the receiver. The latter has no permanent magnet (i.e., is of the "direct current" type, Fig. 4 or 5), and depends on the direct line current from the central office to magnetize its core.

Wall and Desk Sets. — Telephone instruments are assembled for use in two forms, wall sets and desk sets. The latter also are called portable sets, having the receiver, transmitter and hook switch combined into a unit which can be carried about. Wall sets are self-contained, having space for batteries if needed. In local battery and common battery desk sets the ringers are separately mounted, usually upon a wall. Local battery desk sets in addition require housing for batteries, and magneto desk sets require separately mounted hand generators.

A desk set is connected to line and ringer, and to battery and generator, if used, by a flexible cord. The number of strands in the cord depends on the type of circuit used, and on whether the induction or impedance coil, if used, is mounted in the portable set itself or on the wall with the ringer. In Figs. 17, 18 and 19 the parts of the circuit carried by cords between the portable and fixed portions of the set are shown by dotted lines.

Desk sets are more convenient for use but are more costly to maintain than are wall sets. They fall frequently, breaking transmitter mouthpieces, receiver shells, and other parts. Supporting arms for desk sets minimize this breakage.

Extension Set. — An extension set is a set used as an auxiliary to a regular set; the extension set usually has no bell; signals are received on the bell of the terminal set, and if the user of the extension set is wanted, he is notified by a separate signal, such as a buzzer, operated by the person who answered the terminal set. Any number of extension sets may be bridged to a line. Any one of them may make a call. None can receive a call direct from the central office nor from another station on the line unless equipped with a ringer, and not then from other stations on the line unless they are magneto sets.

PARTY LINE CIRCUITS. — Party line sets differ from individual line sets only in methods of signaling. The apparatus used and the method of connecting up depends upon whether the party lines are to be non-selective or selective.

Non-Selective Party Lines. — On such lines a call made by any party on the line operates the ringers at all stations. Each station has a particular group of periods of bell ringing, which group only it is meant to answer. Up to five stations from one to five short rings per station serve well. Beyond five, long and short rings are combined. Magneto sets, both series and bridging, are used on non-selective party lines. The generator of the magneto set at any station can ring all the bells of the line at once, and operate the central office signal also, if the line leads to a central office. The bridging system is so used on many private lines, farmers' co-operative lines, rural toll lines, and, to a degree, on exchange party lines in towns.

Non-selective stations should not answer other calls than their own; this extra listening on lines is an extra drain on local batteries in sets having them; it also destroys privacy. A receiver off the hook impairs signals unless circuits are specialized.

If a condenser of low capacity, say one-half microfarad, be placed in series with the receiver, it will obstruct ringing current (frequency 20 per second) considerably, but voice currents (mean frequency 800 per second) very little.

Selective Party Lines are equipped with ringers arranged to respond each to its own call, the others remaining silent. There are three such methods in use in the United States, and still a fourth method, which has certain very decided advantages, is coming into use.

Metallic Circuit Two Party Lines (Fig. 20.) — On metallic circuits selective two party working is simple. Fig. 20 is the standard system of the

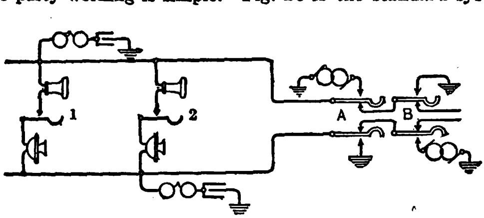


Fig. 20. Two Party System

Associated Bell Telephone Companies in the United States. Each wire of the metallic circuit is used as a private grounded ringing circuit to one station, and the pair of wires is used as a talking circuit by each station. Key *A* connects the ringing generator to the upper wire, ringing station No. 1; key *B* rings

station No. 2. If a station answers while being rung, the other station is not rung falsely, the ground on the non-ringing side of the used key preventing that.

Harmonic Selective Systems (Fig. 21) operate on the principle that a ringer having a spring-supported armature will ring for one frequency of current but not for any other frequency. Frequencies of 16, 33, 50 and 66 per

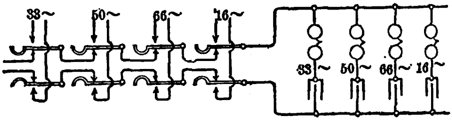
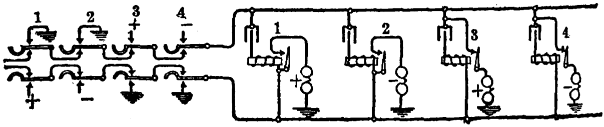


Fig. 21. Four-party Harmonic System

second are used in the Dean four-party harmonic system. Each calling cord in the central office switchboard can be rung upon by one of four keys, as in Fig. 21. The ringers are alike except as to the sizes of tappers, which are cylinders of metal, heaviest for the lowest frequency and graduating to lightest for the highest frequency.

The number of selective stations may be made twice the number of frequencies by connecting harmonic ringers from each wire to ground as in Fig. 20, instead of across the line as in Fig. 21. This enables eight stations to be placed on a line leading to a four-frequency central office.

Biased Ringer Systems (Fig. 22). — The Hibbard party line system uses unidirectional pulsating current for the selection of biased ringers on four-party lines. An improvement on that system (Fig. 22) is used by the American Telephone & Telegraph Company. A biased ringer is an ordinary ringer which has its armature drawn to one side by a spring. Impulses of current



• Fig. 22. Four-party Biased Bell System

in one direction only will operate the ringer. Condensers permanently in series with such ringers would interfere with the selective action; but condensers are necessary to prevent ringer connections from interfering with other functions of common battery lines. In common battery systems, therefore, a relay at each station, as in Fig. 22, connects the ringer to the proper line wire only during ringing. The same current that operates the ringer draws up the ringer relay.

Step-by-Step Party Line Systems select the desired subscriber by the operation one after another of a set of switches at the subscriber's station, controlled from the central office. Step-by-step party line systems have not come into wide use, but the development of automatic switching systems for central office use is increasing knowledge, faith and skill concerning step-by-step mechanisms.

Semi-Selective Party Lines. — Semi-selective ringing is done for more than two stations on a line, as in Fig. 20, by connecting more than one ringer to each wire, ringing code signals on key *A* for the ringers on the upper wire and ringing code signals on key *B* for ringers on the lower wire of the figure.

CENTRAL OFFICE EQUIPMENT. — Telephones are used in two general ways: (1) On a line permanently connecting two or more telephones, this line not being connectible to another, and, (2) On a line carrying one or more telephones, this line being connectible at will to any other of a group of lines. In the second case a switchboard and the necessary adjuncts for producing the various kinds of current required must be provided for connecting any line to any other. Such a switchboard and its adjuncts make up a "central office." A "telephone exchange" is an organization of one or more central offices and the connecting lines and apparatus employed in supplying telephone service to a community. A central office is, therefore, but part of an exchange.

A "manual switchboard" is one in which a human operator connects and disconnects the lines by hand, as instructed by the subscribers through signals and speech. An "automatic switchboard" is one in which machines connect and disconnect the lines in response to electrical changes in the calling line. The functions of manual and automatic switchboards sometimes are combined and named "semi-automatic systems" or "automanual systems."

A switchboard designed to interconnect telephone sets equipped with local batteries and magnetos is called a "magneto switchboard," while a board designed to interconnect telephone sets supplied with current from a single battery in the central office is called a "common battery switchboard." When each of the lines entering a central office is permanently connected to a single "jack" or terminal, the board is called a "simple switchboard." When each line is permanently connected to a plurality of jacks, located at different parts of the board, the board is called a "multiple switchboard." A multiple switchboard is divided into sections, each section accommodating three operators, and containing a jack for each line entering the central office. The group of jacks in each section is referred to as the "multiple." "Simple switchboards" can be used only in case the number of lines connected to the central office is small (less than about 300), unless local trunking in the central office is resorted to. It should be noted that the cost per subscriber of central office switchboard equipment increases as the number of subscribers increases when the total number of subscribers is sufficient to require the use of multiple switchboards.

Cords, Plugs, Jacks and Drops. — As principally used, manual switchboards connect lines by means of flexible conducting "cords" tipped with plugs which fit line-terminal-sockets called jacks. The plug contains some number of parts, electrically insulated from each other but connected to conducting strands of the cord terminating in the plug. There are two strands in the cords of simple magneto switchboards, and therefore two parts in the plug to connect to two conductors of the jack. Fig. 23 shows a jack and plug for a two-stand cord. Few switchboards involve circuits calling for more

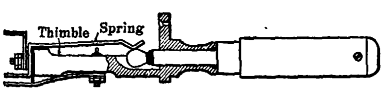


Fig. 23. Jack and Plug

Fig. 24. Diagram of Plug and Jack

than three conductors in regular switching cords. Plugs and jacks for that many conductors can be made in compact and serviceable forms. Fig. 24 shows diagrammatically a three-conductor jack and plug, in which it is intended that the tip of the plug shall engage the short spring, the ring of the plug the long spring, and the sleeve of the plug the tubular sleeve or thimble of the jack.

The cords and plugs are associated with keys, signals, wiring and current sources, and are links which the operator uses to do her work upon the lines made

accessible by the jacks. At D in Fig. 25 is shown diagrammatically the type of signal employed on magneto manual switchboards. This signal, called a "gravity drop," is bridged across the switchboard end of the line in the same manner as the ringer at the subscriber's station. When the core is energized by current from the magneto, at the subscriber's station, the armature is attracted, lifting the catch and releasing the shutter D allowing it to fall forward by gravity. These signals are called gravity drops for this reason.

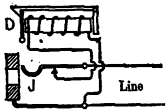


Fig. 25. Magneto Drop and Jack

Power Plants. — The power plant is an organization of devices in a central office to furnish to the lines connected to that office the several kinds of currents required. The principal elements of power plants are storage batteries, charging generators and sources of alternating current for ringing.

In the common battery system one storage battery usually supplies current to all transmitters, whether of subscribers or operators, and actuates all principal relays and lamp signals. In magneto offices storage batteries need only be large enough to supply operators' transmitters and a few auxiliary signals, as magneto equipments usually have primary batteries in subscribers' telephones.

Magneto Manual Switchboards (Figs. 25 and 26). — Fig. 25 represents diagrammatically the central office end of a subscriber's line and Fig. 26 includes the essential elements of a magneto cord circuit. Normally the tip of one plug is connected through the cord to the tip of the other, and the same is true of the plug sleeves. When a subscriber rings, the drop D in Fig. 25 falls. The central operator then inserts the plug P_1 (Fig. 26) of an idle cord circuit in the jack of this subscriber's line; this disconnects the line drop, the shutter of which is returned to its normal position either by hand or automatically. She then connects her telephone set to this cord by closing the switch A and gets the number of the subscriber wanted, and inserts the plug P_2 in the corresponding jack. By closing the switch B she connects the called subscriber's bell to the central office source of ringing current, which usually is running continuously. The called subscriber's bell rings as long as the operator presses the ringing key. When she releases it, direct connection between the two lines results. When the called subscriber answers, conversation may ensue, and at the close of it the subscribers are expected to signal for disconnection by ringing. The gravity drop D (Fig. 25), in this case called a clearing-out drop, is bridged across the cord circuit, and in response to this signal the operator removes both plugs. The line signals of both lines thus are reconnected in readiness to receive calling signals.

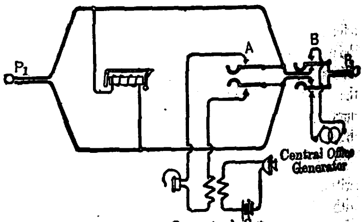


Fig. 26. Magneto Cord Circuit

The operator's set is connectible to the cord circuit by means of a key, enabling her to answer the calling subscriber by telephone, to listen to the conversation, and, by releasing one listening key and pressing another, to shift her telephone set from cord circuit to cord circuit. Depending upon a variety of conditions, an operator can utilize from ten to twenty cord circuits, transferring her telephone set from one to another as required.

In Fig. 26, ringing current can be sent out only from the right-hand plug.

Calls, therefore, are answered by using the left-hand plug of the figure, called lines being taken up by the right-hand plug. Some users require both plugs to be equipped with calling keys, in which case the left-hand plug would be given an arrangement symmetrical with that of the right-hand one.

The shutters of gravity drops used as line or clearing-out signals are restored to their upright position by hand in most simple magneto switchboards. Formerly the drops were placed in one group and the jacks in another, the drops being restored directly by the operator's fingers. Latterly the drop and jack of a line are associated together, so that the plug as inserted into the jack restores the shutter by some thrusting movement on the part of the jack. A simple way is to form a tongue upon the jack spring, this tongue thrusting the shutter directly into its latch.

Large magneto switchboards have been equipped with drops grouped at a distance from the jacks and out of reach of the operators. Such drops are restored after operation by electro-mechanical means, the act of placing the plug in the jack of a line closing circuits to restore the shutter.

Jacks are adapted to be mounted in insulating strips in whatever arrangement is desired, and any jack of a group may be dismounted. When more compact arrangement is desired, jacks are made up in strips of ten or twenty, an entire strip of which must be removed from the framework of the switchboard to enable one jack to be inspected or worked upon.

During hours of light telephone traffic, as at night, it is desirable to use as few operators as can handle the traffic satisfactorily, and, in small exchanges, even to allow the operator to sleep between calls. On a long switchboard a drop may fall many feet away from the night operator and its falling be unheard. A night alarm signal is provided by equipping each drop with a contact closed by the falling of the shutter. These contacts of all the drops of a group are wired in multiple to a battery, bell and switch which is closed at night. When any drop of that group falls, the bell will ring if the switch is closed.

Switchboard plugs, when not in use, stand vertically in a plug shelf, the cords being housed within the switchboard framework and kept reasonably taut by means of weighted pulleys. Weights should be heavy enough to enable cords to return plugs to position after disconnection, and not so heavy as to shorten unduly the lives of the flexible conductors within the cords.

Where grounded circuits are connected to metallic circuit switchboards and require interconnection with metallic circuits, a ground connection must be provided to furnish the second limb of the line, as switchboards now are built entirely for metallic circuits. Such grounds must be connected to jacks in one unvarying way or two grounded circuits may be connected so as not to be able to talk to each other. The usual practice is to connect the line wire to the spring of the jack and the ground to the sleeve.

Simple Common Battery Switchboard (Fig. 27).—The circuits of a representative simple common battery switchboard are shown in Fig. 27 and are those of the Kellogg Switchboard & Supply Company. They are a type of many other makes. The subscribers' lines terminate in cut-off jacks. There is only one jack per line. The calling plugs only are equipped with ringing keys.

The cycle of operations of such a system is as follows: The subscriber removes the receiver from the hook and as there is no plug in the jack of his line at the central office, the line relay is actuated, lighting the line lamp and operating the pilot relay. The pilot relay, in turn, lights a pilot lamp, of which there is one for a considerable number of line lamps, and with which a bell or other night alarm signal can be associated. Responsive to the lighting of the line lamp, which in practice is mounted close to the line jack, the operator inserts an answering plug in the jack, breaking the line relay circuit in two places, and so extin-

guishing the line lamp. Operating her listening key, the operator connects her talking set with the cord circuit, asks for the subscriber's order and executes it by inserting the calling plug in the jack of the line called for, unless that jack already is occupied by a plug. If it is occupied, she tells the calling subscriber

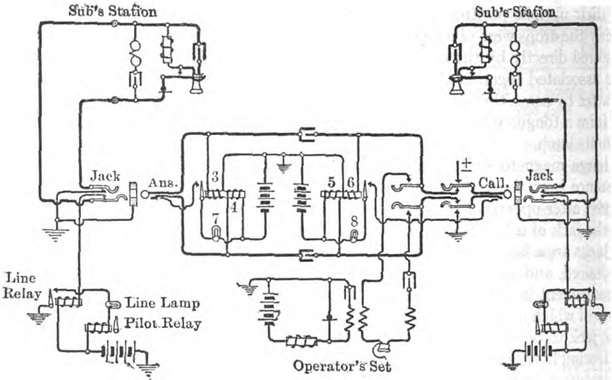


Fig. 27. Circuits of Simple Common Battery Switchboard

that the called line is busy. If it is not occupied, she plugs into it, as stated, and rings. At this stage, the supervisory lamp 7, adjacent to the answering plug, is dark and the supervisory lamp 8, adjacent to the calling plug, is lighted, indicating that the calling subscriber's receiver is off the hook and the called subscriber's receiver is on the hook. When the called subscriber answers in response to the ring, lamp 8 is extinguished. At the close of the conversation, both lamps will re-light as the respective receivers are hung up. In this way the operator can tell at all times the state of the connection and conversation and, responsive to the final lighting of both supervisory lamps, she removes both plugs with the *positive* knowledge that the conversation has been finished.

The heavy lines in Fig. 27 are those over which the subscriber's conversation takes place. In the cord circuit the two conductors which carry voice current are continuous from tip to tip and from sleeve to sleeve of the plugs, except that a condenser is interposed in each of the two conductors. Direct current to actuate subscribers' transmitters is furnished to the calling subscriber through

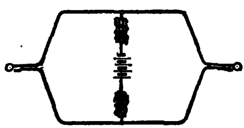


Fig. 28. Impedance Coil Cord Circuit

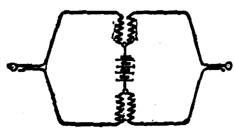


Fig. 29. Repeating Coil Cord Circuit

the windings 3–4 of the answering supervisory relay and to the called subscriber through the windings 5–6 of the other. Each supervisory relay thus serves two purposes, one being to supply the subscriber's transmitter with current, the other being to co-operate with the ground at the *line* jack so as to light the supervisory lamp if no direct current flows through the line, and to extinguish the supervisory lamp while the direct current does so flow.

There are three general ways in which direct current can be supplied from cord circuits to subscribers' lines. In Fig. 28 current is supplied through impedance coils without condensers. Such a plan is suitable where all the lines of a switchboard are of about equal resistance, as in a private exchange. If the lines are of varying resistance, a high resistance line will be denied proper current when connected through such a cord circuit to a low resistance line (low compared to the resistance of the impedance coil). The circuit of Fig. 29, using a repeating coil, is free from this objection and is standard with the American Telephone & Telegraph Company. The circuit of Fig. 30, using impedance coils and condensers, is as efficient as that of Fig. 28, though entirely different in principle. It is widely used in manual and automatic switchboards other than those of the American Telephone & Telegraph Company.

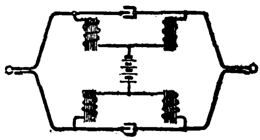


Fig. 30. Condenser Cord Circuit

Comparison of Magneto and Common Battery Systems.—The magneto system is best in small towns and country exchanges. Magneto systems are best for exchanges under 500 lines and common battery systems best above that number. No positive rule can be laid down. For example, the magneto system may be arranged in exchanges which start with little equipment and do not grow beyond 500 lines while the equipment still is in good condition. Conversely, common battery systems may be warranted in exchanges as small as 200 lines, wherein the conditions are favorable and the need of the highest grade of service is great.

Transfer Systems or Local Trunking.—When it is desired to use a simple magneto or common battery system in an office requiring more than three operators, so that there will be lines beyond the reach of each operator, it is possible to provide local trunks or transfer circuits to interconnect lines which cannot be reached directly. These are called transfer systems or local trunking systems. If an operator has a call for a line which she cannot reach directly, she inserts the calling plug in a jack of a trunk which leads to the section carrying the jack of the called line. At that section an operator connects the chosen trunk with the called line and assists in disconnecting it at the end of conversation. These systems have service disadvantages which in most cases more than outweigh the savings, and the best practice is to use multiple switchboards where simple switchboards are outgrown. An important exception is the case of large cities where the majority of calls would have to be completed through a second central office in any event. Under such circumstances multiple jacks for the trunks only are provided, the subscriber's line being provided with but a single jack and practically all calls are trunked irrespective of their designation.

Multiple Switchboards (Fig. 31).—A multiple switchboard is one in which all the lines of the office are brought within reach of each operator by equipping each line with a number of jacks, distributed over the board in such a manner that each operator can reach at least one of these jacks. The usual practice is to equip each line with one answering jack, at which all its calls will be answered, and in addition with a plurality of line jacks into any one of which a plug may be inserted when that line is called. As there is a plurality of places where the called line may be taken, some guard is required to prevent an operator taking the line when it is in use at another part of the switchboard. This guard is called the "busy test" and is provided by changing the electrical state of a metal thimble of each jack of a line whenever a plug is inserted. When idle, all the thimbles of all the jacks of a line are at the same potential as all the tips

of all idle plugs. Touching the tip of a plug to the sleeve of a jack of an idle line produces no electrical result. But when an operator touches the tip of a plug to the sleeve of a line which is busy, difference of potential between tip and sleeve causes current to flow and a click to be audible in her receiver. She then tells the calling subscriber the called line is busy.

Fig. 31 shows the complete circuits of a calling line, a called line, and the connecting cords of the Western Electric Company's relay switchboard No. 1,

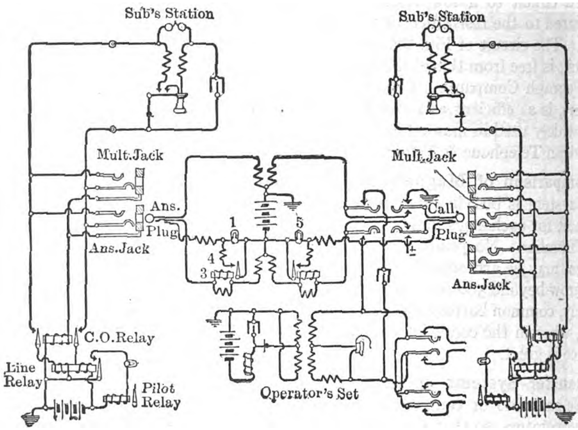


Fig. 31. Circuits of Multiple Battery Switchboard

which is the standard of the American Telephone & Telegraph Company. The cycle of operations is exactly as described with reference to the simple common battery switchboard with the addition of testing the jack of the called line. Current supply to the cord circuit is of the form shown in Fig. 29. The jacks have fewer parts than in the simple switchboard before described. The line relay is disconnected from the line by a cut-off relay instead of by a cut-off jack. The cut-off relay is operated by current from the cord circuit when a plug is placed in any jack of the line. This current also furnishes the busy test potential to all jacks of a line so long as a plug is in any jack. Direct current which actuates the subscriber's transmitter also energizes the relay 3, placing the shunt 4 around the supervisory lamp 1. It results that the lamp 1 or 5 will be lighted or dark, depending upon whether the switchhook of its line is down or up.

All principal common battery multiple switchboard systems now in general use have the common features of an audible busy test, line lamp signals, two supervisory signals per cord circuit — one for each line of a connection — and answering jacks supplementing the multiple (or calling) jacks. None of them requires more than two wires in the subscriber's line outside of the central office, but the system of Fig. 31 requires three wires in the line throughout the switchboard. Other systems require only two wires throughout the switchboard and in these systems the busy test and cut-off relay functions are combined with the other functions of one of the wires of the line. The advantages of the two-wire over the three-wire system lie principally in the greater simplicity of the jacks and the fewer wires necessary in the switchboard cables.

Common battery multiple switchboards for use in offices of from 500 to 1200 lines sometimes are provided with cut-off jacks, avoiding the necessity of using cut-off relays. Some such switchboards use magnetic mechanical signals as a substitute for lamp signals. They are not the best practice. Lamp signals have decided advantages. They are compact, can be mounted closely adjacent to jacks, show great contrast between actuated and non-actuated condition, and are not easily obscured by cords.

A and B Operators. — Manual multiple switchboards are adapted to bring all the lines of the office within reach of each operator, but such switchboards cannot concentrate more than 10,000 lines within such reach unless the jacks are made smaller than is considered the best practice. When an exchange contains more than 10,000 lines, it is customary to provide more than one office. This makes possible a saving in lines and things relative to them, but requires calls to be trunked from one office to another. In an exchange of 100,000 lines, for example, there would be ten switchboards of 10,000 lines each, each switchboard serving a district of the exchange.

Calls *originating* in one district will be *for* lines in all the districts and only those for lines in the originating district can be completed in the switchboard for that district. Calls for subscribers in all the nine other districts must be trunked. To accomplish this, the operators in a multi-office exchange have equipment enabling them not only to complete calls directly in the multiple before them but to connect subscribers with trunk lines leading to the other offices, and in each office furthermore there are operators and equipments adapted to receive and complete calls trunked to them from the offices where they originated.

Operators who answer subscribers are called *A* operators; operators who serve trunk lines incoming from other offices are called *B* operators. The cycle of operations in a call for a line not in the office first called is as follows: subscriber lifts his receiver and his line lamp lights; operator answers and asks number; learning by prefix that call is for a subscriber in a distant office, operator presses a key marked with that prefix, so connecting her telephone set with a line leading directly to a receiver of a *B* operator in the distant office; *A* operator speaks the number desired, following with the *prefix of her own office; B* operator in distant office names back a trunk number; *A* operator inserts calling plug in a multiple jack of that trunk and simultaneously distant *B* operator inserts the plug of that trunk in the called line; method of ringing depends on type of apparatus; at close of conversation *A* operator disconnects, which act lights disconnect lamp before *B* operator, who disconnects in response, extinguishing signal.

Main Distributing Frame. — The main distributing frame is a device upon which are terminated, usually upon opposite sides, the lines from subscribers and from switchboard apparatus. Between these terminals connecting links of wire, called "jumpers," are soldered. The purpose of the main distributing frame is to enable a subscriber's line to be connected semipermanently with a given switchboard line. A subscriber may move anywhere within the district of his central office and may utilize any pair of wires entering that office without relinquishing his particular telephone number, as his switchboard circuit may be connected by jumper to whatever entering cable wires his line may use. Protective apparatus, consisting of sneak current arresters and carbon air-gap arresters, is associated with all lines entering a central office. These protective devices customarily are mounted on the switchboard side of the main distributing frame.

Intermediate Distributing Frame. — The intermediate distributing frame is a device on which lines to multiple jacks and lines to answering jacks

respectively are accessible for inter-connection. Its object is to enable a given line to terminate on an answering jack at any part of the switchboard and this connection also is semi-permanent. By this is meant that when a subscriber's line has been connected by jumper to an answering jack on a certain operating position, all the calls of that line will be answered at that position until the jumper in the intermediate distributing frame has been changed. The purpose is to enable the traffic originated by the subscribers of an office to be divided equally among the operators. This equalization is not automatically done by the intermediate distributing frame, as is the intent in the automanual system.

Automatic Switchboards.—An automatic switchboard is a set of machines in a central office, adapted to connect and disconnect lines under the control of subscribers. These machines do automatically what human operators do on manual switchboards. The objects are to save the cost of the labor of manual operators and to increase the speed and accuracy of connecting and disconnecting. As the only human operating which is done in an automatic system is done by the subscribers themselves, the accuracy of connecting depends on the skill of the subscribers. In practice this is found to be sufficiently high for the rendering of good service. Automatic systems have the considerable advantage that the disconnection of the lines is instantaneous, which is not true in any purely manual system.

Automatic systems are in successful use in many exchanges in the United States and in a few elsewhere. The apparatus is more intricate than that of manual switchboards. The first cost of automatic equipment is greater than manual equipment in small systems and less in large systems. In both manual and automatic systems the larger the traffic the more equipment per subscriber is required in the central office and the greater is its cost.

All automatic systems so far devised utilize the step-by-step principle. In the system of the Automatic Electric Company, which now is in wide use, selector and connector switches take as many steps per movement as there are units in the digit called.

In manual multiple switchboards, all the lines of an office are accessible to each operator. At the time of making a connection the operator and the calling plug of a cord circuit may be considered as a terminal device attached to the calling line and seeking the called line in the multiple of all the lines. In a manual transfer switchboard or in the trunking of a call from one office to another in an exchange having several offices with multiple switchboards, a first and a second operator must co-operate to complete certain connections.

An automatic system represents a third case in which two or more devices must co-operate to complete a connection. In a word, as many separate machines are engaged in making a connection as there are digits in the called number, less one.

Success in automatic telephony came with the abandonment of the complete multiple principle and the adoption of the complete trunking principle, which had been abandoned in manual telephony some time before.

Selector and Connector Switches. — The underlying feature of the automatic trunking system is that the calling subscriber directs the first "selector switch" to seek an idle trunk from a certain group of trunks, all of which lead in a direction determined by the first digit of the called number. This first switch makes as many vertical steps as there are units in the first digit called, and immediately thereafter seeks out and appropriates an idle trunk of that group. On the calling of the next digit, a switch at the end of this selected trunk takes as many vertical steps as there are units in the second digit, and immediately seeks out and appropriates an idle trunk in that group. This process is repeated until all but two digits have been called, which carries

the connection to the last switch which will be used. This switch is called a "connector" and has vertical and rotary motions like the preceding switches, but, unlike them, both vertical and rotary motions are under the subscriber's control. As he calls the next to the last (tens) digit, the switch takes as many vertical steps as there are units in that digit but does not follow with an automatic rotation. It rotates in response to the calling of the last digit, taking as many steps as there are units in that digit and stopping with its wipers in contact with terminals of the called line. Ringing of the called subscriber then is caused by automatic means or by relays responsive to a push button in the calling subscriber's telephone. Disconnection ensues when the calling subscriber hangs up his receiver.

Space does not permit a detailed description herein of the circuit connections for an automatic system.

Semi-automatic Switchboards. — There have been many proposals to introduce automatic switches into manual systems to do certain things better or more cheaply than in complete manual systems. Some of these proposals arrange to distribute incoming calls among operators, the remainder of the process being the same as in a standard multiple manual switchboard. Other proposals arrange to substitute automatic switches for the multiple switchboards with or without any arrangement for distributing the incoming traffic upon the operators.

Both plans have advantages, a chief one being the instantaneous disconnection which is possible with automatic devices. Another is that an automatic system is fundamentally a trunking system and the tendency of growth in exchanges is more and more toward large percentages of trunking. A principal office in New York City, for example, completed 80 per cent of the calls of its subscribers directly in its own multiple twenty years ago, 25 per cent ten years ago, and completes perhaps 5 per cent now. In other words, of the calls of its subscribers the percentage trunked to other offices has risen from 20 per cent of the total to 95 per cent of the total in twenty years. Under such circumstances, it no longer is considered good practice to provide any multiple of subscribers' lines in switchboards which handle calls from subscribers but to trunk all calls whatever may be their destination. Automatic apparatus is particularly adapted to this work.

Automanual System. — The automanual system is an example of semi-automatic systems and is in use in several exchanges in the United States. It consists of a group of automatic switches controlled by keys before an operator. She receives calls orally from subscribers and completes connections by touching a key for each digit called. The automatic apparatus finds and calls the desired line and disconnects the lines when conversation ends. Operators of this system are able to handle several times as many calls per day as in regular manual systems. One operator is reported as having completed 1100 connections in one hour.

PRIVATE EXCHANGES. — A private exchange is a group of telephone lines and switching apparatus subordinate to a central office, and usually on the premises and for the uses of a single business. Private exchanges are widely used in hotels, apartment houses, factories and suites of offices.

Private exchanges are of three types: those using simple switchboards which are attended by operators through whom all local and outside calls pass; those requiring keys at each telephone whereby each user may select the line of any local telephone or a trunk line to a central office, and those having a local equipment of automatic switches. The latter systems usually are connected to automatic central offices.

In all cases the object of the private exchange is to enable the telephones

within them to communicate with each other directly without using lines through the central office at all. A second object is to enable communication from telephones of the private exchange through the central office by the use of fewer trunk lines than there are telephones in the private group. A serviceable ratio of trunk lines to local telephones may be from one-tenth to one-fifth, at a considerable saving in cost over the providing of a line to the central office from each local telephone.

PHANTOM CIRCUITS (Fig. 32.)—In Fig. 32, four wires join two offices. *RR* are transformers with their secondaries arranged as repeating coils. They are designed to be efficient in transforming both talking and ringing currents. Currents from telephones connected to either physical pair of wires pass at any instant in opposite directions in the two wires of the pair. The phantom circuit uses one of the physical pairs as a wire of its line. It does this by tapping the

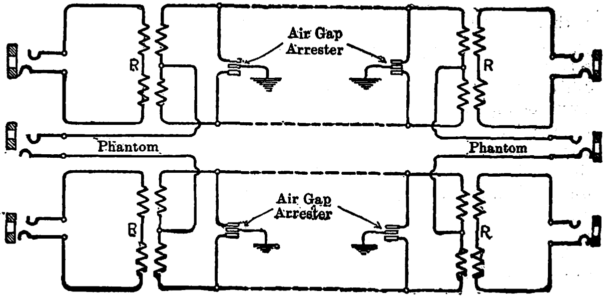


Fig. 32. Phantom Circuits

middle point of the line side of each of the transformers. The currents of the phantom circuit are not heard in the physical circuit because they pass outwardly from the middle points of the secondaries in equal and simultaneous amounts and, therefore, produce no resultant magnetization in the core. The currents of the physical circuits are not heard in the phantom circuit because the former can produce no differences of potential in the phantom circuits, provided all four wires are equal in resistance and insulation; under these conditions no difference of potential can exist between the middle points of the repeating coils at the two ends of the line.

JOINT TELEPHONE AND TELEGRAPH CIRCUITS.—Telephone lines may be equipped so as to permit speech and telegraphy to go on over the same wires at the same time without interference with each other. There are two ways of accomplishing this, being known respectively as the simplex system and the composite system.

Simplex Circuits (Fig. 33) are made from metallic circuit telephone lines as in Fig. 33. The principle is the same as that of a phantom telephone circuit. Nearly the same results can be obtained by using a simple impedance coil (i.e., a single winding with tap brought out from the center) bridged across the telephone line instead of a transformer inserted in it, but in ringing on such a line with a grounded generator the telegraph relays will chatter.

Composite Circuits (Fig. 34) depend upon a different principle. Impedance coils are inserted to oppose alternating currents, and condensers to oppose direct currents. In Fig. 34 one telephone circuit forms two Morse circuits, so

that two wires form one telephone circuit and (with the earth) two telegraph circuits. Each Morse circuit includes in series two 50-ohm impedance coils and condensers are shunted to ground between the Morse sets and the impedance

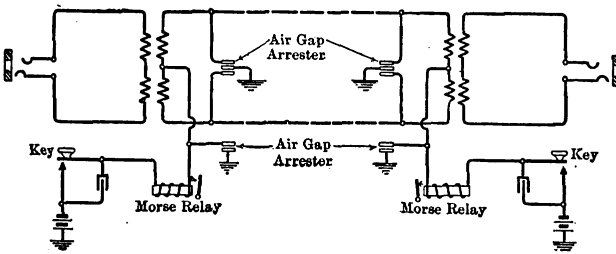


Fig. 33. Simplex Telephone and Telegraph Circuit

coils. The 50-ohm impedance coils are connected differentially and so offer low impedance to the Morse impulses, whose frequency is not high. The coils, however, offer great impedance to voice currents, since they are not differentially connected with respect to the latter. Voice currents can pass through the con-

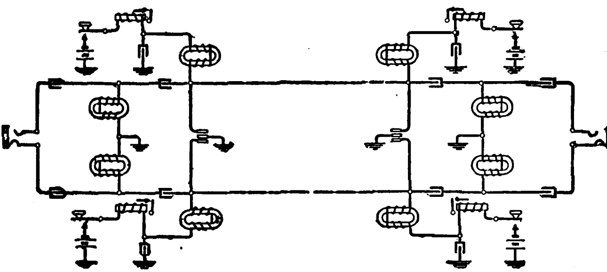


Fig. 34. Composite Telephone and Telegraph Circuit

densers in the telephone line, but direct currents cannot. Impulses due to discharge of coils and capacities in the Morse circuit would make sounds in the telephone but that they are choked off or led off by the 30-ohm impedance coils and the large capacities connected to the Morse sets.

Railway Composite Systems. — The principles of the respective impedances of inductances and capacities to direct and alternating currents, upon which the composite system depends, are applied to the conversion of series telegraph lines into combined telegraph and telephone lines. It sometimes is of advantage to modify a single-wire telegraph line so that telephones may be worked upon it, without abandoning its use as a regular telegraph circuit.

Such an arrangement is known as a railway composite system. Calls between telephone stations on such a circuit are sent by means of high-frequency alternating currents, such as are developed, for example, in an automobile spark coil. The signals are received at the telephone stations on "howlers," which are merely telephone receivers with special horn-shaped mouthpieces. These calling currents are of such high frequency as not to affect the telegraph relays. The paths for the higher-frequency currents in general are supplied by condensers and the

paths for the lower-frequency currents for telegraphy in general are supplied by inductance.

COSTS. — The following prices were current in 1913; they are all f.o.b. factory. These costs are given merely as a rough guide for preliminary estimates. For accurate estimates quotations should be obtained from the manufacturers.

Magneto wall set complete: —	
Series (80-ohm ringer)	$9.00
Bridging (1000- to 2500-ohm ringer)	10.50- 12.25
Magneto desk set complete: —	
Add 75 cents to above prices.	
Common battery wall set complete: —	
Oak or walnut, 500-ohm ringer	8.50
Oak or walnut, 1000-ohm ringer	8.75
Hotel type, 1000-ohm ringer	7.25
Common battery desk set complete with cord: —	
500-ohm ringer	9.25
1000-ohm ringer	9.50
Automatic wall or desk set	13.00
Magneto switchboards: —	
50–200 lines, 1-operator equipment	175–575
250–300 lines, 2-operator equipment	730–865
Simple common battery switchboards: —	
100–600 lines	550–2500
Corresponding cost per line	5.50–4.17
Multiple common battery switchboards: —	
1000–13,000 lines, 160 multiple jacks per position, with power plant	13,000–280,000
Corresponding cost per line	13.50–21.50
Power plants: —	
200 to 1000 lines	200–575

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[S. G. McMeen.]

TELEPHONE LINES. — (*See also Distribution Lines; Transmission Lines; Wires and Cables.*) The line always consists of two conductors, though one of these may be the earth. The standard telephone line consists of two wires of the same size and of the same material. If only one wire is used and the earth forms the return the line is called a "grounded circuit." A line of two wires without earth return is called a "metallic circuit."

LINE CONSTANTS. —(*See also Transmission Lines.*) The transmitting efficiency of a telephone line depends upon its resistance, electrostatic capacity, leakage and inductance.

Leakage. — High insulation, i.e., small leakage, between the two wires of a line and from each wire to earth is possible in cables, and reasonably high insulation is possible in open wire lines. Porcelain insulators in open wire give higher insulation of open wire lines than do glass insulators.

Resistance. — The resistance of the conductors of a line attenuates the telephone current, but does not distort it appreciably, because the losses due to resistance are practically independent of the frequency of the current.

Electrostatic Capacity. — The electrostatic capacity of a line attenuates the current and distorts it also, because the charging current is not independent of the frequency. As the voice current is an alternating one of composite frequency, the higher frequencies are reduced more than the lower. There are two ways of expressing the important and controlling quantity of capacity in cables, i.e., in terms of the "mutual capacity" of the two wires or in terms of the "grounded" or "regular capacity." (*See article on Capacity and Charging Current.*) The mutual expression is the better because it is the amount of mutual capacity which determines the capacity losses. Mutual capacity in telephone cables is about two-thirds the regular capacity of the same wires. A given cable has higher capacity at high temperatures and cable capacities should be referred to a standard temperature in specifying.

Inductance. — The inductance of telephone lines is usually comparatively small, since the wires are usually close together. Insulated wires twisted together for use as lines in cables have very small inductance. (*See article on Inductance and Inductive Reactance.*)

Pupin Coils. — In 1887 Oliver Heaviside pointed out that an increase of inductance would decrease the harmful effects of capacity. In 1900 Prof. M. I. Pupin made public (*Trans. A.I.E.E., Vol. XVII, p. 445*) his method of reducing attenuation and distortion by inserting additional inductance into lines and showed how to determine how much inductance to insert and where. In an ideal method, distributed inductance would offset or neutralize the distributed capacity. In the Pupin method the inductance is inserted in the form of coils at predetermined intervals. The coils ordinarily are a mile or more apart.

CHARACTERISTICS OF OPEN WIRE LINES. — For open wire aerial lines hard drawn copper is now almost universally used, except for short unimportant lines, where iron wire may be employed. The sizes of copper wire employed range from No. 16 B. & S. to No. 8 B. & S., depending upon the length and importance of the line. Where the tensile strength of the wires is unimportant, as when the wires are fastened along walls or fences, with supports close together, smaller sizes may be employed, and soft drawn wire used instead of hard drawn. Aerial wires are usually spaced about a foot apart, horizontally, and from 18 to 24 inches vertically, depending upon the spacing of the cross arms. For the resistance, capacity, and inductance of aerial wires see *Wires and Cables, Bare.* The insulation resistance between a pair of aerial wires depends upon the types of insulator, pin and cross arm, and upon the

condition of the weather. The following values are representative of good practice: —

Very dry,	500 megohms per mile.
Average,	25 megohms per mile.
Very wet,	2 megohms per mile.

Three different gages are employed for specifying the size of telephone wire: the B. & S., the B. W. G., and the N. B. S. (*See article on Gages, Wire.*) The following table gives the diameter, weight and resistance of the various sizes employed in telephone practice.

TABLE I

Number	Diameter in mils	Weight per mile in pounds	Resistance per mile of wire in ohms, 60° F.	Resistance per mile of circuit in ohms, 60° F.
8 B. W. G.	165	435	1.97	3.95
6 B. & S. G.	162	419	2.05	4.10
8 N. B. S. G.	160	409	2.10	4.20
9 B. W. G.	148	350	2.45	4.91
7 B. & S. G.	144.3	331	2.59	5.19
9 N. B. S. G.	144	331	2.59	5.19
10 B. W. G.	134	287	2.98	5.97
8 B. & S. G.	128.5	262	3.28	6.56
10 N. B. S. G.	128	262	3.28	6.56
11 B. W. G.	120	230	3.73	7.47
11 N. B. S. G.	116	215	3.99	7.99
9 B. & S. G.	114.4	208	4.14	8.27
12 B. W. G.	109	190	4.52	9.05
12 N. B. S. G.	104	173	4.97	9.94
10 B. & S. G.	101.9	166	5.17	10.33
13 B. W. G.	95	144	5.96	11.91
13 N. B. S. G.	92	135	6.35	12.70
11 B. & S. G.	90.74	132	6.49	12.98
14 B. W. G.	83	110	7.80	15.61
12 B. & S. G.	80.81	105	8.19	16.39
14 N. B. S. G.	80	102	8.40	16.80

CHARACTERISTICS OF TELEPHONE CABLES. — Telephone cables are groups of pairs of paper insulated wire twisted together and enclosed in a lead sheath. Soft drawn copper is invariably used for the conductors. The size wire usually employed ranges from No. 19 to No. 22 B. & S. For long distance underground lines individual wires as large as No. 10 B. & S. have been employed. For the resistance of the different sizes of conductors see *Wires and Cables*. The capacity of any pair of wires and the capacity of any one wire to ground depends largely upon the way in which the cable is made up. When the wires are twisted tightly together the capacity is greater than when they are loosely assembled. The mutual capacity ranges from 0.067 to 0.090 microfarad per mile, and the regular or grounded capacity from 0.10 to 0.12

microfarad per mile for tightly twisted wires, as against 0.054 to 0.067 and 0.080 to 0.10 respectively for loosely twisted wires. The former range of capacity is usually referred to as "high" and the latter as "low" capacity.

The effective insulation resistance between the two wires of a pair is about ½ megohm per mile at 800 cycles per second in a well-constructed cable. This figure includes the effect of dielectric hysteresis. The corresponding insulation resistance measured by direct current would be about 500 to 1000 megohms per mile.

The inductance of a pair of wires in a telephone cable is practically negligible, being less than 1 millihenry per mile of cable (2 miles of wire).

DISTORTION AND ATTENUATION. — (*See also Transmission Lines.*) Corresponding to any vowel, syllable or word spoken into a transmitter a current wave of a definite shape is sent over the line. This wave is made up of a number of simple sine waves of different frequencies, called harmonics, displaced with reference to one another, i.e., reaching their zero values at different times. Each of the simple sine waves as it progresses along the line decreases in amplitude or is "attenuated." Moreover, the shorter waves are in general attenuated more than the longer ones, with the result that the wave which reaches the receiver is made up of harmonics of relatively different magnitudes than the wave sent out from the transmitter, that is, the resultant wave at the receiver is "distorted." The sound produced by the receiver, consequently, differs from the word spoken into the transmitter, since the quality of the sound depends upon the relative magnitude of the constituent harmonics. There is also a displacement of the harmonics with reference to each other, which displacement, although it changes the shape of the resultant wave, has but little, if any, effect upon the quality of the sound.

Attenuation of a wave without distortion merely changes its amplitude, but since the wave form remains unchanged, only the loudness of the sound produced thereby, were it converted into a sound wave, is affected. Resistance alone, since it produces the same relative attenuation of all wave lengths, is therefore the least troublesome of the line characteristics. In most cases, the capacity of the line is the controlling characteristic, since the attenuation produced thereby depends upon the wave length, and therefore distortion results.

In addition to the distortion produced by the line itself, there is also a distortion in the transformation at the transmitter of the sound wave into a current wave, and again another distortion at the receiver where the current wave is reconverted into a sound wave.

DISTURBANCES DUE TO NEIGHBORING POWER TRANSMISSION LINES.* — (*See also Distribution Lines; Transmission Lines.*) A very small voltage between the wires of the telephone circuit is sufficient to produce noise in the telephone apparatus comparable in volume with the sound produced by the voice currents. The noise is due almost entirely to the harmonics of the power system, especially to those between 150 and 1200 cycles. At these frequencies induced currents in the telephone line equal to a few millionths of an ampere are sufficient to make conversation difficult.

Balanced versus Unbalanced Power Circuits. —In considering the effect of neighboring power circuits on telephone circuits the difference between balanced and unbalanced power circuits must be clearly kept in mind. A *completely balanced* power circuit consists of two or more wires energized in such a way that the vector sum of the currents in all the wires of the circuit is practically zero, and the vector sum of the voltages between the several wires and the ground is practically zero. A *completely unbalanced* power circuit consists of one or more wires with a ground return, the currents in all the wires being practi-

* By H. Pender.

cally in phase and the voltages between the several wires and the ground being practically equal and in phase. In practice power circuits are frequently neither completely balanced nor completely unbalanced, but the currents and voltages may be resolved into completely balanced and completely unbalanced components. Examples of such circuits are a three-phase circuit having one wire grounded and a three-phase circuit with grounded neutral; in the latter case the neutral current may be of fundamental frequency due to an unbalanced load on the system, or the neutral current may be of triple frequency due to a third harmonic in the voltage wave (*see Generators, Alternating Current*).

The inductive effects arising from unbalanced voltages or currents are very much larger than the inductive effects arising from balanced voltages or currents of the same magnitude, because with balanced voltages and currents the effect of one wire is largely neutralized by the near presence of other wires of opposite polarity.

Voltages Induced between Wires and between Wires and Ground. — The power circuits affect the telephone circuits by producing: (1) a voltage between the two wires of the telephone circuit; (2) a voltage between telephone wires and ground.

The voltage between telephone wires and ground is usually large compared with the voltage between wires. Even though the telephone circuit is transposed so that equal voltages are induced between the two wires and ground, the voltage to ground produces a voltage between wires because of the unavoidable minute inequalities in the constants of the two sides of the circuit. If the voltage to ground is high it endangers the users and operators of telephones connected to the circuits and puts the circuits out of commission by operating the protective devices.

Remedies for Inductive Disturbances in Telephone Lines. — It is in practice impossible to perfectly realize any of the following remedies, and no one of them would be sufficient even though perfectly realized. In order to minimize inductive disturbances it is therefore necessary to carry out each of the remedies as far as possible.

1. **Transposition of Power and Telephone Circuits.** — The way in which inductive effects are affected by the transposition of the power and telephone circuits is indicated in the following table:

EFFECT OF TRANSPOSITIONS

Power Line	Voltages Induced in Telephone Line	Affected by Transpositions in	
		Power Circuit	Telephone Circuit
Balanced	To ground	Yes	No
	Between wires	Yes	Yes
Unbalanced	To ground	No	No
	Between wires	No	Yes

In order to balance as far as possible the inductive effects between a three-phase power circuit and a telephone circuit it is necessary to divide each uniform section of the exposure into six or a multiple of six equal sections in which equal voltages of the six possible phase angles are induced. The methods of making transpositions of a telephone line are described below in the section on *Line Construction;* methods of transposing power lines are described in the article on *Transmission Lines*.

2. **Balancing the Power Circuit.** — The voltages induced between the telephone circuits and ground by small unbalanced components of the power voltages and currents are relatively large and are not affected by transpositions. In order to reduce the inductive interference, it is therefore important to construct and operate the power circuit in such a way that it is as far as possible balanced with respect both to voltage and to current.

3. **Balancing the Telephone Circuit.** — In order that the noise caused by the voltage between the telephone wires and ground may be minimized, it is necessary that the telephone circuits be carefully balanced, that is, that the two sides of the circuit be of the same resistance and have as nearly as possible the same insulation resistance and the same capacity to ground. The unbalancing due to unequal insulation and to unequal capacities between cable conductors and ground is frequently important and every effort should be made to minimize it. All apparatus connected to the circuit must be such that the impedances inserted in the two sides of the circuit, or the impedances connected between the two sides of the circuit and ground, are as nearly as possible exactly equal.

COMMERCIAL TRANSMISSION. — LIMITING TRANSMISSION DISTANCES. — Transmission is said to be "commercial" when two persons with normal ears and voices, using standard transmitters and receivers, can converse with reasonable ease. The length of the line over which such a conversation can take place is called the "limiting transmission distance." The limiting transmission distance of a pair of No. 19 B. & S. copper wires in a paper insulated cable is about 30 miles, the resistance per mile of cable or "loop mile" (2 miles of wire) being 88 ohms and the mutual capacity of the two wires 0.06 microfarad per mile. Such a line is taken as a standard to which all other lines may be referred.

For ease in comparing various lines, the qualities of the standard line just described are very closely imitated by assembling resistance coils and condensers in a portable case, the number of coils and condensers being such that the artificial line thus made is equivalent to many miles of the standard line. This artificial line is usually referred to as a standard cable set.

To determine the qualities of an unknown line in terms of standard cable, an observer listens to a distant speaker alternately through the unknown line and the standard cable set. He adjusts the latter until he hears the speech equally and similarly through both. The number of miles of the standard set then gives the limiting transmission distance of the line tested. The quotient of the limiting transmission distance by the actual length of the line is a measure of its transmitting ability.

The limiting transmission distances of various types of lines determined in the manner explained above are given in Table II, below. This table includes no allowance for switchboard and connection losses.

Effect of Bridging Telephone Set Across Line. — The effect of bridging a standard local telephone set across a non-loaded open-wire line is to diminish the limiting transmission distance by approximately 12 per cent. The loss due to bridging such a set across a cable circuit is considerably less, namely, about 7 per cent. These figures also apply roughly to each of several sets bridged across the line, provided these sets are widely separated.

LINE CONSTRUCTION. — In open country and small towns aerial lines with bare wires are almost invariably used. In large cities cables containing from 5 to 600 pairs of wires are usually employed. Cables may be run overhead or underground. Aerial telephone cables, on account of their low tensile strength, are hung from steel messenger wires, the latter being supported from the cross

TABLE II

Gage of wire	Theoretical limiting distance, with no allowance for switchboard losses and without Pupin coils
	Miles
No. 8 B. W. G. copper, open wire line	900
10 B. W. G. copper, open wire line	700
10 B. & S. copper, open wire line	400
12 N. B. S. copper, open wire line	400
12 B.& S. copper, open wire line	240
14 N. B. S. copper, open wire line	240
8 B. W. G. iron, open wire line	135
10 B. W. G. iron, open wire line	120
12 B. W. G., iron, open wire line	90
16 B. & S. cable, copper	40
19 B. & S. cable, copper	30
22 B. & S. cable, copper	20

Wires smaller than those given in this table should not be used on pole lines, on account of their lack of mechanical strength.

arms or fastened directly to the poles. Underground cables are usually run in tile or other type of ducts. (*See Conduits and Conduit Lines, Underground; Wires and Cables.*)

Open Wire Lines.—With the exception of the smaller insulators, usually glass, and the larger number of wires carried on a single pole line (as many as 100), the construction of open wire telephone lines differs but slightly from that of pole lines for power transmission. (*See Cross Arms; Distribution Circuits; Poles for Overhead Lines; Transmission Lines.*)

The size of poles and their spacing usually employed for various classes of lines through level country are given in the following table:

Type of line	Ultimate number of wires	Height, feet	Diameter of top, inches	Number per mile
Short, local	6	22	5 to 6	30
Important routes	20	25	6 to 7	40
Long distance	30 to 40	30 to 35	7 to 8	40

Where special conditions arise the height and size of poles must be selected accordingly. The depth to which a pole is set in the ground ranges from one-fifth to one-eighth of its total height, the larger figure applying to short poles (25 feet), the latter to tall poles (65 feet).

Hard drawn copper wire is now used on all important lines, though galvanized iron wire is still employed on short unimportant lines. Recently copper-clad steel wire has been proposed for telephone lines. See *Wires and Cables* for the properties of wires. The particular size to be employed in any case will depend upon the length of the line; see Table II above. In open wire lines the wires are usually spaced one foot apart horizontally and the cross arms are spaced 18 to 24 inches apart.

Transpositions to Prevent Cross-Talk. — Any varying magnetic flux which may thread the space between the two wires of a telephone line, or any varying electrostatic field about the line, will set up a varying current in the line, and if the frequency is such as to produce in the receiver a sound of audible pitch, this sound will interfere with the proper function of the receiver. Any noise produced in this manner by one telephone line on another is called "cross-talk." Noise may be produced in a similar manner by the induction from other sources, such as railway or lighting (alternating) circuits; see above.

To prevent such cross-talk or noise the wires of a telephone line are usually transposed every quarter of a mile. Fig. 1 shows a common transposition scheme. The vertical lines represent the cross arms of the poles where the

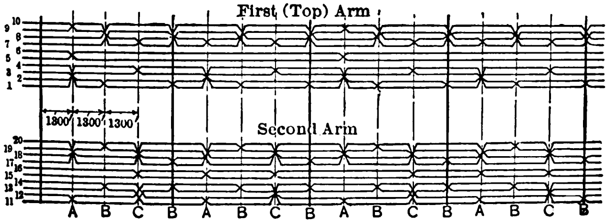

Fig. 1. Transposition Scheme

transpositions are made. The ordinary method of making a transposition is to cut each wire at each transposition point for that wire and to fasten the two ends either to separate insulators or to an insulator with two grooves, and then make the cross-overs with short wires. Transpositions can also be made without cutting the wire by using a single insulator with two grooves one well above the other. This scheme, known as the "single pin transposition," is also cheaper than the ordinary scheme.

The transpositions for the first and second cross arms are not alike, as shown in Fig. 1; when there are more than two cross arms, the wires on the odd numbered arms are usually transposed in the same manner as the first or upper arm, and the wires on the even numbered arms in the same manner as those on the second arm. When phantom circuits are used the wires on the several arms are not transposed alike.

Telephone Cables for either aerial or underground use are made as follows: Soft copper wires are insulated with dry paper laid on spirally; two such insulated wires then are twisted into a pair, the two wires having different colored papers; a number of pairs are laid up spirally to form a cylindrical core. The core then is wrapped spirally with paper tape, is dried thoroughly and a lead sheath molded on it in a lead press. Many users require three per cent of tin in the lead sheath.

The external diameter and weight of a cable containing a given number of pairs depend upon how closely the wires are wrapped together, and this in turn determines the capacity of the wires, the capacity being greater the closer the wires are wrapped together. Table III gives the dimensions and weights of standard cables.

Telephone cables, if used in connecting subscribers' telephones to central offices, usually are formed of 22 B. & S. gauge wires, except for long loops, when No. 19 B. & S. is generally used. For trunk lines between offices in large exchanges, cables are usually formed of No. 19 B. & S. gauge wires. Loading

TABLE III. PROPERTIES OF STRANDED TELEPHONE CABLES

No. pairs	Gage B. & S.	Electro-static capacity	Thickness of sheath, inches	Approximate external diameter, inches	Approximate weight per foot, pounds
5	22	High	1/12	0.48	0.55
10	22	High	1/12	0.59	0.71
15	22	High	1/12	0.66	0.83
20	22	High	1/12	0.72	0.93
25	22	High	1/12	0.77	1.02
50	22	High	1/12	0.97	1.45
50	20	High	3/32	1.10	1.88
100	22	High	3/32	1.32	2.36
100	22	Low	3/32	1.50	2.63
100	20	High	1/8	1.57	3.60
100	20	Low	1/8	1.81	4.11
200	22	High	1/8	1.84	4.43
200	22	Low	1/8	2.11	4.99
200	20	High	1/8	2.11	5.47
200	20	Low	1/8	2.46	6.19
200	19	High	1/8	2.24	6.08
200	19	Low	1/8	2.65	6.94
300	22	High	1/8	2.21	5.71
300	22	Low	1/8	2.51	6.32
300	20	High	1/8	2.53	7.09
300	20	Low	1/8	2.96	7.94
300	19	High	1/8	2.69	7.95
300	19	Low	1/8	3.20	9.04
400	22	High	1/8	2.51	6.84
400	22	Low	1/8	2.86	7.56
400	20	High	1/8	2.89	8.56
400	20	Low	1/8	3.43	9.37
600	22	High	1/8	3.20	9.21
90	16	Low	1/8	2.88	7.2
43	13	Low	1/8	2.88	7.17
50	10	High	1/8	2.88	8.95

Note.—High capacity, 0.067–0.090 mutual, 0.10–0.12 grounded. Low capacity, 0.054–0.067 mutual, 0.080–0.10 grounded.

coils are used in trunk lines, and successful operation of the largest exchanges would be impossible without loading coils.

Cables for long distance purposes have large wires. Where conditions permit the use of loaded cables between cities, the operating conditions are most uniform. Chicago and New York are connected to cities within a radius of two hundred miles by loaded underground cables, and that practice is extending. The principal advantages are that the cables are not disturbed by storms and their insulation does not vary. The insulation resistance affects the attenuation and distortion of the waves sent over the wire. Lines equipped with loading coils are particularly sensitive to changes in insulation resistance. If the leakage in wet weather greatly exceeeds the normal leakage the loading coils may do more harm than good.

Installation of Cables. — (*See also Conduits and Conduit Lines, Underground; Wires and Cables.*) It is of particular importance in all operations connected with telephone cables to keep the core dry. The conductors are insulated only by paper, which is useless if moist and is of value only because it incloses dry air. As soon as the sheath is removed from a cable for splicing or terminating, the paper absorbs moisture from the air and from the hands; therefore expose the core as little as possible and boil out all moisture by pouring hot paraffine over and through the conductors before finally closing the cable.

Splicing. — To splice a telephone cable, a lead sleeve is slipped over one of the ends and the sheath cut off of each of the ends, after scraping bright the part at which the sleeve later is to be soldered with wiped joints. The core now is bound tight with dry muslin just at the end of the sheath and the muslin packed under slightly. The cable ends are boiled in hot paraffine by pouring or immersion. If white fumes arise from the paraffine it is too hot. The wires then are joined by stripping the paper from the ends, twisting the bared parts together so as to include a little of the insulation, then sliding down a paper sleeve previously placed over one wire.

When all the pairs are spliced, the conductors are again boiled out with paraffine. The lead sleeve is slipped into place, its ends dressed down and a wipe joint made at each end. This gives the splice a sheath continuous with the cable itself. A *Y-splice* is one in which one cable branches into two.

Cable Terminals are devices in which the paper insulated wires of a telephone cable are made accessible to wires outside the cable. The fundamental requirement is that the insulation of the conductors shall be maintained, both by keeping moisture from the cable insulation and by insulating the points to which the wires outside the cable attach.

A standard form of cable terminal consists of a box of porcelain, or partly iron and partly porcelain, wherein the end of the cable is sealed by bituminous compound. Binding posts in the porcelain receive the cable wires inside the sealed part and the outer wires on the outside.

Potheads are splices between rubber-insulated wires and paper-insulated cable wires. They are filled with bituminous sealing compound, so that moisture shall not lead to the paper insulation along the rubber-insulated wires.

Potheads are less widely used than before the development of the porcelain terminal. They are necessary for places where more than 50 pairs of wires are to be terminated. At the junction between aerial and underground cables, fuses require to be inserted to complete the protective system. Potheads then are placed on both cables, and the rubber covered wires led to the fuse terminals. It is good practice to house the fuses and pothead ends in wood or metal protecting boxes.

Drop Wires. — The wires used to connect open wire lines or aerial cables to subscribers' premises are called drop wires and are of hard drawn copper insulated with rubber, covered with a braid, then twisted into pairs. Standard sizes are No. 16 B. & S. gage in regions where ice does not form, and No. 14 B. & S. gage in other regions. No. 17 B. & S. copper-clad steel wire is successfully used as a substitute for either.

ELECTROLYSIS of the sheaths of underground telephone cables takes place when the sheaths carry stray direct current from one region to another if the current leaves the sheath in the presence of moisture (*see Electrolysis of Grounded Structures*).

COSTS. — The cost of building a telephone line depends on so many variables that no data of any value can be given in the space here available. Approximate costs of the various constituent items will be found in the articles on *Conduits and Conduit Lines, Cross Arms, Insulators, Poles for Overhead Lines, Wires and Cables, etc.*

BIBLIOGRAPHY. — See *Bibliography* in article on *Telephone Instruments and Circuits.*

[S. G. McMeen.]

TELEPHONE TRAFFIC AND RATES.— (*See also Telephone Instruments and Circuits; Telephone Lines.*) For the purpose of charges against telephone users, the unit of telephone traffic is the "conversation." For the purpose of designing and using telephone equipment the telephone "call" often is the unit. A call does not always result in a conversation. Telephone traffic is subject to general variations closely linked with variations in human activities. Obviously, telephone traffic relative to business is lower on holidays than on working days; on all days telephone traffic varies with the hours, in a way fairly uniform from day to day. It is possible to plot this variation, and Fig. 1 is a load curve representative of what happens each working day in most regular exchanges. The number of calls which subscribers originate depends on the kind of service rendered, whether residence, business, etc., and on the method of charging. Subscribers will call about twice as often under flat rates as under measured rates.

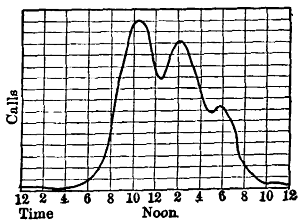

Fig. 1. Load Curve

A knowledge of the amount of traffic in a system, of its distribution as to time and as to divisions of the exchange, is important. By that knowledge the equipment must be designed, modified from time to time and the load distributed upon it as changing circumstances shall require.

Methods of Counting Calls. — Amounts of traffic are observed in three general ways. A peg-count is a record made by the operator actuating some counting device for each call answered or completed. A second way is to determine a ratio existing, for the particular time and place, between the calls in a given period and the average number of cord circuits in use in that period. Knowing these, the probable total can be computed from the cord circuit as counted. The third method is applicable to offices having service meters on all lines and is to associate one master meter per position with all the meters of that position, so that it will count one each time any service meter of the position is operated.

Operator's Speed. —The number of calls an operator can complete in an hour depends on the percentage of total calls which she must trunk to other offices. With standard manual equipment, for example, she can complete 240 calls in an hour if none have to be trunked, and 165 in an hour if 90 per cent of them have to be trunked.

Prompt disconnection is of great importance. A rule should be that disconnect signals shall be given prompt attention by some operator and shall take precedence over a call for connection. A flashing keyboard lamp indicating a recall should be given precedence over all other calls.

TRUNKING. — Traffic studies enable the determination of the number of trunks required for an anticipated traffic. The number of trunks required between two central offices, or between a central office and a private exchange, depends upon the number of calls which must be trunked per maximum or busy hour. The element of the probable coincidence in the time at which the calls will be made must be considered, and for this reason the more trunks installed the greater will be the number of calls which each trunk may be expected to handle. The following illustrate current practice:

Trunked calls per busy hour	35	90	1080
Number of trunks required	5	10	60
Calls per trunk	7	9	18

TELEPHONE RATES. — Rates for telephone service are of two kinds, "flat rates" and "measured service rates." Under flat rates, a fixed sum per month is charged and the subscriber may use his telephone as much as he pleases. Under measured service rates, the subscriber is charged a certain sum for each conversation he originates. Ordinarily also the subscriber guarantees to pay not less than a certain minimum amount per month. Rates for private exchanges often are made up by charging a rate per month for each local telephone, another rate per month for each section of local switchboard, another rate for each trunk line, and a still further rate for each outward call over the trunk lines leading from the private exchange, the monthly charge being the sum of all.

The cost of telephone service depends on the amount of use, though there is a fundamental cost of being prepared to furnish service. There is a tendency to change from the flat-rate to the measured-rate system. The latter is in operation in most large cities.

Flat-rate Systems. — In flat-rate systems there is no continuous effort to record the number of conversations. Counting is done from time to time for statistical purposes.

Measured-rate Systems. — In measured-rate systems it is necessary to record the conversations as they occur, so that each subscriber may be charged for the service he receives. Sometimes this is done by hand on tickets, but that method interferes with the operator's regular work, losing not only her service in operating but wasting switchboard investment by not using it efficiently. The best way is to equip each line with a meter under electrical control which causes it to count one unit for each completed conversation. Such meters are in extensive use in the larger cities. In manual systems, when a conversation takes place, the operator presses a meter key associated with the answering cord, and this makes the record upon the meter. In automatic systems the answering of the called station operates the meter.

In certain offices in London usual calls are charged at a penny each and certain special calls at two-pence each. In trunking calls of the latter kind, the operator is reminded of the higher charge by the lighting of a specially colored lamp as she uses the trunk order key. Being so reminded, she presses the meter key twice when such a conversation occurs, the result being that the calling subscriber pays two-pence for each such special call, because his meter records two units for each.

LONG-DISTANCE CALLS are those which occur between cities, as distinct from those within cities, no matter how large the latter may be. Long-distance calls usually are handled by special operators at special switchboards connected by trunks to the switchboards which handle city service. Long-distance calls usually require that a particular person be found in the called city, and the record of such calls includes the name as well as the number of the called person. On this ticket is recorded the duration of the conversation in minutes and fractions so that the charge may be computed. This elapsed time is best recorded by a time-computing machine which prints a record.

Two-number Method. — Where calls between cities reach a sufficient number, it is found economical to provide for connecting by number only the stations of the subscribers who call frequently, and this is called the "two-number" method. Such connections generally are established without trunking to and from special long-distance switchboards. Operators at regular multiple-switchboard positions make and complete these calls. They are ticketed only at the originating end.

COIN-COLLECTING DEVICES. — Measured service calls, either local or long distance, may be made over telephones equipped with coin-collecting

devices. These are so arranged that the operator making the connection knows in advance of conversation that the proper amount of money has been placed in the machine. Machines of one type require the deposit of the coin before the central office can be signaled at all; the coin is returned if the call is not completed. Machines of the other type require that the coins be deposited after the order has been given but before the conversation occurs.

DEVELOPMENT STUDIES — FACTORS AFFECTING COSTS. — A development study is an inquiry to determine what kind and what amount of construction or reconstruction is warranted by the conditions and probabilities of a region. Some form of development study always precedes such work. Economical construction and efficient operation require constant improvement in development-study methods and their use.

Forecasting Requirements. — In cities a first step is to forecast the future development through a period of from 15 to 20 years. This is done by examining statistics of growth. Ratios of telephones to population then are tabulated for the past, and future ratios estimated. A house count then is undertaken. This means the counting of existing buildings and the estimating of the telephone-using abilities of the occupants. At the same time a forecast is made of the probable future development of each region as it is studied.

Determination of Number of Central Offices. — If the exchange will require switchboards for more than 10,000 lines there will be more than one central office. The more switchboards there are in a town, other things being equal, the shorter will be the average length of subscribers' lines, and the greater will be the mileage of trunk lines. The more offices there are, the greater the cost of owning and maintaining the buildings for them, and the greater the cost of operating the equipment in them. A balance for these conditions must be found.

The economical number of districts having been determined, the office of each district can be located so as to attain the greatest wire economy. The conduit runs and pole lines then are laid out for the subscribers' and trunk lines. Speaking generally, it is good practice to provide a main conduit passing the central office and having cross routes extending from it at right angles, one on each alternate street or in alleys if they exist.

In automatic practice, it is possible further to subdivide exchange districts and to place some of the apparatus in small sub-district offices having trunks leading to the district office, still other trunks interconnecting all the district offices. This lowers still further the cost of subscribers' lines and increases still further the cost of trunk lines.

BIBLIOGRAPHY. — See Bibliography in article on *Telephone Instruments and Circuits*.

[S. G. McMeen.]

TELPHERAGE. — (*See also Conveyors.*) Telpherage is a name given to a system of transporting materials in which the load is suspended from a trolley or small truck running on a cable or overhead rail, and in which the propelling force is obtained from an electric motor carried on the trolley. The trolley, with its motor, is called a "telpher." A historical and illustrated description of the system is given in a paper by C. M. Clark, in *Trans. A.I.E.E.*, 1902. See also circulars of United Telpherage Co., N. Y.

[Wm. Kent.]

TEMPERATURE AND THERMOMETERS. — (*See also Heat and Thermal Properties; Pyrometers; Thermodynamics, Principles of; Units and Conversion Factors.*) The temperature of a body may be defined as its relative hotness or coolness referred to some standard substance under standard conditions. The change in some physical property of a standard substance must be utilized in order to give a number to temperature. Any device which serves this purpose is called a thermometer; if the device is applicable to the measurement of very high temperatures it is also called a pyrometer, q.v.

TEMPERATURE SCALES. — The standard* temperature-measuring device is the constant volume hydrogen thermometer, which consists essentially of a suitable receptacle containing a constant mass of hydrogen gas kept at constant volume, viz., the volume it would have at a pressure of 1000 millimeters of mercury and at the temperature of melting ice, with means provided for measuring any variation that may be caused to take place in the pressure of the gas.

Centigrade Scale. — The temperature of melting ice at a pressure of 760 millimeters of mercury is arbitrarily taken as zero degrees, and the temperature of saturated steam at a pressure of 760 millimeters of mercury is taken as 100 degrees. Calling p_0 the pressure of a constant volume of hydrogen gas when the receptacle is immersed in the melting ice and p_{100} its pressure when immersed in the saturated steam, and p_t its pressure when immersed in any given substance (the pressure in each case being measured after the lapse of a sufficient time for it to reach a constant value), the numerical value of the temperature of the given substance is defined as

$$t = \frac{p_t - p_0}{p_{100} - p_0} \times 100.$$

A degree centigrade is abbreviated deg. cent. or °C.

Fahrenheit Scale. — The Fahrenheit scale of temperature is derived in the same manner, except that the temperature of the melting ice is taken as 32° and the boiling point of water (at 760 millimeters mercury) as 212°. A temperature of t_f degrees Fahrenheit is then equal to

$$t_c = \tfrac{5}{9}(t_f - 32) \qquad \text{degrees centigrade.}$$

Vice versa, a temperature of t_c degrees centigrade is equal to

$$t_f = \tfrac{9}{5}t_c + 32 \qquad \text{degrees Fahrenheit.}$$

A degree Fahrenheit is abbreviated deg. fahr. or °F.

Réaumur Scale. — This scale, which is used to some extent in Europe and in breweries in this country, is defined in the same manner as the centigrade scale, except that the boiling point of water (at 760 millimeters mercury) is taken as 80°. A temperature of t_r degrees Réaumur is then equal to

$$t_c = 1.25\, t_r \qquad \text{degrees centigrade.}$$

Platinum Scale. — See article on *Pyrometers.*

ABSOLUTE TEMPERATURE. — Let p_0 = absolute pressure of a given mass of gas at 0° C., p = absolute pressure of this same mass of gas at any temperature t° C. as above defined. For all values of t between about −150° and +1000° C., i.e., for all values of t within the experimental range, it is found

* Adopted by the Bureau International des Poeds et Mesures.

DEGREES FAHRENHEIT CORRESPONDING TO DEGREES CENTIGRADE

°C.	0	− 1	− 2	− 3	− 4	− 5	− 6	− 7	− 8	− 9
−40	−40.0	−41.8	−43.6	−45.4	−47.2	−49.0	−50.8	52.6	−54.4	−56.2
−30	−22.0	−23.8	−25.6	−27.4	−29.2	−31.0	−32.8	34.6	−36.4	−38.2
−20	− 4.0	− 5.8	− 7.6	− 9.4	−11.2	−13.0	−14.8	−16.6	−18.4	−20.2
−10	14.0	12.2	10.4	8.6	6.8	5.0	3.2	1.4	− 0.4	− 2.2
0	32.0	30.2	28.4	26.6	24.8	23.0	21.2	19.4	17.6	15.8

°C.	+ 0	+ 1	+ 2	+ 3	+ 4	+ 5	+ 6	+ 7	+ 8	+ 9
0	32.0	33.8	35.6	37.4	39.2	41.0	42.8	44.6	46.4	48.2
10	50.0	51.8	53.6	55.4	57.2	59.0	60.8	62.6	64.4	66.2
20	68.0	69.8	71.6	73.4	75.2	77.0	78.8	80.6	82.4	84.2
30	86.0	87.8	89.6	91.4	93.2	95.0	96.8	98.6	100.4	102.2
40	104.0	105.8	107.6	109.4	111.2	113.0	114.8	116.6	118.4	120.2
50	122.0	123.8	125.6	127.4	129.2	131.0	132.8	134.6	136.4	138.2
60	140.0	141.8	143.6	145.4	147.2	149.0	150.8	152.6	154.4	156.2
70	158.0	159.8	161.6	163.4	165.2	167.0	168.8	170.6	172.4	174.2
80	176.0	177.8	179.6	181.4	183.2	185.0	186.8	188.6	190.4	192.2
90	194.0	195.8	197.6	199.4	201.2	203.0	204.8	206.6	208.4	210.2
100	212.0	213.8	215.6	217.4	219.2	221.0	222.8	224.6	226.4	228.2
110	230.0	231.8	233.6	235.4	237.2	239.0	240.8	242.6	244.4	246.2
120	248.0	249.8	251.6	253.4	255.2	257.0	258.8	260.6	262.4	264.2
130	266.0	267.8	269.6	271.4	273.2	275.0	276.8	278.6	280.4	282.2
140	284.0	285.8	287.6	289.4	291.2	293.0	294.8	296.6	298.4	300.2
150	302.0	303.8	305.6	307.4	309.2	311.0	312.8	314.6	316.4	318.2
160	320.0	321.8	323.6	325.4	327.2	329.0	330.8	332.6	334.4	336.2
170	338.0	339.8	341.6	343.4	345.2	347.0	348.8	350.6	352.4	354.2
180	356.0	357.8	359.6	361.4	363.2	365.0	366.8	368.6	370.4	372.2
190	374.0	375.8	377.6	379.4	381.2	383.0	384.8	386.6	388.4	390.2
200	392.0	393.8	395.6	397.4	399.2	401.0	402.8	404.6	406.4	408.2

Example: − 12° C.= 10.4° F.; − 33° C.= − 27.4° F.; 13° C.= 55.4° F.

that, in the case of the so-called permanent gases (i.e., those which are not readily liquefied, such as air, hydrogen and nitrogen), the following relation exists between p_0, p and t provided there is no change in volume

$$p = p_0 (K + t),$$

where K is a constant, approximately equal to 273 to within less than 1 part in 300.* Consequently, assuming this relation to hold when the pressure p becomes zero, the value of t which would correspond to *zero* absolute pressure is

$$t_0 = -273° \text{ C.}$$

This temperature is called the absolute zero. That is, the zero of the centigrade scale is 273° C. above the absolute zero.* Similarly, the zero of the Fahrenheit scale is approximately 460° F. above the absolute zero.

* For hydrogen Callender gives the value 273.10.

The temperature measured above the absolute zero is called the absolute temperature. Let t_c = temperature in ° C. above the centigrade zero, t_f = temperature in ° F. above the Fahrenheit zero, then the absolute temperature in ° C. corresponding to t_c is

$$T_c = 273 + t_c,$$

and the absolute temperature in ° F. corresponding to t_f is

$$T_f = 460 + t_f.$$

Absolute Thermodynamic Temperature. — (*See article on Thermodynamics, Principles of.*) The difference between the temperature scale as defined by the constant-volume hydrogen thermometer and the absolute thermodynamic scale is less than 0.1° C. at ordinary temperatures, and is less than 1° C. throughout the range from − 150° to 1000° C.

THERMOMETERS. — The standard constant-volume hydrogen thermometer is seldom used except for standardizing purposes, and then only for temperatures up to about 500° C. The constant-volume nitrogen thermometer is used for standardizing purposes for temperatures up to about 1500°. For higher temperatures radiation pyrometers are used as standards (*see Pyrometers*).

Mercury-in-Glass Thermometers. — For ordinary temperature measurements, between about − 35° C. and 350° C., the ordinary mercury-in-glass thermometer is almost universally employed. The temperature may be read directly from the position of the end of the mercury column as given by a uniformly divided scale on the stem, provided the 0° and 100° points have been properly located and the stem has a uniform bore.

For accurate measurements the scale on the thermometer should be checked to determine whether the 0° and 100° points are properly located and whether the bore is uniform. Unless the proper quality of glass is used the bulb does not return to exactly the same volume after successive heatings. Even when a high-grade thermometer is employed, the zero should be frequently checked if the thermometer is used for high-temperature measurements.

Methods of calibrating mercury thermometers are described in text books on heat, but at the present day it is more convenient to compare the thermometer with a secondary standard mercury thermometer which has been calibrated by a central standardizing bureau, such as the Bureau of Standards at Washington.

High-range Mercury Thermometers. — The mercury thermometer when made of very hard glass and filled under pressure with the space above the mercury column containing some inert gas, like nitrogen, may be used to measure temperatures up to about 550° C. If a considerable length of stem emerges into the air, a very considerable error, 25° C. or so, may be introduced at high temperatures. This "stem correction" varies slightly with the kind of glass but may be represented very nearly by the formula

$$\text{Correction to be added to reading} = 0.00016\, n(t_b - t),$$

where the temperatures are all in degrees centigrade and n = number of degrees emergent from bath or furnace, t_b = temperature of bath and t = mean temperature of emergent mercury column.

COSTS. — Ordinary mercury thermometers cost from 50 cents to $5.00 apiece, depending upon the accuracy of their calibration. A 100° C. mercury thermometer sufficiently accurate (error not over ½ per cent) for ordinary engineering work costs about $4.00. A 400° C. mercury thermometer costs about $6.00 and a 550° C. mercury thermometer about $12.00.

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[H. PENDER and H. R. RANKEN.]

THERMODYNAMICS, PRINCIPLES OF. — (*See also Electrochemistry, Principles of; Heat and Thermal Properties; Steam; Steam Engines; Temperature and Thermometers.*) "Thermodynamics" is a general name employed to include all problems involving the transfer of energy from one body to another, and the transformations of energy within a body. The word "energetics" is a better general term to designate such problems, but the term "thermodynamics" is the one ordinarily employed, since heat (Greek, "thermos") is developed in practically every case of transfer or transformation of energy. The transfer of energy from one body to another in every known instance is found to be consistent with the following fundamental principles or laws:

FIRST PRINCIPLE OF THERMODYNAMICS. — All known experimental facts are in accord with the principle that, when a body changes from a given state or condition 1 to any other given state or condition 2, the total energy given out by the body is always the same, independent of how the change takes place.

Intrinsic Energy. — Any given body in any given state or condition may, therefore, be considered as having associated with it a definite amount of energy, which, in general, changes when the state or condition of the body changes. This energy is called the "intrinsic energy" of the body, and depends solely upon the state or condition of the body. If the intrinsic energy in a given state or condition 1 is U_1, and in some other state or condition 2 is U_2, then the total energy given out by the body when it changes from the state 1 to the state 2 is $U_1 - U_2$, and this difference depends only upon the initial and final states of the system and is independent of how the body passes from one state to the other.

Heat and Work. — The energy given out by a body may be either heat, mechanical work, electrical, magnetic or other forms of energy. For convenience, all other forms of energy given out by the body than heat, may be called the "external work" done by the body. Hence, calling Q_e the heat *evolved* and, W the external work *done by* a body when it changes from a state 1 to a state 2, the total energy given out by the body may be expressed as $Q_e + W$. In general the amount of heat evolved and the external work done by a body when it changes from a state 1 to a state 2 depend respectively upon the manner in which the change takes place, but the sum of the heat evolved and the work done is always the same for given initial and final states, irrespective of how the change takes place.

Mathematical Expression of First Law. — Equating the two expressions given above for the total energy transferred from a body when it changes from a state 1 to a state 2, gives the relation

$$U_1 - U_2 = Q_e + W,$$

which is the usual mathematical expression of the first law of thermodynamics. If, in the change from state 1 to state 2, heat is actually evolved, then Q_e is positive, while if the heat is absorbed Q_e is negative. It is usually more convenient to consider the heat absorbed by the body as a positive quantity. Hence, putting $Q = -Q_e$ the above expression may be written

$$W = U_1 - U_2 + Q, \tag{1}$$

in which U_1 is the intrinsic energy of the body in any state 1, U_2 the intrinsic energy in any other state 2, Q is the heat *absorbed* and W is the *work done by* the body when it changes from the state 1 to the state 2.

PATH OF CHANGE. — Consider a body which changes from a state or condition 1 to another state or condition 2. In changing from the initial

state to the final state the body passes through a series of successive states, each state differing but infinitesimally from the preceding. The given series of states through which the body passes is called the "path" from state 1 to state 2. In general, a body may pass from a state 1 to a state 2 by an infinite number of such paths; therefore, a change can be completely specified only by stating the "path" of the change as well as the initial and final states.

The *sum* of the work done and the heat given by a body when it changes from a state 1 to a state 2 depends only upon these states, i.e., upon the ends of the path, but the proportions of the energy given out as work and heat respectively depend not only upon the series of successive states through which the body passes (i.e., the "shape" of the path), but also upon the relation of the given body to any external bodies which may in any way affect it.

ADIABATIC PROCESS.—Any process by which a change can be produced in a body under conditions such that the body neither absorbs nor gives out heat to any other body is called an "adiabatic" process. Adiabatic processes can never be completely realized, since no known substance is a perfect heat insulator, but such processes can be closely approximated, e.g., the expansion or compression of a gas in a cylinder with well-insulated walls when the expansion or compression is so rapid that no heat is conducted through the walls.

ISOTHERMAL PROCESS.—Any process by which a change can be produced in a body without changing its temperature is called an "isothermal" process. For example, the melting of ice, when the ice and water are kept well stirred, is an isothermal process.

REVERSIBLE PROCESS.—Whenever a change takes place in a body A, a change also takes place in some other body or bodies B. If the changes in the system formed by A and B are such that the path of each change may be reversed in direction without changing by an appreciable amount the total energy of the system, then the process by which the change takes place is called a "reversible" process.

Since heat can pass only from a hot to a cold body and never in the reverse direction (definition of heat), it follows that, if during any step of a process there is a transfer of heat between bodies whose temperatures differ by a finite amount, then the process is irreversible. Hence, a reversible process can take place under two conditions only: either there must be no transfer of heat (adiabatic process) or the transfer of heat must be between bodies which differ in temperature only by an infinitesimal amount.

Strictly speaking, the last type of process is absolutely reversible only in the limiting case when the bodies between which the transfer of heat takes place are at exactly the same temperature. This condition can be only approximated since there must always be a difference in temperature, though this difference may be infinitely small, in order that a transfer of heat may take place.

SECOND LAW OF THERMODYNAMICS.—The so-called second law of thermodynamics involves three distinct principles, which may be stated as follows:

1. When a body changes from a state 1 to a state 2 by any *reversible* process, and then back from 2 to 1 over the same path, but in the reverse order, then the heat absorbed by the body during the change from 1 to 2 is exactly equal to the heat evolved by this body during the reverse change from 2 to 1.

2. When a body changes from a state 1 to a state 2 by any irreversible process, the heat *absorbed* during this change can never be greater than the heat which it would absorb were the change from 1 to 2 over the same path produced under conditions which would render the process reversible. Similarly, if the body gives out heat during an irreversible change, the heat given out can never be

less than the heat which it would give out were the change over the same path produced under conditions which would render the process reversible.

3. The heat (Q) absorbed by a body during any reversible isothermal process bears the following relation to the temperature (t) of the body during the process,

$$Q = MK\,(t + T_0), \tag{2}$$

where M is the mass of the body, T_0 is a constant which depends *solely upon the scale on which the temperature is measured*, and K is a constant which depends *solely upon the nature of the body and its initial and final states.*

Absolute Thermodynamic Temperature. — It is found impossible by any known means to produce a negative temperature, as measured on any temperature scale, numerically greater than the corresponding value of constant T_0 in the above expression (equation (2)). Hence that temperature below the zero of any given scale equal to this constant T_0 is called the "absolute thermodynamic zero" of this scale, and the temperatures measured from this point are called "absolute thermodynamic temperatures." Equation (2) may therefore be written

$$Q = MKT, \tag{2a}$$

where $T = (t + T_0)$ is the absolute temperature corresponding to the temperature t. The value of the absolute thermodynamic zero is practically the same as that temperature, as measured on the constant-volume hydrogen-gas thermometer, at which the pressure of the gas would be zero, assuming the decrease in pressure per degree ($= 1/273$ of the pressure at 0° C.) to remain constant at all temperatures.

On the centigrade scale the absolute zero is, therefore, $-273°$ (approximately), and on the Fahrenheit scale $-460°$ (approximately).

Entropy. — Since the factor K, in equation (2) above, depends solely upon the nature of the body and its initial and final state, this factor may be looked upon as representing a change in a property of the body. That is, calling this property of the body for the initial state N_1 and for the final state N_2, then K may be put equal to $N_2 - N_1$, and equation (2) may be written

$$Q = M\,(N_2 - N_1)\,T. \tag{2b}$$

This equation, which represents the relation between the heat absorbed and the temperature, for a reversible isothermal process, may also be applied to an adiabatic process, if the property of the body represented by the symbol N is assumed to remain unchanged during such a process. For an adiabatic process the heat absorbed is zero, by definition, and this, on the assumption of no change in N during such a process, is consistent with equation (2b).

In general, the temperature of a body during any process is not constant, but the process may be considered as made up of a series of reversible isothermal and adiabatic steps, and the above definition of the property N may be applied to each step. That is, the change in N for each step composed of a reversible isothermal "tread" and an adiabatic "rise" is

$$dN = \frac{dQ}{MT},$$

where M is the mass of the body, dQ is the heat absorbed, and T is the absolute temperature of the body during this step.

This property N, whose increase during any step in a reversible process is equal to the heat absorbed per unit mass divided by the absolute temperature of the body during this step, is called the "entropy" of the body per unit mass. Since entropy is defined in terms of its change, its absolute value for any given

standard state of a body may be arbitrarily taken as zero. In steam tables, the entropy of water at 32° F. and at atmospheric pressure is usually taken as zero. The entropy of a body can in many cases be calculated from the other properties of the body. (*See article on Steam.*)

In accordance with the above definition of entropy the heat absorbed by a body when it changes from a state 1 to a state 2 by any *reversible* process is then

$$Q = M \int_1^2 TdN, \tag{3}$$

where M is the mass of the body, T its absolute temperature during any step of the process, and dN the increase in its entropy per unit mass during this step.

Principle of the Increase of Entropy. — It can be shown, from the principles above stated, that the only possible changes which can take place in a system of bodies to which no energy is added or subtracted are changes which involve an *increase* in the total entropy of the system. Reversible changes may theoretically take place without increasing the total entropy, but reversible changes never take place in nature nor can they be realized *absolutely* by any known experimental means.

MAXIMUM WORK. — The maximum external work which a body can do when it changes from a state 1 to a state 2 along a given path (i.e., by passing through a given series of states) is equal to the decrease in its intrinsic energy plus the maximum amount of heat it can absorb when it changes along this path. The maximum external work which a body can do in changing from a state 1 to a state 2 along a given path is therefore

$$W_{\max} = U_1 - U_2 + M \int_1^2 TdN. \tag{4}$$

The value of the last term in this expression depends upon the temperature at each step in the change, and this temperature depends upon the "path" along which the change takes place. If the path is such that *all* the heat is absorbed at one given temperature T, then

$$W_{\max} = U_1 - U_2 - TM(N_1 - N_2). \tag{4a}$$

This may also be written

$$W_{\max} = U_1 - U_2 + T\frac{dW_{\max}}{dT}, \tag{5}$$

where $\frac{dW_{\max}}{dT}$ is the rate of increase of the maximum work done by the body with the temperature at which the heat is absorbed.

Free Energy. — The maximum work which a body can do in changing from a state 1 to a state 2 is sometimes referred to as the "free energy" of the body corresponding to this change. This free energy depends not only upon the initial and final states of the body itself but also upon the temperature of the hottest external body available as a source of heat; this is apparent from equation (4).

CYCLIC PROCESSES. — In the theory of the steam and other heat engines the question arises as to what is the maximum amount of external work which can be obtained from a body by alternately heating it to a high temperature and then letting it do work (e.g., by expanding), thereby cooling to a lower temperature, this cycle of operations being repeated over and over again. That is, the body or "working substance" changes from a state 1 to a state 2, then back again to 1, then changes again to 2, and so on for any number of cycles. The intrinsic energy of the body at the beginning and end of any cycle is then the same, and therefore, the net work done by the body during any cycle is

the difference between the heat actually absorbed and that actually given out during the cycle.

Thermal Efficiency.—The ratio of the net work done to the heat absorbed during the cycle gives the proportion of the heat added which is converted into external work, and is called the "thermal efficiency" of the cycle.

Carnot's Cycle.—From the principles stated above it follows that the heat absorbed will be a maximum when this heat is absorbed in a reversible manner under conditions such that the temperature of the working substance is the same as that of the hottest body available as a source of heat; and the heat given out is a minimum when this heat is given out in a reversible manner under conditions such that the temperature of the working substance is the same as that of the coldest body available as an absorber of heat. The "path" along which the body changes (*see* Fig. 1) will then consist of an isothermal change at temperature T_1, an adiabatic change to a lower temperature T_2, an isothermal change at this temperature T_2, and then a second adiabatic change to T_1. Such a path is called a "Carnot's cycle." The figure represents such a path for a perfect gas, the ordinates being pressure and volume.

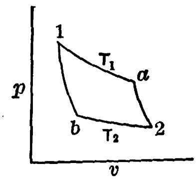

Fig. 1.

From the general expression for maximum work, equation (4), it follows that the efficiency of this cycle is

$$\frac{W}{Q_1} = \frac{T_1 - T_2}{T_1}.$$

For a given amount of heat added to a working substance, during a cyclic change, the maximum possible work which the working substance can do is that done when the cycle is a Carnot's cycle, and this maximum work is directly proportional to the difference between the temperature of the body from which the working substance absorbs heat and the temperature of the body to which it gives out heat, and is inversely proportional to the absolute temperature of the body from which it absorbs heat.

APPLICATIONS OF THE LAWS OF THERMODYNAMICS.—The laws of thermodynamics serve as the basis for the mathematical treatment of the performance of the steam engine, steam turbine, internal-combustion engines, refrigerating machines, air compressors, etc.; they are also involved in many important electrochemical relations (*see Electrochemistry, Principles of*). It is beyond the scope of this book to go into these matters here; see the treatises listed in the following bibliography.

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[H. PENDER.]

THIRD-RAIL, OR CONTACT-RAIL, SYSTEMS. — (*See also Bonds and Bonding; Rails, Track and Third; Standardization Rules of the A.I.E.E.; Trolley Systems, Overhead; Trolley Systems, Underground.*) The following is a brief table of contents of this article:

The contact rail, or third rail, is a conductor supported on insulators near the ground and presenting a continuous contact surface to a collector or shoe attached to the rolling stock. In its commonest form it is a rail of standard section supported at intervals of a few feet by substantial insulators, electrical continuity between adjacent lengths being obtained by copper bonds across the joints.

TERMINOLOGY. — The following terms and definitions are used in connection with third-rail systems.

Contact Shoe. — A third-rail contact shoe is a conductor, fastened to the rolling stock, which is designed to make electrical contact with the third rail. This is hereinafter referred to as the "shoe."

Contact Surface. — The contact surface of a third rail is the surface against which the shoe presses.

Gage of Track. — The minimum clearance between the inside surfaces of the heads of the two track rails, i.e., the distance *A* in Fig. 17. The track gage in this country is 4 feet 8½ inches.

Third-rail Gage. — The distance measured parallel to the plane of the top of both running rails, between the gage line of running rails and the gage line of third rail, i.e., the distance *B* in Fig. 17 of this article. (The Am. Elect. Ry. Assn. 1912.)

Location. — Third-rail locations will be described in terms of the third-rail gage and the vertical distance (*C* in Fig. 17) between the normal contact surface and the normal top of the track rail. See also Table II below.

Top-contact Rail. — A top-contact third rail is one on which the contact surface is on the upper side of the rail.

Under-contact Rail. — An under-contact third rail is one on which the contact surface is on the under side of the rail.

Third-rail Insulator. — A third-rail insulator is that portion of the third-rail support which forms the principal electrical insulation.

Insulator Base or Bracket. — A third-rail insulator base or bracket is a device used to support the third-rail insulator.

Incline. — An incline is a portion of third rail sloped to gradually bring the shoe from its free position into contact with the normal surface of the third rail.

End Incline. — An end incline is an incline at the end of a run of third rail and is made to receive shoes moving in line with the third rail.

Offset-end Incline. — An offset-end incline is an incline *at the end* of a run of third rail and is made to receive shoes moving laterally towards the third rail.

Cross Incline. — A cross incline is a combination in one piece of two end inclines from adjacent third rails which meet at a turn-out.

Side Incline. — A side incline is an incline at *the side of* a third rail and is made to receive shoes moving laterally towards the third rail.

Offset-side Incline. — An offset-side incline is an incline offset from the standard third rail in order to gain clearance between the third rail and the maximum equipment line of the rolling stock.

Maximum Equipment Line. — The maximum equipment line is the boundary which encloses the cross-sectional outlines of all the rolling stock, under all operating conditions.

Third-rail Protection. — A third-rail protection is a partial covering of the third rail which affords some degree of protection from the weather or from accidental contact with foreign bodies.

Third-rail Anchorage. — A third-rail anchorage is a device to prevent the third-rail creeping longitudinally.

Tie Motion. — Unless otherwise specified, tie motion will be understood to mean the vertical play of the ties caused by the passage of trains.

TYPES OF CONSTRUCTION. — Third-rail construction may be classified into the top-contact and under-contact types, each of which is susceptible of important variations in design, especially with reference to the type of protection.

Interborough Top-contact Type (Fig. 1). — One of the most commonly used types is illustrated in Fig. 1. It is often called the "Interborough Type" on account of its use in the subways of the Interborough Rapid Transit Co., of New York. The rail is a standard T-section and rests on reconstructed granite insulators. A board protection is attached to the rail itself by means of clamps and uprights and is thereby kept in perfect alignment.

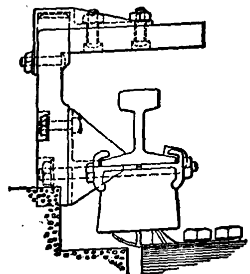


Fig. 1. Interborough Top-contact Type

Pennsylvania Top-contact Type (Fig. 2). — Another type has the protection supported on separate brackets independent of the third rail itself. It is claimed that this reduces the amount of labor which has to be done on the live rail, when repairs are being made, but it cannot be relied upon as well as the Interborough type to keep the rail and protection in perfect alignment.

London Tube Type (Fig. 3). — The mounting and location of the third rail and the negative contact rail of the London Tube Railways is illustrated in Fig. 3.

Under-contact Types (Figs. 4, 5, 6 and 7). — While the top-contact types have given first-class service, they are considered to have certain disadvantages for exposed locations as they cannot be wholly protected from snow, ice and sleet. The lower part is only a few inches from the ties, while holding clips generally reduce this clearance, increasing the danger of grounding from accumulation of wet snow and ashes and from flooding. The occasional suspension of traffic during sleet and snowstorms and floods, on railroads using the top-contact type of third rail, led to the idea of an under-contact third rail loosely clasped

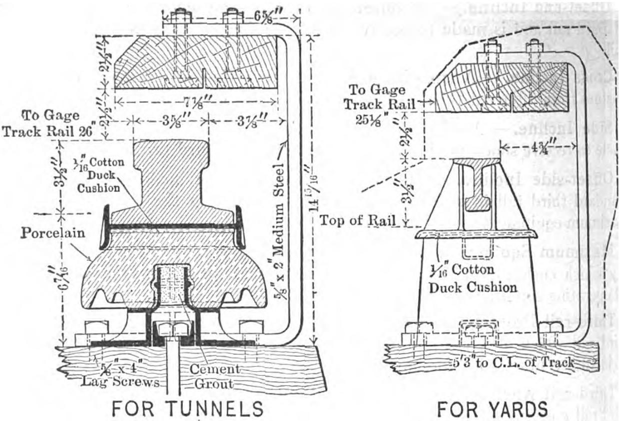


Fig. 2. Pennsylvania Top-contact Type

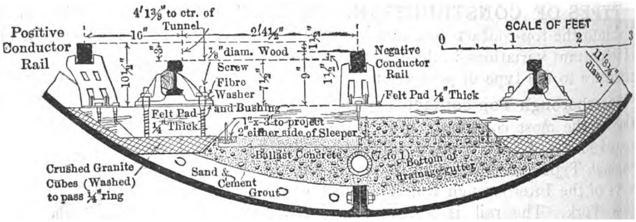


Fig. 3. London Tube Type

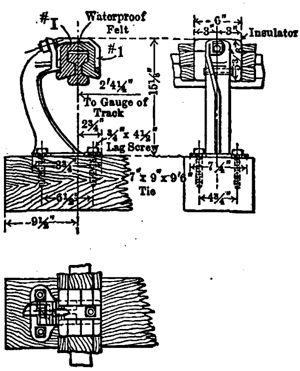


Fig. 4. Type X Bracket, for use on Straight Work

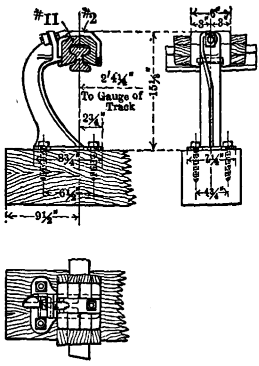


Fig. 5. Type Y Bracket, Applied to End Incline

in insulators by hook bolts hung from brackets, with the top and sides of the rail completely sheathed in a flexible insulating material for protecting the rail from accidental contact with man and beast, and from sleet, snow and spray. With this type of rail (Figs. 4, 5, 6 and 7) the protection is of such character that there is no packing of snow between the sheathing and the contact rail, as in

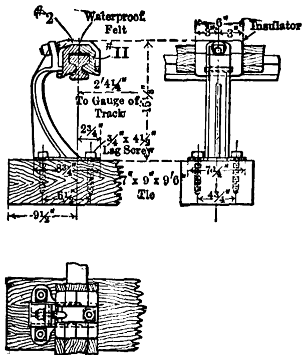


Fig. 6. Type Z Bracket, for use with end inclines A at frogs

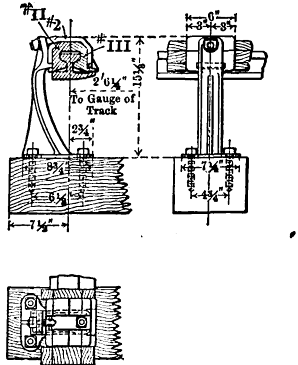


Fig. 7. Type W Bracket, for use with offset end inclines B at frogs

some other forms, and in sleet storms no ice forms on the contact surface; some icicles may form at the edge of the petticoats, but hanging down clear of the edge of the rail, are easily broken off by the passing shoe. A special design, using a standard T-rail, is shown in Fig. 8, and is used for 1200 volts.

Where the rail is buried in snow, the passage of the contact shoe breaks the snow away, leaving the rail surface clear, instead of ironing the snow down on the rail, as may happen with the top-contact type.

Sheathing and Special Work. — The sheathing between the insulator blocks, depending upon local conditions and the price of materials, as well as the potential used, is usually formed of three wooden strips, one grooved on the under side and inclosing the head of the rail, and the other two, attached to and dependent from it, reaching in toward the web of the rail. Where good wood is not available, an alternative protection costing about the same and having a higher electrical resistance, although not so good mechanically, is a semi-flexible shell of indurated fiber conformed to the rail sections.

The special work, i.e., inclines, jumpers, etc., used with the under-contact rail, is shown in Fig. 9,

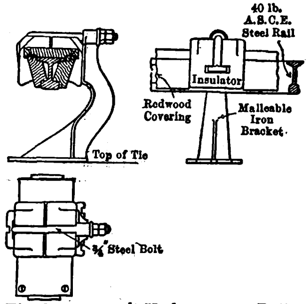
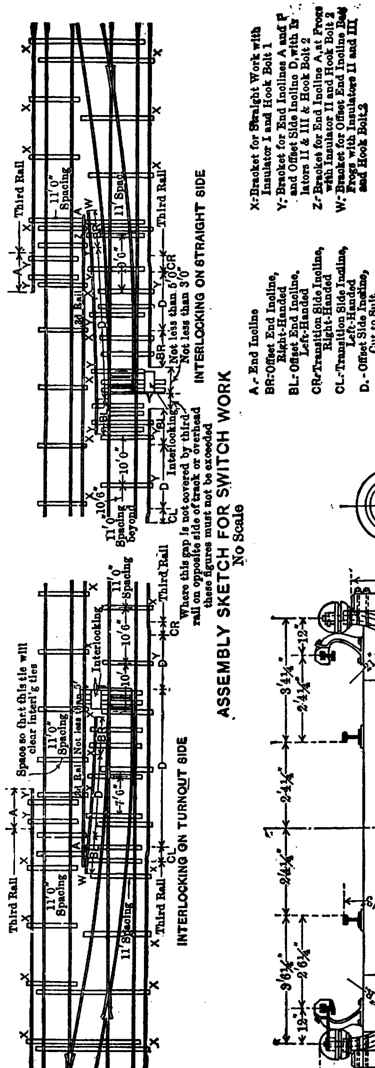
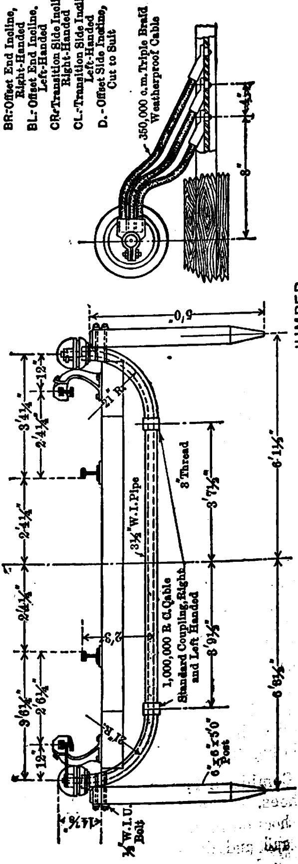


Fig. 8. 1200-volt Under-contact Rail.

Combined Top- and Under-contact Shoes. — The employment of collecting shoes on rolling stock so constructed as to press upwards on the under-contact rail and downwards on the top-contact type solves the question of interchange between railroads not using the same type.



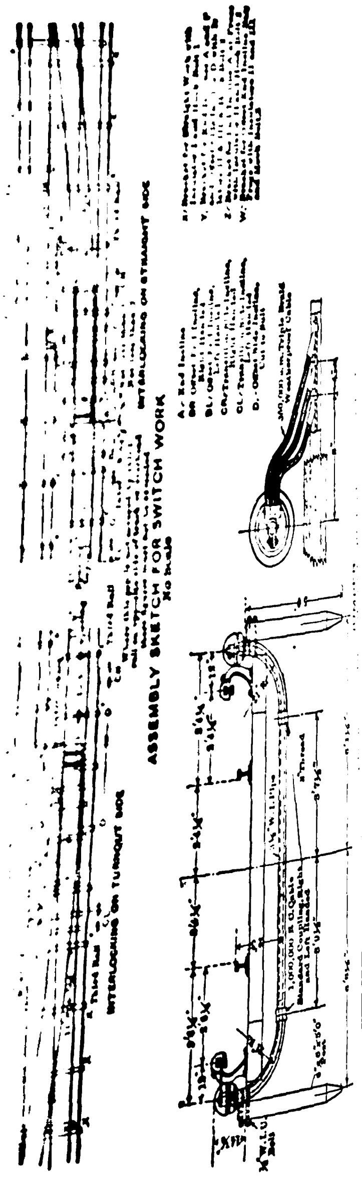
ASSEMBLY SKETCH FOR SWITCH WORK
No Scale



JUMPER

Fig. 9. Special Work used with Under-contact Rail

Third-rail Systems



CONTACT RAILS VERSUS OVERHEAD TROLLEY. — The contact rail is used as a part of the positive conductor system whenever the current to be collected by each collector exceeds the amount which can be taken safely from a trolley wire, or whenever the total current taken by a train exceeds the amount that can economically be carried by conductors of such expensive metal as copper or aluminum.

Positive Contact Rails. — Considered as a part of the positive conductor system, the contact rail and overhead trolley possess the relative qualifications given in Table I.

TABLE I. — RELATIVE QUALIFICATIONS OF THIRD RAIL AND OVERHEAD TROLLEY SYSTEMS.

(*Adapted from Table by C. E. Eveleth.*)

I Protected third rail	II Overhead high-tension bridge, catenary construction	III Overhead side bracket, catenary construction
Interference with track maintenance.	Entirely clear of road bed.	Same as II.
Can be maintained by section gang.	Requires special tools, crews and work trains.	Same as II, but not as important.
Easily cleared up and insulated when derailment occurs.	In the way of boom of derrick car — liable to be knocked down and put all tracks out of service.	Same as II.
Hindrance to coupling freights, etc. With protected rail this is not very serious.	Dangerous to freight brakemen on account of parts hanging down and small bridge clearance. Very difficult to install satisfactory ticklers to warn trainmen when approaching bridges.	This point is of less importance.
Interference with clearing snow between tracks.	Not affected.	Not affected.
Ease of satisfactorily collecting current on account of location, where relative motion between track and rail is small. Collectors may be safely replaced on the road.	Difficult to collect current, as a more complicated mechanism is required on account of the grade of the wire due to low clearances at bridges and high clearances at road crossings.	Similar to II.
May be readily inspected by track walker.	Requires a man with special training.	Similar to II, but of less importance.
Ease of sectionalization. Jumpers may be disconnected at the nearest adjacent road crossings.	Difficult of sectionalization.	Sectionalization not of so much importance.

TABLE I. — RELATIVE QUALIFICATIONS OF THIRD-RAIL AND OVERHEAD TROLLEY SYSTEMS — *Continued*

(*Adapted from Table by C. E. Eveleth.*)

I **Protected third rail**	II **Overhead high-tension bridge, catenary construction**	III **Overhead side bracket, catenary construction**
May be worked on while alive to make track changes or repairs, making system very flexible.	Requires that current be shut off no matter how slight repairs are, making system inflexible.	Similar to II, but not of such importance.
No interference with visual signals.	Signals located and seen with difficulty, as they must be lower than the bridges and even then have the distant bridges as a background. Dangerous to maintain signals in this location, as it must be done from ladders.	Not affected.
Danger of wreck from burning off track rail due to arcing current. This is a possible contingency, but one not very likely to occur.	Danger from dangling overhead work when messenger cable is burned off at a defective insulator.	Same as II.
Little interference with fire-extinguishing apparatus in the car storage yard.	Difficulty in removing cars on account of high-tension wires interfering with firemen.	Will probably have low-tension wires for this type of construction.
Absolute freedom from lightning disturbances.	Very much exposed to lightning.	Same as II.
Entire freedom from telephone and telegraph disturbances, also inductive effects on signal wires.	Difficult problem in connection with these interferences, affecting not only the railroad company's wires, but those belonging to other interests.	Same as II.
No trouble at grade crossings with crossing trolley wires.	Probably trolley crossings will have to be avoided by overhead or undergrade crossings.	Troubles similar to II.
Can add sidings or more tracks with little difficulty.	Can make such additions only at considerable expense.	Difficulty not very great.

TABLE I — RELATIVE QUALIFICATIONS OF THIRD-RAIL [illegible] OVERHEAD TROLLEY SYSTEMS—*Continued*

(*Adapted from Table by C. E. Eveleth.*)

I Protected third rail	II Overhead high-tension bridge, catenary construction	III Overhead [illegible] bracket [illegible] construction
[illegible] be worked on while [illegible] track [illegible] making [illegible] very [illegible]	Requires that current be shut off no matter how slight repairs are, making system inflexible.	Similar [illegible] tance.
[illegible] interference with visual signals.	Signals located and seen with [illegible], as they must be [illegible] the bridges and even then have the distant bridges as a background. Danger as to maintain signals in this location, as it must be done from ladders.	Not affected
[illegible] wreck [illegible] rail due to [illegible] current. [illegible] very likely to [illegible]	Danger from dangling overhead work when messenger cable is burned off at a defective insulator.	Same as II.
[illegible] interference with fire-[illegible]	Difficulty in removing cars on account of high-tension wires interfering with firemen.	Will probably [illegible] for this [illegible] construction
[illegible] from lightning [illegible]	Very much exposed to lightning.	Same as II.
[illegible] from telephone and telegraph disturbances, also [illegible] signal wires.	Difficult problem in connection with these interferences, affecting not only the railroad company's wires, but those belonging to other interests.	Same as II.
[illegible] at grade crossings [illegible] trolley wires.	Probably trolley crossings will have to be avoided by overhead or undergrade crossings.	Troubles similar to II.
[illegible] or more [illegible] with little diffi-[illegible]	Can make such additions only at considerable expense.	Difficulty not great.

Negative Contact Rails. — Negative contact rails are used as part of the negative-feeder system when the drop of potential in the track rails is limited to such a small amount that it is cheaper to use an insulated rail than to reinforce the track rails with feeders. Considered as part of the negative feeder system, the contact rail possesses the following advantages over the track-rail return system. If used in connection with a positive contact rail, the negative rail is placed between the track rails.

(I) When properly insulated, it eliminates every possibility of electrolysis.

(II) It gives the block-signal system complete independence from the electric-traction system. Among the advantages which this entails are, no unbalancing of signal circuits, saving the cost of reactance bonds (*see Signaling, Railway*) and increased economy in signal circuits.

(III) It gives greater safety to passengers.

(IV) It reduces the probability of short circuits.

(V) It decreases the cost of track-rail maintenance by the elimination of bonds.

(VI) It halves the first cost of bonds, as one high-conductivity contact rail usually replaces two track rails.

(VII) It reduces the wear of bonds by saving them from the shock of trains.

These advantages are usually offset by the first cost of the negative contact-rail, by the complications it introduces at special track work, and by the impossibility of protecting a central rail from snow and ice on account of insufficient clearance for any kind of covering.

ELECTRICAL DESIGN. — The calculations of potential drop, network, resistance, etc., are treated under *Trolley Systems, Overhead*. The composition, weight, dimensions, resistance and reactance of rails will be found in the article on *Rails, Track and Third*.

Selection of Suitable Rail. — When the various types of rail are under consideration it is well to arrange a table with the following headings, in order to compare the relative economy of the different types. (1) Circular mils of copper equivalent to rail; (2) Additional circular mils of copper required to equal the rail of highest conductivity; (3) Cost of rail for the entire railroad; (4) Cost of additional copper for entire railroad; (5) Total cost of rail and additional copper for entire railroad.

MECHANICAL DESIGN. — The general design of the rail itself having been settled, the next step is to secure a set of track plans on which to lay out the special work. When an entirely new railway is being projected, the track designer and the third-rail designer can work together, but when an existing line is being converted, the general track plans cannot be used as they are seldom sufficiently accurate for the electrical engineer's purpose. In this case, the contact-rail engineer has to take measurements of the track work in order to make drawings of the cross-overs and other complications.

Location and Weight of Third Rails. — There is no standard gage for contact rails, corresponding to the standard track gage. This unfortunate condition arises from lack of uniformity in the clearance lines of the right-of-way and in the maximum equipment lines of various railroads. The following standard has been recommended (1911 and 1912) by the American Electric Railway Engineering Association:

1. The gage line of the third rail to be located not less than 26 inches and not more than 27 inches from the gage line of the track and the contact surface of the third rail to be not less than 2¾ inches or more than 3½ inches above the plane of the top of the track rail.

2. The clearance lines for third-rail and permanent-way structures and rolling equipment to be as shown in Fig. 10, thus reserving the space within lines *AT, BT, CT, DT, ET, FT* and *AT, JT, KT, LT, MT* for third-rail structures; rolling equipment not to encroach upon the third-rail space under conditions of maximum wear and deflection beyond the line *AE, BE, CE, DE, EE, FE, GE* and permanent-way structures not to encroach upon the third-rail space beyond

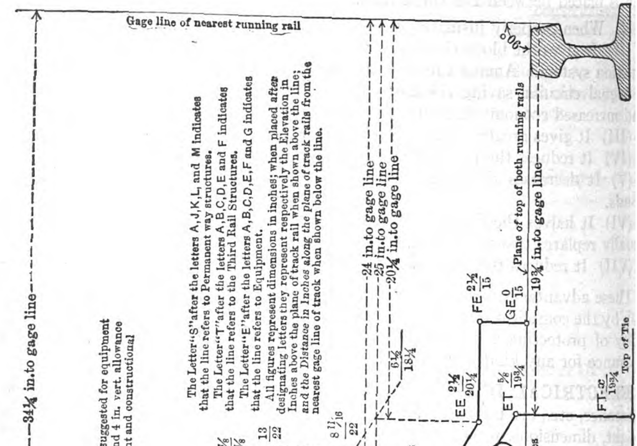


Fig. 10. Clearance Diagram

the line *AS, JS, KS, LS, MS*; this leaves a clearance space or neutral zone of one inch both horizontally and vertically upon which neither the third-rail structures nor equipment shall encroach. On curves of less radius than 800 feet, the third rail must be moved back and the equipment may be allowed to swing outward as indicated by the lines and notes on the diagram.

3. In the design and construction of new rolling stock to be used in interchange service allowance must be made for such horizontal and vertical variations as may in any reasonable probability occur in combination at one time, as

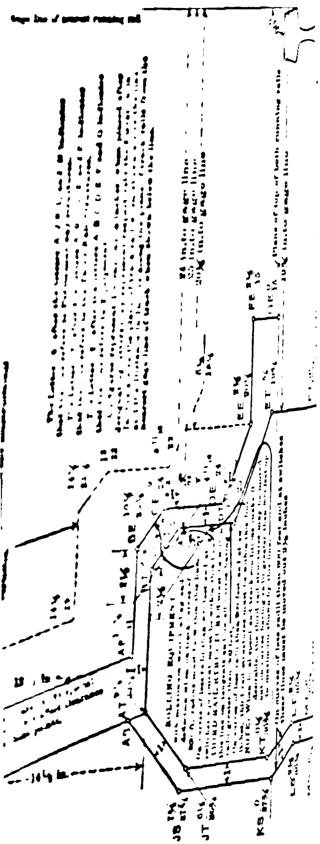


Fig. 10. Clearance Diagram

TABLE II. — LOCATION AND WEIGHT OF THIRD RAIL

Name of railroad	Center of third rail to near gage line, inches	Contact face above top of running rail, inches	Weight, lb. per yard
Albany & Hudson	27	6	....
Aurora, Elgin & Chicago	20⅛	6 5/16	....
Baker St. & Waterloo Ry.	....	....	90
Baltimore & Ohio	30	3½	....
Berlin Elevated and Subway	14⅜	7	....
Boston Elevated and Subway	20⅜	6	....
Brooklyn Rapid Transit	21¾	6	70
Camden & Atlantic City R.R.	26	3½	100
Central London Ry.	Center	1½	....
Columbus, London & Springfield	27	6	....
Columbus & Newark	27	6	....
Fayet-Chamounix	23	9	....
Grand Rapids, Gd. Haven & Muskegon	20⅜	5¾	....
Great Northern Ry., England	19¼	....	80
Hudson Tunnels, New York	26	4	75
Interborough Rapid Transit	26	4	75
Kings County El., Brooklyn	19½	5¼	....
Lackawanna & Wyoming	20⅜	3	....
Lake St. El., Chicago	20⅛	6½	....
Lancashire & Yorkshire Ry.	19¼	3	70
Liverpool Elevated	Center	1½	....
Long Island R.R.	27	3½	100
Manhattan Ry., New York	20¾	7½	100
Mersey Ry.	22	4½	....
Metropolitan & District, London	16	3	100
Metropolitan Elevated, Chicago	20⅛	6¼	....
Milan Gallerate	26⅝	7½	....
*New York Central R.R.	28¼	2¾	70
Northeastern Ry., England	19¼	3¼	80
Northwestern Elevated, Chicago	20⅛	6½	....
North Shore R.R., Cal.	27	6	50–60
Paris Orleans Ry.	25⅝	7⅞	....
Paris Versailles	25⅝	7⅞	....
Pennsylvania R.R.	27	3½	150
*Philadelphia & Western	27	3⅝	....
*Philadelphia Rapid Transit	27	6	70
Seattle & Tacoma R.R.	20	7½	....
South Side Elevated, Chicago	20⅛	6¾	....
Wamseebahn (Berlin)	33½	12⅝	....
Waterloo & City Ry.	28¼	0	....
West Jersey & Seashore	26	3½	100
*West Shore R.R.	32	2¾	70
Wilkesbarre & Hazleton	28	5	....

* Bottom contact surface. All others have top-contact surface.

well as allowance for deflections on curves. Encroachments on normal clearance due to deflection of springs and wear vary with the type of construction used, and the practice of the respective roads as regards permissible wear and deflection before repairs and adjustments are made. For the general guidance of the roads, in designing equipment, but not as a standard, therefore, a dotted line is shown on the diagram located two inches distant horizontally and four inches distant vertically from the limiting clearance line for equipment.

Table II gives the principal characteristics of the contact rails of many of the most important electric railroads.

Turnouts. — (*See also Railways, Location and Permanent Way for.*) A typical turnout is shown diagrammatically in Fig. 11 which shows the lengths which should be measured; F is the distance from the point of the switch to the point of the frog, and is called the "frog distance"; D is the length between the adjacent frogs, and Z the distance apart of the track centers. In such a diagrammatic view the lines represent the gage lines of the track rails. When, however, a contact rail is represented by a line, it is the center line that is given. Having measured the four distances specified above, the radius of the turnout curve may be calculated from the following formulas:

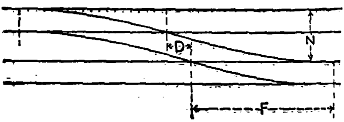


Fig. 11. Typical Turnout

$R = 0.127\ F^2$, when the turnout is from a straight track.

$R = 0.271\ F^2$, when the turnout is from a track of equal radius to the turnout.

Graphical Method of Locating Contact Rail. — The track plans having been drawn from the field sketches, to a scale of say ¼ inch to the foot, the contact rail may be drawn in, as outlined below. An aid in this work is an outline plan of the electric car or locomotive to the same scale as the track work drawings — say ¼ inch to the foot. Such a plan is shown in Fig. 12, in which the points W represent the wheels, S the contact shoes and K the king-pins. This had better be made of celluloid with a large pinhole at each point S. In order to lay out the contact rail the car is drawn along the tracks with the points W on the rails and a pencil stuck in one of the pinholes. The line traced by the pencil represents the center line of the shoe path, and therefore that of the contact rail. Such a line should be made on each side of the car. In cases where there are sharp curves, it will not do to run the wheel points W along the tracks, as an error will arise due to the truck rotation not being represented. In such a case the king-pin points must be run along a track center line.

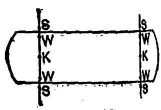


Fig. 12.

Location of Inclines. — The exact location of the contact-rail inclines on each side of a track-rail intersection depends upon a number of conditions, an important one of which is the extent to which the equalizer bar and journal box project outward. If there is a train-bus line connecting all the cars, the end inclines may be situated many feet back from the switch point or frog, but if there is no such bus line, the contact rail will probably have to be terminated in a cross incline extending as near as possible to the switch point or frog.

Isolated Section
Fig. 13.

Sectionalizing the Contact Rail. — Where a train-bus line is not used, it is customary to break the third rail in front of substations and use an isolated section of third rail between the two main sections, as

shown diagrammatically in Fig. 13. The length of this isolated section and of the gaps which bound it should be such that both gaps are never spanned by cars at the same time.

Let L = car length, feet,

T = distance between shoe centers (usually the same as distance between truck centers),

S = shoe length.

Then,

$$\text{Total space must be} > L + T + S.$$
$$\text{Rail section must be} < 2L - T - S.$$
$$\text{Length of each gap must be} < T - S.$$

For example, on the Manhattan Railway, New York, before a train-bus line was adopted, a short isolated section was used. The lengths were as follows:

$$L = 46.37, \quad T = 32.27, \quad S = 1.$$

Total space > 79.64, actually 83 feet.
Section of rail < 59.47, actually 57 feet.
Gap length < 31.27, actually 13 feet.

Cross and End Inclines. — Inclines serve the purpose of assisting the contact shoes to rise (or fall) from their free position to the position of contact with the rail. They should, therefore, have sufficient slope to prevent any undue shock either to the rail or to the shoes. Low-speed lines and sidings may therefore have shorter inclines than high-speed lines. The usual length of straight slope (i.e., without end nose and flat surface at top) is from 60 to 70 inches for high-speed work and 30 or 40 inches for sidings. The slope ranges from 1 in 45 for high-speed work to 1 in 30 for urban railways. Inclines are almost invariably made of cast iron. A typical design is shown in Fig. 14.

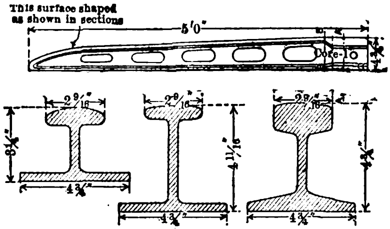


Fig. 14. Typical Incline

Third-rail Insulators. — Third-rail insulators should have the following qualifications. Strength to withstand weight and vibration; surface impervious to moisture; resistance wet shall not be less than one megohm; shall have a drip edge; shall allow free motion of rail laterally, longitudinally and vertically to allow for expansion, contraction and tie motion; and must be capable of easy and quick removal without disturbing the rail.

Spacing of Insulators. — A spacing of 10 feet between insulators is recommended by the American Street & Interurban Assn., Oct., 1908, for 30-foot rails.

INSTALLATION OF CONTACT RAIL. — In the case of an existing railway being electrified, the first step is to replace standard ties at proper intervals by long ones. A man then proceeds along the line with a template and marks the location of the screw holes of the insulator bracket. He is followed by an augur gang which drills the holes. Meanwhile the brackets (in the case of an under-contact rail) or the stool and insulators (in the case of a top-contact rail) are distributed along the line, and a gang following the augur gang screws the brackets or stools to the ties.

Where the rails rest on tie plates which have not yet penetrated the ties, it is not unusual to place shims under the brackets or stool until the tie plates have reached their normal position.

In the case of under-contact rails the brackets must be accurately checked for principal dimensions before being distributed.

The rails and fish-plates, etc., are next distributed along the line on that side of the track where they are to be installed and a gang follows which installs the rail, and, in the case of an under-contact rail, attaches the insulators and hook bolts; in the case of a top-contact rail, another gang follows, attaching the clips or whatever is used to fasten the rail to the insulators. Another gang follows to install the fish plates, expansion joints and anchorages, if any. If there are no anchorages or expansion joints, expansion and contraction are provided against by making an allowance at all joints, depending on the temperature at which the rails are installed. This is done by inserting an "expansion shim" while the rails are being bolted together. The thickness of shim for different temperatures on a 30-foot rail is given in Fig. 15.

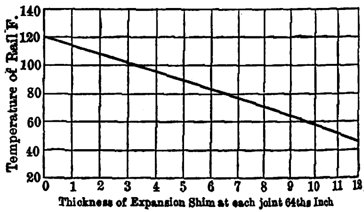


Fig. 15. Thickness of Expansion Shims for 30-ft. Rail

While the rails are being installed, the materials for the protection are distributed along the line and a gang follows installing them. A last gang paints the protection first with a priming and then with a finishing paint.

Inclines have to be carefully located to make sure that the shoes will ride smoothly upon them.

OPERATION AND MAINTENANCE OF CONTACT RAIL.—The wear of the third rails is negligible and even without painting the deterioration due to rust is very slow. The principal items of maintenance are the insulators, protection and bonds. Unless the supports are carefully designed, insulators are shattered by "tie motion," i.e., by the depression and rebound of the ties as the trains pass over them. Every time a wheel passes over a tie carrying an insulator, the latter is pounded against the rail and is likely to be broken unless sufficient play is provided.

Removal of Sleet.—The best protection against sleet is undoubtedly the under-contact third rail and the next best is a top-contact third rail with a wide protecting board. Where these are impracticable the deposit of ice on the rail cannot be prevented and means for its removal have to be adopted. The two principal means are scrapers and hot water. Electrical heating of the rail has also been suggested but found to require too much energy. When the hot-water method is used, some salt such as calcium chloride, or some other substance, which lowers the freezing point of water must be added to the water. There are serious objections to the hot-water method, especially on elevated railways, as the saline solution not only rusts the metal of the structure but when calcium chloride is used in the water, damage also occurs to the roofs of street cars, etc., underneath.

TESTING OF CONTACT RAILS.—Contact rails have to be tested for (1) electrical continuity at the joints, (2) conductivity, (3) insulation from ground and (4) gage.

Tests of Continuity at Joints are described in the article on *Bonds, Railway Track.*

The Conductivity Test is best performed by passing a considerable current through a length, measuring the drop with a milli-voltmeter and calculating the resistance by Ohm's law.

The Insulation Test is best performed by means of a pair of voltmeters, as shown in Fig. 16. The rail A to be tested is connected to a live rail B through a voltmeter, and the voltage V_1 between the two noted. At the same instant the voltage V_2 between the rail B and ground is also read. Let L be the length in miles of the rail A. Then the insulation resistance of the rail A in megohms per mile is

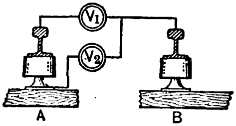


Fig. 16.

$$\frac{rL}{10^6}\left(\frac{V_2}{V_1} - 1\right),$$

where r is the resistance in ohms of the voltmeter on which V_1 is read.

Testing of Gage. — The gage line of a third rail is measured by means of a template which fits over the track rails as shown in Fig. 17.

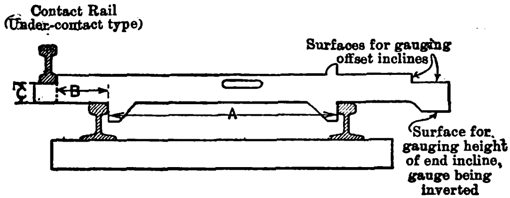


Fig. 17.

COST OF CONTACT-RAIL CONSTRUCTION. — The estimates in Table III include the cost of (1) handling and distributing the material from the storehouse to the place where it is used; (2) the solder, gasoline, etc., used in bonding contact rail; (3) putting three coats of paint on the protection; (4) bending rails on curves; (5) 5 per cent for breakage; (6) foremen's and engineers' salaries. They do not include the cost of tools or of jumpers.

These estimates are approximately correct where existing traffic does not materially impede the work. Under less favorable conditions, the cost may rise 50 per cent or more over the figures given.

The estimate on the top-contact type is based upon the Interborough Rapid Transit Co's. construction, New York (Stillwell-Slater patent), the weight of rail, however, being slightly less than on that railway. The estimate of the under-contact type is based upon construction similar to that used by the New York Central Railroad (Wilgus-Sprague patent).

TABLE III.—COST PER MILE OF CONTACT-RAIL CONSTRUCTION

Item	Top contact		Under contact	
	Amount	Cost	Amount	Cost
Material:				
Rail, 70 lb.	55 tons	$1815	55 tons	$1815
Rail, special	....	....	1.2	40
Inclines	11	47	11	47
Insulators, standard	511	92	1000	165
Insulators, special	....	....	25	13
Brackets or pedestals	515	62	500	250
Brackets, special	....	....	15	7
Bolts	515	10	515	90
Lag screws	1030	20	1515	30
Clips	1030	41	......	....
Drive screws	....	....	80 gross	24
Soldered bonds	350	168	350	168
Splice plates and bolts	350	53	180	31
Protection	....	793	....	642
Paint	....	49	....	82
Felt separator	....	....	....	2
Long ties, excess only *	505	177	505	177
Total material	....	$3327	....	$3583
Labor:				
Installing, bonding and protection of third rail	....	$ 800	....	$1000
Installing long ties	....	101	....	101
Total labor	....	$ 901	....	$1101
Grand total	....	$4228	....	$4684

* This item includes only the *difference* in cost between the long ties which carry the insulators and the cost of the same number of standard ties.

BIBLIOGRAPHY.—(*See also Railways, Systems of Electric Traction for; Electrolysis of Grounded Structures; Endosmose; Trolley Systems, Overhead.*) Anon., *Farnham Protected Third Rail System*, Street Ry. Jour., 1906, Vol. 27, p. 45; Capp, J. A., *Tests of Steel for Electric Conductivity*, Trans. A.I.M.E., 1904, Vol. 34, p. 400; Fortenbaugh, S. B., *Conductor Rail Measurements*, Trans. A.I.E.E., 1908, Vol. 27, p. 1215.

[W. A. DEL MAR.]

TIMBER. — (*See also Structures, Simple; Poles for Overhead Lines.*) The standard names for structural timbers as adopted by the Am. Soc. for Test. Mat. and the Am. Ry. Eng. Assoc. are reprinted below by permission from the present (1913) board of the Am. Soc. for Test. Mat.

Fir, Douglas. — The term "Douglas Fir" is to cover the timber known likewise as yellow fir, red fir, western fir, Washington fir, Oregon or Puget Sound fir or pine, norwest and west coast fir.

Hemlock, to cover Southern or Eastern hemlock; that is, hemlock from all states east of and including Minnesota.

Hemlock, Western, to cover hemlock from the Pacific coast.

Larch, Western, to cover the species of larch or tamarack from the Rocky Mountain and Pacific coast regions.

Pine, Norway, to cover what is known also as "Red Pine."

Pine, Southern Yellow. — Under this heading two classes of timber are used, (*a*) *Longleaf Pine*, (*b*) *Shortleaf Pine.*

It is understood that these two terms are descriptive of quality, rather than of botanical species. Thus, shortleaf pine would cover such species as are now known as North Carolina pine, loblolly pine and shortleaf pine. Longleaf pine is descriptive of quality, and if Cuban, shortleaf, or loblolly pine is grown under such conditions that it produces a large percentage of hard summer wood, so as to be equivalent to the wood produced by the true longleaf, it would be covered by the term "Longleaf Pine."

Pine, Western, to cover the timber sold as white pine coming from Arizona, California, New Mexico, Colorado, Oregon and Washington. This is the timber sometimes known as "Western Yellow Pine," or "Ponderosa Pine," or "California White Pine," or "Western White Pine."

Pine, White, to cover the timber which has hitherto been known as white pine, from Maine, Michigan, Wisconsin and Minnesota.

Pine, Idaho White, the variety of white pine from western Montana, northern Idaho and eastern Washington.

Redwood, to include the California wood usually known by that name.

Spruce, to cover Eastern spruce; that is, the spruce timber coming from points east of Minnesota.

Spruce, Western, to cover the spruce timber from the Pacific coast.

Tamarack, to cover the timber known as "Tamarack," or "Eastern Tamarack," from states east of and including Minnesota.

A fuller description of the timbers used for transmission line poles is given in the article on *Poles for Overhead Lines.*

DEFECTS IN TIMBER. — The standard names for defects in structural timber as adopted by the above associations are the following:

Encased Knot. — An encased knot is one which is surrounded wholly or in part by bark or pitch. Where the encasement is less than ⅛ inch in width on both sides, not exceeding one-half the circumference of the knot, it shall be considered a sound knot.

Large Knot. — A large knot is a sound knot, more than 1½ inches in diameter.

Loose Knot. — A loose knot is one not firmly held in place by growth or position.

Pin Knot. — A pin knot is a sound knot not over ½ inch in diameter.

Pith Knot. — A pith knot is a sound knot with a pith hole not more than ¼ inch in diameter in the center.

Rotten Knot. — A rotten knot is one not as hard as the wood it is in.

Round Knot. — A round knot is one which is oval or circular in form.

Sound Knot. — A sound knot is one which is solid across its face and which is as hard as the wood surrounding it; it may be either red or black, and is so fixed by growth or position that it will retain its place in the piece.

Spike Knot. — A spike knot is one sawn in a lengthwise direction; the mean or average width shall be considered in measuring these knots.

Standard Knot. — A standard knot is a sound knot not over 1½ inches in diameter.

Pitch Pockets. — Pitch pockets are openings between the grain of the wood containing more or less pitch or bark. These shall be classified as *small*, *standard* and *large* pitch pockets.

(*a*) *Small Pitch Pocket.* A small pitch pocket is one not over ⅛ inch wide.

(*b*) *Standard Pitch Pocket.* A standard pitch pocket is one not over ⅜ inch wide, or 3 inches in length.

(*c*) *Large Pitch Pocket.* A large pitch pocket is one over ⅜ inch wide, or over 3 inches in length.

Pitch Streak. — A pitch streak is a well-defined accumulation of pitch at one point in the piece. When not sufficient to develop a well-defined streak, or where the fiber between grains, that is, the coarse-grained fiber, usually termed "Spring wood," is not saturated with pitch, it shall not be considered a defect.

Shakes. — Shakes are splits or checks in timbers which usually cause a separation of the wood between annual rings.

Ring Shake. — An opening between the annual rings.

Through Shake. — A shake which extends between two faces of a timber.

Rot, Dote and Red Heart. — Any form of decay which may be evident either as a dark red discoloration not found in the sound wood, or the presence of white or red rotten spots, shall be considered as a defect.

Wane. — Wane is bark, or the lack of wood from any cause on edges of timbers.

DECAY AND PRESERVATION. — Decay is due usually to a fungus growth. It is prevented by cutting off the air by immersing the timber in water, which so far as is known is a sure preventive. Decay is retarded by thorough seasoning either naturally or in the kiln, or by poisoning the food supply of the fungus by chemical treatment. The latter method usually consists in impregnating the wood with creosote or zinc chloride, the former being more commonly employed. The methods used for preserving timber poles are described in the article on *Poles for Overhead Lines*.

UNIT STRESSES FOR STRUCTURAL TIMBER. — The values of unit stresses in structural timber given in the following table are recommended by the Committee on Wooden Bridges and Trestles of the American Railway Engineering Association. See *Manual* of the Association, Chicago, 1911. For unit values allowable in various cities of the United States see article on *Buildings, Allowable Unit Stresses in.*

UNIT STRESSES FOR STRUCTURAL TIMBER IN POUNDS PER SQUARE INCH

(*See Note under table*)

Kind of timber	Compression: Perpendicular to grain — Elastic limit	Compression: Perpendicular to grain — Working strength	Compression: Parallel to grain — Average ultimate	Compression: Parallel to grain — Working strength	Working strength of columns — Length under 15 diameters	Working strength of columns — Length over 15 diameters	Ratio of length of stringer to depth
Cedar, Red	470	230	2800	900	680	$900\left(1-\frac{L}{60D}\right)$	..
Cypress, Bald	340	170	3900	1100	830	$1100\left(1-\frac{L}{60D}\right)$	..
Fir, Douglas	630	310	3600	1200	900	$1200\left(1-\frac{L}{60D}\right)$	10
Hemlock, Western	440	220	3500	1200	900	$1200\left(1-\frac{L}{60D}\right)$	..
Oak, White	920	450	3500	1300	980	$1300\left(1-\frac{L}{60D}\right)$	12
Pine, Longleaf	520	260	3800	1300	980	$1300\left(1-\frac{L}{60D}\right)$	10
Pine, Norway	....	150	2600*	800	600	$800\left(1-\frac{L}{60D}\right)$	..
Pine, Shortleaf	340	170	3400	1100	830	$1100\left(1-\frac{L}{60D}\right)$	10
Pine, White	290	150	3000	1000	750	$1000\left(1-\frac{L}{60D}\right)$	10
Redwood	400	150	3300	900	680	$900\left(1-\frac{L}{60D}\right)$	..
Spruce	370	180	3200	1100	830	$1100\left(1-\frac{L}{60D}\right)$	..
Tamarack	....	220	3200*	1000	750	$1000\left(1-\frac{L}{60D}\right)$	..

* Partially air dry.

L = unsupported length in inches. D = least side in inches.

NOTE. — These unit stresses are for a green condition of timber and are to be used without increasing the live-load stresses for impact.

The working units given in these tables are intended for railroad bridges and trestles. For highway bridges and trestles the unit stresses may be increased twenty-five (25) per cent. For buildings and similar structures in which the timber is protected from the weather and practically free from impact, the unit stresses may be increased fifty (50) per cent. To compute the deflection of a beam under long-continued loading instead of that when the load is first applied, only fifty (50) per cent of the corresponding modulus of elasticity given in the table is to be employed.

UNIT STRESSES FOR STRUCTURAL TIMBER IN POUNDS PER SQUARE INCH

(*See Note bottom of p. 1589*)

Kind of timber	Bending: Extreme fiber stress		Bending: Modulus of elasticity	Shearing: Parallel to grain		Shearing: Longitudinal shear in beams	
	Average ultimate	Working strength	Average	Average ultimate	Working strength	Average ultimate	Working strength
Cedar, Red	4200	800	860,000	....	....	....	....
Cypress, Bald	4800	900	1,150,000	500	120	....	....
Fir, Douglas	6100	1200	1,510,000	690	170	270	110
Hemlock, Western	5800	1100	1,480,000	630	160	270*	100
Oak, White	5700	1100	1,150,000	840	210	270	110
Pine, Longleaf	6500	1300	1,610,000	720	180	300	120
Pine, Norway	4200	800	1,190,000	590	130	250	100
Pine, Shortleaf	5600	1100	1,480,000	710	170	330	130
Pine, White	4400	900	1,130,000	400	100	180	70
Redwood	5000	900	800,000	300	80	....	....
Spruce	4800	1000	1,310,000	600	150	170	70
Tamarack	4600	900	1,220,000	670	170	260	100

* Partially air dry.

STANDARD SIZES. — The commercial sizes of spruce and yellow pine in the eastern part of the United States are as follows:

Spruce. — Cross-sectional dimensions in inches are:
2 by 3, 2 by 4, 2 by 5, 2 by 6, 2 by 7, 2 by 8, 2 by 10, 2 by 12
3 by 4, 3 by 6, 3 by 8, 3 by 10, 3 by 12
4 by 4, 4 by 6, 4 by 8, 4 by 10, 4 by 12
6 by 6, 6 by 8, 6 by 10, 6 by 12
8 by 8, 8 by 10, 8 by 12.
12 ft. to 22 ft. are ordinary lengths.
23 ft. to 26 ft. are less common.
27 ft. to 32 ft. are obtained with difficulty.

Yellow Pine. — Same cross-sectional dimensions as spruce and also the following (dimensions in inches):

2 by 14, 2 by 16	6 by 14, 6 by 16	12 by 14, 12 by 16
3 by 14, 3 by 16	8 by 14, 8 by 16	14 by 14, 14 by 16
4 by 14, 4 by 16	10 by 14, 10 by 16	16 by 16

Yellow pine sticks are commonly longer than spruce sticks, frequently exceeding 40 ft.

DESIGN OF TIMBER BEAMS. — Timber beams may be designed by the application of the ordinary beam formulas (*see Design of Beams in article on Structures, Simple*). Timber is especially weak in longitudinal shear which is the determining factor in many cases as shown by the preceding tables. It should, however, be noted that the projection of the end of a beam over the end supports often gives the timber a greater resistance to shear than would be allowed by the strict application of theory and the designer must often use considerable discretion in applying the shear formula.

ALLOWABLE LOADS (POUNDS) ON BEAMS OF LONGLEAF YELLOW PINE

(See Note below table)

Allowable uniformly distributed load in pounds per inch width per lineal foot, in excess of weight of beam, for end-supported yellow pine beams, supported laterally at intervals of 12 times their thickness or less. *Tabular values to be multiplied by width of beam in inches to get carrying capacity of beams per lineal ft.*

Span in ft. = distance c. to c. end bearings	Depth of beam in inches									Coefficient of deflection
	4	5	6	7	8	10	12	14	16	
5	91	142	190	221	253	316	380	443	506	0.65
6	63	98	142	184	210	263	315	368	421	0.94
7	46	72	104	142	180	224	270	315	360	1.27
8	35	54	79	108	141	196	235	275	314	1.66
9	27	43	62	85	111	177	209	244	279	2.11
10	22	34	50	68	90	141	188	219	250	2.60
11	18	28	41	56	73	116	167	198	227	3.15
12	15	23	34	47	61	96	140	181	207	3.74
13	12	19	28	39	52	82	119	162	191	4.39
14	10	16	24	33	44	70	102	139	177	5.10
15	9	14	21	29	38	60	88	120	158	5.85
16	....	12	18	25	33	53	77	105	138	6.66
17	....	11	16	22	29	46	67	93	122	7.51
18	....	....	14	19	25	41	59	82	108	8.43
19	....	....	....	17	23	36	53	73	96	9.39
20	....	....	....	....	20	32	47	65	86	10.40
21	....	....	....	....	....	29	43	59	78	11.47
22	....	....	....	....	....	26	38	53	70	12.59
23	....	....	....	....	....	23	35	48	64	13.76
24	....	....	....	....	....	21	32	44	58	14.98
25	....	....	....	....	....	19	29	39	53	16.25
26	....	....	....	....	....	18	26	36	49	17.58
27	....	....	....	....	....	16	24	33	45	18.96
28	....	....	....	....	....	15	22	31	41	20.39
29	....	....	....	....	....	....	20	29	38	21.87
30	....	....	....	....	....	....	19	26	35	23.41

NOTE.—The tables pp. 1591 and 1592 are for railroad structures of green timber and require no allowance for impact.

For highway structures add 25 per cent to tabular values.

For buildings and other structures where timber is protected from weather add 50 per cent to tabular values.

Bold-faced type is used for lengths where shear limits.

Values below zigzag line cause a deflection under intermittent load in excess of 1/360 of span.

For actual deflection in inches, except where strength is limited by shear, divide deflection coefficient by depth of beam in inches. Deflection under permanent load will ultimately equal double this value.

ALLOWABLE LOADS (POUNDS) ON SPRUCE BEAMS

(*See Note, bottom p. 1591*)

Allowable uniformly distributed load in pounds per inch width per lineal foot, in excess of weight of beam, for end-supported spruce beams, supported laterally at intervals of 12 times their thickness or less. ***Tabular values to be multiplied by width of beam in inches to get carrying capacity of beams per lineal ft.***

Span in ft. = distance c. to c. end bearings	Depth of beam in inches									Coefficient of deflection
	4	5	6	7	8	10	12	14	16	
5	70	92	111	130	147	185	221	258	296	0.57
6	48	76	92	108	122	154	184	215	246	0.82
7	35	56	79	92	105	131	157	184	210	1.12
8	27	42	62	81	91	115	137	160	184	1.47
9	21	33	48	66	81	102	121	142	163	1.85
10	17	27	39	54	69	91	109	128	146	2.29
11	14	22	32	44	57	83	99	116	133	2.77
12	11	18	27	37	47	75	90	106	121	3.30
13	10	15	23	31	40	64	83	98	112	3.87
14	8	13	19	27	34	55	77	90	104	4.49
15	7	11	17	23	30	47	68	84	97	5.15
16	....	10	15	20	26	41	60	79	90	5.86
17	....	9	13	18	23	36	52	72	85	6.62
18	....	....	11	16	20	32	46	64	80	7.42
19	....	....	....	14	18	29	41	57	76	8.27
20	....	....	....	....	16	26	37	51	68	9.16
21	....	....	....	....	....	23	33	46	62	10.10
22	....	....	....	....	....	21	30	42	56	11.08
23	....	....	....	....	....	19	27	38	50	12.12
24	....	....	....	....	....	17	25	35	46	13.19
25	....	....	....	....	....	16	23	32	43	14.31
26	....	....	....	....	....	14	21	29	39	15.48
27	....	....	....	....	....	13	19	27	36	16.70
28	....	....	....	....	....	12	17	25	33	17.96
29	....	....	....	....	....	....	16	23	31	19.26
30	....	....	....	....	....	....	15	21	29	20.61

The unit values used in the tables are:

Yellow Pine	Lb. per sq. in.	*Spruce*	Lb. per sq. in.
Fiber stress:		Fiber stress:	
Bending	1,300	Bending	1,000
Longitudinal shear	120	Longitudinal shear	70
Modulus of elasticity	1,500,000	Modulus of elasticity	1,310,000

These unit values used are the same as given in the table on pp. 1589 and 1590, except the modulus of elasticity for Yellow Pine, which is taken as 1,500,000 lb. per sq. in. For other values of stress modify tabular values in proportion to allowable unit stresses.

WEIGHT OF TIMBER. — This is variable, depending upon the moisture in the timber and whether it has been chemically treated or not. It is common to assume yellow pine at 4½ lb. per board foot and spruce at 4 lb. per board foot in estimating the strength of beams. See also section on *Volume and Weight of Poles* in article on *Poles for Overhead Lines*.

COST OF TIMBER. — The price of timber is variable and depends for ordinary stock upon the size and length. Yard quotations in New York in June, 1913, gave the price of spruce as from \$35 to \$37 per 1000 board feet. Similar quotations on longleaf yellow pine were as follows:

COST OF YELLOW PINE IN DOLLARS PER 1000 BOARD FEET AT YARD IN NEW YORK

Cross section	Length of stick			
	20 feet and under	21 to 25 feet	26 to 30 feet	31 to 35 feet
2 in. thick by 8 in. wide, and under..	\$36	\$36	\$36	\$38
10 in. thick by 10 in. wide, and under..	42	42	42	45
12 in. thick by 12 in. wide, and under..	42	42	42	45
14 in. thick by 14 in. wide, and under..	47.50	48	50	50
16 in. thick by 16 in. wide, and under..	50	52	54	60

For regular quotations on timber see the first issue of each month of the *Engineering News*.

The cost of hauling timber and putting in place may be roughly estimated at \$10 per 1000 board-feet for fairly heavy yellow pine timbers which require little labor.

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[C. M. SPOFFORD.]

TOWERS FOR TRANSMISSION LINES. — (*See also Distribution Lines; Insulators for Transmission Lines; Poles for Distribution Lines; Structures, Simple; Transmission Lines.*) Steel towers are used as a substitute for wooden poles for supporting electrical circuits. They have the advantages over poles that they can be of as large size as desired and are much more permanent. Steel towers are especially advantageous for high voltages (over 50,000) where wide wire spacing is necessary; for long spans, such as river crossings, where the mechanical stresses are exceptionally heavy; and in localities where grass, brush or forest fires occur.

GENERAL FEATURES OF DESIGN. — Towers are composed of two parts, the tower proper and the foundation. Towers are usually built of standard structural steel shapes. Angles are used for most members. Channels are used for the larger members of some towers, usually the cross arms, or for posts of flexible towers. Flat pieces are sometimes used for the minimum-sized bracing of light towers. Round rods are used for tension members in some types. The principal members of a tower are the corner posts or legs, which are vertical or approximately vertical, and are usually the heaviest members of the tower proper, and the horizontal and diagonal web members which connect the posts together in vertical planes which constitute the sides or faces of the tower.

The spread of a tower at the base is generally between one-fourth and one-fifth of the height. The greatest economy in cost of tower plus foundation usually requires a little wider base than that which gives the least cost for the tower taken alone.

Towers are usually designed for either one or two three-wire circuits, usually with one or two ground wires above and sometimes with a telephone circuit below. Where two circuits are on one tower they are usually located on opposite sides to reduce the hazard of repairing the line. The three wires of a circuit are occasionally arranged to form an equilateral triangular prism, but frequently lie in a single plane which is usually horizontal for single-circuit towers, and vertical for two-circuit towers.

Types of Towers. — There are two general types of towers, viz., (1) the flexible, having only two legs and (2) the rigid, having three or more legs.

Flexible Towers (Fig. 4). — The flexible tower is set with face at right angles to the line so as to rigidly resist any transverse force, such as a wind blowing across the line, while bending if necessary to relieve any stress in the direction of the line. The stability of the tower in the direction of the line is maintained by the conductors which it supports acting as guys. Where conductors cannot perform this function, due to being attached by suspension insulators, the ground wires are used as the guys. The flexible tower is especially economical in the smaller sizes, where the line conditions approximate those where wooden poles would otherwise be used. On lines of flexible towers, rigid anchor towers are set at intervals.

Rigid Towers. — These are usually triangular, square (Fig. 2) or rectangular (Fig. 3). The square tower is probably the most common. Square towers usually have the four faces framed with the same size members (even though the stresses in the longitudinal and lateral faces rarely figure the same), because of the economy of manufacture and erection which results from the simplicity. This feature has an advantage in design in that the torsional stresses are more simply determined. Rectangular (including square) towers have the disadvantage that the unequal settlement of the foundations may produce high internal stresses not allowed for in the design. Triangular towers avoid internal stresses from unequal foundation settlement, but present diffi-

culties in the joining of standard structural shapes, and stresses in them are difficult to calculate.

Connections of Members. — The members of a tower are usually connected by bolts. By using no rivets the members may be compactly bundled, easily handled even in rough country, erected by less skillful labor and the galvanizing can be done after all shop work is completed. All the bolts of a tower should be of one diameter (5/8 inch is suitable for the members generally used) and of as few different lengths as possible. Bolt holes should be slightly larger than the bolts (1/16 inch is a usual amount). By designing bolted connections so that friction between the surfaces develops the full compressive strength of members, the play in the bolt holes with changing compression and tension in the members is eliminated.

Clearance Between Conductors and Tower. — The clearance between conductor and tower should be sufficient so that the current will not jump to the tower at a lower voltage than is required to arc over the insulator. This is usually determined approximately by making the clearance equal or exceed the length of the string of suspension insulators. Allowance must also be made for change of position of conductor with swing of suspension insulator. The amount of swing will not ordinarily exceed 45 degrees from the vertical, but should be determined for each case with and without ice loading; see *Transmission Lines*. The angle of swing will be greater for small than for large conductors and will be more for aluminum than for copper of the same diameter. Clearance to the cross arm will require special consideration where the angle of swing exceeds 45 degrees from the vertical and also for lesser angles where insulators are used which do not have an arc-over voltage very high compared to the working voltage.

Foundations for Towers (Figs. 5 and 6). — Structural steel, mass (unreënforced) concrete, reënforced concrete or piles may be used for tower foundations. Rock footings are also used in special locations.

Structural Steel Foundations are cheap and easily transported. While their durability has been questioned, they have been widely used without much trouble on this account yet apparent.

Concrete Foundations have an advantage over structural steel in that they can more easily be varied in depth, spread, etc., to accommodate themselves to local conditions of soil. This is especially advantageous where boulders or irregular ledges interfere with the use of a standard-sized foundation.

Mass-concrete foundations are advantageous in those cases where it is necessary or desirable to have a foundation of such weight as to withstand much uplift with little reliance on the holding power of the earth. The towers may be conveniently attached to anchor bolts imbedded in the foundation. To avoid tension in the concrete the bolts must extend to the bottom, with proper plates for distributing the stress. The anchor bolts and plates then become a crude system of reënforcement.

Reënforced concrete foundations are durable and require less material than mass concrete, thereby facilitating transportation.

Piles are used under or for foundations in very marshy ground where the holding power of other foundations is unreliable.

Rock Footings for towers standing on ledges may consist of anchor bolts grouted into holes drilled in rock and extending through level bearing plates grouted to the rough rock surface at the proper elevation.

FORCES ACTING ON TOWERS. — The stresses in towers are caused by: (1) The weight of tower, insulators, clamps, cables (conductors, ground wires,

telephone wires) and ice loads on them; (2) The wind pressure on above;* (3) The unbalanced tension in cables when dead ended or broken on one side; (4) The unbalanced resultant due to cable tension at angles in the line; (5) The loads imposed when erecting towers, stringing wire or repairing line.

A careful study should be made of all the combinations of these loads which are possible or probable. Often no single combination can be found which will produce the maximum stress in all tower members, and therefore several combinations must be used to determine the design.

In a square anchor tower carrying six wires, three on each side, the maximum stress may be expected in the corner posts when all six wires are pulling in the same direction; the maximum stress in the web members will probably be produced by three wires pulling on one side in one direction and the three on the other side in the opposite direction. In the first case the tower is subject to a bending stress and in the second to a torsional stress. In each case the stresses due to weight and wind are to be superimposed. The wind may act as a force along the line or across the line, but generally its longitudinal effect is negligible while its lateral effect is important.

Stresses in Tower Members. — The stresses in the several members of a tower are usually determined graphically from the assumed loadings by means of stress diagrams; see section on *Trusses* in the article on *Structures, Simple*. In most designs the distribution of stress is not fully determinate.

Fundamental Assumptions. — Certain assumptions are, however, commonly made which give a determinate distribution for the purposes of design. Among these assumptions are:

(1) An unbalanced stress on the tower (say a broken wire pulling on one side) can be resolved into an equal stress at the axis of the tower and a torsional moment.

(2) The equivalent stress at the axis of a rectangular tower can be considered as balanced between the two faces parallel with it, each face taking one-half the stress.

(3) The torsional moment can be considered as divided between all four faces of a rectangular tower.

(4) If the tower is square each face takes one-fourth of the torsion.

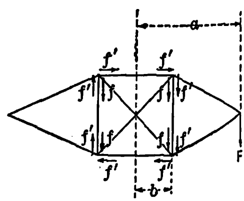

Fig. 1.

The above relations may be expressed as follows, (see Fig. 1):

Let F = unbalanced force in pounds applied at end of cross arm,
a = distance, in feet, from *end of cross-arm* to axis of tower,
b = distance, in feet, from *side of tower body* to axis of tower;

then

$$f = \frac{F}{4} = \text{balanced force in pounds applied at each corner post equivalent to } F \text{ in bending effect on body of tower,}$$

* The wind pressure on a tower is assumed to be uniformly distributed per square foot of surface against which the wind blows, one-half of it consequently being on the windward side of the windward face and the other half on the windward side (inside) of the leeward face. For simplicity in calculation this uniformly-distributed force is replaced by a series of concentrated forces, one at each panel point equivalent to the total distributed force extending over a half panel above and a half panel below the panel point. By panel point is meant a point of intersection of principal members, for example, of horizontal members with vertical members in Fig. 4; and by panel is meant the section of a side between panel points, a panel usually being bounded by two vertical and two horizontal members as in Fig 4.

$f' = \dfrac{Fa}{8b}$ = torsional force in pounds applied at each corner post equivalent to F in twisting effect on body of tower.

(5) In a tower framed with a double system (i.e., diagonals in duplicate and suitable for compression as well as tension) each system may be considered as taking one-half of the stress as far as possible.

Approximations Made in Calculating Tower Stresses.— The stress diagrams are usually simplified by employing certain approximations:

(a) Faces of towers are usually battered so that they deviate slightly from a true vertical plane, but the stress diagrams usually neglect this inclination and are based on the vertical projection of the face.

(b) Where the face of a tower does not lie in one plane (i.e., has a change of batter as occurs frequently at bottom cross arm where a prismatic cage joins a pyramidal base) the change of inclination is neglected and the diagrams are based on a single vertical projection as before.

Subject to the limitations of the assumptions and approximations given above, the four faces of the tower can be regarded as four cantilevers, supported at the base and loaded at the top, which are independent except that the four corner posts are each common to two faces and must contain the resultant of both stresses.

Where a face of a tower or any part of a face has any considerable inclination the above approximations may not be used without danger of serious error.

Unstressed Members. — A tower usually contains members no stress in which is shown by the stress diagram, viz.:

(1) Diagonal members in a horizontal plane do not usually appear in the stress diagram when located below the lowest cross arm. These members play an important part in the distribution of torsion among the faces. In a rectangular tower the torsion will usually redistribute between the four faces at each level where there are horizontal diagonals, therefore the failure of the stress diagram to show stresses in them may be taken to indicate that the assumed distribution of torsion is not quite correct rather than a true absence of stress.

(2) Redundant members are braces which carry no determinate stress but perform the important function of supporting the compression members which do carry stress. The unit stress allowable in a compression member diminishes as the unsupported length increases. The weight of compression members is therefore diminished by dividing their unsupported length by braces applied at one or more intermediate points.

Unit Stresses in Towers. — The factors of safety and unit stresses used in tower design have not been standardized. In some cases it has been assumed that the combination of loading taken for tower design represented conditions which would happen so rarely that a tower which would stand these loadings in test without failure would be sufficient. This is equivalent to taking a severe combination of loads and designing for a factor of safety of but one. The result, however, may be as good a tower as one designed for the same conditions individually (but not in combination) with a factor of safety of two or four. It is probable that few towers are designed for the combination of all loadings, each taken at the most severe condition reasonably possible with an actual factor of safety exceeding two.

For a factor of safety of from $2\frac{1}{2}$ to $1\frac{1}{2}$ (as determined by test of completed tower, not of individual members) the following unit stresses have been used:

Tension: 12,000 to 20,000 lb. per sq. in.
Shear: 12,000 to 20,000 lb. per sq. in.
Bearing: 16,000 to 30,000 lb. per sq. in.
Compression: $12{,}000 \div K$ to $20{,}000 \div K$ lb. per sq. in.

Where

$$K = \frac{L^2}{36{,}000\,R^2},$$

L = unsupported length of member, in inches,
R = least radius of gyration of section, in inches,

L and R are so limited that for main members $L \div R$ does not exceed 125 to 180 and for secondary members $L \div R$ does not exceed 150 to 220.

Eccentricity in Stresses at Joints. — As tower members are ordinarily connected together the stresses in them are slightly eccentric, thereby preventing the full strength of the members being developed. The eccentricity should be eliminated or reduced as much as possible by having the center of gravity of the several members at each connection meet at one point as exactly as possible.

FORCES ACTING ON TOWER FOUNDATIONS.—Foundation stresses are of two classes: first, the foundations resist the tendency of the tower to slide and overturn due to the external forces on it considering the tower as a self-contained structure; second, the foundations resist certain stresses which would be internal tower stresses were the tower framed as a complete self-contained structure, but which become external stresses because the ground is depended on for the function of certain omitted members. The weight of the tower can evidently be reduced by thus substituting the ground for certain members, but the size of foundation is thereby increased. The amount of these latter stresses depends on the outline and framing of the tower and their effect should not be overlooked in determining loadings on foundations.

The magnitude and direction of the forces acting on the tower foundations may be illustrated by taking the case of a rectangular tower, and considering a transmission line which runs north and south.

Let

a = width (feet) of base of tower (east and west);
b = length (feet) of base of tower (north and south);
W = total weight (pounds) of tower, insulators, fittings and one span of all the wires, including ice load, if any;
W' = total weight (pounds) of any unbalanced load, such as a wire c feet off center;
F = resultant force (pounds) of the wind on the tower and a complete span of all the wires (with ice coat, if any), acting at a distance of d feet above the foundation, wind assumed blowing across line from west to east,
P = pull (pounds) of any unbalanced force toward the south applied at a distance of e feet above the foundation and f feet to the west of axis of tower, as for example, a dead-ended wire or when a wire on the north side of the tower is broken.

Then, assuming that the forces divide equally among the four foundations and that the torsional forces are in a circumferential direction, the relations given in the following table hold. These assumptions are reasonably correct for a tower with the four legs joined at the bottom with a horizontal strut in each of the four faces and with horizontal ties across the diagonals, unless the framing which is usually provided in the other faces is inadequate. Probably few towers in use fully meet these requirements. Therefore, there are usually additional stresses of large magnitude due to the foundations performing the function of missing or inadequate members, as pointed out in the notes appended to the table.

FORCES ON TOWER FOUNDATIONS

Magnitude of force on each of the four foundations	Direction at each foundation			
	N. E. corner	S. E. corner	S. W. corner	N. W. corner
$\frac{W}{4}$	Down	Down	Down	Down
$\frac{cW'}{a}$	Down	Down	Up	Up
$\frac{F}{4}$ (Note 1)	East	East	East	East
$\frac{dF}{a}$	Down	Down	Up	Up
$\frac{P}{4}$ (Note 2)	South	South	South	South
$\frac{eP}{b}$	Up	Down	Down	Up
$\frac{afP}{4(a^2+b^2)}$ (Note 3)	North	North	South	South
$\frac{bfP}{4(a^2+b^2)}$ (Note 3)	West	East	East	West

NOTES. — Where there are no struts between the bottoms of the legs, and especially where the bottom panel is framed on the single system, both of which conditions are usual:

(1) The force of the wind F will give a greater force than $F/4$ in an easterly direction on the two west foundations and a correspondingly less force on the other two, and will in addition produce four new forces tending to force the legs apart on the compression side and draw them together on the tension side.

(2) Similarly the pull of the wire P will give a greater force than $P/4$ in a southerly direction on the two north foundations and a correspondingly less force on the other two, and will in addition produce a westerly force at the N. E. and S. W. corners and an easterly force at the S. E. and N. W. corners.

(3) Similarly, the torsional forces due to P will increase in magnitude and change in direction, and the unbalanced pull P may develop new forces tending to raise two diagonally-opposite legs and depress the other two.

Resultant Forces on Foundations. — From the above relations the resultant force on each of the four foundations may be found. In general there will be on each foundation: (1) a downward pressure, (2) a direct uplift, and (3) a horizontal overturning force, producing a tendency to slide and an uplift on one side of the foundation and a downward pressure on the other side.

Downward Pressure. — The downward pressure usually is of little importance in determining the size of the foundation as a foundation large enough for uplift and overturning is unnecessarily safe against downward pressure.

Direct Uplift. — The uplift is very important, as the weight of tower and foundation is rarely sufficient to provide more than a small fraction of the holding-down power required. The excess uplift is usually resisted by the earth in which the foundation is buried. Not only is the weight of the earth directly over the foundation effective but there is an additional resistance due to friction or cohesion of the earth which may be several times greater. These forces are usually computed on the assumption that they are equivalent to the weight of the earth in a frustum of an inverted cone or pyramid covering the founda-

tion and extending to the surface of the earth. The face of this cone is usually taken as making an angle of 30° with the vertical.

Horizontal Overturning Force, Sliding and Indirect Uplift. — The horizontal overturning force is also important. Its effect on the base may be resolved into two components, one a horizontal force tending to slide the foundation and the other a moment tending to rotate the base about a horizontal axis. The resistance of the earth to these forces is an obscure subject, especially if the foundation is of irregular shape. The following discussion which neglects several favorable elements may be considered as a conservative view of the earth resistance.

The resistance to sliding may be considered as due to the friction of the bottom of the base on the earth, and it may further be assumed that any base large enough to resist uplift will also furnish sufficient friction to prevent sliding. The arm of the overturning force is then the same as the height of the foundation (bottom of base to top where tower is attached), and the overturning moment is equal to the horizontal force multiplied by this arm. The resisting moment may be considered as due entirely to the vertical reaction of the earth on the top and bottom of the foundation. These vertical pressures may be taken as varying uniformly from zero at a horizontal neutral axis through the middle of the base, to a maximum at the edges of the base most remote from the axis. The moment of resistance is calculated as for a beam subject to bending and having a cross-section identical with the area of the base. It may be assumed that any unit pressure allowable on the uplift edge will be amply safe on the opposite edge where the pressure is downward, so that calculations for uplift only are necessary. The maximum allowable unit stress on the uplift edge may be taken as equal to the average unit resistance to uplift of the whole foundation, determined as described above under *Direct Uplift*.

Limiting Conditions. — Usually the most severe condition that a tower foundation is required to meet consists of a combination of uplift and overturning. For this condition the unit stress of uplift proper must be added to the maximum unit stress of uplift due to overturning and the sum must be within the allowable average unit resistance to uplift.

Strength of Foundations. — (*See also article on Strength and Elasticity.*) A foundation is subject to stresses from the tower tending to move it and from the resistance of the earth preventing motion. The foundation should of course be strong enough not to break when subject to these opposing forces. As the points of application of the resistance of the earth and the magnitude of the unit stresses transmitted by the earth at any point are subject to great uncertainty, the foundation should be designed for strength for the distribution of earth resistance which is most severe, considering for example that while the holding power is calculated on a uniformly-distributed earth resistance, it may be developed in practice by concentrated pressure from stones or timber located near the outer edge of the base.

Important Points Regarding Design of Foundations. — The following are important conclusions which follow from the above discussion:

(1) The inverted cone theory of resistance to uplift gives a calculated resistance which increases at a rapid rate with the depth (eventually increasing approximately as the cube of the depth). It would however be unsafe to apply the theory for foundations differing much from those of usual dimensions, say for depths much exceeding six feet and for foundations where the spread of base was much less than the depth to which it is buried.

(2) The foot of the tower (top of foundation) should be brought as close to the surface of the ground as possible to reduce the overturning moment.

(3) The tower diagonals of the bottom panel of the tower should intersect the corner posts as low as possible, because this is the actual point of application of the overturning force to the foundation and if this intersection is above the foundation this extra length must be added to the arm of the overturning moment.

(4) By inclining the axis of the foundation approximately in line with the inclined tower leg (i.e., to bring it as near as possible into line with the resultant of the horizontal overturning force and the vertical pressure or uplift) a more economical use of material may be made to resist the combined uplift and overturning.

TESTING OF TOWERS. — Since towers are generally indeterminate structures, new designs are usually tested. Test loads proportional to the loads specified for the design are applied until the required factor of safety has been proven or failure occurs. Towers are usually mounted on a rigid base for testing. This gives the strength which the tower would develop on an "ideal" foundation. In practice the tower must be expected to develop somewhat less strength, as unequal movement of the foundations will ordinarily overstrain certain members. Test loads may be applied by means of weights suspended directly at proper points for vertical loads and applied by means of pulleys attached to a tower-testing structure ("test tower") for horizontal loads. This method of application makes the determination of the test loads easy, but it is inconvenient to have the weights fall any considerable distance if the tower is tested to failure.

Testing of Foundations. — Foundations are occasionally tested, but the test is usually more for determining the holding power of the soil than the strength of the foundation. For the former purpose the test result will depend largely on the character of the soil and its condition (dry, wet or frozen). Tests for holding power are only necessary for uplift and overturning forces. In testing it is important that the testing machine should not press down on the surface of the soil near the foundation. The machine should rest on the ground outside of the base of an inverted cone of angle of 45° from the vertical and enveloping the base of the foundation under test.

SPECIFICATIONS, CONTRACTS AND PROPOSALS. — Different manufacturers of towers use different details of construction so that specifications written for the purpose of obtaining proposals should contain only conditions and requirements. These should state the loadings, factors of safety, maximum allowable stresses, and show outline dimensions of the tower as determined from clearances required.

The specifications should state whether towers are bolted or riveted, galvanized or painted, shipped assembled or partly assembled, tested or not tested, etc., besides containing the usual structural steel specifications; see *Steel.*

The proposals should show the arrangement and sizes of members and typical details of connections.

Contracts are oftentimes let on the basis of furnishing an approximate number of towers when the exact number required cannot be determined in advance, and a unit price per pound is included to cover extensions, special foundations and modifications that may be required.

The galvanizing and sherardizing of towers and parts are usually specified to pass the American Telephone and Telegraph Co's. Specification for galvanized iron wire (*see Galvanizing for Iron or Steel*). All members of towers that are not too heavy, so that there is danger of their buckling in the process, may be specified to be galvanized by the hot dip process. All bolts, nuts, threaded rods and turnbuckles may be specified to be sherardized.

INSTALLATION AND ERECTION. — Towers are generally shipped entirely disassembled, all of the members for one tower being bundled together. The tower is usually assembled lying on its face on the ground adjacent to the foundation. When assembled it is temporarily braced when necessary (usually by struts between the legs at the base of the tower) and up-ended onto the foundations. For the latter operation temporary hinges connecting the two legs lying on the ground to the two adjacent foundations are convenient. Towers too heavy to up-end and towers in inaccessible places are built up in place.

With bolted connections the nuts should be prevented from working loose by checking threads on bolts, riveting over the end of the bolt, or by lock-nut or washer.

Each tower member should have a number which should be shown on the erection drawings and marked on the corresponding member of each tower.

The members should be cut, bent and punched to template so that the parts will be interchangeable and the assembled tower will fit the foundation prepared for it.

Preparation of Foundations. — Foundations should be set with their bases below the frost line. The hole should not be excavated deeper than necessary, so that the foundation may rest on undisturbed earth. If any of the several foundations of a tower rest on loose backfill, unequal settlement may be expected which may greatly weaken the tower. The hole should not be larger than necessary, as the backfill will be less effective than undisturbed earth in resisting uplift. The resistance of concrete foundations can sometimes be increased by digging a hole smaller in diameter than the base and undercutting it at the bottom, while with structural steel foundations large stones can sometimes be placed against the steel to increase its resistance to motion, either vertical or lateral. The backfill should be well tamped in place and especial care should be used if it is probable that the foundation will be subject to heavy stress before the earth has had time to settle. On sloping ground filling may be required on the low side to give the designed weight of earth against uplift. In water or mud the floating power of hydrostatic pressure beneath the foundation must be allowed for. Where towers are raised on extensions the increased foundation stresses due to increased moment must not be overlooked.

The several foundations of a tower should be set accurately by template both as regards spacing and elevation so that towers stand trully vertical and have no initial stresses due to distortion.

MAINTENANCE OF TOWERS. — All bolted connections on towers should be carefully watched and kept tight, and the galvanizing or paint should be inspected regularly and towers must be repainted before any deterioration from rusting occurs. Foundations are the most likely source of trouble in operation. These should be kept properly backfilled and should be watched for unequal settlement of legs.

DIMENSIONS, WEIGHTS, COSTS. — Data on these items for a number of towers are given in the following table. Galvanized towers cost from 2½ to 4 cents per pound, f.o.b. factory. Galvanizing costs from ½ to 1 cent per pound; the cost of galvanizing is included in the figures just given.

DATA ON TYPICAL STEEL TOWERS

Use (a)	Conductors (b)		Number and diam. of ground wires, inches	Kilovolts between wires	Pin or suspension insulators	Height, feet		Base, feet		Weight, pounds
	Number	Size, cir. mils or B. & S.				To lowest cross arm	Over-all	Across line	Along line	
L (2)	6	300,000	1-½	110	*S*	55	79	20	20	6,800
A	6	300,000	1-½	110	*S*	50	75	24	24	10,500
L (3)	3	683,000 (*c*)	1-½	150	*S*	43	47	18	20	4,335
A	3	683,000 (*c*)	1-½	150	*S*	37	41	24	24	6,985
L (4)	3	0000	1-⅜	50	*P*	42	53	9.5	(*d*)	2,150 (*e*)
A	3	0000	1-⅜	50	*P*	42	53	9	9	3,750
L	3	0	2-⅜	102	*S*	43	45	14	13	1,800
L	6	000	1-⅜	32	*S*	51	76	17	17	4,560 (*f*)
L	6	000	1-⅜	100	*S*	51	77	17	17	....

(*a*) *L* = straight-line or suspension tower, *A* = anchor or angle tower; numbers in brackets refer to accompanying cuts, Figs. 2, 3 and 4 respectively. (*b*) All conductors stranded. (*c*) 605,000 circular mils of aluminum with a 78,000 circular-mil steel core. (*d*) Flexible tower. (*e*) Including foundation. (*f*) Average for 197 towers, including anchor towers and hardware.

DATA ON TYPICAL TOWER FOUNDATIONS

(Used in connection with the first four towers listed in above table, respectively)

Type	Fig. No.	Number per tower	Height, in.		Base, in.	Lb. of steel	Cu. yd. of concrete
			Over-all	Projection above earth			
Reënf. concrete.	5	4	78	6	60 by 60	500	4.4
Reënf. concrete.	..	4	96	6	96 by 96	1584	14.4
Steel...........	6	4	90	6	44 by 45	1285	......
Steel...........	..	4	88	6	52 by 52	1865	......

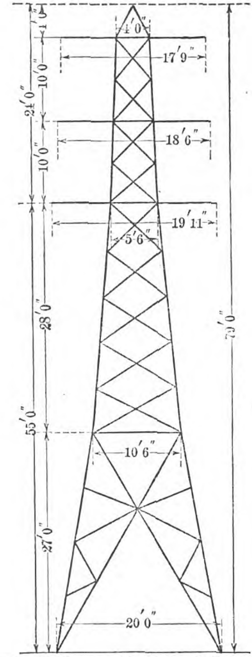

Fig. 2. Square Two-circuit Tower

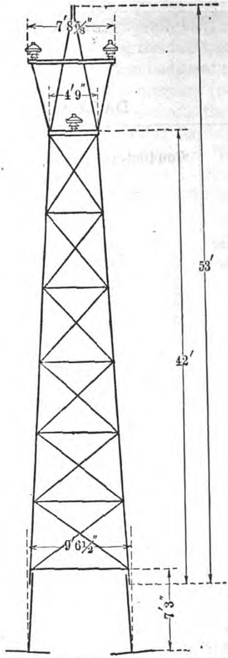

Fig. 4. Flexible Tower with Steel Foundations

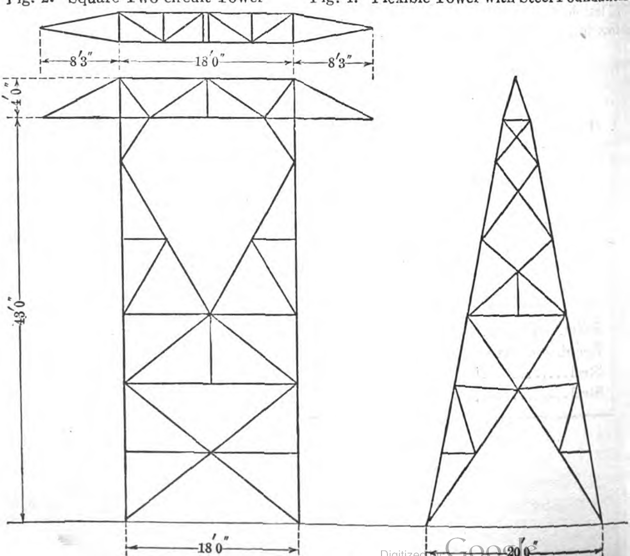

Fig. 3. Rectangular Single-circuit Tower

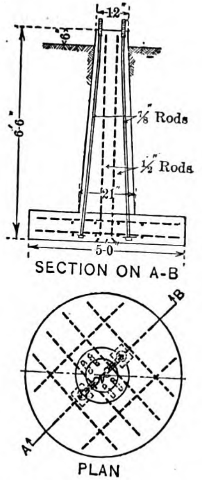

Fig. 5. Reënforced Concrete Foundation

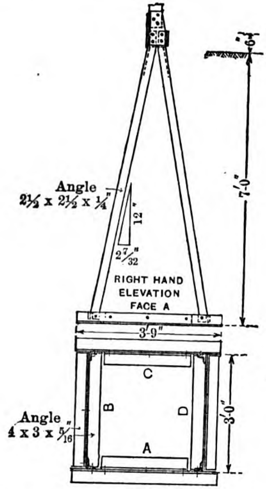

Fig. 6. Structural Steel Foundation

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[R. A. Philip and E. A. Ekern.]

TRANSFORMERS. — (*See also Alternating Currents; Auto-transformers; Electricity and Magnetism, Principles of; Standardization Rules of the A.I.E.E.; Transformers, Instrument.*) The following is a brief table of contents of this article:

General Description. — The electrical transformer, commonly called the static transformer, is a piece of stationary apparatus used to transform alternating-current energy at one voltage to some other higher or lower voltage. The single-phase transformer consists of two electrical circuits, usually of a large number of turns, interlinked with a common magnetic circuit of iron. Since the power is approximately the same in both windings, the currents in the two windings are inversely proportional to the voltages in the windings. The polyphase transformer is essentially two or more single-phase transformers made into a single piece of apparatus, but so designed that at least a part of the magnetic circuit is common to all the phases. A three-phase transformer has three high-tension and three low-tension windings arranged on a single iron core; see Figs. 1 and 2. The auto-transformer is treated in a separate article, viz., *Auto-Transformers.*

Terminology. — The winding by which the energy enters the transformer is logically the "primary" and the one by which the energy leaves the transformer is called the "secondary." Since either winding of the transformer may be connected to the source of energy, these terms are not definite unless the manner of connection is also stated. When referring to the transformer as a separate piece of apparatus the terms "high-tension" winding and "low-tension" winding are used to distinguish the two windings, the high-tension winding being the one with the greater number of turns. When the high-tension winding is connected to the source of supply it is the primary, and the transformer is said to be used as a "step-down" transformer; when the low-tension winding is connected to the source of supply, the high-tension winding is the secondary, and the transformer is said to be used as a "step-up" transformer.

CLASSIFICATION. — Transformers may be classified according to their operating characteristics, to their construction, or to the method of cooling.

Constant Potential and Constant-current Transformers.— Transformers may be either constant potential, such as are intended to give an approximately constant potential on the secondary side, or constant current, intended to give an approximately constant current on the secondary. Both types are intended to operate on a supply circuit of a constant potential.

Series Transformers are connected in series with the main circuit and receive a variable voltage and current in the primary. The secondary circuit is

closed through a path of low impedance and thus the secondary current will be proportional by the ratio of turns to the load current flowing in the primary or supply circuit. They are usually used to supply low-reading ammeters and wattmeters from circuits carrying very heavy currents.

Auto-Transformers or Compensators, sometimes called single-circuit transformers, consist of one electric circuit interlinked with the magnetic circuit and a tap brought off from some part of the winding. The voltage between this tap and either terminal of the electric circuit will be a fraction of the total voltage and thus a fractional voltage may be secured from this piece of apparatus. It is customary to proportion the windings on each side of the tap in accordance with the current to be carried. Auto-transformers are generally used where the ratio of voltages is quite near to unity as in this case they can be constructed with much less copper than the regular transformer. See article on *Auto-transformers*.

Potential Regulators are a form of transformer in which the voltage of one member may be varied from zero to a fixed maximum either by changing the direction of the magnetic flux or by changing the phase of the electromotive force of the secondary with respect to the electromotive force of the primary.

Core and Shell Types.—Two methods of arranging the electric and magnetic circuits are in use, the corresponding construction being designated as the "core type" or the "shell type." At present a large number of lighting transformers of small and medium capacity are constructed in a manner which is a composite of the core and shell types.

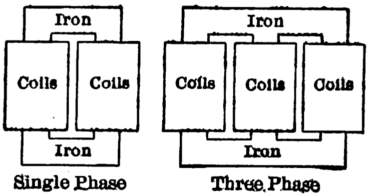


Fig. 1. Core-type Transformers

Core Type (Fig. 1).—The single-phase core-type transformer consists of a single magnetic circuit interlinked with two electric circuits, each consisting of a group of coils as shown in Fig. 1. The three-phase core-type transformer is also shown in Fig. 1.

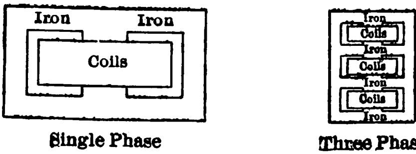


Fig. 2. Shell-type Transformers

Shell Type (Fig. 2).—In the shell type of transformer each electric circuit is interlinked with two magnetic circuits having a common path inside the coils but branching outside of the coils as shown in Fig. 2.

Comparison of Core and Shell Type.—The core type of construction is best adapted for high-voltage low-capacity transformers and the shell type for low-voltage high-capacity transformers. This arises from the fact that the most economical disposition of material in the core type demands a large number of turns and small cross-section of iron, while in the shell type a large cross-section of iron and small number of turns may be used to advantage. In the shell type the coils are usually wound in flat "pan-cakes," this type of construction being particularly well suited to the use of heavy copper rib-

bon or straps. In the core type the coils usually consist of two or more spools, long in comparison with their diameter. The use of the core type is increasing both in number and size of transformers as it is found possible to arrange the coils so they will better withstand the mechanical strain due to short circuits.

Classification According to Method of Cooling. — (*See also below.*) Transformers may be subdivided into classes in accordance with the method used for dissipating the heat due to their internal losses. As a transformer is a very compact piece of apparatus the problem of carrying away the heat is very important and various ingenious means have been devised for the purpose.

Naturally Cooled Transformers. — As this name implies this type of transformer has no special means of cooling but relies upon the ordinary circulation of the air. It is only used in transformers of very small sizes, such as those intended to supply meters.

Oil-cooled Transformers. — In this type of transformer the core and windings are submerged completely in oil in a tank and the windings and core are subdivided by ducts in order that the oil may circulate and carry off the heat from the internal parts. The heat is carried to the surface of the tank which contains the transformer and from there carried off by the surrounding air. The tank is specially designed to provide large air-cooling surfaces, e.g., provided with deep corrugations, or with projecting vanes or tubes.

Air-blast Transformers are designed with special passages through which a current of air is forced by means of a blower, the heat being carried off to the atmosphere in this manner.

Water-cooled Transformers consist of a construction similar to that of the oil-cooled type and in addition a coil of pipe carrying running water is submerged in the oil.

Forced-oil Transformers. — Transformers artificially cooled by circulation of oil are used when the size is too great for the self-cooling oil type and no cooling water is available. The oil circulates through external coils or tanks which give a greater cooling surface.

METHODS OF RATING. — The iron of transformers takes a long time to reach a constant temperature, that of oil-cooled transformers in particular requiring from 10 to 12 hours to reach a constant temperature. In the past, transformers have usually been rated in accordance with one of two following specifications; see, however, the *Standardization Rules of the A.I.E.E.* for the latest recommendations in regard to rating.

A-Rating. — The transformer will have a maximum temperature rise of 40° C. in continuous operation at normal load, and 55° C. rise at 25 per cent overload for two hours after reaching the maximum temperature at continuous normal load.

B-Rating. — The transformer will have a maximum temperature rise of 35° C. in continuous operation at normal load, and 55° C. rise at 50 per cent overload for two hours after reaching the maximum temperature at continuous normal load.

TRANSFORMER PRINCIPLES. — (*See also Electricity and Magnetism, Principles of.*) The essential features of a transformer consist of a primary winding having a number of turns interlinked with a magnetic circuit, and a secondary winding also interlinked with the same magnetic circuit, as shown in Fig. 3.

Simple Theory, Neglecting Leakage Reactance. — In the following discussion the assumption will first be made that all the flux links both primary and secondary windings; the effect of the leakage flux will be discussed later.

In Fig. 3, S_1 is the primary winding, M the magnetic circuit (of iron) and S_2 the secondary winding. If an alternating current is caused to flow in S_1 it will set up a flux in M which at any instant is proportional to the current i_1. Thus

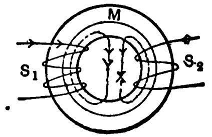


Fig. 3. Elementary Transformer

$$\phi = \frac{4\pi S_1 i_1}{10\,R},$$

where S_1 is the number of turns, i_1 the instantaneous value of the current and R the magnetic reluctance of the path in M.

Thus ϕ alternates with i_1 and since it interlinks with S_2 it will induce a voltage in S_2 at any instant equal to

$$e_2 = -\,S_2 \frac{d\phi}{dt}\,10^{-8} \quad \text{volts}$$

in which S_2 is the number of turns. The negative sign means that any current due to e_2 will tend to diminish ϕ.

No-load Conditions. — When there is no current in the secondary, the vector relations are as shown in Fig. 4. Let E_1 be the voltage impressed upon S_1, then the maximum value of the alternating flux will be

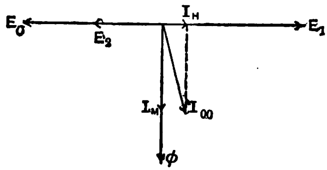


Fig. 4. No-load Relations

$$\phi = \frac{10^8 E_1}{4.44 f S_1},$$

where f is the frequency of alternation of E_1. Strictly, the numerator is $10^8(E_1 - r_1 I_{00})$, where I_{00} is the exciting current and r_1 the resistance of the primary, but the term $r_1 I_{00}$ is practically negligible. This flux will lag 90° behind E_1 or 90° ahead of the counter electromotive force E_c which it induces in S_1. The true magnetizing current I_m will be in phase with the flux and the hysteresis component of current I_H will be in phase with E_1. Hence the total no-load current I_{00} will assume some phase a little less than 90° behind E_1.

The flux ϕ will induce in the secondary turns S_2 an e.m.f. 90° behind the flux, hence 180° behind E_1 or in phase with the primary counter e.m.f. The effective value of this e.m.f. is

$$E_2 = 4.44 f S_2 \phi\, 10^{-8}.$$

Load on Secondary. — When the secondary is closed through a non-inductive external circuit a current will flow. This current will tend to demagnetize the iron, that is, to reduce the flux and hence the counter e.m.f. of the primary. This action, however, allows the current in the primary to increase until the secondary m.m.f. is balanced and there is left an excess m.m.f. in the primary to give sufficient flux to induce the counter e.m.f. E_1'. The vector relations are as shown in Fig. 5, again neglecting the leakage flux.

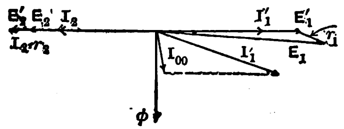


Fig. 5. Full Load, No Leakage

Leakage Reactance. — Due to the fact that a part of the lines of induction set up by the currents in the two windings pass through the air space (X in Fig. 3), the m.m.f.'s set up by the secondary current and the load current in

the primary (the current I_1' in Fig. 5) cannot neutralize each other, since the leakage fluxes established by the two currents are in the same direction, and not in opposition as are the fluxes in the iron. The primary leakage flux is in phase with the total primary current and the secondary leakage flux is in phase with the secondary current; these leakage fluxes are therefore not in phase with the useful flux (i.e., the flux which links both primary and secondary). The result is that the leakage fluxes cause a decrease in the secondary voltage and also a shifting of its phase with respect to the primary voltage.

Since the two leakage fluxes are in phase with the total currents in the primary and secondary respectively the voltages induced by the alternation of these fluxes are in quadrature with the currents. The quotient of the voltage induced in the primary by the alternating primary leakage flux divided by the primary current is called the "primary leakage reactance," and the quotient of the voltage induced in the secondary by the alternating secondary leakage flux divided by the secondary current is called the "secondary leakage reactance." These reactances are practically constants, since the major portion of the leakage path is in the air.

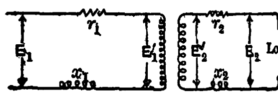


Fig. 6. Diagram of Circuits

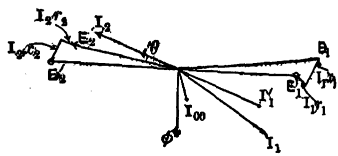


Fig. 7. Complete Vector Diagram

Complete Vector Diagram of Transformer.—Fig. 6 shows diagrammatically the primary and secondary windings of the transformer and Fig. 7 the vector diagram.

E_2 = the secondary terminal e.m.f.;
$\cos\theta$ = power factor of load;
I_2 = secondary current, lagging by angle θ behind E_2;
I_2r_2 = secondary resistance drop in volts; in phase with I_2;
I_2x_2 = secondary reactance drop in volts due to ½ of leakage flux, at 90° to I_2;
E_2' = induced e.m.f. in the secondary (hypothetical);
E_1' = e.m.f. necessary to overcome counter e.m.f. in primary; opposed and proportional to E_2' by ratio u;

$$u = \text{ratio of transformation} = \frac{S_1}{S_2};$$

I_1' = primary load current; opposed and proportional to I_2 by ratio $\frac{1}{u}$;
ϕ = mutual or useful flux;
I_{00} = primary no-load current;
I_1 = total or resultant primary current;
I_1r_1 = primary resistance drop; parallel to I_1;
I_1x_1 = primary reactance drop, at 90° to I_1;
E_1 = the required voltage on the primary, that is, the impressed e.m.f.

"Equivalent" Circuit of Transformer. — In practice it is not possible to measure separately the primary and secondary reactances. They must be measured together and treated as one quantity, called the total reactance x_t of the transformer, which may be expressed in terms of either the primary turns or the secondary turns.

A simple approximate method which is sufficiently accurate for all practical purposes is based on the equivalent circuits shown in Fig. 8. This method is

used in calculating the characteristics of a transformer from constants obtained from tests.

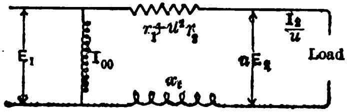


Fig. 8. Equivalent Circuit

Let E_2 = terminal e.m.f. of secondary;
$\cos\theta_2$ = power factor of load;
I_2 = secondary current;
u = ratio of transformation $= \dfrac{S_1}{S_2}$;
$E_2' = uE_2$;
$I_2' = \dfrac{I_2}{u}$;
R = total resistance in terms of primary $= r_1 + u^2 r_2$;
X = total reactance in terms of primary;
I_M = primary magnetizing current;
$I_H = \dfrac{\text{core-loss}}{uE_2}$, to a close approximation;
E_1 = primary impressed e.m.f.;
I_1 = total primary current;
$\cos\theta_1$ = power factor of transformer with load, i.e., power factor at primary terminals;
P_1 = watts input at primary terminals;
η = per cent efficiency.

Then

$$E_1 = \sqrt{(E_2'\cos\theta_2 + RI_1)^2 + (E_2'\sin\theta_2 + XI_1)^2};$$
$$I_1 = \sqrt{(I_2'\cos\theta_2 + I_H)^2 + (I_2'\sin\theta_2 + I_M)^2};$$
$$P_1 = E_2 I_2 \cos\theta_2 + R(I_2')^2 + E_2' I_H;$$
$$\cos\theta_1 = \frac{P_1}{E_1 I_1};$$
$$\eta = \frac{100\,P_2}{P_1};$$
$$P_2 = E_2 I_2 \cos\theta_2;$$
$$\text{Per cent regulation} = \frac{100\,(E_1 - uE_2)}{uE_2}.$$

Constant-current Transformers are used for supplying current to arc lamps connected in a series circuit which requires from 4 to 10 amperes and about 2000 volts. The principle of operation for this type of transformer is based on the existence of the leakage or useless flux which causes a repelling force between the primary and secondary windings. One winding is arranged so that it may move with respect to the other. As the current in the windings increases the repelling force increases and the windings move apart. This allows more flux to leak or pass between the windings and less to thread the secondary. Therefore the voltage induced in the secondary decreases as the current tends to increase. A counterweight is attached to the movable winding to regulate the amount of movement. The current in the primary remains fairly constant, but the power factor decreases with decreasing load on the secondary so the power in the primary is approximately proportional to the output of the secondary.

Series Transformers operate under conditions similar to those in a constant-potential transformer with short-circuited secondary. Under operating conditions the flux is small as the only voltage to be induced in the secondary is that required to overcome the impedance drop in that winding and the instrument which it supplies. The only counter e.m.f. in the primary is that due to impedance drop. The series transformer must be operated with the secondary

short circuited by a low impedance. If the secondary circuit is open there is no counter m.m.f. to balance the primary turns, which, being in series with the main line, carry a current irrespective of the secondary circuit. The primary ampere-turns would then set up a magnetic flux of considerable magnitude. As the transformer is not designed for this condition, the density would become very high in the magnetic circuit of small cross-section, and the transformer is liable to burn up from the heat due to core-loss.

TRANSFORMER CONNECTIONS. — The various transformer connections which are commonly used in lighting and power services are described below; see also section on *Operation* below.

Single-phase System with Three-wire Secondary (Fig. 9). — Standard practice in residence lighting with the a-c. system involves grounding the neutral wire on the low-tension side, the primary side not being grounded. Lamps or motors operating at 110 volts are connected between the neutral and either

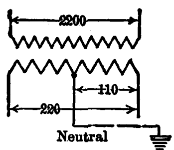


Fig. 9. Single-phase, Three-wire

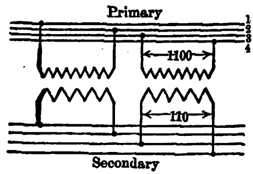


Fig. 10. Two-phase, Four-wire

side. The maximum potential between any secondary and ground is 110 volts but if either of the outside wires becomes grounded it constitutes a short-circuit on that half of the transformer.

Two-phase or Quarter-phase Four-wire System (Fig. 10).—The standard two-phase or quarter-phase system is essentially two independent single-phase systems which are usually independent electrically throughout. When the two phases are not electrically connected inside the generator, either wire of one phase may be connected to either wire of the other phase, without any flow of current resulting. In certain two-phase generators, however, the windings are interconnected, in which case any interconnection of the wires coming from the generator will cause a flow of current through this connection.

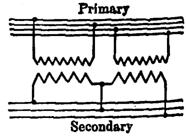


Fig. 11. Two-phase, Three-wire

Two-phase Three-wire System (Fig. 11). — This connection is occasionally used for the distribution of power in small systems. There is a possibility of a slight saving in copper, but the chances of unbalanced voltage and bad regulation, particularly with an inductive load, render it objectionable.

Three-phase Y and Δ Connections (Figs. 12 and 13). — Transformation in a three-phase system with either three independent single-phase transformers or with a three-phase transformer, having three primary coils and three secondary coils on one iron core, is accomplished by connecting the primary either in Y or in Δ and the secondary either in Y or in Δ. With Y or Δ connections there are the following relations between voltage per transformer winding and voltage between lines, and between current in transformer windings and current in lines. The power in the three transformers in any case is $3 \times 0.58 \times EI = 1.73\ EI$,

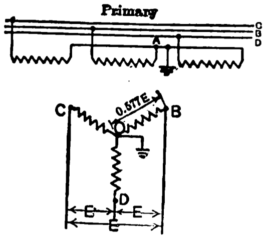


Fig. 12. Three-phase Y

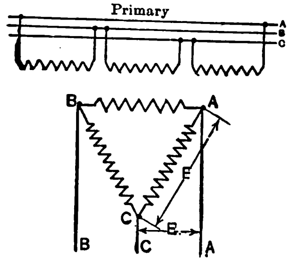


Fig. 13. Three-phase Delta

Connection	Volts bet. lines	Volts per winding	Current per line	Current per winding
Y	E	0.58 E	I	I
Δ	E	E	I	0.58 I

There are four combinations of these connections which may be used on any bank of transformers. These connections are given in the following table, and also the ratio of the voltage between lines for the low-tension and high-tension sides for a given ratio (u) of transformation in each individual transformer connection.

Connection		Low-potential volts bet. lines	High-potential volts bet. lines
Low pot.	High pot.		
Δ	Δ	E	uE
Y	Y	E	uE
Δ	Y	E	1.73 uE
Y	Δ	E	0.58 uE

Parallel Connection of Three-Phase Banks. — If there are several banks of transformers in the same system connected in parallel on one side then to connect the other sides in parallel the connections must be such that the voltage between any two lines on this side will have the same phase in all the banks. From this relation result the following rules:

With $\Delta\Delta$ on one bank, the other bank must be $\Delta\Delta$ or YY.
With YY on one bank, the other bank must be $\Delta\Delta$ or YY.
With ΔY on one bank, the other bank must be ΔY or $Y\Delta$.
With $Y\Delta$ on one bank, the other bank must be $Y\Delta$ or ΔY.

Even when these relations are satisfied a short-circuit will result unless the three phases of each bank are connected in the proper sequence. This can be

readily determined by the polarity test described below in the section on *Testing.*

Relative Advantages of Y and Δ Connections. — There has been much discussion as to the relative advantages of *Y*- and Δ-connected transformers for high-tension transmission and as to the value of a grounded neutral. The general and fundamental arguments may be summed up as follows:

With a ΔΔ-system the advantages are that if one transformer becomes disabled the system may be operated from the other two, operating on open delta (*see below*). If the load is unbalanced the voltages do not become unduly unbalanced. Resonance cannot occur. On the other hand, each transformer must be insulated for full line voltage as there is no neutral to ground, and if one line becomes grounded the voltage strain on the rest of the system becomes 1.73 times the normal strain, and this strain may extend to the low-tension winding and the generator which is connected to the transformers.

With a *YY*-connection there is a very unstable neutral and a possible excessive voltage strain on one phase, unless the neutral is grounded.

With a Δ*Y* arrangement for step-up transformers, the neutral on the high-tension side may be grounded. The voltage strain on any transformer is then limited to 58 per cent of the line voltage. There is a possibility of operating the remaining two transformers, if one becomes damaged, by using the neutral as a third conductor. However, there is the possibility of resonance under certain circumstances and the danger of causing disturbances in nearby telephone and telegraph circuits. Any accidental ground on the system makes a definite short-circuit on one phase.

The arrangement of *Y*Δ for step-up transformers is not desirable on account of the unstable neutral with unbalanced load, but is permissible with a balanced load or with a good connection from neutral of transformers to neutral of generator. This connection, however, is frequently used for step-down transformers particularly when connected to a balanced load.

Grounded Neutral vs. Ungrounded Neutral. — In any system without a grounded neutral there are numerous possibilities of disturbances resulting in a high voltage strain on the various parts of the insulation of the system. With the high voltages now in use for transmission systems these disturbances may give a great deal of trouble by breaking down the insulation, as it is not always possible to employ a large margin of safety and lightning arresters do not always protect from these disturbances. On the other hand, if the neutral is grounded, most of these disturbances will merely result in an excessive current, and if the circuit breakers are installed in the proper places they will open the circuit, so that the only adverse result will be a temporary interruption of service.

The choice is then between a system with ungrounded neutral and a large margin of safety in the insulation, and a system with grounded neutral, moderate insulation and the possibility of occasional interruption of service.

Effect of Higher Harmonics. — Many alternators generate an e.m.f. whose wave shape contains higher harmonics (*see Wave Analysis*) and the magnetizing current of transformers operating at high magnetic densities has a distorted wave shape which contains a prominent third harmonic. If these harmonics are present in a *Y*-connected machine they cause the voltage between each line and the neutral to vary so that there exists an unstable neutral, and the voltage strain on any one phase is indeterminate. If the neutrals of two pieces of apparatus in which these harmonics exist are grounded or joined together, a high-frequency current will flow in the neutral connection. If on the other hand these pieces of apparatus are Δ-connected a third harmonic current will circulate inside the apparatus. This may be measured by con-

necting an ammeter in the Δ between two phases. This current heats the apparatus, does no profitable work, and is therefore to be avoided. High frequency current in the line will cause high voltages to occur at certain points if a large amount of capacity is present, as is the case in any underground line, even of moderate length, and also in long overhead lines. All possible means both in the design of the generators and the connection of the apparatus should be taken to prevent the occurrence of, or to diminish the effect of, these higher harmonics.

Three-phase Open Δ or V Connection (Fig. 14). — This system consists in omitting one transformer from the delta connection, and is used to save expense, particularly in temporary installations or in new installations where the load is not great at first but is expected to increase in time. Thus the

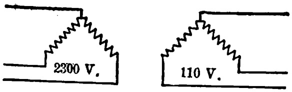


Fig. 14. Three-phase, Open Delta

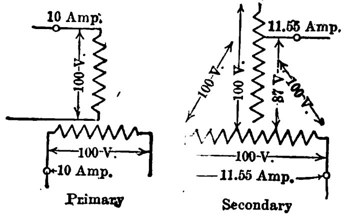


Fig. 15. Two-phase to Three-phase

purchase and installation of the third transformer is postponed until the load requires it. This connection can only be recommended for low voltages, such as 2300, as it is liable to produce dangerous potentials due to electrostatic unbalancing. The regulation and efficiency are also poor, as one phase of the load receives its power from two transformers in series. The aggregate capacity of the two transformers should be 15 per cent greater than the load.

Two-phase to Three-phase Transformation (Fig. 15). — The Scott or *T*-connection, the standard method of transforming from two-phase to three-phase, consists of two transformers which on the two-phase side may be connected in the normal two-phase manner either independently or interlinked. On the three-phase side one transformer has a tap at the middle point and the other a tap giving 87 per cent of full-transformer voltage. Fig. 15 shows the method of connecting and the currents in primary and secondary with balanced loads. Since the total power is $2 E_2 I_2 = \sqrt{3} E_3 I_3$, it follows that $I_3 = \frac{2}{\sqrt{3}} \frac{E_2}{E_3} I_2$, or the three-phase current is 16 per cent greater than it would be for straight single- or two-phase transformation. Thus the total transformer capacity must be 4 per cent greater than the load.

In commercial practice it is customary for the sake of interchangeability to make the capacity of both transformers 16 per cent greater than the load and to put both a 50 per cent and 87 per cent tap on both transformers. For this connection of transformers each half of the main transformer winding must be distributed over both legs of the core in order to prevent flux (and therefore voltage) unbalancing. If this precaution is taken both the primary and secondary voltages are balanced if the load is balanced. Any unbalancing of the secondary causes a like amount of unbalancing on the primary; thus if this connection is used to transform from 3-phase to 2-phase and all the load comes on one phase of the secondary it will also come on one phase of the primary.

Transformer Connection for Synchronous Converters.—For a two-phase converter the connections are simple as shown under *Converters*. For a three-phase converter there is a choice between Δ or *Y* secondaries, which is usually decided in deference to the conditions in the high-tension line. The delta connection is customary in railway substations. A diagram of the connections is given in the article on *Converters, Synchronous*. When the converter is to supply a three-wire d-c. lighting system, the "distributed" *Y* connection for the secondary should be used as shown in Fig. 16. By this arrangement the unbalanced current from the d-c. neutral is divided, and sent through the two halves of each transformer in opposite directions. Thus it does not increase the magnetic densities in the transformers, and avoids the excessive core-loss which would occur with a simple *Y* connection.

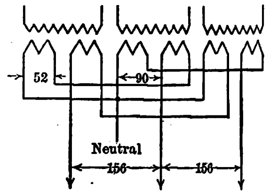


Fig. 16. Distributed Y for Three-wire Converter

For six-phase converters there is the choice between the "diametrical" and "double delta" connections as shown in Figs. 17 and 18. The diametrical is common with railway converters, or where the conditions are such that the delta is not needed to prevent an unstable neutral.

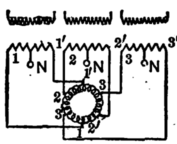


Fig. 17. Six-phase Diametrical

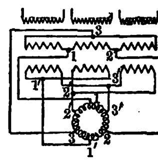


Fig. 18. Six-phase Delta

For regulating pole or split-pole converters certain special conditions must be borne in mind in choosing between the diametrical and the double-Δ connections. With the variation of the direct voltage of the converter the wave shape of the alternating e.m.f. is distorted and this introduces harmonics in either the counter e.m.f. or current waves, among which the third is quite prominent. If the delta connection is used a local harmonic current circulates in the converter armature and transformer secondaries, which increases the losses and heating and lowers the efficiency. If the diametrical connection is used the higher harmonic voltage gives an unstable neutral, and introduces extra potential strains in the high-tension windings and line if *Y*-connected. This may be obviated by grounding the neutral, but then a considerable current may flow in the neutral connection, which may interfere with neighboring circuits. The best solution is to use the diametrical connection in the secondary and ungrounded *Y* in primary, and use transformers having a good margin of safety in the insulation of the high-tension side.

THREE-PHASE TRANSFORMERS. — Considerable space, wiring and first cost may be saved by the use of a three-phase transformer in place of three single-phase transformers. This saving is warranted in large installations, but not in small installations where the convenience of having one interchangeable single-phase transformer as a spare for several banks of three transformers each is an important item.

Core of Three-Phase Transformer. — Three-phase transformers may be of the core type as shown in Fig. 1, in which case all the magnetic paths are of the same cross-section, or of the shell type shown in Fig. 2. In the latter type the horizontal cross pieces and outside vertical pieces may have one-half the cross-section of the central vertical core, providing the coil on the central leg is connected with a definite polarity with respect to the other two coils in order to have the fluxes differ in phase by 60 degrees.

Operation of Damaged Three-phase Transformer. — In a shell-type three-phase transformer if the primary and secondary are delta-connected it is possible to operate with only two phases in open delta at reduced capacity in case of trouble in the third phase of the system. This is accomplished by separating the third phase entirely from the system and short-circuiting both the primary and secondary windings.

DESIGN. — (*See also section below on Cooling of Transformers.*) The methods of design for single-phase and polyphase transformers are similar, the chief difference being that of the magnetic circuit. In the design of polyphase transformers each leg is treated as an independent single-phase transformer. The number of turns is adjusted to the voltage per phase and the cross-section of the conductors to the current per phase. All legs are wound alike and the phases are connected up as described in the preceding section.

The alternating-current transformer is the most efficient piece of electric apparatus. Its efficiency at full load is usually better than 95 per cent and frequently better than 98 per cent. For this reason it is very small in volume and light in weight for a given output compared to other pieces of electrical apparatus and hence the chief difficulty in design is the necessity of providing sufficient surface and a proper means to carry off the heat developed.

The difference in mechanical construction between the core and shell type is that the winding of the core type usually consists of two or more long spools (in which the length is considerably greater than the diameter) surrounding each leg of the transformer. The shell type consists usually of large flat "pan-cake" coils, laid alongside of each other on the central core with proper separating devices to permit ventilation. See Figs. 1 and 2.

The following discussion applies alike to both core and shell types except where specific mention to one or the other is made.

Preliminary Calculation of Main Dimensions. — **Let**

E_1 = primary voltage, effective value;
I_1 = primary full-load current, effective value;
E_2 = secondary voltage, effective value;
I_2 = secondary full-load current, effective value;
f = frequency in cycles per second;
S_1 = number of primary turns;
S_2 = number of secondary turns;
ϕ = maximum value of flux;
$C = \dfrac{\phi}{S_1 I_1}$ = ratio of total flux to full-load primary ampere-turns;
e = volts per turn, effective value;
B = maximum value of magnetic flux density in lines per square inch;
A = cross-section of core in square inches;
f_1 = space factor of winding space;
D = total section of copper in square inches;
U = current density in amperes per square inch;
σ = ampere-conductors per inch length of core.

Determination of Flux (ϕ) and Number of Primary Turns (S_1).—In order to estimate a desirable value of the flux to be used there is employed a factor C which is defined by the relation

$$C = \frac{\phi}{S_1 I_1}. \qquad (1)$$

The proper value of this factor C depends upon the type, capacity and voltage of the transformer, and the relative weights of copper and iron.* Usual values are given in the accompanying table.

VALUES OF C

Form of transformer	Voltage	Core type	Shell type
		Value of C	Value of C
Natural draft	0–6000	55–70	500–700
	6000 up	70–75	500–700
Oil-cooled	0–10,000	75–100	
	10,000 up	100–150	
Air-blast or water-cooled	0–10,000	110–160	600–1000
	10,000 up	160–240	600–1000

The relation between the effective value of the primary voltage and the maximum value of the flux is

$$E_1 = 4.44 f S_1 \phi \times 10^{-8}. \qquad (2)$$

Inserting in equations (1) and (2) the proper values of the primary voltage E_1, the frequency f, the constant C and the primary full-load current I_1, and solving the two equations for ϕ and S_1, reasonable values for the flux ϕ and the number of primary turns S_1 are obtained.

In the preliminary estimation of the full-load primary current I_1 it is sufficiently accurate to ignore the efficiency and power factor and to take the current as the watts output divided by primary voltage.

Volts per Turn (e).—The above value of ϕ should be checked by finding the volts per turn,

$$e = 4.44 f \phi \times 10^{-8},$$

and comparing it with values used in practice. In large transformers a greater voltage between turns is permissible than in small transformers. The accompanying table for core-type transformers is based on the assumption that double-cotton-covered wire is used for the windings. With special insulation higher values may be used.

Kilowatt rating of transformer	Allowable volts per turn for core type
10	2.5
20	3.5
50	5.5
100	7
200	9
500	13
1000	18

Number of Secondary Turns (S_2).—Having determined in a preliminary way the number of primary turns, the number of turns in the secondary is $S_2 = \frac{E_2}{E_1} S_1$. It will considerably simplify the mechanical arrangement of the coils if S_2 is divisible by 4. The nearest multiple of 4 is taken as the final value of S_2 and then S_1 and ϕ are readjusted to correspond to this value of S_2.

* A discussion of this factor C will be found in Arnold's *Transformers* and in S. P. Thompson's *Dynamo Electric Machinery*, Vol. II.

Cross-section of Core (A) **and Magnetic Flux Density** (B). — The core section is determined by the magnetic density which it is desirable to use, which in turn is determined by the core-loss. It is found that in average practice a loss of one watt per pound of iron can be dissipated without excessive rise in temperature. The corresponding flux densities in iron having a thickness of 14 mils are given in the following table. These values are approximate only, since the quality of the steel used is exceedingly variable. It should also be noted for a transformer intended for supplying a load of low power factor that the iron loss should be less than 1 watt per pound for the best distribution of material.

VALUES OF FLUX DENSITY

Size of transformer	Kind of steel	Lines per sq. in.	
		25 cycles	60 cycles
Small	Ordinary transformer sheet	50,000	40,000
Small	Special; silicon-steel	70,000	60,000
Large	Ordinary transformer sheet	75,000	65,000
Large	Special; silicon-steel	90,000	75,000

The proper cross-section of core in square inches is then

$$A = \frac{\phi}{B}.$$

In the core type this cross-section may be assumed as a square whose side is d. Since the core is made up of laminations whose effective length one way is 0.9 d, the value of d is

$$d = \sqrt{\frac{A}{0.9}}.$$

In the shell type this cross-section is a rectangle whose length is from 2 to 3 times its width.

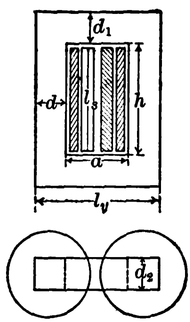


Fig. 19. Dimensions, Core Type

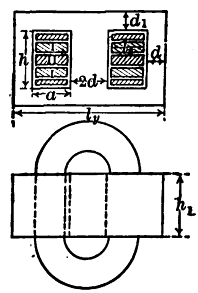


Fig. 20. Dimensions, Shell Type

Dimensions of Winding Space (Figs. 19 and 20). — The area or cross-section of the "window" or winding space depends upon the amount of copper,

insulation and ventilating space. The first can be determined accurately by the chosen current density in the copper. The second and third are allowed for by a space factor (f_1) which gives the ratio of the actual total cross-section of copper (D) to the cross-section of the window. Usual values of f_1 are given in the following table.

SPACE FACTOR IN TRANSFORMERS (f_1)

Size of transformer, Kw.	Core type		Shell type		
	Up to 10,000 volts	33,000 volts	Up to 2000 volts	2000 to 10,000 volts	33,000 volts
0–50	0.22	0.15	....	....	....
50–1000	0.33	0.25	0.40	0.33	0.18
Above 1000	0.38	0.33	0.45	0.36	0.21

In the core type there is one window and it contains $S_1I_1 + S_2I_2$ ampere conductors; in the shell type there are two windows and each contains $S_1I_1 + S_2I_2$ ampere conductors. The cross-section of copper in one window in either case is

$$D = \frac{S_1I_1 + S_2I_2}{U} = \frac{2\,S_1I_1}{U},$$

where U, the current density in amperes per square inch, has the following values:

Condition of transformer	U = amperes per sq. in.
Poorly cooled	850–1200
Ordinary oil cooled, air blast, etc.	1100–1600
Large, well cooled	1500–1900

Then the height h and width a of the window, see Figs. 19 and 20, must be such that

$$ha = \frac{D}{f_1}.$$

Height of Core. — In the core type this depends upon the ampere-turns or ampere-conductors per inch length of core, which in turn is related to the heating. The ampere-conductors per inch length of core is

$$\sigma = \frac{S_1I_1 + S_2I_2}{2\,h} = \frac{S_1I_1}{h},$$

where h is the height of core.

For a maximum rise in temperature of 50° C. σ will have values as given in the accompanying table. A reasonable value of h can then be found.

Form of transformer	σ = amp. conductors per inch of core
Natural cooled	500–750
Oil cooled or air blast	750–1250
Water cooled	1250–2000

In the shell-type transformer the practice is to make the height (h) from 1.5 to 3 times the width of window (a) and the width of window from 0.75 to 1.25 times the width of the central iron core, $2d$ in Fig. 20. See also paragraph above on *Cross-section of Core.*

Details of the Winding. —

$$\text{Cross-section of primary conductor in square inches} = \frac{I_1}{U}.$$

$$\text{Cross-section of secondary conductor in square inches} = \frac{I_2}{U}.$$

For minimum loss the copper density should be the same in both members, but to save space the copper density in the high-potential winding is sometimes greater than that in the low-potential winding.

The low-potential winding is usually placed between the high-potential winding and the iron.

Each coil is now laid out in detail, placing as many turns in a layer as the height and insulation will allow. The voltage per layer must not exceed 150 to 200 volts. A space of 10 mils is to be allowed for insulation between layers when insulated wire is used and 8 mils when insulated strip is used. The space between coils is from 0.04 to 0.30 inch, depending upon the voltage per coil. Space for the air or oil ducts is also provided. Each duct is from ⅛ to ⅜ inch wide and there should be one on each side of each coil. No part of the winding should be more than ⅜ inch from the surface of a duct.

Insulation of Windings.— The insulation of each turn usually consists of the cotton covering if the coils are wound with wire, or mica paper if the coils are wound with strip copper. This is proportioned to withstand the potential between turns as given above. The voltage between conductors on adjacent layers may be equal to twice the voltage per layer. To prevent breaking down between layers, a layer of Fuller board is used as a separator and this should project beyond the windings at the ends to prevent creepage. The maximum voltage between layers should be kept below 400 and to accomplish this it is customary to limit the voltage per coil to about 5000 volts. Between the windings and the core a layer of pressboard and sometimes wood is placed, while between primary and secondary windings a layer of pressboard and micanite may be used. For very high voltages the end turns of the high-tension winding for about 75 feet from the terminals is given a special insulation to withstand the sudden high potentials which occur when there is a sudden change in the potential applied to the transformer. These high potentials result from the distributed capacity of the transformer, between the high-tension winding and the core, frame and other winding.

Terminal Bushings. — At the point where the high potential leads pass through the case there is a very great dielectric stress which must be taken care of by the use of a proper kind of insulation and a proper disposition of the

insulation to prevent a concentration of the dielectric flux at a few points. For voltages below 40,000 a porcelain bushing is customarily used.

For higher voltages it is necessary to supply a large creepage distance and to have the surface submerged in oil to prevent corona effect. This is accomplished in one form of bushing, known as the "condenser type," by surrounding the terminal with layers of insulation and putting sheets of tinfoil between the layers. This arrangement is equivalent to a series of condensers. By properly proportioning the area of the successive layers of tinfoil the drop in potential across the insulation is kept uniform. The whole terminal is inclosed in an oil-filled casing. Another form of bushing known as the "oil-filled type" consists of a long cylinder of composition insulation which surrounds the lead and is filled with oil. The cylinder is divided into compartments to keep the oil properly distributed, and disks or collars project outward from the outside to increase the creepage distance.

End Coils of Shell Type. — With the flat coils customarily used in the shell-type transformer the subdivision of the windings is usually such that there is a half coil of the low-potential winding at each end of the winding space as in Fig. 20. This is to reduce the leakage flux. In order to further reduce the leakage flux all coils may be divided into halves with a ventilating duct between halves. The space between a primary and secondary coil is then reduced to that necessary for the insulation.

Adjustment of Core Dimensions. — After the final details of the windings are arranged the cross-section and length of core is finally settled. Sometimes the cross-section of core in the core type is made cruciform instead of square in order to use more effectively the area inside of circular coils.

PREDETERMINATION OF THE PERFORMANCE OF A TRANSFORMER. — From the above calculations a drawing to scale of the transformer may be laid out. The next step is to calculate its performance, i.e., to predetermine the values of the efficiency, regulation and temperature rise. This last feature is treated in the following section on the *Cooling of Transformers*.

Magnetizing Current (I_M). — The final flux density will probably differ a few per cent from the value assumed earlier in the calculation. The cross-section of core is usually proportioned to give the same magnetic density in all parts, as this condition gives minimum core-loss for a given weight of iron.

The mean length of path in the iron is measured or calculated. If H (found from the magnetization curve of the iron, see article on *Magnetic Properties of Iron*) is the ampere-turns per inch for the given density the magnetizing current of the transformer will be

$$I_M = \frac{H \times (\text{length path})}{\sqrt{2}\, S_1}.$$

Sometimes it is desired to allow for the minute air gaps at the joints of the punchings. Arnold finds that under practical conditions each joint represents a gap of 0.002 inch. Thus if there are n joints (usually 4) there should be added to I_M an amount

$$\frac{0.313 \times 0.002\, Bn}{\sqrt{2}\, S_1}.$$

Core-loss. — The core-loss consists of hysteresis and eddy losses in the steel punchings. These losses have been considerably reduced in recent years

by improvements in the manufacture and quality of the steel (*see article on Magnetic Properties of Iron*).

$$\text{Core-loss} = k_1 f V \left(\frac{B}{1000}\right)^{1.6} 10^{-6} + k_2 V \left(t f \frac{B}{1000}\right)^{2} 10^{-5} \text{ watts,}$$

where f = frequency in cycles per second;
V = volume of iron in cubic inches;
B = magnetic density in lines per square inch;
t = thickness of laminations in inches;
k_1 = a constant, ranging from 8 in ordinary transformer steel to 4 in silicon-steel;
k_2 = a constant, ranging from 4 in ordinary transformer steel to 1.5 in silicon-steel.

The core-loss may be calculated more easily by the method and curves given in the article on *Magnetic Properties of Iron*.

Copper Loss. — The mean length of turn of both primary and secondary windings is obtained from a sketch to scale. The direct-current or ohmic resistance of each member at 60° C. is obtained by substituting the proper values in the formula

$$R = \frac{0.0093\, lS}{12{,}000\, an},$$

where l = mean length of turn of primary or secondary respectively in inches,
S = number of turns in series,
a = cross-section of conductor in square inches,
n = number of conductors or coils in multiple.

To allow for eddy currents caused by the leakage flux, the above resistance is increased by 15 per cent to give the effective resistances r_1 and r_2. The total copper loss is then $I_1^2 r_1 + I_2^2 r_2$.

Transformer Reactance. — As explained in the section above on *Transformer Principles*, the leakage of flux between the primary and secondary windings of a transformer causes a component of voltage in each member which is out of phase with the current; if the current lags very much this voltage may have a considerable component opposed to the useful voltage and cause a loss of voltage or poor regulation. It is, therefore, necessary in designing to estimate the amount of this flux and calculate the voltage it would produce. In practice empirical formulæ are usually used, but a logically deduced formula is desirable as it is more easily adapted to unusual cases.

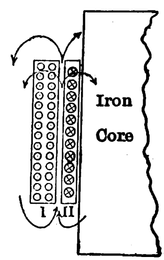


Fig. 21. Leakage Flux

The phase of the current in the secondary coil is nearly opposed to that in the primary and may be assumed to be exactly opposed without any great error. The result as shown in Fig. 21 is that all these ampere-conductors, both primary and secondary, tend to set up a flux in the same direction in the space between the windings and to a lesser degree in the windings themselves. A part of the flux in the intervening space interlinks with the primary turns and another part with the secondary turns. In addition the flux in the windings themselves interlinks with some of the turns. The flux in each part is proportional to the ampere turns producing it and to the permeance of the path. In this case since the path is in air, the permeance is the area divided by the length, where the length is the average length of the flux lines.

The leakage flux passes between the windings and one part closes its path in the iron inside the inner coil and the other part in air outside the outer coil (in the core type). The reluctance of the path between the two coils is large compared to that of the other two portions, because the inside path has a high permeability and the outside path has a large cross-section. The reluctance of the path between the coils is therefore accurately (and easily) calculated and a constant used to allow for the rest of the path.

In order to calculate the inductance it is necessary to have a cross-section of the windings showing their thickness, length and arrangement, as in Figs. 19 and 20.

Two cases must be considered:

(A) Where there are as many primary coils as secondary coils, and all coils are full size (usual core type).

(B) Where there is a "half" secondary coil at each extremity of the group of windings (the usual arrangement in the shell-type transformers).

Arnold's Method of Calculating Reactance. — Arnold calculates the inductance of a single primary coil and its secondary mate or mates, and multiplies this by the number of primary coils in series or divides by the number in multiple. This gives the total "short-circuit" inductance of the transformer in terms of the primary voltage or turns.

Referring to Figs. 19 and 20 for the core and the shell type respectively, let the various quantities be represented as follows, all dimensions being in inches:

q = number of primary coils in series,
p = number of primary coils in parallel,
s_1 = number of turns in one primary coil,
s_2 = number of turns in one whole secondary coil,
l_s = height of coils in cylinder type,
l_s = depth of coils in flat type,
t_1 = thickness of a whole primary coil,
t_2 = thickness of a whole secondary coil,
t = distance between coils,
U_1 and U_2 = mean length of primary and secondary turns respectively,
U_m = average of U_1 and U_2,
k = an empirical constant, varying from 0.95 with flat coils to 1.06 with cylinder coils.

Reactance of Core Type (Fig. 19). — In a core-type transformer with cylindrical coils and an equal number of full-sized primary and secondary coils, the permeance of the path of the flux in the duct which interlinks with a primary coil is $\frac{tU_1}{2\,l_s}$ and the permeance of the path in the winding itself is $\frac{t_1U_1}{3\,l_s}$.

The inductance of one primary coil is

$$L_1 = \frac{3.2\,ks_1^2U_1}{l_s}\left(\frac{t_1}{3}+\frac{t}{2}\right)10^{-8}.$$

The inductance of one secondary coil is

$$L_2 = \frac{3.2\,ks_2^2U_2}{l_s}\left(\frac{t_2}{3}+\frac{t}{2}\right)10^{-8}.$$

Reducing the latter to terms of the primary turns by multiplying by $\frac{s_1^2}{s_2^2}$, adding together and multiplying by $2\pi f$, the total reactance of the transformer in terms of the primary is

$$X = 2\pi f\,\frac{3.2\,qks_1^2U_m}{pl_s}\left(\frac{t_1+t_2}{3}+t\right)10^{-8}.$$

Reactance of Shell Type. — If, as in the shell-type transformer shown in Fig. 20, there is a half secondary coil at each end to give an increased intermixing, then

$$X = 2\pi f \frac{3.2\, qks_1^2 U_m}{2\, pl_s} \left(\frac{l_1 + l_2}{6} + t \right) 10^{-8}.$$

Efficiency and Regulation. — The efficiency and regulation may be calculated as described above under *"Equivalent" Circuit of Transformer*, p. 1610, or by the following method, which is sufficiently accurate for most practical purposes.

Let

E_2 = secondary terminal voltage,
I_2 = secondary current,
$\cos\theta_2$ = power factor of load on secondary,
$P_2 = E_2 I_2 \cos\theta_2$ = secondary output in watts,
A = core-loss in watts = input for no load on secondary, approximately,
u = ratio of number of primary to number of secondary turns,
$R_2 = \dfrac{r_1}{u^2} + r_2$ = total resistance in terms of secondary, r_1 and r_2 being the actual resistances of primary and secondary respectively,
$X_2 = \dfrac{X}{u^2}$ = total reactance in terms of secondary, where X is the total reactance in terms of primary.

Then the per cent efficiency is

$$\eta = \frac{100\, P_2}{P_2 + A + R_2 I_2^2}.$$

Secondary voltage at no load is

$$E_{20} = \sqrt{(E_2 \cos\theta_2 + R_2 I_2)^2 + (E_2 \sin\theta_2 + X_2 I_2)^2}$$

and the regulation is then

$$\frac{100\,(E_{20} - E_2)}{E_2} \text{ per cent.}$$

All-day Efficiency. — The all-day efficiency is the ratio that would exist between the readings of a watt-hour meter connected on the secondary and a similar meter on the primary. It is of importance because a great many transformers, particularly for lighting, operate at full load for only a few hours each day, but the core-loss or iron losses are just as great when the load is very light or when there is no load at all. Hence these transformers may waste a great deal of energy, although their efficiency at full load is very good.

To calculate the all-day efficiency multiply each quantity in the usual formula for efficiency by the number of hours per day that this factor occurs; thus

$$\text{All-day eff.} = \frac{100\, P_2 \times h}{P_2 \times h + h_1 A + h R_2 I_2^2} \text{ per cent,}$$

where h = hours per day of secondary load; h_1 = hours per day that transformer is on line, and the other symbols as above. Since the core-loss occurs 24 hours a day and P_2 only 2 or 3 hours, the importance of making the core-loss low in a transformer for this class of service is apparent.

However, there is another side to the question, since energy may not be worth as much during the day when there is little demand on the central station, as in the evening when the demand is great.

EXAMPLES OF DESIGN. — In the following table are given the chief design characteristics of four different transformers.

EXAMPLES OF SINGLE-PHASE TRANSFORMER DESIGN

American or foreign	Foreign	Foreign	American	American
Form (cooling)	Oil	Air	Oil	Air
Type	Core	Shell	Distributed	Shell
Frequency	50	50	60	25
Kv-a. rating	40	100	20	75
High-tension voltage	3120	2200	2200	2500
Low-tension voltage	230	110	220	320
C (design constant)	56	670	222	1550
High-tension Winding:				
Current at rating, amperes	12.8	45.5	9.1	30
Total turns in series	1408	180	640	219
Coils in series	8	4	2	3
Coils in multiple	1	1	1	1
Size of conductor, inches	$d = 0.183$	0.79×0.059	0.105×0.09	0.34×0.08
Resistance at 25° C., ohms	1.13	0.24	1.43	0.492
Low-tension Winding:				
Total turns in series	104	9	64	28
Coils in series	4	1	4	4
Coils in multiple	1	3	1	1
Size of conductor, inches	0.59×0.13	0.87×0.1	2, each 0.23×0.155	6, each 0.34×0.09
Resistance at 25° C., ohms	0.009	0.00059	0.0204	0.0095
Flux density in core, kilolines per square inch	38.6	52	68	67.5
Core dimens., inches, length	(26 sq. in.)	5.1	4.6	8
Core dimens., inches, width	......	22.8	4.6	21
Window dimens., inches, height	35.5	5.9	9.2	5.5
Window dimens., inches, width	5.45	9.9	3.1	8
Magnetizing current, amperes	0.21	2.12	0.432	1.82
Core-loss, watts	490	1320	164	1520
No-load current, per cent	2.04	4.85	4.8	6.9
Core-loss, per cent	1.22	1.30	0.80	1.96
Primary, RI^2, per cent	0.50	0.52	0.59	0.57
Secondary, RI^2, per cent	0.68	0.48	0.81	0.67
Efficiency, per cent	97.6	97.7	97.8	96.8
Total XI drop, per cent	1	2	2.84	1.36

COOLING OF TRANSFORMERS. — It is necessary to keep the temperature of the various parts of a transformer within such limits that the materials of which it is constructed do not become damaged and deteriorate too rapidly. When subjected to too high a temperature the insulating materials disintegrate and lose their mechanical and dielectric strength, and the iron deteriorates in its magnetic qualities so that the core-loss becomes greater for a given density. This so-called ageing of the iron may cause an increase of as much as 50 per cent in the loss in the iron. The effect varies with the character of the iron and is practically negligible in silicon-steel.

Maximum Rise in Temperature.—The maximum rise in temperature above a room temperature of 25° C. at which the various materials of a transformer should be operated are given in the table.

Material	°C
Iron..................	70–75
Cotton................	60
Paper.................	70
Mica, asbestos.........	90

To guard against this possibility of damage the Standardization Rules of the A.I.E.E. recommend that the windings shall not increase in temperature more than 50° C., as measured by resistance, above the surrounding air, and the other parts shall not increase more than 50° C. as measured by thermometer.

Means of Dissipating Heat. — The problem is to provide a path of low thermal resistance by which the heat energy may pass to the surrounding air. To accomplish this it is necessary, first, to provide sufficient surface in the subdivided transformer to transfer the heat to the cooling agent, air or oil, without too great a difference in temperature; second, to so subdivide the transformer that no part of the iron is more than one inch, and no part of the copper more than ⅜ inch, from a cooling surface; third, to provide a sufficient quantity of the cooling agent, air, oil or water to carry away the heat at the same rate as it is generated; and fourth, to provide sufficient surface on the containing case or tank to transfer the heat from the internal oil to the external air without too great a difference in temperature.

Calculation of Exposed Surface of Transformer.— The practical method of estimating the rise in temperature consists in calculating the drop in temperature in the successive media by finding the watts to be dissipated per square inch of each surface. The first step is to calculate the exposed surface.

The total exposed surface (A_t) of the transformer proper, that is, the surface in contact with the cooling medium, is to be calculated. For core-type transformer, dimensions in inches and symbols as in Fig. 19:

$$A_t = (\text{number of cores}) \times [(\text{outer perimeter of windings})\, l_s + 2\,(\text{end surface of windings}) + h\,(\text{perimeter of core}) + (\text{number of ducts in core}) \times (\text{surface of one duct})] + (2 + \text{number of ducts in yoke}) \times l_y d_1 + 2\,(l_y d_2 + 2 d_1 d_2).$$

For shell-type transformer, dimensions in inches and symbols as in Fig. 20:

$$A_t = d\,[2\,(2\,a + 4\,d) + 2\,h]\,(2 + \text{number of ducts}) + h_1\,[2\,(2\,a + 6\,d) + 2\,\text{h}] + [2\,l_s h_1 + \pi\,(2d + a)\,l_s]\,(2 + \text{number of ducts in windings}).$$

In the preceding formulæ only half the duct surface is used, as with narrow ducts these surfaces are only half as effective as outside surfaces. If the ducts have a width of one inch then the whole surface may be used.

Naturally-cooled Transformers should have from 4.75 to 5.3 square inches of surface (A_t) for each watt loss for 50° C. rise in temperature.

Oil-cooled Transformers. — (*See also Oil, Transformer.*) The transformer is submerged in oil in a tank so that the level of the oil is from 2 to 3 inches above the top of the transformer. The quantity of oil is from 6 to 10 pounds per kv-a. rating, or in small sizes 500 pounds per kilowatt loss. There should be from 1.5 to 2.3 square inches of transformer surface (A_t) per watt loss. The oil ducts should be ¼ inch wide and run vertically. If the tank is smooth there should be from 4 to 8 square inches of tank surface (not including top or bottom) for each watt loss. For sizes greater than 25 kilowatts it is

customary to use fluted or corrugated sides to the tank. In this case there should be from 6 to 10 square inches of radiating surface per watt loss because the air does not circulate as rapidly in the grooves as over a smooth surface and consequently radiation is poor.

For a rise of 50° C. of the transformer there is an average rise of 30° of the oil. The maximum rise of the oil is 1.3 to 1.5 times the average.

The rise in temperature of the windings by resistance or of the iron by thermometer is

$$T = \frac{2\,W}{A_t} + \frac{1.4\,tW}{S},$$

where W = total watts lost;

A_t = radiating surface of transformer in square inches;

S = radiating surface of tank in square inches;

t = from 160 to 200 with smooth tanks

= 200 to 270 with corrugated tanks.

Air-blast Transformers. — This type of transformer is usually set over a large air duct in the floor which is supplied by a blower with air at a pressure of from ½ to 1 ounce (¾ to 1½ inch of water). The air enters the transformer at the bottom and is divided into two streams, one passing vertically through the windings and the other transversely through the iron. There is a damper or valve for each stream so that the proper amount of air is provided to each part independently. Ducts ½ inch wide are provided every 3 to 4 inches. For 50° rise the cooling surface of the transformer (A_t) should be from 1.5 to 2.3 square inches per watt loss. A liberal amount of air for 50° C. rise is 150 cubic feet per minute per kilowatt loss. The air is expected to rise from 15° to 20° C. in its passage through the transformer. The theoretical quantity of air in cubic feet per minute is

$$Q = \frac{1.65\,W}{T_a},$$

where T_a is the rise in temperature of the air, which is usually about half as great as the rise of the transformer.

Water-cooled Transformers. — The cooling coils are suspended in the tank in the oil near the top, usually above the transformer, and carry a continually circulating stream of water. It is customary to provide ⅙ gallon of water per minute for each kilowatt of loss and to allow the outgoing water to rise 25° C. above the incoming. Other requirements are from 1 to 1.5 square inches of surface (A_t) per watt loss for 50° C. rise and 1 square inch surface of water coils per watt loss.

TESTING SINGLE-PHASE TRANSFORMERS. — The customary commercial tests on transformers and the best order of making them are: cold resistance, polarity, ratio and checking of taps, impedance, core-loss and exciting current, parallel run, heat run, insulation tests. The efficiency and regulation are calculated from the results of these tests.

Oil-cooled transformers should never be tested or subjected to potential unless they have been filled with oil from which all moisture has been removed.

Examples of test results are given below.

Cold-resistance Measurement. — The cold resistance must be very carefully made as it is used as a basis of calculating the temperature after the heat run. After the transformer has been standing in one place long enough for all of its parts to have reached the same temperature as the surrounding air, a direct current of from 10 to 15 per cent of the rated current of the coils is sent through the windings and the drop measured with a voltmeter. At the same time the temperature of the windings is measured by a thermometer.

Test of Polarity. — This test gives information necessary for connecting several transformers in a bank and have them operate in parallel or on polyphase circuits. Direct current is sent through one winding of a transformer and a d-c. voltmeter connected across the other winding. If the current is stopped, the voltmeter will give a deflection either positive or negative. For similar connection and direction of current on all the transformers of a bank the deflection should be of the same sign. A small current should be used as otherwise the throw of the voltmeter needle may be sufficient to bend it.

Ratio of Turns. — With no load on a transformer the ratio of voltages is the same as the ratio of the turns. Thus, with a known voltage applied to the low-tension winding, the ratio of turns of the two main windings and of the sections between taps can be checked up by connecting across the other terminals another voltmeter of proper range (using a potential transformer if necessary).

Impedance Test. — This test is important in order to calculate the regulation and in order to determine whether transformers will run in parallel with each other. One winding of a transformer is short circuited and a voltmeter, ammeter and wattmeter are connected in the circuit of the other winding. A voltage of from 1 to 8 per cent of the rated voltage of this winding and of the proper frequency is impressed. The voltage is regulated so that readings are taken at values of current from 50 to 125 per cent of the rated current of the winding. The wattmeter reading will be in the neighborhood of 2 per cent of the rating of the transformer. The total impedance of the transformer will be $Z = \frac{E}{I}$. The total effective resistance will be R = Watts/I^2, which will include the effect of eddy currents, and the total reactance will be $X = \sqrt{Z^2 - R^2}$. This reactance cannot be separated into primary and secondary reactance. The results of this test are usually plotted in two curves, one between volts and amperes and the other between volts and watts.

Core-loss and Exciting Current Test. — The alternator supplying the power for this test should give a sinusoidal e.m.f. wave, as any distortion in the shape may cause a variation of from 5 to 10 per cent in the core-loss. A peaked wave gives a lower core-loss than a sine wave. For this test the high-tension winding is left open and rated voltage at the proper frequency is impressed on the low-tension winding. A voltmeter, ammeter and wattmeter are connected in the low-tension circuit, the voltmeter having a range including the rated voltage of the transformer and the ammeter a range of approximately 15 per cent of the rated current of the machine. Readings are taken with a voltage of from 50 per cent to 125 per cent of the rated voltage of the transformer. Both ampere-volt and watt-volt curves are then plotted. If extreme accuracy is desired, the RI^2 loss should be subtracted to give true core-loss.

Separation of Eddy and Hysteresis Losses. — In certain special cases it is desired to investigate the iron of a transformer by separating the hysteresis from the eddy loss. Let

W_1 = the core-loss at normal voltage and frequency.
W_2 = the core-loss at half voltage and half frequency.
W_e = eddy-current loss at normal voltage and frequency.
W_h = hysteresis loss at normal voltage and frequency.

Then

$$W_e = 2\,W_1 - 4\,W_2,$$
$$W_h = 4\,W_2 - W_1.$$

Parallel Run Test. — The tests for polarity and ratio having shown nothing wrong in the transformers, they are connected two at a time, the low-tension windings being connected in parallel to a generator and the high-tension windings in parallel with each other with an ammeter of about 10 per cent the capacity of the transformer connected in one lead between them. The voltage of the alternator is gradually increased from zero and the current in the ammeter noted. This current should not be greater than 5 per cent of the rated current of that winding at rated voltage. If a transformer has double windings in either member the same test should be made on these windings in parallel.

Heat Run. — The heat run is made at the rated load of the transformer and sometimes at an overload of 25 per cent or 50 per cent, depending upon the guarantees. The run may be made by connecting the transformer to a load such as a water rheostat (*q.v.*) but as there are other equally good methods, avoiding the waste of so much energy, this dead load is seldom used. The other methods require two or three transformers to be tested simultaneously.

Two Transformers "Bucking" (Fig. 22). — The low-tension windings of two transformers are connected in multiple to a voltage of rated value and frequency. The high-tension windings are connected up so that their e.m.f.'s oppose or "buck" each other, and are in series with an adjustable source of e.m.f., of rated frequency having a value of from 2 to 5 per cent of the rated voltage of the windings. This "loss-supply" may be either an alternator and transformer, or a potential regulator. The voltage of this source is adjusted so that full-load current circulates in the high-tension windings and thereby induces full-load current in the low-tension windings. It should be realized that although only 2 to 5 per cent of the rated voltage is needed to send the current through the primaries, yet each primary is generating its rated voltage and if there should be a ground anywhere a dangerous potential strain or shock might result.

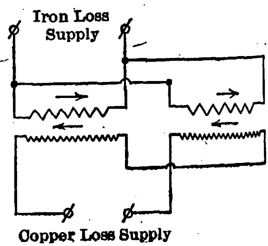


Fig. 22. Connections for Heat Run by Bucking

Three-transformer Arrangement (Fig. 23). — Three single-phase transformers may be connected with their low-tension windings in delta to a three-phase source of supply of proper voltage and with the high-tension windings also connected in delta with one corner open into which the "loss supply" is connected preferably by means of an auxiliary transformer to isolate the dangerous potential of the primaries. The "loss-supply" voltage must be adjusted so that the desired current flows in the primary winding.

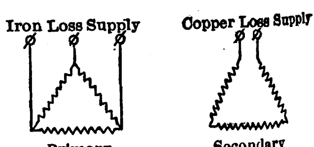


Fig. 23. Heat Run by Open-delta Method

Time for Heat Run. — The time required for a heat run may be considerably shortened by operating at an overload for a short while, or, if the transformer is an air-blast transformer, by operating without the blast until a reasonably high temperature has been attained. Then the proper load is adjusted and the run continued until all temperatures remain practically constant, that is, do not rise more than 1° C. in two hours. The voltage should be cut off once every hour and a resistance measurement quickly made to determine the temperature of the windings.

Temperature Rise by Resistance is calculated from the formula

$$T = (234 + t)\left(\frac{R_h}{R_c} - 1\right),$$

where T = temperature rise in ° C.;
t = temperature in ° C., by thermometer, at which the cold resistance is measured;
R_c = resistance in ohms at temperature t;
R_h = "hot" resistance in ohms at end of heat run.

The number 234 is the reciprocal of the resistivity temperature coefficient (referred to 0° C.), of 100 per cent conductivity copper, i.e., the reciprocal of 0.00427. For any other conductivity, C per cent say, divide 234 by $C/100$. *(See article on Resistance and Conductance.)*

Temperature Rise by Thermometer. — In an air-blast transformer it is desirable to measure by thermometer the temperature of the incoming air, of the outgoing air (from iron and coils), of the primary windings, and of the secondary windings. Spirit thermometers and not mercury thermometers should always be used for measuring the temperature of transformer windings. In an oil-cooled transformer the temperature, by the thermometer, of the tank at top and bottom and of the oil in two or three places near the top should be taken.

Insulation Tests. — To test the insulation between turns and sections of coils double the rated voltage per section is impressed on each section for one minute. This test should be made at a high frequency, preferably at double the rated frequency. After this one and a half times normal voltage at rated frequency is applied for five minutes, to discover the effects of the double voltage test. These voltages should be applied and removed gradually.

High-potential Test on Complete Transformer. — To test the dielectric strength of the insulation as a whole the following high-potential test is made. Connect both terminals of the high-tension winding to one terminal of the high-potential transformer. Ground both ends of the secondary winding to the core and frame and connect to the other terminal of the high-potential transformer. Adjust a needle gap to arc at the desired test voltage and increase the voltage gradually until the gap arcs over. Decrease the voltage till the arcing ceases and hold the voltage as near as possible to the arcing point for one minute. The voltage should then be decreased gradually. The proper testing voltages and spark-gap adjustments are given in the Standardization Rules of the A.I.E.E. (q.v.).

Calculation of Efficiency and Regulation from Test. — From the results of the preceding tests the efficiency and regulation of the transformer at various loads may be calculated by the methods given above, p. 1625 in the paragraph on *Efficiency and Regulation,* using the test data instead of the calculated quantities.

TESTING OF THREE-PHASE TRANSFORMERS. — In testing three-phase transformers the same methods are followed as with single-phase units. The only difference is in testing for polarity; owing to the mixing of the magnetic circuits special care must be exercised. The direct current must be sent in one direction through one primary phase and in the opposite direction through the other two, so that they will not neutralize one another. With a voltmeter similarly connected on the primary and secondary of each phase in turn, break the direct current; if the connections are right the voltmeter on the secondary will deflect in the same direction as the steady deflection on the primary.

Parallel Run. — For the parallel run of two three-phase transformers, connect their low-potential sides in multiple to a source of three-phase potential. Connect together the primary terminals No. 1 of both transformers and bring the pairs of terminals No. 2 close together so they may be connected by a small fuse wire. If no spark is noticeable when the fuse wire spans the connection, then the No. 2 terminals may be permanently connected. The same procedure is followed with the No. 3 terminals.

EXAMPLES OF PERFORMANCE. — Usual values of the performance characteristics of transformers are given below.

Exciting Current ranges from 2 to 6 per cent of full-load current in lighting transformers of the core type and from 5 to 10 per cent in power transformers.

Core-loss ranges from 0.5 per cent to 1.25 per cent of the rated output.

Total I^2R ranges from 0.75 per cent in large sizes to 2 per cent in small sizes of the rated output.

Total Reactance drop ranges from 1.25 to 5 per cent of the voltage, being less for the shell type than for the core type. The value depends largely on the purpose of the transformer, methods of construction and opinion of the designer.

Efficiency and Regulation. — The variation of the efficiency with the load of two small 60-cycle transformers for lighting purposes is shown in Fig. 24. The

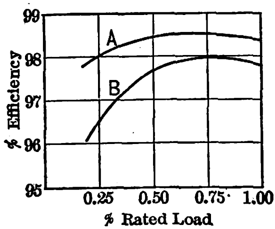


Fig. 24. Efficiency of 60-Cycle Transformers. A = 50 Kv-a.; B = 10 Kv-a.

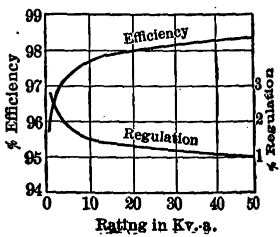


Fig. 25. Efficiency of a Line of 60-Cycle Transformers

efficiency and regulation at full load of a line of these transformers is shown in Fig. 25. These transformers are small and of the core type. Larger transformers would have even better characteristics.

See also above under *Examples of Design*.

SPECIFICATION FOR TRANSFORMERS.* — The following memoranda are intended to assist in writing specifications. See also article on *Specifications*.

Principal Characteristics and Conditions of Service. — Service for which transformer is to be used, e. g., operating synchronous converters, induction motors, lights, etc. High- and low-tension voltages at normal load. Taps for obtaining different voltages. Rating in kilovolt-amperes and in kilowatts at stated power factor. Frequency.

Style and Description: Details of Construction. — Whether oil-, air- or water-cooled. Style and location of terminals. Where line surges are likely to occur, it is usual to specify, for large transformers, that the end turns, say 10 per cent of the total turns at each end, shall have extra heavy insulation.

Work to be Done by Other Contractors. — Who is to supply and install floor framing and supports; high- and low-tension wiring, wiring and supports for delta- or star-connections, if to be used three-phase.

* By W. A. Del Mar.

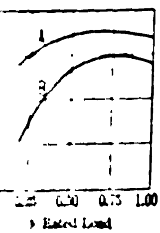
...el Run. — For the parallel run of two three-phase transf... potential sides in multiple to a source of three-phase... the primary terminals No. 1 of both transf... terminals No. 2 close together so they may be connected... If no spark is noticeable when the fuse wire spans the... terminals may be permanently connected. The sam... with the No. 3 terminals.

...MPLES OF PERFORMANCE. — Usual values of the... transformers are given below.

...several ranges from 2 to 6 per cent of full-load current... of the core type and from 5 to 10 per cent in power... ranges from 0.5 per cent to 1.25 per cent of the rated... ranges from 0.75 per cent in large sizes to 2 per cent...

...drop ranges from 1.25 to 5 per cent of the voltage... type than for the core type. The value depends la... the transformer, methods of construction and opinion of the...

...and Regulation. — The variation of the efficiency with... transformers for lighting purposes is shown in Fig...



...of 60 Cycle Transform-... kv-a.; B = 10 kv-a.

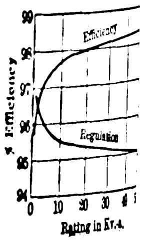


Fig. 25. Efficiency of a Line of Transformers

...regulation at full load of a line of these transformers... These transformers are small and of the core type. Lar... have even better characteristics.

...see under *Examples of Design*.

...ICATION FOR TRANSFORMERS.* — The follo... intended to assist in writing specifications. See also...

Characteristics and Conditions of Service. — Servi... to be used, e. g., operating synchronous converters... etc. High- and low-tension voltages at normal load... voltages. Rating in kilovolt-amperes and in... Frequency.

Description: Details of Construction. — Whether... style and location of terminals. Where line surges... usual to specify, for large transformers, that the... the total turns at each end, shall have extra heavy insul...

...be Done by Other Contractors. — Who is to suppl... and supports; high- and low-tension wiring, wiring... connections, if to be used three-phase.

* By W. A. Del Mar.

Performance and Tests. — (*See Standardization Rules of the A.I.E.E.*) Temperature rises upon which ratings are to be based. Details of overload capacity. Efficiency at 25, 50, 75, 100, and 125 per cent load with stated transformation ratio. High-potential tests of insulation. Requirements regarding effect of moisture and heat on insulation, such as the following: The transformers shall contain no material which will be permanently injured by moisture or by an occasional temperature of 95° C., provided that this temperature is not maintained at any one time for a period greater than 3 hours. The transformers shall be capable of operating continuously at 80° C., without the insulation being damaged thereby. Regulation with rated non-inductive load. Regulation with load of rated kilovolt-amperes at stated power factors, say 100 per cent and 90 per cent. State formula by which regulation is to be calculated. Reactance between primary bus and secondary terminals when transformers are to operate compound-wound synchronous converters. (The required reactance is usually specified by the manufacturers of the synchronous converter.) Amount of air or water for cooling, in cubic feet per minute at stated pressure. After the transformer has been in service for one year, its efficiency at full load shall be not lower than the above guaranteed amount by more than a stated percentage and after two years its efficiency shall be not lower than the guaranteed amount by not more than a stated percentage.

INSTALLATION OF TRANSFORMERS. — Transformers require no special foundations but merely a level floor of sufficient strength to carry the weight. Provision should be made for an electrical connection from the tank of the transformer to the ground.

Oil- and Water-cooled Transformers. — The greatest enemy to successful operation of transformers in general, and high-voltage transformers in particular, is moisture, in that 0.1 per cent of moisture in oil renders it unfit for use. This may result either from rain or dripping water falling into the transformer or onto some of the parts, or from the condensation of the moisture in the atmosphere on the various parts. For this reason all parts of a transformer must be very carefully inspected, cleaned and dried before the transformer is put into service. If the operating potential is 15,000 volts or less an inspection will tell whether the transformer should be dried, but for voltages greater than 15,000 the drying operation should be carried out in every case.

Methods of Drying. — The best method of drying is to send a current of dry air at 90° C. through and around the transformer. This should continue for 24 hours in all transformers, for 72 hours in high-voltage transformers of reasonable capacity and longer in special cases. Another method of drying is to short-circuit one winding of the transformer and to apply to the other member a voltage of from 1 to 2 per cent of the rated voltage of that winding so that a current of from ⅙ to ⅛ of the rated value flows through the winding. This current should be adjusted so that a spirit thermometer placed on the low-tension coils shows a temperature of 80° C. and not greater.

Preparation of Oil. — No potential should be applied to any oil-cooled transformer unless it is supplied with a proper amount of oil as the oil forms the essential insulating medium as well as cooling medium. The oil should be tested before using. Oil is considered in good condition when it will withstand 40,000 volts between disks ½ inch in diameter and $\frac{2}{10}$ inch apart. Transformers for 40,000 volts or less will operate satisfactorily when this dielectric strength has dropped to 25,000 volts, but when this condition has been reached the oil should be dried, purified and strained. Transformers for a voltage greater than 40,000 should not be used with oil which breaks down under 35,000 volts in the above described test apparatus. A special apparatus for

drying and purifying the oil is on the market (*see G. E. Bulletin No. 4134*). See also article on *Oil, Transformer*.

The transformer tanks should be filled by pouring the oil through a fine-cloth strainer and allowing the oil to settle 12 hours before using. Rubber tubing should not be used for carrying the oil as the sulphur in the rubber will eventually cause trouble. The cover of the transformer should prevent any water dripping into the transformer but there should be a free exit allowed for gases which may gather. If the transformer stands in a moist atmosphere a special "breather" containing calcium chloride should be employed. In all transformers the level of the oil should be well above the top of the transformer proper and this should be noted 2 or 3 days after the original filling to see whether the transformer windings have absorbed sufficient of the oil to lower the level a dangerous amount.

Certain small sizes of oil-cooled transformers are designed to be suspended from cross arms on poles or the sides of buildings or to be installed in manholes in a subway. These transformers are especially protected against the weather.

Precaution Against Overheating.—It is most important to be assured of the proper circulation of the cooling medium as the temperature of the transformer will rise quickly to an excessive value in case of a failure of the cooling medium. In this case the load must be reduced and kept at such a value that the temperature of the oil at the top does not exceed 80° C. If at any time the oil reaches an excessive temperature there will be a tendency for a deposit to form on the transformer and coils, which will interfere with the proper cooling. An inspection should be made occasionally for this purpose and the deposit removed. The oil should be sampled and tested each week for the first month and every six months thereafter. In taking samples of the oil great care should be exercised that the vessel in which the sample is contained is perfectly dried. The temperature of the oil in a self-cooled transformer should never exceed 80° C. and in a water-cooled transformer 65° C. Oil-cooled transformers must be in a well-ventilated compartment in which the air should not be more than 5° above the outside atmosphere. An inspection should be made from time to time to see that there is no condensation on the inside walls of the tank.

Precaution for Multiple Operation.—Transformers should never be connected in multiple on the secondary side without proper precautions having been taken to determine not only that the polarity is correct but that the regulation of the two transformers is the same.

Air-blast Transformers; Ducts and Blower Set.—Air-blast transformers should be placed over an air duct of sufficient size to permit the required current of air to flow at a velocity of less than 500 feet per minute. The duct should be of non-combustible material and should have smooth sides in order to offer very little resistance to the current of air. Air is supplied to this duct by means of motor-driven blowers of capacity to supply the proper quantity of air for a group of transformers at pressures from ½ to 1½ ounces (*see Blowers and Compressors; Fans*). Roughly the rating of the blower motor in horse-power is equal to

$$\frac{(\text{Cubic feet of air per minute}) \times (\text{pressure in ounces})}{1200}.$$

The air enters the transformer at the bottom, flows vertically through the coils and passes out at the top, and also flows transversely through the iron, passing out at the side. Separate dampers at each outlet are provided in order to regulate the two currents independently. The dampers are regulated so that the outgoing air has a temperature 20° above the incoming air.

Since the transformers are so dependent on the air blast for their operation and safety, it is customary to provide a reserve blower set.

Location. — Care must be exercised to protect the transformers from moisture and dirt and particularly, since they are open at the top, to place them where water and dirt cannot drop in the open top. All terminals of the air-blast type of transformer are usually brought out below so that the primary and secondary cables may be laid in the air duct and all connections and inspection may be made from below. It is therefore desirable that the air duct be sufficiently large to enable a man to move about therein.

Drying and Cleaning. — In putting the transformer in operation all moisture must be removed before applying potential. This may be accomplished by running the blower set and forcing air through the transformers, which will be sufficient if the air is dry. Another method is to short-circuit one winding of the transformer and apply a low voltage of from 1 to 2 per cent of the rated voltage to the other winding, so that 75 per cent of full-load current flows through the windings. This is maintained for from 24 to 36 hours. All transformers of this type should be cleaned once a month by means of compressed air at 20 pounds pressure.

Measurement of Temperature. — In determining the temperature of the windings only that calculated from a resistance measurement is dependable, as the coils are so thickly wrapped in insulation that a thermometer will not show the true temperature.

OPERATION OF TRANSFORMERS. — Single-phase transformers may be used singly or in parallel on single-phase circuits, and in various combinations on polyphase circuits as described above. The polarity should be determined before making the connections.

Parallel Operation on Single-phase Circuits. — Single-phase transformers are very generally operated with the primaries in parallel on the supply circuit and their secondaries in parallel on the load circuit. In order to successfully operate in this manner two or more transformers, the transformers must have: (1) the same ratio of transformation; (2) the same regulation; and (3) the same value of X/R, where R and X are the total resistance and reactance respectively of any one of the transformers. Transformers divide the load inversely proportional to their reactances, provided the ratio of reactance to resistance is the same in both cases. If transformers are purchased to operate in parallel with others, proper provision must be made in the design of the transformers, and this fact should be stated in the specifications.

In general, a 50-kilowatt transformer will have about twice the resistance and twice the reactance of a 100-kilowatt transformer of the same commercial line, and therefore these transformers will divide the load in proportion to their capacity. But this is not likely to be true of transformers of different lines or made at widely different times.

If transformers not satisfying the above conditions are to be run in parallel, an impedance coil having a proper resistance and reactance may be connected in the circuit of one of them, thus establishing the necessary conditions for the proper distribution of load, provided the ratios of transformation are the same.

Transformers on Polyphase Circuits. — See section above on *Transformer Connections*.

WEIGHTS AND COSTS. — The curves of Figs. 26 to 29 inclusive give approximate values of the weights and costs of several representative lines of single-phase transformers of the most usual common capacities, voltages and frequencies. These values are sufficiently accurate to be used in preliminary

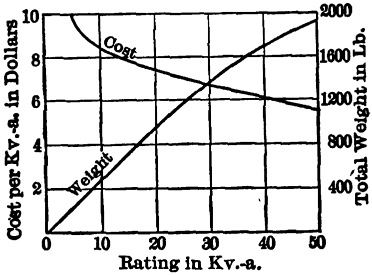


Fig. 26. Oil-cooled, 60-cycle, 2200-volt Lighting Transformers

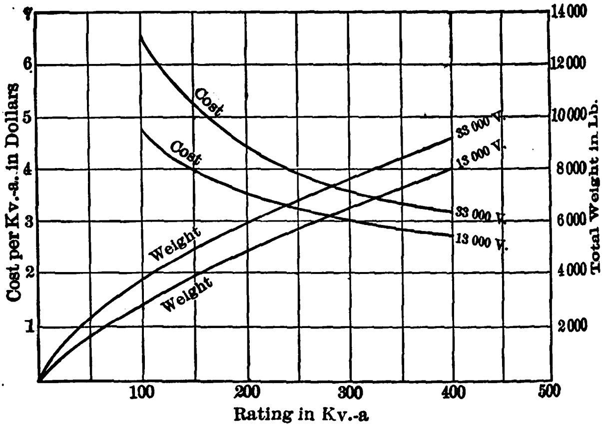


Fig. 27. Oil-cooled, 60-cycle, Single-phase Transformers

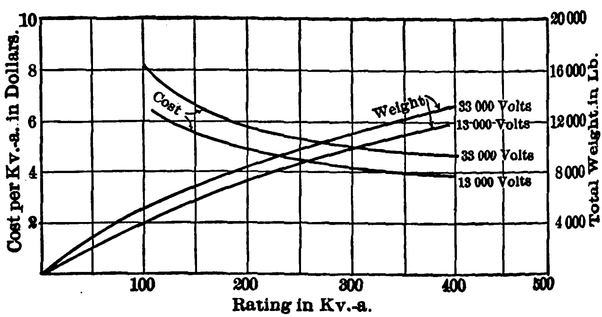


Fig. 28. Oil-cooled, 25-cycle, Single-phase Transformers

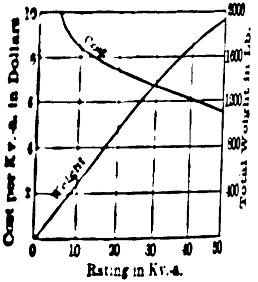


Fig. 26. Oil-cooled 60-cycle, 1100-volt Lighting Transformers

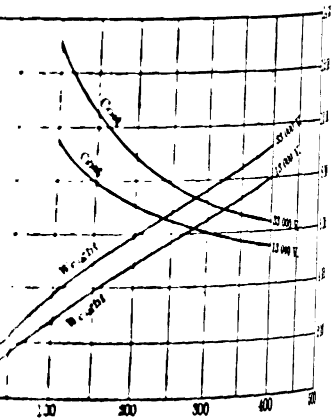


Fig. 27. Oil-cooled, 60-cycle, Single-phase Transformers

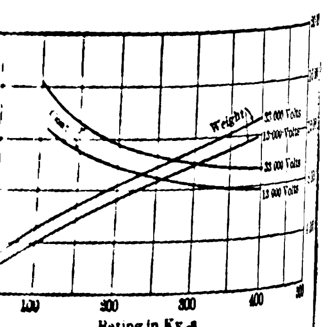


Fig. 28. Oil-cooled, 25-cycle, Single-phase Transformers

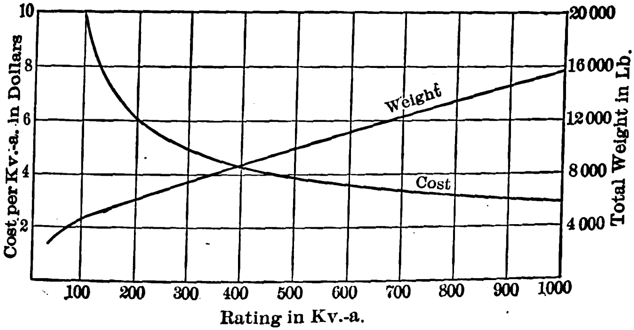


Fig. 29. Air-blast, 25-cycle, 33000-volt, Single-phase Transformers

estimates, plans, theses, etc. The actual prices, however, vary with the price of copper, commercial conditions and special guarantees.

The weight given is that of the transformer complete including case and oil, if any, but does not include the boxing for shipping.

The cost is given in the form of dollars per kv-a. rating and with the exception of the lighting transformers of Fig. 26 this kv-a. means the output which could be obtained in continuous operation with the maximum rise in temperature of 40° C. The transformers of Fig. 26 are of the lighting type and their rating is based on the characteristics of a lighting load, which is roughly a guarantee of a maximum rise in temperature of 50° with a load of this character. The prices include the cost of the oil.

The cost per kv-a. and weight per kv-a. of three-phase transformers are roughly 90 per cent of the cost and weight per kv-a. of 3 single-phase transformers having a combined capacity equal to that of the three-phase transformer.

The voltage of the secondary may vary in all cases from 110 to 440 without appreciably affecting the cost or weight given.

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[W. I. SLICHTER.]

TRANSFORMERS, INSTRUMENT. — (*See also Transformers.*) Instrument transformers are either potential transformers or current transformers. Potential transformers are transformers of comparatively small output arranged for shunt connection to the primary lines, designed to produce a secondary voltage which accurately represents the primary voltage for application to instruments. Current transformers are transformers of comparatively small output arranged for series connection in the primary lines, designed to produce a secondary current which accurately represents the primary current for application to instruments. The insulation of both current and potential transformers is designed with reference to the voltage of the circuit on which the transformer is to be used. The most common voltage for the secondary of the potential transformer is 110 volts. The most common current for secondary full load on the current transformer is 5 amperes. The secondary voltage of the current transformer is usually very low under operating conditions, being only sufficient to force the secondary current through a few instruments of low impedance.

APPLICATIONS. — Instrument transformers are used for three principal purposes: (1) to supply current and voltage to measuring apparatus; (2) to operate regulating devices; (3) to operate circuit protective devices. In each case there are two principal advantages to be attained by the use of transformers: (1) to protect the devices in the secondary circuit against the inconvenience or danger of a direct application of the primary voltage or current; (2) to enable the use of measuring, regulating and protective devices designed for one standard current and voltage for the entire range of currents and voltages used under various operating conditions, thus simplifying design and manufacture, increasing reliability and accuracy and lowering cost.

Accuracy and Reliability Required. — Of the three applications given above that involving measuring instruments requires the highest accuracy in the transformer; in the operation of regulating devices the certainty of continuous operation is more important; in the operation of circuit protective devices, a very moderate degree of accuracy is satisfactory, but certainty of operation is of the highest importance. It is, therefore, frequently desirable to use different transformers for the several purposes, even where convenience and cheapness would suggest the use of a single transformer.

THEORY OF CURRENT TRANSFORMER. — The current transformer consists of a primary winding in which the line current flows, a core, and a secondary winding for connection to a load of instruments and other devices connected in series. The current flowing in the primary is usually unaffected by the characteristics of the transformer or by the amount of secondary load. With the secondary winding short-circuited, the secondary ampere-turns are nearly equal to the primary ampere-turns, the slight difference being due to the relatively small exciting current (*see Electricity and Magnetism, Principles of*). This exciting current is always of low power factor, while the power factor of the secondary current depends on the resistance and inductance of the secondary circuit. While the reversed secondary ampere-turns always equal the primary ampere-turns less the exciting ampere-turns, the subtraction of the exciting ampere-turns is not usually arithmetical but vectorial, and the resulting secondary ampere-turns are less than and not in exact phase opposition to the primary ampere-turns. This is expressed by stating that the transformer current ratio varies from the ratio of turns, and that there is a phase angle between the primary and reversed secondary currents. Since these errors depend on the relative amount of the exciting current and its phase position with respect to the secondary current, they will vary with the imped-

ance of the secondary load and the current flowing through it. When the impedance becomes so great that the transformer core approaches saturation, the exciting current becomes so large that the secondary current is no longer proportional to the primary current and also differs from it appreciably both in wave form and phase position.

Ratio and Phase Angle. — For accurate current measurements through the intermediary of a current transformer the exact ratio of the primary to the secondary current must be accurately known. For power measurements the phase angle between the two currents must also be accurately known.

Let

u = ratio of the number of secondary to the number of primary turns,

I_p = primary current,

I_s = secondary current,

k = ratio of exciting current to the secondary current,

α = phase angle between the exciting current and primary induced e.m.f.,

θ = phase angle between the secondary current and secondary induced e.m.f. (positive for lagging current),

β = the "phase angle" of the transformer, i.e., the angle between the primary current and the secondary current reversed. If the power factor of the secondary impedance load is higher than that of the exciting current, the secondary current (reversed) leads the primary current and β is positive; if the power factor of the secondary impedance load is lower than that of the exciting current, the secondary current (reversed) lags behind the primary current and β is negative.

Then the ratio of transformation is

$$\frac{I_p}{I_s} = u\sqrt{(\cos\theta + k\cos\alpha)^2 + (\sin\theta + k\sin\alpha)^2}$$

and the phase angle is

$$\beta = \tan^{-1}\frac{k\sin(\alpha - \theta)}{1 + k\cos(\alpha - \theta)}.$$

The value of the ratio k for a given primary current depends upon the impedance of the load connected to the secondary; the higher this impedance the greater the value of k and therefore increasing the impedance of the secondary connected load tends to increase the difference between the ratio of turns and the true ratio of the primary and secondary currents; increasing the impedance of the secondary also tends to increase the phase angle β of the transformer. The true ratio $\frac{I_p}{I_s}$ and the phase angle β also depend upon the power factor of the load. For the actual variation of these quantities see Figs. 1 to 4, below.

THEORY OF POTENTIAL TRANSFORMER. — The potential transformer consists of a primary winding, a core and a secondary winding. The primary winding is placed across the line, and the secondary winding is connected to instruments or other devices connected in multiple. The voltage across the primary is usually unaffected by the characteristics of the transformer or of the secondary load. With the secondary open the secondary terminal voltage is approximately equal to the primary impressed voltage divided by the ratio of the number of turns in the primary to the number of turns in the secondary, the difference being due to the impedance drop produced in the primary by the exciting current. The impedance drop in the primary due to exciting current is usually not in phase with the primary voltage; hence its subtraction in a vector relation from the primary voltage results both in a variation of the voltage ratio from the ratio of turns and in a difference of phase

between the primary and reversed secondary voltages. This no-load ratio and phase angle are modified under load conditions by the additional impedance drop in the primary and secondary windings due to the load current. The less the impedance of the load the greater will be the current through the two windings of the transformer, and therefore the greater the impedance drops in these windings; hence the greater will be the discrepancy between the actual ratio of the terminal voltages and the ratio of turns. As the impedance drop due to the load current may be in almost any phase, whereas the drop due to exciting current bears a constant relation to the induced voltage, the phase angle with loads of various power factors may be either less than or greater than that at no load.

Ratio and Phase Angle.—For accurate voltage measurements through the intermediary of a potential transformer the exact ratio of the primary to the secondary terminal voltages must be accurately known. For power measurements the phase angle between the two voltages must also be accurately known. The expressions for these two quantities in terms of the constants of the transformer and the impedance and power factor of the total instrument load connected to the secondary of the transformer are quite complicated. The relations involved in the expression for the true ratio may be expressed as follows:

Let u = ratio of number of turns in primary to number in secondary winding,
E_p = primary impressed voltage,
E_s = secondary terminal voltage,
E = secondary induced e.m.f.,
I_p = primary current,
I_s = secondary current,
Z_p = primary impedance,
Z_s = secondary impedance.

Then

$$\frac{E_p}{E_s} = \frac{uE + Z_pI_p}{E - Z_sI_s},$$

where the numerator and denominator of this fraction are a *vector sum* and *vector difference* respectively, and E, I_p and I_s depend upon the voltage, impedance and power factor of the load, and the value of the exciting current. (*See article on Transformers.*) For actual variations in the true ratio $\frac{E_p}{E_s}$ with the power factor, voltage and volt-amperes of the load on the secondary see Figs. 5 and 7.

The phase angle γ of a potential transformer, i.e., the angle between the primary terminal voltage and the secondary terminal voltage reversed, may range from a positive angle (secondary voltage lagging behind primary voltage) to a considerable negative angle (secondary voltage leading primary voltage), depending on the relations of the impedance and power factor of the secondary connected load, the secondary winding, and the primary winding and on the exciting current. Under the no-load condition this angle is nearly always negative, and at high core densities where the exciting current is large and of very low power factor, it may be very large. The general tendency of non-inductive secondary loads is to cause γ to vary in the positive direction (secondary voltage to lag) and of inductive (lagging current) loads to cause it to vary in the negative direction (secondary voltage to lead). It is frequently possible to bring γ practically to zero for a single voltage and load by adding a non-inductive load in suitable amounts. See Figs. 6 and 8 for actual variations in the phase angle γ with the voltage, power factor and volt-amperes of the load on the secondary.

DESIGN. — The general principles of design of instrument transformers are the same as for power transformers. The special uses of the instrument transformers, however, change the details of design to a considerable degree.

Design of Current Transformers. — A very important purpose of current transformers is the operation of protective devices. For this reason the failure of the current transformer may cause destruction of expensive apparatus, and they are therefore designed to stand a higher insulation test than other types of transformers. Again, the exciting current of the current transformer has direct relation to the phase angle and errors of ratio. The flux density in the core is therefore kept very low for the ordinary metering conditions, in order to keep errors within satisfactory limits. The resistance and leakage reactance of the primary winding do not affect the ratio of the transformer nor the phase angle between primary and secondary currents; their only effect is on the total impedance in the primary line. Resistance and leakage reactance in the secondary constitute definite additions to the secondary load, and affect the phase angle and ratio of the transformer. As the current rating of a current transformer is always greater than the maximum continuous current to be carried by the lines of which it is a part, no considerable overload capacity is required.

A suitable design for current transformers, therefore, should include:

- High factor of safety in insulation,
- Low density of flux under the conditions of metering service (this is best secured by a moderate cross-section of core with a relatively large number of turns),
- Low resistance and leakage reactance in the secondary winding,
- Resistance and reactance of the primary winding low enough so that the impedance will not cause an objectionable drop in voltage in the primary lines, when the transformer is to be used in low voltage circuits.

Special Designs of Current Transformers. — For certain special purposes special designs are useful. In stations where very large power is on the bus-bars while small power circuits, such as station auxiliaries, are drawn from the same bus-bars, current transformers must be used which are adapted to operate on small current, while under enormous extremes of current the transformers must at least remain in service long enough to operate the protective devices. Where the currents are not too low, transformers with a single copper bar as primary may be used. These resist mechanical injury from short-circuit, but the number of ampere-turns under the ordinary load conditions is limited to the number of amperes flowing in the primary line. Hence these transformers have an accuracy suitable for metering purposes only where the rated current of the line is comparatively large. When the normal currents are too low for satisfactory operation with a single primary turn, special transformers of large size, great primary current capacity and reduced accuracy are sometimes used.

A somewhat similar case arises where the cost of current transformers for use in high-tension circuits is prohibitive. A transformer is then placed on the bushing of an oil switch, the lead inside the bushing being the primary, and a secondary being wound on a core which encircles the bushing. As the primary current is usually small, while the length of the magnetic circuit is comparatively large, the accuracy of these transformers is low, and their use is ordinarily confined to tripping devices or specially calibrated ammeters.

Design of Potential Transformers. — The design of a potential transformer is very similar to that of a small power transformer. To obtain accuracy of ratio and small phase angle between primary and secondary voltages, the flux density in the core should be kept comparatively low. The resistance and leakage reactance of the primary and secondary should be kept small. As these trans-

formers are ordinarily connected to the line through fuses, their insulation is sufficient if it equals that of good power transformers for the same voltage. The exciting current is usually a larger fraction of the full-load current than in power transformers, because efficiency is a matter of no moment, and good regulation is necessary. The no-load phase angle, however, is roughly dependent on the product of the exciting current by the primary resistance; hence too large an exciting current or too large a resistance is undesirable.

RATING AND PRECISION. — The watt or volt-ampere rating of instrument transformers as given by many manufacturers is purely formal, and has only the most distant relation to the characteristics of the transformer. There is no general correspondence between accuracies of transformers produced by various manufacturers for the same load. Where transformers are designed with oil immersion for high-voltage circuits, their quality is less likely to be poor than in the small types without oil, because the proportionate difference in cost of manufacture between good and poor transformers is then comparatively small.

Precision of Current Transformer. — The variation of ratio and phase angle with the load on the secondary of two typical 2300-volt, 20 : 1 ratio, cur-

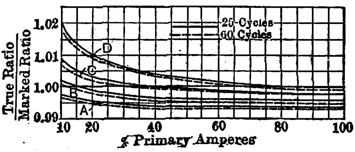

Fig. 1. Ratio

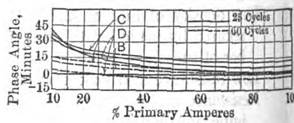

Fig. 2. Phase Angle

Current Transformer No. 1

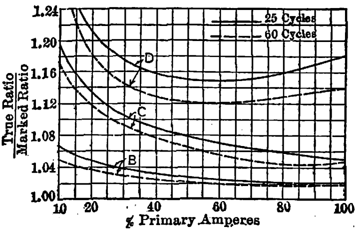

Fig. 3. Ratio

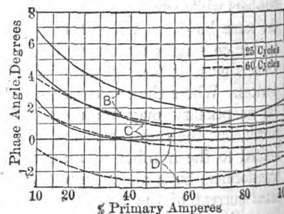

Fig. 4. Phase Angle

Current Transformer No. 2

rent transformers is shown in Figs. 1 to 4. Figs. 1 and 2 show curves of ratio and phase angle obtained from tests on a current transformer (No. 1) of low flux density, high ampere turns, and comparatively low secondary resistance and leakage reactance. Figs. 3 and 4 show the results of similar tests on a current transformer (No. 2) where these values are not so strictly limited. It should be noted that the scales of $\frac{\text{true ratio}}{\text{marked ratio}}$ and of phase angle in Figs. 3 and 4 are four times those used in Figs. 1 and 2.

Each pair of curves was made with a different load on the secondary, loads consisting of a series of combinations of instruments, watt-hour meters, and switchboard devices representing conditions in ordinary practice. Table I gives the volt-amperes and power factor of the loads, designated by the letters

A to *D*, at 60 cycles and 5 amperes and also the corresponding inductance and equivalent resistance components.

TABLE I

LOADS REFERRED TO IN FIGS. 1 TO 4

Load	Volt-amperes, load at 60 cycles and 5 amperes	Power factor at 60 cycles and 5 amperes, per cent	Equivalent resistance, ohms	Inductance, milli-henries
A	4.8	99.8	0.192	0.032
B	10.9	90	0.392	0.504
C	45.1	54	0.972	4.03
D	132.4	38	2.012	13.0

Precision of Potential Transformers.— The variation of ratio and phase angle with the power factor of the load on the secondary and with the secondary voltage of two typical 2200-volt, 25-cycle, 200 volt-ampere, 20 : 1 ratio, poten-

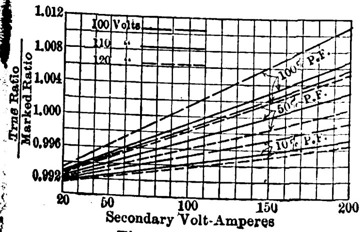

Fig. 5. Ratio

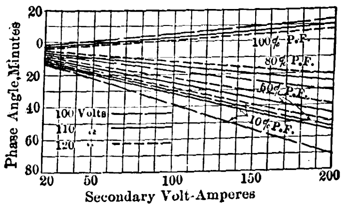

Fig. 6. Phase Angle

Potential Transformer No. 1

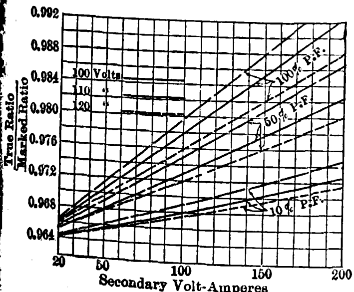

Fig. 7. Ratio

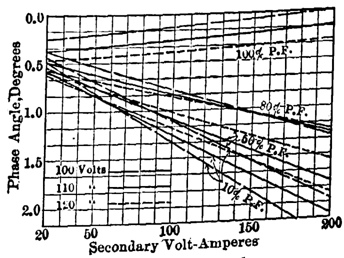

Fig. 8. Phase Angle

Potential Transformer No. 2

tial transformers is shown in Figs. 5 to 8. Figs. 5 and 6 show the curves of ratio and phase angle obtained in test on a potential transformer (No. 1) in which the flux density is low, and the resistances and leakage reactances of the wind-

ings are within moderate limits. Figs. 7 and 8 show the results of similar tests on a transformer (No. 2) of the same rating in which these quantities reach larger limits, resulting in inferior regulation and larger phase angle.

GENERAL TESTS DURING MANUFACTURE. — As the last step in manufacture the insulation of every current or potential transformer should be tested at a voltage greater than that at which it is to operate, the actual voltage selected being determined by the A.I.E.E. Standardization Rules. Some manufacturers find it desirable to apply considerably higher test voltages than these to insure additional safety. A sufficient number of each type should be given heating tests to assure safety of operation for the entire group. A check on accuracy should be made on all transformers to protect against errors in counting turns, short-circuits, etc.

TESTING OF CURRENT TRANSFORMERS FOR RATIO AND PHASE ANGLE. — Where large numbers of current transformers are to be tested to a high degree of accuracy for ratio and phase angle, those methods which use primary and secondary shunts, balancing the voltage drop through a zero reading instrument, are most rapid and satisfactory. The care and expense of such an outfit is justified only where a large number of transformers must be carefully tested. The phase angle may also be determined by the use of two wattmeters or electrodynamometers. Where transformers are in use and require an occasional check to determine that they have not changed in characteristics, a simple method which covers most cases is to compare the ratio with that of a standardized portable transformer. These methods are described below.

The uniformity among transformers of a certain type, make and size is usually very good; in the case of a large lot of transformers detailed tests need be made only on a few representative ones.

Demagnetization of Core Before Testing. — The cores of current transformers should be demagnetized before the test for ratio or phase angle is made. With small low-voltage current transformers demagnetization may be carried out by putting at least one-half-load primary current through the transformer with 10 ohms or more connected to the secondary in series with the instruments to be used. With large high-voltage current transformers having massive cores the resistance should be several times greater to assure perfect demagnetization. This resistance should then be gradually reduced to zero by steps of one ohm or less.

When however current transformers are subjected to test in order to use the results for the correction of observations already taken, the tests should be made in such a way as to avoid changing the magnetic condition of the core before the test results are secured. The secondary should have no greater load than that used in the working condition, and especial care should be taken that current is not allowed to flow in the primary or the secondary while the other winding is open circuited. The current should be brought up to the lowest current point first, and raised to the higher points only after the lower readings have been made.

Frequency of Test Current. — In all current-transformer work where results of test are to be used for correction, the frequency used in test should be the same as that of the circuit in which the transformer is to be employed. When the intention is only to check the condition of the transformer, any commercial frequency may be used.

Shunt Methods of Testing Current Transformers. — The following is a description of the details of one of these methods which has been found satisfactory in use.

Apparatus. — Referring to Fig. 9 a three-phase supply is used to excite a phase-shifting transformer with a single-phase secondary. The supply of current to the transformer under test is obtained from one phase of the same source through a control resistance R_1 and a step-up current transformer. For small ratios the step-up transformer is omitted. Its input side is wound to use up to the voltage of the supply (125 volts). The high-current winding consists of heavy loose cable, of which one or more turns may be wound through the large core opening. By suitably selecting the number of turns, currents from 10 to 4000 amperes are obtained. The primary shunt has a resistance such that full rated current gives a drop of 0.5 volt. It is arranged to carry 50 per cent overload current for a short time for testing purposes. The secondary shunt is adjustable, and is marked in percentage of a normal resistance. By the use of several interchangeable scales the normal resistance corresponding to 100 per cent may be made 0.06, 0.075, 0.08, 0.1, 0.12, or 0.125 ohms. These allow the use of a primary 100-ampere shunt of 0.005 ohms resistance for transformers of 12 : 1, 15 : 1, 16 : 1, 20 : 1, 24 : 1 and 25 : 1 rated ratios, while still obtaining a direct reading on the secondary scale of the $\frac{\text{true ratio}}{\text{marked ratio}}$ of the transformer. A set of primary shunts rated 5, 10, 20, 50, 100, 200, 500, 1000, 2000 and 5000 amperes will thus cover all probable ratios from 1 : 1 to 1250 : 1. M is an adjustable calibrated mutual inductance whose current coil is in series with the secondary of the transformer under test, and whose potential coil is in series with the potential element of the sensitive electrodynamometer D, whose current element in turn is supplied from the secondary of the phase-shifting transformer mentioned above. Z is a double-throw single-pole switch. L is a series of secondary loads suitable to represent the instrument combinations usually found in practice.

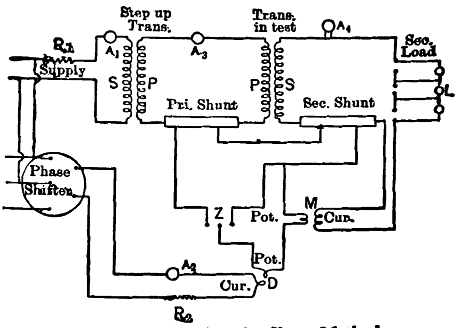

Fig. 9. Connections for Shunt Method

Procedure. — In making the test the procedure is as follows: The proper load is connected to the secondary of the transformer, and the primary current is adjusted to the proper point by means of the resistance R_1. This current may be read by an ammeter and separate current transformer at A_3, or by an ammeter at A_4 which is short-circuited after reading. The switch Z is closed to the right. The mutual inductance then acts as an air-core transformer, supplying to the dynamometer potential element a voltage in quadrature with the secondary current of the transformer under test. The phase of the field current of the dynamometer is then adjusted to the point where the dynamometer reads zero for all positions of the mutual inductance. This is the position of maximum sensibility to changes in the resistance of the secondary shunt. The switch Z is then thrown to the left, and the resistance drop of the secondary shunt is adjusted by moving the drop contact until the dynamometer reads zero. The phase of the dynamometer field current is then shifted through 90 electrical degrees by moving the handle of the phase-shifting transformer a

definite distance. This is the position of maximum sensibility to phase-angle variation. The mutual inductance is then adjusted until the dynamometer reads zero. The phase of the field current is then shifted back to the previous position, and a slight readjustment of the secondary shunt resistance made if necessary.

Calculations. — From the reading of the mutual inductance the phase angle is determined by the formula

$$\beta = \tan^{-1} \frac{2 \pi f M}{R_s},$$

where f is the frequency of the circuit in cycles per second, M is the mutual inductance in henries, and R_s the resistance of the secondary shunt in ohms. Then the true ratio is expressed by the formula

$$\text{Ratio} = \frac{R_s}{R_p} \times \frac{1}{\cos \beta},$$

where R_p is the resistance of the primary shunt.

The correction factor $\frac{1}{\cos \beta}$ for $\beta = 1^0$ is 1.00015, and for $\beta = 2^0$ it is 1.0006. At these values it is negligible in comparison with ordinary errors arising in measurement work involving transformers and instruments. If the phase angle is so great that this correction becomes important, the transformer is so inferior that it should not be used for any purpose involving accurate measurement.

Speed of Operation. — By this means a single operator can take about 30 points of ratio and the same number of phase angle per hour. With a helper to perform necessary calculations and a suitable means of rapidly replacing transformers in the circuit, it is quite practicable to make from 15 to 30 tests of transformers including from 6 to 12 points each of ratio and the same number of determinations of phase angle in an eight-hour day. Checks of single points can be made practically as fast as the transformers can be connected and the current and secondary load adjusted.

Standardized Transformer Method for Determining Ratio of Current Transformer. — A portable transformer with four primary coils arranged for series-multiple connection, giving ratios of 5 : 1, 10 : 1, and 20 : 1 will cover practically everything up to 100 amperes. A second portable transformer without primary, so arranged that one turn through the core gives a rated ratio of 200: 1, and more turns a correspondingly lower ratio, will cover from 100 to 1000 amperes. In testing, the primary windings of the transformer under test and the standardized portable transformer are connected in series. Either ammeters or wattmeters may be used as indicating instruments, the latter being susceptible of giving the higher precision.

Use of Ammeters with Standardized Transformer. — The secondary of the standardized transformer is connected to a standardized 5-ampere ammeter, and that of the transformer under test to a similar ammeter and a suitable secondary load. With a sufficient current flowing to give satisfactory readings on the ammeters, readings are made; the ammeters are interchanged, and readings are repeated, to eliminate difference between the instruments. From the mean of the results and the ratio of the standardized transformer, the ratio of the transformer under test is obtained.

Use of Wattmeters with Standardized Transformer. — The two transformer secondaries are connected to the current coils of the wattmeters and the potential elements are placed in multiple and supplied with a definite

voltage from a phase-shifting transformer. The phase shifter is adjusted at each point to cause the wattmeter connected to the standardized transformer to read a maximum. The instruments then are read as ammeters, using a calibration made with the same voltage applied to the potential element. By this means a larger deflection is obtained for low-current values, and consequently greater accuracy. If only transformers of moderate-current rating are tested, so that the current may be kept nearly in phase with the supply voltage by the use of series resistance, the phase-shifting transformer may be omitted and the wattmeter potential circuits excited from the supply voltage.

This method requires little apparatus or care beyond that necessary for keeping instruments and transformers in good condition. It does not test phase angle, but gives a check on ratio, whose error may readily be kept within 0.3 per cent. This includes the error of comparison, using ordinary care, and the error of the standard transformer. It is reasonably convenient and rapid, and is well adapted for use in the laboratories maintained by most companies supplying light and power.

Two-dynamometer Method of Determining Phase Angle of Current Transformers. — Phase angle may be determined (where primary currents are not too great) by the use of similar types of dynamometer wattmeters whose current elements are connected in the primary and secondary circuits, and whose potential elements are supplied in multiple by a phase-shifting transformer. The phase of the voltage is shifted until the primary wattmeter indicates zero, showing a quadrature relation of its current and voltage. If W is the watt reading of the secondary wattmeter, E the voltage applied to its potential element, and I the current in its current coil

$$\beta = \sin^{-1} \frac{W}{EI}.$$

The sensibility of this method is very low with ordinary portable wattmeters, but quite satisfactory with sensitive reflecting dynamometers. It is the best method in use for the accurate determination of phase angle.

Other Methods of Testing Current Transformers. — Many modifications of the above methods and other entirely different methods are in more or less satisfactory use in various laboratories.

TESTING POTENTIAL TRANSFORMERS FOR RATIO AND PHASE ANGLE. — Where a large number of potential transformers are to be tested, the potentiometer method of testing ratio is very satisfactory. In this method the low-tension voltage is balanced against a part of the drop through a large resistance in which current is maintained by the high-tension voltage. This may be combined with the two-dynamometer method of testing phase angle between primary and secondary voltages. Where the number of transformers to be tested is not large, and where only a few ratios are to be tested, they may be compared with standardized transformers, and the phase angle determined by the use of the two-dynamometer method. The ratio may also be determined by the use of a voltmeter and high-resistance multiplier. These methods are described below.

Potentiometer Method of Testing Potential Transformers. — The following is a description of the apparatus and method of procedure which has given satisfactory results.

Apparatus. — The general connections of such an outfit are shown in Fig. 10. The supply is 3-phase for the excitation of the phase-shifting transformer P. One phase is connected through a voltage regulator Q to the low-tension side of the step-up transformer T_1, which has a sufficient number of

connections to provide the range of voltages required for testing. This supply excites the high-tension side of the transformer T_2 which is under test. r_2 and r_1 are placed across the high-tension circuit in multiple with the high-ten-

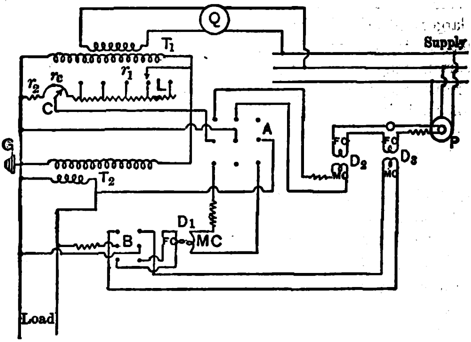

Fig. 10. Connections for Potentiometer Method

sion side of the transformer under test. They consist of non-inductive resistances, containing a non-adjustable portion constituting the greater part of r_2, a middle portion r_c subdivided into equal steps on which a contactor C travels, and a third part constituting the greater part of r_1 on which taps are brought out at points suited to the various voltages to be used in test. One side of the secondary winding is connected to the end of r_2 and the ground G; the other is connected (for ratio test) through switch A, which is in the downward position, through a resistance and the moving element of the dynamometer D_1 to the contactor C. By suitably proportioning the resistances r_1, r_2 and the steps of r_c, the outfit is made direct reading in terms of $\frac{\text{true ratio}}{\text{marked ratio}}$ of the transformer.

Precaution. — Balance can only be secured by a proper connection with regard to polarity of the secondary of the transformer under test to the primary. If the reversed connection is used, double the secondary voltage is applied to the circuit through contactor and dynamometer. A sufficient resistance to protect the dynamometer should therefore be inserted when trying out each transformer to be tested. As soon as an approximate balance is obtained, this resistance may be reduced to secure sensitiveness.

Determination of Ratio. — The fixed coil of the dynamometer D_1 is excited from the low-tension terminals of the transformer under test through the switch B in the downward position. Other secondary load may also be applied to the secondary terminals. With this load and the voltage and frequency suitably adjusted, and with the high-tension lead L connected to the proper tap on r_1, the contactor C is moved until the dynamometer D_1 indicates nearly zero. If the resistance at which the dynamometer indicates zero lies between two steps of the contactor, the nearest may be taken or the readings of dynamometer on both steps may be noted and the result obtained by interpolation. If r_1 and r_2 are respectively the resistances from the contactor C to the high-tension and to the grounded lines, then

$$\text{True ratio} = \frac{r_1 + r_2}{r_2},$$

subject to a small error due to phase angle between primary and secondary voltages, which is negligible for a phase angle of 2 degrees or less, provided

the resistance of the circuit from the contactor C through the dynamometer is approximately equal to r_2.

Determination of Phase Angle. — To obtain the phase angle the switches A and B are thrown upward. The drop of primary voltage across r_2 is thus applied to the potential element of dynamometer D_2, while the secondary voltage is applied to the potential element of dynamometer D_3, both through large non-inductive resistances. The current elements of the dynamometers are placed in series and excited from the phase-shifting transformer P. When the phase of the excitation is shifted so that dynamometer D_2 indicates zero, the phase angle between primary and secondary voltages is

$$\gamma = \sin^{-1}\frac{W}{VA},$$

where W is the watts indicated by the dynamometer D_3, V is the voltage and A the current applied to its potential and current elements respectively.

Speed of Operation. — Ratio points may be taken at the rate of one minute each while operating, and phase-angle points at a somewhat slower rate. Single-point checks can be made practically as rapidly as the transformers can be connected and the voltage, load and frequency adjusted. One observer is required for ratio test, two for phase-angle test.

Standardized Transformer Method for Determining Ratio of Potential Transformer. — The transformer to be tested should be placed in multiple with the standard transformer of the same rated ratio on the high-tension side, and standardized portable voltmeters should be connected to the two secondaries. These should be read, interchanged and read a second time to eliminate errors in calibration of the voltmeters. It is not difficult to maintain an accuracy of about 0.2 per cent on ratio test by this method with carefully standardized apparatus near the rated voltages of the transformer. More accurate determinations of ratio and determinations of phase angle if desired should be made in a thoroughly equipped laboratory, and the check results used simply to verify the unchanged condition of the transformer.

Voltmeter and Resistance Method of Determining Ratio. — Where two transformers of the same rated ratio and a voltmeter with suitable multipliers are available, good checking may be done according to the method shown in Fig. 11. T_1 is a step-up transformer to furnish the voltage for use in test. T_2 and T_3 are potential transformers of similar ratio, T_3 being the transformer under test. V_2 is a portable voltmeter. V_1 is a good portable voltmeter or a laboratory standard instrument. M is a multiplier of such resistance R_m that closing the switch P causes the voltmeter and multiplier to read nearly the same from the primary supply as the voltmeter alone reads from the secondary with the switch S closed. In operation the switch P is closed, and readings made on V_1 and V_2. Then the switch P is opened and S is closed. V_2 is held at the same reading and V_1 is read. If the resistance of V_1 is R_v, its first reading E_1 and its second reading E_2,

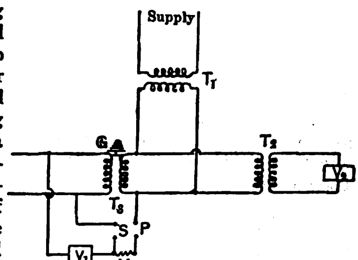

Fig. 11. Connections for Voltmeter and Resistance Method

$$\text{True ratio} = \frac{E_1\,(R_m + R_v)}{E_2 R_v}.$$

If the two readings are close together in the scale of V_1, no specially standardized instrument is necessary. The reading of V_2 does not enter into the result, as it is only used to hold the primary voltage at the same point for the two readings. The accuracy of the method depends on the accuracy of reading of the instruments used, although the absolute values of the readings do not affect it. With care the error should not exceed 0.2 per cent.

Mutual Inductance Method of Determining Phase Angle of Potential Transformer. — The phase angle may be determined by the addition to the ratio-testing outfit described above of a mutual inductance whose primary is placed in series with r_1 or r_2 and whose secondary is in series with the circuit from the contactor C through the dynamometer D_1 in a way similar to that described under current transformers. The primaries of two similar transformers may be placed in multiple, and the difference in their secondary voltages read on a low reading instrument. Where the phase angle is the same in both the transformers, this is an excellent method of comparison if the current drawn in the voltmeter is kept small by the use of a high-resistance instrument.

SPECIFICATIONS. — Heating limits, insulation test, etc., should be in accordance with the Standardization Rules of the A.I.E.E. (q.v.).

Ratio and phase angle under definite conditions of secondary connected load may, for the highest class of product, be defined substantially in accordance with Figs. 1, 2, 5 and 6. Even better accuracy than that shown in these figures can sometimes be secured with considerable increase in cost and amount of material. For less exacting service transformers of lower cost and less accuracy can be procured. The conditions to be fulfilled vary so much that definite general specifications cannot well be given.

For accurate work, especially with current transformers and in cases where much depends on the result, abridged tests covering the performance under operating conditions may be required in connection with each individual transformer. Unless very careful tests are made, however, the information from a curve representing the average of many determinations is apt to be more reliable than the results of one determination.

INSTALLATION.—Instrument transformers previous to installation should be kept in a cool, dry place. All handling should be done in such a manner as to avoid damage to the insulation, particular care being used on those types where the insulation is exposed. Oil-type transformers are usually shipped without oil, and temporary wooden blocking is often placed in the tank. This blocking should be removed before installation. If the transformers have been stored for a considerable time, or if they have been exposed to moisture, they should be thoroughly dried out before installing. (*See Transformers.*) Those of the oil type should be filled with a good quality of dry oil.

Grounding of Transformers.— Both current and potential transformers should be installed in a clean, dry place, preferably at the back of the switchboard in the case of low-voltage transformers, so that all parts, except those that are grounded, will clear all conducting or semi-conducting material by at least twice and preferably by three times the sparking distance of the normal line voltage. The casings and frames should be grounded. The secondary wiring should be grounded at such a point as will not interfere with proper operation of instruments connected to the transformers. See article on *Ground Connections.*

Resistance of Leads. — Secondary leads should be of low resistance and special attention should be given to making all contacts perfect in the wiring connected to the secondary of current transformers. A resistance not exceed-

ing 0.2 ohm (200 feet of No. 10 wire) is usually satisfactory for the secondary circuit of a current transformer operating watt-hour meters or wattmeters. Considerably greater length of leads may be used where ammeters and protective devices only are used because of the lower accuracy required. With potential transformers, the error due to drop in leads is proportional to the amount of current passing through them, which is determined by the number and kind of devices operated. For ordinary circumstances the drop should be less than 0.5 per cent; where special accuracy is desired, the lead resistance should be negligible, or the accuracy test of the transformer should be made with a resistance equivalent to the actual leads placed in series with the testing load. Leads should always be close together, enclosing as little area as possible between them.

Fuses. — Potential transformers should be fused on the primary side. The fuse should be designed to protect the line from disturbance due to failure of the transformer, rather than to protect the transformer against secondary overloads. The fuse should be of such resistance as will not cause appreciable errors in the ratio of transformation.

OPERATION. — After the current transformer has been installed it should need no other care than being kept clean. The secondary of a current transformer must never be left open-circuited. If it becomes desirable to open the secondary winding to insert or remove instruments, the secondary of the current transformer should first be carefully short-circuited. If the secondary has been open even momentarily while current flowed in the primary, the transformer should be demagnetized; see above under *Testing*. Current transformers should always be considered as part of the line circuit and should never be handled except to change connections. Even then only the secondary leads should be touched. When it becomes necessary to change secondary connections the ground connection should be inspected to assure that it is in good condition and is so made that it will not be disconnected in handling the secondary leads. Grounded protective cases should be used where necessary to prevent attendants from coming in accidental contact with current transformers.

Potential transformers should receive the same care in operation as power transformers. No power should be drawn from them except that required for a suitable load of instruments and the usual switchboard devices. The same precautions should be used in handling as with power transformers.

REPAIRS. — In general, replacement or repair by the maker is to be preferred to repairs made by the user where anything more than broken leads or simple external injuries are to be remedied. If repairs to core or windings are attempted, care should be exercised to duplicate as exactly as possible the original construction especially in regard to number of turns and resistance of windings and exact assembly of core. After repairs are completed, the transformer should be carefully tested both for accuracy and for insulation strength.

WEIGHTS, DIMENSIONS AND COSTS. — Current transformers in ordinary use for circuits from 110 to 110,000 volts range from 4 pounds to 1500 pounds weight with oil, and from a maximum dimension of 6 inches to a height of 8 feet with a diameter of 30 inches, depending on the quality of the transformer and the voltage of the circuit on which it is to operate. Potential transformers have nearly the same range of sizes.

The cost of instrument transformers depends on the circuit voltage for which they are designed, and is also affected in the case of current transformers by the ratio of the transformer. There is a considerable range between the prices offered by various manufacturers for transformers of the same rating; the lower prices are, however, associated with less accuracy or with less insulation strength or both.

The approximate cost of an instrument transformer of high grade representing accuracies as per Figs. 1 and 2 for current transformers and Figs. 5 and 6 for potential transformers is as follows:

COST OF POTENTIAL AND CURRENT TRANSFORMERS

Type of Transformer	Voltage of Circuit	Insulation Test Voltage	Price, dollars
Potential	2,200	10,000	25 to 30
"	3,300	10,000	30 to 35
"	4,400	10,000	40 to 50
"	5,500	12,500	50 to 60
"	6,600	15,000	60 to 70
"	11,000	25,000	70 to 80
"	13,200	30,000	
"	22,000	50,000	140 to 150
"	26,400	60,000	
"	33,000	74,000	200 to 220
"	50,000	112,500	400 to 600
"	60,000	135,000	
Current	2,500	10,000	20 to 30
"	15,000	50,000	35 to 45
"	27,000	75,000	90
"	45,000	120,000	125
"	70,000	175,000	250

Current transformers for a 2500-volt circuit with 10,000 volt insulation test and accurate for use with a single instrument or meter, giving a ratio curve approximately like *B* in Fig. 1, cost from $14 to $18.

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[L. T. ROBINSON.]

TRANSIENT ELECTRIC PHENOMENA AND OSCILLATIONS. — (*See also Alternating Currents; Capacity and Charging Current; Electricity and Magnetism, Principles of; Inductance and Inductive Reactance; Transmission Lines.*)

Establishment of a Direct Current in a Coil. — Let E be a constant direct electromotive force, r the resistance of the coil, L the inductance of the coil (assumed constant); then t seconds after closing the circuit (placing the coil and e.m.f. in series) the current in the coil is

$$i = \frac{E}{r}\left(1 - \epsilon^{-ut}\right), \tag{1}$$

where ϵ is the base of the natural logarithms and $u = \frac{r}{L}$. See *Exponential Functions* for values of ϵ^{-x}.

Time Constant of a Coil. — The time required for the exponent $ut = \frac{rt}{L}$ to reach the value unity, and therefore for the current to reach 63.2 per cent of its final value, is called the time constant of the coil, and is equal to $\frac{L}{r}$. The larger the time constant the longer the time required for the current to reach its steady value $\frac{E}{r}$.

Decay of Current in a Coil when Coil is Short-circuited. — Using the same notation as above, the current in the coil t seconds after short-circuiting it is

$$i = I_0 \epsilon^{-ut}$$

where I_0 is the current in it at the instant the short-circuit is made.

Charging a Condenser Through a Resistance from a Source of Constant E.M.F. — Let E be a constant e.m.f., C the capacity of the condenser, r the resistance in series with it, and let the condenser be originally uncharged. Then if the conductance of the condenser and the inductance of the circuit are both negligible, the voltage across the condenser, the current in the circuit and the charge on the condenser t seconds after the circuit is closed (thereby connecting the condenser, resistance and e.m.f. in series), are respectively

$$\left.\begin{aligned} v &= E(1 - \epsilon^{-ut}), \\ i &= \frac{E}{r}\epsilon^{-ut}, \\ q &= CE(1 - \epsilon^{-ut}), \end{aligned}\right\} \tag{2}$$

where $u = \frac{1}{rC}$. For values of ϵ^{-x} see *Exponential Functions.*

Time Constant of a Condenser and Resistance in Series. — The time required for the exponent $ut = \frac{t}{rC}$ to reach the value unity, and therefore for the voltage and charge on the condenser to reach 63.2 per cent of their final values, is called the time constant of this circuit, and is equal to rC. The larger the time constant the longer the time required for the voltage across the condenser to reach the value of the e.m.f. impressed on the circuit.

Discharge of a Condenser Through a Resistance. — Using the same notation as above, and assuming negligible conductance in the condenser and negligible inductance in the circuit, the voltage across the condenser, the current

in the circuit and the charge on the condenser t seconds after short-circuiting it through a resistance r, the condenser being charged to a voltage V_0 at the instant of short-circuit (but no current flowing), are respectively

$$\left.\begin{aligned} v &= V_0 \epsilon^{-ut}, \\ i &= -\frac{V_0}{r}\epsilon^{-ut}, \\ q &= CV_0 \epsilon^{-ut}, \end{aligned}\right\} \tag{3}$$

where $u = \dfrac{1}{rC}$.

GENERAL EQUATIONS FOR LUMPED INDUCTANCE AND CAPACITY. — Let the various quantities be as designated in Fig. 1. The differential equations for this circuit are

$$\left.\begin{aligned} i &= gv + C\frac{dv}{dt}, \\ v &= e - ri - L\frac{di}{dt}, \end{aligned}\right\} \tag{4}$$

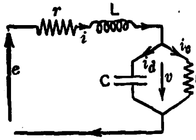

Fig. 1.

where i is the instantaneous current in the impedance coil (equal to the total displacement and conduction current through the condenser), v is the potential drop through the condenser in the direction of the current, and e is the impressed voltage, constant or varying. g represents the "leakance" of the condenser, i.e., the power dissipated in the condenser at any instant is gv^2.

At any given instant, say at $t = 0$, let the impressed e.m.f. be changed from any previous value to a new value $E \sin \theta$, and let this impressed e.m.f. from this instant vary sinusoidally, i.e., at any instant t thereafter, let $e = E \sin (\omega t + \theta)$, where $\omega = 2\pi f$ and f is the frequency of this e.m.f. in cycles per second. (The solution for a constant impressed e.m.f. E can be obtained by putting $f = 0$ and $\theta = \pi/2$ in the equations below.) Let I_0 and V_0 be respectively the values of the current i and of the potential drop v through the condenser at time $t = 0$. I_0 and V_0 may or may not be zero, depending upon the condition of the circuit previous to the change in the impressed e.m.f.

Steady State Relations. — The complete solution of equations (4) shows that the current and voltage at any instant after impressing the e.m.f. on the circuit are each composed of two terms, (1) a term which dies out with time and (2) a term, added to the first, which represents the final or steady (alternating) state of the current and voltage respectively. The "steady state" solution of (4) is readily effected by the ordinary methods of alternating-current calculation (*see Alternating Currents*), and is as follows:

$$\left.\begin{aligned} i_2 &= \frac{E}{Z}\sin(\omega t + \theta - \phi), \\ v_2 &= \frac{E}{yZ}\sin(\omega t + \theta - \phi - \eta), \end{aligned}\right\} \tag{5}$$

where

$$\left.\begin{aligned} Z &= \sqrt{R^2 + X^2} \quad &\text{and} \quad \phi &= \tan^{-1}\frac{X}{R}, \\ R &= r + \frac{g}{g^2 + b^2}, & X &= x - \frac{b}{g^2 + b^2}, \\ b &= \omega C, & x &= \omega L, \\ y &= \sqrt{g^2 + b^2}, & \eta &= \tan^{-1}\frac{b}{g}, \end{aligned}\right\} \tag{6}$$

Free Oscillations. — The transient terms in the complete solution of equations (4) are as follows:

$$\left.\begin{aligned} i_1 &= \epsilon^{-ut}\left[D\cos\omega_0 t - \left(\frac{D'}{\omega_0 L} + \frac{qD}{\omega_0}\right)\sin\omega_0 t\right], \\ v_1 &= \epsilon^{-ut}\left[D'\cos\omega_0 t + \left(\frac{D}{\omega_0 C} + \frac{qD'}{\omega_0}\right)\sin\omega_0 t\right], \end{aligned}\right\} \quad (7)$$

where

$$\left.\begin{aligned} u = \frac{1}{2}\left(\frac{r}{L} + \frac{g}{C}\right) \quad &\text{and} \quad q = \frac{1}{2}\left(\frac{r}{L} - \frac{g}{C}\right), \\ \omega_0 &= \sqrt{\frac{1}{LC} - q^2}\ . \end{aligned}\right\} \quad (8)$$

These three constants, u, q and ω_0 are constants of the circuit, and are independent of the manner in which the transient oscillation is set up. The frequency of this "natural" oscillation is

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - q^2}.$$

When q^2 is small compared with $\frac{1}{LC}$ the natural frequency is $f_0 = \frac{1}{2\pi\sqrt{LC}}$. Due to the resistance of the coil and leakance of the condenser, these natural oscillations die out with time proportionally with ϵ^{-ut}, or the amplitudes of both the transient current and the transient voltage decrease by the fraction ϵ^{-u} each second.

The constants D and D', which determine the amplitudes of the transient terms (equations 7) are given by the following relations:

$$\left.\begin{aligned} D &= I_0 - \frac{E}{Z}\sin(\theta - \phi), \\ D' &= V_0 - \frac{E}{yZ}\sin(\theta - \phi - \eta), \end{aligned}\right\} \quad (9)$$

where I_0 is the current in the coil and V_0 the voltage across the condenser at the instant ($t = 0$) at which any change is made in the circuit conditions, i.e., switch opened or closed, or e.m.f. short circuited.

Solution when Damping is Great. — When q^2 is greater than $1 \div (LC)$, ω_0 becomes imaginary and the oscillation therefore has an imaginary frequency, which means that the transient terms die out without oscillating. Equations (7), (8) and (9) still hold in this case and will give the *real* solution *provided* that in equation (7) "cos" is changed to "cosh" (hyperbolic cosine), and "sin" is changed to "sinh" (hyperbolic sine) and for ω_0 is taken the value $\sqrt{q^2 - \frac{1}{LC}}$. See *Hyperbolic Functions* for tables of "sinh" and "cosh." The expressions for u, q, D and D' remain unaltered.

Critical Damping. — Where $\omega_0 = 0$, that is, when $q^2 = 1 \div (LC)$, equations (7) for the transient current and voltage become

$$\left.\begin{aligned} i_1 &= \left[D\left(1 - qt\right) - \frac{D't}{L}\right]\epsilon^{-ut} \\ v_1 &= \left[\frac{Dt}{C} + D'\left(1 + qt\right)\right]\epsilon^{-ut} \end{aligned}\right\} \quad (10)$$

where D and D' are given by equations (9).

Complete Solution. — The complete expressions for the current and voltage at any instant are in all cases

$$\left.\begin{aligned} i &= i_1 + i_2, \\ v &= v_1 + v_2, \end{aligned}\right\} \qquad (11)$$

where i_2 and v_2 are given by equations (5) and i_1 and v_1 are given by equations (7), the latter changed as noted above when q^2 is greater than $1 \div (LC)$ or reducing to equation (10) when $q^2 = 1 \div (LC)$.

Discharge of a Condenser through an Impedance Coil. — The above equations may be used to obtain the solution of the special case of a condenser charged to a voltage V_0 and short-circuited at time $t = 0$ through a coil having both resistance and inductance. Assume that at time $t = 0$ the current through the condenser is zero. The "steady state" terms in this case are zero, and only the transient terms appear, viz.,

$$\left.\begin{aligned} i &= -\frac{V_0}{\omega_0 L}\epsilon^{-ut}\sin\omega_0 t, \\ v &= V_0\epsilon^{-ut}\left(\cos\omega_0 t + \frac{q}{\omega_0}\sin\omega_0 t\right), \end{aligned}\right\} \qquad (12)$$

where the values of u, q and ω_0 are as given above, equations (8). When $1 \div (LC)$ is greater than q^2, equations (12) apply directly; the current-time curve in this case is shown in Fig. 2. When $1 \div (LC)$ is less than q^2 the circular functions become hyperbolic functions and $\omega_0 = \sqrt{q^2 - \frac{1}{LC}}$ and the current-time curve is as shown in Fig. 3. When $q^2 = 1 \div (LC)$, equations (12) reduce to

$$i = -\frac{V_0}{L}t\epsilon^{-ut} \quad \text{and} \quad v = V_0\epsilon^{-ut}(1 + qt). \qquad (13)$$

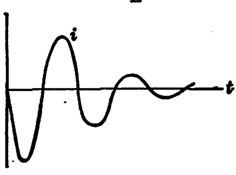

Fig. 2. $q < \frac{1}{\sqrt{LC}}$

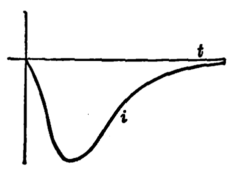

Fig. 3. $q > \frac{1}{\sqrt{LC}}$

OSCILLATIONS IN TRANSMISSION LINES. — Oscillations similar in nature to those discussed above occur in a transmission line when any change takes place in the load connections and particularly when high voltages are suddenly induced in the line by lightning discharges or when heavy currents are suddenly interrupted. The mathematical analysis of such cases is extremely involved, since account must be taken of the distributed nature of the inductance and capacity, and also of the fact that the possible periods of oscillations depend not only upon the line constants but also upon the constants of the apparatus at the two ends of the line. See article on *Transmission Lines*.

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[H. PENDER.]

TRANSMISSION LINES. — (*See also Capacity and Charging Current; Conduits and Conduit Lines, Underground; Copper; Corona, Electric; Cross Arms; Distribution Lines; Distribution and Transmission Systems; Inductance and Inductive Reactance; Insulators for Overhead Lines; Insulator Pins; Trolley Systems, Overhead; Wind Pressure; Wires and Cables, Bare; Wires and Cables, Insulated; Wiring of Buildings.*) Circuits designed for transmitting relatively large amounts of power from one fixed point to another are called transmission lines, while those for delivering small amounts at numerous points are called distribution circuits. The same type of construction is used for both kinds of lines when designed for operation at the same voltage. The formulas used in the electrical design of both distribution and transmission lines and in the mechanical design of wire spans are included in this article; see also the articles on *Trolley Systems, Overhead*, for formulas for railway distribution, and *Wiring of Buildings* for formulas for interior wiring. The construction details given in this article refer primarily to three-phase lines designed to operate at or above 13,200 volts; for the construction of low-voltage lines see *Distribution Lines.* For a discussion of the general features of the various types of distribution and transmission systems see the article on *Distribution and Transmission Systems.*

The following is a brief table of contents of this article:

DEFINITIONS AND FUNDAMENTAL RELATIONS. — The following definitions and relations apply to all types of transmission and distribution lines.

Generator End and Load End. — By the generator end of the line is meant the end which is connected to the source of power (either directly or through transformers), and by the load end is meant the end which is connected to the load or substation which is supplied with power over the line.

Per cent Power Loss (Q). — By per cent power loss as used in this article is meant the percentage ratio

$$Q = 100 \cdot \frac{\text{total power lost in the line}}{\text{total power delivered at load end}}. \qquad (1)$$

Hence if P is the power delivered, then the total power supplied to the line and load is

$$P_0 = P\left(\frac{100 + Q}{100}\right). \tag{2}$$

Per cent Voltage Loss (D). — By per cent voltage loss as used in this article is meant the percentage ratio*

$$D = 100 \cdot \frac{\text{(voltage at generator end)} - \text{(voltage at load end)}}{\text{voltage at load end}}. \tag{3}$$

Hence calling E the voltage at the load end, the voltage at the generator end is

$$E_0 = E\left(\frac{100 + D}{100}\right). \tag{4}$$

The per cent voltage loss allowed under various conditions is discussed in detail in the article on *Distribution and Transmission Systems;* the allowable voltage loss is usually between 2 and 20 per cent, the most common figure being 10 per cent.

In the case of a direct-current line with a single load at its far end, the per cent power loss and the per cent voltage loss are always equal, but in the case of an alternating-current line the per cent voltage loss may be either greater or less than the per cent power loss, depending upon the constants of the line and the power factor of the load, or may even be negative, i.e., there may be an actual rise of voltage at the load end above the voltage at the generator end; see below.

Efficiency of Transmission. — By the efficiency of transmission is meant the percentage ratio

$$100 \cdot \frac{\text{power output of line at load end}}{\text{total power input to line at generator end}}. \tag{5}$$

The per cent efficiency is related to the per cent power loss Q as follows:

$$\text{Per cent efficiency} = \frac{10{,}000}{100 + Q}. \tag{6}$$

ELECTRICAL DESIGN OF DIRECT-CURRENT LINES. — Two types of problems arise: (1) given a definite line with known constants, what is the power loss and voltage loss for a given load, and (2) to transmit a given amount of power a given distance with a given allowable loss, what will be the size and weight of the conductor required? In the following paragraphs are given the necessary formulas for the several cases.

Two-wire Line; Concentrated Load at Far End. — Let

E = volts between wires at the load end of the line,

$P = \dfrac{EI}{1000}$ = kilowatts taken by load,

$I = \dfrac{1000\,P}{E}$ = amperes taken by load,

l = length of each line wire in feet,

r = ohms per 1000 feet of conductor; see tables in article on *Wires and Cables, Bare,*

$R = \dfrac{rl}{500}$ = total resistance of line (both conductors), in ohms.

* In the case of an alternating-current line the difference in the numerator of this ratio is the algebraic difference between r.m.s. values of the two voltages, not the vector difference.

The following relations then hold:

$$\text{Total kilowatts lost} = p = \frac{RI^2}{1000} = \frac{rlI^2}{500{,}000} = \frac{2\,rlP^2}{E^2}, \tag{7}$$

$$\text{Total volts lost} = v = RI = \frac{rlI}{500} = \frac{2\,rlP}{E}, \tag{8}$$

$$\text{Per cent power loss} = Q = \frac{100\,p}{P} = \frac{rlI^2}{5000\,P} = \frac{200\,rlP}{E^2}, \tag{9}$$

$$\text{Per cent voltage loss} = D = \frac{100\,v}{E} = \frac{rlI}{5\,E} = \frac{200\,rlP}{E^2}, \tag{10}$$

$$\left.\begin{array}{l}\text{Resistance of each con-}\\ \text{ductor per 1000 feet}\end{array}\right\} = r = \frac{500\,v}{lI} = \frac{QE^2}{200\,lP} = \frac{DE^2}{200\,lP}. \tag{11}$$

Calculation of Size and Weight of Conductor for Concentrated Load. — From the value of r calculated from any one of the relations given in equation (11), the size of wire may be found from the tables in the article on *Wires and Cables, Bare;* the next larger size of wire (next smaller gauge number) should usually be chosen when the calculated resistance lies between that of two commercial sizes. The wire selected must also have sufficient current-carrying capacity; see *Wires and Cables, Bare*; *Wires and Cables, Insulated;* and *Wiring of Buildings*. For outside lines, however, the current-carrying capacity will in general be ample unless the allowable voltage loss is excessive. For an outside overhead line a wire smaller than No. 6 A.W.G. (or B. & S.) gauge is seldom used, chiefly on account of its lack of mechanical strength.

Let w = weight per 1000 feet of the wire finally selected, then

$$\text{Total weight of conductor in pounds} = W = \frac{wl}{500}. \tag{12}$$

Direct Calculation of Total Weight of Conductor (W); Two-wire Line. — For preliminary estimates it is sometimes convenient to calculate the total weight of conductor directly, without reference to a wire table. The total weight of conductor for a two-wire line with concentrated load at its end is given by the formula

$$W = \frac{KP}{Q}\left(\frac{l}{E}\right)^2 \text{ pounds}, \tag{13}$$

where P is the power taken by the load, l the length of the line (length of each wire), E the voltage at the load, Q the per cent power loss (= per cent voltage drop for 2-wire d-c. line) and K a constant depending upon the material of the conductor and the units in which P, l and E are expressed, viz.,

VALUES OF K IN FORMULA 13

Material	E in volts, l in feet, P in kilowatts	E in kilovolts, l in miles, P in kilowatts
Copper (98 per cent conductivity).......	13.5	380
Aluminum (61 per cent conductivity)...	6.5	185
Any material of specific gravity δ having a conductivity of c per cent at 20° C...	$\frac{141\,\delta}{c}$	$\frac{3940\,\delta}{c}$

NOTE. — The values of K given for copper and aluminum are about 5 per cent greater than their theoretical values to allow for stranding, higher working temperature, etc.; 100 per cent conductivity corresponds to 1.724 microhms per centimeter cube at 20° C.

Example: Two-wire D-C. Line, Concentrated Load.—A load of 100 kw. is to be transmitted over a two-wire line to a motor operating at 230 volts, the motor being 1000 feet from the power-house switchboard. For a 10 per cent power loss or voltage drop in the line, the approximate total weight of copper required is, from equation (13),

$$W = \frac{13.5 \times 100}{10}\left(\frac{1000}{230}\right)^2 = 2550 \text{ pounds.}$$

From equation (11) the resistance per 1000 feet is

$$r = \frac{10 \times (230)^2}{200 \times 1000 \times 100} = 0.0264 \text{ ohm per 1000 feet.}$$

The nearest even circular mil size is 400,000 circular mils (stranded), which has a resistance of 0.0270 ohm per 1000 feet at 77° F. (*see Wires and Cables, Bare*), and a weight of 1240 pounds per 1000 feet. From equation (12) the total weight of conductor is then

$$W = \frac{1240 \times 1000}{500} = 2480 \text{ pounds.}$$

This wire, if bare, weather-proofed, or insulated with paper or varnished cambric, will safely carry the required current of 100,000/230 = 435 amperes, but if rubber insulated and mounted indoors, a larger wire should be required, viz., 600,000 circular mils, according to the National Electric Code (*see Wires and Cables, Bare*).

Calculation of Two-wire Direct-current Line in Terms of Voltage at Generator End.—When the volts E_0 at the generator end are given instead of the volts E at the load end, the calculations for a line of given total resistance of R ohms with a concentrated load of P kilowatts at the load end may be made in the same manner as above by first finding the volts E at the load by the formula

$$E = \frac{E_0}{2}\left[1 + \sqrt{1 - \frac{4000\,RP}{E_0^2}}\right]. \tag{14}$$

For an efficiency of transmission of less than 50 per cent, the sign before the radical should be − instead of +, but an efficiency of less than 50 per cent practically never occurs in power transmission. It is of interest to note that for an efficiency of 50 per cent $P = E_0^2 \div (4000\,R)$ which is the maximum power which can be delivered at the far end of the line for a given impressed voltage at the generator end.

When E has been calculated by this formula (14), the formulas (7) to (13) above may be applied directly.

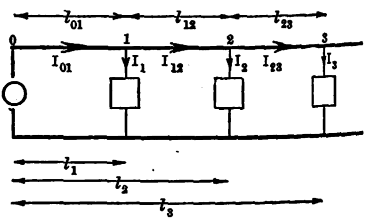


Fig. 1.

Two-wire Line; Distributed Load.—When a line supplies a number of loads at different distances from the generator end, the voltage loss to the far end of the line is the same as would be produced by a load, concentrated at the "center of gravity" of the line and taking a current equal to the total current taken by all the loads. The center of gravity of the line is defined as follows: Let I_1, I_2, I_3, etc., be the currents taken from the line by the various loads, Fig. 1, and let R_1, R_2, R_3, etc., be the total line resistances (both wires) from the generator end to the respective loads, and put

$$I = I_1 + I_2 + I_3 + \ldots \tag{15}$$

Then the center of gravity is that point between which and the generator end of the line the total line resistance is

$$R_g = \frac{R_1I_1 + R_2I_2 + R_3I_3 + \ldots}{I}. \tag{16}$$

When the line conductor has the same cross-section throughout its length, then the center of gravity is at the distance,

$$l_g = \frac{l_1I_1 + l_2I_2 + l_3I_3 + \ldots}{I} \tag{16a}$$

from the generator end, where l_1, l_2, l_3, etc., are the distances of the respective loads from the generator end.

The total voltage loss to the far end of the line is then

$$v = R_gI = \frac{rl_gI}{500}, \tag{17}$$

where the distance l_g is in feet and r is the resistance of the conductor per 1000 feet, the second relation in (17) holding only when the conductor has the same cross-section throughout its length.

The total kilowatts lost in the line are

$$p = \frac{1}{1000}\left(R_{01}I_{01}^2 + R_{12}I_{12}^2 + R_{23}I_{23}^2 + \ldots\right), \tag{18}$$

where, referring to Fig. 1, R_{01} = total resistance (both wires) from 0 to 1, R_{12} = total resistance (both wires) from 1 to 2, etc., and $I_{01} = I_1 + I_2 + I_3 + \ldots$ = the current in the line from 0 to 1, $I_{12} = I_2 + I_3 + \ldots$ = the current in the line from 1 to 2, etc. When the cross-section of the line conductor is the same throughout its length equation (18) may be also written

$$p = \frac{r}{500{,}000}\left(l_{01}I_{01}^2 + l_{12}I_{12}^2 + l_{23}I_{23}^2 + \ldots\right), \tag{18a}$$

where the distances l_{01}, l_{12}, l_{23}, etc., are as shown in Fig. 1, and are measured in feet, and r is the resistance of the line conductor per 1000 feet.

Calculation of Size and Weight of Conductor for Distributed Load. — For a conductor of the same cross-section throughout, the required resistance per 1000 feet for a given voltage loss of v volts to the end of the line may be calculated from the formula

$$r = \frac{500\,v}{l_gI}, \tag{19}$$

where l_g, expressed in feet, and I are given by formulas (16a) and (15). The size and weight of conductor can then be found by reference to the wire tables in the article on *Wires and Cables, Bare.*

When the loads are far apart and the smaller loads are farthest from the generator, it is sometimes advisable to use different sizes of conductors for the various portions of the line. For a given voltage loss v to the end of the line, the minimum weight of conductor is obtained when the volts lost per unit length of conductor in each section of the line is proportional to the square root of the current in this portion of the line. For minimum total weight of conductor then, referring to Fig. 1, the resistance per 1000 feet *of wire* for the section between 1 and 2, say, must be

$$r_{12} = \frac{1}{\sqrt{I_{12}}} \cdot \frac{500\,v}{l_{01}\sqrt{I_{01}} + l_{12}\sqrt{I_{12}} + l_{23}\sqrt{I_{23}} + \ldots}, \tag{20}$$

where the lengths are in feet; and similarly for the other sections. The weight of wire for each section may then be found by reference to the wire tables in the article on *Wires and Cables, Bare,* and the total weight W can then be computed. Vice versa, for a line proportioned in this manner the voltage loss to the end of the line for a given total weight of copper will be a minimum.

A line proportioned in this manner, however, does not give minimum power loss for the total weight of conductor used. For a given total weight of conductor the total power loss will be a minimum when the power loss per unit length of conductor in each section of the line is directly proportional to the current in this section, i.e., when the

voltage loss per unit length in each section is the same, which means that the weight per unit length of each section must be proportional to the line current in this section. Whence letting W be the total weight of the conductor in pounds, then the power loss will be a minimum when the section from 1 to 2, say, has a weight in pounds per 1000 feet of

$$w_{12} = I_{12} \frac{500\,W}{l_{01}I_{01} + l_{12}I_{12} + l_{23}I_{23} + \ldots}, \tag{21}$$

where the lengths are in feet; and similarly for the other sections. The size of wire for each section may be obtained from the wire tables in the article on *Wires and Cables, Bare*. When the sizes as found by the two formulas (20) and (21) differ considerably, the choice will depend upon which is the more important, minimum power loss or minimum voltage loss.

Three-wire Direct-current Line. — When a three-wire circuit is exactly balanced, i.e., when the loads between each of the two outer wires and the neutral are the same and are connected to the neutral at the same point or points, no current flows in the neutral wire. The formulas given above for a two-wire line then apply directly to the case of a balanced three-wire line, noting however that the E (= volts between wires) in these formulas is to be taken as the volts between the *outer* wires, and that the weight as calculated by the above formulas is the weight of the two outer wires. The neutral wire is usually made equal in size to each outer wire, but when only slight unbalancing is expected it is sometimes made smaller. When the neutral is made equal in size to the outer wire the total weight of the three conductors will be 50 per cent more than that given by formula (13), when E in this formula is taken equal to the volts between the outer wires.

The exact calculation of the voltage loss and power loss when the loads on the two sides of the system are different and are connected at different points is somewhat complicated, but can always be effected by the application of Kirchhoff's Laws for an electrical network; see *Electricity and Magnetism, Principles of*.

SIZE AND WEIGHT OF CONDUCTORS FOR ALTERNATING-CURRENT LINES. — As a rough guide in fixing upon a preliminary design, the following facts should be noted; complete formulas for the various calculations required are given later.

1. A power loss of approximately 10 per cent of the delivered power is usually allowed.

2. A line voltage of approximately 1000 volts per mile of line is common practice for long-distance lines not over 150 miles in length; that is, for a 10-mile line a line voltage of 10,000 volts would be employed; for a 100-mile line a line voltage of 100,000 volts would be used. The maximum line voltage at present (1914) employed is 150,000 volts and the maximum distance of transmission is 240 miles.

3. On the basis of 1000 volts per mile of line, unity power factor at the load, a 10 per cent power loss, and copper at 15 cents per pound, the cost of the copper required for a three-phase line is \$4.00 per kilowatt delivered, and for a single-phase or two-phase four-wire line \$5.33 per kilowatt delivered.

Calculation of Total Weight of Conductor for A-C. Lines. — The size and total weight of the conductor required for any conditions * of length, power delivered, power factor, line voltage and power loss may be calculated as follows:

* These formulas are based on the assumption that the charging current is negligible in comparison with the load current, which condition is practically realized in all but the longest high-voltage lines; formulas for power loss taking the charging current into account are given later.

Let

E = voltage between wires at the load end of the line,
P = total power taken by all phases of the load,
$\cos\phi$ = power factor of load, as a decimal,
l = length of line (= length of each line wire),
Q = allowable total power loss in per cent of delivered power.

Then the total weight of all conductors is given by the formula

$$W = \frac{KP}{Q}\left(\frac{l}{E\cos\phi}\right)^2 \quad \text{pounds,} \qquad (22)$$

where K is a constant depending upon the number of phases and wires, the material of the conductor and the units in which the various quantities are expressed, viz.,

VALUES OF K IN FORMULA 22

Material and units	Single-phase or balanced 4-wire 2-phase*	Balanced 3-wire 3-phase
Copper (98 per cent conductivity):		
E in volts, l in feet, P in kilowatts ...	13.5	10
E in kilovolts, l in miles, P in kilowatts	380	285
Aluminum (61 per cent conductivity):		
E in volts, l in feet, P in kilovolts....	6.5	4.9
E in kilovolts, l in miles, P in kilowatts	185	140
Any material of specific gravity δ having a conductivity of c per cent at 20° C.:		
E in volts, l in feet, P in kilowatts....	$\frac{141\,\delta}{c}$	$\frac{106\,\delta}{c}$
E in kilovolts, l in miles, P in kilowatts	$\frac{3940\,\delta}{c}$	$\frac{2950\,\delta}{c}$

NOTE. — The values of K given for copper and aluminum are taken about 5 per cent greater than their theoretical values to allow for stranding, higher working temperatures, etc. 100 per cent conductivity corresponds to 1.724 microhms per centimeter cube at 20° C.

* For a 3-wire 2-phase system with the middle conductor having a cross-section equal to $\sqrt{2}$ times the cross-section of either outer wire multiply these constants by 0.85, taking for E the voltage (volts or kilovolts) between the middle wire and either outer wire.

Calculation of Commercial Size of Conductor and Corresponding Total Weight. — Formula (22) takes no account of the available commercial sizes of wire. These sizes differ successively by approximately 25 per cent in cross-section. The weight can also be determined by calculating the resistance of the required conductor per 1000 feet or per mile, and taking from the wire tables in the article on *Wires and Cables, Bare,* the nearest commercial size. Neglecting the charging current, the required resistance per unit length of wire is

$$r = \frac{K_1 Q\,(E\cos\phi)^2}{lP} \quad \text{ohms,} \qquad (23)$$

where K_1 is a constant depending on the number of phases and wires and the units in which the other quantities are expressed, as given in the table below.

From the wire table the corresponding weight (w) per unit length of conductor having a resistance nearest to that calculated by formula (23) is obtained; the total weight of conductor, including all wires, is then

$$W = K_2wl \qquad \text{pounds,} \tag{24}$$

where K_2 is a constant depending upon the number of phases and wires and the units in which w and l are expressed, as given in the table below.

VALUES OF K_1 AND K_2 IN FORMULAS 23 AND 24

Units	Single-phase	Balanced 4-wire 2-phase*	Balanced 3-wire 3-phase
E in volts, l in feet, P in kilowatts, r in ohms per 1000 feet and w in pounds per 1000 feet.	$K_1 = 0.005$ $K_2 = 0.002$	$K_1 = 0.01$ $K_2 = 0.004$	$K_1 = 0.01$ $K_2 = 0.003$
E in kilovolts, l in miles, P in kilowatts, r in ohms per mile and w in pounds per mile.	$K_1 = 5$ $K_2 = 2$	$K_1 = 10$ $K_2 = 4$	$K_1 = 10$ $K_2 = 3$

* The values of K_1 given in this column when used in formula (23) will give the resistance per 1000 feet or per mile of either outer wire in a 3-wire 2-phase system; the middle wire should, for the same energy loss per pound, have a cross-section 41 per cent greater than either outer, but when commercial sizes (B. & S. gage) are used, either a wire one gage number smaller (25 per cent greater cross-section) or two gage numbers smaller (60 per cent greater cross-section) may be used; in the first case the corresponding value of K_2 is 0.81 times the values given in this column and in the second case 0.90 times the values given in this column.

Current per Wire; Heating of Line Conductors. — The size of wire as determined from formula (23) must be ample to carry the required current without overheating. Heating of the line conductors is seldom a limitation in outside overhead lines, but for inside wiring or underground cables the temperature rise may set a limit to the size of wire which may be used. It is therefore always wise to determine the current which the conductor must carry, and make sure that the wire is sufficiently large not to overheat; see articles on *Wires and Cables, Bare; Wires and Cables, Insulated;* and *Wiring of Buildings,* for tables of current-carrying capacity under various conditions.

The current per line wire in amperes may be calculated from the following formulas, in which E *is the kilovolts* between wires at the load end, P *the total kilowatts* (all phases) delivered to the load, and $\cos\phi$ the power factor of the load as a fraction.

$$\left.\begin{array}{ll} \text{Single-phase:} & I = \dfrac{P}{E\cos\phi}. \\[2ex] \text{Two-phase,* 4-wire, balanced:} & I = \dfrac{P}{2\,E\cos\phi}. \\[2ex] \text{Three-phase, 3-wire, balanced:} & I = \dfrac{P}{\sqrt{3}E\cos\phi}. \end{array}\right\} \tag{25}$$

Example: Calculation of Weight and Size of Conductor for a Three-phase Line. — A load of 20,000 kilowatts is to be transmitted by means

* E is here the volts between the two wires of the same phase. This formula also gives the current in each outer of a balanced 2-phase 3-wire line, E being the kilovolts between either outer and the middle wire; the current in the middle is $\sqrt{2}$ times the current in each outer.

of an overhead three-phase line of copper wire to a substation 50 miles away operating at 60,000 volts between wires, the frequency being 25 cycles per second, and the power factor of the load 80 per cent with the current lagging; a power loss of 10 per cent of the delivered power to be allowed. From formula (22) the required total weight of copper is

$$W = \frac{285 \times 20{,}000}{10}\left(\frac{50}{60 \times 0.8}\right)^2 = 618{,}000 \text{ pounds}.$$

From formula (23) the required resistance per mile of conductor is

$$r = \frac{10 \times 10(60 \times 0.8)^2}{50 \times 20{,}000} = 0.231 \text{ ohm per mile}.$$

The nearest commercial size is 250,000 circular mils (stranded), which has a resistance of 0.228 ohm per mile at 77° F. (*see Wires and Cables, Bare*), and a weight of 4080 pounds per mile. From formula (24) the total weight is then

$$W = 3 \times 4080 \times 50 = 612{,}000 \text{ pounds}.$$

The current corresponding to the given load is, from formula (25),

$$I = \frac{20{,}000}{\sqrt{3} \times 60 \times 0.8} = 241 \text{ amperes},$$

which will give a negligible temperature rise in the wire. See section on *Current-carrying Capacity* in article on *Wires and Cables, Bare*.

Calculation of Size and Weight of Conductors for a Given Per cent Voltage Loss. — The voltage loss in an alternating-current line depends not only upon the resistance of the line, but also upon the line reactance, and in the case of long lines upon the electrostatic capacity of the line. It is therefore impossible to express directly in a simple formula the size or weight of the wire in terms of the voltage loss. The most practical method of making such calculations is to assume first that the per cent power loss is equal to the given per cent voltage loss, and calculate the size by formula (23); then using this size of wire calculate the per cent voltage loss by the formulas given below. If this calculated voltage loss differs appreciably from the given voltage loss, choose the next larger or smaller size of wire (accordingly as the calculated loss is greater or less than the given loss) and recalculate the voltage loss, and so on, until the proper size of wire has been found.

FACTORS WHICH AFFECT THE VOLTAGE AND POWER LOSS IN A-C. LINES. — Due to the inductance and electrostatic capacity the per cent voltage loss in an alternating-current line is not so easily calculated as the voltage loss in a direct-current line and is in general different from the per cent power loss, by an amount dependent upon the inductance and capacity of the line, the frequency and the power factor of the load.

Determination of Line Constants. — The four fundamental line constants are the resistance (r) and inductance (L) of the line conductors per unit length and the capacity (C) and leakage conductance (G) per unit length. For all but the shortest transmission lines the mile is usually the most convenient unit of length, and this unit will be used throughout the remainder of this article unless distinctly stated otherwise. Tables of resistance, inductance and capacity both per mile and per 1000 feet are given respectively in the articles on *Wires and Cables, Bare; Inductance and Inductive Reactance;* and *Capacity and Charging Current.* From the inductance and capacity per mile or per 1000 feet may be calculated for any given frequency the reactance x ($= 2\pi fL$) and the capacity

susceptance* b ($= 2\pi fC$) for the corresponding unit of length. In the last two articles just mentioned are given full tables of reactance and of capacity susceptance per mile for frequencies of 60 and 25 cycles per second; dividing the numerical values given in these tables by 5.28 will give the corresponding quantity per 1000 feet. For any other frequency of f cycles per second, multiply the numerical values given in the tables for 25 cycles by the ratio $f/25$; i.e., for 40 cycles the reactance is 1.6 times the reactance for 25 cycles.

Allowance for Skin Effect in Conductors. — For non-magnetic wires the increase in the conductor resistance due to the so-called skin effect is equal to 2 per cent when the quotient

$$\text{(cycles per sec.)} \div \text{(ohms per mile of conductor)} = 485. \tag{26}$$

For smaller values of this quotient the increase of resistance due to skin effect diminishes very rapidly; see article on *Skin Effect.* The skin effect is therefore practically negligible at 25 cycles for all copper conductors smaller than 1,000,000 circular mils, and at 60 cycles for all copper conductors smaller than 450,000 circular mils. The corresponding limiting sizes for aluminum are about 30 per cent larger. The skin effect is quite appreciable in copper or aluminum cables with a steel core; it is usual to neglect the conductivity of the steel core entirely in calculating the resistance of such cables.

Apparent Resistance and Reactance in Unsymmetrical Arrangements of Wires. — When the three wires of a three-phase line are so arranged that they form the three edges of an equilateral prism the reactance of each wire is the same as for one wire of a two-wire line. However, when the wires are arranged all in one plane, as is frequently done, the unequal mutual induction sets up a reactive electromotive force in each outer wire which is not in quadrature with the current in this wire; see equation (20) in the article on *Inductance and Inductive Reactance.* As a result, both the apparent resistance and the apparent reactance of each outer wire is different from its true resistance and reactance. Let r = the true resistance per mile of each wire in ohms, x = the reactance per mile of each wire in ohms, as given in the tables in the article on *Inductance and Inductive Reactance;* and f = the frequency in cycles per second, then the apparent resistances and reactances per mile of the three wires, No. 2 being the middle wire, are:

$$\left.\begin{array}{lll} r_1 = r + 0.00121f, & r_2 = r, & r_3 = r - 0.00121f, \\ x_1 = x - 0.00070f, & x_2 = x, & x_3 = x - 0.00070f. \end{array}\right\} \tag{27}$$

The changes in the apparent resistances do not indicate any change in the power dissipated as heat in the wires but a transfer of energy from one wire to the other by the magnetic field surrounding the wires. These relations are deduced from equation (20) in the article on *Inductance and Inductive Reactance*, assuming sine-wave currents equal in effective value and differing in phase by exactly 120°. The assumption that the currents are exactly balanced cannot be strictly true, since the inequality in the apparent resistances and reactances of the three wires tends to unbalance the system, but the values just given may be taken as a fair approximation when the voltage loss in the line is not over 10 per cent, say. When the line wires are transposed these mutual inductance effects are eliminated from the line as a whole, though the apparent impedances of the three wires in any one "exposure" of the transposition will be different; the transpositions, however, keep the currents balanced.

Similar effects take place in a two-phase three-wire line, see p. 1671.

Leakage Conductance. — The leakage current, even at very high voltages, is usually negligible in power transmission lines, but for telephone lines, the leakage is much greater, due to the large number of small insulators used, and has a very appreciable effect on both the attenuation and distortion of the voice currents. McMeen (*see article on Telephone Lines*) gives the fol-

* The capacity susceptance to neutral as given in the tables (i.e., in micromhos per mile) is equal to the charging current in amperes per mile per million volts between wire and neutral; the charging current for any other voltage to neutral is in proportion.

lowing values of the leakage conductance of telephone lines, the conductance being from one wire to neutral,

$$\left.\begin{array}{lll} \text{Very dry} & g = 0.004 & \text{micromhos per mile.} \\ \text{Average} & g = 0.08 & \text{micromhos per mile.} \\ \text{Very wet} & g = 1.0 & \text{micromhos per mile.} \end{array}\right\} \quad (28)$$

The leakage conductance from one wire to the other is one-half these values. Note that A megohms per mile equals $1/A$ micromhos per mile.

When the voltage is sufficiently high on a power line to cause the formation of corona (*see article on Corona*), an appreciable leakage current passes from one wire to the other. Even for a sine-wave voltage this leakage current is by no means sinusoidal, since its instantaneous values are practically zero except during the peak of the voltage wave, and consequently the corona loss cannot be accurately represented by a constant leakage conductance. Roughly, however, calling p_c the average value of the corona loss from each wire in watts per mile, corresponding to the given line voltage, the leakage conductance to neutral in micromhos per mile due to the corona may be taken equal to

$$g_c = \frac{p_c}{V^2}, \qquad (29)$$

where V is the effective (r.m.s.) kilovolts to neutral.

Rise of Voltage at Load End of Line on Open Circuit. — In every alternating-current transmission line the voltage at the load end when this end is open is higher than at the generator end, although in short low-frequency lines this rise is inappreciable. In overhead lines for which the product

$$(\text{cycles per sec.}) \times (\text{length of line* in miles}) < 10{,}000, \qquad (30)$$

this no-load rise as a percentage of the delivered voltage is, to a close approximation when the resistance of the wire is less than that of a No. 0 B. & S. copper wire, equal to $\left(\frac{fl}{4000}\right)^2$, where f is the frequency in cycles per second, and l the length of the line in miles. For example, in a 25-cycle line 160 miles long this no-load rise is 1 per cent of the delivered voltage, in a 60-cycle line of the same length the no-load rise in voltage is 5.8 per cent of the delivered voltage. The relation expressed by the above formula under the conditions stated is independent of the value of the delivered voltage and of the size and spacing of the wires, at least for all practical cases.

This rise, which is due to the charging current taken by the line, may be looked upon as present at all loads, but when the load is appreciable the voltage drop, due to the load current, unless leading, more than offsets this voltage rise. A leading current may increase the rise in voltage at the load end as the load comes on.

Use of Synchronous Condensers (or Phase Modifiers) to Maintain Constant Voltage at Load End. — By making the line current at the load end of the line lead the line voltage by the proper phase angle, it is possible to compensate entirely for the change in the load voltage which normally takes place as the load current increases. An overexcited synchronous motor connected in parallel with the load is sometimes used for this purpose, as described in detail in the article on *Motors, Synchronous*. A synchronous motor so used is commonly called a synchronous condenser, since the current taken by it leads the voltage impressed on its terminals. A striking example of the use of synchronous motors for this purpose is in connection with the 150,000-volt, 240-mile

* Length of each conductor.

line from Big Creek, Cal., to Los Angeles, constructed in 1913. Two separate lines are used, the total generator output is 70,000 kilovolt-amperes, and the voltage at the load end is controlled by two 15,000-kilovolt-ampere overexcited synchronous motors.

METHODS OF CALCULATING VOLTAGE AND POWER LOSS IN A-C. LINES.—The absolutely rigorous calculation of an alternating-current line requires that the distributed nature of the inductance and capacity be considered, i.e., that the line be considered as made up of an infinite number of sections such as shown in Fig. 7. However, simpler approximate methods may be employed for nearly all power lines such as are now used, and give results sufficiently accurate for all practical purposes. The accuracy of these approximate methods depends primarily upon the frequency and length of the line; the less the value of the product of these two quantities the simpler is the method which may be employed. Accurate calculation of even a short a-c. line with distributed load can be effected only by rather complicated network equations; see under *Kirchhoff's Laws* in the article on *Alternating Currents*.

Limitations of Approximate Methods of A-C. Line Calculations.—The approximate methods of calculating a-c. lines, in the order of their simplicity, may be designated as (1) the simple impedance method; (2) the single end-condenser method; (3) the middle condenser or "T" method and (4) the split condenser or "π" method; the "π" method is also called the "U" method. For short low-voltage lines, the simple impedance method, which neglects entirely the charging current, is usually sufficiently accurate. The single end-condenser method takes the charging current into account in a manner usually sufficiently accurate for all but the longest high-voltage lines. The "T" and "π" methods are still more accurate, but for exact calculations the rigorous method given on p. 1681ff should be used. In fact, it is always well to check an approximate solution by this exact method.

Fundamental Assumptions on which Formulas Given Below are Based.—All the formulas given below are based on the assumptions of pure sine-wave currents and voltages and a perfectly balanced* system. By using the voltage to neutral instead of the voltage between wires, the formulas are also put in such shape that they may be applied directly to either a single-phase, a two-phase four-wire line or to a three-phase three-wire line, a two-phase four-wire line being considered as two separate single-phase lines. The fundamental idea in this method of treatment is that each line wire is considered as a separate circuit. The return wire shown in the various diagrams is therefore to be considered as having no impedance. This method of treatment of *balanced* polyphase circuits is strictly accurate (*see Alternating Currents*), but when the system is not balanced the circuit must be treated by the more general methods of network calculations; see section on *Kirchhoff's Laws in Symbolic Notation* in the article on *Alternating Currents*. Also, when the voltage and current waves are not sinusoidal, each harmonic must be treated separately as described in the article on *Alternating Currents*. The calculation of a two-phase three-wire line is treated separately, p. 1671.

SIMPLE IMPEDANCE METHOD.—This method is based upon the assumption that the electrostatic capacity of the line may be neglected entirely, and that the line may be considered simply as an impedance in series with the load, this impedance having a resistance equal to the total resistance of the line conductor and a reactance equal to the total inductive reactance of the line

* The meaning of a "balanced" system is fully explained in the article on *Alternating Currents* (q.v.).

conductor. Fig. 2 is a diagram of the circuit, and Fig. 2a is a complete vector diagram of the current and voltage. When the wires are unsymmetrically arranged transpositions are assumed.

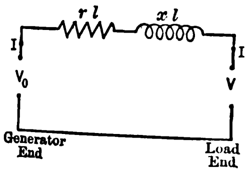


Fig. 2.

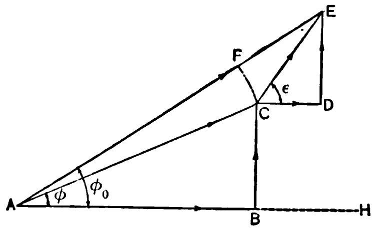


Fig. 2a.

$\overline{AH} = I,\quad \overline{BC} = V \sin\phi,\quad \overline{CE} = zlI,$
$\overline{AC} = V,\quad \overline{CD} = rlI,\quad \overline{AE} = V_0,$
$\overline{AB} = V\cos\phi,\quad \overline{DE} = xlI,\quad \overline{FE} = V_0 - V$

Let V = volts to neutral at load end of the line (= volts between wires divided by 2 in case of a single-phase line, and volts between wires divided by $\sqrt{3}$ in the case of a three-phase line),
I = amperes per wire; see formula (25) above,
l = length of each line wire in miles,
$Z = \dfrac{V}{lI}$ = "equivalent" impedance of the load per mile of line,
$\cos\phi$ = power factor of the load at end of line,
$\sin\phi = \sqrt{1 - \cos^2\phi}$ = the reactive factor of the load; $\sin\phi$ is to be taken positive for a lagging and negative for a leading current,
V_0 = volts to neutral at the generator end of the line,
r = conductor resistance per mile of line in ohms; see tables in article on *Wires and Cables, Bare,*
x = conductor reactance per mile of line, in ohms; see tables in article on *Inductance and Inductive Reactance,*
$z = \sqrt{r^2 + x^2}$ = conductor impedance* per mile of line, in ohms,
Q = per cent power loss, as a percentage of delivered power,
D = per cent voltage loss, as a percentage of delivered voltage.

From the vector diagram it is evident that the voltage at the generator end is

$$V_0 = \sqrt{(V\cos\phi + rlI)^2 + (V\sin\phi + xlI)^2}, \tag{31}$$

* A convenient way of calculating an expression of the form $\sqrt{a^2 + b^2}$ is to write it $a\sqrt{1 + \left(\frac{b}{a}\right)^2}$ or $b\sqrt{1 + \left(\frac{a}{b}\right)^2}$ accordingly as a is greater or less than b; the expression under the radical will then always lie between the numbers 1 and 2, and no difficulty will be experienced with decimal points. When b/a is less than 0.3, then the expression $\sqrt{a^2 + b^2} = a + \dfrac{b^2}{2a}$ with an error of less than 0.1 per cent, and when a/b is less than 0.3, then $\sqrt{a^2 + b^2} = b + \dfrac{a^2}{2b}$ with an error of less than 0.1 per cent. The error in the approximate expressions, diminishes very rapidly as the ratio of b/a or a/b, as the case may be, decreases, being only 0.02 per cent when the ratio is 0.2.

which may also be written

$$V_0 = V\sqrt{\left(\cos\phi + \frac{r}{Z}\right)^2 + \left(\sin\phi + \frac{x}{Z}\right)^2}, \tag{32}$$

where r and x are per mile of line.

The current at the generator end is the same as at the load end.

The per cent power loss is

$$Q = \frac{100\,rlI}{V\cos\phi} = \frac{100\,r}{Z\cos\phi}, \tag{33}$$

and the per cent voltage loss is

$$D = \frac{100\,(V_0 - V)}{V} = 100\left[\sqrt{\left(\cos\phi + \frac{r}{Z}\right)^2 + \left(\sin\phi + \frac{x}{Z}\right)^2} - 1\right]. \tag{34}$$

The power factor at the generator end is

$$\cos\phi_0 = \left(\frac{100 + Q}{100 + D}\right)\cos\phi. \tag{35}$$

Relation between Impedance Drop and Voltage Loss.—The total impedance drop, which is zlI volts, should be carefully distinguished from the voltage loss, which is $v = V_0 - V$ volts. The vector diagram, Fig. 2a, will make the difference clear. For a given impedance drop of, say, A per cent, the voltage at the load end of the line may be anything from A per cent less than the voltage at the generator end to A per cent greater than the voltage at the generator end. The determining factor is the difference between the power-factor angle (ϕ) of the load and the power-factor angle $\left(\epsilon = \tan^{-1}\frac{x}{r}\right)$ of the line; only when $\epsilon - \phi = 0$ are the voltage loss and impedance drop the same. When $\epsilon - \phi$ is greater than 90° (which may occur for a leading current, since ϕ is then negative) the voltage at the load end will in general be higher than at the generator end although the impedance drop in the line may be very large. As a fair approximation, when the impedance drop is less than 20 per cent, that is, when z/Z is less than 0.2, the percentage voltage loss may be written

$$D = \frac{100\,z}{Z}\cos(\epsilon - \phi), \tag{36}$$

z and Z being the impedances of the line and load respectively, and ϵ and ϕ the power-factor angles of the line and load respectively.

Example of Calculation by Simple Impedance Method.—Take the case of a three-phase, 60-cycle line 50 miles long, the wires being No. 0000 A.W.G. (or B. & S.) stranded copper spaced symmetrically with 6 feet between centers, and let the load be 15,000 kilowatts at 60,000 volts between wires and at a power factor of 80 per cent with lagging current. The voltage to neutral is then $60{,}000 \div \sqrt{3} = 34{,}600$ and the current per wire $15{,}000 \div (\sqrt{3} \times 60 \times 0.8) = 180$ amperes. The resistance of each wire per mile is 0.269 ohm at 77° F., and the reactance 0.728 ohm. The equivalent impedance of the load per mile of line is $Z = 34{,}600 \div (50 \times 180) = 3.84$. $\cos\phi = 0.8$ and $\sin\phi = \sqrt{1 - \overline{0.8}^2} = 0.6$. Whence

$$\text{Per cent power loss} = Q = \frac{100 \times 0.269}{3.84 \times 0.8} = 8.76 \text{ per cent},$$

$$\text{Per cent voltage loss} = D = 100\left[\sqrt{\left(0.8 + \frac{0.269}{3.84}\right)^2 + \left(0.6 + \frac{0.728}{3.84}\right)^2} - 1\right] = 17.5\%,$$

$$\text{Power factor at generator end} = \cos\phi_0 = \left(\frac{100+8.76}{100+17.5}\right)\times 0.80 = 74.1 \text{ per cent,}$$

$$\text{Per cent impedance drop} = \frac{100\sqrt{0.269^2+0.728^2}}{3.84} = 20.2 \text{ per cent.}$$

Graphical Determination of Voltage Loss; Mershon's Chart (Fig. 3). — The voltage loss may also be determined by means of the chart shown in Fig. 3, which was devised by R. D. Mershon. This chart is nothing more than a means of solving equation (32) graphically. To use the chart calculate

$$\text{the per cent resistance drop} = r\left(\frac{100\, lI}{V}\right), \qquad (37)$$

$$\text{the per cent reactance drop} = x\left(\frac{100\, lI}{V}\right), \qquad (38)$$

and from the point on the curve marked "0" where this curve cuts the vertical line corresponding to the load power factor lay off horizontally the per cent resistance drop, and from the end of this horizontal line lay off vertically upward the per cent reactance drop; the per cent voltage loss is then given by the number on the circle through the end of this vertical line.

Taking the same example as given in the preceding section, $(100\, lI) \div V = (100 \times 50 \times 180) \div 34{,}600 = 26.0$, whence the per cent resistance drop is $0.269 \times 26.0 = 7.0$ per cent and the per cent reactance drop is $0.728 \times 26.0 = 18.9$ and therefore from the chart the per cent voltage loss is 17.6 per cent.

Calculations in Terms of Voltage at Generator End. — When the volts to neutral V_0 at the generator end are given instead of the volts to neutral V at the load end, the calculations for a line of total resistance of R ohms per wire, and total reactance of X ohms per wire for a load of P kilowatts per wire* at a power factor of $\cos\phi$ at the load end may be made in the same manner as above by first finding V by the formula,

$$V = A\sqrt{1 \pm \sqrt{1 - \frac{(R^2+X^2)\,P^2 \times 10^6}{A^4\cos^2\phi}}}, \qquad (39)$$

where

$$A = V_0\sqrt{\frac{1}{2} - \frac{1000\,P\,(R\cos\phi + X\sin\phi)}{V_0^2\cos\phi}}. \qquad (40)$$

The plus or minus sign in the formula for V arises from the fact that for a given amount of delivered power and given voltage at the generator end two different voltages at the load end are possible; for an inductive load the voltage corresponding to the minus sign will in general be less than 50 per cent of the voltage at the generator end, and the "minus sign" solution is therefore usually of no practical importance. The maximum power which can be delivered at the load end for a given voltage at the generator end is

$$P_m = \frac{V_0\cos\phi}{2000\,(\sqrt{R^2+X^2} + R\cos\phi + X\sin\phi)}. \qquad (41)$$

Voltage and Power Loss in Two-phase Three-wire Line. — Take the usual case of the three wires in the same plane, with the common wire midway between the two outers. Let r be the resistance per mile of each outer, and r_1 the resistance per mile of the common wire, taken from the tables in the article on *Wires and Cables, Bare*, and x and x_1 the corresponding reactances per mile taken from the tables in the article on *Inductance and Inductive Reactance*, for a spacing equal to the distance between the middle wire and each outer. Let E be the voltage between each outer and the middle wire, I the amperes in each outer, $\cos\phi$ the power factor of the load, and l the length of the line. Assuming a balanced load, then the voltage between the two outers is $\sqrt{2}\,E$ and the current in the middle wire is $\sqrt{2}\,I$, and the total power delivered to the load is $(2\,EI\cos\phi) \div 1000$ kilowatts. The per cent power loss is then

$$Q = \frac{100\,(r + r_1)\,lI}{E\cos\phi}.$$

* Calling P_t the total kilowatts delivered to the load, then $P = P_t/2$ for a single-phase line, $P = P_t/3$ for a three-phase line and $P = P_t/4$ for a two-phase four-wire line,

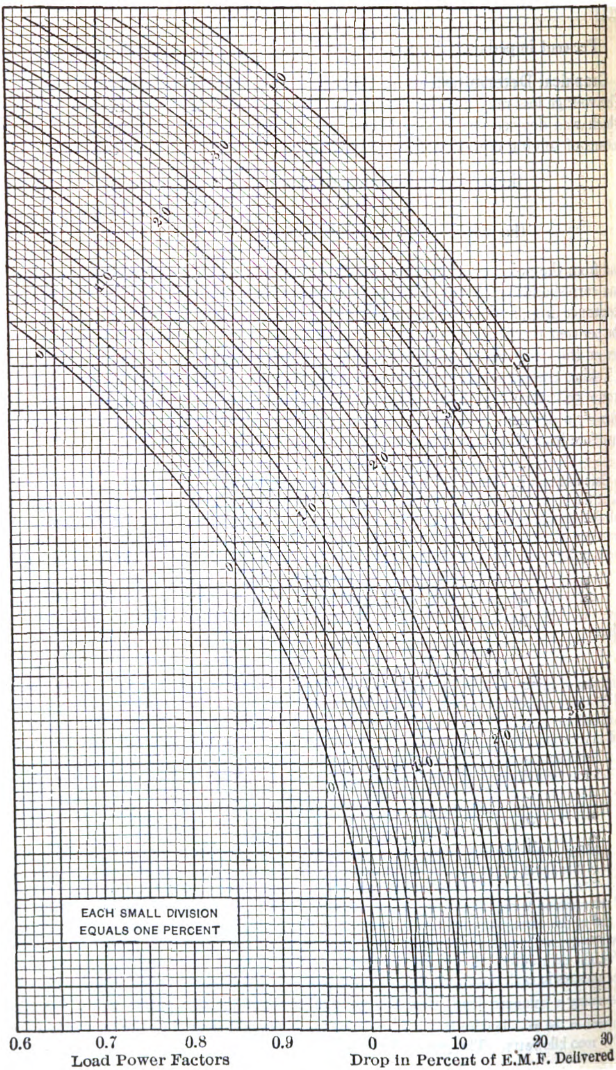


Fig. 3. Mershon's Chart for Calculating Drop in A-C. Line

The equivalent resistances and reactances of the line for the two phases, assuming the currents in the two outer wires to be exactly equal in effective value and to differ in phase by exactly 90°, are

$$R_1 = (r + r_1 - x_1 + 0.0014f)\,l, \qquad X_1 = (x + x_1 + r_1)\,l,$$
$$R_2 = (r + r_1 + x_1 - 0.0014f)\,l, \qquad X_2 = (x + x_1 - r_1)\,l,$$

where f is the frequency in cycles per second. When the wires are transposed only the term $0.0014\,f$ in the expressions for R_1 and R_2 go out, the other terms in the R's and X's remaining unaltered. The voltages at the generator end for the two phases are then

$$E_1 = E\sqrt{\left(\cos\phi + \frac{R_1 I}{E}\right)^2 + \left(\sin\phi + \frac{X_1 I}{E}\right)^2},$$
$$E_2 = E\sqrt{\left(\cos\phi + \frac{R_2 I}{E}\right)^2 + \left(\sin\phi + \frac{X_2 I}{E}\right)^2}.$$

Of course, if the voltages E_1 and E_2 are maintained equal at the generator end, which is usually the case, the assumptions of equal currents and equal voltages at the load end are incorrect, but the above formulas may be used to obtain an approximate value of the voltage loss in the two phases. The unbalancing due to the unequal equivalent impedances of the two phases renders the two-phase three-wire system undesirable except for lines in which the voltage loss is a small percentage of the voltage at the load, and the unbalancing therefore small.

SINGLE END-CONDENSER METHOD. — This method assumes that the total current at the load end is equal to the actual load current plus (vectorially) the charging current which would be taken by a single condenser shunted across the line at the load end, the capacity of this condenser being taken equal to the total capacity of the line. This method gives too low a voltage at the generator end by approximately the same amount that the straight impedance

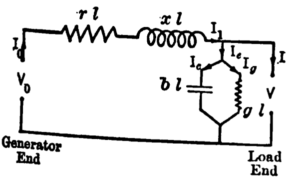


Fig. 4.

$\overline{AH} = I,$
$\overline{AC} = V,$
$\overline{AG} = I\cos\phi,$
$\overline{GH} = I\sin\phi,$
$\overline{HJ} = blV \times 10^{-6},$
$\overline{JK} = glV \times 10^{-6},$
$\overline{HK} = ylV \times 10^{-6},$
$\overline{AK} = I_1 = I_0,$

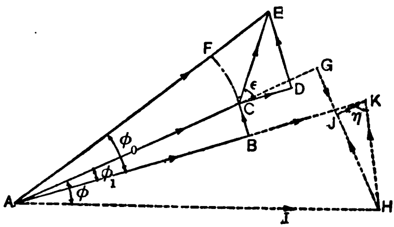


Fig. 4a.

$\overline{AB} = V\cos\phi_1,$
$\overline{BC} = V\sin\phi_1,$
$\overline{CD} = rlI_1,$
$\overline{DE} = xlI_1,$
$\overline{CE} = zlI_1,$
$\overline{AE} = V_0,$
$\overline{FE} = v,$

method gives it too high, and also gives the power loss too low by approximately the same amount that the straight impedance method gives it too high. By averaging the losses obtained by the two methods a close approximation to their true values is obtained.

Fig. 4 is a diagram of the circuit and Fig. 4a is a complete vector diagram of the voltage and current; voltages are shown by full lines and currents by dotted lines. The diagrams and formulas are for the general case of a line with leakage, but for nearly all practical cases the leakage may be neglected.

The effect of the electrostatic capacity of the line is to change both the numerical value and the phase angle of the line current. Or, the condenser and the load may be looked upon as forming together an equivalent load taking a current I_1 at a power factor $\cos\phi_1$ differing from the actual current and power factor of the load. Let

V = volts to neutral at the load end of the line,
I = actual amperes per wire at the load end,
$\cos\phi$ = actual power factor at the load end,
$\sin\phi = \sqrt{1-\cos^2\phi}$ = actual reactive factor at the load end,
l = length of each line wire in miles,
b = capacity susceptance to neutral per mile of line, in micromhos, see tables in the article on *Capacity and Charging Current*,
g = leakage conductance to neutral per mile of line in micromhos, usually taken equal to zero in power lines, as explained above,
$y = \sqrt{g^2+b^2}$ = dielectric admittance to neutral per mile of conductor, in micromhos. Note that for no leakage $y = b$.

The total leakage current, total charging current and total exciting current of the line are then respectively

$$I_g = glV \times 10^{-6}, \qquad I_c = blV \times 10^{-6}, \qquad I_e = \sqrt{I_g^2 + I_c^2}. \tag{42}$$

The total line current, i.e., the resultant of the actual load current and the exciting current, is

$$I_1 = I\sqrt{\left(\cos\phi + \frac{I_g}{I}\right)^2 + \left(\sin\phi - \frac{I_c}{I}\right)^2}. \tag{43}$$

On the assumptions of this method of calculation I_1 is also the current at the generator end.

The power factor of the equivalent load formed by the actual load and the condenser is then

$$\cos\phi_1 = \frac{I\cos\phi + I_g}{I_1}, \tag{44}$$

and the reactance factor of this equivalent load is

$$\sin\phi_1 = \frac{I\sin\phi - I_c}{I_1}. \tag{45}$$

The formulas given above for the straight impedance method are then directly applicable, using for I, $\cos\phi$ and $\sin\phi$ in those formulas the values of I_1, $\cos\phi_1$, and $\sin\phi_1$ just calculated; i.e., the straight impedance method is to be applied not to the actual load but to the equivalent load formed by the actual load and a condenser having an admittance equal to the total admittance of the line.

Example of Calculation by Single End-Condenser Method. — Three-phase line, 50 miles long, No. 0000 A.W.G. stranded copper, 6 feet between centers, frequency 60 cycles, load 15,000 kilowatts, 60,000 volts between wires at load end, 80 per cent power factor at load. This is the same example as used above for the straight impedance method. Then $V = 34{,}600$, $I = 180$, $\cos\phi = 0.8$, $\sin\phi = 0.6$, $l = 50$, $b = 6.03$, $g = 0$, $r = 0.269$, $x = 0.728$. Then,

$$\text{Charging current} = I_c = 6.03 \times 50 \times 34{,}600 \times 10^{-6} = 10.4 \text{ amperes},$$

$$\text{Resultant current} = I_1 = 180\sqrt{(0.8)^2 + \left(0.6 - \frac{10.4}{180}\right)^2} = 174 \text{ amperes},$$

$$\text{Power factor of equivalent load} = \cos\phi_1 = \frac{180 \times 0.8}{174} = 0.828.$$

$$\text{Reactive factor of equivalent load} = \sin\phi_1 = \frac{180 \times 0.6 - 10.4}{174} = 0.561,$$

$$\text{Impedance of equivalent load per mile of line} = Z_1 = \frac{34{,}600}{50 \times 174} = 3.98,$$

$$\text{Per cent power loss} = Q = \frac{100 \times 0.269}{3.98 \times 0.828} = 8.16 \text{ per cent},$$

Per cent voltage loss $= D$

$$= 100\left[\sqrt{\left(0.828 + \frac{0.269}{3.98}\right)^2 + \left(0.561 + \frac{0.728}{3.98}\right)^2} - 1\right] = 16.30 \text{ per cent},$$

$$\text{Power factor at generator end} = \cos\phi_0 = \left(\frac{100 + 8.16}{100 + 16.3}\right) \times 0.822 = 76.4 \text{ per cent.}$$

The per cent power loss and voltage loss obtained by the straight impedance method neglecting the line capacity are respectively 8.76 and 17.6 per cent. As noted above the single end-condenser method gives these losses too low (for inductive loads) by approximately the same amount that the straight impedance method gives them too high, whence closer approximations to the true losses are: per cent power loss = $(8.16 + 8.76) \div 2 = 8.46$ and per cent voltage loss = $(16.3 + 17.6) \div 2 = 17.0$.

Calculation of Effect of Synchronous Condenser. — Formulas (43) to (45) apply directly to the calculation of the effect of a synchronous condenser at the end of the line taking a current having an in-phase or energy component equal to I_g and a quadrature leading component equal to I_c. Fig. 4a then represents the vector relations of the currents and voltages, the vector JK being the in-phase component of the current taken by the synchronous condenser and HJ the quadrature component.

MIDDLE-CONDENSER OR "T" METHOD. — This method assumes that the line may be considered equivalent to the circuit shown in Fig. 5, which represents a single condenser, having a capacity and leakage conductance equal respectively to the total capacity and leakage conductance of the line, shunted across the line at its middle point, the resistance and inductance on each side of this condenser being equal respectively to half the total conductor resistance and inductance.

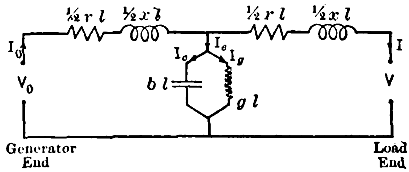


Fig. 5.

An inspection of Fig. 5 will show that the half of the line nearest the load is represented by a straight impedance in series with the load. Hence the voltage, current and power factor at the middle point of the line may be figured by the straight impedance method given above. Then, considering the voltage, current and power factor thus calculated as forming a load at the middle point of the line, the second half of the line may be calculated by the single end-condenser method just described.

By assigning proper values (which are somewhat complicated functions of the actual line constants and the frequency) to the constants of the "T" circuit shown in Fig. 5, this circuit may be made *exactly equivalent*, as far as the relations between the voltage, current and power at the two ends are concerned, *to a line of any length and for any frequency;* the "corrected" constants are entirely independent of the voltage, current and power taken by the load; see p. 1683.

Solution of the "T" Circuit in Symbolic Notation. — The solution of the circuit shown in Fig. 5 is also given below in terms of complex quantities (*see under Symbolic Notation in the article on Alternating Currents*). Let

V = numerical value of volts to neutral at the load end of the line,
I = amperes per wire at the load end of the line, taken as the vector of reference,
$\cos\phi$ = power factor at the load end of the line,
$V = V\cos\phi + jV\sin\phi$ = volts to neutral at the load end of the line, referred to the current at the load end,
$z = r + jx$ = conductor impedance in ohms per mile of wire, r being the resistance per mile and x the reactance per mile,
$y = g + jb$ = dielectric admittance, one wire to neutral, in micromhos per mile of line, where g is the leakage conductance per mile and b the capacity susceptance per mile,
l = length of each wire in miles.

Put

$$M = 10^{-6}yzl^2 = [(gr - bx) + j(br + gx)]\,10^{-6}l^2. \tag{46}$$

Then the current at the generator end, in symbolic notation, is

$$I_0 = (1 + \tfrac{1}{2}M)\,I + 10^{-6}ylV, \tag{47}$$

and the volts to neutral at the generator end, in symbolic notation, are

$$V_0 = (1 + \tfrac{1}{2}M)\,V + zl\,(1 + \tfrac{1}{4}M)I. \tag{48}$$

Calling I_0' and I_0'' the real and "j" components respectively of the current I_0, and V_0' and V_0'' the real and "j" components of the voltage V_0, then the numerical values of the current and voltage at the generator end are

$$I_0 = \sqrt{(I_0')^2 + (I_0'')^2}, \quad \text{and} \quad V_0 = \sqrt{(V_0')^2 + (V_0'')^2}, \tag{49}$$

and the power input per wire at the generator end is

$$P_0 = V_0'I_0' + V_0''\,I_0''. \tag{50}$$

In applying this last formula particular attention must be paid to the signs of the quantities involved, e.g., if $V_0 = 1000 + j\,300$ and $I_0 = 100 - j\,20$, then $V_0' = 1000$, $V_0'' = 300$, $I_0' = 100$ and $I_0'' = -20$, and $P_0 = 1000 \times 100 - 300 \times 20$. The power factor at the generator end is

$$\cos\phi_0 = \frac{P_0}{V_0 I_0}. \tag{51}$$

SPLIT-CONDENSER OR "π" OR "U" METHOD. — This method assumes that the line may be considered equivalent to the circuit shown in Fig. 6, which represents a single impedance in series with the load, the resistance and inductance of this impedance being equal respectively to the total conductor resistance and inductance of the line, and a condenser shunted across the line at each end, each condenser having a capacity and leakage conductance equal respectively to half the total capacity and half the total leakage conductance of the line.

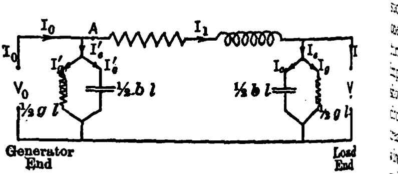


Fig. 6.

An inspection of Fig. 6 will show that the calculation of the line up to the point A at which the condenser at the generator end is connected is exactly the same as for the single end-condenser method described above, except that the capacity and leakage of the end condenser are taken respectively as half the total capacity and half the total leakage of the line. The voltage at the point A is the same as the voltage at the generator end, but to find the current I_0 at the generator end the exciting current I_e' of the condenser shunted across the line at the generator end must be added (vectorially) to the current I_1 at A. Using the same notation as in the section above on the single end-condenser method, the total generator current is then

$$I_0 = I_1\sqrt{\left(\cos\phi_0 + \frac{I_g'}{I_1}\right)^2 + \left(\sin\phi_0 - \frac{I_e'}{I_1}\right)^2},$$

where

$$I_g' = \tfrac{1}{2}glV_0 \times 10^{-6} \quad \text{and} \quad I_e' = \tfrac{1}{2}blV_0 \times 10^{-6}.$$

As in the case of the "T" circuit, a "π" circuit, such as shown in Fig. 6, may be given such "corrected" constants as will make it exactly equivalent for any one frequency, as far as the terminal conditions are concerned, to a line of any length whatever no matter how high this frequency may be; see p. 1683.

Solution of the "π" Circuit in Symbolic Notation. — Using the same notation as in the section above on the *Solution of the "T" Circuit in Symbolic Notation,* and calculating the quantity $\dot{M}$ by the same formula, viz., equation (46), the current and voltage at the generator end in symbolic notation are respectively

$$\dot{I}_0 = (1 + \tfrac{1}{2}\dot{M})\,\dot{I} + 10^{-6}\,\dot{y}l\,(1 + \tfrac{1}{4}\dot{M})\,\dot{V}, \qquad (52)$$

$$\dot{V}_0 = (1 + \tfrac{1}{2}\dot{M})\,\dot{V} + \dot{z}l\dot{I}. \qquad (53)$$

The numerical values of these quantities, the power and the power factor are then to be calculated in the same manner as for the "T" circuit, viz., by formulas (49) to (51).

EXACT CALCULATION OF A-C. LINES OF ANY LENGTH AND FREQUENCY.* — The charging current (and also the leakage current) for any element of a transmission line passes through only that portion of the line conductor between the given element and the generator end. The exact determination of the line current, voltage and power at any point therefore requires that this fact be taken into account; in other words the capacity and leakage of the line are distributed and not lumped as assumed in the above methods of calculation. In Fig. 7 are shown three successive elements of one wire of a line, a return or neutral of zero impedance being also shown to complete the circuit. This method of treating separately each wire of a line is fully explained above, p. 1668.

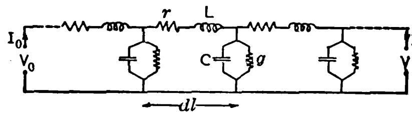


Fig. 7.

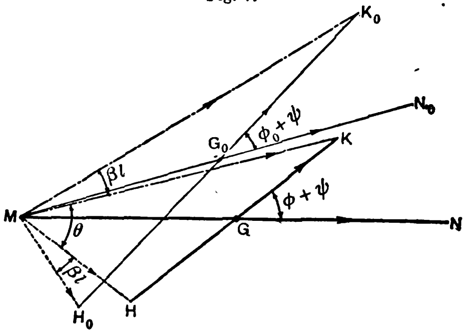


Fig. 7a.

$\overline{MN} = I$, $\quad \overline{MK_0} = A\epsilon^{\alpha l}$,

$\overline{HK} = YV$, $\quad \overline{MH_0} = B\epsilon^{-\alpha l}$,

$\overline{MK} = A$, $\quad \overline{MN_0} = I_0$,

$\overline{MH} = B$, $\quad \overline{H_0K_0} = YV_0$.

In order to make clear the physical meaning of the various terms employed, the general solution is first given in terms of instantaneous values. Following this are given the working formulas (1) in terms of exponentials, viz., ϵ^{x} and ϵ^{-x}, (2) in terms of real hyperbolic functions, and (3) in terms of hyperbolic functions of complex angles.

General Equations of Transmission Line. — Let l = the distance in miles of any point along a transmission line, measured from any arbitrarily chosen point (say from the load end); let i = the instantaneous current in amperes in the conductor at this point, taken positive in the direction *opposite* to that in which l is measured (i.e., positive when *toward* the load end); let v = the potential drop in volts between this point and the neutral; and let r = the conductor resistance in ohms per mile, L = the inductance of the conductor in henries per mile, C = the capacity to neutral in microfarads per mile

* This section is abstracted from the lecture notes of Dr. H. Pender.

and g = the leakage conductance to neutral in micromhos per mile. The following relations then hold at any point along the line, t being time in seconds measured from any arbitrarily chosen instant:

$$\frac{dv}{dl} = ri + L\frac{di}{dt}, \tag{54}$$

$$10^6\frac{di}{dl} = gv + C\frac{dv}{dt}. \tag{55}$$

If the circuit is composed of two or more sections of different constants (e.g., an overhead section and an underground section, or a circuit formed by a step-up transformer, transmission line and a step-down transformer) then a similar set of equations holds for each section of the circuit, the constants r, L, g, and C being in general different for the several sections.

The complete solution of any two equations of the form given by (54) and (55) consists of an infinite series of terms for both v and i, corresponding terms in the two series having the following values:

$$i = \sqrt{2}\,\epsilon^{-(u-s)t}\left[A\epsilon^{\alpha l}\sin(\omega t + \beta l + \theta_1) + B\epsilon^{-\alpha l}\sin(\omega t - \beta l + \theta_2)\right], \tag{56}$$

$$v = \frac{\sqrt{2}}{Y}\epsilon^{-(u-s)t}\left[A\epsilon^{\alpha l}\sin(\omega t + \beta l + \theta_1 - \psi) - B\epsilon^{-\alpha l}\sin(\omega t - \beta l + \theta_2 - \psi)\right]. \tag{57}$$

Physical Interpretation and Names Given to the Various Constants.—The constant ω in equations (56) and (57) is equal to $2\pi f$, where f is the frequency of the oscillation represented by these equations. In the most general case any change in the circuit conditions, such as closing or opening a switch, or a lightning stroke in the vicinity of the circuit, may set up an infinite number of oscillations of different frequencies, their frequency being determined by the initial conditions at the instant the change is made. The current and voltage set up by each oscillation is represented by a set of terms of the form given by (56) and (57), and the resultant current and voltage will be respectively the sum of all the current terms and the sum of all the voltage terms. The oscillation of any given frequency, however, may be considered separately, as it is uninfluenced by the presence of the other oscillations. Moreover, in the case of a composite circuit, consisting say of a step-up transformer, transmission line and step-down transformer, if an oscillation of frequency f is set up in one part of the circuit it will also appear in all other sections of the circuit, though it may be greatly damped in these sections and therefore produce no appreciable effect.

Attenuation Constant (α), Wave Length Constant (β), Wave Length (λ) and Velocity of Propagation (U).—Referring to equations (56) and (57) each oscillation sets up in each section of the circuit two waves, each of which has a wave length $\lambda = \frac{2\pi}{\beta}$; the constant β is therefore called the "wave length constant;" it is a function of the frequency and of the constants r, L, g and C of the circuit. The two waves travel along the line in opposite directions each with a velocity $U = \frac{\omega}{\beta}$; in a composite circuit this velocity U is in general different for each section of the circuit. One wave may be looked upon as the "incident" and the other as the "reflected" wave. The amplitude of each wave diminishes by the factor ϵ^{α} as the wave travels unit distance; the factor ϵ^{α} is called the "attenuation factor" and the constant α is called the "attenuation constant." The attenuation constant is a function of the frequency and the constants r, L, g and C of the circuit; see p. 1680.

Surge Admittance (Y) and its Power-factor Angle (ψ).—The constant Y, which is equal to the quotient of the amplitude (or r.m.s. value) of the incident current wave by the incident voltage wave, is called the "surge admittance" and its reciprocal is called the "surge impedance"; it is a function of the frequency and the constants r, L, g and C of the circuit; see p. 1680. The constant ψ, which is equal to the angle by which the incident current wave leads the incident voltage wave, is called the "power-factor angle of the surge admittance"; it is a function of the frequency and the constants r, L, g and C; see p. 1680.

Amplitude Constants (A and B) and Phase-Angle Constants (θ_1 and θ_2).—The constants are equal to the amplitudes of the incident and reflected current waves at the point from which the distance l is measured, and the constants θ_1 and θ_2 give the

phase of these two waves at this point ($l = 0$) at the instant from which time is measured ($t = 0$). Note that the incident current wave at $l = 0$ leads the reflected current wave by the angle $\theta = \theta_1 - \theta_2$. The determination of these constants for steady state conditions is given on p. 1681.

Natural Damping Constant (u), Energy Transfer Constant (s) and Composite Damping Constant ($u - s$). — In the general case of a natural oscillation in a composite circuit the amplitude of each wave diminishes in unit time by the factor $\epsilon^{(u-s)}$; this factor is called the "composite damping factor," and the constant $(u - s)$ is called the "composite damping constant." The composite damping constant, like the frequency f, is the same for all sections of a composite circuit. In the case of a line of uniform constants throughout, not connected to any terminal apparatus, it can readily be shown that $s = 0$, in which case the amplitude of the oscillations diminishes in unit time by the factor ϵ^{u}; this factor is therefore called the "natural damping factor" and the constant u, which for a section having the constants r, L, g and C per unit length, is equal to

$$u = \frac{1}{2}\left(\frac{r}{L} + \frac{g}{C}\right), \tag{58}$$

is called the "natural damping constant." Any section of a composite circuit for which the actual damping $\epsilon^{(u-s)}$ is less than the natural damping ϵ^{u} must receive energy from some other section; consequently a positive value of s for a given section means that energy is transferred into this section from some other section of the circuit. Similarly, a negative value of s for a given section means that energy is transferred from this section to some other section. The constant s may therefore be called the "energy transfer constant." Since the voltage and current in the circuit cannot increase indefinitely the energy transfer constant s can never have a positive value greater than u.

Since the composite damping constant $(u - s)$ is the same for all sections of a circuit, it follows that the transfer of energy from one section to another by oscillations in a composite circuit will always be into the section in which u is the larger from the section in which u is the smaller. Neglecting the leakage conductance g, this means that energy will be transferred into section 1 from section 2, when $\frac{r_1}{L_1}$ is greater than $\frac{r_2}{L_2}$, that is, energy is transferred from the section of the larger "time constant" $\left(\frac{L_2}{r_2}\right)$ to that of the smaller "time constant" $\left(\frac{L_1}{r_1}\right)$. When the resistances are small, this means that in the limiting case all the energy ($= \frac{1}{2} L_2 I^2$) of the magnetic field of the second section may go into electrostatic energy ($= \frac{1}{2} C_1 V^2$) in the first section, producing therefore a very high voltage at the junction point when the inductance L_2 of the second section is large compared with the capacity C_1 of the first section. This accounts for the very high voltages sometimes set up during switching operations at the junction point of an overhead line with an underground line, or in a transformer connected to a long overhead line.

"Steady State" Conditions in a Transmission Line. — From the above discussion it is evident that when a sufficient time (usually a small fraction of a second) has elapsed after any change in the circuit conditions, the only terms left in the general equations of a transmission line for a given impressed sine-wave voltage of frequency f are those for which $s = u$, viz.,*

$$i = \sqrt{2}\left[A\epsilon^{\alpha l} \sin(\omega t + \beta l) + B\epsilon^{-\alpha l} \sin(\omega t - \beta l - \theta)\right], \tag{59}$$

$$v = \frac{\sqrt{2}}{Y}\left[A\epsilon^{\alpha l} \sin(\omega t + Bl - \psi) - B\epsilon^{-\alpha l} \sin(\omega t - \beta l - \theta - \psi)\right]. \tag{60}$$

The effective value of the current at any point is then equal to the sum of two vectors having the lengths $A\epsilon^{\alpha l}$ and $B\epsilon^{-\alpha l}$, the former leading the latter by the angle $(2\beta l + \beta)$, and the effective value of the voltage is equal to the difference of these same two vectors divided by Y. The phase angle between the voltage and current is equal to the phase angle between the sum and difference of the A and B vectors less the angle ψ.

* For steady state conditions time may be counted from any arbitrarily chosen interval, i.e., θ_1 in equations (56) and (57) may be put equal to zero, and for convenience $-\theta$ may be used for θ_2.

Notation for Steady State Conditions. — These relations are clearly shown in the vector diagram, Fig. 7a, which is a complete vector diagram of a transmission line with distributed capacity and leakage. The four constants α, β, Y and ψ are constants of the line, independent of the load, and are expressed in terms of the ordinary line constants as follows: Let

f = frequency in cycles per second,

r = conductor resistance per mile, in ohms; see tables in article on *Wires and Cables, Bare,*

$x = 2\pi fL$ = conductor reactance per mile, in ohms, corresponding to the impressed frequency f; see tables in article on *Inductance and Inductive Reactance,*

$z = \sqrt{r^2 + x^2}$ = conductor impedance per mile, in ohms,

g = leakage conductance to neutral per mile of line, in micromhos; see above, p. 1667. For power lines g is usually taken equal to zero,

$b = 2\pi fC$ = capacity susceptance to neutral per mile of line, in micromhos, corresponding to the impressed frequency f; see tables in article on *Capacity and Charging Current,*

$y = \sqrt{g^2 + b^2}$ = dielectric admittance per mile, in micromhos; when $g = 0$, then $y = b$,

$\alpha = 10^{-3}\sqrt{\dfrac{yz - bx + gr}{2}}$ = the attenuation constant; for r and g small compared with x and b this reduces to $\dfrac{10^{-3}}{2}\left(r\sqrt{\dfrac{C}{L}} + g\sqrt{\dfrac{L}{C}}\right)$,

$\beta = 10^{-3}\sqrt{\dfrac{yz + bx - gr}{2}}$ = the wave length constant; for r and g small respectively compared with x and b this reduces to $2\pi f \times 10^{-3}\sqrt{LC}$, which for an overhead line equals approximately $\dfrac{2\pi f}{180{,}000}$,

$Y = 10^{-3}\sqrt{\dfrac{y}{z}}$ = surge admittance; for r and g small respectively compared with x and b, this reduces to $10^{-3}\sqrt{\dfrac{C}{L}}$. The reciprocal of the surge admittance is called the "surge impedance,"

$\psi = \tan^{-1}\sqrt{\dfrac{yz - bx - rg}{yz + bx + rg}}$ = the power-factor angle of the surge admittance, taken positive for $gx < br$ and negative for $gx > br$. For r and g small compared with x and b, then $\psi = 28.7\left(\dfrac{r}{x} - \dfrac{g}{b}\right)$ degrees,

$U = \dfrac{2\pi f}{\beta}$ = velocity of propagation in miles per second; for a frequency f sufficiently high to make r negligible compared with x, and g negligible compared with b, this reduces to $\dfrac{10^3}{\sqrt{LC}}$, which for an overhead line with wires far apart is equal to the velocity of light in air, viz., 180,000 miles per second, approximately,

$\lambda = \dfrac{2\pi}{\beta} = \dfrac{U}{f}$ = wave length of each wave in miles; for a frequency f sufficiently high to make r negligible compared with x, and g negligible

compared with b, this reduces to $\frac{10^3}{f\sqrt{LC}}$, which for an overhead line is equal approximately to $\frac{180{,}000}{f}$ miles.*

In the vector diagram and in the formulas given below, let

l = length of the line in miles,

I = effective (r.m.s.) value of the amperes per wire at the load end,

V = effective (r.m.s.) value of the volts to neutral at the load end,

ϕ = the power-factor angle at the load end, i.e., $\cos\phi$ is the power factor at the load end. ϕ is taken positive for a lagging and negative for a leading current,

I_0, V_0, ϕ_0 = corresponding quantities at the generator end.

Solution by Vector Diagram (Fig. 7a). — Having calculated the constants α, β, Y and ψ, and knowing the current I, voltage V and power-factor angle ϕ of the load,

Lay off $\overline{MN} = I$ as the base line, and bisect it at G,

At the angle $(\phi + \psi)$ ahead of $\overline{MN}$ lay off the line $\overline{HK}$ equal in length to YV, so that it is also bisected by G,

Then measure off $\overline{MK} = A$ and $\overline{MH} = B$,

Lay off at the angle $57.3\,\beta l$ degrees ahead of A the line $\overline{MK_0}$ equal in length† to $A\epsilon^{\alpha l}$,

Lay off at the angle $57.3\,\beta l$ degrees behind B the line $\overline{MH_0}$ equal in length to $B\epsilon^{-\alpha l}$.

Then the line $\overline{MN_0} = 2\,\overline{MG_0}$ is equal to the current at the generator end,

The line $\overline{H_0K_0}$ divided by Y is equal to the voltage at the generator end,

The angle between $\overline{G_0N_0}$ and $\overline{G_0K_0}$ less the angle ψ is the power-factor angle at the generator end.

Note that the voltage at the load end if drawn in the diagram would be at the angle ψ behind the vector $\overline{GK}$, and at the generator end would be at the angle ψ behind $\overline{G_0K_0}$.

Algebraic Solution for Steady-state Conditions. —The vector diagram may be solved algebraically as follows: Calculate first the constants A, B and θ from the formulas:

$$A = \tfrac{1}{2}\sqrt{I^2 + (YV)^2 + 2\,YVI\cos(\phi + \psi)}, \tag{61}$$

$$B = \tfrac{1}{2}\sqrt{I^2 + (YV)^2 - 2\,YVI\cos(\phi + \psi)}, \tag{62}$$

$$\theta = \tan^{-1}\left[\frac{2\,YVI\sin(\phi + \psi)}{I^2 - (YV)^2}\right]. \tag{63}$$

Note that $\sin\theta$ has the same algebraic sign as the numerator of this fraction and $\cos\theta$ has the same algebraic sign as the denominator of this fraction; this fixes the quadrant in which θ lies.

Put

$$A_0 = A\epsilon^{\alpha l}, \quad \text{and} \quad B_0 = B\epsilon^{-\alpha l}. \tag{64}$$

* The above formulas for z, y, α, β, Y, ψ, U and λ also hold for the transient or free oscillations in a single circuit, and also for each section of a composite circuit, provided in these formulas $r_1 = r\,(u - s)\,L$ is substituted for r and $g_1 = g + (u - s)\,C$ is substituted for g.

† See the table in the article on *Exponential Functions* for values of ϵ^x and ϵ^{-x}, where x is any number.

The current, voltage and power-factor angle at the generator end are, then, expressing all angles in degrees,

$$I_0 = \sqrt{A_0^2 + B_0^2 + 2\,AB\cos(114.6\,\beta l + \theta)}, \tag{65}$$

$$V_0 = \frac{1}{Y}\sqrt{A_0^2 + B_0^2 - 2\,AB\cos(114.6\,\beta l + \theta)}, \tag{66}$$

$$\phi_0 = \tan^{-1}\left[\frac{2\,AB\sin(114.6\,\beta l + \theta)}{A_0^2 - B_0^2}\right] - \psi. \tag{67}$$

Note that the quadrant in which $(\phi_0 + \psi)$ lies is determined by the algebraic signs of the numerator and denominator of the fraction in the brackets, just as in the case of the angle θ.

Solution of Steady State Conditions in Terms of Hyperbolic Functions.* — The above expressions for the current, voltage and power factor at the generator may also be put in the form,

$$I_0 = I\sqrt{\frac{\cosh(2\,\alpha l + \gamma) + \cos(114.6\,\beta l + \theta)}{\cosh\gamma + \cos\theta}}, \tag{68}$$

$$V_0 = V\sqrt{\frac{\cosh(2\,\alpha l + \gamma) - \cos(114.6\,\beta l + \theta)}{\cosh\gamma - \cos\theta}}, \tag{69}$$

$$\phi_0 = \tan^{-1}\left[\frac{\sin(114.6\,\beta l + \theta)}{\sinh(2\,\alpha l + \gamma)}\right] - \psi, \tag{70}$$

where θ and γ are given by the formulas

$$\gamma = \tanh^{-1}\left[\frac{2\,YVI\cos(\phi + \psi)}{I^2 + (YV)^2}\right], \tag{71}$$

$$\theta = \tan^{-1}\left[\frac{2\,YVI\sin(\phi + \psi)}{I^2 - (YV)^2}\right]. \tag{72}$$

The other quantities are as above defined. Note that θ is the same angle as given by equation (63) and the quadrant in which it lies is to be determined as described in the note under (63). Also note that the constant γ given by (71) may be expressed in terms of A and B, given by (61) and (64), by means of the formula

$$\gamma = \log_\epsilon\left(\frac{A}{B}\right) = 2.302\log_{10}\left(\frac{A}{B}\right). \tag{73}$$

Formulas for Open Circuit and Short Circuit at Load End. — When the line is open at the load end, $I = 0$, whence from equations (61) to (63), $A = \frac{1}{2}\,YV$, $B = \frac{1}{2}\,YV$, and $\theta = 180°$ (since the denominator of the fraction is negative). When the line is short-circuited at the load end, $V = 0$, and $A = \frac{1}{2}\,I$, $B = \frac{1}{2}\,I$, and $\theta = 0°$ (since the denominator of the fraction is positive). The current, voltage and power-factor angle at any point along the line may then be found in either case by substituting these values in equations (65) to (67) which reduce to the simple hyperbolic forms:

On open circuit	On short circuit
$I_0 = YV\sqrt{\sinh^2(\alpha l) + \sin^2(57.3\,\beta l)}$	$I_0 = I\sqrt{1 + \sinh^2(\alpha l) - \sin^2(57.3\,\beta l)}$
$V_0 = V\sqrt{1 + \sinh^2(\alpha l) - \sin^2(57.3\,\beta l)}$	$V_0 = \frac{I}{Y}\sqrt{\sinh^2(\alpha l) + \sin^2(57.3\,\beta l)}$
$\phi_0 = -\tan^{-1}\left[\frac{\sin(114.6\,\beta l)}{\sinh(2\,\alpha l)}\right] - \psi$	$\phi_0 = +\tan^{-1}\left[\frac{\sin(114.6\,\beta l)}{\sinh(2\,\alpha l)}\right] - \psi$

* Tables of hyperbolic functions are given in the article on *Hyperbolic Functions.*

Solution of Steady-state Equations in Complex Hyperbolic Functions.* — Expressing all quantities other than the length l in symbolic notation (*see Alternating Currents*), the equations of the transmission line for *steady state conditions only*, may be written

$$\dot{I}_0 = \dot{I}\,\frac{\sinh\,(l\sqrt{\dot{y}\dot{z}} + \dot{B})}{\sinh \dot{B}}, \tag{74}$$

$$\dot{V}_0 = \dot{V}\,\frac{\cosh\,(l\sqrt{\dot{y}\dot{z}} + \dot{B})}{\cosh \dot{B}}, \tag{75}$$

where the symbols other than B have the same meanings as above, except that they are all expressed as complex quantities and

$$\dot{B} = \tanh^{-1}\left(\frac{\dot{V}}{\dot{I}}\sqrt{\frac{\dot{y}}{\dot{z}}}\right). \tag{76}$$

The real part of $\sqrt{\dot{y}\dot{z}}$ is the attenuation constant and the imaginary or "j" part is the wave length constant.

Rigorously Equivalent "T" and "π" Circuits. — As noted above the middle condenser or "T" circuit, Fig. 5, or the split condenser or "π" circuit, Fig. 6, is rigorously equivalent to the actual transmission line when proper values are assigned to the constants of these circuits. The corrected values z' and y' of the impedance per mile and of the admittance per mile are, in symbolic notation, as follows:

"T" circuit	"π" circuit
$\dot{z}' = \dot{z}\,\dfrac{\tanh\left(\dfrac{l\sqrt{\dot{y}\dot{z}}}{2}\right)}{\dfrac{l\sqrt{\dot{y}\dot{z}}}{2}}$ $\dot{y}' = \dot{y}\,\dfrac{\sinh\,(l\sqrt{\dot{y}\dot{z}})}{l\sqrt{\dot{y}\dot{z}}}$	$\dot{z}' = \dot{z}\,\dfrac{\sinh\,(l\sqrt{\dot{y}\dot{z}})}{l\sqrt{\dot{y}\dot{z}}}$ $\dot{y}' = \dot{y}\,\dfrac{\tanh\left(\dfrac{l\sqrt{\dot{y}\dot{z}}}{2}\right)}{\dfrac{l\sqrt{\dot{y}\dot{z}}}{2}}$

In Dr. Kennelly's tables above referred to are given tables of these correction fractions.

FACTORS AFFECTING THE MECHANICAL DESIGN. — In designing a transmission line the following factors, in addition to the line losses, must be considered: (1) right-of-way, (2) telephone circuits, (3) ground wires, (4) clearances, (5) type of supporting structure, (6) type of insulators, (7) conductors, (8) temperature range, (9) collection of ice, (10) wind velocity and wind pressure. The requirements imposed by these several conditions are noted below in the order mentioned; this is followed by the formulas and curves necessary to make the required calculations.

Location and Width of Right-of-way. — The right-of-way should be as short and as straight as practicable. Its width for a single line should be equal to the width of space over which the conductors normally hang, plus twice the side swing of conductors under maximum wind, plus twice the safe clearance from conductor to possible buildings adjacent to the right-of-way. In computing the side swing allowance must be made for swing of suspension insulators. Side swing of conductor should be based on the longest span ordinarily

* Complete tables and charts of such functions have recently been published by Prof. A. E. Kennelly, *Tables of Complex Hyperbolic and Circular Functions*, Harvard University Press, 1914.

used, and extra width should be secured wherever extraordinarily long spans are made. For telephone lines on separate poles extra width will be required unless the power wires are high enough to swing safely over them. Where two power lines are on one right-of-way the towers for the two lines should be located directly opposite each other, especially on long spans, since the two lines may then be placed close together with less danger of the wires of the two lines swinging together. However, it is usually desirable to have the lines far enough apart so that the towers of one line may fall without striking those of the other line. The right-of-way should be passable (or at least accessible) for patrolling as well as for construction.

When it is necessary that the right-of-way cross railroads, roads or other lines the length of crossing should be reduced to a minimum by making the crossing at as near right angles as practicable. Rights-of-way through swamps often require expensive road building and expensive tower foundations, though small swamps (up to about 1000 feet across) can often be crossed in a single span. Steep side hills require extra expense for foundations and tower extensions, and introduce a hazard of injury to tower from sliding earth, rocks, trees or snow. A right-of-way through forests requires expensive clearing.

It is usually advantageous to own and fence the right-of-way. However, when the right-of-way passes through farm lands, it is sometimes advantageous not to fence it in, but to have it cultivated and kept free from brush. Instead of purchasing a right-of-way, it is often sufficient to obtain easements covering the location of towers and suspension of wires. Easements should include the right to remove and trim trees under and adjacent to the line.

Telephone Circuits for Power Lines. — When the line voltage does not exceed 66,000 volts, the telephone circuits are usually carried on the same poles or towers as the power circuits, being placed below the power conductors. A separate line of wooden poles on the same right-of-way is usually employed when the voltage is higher than 66,000.

Where the telephone wires are on the same supporting structure as the power wires, sufficient clearance between the power and telephone circuits must be allowed to make the telephone line accessible for repairs and also to prevent the two circuits touching under abnormal conditions. On wood pole lines of short span (100 to 125 feet) the vertical clearance at the poles or towers ranges from 4 feet for 22,000 volts to 6 feet for 66,000 volts. On long span lines greater spacing is necessary to allow for safe clearance in the middle of the longest span due to the change in the sag of the power and telephone wires under all conditions of unequal ice loading and all variations of side deflection due to wind.

Telephone wires are ordinarily of copper, though copper-clad steel is sometimes used for long spans. For spans up to 125 feet No. 10 B. & S. copper may be used (though No. 8 is preferable) while for longer spans larger sizes (usually No. 8 or No. 6 or even No. 4) are necessary to allow for sleet load. A spacing of 12 inches between wires may be used for 125-foot spans, but a wider spacing is necessary for longer spans. Where inadequate spacing is used the telephone lines will frequently become crossed by the wind, unless they are strung with little sag, in which case they are overloaded and broken by sleet. With wide spacing and large sag higher poles must be used.

Ground Wires. — Grounded cables or wires are placed above the transmission line circuits to protect the latter from lightning discharges (*see Lightning Protectors*). They are usually grounded at each supporting structure except where short spans are used. The same care must be exercised to obtain clearances between conductors and ground cables or wires, as outlined above under telephone circuits. For tower lines having flexible towers and single-circuit towers having conductors arranged in a horizontal plane, two ground cables are

preferable, but for double-circuit tower lines either one or two may be used. As a general rule a line drawn through the ground cable and any conductor should not make an angle of more than 45 degrees with the vertical.

Clearances. — The following clearances require determination: conductor to ground; to edge of right-of-way; to tower; to ground wire; to telephone wire; to other conductors.

The minimum clearance from the high-voltage wires to the surface of the ground is ordinarily 20 feet or more, and from the telephone wires on towers used for high-voltage wires is 18 feet to ground. The minimum clearance from the wires to the edge of the right-of-way or to the tower, under maximum swing of insulators and cables in the wind, should exceed the striking distance corresponding to the arc-over voltage of the insulators used. The minimum clearance between conductors, under extreme conditions of unequal ice loading (*see below*), should exceed the striking distance of the normal voltage by a factor of safety of at least two. The normal clearance between conductors should be much greater than this, and is ordinarily from 10 inches to 1 foot per 10,000 volts between wires.

Type of Supporting Structure. — (*See also Poles for Overhead Lines; Cross Arms; Towers.*) Wood poles are usually the cheapest form of supporting structure for lines up to about 66,000 volts. Most lines of voltage over 66,000 and some lines of lower voltage are supported on steel towers.

A single line of poles or towers may support one, two and occasionally more circuits. Where several circuits are required two or more single-circuit tower lines with conductors arranged in a horizontal plane have the advantage that the effects of an accident will usually be confined to one circuit, and that one circuit may safely be repaired with another circuit alive, but this arrangement requires the maximum width of right-of-way. Where it is important to use the right-of-way economically two-circuit towers having the conductors of each circuit arranged in vertical planes on each side of the tower are used.

Wood poles are ordinarily placed from 100 to 200 feet apart and steel towers from 400 to 800 feet apart. The span for steel tower lines is ordinarily chosen so as to render the sum of the costs of all of the items a minimum, and is usually found to lie within the limits mentioned.

The poles or towers ordinarily used where the line is straight are designed to resist the weight of the conductors with sleet and the side pressure of wind on them, but not to resist the tension in the conductors which is assumed to be wholly or partially balanced. Such towers are called standard, straight line or suspension towers. Heavier towers designed to resist the unbalanced tension of all of the cables (which may occur if the cables are broken on the other side of the tower), called anchor or dead-end towers, are used at intervals of from five to ten spans where the line is straight. Heavier towers designed to resist the unbalanced lateral resultant of the tension in the cables are also used at angles and are known as angle towers. For economy in design and manufacture anchor and angle towers (for any angle up to about 60°) are usually made interchangeable. Where towers are not very high, economy is often secured by using flexible towers intermediate between square anchor and angle towers. On wood pole lines extra strong poles guyed or braced in all four directions, with double cross arms and double pins and insulators, are used for anchoring the line and at angles.

Type of Insulators. — (*See also Insulators.*) Pin insulators are usually used for voltages up to 50,000 and suspension insulators for higher voltages. Suspension insulators are sometimes used for lower voltage where a high factor of safety is desired. For the lower voltages the cost of the line is usually less for pin than for suspension insulators. The suspension insulator requires a higher tower

than the pin insulator, for the conductor is below the cross arm, and a longer cross arm is also required to allow for the swing of the insulators in the wind.

Conductors for Overhead Lines. — (*See also Wires and Cables.*) Copper, aluminum or steel, or combinations of the two former with steel, are used for line conductors. The use of an all-steel conductor is limited to those cases where great mechanical strength is required, although some economy is claimed for it when high voltages are used and a conductor of large diameter is required on account of corona (*see Corona*). Conductors consisting of a steel center with copper or aluminum outside are used where long spans are desirable and mechanical strength of conductor is required, and may also be used where large diameter is required to prevent corona formation. The maximum stress in wires and cables should not exceed the elastic limit. This condition is ordinarily met if a factor of safety of two, based on the ultimate strength, is used.

Distribution of Stress in Stranded Conductors. — Conductors for long spans are usually stranded when larger than No. 6 B. & S. gage, on account of the tendency to crystallize at the points of support, due to swinging in the wind. However, the maximum size of solid conductor which may be used is dependent largely upon the kind of support; if these are designed to prevent sharp bending, solid conductors, larger than No. 6, may be used. One difficulty in the use of a stranded cable is that for a given total tension in the cable the wires in the various layers are not all stressed equally. In particular, the center wire or core, if of metal, takes more than its share of the tension. On this account the actual sag in the cable when suspended may differ from that calculated, unless an allowance is made for the unequal tension in the various wires composing the cable. One method of avoiding this difficulty is to stress the cable, before stringing it, to a tension per square inch corresponding to the elastic limit of the component wires; the center wire will then be given a permanent set and when the cable is again stressed each of the component wires will tend to take the same proportion of the total tension.

Cables having centers or cores of a different material than the outside strands should be so designed that both materials are stressed to their elastic limit at the same time at the particular temperature at which maximum stress of conductors will occur. The two materials should also share the stress over the operating range of temperatures, to such extent as is possible.

The distribution of stress among the wires of which conductors may be composed applies more particularly to long spans where clearances are important and it is necessary to use small deflections compared to lengths of spans. For all ordinary construction, having spans of 500 to 600 feet or less, a very slight change in either thickness of ice or amount of wind pressure on the conductor will affect results as much as the error resulting from not taking into consideration the distribution of stresses in conductors.

Mechanical Characteristics of Conductor Materials. — For purposes of design both elastic limit and modulus of elasticity of cables should be obtained from tests on samples of the cables actually to be used, since errors in these materially affect the accuracy of the calculations of deflections in spans. The modulus of elasticity of cables varies widely with the number of wires and their pitch or lay in the different layers. Table I shows the values of these quantities and also of the other mechanical properties of conductors commonly used in design, when specific test results are not available. See also the articles on *Aluminum; Copper; Strength and Elasticity; Wires and Cables, Bare.*

Temperature Range. — The maximum and minimum air temperatures which have occurred in any locality for a period of years can be obtained from the records of the United States Weather Bureau, or from similar records for other countries. The minimum air temperature recorded (which may be as

TABLE I

MECHANICAL CHARACTERISTICS OF WIRES AND CABLES

Item	Copper	Aluminum	Steel
Ultimate strength, in lb. per sq. in.	60,000–65,000	25,000–50,000	60,000–80,000
Elastic limit, in lb. per sq. in.	30,000–35,000	11,000–14,000	35,000–40,000
Modulus of elasticity, in pound-inch units	12×10^6–16×10^6	$7\times10^6-10\times10^6$	22×10^6–28×10^6
Coefficient of linear expansion per ° F.	9.6×10^{-6}	12.8×10^{-6}	6.6×10^{-6}
Weight in pounds of a 1-ft. length having a cross section of 1,000,000 circ. mils (stranded)	3.09	0.92	2.67

low as − 40° F. in some of the northern states) will be the minimum temperature which the conductor may be expected to reach. However, since the conductors are exposed to the direct rays of the sun, they will reach a maximum temperature in the summer considerably in excess of the Weather Bureau records, which give the temperatures in the shade.

Another important temperature which should be determined is that of the wire when coated with ice. As noted below, a sleet storm is usually followed by a fall in temperature, and although the ice forms at 32° F., the wire may reach a much lower temperature while the ice is on it.

The following temperature ranges have been used in the design of certain lines:

	Maximum	Minimum
Eastern Canada	+ 120° F.	− 40° F.
Mississippi Valley	+ 120° F.	− 20° F.
Southern California	+ 140° F.	+ 10° F.

Collection of Ice on Wires. — Investigation of the records of the Weather Bureau leads to the conclusion that sleet and ice storms are generally followed by falling temperatures and high winds, and transmission lines should be designed to meet these conditions. Records indicate that under favorable conditions ice and sleet will collect on wires and cables to the same amount in any climate where freezing temperatures are obtained. Mild, moderate and cold climates differ in the frequency with which conditions are favorable. In general, sleet storms are most frequent in the moderate climates, since precipitation takes place more often at freezing temperatures. Destructive sleet storms occur in the eastern part of the United States at least as far south as Atlanta. One-half inch thickness of solid ice on wires and cables is generally assumed in designing transmission lines, but thicknesses of one-quarter and three-quarter inch are also assumed in the more favorable and unfavorable localities.

Ice and sleet generally collect quite uniformly on wires throughout their length. The collection is sometimes in the form of icicles but more often is egg-shaped in cross section, with the wire in the small end of the section. It frequently falls off non-uniformly in sections.

Clear solid ice weighs 57 pounds per cubic foot or 0.033 pound per cubic inch, but sleet or frozen snow such as often collects on wires weighs much less, sometimes as little as 8 pounds per cubic foot.

Effect of Surface and Electric Potential on Collection of Ice.— Local observations of single ice or sleet storms have shown that ice will sometimes collect on one wire or cable and not on an adjoining one. This has led to the conclusion that ice will not collect on certain kinds of wires, but more extensive observations indicate that ice will collect on any kind of wire under favorable conditions. Observations on wires carrying sufficient current to heat them and wires having potential near the critical corona voltage are not sufficiently extensive to warrant the conclusion that they will not collect ice and sleet.

Wind Velocity and Wind Pressure on Wires. — As noted in the article on *Wind Pressure*, the records of the United States Weather Bureau give nominal average velocities for five minute intervals, the true average velocities differing from the recorded velocities as follows:

Recorded velocity in miles per hour	10	20	30	40	50	60	70	80	90	100
Actual velocity in miles per hour	9.6	17.8	25.7	33.3	40.8	48.0	55.2	62.2	69.2	76.2

The Weather Bureau records give no indication of the "gust" velocities which may occur during the five minute periods, and which may greatly exceed the average velocity. Tests with a Dines pressure tube anemometer have shown that the extreme maximum is about 50 per cent greater than the average for short periods.

The extreme maximum wind velocity observed in Chicago in the whole thirty-six year period from 1873 to 1910 was 84 miles per hour (uncorrected) in February, 1894. A velocity of 76 miles per hour (uncorrected) was observed once in November, 1898, and a velocity of 72 miles per hour (uncorrected) was observed seven times. During the ten year period from 1894 to 1903 the maximum wind velocity in a few other representative localities was as follows, all velocities being the observed or uncorrected velocities: Bismark, N. D., 72; Eastport, Me., 78; Buffalo, N. Y., 90; New York City, N. Y., 78; Galveston, Tex., 84; Savannah, Ga., 76; Salt Lake City, Utah, 60. All the maxima range between 60 miles and 90 miles per hour.

The wind pressure on a cable is usually calculated from the formula

$$p = KV^2, \tag{1}$$

where p is the pressure in pounds per square foot of the projected area of the cable (including sleet and insulation, if any), and V is the actual velocity of the wind in miles per hour blowing perpendicularly across the span. The projected area, in square feet, of a one-foot length of cable is equal to the over-all diameter in inches divided by 12, where by over-all diameter is meant the diameter over the ice and insulation, if any. Buck's formula (*see article on Wind Pressure*) gives a value of 0.0025 for the constant K, whereas a value of 0.002 derived from the work of the Weather Bureau and experiments of Borda is also used.

Maximum Loading. — There is considerable difference of opinion as to what should be taken as the maximum loading in designing a transmission line. The Committee on Overhead Line Construction of the N.E.L.A. have proposed three different loadings, viz.: (A) No ice and a wind pressure of 15 pounds per square foot; (B) ice ½-inch thick and a wind pressure of 8 pounds per square foot; and (C) ice ¾-inch thick and a wind pressure of 11 pounds per square foot. Class B loading gives greater stress than Class A loading for all sizes of wire in use. The difference is greatest for small sizes.

Several important lines have been designed on the assumption of a maximum loading of ½ inch of ice and wind pressure at 6 pounds per square foot, the temperature corresponding to this loading being taken as 0° F. In these same lines the clearance under unequal loadings was calculated on the assumption of five

spans between anchor towers, with no ice on the third span of the lower conductor, and no ice on the first, second, fourth and fifth spans of the upper conductor, but with a loading of ¼ inch of ice on all other spans the temperature being taken as 32° F. and no wind assumed.

VERTICAL AND TRANSVERSE FORCES ON A SUSPENDED WIRE. — The resultant force acting on one foot of a suspended wire is in general made up of three components, viz.:

c = weight of the conductor (including insulation, if any) per foot length, in pounds;
i = weight of the ice coating per foot length of the conductor, in pounds;
h = wind pressure per foot length of the conductor, in pounds.

The weight of the conductor per foot length may be taken directly from the tables in the articles on *Wires and Cables, Bare,* as can also the diameter of the conductor (over the insulation, if any). Let d be this diameter in inches and let t be the thickness of the ice coating, then the weight of the ice coating per foot length of the conductor is

$$i = 1.24\, t\,(d + t). \tag{2}$$

Let p be the wind pressure per square foot of projected area, assumed or calculated from equation (1); then the wind pressure per foot length of the conductor, i.e., the horizontal component of the resultant force, is

$$h = \frac{p\ (d + 2t)}{12}. \tag{3}$$

The vertical component of the resultant force per foot length of conductor, which is equal to the resultant force for no wind, is

$$v = c + i. \tag{4}$$

Values of v and h for various ice thicknesses and various sizes of wires are given in Table II. The values of h are given in the table for a wind pressure of 10 pounds per square foot; these values of h are designated h_0; for any other wind pressure of say p pounds multiply these values by $\frac{p}{10}$. Knowing v and h the resultant force w for any combination of wind and ice loads is readily determined by the formula

$$w = \sqrt{v^2 + h^2}. \tag{5}$$

TABLE II. VERTICAL AND HORIZONTAL LOADING FORCES*

Pounds per Foot of Conductor

The horizontal component h_0 is given for a wind pressure of 10 lb. per sq. ft.; for any other wind pressure of p lb. per sq. ft., $h = \frac{ph_0}{10}$.

The resultant force for any ice thickness and wind pressure is $w = \sqrt{v^2 + h^2}$

Wire, size, B. & S. (A. W. G.) and circular mils		No Ice		¼-inch ice		½-inch ice		¾-inch ice	
		Vertical v	Horizontal h_0	Vertical v	Horizontal h_0	Vertical v	Horizontal h_0	Vertical v	Horizontal h_0
Aluminum, stranded									
	500,000	0.460	0.679	0.791	1.096	1.280	1.511	1.919	1.928
	450,000	0.414	0.643	0.731	1.060	1.205	1.476	1.834	1.893
	400,000	0.368	0.604	0.670	1.021	1.130	1.438	1.744	1.855
	350,000	0.322	0.566	0.610	0.983	1.055	1.399	1.655	1.815
	300,000	0.276	0.517	0.546	0.934	0.973	1.351	1.555	1.767
	250,000	0.230	0.473	0.483	0.890	0.894	1.306	1.459	1.723
0000	211,600	0.195	0.435	0.434	0.852	0.831	1.269	1.382	1.685
000	167,800	0.155	0.387	0.376	0.804	0.755	1.220	1.288	1.636
00	133,100	0.122	0.345	0.328	0.762	0.691	1.179	1.208	1.595
0	105,500	0.097	0.307	0.289	0.724	0.637	1.140	1.140	1.556
1	83,690	0.077	0.273	0.256	0.690	0.592	1.106	1.082	1.524
2	66,370	0.061	0.243	0.229	0.660	0.533	1.076	1.032	1.493
3	52,640	0.049	0.217	0.207	0.634	0.522	1.051	0.992	1.467
4	41,740	0.039	0.193	0.188	0.610	0.494	1.026	0.954	1.443
Copper, stranded									
	500,000	1.525	0.683	1.856	1.100	2.345	1.516	2.989	1.933
	450,000	1.373	0.642	1.689	1.059	2.165	1.475	2.791	1.892
	400,000	1.220	0.607	1.523	1.024	1.984	1.440	2.599	1.856
	350,000	1.068	0.566	1.356	0.983	1.801	1.399	2.401	1.815
	300,000	0.915	0.525	1.188	0.942	1.618	1.359	2.203	1.775
	250,000	0.762	0.492	1.022	0.909	1.440	1.325	2.012	1.742
0000	211,600	0.645	0.442	0.887	0.859	1.286	1.275	1.831	1.692
000	167,800	0.513	0.392	0.736	0.809	1.116	1.225	1.651	1.642
00	133,100	0.406	0.350	0.614	0.767	0.978	1.184	1.498	1.600
0	105,500	0.322	0.313	0.516	0.728	0.866	1.146	1.372	1.563
1	83,690	0.255	0.275	0.435	0.692	0.771	1.109	1.263	1.525
2	66,370	0.203	0.243	0.371	0.660	0.695	1.076	1.174	1.493
3	52,640	0.160	0.217	0.318	0.634	0.633	1.051	1.103	1.467
4	41,740	0.127	0.193	0.276	0.610	0.582	1.026	1.042	1.443
5	33,100	0.101	0.172	0.242	0.589	0.540	1.005	0.992	1.422
6	26,250	0.080	0.153	0.215	0.570	0.505	0.986	0.951	1.403

* This table is in agreement with the *Report of the Committee on Overhead Line Construction*, Proc. N.E.L.A., May, 1910, Vol. 1, p. 472; the diameters used are slightly greater than those given in the article on *Wires and Cables, Bare*.

TABLE II. — VERTICAL AND HORIZONTAL LOADING FORCES — *Continued*

Wire, size, B. & S. (A. W. G.) and circular mils		No ice		¼-inch ice		½-inch ice		¾-inch ice	
		Vertical v	Horizontal h_0	Vertical v	Horizontal h_0	Vertical v	Horizontal h_0	Vertical v	Horizontal h_0
Copper, solid bare									
0000	211,600	0.641	0.383	0.911	0.800	1.238	1.216	1.770	1.634
000	167,800	0.509	0.341	0.713	0.758	1.074	1.175	1.591	1.591
00	133,100	0.403	0.304	0.594	0.724	0.940	1.138	1.443	1.554
0	107,500	0.320	0.271	0.498	0.688	0.833	1.104	1.323	1.521
1	83,690	0.253	0.241	0.420	0.658	0.744	1.075	1.223	1.491
2	66,370	0.202	0.215	0.359	0.632	0.673	1.048	1.142	1.465
3	52,640	0.159	0.191	0.308	0.608	0.613	1.025	1.073	1.441
4	41,740	0.126	0.170	0.267	0.587	0.564	1.004	1.016	1.425
5	33,100	0.100	0.151	0.234	0.568	0.524	0.985	0.969	1.402
6	26,250	0.079	0.135	0.207	0.552	0.491	0.969	0.930	1.385
Copper, solid, triple braid, weatherproof									
0000	211,600	0.767	0.533	1.043	0.950	1.476	1.366	2.064	1.783
000	167,800	0.629	0.494	0.890	0.911	1.309	1.328	1.882	1.744
00	133,100	0.502	0.429	0.739	0.846	1.133	1.263	1.682	1.679
0	105,500	0.407	0.417	0.639	0.834	1.029	1.250	1.573	1.666
1	83,690	0.316	0.378	0.534	0.795	0.909	1.210	1.438	1.627
2	66,370	0.260	0.364	0.473	0.781	0.843	1.198	1.367	1.614
3	52,640	0.199	0.338	0.403	0.755	0.763	1.171	1.278	1.588
4	41,740	0.164	0.299	0.353	0.716	0.698	1.133	1.199	1.549
5	33,100	0.135	0.287	0.319	0.704	0.660	1.120	1.146	1.536
6	26,250	0.112	0.273	0.291	0.690	0.627	1.106	1.118	1.523
Steel, stranded, galvanized									
⅞ in.	575,000	1.540	0.730	1.888	1.147	2.393	1.563	3.050	1.980
13/16	500,000	1.336	0.677	1.666	1.094	2.151	1.510	2.789	1.927
¾	425,000	1.138	0.625	1.448	1.042	1.903	1.459	2.533	1.875
11/16	357,000	0.958	0.572	1.249	0.989	1.694	1.406	2.295	1.822
⅝	295,000	0.791	0.522	1.062	0.939	1.489	1.354	2.070	1.772
9/16	250,000	0.668	0.469	0.920	0.886	1.392	1.303	1.889	1.719
½	190,000	0.510	0.417	0.742	0.834	1.132	1.250	1.672	1.667
7/16	145,000	0.415	0.364	0.628	0.781	0.998	1.198	1.519	1.614
⅜	106,000	0.295	0.312	0.489	0.779	0.839	1.146	1.341	1.562
5/16	74,000	0.210	0.260	0.384	0.677	0.715	1.094	1.200	1.510

CALCULATION OF SAG AND TENSION. — A wire or cable suspended between towers takes the form of a catenary curve. In transmission-line practice the maximum deflection or sag is always small compared to the span, that is, the curve is very flat. The shape of such a flat catenary curve does not differ appreciably from a parabola and, as the approximate parabolic formulas are much simpler than the more exact catenary formulas, they are used instead. The flatness of the curve allows of some further simplifications even in the parabolic formulas, viz., (1) the tension is considered uniform throughout the span, the slight excess of tension at the ends over that at middle being neglected; (2) the change of length of the wire due to elastic stretch or temperature expansion is taken as equal to the change of length of a wire equal in length to the horizontal distance between the points of support.

Notation Used in Sag-tension Formulas. — The following notation, listed alphabetically, will be used throughout the discussion of sag and tension:

A = cross section of the conductor (actual metal cross section) in circular inches = square of diameter in inches; 1 circular inch = 1,000,000 circular mils.

a = coefficient of linear expansion of the conductor per °F.; see Table I above.

D = deflection, in feet, of the lowest point of the conductor when suspended from two points of support at the same elevation and at a distance L apart. (D is measured in the direction of the resultant transverse force.)

e = difference in elevation of the two points of support, in feet.

F = longitudinal *horizontal* component of the stress in the conductor, in pounds. (The resultant stress in the wire at the insulator is equal to $\sqrt{F^2 + H^2 + V^2}$, where V is the weight of the conductor and ice from the insulator to the lowest point of the span, and H is the total wind pressure on half the length of span; H and V in this expression are usually negligible compared with F.)

h = wind pressure in pounds per foot length of conductor assumed perpendicular to the vertical plane through the two points of support; see equation (3) and Table II above.

L = length of span in feet, i.e., the horizontal distance between the two points of support in feet.

l = length in feet of the arc of the curve in which the conductor hangs, i.e., the length of stretched conductor between the two points of support.

M = modulus of elasticity of the conductor in pound-inch units; see Table I above.

$S = \dfrac{vD}{w}$ = sag of the lowest point of the conductor below the horizontal line through the lower point of support; for no wind $S = D$.

T_0 = maximum allowable tension in the conductor in pounds per square inch of its cross section; T_0 is usually taken as one-half the ultimate strength of the conductor; see Table I.

v = vertical force in pounds on a one-foot length of the conductor, including the weight of conductor and the weight of the ice, if any, on it; see equation (4) and Table II above.

$w = \sqrt{v^2 + h^2}$ = resultant load in pounds on a one-foot length of the conductor.

$Z = \dfrac{hD}{w}$ = side swing, in feet, of the middle point of the conductor, measured perpendicularly to the vertical plane through the two points of support.

The various symbols with the subscript "o" will be used to designate the values of the various quantities under the conditions of maximum assumed loading; see paragraph above on *Maximum Loading*.

Fundamental Equations of a Wire Span. — As noted above, a perfectly flexible wire suspended between two points of support hangs in a catenary. The assumption that the wire hangs in a parabola instead of in a catenary is sufficiently accurate for all practical calculations of wire spans, the error in the sag calculated on this assumption being less than 2 per cent of its true value when this sag is less than 0.06 times the length of the span (e.g., less than 60 feet in a 1000-foot span), and the error in the length of the wire calculated on this assumption being less than 0.002 per cent of its true value for the same limiting conditions. The formulas given below are all based on the assumption of a parabola.

Deflection, Sag and Side Swing. — For a given length of span L, loading w, and stress F, the deflection D for the points of support at the same elevation is given by the relation

$$D = \frac{wL^2}{8F}. \tag{6}$$

When there is no wind this is also equal to the vertical sag, that is $S = D$. When there is wind, w is greater than the vertical loading v, and the vertical sag for the points of support at the same elevation is

$$S = \frac{vD}{w} = \frac{vL^2}{8F}. \tag{7}$$

D in equation (7) has the value given by equation (6). When one point of support is at an elevation e above the other, then the vertical sag of the lowest point of the conductor below the lower point of support is

$$S' = S\left(1 - \frac{e}{4S}\right)^2, \tag{8}$$

where S is given by equation (7). The horizontal distance of the lowest point of the conductor from the lower point of support is

$$L' = \frac{L}{2}\left(1 - \frac{e}{4S}\right). \tag{9}$$

The side swing Z of the middle point of the conductor, which is the point which is deflected the maximum distance from the vertical plane through the two points of support, is

$$Z = \frac{hD}{w} = \frac{hL^2}{8F}. \tag{10}$$

D in equation (10) has the value given by equation (6).

Length of Stretched Conductor. — The length of conductor between the two points of support for a given length of span L, loading w, stress F and difference of elevation e, is

$$l = L + \frac{8}{3}\frac{D^2}{L} + \frac{e^2}{2L}, \tag{11}$$

where D has the value given by equation (6), that is, D is the deflection for the same length of span, loading and tension, but for the points of support at the same elevation.

Effect of Changes in Loading and Temperature. — When the loading or the temperature changes, the stress in the conductor will change to some new value, F_0 say, and the deflection will change to some new value, say D_0. Let the new loading be w_0 and the new temperature be t_0, the initial temperature being t; also let a be the coefficient of linear expansion, M the modulus of elasticity, and A the cross section of the conductor in millions of circular mils. Then, when the points of support remain fixed, the following relation must hold

$$\frac{8}{3L^2}(D^2 - D_0^2) = a(t - t_0) + \frac{1.273}{MA}(F - F_0). \qquad (12)$$

The D's in this equation are the same as given by equation (6) for a loading of w and w_0 respectively and stresses of F and F_0 respectively. Note that equation (12) is independent of the difference in elevation of the two points of support; also that the two sets of symbols, with and without the subscripts, refer to *any* two sets of conditions.

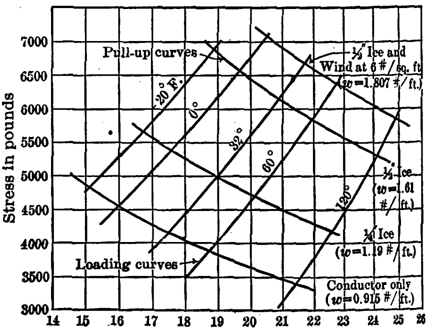


Fig. 8. Stress-Deflection Chart for a 300,000-Circular-Mil, 19-Strand Copper Conductor on an 800-ft. Span

Stress - Deflection Charts (Fig. 8). — In order to apply the above equations to the calculation of clearances under various conditions equation (6), namely

$$D = \frac{wL^2}{8F}, \qquad (13)$$

and equation (12), which may be written

$$F = F_0 - \frac{2.10\,MA}{L^2}D_0^2 - \frac{MAa}{1.273}(t - t_0) + \frac{2.10\,MA}{L^2}D^2, \qquad (14)$$

may be plotted for any given length of span as shown in Fig. 8, with deflection D as abscissas and the stress F as ordinates.

Pull-up Curves. — The curves representing the relation between D and F in equation (13) are equilateral hyperbolas, as many of these curves being drawn as there are loadings to be considered. These hyperbolas may be called "pull-up" curves, since they give the stress to which the wire must be pulled up for any given deflection. In Fig. 8 four pull-up curves are shown, one for the conductor only ($w = 0.915$), conductor coated with ¼ inch of ice but no wind ($w = 1.19$), conductor coated with ½ inch of ice but no wind ($w = 1.61$), and conductor coated with ½ inch of ice and a wind pressure of 6 pounds per foot of projected area of wire ($w = 1.807$), this last being taken as the maximum loading.

Loading Curves. — The curves representing the relation between D and F in equation (14) are parabolas with vertical axes, as many of these curves being drawn as there are temperatures to be considered. These parabolas may be called "loading curves," since for a change in loading (at constant temperature) they give the relation which must exist between the new stress and new deflection produced. In order to plot these curves it is necessary to assume a maximum allowable tension, say T_0 pounds per square inch, a maximum load-

ing w_0, and the minimum temperature t_0 at which this loading is assumed to occur. Then

$$F_0 = \frac{AT_0}{1.273} \quad \text{and} \quad D_0 = \frac{w_0 L^2}{8 F_0},$$

and for any given temperature t all the terms in the right-hand member of (14) are constants except the last term, which varies as D^2. In Fig. 8 the maximum allowable tension T_0 is taken as 30,000 pounds per square inch, the maximum loading w_0 as 1.807 pounds per foot length, and the minimum temperature under maximum loading as 0° F. Then $F_0 = 7065$ pounds and $D_0 = 20.5$, which fixes one point on the 0° loading curve, and the other points on this curve are calculated directly from (14) by putting $t = 0$. For any other temperature, equation (14) gives the same shaped parabola as for 0°, but each point of the curve is shifted vertically downward a distance $\frac{MAa}{1.273}$ for each degree increase of temperature. Hence it is only necessary to calculate the 0° loading curve, and the loading curve for any other temperature can be readily plotted by the use of a pair of dividers.

Stringing Stresses and Deflections. — In stringing cable it is essential that the stress or deflection be correct for the stringing temperature, otherwise the maximum stress will be more than safe or the clearance to ground less than safe due to maximum deflection exceeding the designed amount. To insure proper stringing, curves of temperature-sag and temperature-stress are plotted from the stress-deflection curves. These curves are for no wind and no ice. Fig. 9 shows such curves for the 800-foot span plotted from Fig. 8. For example, if the cable is strung at 70° F., then it should be given a deflection (vertical sag) of 19.25 feet, provided the points of support are at the same elevation, or at a stress of 3800 pounds, this stress being independent of the difference of elevation of the two points of support. If there is a difference of elevation of 30 feet, say, between the two points of support, then from equation (8a), the cable should be drawn up until the sag of the lowest point below the lower point of support is

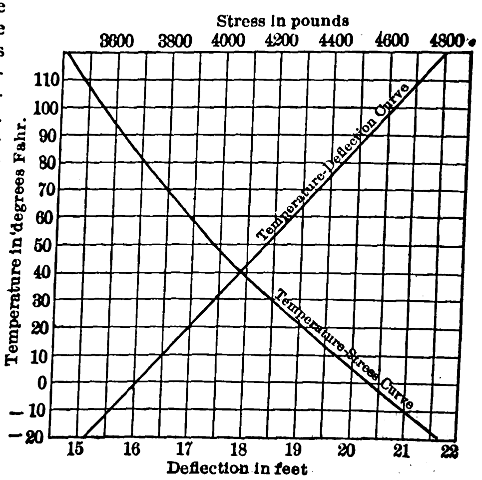


Fig. 9. Stringing Chart

$$S' = 19.25\left(1 - \frac{30}{4 \times 19.25}\right)^2 = 7.17 \text{ feet},$$

and the stress will then be 3800 pounds.

Stress and Deflection in Spans of Unequal Length. — By stringing cables according to the stress determined for each particular length of span, the maxi-

mum allowable tension will be reached in all spans under maximum loading conditions. Under other loading conditions the tension will be unequal where span lengths are not the same. This is shown in Fig. 10 which is plotted for the same conditions as stated in the title of Fig. 8, but for spans of from 600 to 900 feet in length. Fig. 11 shows the corresponding deflections.

The unequal stresses on the two sides of the insulator will tend to bend the tower or insulator pin, but any motion of the point of support will tend to equalize the stresses. When suspension insulators are used, the stresses are practically equalized, since the insulator is free to move. It should be noted that the motion of the insulator necessary to equalize the stresses is small. When suspension insulators are used the cable is therefore strung at a tension corresponding to the average length of span. When the cable is thus strung the tension under minimum temperature and maximum loading will usually exceed the assumed tension in the shorter spans and be less than the assumed tension in the longer spans. Similarly, under maximum temperature conditions, the longer spans will have a sag in excess of the value calculated on the assumption of the maximum allowable tension being reached with maximum loading, and the shorter spans will have a sag less than the calculated value. The actual stresses and deflections can be calculated by the method given in the following section.

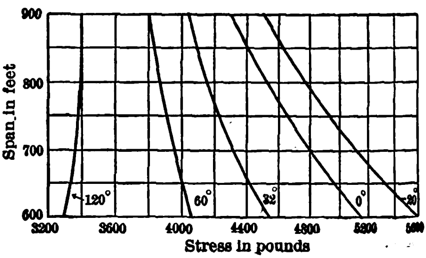


Fig. 10.

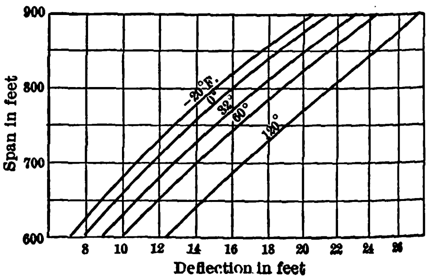


Fig. 11.

Calculation of Stresses in Unequally Loaded Spans.* — When suspension insulators are used, any tendency of the stresses in two adjacent spans to become unequal will produce such a deflection of the insulator, in the direction of the span with greater stress, as will establish equilibrium in the line. This state of affairs will occur (1) when adjacent spans carry unequal ice loads, (2) when the wire on one side of the insulator breaks, and (3) to a slight extent with changes in temperature when the adjacent spans are unequal in length, as noted above. The following method of calculating the "equilibrium" stresses in the wires and corresponding sags is applicable to all cases of initially unbalanced stresses, irrespective of their cause. The method may also be used to calculate the stress and sag in spans supported on pin insulators, provided the moment of bending of the pin and of the pole or tower is known or can be calculated.

* From lecture notes by Dr. H. Pender.

Change of Stress Due to Change in Length of Span. — When the length L of the span (i.e., distance between points of support) increases by λ inches, due to a horizontal displacement of the insulators (without slipping of the wire), the stress in the wire is increased by the same amount as would be produced by a fall in temperature of

$$t - t' = \frac{\lambda}{12\, aL} \qquad \text{or} \qquad t' = t - \frac{\lambda}{12\, aL}, \tag{15}$$

degrees Fahrenheit, where t is the actual temperature and t' may be called the "equivalent" temperature corresponding to the change in length λ. In this equation λ is the actual increase in the distance between the points of support in inches, a the temperature coefficient of linear expansion per °F., and L the original length of the span in feet. For example, in an 800-foot span of copper wire an increase of 1 inch in L corresponds to a drop of temperature of $1 \div (12 \times 9.6 \times 10^{-6} \times 800) = 10.85$ degrees, and an increase in length of λ inches corresponds to a drop of temperature of $t - t' = 10.85\,\lambda$ degrees.

Hence the stress-deflection chart for any given length of span, see Fig. 8, may be used directly to determine the stress in the wire after any change in the length of the span, due to the deflection of the insulator. For example, consider an 800-foot span of 300,000-circular-mil, bare, copper conductor, at 32° F., without ice or wind, initially stressed to 4170 pounds, and let the length of the span be increased 4 inches as the result of the deflection of the insulators by this amount. This increase in length of span will then give the same stress in the wire as would be produced if the temperature fell from 32° to $t' = 32 - 4 \times 10.85 = -11.4°$. The point at which the loading curve (Fig. 8) corresponding to a temperature of $-11.4°$ crosses the pull-up curve for conductor only gives the new stress, viz., 4700 pounds.

Horizontal Pull of Insulator. — Referring to Fig. 12, let m = the horizontal distance in inches (measured along the span) which any insulator is deflected from the vertical, taken positive when to the right, say, and negative when to the left. Let x = the length of the insulator string in inches, i.e., distance from point of attachment to tower to point of attachment to wire; V = total weight of wire and ice between the lowest point of the wire in the span to the left of the insulator and the lowest point of the wire in the span to the right of the insulator plus one-half the weight of the insulator; H = total wind pressure on the length of wire between the middle points of the two adjacent spans, plus half the wind pressure on the insulator; and put $W = \sqrt{V^2 + H^2}$. Then the horizontal component of the pull of the insulator toward the left along the line of the span is *

$$P = \frac{m}{\sqrt{x^2 - m^2}} \cdot W. \tag{16}$$

For example, consider two adjacent spans of 300,000-circular-mil copper, each 800 feet long and with points of support at the same elevation. Let the span to the left be free of ice and let the one to the right have a ¼-inch ice coating; assume the insulator to be 60 inches long and to weigh 100 pounds. Then for no wind $H = 0$, $V = 1.19 \times 400 + 0.915 \times 400 + 100/2 = 892$ pounds. Whence for deflections of the insulator of less than 12 inches the horizontal pull of the insulator is $P = (892 \times m) \div 60 = 14.9\, m$, or 14.9 pounds per inch deflection.

* When m is less than 20 per cent of x this may be written, with an error of less than 2 per cent,

$$P = \frac{m}{x} W. \tag{16a}$$

Stresses in a Series of Spans When Points of Support are not Fixed. — Referring to Fig. 12, let the left-hand end of span No. 1 be anchored, and assume the insulator at the right-hand end to be deflected a horizontal distance of m_1 inches, due, for example, to a change in the loading on the succeeding

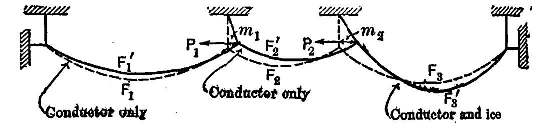


Fig. 12.

spans (or to a change in temperature when the spans are of unequal length). From equation (15) calculate the "equivalent" temperature t_1' corresponding to this change in length, and from the pull-up curve in Fig. 8 corresponding to the assumed loading w_1 of this span find the stress on this curve corresponding to a temperature of t_1' degrees; call this stress F_1'.

Next calculate the transverse and vertical loads on the insulator, viz., H_1 and V_1, and the resultant load $W_1 = \sqrt{V_1^2 + H_1^2}$, as explained in the preceding section. Then from equation (16) or (16a) calculate the horizontal pull P_1 of the insulator. The stress in the second span, assuming the value of m_1 chosen at the start is correct, must then be

$$F_2' = F_1' + P_1. \tag{17}$$

t_2' is then determined from F_2' on the pull-up curve. From equation (15) the corresponding increase in the length of span No. 2 must then be

$$\lambda_2 = 12\, aL_2\,(t - t_2'), \tag{18}$$

where L_2 is the length of the second span. The corresponding deflection of the insulator at the right-hand end of span No. 2 must then be

$$m_2 = m_1 + \lambda_2, \tag{19}$$

always taking the insulator deflection positive when to the right, say.

Using the values of λ_2 and m_2 thus found, calculate the λ_3 and m_3 in exactly the same manner as λ_2 and m_2 were calculated, and similarly for the succeeding spans until the next anchor tower is reached. For the anchor tower at the right-hand end of the n-th span, say, the deflection of the insulator must be zero, viz.,

$$m_n = 0. \tag{20}$$

If m_n as calculated comes out greater than zero, then the assumed value of m_1 is too great; if m_n comes out less than zero the assumed value of m_1 is too small. By calculating m_n for two or three assumed values of m_1, and plotting m_n as ordinates against m_1 as abscissas, the correct value of m_1 will be where this curve crosses the axis of abscissas. Using this correct value of m_1, the stresses and deflections in each span may then be accurately calculated by the process just given, using the Stress-Deflection Chart, Fig. 8. The complete process is best shown by an example.

Example. — Consider the case of three spans between anchor towers (Fig. 12) all of the same length, 800 feet, and all supports at the same elevation, 300,000-circular-mil copper being used for the conductor. Let the temperature be 32° F., and let the middle span have a ¼-inch ice coating but the other two spans have no ice on them; also assume no wind. The Stress-Deflection Chart given in Fig. 8 then applies directly, provided the wires are strung in accordance therewith. Assume that each insulator weighs 100 pounds and has a length of

60 inches. Then for an increase of λ inches in the length of any span, the "equivalent" temperature is, from equation (15),

$$t' = 32 - 10.85\lambda$$

or if the equivalent temperature rise t' is known

$$\lambda = 0.092\,(32 - t').$$

The horizontal pull of any insulator for a deflection of m inches (small compared with the length of the insulator) is, from equation (16a),

$$P = 14.9\,m.$$

In the following table are given the calculations for assumed values of m_1 of 1, 2, 3 and 4 inches, and in Fig. 13 are plotted the corresponding calculated values of m_3 against m_1. It is seen that the relation between m_3 and m_1 is practically a straight line cutting the horizontal axis at $m_1 = 2.4$, which is therefore the correct value of m_1. The calculations for $m_1 = 2.4$ inches are given in the last column of the table. Hence the stresses and deflections in the two end spans (without ice) are $F_1' = F_3' = 4460$ pounds and $D_1' = D_3' = 16.3$ feet respectively, and the stress and deflection in the middle span loaded with ¼-inch of ice are $F_2' = 4496$ pounds and $D_2' = 21.1$ feet respectively, the deflections being read directly from Fig. 8 corresponding to the proper stresses and loadings (w).

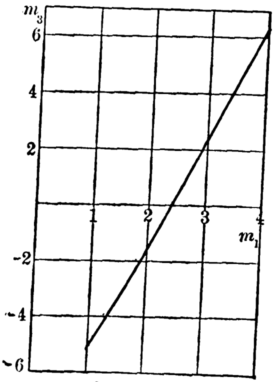


Fig. 13.

m_1	= assumed	1	2	3	4	2.4
t_1'	$= 32 - 10.85\,m_1$	21.1	10.3	−0.6	−11.4	6
F_1'	From Fig. 8 ($w = 0.915$)	4300	4410	4550	4700	4460
P_1	$= 14.9\,m_1$	15	30	45	60	36
F_2'	$= F_1' + P_1$	4315	4440	4595	4760	4496
t_2'	From Fig. 8 ($w = 1.19$)	105	90	74	55	84
λ_2	$= 0.092\,(32 - t_2')$	−6.72	−5.33	−3.86	−2.12	−4.78
m_2	$= m_1 + \lambda_2$	−5.72	−3.33	−0.86	1.88	−2.38
P_2	$= 14.9\,m_2$	−85	−50	−13	28	−36
F_3'	$= F_2' + P_2$	4230	4390	4582	4788	4460
t_3'	From Fig. 8 ($w = 0.915$)	26	13	−3	−16	6
λ_3	$= 0.092\,(32 - t_3')$	0.55	1.75	3.22	4.42	2.39
m_3	$= m_2 + \lambda_3$	−5.17	−1.58	2.36	6.30	0.01

LOCATION OF TOWERS AND DETERMINATION OF CLEARANCES.—The height of towers is determined so as to give some specified minimum clearance from conductor to ground for some length of span chosen as a nominal standard on the basis of level ground. In practice the ground is rarely level and the towers are actually located to conform to the irregularities of the ground. In locating towers of a given height the spans are made as long as possible consistent with maintaining the ground clearance. The irregularities of the ground are ordinarily advantageous and permit of slightly longer spans on the average than could be obtained with the same height of towers on level ground.

Profile and Plan of Right-of-Way. — In order to locate towers properly it is necessary to have a profile of the right-of-way. Profiles are conveniently plotted on standard ruled profile section paper to a vertical scale of 20 feet to the inch and a horizontal scale of 200 feet to the inch. Three profiles are desirable, one along the center of the tower line and one on each side, say at each edge of right-of-way, as shown on Fig. 14 at *A*, *B* and *C*. The two side profiles indicate the amount and direction of the slope of the ground across the line, which must be allowed for in determining ground clearance and foundation or tower extensions.

A plan of the right-of-way is of course also necessary for determining the construction at angles in the line, and the clearances from the conductor to the edge of the right-of-way when the conductor is deflected horizontally by the wind. Such a plan is shown at the bottom of Fig. 14.

Templates for Locating Towers. — Three templates are required, one for ground clearance with maximum sag, marked *M* in Fig. 14, one for uplift at times of minimum sag, marked *N*, and one for maximum side swing, marked *Z*. These are cut from thin celluloid and are to the same horizontal and vertical scales as used for the profile and plan of the right-of-way.

Since the curvature of the catenary or parabola in which the wire hangs depends only on the tension and loading and not on the length of the span or on the difference in elevation of the points of support, all spans having the same tension and loading can be drawn (for any one predetermined scale) from a single template, irrespective of their lengths or of the differences in elevation of the points of support. However, when the elevations of the points of support are not the same, the lowest point of the curve is shifted from the middle of the span toward the lower support, but the axis of the curve remains vertical.

Construction of Maximum Sag Template *M*. — The maximum sag is found from the Stress-Deflection Chart, Fig. 8, and may be the deflection corresponding to the maximum temperature, and conductor only, e.g., 21.4 feet in Fig. 8, or may be the deflection corresponding to 32° F. and the maximum ice loading. Wind will increase this deflection but will not increase the *vertical sag*. Call S_m this maximum sag. Then the equation of the maximum sag template, or template *M*, is

$$y = \left(\frac{4\,S_m}{L^2}\right) x^2, \tag{21}$$

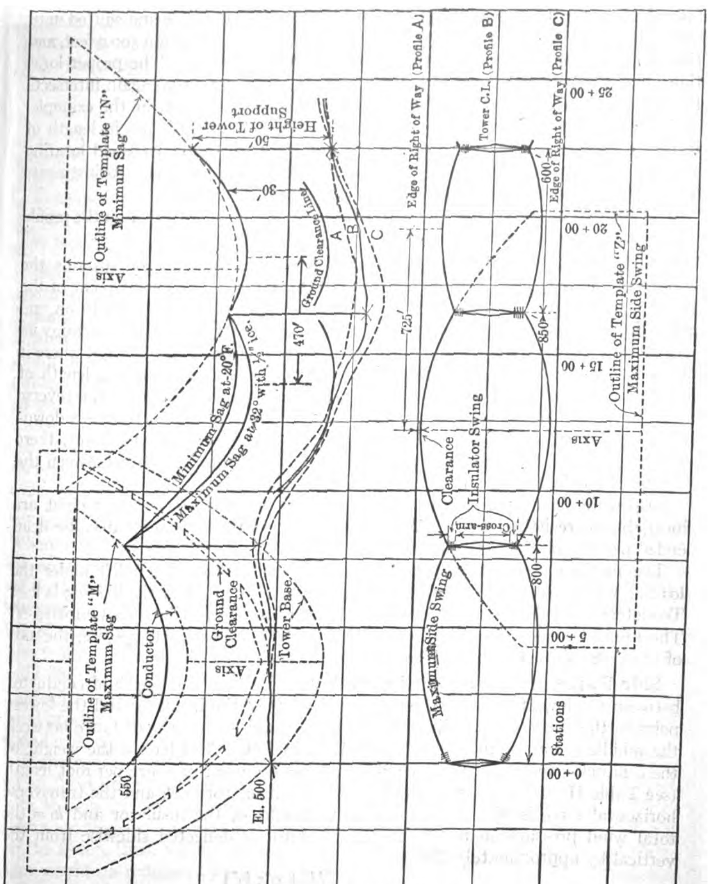
where L is the length of the particular span for which the maximum sag is S_m, and the origin of the curve is its lowest point and the axis of y is vertical; all dimensions are in feet. Three parabolic curves are given by this template; the top curve represents the position of the cable and is drawn on the basis of the average span under the maximum load; the middle curve is a similar parabola below the upper curve a distance equal to the minimum allowable clearance to ground; the bottom curve is another similar parabola below the upper curve by a distance equal to the height of cable above ground at the support.

Construction of Minimum Sag Template *N*. — The minimum sag is also found from the Stress-Deflection Chart, and is usually the deflection corresponding to the minimum temperature and conductor only, e.g., 15.1 feet in Fig. 8. Call this sag S_n; then the equation of the minimum sag template, or template *N*, is

$$y = \left(\frac{4\,S_n}{L^2}\right) x^2. \tag{22}$$

Construction of Maximum Side-swing Template *Z*. — The maximum side swing occurs at time of maximum wind pressure and may be at

Fig. 14. Chart Showing Use of Templates in Locating Towers and Determining Clearances



maximum temperature or may be at 32° F. when covered with ice. In the latter case the side swing depends on the shape (circular or elliptical) of the ice covering and its specific gravity. For a circular covering of solid ice one particular thickness (usually but not necessarily the maximum thickness) gives the greatest side swing. For example, in Fig. 8, the maximum side swing occurs at $D = 21.7$ and $w = 1.807$, and its value is, see equation (10), $Z = (0.815 \times 21.7) \div 1.807 = 9.8$ feet, 0.815 being the wind pressure per foot of wire. Calling the maximum side swing Z_m, then the equation of the side swing template, or template Z, is

$$y = \left(\frac{4 Z_m}{L^2}\right) x^2. \tag{23}$$

Locating Towers by Means of Template *M*.—Choose a starting point, as shown for example in Fig. 14, at station 0 + 00, elevation 500.0 feet for the first tower location. The template *M* is then placed over the profile and shifted until its axis is vertical and the lower curve is at station 0 + 00, elevation 500.0 feet, and the middle curve is tangent to the ground profile as shown. The proper location for the second tower is at the point where the lower curve again intersects the ground profile, or at station 8 + 00, elevation 506.0 feet, in the example. The operation is then repeated for the next tower. Adjustments in length of span are usually necessary to meet local conditions, in order to avoid locating towers in roads or swamps and to bring towers at angle points. Adjustments which increase the ground clearance are of course allowable.

The position of the conductors with maximum sag may be drawn on the profile from the top curve of the template.

Uplift on Insulator; Use of Template *N*.—An insulator sustains the weight of the lengths of conductor from the insulator to the lowest point of the span on each side. If the conductor leaves the insulator horizontally on one side, the lowest point of that span is at the insulator, which then sustains no weight due to that span. If the conductor has an upward inclination where it leaves an insulator, it is exerting an uplift equal to the weight of a length of conductor extending from the insulator along the span produced in the reverse direction to the lowest point of the parabola. Where the conductor has a downward inclination on one side and upward on the other side of the insulator, there will be a weight or uplift on the insulator equal to the difference between the weight of conductor on one side and uplift on the other.

Suspension insulators when used hanging downward to sustain weight are incapable of resisting uplift. Where uplift occurs the conductor may be dead ended or may be tied down or weighted down.

The method used for locating towers ordinarily precludes uplift under the loading which gives maximum sag, but uplift may occur when the loading is less. To determine this the minimum sag is drawn on the profile with template *N*. The minimum sag curve is drawn between points of support, keeping the axis of the parabola vertical as before.

Side Swing of Suspension Insulators.—Let l_1 = the length of conductor between the lowest point in the span to the left of the insulator and the lowest point in the span to the right of the insulator, and let l_2 = the distance between the middle points of these two spans, both in feet. Also let v = the weight of the conductor and ice per foot length, and h = the wind pressure per foot length (see Table II). Then the vertical pull on the insulator is vl_1 and the transverse horizontal force is hl_2. Also let v_1 = the weight of the insulator and h_1 = the total wind pressure on it. Then the insulator is deflected sidewise from the vertical by approximately the angle

$$\theta = \tan^{-1}\left[\frac{hl_2 + 0.5\, h_1}{vl_1 + 0.5\, v_1}\right]. \tag{24}$$

Usually the weight of the insulator and the wind pressure on it are negligible compared with the weight and wind pressure on the conductor, in which case

$$\theta = \tan^{-1}\left(\frac{hl_2}{vl_1}\right). \qquad (24a)$$

When the points of support are at the same elevation $l_1 = l_2$ and

$$\theta = \tan^{-1}\frac{h}{v}. \qquad (24b)$$

Calling X the length of the insulator in feet then the transverse horizontal deflection of the insulator is $X \sin \theta$ feet.

For example, consider the side swing of the third insulator (from the left) in Fig. 14. Then $l_1 = 470$, $l_2 = 850/2 + 600/2 = 725$, $v = 1.61$ (for 300,000-circular-mil conductor with ½ inch of ice), and $h = 0.82$ (for wind pressure of 6 pounds per square foot). Whence, neglecting the weight of the insulator and the wind pressure on it,

$$\theta = \tan^{-1}\frac{0.82 \times 725}{1.61 \times 470} = 38°.$$

If the insulator is 5 feet long, the transverse horizontal deflection is then $5 \sin 38° = 3.1$ feet.

If the angle of swing as thus determined is excessive the cables and insulators will be lifted up into the cross arms at times of low temperatures and high winds. The remedy is the same as in case of direct uplift.

Side Clearance; Use of Template *Z*. — Where a right-of-way of definite width is obtained it is necessary to determine whether the conductor will swing beyond the edge of the right-of-way. Therefore after the towers have been located by the use of the profile they should be marked on the plan and the side swing marked in from template *Z*, as shown in Fig. 14. In determining side swing, the swing of the insulator (if suspension type) must be allowed for, as well as the side swing of the conductor. Adequate margin should be allowed between the extreme position of conductor and the edge of the right-of-way, so that a safe clearance will be preserved from any structures erected adjacent thereto. Where extraordinarily long spans must be used, an adequate extra width of right-of-way should be obtained in the first place.

Loss of Clearance Between Conductors Due to Unequal Ice Loading.— Where one span is loaded with ice and the one immediately below it is not, the clearance is reduced. This condition may sometimes arise due to the ice falling off the lower wire before it falls off the upper wire. Where the wires are directly over each other the normal clearance must be great enough to prevent the crossing of wires under these conditions. For ice loading without wind, clearance under unequal loading is most easily obtained by offsetting the wires horizontally for the required clearance instead of increasing the vertical clearance. However, to prevent crossing of the unequally loaded wires when deflected by wind pressure, this horizontal offset must be considerable, as the clearance must then be obtained between the wires in their inclined positions.

If the conductors to which Fig. 8 refer are normally 10 feet apart vertically on an 800-foot span, the sag would be 17.5 feet without ice at 32° F., 19.0 feet with ¼-inch ice, and 20.9 feet with ½-inch ice at 32° F. Consequently, if two cables are used one above the other, and ice should form on the upper but not on the lower, then, assuming fixed points of support (pin insulators), the clearance would be reduced by 3.4 feet for ½-inch ice, and 1.5 feet for ¼-inch ice, making the clearances 6.6 feet and 8.5 feet respectively, instead of 10 feet.

Where suspension insulators are used the reduction of clearance from unequal ice loading is greater. If one span is loaded with ice and the adjacent spans of the same wire are not loaded, the sag of the loaded span will be increased because the insulators will swing toward that span. Similarly if one span is unloaded and adjacent spans are loaded, the sag will be decreased. The minimum clearance occurs where only one span of the upper wire is loaded and is immediately over the only unloaded span of the lower wire. The actual reduction is readily calculated by the method given above in the section on *Calculation of Stresses in Unequally Loaded Spans*, p. 1696. The amount of reduction depends on the number of spans between anchor towers, and the distance from the anchor towers at which the unbalanced loading occurs. For a 300,000-circular-mil copper cable on 800-foot spans at 32° F. with unequal loadings on sections of one, two, three and five spans (see Fig. 15) the assumed conditions of unequal loading and the loss of clearance are as follows:

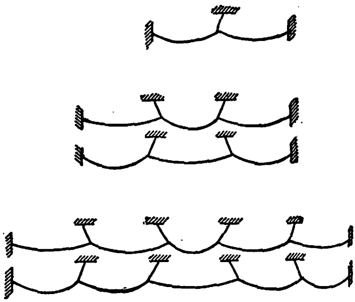


Fig. 15.

Number of spans between anchor towers	1	2	3	5
Upper conductor; ¼-inch ice on spans Nos.:*	1	1	2	3
Lower conductor; ¼-inch ice on spans Nos.:*	...	2	1, 3	1, 2, 4, 5
Sag of middle span of upper conductor, feet	19.0	20.5	21.1	21.6
Sag of middle span of lower conductor, feet	17.5	15.9	15.5	15.2
Loss of clearance in middle span, feet	1.5	4.6	5.6	6.4

* The other spans assumed to have no ice load.

Stresses and Deflections Due to Broken Conductors.—When a conductor breaks in a span supported by suspension insulators, the insulators adjacent to the broken span swing up into line with the cable, throwing increased slack into the unbroken part of the cable equal to the length of the insulator. This slack divides between the unbroken spans, increasing the deflection of each. The stresses and deflections of the unbroken spans may be determined by the method given above in the section on *Calculation of Stresses in Unequally Loaded Spans*, p. 1696, calling the span in which the break occurs span No. 1.

ERECTION OF TRANSMISSION LINES.—The preliminary work in constructing a transmission line includes the clearing of the right-of-way, moving of buildings (if necessary), building roads, bridges, fences, gates, etc., to make the right-of-way passable. If the telephone line is to be supported on separate structures, this is generally built in advance of the main line and used during the rest of the construction.

After the towers are in place and the insulators placed, the wires and cables are strung. The reels are distributed and spaced according to the length of cable on each. The reels of heavy cable are supported on reel jacks. The lighter wire and cable is usually drawn out on the ground and carried or hauled up on the supporting structures. Heavy cables are drawn over rollers or sheaves attached to the towers at approximately the final point of support. The pulling

out of wires and cables is usually done with horses, but under favorable conditions a traction engine or hoisting engine can be used. Where lines parallel railroads they have been pulled out successfully with locomotives. Care should be taken in drawing out the cable so as not to scratch or injure it by sharp bends or faulty cable grips, or by dragging it over sharp stones. See also the article on *Wires and Cables, Bare.*

After the cable is drawn out and is in place on the sheaves on the towers, it should be adjusted to the proper sag by using dynamometers for measuring the tension. In order to have uniform tension in all spans when pulling several at one time, the cable should be free to move at all points of support. Wires and cables should be clamped or tied in place while under the proper tension corresponding to the temperature at time of stringing. Wind loads on cables or wires are generally not of an amount during wire stringing to require allowance for additional tension. Splicing sleeves are twisted by hand wrenches in the smaller sizes and by splicing machines for the larger sizes.

At anchor towers loose jumpers must be bent to shape and not left so that they may ground on the tower due to twist of cable, pressure of wind or weight of sleet.

Where ground wires or telephone wires are also on the tower, it is equally important that they be strung at the proper tension; otherwise they may cross and ground the conductors.

Suspension towers (i.e., those intermediate between the dead-end towers) are ordinarily not strong enough to stand the strain of dead-ended cables during high winds and heavy sleet storms; consequently care must be used if cables are temporarily dead ended on them during construction.

Transposition of Transmission Lines. — (*See also Telephone Lines.*) Transpositions are not necessary for the proper operation of an isolated transmission line, but are used to diminish the inductive effects of the transmission line on neighboring circuits. The number of transpositions required depends principally on the sensitiveness and proximity of other circuits, especially telephone lines and on the distance that such lines parallel the transmission line, and also to a lesser degree on the current and voltage of the transmission line.

Where the voltage of transmission is 2200 or less the length of exposure (i.e., distance between two successive transposition points) is usually insufficient to require transpositions and, when the voltage is over 60,000, the lines are often at sufficient distance from nearest telephone line to make transpositions unnecessary. For voltages above 2200 and not over 60,000 a telephone line is ordinarily run the whole distance on the same poles or towers with a transmission line, thereby making transpositions necessary. Transpositions on such transmission lines are ordinarily located three miles or more apart, the telephone line also being transposed every 500 or 600 feet.

Each transposition of a three-phase line ordinarily consists of a spiral of one-third of a turn. Three transpositions give a complete spiral and bring the phases back to their original position. A line is ordinarily transposed by giving it one or more complete spirals, the transpositions being located so as to divide the line into approximately equal sections. For one complete spiral two transpositions dividing the line into thirds are sufficient, though the equivalent of a third transposition is necessary at one end if the phases are to have the same relative position in the station wiring at each end.

Special poles or towers are usually necessary at transposition points.

TESTS AND INSPECTION OF TRANSMISSION LINES. — Tests on transmission lines include testing of the several parts; conductors, insulators, pins, clamps, ties, towers, etc., and are described in the articles on ***Wires and***

Cables; Insulators for Overhead Lines; Poles for Overhead Lines, Cross Arms; Towers. The efficiency and other electrical characteristics of transmission lines can be computed with such certainty that tests are not necessary to determine these features, though the calculations are occasionally checked by observations made during actual operation.

When a line is completed it should be carefully inspected to see that all joints have been made, all insulators put on, etc. Moreover, before starting continuous operation, it is advisable to apply voltage in order to make sure that the line is not open circuited, short-circuited or grounded.

Where two or more transmission lines are to be in parallel, or where a new line connects two points already connected by other lines, it is necessary to test out to find corresponding phases before connecting in multiple, especially if any of the lines have transpositions.

OPERATION. — Patrolmen are usually located between ten and twenty miles apart, depending on the character of the country. The coöperation of people living along the line must be obtained in order to prevent damage by breaking of insulators, throwing wires over the conductors, blasting rocks or stumps near the line, etc. Patrolmen should inspect the line regularly and keep weeds, brush and inflammable material away from the poles and towers. They should note the condition of the foundations, towers, poles, insulators and conductors. In addition they should make minor and emergency repairs on the line.

Section switches are often installed at each patrolman's house. By manipulating these switches any section of line may then be tested for faults.

The proper maintenance of a line includes resetting foundations that have settled; covering of foundations with earth to proper depth after heavy rains; repainting of towers before they are affected by rust; renewal of rusted ground cables; replacement of cracked or partially defective insulators that have not failed; correcting sag of any cable where sag has changed due to stretch of cable, change of length during emergency repairs, etc.

Telephone Connections. — A good telephone line is essential and the patrolmen should report by telephone at regular intervals. As a convenience in operation, telephones are installed or connections are provided for a portable telephone set at intervals of three or four miles along the line. For the higher potential lines protection must be provided for the person using telephone. The usual protection includes insulated stools, telephone insulating transformers, drainage coils, telephone lightning arresters and fuses.

COSTS OF TRANSMISSION LINES. — To obtain an accurate estimate of the cost of a transmission line the cost of the various elements should be separately determined. The following over-all costs are given as a rough guide in preliminary estimating.

Cost of Right-of-Way. — The cost of the right-of-way for a low-voltage line ranges from a nominal amount for a line on public roads, to $10,000 per mile or more for a private right-of-way in thickly settled districts. A right-of-way 100 feet wide requires approximately 12 acres per mile, which for farming land at $200 per acre, amounts to $2400 per mile. Land values may be expected to average considerably higher for right-of-way than for farming, especially if a strip of land is desired crossing a farm diagonally.

The right-of-way for high-voltage tower lines should be estimated liberally as such lines should have long spans and few angles, and cannot be diverted around expensive property and obstructions as readily as short-span wooden-pole lines.

Effect of Size and Cost of Conductor on Total Cost. — The size and cost per pound of conductors greatly affect the cost of a line. A circuit of 3

No. 0000 cables costs about $1500 per mile with copper at 14.4 cents per pound and $2000 per mile at 19.2 cents per pound. The cost of smaller cables is proportionally less; No. 0 being about one-half, No. 3 about one-fourth and No. 6 about one-eighth the cost of No. 0000. Thus the copper cost of a three-wire line will range from about $200 per mile for No. 6 at 15.6 cents per pound to $2000 per mile for No. 0000 at 19.2 cents per pound.

For the same power loss in the line aluminum conductors usually cost about 10 per cent less than copper.

Total Structural Cost of Wooden Pole Lines. — The structural cost of a wooden-pole transmission line ordinarily ranges from about $1500 per mile for a 11,000-volt line with 3 No. 6 wires to about $4000 per mile for a 55,000-volt line with 3 No. 0000 wires, excluding the cost of right-of-way. To the structural cost and cost of right-of-way should be added the charges for engineering, contractor's services, superintendence, tools and equipment, etc., which may amount to from 20 to 40 per cent of the structural cost.

Total Structural Cost of Tower Lines. — The cost of light tower lines for 55,000 volts with 3 No. 0000 wires may be as low as for a wooden pole line, i.e., about $4000 per mile. For 110,000 volts with three 300,000-circular-mil wires the cost may be about $8000 per mile or more. The cost of towers proper varies greatly according to the views of the designers as to what risks are proper in design, that is, the severity of assumed wind and ice loadings, and the factors of safety. The cost of foundations is very small when simple steel stubs are used in very firm ground, but foundations add greatly to the cost of a line when concrete or steel structures are used, so designed as to make the strength, rigidity and holding power of the foundations actually equal to the strength of the tower under reasonably unfavorable conditions of ground. See article on *Towers for Transmission Lines*.

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[R. A. PHILIP, H. F. THOMSON and E. A. EKERN.]

TRIGONOMETRIC FUNCTIONS. — (*See also Derivatives; Integrals; Series, Mathematical; Trigonometry.*) The trigonometric functions of an angle are the ratios to one another of the various sides of a right triangle having the given angle as one of its angles. Referring to Fig. 1, let B, P and H be the three sides of a triangle. Then the trigonometric functions of the angle x are

sine of x, abbreviated $\sin x = \frac{P}{H}$; cotangent of x, abbreviated $\cot x = \frac{B}{P}$;

cosine of x, abbreviated $\cos x = \frac{B}{H}$; secant of x, abbreviated $\sec x = \frac{H}{B}$;

tangent of x, abbreviated $\tan x = \frac{P}{B}$; cosecant of x, abbreviated $\csc x = \frac{H}{P}$.

When B, P and H are limited to the three sides of a right triangle, the above definitions are directly applicable only to angles lying between 0 and 90°. The definitions, however, may be extended by considering the point A (Fig. 2) as

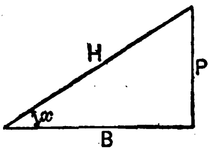

Fig. 1.

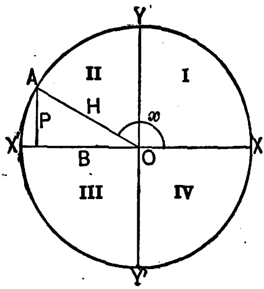

Fig. 2.

describing a circle of radius OA with the center at O. Let XX' be the horizontal diameter and YY' the vertical diameter of this circle, and call P the perpendicular distance from A to the line XX' and B the horizontal distance from A to YY'. P is to be considered positive when A lies above XX', negative when below. B is considered positive when A is to the right of YY' and negative when to the left. The four quarters of the circle are called quadrants, and are designated as the first, second, third and fourth quadrants as indicated. The angle is said to lie in the quadrant in which the point A lies. In Fig. 2 the angle x is in the second quadrant.

ALGEBRAIC SIGNS OF THE FUNCTIONS

	Sine	Cosine	Tangent
Angle in first quadrant....................	+	+	+
Angle in second quadrant..................	+	−	−
Angle in third quadrant....................	−	−	+
Angle in fourth quadrant..................	−	+	−

Period. — From the above definitions it is evident that adding 2π radians or 360° to an angle does not change the value of any of its functions, that is, these functions repeat themselves every time the angle increases by the 2π radians or 360°. They are therefore said to have a period equal to 2π radians or 360°.

Functions of Angles in Any Quadrant in Terms of Angles in First Quadrant.—

$$\sin(-x) = -\sin x, \qquad \sin(90+x) = \cos x,$$
$$\cos(-x) = \cos x, \qquad \cos(90+x) = -\sin x,$$
$$\tan(-x) = -\tan x, \qquad \tan(90+x) = -\cot x,$$

$$\sin(180-x) = \sin x, \qquad \sin(180+x) = -\sin x,$$
$$\cos(180-x) = -\cos x, \qquad \cos(180+x) = -\cos x,$$
$$\tan(180-x) = -\tan x, \qquad \tan(180+x) = \tan x,$$

$$\sin(270-x) = -\cos x, \qquad \sin(270+x) = -\cos x,$$
$$\cos(270-x) = -\sin x, \qquad \cos(270+x) = \sin x,$$
$$\tan(270-x) = \cot x, \qquad \tan(270+x) = -\cot x.$$

Table of Trigonometric Functions. — By making use of the above relations the functions of any angle may be obtained from a table giving the values of the functions for angles between 0 and 90°. Such a table is given below. For the cot, sec and csc take the reciprocals of the tan, cos and sin respectively.

Example of Use of Table. — sin 21.6° = 0.3681, cos 21.6° = 0.9298, tan 21.6° = 0.3959; sin 107° = sin (180° − 107°) = sin 73° = 0.9563, cos 107° = − cos 73° = − 0.2924, tan 107° = − tan 73° = − 3.2709.

TRIGONOMETRIC FUNCTIONS 0.0°–6.9°

Angle in degrees	Name of function	Value of function for each tenth of a degree									
		0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	sin	0.0000	0.0017	0.0035	0.0052	0.0070	0.0087	0.0105	0.0122	0.0140	0.0157
	cos	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9999
	tan	0.0000	0.0017	0.0035	0.0052	0.0070	0.0087	0.0105	0.0122	0.0140	0.0157
1	sin	0.0175	0.0192	0.0209	0.0227	0.0244	0.0262	0.0279	0.0297	0.0314	0.0332
	cos	0.9998	0.9998	0.9998	0.9997	0.9997	0.9997	0.9996	0.9996	0.9995	0.9995
	tan	0.0175	0.0192	0.0209	0.0227	0.0244	0.0262	0.0279	0.0297	0.0314	0.0332
2	sin	0.0349	0.0366	0.0384	0.0401	0.0419	0.0436	0.0454	0.0471	0.0488	0.0506
	cos	0.9994	0.9993	0.9993	0.9992	0.9991	0.9990	0.9990	0.9989	0.9988	0.9987
	tan	0.0349	0.0367	0.0384	0.0402	0.0419	0.0437	0.0454	0.0472	0.0489	0.0507
3	sin	0.0523	0.0541	0.0558	0.0576	0.0593	0.0610	0.0628	0.0645	0.0663	0.0680
	cos	0.9986	0.9985	0.9984	0.9983	0.9982	0.9981	0.9980	0.9979	0.9978	0.9977
	tan	0.0524	0.0542	0.0559	0.0577	0.0594	0.0612	0.0629	0.0647	0.0664	0.0682
4	sin	0.0698	0.0715	0.0732	0.0750	0.0767	0.0785	0.0802	0.0819	0.0837	0.0854
	cos	0.9976	0.9974	0.9973	0.9972	0.9971	0.9969	0.9968	0.9966	0.9965	0.9963
	tan	0.0699	0.0717	0.0734	0.0752	0.0769	0.0787	0.0805	0.0822	0.0840	0.0857
5	sin	0.0872	0.0889	0.0906	0.0924	0.0941	0.0958	0.0976	0.0993	0.1011	0.1028
	cos	0.9962	0.9960	0.9959	0.9957	0.9956	0.9954	0.9952	0.9951	0.9949	0.9947
	tan	0.0875	0.0892	0.0910	0.0928	0.0945	0.0963	0.0981	0.0998	0.1016	0.1033
6	sin	0.1045	0.1063	0.1080	0.1097	0.1115	0.1132	0.1149	0.1167	0.1184	0.1201
	cos	0.9945	0.9943	0.9942	0.9940	0.9938	0.9936	0.9934	0.9932	0.9930	0.9928
	tan	0.1051	0.1069	0.1086	0.1104	0.1122	0.1139	0.1157	0.1175	0.1192	0.1210

TRIGONOMETRIC FUNCTIONS

Angle in degrees	Name of function	Value of function for each tenth of a degree									
		0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
7	sin	0.1219	0.1236	0.1253	0.1271	0.1288	0.1305	0.1323	0.1340	0.1357	0.1374
	cos	0.9925	0.9923	0.9921	0.9919	0.9917	0.9914	0.9912	0.9910	0.9907	0.9905
	tan	0.1228	0.1246	0.1263	0.1281	0.1299	0.1317	0.1334	0.1352	0.1370	0.1388
8	sin	0.1392	0.1409	0.1426	0.1444	0.1461	0.1478	0.1495	0.1513	0.1530	0.1547
	cos	0.9903	0.9900	0.9898	0.9895	0.9893	0.9890	0.9888	0.9885	0.9882	0.9880
	tan	0.1405	0.1423	0.1441	0.1459	0.1477	0.1495	0.1512	0.1530	0.1548	0.1566
9	sin	0.1564	0.1582	0.1599	0.1616	0.1633	0.1650	0.1668	0.1685	0.1702	0.1719
	cos	0.9877	0.9874	0.9871	0.9869	0.9866	0.9863	0.9860	0.9857	0.9854	0.9851
	tan	0.1584	0.1602	0.1620	0.1638	0.1655	0.1673	0.1691	0.1709	0.1727	0.1745
10	sin	0.1736	0.1754	0.1771	0.1788	0.1805	0.1822	0.1840	0.1857	0.1874	0.1891
	cos	0.9848	0.9845	0.9842	0.9839	0.9836	0.9833	0.9829	0.9826	0.9823	0.9820
	tan	0.1763	0.1781	0.1799	0.1817	0.1835	0.1853	0.1871	0.1890	0.1908	0.1926
11	sin	0.1908	0.1925	0.1942	0.1959	0.1977	0.1994	0.2011	0.2028	0.2045	0.2062
	cos	0.9816	0.9813	0.9810	0.9806	0.9803	0.9799	0.9796	0.9792	0.9789	0.9785
	tan	0.1944	0.1962	0.1980	0.1998	0.2016	0.2035	0.2053	0.2071	0.2089	0.2107
12	sin	0.2079	0.2096	0.2113	0.2130	0.2147	0.2164	0.2181	0.2198	0.2215	0.2232
	cos	0.9781	0.9778	0.9774	0.9770	0.9767	0.9763	0.9759	0.9755	0.9751	0.9748
	tan	0.2126	0.2144	0.2162	0.2180	0.2199	0.2217	0.2235	0.2254	0.2272	0.2290
13	sin	0.2250	0.2267	0.2284	0.2300	0.2317	0.2334	0.2351	0.2368	0.2385	0.2402
	cos	0.9744	0.9740	0.9736	0.9732	0.9728	0.9724	0.9720	0.9715	0.9711	0.9707
	tan	0.2309	0.2327	0.2345	0.2364	0.2382	0.2401	0.2419	0.2438	0.2456	0.2475
14	sin	0.2419	0.2436	0.2453	0.2470	0.2487	0.2504	0.2521	0.2538	0.2554	0.2571
	cos	0.9703	0.9699	0.9694	0.9690	0.9686	0.9681	0.9677	0.9673	0.9668	0.9664
	tan	0.2493	0.2512	0.2530	0.2549	0.2568	0.2586	0.2605	0.2623	0.2642	0.2661
15	sin	0.2588	0.2605	0.2622	0.2639	0.2656	0.2672	0.2689	0.2706	0.2723	0.2740
	cos	0.9659	0.9655	0.9650	0.9646	0.9641	0.9636	0.9632	0.9627	0.9622	0.9617
	tan	0.2679	0.2698	0.2717	0.2736	0.2754	0.2773	0.2792	0.2811	0.2830	0.2849
16	sin	0.2756	0.2773	0.2790	0.2807	0.2823	0.2840	0.2857	0.2874	0.2890	0.2907
	cos	0.9613	0.9608	0.9603	0.9598	0.9593	0.9588	0.9583	0.9578	0.9573	0.9568
	tan	0.2867	0.2886	0.2905	0.2924	0.2943	0.2962	0.2981	0.3000	0.3019	0.3038
17	sin	0.2924	0.2940	0.2957	0.2974	0.2990	0.3007	0.3024	0.3040	0.3057	0.3074
	cos	0.9563	0.9558	0.9553	0.9548	0.9542	0.9537	0.9532	0.9527	0.9521	0.9516
	tan	0.3057	0.3076	0.3096	0.3115	0.3134	0.3153	0.3172	0.3191	0.3211	0.3230
18	sin	0.3090	0.3107	0.3123	0.3140	0.3156	0.3173	0.3190	0.3206	0.3223	0.3239
	cos	0.9511	0.9505	0.9500	0.9494	0.9489	0.9483	0.9478	0.9472	0.9466	0.9461
	tan	0.3249	0.3269	0.3288	0.3307	0.3327	0.3346	0.3365	0.3385	0.3404	0.3424
19	sin	0.3256	0.3272	0.3289	0.3305	0.3322	0.3338	0.3355	0.3371	0.3387	0.3404
	cos	0.9455	0.9449	0.9444	0.9438	0.9432	0.9426	0.9421	0.9415	0.9409	0.9403
	tan	0.3443	0.3463	0.3482	0.3502	0.3522	0.3541	0.3561	0.3581	0.3600	0.3620
20	sin	0.3420	0.3437	0.3453	0.3469	0.3486	0.3502	0.3518	0.3535	0.3551	0.3567
	cos	0.9397	0.9391	0.9385	0.9379	0.9373	0.9367	0.9361	0.9354	0.9348	0.9342
	tan	0.3640	0.3659	0.3679	0.3699	0.3719	0.3739	0.3759	0.3779	0.3799	0.3819

TRIGONOMETRIC FUNCTIONS 21.0°–34.9°

Angle in degrees	Name of function	Value of function for each tenth of a degree									
		0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
21	sin	0.3584	0.3600	0.3616	0.3633	0.3649	0.3665	0.3681	0.3697	0.3714	0.3730
	cos	0.9336	0.9330	0.9323	0.9317	0.9311	0.9304	0.9298	0.9291	0.9285	0.9278
	tan	0.3839	0.3859	0.3879	0.3899	0.3919	0.3939	0.3959	0.3979	0.4000	0.4020
22	sin	0.3746	0.3762	0.3778	0.3795	0.3811	0.3827	0.3843	0.3859	0.3875	0.3891
	cos	0.9272	0.9265	0.9259	0.9252	0.9245	0.9239	0.9232	0.9225	0.9219	0.9212
	tan	0.4040	0.4061	0.4081	0.4101	0.4122	0.4142	0.4163	0.4183	0.4204	0.4224
23	sin	0.3907	0.3923	0.3939	0.3955	0.3971	0.3987	0.4003	0.4019	0.4035	0.4051
	cos	0.9205	0.9198	0.9191	0.9184	0.9178	0.9171	0.9164	0.9157	0.9150	0.9143
	tan	0.4245	0.4265	0.4286	0.4307	0.4327	0.4348	0.4369	0.4390	0.4411	0.4431
24	sin	0.4067	0.4083	0.4099	0.4115	0.4131	0.4147	0.4163	0.4179	0.4195	0.4210
	cos	0.9135	0.9128	0.9121	0.9114	0.9107	0.9100	0.9092	0.9085	0.9078	0.9070
	tan	0.4452	0.4473	0.4494	0.4515	0.4536	0.4557	0.4578	0.4599	0.4621	0.4642
25	sin	0.4226	0.4242	0.4258	0.4274	0.4289	0.4305	0.4321	0.4337	0.4352	0.4368
	cos	0.9063	0.9056	0.9048	0.9041	0.9033	0.9026	0.9018	0.9011	0.9003	0.8996
	tan	0.4663	0.4684	0.4706	0.4727	0.4748	0.4770	0.4791	0.4813	0.4834	0.4856
26	sin	0.4384	0.4399	0.4415	0.4431	0.4446	0.4462	0.4478	0.4493	0.4509	0.4524
	cos	0.8988	0.8980	0.8973	0.8965	0.8957	0.8949	0.8942	0.8934	0.8926	0.8918
	tan	0.4877	0.4899	0.4921	0.4942	0.4964	0.4986	0.5008	0.5029	0.5051	0.5073
27	sin	0.4540	0.4555	0.4571	0.4586	0.4602	0.4617	0.4633	0.4648	0.4664	0.4679
	cos	0.8910	0.8902	0.8894	0.8886	0.8878	0.8870	0.8862	0.8854	0.8846	0.8838
	tan	0.5095	0.5117	0.5139	0.5161	0.5184	0.5206	0.5228	0.5250	0.5272	0.5295
28	sin	0.4695	0.4710	0.4726	0.4741	0.4756	0.4772	0.4787	0.4802	0.4818	0.4833
	cos	0.8829	0.8821	0.8813	0.8805	0.8796	0.8788	0.8780	0.8771	0.8763	0.8755
	tan	0.5317	0.5340	0.5362	0.5384	0.5407	0.5430	0.5452	0.5475	0.5498	0.5520
29	sin	0.4848	0.4863	0.4879	0.4894	0.4909	0.4924	0.4939	0.4955	0.4970	0.4985
	cos	0.8746	0.8738	0.8729	0.8721	0.8712	0.8704	0.8695	0.8686	0.8678	0.8669
	tan	0.5543	0.5566	0.5589	0.5612	0.5635	0.5658	0.5681	0.5704	0.5727	0.5750
30	sin	0.5000	0.5015	0.5030	0.5045	0.5060	0.5075	0.5090	0.5105	0.5120	0.5135
	cos	0.8660	0.8652	0.8643	0.8634	0.8625	0.8616	0.8607	0.8599	0.8590	0.8581
	tan	0.5774	0.5797	0.5820	0.5844	0.5867	0.5890	0.5914	0.5938	0.5961	0.5985
31	sin	0.5150	0.5165	0.5180	0.5195	0.5210	0.5225	0.5240	0.5255	0.5270	0.5284
	cos	0.8572	0.8563	0.8554	0.8545	0.8536	0.8526	0.8517	0.8508	0.8499	0.8490
	tan	0.6009	0.6032	0.6056	0.6080	0.6104	0.6128	0.6152	0.6176	0.6200	0.6224
32	sin	0.5299	0.5314	0.5329	0.5344	0.5358	0.5373	0.5388	0.5402	0.5417	0.5432
	cos	0.8480	0.8471	0.8462	0.8453	0.8443	0.8434	0.8425	0.8415	0.8406	0.8396
	tan	0.6249	0.6273	0.6297	0.6322	0.6346	0.6371	0.6395	0.6420	0.6445	0.6469
33	sin	0.5446	0.5461	0.5476	0.5490	0.5505	0.5519	0.5534	0.5548	0.5563	0.5577
	cos	0.8387	0.8377	0.8368	0.8358	0.8348	0.8339	0.8329	0.8320	0.8310	0.8300
	tan	0.6494	0.6519	0.6544	0.6569	0.6594	0.6619	0.6644	0.6669	0.6694	0.6720
34	sin	0.5592	0.5606	0.5621	0.5635	0.5650	0.5664	0.5678	0.5693	0.5707	0.5721
	cos	0.8290	0.8281	0.8271	0.8261	0.8251	0.8241	0.8231	0.8221	0.8211	0.8202
	tan	0.6745	0.6771	0.6796	0.6822	0.6847	0.6873	0.6899	0.6924	0.6950	0.6976

TRIGONOMETRIC FUNCTIONS 35.0°–48.9°

Angle in degrees	Name of function	Value of function for each tenth of a degree									
		0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
35	sin	0.5736	0.5750	0.5764	0.5779	0.5793	0.5807	0.5821	0.5835	0.5850	0.5864
	cos	0.8192	0.8181	0.8171	0.8161	0.8151	0.8141	0.8131	0.8121	0.8111	0.8100
	tan	0.7002	0.7028	0.7054	0.7080	0.7107	0.7133	0.7159	0.7186	0.7212	0.7239
36	sin	0.5878	0.5892	0.5906	0.5920	0.5934	0.5948	0.5962	0.5976	0.5990	0.6004
	cos	0.8090	0.8080	0.8070	0.8059	0.8049	0.8039	0.8028	0.8018	0.8007	0.7997
	tan	0.7265	0.7292	0.7319	0.7346	0.7373	0.7400	0.7427	0.7454	0.7481	0.7508
37	sin	0.6018	0.6032	0.6046	0.6060	0.6074	0.6088	0.6101	0.6115	0.6129	0.6143
	cos	0.7986	0.7976	0.7965	0.7955	0.7944	0.7934	0.7923	0.7912	0.7902	0.7891
	tan	0.7536	0.7563	0.7590	0.7618	0.7646	0.7673	0.7701	0.7729	0.7757	0.7785
38	sin	0.6157	0.6170	0.6184	0.6198	0.6211	0.6225	0.6239	0.6252	0.6266	0.6280
	cos	0.7880	0.7869	0.7859	0.7848	0.7837	0.7826	0.7815	0.7804	0.7793	0.7782
	tan	0.7813	0.7841	0.7869	0.7898	0.7926	0.7954	0.7983	0.8012	0.8040	0.8069
39	sin	0.6293	0.6307	0.6320	0.6334	0.6347	0.6361	0.6374	0.6388	0.6401	0.6414
	cos	0.7771	0.7760	0.7749	0.7738	0.7727	0.7716	0.7705	0.7694	0.7683	0.7672
	tan	0.8098	0.8127	0.8156	0.8185	0.8214	0.8243	0.8273	0.8302	0.8332	0.8361
40	sin	0.6428	0.6441	0.6455	0.6468	0.6481	0.6494	0.6508	0.6521	0.6534	0.6547
	cos	0.7660	0.7649	0.7638	0.7627	0.7615	0.7604	0.7593	0.7581	0.7570	0.7559
	tan	0.8391	0.8421	0.8451	0.8481	0.8511	0.8541	0.8571	0.8601	0.8632	0.8662
41	sin	0.6561	0.6574	0.6587	0.6600	0.6613	0.6626	0.6639	0.6653	0.6665	0.6678
	cos	0.7547	0.7536	0.7524	0.7513	0.7501	0.7490	0.7478	0.7466	0.7455	0.7443
	tan	0.8693	0.8724	0.8754	0.8785	0.8816	0.8847	0.8878	0.8910	0.8941	0.8972
42	sin	0.6691	0.6704	0.6717	0.6730	0.6743	0.6756	0.6769	0.6782	0.6794	0.6807
	cos	0.7431	0.7420	0.7408	0.7396	0.7385	0.7373	0.7361	0.7349	0.7337	0.7325
	tan	0.9004	0.9036	0.9067	0.9099	0.9131	0.9163	0.9195	0.9228	0.9260	0.9293
43	sin	0.6820	0.6833	0.6845	0.6858	0.6871	0.6884	0.6896	0.6909	0.6921	0.6934
	cos	0.7314	0.7302	0.7290	0.7278	0.7266	0.7254	0.7242	0.7230	0.7218	0.7206
	tan	0.9325	0.9358	0.9391	0.9424	0.9457	0.9490	0.9523	0.9556	0.9590	0.9623
44	sin	0.6947	0.6959	0.6972	0.6984	0.6997	0.7009	0.7022	0.7034	0.7046	0.7059
	cos	0.7193	0.7181	0.7169	0.7157	0.7145	0.7133	0.7120	0.7108	0.7096	0.7083
	tan	0.9657	0.9691	0.9725	0.9759	0.9793	0.9827	0.9861	0.9896	0.9930	0.9965
45	sin	0.7071	0.7083	0.7096	0.7108	0.7120	0.7133	0.7145	0.7157	0.7169	0.7181
	cos	0.7071	0.7059	0.7046	0.7034	0.7022	0.7009	0.6997	0.6984	0.6972	0.6959
	tan	1.0000	1.0035	1.0070	1.0105	1.0141	1.0176	1.0212	1.0247	1.0283	1.0319
46	sin	0.7193	0.7206	0.7218	0.7230	0.7242	0.7254	0.7266	0.7278	0.7290	0.7302
	cos	0.6947	0.6934	0.6921	0.6909	0.6896	0.6884	0.6871	0.6858	0.6845	0.6833
	tan	1.0355	1.0392	1.0428	1.0464	1.0501	1.0538	1.0575	1.0612	1.0649	1.0686
47	sin	0.7314	0.7325	0.7337	0.7349	0.7361	0.7373	0.7385	0.7396	0.7408	0.7420
	cos	0.6820	0.6807	0.6794	0.6782	0.6769	0.6756	0.6743	0.6730	0.6717	0.6704
	tan	1.0724	1.0761	1.0799	1.0837	1.0875	1.0913	1.0951	1.0990	1.1028	1.1067
48	sin	0.7431	0.7443	0.7455	0.7466	0.7478	0.7490	0.7501	0.7513	0.7524	0.7536
	cos	0.6691	0.6678	0.6665	0.6652	0.6639	0.6626	0.6613	0.6600	0.6587	0.6574
	tan	1.1106	1.1145	1.1184	1.1224	1.1263	1.1303	1.1343	1.1383	1.1423	1.1463

TRIGONOMETRIC FUNCTIONS 49.0°–62.9°

Angle in degrees	Name of function	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
49	sin	0.7547	0.7559	0.7570	0.7581	0.7593	0.7604	0.7615	0.7627	0.7638	0.7649
	cos	0.6561	0.6547	0.6534	0.6521	0.6508	0.6494	0.6481	0.6468	0.6455	0.6441
	tan	1.1504	1.1544	1.1585	1.1626	1.1667	1.1708	1.1750	1.1792	1.1833	1.1875
50	sin	0.7660	0.7672	0.7683	0.7694	0.7705	0.7716	0.7727	0.7738	0.7749	0.7760
	cos	0.6428	0.6414	0.6401	0.6388	0.6374	0.6361	0.6347	0.6334	0.6320	0.6307
	tan	1.1918	1.1960	1.2002	1.2045	1.2088	1.2131	1.2174	1.2218	1.2261	1.2305
51	sin	0.7771	0.7782	0.7793	0.7804	0.7815	0.7826	0.7837	0.7848	0.7859	0.7869
	cos	0.6293	0.6280	0.6266	0.6252	0.6239	0.6225	0.6211	0.6198	0.6184	0.6170
	tan	1.2349	1.2393	1.2437	1.2482	1.2527	1.2572	1.2617	1.2662	1.2708	1.2753
52	sin	0.7880	0.7891	0.7902	0.7912	0.7923	0.7934	0.7944	0.7955	0.7965	0.7976
	cos	0.6157	0.6143	0.6129	0.6115	0.6101	0.6088	0.6074	0.6060	0.6046	0.6032
	tan	1.2799	1.2846	1.2892	1.2938	1.2985	1.3032	1.3079	1.3127	1.3175	1.3222
53	sin	0.7986	0.7997	0.8007	0.8018	0.8028	0.8039	0.8049	0.8059	0.8070	0.8080
	cos	0.6018	0.6004	0.5990	0.5976	0.5962	0.5948	0.5934	0.5920	0.5906	0.5892
	tan	1.3270	1.3319	1.3367	1.3416	1.3465	1.3514	1.3564	1.3613	1.3663	1.3713
54	sin	0.8090	0.8100	0.8111	0.8121	0.8131	0.8141	0.8151	0.8161	0.8171	0.8181
	cos	0.5878	0.5864	0.5850	0.5835	0.5821	0.5807	0.5793	0.5779	0.5764	0.5750
	tan	1.3764	1.3814	1.3865	1.3916	1.3968	1.4019	1.4071	1.4124	1.4176	1.4229
55	sin	0.8192	0.8202	0.8211	0.8221	0.8231	0.8241	0.8251	0.8261	0.8271	0.8281
	cos	0.5736	0.5721	0.5707	0.5693	0.5678	0.5664	0.5650	0.5635	0.5621	0.5606
	tan	1.4281	1.4335	1.4388	1.4442	1.4496	1.4550	1.4605	1.4659	1.4715	1.4770
56	sin	0.8290	0.8300	0.8310	0.8320	0.8329	0.8339	0.8348	0.8358	0.8368	0.8377
	cos	0.5592	0.5577	0.5563	0.5548	0.5534	0.5519	0.5505	0.5490	0.5476	0.5461
	tan	1.4826	1.4882	1.4938	1.4994	1.5051	1.5108	1.5166	1.5224	1.5282	1.5340
57	sin	0.8387	0.8396	0.8406	0.8415	0.8425	0.8434	0.8443	0.8453	0.8462	0.8471
	cos	0.5446	0.5432	0.5417	0.5402	0.5388	0.5373	0.5358	0.5344	0.5329	0.5314
	tan	1.5399	1.5458	1.5517	1.5577	1.5637	1.5697	1.5757	1.5818	1.5880	1.5941
58	sin	0.8480	0.8490	0.8499	0.8508	0.8517	0.8526	0.8536	0.8545	0.8554	0.8563
	cos	0.5299	0.5284	0.5270	0.5255	0.5240	0.5225	0.5210	0.5195	0.5180	0.5165
	tan	1.6003	1.6066	1.6128	1.6191	1.6255	1.6319	1.6383	1.6447	1.6512	1.6577
59	sin	0.8572	0.8581	0.8590	0.8599	0.8607	0.8616	0.8625	0.8634	0.8643	0.8652
	cos	0.5150	0.5135	0.5120	0.5105	0.5090	0.5075	0.5060	0.5045	0.5030	0.5015
	tan	1.6643	1.6709	1.6775	1.6842	1.6909	1.6977	1.7045	1.7113	1.7182	1.7251
60	sin	0.8660	0.8669	0.8678	0.8686	0.8695	0.8704	0.8712	0.8721	0.8729	0.8738
	cos	0.5000	0.4985	0.4970	0.4955	0.4939	0.4924	0.4909	0.4894	0.4879	0.4863
	tan	1.7321	1.7391	1.7461	1.7532	1.7603	1.7675	1.7747	1.7820	1.7893	1.7966
61	sin	0.8746	0.8755	0.8763	0.8771	0.8780	0.8788	0.8796	0.8805	0.8813	0.8821
	cos	0.4848	0.4833	0.4818	0.4802	0.4787	0.4772	0.4756	0.4741	0.4726	0.4710
	tan	1.8040	1.8115	1.8190	1.8265	1.8341	1.8418	1.8495	1.8572	1.8650	1.8728
62	sin	0.8829	0.8838	0.8846	0.8854	0.8862	0.8870	0.8878	0.8886	0.8894	0.8902
	cos	0.4695	0.4679	0.4664	0.4648	0.4633	0.4617	0.4602	0.4586	0.4571	0.4555
	tan	1.8807	1.8887	1.8967	1.9047	1.9128	1.9210	1.9292	1.9375	1.9458	1.9542

Angle in degrees	Name of function	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
		Value of function for each tenth of a degree									
63	sin	0.8910	0.8918	0.8926	0.8934	0.8942	0.8949	0.8957	0.8965	0.8973	0.8980
	cos	0.4540	0.4524	0.4509	0.4493	0.4478	0.4462	0.4446	0.4431	0.4415	0.4399
	tan	1.9626	1.9711	1.9797	1.9883	1.9970	2.0057	2.0145	2.0233	2.0323	2.0413
64	sin	0.8988	0.8996	0.9003	0.9011	0.9018	0.9026	0.9033	0.9041	0.9048	0.9056
	cos	0.4384	0.4368	0.4352	0.4337	0.4321	0.4305	0.4289	0.4274	0.4258	0.4242
	tan	2.0593	2.0594	2.0686	2.0778	2.0872	2.0965	2.1060	2.1155	2.1251	2.1348
65	sin	0.9063	0.9070	0.9078	0.9085	0.9092	0.9100	0.9107	0.9114	0.9121	0.9128
	cos	0.4226	0.4210	0.4195	0.4179	0.4163	0.4147	0.4131	0.4115	0.4099	0.4083
	tan	2.1445	2.1543	2.1642	2.1742	2.1842	2.1943	2.2045	2.2148	2.2251	2.2355
66	sin	0.9135	0.9143	0.9150	0.9157	0.9164	0.9171	0.9178	0.9184	0.9191	0.9198
	cos	0.4067	0.4051	0.4035	0.4019	0.4003	0.3987	0.3971	0.3955	0.3939	0.3923
	tan	2.2460	2.2566	2.2673	2.2781	2.2889	2.2998	2.3109	2.3220	2.3332	2.3445
67	sin	0.9205	0.9212	0.9219	0.9225	0.9232	0.9239	0.9245	0.9252	0.9259	0.9265
	cos	0.3907	0.3891	0.3875	0.3859	0.3843	0.3827	0.3811	0.3795	0.3778	0.3762
	tan	2.3559	2.3673	2.3789	2.3906	2.4023	2.4142	2.4262	2.4383	2.4504	2.4627
68	sin	0.9272	0.9278	0.9285	0.9291	0.9298	0.9304	0.9311	0.9317	0.9323	0.9330
	cos	0.3746	0.3730	0.3714	0.3697	0.3681	0.3665	0.3649	0.3633	0.3616	0.3600
	tan	2.4751	2.4876	2.5002	2.5129	2.5257	2.5386	2.5517	2.5649	2.5782	2.5916
69	sin	0.9336	0.9342	0.9348	0.9354	0.9361	0.9367	0.9373	0.9379	0.9385	0.9391
	cos	0.3584	0.3567	0.3551	0.3535	0.3518	0.3502	0.3486	0.3469	0.3453	0.3437
	tan	2.6051	2.6187	2.6325	2.6464	2.6605	2.6746	2.6889	2.7034	2.7179	2.7326
70	sin	0.9397	0.9403	0.9409	0.9415	0.9421	0.9426	0.9432	0.9438	0.9444	0.9449
	cos	0.3420	0.3404	0.3387	0.3371	0.3355	0.3338	0.3322	0.3305	0.3289	0.3272
	tan	2.7475	2.7625	2.7776	2.7929	2.8083	2.8239	2.8397	2.8556	2.8716	2.8878
71	sin	0.9455	0.9461	0.9466	0.9472	0.9478	0.9483	0.9489	0.9494	0.9500	0.9505
	cos	0.3256	0.3239	0.3223	0.3206	0.3190	0.3173	0.3156	0.3140	0.3123	0.3107
	tan	2.9042	2.9208	2.9375	2.9544	2.9714	2.9887	3.0061	3.0237	3.0415	3.0595
72	sin	0.9511	0.9516	0.9521	0.9527	0.9532	0.9537	0.9542	0.9548	0.9553	0.9558
	cos	0.3090	0.3074	0.3057	0.3040	0.3024	0.3007	0.2990	0.2974	0.2957	0.2940
	tan	3.0777	3.0961	3.1146	3.1334	3.1524	3.1716	3.1910	3.2106	3.2305	3.2506
73	sin	0.9563	0.9568	0.9573	0.9578	0.9583	0.9588	0.9593	0.9598	0.9603	0.9608
	cos	0.2924	0.2907	0.2890	0.2874	0.2857	0.2840	0.2823	0.2807	0.2790	0.2773
	tan	3.2709	3.2914	3.3122	3.3332	3.3544	3.3759	3.3977	3.4197	3.4420	3.4646
74	sin	0.9613	0.9617	0.9622	0.9627	0.9632	0.9636	0.9641	0.9646	0.9650	0.9655
	cos	0.2756	0.2740	0.2723	0.2706	0.2689	0.2672	0.2656	0.2639	0.2622	0.2605
	tan	3.4874	3.5105	3.5339	3.5576	3.5816	3.6059	3.6305	3.6554	3.6806	3.7062
75	sin	0.9659	0.9664	0.9668	0.9673	0.9677	0.9681	0.9686	0.9690	0.9694	0.9699
	cos	0.2588	0.2571	0.2554	0.2538	0.2521	0.2504	0.2487	0.2470	0.2453	0.2436
	tan	3.7321	3.7583	3.7848	3.8118	3.8391	3.8667	3.8947	3.9232	3.9520	3.9812
76	sin	0.9703	0.9707	0.9711	0.9715	0.9720	0.9724	0.9728	0.9732	0.9736	0.9740
	cos	0.2419	0.2402	0.2385	0.2368	0.2351	0.2334	0.2317	0.2300	0.2284	0.2267
	tan	4.0108	4.0408	4.0713	4.1022	4.1335	4.1653	4.1976	4.2303	4.2635	4.2972

TRIGONOMETRIC FUNCTIONS 77.0°–89.9°

Angle in degrees	Name of function	Value of function for each tenth of a degree									
		0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
77	sin	0.9744	0.9748	0.9751	0.9755	0.9759	0.9763	0.9767	0.9770	0.9774	0.9778
	cos	0.2250	0.2232	0.2215	0.2198	0.2181	0.2164	0.2147	0.2130	0.2113	0.2096
	tan	4.3315	4.3662	4.4015	4.4374	4.4737	4.5107	4.5483	4.5864	4.6252	4.6646
78	sin	0.9781	0.9785	0.9789	0.9792	0.9796	0.9799	0.9803	0.9806	0.9810	0.9813
	cos	0.2079	0.2062	0.2045	0.2028	0.2011	0.1994	0.1977	0.1959	0.1942	0.1925
	tan	4.7046	4.7453	4.7867	4.8288	4.8716	4.9152	4.9594	5.0045	5.0504	5.0970
79	sin	0.9816	0.9820	0.9823	0.9826	0.9829	0.9833	0.9836	0.9839	0.9842	0.9845
	cos	0.1908	0.1891	0.1874	0.1857	0.1840	0.1822	0.1805	0.1788	0.1771	0.1754
	tan	5.1446	5.1929	5.2422	5.2924	5.3435	5.3955	5.4486	5.5026	5.5578	5.6140
80	sin	0.9848	0.9851	0.9854	0.9857	0.9860	0.9863	0.9866	0.9869	0.9871	0.9874
	cos	0.1736	0.1719	0.1702	0.1685	0.1668	0.1650	0.1633	0.1616	0.1599	0.1582
	tan	5.6713	5.7297	5.7894	5.8502	5.9124	5.9758	6.0405	6.1066	6.1742	6.2432
81	sin	0.9877	0.9880	0.9882	0.9885	0.9888	0.9890	0.9893	0.9895	0.9898	0.9900
	cos	0.1564	0.1547	0.1530	0.1513	0.1495	0.1478	0.1461	0.1444	0.1426	0.1409
	tan	6.3138	6.3859	6.4596	6.5350	6.6122	6.6912	6.7720	6.8548	6.9395	7.0264
82	sin	0.9903	0.9905	0.9907	0.9910	0.9912	0.9914	0.9917	0.9919	0.9921	0.9923
	cos	0.1392	0.1374	0.1357	0.1340	0.1323	0.1305	0.1288	0.1271	0.1253	0.1236
	tan	7.1154	7.2066	7.3002	7.3962	7.4947	7.5958	7.6996	7.8062	7.9158	8.0285
83	sin	0.9925	0.9928	0.9930	0.9932	0.9934	0.9936	0.9938	0.9940	0.9942	0.9943
	cos	0.1219	0.1201	0.1184	0.1167	0.1149	0.1132	0.1115	0.1097	0.1080	0.1063
	tan	8.1443	8.2636	8.3863	8.5126	8.6427	8.7769	8.9152	9.0579	9.2052	9.3572
84	sin	0.9945	0.9947	0.9949	0.9951	0.9952	0.9954	0.9956	0.9957	0.9959	0.9960
	cos	0.1045	0.1028	0.1011	0.0993	0.0976	0.0958	0.0941	0.0924	0.0906	0.0889
	tan	9.5144	9.6778	9.8448	10.02	10.20	10.39	10.58	10.78	10.99	11.20
85	sin	0.9962	0.9963	0.9965	0.9966	0.9968	0.9969	0.9971	0.9972	0.9973	0.9974
	cos	0.0872	0.0854	0.0837	0.0819	0.0802	0.0785	0.0767	0.0750	0.0732	0.0715
	tan	11.43	11.66	11.91	12.16	12.43	12.71	13.00	13.30	13.62	13.95
86	sin	0.9976	0.9977	0.9978	0.9979	0.9980	0.9981	0.9982	0.9983	0.9984	0.9985
	cos	0.0698	0.0680	0.0663	0.0645	0.0628	0.0610	0.0593	0.0576	0.0558	0.0541
	tan	14.30	14.67	15.06	15.46	15.89	16.35	16.83	17.34	17.89	18.46
87	sin	0.9986	0.9987	0.9988	0.9989	0.9990	0.9990	0.9991	0.9992	0.9993	0.9993
	cos	0.0523	0.0506	0.0488	0.0471	0.0454	0.0436	0.0419	0.0401	0.0384	0.0366
	tan	19.08	19.74	20.45	21.20	22.02	22.90	23.86	24.90	26.03	27.27
88	sin	0.9994	0.9995	0.9995	0.9996	0.9996	0.9997	0.9997	0.9997	0.9998	0.9998
	cos	0.0349	0.0332	0.0314	0.0297	0.0279	0.0262	0.0244	0.0227	0.0209	0.0192
	tan	28.64	30.14	31.82	33.69	35.80	38.19	40.92	44.07	47.74	52.08
89	sin	0.9998	0.9999	0.9999	0.9999	0.9999	1.000	1.000	1.000	1.000	1.000
	cos	0.0175	0.0157	0.0140	0.0122	0.0105	0.0087	0.0070	0.0052	0.0035	0.0017
	tan	57.29	63.66	71.62	81.85	95.49	114.6	143.2	191.0	286.5	573.0

Anti-functions. — If $a = \sin x$, then x is the angle whose sine is a; this may be expressed symbolically $x = \sin^{-1} a$, which is read "x equals the angle whose sine is a." The angle x is also called the "anti-sine" or the "inverse sine" of a. Similar notation is used for the other functions; for example, $x = \cos^{-1} b$ is used to express the relation that x is the angle whose cosine is b. At least two "anti-functions" must be known to completely determine the quadrant in which an angle lies; for example, if $x = \sin^{-1} 0.5$ then x may be either $30°$ or $150°$, but if we also have $x = \cos^{-1} 0.866$, then x must equal $30°$, while if $x = \cos^{-1}(-0.866)$, then x must equal $150°$.

Anti-functions may be taken from the table given above by finding the angle in the margin corresponding to the function in the table. *Example:* $\sin^{-1} 0.319 = 18.6°$ or $180° - 18.6° = 161.4°$.

Versine. — The expression $(1 - \cos x)$ is called the "versine" of x.

Relations Among Functions of the Same Angle. —

$$\tan x = \frac{\sin x}{\cos x} = \frac{1}{\cot x}, \qquad \sin^2 x + \cos^2 x = 1,$$

$$\sec x = \frac{1}{\cos x}, \qquad 1 + \tan^2 x = \frac{1}{\cos^2 x},$$

$$\csc x = \frac{1}{\sin x}, \qquad 1 + \cot^2 x = \frac{1}{\sin^2 x},$$

$$\sin(90 - x) = \cos x, \qquad \sin(-x) = -\sin x,$$

$$\cos(90 - x) = \sin x, \qquad \cos(-x) = \cos x,$$

$$\tan(90 - x) = \cot x, \qquad \tan(-x) = -\tan x.$$

Sum and Difference of Two Angles. —

$$\sin(x + y) = \sin x \cos y + \cos x \sin y,$$

$$\cos(x + y) = \cos x \cos y - \sin x \sin y,$$

$$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y},$$

$$\sin(x - y) = \sin x \cos y - \cos x \sin y,$$

$$\cos(x - y) = \cos x \cos y + \sin x \sin y,$$

$$\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}.$$

Product of the Functions of Two Angles. —

$$\sin x \sin y = \tfrac{1}{2}[\cos(x - y) - \cos(x + y)],$$

$$\sin x \cos y = \tfrac{1}{2}[\sin(x + y) + \sin(x - y)],$$

$$\cos x \sin y = \tfrac{1}{2}[\sin(x + y) - \sin(x - y)],$$

$$\cos x \cos y = \tfrac{1}{2}[\cos(x + y) + \cos(x - y)].$$

Functions of Twice an Angle. —

$$\sin 2x = 2 \sin x \cos x, \qquad \cos 2x = \cos^2 x - \sin^2 x,$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}.$$

Functions of Half an Angle. —

$$\sin \frac{x}{2} = \sqrt{\frac{1 - \cos x}{2}}, \qquad \cos \frac{x}{2} = \sqrt{\frac{1 + \cos x}{2}}, \qquad \tan \frac{x}{2} = \sqrt{\frac{1 - \cos x}{1 + \cos x}}.$$

Functions of Three Times an Angle. —

$$\sin 3x = 3 \sin x - 4 \sin^3 x, \qquad \cos 3x = 4 \cos^3 x - 3 \cos x,$$

$$\tan 3x = \frac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x}.$$

[W. A. Del Mar.]

TRIGONOMETRY. — (*See also Trigonometric Functions.*) Any triangle is completely defined when, (1) two sides and the included angle are known, (2) one side and two angles are known, (3) three sides are known. Let the sides and angles of a triangle be designated as in Fig. 1.

1. Given two sides a **and** b**, and the included angle** γ**.** Then

$$c = \sqrt{a^2 + b^2 - 2\,ab\cos\gamma}$$

$$\sin\alpha = \frac{a}{c}\sin\gamma$$

$$\beta = 180 - \alpha - \gamma.$$

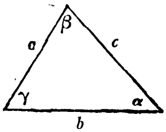

Fig. 1.

2. Given the side a **and the two angles** β **and** γ**.** Then

$$\alpha = 180 - \beta - \gamma$$

$$b = a\frac{\sin\beta}{\sin\alpha}$$

$$c = a\frac{\sin\gamma}{\sin\alpha}.$$

3. Given the three sides a**,** b **and** c**.** Put

$$s = \tfrac{1}{2}(a+b+c)$$

Then

$$\sin\alpha = \frac{2}{bc}\sqrt{s(s-a)(s-b)(s-c)}$$

$$\sin\beta = \frac{b}{a}\sin\alpha$$

$$\gamma = 180 - \alpha - \beta.$$

Relations Between Sides and Angles. — The following relations between the sides and angles of a triangle are sometimes useful:

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

$$\cos\alpha = \frac{b^2 + c^2 - a^2}{2\,bc}$$

$$\sin\frac{\alpha}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$$

$$\cos\frac{\alpha}{2} = \sqrt{\frac{s(s-a)}{bc}}$$

and similar relations for the other two angles.

[W. A. Del Mar.]

TROLLEY SYSTEMS, OVERHEAD. — (*See also Cars, Electric; Cross Arms; Locomotives, Electric; Poles for Overhead Lines; Rails, Track and Third; Railways, Electric, Traction Systems for; Railways, Location and Permanent Way for; Third-rail Systems; Transmission Lines; Trolley Systems, Underground; Wires and Cables, Bare.*) The following is a brief table of contents of this article:

The trolley wire is usually of hard-drawn copper but sometimes of steel, which is suspended from insulators some 16 to 30 feet above the ground, and presents a continuous contact surface to a trolley wheel or bow attached to the rolling stock. There are two classes of trolley construction, the span wire and the side bracket; each may have either simple or catenary suspension.

Span-wire and Side-bracket Construction. — In the simple span wire construction the trolley wire is supported by wires stretched across the tracks between poles or building walls. The side-bracket construction resembles the span wire except that instead of the supporting wire being stretched between two poles, it is stretched between two supports on the same pole. In both of these types of construction, the trolley wire is supported at intervals of 100 feet or more and sags considerably between supports, making it necessary for the trolley to be in constant vertical vibration as the cars move.

Catenary Construction. — The speed attained upon modern electric roads makes it difficult to obtain satisfactory service with a trolley wire which dips between each support and sags and sways with every impulse. The catenary construction was devised to meet this condition. In general, it consists of a grooved copper trolley wire suspended horizontally from a sagging messenger cable, which is suitably insulated and firmly held in place. The supporting structure preferably employed for interurban single- or double-track roads is of the side-bracket type, but for some conditions cross-span construction becomes necessary. The latter method of support differs only in the substitution of a catenary cross span for the bracket arm and doubling the number of poles required for single track.

APPLICATIONS OF VARIOUS TYPES OF CONSTRUCTION. — The overhead trolley system is used on urban railways, wherever the unsightliness or danger of its exposed construction is not considered objectionable. It is used on interurban and suburban lines wherever the current taken by the trains is not too great to be economically carried on copper wires. In recent years it has been used in conjunction with the alternating-current systems of electric traction on electric trunk lines.

Center-pole construction is the most sightly for double-track city railways, especially if ornamental brackets are used, side-pole construction being generally used for single-track lines. Span-wire construction is used where, for any reason, it is impracticable to have the poles near the tracks or where, as is commonly the case in Europe, the span wires are supported from the walls of buildings. Prejudice against overhead lines is often due to the excessive loading of poles, which is both unsightly and dangerous.

ELECTRICAL DESIGN OF CONDUCTORS. — In designing the trolley system careful attention must be given to both the electrical and mechanical features. In this section will be treated the electrical features, and in the following section the mechanical features. The electrical design of railway distribution systems involves the consideration of potential drop, heating of conductors and feeder economy. The heating of conductors is seldom an important factor in railway feeder design, as it is usually necessary to use a low current density to keep down the drop of potential. A discussion of the heating of conductors will be found under *Wires and Cables, Bare.* The feeder system being sufficient to meet the conditions imposed by the allowable potential drop (*see below*), it will be economical to make it greater if the saving in the cost of energy which will result, is greater than the increase in interest and other charges on the additional investment.

Allowable Potential Drop. — The total potential drop is limited by the necessity of running the cars at a certain speed and by the need of keeping the car lights brilliant. The potential drop in the ground conductors, i.e., the track rails and bare negative feeders, is further limited by the danger of electrolysis by current leaking into the earth (*see article on Electrolysis*).

The drop in the grounded conductors under maximum-load conditions is limited by law in Great Britain to 7 volts between any two points of the system. In Germany the maximum drop in the grounded conductors is limited to 1 volt per kilometer (1.61 volts per mile). In the United States the legal limit is a matter of local option and is, in general, less severe than in Europe.

Calculation of Potential Drop. — The method of calculation of potential drop depends upon the following conditions: (1) whether the current is direct or alternating; (2) whether the load is concentrated at one point, sparsely distributed or evenly distributed; (3) the distribution of metal in the feeder circuits, and (4) whether the section is being fed by one or by two or more substations. Calculations for alternating-current lines differ from those for direct-current lines only in taking into account the inductance, as described below. On city railways it is usual to assume the load to be evenly distributed, it being stated as a given number of amperes per foot (*see article on Railways, Energy Requirements and Motor Capacity for*). If the load is actually concentrated at n equidistant points, the drop will exceed that calculated on the assumption of uniform distribution by about $\frac{100}{n}$ per cent. On interurban and trunk lines, the cars are usually concentrated at one or two points between substations, making the assumption of uniform distribution impracticable. In such cases the loads should be located so as to give the worst conditions, and calculations made as for any network.

Where electrolytic damage is to be guarded against, the drop of potential in the track rails themselves has to be calculated, as well as the total drop in the rails and feeders.

Resistance of Trolley and Track. — Values of the resistance of trolley wires to direct current will be found in the article on *Wires and Cables, Bare,* and values of the resistance of rails to direct current will be found in the article on *Rails, Steel.* It should be noted that the resistances of the trolley and positive feeders are in parallel and that the track rails and negative feeders are in parallel. Also note that in the case of high-voltage systems a considerable portion of the current returns through the earth and not through the rails, and consequently the drop in the rails is due only to that part of the current which returns through them. For preliminary calculations, however, the full current may be assumed as returning through the rails.

Formulas for Direct-current Trolley Circuits. — The following formulas apply to certain typical circuits which frequently occur in practice. Let

I = total current in amperes taken by all cars on section considered,
L = total length of section in 1000 feet,
V_p = total drop in volts, in positive conductors between substation bus and far end of line,
V_n = total drop in volts in negative conductors between substation bus and far end of line,
$V = V_p + V_n$ = total drop in volts in both positive and negative conductors,
r_p = resistance in ohms of all the positive conductors in multiple per 1000 feet of line,
r_n = resistance in ohms of all the negative conductors in multiple per 1000 feet of line,
$r = r_p + r_n$ = total resistance in ohms per 1000 feet of line,
l = distance in 1000 feet, from far end of line to any point P,
v = drop to the point P, subscripts used as for V.

Uniformly Distributed Load, Uniform Conductor, Fed from One Substation. — Then

$$v = \frac{rIl^2}{2L},$$

$$V = \frac{rIL}{2}.$$

These formulas are applicable to either the positive or negative conductors considered separately or to both in series.

Uniformly Distributed Load, Conductor Tapered to give Minimum Weight of Metal, Fed from One Substation. — For minimum weight the tapering must be such that at any point P

$$r = \frac{3V}{2I\sqrt{L}\sqrt{l}},$$

i.e., the cross-section, if all the conductors are of the same metal, must increase directly as the square root of l. The drop to the point P is

$$v = \frac{V\sqrt{l^3}}{L^3}.$$

These formulas also apply to either the positive or negative conductors separately or to both in series.

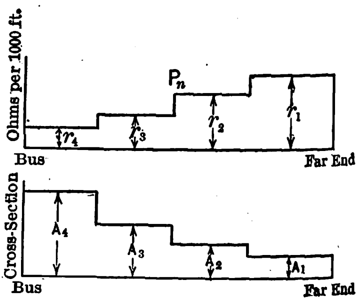

Fig. 1.

Uniformly Distributed Load, Conductor Divided into Sections (Fig. 1); Each Section of Constant Resistance, Fed from One Substation. — The drop from P_n to the far end of line is

$$\frac{I}{2L}\sum_{n=1}^{n=n} r_n[L^2{}_n - L^2{}_{(n-1)}].$$

This formula is applicable to either the positive or negative conductors.

Concentrated Load, Section Fed from One End.—Fig. 2 shows a 4-track road with 4 trolley wires and 3 feeders with the tracks cross-bonded at intervals. The solution given below is a general one, and may be applied to any case from 1 track and 1 trolley wire up. Let all distances be expressed in 1000 feet and let

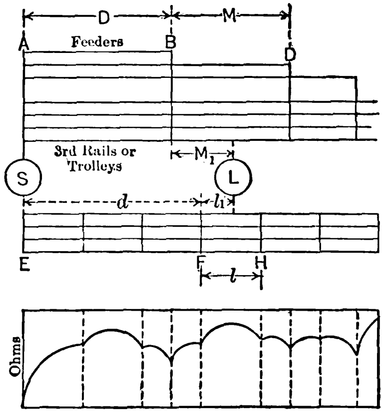

Fig. 2.

N_f = number of feeders in section considered, e.g., for the section AB, $N_f = 3$, and for the section BD, $N_f = 2$,

N_c = number of contact conductors, trolley wires or third rails in section considered, e.g., four are shown in Fig. 2,

N_t = number of tracks in section considered,

R_f = resistance of each feeder per 1000 feet,

R_c = resistance of each contact conductor per 1000 feet,

R_t = resistance of each track per 1000 feet (1 rail or 2 rails in multiple depending upon whether 1 or 2 rails are used for return conductor),

$n = N_c + \frac{R_c}{R_f} N_f$ for the section considered.

Then for the section in which the load may be, the resistance of the positive conductors from the load to the end of that section in the direction of the substation, e.g., the resistance from L to B, is

$$R_c M_1 \left[1 - \frac{(n-1) M_1}{nM} \right].$$

The resistance of the positive conductors in any section such as BA is

$$\frac{R_c D}{n}.$$

(Note that the value of n for this section is not the same as for the section BD.)

The resistance of the negative conductors from the load to the first cross bond in the direction of the substation, e.g., the resistance from L to F, is

$$R_t l_1 \left[1 - \frac{(N_t - 1) l_1}{N_t l} \right].$$

The resistance of the remaining portion of the negative conductors, e.g., from F to E, is

$$\frac{R_t d}{N_t}.$$

(Note that if negative feeders are used, each negative feeder having a resistance of R_f' per 1000 feet, then for N_t in the last two formulas substitute $n' = N_t + \frac{R_t}{R_f'} N_f'$, where N_f' is the number of negative feeders for that section.)

The total resistance from the load to the substation is the sum of the resistances as above calculated.

Concentrated Load, Section Fed from Both Ends, Substation Voltage at the Two Ends the Same.—The most convenient method of treating such problems is to plot an "equivalent-resistance-distance" curve such as shown in Figs. 2 to 5. By "equivalent resistance" is here meant that resistance by which the total current taken by the load must be multiplied to give the total drop in voltage between the load and either substation. For example, if the substation voltage is 600 at each end, the voltage across the load is 550 and the current taken by the load is 200 amperes, then the equivalent resistance is $R = \dfrac{600-550}{200} = 0.25$. This method avoids the determination of the distribution of the current in the various parts of the network, and the resistance when once determined can be applied to any load.

1. In Fig. 3 is shown a single track and single trolley. S_1 and S_2 are substations; L is a load placed arbitrarily between corresponding points P_1 and P_2 on the positive and negative conductors respectively. Let

a = resistance of the conductors between the points $S_1P_1P_2S_1$;

b = resistance of the conductors between the points $S_2P_1P_2S_2$.

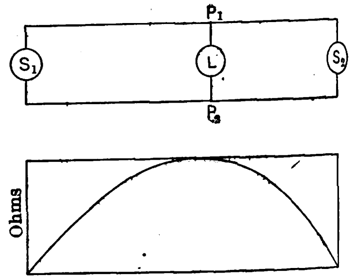

Fig. 3.

These resistances are the resistances of the transmitting conductors and do not include the internal resistances of the substations and load. Then the equivalent resistance is

$$R = \frac{ab}{a+b}.$$

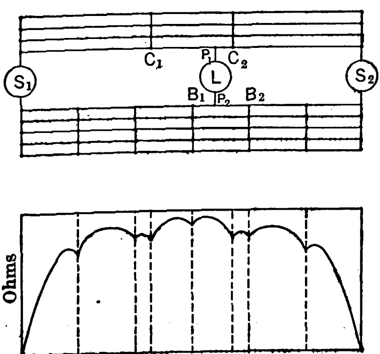

Fig. 4.

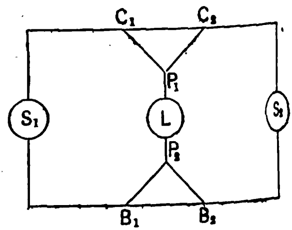

Fig. 4a.

2. In Fig. 4 is shown a 4-track road with 4 trolleys, both track and trolley cross-bonded. Fig. 4a is a simplified diagram of Fig. 4, corresponding points being designated by identical letters.

S_1 and S_2 are substations; L is a load placed arbitrarily between points P_1, on the positive system and P_2 on the negative system; C_1 and C_2 are ties between positive conductors and B_1 and B_2 ties between negative conductors.

Let

a = resistance of loop $S_1C_1P_1LP_2B_1S_1$,
b = resistance of loop $C_1P_1C_2C_1$,
c = resistance of contact conductor C_1P_1,
d = resistance of contact conductor C_2P_1,
e = resistance of loop $B_1P_2B_2B_1$,
f = resistance of track B_1P_2,
g = resistance of track B_2P_2,
h = resistance $\frac{d^2}{b} + \frac{g^2}{e}$,
i = resistance $\frac{c^2}{b} + \frac{f^2}{e}$,
j = resistance $S_2C_2P_1LP_2B_2S_2$.

These resistances are the resistances of the transmitting conductors and do not include the internal resistances of the substations and load. The equivalent resistance is

$$R = \frac{aj - ah - ji - 2\left(\frac{fg}{e} \times \frac{cd}{b}\right) + \frac{f^2d^2 + c^2g^2}{eb}}{a + j - h - i - 2\left(\frac{fg}{e} + \frac{cd}{b}\right)}.$$

Concentrated Load, Section Fed by Feeders from Both Ends. — Fig. 5 shows a simple case with feeders for the positive conductors only. No general formula is available for such a circuit. Any such network can, however, be calculated by Kirchhoff's Laws (q.v.), using the numerical values of the resistances for the various resistances. The equivalent resistance from the substations to the load is then the drop in voltage for a load of one ampere.

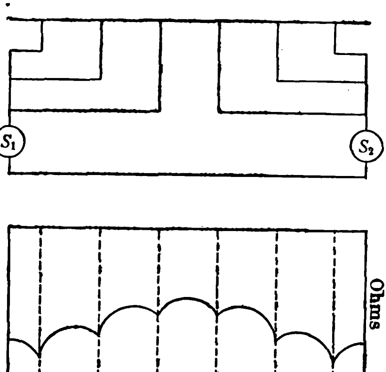

Fig. 5.

Ways of Reducing Total Drop. — Three methods are employed for reducing the total drop in the trolley and rails, viz:

1. Plain or "Non-boosted" Feeders. — Copper feeders, usually bare, may be connected to the trolley wire, third rail or track rails at frequent intervals. If the resistance of the trolley is high compared with that of the rails, which is usually the case, the feeders should be used to reënforce the trolley wire and not the track. In general, the most economical use of additional copper is to connect it in parallel with that side of the circuit having initially the higher resistance.

2. Boosted Feeders. — An insulated feeder may be tapped into the trolley wire or third rail at one point, and connected to a booster at the substa-

tion. The principle of this method is to increase the activity of the feeder, i.e., to make it carry more current and thereby reduce the current in the trolley wire. This is necessary only where the resistance of the line is so low that a moderate-sized feeder does not take its full share of current. It is not much used because if the line resistance is already low and the load is small, the drop will usually be small enough naturally, and if the load is great, the booster and feeder will usually have to be so large in order to produce any effect, as to be prohibitive in first cost and cost of operation.

3. **Floating Battery,** i.e., a battery connected across the line so as to charge when the line potential is high and discharge when it is low. This scheme is very little used on account of the high cost of operating and maintaining a battery. It is difficult to imagine under what conditions such a system would be commercially practicable, but if anywhere, it would be on a long line with light load and small feeders. Typical calculations for floating batteries are given in Lyndon's *Storage Battery Engineering*, Chap. XLIV.

Ways of Reducing Drop in the Negative Side of Circuit.—Owing to the necessity of preventing the leakage of current from track rails into the earth, it may be necessary to reduce, not the total drop in the negative system, but the drop in the grounded portion of the negative. This may be accomplished by the use of insulated feeders connected to the track rails so as to drain the current from the latter and thus reduce the drop in the rails, regardless of the drop in the feeders themselves. Such feeders may be used with or without a booster. Used with boosters, this method is probably the most economical way of reducing the drop in return rails where a large reduction is necessary to prevent electrolysis.

Negative Feeder with Booster.—In the case where the load is uniformly distributed over the line, the size of booster which must be installed in the substation may be determined as follows:

Let a = amperes entering negative feeder system per foot of line,

r = resistance of negative feeder system per foot,

R = total resistance of the feeder connecting the booster to the negative feeder system,

I = total amperes entering negative feeder system from the motors,

I_0 = amperes to be taken off by booster,

$l = \dfrac{I - I_0}{a}$ = distance from substation to the point at which the current in the negative feeders is to be zero,

l_1 = distance from booster tap to point in negative feeders where current is zero,

l_2 = the distance from the booster to the end of the line.

Then the booster voltage is

$$RI_0 - \tfrac{1}{2}\,ar(l^2 - l_1^2).$$

The output of the booster is, of course, the product of the current and voltage. The total drop is

$$\tfrac{1}{2}\,ar(l^2 - l_1^2 + l_2^2).$$

See Del Mar's *Electric Power Conductors*, Chap. V, for a more complete treatment.

Formulas and Constants for Alternating Current Trolley Circuits.—Little information is available on the effective or a-c. resistance (*see Alternating Currents*) and reactance of steel rails. As a rough approximation the a-c. resistance of a rail at 25 cycles per second may be taken as 5 times its d-c. resistance and the reactance of the rail as approximately the same as that of one wire of a pair of No. 0000 wires at a distance apart equal to the height of the

trolley wire above the track, i.e., roughly as 0.4 ohm per mile. As a matter of fact the a-c. resistance depends upon the shape of the rail, its magnetic quality, the value of the current and the frequency; to a lesser extent the inductance, and therefore the reactance, depends upon these same items.

Data from Tests on N. Y., N. H. & H. R.R. — The most reliable data available are those obtained by A. W. Copley from tests on the New York, New Haven and Hartford Railroad and reported in the Trans. A.I.E E., 1908, Vol. 27(2) p. 1171. An analysis of these tests is also given by H. Pender in the Elec. World, 1909, Vol. 53, p. 1457. Copley's results showed that a considerable portion of the current entering the track from the load leaked to the earth. The combined resistance r of any number of trolley wires, and track rails and earth, as determined by Copley, can be represented by the following formula

$$r = \frac{p_1 r_1 + p_2 r_2}{100},$$

where r_1 = the resistance of each trolley wire separately, p_1 = the per cent of the total current in this wire, r_2 = the a-c. resistance of each rail separately and p_2 = the per cent of the total current in this rail. Similarly, the combined reactance x of any number of trolley wires and track rails and earth return can be represented approximately by the formula

$$x = \frac{p_1 x_1 + p_2 x_2}{100},$$

where x_1 = the reactance of a given trolley wire, x_2 = the reactance of a given rail and p_1 and p_2 as above.

Pender gives the following values for trolley wire and rail separately, based upon Copley's tests:

Item	Resistance, ohms per mile		Reactance,* ohms per mile	
	25 cycles	15 cycles	25 cycles	15 cycles
Single trolley wire, No. 0000 B. & S....	0.26	0.26	0.38	0.23
000 B. & S....	0.33	0.33	0.38	0.23
00 B. & S....	0.42	0.42	0.39	0.23
Single rail, 100 pounds to the yard.....	0.16	0.13	0.44	0.26

* For the trolley wire 25 ft. above the track, changing the height of the trolley wire 5 ft. up or down changes these values by less than 5 per cent.

The combined resistance and reactance respectively of the trolley wires, rails and earth are given in the following table:

For any other weight of rail the a-c. resistance of the rail itself may be taken as roughly inversely proportional to the weight per yard. The reactance of the rail, however, is practically independent of its cross-section, for the magnetic flux within the rail is relatively small compared with the total flux surrounding the rail (*see Inductance*).

Formulas for Single-phase Calculations. — The calculation of the power lost and drop in voltage between a load and the power house are calculated in the same manner as in the case of an ordinary single-phase a-c. trans-

RESISTANCE AND REACTANCE PER MILE OF ROAD

100-pound Rails

Number of tracks	....		1		2		4	
Number of trolley wires	1		1		2		4	
Number of rails	1		2		4		8	
Per cent of total current returning through rails†	25		40*		58*		75*	
Cycles per second	25	15	25	15	25	15	25	15
Combined resistance of trolley wires, track and earth per mile of road								
No. 0000 B. & S. trolley	0.30	0.29	0.29*	0.28*	0.16*	0.15*	0.086*	0.082*
000 B. & S. trolley	0.37	0.36	0.36	0.35	0.20	0.19	0.11	0.10
00 B. & S. trolley	0.46	0.45	0.45	0.44	0.24	0.23	0.13	0.12
Combined reactance of trolley wires, track and earth per mile of road, all three sizes of trolley wire	0.49	0.30	0.47*	0.28*	0.27*	0.16*	0.17*	0.10*

* The figures marked thus * are taken directly from the paper by Copley, whose tests were confined to No. 0000 trolley wires; the other values are calculated by Pender.

† The percentages of total current returning through the rails are test results on the N.Y., N.H. & H.R.R., and refer to relatively long sections (over 3 miles); a greater proportion of current flows in the rails in the immediate vicinity of the load and power house. The proportion of the total current which will return through the earth will depend upon the nature of the soil and ballast; for any other division of current the resistance and reactance can be obtained from the approximate formulas given above.

mission line (*see Transmission Lines*). For a concentrated load (e.g., a single train) the calculation is as follows: Let

V = volts between trolley wire and track;
P = kilowatts at the locomotive delivered to locomotive;
$\cos\phi$ = power factor of load taken by locomotive;
l = distance in miles, between power house and locomotive;
r = combined resistance in ohms, per mile of trolley wire, rails and earth return;
x = combined reactance, in ohms, per mile of trolley wire, rails and earth returns;
$R = rl$;
$X = xl$.

Then the current taken by the locomotive is

$$I = \frac{1000\,P}{V\cos\phi} \quad \text{amperes,}$$

the power lost is

$$p = \frac{RI^2}{1000} \quad \text{kilowatts,}$$

the drop in voltage between the power house and locomotive is

$$v = \sqrt{(V\cos\phi + RI)^2 + (V\sin\phi + XI)^2} - V \quad \text{volts,}$$

and the power factor at the power house is

$$\cos\phi_0 = \frac{1000\,(P + p)}{I\,(V + v)}.$$

Example. — 2000 kw. at 25 cycles per second are to be supplied to a locomotive at 90 per cent power factor and 10,000 volts at a distance of 20 miles from the power house. The circuit consists of a No. 0000 trolley wire and two 100-lb. track rails. Then $V = 10{,}000$, $P = 2000$, $\cos\phi = 0.9$, $\sin\phi = 0.435$, $l = 20$, $r = 0.29$ (assuming only 40 per cent of the current returning through the rails), $x = 0.47$ (same assumption), $R = 0.29 \times 20 = 5.8$, $X = 0.47 \times 20 = 9.4$. Whence

$$I = \frac{1000 \times 2000}{10{,}000 \times 0.9} = 222 \text{ amperes},$$

$$p = \frac{5.8 \times (222)^2}{1000} = 286 \text{ kilowatts},$$

$$v = \sqrt{(10{,}000 \times 0.9 + 5.8 \times 222)^2 + (10{,}000 \times 0.435 + 9.4 \times 222)^2} - 10{,}000 = 2130 \text{ volts},$$

$$\cos\phi_0 = \frac{1000\,(2000 + 286)}{222\,(10{,}000 + 2130)} = 0.848.$$

Scheme of Connections Used by N.Y., N.H. & H.R.R. — The distribution system is shown diagrammatically in Fig. 6 where A is the trolley

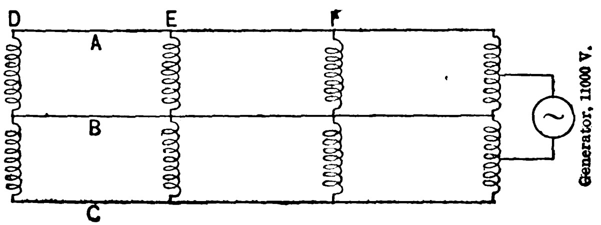

Fig. 6.

wires, B the track rails and C a feeder. These three sets of conductors are connected at intervals of about two miles through auto-transformers D, E and F. The difference of potential between A and B, and between B and C is 11,000 volts and between A and C is 22,000 volts. The effect of this arrangement is to practically give a transmission voltage of 22,000 with 11,000 volts at the trolley, and to greatly reduce the current in the grounded conductors with consequent reduction of disturbances to telephone lines.

MECHANICAL DESIGN OF OVERHEAD SYSTEM. — The various parts of the overhead system will first be described, and then the various types of construction, span wire, catenary, etc.

Parts of Overhead Trolley Construction. — The trolley wire is secured to an "ear" by soldering, clamps or other means, the ear is bolted to a "suspension" which may or may not be provided with an insulating portion. These suspensions are carried by span wires which in turn are fastened to the poles or brackets, or may be fastened directly to the bracket. If the "ear" is not insulated, "strain" insulators are inserted in the span wire between the ear and its point of attachment to the poles or brackets. A slightly different form of suspension, called a "pull-off," is used on curves. At turnouts "trolley frogs" must be used to guide the trolley wheel. These various parts are illustrated in the accompanying cuts.

Trolley Ears. — Figs. 7 to 10 are for round wire. Fig. 7 shows an ear with flaps, which are bent around the wire to hold it in place; this type is seldom

Fig. 7.

Fig. 8.

Fig. 9.

Fig. 10.

Fig. 11.

Types of Trolley Ears for Round Wire

used on account of arcing at the flaps. The ear shown in Fig. 8, which has a deep groove into which the wire is soldered, is the type in common use. These ears may be provided with rings as in Fig. 9 to which guy wires are attached to relieve the strain at curves and for steadying the line at intervals. Fig. 10 shows the type of ear used at points where the trolley is spliced, which should always be at an ear. Fig. 11 shows an ear with a terminal for a feeder connection.

Figs. 12 to 14 are designed for grooved or figure "8" trolley wire. Special

Fig. 12.

Fig. 13.

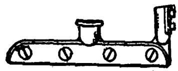

Fig. 14.

Types of Trolley Ears for Figure "8" Wires

grooved or figure-8 wire affords a smoother running surface for the trolley wheel or bow as the ear only grips the upper part, leaving the lower part absolutely even. This construction is practically essential for bow trolleys.

Soldering Trolley Wire to Ears. — The strain should be taken off the wire by a U-shaped clamp catching hold of it on each side of the ear. The soldering iron should weigh about 8 or 9 pounds, and should have a groove fitting half way round the wire. Special precautions should be taken not to have the iron unnecessarily hot.

Suspensions for Straight Line Work. — Fig. 15 shows an uninsulated suspension with ear attached for straight line work. Fig. 16 shows a suspension with strain insulators at each end; this is also used on curves for double-track

Fig. 15.

Fig. 16.

Fig. 17.

Fig. 18.

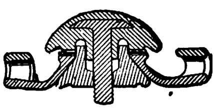

Fig. 19.

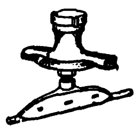

Fig. 20.

Types of Suspensions

work. Figs. 17 and 18 show solid insulated suspensions for span wire and side-bracket construction, respectively. These types are frequently used, and are quite satisfactory if the insulating material used in their construction is properly made. Fig. 19 shows a section of an assembled cap and cone suspension. Fig. 20 shows a cap and cone suspension with ear attached. The cap and cone are made separate. This type is preferred by some engineers on account of the

possibility of replacing injured bolts and insulation without removing the whole suspension. This advantage is partly offset by the greater liability to trouble due to multiplicity of parts.

Strain Insulators. — Fig. 21 shows a "globe" strain insulator, the type most commonly used. Fig. 22 shows a "Brooklyn" strain insulator. This type is used on wooden pole and light iron pole construction to draw span wires taut. It is also required even for heavy iron pole construction if spans are long and temperature variation great. Bolts may be provided at both ends if an extra amount of adjustment is required. A globe strain and a Brooklyn strain may be used in series where extra insulation is required.

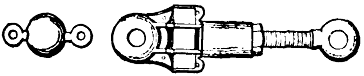

Fig. 21. Fig. 22.
Strain Insulators

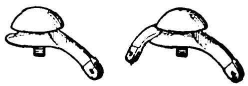

Fig. 23. Fig. 24.
Pull-offs

Pull-offs. — Fig. 23 shows a cap and cone pull-off for single-curve construction and Fig. 24 a cap and cone pull-off for double-track-curve construction. Pull-offs of the same type as the uninsulated and solid insulated suspension shown in Figs. 15, 17 and 18 are also used.

Trolley Frogs. — A trolley frog is a malleable iron casting used at switches or crossovers where trolley wires from different tracks unite. Its function is to hold the diverging wires together and afford a smooth running path to the trolley wheel when a car passes or enters a switch. A common type is illustrated in Fig. 25. Frogs are made for various angles of divergence, and both right and left handed. The usual angles are 8, 15 and 20 degrees.

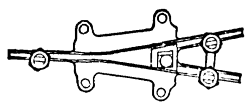

Fig. 25. Trolley Frog

Sag and Tension in Overhead System. — Particular attention must be paid to designing the overhead structure in such a manner that it will safely stand the extra tension due to the contraction of the wires at low temperatures and the extra loads due to wind and sleet, and a sufficient allowance should be made in the height of the trolley wire to take care of the extra sag which it experiences at high temperatures. With a simple trolley construction the variations in tension and sag may be calculated by a direct application of the rules given in the article on *Transmission Lines;* in the case of catenary construction, the additional load due to trolley wire, clips, etc., is treated in the same way as a sleet or wind load.

Direct Suspension. — The following discussion applies primarily to span-wire and side-bracket construction; catenary construction is treated in a separate section below. Modern practice regarding the dimensions of poles, size of trolley and span wire, height of trolley wire, etc., is illustrated in Tables I and II.

TABLE I. — DETAILS OF CITY RAILWAY CONSTRUCTION. DIRECT SUSPENSION

Names of companies	Spacing of poles, feet	Size of trolley wire, A. W. G.	Span wire	
			Stranding	Diameter, inches
Birmingham (Ala.) Ry. Lgt. & Power Co.	100	000 G	7	1/4*
Boston Elevated Co.	...	00 R	7	5/16
Brooklyn Rapid Transit Co.	...	....	..	...
Denver City Tramway Co.	100	0000 G	7	5/16
Detroit United Ry. Co.	110	00 R	7	5/16
Indianapolis Traction & Terminal Co.	100	00 R	7	3/8
Little Rock Ry. & Lgt. Co.	...	00 R G	7	1/4
Louisville Ry. Co.	100	00 R	..	1/4
Milwaukee E. Ry. & Lgt. Co.	110	0000 G	..	...
Pacific El. Co. (Los Angeles)	115	0000 E	..	3/8 & 5/16
Philadelphia Rapid Transit Co.	100	0000 G	7	5/16
Twin City Rapid Transit	100-110	00 E	3	...

* 3/8-inch for double trolley.
G = grooved; *R* = round; *E* = figure eight.

Poles range in height from 30 to 40 feet, with diameters at top ranging from 6 to 8 inches. Cedar, chestnut and cypress poles as well as iron poles are in common use. Span construction is usually employed, except in unusually wide streets, where double-bracket poles may be used. See also Table II.

TABLE II. — DETAILS OF INTERURBAN RAILWAY CONSTRUCTION DIRECT SUSPENSION

Names of companies	Single or double trolley	Bracket or span suspension	Height of trolley wire above rail in feet	Shape and size of trolley wire A. W. G. or B. & S.	Number of poles to the mile	Length of poles in feet	Diameter of poles at top in inches	Material of poles
Northern Ohio								
Cleveland & Southwestern	Both	B	18	00 GR	53	35	8	Ce & Ch
Lake Shore Electric	D	Both	19	000 E	53 & 58	35	7 & 8	Ce & Ch
Eastern Ohio	D	B	18	0000 E	52	35	7 & 8	Ce
Toledo & Indiana	D	B	17	000 G	60	35	6½ & 7	Ch
Toledo & Western	S	B	17½	000 E	52	35	6½ & 7	Cy
Toledo, Pt. Clinton & Lakeside	S	Both	19	000 E	52	35	7	Ch
Stark Electric	D	Both	19	000 E	52	35	7	Ch
Canton-Akron	S	B	18	0000 T	52	35	7	Ch

TABLE II. — DETAILS OF INTERURBAN RAILWAY CONSTRUCTION DIRECT SUSPENSION — *Continued*

Names of companies	Single or double trolley	Bracket or span suspension	Height of trolley wire above rail in feet	Shape and size of trolley wire A. W. G. or B. & S.	Number of poles to the mile	Length of poles in feet	Diameter of poles at top in inches	Material of poles
Central and Southern Ohio								
Western Ohio......	D	B	18	00 R	52	40	7	Cy
Ft. Wayne, Van Wert & Lima....	S	B	18	000 R	52	40	7	Ce
Dayton & Troy...	D	Both	21	00 E	48	35	7	Ce
Dayton, Covington & Piqua.........	D	B	18	00 R	52	35	7	Ch
Cincinnati & Columbus..........	S	B	16	0000 G	52	35	7	Ch
Cincinnati, Milford & Loveland	D	Both	18	0000 R	52	35	7	Ch
Interurban Ry. & T. Cincinnati....	D	Both	20	00 R	55	35	7	Ce
Cincinnati, Georgetown & Portsmouth..........	D	B	18	000 R	52	35	6	Ch & Ce
Indiana								
Indiana Union.....	S	Both	20	000 E	52	30	7	Ce
Indianapolis & Northwestern...	S	B	18	000 G	52	36 & 32	6	Ce
Indianapolis & Cincinnati..........	S	B	18	000 G	52 & 44	40	7	Ce
Indianapolis, Columbus & Southern.............	D	B	18	0000 R	52	35	8	Ce
Terre Haute Tr. & Lgt...........	S	B	19	00 R	52	40	..	Ce
Kokomo, Marion & Western.........	D	B	19	000 R	52	40	..	Ce
Ft. Wayne & Wabash Valley..	Both	B	20	000 R	52	40	7	......
Michigan								
United Railway System, Detroit.	D	B	19	00 & 000 R E	48	40	..	......
Detroit, Ypsilanti, A. A. & J........	D	B	19–20	00 E	52	40	..	Ce

Abbreviations: *G* = grooved, *R* = round, *E* = figure 8, *Ch* = chestnut, *Ce* = cedar, *Cy* = cypress, *B* = bracket, *D* = double, *S* = single.

Height of Trolley Wire Above Rail. — The height of wire varies between a minimum of 16 feet and a maximum of 22 feet, the usual height being about 18 feet. It is usual to raise the wire at railroad crossings to a height of 22 feet or more.

Rake of Poles. — Bracket-arm poles on tangent construction should have a rake backwards not exceeding 3 inches, and span-wire poles in hard ground a rake of from 4 to 5 inches. In soft ground a rake of 12 inches is not uncommon. Center poles should be set vertically except at curves, where they should bend away from the curve along the perpendicular to the tangent at that point of the curve.

Anchorage. — At both ends of every grade and curve, there should be a permanent anchorage. If there are not many grades and curves, anchorages should be provided at intervals of from ¼ to ¾ of a mile. An anchorage is made by means of a steel cable running from the trolley wire to one or more anchor poles through an anchor ear and strain insulators.

Curves. — At curves in the track the trolley wire should be made to follow the curve by means of pull-offs or wires pulling the trolley wire outward

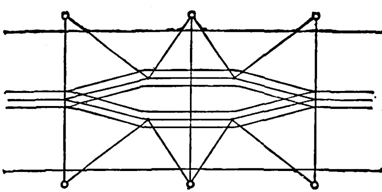

Fig. 26. Simple Pull-off Arrangement

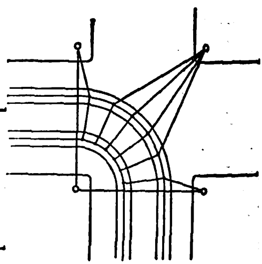

Fig. 27. Simple Pull-off Arrangement

as shown in Figs. 26 and 27.

Wherever possible the pull-off wires should be radial to the trolley wire. This, however, requires a large number of poles, and is therefore impracticable in cities. In such cases, the bridle, bow-string or backbone construction shown in Figs. 28, 29, and 30 is resorted to. In this construction the pull-off wires

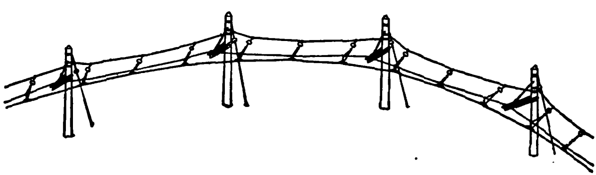

Fig. 28. Bridle Construction at Curve

instead of being anchored to individual poles are fastened to a wire which is stretched between poles. While this construction is almost universally used in cities, it is more expensive to maintain than single pull-offs, and is therefore less favored for interurban lines.

A combination of the two types of construction is shown in Fig. 31. Figs. 28 to 31 are from G. E. Co. publications.

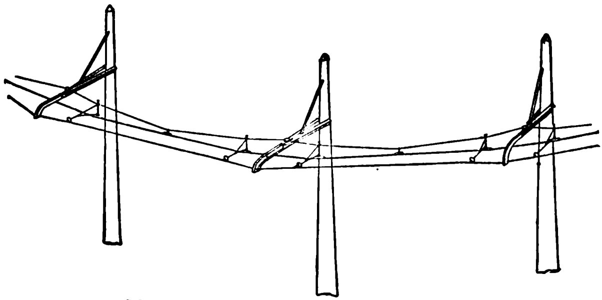

Fig. 29. Bow-string Construction at Curve

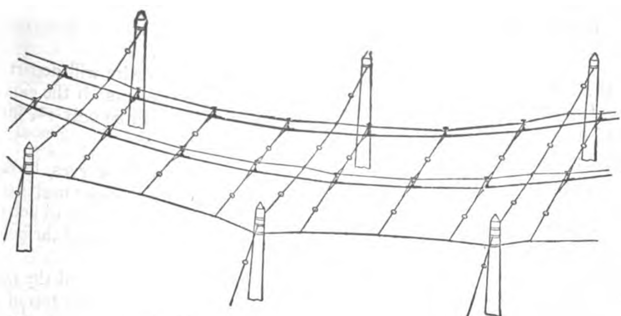

Fig. 30. Backbone Construction at Curve

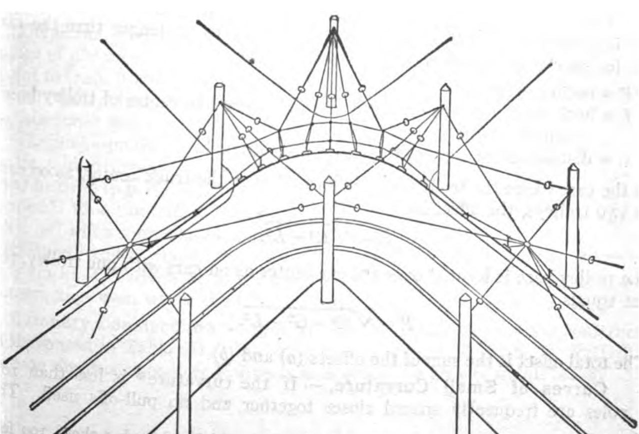

Fig. 31. Combination Construction at Curve

Spacing of Pull-offs at Curves. — The number of pull-offs should be sufficient to keep the wire within about 2½ inches from the theoretical curve. This may be accomplished by spacing the successive ears in accordance with the following relations:

Let L = distance between pull-offs in feet,
R = radius of curve in feet,
a = offset of wire in inches from theoretical curve, midway between pull-offs, the ears being assumed to lie on the theoretical curve.

Then,

$$L = \sqrt{\frac{2aR}{3} - \left(\frac{a}{6}\right)^2},$$

or

$$L = 0.815\sqrt{aR},$$

with an error less than ¼ per cent for all radii greater than 40 feet.

If a is to be 2½ inches, then

$$L = 1.29\sqrt{R} \text{ approximately.}$$

If the ears are set exactly on the theoretical curve, the wire will depart 2½ inches from the correct position half way between the two ears; if the ears are set 1¼ inches from the curve, the mid-point of the wire will be only 1¼ inches when L is taken equal to $1.29\sqrt{R}$.

Offset of Trolley Wire at Curves. — It is usual, at curves, to offset the trolley wire from the center of the track (a) because the trolley wheel is tilted inward due to the elevation of the outer rail, and (b) because in order to keep the wheel on the wire, the latter must be so placed that the projection of the pole on the plane of the track is always tangential to the wire.

(a). The former offset, measured horizontally toward the inside of the curve, equals $h \tan \theta$, where h is the normal height of the wire above the top of rail, and θ is the angle of elevation between the plane of the track and the horizontal. For standard gauge and wire 18 feet above the top of rail this offset is about 4 inches for every inch of elevation.

(b). The latter offset, also measured horizontally toward the center of the curve, is calculated as follows, assuming the curve to be longer than the car itself; for shorter curves the offset will be less. Let

R = radius of curve,
L = horizontal distance of center of trolley wheel to center of trolley base, usually about 11 feet,
G = distance from center of car to center of truck.

In the case where the trolley base is located over the truck center, as on cars with two trolleys, the offset is

$$R - \sqrt{R^2 - L^2}.$$

If the trolley base is located over the car center as on cars with one trolley, the offset equals

$$R - \sqrt{R^2 - G^2 - L^2}.$$

The total offset is the sum of the offsets (a) and (b).

Curves of Small Curvature. — If the curvature* is less than 10°, the poles are frequently spaced closer together and no pull-offs used. The

* The degree of curvature of a track is the angle which subtends a chord 100 feet long, between points on the center line of the track.

following table gives the practice of the Denver and Interurban R.R. Co., which may be considered typical, the divergence of the wire from the theoretical curve being kept within 3½ inches.

SPACING OF POLES ON CURVES, INTERURBAN ROAD

Degree of curvature of track	Pole spacing, feet	Divergence of trolley wire from track center, inches	Degree of curvature of track	Pole spacing, feet	Divergence of trolley wire from track center, inches
Tangent	120	0	6	60	2.87
1	120	1.87	7	50	2.34
2	110	3.37	8	50	2.64
3	90	3.06	9	50	2.94
4	80	3.37	10	50	3.24
5	70	3.06			

Turnouts. — (*See also Railways, Location and Permanent Way for.*) The location of the trolley frog at turnouts may be determined as follows. Referring to Fig. 32, from switch point *A* draw a line to center point *D* of track frog distance *BC* and from switch point *B* draw a line to center point *E* of arc *AEC*. The intersection of these two lines at *F* will be the proper location of the frog. While certain variables, such as superelevation of the outer rail on the curve, length of wheel base and projection of trolley pole rearward from center of car, may necessitate slight variation of setting, this location will be found so nearly correct that a very small alteration, which must be determined by experiment, will compensate for the variable conditions.

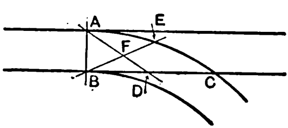

Fig. 32.

The accompanying table gives the range of distance from track switch point to track frog with which each set of trolley frogs may be most satisfactorily used:

Track-frog distance	Divergence angle of trolley frog
Up to 22 feet..............	20°
From 20 to 30 feet.........	15°
Above 28 feet..............	8°

The minimum frog distance given in the table with which the 15° frogs may be used to best advantage corresponds to a turnout radius of 40 feet, but when suburban cars, using high-speed trolley wheels, run over city tracks it is advisable to use 15° rather than 20° frogs throughout the city construction even where the minimum frog distance is less than 20 feet.

Catenary Construction. — Modern practice regarding catenary construction is illustrated in Table III. The construction on the N. Y., N. H. & H. R.R., and on the Austrian State Railways is described in greater detail below.

TABLE III. — DETAILS OF TRUNK & INTERURBAN RAILWAY CONSTRUCTION. CATENARY SUSPENSION.

Railway	Voltage	Type of support	Tangent spacing of poles	Size of trolley wire, A. W. G. or B. & S.	Size of messenger wire, in.	Suspensions per span
Boston & Westchester Ry. Co..	11,000	Bridges	300	0000	...	..
Denver & Interurban Ry.......	11,000	Bracket	120	0000	7/16	12
Illinois Traction System.......	.....	Bracket	140	....	...	3
Indianapolis & Cincinnati Co...	3,300	Bracket	120	....	...	..
Milwaukee El. Ry. & Lgt. Co..	.....	Center pole	110	0000	9/16	3
N. Y., N. H. & H. R.R.......	11,000	Bridges	300	0000	....	30
Spokane & Inland Empire......	.....	Span wire	100	000	5/16	..
Syracuse, Lake Shore & Northern..........................	6,600	Bridges	300	0000	7/16	10
Texas Traction Co............	600	Bracket	150	000	3/8	3
Washington, Baltimore & Annapolis Ry. Co...............	6,000	Bracket	150	0000	3/8	9

(Messenger cables composed of 7 strands in all cases.)

Simple and Compound Catenary Construction. — Catenary constructions may be divided into simple and compound. In the former the trolley wire or wires are carried by one or two messenger cables which are supported only at the poles, bents or span wires; in the latter, the horizontal wire or wires are carried by a messenger cable, which is itself suspended from another messenger cable which is supported at the poles, bents or span wires. The advantages of the compound catenary are greater flexibility, reduced stresses in the supporting wires, shorter hangers, better lightning protection and superior curve construction. (*See description of N. Y., N. H. & H. R.R., construction below.*) The simple catenary is sometimes made with two messenger cables from which the horizontal wire or wires are suspended by triangular frames.

To obtain a line which will not require frequent readjustment, the messenger cable must be installed with practically uniform tension throughout its length, making it necessary to have less sag in the shorter spans. For this reason certain definite pole spacings and corresponding hanger lengths have been standardized (*see next section below*).

Suspensions. — (*See also p. 1738.*) The number of suspensions depends upon the speed at which the cars are to be run, and upon whether a bow or wheel trolley is used. The three-point suspension in which, with 150 feet spacing, the hangers are 50 feet apart has been found ample for wheel collectors at speeds up to 65 m.p.h. With the sliding pantagraph or bow trolley an eleven-point suspension has been found sufficient, with 150 feet pole spacing.

Hangers. — (*See also p. 1739.*) Where only one horizontal wire is suspended from the messenger cable, the hangers should hang loosely from the cable and should be screwed fast to the trolley wire. This permits the trolley wire to rise slightly as the trolley passes under it, thereby making the wire equally flexible along its entire length. Where a steel contact wire is clipped to the horizontal conductor, the hangers are usually rigidly attached at both ends, as the duplication of horizontal wires assures uniform flexibility, regardless of the hangers.

TANGENT CONSTRUCTION

Number of Hangers per Span. Pantagraph or Bow Trolleys

Length pole spacing, feet	Number of points of suspension	Length of hangers, inches										
		6	6¾	8½	11	12	13½	14¾	16	17½	19¼	20½
150	11	1	2	2	2	..	..	2	..	..	2	..
125	9	..	..	..	1	2	2	..	2	..	2	..
110	8	..	..	..	..	..	2	2	..	2	..	2
95	7	..	..	..	..	..	..	..	3	2	..	2
80	6	..	..	..	..	..	..	..	..	2	2	2
70	5	..	..	..	..	..	..	..	..	..	3	2
55	4	..	..	..	..	..	..	..	..	..	..	4

Number of hangers per span. Wheel trolleys

Length pole spacing, feet	Number of points of suspension	Length of hangers, inches							
		6	11	13½	14¾	16	17½	19¼	20½
150	3	1	..	..	2	..	..	..	..
125	3	..	1	..	..	..	2	..	..
110	3	..	..	1	..	..	2	..	..
95	3	..	..	..	..	1	..	2	..
80	3	..	..	..	..	..	1	2	..
70	2	..	..	..	..	..	..	..	2
55	2	..	..	..	..	..	..	..	2

Labor and Equipment for Erection. — The following description from a paper on the Denver and Interurban Ry. in the *Proc. Am. Soc. Civil Eng.*, August, 1908, p. 540, is typical of the best practice. The line is for 11,000-volt single-phase operation.

The overhead line was put up rapidly with the assistance of a steam construction train and in one day's work 26 men installed 117 poles, including digging holes and tamping. A different construction train was used for erecting the brackets. For this service a number of box freight cars were equipped with a scaffold on the tops of the cars. With a train of five cars work could be done on four or five poles simultaneously, and 150 brackets were erected per day with 18 men. For stringing the catenary a derrick car was used, arranged to carry the reels and pay the wire out over elevated rollers. The best day's run on messenger wire was 7 miles strung and tied in. This took 17 workmen and two train crews. The best day's run on stringing and splicing trolley wire was also 7 miles, with 12 workmen and one train crew. The best day's record of clipping up messenger and trolley wire was 3½ miles with 13 workmen and one train crew. A working day consisted of nine hours, with transportation from headquarters each day on the company's time, leaving about seven working hours. The ground wires and feeder wires were placed before the brackets were erected, and the wire was paid out over the end of the boom of the derrick.

Equalization of Tension in Adjacent Spans. — The calculated sag table was verified by a special dynamometer consisting of a calibrated car spring. The use of this dynamometer disclosed a point of interest in connection with the erection of long lengths of wire and cable. For an average sag of the messenger cable, which according to the sag table should result in a tension of 2300 pounds, the dynamometer showed an initial tension of 6000 pounds. This initial strain decreased to 4000 pounds before quitting, and after expansion and contraction over night came down to the figure in the sag table. This excessive initial strain occurred on a portion of the line which is full of curves and was due to the lack of equilization between spans of the 1-mile section. On tangents the initial strain was between 3500 and 4000 pounds. This is a point to be borne in mind in calculating the proper elastic limit for such wires and cables.

TYPICAL EXAMPLES OF CATENARY CONSTURCTION. — The following examples are given to illustrate in greater detail the methods employed in catenary construction.

New York, New Haven & Hartford R.R. — The overhead construction of this railroad represents the latest and most substantial work of its type in America, and operates the heaviest and fastest type of trunk-line trains. The original overhead construction consisted of a rigid double-catenary system with equilateral triangular steel-pipe frames of different lengths joining two steel messenger cables to the copper trolley wire. This, being found too rigid, was modified by the addition of a steel contact wire suspended from the copper by clips midway between hangers. Considerations of economy and the desirability of securing better lightning protection led to the construction described below, wherein the live conductors are protected by grounded cables above them.

Tangent Construction. — Steel lattice bents are spaced 300 feet apart to support the wires. They consist of girders resting on and rigidly attached to columns which, when viewed along the track, taper to a minimum width at the base, and viewed across the track have parallel sides. A cast-iron saddle bolted to the top of the girders over each track carries a ⅞-inch galvanized steel 19-wire messenger cable which runs continuously from bent to bent with a normal sag of 6.42 feet at 60° F. At points 75 feet each side of every bent these messenger cables are joined by a 3-inch 5½-pound I-beam spanning all the tracks. These I-beams are attached to the messengers by means of clamps as shown in Fig. 33. The system thus far described is grounded and serves the double purpose of a support for the catenary construction and a lightning protection for the conductors.

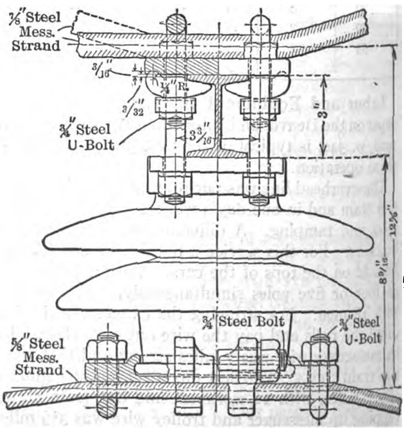

Fig. 33. N. Y., N. H. & H. Tangent Construction

Over the center of each track there is bolted to the bottom of the I-beam a porcelain insulator of the type shown in Fig. 33. These insulators occur every 150 feet along the track and serve to support a ⅝-inch steel 19-wire cable which runs continuously over the center of each track. Hangers of various lengths, such as shown in Fig. 34, depend from this steel cable at intervals of 10 feet and support a 0000 B. & S. grooved trolley wire in an approximately horizontal position. Midway between these hangers clips are attached, supporting a 0000-grooved steel contact wire, the whole construction being shown in elevation in Fig. 35. The object of placing the clips between hangers is to give flexibility to the contact wire by reducing the resistance to upward spring.

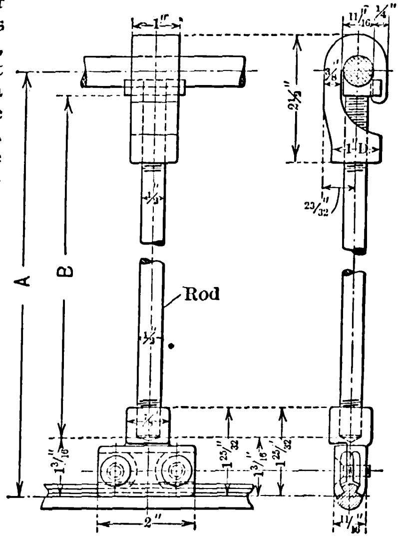

Fig. 34. N. Y., N. H. & H. Hangers

Curve Construction. — On curves up to 3 or 4 degrees, the use of pull-offs and bridle construction is avoided by a modification of the tangent construction, whereby the ⅝-inch messenger cable itself acts as a bridle and the hangers do duty as pull-offs. This is accomplished by installing the ⅝-inch messenger cable well outside of the center line of the pantograph and using slanting hangers bolted to the clips, instead of the usual vertical hangers located between clips. A plan view of this construction looks like a bridle with pull-

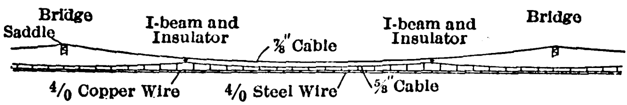

Fig. 35. N. Y., N. H. & H. Compound Catenary

offs and an elevation looks like the standard tangent construction. The inclined position of the hangers gives the requisite flexibility to the contact wire in spite of the clips being attached to the hangers. The spacing of the bents is reduced on curves to between 180 and 200 feet.

Curves greater than 3.5 degrees require pull-offs to keep the contact wire over the pantagraph shoe. These pull-offs are attached to extra clips between the hanger clips and are anchored either to a bridle construction or to poles, according to local requirements.

Yard Construction. — In yards a wire span between two poles replaces the bent used in main-line construction, the insulators being suspended from the cross-span by 5/16-inch soft-steel cable. A horizontal steady-wire insulated by strains slightly above the level of the contact wire supports the steady-hangers to which the contact wire is clipped. On curves, an additional insulated steady-wire is attached to the messenger wire at the poles. The messenger

wire is often reduced from 5/8 inch to 3/8 inch and the hangers are spaced 15 feet on tangents and 10 feet on curves. No copper trolley is used.

Insulated Sections. — The main-line conductors are divided into sections from cross-over to cross-over. Wherever conditions permit, sections are insulated by staggering the contact wires laterally and running the two wires parallel, 18 inches apart, each to a separate insulator. The ends are turned up to avoid catching the pantograph.

Turnouts. — Turnouts are made by means of steel deflectors or frames which keep the diverging wires at a fixed angle apart.

Stresses in Spans. — The stresses in the 7/8-inch cable are as follows on a tangent 300-foot span.

Condition	Temperature, ° F.	Sag, ft.	Stress, lb.
Normal	60	6.42	7,550
Normal	−10	5.71	8,550
Sleet and wind	−10	6.46	14,750

The stresses in the 5/8-inch cable are as follows on a tangent 140-foot span.

Condition	Temperature, ° F.	Stress, lb.
Normal	60	4930
Normal	−10	7100
Sleet and wind	−10	8400

The stresses in the copper wire and contact wire are negligible. Sleet is assumed to cover the wire to a depth of one-half inch and the wind is assumed to exert a pressure of 2/3 of 8 pounds per square foot.

Hangers. — The longest hangers (140 feet apart) have a length of 15 9/16 inches measured between centers of wires. The shortest hangers are 4½ inches long. For main-line construction they are made of ½-inch rods, for yards, of 3/8-inch rods. (*The above details of the N. Y., N. H. & H. R.R. published by courtesy of Mr. W. S. Murray.*)

Catenary Construction on Austrian State Railways. — The working conductor is suspended from a compound catenary. It is of hard-drawn copper, has a cross-section of 154,000 circular mils, and is supported at intervals of 10 feet by clamps loosely hung on the auxiliary catenary. The auxiliary catenary is a steel wire 0.24 inch in diameter suspended from the main catenary at intervals of about 19½ feet by hangers which are rigidly attached at both ends. The main catenary is a seven-strand steel cable having a total cross-section of 68,000 circular mils. In the tunnels where there is much moisture, bronze is substituted for steel in the catenary construction. The working conductor is prevented from swinging by gas-pipe braces insulated with two petticoat insulators in series. On curves the braces are so arranged that they are always under tension. The height of the working conductor over the top of the rail is 18 feet

in the open. An even tension is maintained on the trolley wire by inserting an automatic tension device at intervals of about 0.6 mile. The construction is carried by iron poles either of the lattice type of construction or of broad-flange I-beams. In the open where there is only one track the catenary is carried on brackets, the poles being spaced at 165 feet. In the stations where there are several tracks the overhead work is carried by a channel iron supported by two poles. All poles are grounded to the rails.

Catenary Construction in Tunnel. — In the tunnels special construction had to be used in order to carry out the compound catenary type of construction within the clearance limits imposed by the profile of the tunnel. The maximum height of the tunnel over the rails is 13.5 feet and the height of the clearance profile is 12 feet over the top of the rails. Therefore there is only 1.5 feet available for the overhead construction. The working conductor was brought within 2 inches of the clearance profile — that is, 12 feet 2 inches over the top of the rail, — and the construction was supported at intervals of about 60 feet by three-part insulators mounted in the upper part of the tunnel walls. One of the advantages of mounting insulators on the side walls was the freedom from trouble due to locomotive smoke during the period when steam locomotives were still used on the road.

Insulated Sections. — The overhead conductor is provided with a section insulator at both ends of all tunnels and at all stations so that these portions of the road can be isolated in case of trouble. Then, in order not to interfere with the operation of the road as a whole, by-pass lines are carried around the tunnels and around the stations. The switches connected to these by-pass lines are mounted on the poles and operated from the ground by rods. The section insulators themselves are carried by two brackets spaced about 33 feet apart, between which two working conductors are strung parallel to each other and about 1 foot apart. Between these points the two conductors are inclined horizontally in opposite directions, and therefore in passing along the conductor the current collector is transferred from one to the other without shock. At stations section insulators are very much simplified, and instead of overlapping the sections, as described above, the insulator is built right into the line itself. (*El. W., 1912, Vol. 59, p. 1253, with dimensions corrected.*)

SPECIFICATIONS. — (*See also article on Specifications.*) Specifications for trolley wire are given under *Wires and Cables, Bare,* and for insulators under *Insulators for Overhead Lines.*

Globe and Brooklyn Strains. — The following data should be given:

Dimensions, size of eye, etc.

All samples tested to destruction shall break in the eye.

The average ultimate tensile strength of all samples subjected to mechanical test shall not be less than a stated number of pounds, and no individual sample shall show a tensile strength of less than 85 per cent of the average tensile strength of all the samples that are tested.

The average breakdown voltage for samples which have been broken in the eye in the mechanical test shall be not less than a stated number of volts, and no individual sample shall break down at less than 90 per cent of that voltage.

Round-top Hangers. — Round-top hangers, when suspended free from draught in an inverted position by means of a bronze ear weighing 8 ounces and being 5½ inches long, the ear clamping the middle of a round rod of soft iron ¼ inch in diameter, and of at least 20 inches length between connectors, must be able, without breaking down or becoming permanently deformed by more than 1/16 inch, to sustain a weight of 200 pounds from the cap for one hour, a

current of 200 amperes being passed continually through the iron rod, the rod being cold at the start. (*Sheldon and Keiley, Vol. 22, Trans. A.I.E.E., 1903.*)

OPERATION. — The operation of overhead lines for low-speed cars presents few difficulties owing to the adaptability of the trolley pole and wheel at such speeds. With high-speed cars the case is different, as the collector vibrates rapidly and the line must be kept uniformly level, smooth and elastic throughout and all special work must be maintained in first-class condition. If the collector vibrates in resonance with the loops in the contact wire, an increase in the number of supports will usually cure the trouble.

Prevention of Formation of Sleet. — Sleet may be prevented from forming on the wires by greasing the latter with petroleum jelly and if it does form it may be easily removed by any of the numerous commercial forms of sleet cutters.

Wear of Trolley Wheel, Bow and Trolley Wire. — The wear of the trolley wires is not serious either with wheel or bow collectors. On the lines of the Indianapolis & Cincinnati Traction Co., a copper trolley wire lost less than 1 per cent in weight after it had experienced 39,000 car movements, each car taking an average of about 40 amperes by an aluminum slider.

The vertical wear of the steel contact wire on the N. Y., N. H. & H. R.R. was 0.028 inch in thirty months, which is practically 4.5 per cent per year of the half diameter of the wire (one-half taken to permit wire to be held in clips) which, even on this vertical diameter basis, indicates a life of over 20 years; but as a matter of fact it will be much more than this, for the reason that as the vertical diameter lessens the breadth of contact increases throughout, thus diminishing the rate of vertical wear. Of further interest is the fact that there is practically no corrosion of the wire, for like the traffic rails in service, the wire is covered with a film of grease deposited by the pantograph shoe (*W. S. Murray*).

Change in Length of Trolley Wire. — The change in length of copper due to changes in temperature is one of the greatest difficulties in the maintenance of overhead work. A drop of 100° F. in temperature will cause a copper bar to contract approximately 1 inch for every 100 feet of length. If it be restrained at the ends this will cause an additional stress of 2500 pounds in a No. 0000 trolley. (*See Transmission Lines.*)

European catenary lines are usually maintained at constant tension by means of weights pulling on the free ends of the trolley wire at the end of every section.

TESTING OF TROLLEY CIRCUITS. — Trolley lines have to be tested, (1) for resistance, and (2) for insulation from ground. The resistance may be measured by grounding one end of the wire to the track rails and circulating current at reduced voltage. The drop in the overhead lines may be measured using another trolley wire, a telephone wire or the earth as a potential lead. If the earth is used, the connections to ground should be sufficiently far from the track rails to be outside the zone of potential disturbance (*see Electrolysis of Underground Structures*).

REPAIRS OF TROLLEY CIRCUITS. — The repair methods of four typical railways are given below.

Philadelphia Rapid Transit Co. — The maintenance department is divided into regular maintenance crews and emergency crews and a pole gang, which are under separate foremen. The regular maintenance foreman has two crews consisting of two linemen, one helper and a driver, and the pole gang consists of the foreman and five helpers. The emergency foreman is in charge of seven emergency stations in winter and eight in summer. The crews in these stations consist of two linemen, two helpers and two drivers divided into day and night

shifts on duty 12 hours each. The eight stations care for approximately 75 miles of line each, although when necessary the crews answer calls outside of their own districts. The equipment of the stations consists of one two-horse telescopic tower wagon and four horses. In addition, three of the stations are equipped with telescopic tower cars for heavy work. (*Elect. Ry. Journal, 1908, Vol. 22, p. 856.*)

Detroit United Railway Co. — The maintenance force of this company consists of 23 men under the direction of a city line foreman reporting to the superintendent of power. Three construction tower wagons are kept on regular maintenance work, and for heavy emergency work two tower line cars are used. These cars distribute line supplies and material and are called on to aid in moving disabled cars or other heavy work. The crews of these cars consist of a motorman and two linemen.

One emergency station in the center of the city serves 186 miles of city lines. Its equipment includes two one-horse, ball-bearing, light tower wagons, a hose jumper wagon, wreck wagon and four horses. Two men are on duty night and day, and in addition the two line cars and their crews are stationed in the same building when not engaged in outside work. This emergency station answers approximately 300 calls a month for trolley-wire breakdowns. The horses are not kept harnessed, but the wagons have quick-hitch harness. (*Elec. Ry. Journ., 1908, Vol. 22, p. 858.*)

Birmingham Railway, Light & Power Co. — The maintenance force for the overhead lines consists of two foremen and 12 men, who are under the superintendent of the electric department. Only one emergency station is used for the entire system. Its equipment includes one two-horse tower wagon, one one-horse buggy on which a ladder is carried and one motor tower car for heavy repairs and emergency work on the suburban lines. This station answers as many as 350 calls a month for all kinds of emergency repairs.

Illinois Traction System. — The line is divided, for overhead maintenance purposes, into districts averaging 90 miles in length. A line foreman and three helpers are assigned to each district and are given a tower line car with a regular crew. The members of the crew act as groundmen on heavy repair work.

COST OF TROLLEY CONSTRUCTION. — The costs given in the following tables will serve as a rough guide in making preliminary estimates.

Extras for Curves. — Under ordinary conditions curves add about 10 per cent to the cost of direct-suspension construction and about 15 per cent to the cost of catenary construction.

Extras for 1200-volt Construction. — The following amounts should be added to give proper values for 1200-volt construction:

Direct suspension:	
Bracket construction	$40.00
Span construction	40.00
Catenary suspension:	
Bracket construction	$10.00
Span construction	10.00

Cost of 11,000-volt Catenary Construction. — Under favorable conditions, an 11,000-volt catenary construction, such as that of the Denver and Interurban Ry., with sufficient conductors for a half-hourly operation of two-car trains, including track bonding, costs from $3500 to $5000 per mile of single track. (*O. S. Lyford, Proc. Am. Soc. Civil Eng., August, 1908, p. 540.*)

COST PER MILE OF SPAN-WIRE TROLLEY CONSTRUCTION (600 VOLTS)

(Exclusive of Track Work and Bonding.)

Item	Unit price	Single track		Double track	
		Quantity	Total cost	Quantity	Total cost
Material (incl. 2 double curves):					
Yellow-pine poles, octagon..	$6.00	88	$528	....	....
Iron poles, No. 2...........	19.00	....	....	88	$1672
Iron poles, No. 4...........	36.00	....	....	4	144
Cement....................	2.35 & 2.15	22 bbl.	52	33 bbl.	71
Broken stone...............	0.95	14 cu. yd.	13	14 cu. yd.	13
Black paint.................	0.90	20 gal.	18	11 gal.	10
Span wire..................	0.012	1250 ft.	15	2500 ft.	30
Pull-off wire................	0.006	1250 ft.	8	2500 ft.	15
No. 000 copper wire, per lb..	0.18	1 mi.	483	2 mi.	966
Straight-line suspensions....	0.285	36	10	72	21
Side-feed suspensions........	0.57	8	5	16	9
Deep groove ears...........	0.235	56	13	112	26
Frogs......................	3.25	2	7	4	13
Diagonals..................	3.60	....	....	2	7
Brooklyn strains...........	0.71	....	....	110	78
Frog pull-offs...............	0.36	6	2	12	4
Pole clamps................	0.12	9	1	18	2
Globe strains...............	0.31	15	5	30	9
Side-feed wire (No. 0, inches)	0.102	120 ft.	12	240 ft.	24
Double bodies..............	0.93	6	6	12	11
Single bodies...............	0.53	6	3	12	6
Miscellaneous..............	....	....	1	....	7
Total material...........	....	....	$1182	....	$3138
Labor (incl. 2 double curves):					
Setting poles...............	....	....	$156	....	$138
Trucking..................	....	....	25	....	25
Painting, one coat.........	....	....	9	....	12
Running trolley wire.......	....	....	50	....	75
Building 2 double curves....	....	....	34	....	50
Putting up span wire........	....	....	20	....	20
Total labor...............	....	....	$ 294	....	$320
Grand total per mile..........	....	....	$1476	....	$3458

For heavy catenary construction, such as used on trunk-line railways, the cost depends entirely upon the standards selected, which are inclusive of the consideration of importance of track, in turn bringing into consideration the advisability of wood- and steel-post construction, cross-catenary and bridge-span construction, single or compound catenaries, etc. The cost of overhead yard construction can vary from $1500 to $3000 a mile of single track, depending upon number of tracks spanned and type of construction selected.

COMPARATIVE COST PER MILE OF SINGLE-TRACK DIRECT SUSPENSION AND CATENARY CONSTRUCTION

Adapted from (*G. E. Review, 1910, Vol. 13, p. 516.*)

600-volt Line, Tangent Track

Item	Direct suspension		Catenary, three-point	
	Bracket	Span	Bracket	Span
Material:				
Poles, 8 inches by 30 feet	$265	$530	$180	$360
Anchor, guy and span cable	45	150	21	100
Messenger cable	....	....	92	92
No. 0000 trolley wire	540	540	540	540
Other line material	145	99	144	101
Total material	995	1319	977	1193
Labor:				
Erecting poles	185	371	126	252
Mounting brackets	13	....	9	....
Installing span wire and guys	....	212	....	144
Stringing and clamping wire	75	75	200	200
Installing anchors	100	100	50	60
Total labor	373	758	385	656
Miscellaneous extras	150	150	150	150
Grand total	$1518	$2227	$1512	$1999

The following figures are representative of modern 11,000-volt trunk-line catenary construction, using steel bents similar to the recent construction on the N. Y., N. H. & H. R.R.

Number of tracks	Cost of construction per mile	
	Of right-of-way	Of single track
1	$4,000–$7,000	$4,000–$7,000
2	8,000–15,000	4,000–7,500
4	25,000–40,000	6,250–10,000

Sidings, with wooden pole construction, cost from $2500 to $3500 per mile, and yard construction from $1500 to $3000 per mile of track.

ESTIMATED COST OF TRIANGULAR CATENARY CONSTRUCTION FOR 11,000-VOLTS, ORIGINAL N. Y., N. H. & H. TYPE

(*Elec. Age, April, 1908, p. 96.*)

Contact Line and Supports

Item	Quantity, unit	Price	Per mile of single track	Per mile of four tracks
Steel bridges, intermediate; every 300 ft., wgt. 13,000 lb.	115 tons	$100.00	....	$11,500
Steel bridges, anchor; every 2 miles, wgt. 23,000 lb.	5¾ tons	100.00	....	575
Foundations for intermediate bridge, 9 cu. yds. each side; 34 per mile	306 yds.	10.00	....	3,060
Foundations for anchor bridge, 12 cu. yds., 1 per mile	12 yds.	10.00	....	120
Special foundations	....	......	....	775
Trolley wire, No. 0000 B. & S., 5280 ft.	3380 lb.	0.18	$608	......
Messenger wires, two ⅝-in. steel, 10,900 ft.	9150 lb.	0.08	732	......
Hangers, 10 ft. apart	528	0.75	395	......
Insulators, two every 300 ft.	34	0.50	17	......
Pins and yokes for above	34	0.75	26	......
Strain insulator and accessories, 16 every two miles	8	6.00	48	......
Trolley strain insulators and section breaks, 4 every 2 miles	2	16.00	32	......
Circuit breakers, 8 per section	4	500.00	2000	......
Linemen's materials.	....	......	20	......
Labor on trolley, messengers and supports	....	......	1200	......
			5078	20,308
Total for contact system	....	......	....	36,338

Feeder System

Item	Quantity, unit	Price	Per mile of single track	Per mile of four tracks
Feeder wires, No. 0 B. & S. (two), 10,900 ft.	3,380 lb.	$ 0.18	....	608
Insulators	35	0.50	....	18
Pins	35	0.50	....	17
Circuit breakers	1	500.00	....	500
Control wire and pipe	500 ft.	0.50	....	250
Control transformers, 5 kw., 2 per section	1	100.00	....	100
Lightning arresters	....	......	....	50
Miscellaneous material	....	......	....	20
Labor on feeders	10,900 ft.	0.03	....	327
Total for feeder system				1890

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[W. A. DEL MAR.]

TROLLEY SYSTEMS, UNDERGROUND.— (*See also Trolley Systems, Overhead.*) This system is little used on account of its cost, the installations at Buda-Pest, New York, Washington and London being the only notable ones. The essential feature of all these systems is the underground conductor which is reached from the car by a "Plough" extending through a continuous slot parallel to the tracks.

NEW YORK SYSTEM. — The most successful construction is the latest New York type, a brief description of which is given below.

General Description. — The street is excavated and cleared of obstructions for a width of about 5 feet 6 inches and a depth of about 3 feet and cast-iron yokes (Fig. 1) set in the excavation about 5 feet apart. The track and slot rails are supported on these yokes and the whole system made solid with concrete which fills the excavation from the foundation to near the top of the rails leaving only a tunnel under the slot free from masonry. The conductors are suspended in this tunnel, by special strain insulators, no part of the electrical system being grounded. The use of two insulated conductors avoids trouble in case of accidental grounding.

Yoke. — The yoke shown in Fig. 1 is made in three parts. The lowest piece which rests on the floor of the excavation is a 6-inch steel I-beam, 4 feet 8 inches long. Two castings 6½ inches wide are riveted to this, each one serving to support one track rail and one slot rail. Fig. 1 shows both slot rails in place but only one track rail. The track rail is carried on a timber stringer which extends from yoke to yoke along the whole line. The stringers are held to the yokes by countersunk iron plates the ends of which project beyond the stringer and have holes which accommodate bolts running through the yoke. These bolts serve the additional purpose of fastening the rails to the stringers as they also pass through clips which bear upon the foot of the rail. The rails are further secured by long bolts running to the center of the yoke. The slot rails are bolted direct to the yoke and are connected to the track rails by long bolts every 30 inches.

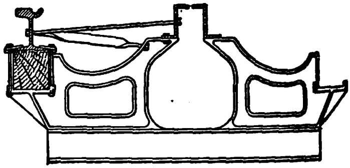

Fig. 1.

Insulator Boxes. — A pair of cast-iron boxes are laid across from the slot rails to the track rails and bolted to them every 15 feet as shown in Fig. 2. These contain the insulators which support the contact conductors. Inside each box is a pair of shelves running normal to the track; resting on these shelves and bolted to them is a cast-iron bridge which holds the insulator. The top of the box is provided with a cast-iron cover which is flush with the street surface.

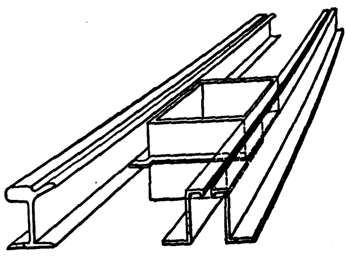

Fig. 2.

Concrete Work. — When the iron work is laid the tunnel which is to protect the conductors and do the draining is formed with collapsible sheet iron, fitting closely to the center opening of the yokes and run from yoke to yoke. The space from the bottom

of the insulator boxes to the road foundation is formed with boards. Concrete is then poured into the excavation completely filling every part but the tunnel and its offsets at the insulator boxes. The concrete is poured to within the height of a paving block of the rail tops. When the concrete is hard the sheet-iron form is collapsed and pulled along to the next section. The forms at the bottom of the insulator boxes being collapsible are drawn out through

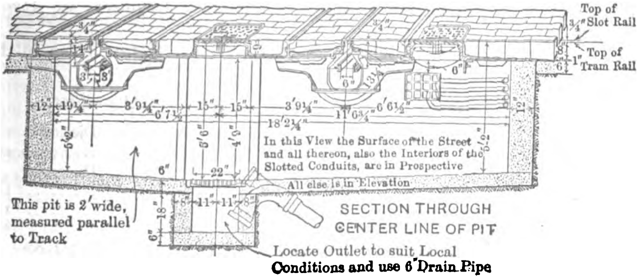

Fig. 3.

the handholes and are used again elsewhere. The depth of the completed tunnel is 18 inches from the base of the slot rails. A view of an older New York City construction is shown in Fig. 3, which is taken from an article in the *Street Railway Journal*.

THE LONDON SYSTEM has alternate long and short yokes, the long ones fulfilling the same function as the New York ones, the short ones serving merely to give additional support to the slot rails. No stringers are used, the rails resting on hard-wood blocks at the yokes only.

THE BUDA-PEST CONSTRUCTION has the slot in the track rail, thus saving considerable iron, but necessitating an excessively wide slot to accommodate the wheel flanges.

COSTS. — The type of construction in New York City cost from $60,000 to $100,000 per mile of track, exclusive of feeders, depending upon the conditions under which it was installed.

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[W. A. Del Mar.]

UNITS AND CONVERSION FACTORS. — In this article are given the numerical interrelations, or "conversion factors" for all the commonly employed English and metric units. For the definitions of the various quantities see *Mechanics, Principles of; Heat and Thermal Properties; Temperature; Electricity and Magnetism, Principles of; Alternating Currents; Units, Practical Electrical;* and *Photometric Quantities*. For a list of the abbreviations and symbols adopted by the American Institute of Electrical Engineers see *Abbreviations and Symbols*.

The following is a list of the tables of conversion factors given in this article:

SYSTEMS OF UNITS. — By the magnitude of any quantity is meant the *relative* magnitude of this quantity as compared with some other quantity *of the same nature;* e.g., a length of 10 inches means a length which is ten times the length of one inch. To measure a quantity, then, it is necessary either (1) to adopt first some fixed magnitude of the same nature which may be used as a standard of reference, or unit, or (2) to define the quantity in such a way that the unit of measurement may be derived from the arbitrarily selected units of the quantities involved in the definition.

Fundamental and Derived Units. — Units which are chosen arbitrarily are called "fundamental" units; those which are derived from a set of fundamental units are called "derived" units. All mechanical units can be derived from a set of three fundamental units; the three units ordinarily chosen as fundamental are those of length, mass and time. In relations involving temperature changes, an arbitrary unit of temperature is also chosen; to express electrical and magnetic quantities at least one electrical or magnetic unit is arbitrarily chosen; to express photometric quantities the unit of luminous intensity is arbitrarily chosen. These arbitrarily chosen non-mechanical units are called "auxiliary fundamental" units.

English and Metric Systems of Units. — There are two systems of mechanical units in use in English speaking countries, the English and the French, or metric, systems. The latter system is used universally by physicists in all civilized countries and also forms the basis of the electrostatic and elec-

tromagnetic systems of electric units used by engineers as well as physicists. The English system of mechanical units, however, is used almost universally by English speaking peoples, in engineering, in commerce and in the arts; it is also the basis of the system of thermal units used largely by engineers. Engineers, however, are coming to use the metric system to a greater and greater extent for all purposes.

In both the English and the metric systems length, mass and time are chosen as the fundamental quantities. In the English system the fundamental units for these three quantities are respectively the foot, the pound (avoirdupois) and the second; this system is therefore also called the foot-pound-second system. In the metric system the corresponding units are the centimeter, gram and the second; this system is therefore also called the centimeter-gram-second, or c.g.s., system. The magnitudes of these fundamental units have been fixed arbitrarily by law; see below.

Absolute and Gravitational Units. — Two different units of force (and units derived therefrom) are used in both systems, viz.: (1) the unit of force is defined as that force which will give unit acceleration to unit mass, and (2) the unit of force is defined as that force which will give to unit mass an acceleration equal to the acceleration of a freely falling body under the influence of "gravity" or the pull of the earth alone (*see Mechanics, Principles of*). The first unit is called the "absolute" unit of force and the second the "gravitational" unit of force, and derived units based on these two units are called absolute and gravitational units respectively. The gravitational units are not definite unless the latitude and elevation of the point at which the acceleration due to gravity, usually represented by the symbol "g," is specified. By international agreement (*Troisème Conf. Gen. des Poids et Mes., 1901, p. 66*) the value $g_0 = 980.665$ cm. per sec. per sec. (= 32.1739 feet per sec. per sec.) has been chosen as the standard value for the acceleration due to gravity; this value of g may be called the "gravitational acceleration constant." The value 980.665 was chosen to represent the value of g at 45° latitude and sea level, but later measurements give a slightly different value of g at this latitude and elevation; the value 980.665, however, is retained as the *standard* value of g_0.

The gravitational unit of force in the English system is called the pound and in the metric system the gram, the same names being used for the unit of force as for the unit of mass. This arises from the fact that unit mass has a force exerted upon it by the earth, i.e., has a *weight*, numerically equal to its mass, when the force is expressed in terms of this gravitational unit. (This is exactly true only at the latitude and elevation for which $g = g_0$, but the variation with both latitude and elevation is usually negligible in engineering work.*) The absolute unit of force in the English system is called the poundal, but is practically never used; the absolute unit of force in the metric system is called the dyne, and is the unit of force commonly used by scientists. In engineering work the common units of force are the gravitational units, pound or gram, and their multiples and submultiples.

The relation between the absolute and gravitational units of force are:

1 pound = 32.1739 poundals,
1 gram = 980.665 dynes.

Weight and Mass. — The word weight is used in two different senses, viz.: (1) to mean the force with which the earth attracts a piece of matter,

* The force acting on a mass of 1 pound at sea level at the equator is 0.997363 pound, and at sea level at either pole of the earth is 1.002651 pound. An increase in elevation of 10,000 feet diminishes the force acting on a mass of 1 pound by only 0.00096. See Landolt, Börnstein and Roth, *Physikalish-Chemische Tabellen.*

and (2) to designate a given mass or quantity of matter; in other words, weight is used to designate both force and mass. As commonly employed the term almost invariably has its second meaning; that is, by a "weight of 10 pounds" is meant a piece of matter which has a mass of 10 pounds. A weight (2nd sense) of 10 pounds has a weight (1st sense) of 10 pounds at 45 degrees latitude and sea level (practically), but at any other latitude and elevation it has a weight (1st sense) slightly different from 10 pounds, and on the sun, for example, would have a weight (1st sense) many times 10 pounds.

In this connection it should be noted that a beam-balance when used in the ordinary way to compare two pieces of matter measures weight in the second sense (i.e., it would "read" the same whether the measurement were made on the earth or on the sun), that is, a beam-balance as ordinarily used compares masses and not forces. A beam-balance, however, is frequently used in testing (e.g., in connection with a Prony brake, see ***Index***) to compare a force produced by a machine with the pull on a standard mass (*or* "*weight*") due to gravity. If great precision is desired the reading of the balance should then be corrected for the value of "*g*" at the place of observation, but in ordinary measurements the inaccuracy of the balance is likely to be much greater than this small correction factor. For values of "*g*" at various places see reference in footnote, bottom of preceding page.

STANDARDS OF THE FUNDAMENTAL UNITS. — The physical standards upon which the c.g.s. system of units is based, and the legalized standards of the foot and pound used in Great Britain and the United States are described below.

Standard of Length. — The standard meter (100 centimeters) is the distance between two lines on a platinum-iridium bar carefully preserved at the Bureau of Weights and Measures, at Sevres, France, when the bar is kept at a uniform temperature of zero degrees centigrade throughout. In the United States the yard (3 feet) was defined by Act of Congress, July 28, 1866, as

$$\mathbf{1\ U.S.\ yard} = \frac{3600}{3937}\ \mathbf{meter},$$

and similarly the British imperial yard is defined by law as

$$1 \text{ British imperial yard} = \frac{3600}{3937.079} \text{ meter.}$$

For engineering purposes the U. S. and British yards may be considered as identical.

Standard of Mass and Force. — The standard kilogram (1000 grams) as a unit of mass is a cylinder of platinum preserved at the Bureau of Weights and Measures, at Sevres, France. The U. S. pound avoirdupois is defined by law (*Act of Congress, 1866*) as $\frac{1}{2.2046}$ kilogram, but in 1893, the Superintendent of Weights and Measures, with the approval of the Secretary of the Treasury, declared* the pound to be

$$\mathbf{1\ U.\ S.\ pound} = \frac{1}{2.204622}\ \mathbf{kilogram.}$$

The British imperial pound has the same value.

The same relations between pound and kilogram hold whether these units be taken as units of mass or as units of force, the unit of force being defined in

* Bull. Bureau Standards, 1904, Vol. 1, p. 380.

both cases as the pull of the earth on unit mass at 45 degrees latitude and sea level.

Standard of Time.—The standard second universally adopted is the $\frac{1}{86,400}$ th part of a mean solar day. The solar day is the interval of time between two successive transits of the sun across a meridian of the earth at the point of observation; this interval varies in length at different times during the year, but the average length of the interval for one year is constant as far as can be determined by any known methods of observation.

STANDARDS OF THE AUXILIARY FUNDAMENTAL UNITS.— The standards of temperature, dielectric coefficient, magnetic permeability and luminous intensity ordinarily used are defined below.

Standard of Temperature. — Two units of temperature, or temperature scales, are commonly employed, viz., the centigrade and the Fahrenheit units. The relation between these two units results solely from the manner in which they are defined.

$$1 \text{ degree centigrade} = \frac{9}{5} \text{ degrees Fahrenheit.}$$

Due to the difference in the zeros of the two scales, a temperature of t_f degrees Fahrenheit corresponds to a temperature of

$$t_c = \frac{5}{9}(t_f - 32) \text{ degrees centigrade,}$$

and vice versa,

$$t_f = \frac{9}{5} t_c + 32 \text{ degrees Fahrenheit.}$$

Standards of the Auxiliary Fundamental Electrical Units. — Three systems of electrical and magnetic units are in use, viz., (1) the c.g.s. electrostatic system, (2) the c.g.s. electromagnetic system, and (3) the practical system. In the c.g.s. electrostatic system the dielectric coefficient k of air at 0° C. and 760 mm. mercury pressure is arbitrarily chosen as unity. In the c.g.s. electromagnetic system the magnetic permeability of air under the same standard conditions is arbitrarily chosen as unity. In the practical system a concrete standard of the unit of resistance (called the ohm) is arbitrarily chosen (*see Units, Practical Electrical*); this unit resistance was originally designed to be equal to 10^9 times the unit of resistance in the c.g.s. electromagnetic system, and within the limits of ordinary experimental error this relation may still be considered exact.

Use of the Prefixes "Stat" and "Ab."— To designate the electrical and electromagnetic units in the electrostatic and electromagnetic systems of units respectively the prefixes "stat" and "ab" may be used with the name of the corresponding practical unit. For example, the c.g.s. electrostatic unit of quantity may be called the statcoulomb and the c.g.s. electromagnetic unit of quantity may be called the abcoulomb. For the names and definitions of the various electrical and magnetic units see the article on *Electricity and Magnetism, Principles of*.

Standards of Luminous Intensity. — Two standards are in use, the international candle and the hefner, the latter being used chiefly in Germany. See article on *Photometric Quantities* for the specifications for these units.

EXPERIMENTALLY DETERMINED CONVERSION FACTORS.— By conversion factor is meant the numerical factor which gives the magnitude of one unit for any given quantity in terms of any other unit for the same quantity.

For example, in the expression 1 yd. = 3600/3937 meter, the factor 3600/3937 is the conversion factor between the yard and the meter.

In the case of two units for the same quantity based on two different sets of arbitrarily chosen units which are defined independently, the conversion factor can be obtained only by experiment. The more important experimentally determined conversion factors are given in the paragraphs immediately following.

Mercury- and Water-column Pressure. — To convert pressure per unit area into height of mercury column the density of mercury must be known. Thiesen and Scheel (*Zeitsch. f. Instrk. de., 1898, Vol. 18, p. 138*) give the density of mercury at 0° C. as 13.59545 grams per (cm.)^3. Using the standard value of the gravitational acceleration constant, $g_0 = 980.665$, the relation between the dyne per sq. cm. and 1 cm. of mercury column at 0° C. is then

1 cm. mercury column at 0° C. = 13332.6 dynes per sq. cm.

Taking the density of water as 1 gram per (cm.)^3 at 4° C.,

1 cm. water column at 4° C. = 980.665 dynes per sq. cm.

Mechanical Equivalent of Heat. — Very useful units of energy are those based upon the mass of a standard substance and temperature, viz., the heat (which is energy) required to raise the temperature of a specified mass of water a specified number of degrees; see *Heat and Thermal Properties*. The experimentally determined relation between the mean small calorie (i.e., the one-hundreth part of the heat required to raise the temperature of 1 gram of water from 0° C. to 100 C. at 760 mm. mercury pressure) and the erg is

1 mean small calorie = 4.1834×10^7 ergs.

See Marks and Davis, *Steam Tables and Diagrams*, N. Y., 1912. From this relation and the relations (fixed by definition) given above between the yard and the meter, the pound and the kilogram, the value of the gravitational acceleration constant g_0, and the two temperature scales, Marks and Davis, deduce the relation

1 British thermal unit = 777.52 foot-pounds.

The British thermal unit as here used is defined as the $\frac{1}{180}$ th part of the heat required to raise the temperature of 1 pound of water from 32° F. to 212° F. at 760 mm. mercury pressure. See also *Heat and Thermal Properties*.

The experimentally determined conversion factor between the energy unit based on the units of temperature and mass, and the energy unit based on the units of length, mass and time, i.e., the unit of mechanical work, is called the "mechanical equivalent of heat."

Relations between the Three Systems of Electrical Units. — The fundamental relation, experimentally determined, between the c.g.s. electrostatic and the c.g.s. electromagnetic system is that

1 abfarad = 9×10^{20} statfarads,

which as a consequence of the definitions of the various terms is equivalent to

1 abcoulomb = 3×10^{10} statcoulombs, the erg being the unit of energy in both systems.

The fundamental relations between the c.g.s. electromagnetic system and the practical system are

1 abcoulomb = 10 coulombs,

1 erg = 10^{-7} watt-seconds, or joules,

the erg being the unit of energy in the c.g.s. electromagnetic system and the watt-second or joule the unit of energy in the practical system.

Relation between the International Candle and the Hefner. — The experimentally determined relation is

$$1 \text{ hefner} = 0.9 \text{ international candle.}$$

MULTIPLES AND SUBMULTIPLES OF UNITS. — Multiples and submultiples of the metric units are designated by the following prefixes; the relations are *definitions* and are therefore absolutely exact.

$$\text{micro} = \frac{1}{1{,}000{,}000} \quad \text{or} \quad 10^{-6}$$

$$\text{milli} = \frac{1}{1{,}000} \quad \text{or} \quad 10^{-3}$$

$$\text{centi} = \frac{1}{100} \quad \text{or} \quad 10^{-2}$$

$$\text{deci} = \frac{1}{10} \quad \text{or} \quad 10^{-1}$$

$$\text{deka} = 10 \quad \text{or} \quad 10$$

$$\text{hecto} = 100 \quad \text{or} \quad 10^{2}$$

$$\text{kilo} = 1{,}000 \quad \text{or} \quad 10^{3}$$

$$\text{myria} = 10{,}000 \quad \text{or} \quad 10^{4}$$

$$\text{mega} = 1{,}000{,}000 \quad \text{or} \quad 10^{6}$$

The multiples and submultiples of the English units are given in bold-face type in the tables below.

CALCULATION OF CONVERSION FACTORS. — The conversion factors noted above, which are either definitions or results of experiment, form the basis for the calculation of the conversion factors for the various derived units. The method of procedure is first to express one of the pair of units, say *A*, in terms of its component units (length, mass, time, temperature, capacity or resistance, and luminous intensity), then express the magnitude of each of these units in terms of the corresponding component units of *B*; the value of the resultant numerical factor is the conversion factor. For example, let it be required to find the conversion factor between pressure in pounds per square foot and pressure in dynes per square centimeter.

$$1 \text{ lb. per sq. ft.} = \frac{(1 \text{ lb. force})}{(1 \text{ ft.})^2}.$$

$$1 \text{ lb. force} = \frac{1}{2.204622} \times (1 \text{ kilogram force}).$$

$$1 \text{ kilogram force} = 1000 \times (1 \text{ gram force}).$$

$$1 \text{ gram force} = 980.665 \times (1 \text{ dyne}).$$

Therefore

$$1 \text{ lb. force} = \frac{1000 \times 980.665}{2.204622} \times (1 \text{ dyne}).$$

Also

$$1 \text{ ft.} = \frac{1}{3} \times (1 \text{ yd}).$$

$$1 \text{ yd.} = \frac{3600}{3937} \times (1 \text{ meter}).$$

$$1 \text{ meter} = 100 \times (1 \text{ cm.}).$$

Therefore

$$1 \text{ ft.} = \frac{1}{3}\left(\frac{3600}{3937}\right) \times (100) \times (1 \text{ cm.}).$$

Whence

$$\frac{1 \text{ lb. force}}{(1 \text{ ft.})^2} = \frac{1000 \times 980.665 \times (3 \times 3937)^2}{2.204622 \times (3600 \times 100)^2} \times \frac{(1 \text{ dyne})}{(1 \text{ cm.})^2}$$

$$= 478.799 \times \frac{(1 \text{ dyne})}{(1 \text{ cm.})^2}.$$

But $\dfrac{(1 \text{ dyne})}{(1 \text{ cm.})^2} = 1$ dyne per sq. cm.

Therefore

1 pound per sq. ft. = 478.799 dynes per sq. cm.

DIMENSIONAL FORMULAS. — Instead of writing the expression for a derived unit in words, it is frequently convenient to express it in algebraic symbols, using a specific symbol for each of the fundamental or auxiliary fundamental units. For example, let L stand for a length of 1 centimeter, M for a mass of 1 gram, and T for a time interval of 1 second; then the unit of force in the c.g.s. absolute system (the dyne) may be written

$$F = MLT^{-2},$$

which is the same as 1 dyne = (1 gram) $\times \dfrac{(1 \text{ cm.})}{(1 \text{ sec.})^2}$.

A formula, such as $F = MLT^{-2}$, where each letter represents the magnitude of a *single* unit of the quantity for which it stands, is called a "dimensional formula." A pure number, or a quantity which is the ratio of two units of the same dimensions, such as an angle, has zero dimensions and does not appear in a dimensional formula.

Dimensional formulas are useful for two purposes, viz., (1) as a systematic method for calculating conversion factors and (2) as a check of algebraic formulas expressing various relations. Unless one has a large number of conversion factors to calculate it is probably simpler to use the direct method given in the preceding section. Certain quantities, for example, work and torque, may have the same dimensions although they are physically entirely different. Although such quantities are physically different their conversion factors are identical. The second use of dimensional formulas is based on the fact that every term in any set of terms connected either by equal signs, plus signs or minus signs, must have the same dimensions.

For a complete list of the dimensional formulas for the various mechanical, electrical, thermal, and photometric quantities see Hering, C., *Conversion Tables*, N. Y., 1904. In the tables below are given, in the parentheses after the titles, the dimensional formulas of the mechanical quantities in terms of length, mass and time, designated by L, M and T respectively. For the electrical and magnetic quantities the dimensions are expressed in terms of electric potential difference V, quantity of electricity Q and time T. In the *c.g.s. electromagnetic system* of units Q may be expressed in terms of L, M, and T by the formula

$$Q = L^{\frac{1}{2}}M^{\frac{1}{2}}\mu^{-\frac{1}{2}};$$

in the *c.g.s. electrostatic system* by the formula

$$Q = L^{\frac{3}{2}}M^{\frac{1}{2}}T^{-1}k^{\frac{1}{2}}.$$

In both systems $VQ = L^2MT^{-2}$ = energy.

TABLES OF CONVERSION FACTORS.* — The following tables give the most commonly required conversion factors for mechanical, electrical and magnetic quantities. See *Heat and Thermal Properties* for special heat units (other than energy) and *Photometric Quantities* for the special photometric units.

LENGTH (L)

cm. = *centimeter.* *m.* = *meter.*
ft. = *foot.* *mi.* = *mile.*
in. = *inch.* *mm.* = *millimeter.*
km. = *kilometer.* *yd.* = *yard.*

1 centimeter	1 foot	1 inch	1 kilometer	1 knot
0.3937 in.	30.48 cm.	2.540 cm.	10^5 cm.	6080 ft.
0.01 m.	12 in.	0.02540 m.	3281 ft.	1.853 km.
393.7 mil	0.3048 m.	10^3 mil	1000 m.	1853 m.
10 mm.	304.8 mm.	25.40 mm.	0.6214 mi.	1.152 mi.
....	0.3333 yd.	....	1094 yd.	2027 yd.

1 meter	1 mil	1 mile	1 millimeter	1 yard
100 cm.	0.002540 cm.	1.609×10^5 cm.	0.1 cm.	91.44 cm.
3.281 ft.	0.001 in.	5280 ft.	0.03937 in.	3 ft.
39.37 in.	0.02540 mm.	1.609 km.	0.001 meter	36 in.
0.001 km.	....	1609 m.	39.37 mil.	0.9144 meter
1000 mm.	....	1760 yd.	....	....
1.094 yd.	....	....	....	....

* **Example.** — 7 yards equals how many meters? *Answer*, 1 yard = 0.9144 meters, therefore 7 yards = $7 \times 0.9144 = 6.4008$ meters.

AREA (L^2)

C.M. = *circular mil.*
hect. = *hectare.*
sq. cm. = *square centimeter.*
sq. ft. = *square foot.*
sq. in. = *square inch.*
sq. km. = *square kilometer.*
sq. m. = *square meter.*
sq. mil. = *square mil.*
sq. mi. = *square mile.*
sq. mm. = *square millimeter.*
sq. yd. = *square yard.*

1 acre	**1 are**	**1 circular mil ***	**1 hectare**
40.47 ares	0.02471 acre	5.067×10^{-6} sq. cm.	2.471 acres
0.4047 hect.	0.01 hect.	7.854×10^{-7} sq. in.	100 ares
43,560 sq. ft.	1076 sq. ft.	0.7854 sq. mil.	1.076×10^{5} sq. ft.
4.047×10^{-3} sq. km.	10^{-4} sq. km.	5.067×10^{-4} sq. mm.	0.01 sq. km.
4047 sq. m.	**100 sq. m.**	....	10^{4} sq. m.
1.562×10^{-3} sq. mi.	3.861×10^{-5} sq. mi.	....	3.861×10^{-3} sq. mi.
4840 sq. yd.	119.6 sq. yd.	....	1.196×10^{4} sq. yd.

1 sq. centimeter	**1 square foot**		**1 square inch**
1.973×10^{5} C.M.	2.296×10^{-5} acre	9.290×10^{-8} sq. km.	1.273×10^{6} C. M.
1.076×10^{-3} sq. ft.	9.290×10^{-4} are	0.09290 sq. m.	6.452 sq. cm.
0.1550 sq. in.	1.833×10^{8} C. M.	1.44×10^{8} sq. mil.	6.944×10^{-3} sq. ft.
1.550×10^{5} sq. mil.	9.290×10^{-6} hect.	3.587×10^{-8} sq. mi.	10^{6} sq. mil.
10^{-6} sq. m.	929.0 sq. cm.	9.290×10^{4} sq. mm.	645.2 sq. mm.
100 sq. mm.	**144 sq. in.**	0.1111 sq. yd.	....

1 square kilometer	**1 square meter**	**1 square mil †**
247.1 acres	2.471×10^{-4} acre	1.273 C. M.
10^{4} ares	0.01 are	6.452×10^{-6} sq. cm.
100 hect.	10^{-4} hect.	10^{-6} sq. in.
10.76×10^{6} sq. ft.	10.76 sq. ft.	6.452×10^{-4} sq. mm.
10^{6} sq. m.	10^{-6} sq. km.	....
0.3861 sq. mi.	3.861×10^{-7} sq. mi.	....
1.196×10^{6} sq. yd.	1.196 sq. yd.	....

1 square mile	**1 square millimeter**	**1 square yard**
640 acres	1.973×10^{3} C. M.	2.066×10^{-4} acre
2.590×10^{4} ares	**0.01 sq. cm.**	8.361×10^{-3} are
2.590×10^{2} hect.	1.076×10^{-5} sq. ft.	8.361×10^{-5} hect.
27.88×10^{6} sq. ft.	1.550×10^{-3} sq. in.	**9 sq. ft.**
2.590 sq. km.	1.550×10^{3} sq. mil.	8.361×10^{-7} sq. km.
2.59×10^{6} sq. m.	....	0.8361 sq. m.
3.098×10^{6} sq. yd.	....	3.228×10^{-7} sq. mi.

* A circular mil is the area of a circle 1 mil, or 0.001 in., in diameter.
† A square mil is the area of a square 1 mil, or 0.001 in., on each side.

VOLUME (L^3)

bu. = *bushel.*
cu. cm. = *cubic centimeter.*
cu. ft. = *cubic foot.*
cu. in. = *cubic inch.*
cu. m. = *cubic meter.*
cu. yd. = *cubic yard.*
gal. = *gallon.*
lit. = *liter.*
pt. = *pint.*
qt. = *quart.*

1 bushel (dry*)	1 cu. centimeter	1 cubic foot	1 cubic inch
1.244 cu. ft.	3.531×10^{-5} cu. ft.	2.832×10^{4} cu. cm.	16.39 cu. cm.
2150 cu. in.	6.102×10^{-2} cu. in.	**1728 cu. in.**	5.787×10^{-4} cu. ft.
0.03524 cu. m.	**10^{-6} cu. m.**	0.02832 cu. m.	1.639×10^{-5} cu. m.
4 pk. (dry)	1.308×10^{-6} cu. yd.	0.03704 cu. yd.	2.143×10^{-5} cu. yd.
64 pt. (dry)	2.642×10^{-4} gal.	7.481 gal.	4.329×10^{-3} gal.
32 qt. (dry)	10^{-3} lit.	28.32* lit.	1.639×10^{-2} lit.
....	2.113×10^{-3} pt.	59.84 pt.	0.03463 pt.
....	1.057×10^{-3} qt.	29.92 qt.	0.01732 qt.

1 cubic meter	1 cubic yard	1 gallon (liq. *)
10^6 cu. cm.	7.646×10^{5} cu. cm.	3785 cu. cm.
35.31 cu. ft.	**27 cu. ft.**	0.1337 cu. ft.
61,023 cu. in.	46,656 cu. in.	**231 cu. in.**
1.308 cu. yd.	0.7646 cu. m.	3.785×10^{-3} cu. m.
264.2 gal.	202.0 gal.	4.951×10^{-3} cu. yd.
10^3 lit.	764.6 lit.	3.785 lit.
2113 pt.	1616 pt.	8 pt.
1057 qt.	807.9 qt.	4 qt.

1 liter	1 pint (liq.*)	1 quart (liq. *)
10^3 cu. cm.	473.2 cu. cm.	946.4 cu. cm.
0.03531 cu. ft.	0.01671 cu. ft.	0.03342 cu. ft.
61.02 cu. in.	28.87 cu. in.	57.75 cu. in.
10^{-3} cu. m.	4.732×10^{-4} cu. m.	9.464×10^{-4} cu. m.
1.308×10^{-3} cu. yd.	6.189×10^{-4} cu. yd.	1.238×10^{-3} cu. yd.
0.2642 gal.	0.125 gal.	**0.25 gal.**
2.113 pt.	0.4732 lit.	0.9464 lit.
1.057 qt.	**0.5 qt.**	**2 pt.**

* Dry measure units = 1.164 × (liquid measure units). Quarts, pints, bushels and pecks as here used are United States measures; see Hering's *Conversion Tables* for English measures.

PLANE ANGLE (*Zero Dimensions*)

deg. or ° = *degree.* *rad.* = *radian.*
min. or ′ = *minute.* *rev.* = *revolution.*
quad. = *quadrant.* *sec.* or ″ = *second.*

1 degree	1 minute	1 quadrant
60 min.	2.909×10^{-4} rad.	90 deg.
0.01745 rad.	60 sec.	5400 min.
3600 sec.	....	1.571 rad.
....	....	324,000 sec.

1 radian *	1 revolution	1 second
57.30 deg.	360 deg.	4.848×10^{-6} rad.
3438 min.	21,600 min.	....
0.637 quad.	4 quad.	....
206,265 sec.	6.283 rad.	....
....	1.296×10^{6} sec.	....

* 2π radians = 360 degrees by definition.

SOLID ANGLE (*Zero dimensions*)

1 hemisphere	1 sphere *	1 sph. right angle	1 steradian †
0.5 sphere	2 hem. sp.	0.25 hem. sph.	0.1592 hem. sph.
4 sph. rt. ang.	8 sph. rt. ang.	0.125 sphere	0.07958 sphere
6.283 steradians	12.57 steradians	1.571 steradians	0.6366 sph. rt. ang.

* A sphere is the total solid angle about a point.
† 4π steradians = 1 sphere, by definition.

TIME (T)

hr. = *hour.* *mo.* = *month.* *wk.* = *week.*
min. = *minute.* *sec.* = *second.* *yr.* = *year.*

1 day	1 hour	1 week	1 aver. month	1 civil year	1 leap year
24 hr.	60 min.	7 days	1/12 civil yr.	365 days	366 days
1,440 min.	3600 sec.	168 hr.	30.42 days	8,760 hr.	8,784 hr.
86,400 sec.	....	10,080 min.	730 hr.	525,600 min.	527,040 min.
....	....	604,800 sec.	43,800 min.	12 mo.	31,622,400 sec.
....	....	....	2,628,000 sec.	31,536,000 sec.	52.27 wk.
....	....	....	....	52.14 wk.	

LINEAR VELOCITY AND SPEED (LT^{-1})

1 centimeter per sec.	1 foot per minute	1 foot per second
1.969 ft. per min.	0.5080 cm. per sec.	30.48 cm. per sec.
0.03281 ft. per sec.	0.01667 ft. per sec.	1.097 km. per hr.
0.036 km. per hr.	0.01829 km. per hr.	0.5921 knot per hr.
0.6 m. per min.	0.3048 m. per min.	18.29 m. per min.
0.02237 mile per hr.	0.01136 mile per hr.	0.6818 mile per hr.
3.728×10^{-4} mile per min.	....	0.01136 mile per min.

1 kilometer per hour	1 kilometer per minute	1 knot per hour
27.78 cm. per sec.	54.68 ft. per sec.	51.48 cm. per sec.
54.68 ft. per min.	32.38 knots per hr.	1.689 ft. per sec.
0.9113 ft. per sec.	16.67 m. per hr.	1.853 km. per hr.
0.5396 knot per hr.	37.28 miles per hr.	1.152 miles per hr.
16.67 m. per min.	0.6214 mile per min.	....
0.6214 mile per hr.	....	....

1 meter per minute	1 meter per second
1.667 cm. per sec.	196.8 ft. per min.
3.281 ft. per min.	3.281 ft. per sec.
0.05468 ft. per sec.	3.6 km. per hr.
0.06 km. per hr.	0.06 km. per min.
0.03728 mile per hr.	2.237 miles per hr.
....	0.03728 mile per min.

1 mile per hour	1 mile per minute
44.70 cm. per sec.	2682 cm. per sec.
88 ft. per min.	88 ft. per sec.
1.467 ft. per sec.	1.609 km. per min.
1.609 km. per hr.	0.8684 knot per min.
0.8684 knot per hr.	60 miles per hr.
26.82 m. per min.	....

ANGULAR SPEED (T^{-1})

1 deg. per sec.	1 rad. per sec.	1 rev. per min.	1 rev. per sec.
0.01745 rad. per sec.	57.30 deg. per sec.	6 deg. per sec.	360 deg. per sec.
0.1667 rev. per min.	0.1592 rev. per sec.	0.1047 rad. per sec.	6.283 rad. per sec.
0.002778 rev. per sec.	9.549 rev. per min.	0.01667 rev. per sec.	60 rev. per min.

LINEAR ACCELERATION (LT^{-2})

1 cm. per sec. per sec.	**1 foot per sec. per sec.**	**Gravity-g_0**
0.03281 ft. per sec. per sec. 0.001020 gravity 0.036 km. per hr. per sec. 0.01 m. per sec. per sec. 0.02237 mile per hr. per sec.	30.48 cm. per sec. per sec. 0.03108 gravity 1.097 km. per hr. per sec. 0.3048 m. per sec. per sec. 0.6818 mile per hr. per sec.	980.7 cm. per sec. per sec. 32.17 ft. per sec. per sec. 35.30 km. per hr. per sec. 9.807 m. per sec. per sec. 21.94 miles per hr. per sec.
1 kilometer per hour per sec.	**1 meter per sec. per sec.**	**1 mile per hour per sec.**
27.78 cm. per sec. per sec. 0.9113 ft. per sec. per sec. 0.02833 gravity 0.2778 m. per sec. per sec. 0.6214 mile per hr. per sec.	100 cm. per sec. per sec. 3.281 ft. per sec. per sec. 0.1020 gravity 3.6 km. per hr. per sec. 2.237 miles per hr. per sec.	44.70 cm. per sec. per sec. 1.467 ft. per sec. per sec. 0.04559 gravity 1.609 km. per hr. per sec. 0.4470 m. per sec. per sec.

ANGULAR ACCELERATION (T^{-2})

rad. = *radian* *rev.* = *revolution*

1 rad. per sec. per sec.	**1 rev. per min. per min.**
573.0 rev. per min. per min. 9.549 rev. per min. per sec. 0.1592 rev. per sec. per sec.	1.745×10^{-3} rad. per sec. per sec. 0.01667 rev. per min. per sec. 2.778×10^{-4} rev. per sec. per sec.
1 rev. per min. per sec.	**1 rev. per sec. per sec.**
0.1047 rad. per sec. per sec. 60 rev. per min. per min. 0.01667 rev. per sec. per sec.	6.283 rad. per sec. per sec. 3600 rev. per min. per min. 60 rev. per min. per sec.

MASS (M) AND WEIGHT *

kg. = *kilogram.* *oz.* = *ounce.*
mg. = *milligram* *lb.* = *pound* †

1 grain †	1 gram	1 kilogram	
0.06480 gram	15.43 grains	15,432 grains	2.205 lb.
64.80 mg.	10^{-3} kg.	10^3 grams	9.842×10^{-4} long ton
2.286×10^{-3} oz.	10^3 mg.	10^6 mg.	10^{-3} metric ton
....	0.03527 oz.	35.27 oz.	1.102×10^{-3} short ton
....	2.205×10^{-3} lb.	....	....

1 milligram	1 ounce †	1 pound †
0.01543 grain	437.5 grains	7000 grains
10^{-3} gram	28.35 grams	453.6 grams
10^{-6} kg.	0.02835 kg.	0.4536 kg.
....	28,350 mg.	4.536×10^5 mg.
....	0.06250 lb.	16 oz.

1 long ton	1 metric ton	1 ton (short)
1016 kg.	10^3 kg.	907.2 kg.
2240 lb.	2205 lb.	2000 lb.
1.016 metric tons	0.9842 long ton	0.8929 long ton
1.120 short tons	1.102 short tons	0.9072 metric ton

* These same conversion factors apply to the *gravitational* units of force having the corresponding names. The dimensions of these units when used as gravitational units of force are MLT^{-2}; see table for *Force* on next page.
† Avoirdupois pound and its subdivisions used throughout.

DENSITY OR MASS PER UNIT VOLUME (ML^{-3})

1 gram per cu. cm.	1 kg. per cu. meter
10^3 kg. per cu. m.	10^{-3} g. per cu. cm.
62.43 lb. per cu. ft.	0.06243 lb. per cu. ft.
0.03613 lb. per cu. in.	3.613×10^{-5} lb. per cu. in.
3.405×10^{-7} lb. per mil ft.	3.405×10^{-10} lb. per mil ft.

1 pound per cu. ft.	1 pound per cu. in.
0.01602 g. per cu. cm.	27.68 g. per cu. cm.
16.02 kg. per cu. m.	2.768×10^4 kg. per cu. m.
5.787×10^{-4} lb. per cu. in.	1728 lb. per cu. ft.
5.456×10^{-9} lb. per mil ft.	9.425×10^{-6} lb. per mil ft.

FORCE (MLT^{-2})

kg. = *kilogram.* *lb.* = *pound.*

1 dyne *	1 gram	1 kilogram
1.020×10^{-3} gram 1.020×10^{-6} kg. 2.248×10^{-6} lb. 7.233×10^{-5} poundal	980.7 dynes 10^{-3} kg. 2.205×10^{-3} lb. 0.07093 poundal	980,665 dynes 10^3 grams 2.205 lb. 70.93 poundals

1 pound	1 poundal †
444,823 dynes 453.6 grams 0.4536 kg. 32.1739 poundals	13,826 dynes 14.10 grams 0.01410 kg. 0.03108 lb.

* Force required to give a mass of 1 gram an acceleration of 1 cm. per sec. per sec.
† Force required to give a mass of 1 pound an acceleration of 1 ft. per sec. per sec.

TORQUE OR MOMENT OF FORCE (L^2MT^{-2}) *

cm.-dyne = *centimeter-dyne.* *m.-kg.* = *meter-kilogram.*
cm.-gram = *centimeter-gram.* *lb.-ft.* = *pound-foot.*

1 centimeter-dyne	1 centimeter-gram
1.020×10^{-3} cm-gram 1.020×10^{-8} m-kg. 7.376×10^{-8} lb-ft.	980.7 cm-dynes 10^{-5} m-kg. 7.233×10^{-5} lb-ft.

1 meter-kilogram	1 pound-foot
9.807×10^7 cm-dynes 10^5 cm-grams 7.233 lb-ft.	1.356×10^7 cm-dynes 13,825 cm-grams 0.1383 m-kg.

* Same dimensions as energy; see below.

PRESSURE OR FORCE PER UNIT AREA ($L^{-1}MT^{-2}$)

barie = *dyne per sq. cm.*
mercury = *column of mercury.*
ton = *2000 lb.*
water = *column of water.*

1 atmosphere	1 barie or dyne per sq. cm.	1 cm. of mercury
76.0 cm. mercury	9.870×10^{-7} atmosphere	0.01316 atmosphere
29.92 in. mercury	0.01020 kg. per sq. m.	0.4461 ft. water
33.90 ft. water	2.089×10^{-3} lb. per sq. ft.	136.0 kg. per sq. m.
10,333 kg. per sq. m.	1.450×10^{-5} lb. per sq. in.	27.85 lb. per sq. ft.
14.70 lb. per sq. in.	....	0.1934 lb. per sq. in.
1.058 tons per sq. ft.	....	....

1 inch of mercury	1 inch of water	1 foot of water
0.03342 atmosphere	0.002458 atmosphere	0.02950 atmosphere
1.133 ft. water	0.07355 in. mercury	0.8826 in. mercury
345.3 kg. per sq. m.	25.40 kg. per sq. m.	304.8 kg. per sq. m.
70.73 lb. per sq. ft.	5.204 lb. per sq. ft.	62.43 lb. per sq. ft.
0.4912 lb. per sq. in.	0.03613 lb. per sq. in.	0.4335 lb. per sq. in.

1 kilogram per square meter	1 kilogram per sq. mm.	1 pound per square foot
9.678×10^{-5} atmosphere	96.78 atmospheres	4.725×10^{-4} atmosphere
98.06 baries	98.06×10^{6} baries	478.8 baries
3.281×10^{-3} ft. water	3.281×10^{3} ft. water	0.01602 ft. water
2.896×10^{-3} in. mercury	2.048×10^{5} lb. per sq. ft.	0.01414 in. mercury
0.2048 lb. per sq. ft.	1.422×10^{3} lb. per sq. in.	4.882 kg. per sq. m.
1.422×10^{-3} lb. per sq. in.	....	6.944×10^{-3} lb. per sq. in.

1 pound per sq. inch	1 short ton per sq. ft.	1 short ton per sq. in.
0.06804 atmosphere	0.9450 atmosphere	136.1 atmospheres
2.307 ft. water	32.04 ft. water	4613 ft. water
2.036 in. mercury	28.28 in. mercury	1.406×10^{6} kg. per sq. m.
703.1 kg. per sq. m.	9765 kg. per sq. m.	2.88×10^{5} lb. per sq. ft.
144 lb. per sq. ft.	2000 lb. per sq. ft.	2000 lb. per sq. in.
....	13.89 lb. per sq. in.	....

ENERGY AND WORK (L^2MT^{-2})*

B.t.u.	= *British thermal unit.*	*hp-hr.*	= *horse-power-hour.*
ft-lb.	= *foot-pound.*	*kg-m.*	= *kilogram-meter.*
kg-cal.	= *kilogram-calorie.*	*kw-hr.*	= *kilowatt-hour.*
gram-cal.	= *gram-calorie.*	*watt-hr.*	= *watt-hour.*
gram-cm.	= *gram-centimeter.*		

1 British thermal unit	1 erg or dyne-centimeter	1 foot-pound
0.2520 kg-cal.	9.486×10^{-11} B.t.u.	1.286×10^{-3} B.t.u.
777.5 ft-lb.	2.390×10^{-11} kg-cal.	3.241×10^{-4} kg-cal.
3.927×10^{-4} hp-hr.	7.376×10^{-8} ft-lb.	1.356×10^{7} ergs
1054 joules	1.020×10^{-3} gram-cm.	5.050×10^{-7} hp-hr.
107.5 kg-m.	10^{-7} joule	1.356 joules
2.928×10^{-4} kw-hr.	1.020×10^{-8} kg-m.	0.1383 kg-m.
....	....	3.766×10^{-7} kw-hr.

1 kilogram-calorie †	1 gram-centimeter	1 horse-power-hour
3.968 B.t.u.	9.302×10^{-8} B.t.u.	2547 B.t.u.
3086 ft-lb.	2.344×10^{-8} kg-cal.	641.7 kg-cal.
1.558×10^{-3} hp-hr.	980.7 ergs	1.98×10^{6} ft-lb.
4183 joules	7.233×10^{-5} ft-lb.	2.684×10^{6} joules
426.6 kg-m.	9.807×10^{-5} joule	2.737×10^{5} kg-m.
1.162×10^{-3} kw-hr.	10^{-5} kg-m.	0.7457 kw-hr.

1 joule or watt-second	1 kilogram-meter
9.486×10^{-4} B.t.u.	9.302×10^{-3} B.t.u.
2.390×10^{-4} kg-cal.	2.344×10^{-3} kg-cal.
10^{7} ergs	9.807×10^{7} ergs
0.7376 ft-lb.	7.233 ft-lb.
0.1020 kg-m.	9.807 joules
2.778×10^{-4} watt-hr.	2.724×10^{-6} kw-hr.

1 kilowatt-hour	1 watt-hour
3415 B.t.u.	3.415 B.t.u.
860.5 kg-cal.	0.8605 kg-cal.
2.655×10^{6} ft-lb.	2655 ft-lb.
1.341 hp-hr.	1.341×10^{-3} hp-hr.
3.6×10^{6} joules	367.1 kg-m.
3.671×10^{5} kg-m.	10^{-3} kw-hr.

* See note at bottom of next table.

† 1 gram-calorie = 10^{-3} kilogram-calorie; 1 Ostwald calorie = 10^{-2} kilogram-calorie.

POWER OR RATE OF DOING WORK (L^2MT^{-3})*

B.t.u. = British thermal unit; met. h.p. = metric horse-power.

1 B.t.u. per minute	1 erg per second	1 foot-pound per minute
777.5 ft-lb. per min.	5.692×10^{-9} B.t.u. per min.	1.286×10^{-3} B.t.u. per min.
12.96 ft-lb. per sec.	1.434×10^{-9} kg-cal. per min.	3.241×10^{-4} kg-cal. per min.
0.02356 h.p.	4.426×10^{-6} ft-lb. per min.	0.01667 ft-lb. per sec.
0.01757 kw.	7.376×10^{-8} ft-lb. per sec.	3.030×10^{-5} h.p.
0.02389 met. h.p.	1.341×10^{-10} h.p.	2.260×10^{-5} kw.
17.57 watts	10^{-10} kw.	3.072×10^{-5} met h.p.
....	1.360×10^{-10} met. h.p.	....

1 foot-pound per second	1 horse-power	1 kg-cal. per minute
7.717×10^{-2} B.t.u. per min.	42.44 B.t.u. per min.	3086 ft-lb. per min.
1.945×10^{-2} kg-cal. per min.	10.70 kg-cal. per min.	51.43 ft-lb. per sec.
1.818×10^{-3} h.p.	33,000 ft-lb. per min.	0.09351 h.p.
1.356×10^{-3} kw.	550 ft-lb. per sec.	0.06972 kw.
1.843×10^{-3} met. h.p.	0.7457 kw.	0.09481 met. h.p.
1.356 watts	1.014 met. h.p.	69.72 watts
....	*745.7 watts	....

1 kilowatt	1 metric horse-power	1 watt
56.92 B.t.u. per min.	41.86 B.t.u. per min.	0.05692 B.t.u. per min.
14.34 kg-cal. per min.	10.55 kg-cal. per min.	0.01434 kg-cal. per min.
4.425×10^4 ft-lb. per min.	3.255×10^4 ft-lb. per min.	10^7 erg per sec.
737.6 ft-lb. per sec.	542.5 ft-lb. per sec.	44.26 ft-lb. per min.
1.341 h.p.	0.9863 h.p.	0.7376 ft-lb. per sec.
1.360 met. h.p.	0.7354 kw.	1.341×10^{-3} h.p.
10^3 watts	735.4 watts	10^{-3} kw.
....	....	1.360×10^{-3} met. h.p.

* The value 746 watts, has been adopted by the Bureau of Standards as the exact equivalent of one horse-power; this, however, is not consistent with the use of 980.665 cm. per sec. per sec. as the standard value of g, which latter is used throughout these tables.

QUANTITY OF ELECTRICITY; DIELECTRIC FLUX (Q)

Abcoul. = abcoulomb; coul. = coulomb; statcoul. = statcoulomb.

1 abcoulomb	1 coulomb	1 statcoulomb
10 coul.	1/10 abcoul.	$\frac{1}{3}\times10^{-10}$ abcoul.
3×10^{10} statcoul.	3×10^9 statcoul.	$\frac{1}{3}\times10^{-9}$ coul.

CHARGE PER UNIT AREA; DIELECTRIC FLUX DENSITY (QL^{-2})

1 abcoulomb per square centimeter	1 coulomb per square centimeter
10 coul. per sq. cm. 64.52 coul. per sq. in. 3×10^{10} statcoul. per sq. cm.	$1/10$ abcoul. per sq. cm. 6.452 coul. per sq. in. 3×10^{9} statcoul. per sq. cm.
1 coulomb per square inch	**1 statcoulomb per square centimeter**
0.01550 abcoul. per sq. cm. 0.1550 coul. per sq. cm. 4.650×10^{8} statcoul. per sq. cm.	$1/3\times10^{-10}$ abcoul. per sq. cm. $1/3\times10^{-9}$ coul. per sq. cm. 2.150×10^{-9} coul. per sq. in.

ELECTRIC CURRENT (QT^{-1})

1 abampere	1 ampere	1 statampere
10 amperes 3×10^{10} statamp.	$1/10$ abamp. 3×10^{9} statamp.	$1/3\times10^{-10}$ abamp. $1/3\times10^{-9}$ ampere

CURRENT DENSITY ($QT^{-1}L^{-2}$)

1 abampere per square centimeter	1 ampere per square centimeter
10 amperes per sq. cm. 64.52 amperes per sq. in. 3×10^{10} statamp. per sq. cm.	$1/10$ abampere per sq. cm. 6.452 amperes per sq. in. 3×10^{9} statamp. per sq. cm.
1 ampere per square inch	**1 statamp. per square centimeter**
0.01550 abamp. per sq. cm. 0.1550 ampere per sq. cm. 4.650×10^{8} statamp. per sq. cm.	$1/3\times10^{-10}$ abamp. per sq. cm. $1/3\times10^{-9}$ ampere per sq. cm. 2.150×10^{-9} ampere per sq. in.

ELECTRIC POTENTIAL DIFFERENCE; ELECTROMOTIVE FORCE (V)

1 abvolt	1 statvolt	1 volt
$1/3\times10^{-10}$ statvolt 10^{-8} volt	3×10^{10} abvolts 300 volts	10^{8} abvolts $1/300$ statvolt

ELECTRIC POTENTIAL GRADIENT; ELECTROSTATIC FIELD INTENSITY (VL^{-1})

1 abvolt per centimeter	1 statvolt per centimeter
$\frac{1}{3}\times 10^{-10}$ statvolt per cm. 10^{-8} volt per cm. 2.540×10^{-8} volt per in.	3×10^{10} abvolts per cm. 300 volts per cm. 762.0 volts per in.
1 volt per centimeter	**1 volt per inch**
10^{8} abvolts per cm. $\frac{1}{300}$ statvolt per cm. 2.540 volts per in.	3.937×10^{7} abvolts per cm. 1.312×10^{-3} statvolt per cm. 0.3937 volt per cm.

ELECTRIC RESISTANCE ($Q^{-1}VT$)

1 abohm	1 megohm	1 microhm
10^{-15} megohm 10^{-3} microhm 10^{-9} ohm $\frac{1}{9}\times 10^{-20}$ statohm	10^{15} abohms 10^{12} microhms 10^{6} ohms $\frac{1}{9}\times 10^{-5}$ statohm	10^{3} abohms 10^{-12} megohm 10^{-6} ohm $\frac{1}{9}\times 10^{-17}$ statohm

1 ohm	1 statohm
10^{9} abohms 10^{-6} megohm 10^{6} microhms $\frac{1}{9}\times 10^{-11}$ statohm	9×10^{20} abohms 9×10^{5} megohms 9×10^{17} microhms 9×10^{11} ohms

ELECTRIC RESISTIVITY † ($Q^{-1}VTL$)

See also *Resistance and Conductance*

1 abohm per cm. cube	1 microhm per cm. cube	1 microhm per in. cube
10^{-3} microhm per cm. cube	10^3 abohms per cm. cube	2.540×10^3 abohms per cm. cube
3.937×10^{-4} microhm per in. cube	0.3937 microhm per in. cube	2.540 microhms per cm. cube
6.015×10^{-3} ohm per mil-ft.	6.015 ohms per mil-ft.	15.28 ohms per mil-ft.
$10^{-5}\,\delta$ ohm per meter-gram	$10^{-2}\,\delta$ ohm per meter-gram	$2.540 \times 10^{-2}\,\delta$ ohm per meter-gram

1 ohm per mil-foot	1 ohm per meter-gram *
1.662×10^2 abohms per cm. cube	$\frac{10^5}{\delta}$ abohms per cm. cube
0.1662 microhm per cm. cube	$\frac{10^2}{\delta}$ microhms per cm. cube
0.06524 microhm per in. cube	$\frac{39.37}{\delta}$ microhms per in. cube
$1.662 \times 10^{-3}\,\delta$ ohm per meter-gram	$\frac{6.015 \times 10^2}{\delta}$ ohms per mil-ft.

* See *p. 1755 for multiples and submultiples.*

† In this table δ is density expressed as a decimal fraction.

ELECTRIC CONDUCTIVITY † ($QV^{-1}T^{-1}L^{-1}$)

1 abmho per cm. cube	1 mho per meter-gram	1 mho per mil-foot
$\frac{10^5}{\delta}$ mhos per meter-gram	$10^{-5}\,\delta$ abmho per cm. cube	6.015×10^{-9} abmho per cm. cube
1.662×10^8 mhos per mil-ft.	$1.662 \times 10^{-3}\delta$ mho per mil-foot	$\frac{601.5}{\delta}$ mhos per meter-gram
10^3 megmhos per cm. cube	$10^{-2}\delta$ megmho per cm. cube	6.015 megmhos per cm. cube
2.540×10^3 megmhos per in. cube	$2.540 \times 10^{-2}\delta$ megmho per in. cube	15.28 megmhos per in. cube

1 megmho per cm. cube	1 megmho per in. cube
10^{-3} abmho per cm. cube	3.937×10^{-4} abmho per cm. cube
$\frac{10^2}{\delta}$ mhos per meter-gram	$\frac{39.37}{\delta}$ mhos per meter-gram
0.1662 mho per mil-foot	0.06524 mho per mil-foot
2.540 megmhos per in. cube	0.3937 megmho per cm. cube

† In this table δ is density expressed as a decimal fraction.

CAPACITY (QV^{-1})

Abf. = *abfarad*; *mf.* = *microfarad*; *statf.* = *statfarad*

1 abfarad	1 farad	1 microfarad	1 statfarad
10^9 farads 10^{15} mf. 9×10^{20} statf.	10^{-9} abf. 10^6 mf. 9×10^{11} statf.	10^{-15} abf. 10^{-6} farad 9×10^5 statf.	$\frac{1}{9}\times10^{-20}$ abf. $\frac{1}{9}\times10^{-11}$ farad. $\frac{1}{9}\times10^{-5}$ mf.

INDUCTANCE $(VQ^{-1}T^2)$

Abh. = *abhenry*; *mh.* = *millihenry*; *stath.* = *stathenry*

1 abhenry	1 henry	1 millihenry	1 stathenry
10^{-9} henry 10^{-6} mh. $\frac{1}{9}\times10^{-20}$ stath.	10^9 abh. 10^3 mh. $\frac{1}{9}\times10^{-11}$ stath.	10^6 abh. 10^{-3} henry $\frac{1}{9}\times10^{-14}$ stath.	9×10^{20} abh. 9×10^{11} henries 9×10^{14} mh.

MAGNETIC FLUX (VT)

1 maxwell or "line"*	1 kiloline	1 volt-second*
10^{-3} kiloline * 10^{-8} volt-sec.*	10^3 maxwells 10^{-5} volt-sec.*	10^8 maxwells 10^5 kilolines

* Or abvolt second.

MAGNETIC FLUX DENSITY (VTL^{-2})

1 gauss, or line per square centimeter	1 line per square inch
6.452 lines per sq. in. 10^{-8} volt-sec.* per sq. cm. 6.452×10^{-8} volt-sec.* per sq. in.	0.1550 gauss 1.55×10^{-9} volt-sec.* per sq. cm. 10^{-8} volt-sec.* per sq. in.
1 volt-sec. per square centimeter	**1 volt-sec. per square inch**
10^8 gausses 6.452×10^8 lines per sq. in. 6.452 volt-sec.* per sq. in.	1.550×10^7 gausses 10^8 lines per sq. in. 0.1550 volt-sec.* per sq. cm.

* By volt-second is meant the unit of flux which must be used in the equation $e = \frac{d\phi}{dt}$ in order to obtain e in volts when t is in seconds; this unit is sometimes called a "weber."

MAGNETIC POTENTIAL DIFFERENCE; MAGNETOMOTIVE FORCE
(QT^{-1})

Abamp-turns = abampere-turns; amp-turns = ampere-turns; statamp-turns = statampere-turns

1 abampere-turn	1 ampere-turn	1 gilbert
10 amp-turns 12.57 gilberts	1/10 abamp-turn 1.257 gilberts	0.07958 abamp-turn 0.7958 amp-turn

MAGNETIC POTENTIAL GRADIENT; MAGNETIZING FORCE $(QL^{-1}T^{-1})$

1 abamp-turn per cm.	1 ampere-turn per cm.
10 amp-turns per cm. 25.40 amp-turns per inch 12.57 gilberts per cm.	1/10 abamp-turn per cm. 2.540 amp-turns per inch 1.257 gilberts per cm.
1 amp-turn per inch	**1 gilbert per cm.**
0.03937 abamp-turn per cm. 0.3937 amp-turn per cm. 0.4950 gilbert per cm.	0.07958 abamp-turn per cm. 0.7958 amp-turn per cm. 2.021 amp-turns per inch

BIBLIOGRAPHY. — A more complete set of conversion factors is given in a book by Hering called *Conversion Tables*, N. Y., 1904. See also circular of the Bureau of Standards entitled *Tables of Equivalents*, 1913.

[H. PENDER and R. G. HUDSON.]

UNITS, PRACTICAL ELECTRIC. — (*See also Electricity and Magnetism, Principles of; Units and Conversion Factors.*) Following the recommendation of the Chicago International Electrical Congress of 1893, the following Act was passed by Congress. (Act approved July 12, 1894.) *Be it enacted, etc.* That from and after the passage of this act the legal units of electrical measure in the United States shall be as follows:

First. The unit of resistance shall be what is known as the international ohm, which is substantially equal to one thousand million units of resistance of the centimeter-gram-second system of electromagnetic units, and is represented by the resistance offered to an unvarying electric current by a column of mercury at the temperature of melting ice fourteen and four thousand five hundred and twenty-one ten-thousandths (14.4521) grams in mass, of a constant cross-sectional area, and of the length of one hundred and six and three-tenths (106.3) centimeters.

Second. The unit of current shall be what is known as the international ampere, which is one-tenth of the unit of current of the centimeter-gram-second system of electromagnetic units, and is the practical equivalent of the unvarying current, which, when passed through a solution of nitrate of silver in water in accordance with standard specifications, deposits silver at the rate of one thousand one hundred and eighteen millionths (0.001118) of a gram per second.

Third. The unit of electromotive force shall be what is known as the international volt, which is the electromotive force that, steadily applied to a conductor whose resistance is one international ohm, will produce a current of an international ampere, and is practically equivalent to $\frac{1000^*}{1434}$ of the electromotive force between the poles or electrodes of the voltaic cell known as Clark's cell, at a temperature of fifteen degrees centigrade (15° C.), and prepared in the manner described in the standard specifications.

Fourth. The unit of quantity shall be what is known as the international coulomb, which is the quantity of electricity transferred by a current of one international ampere in one second.

Fifth. The unit of capacity shall be what is known as the international farad, which is the capacity of a condenser charged to a potential of one international volt by one international coulomb of electricity.

Sixth. The unit of work shall be the joule, which is equal to ten million units of work in the centimeter-gram-second system, and which is practically equivalent to the energy expended in one second by an international ampere in an international ohm.

Seventh. The unit of power shall be the watt, which is equal to ten million units of power in the centimeter-gram-second system, and which is practically equivalent to the work done at the rate of one joule per second.

Eighth. The unit of induction shall be the henry, which is the induction in a circuit when the electromotive force induced in this circuit is one international volt while the inducing current varies at the rate of one ampere per second.

SECTION 2. That it shall be the duty of the National Academy of Sciences to prescribe and publish, as soon as possible after the passage of this act, such specifications of detail as shall be necessary for the practical application of the definitions of the ampere and volt hereinbefore given, and such specifications shall be the standard specifications herein mentioned.

NOTES ON THE ABOVE DEFINITIONS. — The specifications mentioned above were prepared by a committee, and their report, based on the best

* Later experiments give an e.m.f. of 1.4328 volts at 15° C.

work that had been done up to that time, was accepted and adopted by the National Academy of Sciences on February 9, 1895.

Attention is called to the wording of the definitions of the three fundamental quantities:

1. The international ohm is based upon, but *not stated to be equal* to, 10^9 c.g.s. units of resistance; it is, however, *definitely defined* in terms of a column of mercury.

2. The international ampere *is* one-tenth of the c.g.s. unit of current, and is stated to be represented *sufficiently well for practical purposes* by the current which deposits a definite mass of silver per second under specified conditions.

3. The international volt is the e.m.f. which will cause an international ampere to flow through an international ohm, and is stated to be represented *sufficiently well for practical purposes* by a definite fractional part of the e.m.f. of the Clark cell.

Subsequent investigation has shown that the figures for the silver voltameter and the Clark cell are not equivalent. The e.m.f. of the Clark cell is really 1.4328 international volts at 15°.

PRACTICAL STANDARDS. — The Act of July 12, 1894, is still in force but advantage has been taken of the manner of stating how the international volt shall be practically realized to use as a standard of e.m.f. the Weston normal cell instead of the Clark cell (*see Cells, Standard*).

Standard resistance coils (*see Resistors, Standard*) are used as secondary standards of resistance. These are usually accurately adjusted by the makers. When great accuracy is required these standard resistances should be sent to the Bureau of Standards for calibration. Much more accurate results can be obtained in an ordinary laboratory by using properly calibrated secondary standards than by attempting to construct primary standards in accordance with the definitions in the above Act.

The use of a voltameter for the calibration of current-measuring instruments is also now seldom employed. A more accurate method for ordinary laboratory work is to determine, by means of a properly calibrated potentiometer (q.v.), the voltage drop produced by the current through a standard resistance.

BIBLIOGRAPHY. — *The so-called International Electric Units*, Bull. Bur. Stand., 1904, Vol. 1, p. 30; *Selection and Definition of the Fundamental Electrical Units*, Bull. Bur. Stand., 1908, Vol. 5, p. 243; *Announcement of Change in Value of the International Volt*, Circ. No. 29, Bureau of Standards.

[H. PENDER.]

UNLOADERS, COAL AND ORE. — (*See also Cranes; Motors, Industrial Applications of; Power Stations.*) Coal and ore unloaders may be divided into two distinct classes, namely, the stiff-leg type and the bridge type. Both are adapted to electric drive and each possesses certain advantages under the conditions for which it is designed. Unloaders range in capacity from three to fifteen tons per bucket load. The capacity in tons per hour varies within wide limits, but records have been established where under normal operating conditions four 15-ton equipments unloaded 10,000 tons in five hours. The power required naturally also varies under different conditions, but an average of from 0.4 to 0.5 kilowatt hour per ton is not uncommon.

STIFF-LEG UNLOADERS (Fig. 1) are mainly used at receiving docks for unloading ore from the boat to railway cars or to a large concrete trough from which it is in turn transferred to the storage yards by means of ore bridges. The machine, as seen from Fig. 1, consists essentially of a massive pantograph, the short leg of which forms an integral part of the carriage *C*. This leg is rigidly vertical, hence the stiff leg *L* which carries the bucket *B* must always be vertical. The weight of the moving parts of the link motion is nearly counterbalanced at *W* by the main hoist, rotation and bucket motors and their respective drums. The motor house on the carriage *C* contains the trolley motor and the magnetic control panels for all the above motors. The carriage is mounted on trucks and moves back and forth on the girder runway *G*. The girder structure supports a hopper with rotating gates which receives the ore from the bucket *B* and distributes it slowly to the weighing larry which in turn deposits it in the ore car underneath or in the concrete trough *I*. Cables from the main hoist drum at *W* are attached to the rear of the carriage. The operator rides in the bucket leg and has absolute control of lowering, opening, closing, bucket rotation, hoisting and trolleying movements. The entire girder structure is mounted on trucks and can be run along the dock at will by means of the bridge movement motor. Magnetic control is employed throughout and all motors except leg-rotation and bridge-movement motors are arranged for dynamic braking.

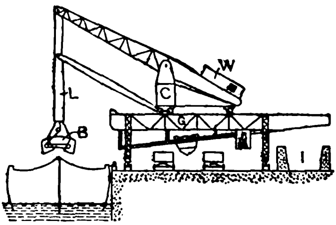

Fig. 1. Stiff-leg Unloader

Cycles of Motor Movements. — The service of the main hoist and opening and closing motor is intermittent with continuous repetition of cycle often at the rate of one round trip per minute for several consecutive hours. The leg rotation motor is used chiefly in cleaning up the bottom of the boat between hatches and involves rapid and frequent reversals at irregular intervals. The trolley-motor cycle includes acceleration, free running, retardation, reversal and repeat with every bucket load hoisted. The ore-gate motor operates intermittently whenever ore is distributed to the weighing larry. The larry travel and hopper-gate motors both operate on a continuously repeated cycle of short duration. The bridge-movement motor is used only at irregular intervals for locating the bucket over the hatches.

Motor Equipment. — The following table shows the motor equipment of a modern unloader of the stiff-leg type. The main-hoist motor has a continuous rating, but all the others are rated on an intermittent basis.

MOTOR EQUIPMENT, 15-TON STIFF-LEG UNLOADER

No. of motors	H.P.	R.p.m.	Volts, d-c.	Application	Type of motor
1	135	350	230	Main hoist	Compound wound
1	75	500	230	Trolley	Compound wound
1	75	500	230	Bucket { Opening Closing	Compound wound
1	100	450	230	Ore gates	Compound wound
1	100	450	230	Larry	Compound wound
1	100	450	230	Bridge movement	Compound wound
1	20	750	230	Leg rotation	Series wound
1	30	750	230	Larry hopper	Series wound

BRIDGE-TYPE UNLOADERS (Figs. 2 and 3) can be divided into two general classes, viz., the "man-trolley" and the "rope-trolley." In the former all hoisting and conveying motors together with a cab containing the control equipment and operator are mounted on a carriage which runs the length of the bridge, as shown in Fig. 2. In the rope-trolley unloader the motors are located in a stationary motor house and power is transmitted to the hoisting drum and carriage by means of cables, as shown in Fig. 3. The operator is also located in a stationary cab as seen.

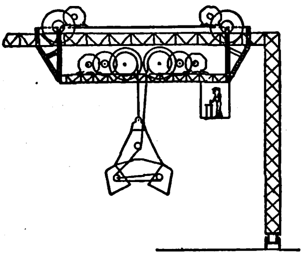

Fig. 2. "Man-trolley" Bridge Unloader

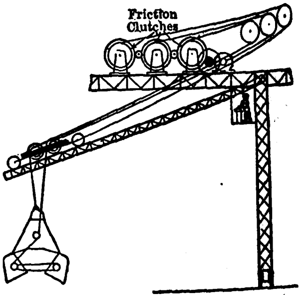

Fig. 3. "Rope-trolley" Bridge Unloader

MOTOR EQUIPMENT, 625-FOOT "MAN-TROLLEY" BRIDGE UNLOADER

No. of motors	H.P.	R.p.m	Volts	Application	Type of motor
4	125	450	230	Main hoist	Series wound
4	50	500	230	Main trolley	Series wound
4	40	460	230	Bridge movement	Series wound

MOTOR EQUIPMENT, 7-TON ROPE-OPERATED BRIDGE-TYPE UNLOADER

No. of motors	H.P.	R.p.m.	Volts	Application	Type of motor
2	55	430	250	Opening lines	Series wound
3	55	430	250	Closing and hoisting	Series wound
1	25	430	250	Travel	Series wound
1	11	...	250	Bucket twist	Series wound
1	15	...	250	Car-loading drums	Shunt wound

TYPE OF MOTOR.—The selection of the most suitable type of motor for coal and ore bridge work must be determined for each specific installation by local considerations. In general the requirements are not quite as severe as those commonly included under the term "steel mill service," and standard industrial motors in a few instances have been used for high-speed bridges with entire success. As a rule, however, especially for ore bridges, the mill-type motor is recommended, since even under the best conditions the short cycle, rapid and frequent acceleration, severe vibration, dirt and moisture, all demand the most rugged construction.

The question of alternating- or direct-current motors is usually one of comparative first costs, costs of maintenance and operation. Where adjustable speeds under variable load are required the induction motor with its constant-speed characteristics is somewhat at a disadvantage if compared with the direct-current motor equipped with interpoles and best available control. For dynamic braking the direct-current motor also offers fewer complications in the control. Furthermore, for the rack movement or trolley travel the acceleration losses are less for the series direct-current motor. On the other hand, the efficiency of transformation and distribution of power is considerably higher for alternating- than for direct-current systems so that the apparent gain due to the characteristic of the series motor is more than offset by the simplicity and ruggedness of the induction motor, the elimination of motor-generator sets, equalizer sets and in some cases even of static transformers.

BIBLIOGRAPHY. — Stephan, W. G., *Ore Handling at Lower Lake Ports,* Ir. Tr. Rev., 1911, Vol. 40, p. 641; Ryerson & Crane, *Notes on the Use of Alternating Current in Unloading Coal,* A.I.E.E., 1912, Vol. 31, p. 231; *Modern Iron Ore Dock and Unloading Plant,* Ir. Tr. Rev., 1912, Vol. 50, p. 845; McLain, R. N., *Electrically Operated Coal Dock,* G. E. Rev., 1911, Vol. 14, p. 523.

[D. B. Rushmore, assisted by E. A. Lof.]

VALVES. — (*See also Boilers; Gas Engines; Pipes; Power Stations; Steam Engines; Steam Turbines.*) The following types of valves are used to control the flow of fluids in pipes.

Gate Valve. — The opening in this valve is perpendicular to the axis of the pipe, and is closed by a disk which moves across the opening. A gate valve offers but little resistance to the flow of the fluid, since when fully opened it provides a straight passage the full diameter of the pipe (*see Pipe and Piping*).

Globe Valve. — In this valve the opening is in a partition parallel to and passing through the axis of the pipe. The opening is closed by a mushroom-shaped disk which is screwed down against the partition. The name "globe" valve arises from the globular form of the casing. The passage through a globe valve has the general shape of the letter S, and consequently offers considerable resistance to the flow (*see Pipe and Piping*). It is simpler in construction and cheaper than a gate valve, and the contact surfaces are more easily ground.

Angle Valve. — This valve has the inlet and outlet at right angles to each other; it may therefore be installed in place of an elbow. The opening is at right angles to the inlet. The closing mechanism may be either of the gate or of the globe type.

Cocks. — This type of valve consists of a conical plug with a hole through it perpendicular to its axis, mounted in a suitable casing. The valve is opened by turning the plug so that the hole is in line with the pipe and is closed by giving the plug a quarter turn. Cocks are frequently used on blow-off piping.

Stop Valves. — A stop valve is any valve that is controlled by hand or other external means, and used to stop the flow of fluid in a pipe.

Check Valves. — A check valve is any valve which opens automatically when the pressure in the normal direction reaches a predetermined value, but remains closed when the pressure is less than this amount or is in the opposite direction. Ordinary forms of check valves resemble a globe valve without a valve spindle. The closing device may be a disk, ball or cup. When the disk is arranged to swing about an axis like a hinge the valve is called a "swing check," or "flap valve," or "butterfly valve." When the disk or other device is lifted vertically, the valve is called a "lift-check valve." A check valve placed at the base of the suction pipe of a pump is called a "foot valve."

Safety Valves. — Lift check valves provided either with a helical spring or with a ball and lever mechanism to hold the valve closed under normal pressure are used as safety valves; the former type is called a pop safety valve, the latter a lever safety valve. The pop safety valve has practically supplanted the lever type, since it closes more promptly and is less liable to leak.

Relief Valves. — Check valves used to relieve excessive back pressure in atmospheric exhaust pipes are called "back-pressure valves," and when used in the piping to condensers to relieve excessive pressure are called "atmospheric relief valves."

Reducing Valves are used when it is desired to obtain steam at a pressure less than that of the boiler. In this type of valve the opening is relatively small, and the steam in expanding through this small opening has its pressure reduced. The proper size of opening for various rates of flow is maintained automatically by balancing the force produced on the closing mechanism by the pressure on the low-pressure side of the valve against the force produced on this mechanism by a helical spring. The size of the opening then adjusts itself automatically to maintain this balance irrespective of the rate of flow.

ELECTRICAL OPERATION OF VALVES.* — (*See also Motors, Industrial Applications of.*) The advantages of electrical motor operation for large valves is emphasized not only by the fact that such valves must be closed in a very short time, which would be impossible with hand control, but also by the fact that remote control is very often essential, as, for example, in hydroelectric power developments where it becomes desirable to control the gate valves from the control switchboard. The service of valve motors is exceedingly intermittent and may vary from comparatively short intervals, such as once every hour, to weeks or even months. Due to the intermittent nature of the service, efficiency or power factor need not be considered in this kind of motor application, the main consideration being the most reliable system of operation.

Size of Motor. — The proper size of a motor for driving a valve will vary with the duty and conditions under which the valve operates. A small valve may only require a one-horse-power motor, whereas very large valves, such as the Stoney-Gate valves at Panama, require 40-horse-power motors. The required motor capacity also depends to a large extent on the pressure on the valve. When opening the valve, the torque is a maximum shortly after the time of unseating, that is, after the wedges have been released and the actual motion begins. The torque then drops some until the valve has opened about one-fourth, after which it takes comparatively little power to complete the opening. When closing the valve, friction only needs to be overcome in starting as there is no pressure on the valve until it has begun to close. After the valve is about three-fourths closed the pressure causes the torque to increase rapidly. At the end of the closing cycle the torque does not, however, reach the value it did during the period of starting.

Type of Motor. — Either direct- or alternating-current motors may be used. With the former the series- or compound-wound type is generally used as it gives a high starting torque. With the latter the squirrel-cage induction motor is most widely used for small- and medium-size valves, principally on account of its simplicity. To overcome the sticking when opening, the drive is sometimes provided with a "lost motion" so as to give a hammer blow. For alternating-current motors this is furthermore of value in that it permits the motor to speed up some and gain in torque before the load comes on.

Control Equipment. — Where the size of the motor permits, the simplest method of control is to throw the motor directly on the line without the use of starting resistances. The automatic overload circuit breaker which should afford protection under normal operating conditions, must be prevented from tripping during the rush of starting current. This is readily accomplished either by short-circuiting the overload coil of the circuit breaker or by opening the connections to the coil. The former is the preferable method as this will leave some overload protection even though the coil is short-circuited. A push button is generally provided for this purpose.

Limit switches which will open the circuit when the gate has reached its limit of travel should always be provided. As a further precaution against over-travel or too high closing torque the motors should preferably be geared to the valve stems through efficient and reliable friction clutches.

When the motors are too large to be thrown directly on the line, starting resistances must be provided. With direct-current series motors a permanent resistance and a running resistance are often provided. The permanent resistance remains fixed in the circuit and should be so dimensioned as to give the maximum torque required. The running resistance is short-circuited at starting until the motor current drops to any desired value, at which time a current limit relay opens the shunt circuit of the contactor used for short-circuiting

* By D. B. Rushmore.

the resistance. This running resistance should be such as to prevent excessive speed at light load and to limit the torque should the valve seat itself before the motor is disconnected. The same general scheme may be used with alternating-current motors, but with these there is no danger of exceeding the synchronous speed.

Devices for Indicating Valve Positions. — When it is desired to indicate on the switchboard the position of the gate throughout its range of travel, this can readily be done by placing a number of sliding contacts on the inside of the limit-switch housing. These contacts are then connected with a row of incandescent lamps installed on the switchboard, and as the limit switch travels along, the lamps on the board will light successively.

Another system, much more comprehensive, is used in connection with the valve motors at the Panama Canal. In connection with each limit switch there is installed a "synchronizing transmitting device." This consists merely of a small generator which is mechanically connected to and revolves with the limit switch. It is electrically connected to a small motor which is mounted on the control switchboard and there mechanically connected to some sort of indicating device such as a pointer. When the generator at the valve revolves, the motor on the board will follow in synchronism thus indicating the exact position of the valve.

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[WM. KENT.]

VECTORS. — (*See also Complex Quantities.*) Any quantity which requires for its complete specification a magnitude and a direction is called a vector quantity. Such a quantity may be represented graphically by a line having a length equal to the magnitude of the quantity and a direction parallel to the direction of the quantity. Such a line is called a vector. Its direction is, in general, specified by the angles which it makes with three arbitrarily chosen lines or axes of reference. Vectors which lie in the same or in parallel planes are called co-planar vectors; the direction of a vector in the plane in which it lies (or parallel planes) may be specified by the angle which it makes with a single line of reference in this plane.

Quantities which are completely specified by magnitude and "sense" (i.e., whether positive or negative) are called scalar quantities. Forces, velocities, displacements, etc., are vector quantities, while time, work, mass, etc., are scalar quantities.

Vector Addition and Subtraction. — Since vector quantities cannot be completely specified by ordinary numbers, the various operations of arithmetic such as addition, subtraction, multiplication and division have no meaning when applied to such quantities. Analogous processes, however, are of great value in dealing with them. For example, the resultant of two forces which make an angle with each other is, by the ordinary parallelogram of forces, equal to the diagonal of the parallelogram formed by drawing from the free ends of the two lines representing the forces, lines parallel to these forces. That is, the resultant of OA and OB in Fig. 1 is the diagonal OC. Both the length OC and direction (the angle AOC) of the resultant is fixed by this construction. The resultant OC is called the vector sum of OA and OB. Similarly, the vector sum of OA and AB is equal to OB, or vice versa, AB may be called the vector difference of OB and OA. The angle XAB gives the direction of this vector difference when OA is subtracted vectorially from OB. When OB is subtracted vectorially from OA the vector difference also has the same length or magnitude, but is in the opposite direction, i.e., makes the angle $180° - \angle XAB$ with OA.

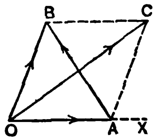

Fig. 1

Components of a Vector. — Any vector may be considered as made up of any number of vectors which when added vectorially, as described above, give a resultant vector equal to the given vector. It is frequently convenient in analyzing problems involving vectors to resolve each vector into two components, one parallel to and the other perpendicular to the axis of reference. The first component is usually referred to as the horizontal component and the second as the vertical component. It is readily proved that the horizontal component H of the resultant (or vector sum) of any number of vectors is equal to the algebraic sum of the horizontal components h_1, h_2, etc., of the individual vectors, i.e.,

$$H = h_1 + h_2 + \text{etc.}$$

and the vertical component V of the resultant is equal to the algebraic sum of the vertical components v_1, v_2, etc., of the individual vectors, i.e.,

$$V = v_1 + v_2 + \text{etc.}$$

The length of the resultant is then

$$S = \sqrt{H^2 + V^2},$$

and it makes with the axis of reference the angle σ where

$$\tan \sigma = \frac{V}{H}.$$

Analytical Representation of a Vector. — The geometrical representation of a complex quantity (*see Complex Quantities*) is a vector, i.e., the line representing a complex quantity has both magnitude and direction. Any vector quantity may, therefore, be represented by a complex quantity, and the operations of vector addition and subtraction may be carried out by the ordinary processes of algebraic addition and subtraction of complex quantities. For example, two vectors A and B may be represented by the algebraic expressions

$$\underset{\cdot}{A} = h_1 + jv_1,$$
$$\underset{\cdot}{B} = h_2 + jv_2,$$

where h_1 and h_2 and v_1 and v_2 are the horizontal and vertical components respectively, and the dots under A and B signify that $\underset{\cdot}{A}$ and $\underset{\cdot}{B}$ are vector quantities. The vector sum of $\underset{\cdot}{A}$ and $\underset{\cdot}{B}$ is then

$$\underset{\cdot}{S} = \underset{\cdot}{A} + \underset{\cdot}{B} = (h_1 + h_2) + j\,(v_1 + v_2),$$

which has the magnitude

$$S = \sqrt{(h_1 + h_2)^2 + (v_1 + v_2)^2},$$

and the angle

$$\sigma = \tan^{-1} \frac{v_1 + v_2}{h_1 + h_2}.$$

Multiplication and Division of a Vector by a Number. — Multiplying a vector by a real number n means taking a vector n times as long; dividing a vector by n means taking a vector $\frac{1}{n}$ of the length of the original vector. The result of multiplying a vector by an imaginary number jn is defined as a vector n times the length of the original vector and making a positive angle of 90° with the original vector. The result of dividing a vector by an imaginary number jn is defined as equivalent to multiplying the vector by $\frac{1}{jn} = -j\frac{1}{n}$ that is, as equivalent to a vector $\frac{1}{n}$ of the length of the original vector and making a negative angle of 90° with the given vector. Hence the multiplication or division of a vector $h + jv$ by a complex number $n + jn$ is equivalent to ordinary algebraic multiplication or division of two complex numbers (*see Complex Quantities*).

Scalar Product of Two Vectors. — Multiplication, in the ordinary sense, of one vector by another has no meaning, since a vector is not a number but involves both magnitude and direction. In the analysis of the more complicated problems of mechanics and electricity, certain expressions arise, however, which are analogous to ordinary multiplication. One of these expressions is a scalar quantity equal to the product of the magnitude of the two vectors by the cosine of the angle between them. This product is called the scalar product of the two vectors. The scalar product of the two vectors A and B is $AB \cos\theta$ where θ is the angle between them. If the vectors are expressed as complex quantities

$$\underset{\cdot}{A} = h_1 + jv_1,$$
$$\underset{\cdot}{B} = h_2 + jv_2,$$

the scalar product is

$$h_1h_2 + v_1v_2.$$

Note that this has no relation to the algebraic product of the two expressions $h_1 + jv_1$ and $h_2 + jv_2$.

Vector Product of Two Vectors. — Another expression which arises is a vector quantity at right angles to the plane of the two vectors and equal in magnitude to the product of the magnitudes of the two vectors by the sine of the angle between them. This product is called the vector product of the two vectors.

Example. — The vector product of the vector A by the vector B (Fig. 1) is $AB \sin \theta$, and is in the direction in which a right-handed screw, to the head of which A is conceived to be fixed, is advanced when A is turned through an angle of less than 180° into coincidence with B. From this definition it follows that the vector product of B times A is equal but *opposite* to the vector product of A times B. When A and B are expressed as complex quantities the vector product of A times B is

$$h_1v_2 - h_2v_1.$$

Note that the algebraic product of the expressions

$$h_1 + jv_1 \quad \text{and} \quad h_2 + jv_2 \quad \text{is} \quad h_1h_2 - v_1v_2 + j\,(h_1v_2 + h_2v_1).$$

The real and imaginary parts of this expression are *not* equal to the scalar and vector products.

[W. A. Del Mar.]

VISION, LAWS OF.— (*See also Illumination, Laws of; Photometric Quantities; Photometry.*) Light enters the eye through the *cornea*, a thin, transparent, curved wall, and passes successively through the *aqueous humor*, the *pupil*, the *biconvex lens*, and the *vitreous humor* to an image on the rear wall or *retina*. The cornea and lens possess the image-forming function, and accommodation to distance is given by the varying curvature of the lens. The humors give optical contact and preserve the size and form of the eye. The contraction and dilation of the pupil gives automatic accomodation to the intensity and quantity of light in the field of view. The retina comprises an elaborate structure of microscopic nerve terminals known as rods and cones. In its periphery rods predominate, but the ratio of cones to rods increases steadily toward the axial region of *fovea*, where the cones greatly predominate. The directly visualized image falls on the fovea and the sharpness of perception diminishes markedly in passing to the periphery. At very low intensities rod vision predominates. At moderate and high intensities cone vision predominates. In the former state the relative sensibility to the blue end of the spectrum is emphasized, in the latter state the relative sensibility to the red. As an optical system the eye has a chromatic aberration of approximately two diopters.

ELEMENTS OF VISION. — Visual sensations in general are characterized by intensity, extensity, quality and duration. Intensity refers to the aspects of vision measurable photometrically, as candle-power, brilliancy and illumination. Extensity involves the perception of contour, perspective, relief, detail and distance. Quality discriminates between sensations of color and non-color. Duration relates to the growth, fatigue, persistence and intermittency of visual sensations.

Intensity Relations. — All photometric units are essentially intensity measures of sensation. The luminous equivalent of radiation as determined by Ives (*see Phil. Mag., Vol. 24*) for the average eye is shown in Fig. 1. The lumens per watt scale shown is not final, but embodies the best available estimate. There is obviously no simple mechanical equivalent of light and the efficiency of light production depends to a marked degree on its spectral composition. Ives suggests two measures of luminous efficiency, viz., (1) the ratio of actual light emitted to that produced by the same power concentrated at the wave length of greatest luminosity, and (2) the ratio of the light emitted to that produced by the same power so distributed as to produce white light.

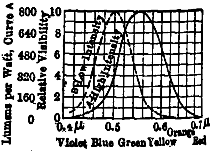

Fig. 1.

Purkinje Effect. — Transition from vision at low intensity to high involves relative loss of sensibility to blue and gain of sensibility to red, so that two lights of reddish and bluish hue respectively show a ratio of intensities which varies with the brightness of the field of view. This is known as the Purkinje effect. In fields above a brightness of one lumen per square foot the shift is slight. Photometric comparisons involving color differences should be made only in bright fields.

Sensibility. — The discriminating power of the eye for brightness, measured by the least perceptible percentage of increment, is practically constant at 1 per cent over a very wide range of intensity and is practically independent of color. Within this range the sensation of brightness varies as the logarithm of the stimulus. In so far as good seeing is a matter of perceiving differences in brightness the effectiveness of illumination does not increase in direct ratio to its intensity, but in a logarithmic ratio. It follows that if a

reasonable minimum of field brightness is maintained, say from one to two lumens per square foot, the gain from more intense illumination though real is relatively small.

Extensity Relations. — Extensity is perceived largely through differences in brightness and color, by varying visual angles and by acuity, or the power of resolving detail. Acuity follows a logarithmic relation to brightness similar to the strength of sensation. The same practical lower limit, say from one to two lumens per square foot, affords a fair normal basis of acuity. Above this the gain in acuity is relatively much less than the increase in illumination. Acuity, however, is not independent of color, due to the chromatic aberration of the eye. Retinal images formed by approximately monochromatic light are sharper than those due to a rich spectrum. Luckeish (*see Elec. W., Vol. 58, p. 1252*) gives the data in Fig. 2 for the relative acuity with various colors. For near vision blue light is more readily focussed, whereas for distant vision red light is more readily focussed.

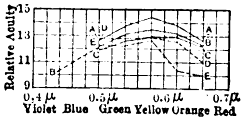

Fig. 2.

Glare. — The effectiveness of illumination is depressed to a marked degree by disturbing influences grouped under the term glare, which may be approximately defined as the relative overbrightness of part of the field of view. Transient glare, caused by the slowly changing adaptation of the eye when emerging from a dim region into a brilliantly lighted one, causes only temporary discomfort. Persistent glare, in addition to the depression of visual functions, may be highly injurious. No complete quantitative analysis of glare has yet been made. The following qualitative relations have been well established:

(*a*) Glare effect, or reduction of visual effectiveness, increases as the glaring source approaches the eye. Glare increases as the ratio of the distance of the glaring source to that of the visualized object from the eye diminishes. The distance at which a light source ceases to be glaring depends on its intensity, brilliancy and position in the field of view.

(*b*) Glare increases with the quantity of light received from the source.

(*c*) Glare increases with the brilliancy of the glaring source and with its degree of contrast to objects visualized. An automobile headlight is exceedingly glaring at night, but not glaring by day. Any light source giving an after-image is excessive in brilliancy. Glare due to this cause is accentuated by the reflex tendency of the eye to wander from the object of vision and fix on the brilliant source, causing a fatigue of attention.

(*d*) Glare increases as the retinal image of the glaring source approaches the center of the field of view. Depression of vision from side light is due in part to contrast and in part to the dilution of the central image by light scattered in the eye. Glare due to bright walls and backgrounds is largely of the latter class. The glare from bright sources situated at an angle from the line of vision exceeding 26 degrees is generally negligible.

The chief practical cases of glare are due to directly visible light sources of high power and brilliancy, to scattered light from side sources and backgrounds and the direct reflection of bright images by glazed surfaces in the immediate field of view. Prevention should be sought by the proper location and shading of light sources, by creating suitable constrasts between the field and its surroundings and by avoiding the use of glazed surfaces, especially highly-sized paper.

Duration. — Below certain frequencies intermittent sensations retain a measure of distinctness and produce a flicker effect. If a critical frequency is exceeded the sensations blend into a continuous effect equal in intensity to the

mean intensity over the complete cycle. Examples are the rotating sectored disk and the electric lamp operated by alternating current. The vanishing frequency of flicker increases with the degree of variation from maximum to minimum, with the solid angle subtended by the object viewed, and with the intensity of its brightness. The vanishing frequency of flicker is higher for white than for colored light. Within the common limits of practice the vanishing frequency is between 30 and 45 cycles per second. Electric lamps display no observable flicker in themselves or in illuminated objects at frequencies exceeding 40 cycles.

Quality or Color Relations. — Non-colored sensations are white, gray and black. Color sensations differ in brightness, in hue and in tone. The hue of a light is determined by the spectral position of its predominant component. Its tone or tint depends on the degree of dilution with white or black. Thus spectral red, pink and claret may agree in hue, but differ essentially in tone. Color sensations are equal in brightness when they may be rapidly alternated in the field of view without the appearance of flicker.

Synthesis of Colors may be accomplished by mixing red, green and blue in proper proportions, or by diluting the predominant hue with the proper amount of white or black. A color may be specified by the synthesis necessary to reproduce it. Tri-chromatic specification gives the mixing proportions of red, green and blue. Mono-chromatic specification gives the spectral hue and the degree of dilution. The tri-chromatic analysis as determined by the colorimeter (*see Photometry*) is of greatest practical utility. In this system the proportions of red, green and blue found in average daylight, though not equal, are taken as 33.3 per cent each to fix three color scales. The mixing proportions of these colors on the scales so established which reproduce any given light serve to specify it and to compare its color composition with daylight. Ives gives the following color analyses of common light sources.

Source	Red	Green	Blue
Black body at 5000° F. abs. (white)	33.3	33.3	33.3
Overcast sky	34.6	33.9	31.5
Blue sky	32.0	32.2	35.8
Moore carbon dioxide tube	31.3	31.0	37.7
Afternoon sunlight	37.7	37.3	25.0
Mercury vapor arc	29.0	30.3	40.7
Direct-current carbon arc	41.0	36.3	22.7
Welsbach mantle, ¼% cerium	42.5	40.8	16.7
Welsbach mantle, ¾% cerium	45.5	42.0	12.5
Welsbach mantle, 1¾% cerium	47.2	41.8	11.0
Tungsten filament, 1.25 w. p. c.	48.7	40.5	10.9
Nernst glower	49.2	40.7	10.1
Acetylene flame	49.1	40.5	10.5
Yellow flame arc	52.0	37.5	10.5
Carbon filament, 3.1 w. p. c.	51.3	40.4	8.3
Hefner standard lamp	55.0	38.8	6.2

Relation of Color of Light to Appearance of Objects. — Objects appear in the colors which they reflect or transmit to the eye. To produce true color effects light must possess all the components necessary to reveal objects and possess them in proper proportions. If the spectrum is deficient in

parts or its components differ greatly from the proportions in daylight it will in general produce distorted color effects.

Artificial White Light. — With the exception of the carbon dioxide vacuum tube no artificial illuminant approaches closely the color value of daylight. White light may be approximated by synthesis by combining the light of two sources, one excessive in red and the other excessive in blue, as the incandescent electric lamp and the mercury arc. A closer realization may be obtained from any incandescent lamp by means of a selective absorption screen which removes the excess of red and green light. The latter method involves a large sacrifice of efficiency.

SUMMARY OF CONDITIONS FOR BEST VISION. — The surface brightness of objects viewed should not be less than 0.0015 candle per square inch nor more than 1.5 candles. In terms of the illumination of ordinary white paper these values correspond roughly to limits of 1 to 1000 foot-candles. The value best adapted to any condition varies with the individual, the closeness of application, the nature of the surroundings and the general degree of diffusion. For reading by daylight 100 foot-candles is approximately the best condition, by artificial light approximately from 4 to 10. The degree of illumination required is strongly influenced by the contrast between objects visualized and their surroundings. The high diffusion of daylight and the flatness of the contrasts which it gives account in large measure for the higher illumination desired. For best seeing the objects viewed should be brighter than the surroundings, but the contrast should not exceed 10 : 1. The illumination provided for work requiring close application should be in inverse ratio to the reflecting power of the predominant surfaces. Directly visible light sources may generally be shaded down to a brightness of 2.5 candles per square inch with advantage to vision. Precautions outlined under *Glare* should be observed.

The accompanying table indicates the range of artificial illumination in foot-candles which has been found satisfactory under practical conditions.

Location	Desirable illumination in foot-candles	Location	Desirable illumination in foot-candles
Auditoriums	1.5-3.0	Residences:	
Art-gallery walls	3.0-5.0	Living rooms	2.0-3.0
Corridors	0.5-1.5	Bedrooms	1.0-2.0
Desks	3.0-5.0	Halls	1.0-2.0
Drafting rooms	5.0-8.0	Basements	0.5-1 0
Factories:		Restaurants	2.0-3.0
General lighting	1.0-2.0	School rooms	3.0-4.0
Specific lighting	4.0-6.0	Signs	2.0-3.0
Games and amusements:		Show windows:	
General	1.5-3.0	Dark goods	20-30
Specific	4.0-5.0	Medium goods	12-25
Laboratories	3.0-5.0	Light goods	8-15
Passenger cars	2.0-3.0	Stores and show rooms:	
Postal service	4.0-6.0	Dark goods	5-8
Reading rooms	3.0-4.0	Medium goods	3-5
		Light goods	2-4

Definition. — Vision is aided by the sharp definition of details down to an angular range of 1 in 10,000. Reading is easiest with type subtending an angle of about 1 in 100. For comfort it should not be larger than 1 : 20 nor less than 1 : 300.

Contrasts in intensity are best at about 1 : 10, but should not be less than 1 : 100. Contrasts as slight as 98 : 100 are readily perceivable in good light.

Uniformity of light on the working area is important. Excessively bright spots constitute an annoying glare and interfere with the fixation of vision.

Steadiness of light is essential, as flicker causes rapid fatigue.

Color. — A well-balanced spectrum is required for the true revelation of surface colors. Fatigue is most rapid in the extreme blue end of the spectrum and ultra-violet radiations may cause injury if present in unusual quantity. Light having a strongly dominant yellow-green hue gives the highest visual acuity.

Diffusion. — Complete diffusion tends to eliminate shadows, with the resulting loss of relief. The value of moderate shadows varies greatly with the circumstances, but is greatest where color distinctions are lacking.

VISUAL EFFICIENCY. — The resultant of all conditions affecting vision is included in the somewhat loose term visual efficiency. This concept must take account of the fundamental discriminations of brightness, detail and color and of the maintenance of these faculties in use. Tests of visual acuity have been largely used as a working criterion of visual efficiency. As applied to the momentary state of vision this criterion is unsatisfactory as it invites a semi-conscious mental spur to offset fatigue. An ideal criterion must take account of the maintenance of acuity in work, of the ease with which fixation of vision and attention are sustained, and of the general requirements of ocular hygiene. (*See Trans. Ill. Eng. Soc., Vol. 8, p. 40.*)

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[W. E. WICKENDEN.]

VOLTMETERS. — (*See also Ammeters; Wattmeters.*) A voltmeter is an instrument for measuring the potential difference, or voltage, between the two points to which its two terminals are connected. Any type of ammeter (q.v.) may be used as a voltmeter, but when designed for this purpose the coil which carries the current is made of a number of turns of fine wire, so that the moving element will be deflected by a very small current (from 10 to 200 milliamperes for full-scale deflection, depending upon the type of meter) and to limit the current to this value a high resistance, contained within the case of the instrument, is usually inserted in series with the coil. The high resistance may be provided with taps brought out to additional binding posts, so that the same instrument will give full-scale deflection for say 75, 150 and 300 volts, depending upon which binding post is used in conjunction with the terminal marked "0" or "+." The range of a voltmeter may also be indefinitely extended by the use of multipliers (*see below*) or potential transformers (*see Transformers, Instrument*).

Current Taken by, and Resistance of, Voltmeters. — The current taken by voltmeters of various types for full-scale deflection, and the resistance per volt of full-scale deflection are usually as follows:

Type of voltmeter	Milliamperes for full-scale deflection	Ohms per volt of full-scale deflection*
Moving magnet type	83 to 60	12 to 17
Iron-vane type	83 to 60	12 to 17
Moving-coil type	10	100
Electrodynamometer type	83 to 40	12 to 25
Hot-wire type	250 to 187	4 to 5.3

* For example, an iron-vane type voltmeter having a 150-volt scale would have a resistance of from $12 \times 150 = 1800$ to $17 \times 150 = 2550$ ohms. The resistance corresponding to a 15-volt scale would be from 180 to 255 ohms. The current taken by the instrument for either scale would be from 83 to 60 milliamperes.

Temperature and Inductance Errors. — In order that a voltmeter calibrated at one temperature shall read correctly at all other ordinary temperatures, it is necessary that the total resistance within the instrument shall not change appreciably with change in temperature, due either to variations in room temperature or to the heating of the windings by the current. To secure this condition, the resistance included in series with the active winding (which is usually made of copper wire) must have a practically negligible temperature coefficient. Various alloys may be used for this purpose; see *Wires, Resistance.* Precision voltmeters are sometimes provided with a thermometer, with the bulb within the case, and a small adjustable resistance to compensate for change in resistance due to temperature variations.

The high resistance within an a-c. voltmeter is wound non-inductively, i.e., the wire is doubled back on itself before being wound on the spool, or otherwise suitably arranged; the total impedance of the meter is then practically equal to its total resistance, and consequently its indications are affected but slightly, if at all, by the frequency of the voltage, at least up to 100 or 125 cycles per second. Some voltmeters of the soft-iron vane type, however, should be calibrated with voltage of the same frequency as that of the circuit on which they are to be used; for explanation see article on *Ammeters.*

Multipliers and Potential Transformers. — A voltmeter multiplier is simply an external resistor placed in series with the voltmeter. These resistors are made of high-resistance alloys having a low temperature coefficient, and are wound non-inductively. Let

r = resistance of voltmeter, in ohms,
R = resistance of multiplier, in ohms,
v = value of 1 scale division in volts without multiplier,
V = value of 1 scale division in volts when a multiplier having a resistance R is used.

Then

$$V = \frac{R+r}{r} \cdot v,$$

or the multiplier multiplies the voltmeter reading by the fraction $(R+r)/r$. This ratio is frequently called the "multiplying power" of the multiplier for the voltmeter in question.

For voltages above 750 volts potential transformers are to be preferred to multipliers for a-c. circuits; see *Transformers, Instrument.*

Low-voltage A-C. Voltmeters. — In voltmeters for measuring small alternating voltages the extra resistance in series with the active winding must necessarily be small or eliminated entirely for the lowest voltages, in which case the errors due to inductance, heating and change in room temperature may be quite large, 1 per cent to 3 per cent or even larger. When the voltmeter may be permitted to take a larger current than that given in the above table, without interfering with the actual voltage to be measured, a special low-resistance voltmeter may be used having a total resistance much lower than that of the ordinary voltmeter, the resistance of the active winding and the external resistance being reduced in the same proportion. For usual switchboard service and large power work satisfactory instruments of this kind may be had, giving a full-scale deflection for 15 volts or less.

Electrostatic Voltmeters. — When the current taken by the ordinary form of voltmeter is an appreciable fraction of the load current or when the frequency is very high, as in wireless work, an electrometer (*see article on Electrometers*) or electrostatic voltmeter may be used. The ordinary electrostatic voltmeter is in principle the same as an electrometer, but is provided with a pointer and scale and is so constructed as to be more readily portable than the ordinary electrometer which is read by means of a telescope and scale. Repulsion electrostatic voltmeters are also used, the principle of operation being the repulsion of two conductors charged with electricity of the same sign, as in a gold-leaf electroscope.

Condenser multipliers are sometimes used with electrostatic voltmeters; see *Electrometers*.

Calibration of Voltmeters. — A voltmeter may be calibrated by comparing it with a standard voltmeter whose calibration curve is known, or it may be calibrated directly by means of a potentiometer and standard cell; see *Potentiometers*. Hot-wire and electrodynamometer voltmeters for alternating-current work are calibrated on direct or alternating current; an iron-vane voltmeter should be calibrated by comparing it, on alternating current, with one of the other types which has been previously calibrated on direct current. Electrostatic voltmeters are usually employed commercially only for high voltages, and therefore must be calibrated on alternating current.

Precautions in Use and Installation, Costs, Bibliography, etc. — See the article on *Ammeters*.

[L. T. Robinson.]

WATER WHEELS AND THEIR SETTINGS.* — (*See also Hydraulics, Principles of; Power Stations, Hydroelectric; Water Wheels, Speed Regulation of.*) A water wheel is a machine for converting the energy of falling water into mechanical work.

CLASSIFICATION. — The old-fashioned under-shot and over-shot wheels (see any dictionary) are now seldom used, although a few over-shot wheels of very high efficiency have been constructed recently. Modern wheels may be classified as (*a*) impulse wheels (*b*) reaction wheels and (*c*) turbines.

Impulse Wheels are those in which the energy of the water is transformed into kinetic energy before reaching the wheel. The water is usually directed against the wheel in a single jet. The line of demarcation between turbines and impulse wheels is not sharply drawn.

Reaction Wheels are those in which the work is done by the reactive energy of the jets of water as they issue from the wheel.

Turbines may be defined as water wheels in which the energy of the water is gradually transformed into kinetic energy as it passes through the guides and wheel. Turbines in general have a series of curved moving vanes called "buckets" around their circumference into which the water is directed through curved "guide vanes." These vanes are so proportioned that they first transform the energy of the water into kinetic energy and then use this kinetic energy in driving the wheel. The following classification depends upon the action of the water:

Reaction Turbines are those in which the energy of the water is transformed into kinetic energy partly in the guides and partly in the wheel. They must run full of water and may be submerged without greatly reducing their efficiency.

Impulse Turbines are those in which the energy of the water is transformed into kinetic energy wholly in the guides. The wheel may run partially empty without particularly affecting the efficiency. The discharge must be into the atmosphere. If the wheel becomes submerged a great loss of efficiency takes place.

Limit Turbines which are on the dividing line between impulse and reaction turbines and may act efficiently as either.

Turbines may be classified also in accordance with the direction of flow as (*a*) radial flow, either outward or inward, (*b*) axial flow (parallel to the shaft) and (*c*) mixed flow, in which the flow is partly radial and partly axial. Again, they may be classified in accordance with the position of the wheel shaft as (*a*) vertical and (*b*) horizontal. Again, they may be classified according to whether the nozzles or guides extend partially or entirely around the wheel as (*a*) complete turbines and (*b*) partial turbines.

APPLICATIONS OF VARIOUS TYPES. — The following is intended to serve as a broad general guide in the choice of the particular type of wheel for a given set of conditions. It is to be understood that these rules are subject to considerable variation under varying conditions.

Impulse Wheel to be used for high heads and small quantities of water.

Reaction Turbine to be used for all heads and large quantities of water when the flow is fairly uniform at all times.

* Much of the information in this article is from the S. Morgan-Smith Co., manufacturers of wheels, who also furnished most of the drawings from which the cuts were made.

Impulse Turbine to be used for high heads and large quantities of water when flow is variable.

Setting and Position for Various Heads. — The methods of setting and style of wheel used (whether horizontal or vertical) under different conditions are about as follows:

Up to 12 ft. Head. — Vertical wheels in open or closed flumes are used.

12 ft. to 30 ft. Head. — Either horizontal or vertical wheels are used, more often horizontal; either open or closed flume is used, more often open.

30 ft. to 90 ft. Head. — Usually horizontal wheels in steel cases are used, sometimes in concrete cases.

90 ft. to 600 ft. Head. — Either horizontal or vertical wheels are used, more often horizontal, usually in a cast-iron scroll case.

DESIGN AND CONSTRUCTION. — The actual design of the details of the guides and runners is too complicated and involved to be treated here in detail. Certain general principles should be observed of which the most important is that the shape of the guides and runners should be such that the water will enter the wheel without shock and leave without velocity. It is possible to fulfil the first condition very closely. The last can only be approached because the leaving water must have velocity enough to carry it away from the turbine. The design of American turbines has been largely the outgrowth of some 70 years of practical experience and tests.

Runners. — The runner is the revolving part that directly transforms the energy of the water into mechanical work. It is either cast solid or built up. The parts of a runner are the hub, the crown, the ring, and the buckets or vanes. Fig. 1 shows a typical runner. All parts of the wheel, but more especially the buckets, must be smooth and free from projections, as all inequalities of surface in a flowing stream of water cause appreciable losses of energy. For this reason the majority of cast wheels are made in one solid casting of soft iron to reduce shrinkage stresses; cast steel or bronze wheels are also used for high heads. The buckets may also be cast separately and afterward put in a mold and the hub, crowns and rings cast about them. The curve of the buckets may be almost any smooth curve which will direct the water backward sufficiently relative to the wheel as it leaves.

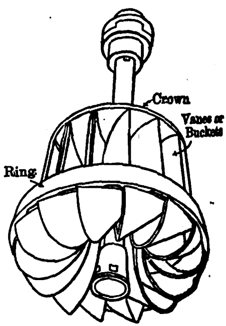

Fig. 1. Typical Runner

Diameter of Runners. — Reaction runners are measured at inlet. For buckets like Fig 2A this is simple. Buckets are often however cut back as shown in Fig. 2B to give greater speed and capacity. The buckets are usually tapered as shown. Different manufacturers measure and list their wheels at different points. This lack of uniformity in practice partially accounts for apparent variations between wheels of the same nominal size made by different manufacturers. Mead's *Water Power Engineering, p. 256,* gives a table showing the practice of some American manufacturers in measuring and listing their wheels.

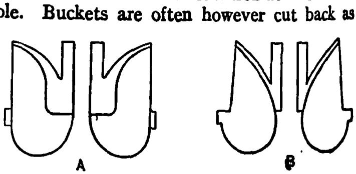

Fig. 2. Typical Bucket (after Mead)

Peripheral Speed of Runner. — The efficient speed of a wheel depends upon the ratio of the peripheral velocity of the wheel to the spouting velocity of water under the working head. This spouting velocity is equal to $\sqrt{2gh}$ where h is the working head and g is the acceleration due to gravity (32.2 feet per sec.^{2}). The ratio is practically 0.50 for impulse wheels; for reaction turbines it ranges from 0.65 to 0.95; for impulse turbines it ranges from 0.40 to 0.50.

Bearings. — The bearings in a turbine are often subjected to axial thrust and are quite often located of necessity in inaccessible places, and are sometimes even submerged. The forces causing thrust are (1) a reaction from the flowing water which acts in a direction opposite to the flow, and is of importance only in turbines which discharge axially and in one direction; and (2) thrust due to leakage into the space back of the runner, which may be relieved by leaving holes in the runner close to the shaft. In large turbines these pressures are carefully provided for and a thrust bearing is used to take care of them. In all but the large vertical units a simple step bearing, consisting of a lignum vitæ block located at the bottom of the shaft, is generally used. Wood is used because of the difficulty of lubricating a bearing in water. Wet wood is quite slippery and when hard enough to wear very slowly makes an excellent submerged bearing. A bearing of the ordinary type above the turbine serves to keep it in alignment. In some turbines the runner is suspended from above. The suspended type of bearing is easier to lubricate and to inspect but is more expensive than the step bearing. When twin horizontal turbines are used they are located on the shaft so that the water flows through them in opposite directions. The thrust on each wheel is balanced by that on the other. It is usual to provide a thrust bearing in such a case so that one turbine may be operated if necessary without the other.

Chute Case. — The chute case is the fixed portion of the turbine to which the bearings, guide vanes, and gates are attached. It surrounds the wheel and in it are formed the lower end of the penstock and the upper end of the draft tube. The chute case is generally made of cast iron or cast steel and sometimes of steel plates riveted together. The cast case is preferable because the passages, which are usually more or less curved, can be cast much more readily than they can be shaped in plates. The plates have the further objection of the obstruction caused by the rivet heads. In the most up-to-date practice a scroll case is used. It is so constructed that the cross-sectional area of the case decreases gradually as the water leaves the case to enter the wheel. This keeps the water moving at a constant velocity at all points in the case, which results in its entering the wheel with the same velocity at all points on its circumference and also obviates eddies and foam in the case. It is one of the important elements which influence the efficiency of the wheel.

Gates. — Three forms of gates are used in connection with reaction turbines, viz., the cylinder gate, the wicket gate, and the register gate. The cylinder gate is the cheapest and the wicket gate is best when properly designed and constructed.

Cylinder Gate. — This gate is a cylinder closely fitting the guides and located between them and the wheel. It moves parallel to the axis of the turbine and controls the admission of water by its position. This type of gate is sometimes used over the discharge. A gate of this type when partially closed causes a sudden contraction and expansion of the stream of entering water. This causes eddying and consequently decreased efficiency of the turbine at part gate.

Wicket Gate. — This type of gate, which is illustrated in Fig. 3, is very satisfactory. The quantity of water admitted is regulated by rotating each

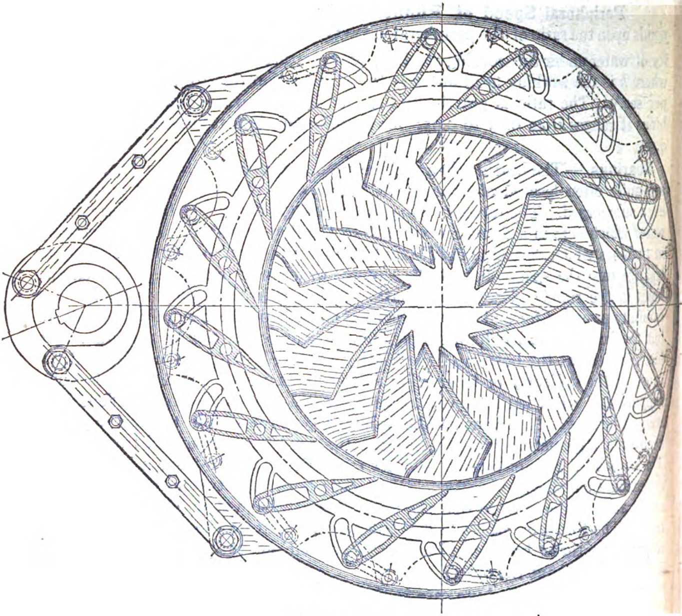

Fig. 3. Wicket Gate

wicket or "slat" about its axis so that the cross-sectional area of the passage through the guides is reduced. The wickets are connected together so that all are operated at once. Eddying is much reduced with this type of gate.

Register Gate.—This gate is a cylindrical case with openings in it to correspond with those in the guides. When running at full gate the openings in gate and guide register. By rotating the gate about the axis of the turbine the openings can be partially or wholly closed as desired. There is considerable eddying and consequent decreased efficiency under part gate.

Gates for Impulse Turbines.—These are often arranged so that the guide passages are opened one at a time. This causes less loss due to eddying at part gate.

Nozzles for Impulse Wheel.—In the tangential impulse wheel the best method for varying the opening is with a needle nozzle. When properly designed such a nozzle gives a very high velocity coefficient under a wide range of opening.

The Draft Tube.—The draft tube is in principle a suction pipe attached to the discharge of the turbine with its end submerged in the water of the tail race. It enables the full difference in head between the fore-bay and the tail-bay to be utilized. If the draft tube is not used it is still possible, with a reaction turbine, to utilize the full head by submerging the wheel. The submerged wheel is difficult of access for repairs and is likely to be injured by ice in case of a shutdown in weather cold enough to freeze the water in the tail-bay. The water in passing through the draft tube exhausts the air from it and the

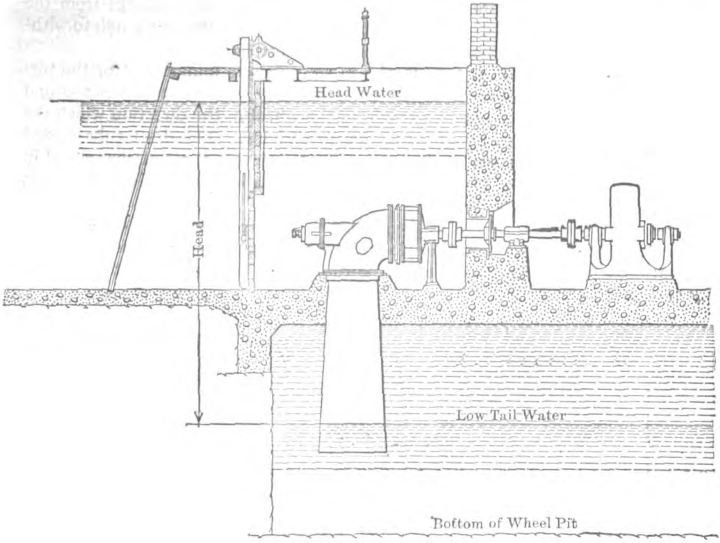

Fig. 4. Open-flume Setting for Horizontal Turbine

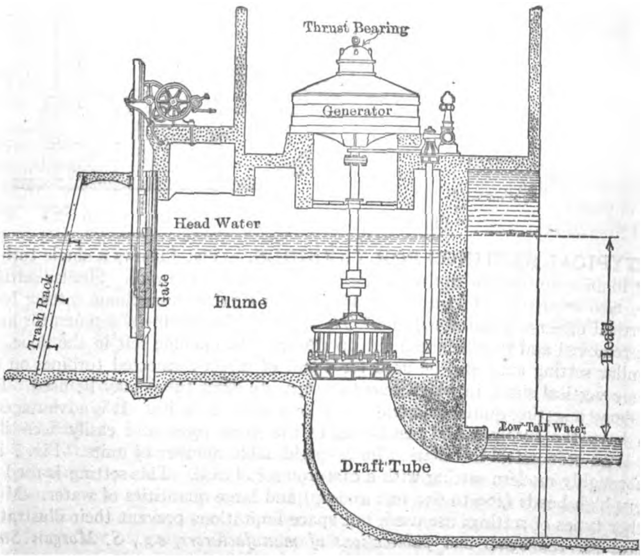

Fig. 5. Open-flume Setting for Vertical Turbine

column of water in the tube then exerts a suction on the discharge from the wheel. A vacuum exists in the draft tube and it must be strong enough to withstand the collapsing pressure of the atmosphere upon it.

In many plants the draft tube is cast in the concrete foundation for the turbine. Water should leave the wheel with a velocity of about 1.5 ft. per second and the draft tube should start away from the wheel in the direction which the water takes, in order to avoid losses due to eddying. The draft tube should gradually enlarge to as great a diameter as is practicable at its discharge end in order to reduce the velocity head as much as practicable, which in turn increases the head available at the wheel.

The draft tube should not be longer than about 20 feet, both because it is difficult to hold the vacuum in a longer tube under the varying conditions of operation, and because the water in the tube is apt to surge when the position of the gate is changed.

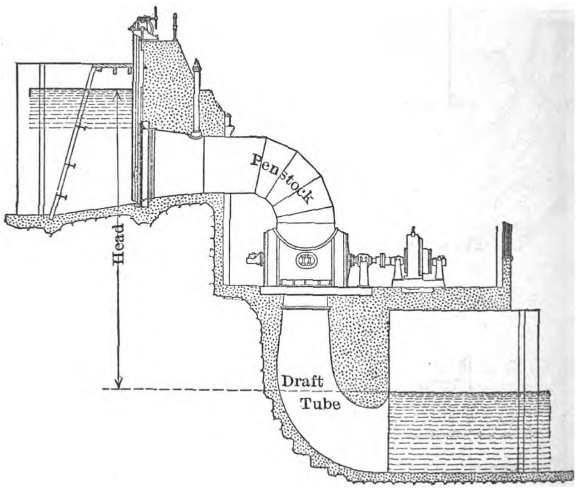

Fig. 6. Common Form of Setting

TYPICAL SETTINGS FOR TURBINES.—Fig. 4 shows a single turbine set in an open flume with the shaft extending through the wall. Similar settings are also used for twin turbines. Fig. 5 is a typical open flume setting for a vertical direct-connected turbine. To take out the turbine the generator must be removed and the turbine lifted up through the opening left in the floor. A similar setting may also be used for a pair of direct-connected turbines on the same vertical shaft, in which case two separate draft tubes may be used. Fig. 6 shows a setting quite commonly used for a series of units. It is advantageous to have the turbines and generators in the same room and easily accessible. It is an economical installation for a considerable number of units. Fig. 7 is a thoroughly modern setting with a cast-iron scroll case. This setting is used for very high heads (400 to 600 feet and up) and large quantities of water. Many other types of settings are used, but space limitations prevent their illustration here; see *catalogues and publications of manufacturers, e.g., S. Morgan Smith Co., Holyoke Machine Co., etc.*

TESTING OF WATER WHEELS. — The only practicable way to test a turbine is after it is set up and ready to run. The object of the test is to determine efficiency (ratio of output to input) under different conditions of gate opening and load. To determine input the head and quantity of water flowing must be known. In computing the efficiency of the turbine it is not fair to take the head as the difference in level between the fore-bay and the tail-bay because the water must have some velocity in order to escape from the turbine. The velocity head corresponding to the velocity of the water at the exit to the draft tube should be subtracted from the difference in level between the fore-bay and the tail-bay. The difference in level between the pond and the fore-bay should not be charged against the turbine as this is a loss incurred in bringing the water to it.

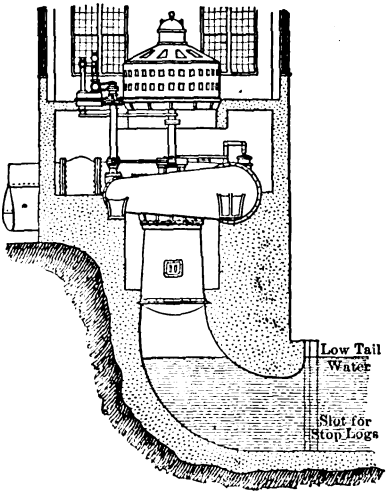

Fig. 7. Setting with Scroll Case

Determination of Input. — The greatest difficulty in testing lies in determining accurately the quantity of water which flows through the turbine. For this purpose a Pitot tube may be inserted in the pipe or a current meter may be used to measure the velocities at different points on a cross-section of either the fore-bay or tail-bay; the Pitot tube and current meter are described in the article on *Hydraulics*. Multiplying the average velocity in ft. per min. on a cross-section by the area of the cross-section will give the cu. ft. per minute. This quantity multiplied by the head and by 62.4 (the weight of a foot of water) and divided by 33,000 will give the horse-power input.

In some cases, when a penstock is used, it has a Venturi meter (*see Hydraulics*) built in it. This gives an excellent means of determining accurately the quantity flowing. This meter is not accurate below 20 per cent of its rated capacity, but results of tests below that point are seldom desired.

Determination of Output. — The horse-power output up to 1500 horse-power is best determined by means of an Alden brake; this is a special form of water-cooled Prony brake (*see Index*). If the turbine is connected to a generator the losses in which are known the generator may be used to measure the output.

Example of Test. — The appended table gives the results (for a number of points) of a complete test of a 33-inch special Smith turbine made by the S. Morgan Smith Co., who have kindly permitted its use. The test was made at the Holyoke Testing Flume, Holyoke, Mass. Study of the test shows that the highest efficiencies occur at about 0.8 gate opening. This is as it should be, because when there is abundant water and the wheel is running at full gate the efficiency is not so important as it is when running at part gate with a more or less restricted flow of water. Under the latter circumstances as much power as possible must be obtained from every cubic foot of water. Moreover, under

(PARTIAL) REPORT OF TESTS OF A 33-INCH L. H. SPECIAL SMITH TURBINE WHEEL (August 14-15, 1913)

Number of the experiment	Swing-gate: Proportional part of full gate opening,	Swing-gate: Proportional part of full discharge	Head acting on the wheel, feet	Duration of the experiment, minutes	Revolutions of the wheel per minute	Conical draft tube: Water discharged by wheel cu. ft. per sec.	Conical draft tube: Power developed by the wheel, h.p.	Conical draft tube: Efficiency of the wheel, per cent
44	1.000	0.989	14.02	3	140.00	122.03	157.18	81.15
43	1.000	0.990	14.03	3	150.67	122.28	160.48	82.63
42	1.000	0.991	14.02	4	161.25	122.28	162.46	83.71
38	1.000	0.996	14.07	4	174.00	123.15	165.29	84.27
39	1.000	1.004	14.03	3	187.00	123.90	166.88	84.80
41	1.000	1.007	13.96	4	197.00	124.02	164.46	83.91
93	1.000	1.001	16.11	4	208.75	132.37	204.31	84.63
94	1.000	1.003	16.08	3	214.33	132.63	203.60	84.33
11	0.857	0.855	15.16	3	112.67	109.67	132.98	70.65
10	0.857	0.868	15.14	4	129.50	111.36	149.12	78.13
9	0.857	0.883	15.08	5	151.80	113.06	166.05	86.03
6	0.857	0.890	15.11	5	172.20	114.03	173.50	88.95
7	0.857	0.894	15.09	4	184.00	114.52	174.79	89.35
3	0.857	0.882	15.16	4	193.50	113.18	167.10	86.03
1	0.857	0.843	15.28	6	209.50	108.59	144.74	77.05
87	0.806	0.819	16.61	4	133.50	110.03	162.18	78.38
86	0.806	0.831	16.59	4	148.50	111.60	175.27	83.62
85	0.806	0.844	16.57	4	166.00	113.18	186.36	87.78
82	0.806	0.845	16.58	6	172.50	113.42	188.70	88.64
83	0.806	0.847	16.58	4	177.75	113.67	191.37	89.70
89	0.806	0.848	16.52	5	181.00	113.67	191.74	90.20
80	0.806	0.842	16.60	5	186.40	113.06	187.80	88.39
78	0.806	0.821	16.66	4	198.00	110.51	176.69	84.77
77	0.806	0.805	16.68	4	207.50	108.36	167.25	81.74
76	0.806	0.781	16.70	4	216.00	105.15	149.23	75.07
23	0.762	0.782	15.08	3	127.33	100.11	133.42	78.07
22	0.762	0.797	15.06	4	142.25	101.89	144.96	83.45
20	0.762	0.807	15.06	4	155.50	103.26	153.99	87.47
25	0.762	0.811	14.95	4	165.25	103.38	156.98	89.72
16	0.762	0.777	15.23	4	182.50	99.99	147.10	85.33
15	0.762	0.762	15.25	4	193.25	98.02	139.08	82.18
14	0.762	0.737	15.31	5	201.80	95.04	127.80	77.59
74	0.499	0.549	14.18	3	128.67	68.16	85.19	77.86
72	0.499	0.537	14.27	3	150.00	66.84	86.36	79.98
75	0.499	0.534	14.18	4	159.50	66.34	85.40	80.19
70	0.499	0.518	14.35	3	184.00	64.73	74.15	70.52
69	0.499	0.506	14.39	4	194.25	63.24	61.51	59.71
68	0.499	0.495	14.45	4	204.25	62.06	47.04	46.33

NOTES. — The turbine and dynamometer carried during test on ball-bearing step. With the flume empty a strain of 3 lb., applied at a distance of 3.0 ft. from the center of the shaft, sufficed to start the wheel.

ordinary conditions the load should be such that somewhat less then full gate opening will take care of it, thus leaving something in reserve for a sudden excess demand upon the wheel. This is an exceptionally efficient wheel giving an efficiency of 90.2 per cent under certain conditions.

Usual Efficiencies. — A water wheel is usually designed so that the maximum efficiency occurs at about 80 per cent gate opening. For tangential wheels this maximum efficiency is usually from 75 to 85 per cent and for reaction turbines from 80 to 85 per cent. The test given in the preceding paragraph shows much higher efficiencies than usually obtained, the wheel in this case being an unusually good one.

COST OF WATER WHEELS. — The following figures give some idea of the cost of the different sizes of wheels exclusive of settings, except as noted; the necessary pulleys and gears are included. Freight and cost of installation not included. The cost per horse-power, which varies from $2 to $20, decreases both as the head and the size of the unit increase. See also article on *Power Stations*.

Style of wheel	Head, feet	Horse-power	Cost per horse-power
Vertical, open flume	10	100	$ 8.00
Same except with steel case and steel supports	10	100	13.00
Pairs of horizontal wheels in open flume.	20	500	7.00
Same conditions, steel case	20	500	10.00
Pairs of horizontal wheels, open flume	40	5000	5.00
Horizontal wheel, single runner, cast-iron scroll case	90	1400	6.00
Horizontal or vertical single-wheel cast-iron scroll case	200	5000	2.00 to 3.00

BIBLIOGRAPHY. — A complete bibliography up to 1908 is given in Mead's *Water Power Engineering*. Among the books in English there listed may be noted Church, I. P., *Hydraulic Motors*, N. Y., 1905; Thurso, J. W., *Modern Turbine Practice*, N. Y., 1905; Wood, DeV., *Turbines, Theoretical and Practical*, N. Y., 1901; Horton, R. E., *Turbine Water Wheel Tests, Water Supply and Irrigation Paper*, No. 180, *U.S. Geological Survey*, 1906. Some more recent books on water wheels are Russell, G. E., *Text Book on Hydraulics*, N. Y., 1912; Daugherty, R. L., *Hydraulic Turbines*, N. Y.; Gelpke and Van Cleve, *Hydraulic Turbines*, N. Y., 1911; Von Schon, H.A.E.C., *Hydro-electric Practice*, Phila., 1908; Koester, F., *Hydroelectric Developments and Engineering*, N. Y., 1909; Lyndon, L., *Development and Electric Distribution of Water Power*, N. Y., 1908. Some recent articles in the technical press are: *Testing Water Wheels after Installation*, A.S.M.E., April, 1910; *Prime Mover and Generator Capacity*, Trans. A.I.E.E., March, 1913; *The Runaway Speed of Water Wheels*, Trans. A.I.E.E., June, 1912; *Selection of a Water Wheel Unit*, Trans. A.I.E.E., Apr., 1912; Several articles in the *Jour. Elec. Power and Gas*, 1911 and 1912.

[L. E. MOORE.]

WATER WHEELS, SPEED REGULATION OF.* — (*See also Hydraulics; Power Stations, Hydroelectric; Water Wheels and Their Settings.*) In a few special cases close speed regulation of water wheels is unnecessary, for example, in pulp grinding, certain chemical processes and operating air compressors. Even in these cases experience has shown that better results may be obtained when wheels are controlled by properly designed governors. For factory work, where the power is applied through the medium of belting, the problem of regulation is simplified by the large inertia effect of the pulleys and belting.

SPEED VARIATION IN HYDROELECTRIC PLANTS. — For electrical generators, which perhaps comprise the majority of installations, the utmost nicety of speed regulation is required because the value of the electrical energy produced is greatly lessened if the voltage and frequency are not uniform. In the best large modern power plants speed variations greater than one or two per cent are exceptional. In smaller plants, since there are usually larger proportional load fluctuations, it is more difficult to maintain close speed regulation.

Calculation of Speed Regulation. — When the gates are controlled by a governor (*see below*) the percentage change in speed of the wheel due to a change in load, when the penstock, if any, is short, is given approximately by the formula:

$$s = 81{,}000{,}000 \frac{TL}{IN^2},$$

where

s = percentage change in speed,
T = time required to reset gates, in seconds,
L = change in load, in horse-power,
I = flywheel effect of wheel and generator, in foot and pound units, i.e., I is the weight in pounds which at a radius of one foot would have the same moment of inertia as that of the wheel and generator combined,
N = number of revolutions per minute.

When the penstock is long, the calculation is more complicated; see section below under *Long Penstocks and Auxiliary Devices.*

DIFFICULTIES OF REGULATION. — In an open flume installation under low heads the regulation is a very simple problem. Where the water is conducted to the wheel through a long pipe, or penstock, regulation becomes a very serious matter. Regulation is effected, in general, by varying the quantity of water flowing to the wheel. This requires a change in the velocity of the water flowing in the penstock, when one is used.

The difficulty of regulating turbines at the end of long pipe lines becomes less in proportion as the flywheel effect per horse-power becomes greater, because an increase in the flywheel effect permits slower governor action for a given per cent change in speed. A slow governor action insures slower acceleration and retardation of the water column, and consequently smaller pressure changes. The necessity for large flywheel effects for a given head is greater the less the slope of the pipe line.

A successful *governor must anticipate the effect* of any gate movement. It must move the gate to, or only slightly beyond, the position which will give normal speed when the flow in the penstock has become adjusted to the new conditions. The gates are heavy and not infrequently bind; hence moving them with certainty and precision requires a powerful governor.

* The material for this section was obtained from Mr. Henry E. Warren, *Consulting Engineer* of the Lombard Governor Co., Ashland, Mass.

METHODS OF REGULATING. — Under small heads the regulation is easily and safely accomplished, and depends almost solely upon the power and sensitiveness of the governor. Under large heads the practical impossibility, in most cases, of safely changing the velocity of the water column with the requisite quickness leads to various expedients. With impulse wheels a "deflecting nozzle" (*see below*) is used in connection with a needle valve. In all turbines the regulation is accomplished by moving the gates. The difficulties met with in connection with high heads are taken care of by auxiliary devices which operate in conjunction with the governor. They may or may not be operated by mechanical connection with the governor. The devices are: (1) by pass valves, (2) relief valves, (3) stand pipes, (4) surge tanks, all of which are treated below.

WATER-WHEEL GOVERNORS. — The governor proper is the device for automatically opening and closing the gates as the load on the wheel varies. The requirements to be met are (1) sensitiveness to small variations in speed, (2) quickness in action, (3) steadiness and accuracy in moving the gates the proper amount, (4) ability to operate continuously with minimum amount of attention.

Types of Governors. — There are two general classes of governors; (1) the "mechanical" type, which takes energy from the water wheel itself, as required, through a belt or gearing, and by means of friction clutches applies this energy at suitable times to the operation of the water-wheel gates; (2) the "hydraulic" type, in which stored energy in the form of compressed air is made available when required to move the water-wheel gates through the medium of liquid acting against the piston in a hydraulic cylinder. The energy in the compressed air is thus transmitted by the liquid through a suitably controlled valve to do the work of moving the water-wheel gates, when necessary, and the energy of the air is afterwards restored by pumping the same liquid back into the air tank. The operation of restoring the energy by pumping is gradual; while the operation of moving the water-wheel gates, by utilizing the energy of the compressed air, may be as rapid as desired.

Governors of the mechanical type were the first in the field, and, being simpler and cheaper to construct, have been very widely used. They have certain objectionable features, however, which have prevented them from being adopted under any conditions which require extreme nicety in speed regulation. Consequently, governors of the hydraulic type are almost exclusively used in large and important power plants. When properly designed, they are capable of giving extremely accurate speed regulation and are thoroughly reliable. Such governors are sometimes operated continuously night and day for weeks, or even months.

Connections Between Wheel and Governor. — The connections between a water-wheel governor and the gates of a turbine must be strong, direct and free from lost motion; otherwise the governor cannot perform its functions properly. Governors are built nowadays in many different types, so as to be readily adapted for connections to any standard make of turbine. It is always desirable to submit plans of the water wheels, with surrounding portions of the power plant, to the governor builder so that the two machines may be fitted together to the best advantage. It is generally advantageous to have the hydraulic cylinder of the governor located near the water-wheel gates, although for very large units the control mechanism of the governor may be more conveniently placed on the floor above. For small water wheels, which are occasionally located in comparatively inaccessible places, it is sometimes necessary to place the governors at a considerable distance and connect them with the water-wheel gates by means of special flexible steel cable. Occasionally several water wheels are connected in this manner to a single governor.

Distant Control. — Large governors are frequently provided with suitable electric devices by means of which the speed of the wheel may be adjusted from any distant point, thus permitting alternators driven by independent water wheels being brought to the same speed and angular position for multiple connection.

Rating of a Governor.— A hydraulic water-wheel governor is usually rated in terms of the energy, expressed in foot-pounds, which is developed by one stroke of the piston, and the time, in seconds, required for a complete stroke (i.e., to completely close or open the gates). The size of governor required for a given size of wheel depends so largely upon the type of gate, head, etc., that no general rules can be given here.

Specifications for Governor. — The following is taken from the standard specifications of one of the large governor manufacturers:

The governors are each guaranteed to develop the energy, expressed in foot pounds stated below, and if not called upon to develop a greater amount of energy than that named, will completely open or close the gates to which they are attached in the following number of seconds:

Type,........Governor,........Foot Pounds,........Seconds,.......

Said governors are further guaranteed: (*a*) to stand substantially steady when the speed does not vary; (*b*) to be dead beat in action and not to hunt; (*c*) to correct with maximum promptness for all load changes within the capacity of the water wheels to which they are attached; (*d*) to maintain the speed steady within half of one per cent under uniform load upon the water wheels to which they are attached; (*e*) to begin to adjust the water-wheel gates when the speed has varied half of one per cent; (*f*) after sudden decrease of load to bring the speed back to normal in...... seconds; (*g*) to operate perfectly in parallel with other governors of the same make.

Dimensions, Weight and Cost of Governors. — On account of the very great variation in the design of governors, it is only possible to give some approximate dimensions and costs which are subject to very large variations for special machines. The following figures pertain to a few standard hydraulic governors.

HYDRAULIC GOVERNORS

Capacity, foot-pounds per stroke	Horse-power of turbine*	Floor space, square feet	Extreme height, feet and inches	Shipping weight, pounds	Cost,* F.O.B. factory
			Ft. In.		
2,500	10 to 400	13	4 11	2,400	$350
10,000	1000	22	7 3	3,700	650 to 1150
30,000	....	30	6 5	6,200	1800
60,000	5000	43	6 7	17,030	2800

* The design of the turbine gates regulates necessary capacity of governor. For the same capacity of governor the cost increases with the speed of the stroke.

LONG PENSTOCKS AND AUXILIARY DEVICES. — When the power required from a wheel varies, the velocity of the water in the penstock must be changed. If the velocity be increased, no trouble will ensue, unless it be increased so rapidly that the water column breaks. If this happens the reuniting of the parts of the water column will prove disastrous. The sudden checking of the velocity of the water may result in very large pressures, which throw severe strains upon the penstocks and wheel cases and may even rupture them.

Calculation of Speed Regulation at End of Long Penstock. — To compute the speed regulation obtainable at the end of a long pipe line proceed as follows: Find the excess energy, above the load requirements, passing through the water wheel gates while they are being closed by the governor in a definite time interval T. This excess energy will be due (1) to the extra water flowing during the time interval T, plus (2) the portion of the kinetic energy of the water column suddenly released by changing the velocity of the stream, minus (3) the energy absorbed in the standpipe, surge tank or other pressure-regulating device. Most of this excess energy will be added to the kinetic energy of the revolving parts, and the final speed will be such as to represent the initial kinetic energy of the revolving parts plus the added energy received during the time of closure. For small increments of energy the per cent increase in speed will be approximately one-half as great as the per cent increase in kinetic energy. There are so many variable factors, namely, length of pipe line, total effective head, velocity of flow, permissible rise in pressure, minimum time of governor action, flywheel effect per horse-power and speed, that it is not feasible to give a general equation for all cases. Each problem must be worked out by itself with the assistance of the water-wheel builder, the manufacturer of the generator, and the builder of the governor.

Water Hammer in Long Penstock. — The following formula, developed by A. H. Gibson of Manchester University (*Water Hammer in Hydraulic Pipe Lines*, London, 1908, p. 12) gives the water-hammer pressure (i.e., the rise in pressure due to closing the gates) under working conditions where there is no standpipe or other relief device.

$$p' = \frac{w}{g}\left[\left(\frac{la'}{aT}\right)^2 + \frac{la'}{aT}\sqrt{2\,gh + \left(\frac{la'}{aT}\right)^2}\right],$$

Where p' = water-hammer pressure in lb. per sq. ft.,
w = weight of one cu. ft. of water (62.4 lb.),
g = acceleration of gravity (32.2 ft. per sec. per sec.),
l = length of pipe line in feet,
a = cross-sectional area of pipe line in sq. ft.,
a' = cross-sectional area of turbine gate in sq. ft.,
T = time in seconds to close gate at a uniform rate,
h = static head in feet.

To obtain the pressure indicated by the formula it is not necessary that the gate be completely closed; closing it partially at such a rate that T is the time which it would require to completely close it gives the same rise in pressure.

Effect of Elasticity of Water and Pipe. — In the case of extremely high head plants the amount of energy which may be absorbed by the compression of the water and the expansion of the pipe near the lower end should be taken into account in computing variations of pressure due to sudden velocity changes. Under certain conditions the elasticity of the water is sufficient to absorb all the kinetic energy even if the water column be very long and stopped very suddenly without causing a dangerously great rise in pressure compared to the normal pressure. This is only true where the normal pressure is very great.

Standpipes. — Under favorable conditions a standpipe may be used to equalize the pressure fluctuations. A standpipe is a vertical pipe in which the water stands normally at the same level as in the reservoir. If the standpipe has an overflow, increase in pressure beyond a certain point can be prevented, but there remains an effect upon the speed regulation due to the diminution of pressure which follows a sudden opening of the gates. At such times the water in the standpipe will fall, while the water column as a whole is accelerating. This temporary fall in pressure will be less the greater the area of

the cross-section of the standpipe near the normal water level. If the standpipe has a very large cross-section the changes in gate opening caused by the governor will not seriously disturb the pressure at the turbine. The diameter of the standpipe may be a matter of relative costs. It does not pay to spend money on closer regulation than the character of the service requires.

For low head developments where long pipe lines are necessary, open gravity standpipes of liberal dimensions, preferably enlarged at the upper end, so as to contain enough water to operate the turbines for several seconds while the main water column is being accelerated, are preferable to other forms of pressure-equalizing devices, from the standpoints of both efficiency and economy. For pipe lines several miles in length it is desirable to provide equalizing reservoirs at intervals along the pipe line, so that the changes in velocity of the water column will be as gradual as possible. If such equalizing reservoirs are not provided it will probably be necessary to arrange the governing mechanism so that the velocity of the water will be increased or decreased very gradually, speed regulation being effected by diverting a portion of the issuing stream, either by a deflecting nozzle, a movable shield, an auxiliary nozzle, or very slow closing by-pass valves.

Calculation of Rise of Water in Standpipe. — The maximum possible rise of water in a standpipe may be determined by computing the total kinetic energy in the moving water column and assuming that it is wholly utilized in raising the water level in the standpipe, which is the assumption equivalent to assuming that the water-wheel gates are closed instantly. Let

A = area of standpipe at and above the normal water level in sq. ft.,
a = area of penstock, assumed to have uniform cross-section in sq. ft.,
l = length of penstock in ft.,
V = velocity of water column in ft. per sec.,
g = acceleration due to gravity = 32.16 ft. per sec. per sec.

Then the momentary rise of water in the standpipe, in feet, upon the instantaneous closure of the turbine gate, is

$$H = V\sqrt{\frac{al}{Ag}}.$$

This formula takes no account of the loss of energy due to friction of water in the water column or standpipe, nor of the absorption of energy in compressing the water, the last of which is regained when the water expands; the error involved in neglecting these losses is small.

Surge Tanks. — A surge tank is a closed tank containing air under pressure and communicating with the penstock. A sudden increase in pressure in the penstock will force the water into the surge tank and compress the air, thus transferring the energy of the water to the air. A decrease in pressure in the penstock will permit the air in the surge tank to expand, returning its energy to the water. Some form of throttling valve or equivalent device must be used between the penstock and the surge tank to dampen the pressure waves which may otherwise become periodic.

Relief Valves. — These are valves similar in purpose to the safety valves on a steam boiler. They allow excess pressure to escape but do not restore energy when the pressure drops. In order to prevent the setting up of pressure waves the relief valves must be so constructed that they will close very slowly after they have opened.

There are many types of relief valves, some depending altogether on the pressure in the water column itself, and others being opened by the same movement of the governor which closes the gates. The latter class of valves

are, on the whole, preferable because they provide a passage through which the surplus kinetic energy of the water column may be discharged without waiting for a rise in pressure. These governor-actuated valves are generally designed to close slowly after the initial opening movement.

By-pass Valves. — By-pass valves may be utilized to maintain constant velocity of the water in the pipe line by opening the by-pass the same amount as the passageway though the water-wheel gate is closed and vice versa. Such valves are very wasteful of water, but their use is justified in certain cases where the stream is utilized for irrigating purposes. Deflecting nozzles used for impulse wheels belong to this class of devices. To conserve water the by-pass may be arranged so as to close gradually after it has opened. This closing should be so gradual that no water hammer is produced in the penstock.

Deflecting Nozzles. — When impulse wheels are used, the regulation is commonly effected by suddenly diverting the stream issuing from the nozzle so that a less or greater portion of the stream strikes the wheel. Such devices are preferably arranged so that when the deflection of the nozzle becomes greater than a certain predetermined amount, the cross section of the stream through the nozzle is slowly varied by the axial movement of a needle valve inside the nozzle. This is best accomplished by electrical contacts which are touched by some part of the nozzle when it is deflected more than a certain amount from the normal. These contacts permit current to flow to a small motor which actuates the needle valve.

BIBLIOGRAPHY. — A complete bibliography up to 1908 is given in Mead's *Water Power Engineering*. Some recent books and articles dealing with speed regulation of water wheels are: Koester, F., *Hydroelectric Developments and Engineering*, N. Y., 1909; Von Schon, H. A. E. C., *Hydro-Electric Practice*, Phila., 1908; *Speed Regulation in Hydro-Electric Plants*, A.S.M.E., Feb., 1911; *Oil Pressure Governor for Water Wheels*, Elec. World, Jan. 6 and Feb. 17, 1912.

[L. E. Moore.]

WATTHOUR METERS. — (*See also Amperehour Meters; Electrodynamometers; Wattmeters.*) A watthour meter consists essentially of (1) a small electric motor, which may be either of the commutator type, mercury and disc type, or induction type, (2) a brake system composed of a disc of non-magnetic material (usually copper or aluminum) mounted on the armature spindle and so arranged that its edge rotates between the poles of one or more permanent magnets, and (3) a system of gears with numbered dials forming a suitable registering mechanism for indicating the number of revolutions of the armature or disc. One winding, called the "potential" coil, is connected across* (in shunt with) the load and the other winding, called the "current" coil, is connected in series with the load, the connections being the same as for an indicating wattmeter; see *Wattmeters.*

Principle of Operation. — Motor-type meters are so constructed that the average torque exerted by the motor is proportional to the average power taken by the load. The brake system is so designed that the opposing torque, due to the eddy or Foucault currents induced in the disc as it rotates between the poles of the permanent magnets, is proportional to the speed of the disc. When the disc acquires a given speed the driving torque must be just equal to the opposing torque, and must therefore be proportional to the speed. Hence the speed of the disc is proportional to the average power, and therefore the total number of revolutions which the disc makes during any interval must be proportional to the total energy input during this interval, whether the power remains constant or varies. To determine the energy input to the load in watt-hours or kilowatt-hours it is therefore only necessary to take the difference between the dial readings at the beginning and end of the given interval, and multiply by the proper constant if the meter is not direct reading.

Sources of Error. — In a d-c. watthour meter the chief source of error is the friction of the brushes (which is more or less variable), friction of the bearings of the motor and gear train, and air friction, the brush friction being by far the most important. In a-c. watthour meters, the lack of exact 90° phase relation between the impressed voltage and the magnetic flux due to the current in the potential coil may cause additional errors which vary with the frequency, power factor and also with the distortion of the wave form; in modern meters these errors are practically negligible. Instrument transformers (*see Transformers, Instrument*), unless properly designed, introduce additional sources of error. Devices and methods for overcoming, or of correcting for, these errors are described below.

Classification according to Service. — Watthour meters may be classified according to the service in which they are used, viz., (1) house meters for use in residences or factories, (2) switchboard meters for use in central stations, and (3) meters for use in individual power installations and isolated plants. Meters for house service are generally front-connected, separately-sealed devices which can be installed and sealed to prevent tampering. Switchboard meters are generally back connected and of such design as to match other switchboard devices. "Meter-boards," equipped with the necessary auxiliary or protective devices, are frequently used with house meters, in order to insure proper connections and sealing.

COMMUTATOR TYPE OF WATTHOUR METER. — The commutator type of meter is now used only on direct-current circuits, although before the induction meter was so highly perfected large numbers were used for alter-

* Either directly or through suitable instrument transformers; see *Transformers, Instrument.*

nating current as well as for direct current. The motor consists of a set of stationary coils, commonly called field coils, which carry the current, and an armature wound with small wire which is connected across the terminals of the load or in shunt with the supply circuit; generally a series resistance is used to absorb part of the line voltage. The connection to the armature is by a commutator and brushes, as in an ordinary commutator type of shunt motor. Special attachments are sometimes added for very large capacities, such as double armatures astatically arranged, and damping magnets inclosed in a laminated iron shield in order to reduce stray-field errors where heavy currents are used and heavy short-circuits are frequent.

It should be noted that the operation differs from that of an ordinary shunt motor in that the speed increases with increase of field strength, since its back e.m.f. is much less than 50 per cent of the line voltage.

On account of the inductance of the windings of the commutator meter, it does not record accurately on a-c. circuits unless properly compensated, the error being greater the less the power factor.

Compensation for Friction; Light-load Adjustment.—Friction in a well-designed watthour meter will be very small and will be noticeable only on light loads of 10 per cent and below. To compensate for this friction, a "light-load adjusting coil" is added, which is an auxiliary or compounding coil. This light-load coil is connected in series with the armature. It is placed adjacent to the field coils so that its field strengthens the main field and produces a slight torque independent of the power and just sufficient to compensate for friction.

Details of Construction, Bearings, etc. — In the design of the modern direct-current commutator type of watthour meter, great thought has been given to the mechanical construction in order to obtain small air gaps between the armature and field coils, light-weight armatures and commutators and brushes which will have very small friction and yet stand the wear and carry the current to the armature without undue sparking and pitting of the contact parts. In a watthour meter of the most approved design the field coils are wound with enameled copper strips and the armature winding is enamel-covered wire wound on a paper sphere mounted on a spindle of light steel tubing. The lower bearing carries all of the weight and the upper bearing is a guide bearing, as all meters have vertical shafts. The lower bearing consists of a steel pivot mounted in the end of the spindle or armature shaft, running on a jewel bearing. In most cases a good grade of sapphire is used for the jewel and is cupped and polished so as to provide a bearing with as little friction as possible. Of late years a great deal of attention has been given to the bearings and in some cases, particularly on high-capacity meters with astatic armatures and a correspondingly heavy weight, diamond jewels are being used with excellent results. The diamond jewel will last much longer than the sapphire before wearing sufficiently to increase the friction and thereby change the accuracy of the meter, particularly at light loads. See also section below on *Data on House-Type Meters*.

Capacity of Commutator Watthour Meters. — Commutator-type direct-current meters are built in capacities ranging from 5 to 10,000 amperes, 100 to 600 volts. Meters of this type are furnished with double-current circuits for three-wire circuits, the maximum ampere capacity for three-wire meters being about 6000 amperes. Special meters have been supplied for 1200- and 2400-volt railway circuits.

Accuracy. — Tests on direct-current commutator meters show that a meter of good design should start on two per cent of full load and should give accuracies about as follows: — To within 3½ per cent from 5 per cent to ¼ load,

2 per cent from ¼ to full load. The Public Service Commission of New York State allow no d-c. watthour meters to be used which are not accurate within the following limits:

Per cent rated current.....	5	10	20	50	100	150
Maximum deviation, per cent.	7.5	3.0	2.0	2.0	2.0	2.0

MERCURY-MOTOR WATTHOUR METER. — The mercury-motor watthour meter has been manufactured for both alternating and direct current, but at the present time its use is practically limited to direct current, as the induction type of meter has been found superior for alternating current. The motor element consists of a mercury well or reservoir in which is partially floated a metal armature disc or drum, usually of copper. The chamber is filled with mercury and fitted with a non-spillable opening at the top through which the spindle is attached. The main current is led into the mercury by means of electrodes, and since mercury has about forty times the resistance of copper the major part of the current traverses the armature disc or drum, passing out through the mercury to the opposite electrode. The potential or shunt coil is wound with many turns and is mounted on a laminated iron core so placed that the flux set up will cut the armature disc or drum.

The torque exerted on the disc or drum is proportional to the product of the current through the disc and the current in the potential coil, and therefore to the power supplied to the load. A brake system similar to that used in the commutator type of watthour meter renders the speed proportional to the torque and therefore to the power supplied to the load.

Compensation for Friction, Light-load Adjustment. — For compounding or light-load compensation for friction, a thermal device is used in some cases, and it is also possible to obtain compensation by shunting part of the potential circuit through the armature disc. The thermal device employed is a thermocouple shunted around the mercury chamber, and heated from a resistance coil connected across the line. The small current generated in the thermal couple forces a slight current through the armature and produces a compounding effect.

Application of Mercury Meters. — Mercury-motor meters, on account of the low resistance of the current circuit (disc or drum), are particularly well adapted for use with external shunts, since but a small potential drop in the shunt is required. They are therefore used in comparatively large numbers for switchboard work where large currents are used and where external shunts can be conveniently installed.

Details of Construction. — The several designs of mercury-motor meters on the market differ in mechanical construction and in methods for obtaining light-load adjustments. The chief difference in mechanical construction is in the motor element, where a mercury chamber of non-metallic compound is used in some cases, and a german silver chamber enameled on the inside in other designs. Armatures differ in that some designs use a copper disc and others a copper drum or thimble. See also section below on *Data on House-Type Meters*.

Capacity of Mercury Watthour Meters. — Direct-current mercury meters are in themselves independent of ampere capacity, as external shunts are used. Shunts giving a capacity as high as 60,000 amperes have been furnished. Meters designed for voltages up to 600 volts can be furnished without any difficulty and special meters have been supplied for 1200- and 2400-volt direct-current railway circuits.

Accuracy. — The light-load accuracy of a mercury-motor meter may vary on account of varying friction of the armature disc in the mercury, and

there is an overload droop in the accuracy curve due probably to heating of the mercury and a slight buckling of the armature disc by the current; see Fig. 1. The mercury meter also shows hysteresis errors due to the fact that iron is used in the potential circuit. For this reason the meter shows a small

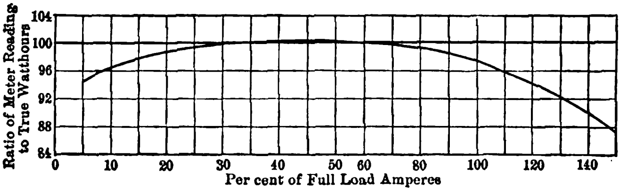

Fig. 1. Calibration Curve of Mercury Motor Meter

difference in reading when the voltage is raised and then lowered even though through a comparatively small range. Direct-current mercury-motor meters for house service should have an accuracy within the limits given above, for commutator meters.

INDUCTION WATTHOUR METER (Fig. 2).—This type of meter, the essential elements of which are illustrated in Fig. 2, has a laminated soft-iron core on which is mounted the current and potential windings. The current winding consists of a few turns of coarse wire, while the potential coil has many turns of fine wire. The flux due to the current coil is in phase with the load current, and the flux due to the potential coil is approximately in quadrature with the voltage across the load, since the potential coil is highly inductive; see *Alternating Currents*. The poles from the two windings are arranged so that the armature disc passes between them and is cut by the alternating flux due to each winding.

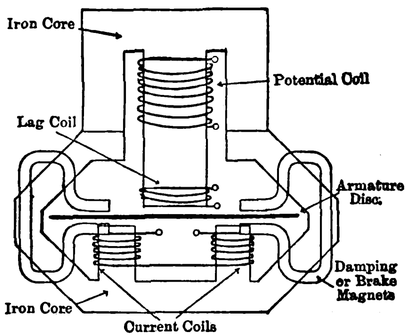

Fig. 2. Diagram of Induction Watthour Meter

Principle of Operation. — There is thus set up in the disc currents which flow about each pole in approximately concentric circles, the induced currents due to the two windings being in quadrature for a load of unity power factor (approximately only, unless a suitable phase compensator is used; see below). Part of the current induced by the current coil passes under the pole of the potential coil and therefore through a magnetic field which is approximately in phase with this current, and similarly part of the current induced by the potential coil passes under the pole of the current coil and therefore through a magnetic field approximately in phase with it; for power factors less than unity the phase difference between these currents and the magnetic fields through which they pass is the same as the difference in phase between the load current and voltage. Hence a torque is produced on the disc having an average value proportional to the load supplied through the meter; see *Electricity and Magnetism*,

Principles of. A braking system similar to that employed in the commutator type of meter (see above) causes the speed of the disc to be proportional to this average torque, and therefore the number of revolutions is proportional to the energy supplied.

Phase Compensation; Lag Coil. — In order to make the induction watthour meter record correctly, especially for power factors less than unity, it is necessary to cause the current in the potential coil to lag *exactly* 90 degrees behind the current in the current coils. A common method of securing this condition is to mount on the potential pole a short-circuited winding the resistance of which can be varied by a resistance wire soldered to the terminals.

Compensation for Friction; Starting Plate. — The light-load adjustment can be obtained by placing a short-circuited loop of copper adjacent to the potential pole so that it can be shifted in a plane at right angles to the axis of the potential coil. This loop has induced in it currents which produce a field out of phase with the flux from the potential coil, and the reaction between these two produces on the disc a slight turning moment independent of the current in the current coil. The amount of compensation can be varied by shifting the position of the light-load coil, or starting plate, as it is sometimes called.

Details of Construction. — The induction type of watthour meter is manufactured in several different types all based on the same principle of operation but differing in mechanical construction and electrical characteristics. The general type of construction is to mount the iron core and windings on a metal frame, which carries the bearings for the armature, the core and windings being combined as a single unit in some designs, whereas in others the current and potential coils with their iron cores are mounted separately on a common frame or on the meter case. The other details of design consist of mounting the registering train and damping magnets and arranging the full-load, light-load and power-factor adjustments so that they are readily accessible. See also section below on *Data on House-Type Meters*.

Polyphase Induction Meters. — The polyphase induction meter in commercial use is nothing more than two single-phase meters with a common spindle connecting the two armature discs. The measurement of power with the meter is based on the two-wattmeter method for three-wire, three-phase and quarter-phase work (*see Wattmeters*), and a slight modification of the two-element meter is used quite extensively for four-wire, three-phase work. The modification consists of making a third current circuit by adding a winding to the current coils of both elements. Such a construction is quite accurate except on badly unbalanced voltages, the most accurate arrangement for such work being three separate single-phase meters, or a single meter which contains three single-phase elements.

Capacity of Induction Watthour Meters. — Induction type meters are supplied in standard capacities ranging from 5 to 300 amperes both two- and three-wire single-phase and up to 150 amperes polyphase. Meters are generally used with self-contained potential circuits up to 600 volts, and for higher voltages and current capacities potential and current transformers are used with 5-ampere, 110-volt meters. When transformers are used, either a multiplying constant for the dial alone can be used in connection with the ratio of transformation of the transformers, or the ratio of the transformer can be included in the meter constant (see below), so that the register is direct reading with the transformers of proper ratio. For three-wire, single-phase circuits it is possible to obtain a special form of current transformer with double primary windings and a single secondary winding for connection to the meter.

Accuracy. — Modern induction watthour meters are susceptible of a higher degree of accuracy than the commutator or mercury-motor meter. Fig. 3 shows

a typical characteristic curve of an induction watthour meter. It is possible to obtain by special calibrations a combination of induction watthour meters and

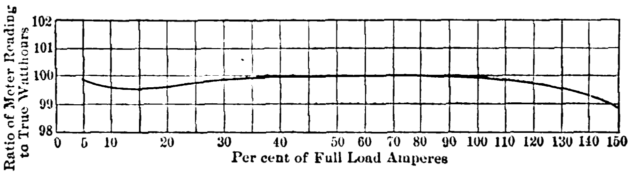

Fig. 3. Calibration Curve of Induction Watthour Meter

instrument transformers which will be accurate to within 1 per cent over the ordinary range of commercial operation.

The requirements of the New York Public Service Commission, Second District, are cited as showing accuracies which are necessary and which are met by the modern meter of the induction type:

Per cent rated current....	5	10	20	50	100	150
Maximum deviation......	3.0	1.5	2.0	2.0	1.5	3.0

The characteristic curve usually crosses the 100 per cent line at about 10 per cent and 100 per cent load; the meter can therefore be held to closer limits at these points. Meters must also be adjusted for measurement on power factors of 75 per cent and 50 per cent to within 2 per cent and 4 per cent respectively. Other requirements of accuracy can be found in the rules and regulations governing watthour meters issued by Public Service Commissions.

PREPAYMENT METERS. — Standard watthour meters are sometimes fitted with a device whereby the circuit through the meter and load is closed only upon the insertion of a coin into the device, and remains closed only until a certain predetermined amount of energy has been recorded by the meter, when the circuit again opens automatically. When so equipped the meter is called a prepayment meter. The prepayment device may be either inserted in the watthour meter, or it may be placed in a separate attachment electrically connected to the meter.

OTHER TYPES OF WATTHOUR METERS. — The descriptions of watthour meters given above refer particularly to the types manufactured in the United States. Meters of the same general types are manufactured abroad, as well as certain other types. Among the latter, the Aron clock meter is worthy of notice. In this meter the potential coils are carried on two pendulums arranged in such a manner that they swing close to the coils carrying the line current. One pendulum must be retarded as the other is accelerated and to do this connections are so made that the instantaneous polarity of the pendulum coils is always opposite while the current coils are always alike. The meter contains no iron and has no commutator and is, therefore, a very accurate measuring device, although somewhat complicated and delicate of adjustment for ordinary commercial work.

DATA ON HOUSE-TYPE METERS. — General data on house-type watthour meters of different types is tabulated below. These data are approximate, but are representative of modern practice in this country.

COMMERCIAL SINGLE-PHASE HOUSE-TYPE METERS

Item	Commutator type	Mercury type*	Induction type
Total weight, pounds	14 to 15	10 to 15	8 to 10
Height, inches	12	6.5	6
Width, inches	7.5	7	7
Depth, inches	6.5	4	4.5
Torque of motor element at full load, gram-millimeters	170	20 to 30	30 to 70
Weight of armature, grams	100	60 to 70†	11 to 30
Watts lost in potential circuit at 110 volts	5	5	1 to 3
Watts lost in current circuit at full load	5.5	1‡	0.5 to 1.5
Resistance of potential circuit, ohms	2100	....	....
Drop across shunt, millivolts	....	60 to 100	....

* The data on the mercury type also apply to meters used with shunt of any capacity.
† Partially supported by mercury. ‡ Not including shunt.

TESTING OF WATTHOUR METERS. — For laboratory tests the meter should be mounted on a firm support and set level. These conditions should also obtain when the meter is permanently mounted for service; see below under *Installation*. For laboratory tests a bank of lamps or some form of rheostat may be used for a load. The instructions given below apply to both laboratory and service tests.

Testing Instruments. — For d-c. measurements a portable ammeter and voltmeter or a standard watthour test meter may be used; for a-c. measurements an indicating wattmeter or standard watthour test meter may be used. Two instruments of different capacities should be provided, one of sufficient capacity to take care of the full load of the meter and the other of small enough capacity to read the light load with a proper degree of accuracy.

The use of a portable watthour test meter is now recognized as the standard method for testing watthour meters up to 100-ampere capacity. There is no question as regards the accuracy of this method, since it is not necessary that the load be constant. With the portable test meter, indicating instruments and stop watches are unnecessary, and personal errors in reading are thereby reduced.

When indicating instruments are employed, a reliable stop watch is essential for taking the meter speed. As a matter of safety, it is advisable to have at least two stop watches, one to serve as a check on the other in test. Too much care cannot be used in the handling of these watches and in giving them careful checks with some local jeweler's standards. In checking the stop watch it is advisable to check at different points of the dial.

Precautions. — In order to insure accuracy, d-c. meters should not be tested until they have been connected in circuit and the current on for fully 15 to 20 minutes before the test is made. This enables the heating effect of the current in the potential circuit to become constant. If the meters are tested before this consecutive readings may not check.

The testing instruments should not be allowed to measure the losses of the potential circuit of the meter under test, or vice versa. In order to avoid this the current circuits of the testing instrument should be connected in series with the meter under test, while the potential circuits of the testing instrument and of the meter under test must both be connected across the line at the same place and between the generator and the nearer instrument.

A number of revolutions should be taken so that the time of observation will be at least from 40 to 60 seconds. If the time is materially less than 40 seconds, errors in the measurement of time are probable; a time interval longer than 120 seconds is generally unnecessary.

Meter Constant. — In checking a watthour meter the speed of the disc is timed directly, since too long a time would be required to obtain a suitable reading on the dial of the instrument. The relation between the number of revolutions of the disc and the corresponding dial reading may be expressed by a constant multiplier, called the "meter constant," which depends only upon the gearing between the disc and dial. Various manufacturers express this constant differently, viz:

G. E. Meter Constant (K). — The number of watthours indicated by the register per revolution of disc; its value is marked on the disc.

Westinghouse Meter Constant (K). — Meters of Westinghouse manufacture are all adjusted for a certain speed at full load or rated capacity, which is stamped on the nameplate. Meters of modern manufacture have a speed of 25 r.p.m., and older forms of 50 r.p.m. Let N be the number of revolutions per minute corresponding to a rating of P kilowatts, then the meter constant K (the number of watt-seconds per revolution of disc), as given in the instruction books issued by this company, is $K = \frac{60{,}000\,P}{N}$.

Fort Wayne Meter Constant (C). — For type K meters the number of watthours for 36 revolutions of the disc; for types K_1, K_2, K_3 and K_4 the number of watthours for 1 revolution of the disc; the constant is marked on the disc.

Duncan Meter Constant (K). — The number of watthours per revolution of the disc; it is marked on the disc.

Sangamo Meter Constant (K). — The number of watt-seconds per revolution of disc; it is marked on the register in the mercury-motor meters and on the disc of the induction meters.

Calibration Formulas. — Let R = number of revolutions of disc, S or T = number of seconds for R revolutions, K or C = meter constant as defined above: Then the corresponding reading of the meter in watts is:

Type of meter	Meter reading in watts
G. E.	$\frac{3600\,KR}{S}$
Westinghouse	$\frac{KR}{T}$
Fort Wayne, Type K	$\frac{100\,CR}{S}$
Fort Wayne, Types K_1, K_2, K_3 and K_4	$\frac{3600\,CR}{S}$
Duncan	$\frac{3600\,KR}{S}$
Sangamo	$\frac{KR}{S}$

Per Cent Accuracy. — The meter reading as thus calculated divided by the true watts, as read on the test meter, and multiplied by 100, gives the per cent accuracy. If the per cent accuracy is less than 100 the meter is running slow; if greater than 100 it is running fast.

Adjusting Speed of Meter. — If the error in the meter reading at full load is in excess of the permissible error the speed should be altered by adjusting the position of the permanent magnets or shunting part of their flux by the means provided therefor. If the error at light load, say 5 per cent load, is excessive, the light-load adjustment should be altered to bring the speed to its correct value.

Method of Using the Portable Watthour Test Meter. — The dial of the test meter reads directly in revolutions of the meter disc. At the instant the mark on the disc of the service meter passes some arbitrarily chosen fixed point, note the dial reading of the test meter. Count the number of revolutions of the disc of the service meter, and at the instant when a suitable whole number of revolutions are completed note the dial reading of the test meter again. Let R_0 be the difference in the two dial readings of the test meter, K_0 the meter constant of the test meter, R the number of revolutions of the service meter and K its meter constant. Then the per cent accuracy is $\frac{KR}{K_0R_0}$. Of course the meter constants of the two meters must be expressed in the same manner, i.e., as watt-hours per revolution or as watt-seconds per revolution.

Testing of Three-wire and Polyphase Meters. — A three-wire meter can be tested as a two-wire meter by connecting the current coils in series. The three-wire meter will then read twice the number of watts actually supplied to the test load. By using two sets of instruments three-wire meters may be tested as actually used.

A polyphase meter is most readily tested as a single-phase meter with the current circuits in series and the potential circuits in parallel. The reading of the meter is then twice the number of watts actually supplied to the test load.

INSTALLATION OF WATTHOUR METERS. — A watthour meter should always be mounted on a firm support, should be set level, and in a place where it will not be liable to be injured or subjected to weather. A firm support is particularly essential, as continual vibration of the meter may cause it to "creep," since vibration tends to lessen the friction, and the friction compensation may then be excessive.

Meters should not be installed closer than about fifteen inches between centers, and should not be too close to conductors carrying heavy currents or in the vicinity of iron girders, posts, water or steam pipes.

A check on its accuracy should always be made after the installation of a meter.

Determination of the Capacity of Meter. — Before installing a meter some tests or accurate estimates should be made to determine the correct capacity of meter to install, as it is not advisable to run a meter continuously above its rated capacity, and too large a capacity of meter means in general poor accuracy on account of running it at light load. As a rough guide, the capacity of the meter should be one-half the combined capacity of the total connected load for ordinary residences, three-fourths the total connected load for stores and offices, and 1½ times the connected load for elevator, hoist and other motors taking a large starting current.

Precautions in Connecting Polyphase Meters. — Direct-current and single-phase alternating-current meters are comparatively simple to install and connect properly, but all installations on polyphase circuits require considerably more care and knowledge to insure proper connections. Polyphase induction meters must be connected properly in regard to phase relations, as otherwise very inaccu-

rate results will be obtained, which may not be apparent on casual examination of the meter. The manufacturers instructions should be followed very carefully and the phase relations traced out to make sure that they are correct. Practically all instrument transformers are now marked in some distinctive manner to indicate the relative instantaneous polarity of the primary and secondary windings, in order to facilitate making connections.

MAINTENANCE, REPAIRS AND DEPRECIATION. — It has been found from experience that except for the very smallest central stations or distributing companies that it is economy to maintain some systematic method of testing and caring for all meters on the system. For ordinary house service and all except very large installations, readings are taken about once a month and meters are examined and tested about once a year. Commutator-type and mercury-motor meters should undoubtedly be tested and put in good condition at least once a year; induction meters may be left for two years and perhaps more before examination, as the very slight change in accuracy is not enough to warrant the expense of the test. With very large capacity meters, where the amount of money involved is large, frequent tests are made and in some cases a monthly check and adjustment is made, and whenever necessary the meter is given a complete overhauling and a spare meter installed in the meantime.

Repairs. — In making repairs on watthour meters, it is practically necessary to maintain considerable equipment and a department for this class of work. Unless such a department is available it is preferable to return meters to the manufacturer for all repairs except those of a minor nature, such as cleaning of commutators on direct-current meters and inserting new jewels and pivots for the lower bearing. Manufacturers will usually supply repair parts at reasonable prices and quite extensive repairs can be made by the central stations if they are properly equipped. In service the attention and care necessary vary with the type of meter, induction meters being by far the cheapest to repair and under most conditions such meters require the least attention of any type.

Cost of Watthour Meters. — The ordinary house type of commutator type meters for direct-current service range in price from $12.00 to $100.00. Large-capacity switchboard commutator meters in capacities from 1000 to 10,000 amperes are sold at prices ranging from $100.00 to $500.00.

Mercury-motor meters usually have external shunts and the meter itself is of about 10 ampere capacity and sells for about $10.00 in the house type and for about $50.00 to $60.00 in the switchboard type. Prices of shunts for these meters vary, being roughly $2.00 for a 100-ampere, $20.00 for a 500-ampere, $30.00 for a 1000-ampere and $50.00 for a 5000-ampere meter.

At the present time small-capacity single-phase induction meters for house service are sold for about $7.00 to $10.00 each; larger capacities and polyphase meters vary in price from $30.00 to $50.00. The commercial forms of switchboard induction meters of large capacity sell for about $40.00 to $75.00; they are 5-ampere meters designed for use with instrument transformers, q.v.

Besides the original expense of the cost of the meter itself, there is the cost of installation and necessary attachments to mount and connect the meter. In most cases, except for very large installations, the expense of the meter itself is a small part of the total installation cost. It has been estimated that the installation cost of an ordinary house meter is about $5 or $6, not including the cost of the meter itself.

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WATTMETERS. — (*See also Ammeters; Braun Tube; Electrodynamometers; Electrometers; Voltmeters; Watthour Meters.*) A wattmeter is a device for measuring electric power and is particularly useful for measuring alternating-current power. Usually some form of electrodynamometer is used in which the current in the circuit or a known part of it is sent through one winding, the current coil; and a small current in phase with the e.m.f. across the terminals of the circuit is sent through the other winding, or potential coil; see Figs. 1 and 2. The small current in the potential circuit is drawn through a resistance as in a voltmeter. For voltages above 150 to 750 volts, potential transformers (*see Transformers, Instrument*) are usually employed. In some cases specially-constructed resistances, or multipliers, are employed in testing work up to 25,000 volts or even higher. The moving coil is generally used for the potential circuit as it is more convenient to carry through it a small current than a large one.

Principle of Operation. — In an instrument constructed and used as above described, the current in the current coil is practically proportional to the load current i at each instant and the current in the potential coil is practically proportional to the voltage v across the load at that instant. Hence the instantaneous torque acting on the moving element at this instant is proportional to vi or to the instantaneous power. The average torque acting on the moving element during each cycle of the current and voltage is then proportional to the average power. If the free period of the moving system is large compared with the period of the current and voltage, then the steady reading will also be proportional to the average power.

CONSTRUCTION OF WATTMETERS. — The electrodynamometer type of instrument is built in several forms, e.g., the original Siemens electrodynamometer in various forms in which the moving coil is brought back to zero by means of a torsion head from which the reading is obtained; deflection instruments in which the moving coil is allowed to advance through an angle until the force exerted on it by the currents in the instrument windings is balanced by the restoring torque developed in a spring or equivalent device. In the latter form of instrument the position of the moving coil is usually read by means of an attached pointer passing over a scale, graduated to read directly in watts or decimal multiples. In very sensitive instruments for laboratory use a beam of light or a telescope and scale like those used with mirror galvanometers is employed; see also *Electrodynamometers.*

In addition to the electrodynamometer type of instrument, induction wattmeters, operating on the same general principle as the induction watthour meter (*see Watthour Meters*), are sometimes used for switchboard and even for portable work, but they are not so generally useful through wide ranges of frequency and voltage as electrodynamometer instruments.

For certain limited conditions electrostatic wattmeters (*see Electrometers*) have been successfully used; in this general class of wattmeters for special purposes may be included the cathode-ray oscillograph or Braun tube (*see Braun Tube*).

Polyphase wattmeters, having two or more sets of current and potential windings contributing torque to a common shaft, are made for use on polyphase circuits; see below under *Measurement of Three-phase Power.*

Range of Different Types of Wattmeters. — Reflecting electrodynamometer wattmeters for laboratory use are made for currents up to 100 or 200 amperes and for use on circuits from a fraction of a volt up to 150 volts. Such instruments have been specially constructed for larger currents and higher voltages. See article on *Electrodynamometers.* Such instruments are chiefly useful for measurements on circuits of low voltage and of low power factor where even

the small power required to operate the instrument makes the use of ordinary portable instruments inconvenient or impossible.

Portable wattmeters are usually made to give a full-scale deflection with two-thirds of the product of their rated amperes and volts, i.e., an instrument rated as 5 amperes and 150 volts would usually have a 500-watt scale. For determining core-losses and other service where the power factor is low, instruments may be found giving full-scale reading with one-third or less of the product of the rated amperes and volts. It is customary to supply such wattmeters with a certificate showing the values of the equivalent phase angle α (*see below*) that have been determined, so that they may be used where a high degree of precision is required on low power factor work.

Portable instruments are made in sizes up to 200 amperes and 750 volts. Extra potential terminals are sometimes supplied so that the instruments may be used on more than one voltage, e.g., 125 or 250 volts. On the low power factor instruments this makes it possible to measure the watts with a good scale reading on either low or high power factor, the 250 terminal being used on a 125-volt circuit and the reading multiplied by two. Double-current windings are also supplied with a paralleling switch, thus extending the current range, and in some sizes it is possible to procure double range for volts and current in the same instrument.

Portable instruments for larger currents than 200 amperes and higher voltages than 750 have been successfully constructed but are not generally used, because the higher ranges can usually be taken care of with 5-ampere 150-volt instruments in connection with calibrated instrument transformers.

Switchboard instruments are made single phase up to 200 amperes and 750 volts, and polyphase up to 50 or 100 amperes and up to 750 volts. Higher ranges are usually taken care of by using 5-ampere instruments in connection with instrument transformers.

Switchboard wattmeters are constructed in various sized cases and forms to match other switchboard instruments.

METHODS OF CONNECTING WATTMETER TO LOAD; CORRECTION FOR POWER LOST IN WATTMETER. — A wattmeter may be connected to the load which it is to measure in either of the two ways illustrated in Figs. 1 and 2. When connected as in Fig. 1 the current through the

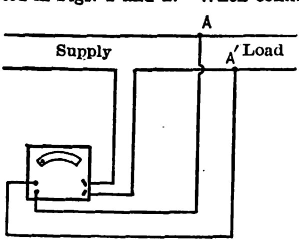


Fig. 1. Includes Loss in Potential Circuit

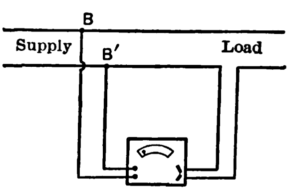


Fig. 2. Includes Loss in Current Coil

current circuit is equal to the sum (vector sum in the case of alternating currents) of the current taken by the load and that taken by the potential circuit of the wattmeter; hence the wattmeter will read the sum of the watts taken by the load and the watts lost in the potential circuit. The loss in the potential circuit is equal to $\frac{E^2}{R_p}$, where R_p is the resistance of the potential circuit and E the voltage across the load. For precise work this correction should always be made, unless the wattmeter is "compensated"; see below.

In the second scheme of connection, Fig. 2, the current through the current circuit is the same as that taken by the load, but the voltage across the potential circuit is higher than the voltage across the load by the voltage drop through the current circuit of the instrument, and the wattmeter reads too high by an amount equal to the watts lost in the current circuit. This loss is equal to R_cI^2, where R_c is the resistance of the current circuit and I the load current. This loss is usually less than the loss in the potential circuit which occurs when the first scheme of connection is used. If no correction is made for the wattmeter loss the scheme of connections shown in Fig. 2 should therefore be used. If the highest precision is required, especially when a small-capacity wattmeter is used, the connections shown in Fig. 1 should be used, and a correction should be applied. The resistance of the potential coil is usually stated on the case of the instrument.

When a voltmeter is connected to the load across AA' in Fig. 1, while the wattmeter reading is being taken, a correction should also be made, if appreciable, for the power taken by the voltmeter. Calling R_v the resistance of the voltmeter and multiplier, if any, and E the voltage across the load, the loss in the voltmeter is $\frac{E^2}{R_v}$. In most cases the potential circuit losses in the wattmeter and voltmeter are best determined by a direct measurement with the wattmeter using the test voltage and leaving the current circuit open. If the voltmeter is connected across BB' in Fig. 2, then the power lost in it is not read by the wattmeter, but the voltmeter reading is not exactly equal to the voltage across the load, but equals the load voltage plus the resistance drop (or impedance drop, added vectorially, in the case of an alternating-current circuit) through the current circuit of the wattmeter.

Compensation for Wattmeter Loss. — In the so-called compensated wattmeters a stationary compensating coil inside the instrument is connected in series with the potential coil, this compensating coil being so placed that the current through it produces on the moving element a torque equal and opposite to that produced by this same current when it passes through the current coil. A compensated wattmeter should always be connected to the circuit, as shown in Fig. 1; when so connected no correction for the loss in the wattmeter is made.

SOURCES OF ERROR. — Due to the unavoidable inductance in the potential circuit of a wattmeter, the current in the moving system is reproduced not exactly in phase with the e.m.f. impressed on the circuit; in addition there are other small inherent defects in wattmeters, such as eddy currents in windings, supports, etc., which tend to make commercial wattmeters more or less imperfect. These imperfections are more serious as the frequency of the circuit is raised and the power factor is lowered.

For precision testing under service conditions when the best obtainable accuracy is required, it is often desirable to apply corrections to wattmeter observations to eliminate the errors due to phase displacement, or its equivalent, in the potential circuit; when instrument transformers are used, additional phase displacement occurs, rendering such corrections especially important. In other cases where corrections cannot well be applied, it is useful to consider the magnitude of the error that will occur under any given conditions, in order to select suitable instruments and transformers and to arrange the loads connected to their secondaries in such a way as to produce as near as may be the desired results. In many cases indications that would otherwise be considerably in error may be made to give results of satisfactory accuracy by correcting for phase displacement in the wattmeter, and particularly in the instrument transformers.

There are many ways in which the necessary corrections may be determined and applied but the following has been found satisfactory in practical work.

Equivalent Phase Angle of Wattmeter. — All the errors in wattmeters may be considered as due to a certain phase displacement between the current in the potential coil and the impressed e.m.f. of the circuit. This equivalent angle of phase displacement may be due to a variety of causes, but can be determined for any given instrument from the following relations. Consider a wattmeter in which the current in the potential circuit lags α degrees behind the e.m.f. impressed on the potential circuit, the meter being in all other respects perfect. If such a meter is used to measure the watts P supplied by a current I at a voltage E, this current lagging behind E by an angle θ, the wattmeter will read $P_2 = EI \cos \theta_2$ watts, where $\theta_2 = \theta - \alpha$. Hence if a wattmeter is used to measure a known load of P watts with I amperes and E volts, and the wattmeter reads P_2 watts, the equivalent phase angle of the wattmeter may be defined by the relation

$$\alpha = \cos^{-1}\left(\frac{P}{EI}\right) - \cos^{-1}\left(\frac{P_2}{EI}\right).$$

This angle α is to be taken positive when the wattmeter reads too high on an inductive load and negative when it reads too low on an inductive load.

For a given wattmeter and given potential coil this equivalent phase angle is practically constant for all currents, voltages and load power factors, provided the frequency remains constant.

In the best portable and reflecting instruments α is usually + and very small. In a portable instrument built for a 125- to 150-volt circuit, when used on a 60-cycle circuit, α may have a value of + 5 minutes or + 6 minutes and possibly as much as + 10 minutes; for higher voltages and lower frequencies α is correspondingly less. For example, in a particular wattmeter this equivalent phase angle equals 3′ for the 125-volt coil and 1.5′ for the 250-volt coil, both at 25 cycles per second, and equals 7′ for the 125-volt coil and 3½′ for the 250-volt coil, both at 60 cycles per second.

Correction for Phase Angle of Wattmeter and of Instrument Transformers. — Let α = the phase angle of the wattmeter, as defined above, β = the phase angle of the current transformer, γ = the phase angle of the potential transformer (*see Transformers, Instrument*), P_2 = wattmeter reading corrected for scale error and multiplied by the product of the corrected ratios of the current and potential transformers, E = voltmeter reading corrected for scale error and multiplied by the ratio of the potential transformer, and I = ammeter reading corrected for scale error and multiplied by the corrected ratio of the current transformer. Then the apparent power factor is

$$\cos \theta_2 = \frac{P_2}{EI},$$

and the true power is

$$P = P_2 \frac{\cos (\theta_2 + \alpha + \beta + \gamma)}{\cos \theta_2}.$$

The angle θ_2 is to be taken positive when the current lags behind the voltage, and negative when the current leads. Note that α, β and γ may also be positive or negative angles. Values of the correction factor $\frac{\cos (\theta_2 + \alpha + \beta + \gamma)}{\cos \theta_2}$ are given in Tables I and II. Note carefully the conditions, stated at the head of each table, to which each table applies.

On three-phase systems (*see pp. 1824 and 1825*), when badly unbalanced, corrections should be applied separately, as above, to each wattmeter. When the circuit is balanced a single correction based on $\cos \theta_2$ for the whole circuit may be used.

TABLE I. CORRECTION FACTORS FOR PHASE ANGLE

$$\frac{\cos(\theta_2 + \alpha + \beta + \gamma)}{\cos \theta_2}$$

For lagging current when $(\alpha + \beta + \gamma)$ is positive
For leading current when $(\alpha + \beta + \gamma)$ is negative

$\alpha+\beta+\gamma$	Apparent power factor $\cos\theta_2$										
	0.10	0.15	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.80	0.90
5′	0.9855	0.9904	0.9928	0.9943	0.9953	0.9966	0.9974	0.9980	0.9985	0.9989	0.9993
10′	0.9710	0.9808	0.9857	0.9887	0.9907	0.9933	0.9949	0.9961	0.9970	0.9978	0.9985
15′	0.9565	0.9712	0.9786	0.9831	0.9860	0.9899	0.9924	0.9941	0.9955	0.9967	0.9978
20′	0.9420	0.9616	0.9714	0.9774	0.9814	0.9866	0.9898	0.9922	0.9940	0.9956	0.9971
25′	0.9276	0.9520	0.9643	0.9718	0.9768	0.9832	0.9873	0.9902	0.9925	0.9945	0.9964
30′	0.9131	0.9424	0.9571	0.9661	0.9722	0.9799	0.9848	0.9883	0.9910	0.9934	0.9956
40′	0.8841	0.9232	0.9429	0.9548	0.9629	0.9732	0.9797	0.9844	0.9880	0.9912	0.9942
50′	0.8552	0.9040	0.9286	0.9436	0.9537	0.9665	0.9747	0.9805	0.9850	0.9890	0.9928
1°	0.8262	0.8848	0.9143	0.9323	0.9444	0.9598	0.9696	0.9766	0.9820	0.9868	0.9914
10′	0.7971	0.8655	0.8999	0.9210	0.9350	0.9530	0.9645	0.9726	0.9789	0.9845	0.9899
20′	0.7681	0.8463	0.8857	0.9097	0.9257	0.9463	0.9594	0.9687	0.9759	0.9822	0.9884
30′	0.7391	0.8271	0.8713	0.8984	0.9164	0.9396	0.9543	0.9647	0.9729	0.9800	0.9869
40′	0.7101	0.8079	0.8570	0.8872	0.9071	0.9329	0.9491	0.9608	0.9699	0.9777	0.9854
50′	0.6811	0.7887	0.8427	0.8759	0.8978	0.9261	0.9440	0.9568	0.9668	0.9755	0.9839
2°	0.6521	0.7695	0.8284	0.8646	0.8885	0.9194	0.9389	0.9529	0.9638	0.9732	0.9824
10′	0.6230	0.7502	0.8140	0.8531	0.8791	0.9126	0.9337	0.9488	0.9607	0.9709	0.9808
20′	0.5940	0.7309	0.7996	0.8417	0.8697	0.9058	0.9286	0.9448	0.9576	0.9686	0.9793
30′	0.5649	0.7116	0.7853	0.8303	0.8604	0.8990	0.9234	0.9408	0.9545	0.9663	0.9778
40′	0.5359	0.6924	0.7709	0.8189	0.8510	0.8922	0.9183	0.9368	0.9514	0.9640	0.9763
50′	0.5068	0.6731	0.7566	0.8074	0.8417	0.8855	0.9131	0.9328	0.9483	0.9617	0.9748
3°	0.4778	0.6538	0.7422	0.7960	0.8323	0.8787	0.9080	0.9288	0.9452	0.9594	0.9733
10′	0.4487	0.6343	0.7277	0.7845	0.8228	0.8718	0.9028	0.9247	0.9420	0.9570	0.9717
20′	0.4196	0.6148	0.7133	0.7731	0.8134	0.8650	0.8976	0.9207	0.9389	0.9547	0.9701
30′	0.3906	0.5953	0.6989	0.7617	0.8040	0.8582	0.8924	0.9166	0.9358	0.9523	0.9685
40′	0.3615	0.5759	0.6845	0.7503	0.7946	0.8514	0.8871	0.9126	0.9327	0.9500	0.9670
50′	0.3325	0.5564	0.6701	0.7388	0.7852	0.8445	0.8819	0.9085	0.9295	0.9476	0.9654
4°	0.3034	0.5369	0.6557	0.7274	0.7758	0.8377	0.8767	0.9045	0.9264	0.9453	0.9638
10′	0.2743	0.5177	0.6412	0.7159	0.7663	0.8308	0.8714	0.9004	0.9232	0.9428	0.9621
20′	0.2452	0.4985	0.6268	0.7045	0.7569	0.8239	0.8662	0.8963	0.9200	0.9404	0.9605
30′	0.2161	0.4793	0.6124	0.6930	0.7474	0.8171	0.8609	0.8922	0.9168	0.9380	0.9590
40′	0.1871	0.4602	0.5980	0.6816	0.7380	0.8102	0.8557	0.8881	0.9136	0.9356	0.9573
50′	0.1580	0.4410	0.5836	0.6701	0.7285	0.8034	0.8504	0.8841	0.9104	0.9332	0.9557
5°	0.1289	0.4218	0.5692	0.6587	0.7191	0.7965	0.8452	0.8800	0.9072	0.9308	0.9540
10′	0.0998	0.4026	0.5548	0.6472	0.7096	0.7896	0.8399	0.8759	0.9040	0.9284	0.9524
20′	0.0707	0.3834	0.5404	0.6358	0.7002	0.7828	0.8347	0.8718	0.9008	0.9260	0.9507

TABLE II. CORRECTION FACTORS FOR PHASE ANGLE

$$\frac{\cos(\theta_2 + \alpha + \beta + \gamma)}{\cos \theta_2}$$

For lagging current when $(\alpha + \beta + \gamma)$ is negative
For leading current when $(\alpha + \beta + \gamma)$ is positive

$\alpha+\beta+\gamma$	Apparent power factor $\cos \theta_2$										
	0.10	0.15	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.80	0.90
5′	1.0145	1.0096	1.0071	1.0057	1.0046	1.0033	1.0025	1.0020	1.0015	1.0011	1.0007
10′	1.0289	1.0192	1.0142	1.0130	1.0092	1.0066	1.0050	1.0039	1.0029	1.0022	1.0014
15′	1.0434	1.0288	1.0213	1.0170	1.0138	1.0099	1.0075	1.0059	1.0044	1.0033	1.0021
20′	1.0578	1.0383	1.0284	1.0225	1.0185	1.0133	1.0100	1.0077	1.0059	1.0043	1.0028
25′	1.0723	1.0479	1.0355	1.0282	1.0231	1.0166	1.0125	1.0097	1.0074	1.0054	1.0035
30′	1.0867	1.0575	1.0426	1.0338	1.0277	1.0199	1.0150	1.0116	1.0088	1.0065	1.0041
40′	1.1156	1.0766	1.0568	1.0450	1.0369	1.0265	1.0200	1.0154	1.0117	1.0086	1.0055
50′	1.1445	1.0958	1.0711	1.0563	1.0462	1.0332	1.0250	1.0193	1.0147	1.0108	1.0069
1°	1.1734	1.1150	1.0853	1.0675	1.0554	1.0398	1.0300	1.0231	1.0176	1.0129	1.0083
10′	1.2023	1.1342	1.0995	1.0787	1.0646	1.0464	1.0350	1.0269	1.0205	1.0150	1.0096
20′	1.2311	1.1533	1.1136	1.0899	1.0737	1.0530	1.0399	1.0307	1.0234	1.0171	1.0110
30′	1.2600	1.1725	1.1278	1.1010	1.0829	1.0595	1.0449	1.0345	1.0262	1.0192	1.0123
40′	1.2888	1.1916	1.1419	1.1122	1.0920	1.0661	1.0498	1.0383	1.0291	1.0213	1.0136
50′	1.3177	1.2108	1.1561	1.1234	1.1012	1.0727	1.0548	1.0421	1.0320	1.0235	1.0150
2°	1.3466	1.2300	1.1703	1.1346	1.1104	1.0793	1.0598	1.0459	1.0349	1.0256	1.0163
10′	1.3754	1.2492	1.1844	1.1462	1.1195	1.0858	1.0647	1.0497	1.0377	1.0277	1.0176
20′	1.4042	1.2683	1.1985	1.1578	1.1286	1.0924	1.0696	1.0534	1.0406	1.0297	1.0189
30′	1.4329	1.2875	1.2126	1.1693	1.1377	1.0989	1.0745	1.0572	1.0435	1.0318	1.0201
40′	1.4617	1.3066	1.2267	1.1809	1.1468	1.1054	1.0794	1.0609	1.0463	1.0338	1.0214
50′	1.4905	1.3258	1.2409	1.1925	1.1560	1.1120	1.0844	1.0647	1.0492	1.0359	1.0227
3°	1.5193	1.3449	1.2550	1.2041	1.1651	1.1185	1.0893	1.0684	1.0520	1.0379	1.0240
10′	1.5480	1.3637	1.2690	1.2147	1.1742	1.1250	1.0942	1.0721	1.0548	1.0399	1.0252
20′	1.5769	1.3824	1.2831	1.2253	1.1832	1.1315	1.0990	1.0758	1.0576	1.0419	1.0264
30′	1.6056	1.4012	1.2971	1.2359	1.1923	1.1379	1.1039	1.0795	1.0603	1.0438	1.0276
40′	1.6344	1.4199	1.3111	1.2465	1.2013	1.1444	1.1087	1.0832	1.0631	1.0458	1.0288
50′	1.6632	1.4387	1.3252	1.2572	1.2104	1.1509	1.1135	1.0869	1.0660	1.0478	1.0300
4°	1.6920	1.4575	1.3392	1.2678	1.2194	1.1574	1.1184	1.0906	1.0687	1.0498	1.0313
10′	1.7206	1.4764	1.3532	1.2788	1.2284	1.1638	1.1232	1.0942	1.0714	1.0518	1.0325
20′	1.7491	1.4952	1.3672	1.2898	1.2374	1.1703	1.1280	1.0979	1.0742	1.0537	1.0337
30′	1.7777	1.5141	1.3811	1.3007	1.2463	1.1767	1.1328	1.1015	1.0769	1.0557	1.0348
40′	1.8062	1.5329	1.3951	1.3117	1.2553	1.1831	1.1376	1.1051	1.0796	1.0576	1.0360
50′	1.8348	1.5518	1.4091	1.3227	1.2643	1.1896	1.1424	1.1088	1.0824	1.0596	1.0372
5°	1.8634	1.5707	1.4231	1.3337	1.2733	1.1960	1.1472	1.1124	1.0851	1.0616	1.0384
10′	1.8920	1.5896	1.4371	1.3447	1.2823	1.2024	1.1520	1.1160	1.0878	1.0636	1.0396
20′	1.9205	1.6084	1.4511	1.3557	1.2913	1.2089	1.1568	1.1196	1.0906	1.0655	1.0408

Example No. 1. — Given a single-phase circuit with lagging current in which the wattmeter reading corrected for scale error and multiplied by the corrected ratios of current and potential transformers equals 24,520 watts, and the product of the voltmeter and ammeter readings, similarly corrected, equals 35,600 volt-amperes. Then $\cos \theta_2 = \frac{24,520}{35,600} = 0.688$ if the equivalent phase angle α of the wattmeter is $+4'$ and if from examination of characteristic curves the current transformer phase angle β is found to be $+48'$ and the potential transformer phase angle γ is $-10'$, then $\alpha + \beta + \gamma = +42'$, and from Table I, the correction factor is 0.9870. Whence the true power equals $24,520 \times 0.9870 = 24,201$ watts.

Example No. 2. — Given a single-phase circuit with leading current, in which the wattmeter reading, corrected as in example No. 1, equals 12,266 watts and the product of the voltmeter and ammeter readings, similarly corrected, equals 24,532 volt-amperes. Then $\cos \theta_2 = \frac{12,266}{24,532} = 0.5$. If the equivalent phase angle α of the wattmeter is $+5'$, the phase angle β of the current transformer $+2° 33'$ and the phase angle γ of the potential transformer is $+38'$, then $\alpha + \beta + \gamma = +3° 16'$ and therefore from Table II the correction factor to be used is 1.0971. Whence the true power equals $12,266 \times 1.0971 = 13,457$ watts.

Errors Due to Stray Fields. — Stray magnetic fields influence the reading of a wattmeter in much the same way as such disturbances influence the readings of other electrical instruments; see *Ammeters*. In unshielded instruments errors due to stray fields may be quite large. Considerable errors are often caused by the effect of neighboring instruments on one another. To avoid these errors some wattmeters are magnetically shielded.

ACCURACY OF WATTMETERS. — The question of the accuracy of electrical instruments is discussed in detail in the article on *Ammeters*. High-grade portable wattmeters may be obtained having a stated accuracy of 0.4 per cent of full-scale deflection, and when the readings are corrected in accordance with the calibration curve furnished with the meter, they may be relied upon to within 0.2 per cent of full-scale deflection. For example, if a single reading is 150 watts and the full-scale deflection is 500 watts, then the true watts will be between 149 and 151 watts. If the average of a number of readings is taken, this average will have a much greater degree of precision than the stated accuracy of the meter.

High-grade switchboard wattmeters may be obtained having a stated accuracy of 1 per cent of full-scale deflection. Of course the higher the degree of precision demanded the more costly is the meter.

TESTING OF WATTMETERS. — Wattmeters are usually tested by being compared with other standard wattmeters, which have in turn been examined by special methods and their behavior under various conditions of use determined. For examining the effect of low power factor on wattmeters phase-shifting transformers or generators having shifting fields are conveniently employed. By these means low power factor conditions may be produced on instruments without the necessity of troublesome adjustments of reactances, condensers, etc.

After a given instrument or type of instrument is known with certainty to give no sensible error on alternating current due to structural defects, it is usually more convenient to make subsequent checks to detect any change in the instrument by using direct current. The volts and amperes are in this case determined by means of suitable standard ammeter and voltmeter or by direct reference to potentiometer and standard cells; see *Potentiometers*.

MEASUREMENT OF THREE-PHASE POWER. — (*See also article by L. T. Robinson in General Electric Review, 1912, Vol. 15, p. 350.*) The measurement of the power supplied to a three-phase load may be effected in one or more of the following ways. The connections in each case may be made either directly or through proper instrument transformers, the latter being used for heavy currents and high voltages.

Single (One-element) Wattmeter on Three-wire System. — The current circuit of the wattmeter is connected in series with one of the mains supplying the load and the potential circuit of the wattmeter is connected between one terminal of the load and the neutral. If the neutral point of the load, or of the transformers supplying the load, is not available, a "Y-box" (see below) can be used to establish an artificial neutral. If the load is perfectly balanced (*see Alternating Currents*) the power input is then three times the wattmeter reading. However, a three-phase load is seldom sufficiently well balanced, even in the case of a three-phase motor load, to render this method of measurement an accurate one. In most practical cases it gives merely a rough approximation to the true power.

Y-Box. — The simplest form of Y-box consists of two equal non-inductive resistors connected in series, each of the free ends and the junction point being connected to a binding post. Each resistor has a resistance equal to that of the potential circuit of the wattmeter. One terminal of the potential circuit of the wattmeter is connected to the junction terminal of the Y-box and the other terminal of the potential circuit to the line wire in which the current circuit of the wattmeter is connected. The other two terminals of the Y-box are connected to the other two line wires respectively.

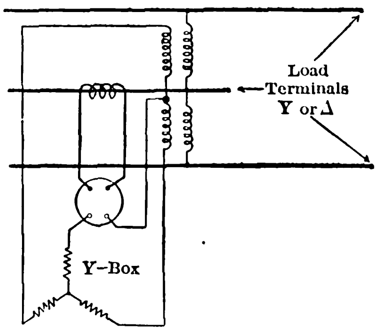


Fig. 3. Single-phase Wattmeter and Y-box for Three-phase Circuit

In the case of wattmeters designed especially for use with a Y-box, part of the resistance of the potential circuit of the wattmeter is placed in the Y-box, being connected permanently to the junction point between the other two resistors. A similar arrangement may be used as a multiplier. The connections of such a Y-box, wattmeter and instrument transformers are shown diagrammatically in Fig. 3.

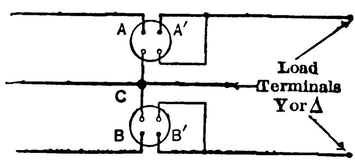


Fig. 4. Two Single-phase Wattmeters for Three-phase Circuits

Two-wattmeter Method for a Three-wire System. — The simple arrangement of two wattmeters shown in Fig. 4 will give exactly the total power in *any three-wire* circuit, provided each meter by itself gives accurate indications. Aside from the sources of error, as noted above, which may affect the accuracy of a meter on a single-phase circuit, the arrangement shown in Fig. 4 will give the true power for any condition of unbalancing, wave-form, frequency, etc. It is also immaterial whether the load be Y or Δ connected. The connections may be made directly as shown or through two

current transformers and two potential transformers, the connections being the same as in Fig. 5, except that two separate meters instead of a polyphase meter are used.

Rule for Adding or Subtracting Readings.—With the arrangement shown in Fig. 4 the total power is always the *algebraic* sum of the readings of the two wattmeters. Since a wattmeter reads only in one direction, usually to the right, the potential terminals of the two instruments must be so connected to the line that the needle of each instrument is deflected over the scale and not against the stop. For a balanced load having a power factor greater than 50 per cent, the sum of the two wattmeter readings, when the connections are thus made, gives the total power; for a power factor less than 50 per cent the difference of the two readings must be taken. When the power factor is not known, one can determine whether the readings should be added or subtracted by interchanging the two meters, leaving unaltered the potential connections to the third wire (C in Fig. 4); if the pointers of the two meters deflect in the same direction as before add the two readings, if the pointers deflect in the opposite direction (i.e., against the stop) subtract.

When the load is *balanced*, and the two wattmeters are connected, as in Fig. 4, so that each gives a positive reading, the question as to whether the power factor is above or below 50 per cent can be determined by changing the potential lead of the lower reading meter from the common connection C to the line in which the current coil of the other wattmeter is connected. If the reading thus obtained is positive, the power factor is more than 50 per cent and the readings should be added; if the reading is negative the power factor is less than 50 per cent and the readings should be subtracted.

Phase Difference between Voltage and Current in the Meter Windings.—In case the load is balanced it can be shown that if θ is the power-factor angle of the load, I the line current (current per wire) and E the line voltage (voltage between wires), then the power read by one wattmeter is

$$P_1 = EI\cos(\theta - 30),$$

and the power read by the other is

$$P_2 = EI\cos(\theta + 30).$$

The performance of the first wattmeter is the same as it would be on a single-phase load having a power factor of $\cos(\theta - 30)$ and the performance of the second meter is the same as it would be on a single-phase load having a power factor of $\cos(\theta + 30)$. These relations should be borne in mind when making any corrections for errors due to low power factor (see above).

Measurement of Power Factor by Two-wattmeter Method.—From the above relations it can be shown that the power factor of the load, when the load is balanced, may be calculated from the two wattmeter readings P_1 and P_2 by the formula

$$\text{Power factor} = 100\cos\left[\tan^{-1}\left(\sqrt{3}\,\frac{P_1 - P_2}{P_1 + P_2}\right)\right] \qquad \text{per cent,}$$

when the power factor is greater than 50%; and by the formula

$$\text{Power factor} = 100\cos\left[\tan^{-1}\left(\sqrt{3}\,\frac{P_1 + P_2}{P_1 - P_2}\right)\right] \qquad \text{per cent,}$$

when the power factor is less than 50%, P_1 and P_2 both being taken as positive in both cases and P_2 being the smaller reading.

Two-element Polyphase Wattmeter on Three-wire System. — Instead of using two separate wattmeters, as shown in Fig. 4, the two meters may be combined into a single meter with but one shaft and pointer. The connections for such a two-element meter, when instrument transformers are used, are shown in Fig. 5. For measurements where the power is badly fluctuating and especially when accompanied by low power factor, polyphase wattmeters may be more accurate and convenient. For ordinary polyphase service a polyphase wattmeter is not capable of as high accuracy as two single-phase wattmeters.

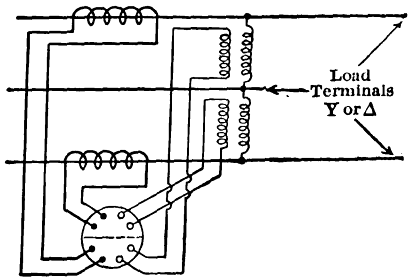


Fig. 5. Two-element Polyphase Wattmeter with Instrument Transformers

Determination of Power Factor with Two-element Wattmeter. — The two-element wattmeter reads directly the sum $P_1 + P_2$, using the notation in the previous paragraph. By reversing the connections of one set of potential terminals to the corresponding potential transformer the meter may be made to read $P_1 - P_2$. Hence by the use of a suitable reversing switch the power factor of the load may be readily determined by taking two readings; see preceding paragraph.

Three Wattmeters for Three-wire System. — As noted above, where two wattmeters, or a two-element polyphase wattmeter, are used on a three-wire system, the phase difference between the voltage and current in each meter or element may differ greatly from the power factor angle of the load. For example, in the case of a balanced load having a 50 per cent power factor, the current in one meter differs by 90° from the voltage on the potential circuit of this meter, and this meter should read zero watts, i.e., this meter operates under the worst possible conditions as regards power factor, and consequently the phase-angle error (see above), particularly when instrument transformers are used, may be quite large. It should be noted, however, that the reading of the meter or element in which the large phase difference occurs contributes proportionally less to the total reading as this phase difference approaches 90°, and consequently the percentage error of the total reading will be only a correspondingly small part of the error in this meter or element.

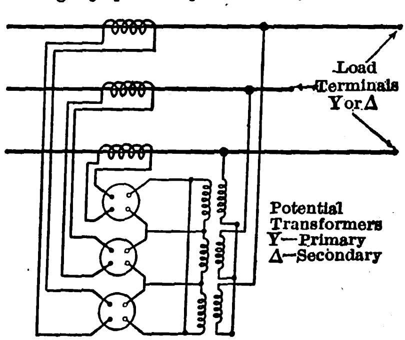


Fig. 6. Y–Δ Connections for Three Wattmeters on Three-wire System

Where the highest degree of precision is demanded, it is therefore advisable to use three meters connected as shown in Fig. 6. It should be noted, however, that for most practical purposes the very slight gain in accuracy by such an arrangement over the two-element meter does not justify the expense of the

additional meters and instrument transformers. Again, a precision measurement of power with the three-meter arrangement requires the accurate calibration and the accurate reading of three meters instead of one. The three-meter arrangement, however, possesses a practical advantage in that a failure of one or even of two of the meters may occur and still a fairly accurate determination of the power may be obtained from the meters or meter left in service.

Measurement of Power in Four-wire Circuits. — When part of the load current returns through a fourth wire, e.g., in an unbalanced three-phase system having a neutral wire, at least three wattmeters or wattmeter elements are required to obtain a theoretically accurate measurement of the power. A two-element meter, having three current circuits may be used for the measurement of power on a four-wire system, and under most circumstances will give results having as high a degree of precision as can be obtained with a two-element meter on a three-wire system.

When three meters are used, they should be connected in a manner similar to that used for two meters on a three-wire system, i.e., each of the three potential circuits should be connected between a main wire and the neutral wire, and each of the three current circuits should be connected in series with a main wire. When instrument transformers are used the primaries of the potential transformers are connected in Y, with the neutral point connected to the neutral wire, the three branches being connected to the main wires; the secondaries and the potential circuits of the meters are also connected in Y, with the meters Y-connected to the Y made by the secondaries.

Two-element Wattmeter on Four-wire System. — When used on a four-wire system the current coil of each element is divided into two sections, and one section of each current coil is connected in series with one section of the other coil, thus forming three groups of current coils. Each group is connected to a current transformer as shown in Fig. 7, which also shows the connections of the potential transformers, only two of which are required. With this arrangement the instrument measures correctly the power supplied by two branches of the system, but for the third branch the power measured is that corresponding to the current in this branch and the resultant (vector sum) of the voltages to neutral in the other two branches. Unbalancing of the system or badly distorted current and voltage waves will cause this resultant voltage to differ from the actual voltage to neutral in the third branch, producing an error in the measurement of this third of the total power. As noted above, however, this error is practically negligible under ordinary conditions unless the power must be measured with the highest degree of precision.

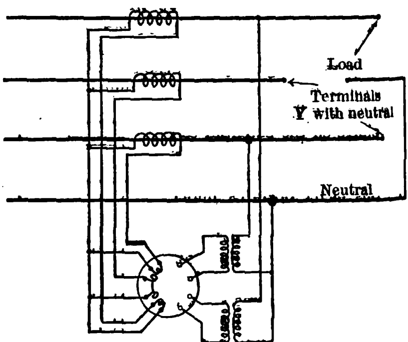


Fig. 7. Connections for Two-element Wattmeter on Four-wire System

GRAPHIC WATTMETERS. — These instruments are designed to draw on a chart a curve of power against time, without any appreciable time lag.

All instruments of this type consist of a wattmeter element and of a time element. The curve-drawing pointer is actuated either directly by the motion of the moving part of the wattmeter element, or indirectly on the relay principle through a system of contacts and solenoids. The purpose of the time element is to produce a steady motion of the paper at the point where the curve-drawing pointer (pen) is in contact with it. The clock, which turns the paper rolls, is either of the hand-wound type or of the electric self-winding type. In several types the rate of the paper feed may be varied from ¾ inch per hour to as high as 6 inches per minute.

Semi-portable forms of graphic wattmeters are on the market, but the more accurate instruments are for switchboard use only. Large errors due to friction between paper and curve-tracing pen or pointer, or due to change in level of the instrument, are not uncommon with certain types of graphic wattmeters.

COSTS, WEIGHTS AND DIMENSIONS OF WATTMETERS. — The following figures give a rough idea of the cost of wattmeters:

Indicating wattmeters	Portable	Switchboard
Single-phase:		
Low-grade	$25–$50	....
High-grade	$50–$100	$35– $50
Polyphase (two-element)	$100–$150	$60–$125

Current transformers are required when the current exceeds 200 amperes for single-phase or 50 amperes for polyphase wattmeters, and potential transformers are required when the voltage exceeds 600 volts for portable or 750 volts for switchboard wattmeters. Instrument transformers are also frequently used with lower voltages and currents.

A shielded single-phase switchboard wattmeter weighs about 10 lb., and has roughly the dimensions 8 in. by 8 in. by 5½ in. A shielded polyphase switchboard wattmeter weighs about 20 lb., and has roughly the dimensions 10 in. by 9 in. by 8 in. An unshielded single-phase wattmeter weighs about 4.5 lb. and has roughly the dimensions 7 in. by 6 in. by 4.5 in. Weights and dimensions of course vary with the make of the instrument.

BIBLIOGRAPHY. — See the Bibliographies in the articles on *Ammeters* and *Watthour Meters;* also Robinson, L. T., *Metering on Three-phase Systems,* Gen. Elec. Rev., 1912, Vol. 15, p. 350.

[L. T. Robinson.]

WAVE ANALYSIS. — (*See also Alternating Currents.*) Any sine function y of a variable t, that is, any function of the form $y = Y \sin(\omega t + \phi)$, may be plotted as a wave, as shown in Fig. 1. The constant Y is called the

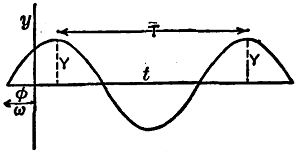


Fig. 1.

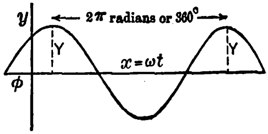


Fig. 2.

"maximum value" of the wave, the constant ϕ the "phase" of the wave, and the constant ω is called the "periodicity" of the wave. The distance between successive positive maxima (or between successive negative maxima) of such a wave is called the "period" of the wave, and is usually represented by the symbol T. The reciprocal of the period is called the "frequency," and is usually represented by the symbol $\frac{1}{T}$.

Period, frequency and periodicity are related as follows:

$$\omega = 2\pi f = \frac{2\pi}{T}.$$

The phase (ϕ) of the wave is also equal to the product of the periodicity by the distance to the left of the origin at which the wave first crosses the t axis in a *rising* direction; see Fig. 1.

In plotting a wave it is usually more convenient to take as the distance along the horizontal axis, not the value of the variable t, but the value of ωt. That is, putting $x = \omega t$, the equation for the wave may be written $y = Y \sin(x + \phi)$ and Fig. 1 then reduces to Fig. 2. When the wave is thus plotted the distance between successive positive maxima (or between successive negative maxima) is equal to 2π radians or 360°, and the phase is the distance to the left of the origin at which the wave first crosses the axis of x in the rising direction.

FOURIER'S SERIES. — Any periodic function y of a variable t may be plotted as a wave as shown in Fig. 3. If the function y and its first derivative with respect to t (that is, dy/dt) are finite and continuous for all values of t, then the function y may be represented by a series of terms of the form

$$y = Y_1 \sin(\omega t + \phi_1) + Y_2 \sin 2(\omega t + \phi_2) + Y_3 \sin 3(\omega t + \phi_3) + \text{etc.},$$

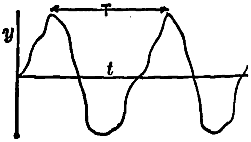


Fig. 3.

in which $\omega = 2\pi/T$, where T is the period of the given wave (i.e., the distance between successive positive, or between successive negative, maxima) and the Y's and ϕ's are constants. That is, the wave, no matter how complex, may be represented by the sum of a number of sine waves whose frequencies are even multiples of the period of the given wave.

This theorem is of great value in the theory of alternating currents (q.v.), since any alternating current or voltage satisfies the above conditions. The series is known as Fourier's Series.

As in the case of simple sine waves, it is usually more convenient in plotting

to take the distance along the horizontal, not the variable t, but the value of ωt. That is, putting $x = \omega t$, the series for y may be written

$$y = Y_1 \sin (x + \phi_1) + Y_2 \sin 2 (x + \phi_2) + Y_3 \sin 3 (x + \phi_3) + \text{etc.}$$

Fundamental and Harmonics. — The first term of the series, which has a frequency equal to that of the given complex wave, is called the fundamental. The succeeding terms, having frequencies which are multiples of the fundamental frequency, are called harmonics. For example, the third term, or the wave representing this term, has a frequency three times that of the fundamental and this term or sine wave is called the third harmonic, etc.

If the complex wave is such that the successive positive values of y for a positive half wave are numerically equal to the successive negative values of y for the succeeding negative half wave, only the *odd* harmonics occur. Such a wave is called a symmetrical wave. Voltage and current waves are usually symmetrical, and therefore contain only the odd harmonics.

DETERMINATION OF MAXIMUM VALUES AND PHASES OF THE HARMONICS BY INTEGRATION. — Consider the n th harmonic in the series

$$y = Y_1 \sin (x + \phi_1) + Y_2 \sin 2 (x + \phi_2) + Y_3 \sin 3 (x + \phi_3) + \text{etc.}$$

Plot the value of $u_n = y \sin nx$ between the limits $x = 0$ and $x = 2\pi$, and integrate the resultant curve, by planimeter, between these limits 0 and 2π. Call this area U_n. Similarly, plot the value of $v_n = y \cos nx$ between the limits $x = 0$ and $x = 2\pi$, and integrate the resultant curve, by planimeter, between these limits 0 and 2π. Call this area V_n. Then

$$Y_n = \frac{1}{\pi} \sqrt{U_n^2 + V_n^2}$$

and

$$\phi_n = \frac{1}{n} \tan^{-1} \left(\frac{V_n}{U_n} \right),$$

where the wave length of the fundamental corresponds to 360°. If the wave contains only the *odd* harmonics, as is usually the case in current and voltage waves (*see preceding paragraph*), U_n and V_n need be plotted only between the limits 0 and π. Calling U_n' and V_n' the corresponding areas, then

$$Y_n = \frac{2}{\pi} \sqrt{(U_n')^2 + (V_n')^2},$$

$$\phi_n = \frac{1}{n} \tan^{-1} \left(\frac{V_n'}{U_n'} \right).$$

This method, though cumbersome, is applicable to the determination of the maximum value and phase of any harmonic.

FISCHER-HINNEN METHOD OF ANALYSIS.— (*Elec. Journal, 1908, Vol. 5, page 386; Electrotechnische Zeitschrift, 1901, Vol. 22, page 396.*) This method is quite simple when the wave contains only the fundamental and the third, fifth and seventh harmonics. When even harmonics are present, or when there exist higher odd harmonics than the seventh, certain corrections must be applied. Since voltage and current waves usually contain only odd harmonics, and seldom contain higher harmonics than the seventh, the simple method without corrections is usually sufficiently accurate.

Waves Containing only the Third, Fifth and Seventh Harmonics. — To determine the nth harmonic (n equals 3, 5 or 7) divide the base of a half

wave into $2n$ equal parts and measure the ordinates of the wave at the beginning of each of these sections of the base. Call these ordinates y_1, y_2, y_3 . . . y_{2n}, taking the y's positive if above the base line, negative if below. y_1 will be zero, since the first section begins where the resultant wave crosses the base line.

Then the ordinates of this harmonic at the points 1 and 2 are, respectively,

$$A_n = \frac{1}{n}[(y_1 + y_5 + \ . \ . \ . \ y_{2n-1}) - (y_3 + y_7 + \ . \ . \ . \ y_{2n-3})],$$

$$B_n = \frac{1}{n}[(y_2 + y_6 + \ . \ . \ . \ y_{2n}) - (y_4 + y_8 + \ . \ . \ . \ y_{2n-2})].$$

The maximum value of this harmonic is

$$Y_n = \sqrt{A_n^2 + B_n^2},$$

and the phase angle, calling the wave length of the fundamental 360°, is

$$\phi_n = \frac{1}{n}\tan^{-1}\frac{A_n}{B_n}.$$

These formulas give the third, fifth and seventh harmonics ($n = 3$, 5 and 7 respectively). The fundamental is found by calculating

$$A_1 = -(A_3 + A_5 + A_7),$$
$$B_1 = y_0 + B_3 - B_5 + B_7,$$

where y_0 is the mid-ordinate of the half wave. Then

$$Y_1 = \sqrt{A_1^2 + B_1^2},$$
$$\phi_1 = \tan^{-1}\frac{A_1}{B_1}.$$

The equation of the given wave is then

$$y = Y_1 \sin(x + \phi_1) + Y_3 \sin 3(x + \phi_3) + Y_5 \sin 5(x + \phi_5) + Y_7 \sin 7(x + \phi_7).$$

The *effective* value of the given wave is

$$Y = \sqrt{\frac{Y_1^2 + Y_3^2 + Y_5^2 + Y_7^2}{2}},$$

and the *average* value is

$$Y_{\text{av'ge}} = \frac{2}{\pi}\left[Y_1 \cos\phi_1 + \frac{1}{3}Y_3 \cos 3\phi_3 + \frac{1}{5}Y_5 \cos 5\phi_5 + \frac{1}{7}Y_7 \cos 7\phi_7\right].$$

In using the above formulas strict attention must be paid to algebraic signs.

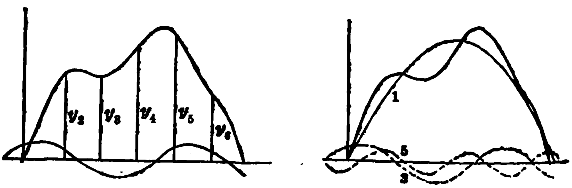


Fig. 4.

Example.—Find the third harmonic in the wave shown in Fig. 4. The values of the six ordinates are found by measurement to be 0, 676, 660, 940, 1004 and 554 respectively. Then

$$A_3 = \tfrac{1}{3}(1004 - 660) = 114.7,$$
$$B_3 = \tfrac{1}{3}(676 + 554 - 940) = 96.7,$$
$$Y_3 = \sqrt{(114.7)^2 + (96.7)^2} = 150,$$
$$\phi_3 = \tfrac{1}{3}\tan^{-1}\frac{114.7}{96.7} = 16.6^\circ.$$

Similarly, for the fifth harmonic,

$$A_5 = -92.8 \text{ and } B_5 = 37.4,$$
$$Y_5 = 100 \text{ and } \phi_5 = -13.6^\circ.$$

For the fundamental,

$$A_1 = -114.7 + 92.8 = -21.9,$$
$$B_1 = 940 + 96.7 - 37.4 = 999.3,$$
$$Y_1 = \sqrt{(21.9)^2 + (999.3)^2} = 1000,$$
$$\phi_1 = \tan^{-1}\frac{-21.9}{999.3} = -1.25^\circ.$$

Hence the complete expression for the given wave, taking as the origin the point at which the resultant wave crosses the base line in the rising direction, is

$$y = 1000\sin(x - 1.25^\circ) + 150\sin 3(x + 16.6^\circ) + 100\sin 5(x - 13.6^\circ).$$

The *effective* value is then 718 and average value 673.

Waves Containing Any Number of Harmonics, Odd or Even.—See *Electric Journal*, 1908, Vol. 5, p. 386.

[W. A. Del Mar and H. Pender.]

WAVEMETERS. — (*See also Wireless Telegraphy.*) By the wave length of an oscillation is meant the length of the wave which is set up by the oscillation in free space. Let λ be this wave length, f the frequency of the oscillation, and v the velocity of light in free space. Then

$$\lambda = \frac{v}{f}.$$

The velocity of light in free space is almost exactly 3×10^{10} (the average of the best measurements is 2.998×10^{10}) centimeters per second, or 300,000,000 meters per second, or 300,000 kilometers per second, or 984,000,000 feet per second, or 186,000 miles per second. Dividing this velocity by the frequency in cycles per second gives the wave length in the corresponding unit of length. For example, a frequency of 500,000 cycles per second gives a wave length of 600 meters or 1968 feet.

Practically all of the wavemeters, or high frequency meters, consist of a circuit containing inductance and capacity and an indicating instrument. By variation of the inductance or capacity, the wavemeter circuit can be brought to resonance (*see Alternating Currents*) with the oscillations the wave length of which is to be determined, and the indicating instrument tells when the resonant adjustment is attained. The meter may be made direct reading by utilizing a pointer moving over a properly calibrated scale.

Essential Elements of a Wavemeter. — Figs. 1, 2, 3 show the essential elements of a wavemeter. *C* is a variable capacity, *L* an inductance (which may also be variable) and *D* an indicating instrument. As an indicator we may use a low-resistance current detector (e.g., a hot wire ammeter, a thermo-

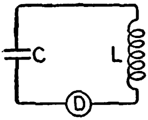


Fig. 1.

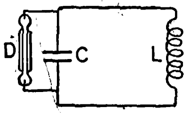


Fig. 2.

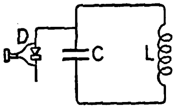


Fig. 3.

electric junction in shunt with a galvanometer, a low-resistance dynamometer, etc.), in series with the circuit as in Fig. 1, or a high-resistance detector (e.g., a Geissler tube) shunted around the condenser as in Fig. 2; or a high-resistance detector (e.g., a crystal detector in shunt with a high-resistance telephone) with but a single terminal connected to the condenser as in Fig. 3. In the latter case the capacity between the free terminal of the detector and the other terminal of the condenser is sufficient to permit the necessary current to flow through the detector.

To determine the frequency of the oscillation given out by an oscillation circuit, the wavemeter circuit is placed near the oscillation circuit in such a manner that a current is induced in the latter, and the capacity (or inductance) of the wavemeter is adjusted until the indicating instrument gives a maximum deflection or a maximum volume of sound or a maximum of light.

The wavemeter is usually calibrated by bringing it to resonance with various oscillation circuits of known frequency, the known frequency being obtained by calculation (*see Transient Electric Phenomena*), or preferably by revolving-mirror photographs of the spark of the oscillation circuit.

BIBLIOGRAPHY. — See *Wireless Telegraphy;* also Mauborgne, *Practical Uses of Wavemeters in Wireless Telegraphy*, N. Y., 1913.

[G. W. PIERCE.]

WAVES, ELECTROMAGNETIC. — (*See also Detectors, Electric Wave; Wavemeters; Wireless Telegraphy; Wireless Telephony.*) Every variation of the current in an electric circuit produces a modification of the electric and magnetic forces in the neighborhood of the circuit. The modification of the electric and magnetic forces is propagated away from the source of the disturbance with a finite velocity and constitutes an "electromagnetic wave." If the variation of the current at the source takes place in a periodic manner, the modification of the surrounding electric and magnetic fields also occurs in a periodic manner, and gives rise to a series or "train of electromagnetic waves."

Electromagnetic waves may be guided by two parallel wires as in telephony and in alternating-current transmission, in which case the waves manifest themselves chiefly on the wires or in the immediate neighborhood of the wires; or the electromagnetic waves may be set free into space, as by the use of a Hertz "oscillator," in which case they become completely disconnected from all electric conductors and travel away as "electromagnetic radiation" in the dielectric; or, as a third possibility, the waves may be sent out over a partially conducting surface, as in the case of the electromagnetic waves of wireless telegraphy traveling over the surface of the earth, in which case the energy of the waves exists chiefly in the dielectric above the earth, but at the same time there are periodic electric currents induced in the earth.

APPLICATIONS. — Electromagnetic waves in dielectric media meet with their chief direct application to wireless telegraphy and wireless telephony, while electric waves on wires are the primary transmission factors of wire telephony. The electromagnetic wave phenomena associated with the transmission of electric power must be reckoned with in consideration of power losses and abnormal resonance rises of voltage.

THEORY. — Consider any point P in space, and let the coördinates of this point referred to three mutually perpendicular axes OX, OY and OZ be x, y and z, respectively. Let the positive sense of these three axes be fixed by the convention that OX points to the right of the page, OY to the top of the page and OZ out from the page toward the reader. Let

H = magnetic intensity at P,
B = flux density of magnetic induction at P,
F = electric intensity at P,
D = flux density of electrostatic induction at P,
u = current density at P,
ρ = volume density of electric charge at P,
k = dielectric constant at P,
μ = magnetic permeability at P,
c = velocity of light in centimeters per second in empty space $= 3 \times 10^{10}$,
t = time in seconds.

In addition, let all magnetic quantities be measured in abs. c.g.s. electromagnetic units and all electrical quantities in abs. c.g.s. electrostatic units, and let the subscripts x, y, z refer to the components of the various quantities in the direction of the three axes. Then, for any non-crystalline medium surrounding P, Maxwell (*Electricity and Magnetism*) shows that the following relations hold

$$\left.\begin{aligned} \frac{1}{c}\frac{\partial D_x}{\partial t} + \frac{4\pi u_x}{c} &= \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}, \\ \frac{1}{c}\frac{\partial D_y}{\partial t} + \frac{4\pi u_y}{c} &= \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}, \\ \frac{1}{c}\frac{\partial D_z}{\partial t} + \frac{4\pi u_z}{c} &= \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}, \end{aligned}\right\} (1) \qquad \left.\begin{aligned} -\frac{1}{c}\frac{\partial B_x}{\partial t} &= \frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}, \\ -\frac{1}{c}\frac{\partial B_y}{\partial t} &= \frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}, \\ -\frac{1}{c}\frac{\partial B_z}{\partial t} &= \frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} \end{aligned}\right\}. \quad (2)$$

$$\frac{\partial D_x}{\partial x}+\frac{\partial D_y}{\partial y}+\frac{\partial D_z}{\partial z}=4\pi\rho. \quad (3) \qquad \frac{\partial B_x}{\partial x}+\frac{\partial B_y}{\partial y}+\frac{\partial B_z}{\partial z}=0. \quad (4)$$

$$\left.\begin{aligned} D_x &= kF_x, \\ D_y &= kF_y, \\ D_z &= kF_z. \end{aligned}\right\} \quad (5) \qquad \left.\begin{aligned} B_x &= \mu H_x, \\ B_y &= \mu H_y, \\ B_z &= \mu H_z. \end{aligned}\right\} \quad (6)$$

Wave Equation in Dielectric. — Assuming that the medium at the point x, y, z, is insulating, uncharged and at rest, and is non-crystalline, it may be shown by elimination among the equations (1) to (6) that each component of electric force and each component of magnetic force satisfies an equation of the form

$$\frac{k\mu}{c^2}\frac{\partial^2 M}{\partial t^2}=\frac{\partial^2 M}{\partial x^2}+\frac{\partial^2 M}{\partial y^2}+\frac{\partial^2 M}{\partial z^2}, \quad (7)$$

in which M is a general symbol for either of the quantities $H_x, H_y, H_z, F_x, F_y, F_z$. Equation (7) is called the "wave equation." Equation (7) may be simplified by restricting the disturbance to a plane distribution.

Plane Wave. — The assumption that each of the force components (indicated by the general symbol M) is a constant from point to point of a plane perpendicular to any given direction OP (Fig. 1) reduces the problem to a study of a plane wave. Let the position of the plane be designated by its distance x from an arbitrary origin O. Equation (7) then reduces to the form

$$\frac{k\mu}{c^2}\frac{\partial^2 M}{\partial t^2}=\frac{\partial^2 M}{\partial x^2}. \quad (8)$$

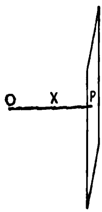


Fig. 1.

This equation is called the "plane-wave equation."

Solution of the Plane-wave Equation. — The general solution of equation (8) is of the same form as the solution of the equations of a transmission line (see p. 1678) having no resistance and no leakage, i.e., in which there is no attenuation. Without finding the form of the functions several important properties of the electric waves may be deduced. Some of these properties are here enumerated.

Velocity. — In any homogeneous dielectric medium at rest the electric and magnetic forces each travel with the velocity

$$v=\frac{c}{\sqrt{\mu k}}.$$

In empty space, $k = 1$ and $\mu = 1$; therefore in empty space the velocity

$v_0 = c$, the velocity of light.

Index of Refraction. — If a plane electric wave passes from empty space into a homogeneous medium of dielectric constant k (the medium being non-magnetic, i.e., $\mu = 1$) the velocity is changed from $v_0 = c$ to $v=\frac{c}{\sqrt{k}}$; therefore the index of refraction of the second medium for the electric waves is

$$n=\frac{v_0}{v}=\sqrt{k}.$$

Orientation of Electric and Magnetic Force. — In a homogeneous dielectric medium the electric and magnetic forces in a polarized electromag-

netic wave are perpendicular to each other and perpendicular to the direction of propagation of energy.

Reflection. — Perfect conductors are perfect reflectors of electromagnetic waves. For electromagnetic waves of a frequency of one million or more cycles per second, all ordinary metals are practically perfect reflectors.

Energy. — In a non-crystalline dielectric medium the energy of the wave as it travels forward is half electrostatic and half magnetic.

ELECTROMAGNETIC WAVES PRODUCED BY A HERTZ OSCILLATOR. — When the electric current is flowing in the direction from B to A in Fig. 2, the lines of electric force, such as are represented by the shaded loops, are in planes through the axis of the oscillator, while the lines of magnetic force are circles in planes perpendicular to the axis of the oscillator.

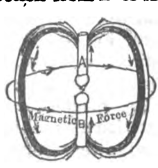


Fig. 2.

By treating the oscillator as a doublet, Hertz (*Electric Waves*) has computed the field of force around the doublet and has constructed the lines of induction from such an oscillator. A simplification of Hertz's diagram, in which only a few of the lines are drawn, is given in Fig. 3. This diagram represents approximately the condition of things at a particular instant. Up near the oscillator the lines of electric induction terminate on a plus charge on one half and a negative charge on the other half of the oscillator. Out farther from the oscillator the lines of electric induction are closed loops. These loops have been produced by successive oscillations of the current on the oscillator and have been liberated from the oscillator and are moving freely away. As these loops move away from the oscillator, they elongate and the force becomes less intense, but the width of the loops in the direction PP remains practically the same, so that if a receiver be placed in a fixed position in the equatorial plane the inductive action of the loops, as they pass, changes continuously from one direction to the other with a period equal to that of the oscillator. Associated with these lines of electric induction, and at right angles to them, are the lines of magnetic induction; and both the electric and the magnetic induction act together on the receiving resonator to produce electromotive force in it. The best position of a linear resonator for receiving the waves is parallel to the electric induction from the oscillator.

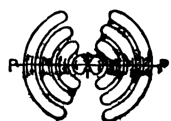


Fig. 3.

At a considerable distance from the oscillator the electric and magnetic inductions are in intensity inversely proportional to the distance from the oscillator. The factor of proportionality is, however, different along radial vectors to the oscillator making different angles with the axis of the oscillator.

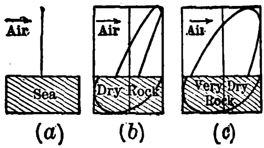
Very near the oscillator the electric and magnetic forces obey a different law with respect to distance.

PROPAGATION OF ELECTROMAGNETIC WAVES OVER A PERFECT CONDUCTOR. — The oscillation of an oscillator grounded to a plane perfect conductor of infinite extent and the radiation from such an oscillator is the same as the oscillation and radiation of the upper half of a symmetrical Hertz oscillator. Fig. 4 shows the lines of electric induction from the perfectly grounded oscillator. Since a line of electric induction must be a closed loop or must terminate upon a charge, there are induced in the surface of the conductor a series of positive and negative charges as shown in the figure. These positive and negative charges move away with the loops of induction and constitute cur-

Fig. 4.

rents in the surface of the conductor. The earth, where perfectly conductive, serves as a guiding conductor for the waves.

PROPAGATION OVER POOR CONDUCTORS. — A comparison of the conditions at points where the earth is a good conductor with conditions where it is a poor conductor has been made in a theoretical treatment of the subject by Professor Zenneck (*Ann. der Phys.*, *23, p. 846, 1907*). The result arrived at by Zenneck is typified in Fig. 5. Where the earth is a good conductor (for example, *sea water*) the electric force in the air above the water is perpendicular to the surface, as is shown in diagram (*a*), Fig. 5. This figure represents the direction of the electric induction (which in air is the same as the electric intensity). The magnitude of the induction at a stationary position is obtained by supposing the length of the line of diagram (*a*) to vary sinusoidally with the period of the waves. There would thus arrive at a station at sea a train of electric waves, and the force would be vertical and would go through a series of continuous oscillations. The force in the sea water below the surface would be zero.



(*a*) (*b*) (*c*)
Fig. 5.

Let us next suppose the waves to be traveling over a surface of a particular kind of dry rock. Professor Zenneck finds for this case that the electric force in the air above is by no means perpendicular to the surface and that not only the magnitude changes but the direction of the intensity also changes as the wave progresses. There is a similar intensity in the earth below, but different in magnitude and inclination. The condition is represented by the two semi-ellipses of diagram (*b*), Fig. 5. The electric intensity is obtained in magnitude and direction from (*b*) by taking the radius vector drawn from the center of the ellipse to a point moving around the ellipse with the frequency of the wave. The length and direction of the radius vector so drawn would represent the changing magnitude and direction of the electric intensity at a particular point with change of time. It is seen that such a radius vector has in general a vertical component and a horizontal component. The latter produces currents in the earth, and, if the earth is a poor conductor, causes absorption of energy.

Diagram (*c*) of Fig. 5 is obtained with a different quality of rock; that is, a rock of different conductivity of different specific inductive capacity, since both these constants enter into the treatment of the problem.

PROPAGATION OF WAVES ALONG WIRES. — See *Transient Electric Phenomena*.

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[G. W. PIERCE.]

WEIGHTS (MASSES) OF MATERIALS. — (*See also Mechanics, Principles of.*) The density of any substance is the mass of that substance per unit volume. Or, using weight in the ordinary sense as equivalent to mass, the density may also be defined as the *weight* per unit volume. The numerical value of the density of any substance depends upon the unit in which the mass or weight is expressed and also upon the unit of volume used; see *Units and Conversion Factors.* However, it is quite common to state the density of a substance in grams per cubic centimeter, without naming the units, since when so expressed the density is numerically equal (practically) to the specific gravity.

The "specific gravity" of a substance is defined as the ratio of the weight (mass) per unit volume of that substance to the weight (mass), expressed in the same unit of an equal volume of water. To make such a statement exact the temperature of the water should be specified. There is no general agreement as to the temperature of reference, though water at 0° C. is commonly taken as the reference temperature. For gases, air at 0° C. and 760 mm. mercury pressure is frequently taken as the reference substance instead of water.

Variation of Density of Water with Temperature. — The following table gives the results of measurements by Thiesen, Scheel and Diesselhorst (*Landolt, Börnstein and Roth, Physikalisch-chemische Tabellin, 1913*).

DENSITY OF WATER; GRAMS PER CU. CM.

Degrees Cent.	0	1	2	3	4	5	6	7	8	9
0	0.99987	0.99993	0.99997	0.99999	1.00000	0.99999	0.99997	0.99993	0.99988	0.99981
10	0.99973	0.99963	0.99953	0.99940	0.99927	0.99913	0.99897	0.99880	0.99862	0.99843
20	0.99823	0.99802	0.99780	0.99756	0.99732	0.99707	0.99681	0.99654	0.99626	0.99597
30	0.99567	0.99537	0.99505	0.99473	0.99440	0.99406	0.99371	0.99336	0.99299	0.99262
40	0.99224	0.99186	0.99147	0.99107	0.99066	0.99025	0.98982	0.98940	0.98896	0.98852
50	0.98807	0.98762	0.98715	0.98669	0.98621	0.98573	0.98525	0.98475	0.98425	0.98375
60	0.98324	0.98272	0.98220	0.98167	0.98113	0.98059	0.98005	0.97950	0.97894	0.97838
70	0.97781	0.97723	0.97666	0.97607	0.97548	0.97489	0.97429	0.97368	0.97307	0.97245
80	0.97183	0.97121	0.97057	0.96994	0.96930	0.96865	0.96800	0.96734	0.96668	0.96601
90	0.96534	0.96467	0.96399	0.96330	0.96261	0.96192	0.96122	0.96051	0.95981	0.95909
100	0.95838	0.95765	0.95693	.....	.....	.....	.....	.....	.....	.....

* Example: The density of water at 33° C. is 0.99473.

Weights per Cubic Foot and Specific Gravity. — In the following table are given the values of the density in pounds per cubic foot of the more commonly used substances. The specific gravity, or density in grams per cubic centimeter, corresponding to any weight per cubic foot w is equal to $w/62.43$; for the conversion factors necessary to convert these figures into densities for other units of mass and volume, see *Units and Conversion Factors.*

SPECIFIC GRAVITY AND POUNDS PER CUBIC FOOT OF VARIOUS MATERIALS AT ROOM TEMPERATURES

Specific Gravities all referred to water at 0° C.

(*See References at end of table*)

Material	Lb. per cu. ft. From	Lb. per cu. ft. To	Average spec. grav.
Air* (2)	0.0809	...	0.00129
Acetylene gas* (2)	0.0733	...	0.00117
Aluminum, cast (1)	160	161	2.57
" wire (6)	168	...	2.70
Ammonia* (2)	0.0482	...	0.000771
Antimony (1)	414	...	6.64
Asbestos (2)	125	175	2.40
Asphaltum (1)	69	94	1.30
Basalt (3)	176	181	2.86
Bismuth (2)	604	618	9.78
Brass (1)	511	542	8.45
Brick, red (3)	111	128	1.92
" fire (3)	110	...	1.76
Bronze (1)	545	555	8.80
Carbon (2)	125	144	2.15
" dioxide* (2)	0.124	...	0.00199
" monoxide* (1)	0.0782	...	0.00125
Caoutchouc (1)	57	62	0.955
Cement, loose (1)	72	105	1.42
" set (1)	168	187	2.85
Charcoal (1)	17	35	0.421
Clay, hard (3)	129	133	2.10
" soft (3)	118	...	1.89
Coal, anthracite (5)	81	106	1.50
" anthracite piled loose (5)	47	58	0.84
Coal, bituminous (5)	78	88	1.33
" bituminous, piled loose (5)	44	54	0.79
Coal, lignite (3)	52	...	0.83
Cobalt (2)	530	563	8.77
Coke (1)	62	105	1.34
" piled loose (5)	23	32	0.45
Concrete, 1:2:4 (3)	146	...	2.34
" 1:1½:3 (3)	139	...	2.23
" 1:3:6 (3)	156	...	2.50
Copper, cast (1)	549	558	8.87
" wrought (1)	552	558	8.90
" wire (1) (6)	555	558	8.89†
Cork (5)	15.6	...	0.25
Ebonite (1)	72	...	1.15
Flint (1)	162	...	2.61
German silver (1) (52 Cu+26 Zn+22 Ni)	527	...	8.44
Glass, common (1)	150	175	2.6
" flint (1)	180	280	3.7
Gold, cast (2)	1200	...	19.3
Granite (1)	125	187	2.5
Gravel (5)	90	147	1.9
Gutta percha (5)	61.1	...	0.980
Gypsum or plaster of Paris (5)	142	143	2.28
Hydrogen* (2)	0.00562	...	0.0000900
Ice (1)	55	57	0.895
Iridium (2)	1399	...	22.4
Iron, pure (1)	490	492	7.86
" gray cast (1)	439	445	7.08
" white cast (1)	473	482	7.65
" wrought (1)	487	492	7.85
" steel (1)	474	494	7.76
Lead (1)	710	...	11.4
Leather, dry (1)	54	...	0.86
" greased (1)	64	...	1.02
Lime (5)	53	75	1.03
Limestone (3)	156	162	2.55
Loam (3)	65	88	1.23
Marble (1)	157	177	2.68
Masonry (5)	100	165	2.12
Mercury at 0° C. (1)	849	...	13.6
Mercury at 20° C. (1)	846	...	13.5
Mica (1)	165	200	2.9
Molybdenum (2)	524	536	8.50
Mortar, hard (5)	103	...	1.75
Muck (3)	40	74	0.915
Mud (5)	80	130	1.68
Nickel (1)	540	550	8.75
Nitrogen* (2)	0.0782	...	0.00125
Nitrous oxide* (2)	0.0838	...	0.00134

* At a temperature of 0° C. and a pressure of 760 mm. mercury.

† This value has been adopted internationally as representing the average density at 20° C.; see reference.

SPECIFIC GRAVITY AND POUNDS PER CUBIC FOOT OF VARIOUS MATERIALS AT ROOM TEMPERATURES

Specific Gravities all referred to water at ° C.

(See References at end of table)

Material	Lb. per cu. ft.		Average spec. grav.
	From	To	
Oil, cotton-seed (1)	60.2	....	0.962
" gasoline (1)	41	43	0.675
" lard (1)	57.4	....	0.920
" linseed (1)	58.8	....	0.942
" mineral, lubricating (1)	56.2	57.7	0.912
Oil, petroleum (1)	54.8	....	0.878
" turpentine (1)	54.2	....	0.873
" whale (1)	57.3	....	0.918
Osmium (2)	1400	....	22.5
Oxygen* (2)	0.0895	....	0.00143
Palladium (2)	686	749	11.4
Paper (1)	44	72	0.92
Paraffine (1)	54	57	0.89
Pitch (1)	67	....	1.07
Platinum (1)	1320	1350	21.4
Porcelain (1)	143	156	2.4
Pumice stone (1)	23	56	0.63
Quartz (1)	165	....	2.65
Rhodium (1)	686	755	11.5
Salt (5)	50	70	0.965
Sand (5)	90	120	1.68
Sandstone (1)	124	200	2.6
Selenium (2)	300	....	4.8
Silver (1)	650	657	10.5
Slate (1)	162	205	2.85
Snow, fresh fallen (5)	5	12	0.136
" wet compact (5)	15	50	0.520
Soapstone (1)	162	175	2.7
Steel (*see Iron*)	...	....	....
Sulphur (1)	120	130	2.05
Tantalum (2)	1040	....	16.7
Tar (5)	62.4	....	1.00
Tile, hollow terra cotta, building block (3)	26	38	0.51
Tile, flat and segmental arches (3)	31	45	0.608
Tile partitions† (3)	12	26	....
Tin (1)	455	....	7.29
Trap rock (5)	187	190	3.02
Tungsten (1)	1160	1190	18.8
Turf (5)	20	30	0.400
Water, max. density (2)	62.4	....	1.00
" sea (5)	64.0	64.3	1.03
Wax, bees (1)	60.5	....	0.965
Wood, ash (4)	45	47	0.737
" butternut (4)	28	....	0.448
" cedar (4)	37	38	0.600
" chestnut (4)	38	41	0.633
" cypress (4)	32	37	0.553
" elm (4)	35	36	0.569
" fir (4)	34	35	0.553
" hemlock (4)	25	29	0.432
" hickory (4)	53	58	0.890
" lignum vitae (4)	78	83	1.29
" mahogany (4)	32	40	0.577
" maple (4)	49	50	0.793
" oak (4)	37	56	0.745
" pine, white (4)	24	25	0.392
" " yellow (4)	34	45	0.633
" poplar (4)	24	27	0.424
" red wood (4)	30	....	0.481
" spruce (4)	25	32	0.457
" walnut (4)	38	45	0.649
Zinc (1)	394	447	7.10

* At a temperature of 0° C. and a pressure of 760 mm. of mercury.
† Including air spaces.

REFERENCES. — (1) *Smithsonian Physical Tables,* (2) *Physikalisch-Chemische Tabellen,* Landolt-Börnstein-Roth, Berlin, 1912; (3) *Investigation of Weights of Building Material,* Thesis, Mass. Inst. of Tech., 1913, Orr, S. W., and Mutersbaugh A. M.; (4) *Publications of Forestry Division, U. S. Dept. of Agriculture,* Bulletin No. 10, 1893; Circular No. 32, 1904; Circular No. 115, 1907; (5) Trautwine, J. C., *Civil Engineers' Pocket Book,* Philadelphia, 1911; (6) *Copper Wire Tables,* Cir. No. 31, Bureau of Standards, 1914.

[H. PENDER and R. G. HUDSON.]

WIND PRESSURE. — (*See also Structures, Simple; Transmission Lines.*) Wind pressure is a subject upon which little exact information exists, although many experiments have been made and much study given to the subject by engineers and scientists. Among the unsettled questions are:

a. The relation between pressure and velocity.
b. The variation of pressure with size and shape of exposed plane surfaces.
c. The direction and intensity of pressure upon non-vertical surfaces.
d. The intensity of pressure upon non-planar surfaces.
e. The total pressure upon a number of parallel bars or other members placed side by side.
f. The decrease of pressure upon leeward surfaces.
g. The lifting power of the wind.

RELATION BETWEEN INDICATED (U. S. WEATHER BUREAU) WIND VELOCITY AND ACTUAL VELOCITY.—The indications of the anemometers used by the U. S. Weather Bureau do not give the *actual* wind velocity, but give values considerably higher than the actual velocities, as shown in the following table:

RELATION BETWEEN INDICATED AND ACTUAL WIND VELOCITY

Indicated Velocity, mi. per hr.	Actual Velocity, mi. per hr.	Indicated Velocity, mi. per hr.	Actual Velocity, mi. per hr.
10	9.6	60	48.0
20	17.8	70	55.2
30	25.7	80	62.2
40	33.3	90	69.2
50	40.8	100	76.2

In the U. S. Weather Bureau reports the indicated and not the actual wind velocities are given. However, as the anemometers used give the *average* velocity for several minutes, the instantaneous velocities due to sudden gusts may be considerably greater than the indicated velocities; the indicated velocity probably more nearly represents the "gust" velocity than the actual average velocity. In all calculations of maximum wind pressure it is therefore recommended that the *indicated* velocity be used.

RELATION BETWEEN PRESSURE AND VELOCITY. — The pressure varies about as the square of the velocity, the results given by different experimenters for the pressure due to a *normal wind on a plane surface* ranging from

$$P = 0.005\,V^2 \quad \text{to} \quad P = 0.0032\,V^2,$$

where P = pressure in pounds per square foot,
V = actual wind velocity in miles per hour.

The latter of these values represents the results of unusually careful experiments by Stanton (*see Minutes of Proceedings of the Institute of Civil Engineers, Vols. 156 and 171*) upon the intensity of pressure on plates varying in size from 25 to 100 square feet and is probably more nearly correct than the higher value. In the Stanton formula the values are reduced to correspond to a temperature of 60° F. and an atmospheric pressure of 14. 7 pounds per square inch.

The influence of size and shape of exposed surface is an important question and is not well understood, although it is known that the resultant pressure on a large surface may be taken as less per square foot than that on a small

surface, since the maximum intensity of the wind is due to gusts of comparatively small cross-section.

Formulas for Pressure on Plane Surfaces when Wind is not Normal. — The pressure upon vertical plane surfaces may be taken as normal to the surface and equal in intensity to the assumed wind pressure. Upon surfaces which are not vertical, the pressure is usually considered to be normal to the surface but lower in intensity than upon vertical surfaces. The variation in pressure with respect to the slope is not well understood and a number of empirical formulas are in use, among which are the Duchemin formula

$$P_n = P\frac{2 \sin i}{1 + \sin^2 i},$$

and the Hutton formula

$$P_n = P\,(\sin i)^{(1.84 \cos i - 1)},$$

where P = intensity of normal pressure upon the vertical surface,
P_n = intensity of normal pressure upon the given surface,
i = angle made by surface with the horizontal.

The following theoretical formula results from the assumption that the wind always blows in horizontal lines, and that if the pressure be resolved into normal and tangential components, the tangential component may be neglected:

$$P_n = P \sin^2 i.$$

This formula gives lower values than the empirical formulas and probably gives too low results since it makes no allowance for the reduction in pressure on the leeward side which is known to occur, and which may in part be attributed to the influence of the tangential component. It should also be noted that the wind does not blow uniformly in horizontal lines but may deviate considerably from the horizontal.

The values given by these three formulas are tabulated for comparison, using an assumed value of 30 pounds per square foot for P. In the absence of further experience upon this phase of wind pressure it would seem wise to use one of the empirical formulas instead of the theoretical one. The Hutton formula is used quite generally by structural engineers in England and the United States.

Pressure on Non-Planar Surfaces. — The pressure upon non-planar surfaces is important in the case of chimneys, standpipes, and other similar objects.

Upon the same assumptions as made in the preceding paragraph it may be demonstrated that theoretically the pressure on a cylinder is two-thirds of the total pressure on a plane diametrical section. This value is quite generally used. The pressures thus obtained lack experimental proof but are probably more nearly correct than the pressure obtained by the same method upon plane surfaces.

Effect of Reduction of Pressure on Leeward Side. — The pressure upon the windward side of an exposed surface is a function of the density and velocity of the air currents. The pressure on the leeward side is also a function of the shape of the surface, and has been shown by numerous experiments to be less than the static pressure of the air current. The resultant total pressure upon a surface is in consequence not only a function of the direct pressure on the windward side, but also of the pressure on the leeward side, which in turn is a function of the form of the surface. No algebraic formula can be given which will give the pressure on surfaces of varying shape with any considerable degree of precision.

Wind Pressure on Wires. — H. W. Buck (*Trans. Int. Elec. Cong., St. Louis, 1904, Vol. 2, p. 318*) gives the following formula for the pressure due to a normal wind on a stranded wire:

$$P = 0.0025V^2,$$

WIND PRESSURE IN POUNDS PER SQUARE FOOT

$P = 30$ pounds per square foot

Angle i, degrees	Theoretical $P \sin^2 i$	Duchemin $P \frac{2 \sin i}{1 + \sin^2 i}$	Hutton $P(\sin i)^{(1.84 \cos i - 1)}$
5	0.0	5.2	3.9
10	0.9	10.1	7.3
15	2.0	14.6	10.5
20	3.5	18.4	13.7
25	5.3	21.5	16.9
30	7.5	24.0	19.9
35	9.9	25.8	22.6
40	12.4	27.3	25.1
45	15.0	28.3	27.0
50	17.6	29.0	28.6
55	20.1	29.4	29.7
60	22.5	....	....
65	24.6	Above 60 deg.	Above 60 deg.
70	26.4	use 30 lb.	use 30 lb.
75	28.0		
80	29.1		
85	29.7		
90	30.0		

where P is the pressure in pounds per square foot of projected area of wire (length times diameter) and V is the velocity in miles per hour. This formula is based upon tests made on a 950-foot span at Niagara Falls; the wind velocities were measured by a U. S. Weather Bureau anemometer corrected to give actual average velocities.

PRACTICAL RULES FOR WIND PRESSURE ALLOWANCE.— The many uncertainties connected with wind pressure make worthless the attempts to specify with precision its magnitude and direction. In the lack of additional information and further theoretical studies there seems to be no reason for deviating from the common rules which have been used for many years with satisfactory results.

Bridges.—The portal, vertical and horizontal bracing are usually proportioned for a wind pressure of 30 lb. per sq. ft. on the surface of the applied load, and on the exposed surfaces of the floor system and both trusses. The pressure on the applied load is considered as a moving live load, and the other pressure as a dead load. For structures of ordinary spans the wind stresses are computed upon the unloaded structure for a pressure of 50 lb. per sq. ft. In the design the maximum stress computed by either of the above methods is used.

Buildings.—For wind pressure on roofs and buildings it is common practice to allow 30 lb. per sq. ft. acting horizontally upon the sides and ends of buildings, or on the vertical projection of roofs. It is also very important to figure the wind stresses on the steel frame considering it as an independent structure without walls, floors or partitions, since failures often occur in erection.

Transmission Poles and Towers.—For transmission towers a pressure as low as 13 lb. per sq. ft. has been used. This is perhaps warranted by the

fact that such towers are comparatively low and not exposed to the highest wind pressure. In the report of the Joint Committee on Overhead Line Construction (*Trans. N.E.L.A., 1911, Vol. 2, p. 521*) the following is recommended: "The wind pressure on the poles, or towers, shall be assumed at 13 lb. per sq. ft. of the projected area of solid or closed structures and 1½ times the projected area of latticed structures."

Wire Spans. — In the report of the Joint Committee just referred to it is recommended that the spans be designed for "a wind pressure of 8.0 lb. per sq. ft. on the ice-covered diameter (ice coating ½ inch in thickness), at a temperature of 0° F."

LIFTING POWER OF WIND. — In the case of a very rapid reduction of atmospheric pressure, as in a tornado, it is often observed that building roofs are lifted and walls blown outward. This phenomenon is due to the air in the building which is under more or less restraint, changing pressure less rapidly than the outside air and thereby producing a difference in pressure. This lifting action doubtless occurs to a greater or less degree whenever the external pressure is reduced, and should be guarded against by anchoring roofs securely to the walls.

BIBLIOGRAPHY. — *Minutes of the Proceedings of the Institute of Civil Engineers*, Vol. 156, 1903, and 171, 1907; Eiffel, *La Resistance de L'Air et L'Aviation*, Paris, 1911 (English Translation, by Hunsaker, Boston and London, 1913); Buck, H. W., *Trans. Int. Elec. Cong.*, St. Louis, 1904, Vol. 2, p. 318.

[C. M. SPOFFORD.]

WIRELESS TELEGRAPHY.—(*See also Detectors, Electric Wave; Wave-meters; Waves, Electromagnetic; Wireless Telephony.*) In addition to the ordinary commercial and social correspondence between stations on land and sea, well-organized systems of time signals, storm warnings, and facilities for the collection and dissemination of weather statistics are maintained by the various governments, through the agency of wireless telegraphy.

Whenever an oscillating electric or magnetic field is set up electromagnetic waves are radiated into space. The amount of energy radiated will depend upon the form of the radiating circuit. It has been found that the best practicable form of radiating circuit consists of one or more vertical wires, from 50 to 1000 feet long, grounded through an inductance. The upper end of the vertical wire is also frequently connected to one or more horizontal wires, particularly for wireless stations on board ship. The vertical wire, together with the horizontal cradle, if any, is called the "antenna," or "aerial." The aerial and the inductance form together an "electric oscillator." The antenna and the earth form the two "plates" of a condenser, the dielectric between the two being the surrounding air. Such an oscillating circuit is therefore essentially a condenser and inductance connected in series.

Transmitting Circuits (Figs. 1 and 2).—In order to charge such an oscillator, a second coil is placed close to the coil forming the inductance, the two coils together forming an "oscillation transformer." This oscillation transformer

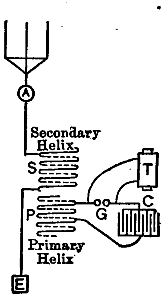


Fig. 1. Inductively-coupled Transmitting Station

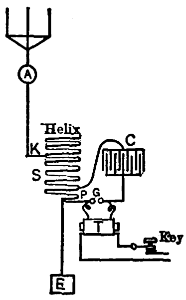


Fig. 2. Direct-coupled Transmitting Station

consists of but a few turns of heavy wire and is without iron. The two coils may be conductively independent, in which case the circuit is said to be "inductively coupled," as Fig. 1; or a single helix may be used, the entire helix forming the secondary and the primary being formed by a few turns tapped off from the helix as in Fig. 2, in which case the circuit is said to be "direct coupled." The primary of the oscillation transformer in either case is connected in series with a spark gap *G* and condenser *C*.

For the purpose of repeatedly charging the condenser *C* of either system, a step-up transformer *T* is used. The primary of *T* is connected through a key and through suitable regulating resistances or inductances with an alternating-current source of power. The secondary of the transformer *T* is connected about the spark gap *G*, or, what is the same thing, about the terminals of the condenser *C*.

Receiving Apparatus (Figs. 3 and 4).—The circuits of the receiving apparatus (of the inductively coupled type) are shown in Figs. 3 and 4. The corresponding

direct-coupled receiving circuits are not drawn. In Fig. 3, it is seen that the antenna is in series with an antenna inductance, a primary coil of an oscillation transformer, a series adjustable condenser and a ground. Oscillations produced in this antenna circuit by incident electric waves act inductively on the second-

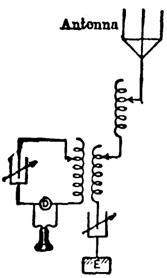


Fig. 3. Inductively-coupled Receiving Station with Detector in Secondary Circuit

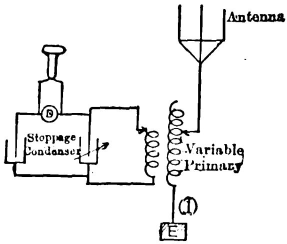


Fig. 4. With Detector in Shunt to Secondary Condenser

ary circuit containing the detector (*see Detectors, Electric Wave*), either in series with a condenser (Fig. 3) or in shunt with a variable condenser (Fig. 4). In this latter case another condenser, called a "stoppage condenser," shown at left of figure, is used to prevent the secondary of the receiving transformer from acting as a low-impedance shunt on the telephone receiver.

The various elements marked with arrows or arrow heads are adjustable for tuning.

THEORY. — The signals of wireless telegraphy are transmitted by electromagnetic waves sent out from the antenna of the transmitting station, and received by the antenna of the receiving station. The theory, therefore, comprises the theory of the oscillations that occur in the circuits, the theory of the propagation of the waves, and the theory of detectors. The last two subjects are discussed in the articles on *Waves, Electromagnetic*, and *Detectors, Electric Wave*. The theory of the oscillations occurring in the transmitting and receiving circuits is given briefly below.

Relation of Wave Length to Period. — If a source of oscillations has a period T of complete vibration and radiates energy in the form of waves, the wave length λ is the distance that the wave goes during one oscillation; whence

$$\lambda = vT, \tag{1}$$

in which $v =$ velocity of the waves. The velocity of electric waves in free space is the velocity of light, 3×10^8 meters per second, so that the wave length of a free wave in space is

$$\lambda = 3 \times 10^8\, T \text{ meters}, \tag{2}$$

where T is the period of the source in seconds.

In the case of an actual radiation of waves, this wave length λ will be the actual distance between two adjacent positions of similar phase in the train of waves radiated.

It has become customary in wireless-telegraphic practice to describe the period of oscillation of oscillatory circuits in terms of this free-space wave length, and this is done even though the circuit be of a form that radiates only an in-

considerable amount of its energy in the form of electric waves. For example, if a circuit has a period of 10^{-6} seconds, it is said to have a wave length of 300 meters (i.e., $3 \times 10^8 \times 10^{-6}$), even though the circuit be of such a form as to radiate practically no energy. See also article on *Wavemeters*.

Free Oscillation of a Circuit formed by a Lumped Inductance, Lumped Capacity and Resistance. — When a condenser of capacity C, and negligible leakage, discharges through an inductance L and a resistance R connected in series and all measured in the same set of units, the current oscillates with the period

$$T = \frac{2\pi\sqrt{LC}}{\sqrt{1 - \frac{R^2C}{4L}}} \text{ seconds,} \tag{3}$$

provided $\frac{R^2C}{4L} < 1$.

The equation for the current as a function of the time is, calling V_0 the initial p.d. of the condenser,

$$i = \sqrt{\frac{C}{L}} \frac{V_0}{\sqrt{1 - \frac{R^2C}{4L}}} \epsilon^{-\frac{\delta \cdot t}{T}} \sin 2\pi \frac{t}{T}, \tag{4}$$

where

$$\delta = \frac{RT}{2L}. \tag{5}$$

The constant δ is called the "logarithmic decrement." $100(1 - \epsilon^{-\delta})$ is the percentage decrease in the amplitude of the oscillation in one complete period.

Usually the effect of the resistance on the period of oscillation may be neglected, and equations (3) and (4) become, provided $\frac{R^2C}{4L}$ is negligible in comparison with unity,

$$T = 2\pi\sqrt{LC}, \tag{6}$$

$$i = \sqrt{\frac{C}{L}} V_0 \epsilon^{-\frac{\delta \cdot t}{T}} \sin 2\pi \frac{t}{T}, \tag{7}$$

where

$$\delta = \pi R \sqrt{\frac{C}{L}}. \tag{8}$$

It should be carefully noted that formulas (3) to (8) hold only for a circuit in which the capacity is "lumped." Also the formulas hold accurately only provided the circuit does not contain a spark gap (*see next section*).

Effect of Spark Gap on Period and Damping. — A spark gap is a resistance which varies with the current that flows through it. The greater the current through the gap, the less its resistance. In fact, the gap acts in such a manner as to oppose approximately a constant voltage to the discharge, instead of a constant resistance. This does not have much effect on the period of the discharge, but it materially changes the character of the damping, so that, in case the spark is the principal resistance in the circuit, the amplitude of the current of the condenser decreases linearly with the time, instead of

decreasing exponentially as is the case with constant resistance in the discharge circuit.

Free Oscillation of a Rectilinear Oscillator. — The antenna of a transmitting station has not only resistance, but also capacity and inductance, but the capacity and inductance are "distributed" instead of lumped. Such an antenna is approximately equivalent to a long, straight wire removed from all other conductors, and of length equal to twice the length of the antenna.

According to a theoretical investigation by Prof. Max Abraham (*Wied. Ann., Vol. 66, p. 435, 1898*), if such a rectilinear oscillator, long in comparison with its diameter and far removed from other conductors, is excited, by giving it an unequal distribution of charge, the free flow of current in it will have a period

$$\text{(Abraham's theory)} \qquad T = \frac{2l}{v} \text{ seconds,} \tag{9}$$

where l is the length of the oscillator in meters, and v is the velocity of light in meters per second $= 3 \times 10^8$.

Also, according to Abraham, the logarithmic decrement by radiation alone per complete period of such a rectilinear oscillator is

$$\text{(Abraham's theory)} \qquad \delta = \frac{9.74}{4 \log_e \frac{l}{d}} = \frac{2.435}{\log_e \frac{l}{d}}, \tag{10}$$

in which l/d is the ratio of the length to the diameter of the oscillator.

Several experimental tests of Abraham's formula (9) have been made, and such experiments, performed with or without a spark gap at the center of the oscillator, have given consistently values for the period which are somewhat higher than those calculated by formula (9). The experimental results are better represented by the equation

$$\text{(Experimental)} \qquad T = \frac{2.1\, l}{v} \text{ seconds.} \tag{11}$$

The departure of the experimental results from the theoretical values may be due to the fact that the theory presupposes that all other bodies are so far removed as not to have any influence on the period of the oscillator, — a condition which has not been attained in the experiments.

Free Oscillation of a Grounded Oscillator. — If the oscillator consists of a straight vertical wire grounded to a good conductor of practically infinite extent, the flow of current in the antenna (Fig. 5) is theoretically the same as if the ground were removed and replaced by a wire (dotted line) which is the reversed duplicate or image, of the antenna. The period is, therefore, according to equation (9), since the length of the antenna l is half the length of the equivalent free oscillator,

$$T = \frac{4l}{v}, \tag{12}$$

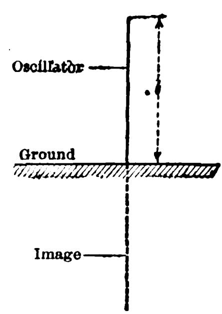


Fig. 5. Grounded Oscillator and Image

and the wave length is, theoretically,

$$\lambda = 4l. \tag{13}$$

The logarithmic decrement, by radiation alone, for such a grounded antenna,

if we accept Abraham's value of equation (10), and assume that the antenna radiates but that the image does not, is approximately

$$\delta = \frac{1.22}{\log_e \frac{2l}{d}}, \qquad (14)$$

in which l is length and d diameter of the oscillator.

The following table contains values of the radiation logarithmic decrement calculated from formula (14) for various lengths l and diameters d of vertical rectilinear oscillator.

TABLE I. — LOGARITHMIC DECREMENT BY RADIATION

l, meters	l, cm.	Diameter of wire in cm. = d				
		0.1	0.2	0.3	0.4	0.5
10	1000	0.123	0.134	0.140	0.142	0.147
20	2000	0.115	0.123	0.129	0.133	0.136
30	3000	0.111	0.118	0.123	0.127	0.130
40	4000	0.108	0.116	0.120	0.123	0.126
50	5000	0.106	0.113	0.117	0.121	0.123

These values give approximately the logarithmic decrement of a single grounded vertical wire calculated by equation (14). This decrement is due solely to radiation of electromagnetic waves of wave length approximately four times the length of the antenna. This decrement will be increased by the heat losses from the wire due to the heating of the conductor by the currents in it. These heat losses are in the form of electromagnetic waves of small wave length; i.e., wave length of the order of 0.01 to 0.0001 millimeter. The decrement due to heat losses may, however, be small in comparison with the decrement due to the long electromagnetic waves.

Equivalent Lumped Inductance and Lumped Capacity of an Antenna. — Although the inductance and capacity of the aerial circuit is distributed, it has been shown by experiment that as far as the period, amplitude and logarithmic decrement of the oscillations are concerned, the aerial circuit may be considered as replaced by an equivalent lumped inductance and capacity, and the formulas given above ((6) to (8)) for lumped inductance and capacity may be employed. To determine this equivalent inductance L_0 and capacity C_0, an inductance L is inserted in the aerial circuit near the ground (i.e., at the loop of current) and the period of the waves sent out from the oscillator with this added inductance is measured by a wavemeter (*see Wavemeters*). Call this period T; then, from (6),

$$T = 2\pi\sqrt{(L + L_0)\,C_0}.$$

The added inductance is then changed to a new value L_1, and the corresponding period T_1 measured by the wavemeter. From (6)

$$T_1 = 2\pi\sqrt{(L_1 + L_0)\,C_0}.$$

From these two equations L_0 and C_0 may be calculated.

Effect of Introducing Inductance or Capacity in Series with an Antenna. — From formulas (6) and (8) it is apparent that introducing an inductance into the antenna near the ground increases the period and wave length and decreases the logarithmic decrement. Inserting a condenser C in series in the aerial circuit near the ground decreases the resultant capacity, since this resultant capacity becomes (two condensers in series) $\frac{C_0C}{C_0+C}$. Therefore, from (6) and (8), it is apparent that the insertion of a condenser in the aerial decreases the period and wave length and increases the logarithmic decrement. The substitution of the resultant capacity $\frac{C_0C}{C_0+C}$ in (6) and (8) gives the new period and logarithmic decrement only to a rough approximation, since the introduction of the capacity C also changes the equivalent capacity of the antenna.

Free Oscillations in Coupled Circuits. — The free oscillation of two circuits coupled together as in Fig. 6 is doubly periodic; the current in the primary circuit and also the current in the secondary circuit both have two periods, resulting in beats, as shown in Fig. 7.

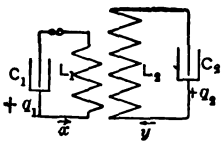
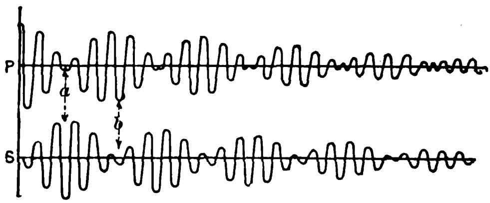


Fig. 6. Coupled Condenser Circuits.



Primary and Secondary, Ordinary Spark

Fig. 7. Showing Double Periodicity

As is seen in Fig. 7, the current in the primary oscillates at the beginning with a large amplitude, and that in the secondary has at the beginning a small amplitude; then at each oscillation the current in the primary decreases in amplitude, and that in the secondary increases until the condition at a is reached, in which the current in the primary is practically zero, and all of the energy is in the secondary. Then, as the oscillations continue, the energy comes back into the primary (condition b), etc.

If T_1 is the period of one of the circuits called the *primary* when alone, T_2 the period of the other circuit (the *secondary*) alone, then, if we neglect the influence of damping on period, the two periods of *each circuit when coupled together* are

$$T_1' = \sqrt{\frac{T_1^2 + T_2^2 + \sqrt{(T_1^2 - T_2^2)^2 + 4\tau^2 T_1^2 T_2^2}}{2}},$$

$$T_2' = \sqrt{\frac{T_1^2 + T_2^2 - \sqrt{(T_1^2 - T_2^2)^2 + 4\tau^2 T_1^2 T_2^2}}{2}}, \qquad (15)$$

in which τ, called the "coefficient of coupling," is

$$\tau = \sqrt{\frac{M^2}{L_1 L_2}}, \qquad (16)$$

where

M = mutual inductance between the circuits.
L_1 = self inductance of the primary,
L_2 = self inductance of the secondary.

The equations (15) have been obtained theoretically by neglecting th effect of the damping of the circuits in modifying their periods, and hav confirmed experimentally.

The wave lengths of the two circuits may be obtained by multiply periods by the velocity of light, and give the result

$$\lambda_1' = \sqrt{\frac{\lambda_1^2 + \lambda_2^2 + \sqrt{(\lambda_1^2 - \lambda_2^2)^2 + 4\tau^2\lambda_1^2\lambda_2^2}}{2}},$$

$$\lambda_2' = \sqrt{\frac{\lambda_1^2 + \lambda_2^2 - \sqrt{(\lambda_1^2 - \lambda_2^2)^2 + 4\tau^2\lambda_1^2\lambda_2^2}}{2}},$$

in which λ_1 and λ_2 are the wave lengths of the two circuits respectivel standing apart so as not to influence each other, and λ_1' and λ_2' are derived wave lengths obtained when the circuits are coupled together.

An interesting special case arises when the two circuits are equiperiod standing alone; that is when $\lambda_1 = \lambda_2 = \lambda$ (say). In this case, equatio become

$$\lambda_1' = \lambda\sqrt{1+\tau},$$

$$\lambda_2' = \lambda\sqrt{1-\tau}.$$

Free Oscillation of an Antenna Circuit Coupled with a Con Circuit. — Likewise, when a condenser circuit is coupled with an ante cuit, as in Figs. 1 and 2, the oscillation is generally doubly periodic, with periods of oscillation ~~and~~ wave lengths given approximately by the ec (15), (16), (17); and if the two circuits, when standing alone, have t period as each other, the derived periods are approximately related to th period by the equations (18).

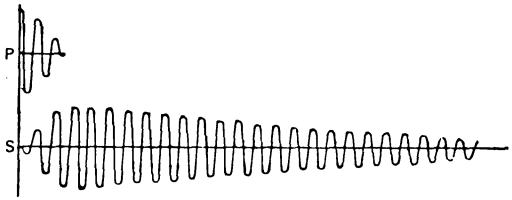
These several formulas are derived by considering the inductance and of the antenna as lumped, and the approximation is fairly close when t comparatively large inductance in the antenna helix (cf. Figs. 1 and 2), the distributed inductance is small in comparison. In circuits where t dition is not fulfilled the quantitative relations of equations (15) to (be used only to get a general insight into the manner in which the together of circuits affects their oscillation.

Effect of Quenched Spark. — If the condenser circuit of a coupled station contains a quenched spark gap (described below), the oscill doubly periodic only for the short interval that the spark persists. W oscillation reaches the primary minimum, at *a* in Fig. 7, the spark of the q gap becomes extinguished and does not relight. In consequence the circuit ceases to influence the oscillation of the antenna, which goes on ing with its own free period.

This condition is shown in Fig. 8. The quenched-gap method of e possesses the advantage of avoiding the successive return of energ primary where it is continually dissipated as heat in the gap and in the circuit. It possesses also the further advantage that the system em a single wave length and thereby is adapted to sharper tuning than system emitting two waves. A third advantage of the quenched gap

the fact that it operates at low potential and high spark frequency, thereby diminishing the breakage of transmitting condensers.

The quenched gap consists of a series of very short discharge spaces between



Primary and Secondary, Quenched Spark

Fig. 8. Current Plotted against Time with Quenched-spark Excitation

heavy copper disks held apart by thin gaskets of paper, or rubber or other insulating material. The large metal disks afford rapid cooling of the gap, and the exclusion of air from the discharge spaces seems to facilitate the rapid extinction of the spark when the beat-zero of oscillation is reached in the primary.

POWER REQUIRED TO OPERATE TRANSMITTING STATION. — The primary condenser (C, Fig. 1) of the sending station is charged from a step-up transformer. The power at the terminals of secondary of the transformer required to operate the station is

$$P = \frac{nCV^2}{2} \text{ watts,} \tag{19}$$

in which C is the capacity of the condenser in farads, n is the number of charges of the condenser per second, and V is the p.d. of the condenser at the beginning of the spark. If one spark occurs at each alternation of the supply current, n is twice the number of cycles per second f of the supply current; i.e., $n = 2f$. If, on the other hand, more than one discharge takes place during each half cycle, n is greater than twice the number of cycles per second. In some of the largest transmitting stations the power required amounts to as much as 60 kilowatts. In an ordinary ship's installation from 3 to 15 kilowatts are used.

The power given by equation (19) is that required to operate the station. The power actually radiated is very much less than this, ranging from 5 per cent to 50 per cent of the power input. Because the maximum sensitiveness of the telephone receiver used in receiving signals lies in the neighborhood of 1000 cycles per second, it is now good practice in wireless-telegraphic engineering to use alternating-current generators of about 500 cycles and arrange the spark gap to give one spark at each alternation of the generator; that is, 1000 sparks per second.

However, on account of the economy in using sources of power already available, which are usually of 60 or 120 cycles, these frequencies of alternating current are still extensively used in wireless telegraphy.

DESIGN OF CONDENSERS FOR TRANSMITTING STATION. — The following table (Table II) contains values of transmitting condenser capacity computed from equation (19) for use in a 5-kilowatt sending station, with various cycles and sparking voltages across the gap. Two sparks per cycle are assumed and the discharge voltage of the first column is the instantaneous voltage at which the discharges occur, when the key is held down.

TABLE II

CAPACITY OF CONDENSER FOR 5-KW. STATION

Discharge voltage	Capacity in microfarads required for various cycles per second					
	60 ~	120 ~	200 ~	300 ~	400 ~	500 ~
10,000	0.833	0.417	0.250	0.167	0.125	0.100
15,000	0.371	0.186	0.111	0.0743	0.0556	0.0445
20,000	0.208	0.104	0.0625	0.0417	0.0312	0.0250
25,000	0.133	0.067	0.0400	0.0267	0.0200	0.0160
30,000	0.093	0.0465	0.0278	0.0185	0.0139	0.0111
35,000	0.068	0.0340	0.0204	0.0136	0.0102	0.0082
40,000	0.052	0.0260	0.0156	0.0104	0.0078	0.0062
45,000	0.041	0.0206	0.0123	0.0082	0.0062	0.0049
50,000	0.033	0.0166	0.0100	0.0067	0.0050	0.0040
55,000	0.028	0.0137	0.0082	0.0055	0.0041	0.0033
60,000	0.023	0.0116	0.0070	0.0046	0.0035	0.0028

Since the capacity required is proportional to the power, the value of the capacity required for a station of any other power, *P* kilowatts, is equal to that given in the table multiplied by $P/5$. In designing condensers for the transmitting station it is necessary to allow for abnormal voltages in starting the spark. Good Bohemian glass may be submitted to a potential difference of about 11,000 volts per millimeter of thickness. Ordinary window glass is capable of standing about the same electrical stress. Plate glass, which is usually a lead glass, is much weaker. Assuming the breaking strength of a good glass to be 11,000 volts per millimeter of thickness, the following table (Table III) is computed for the physical dimensions of condensers for a 5-kilowatt station, with the cycles and discharge voltages of Table II. The table is given in terms of the number of sheets of dielectric required, when the metallic coating of the two opposed plates has an area of one square foot, the thickness of the dielectric plates being computed to stand a potential difference of 2.5 times the operating potential.

These condenser values are for a 5-kilowatt station. The number of plates for a station of any other power, *P* kilowatts, can be obtained by multiplying the numbers in the appropriate columns of Table III by $P/5$.

The glass for these condensers should be selected for absence of blowholes and other defects, and, if the plates are piled up, the glass plates, which are likely to be curved, should be placed with convex side of one piece against concave side of adjacent piece to avoid breakage from mechanical pressure. The metallic coatings of the glass should preferably be of copper and should be plated upon the glass.

Leyden jars, copper plated, are on the market, and have about the same breaking strength as the plates above described and may be assembled to give the required capacity and strength.

The last column of Table III contains the width of margin required for the condensers when the plates are in air. If the condensers are submerged in castor oil, having been built up under the oil, the margin may be reduced to about one-half the values given in the table.

TABLE III

DIMENSIONS OF CONDENSER FOR 5-KW. STATION

Discharge potential in volts	Potential at start (assumed) in volts	Required thickness of glass		Capacity per sq. ft. of conductor in microfarads	No. of these plates required at various cycles							Width of margin on each side of plate in in.
		mm.	in.		~ 60	~ 100	~ 120	~ 200	~ 300	~ 400	~ 500	
10,000	25,000	2.3	0.090	0.00250	333	200	167	100	67	50	40	1.0
15,000	37,000	3.45	0.135	0.00167	222	133	111	67	44	33	27	1.5
20,000	50,000	4.60	0.180	0.00125	167	100	84	50	34	25	20	2.0
25,000	63,000	5.75	0.225	0.00100	133	80	67	40	28	20	16	2.6
30,000	75,000	6.90	0.270	0.00083	110	67	56	33	22	16	13	3.2
35,000	87,000	8.05	0.315	0.00072	95	57	48	29	19	14	12	4.0
40,000	100,000	9.20	0.360	0.00063	84	50	42	25	17	13	10	4.8
45,000	112,000	10.35	0.405	0.00056	75	45	38	22	15	11	9	5.5
50,000	125,000	11.50	0.450	0.00050	67	40	31	20	13	10	8	6.2
55,000	137,000	12.65	0.495	0.00046	61	36	30	18	12	9	7	7.0
60,000	150,000	13.80	0.540	0.00042	55	34	28	17	11	8	7	8.0

DESIGN OF CHARGING TRANSFORMER. — Accurate engineering data for the design of the transformer for charging the sending condenser are, so far as the writer knows, not available. A closed iron core without magnetic leakage should not be used, for the reason that such a transformer draws more power on short circuit than it does when its secondary is charging the condenser; and, in consequence, when the spark is started the power input increases and maintains the spark as an arc across the gap, so that the condenser does not again charge. What is required is a resonance transformer, — one with magnetic leakage, so designed that when the spark starts, the transformer circuit is thrown out of resonance by the spark and thus cuts off the power supply, so that the spark may extinguish and the condensers again charge. This is attained by an open-core transformer, or by one with an E-shaped core designed to have magnetic leakage between the primary and secondary windings. A calculation of the current-carrying capacity required of the secondary windings may be made as follows:

The average value of the current required to charge the condenser n times per second to a voltage V is nVC, where C is in farads. Substituting for C its value in terms of P from equation (19), the average secondary current in amperes is $\frac{2P}{V}$.

It is difficult to say just what is the form of cycle of the current flowing into the condenser, and it is therefore difficult to estimate the effective value of the secondary current. Assuming that the current is sinusoidal for 90° then zero for 90°, the effective value I_s of this current is found to be $\frac{\pi}{2}$ times the average value, or

$$I_s = \frac{\pi P}{V} \text{ amperes,} \tag{20}$$

if the sending key is held down continuously. In equation (20) P is power in watts supplied to the condenser, V is volts at which condenser discharges, and I_s is the effective current in the secondary of the transformer.

Since in general the sending key will be held down only at intermittent intervals, the actual effective current will be considerably less than that given by equation (20). This equation should therefore be used as the limiting maximum value of this current.

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[G. W. PIERCE.]

WIRELESS TELEPHONY. — (*See also Wireless Telegraphy.*) The circuits employed in wireless telephony resemble closely those used in wireless telegraphy. The receiving circuits and their appendages are identical for telephony and for telegraphy, so that a receiving wireless-telegraph station, when operating with any of the rectifier detectors (with a telephone receiver as indicating instrument), is suitable for receiving either telegraph or telephone messages. The high-frequency circuits at the transmitting station are also of the same general character for telephony and for telegraphy, but the ordinary spark-discharge method of excitation at the transmitting station is not applicable for wireless telephony, for such excitation produces discrete trains of oscillations, which, arriving at the receiving station, are rectified by the detectors, and give discrete impulses of rectified current to the telephone receiver. These discrete impulses used in wireless telegraphy are usually within the range of audibility and produce a musical note at the receiver. This musical note would predominate over any modifications impressed by speech at the transmitting station.

In order to adapt the transmitting station of wireless telegraphy to wireless telephony, it is necessary to change the excitation at the sending station, so that the waves emitted shall be a continuous sequence, or, if discrete, shall follow one another at a frequency above the limit of audibility.

When such a persistent sequence of waves arrives at the receiving station, it is rectified by the detector, and gives a practically continuous pull upon the diaphragm of the telephone receiver, and is not audible except for such accidental irregularities of audible frequency as arise in imperfections of the exciting generators. Now, if such persistent oscillations at the transmitting station are caused to pass through a carbon microphone, or similar instrument, and the resistance of the microphone is caused to vary by the voice, the sequence of waves is modified in intensity, so that the amplitude of the high-frequency sequence of waves carries with it a series of modifications, or modulations, corresponding to the pitch, intensity and quality of the voice. These modulations persist also in the rectified current through the receiving telephone and are heard as spoken words.

The main problem of wireless telephony, is, therefore, the production of a sequence of high-frequency oscillations at the transmitting station of such persistence and regularity as not to have any variations of audible frequency.

GENERATORS FOR PERSISTENT HIGH-FREQUENCY OSCILLATIONS. — Among the generators for persistent high-frequency oscillations may be mentioned: (1) the high-frequency alternator, (2) the Poulsen arc, and (3) the Chaffee gap.

High-frequency Alternators. — A high-frequency alternator capable of giving 100,000 cycles per second is manufactured by the General Electric Company. It is driven by a 10-horse-power electric motor geared to the alternator by a De Laval turbine gear, giving the inductor of the alternator a speed of 20,000 revolutions per minute. The machine contains a stationary field, a stationary armature and a rotating inductor. The air gap of the machine may be varied by means of a micrometer screw attached to the frame of the machine. The machine is intended to be used with a resonant condenser, and gives, when so used, a terminal voltage of from 150 to 300 volts. The power output is about 5 kilowatts.

The Goldschmidt High-frequency Alternator (Fig. 1). — Another type of high-frequency alternator has recently been developed by R. Goldschmidt in Germany for giving from 30,000 to 100,000 cycles per second with a much slower speed of the rotating part. This is done by a combination of the effects of rotation with the effects of resonance, and results in raising the frequency of the

generator to a multiple of its normal frequency. In Fig. 1, S represents the windings on the stator and R the windings on the rotor. Suppose that the number of poles of the stator and rotor are designed so as to give a frequency f when the machine is used as an ordinary alternating-current generator with direct current in the stator. The frequency f may be of the order of 10,000 cycles without excessive speed. In order to get higher frequencies from the machine, a choking coil D is placed in the direct-current feeding circuit, B, D, S, and condensers C_1 and C_2 are shunted with the field, and other condensers C_3, C_4 and C_5 are attached to the slip rings of the rotor.

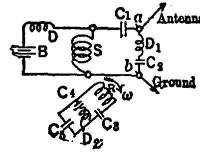


Fig. 1. Diagram of Circuits in Goldschmidt's Alternator

The machine operates on the principle that when an alternating current flows in the field coils the frequency of the rotor e.m.f. is the sum of the frequency of the alternating field and the frequency which would be induced in the rotor were a steady-current field used. This principle is used in connection with resonance methods for accentuating the required multiple frequencies.

The inventor claims that a machine of this type has been in operation since 1910, and that it furnishes 12.5 kilowatts at 30,000 cycles with an efficiency of about 80 per cent, and that it delivers about 8 to 10 kilowatts at 60,000 cycles. He claims that there is no difficulty in constructing machines on the same principle capable of giving 60 to 80 kilowatts.

Poulsen's Arc Generator for High-frequency Oscillations (Fig. 2).—This device consists of a copper-carbon arc in an atmosphere of coal gas or hydrogen. Details of the arc and its circuits are shown in Fig. 2. The copper terminal, which is usually hollow and water-cooled, is the positive terminal of the arc. The negative terminal of carbon is slowly rotated by clockwork. The gas receptacle surrounding the arc is provided with a worm carrying water for cooling the walls of the receptacle. The poles of a powerful electromagnet are inserted gastight, into the chamber, and placed so as to give a strong magnetic field transverse to the arc. The circuits are as follows: The leads from a direct-current dynamo of from 200 to 500 volts are brought through a control resistance and around the coils of the electromagnet and connected to the terminals of the arc, so that the copper terminal is positive. About the arc a circuit consisting of a condenser C and self inductance P are shunted. Oscillations are set up in this circuit and act inductively upon a secondary coil S attached to an antenna and ground. (*See also Arc, Electric.*)

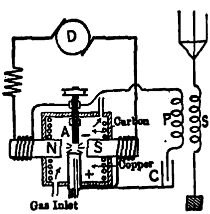


Fig. 2. Poulsen Arc-generator

At 100,000 cycles the Poulsen arc is capable of delivering several kilowatts of high-frequency power. It is difficult to get frequencies much above 200,000 cycles with this form of generator.

The Chaffee Gap for High-frequency Oscillations (Fig. 3).—A gap devised by E. L. Chaffee for producing high-frequency oscillations consists of a minute gap in hydrogen between a copper anode and an aluminum cathode. The copper terminal is shown in section at e in Fig. 3. A terminal of the same size but made of aluminum is shown facing e. Except for the difference in the metal of the two terminals, the two halves of the gap and mounting are symmetrical. Each terminal is inserted in a heat-radiating support t. The two supports are separated by an insulating block, which contains a small cavity

in which the arc is surrounded by an atmosphere of hydrogen. The gap operates on from 220 to 500 volts, and is fed from a direct-current source through a choking coil. A condenser and inductance are shunted about the gap. This condenser and inductance with the gap constitute the primary of the oscillation system. The secondary of the system consists of a secondary coil and an antenna and ground as in Poulsen's arrangement of circuits. However, in the Chaffee device the primary circuit is not an oscillation circuit, for the gap is a rectifier and permits only direct-current impulses flowing in a direction from the copper to the aluminum through the gap. One of these impulses starts the secondary circuit oscillating, and the secondary oscillations impose potential ripples on the primary so as to trigger off successive primary impulses at just the proper time for maintaining the secondary current. The recurrence of the primary impulses may be made of any desired frequency with respect to the secondary current by adjusting the primary condenser and inductance and the main supply current. The Chaffee gap may be used to develop any frequency up to 30,000,000 cycles per second. At 1,000,000 cycles a single gap will give about ½ kilowatt of high-frequency power.

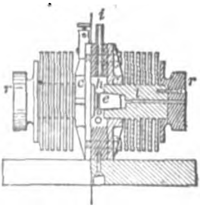


Fig. 3. Chaffee Gap

CONNECTIONS FOR TRANSMITTING STATION. — Figs. 4, 5 and 6 show connections for a wireless telephone transmitting station with an arc as source of oscillations. When one of the high-frequency alternators is to be

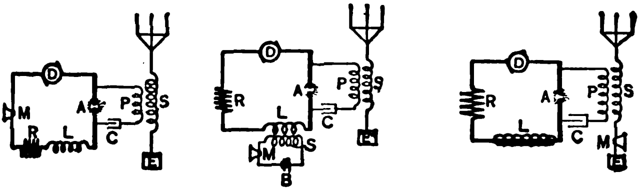


Fig. 4. Fig. 5. Fig. 6.

Types of Connections for Wireless Telephone Transmitting Stations

used, the alternator terminals are connected to the leads to *C* and *P* respectively. These diagrams are introduced particularly to show the disposition of the microphonic transmitter *M*. The best position for this transmitter is directly in series between the secondary coil *S* and the ground *E*, as in Fig. 6. Other arrangements are with the transmitter in series with the dynamo circuit through the arc, as in Fig. 4, or in a circuit *B*, *M*, *S* (Fig. 5), arranged to act inductively upon the dynamo circuit. Another disposition of the microphone, not shown in the figures, is in a tertiary circuit inductively coupled with the antenna circuit.

The microphone for wireless telephony must be of large current-carrying capacity and in consequence water cooling in the transmitter is sometimes resorted to.

TRANSMISSION DISTANCE. — The wireless telephone, though not in general use, is easily practicable up to distances of 100 miles.

BIBLIOGRAPHY. — See references under *Wireless Telegraphy;* also Chaffee, *Proc. Am. Academy*, Vol. 47, No. 9, pp. 265–312.

[G. W. Pierce.]

WIRES AND CABLES, BARE. — *(See also Aluminum; Copper; Gages, Wire; Resistance and Conductance; Standardization Rules of the A.I.E.E; Wires and Cables, Insulated; Wires, Resistance.)*

The following is a brief table of contents of this article:

A wire may be either solid or stranded, i.e., made up of a number of smaller wires twisted or braided together. A large bare stranded wire is usually called a bare cable. Data on the insulation and protection of wires and cables will be found in the article on *Wires and Cables, Insulated*. Data on resistance wires will be found in the article on *Wires, Resistance*, and data on the properties of the various metals will be found in the articles on *Aluminum, Copper*, and *Steel*.

PER CENT CONDUCTIVITY. — The definitions, constants and tables in this article are all in accord with the 1914 edition of the *Standardization Rules of the A.I.E.E.* (q.v.). Per cent conductivity refers to the "International Annealed Copper Standard." On the assumption of a resistivity temperature coefficient of 0.00393 at 20° C. this per cent conductivity is only 0.283 per cent higher than the conductivity referred to Matthiessen's Standard (*see Resistance and Conductance*. If the length of a given wire is L cm., its cross-section A sq. cm. and its resistance at 20° C. is R_{20} ohms then the per cent conductivity of this wire is

$$C = \frac{15.328\,L}{88{,}900\,A R_{20}} \quad \text{per cent.}$$

Annealed or soft-drawn copper usually has a conductivity of 100 per cent, hard-drawn copper a conductivity of about 97 per cent. Ordinary hard-drawn aluminum has a conductivity of 61 per cent. The conductivity of iron or steel wire ranges from 8 to 16 per cent.

DIMENSIONS, RESISTANCE AND WEIGHT. — The following tables for copper and aluminum are compiled from the tables in *Circular No. 31 of the Bureau of Standards (1914 ed.)*, except that for stranded aluminum on p. 1868.. The table for steel wire is compiled from that published by the American Steel and Wire Co., and the table for copper-clad steel wire is based on data published by the Duplex Metals Co.

SOLID COPPER WIRE

A. W. G. or B. & S. Gage; English Units

100 Per Cent Conductivity; Density 8.89 at 20° C.

Gage No.	Diameter in mils.	Cross-section: Circular mils	Cross-section: Square inches	Resistance at 20° C. or 68° F.*: Ohms per 1000 feet	Resistance at 20° C. or 68° F.*: Ohms per mile	Weight in pounds: per 1000 feet	Weight in pounds: per mile	Feet per pound
0000	460.0	211,600	0.1662	0.04901	0.259	640.5	3380	1.561
000	409.6	167,800	0.1318	0.06180	0.326	507.9	2680	1.968
00	364.8	133,100	0.1045	0.07793	0.411	402.8	2130	2.482
0	324.9	105,500	0.08289	0.09827	0.519	319.5	1680	3.130
1	289.3	83,690	0.06573	0.1239	0.654	253.3	1340	3.947
2	257.6	66,370	0.05213	0.1563	0.825	200.9	1060	4.977
3	229.4	52,640	0.04134	0.1970	1.04	159.3	841	6.276
4	204.3	41,740	0.03278	0.2485	1.31	126.4	667	7.914
5	181.9	33,100	0.02600	0.3133	1.65	100.2	529	9.980
6	162.0	26,250	0.02062	0.3951	2.09	79.46	420	12.58
7	144.3	20,820	0.01635	0.4982	2.63	63.02	333	15.87
8	128.5	16,510	0.01297	0.6282	3.32	49.98	264	20.01
10	101.9	10,380	0.008155	0.9989	5.28	31.43	166	31.82
12	80.81	6,530	0.005129	1.588	8.38	19.77	104	50.59
14	64.08	4,107	0.003225	2.525	13.3	12.43	63.3	80.44
15	57.07	3,257	0.002558	3.184	16.8	9.858	52.0	101.4
16	50.82	2,583	0.002028	4.015	21.2	7.818	41.3	127.9
17	45.26	2,048	0.001609	5.064	26.7	6.200	32.7	161.3
18	40.30	1,624	0.001276	6.385	33.7	4.917	26.0	203.4
19	35.89	1,288	0.001012	8.051	42.5	3.899	20.6	256.5
20	31.96	1,022	0.0008023	10.15	53.6	3.092	16.3	323.4
21	28.46	810.1	0.0006363	12.80	67.6	2.452	12.9	407.8
22	25.35	642.4	0.0005046	16.14	85.2	1.945	10.3	514.2
23	22.57	509.5	0.0004002	20.36	108	1.542	8.14	648.4
24	20.10	404.0	0.0003173	25.67	135	1.223	6.46	817.7
25	17.90	320.4	0.0002517	32.37	171	0.9699	5.12	1,031
26	15.94	254.1	0.0001996	40.82	216	0.7692	4.06	1,300
27	14.20	201.5	0.0001583	51.46	272	0.6100	3.22	1,639
28	12.64	159.8	0.0001255	64.90	343	0.4837	2.55	2,067
29	11.26	126.7	0.00009953	81.84	432	0.3836	2.03	2,607
30	10.03	100.5	0.00007894	103.2	545	0.3042	1.61	3,287
31	8.928	79.70	0.00006260	130.1	687	0.2413	1.27	4,145
32	7.950	63.21	0.00004964	164.1	866	0.1913	1.01	5,227
33	7.080	50.13	0.00003937	206.9	1090	0.1517	0.814	6,591
34	6.305	39.75	0.00003122	260.9	1380	0.1203	0.635	8,310
35	5.615	31.52	0.00002476	329.0	1740	0.09542	0.504	10,480
36	5.000	25.00	0.00001964	414.8	2190	0.07568	0.400	13,210
38	3.965	15.72	0.00001235	659.6	3480	0.04759	0.251	21,010
40	3.145	9.888	0.000007766	1049	5540	0.02993	0.158	33,410

*Let C = per cent conductivity, R_{20} = resistance of 100 per cent conductivity wire at 20° C. (from table), R_t = resistance of wire of conductivity C at any temperature t° C, then

$$R_t = R_{20}\left[\frac{100}{C} + 0.00393\,(t - 20)\right].$$

COPPER CABLES, CONCENTRIC-LAY

Cir. Mils and A. W. G. or B. & S. Gage; English Units

100 Per Cent Conductivity; Density 8.89 at 20° C. (*See p. 1873.*)

Circular mils and A. W. G.	Resistance at 25° C. or 77° F.*		Weight in pounds, bare		Standard strands			Flexible strands		
	Ohms per 1000 feet	Ohms per mile	per 1000 feet	per mile	Number of wires	Diameter of wires in mils	Outside diameter, in mils	Number of wires	Diameter of wires, in mils	Outside diameter, in mils
2,000,000	0.00539	0.0285	6180	32600	127	125.5	1631	169	108.8	1632
1,900,000	0.00568	0.0300	5870	31000	127	122.3	1590	169	106.0	1590
1,800,000	0.00599	0.0316	5560	29300	127	119.1	1548	169	103.2	1548
1,700,000	0.00634	0.0335	5250	27700	127	115.7	1504	169	100.3	1504
1,600,000	0.00674	0.0356	4940	26100	127	112.2	1459	169	97.3	1460
1,500,000	0.00719	0.0380	4630	24500	91	128.4	1412	127	108.7	1413
1,400,000	0.00770	0.0407	4320	22800	91	124.0	1364	127	105.0	1365
1,300,000	0.00830	0.0438	4010	21200	91	119.5	1315	127	101.2	1315
1,200,000	0.00899	0.0475	3710	19600	91	114.8	1263	127	97.2	1264
1,100,000	0.00981	0.0518	3400	17900	91	109.9	1209	127	93.1	1210
1,000,000	0.0108	0.0570	3090	16300	61	128.0	1152	91	104.8	1153
950,000	0.0114	0.0600	2930	15490	61	124.8	1123	91	102.2	1124
900,000	0.0120	0.0633	2780	14670	61	121.5	1093	91	99.4	1094
850,000	0.0127	0.0670	2620	13860	61	118.0	1062	91	96.6	1063
800,000	0.0135	0.0712	2470	13040	61	114.5	1031	91	93.8	1031
750,000	0.0144	0.0759	2320	12230	61	110.9	998	91	90.8	999
700,000	0.0154	0.0814	2160	11410	61	107.1	964	91	87.7	965
650,000	0.0166	0.0876	2010	10600	61	103.2	929	91	84.5	930
600,000	0.0180	0.0949	1850	9780	61	99.2	893	91	81.2	893
550,000	0.0196	0.1036	1700	8970	61	95.0	855	91	77.7	855
500,000	0.0216	0.1139	1540	8150	37	116.2	814	61	90.5	815
450,000	0.0240	0.1266	1390	7340	37	110.3	772	61	85.9	773
400,000	0.0270	0.1424	1240	6520	37	104.0	728	61	81.0	729
350,000	0.0308	0.1627	1080	5710	37	97.3	681	61	75.7	682
300,000	0.0360	0.1899	926	4890	37	90.0	630	61	70.1	631
250,000	0.0431	0.228	772	4080	37	82.2	575	61	64.0	576
0000	0.0509	0.269	653	3450	19	105.5	528	37	75.6	533
000	0.0642	0.339	518	2735	19	94.0	470	37	67.3	471
00	0.0811	0.428	411	2170	19	83.7	418	37	60.0	420
0	0.102	0.540	326	1720	19	74.5	373	37	53.4	374
1	0.129	0.681	258	1364	19	66.4	332	37	47.6	333
2	0.162	0.858	205	1082	7	97.4	292	19	59.1	296
3	0.205	1.082	163	858	7	86.7	260	19	52.6	263
4	0.259	1.365	129	680	7	77.2	232	19	46.9	234
5	0.326	1.721	102	540	7	68.8	206	19	41.7	209
6	0.410	2.170	81.0	428	7	61.2	184	19	37.2	186
7	0.519	2.74	64.3	339	7	54.5	164	19	33.1	166
8	0.654	3.45	51.0	269	7	48.6	146	19	29.5	147

*Let C = per cent conductivity, R_{25} = resistance of 100 per cent conductivity cable at 25° C. (from table), R_t = resistance of cable of conductivity C at any temperature t° C., then

$$R_t = R_{25}\left[\frac{100}{C} + 0.00385\,(t - 25)\right].$$

SOLID COPPER WIRE

A. W. G. or B. & S. Gage in Metric Units

100 Per Cent Conductivity; Density 8.89 at 20° C.

Gage No.	Diameter in mm.	Cross-section in sq. mm.	*Ohms per kilometer, 20° C*	Kilograms per kilometer
0000	11.68	107.2	0.1608	953.2
000	10.40	85.03	0.2028	755.9
00	9.266	67.43	0.2557	599.5
0	8.252	53.48	0.3224	475.4
1	7.348	42.41	0.4066	377.0
2	6.544	33.63	0.5126	299.0
3	5.827	26.67	0.6464	237.1
4	5.189	21.15	0.8152	188.0
5	4.621	16.77	1.028	149.1
6	4.115	13.30	1.296	118.2
7	3.665	10.55	1.634	93.78
8	3.264	8.366	2.061	74.37
10	2.588	5.261	3.277	46.77
12	2.053	3.309	5.211	29.42
14	1.628	2.081	8.285	18.50
15	1.450	1.650	10.45	14.67
16	1.291	1.309	13.18	11.63
17	1.150	1.038	16.61	9.226
18	1.024	0.8231	20.95	7.317
19	0.9116	0.6527	26.42	5.803
20	0.8118	0.5176	33.31	4.602
21	0.7230	0.4105	42.00	3.649
22	0.6438	0.3255	52.96	2.894
23	0.5733	0.2582	66.79	2.295
24	0.5106	0.2047	84.22	1.820
25	0.4547	0.1624	106.2	1.443
26	0.4049	0.1288	133.9	1.145
27	0.3606	0.1021	168.8	0.9078
28	0.3211	0.08098	212.9	0.7199
29	0.2859	0.06422	268.5	0.5709
30	0.2546	0.05093	338.6	0.4527
31	0.2268	0.04039	426.9	0.3590
32	0.2019	0.03203	538.3	0.2847
33	0.1798	0.02540	678.8	0.2258
34	0.1601	0.02014	856.0	0.1791
35	0.1426	0.01597	1079	0.1420
36	0.1270	0.01267	1361	0.1126
38	0.1007	0.007967	2164	0.07083
40	0.07987	0.005010	3441	0.04454

*Let C = per cent conductivity, R_{20} = resistance of 100 per cent conductivity wire at 20° C. (from table), R_t = resistance of wire of conductivity C at any temperature t° C, then

$$R_t = R_{20}\left[\frac{100}{C} + 0.00393\,(t-20)\right].$$

COPPER CABLES, CONCENTRIC-LAY

Cir. Mils and A.W.G. or B. & S. Gage in Metric Units

100 Per Cent Conductivity; Density 8.89 at 20°C. (*See p. 1873.*)

Circular mils and A.W.G.	Total cross-section in mm.²	Ohms per kilometer at 25°C.*	Kilograms per kilometer, Bare	Standard strands: Number of wires	Standard strands: Diameter of wires, in mm.	Standard strands: Outside diameter, in mm.	Flexible strands: Number of wires	Flexible strands: Diameter of wires, in mm.	Flexible strands: Outside diameter, in mm.
2,000,000	1013	0.0177	9190	127	3.19	41.4	169	2.76	41.4
1,900,000	963	0.0186	8730	127	3.11	40.4	169	2.69	40.4
1,800,000	912	0.0197	8270	127	3.02	39.3	169	2.62	39.3
1,700,000	861	0.0208	7810	127	2.94	38.2	169	2.55	38.2
1,600,000	811	0.0221	7350	127	2.85	37.1	169	2.47	37.1
1,500,000	760	0.0236	6890	91	3.26	35.9	127	2.76	35.9
1,400,000	709	0.0253	6430	91	3.15	34.7	127	2.67	34.7
1,300,000	659	0.0272	5970	91	3.04	33.4	127	2.57	33.4
1,200,000	608	0.0295	5510	91	2.92	32.1	127	2.47	32.1
1,100,000	557	0.0322	5050	91	2.79	30.7	127	2.36	30.7
1,000,000	507	0.0354	4590	61	3.25	29.3	91	2.66	29.3
950,000	481	0.0373	4370	61	3.17	28.5	91	2.60	28.5
900,000	456	0.0393	4140	61	3.09	27.8	91	2.53	27.8
850,000	431	0.0416	3910	61	3.00	27.0	91	2.45	27.0
800,000	405	0.0442	3680	61	2.91	26.2	91	2.38	26.2
750,000	380	0.0472	3450	61	2.82	25.3	91	2.31	25.4
700,000	355	0.0506	3220	61	2.72	24.5	91	2.23	24.5
650,000	329	0.0544	2990	61	2.62	23.6	91	2.15	23.6
600,000	304	0.0590	2760	61	2.52	22.7	91	2.06	22.7
550,000	279	0.0643	2530	61	2.41	21.7	91	1.97	21.7
500,000	253	0.0708	2300	37	2.95	20.7	61	2.30	20.7
450,000	228	0.0786	2070	37	2.80	19.6	61	2.18	19.6
400,000	203	0.0885	1840	37	2.64	18.5	61	2.06	18.5
350,000	177	0.101	1610	37	2.47	17.3	61	1.92	17.3
300,000	152	0.118	1380	37	2.29	16.0	61	1.78	16.0
250,000	127	0.142	1150	37	2.09	14.6	61	1.63	14.6
0000	107	0.167	972	19	2.68	13.4	37	1.93	13.5
000	85	0.211	771	19	2.39	11.9	37	1.71	12.0
00	67.4	0.266	611	19	2.13	10.6	37	1.52	10.7
0	53.5	0.334	485	19	1.89	9.46	37	1.36	9.50
1	42.4	0.423	385	19	1.69	8.43	37	1.21	8.46
2	33.6	0.533	305	7	2.47	7.42	19	1.50	7.51
3	26.7	0.673	242	7	2.20	6.61	19	1.34	6.68
4	21.2	0.849	192	7	1.96	5.88	19	1.19	5.95
5	16.8	1.07	152	7	1.75	5.24	19	1.06	5.30
6	13.3	1.35	121	7	1.56	4.67	19	0.944	4.72
7	10.5	1.70	95.7	7	1.39	4.16	19	0.841	4.20
8	8.37	2.14	75.9	7	1.23	3.70	19	0.749	3.74

* Let C = per cent conductivity, R_{25} = resistance of 100 per cent conductivity cable at 25° C. (from table), R_t = resistance of cable of conductivity C at any temperature $t°$ C., then

$$R_t = R_{25}\left[\frac{100}{C} + 0.00385\,(t - 25)\right].$$

SOLID COPPER WIRE

British Standard Wire Gage; English Units

100 Per Cent Conductivity; Density 8.89 at 20° C.

Gage No.	Diameter in mils	Cross-section		Ohms per 1000 feet, 15.6° C. or 60° F.*	Pounds per 1000 feet
		Circular mils	Square inches		
7-0	500	250,000	0.1964	0.04077	756.8
6-0	464	215,300	0.1691	0.04734	651.7
5-0	432	186,600	0.1466	0.05461	564.9
4-0	400	160,000	0.1257	0.06370	484.3
3-0	372	138,400	0.1087	0.07365	418.9
2-0	348	121,100	0.09512	0.08416	366.6
0	324	105,000	0.08245	0.09709	317.8
1	300	90,000	0.07069	0.1132	272.4
2	276	76,180	0.05983	0.1338	230.6
3	252	63,500	0.04988	0.1605	192.2
4	232	53,820	0.04227	0.1894	162.9
5	212	44,940	0.03530	0.2268	136.0
6	192	36,860	0.02895	0.2765	111.6
7	176	30,980	0.02433	0.3290	93.76
8	160	25,600	0.02011	0.3981	77.49
9	144	20,740	0.01629	0.4915	62.77
10	128	16,380	0.01287	0.6221	49.59
11	116	13,460	0.01057	0.7574	40.73
12	104	10,820	0.008495	0.9423	32.74
13	92	8,464	0.006648	1.204	25.62
14	80	6,400	0.005027	1.592	19.37
15	72	5,184	0.004072	1.966	15.69
16	64	4,096	0.003217	2.488	12.40
17	56	3,136	0.002463	3.250	9.493
18	48	2,304	0.001810	4.424	6.974
19	40	1,600	0.001257	6.370	4.843
20	36	1,296	0.001018	7.864	3.923
22	28	784.0	0.0006158	13.00	2.373
24	22	484.0	0.0003801	21.06	1.465
26	18	324.0	0.0002545	31.46	0.9807
28	14.8	219.0	0.0001720	46.54	0.6630
30	12.4	153.8	0.0001208	66.28	0.4654
32	10.8	116.6	0.00009161	87.38	0.3531
34	9.2	84.64	0.00006648	120.4	0.2562
36	7.6	57.76	0.00004536	176.5	0.1748
38	6.0	36.00	0.00002827	283.1	0.1090
40	4.8	23.04	0.00001810	442.4	0.06974
42	4.0	16.00	0.00001257	637.0	0.04843
44	3.2	10.24	0.000008042	995.3	0.03100
50	1.0	1.000	0.0000007854	10,190	0.003027

*Let C = per cent conductivity, R_{60} = resistance of 100 per cent conductivity wire at 60° F. (from table), R_t = resistance of wire of conductivity C at any temperature t° F., then

$$R_t = R_{60}\left[\frac{100}{C} + 0.00223\,(t-60)\right].$$

SOLID COPPER WIRE

"Millimeter Gage"; Metric Units

100 Per Cent Conductivity; Density 8.89 at 20° C.

Diameter in mm.	Cross-section in sq. mm.	Ohms per kilometer, 20° C.*	Kilograms per kilometer
10.0	78.54	0.2195	698.2
9.0	63.62	0.2710	565.6
8.0	50.27	0.3430	446.9
7.0	38.48	0.4480	342.1
6.0	28.27	0.6098	251.4
5.0	19.64	0.8781	174.6
4.5	15.90	1.084	141.4
4.0	12.57	1.372	111.7
3.5	9.621	1.792	85.53
3.0	7.069	2.439	62.84
2.5	4.909	3.512	43.64
2.0	3.142	5.488	27.93
1.8	2.545	6.775	22.62
1.6	2.011	8.575	17.87
1.4	1.539	11.20	13.69
1.2	1.131	15.24	10.05
1.0	0.7854	21.95	6.982
0.90	0.6362	27.10	5.656
0.80	0.5027	34.30	4.469
0.70	0.3848	44.80	3.421
0.60	0.2827	60.98	2.514
0.50	0.1964	87.81	1.746
0.45	0.1590	108.4	1.414
0.40	0.1257	137.2	1.117
0.35	0.09621	179.2	0.8553
0.30	0.07069	243.9	0.6284
0.25	0.04909	351.2	0.4364
0.20	0.03142	548.8	0.2793
0.15	0.01767	975.6	0.1571
0.10	0.007854	2195	0.06982
0.05	0.001964	8781	0.01746

*Let C = per cent conductivity,
R_{20} = resistance of 100 per cent conductivity wire at 20° C. (from table),
R_t = resistance of wire of conductivity C at any temperature t° C.,
then

$$R_t = R_{20}\left[\frac{100}{C} + 0.00393\,(t - 20)\right].$$

SOLID COPPER WIRE; OHMS PER UNIT WEIGHT

A. W. G. or B. & S. Gage; English and Metric Units

100 Per Cent Conductivity; Density 8.89 at 20° C.

Gage No.	Ohms per pound			Ohms per kilogram		
	0° C. 32° F.	20° C. 68° F.	50° C. 122° F.	0° C.	20° C.	50° C.
0000	0.00007051	0.00007652	0.00008554	0.0001554	0.0001687	0.0001886
000	0.0001121	0.0001217	0.0001360	0.0002472	0.0002682	0.0002999
00	0.0001783	0.0001935	0.0002163	0.0003930	0.0004265	0.0004768
0	0.0002835	0.0003076	0.0003439	0.0006249	0.0006782	0.0007582
1	0.0004507	0.0004891	0.0005468	0.0009936	0.001078	0.001206
2	0.0007166	0.0007778	0.0008695	0.001580	0.001715	0.001917
3	0.001140	0.001237	0.001383	0.002512	0.002726	0.003048
4	0.001812	0.001966	0.002198	0.003995	0.004335	0.004846
5	0.002881	0.003127	0.003495	0.006352	0.006893	0.007706
6	0.004581	0.004972	0.005558	0.01010	0.01096	0.01225
7	0.007284	0.007906	0.008838	0.01606	0.01743	0.01948
8	0.01158	0.01257	0.01405	0.02553	0.02771	0.03098
9	0.01842	0.01999	0.02234	0.04060	0.04407	0.04926
10	0.02928	0.03178	0.03553	0.06456	0.07006	0.07833
11	0.04656	0.05053	0.05649	0.1026	0.1114	0.1245
12	0.07404	0.08035	0.08983	0.1632	0.1771	0.1980
13	0.1177	0.1278	0.1428	0.2595	0.2817	0.3149
14	0.1872	0.2032	0.2271	0.4127	0.4479	0.5007
15	0.2976	0.3230	0.3611	0.6562	0.7121	0.7961
16	0.4733	0.5136	0.5742	1.043	1.132	1.266
17	0.7525	0.8167	0.9130	1.659	1.800	2.013
18	1.197	1.299	1.452	2.638	2.863	3.201
19	1.903	2.065	2.308	4.194	4.552	5.089
20	3.025	3.283	3.670	6.670	7.238	8.092
21	4.810	5.221	5.836	10.60	11.51	12.87
22	7.649	8.302	9.280	16.86	18.30	20.46
23	12.16	13.20	14.76	26.81	29.10	32.53
24	19.34	20.99	23.46	42.63	46.27	51.73
25	30.75	33.37	37.31	67.79	73.57	82.25
26	48.89	53.06	59.32	107.8	117.0	131.8
27	77.74	84.37	94.32	171.4	186.0	207.9
28	123.6	134.2	150.0	272.5	295.8	330.6
29	196.6	213.3	238.5	433.3	470.3	525.7
30	312.5	339.2	379.2	689.0	747.8	836.0
31	497.0	539.3	602.9	1,096	1,189	1,329
32	790.2	857.6	958.7	1,742	1,891	2,114
33	1,256	1,364	1,524	2,770	3,006	3,361
34	1,998	2,168	2,424	4,404	4,780	5,344
35	3,177	3,448	3,854	7,003	7,601	8,497
36	5,051	5,482	6,128	11,140	12,080	13,510
38	12,770	13,860	15,490	28,150	30,560	34,160
40	32,290	35,040	39,170	71,180	77,260	86,360

SOLID COPPER WIRE; WEIGHT PER OHM
A. W. G. or B. & S. Gage; English and Metric Units
100 Per Cent Conductivity; Density 8.89. at 20° C.

Gage No.	Meters per ohm		Feet per ohm		Pounds per ohm		Grams per ohm	
	20° C.	50° C.	20° C. 68° F.	50° C. 122° F.	20° C. 68° F.	50° C. 122° F.	20° C.	50° C.
0000	6219	5563	20,400	18,250	13,070	11,690	5,928,000	5,302,000
000	4932	4412	16,180	14,470	8,219	7,352	3,728,000	3,335,000
00	3911	3499	12,830	11,480	5,169	4,624	2,344,000	2,097,000
0	3102	2774	10,180	9,103	3,251	2,908	1,474,000	1,319,000
1	2460	2200	8,070	7,219	2,044	1,829	927,300	829,500
2	1951	1745	6,400	5,725	1,286	1,150	583,200	521,700
3	1547	1384	5,075	4,540	808.6	723.3	366,800	328,100
4	1227	1097	4,025	3,600	508.5	454.9	230,700	206,300
5	972.8	870.2	3,192	2,855	319.8	286.1	145,100	129,800
6	771.5	690.1	2,531	2,264	201.1	179.9	91,230	81,610
7	611.8	547.3	2,007	1,796	126.5	113.2	57,380	51,330
8	485.2	434.0	1,592	1,424	79.56	71.16	36,090	32,280
9	384.8	344.2	1,262	1,129	50.03	44.75	22,690	20,300
10	305.2	273.0	1,001	895.6	31.47	28.15	14,270	12,770
11	242.0	216.5	794.0	710.2	19.79	17.70	8,976	8,030
12	191.9	171.7	629.6	563.2	12.44	11.13	5,645	5,050
13	152.2	136.1	499.3	446.7	7.827	7.001	3,550	3,176
14	120.7	108.0	396.0	354.2	4.922	4.403	2,233	1,997
15	95.72	85.62	314.0	280.9	3.096	2.769	1,404	1,256
16	75.90	67.90	249.0	222.8	1.947	1.742	883.1	790.0
17	60.19	53.85	197.5	176.7	1.224	1.095	555.3	496.8
18	47.74	42.70	156.6	140.1	0.7701	0.6888	349.3	312.4
19	37.86	33.86	124.2	111.1	0.4843	0.4332	219.7	196.5
20	30.02	26.86	98.49	88.11	0.3046	0.2725	138.2	123.6
21	23.81	21.30	78.11	69.87	0.1915	0.1713	86.89	77.72
22	18.88	16.89	61.95	55.41	0.1205	0.1078	54.64	48.88
23	14.97	13.39	49.12	43.94	0.07576	0.06777	34.36	30.74
24	11.87	10.62	38.96	34.85	0.04765	0.04262	21.61	19.33
25	9.417	8.424	30.90	27.64	0.02997	0.02680	13.59	12.16
26	7.468	6.680	24.50	21.92	0.01884	0.01686	8.548	7.647
27	5.922	5.298	19.43	17.38	0.01185	0.01060	5.376	4.809
28	4.696	4.201	15.41	13.78	0.007454	0.006668	3.381	3.022
29	3.725	3.332	12.22	10.93	0.004688	0.004193	2.126	1.902
30	2.954	2.642	9.691	8.669	0.002948	0.002637	1.337	1.196
31	2.342	2.095	7.685	6.875	0.001854	0.001659	0.8410	0.7523
32	1.858	1.662	6.094	5.452	0.001166	0.001043	0.5289	0.4731
33	1.473	1.318	4.833	4.323	0.0007333	0.0006560	0.3326	0.2976
34	1.168	1.045	3.833	3.429	0.0004612	0.0004126	0.2092	0.1871
35	0.9264	0.8288	3.040	2.719	0.0002900	0.0002595	0.1316	0.1177
36	0.7347	0.6572	2.410	2.156	0.0001824	0.0001632	0.08275	0.07402
38	0.4621	0.4133	1.516	1.356	0.00007216	0.00006454	0.03273	0.02927
40	0.2906	0.2600	0.9534	0.8529	0.00002854	0.00002553	0.01294	0.01158

SOLID ALUMINUM WIRE

A. W. G. or B. & S. Gage; English Units

61 Per Cent Conductivity; Density 2.70

Gage No.	Diameter in mils	Cross-section		Resistance at 20° C. or 68° F.*		Weight in pounds		Feet per pound
		Circular mils	Square inches	Ohms per 1000 feet	Ohms per mile	per 1000 feet	per mile	
0000	460.0	211,600	0.1662	0.0804	0.424	195	1027	5.14
000	409.6	167,800	0.1318	0.101	0.535	154	815	6.48
00	364.8	133,100	0.1045	0.128	0.675	122	646	8.17
0	324.9	105,500	0.08289	0.161	0.851	97.0	512	10.31
1	289.3	83,690	0.06573	0.203	1.073	76.9	406	13.00
2	257.6	66,370	0.05213	0.256	1.353	61.0	322	16.39
3	229.4	52,630	0.04134	0.323	1.706	48.4	255	20.7
4	204.3	41,740	0.03278	0.408	2.15	38.4	203	26.1
5	181.9	33,100	0.02600	0.514	2.71	30.4	160.7	32.9
6	162.0	26,250	0.02062	0.648	3.42	24.1	127.4	41.4
7	144.3	20,820	0.01635	0.817	4.31	19.1	101.0	52.3
8	128.5	16,510	0.01297	1.03	5.44	15.2	80.2	65.9
10	101.9	10,380	0.008155	1.64	8.65	9.55	50.4	104.8
12	80.81	6,530	0.005129	2.61	13.76	6.00	31.7	166.6
14	64.08	4,107	0.003225	4.14	21.9	3.78	19.93	265
15	57.07	3,257	0.002558	5.22	27.6	2.99	15.81	334
16	50.82	2,583	0.002029	6.59	34.8	2.37	12.54	421
17	45.26	2,048	0.001609	8.31	43.8	1.88	9.94	531
18	40.30	1,624	0.001276	10.5	55.3	1.49	7.89	670
19	35.89	1,288	0.001012	13.2	69.7	1.18	6.25	844
20	31.96	1,022	0.0008023	16.7	87.9	0.939	4.96	1,065
21	28.46	810.1	0.0006363	21.0	110.9	0.745	3.93	1,343
22	25.35	642.4	0.0005046	26.5	139.8	0.591	3.12	1,693
23	22.57	509.5	0.0004002	33.4	176.3	0.468	2.47	2,130
24	20.10	404.0	0.0003173	42.1	222	0.371	1.961	2,690
25	17.90	320.4	0.0002517	53.1	280	0.295	1.556	3,390
26	15.94	254.1	0.0001996	67.0	353	0.234	1.233	4,280
27	14.20	201.5	0.0001583	84.4	446	0.185	0.978	5,400
28	12.64	159.8	0.0001255	106	562	0.147	0.776	6,810
29	11.26	126.7	0.00009953	134	709	0.117	0.615	8,580
30	10.03	100.5	0.00007894	169	894	0.0924	0.488	10,820
31	8.928	79.70	0.00006260	213	1127	0.0733	0.387	13,650
32	7.950	63.21	0.00004964	269	1421	0.0581	0.307	17,210
33	7.080	50.13	0.00003937	339	1792	0.0461	0.243	21,700
34	6.305	39.75	0.00003122	428	2260	0.0365	0.1929	27,400
35	5.615	31.52	0.00002476	540	2850	0.0290	0.1530	34,510
36	5.000	25.00	0.00001964	681	3590	0.0230	0.1214	43,500
38	3.965	15.72	0.00001235	1080	5710	0.0145	0.0763	69,200
40	3.145	9.888	0.000007766	1720	9080	0.0091	0.0480	110,000

*Let C = per cent conductivity, R_{20} = resistance of 61 per cent conductivity wire at 20° C. (from table), R_t = resistance of wire of conductivity C at any temperature t° C., then

$$R_t = \frac{61\ R_{20}}{C}[1 + 0.004\ (t - 20)].$$

ALUMINUM CABLES.

The commercial sizes of stranded aluminum cables, made by the Aluminum Company of America, are not even circular mil and B. & S. sizes, but are of such cross sections as to give the same conductivity as even circular mil and B. & S. sizes of copper cables of 97 per cent conductivity. In the following table the first four and the seventh columns are taken from a pamphlet entitled "Instructions for Installation and Maintenance of Aluminum Electrical Conductors," issued by the Aluminum Company of America, in 1914.

B. & S. gage or circular mils		Usual number of strands	Diameter of bare cable, inches	Resistance at 25° C. or 77° F.*		Weight in pounds	
Copper (97 per cent) equivalent	Aluminum 61 per cent			Ohms per 1000 feet	Ohms per mile	per 1000 feet	per mile
1,000,000	1,590,000	61	1 7/16	0.0111	0.0588	1462	7719
950,000	1,515,000	61	1 13/32	0.0118	0.0619	1393	7355
900,000	1,431,000	61	1 23/64	0.0124	0.0653	1317	6954
850,000	1,351,500	61	1 21/64	0.0131	0.0691	1243	6563
800,000	1,272,000	61	1 9/32	0.0139	0.0734	1171	6189
750,000	1,192,500	37	1 1/4	0.0148	0.0783	1098	5797
700,000	1,113,000	37	1 13/64	0.0159	0.0839	1025	5412
650,000	1,033,500	37	1 5/32	0.0171	0.0903	950	5016
600,000	954,000	37	1 7/64	0.0186	0.0978	877	4631
550,000	874,500	37	1 1/16	0.0202	0.1068	805	4250
500,000	795,000	37	1 1/64	0.0223	0.1174	732	3865
450,000	715,500	37	31/32	0.0247	0.1305	658	3474
400,000	636,000	37	29/32	0.0278	0.1468	585	3089
350,000	556,500	19	55/64	0.0318	0.1677	512	2703
300,000	477,000	19	25/32	0.0371	0.1958	439	2318
250,000	397,500	19	23/32	0.0444	0.2351	365	1927
0000	336,420	7	21/32	0.0525	0.277	310.2	1638
000	266,800	7	37/64	0.0662	0.350	245.7	1297
00	211,950	7	33/64	0.0836	0.441	195	1030
0	167,800	7	15/32	0.105	0.557	155	818.4
1	133,220	7	13/32	0.133	0.702	122.6	647.3
2	105,530	7	23/64	0.167	0.885	97.2	513.2
3	83,640	7	21/64	0.211	1.116	77	406.6
4	66,370	7	19/64	0.267	1.407	61.2	323.1
5	52,630	7	17/64	0.336	1.774	48.5	256.1
6	41,740	7	15/64	0.423	2.237	38.5	203.3

* These resistances are taken equal to those given on page 1860 divided by 0.97.

SOLID ALUMINUM WIRE

A. W. G. or B. & S. Gage in Metric Units

61 Per Cent Conductivity; Density 2.70; Temperature 20° C. or 68° F.*

Gage No.	Diameter in mm.	Cross-section in sq. mm.	Ohms per kilometer	Kilograms per kilometer
0000	11.68	107.2	0.264	289
000	10.40	85.03	0.333	230
00	9.266	67.43	0.419	182
0	8.252	53.48	0.529	144
1	7.348	42.41	0.667	114
2	6.544	33.63	0.841	90.8
3	5.827	26.67	1.06	72.0
4	5.189	21.15	1.34	57.1
5	4.621	16.77	1.69	45.3
6	4.115	13.30	2.13	35.9
7	3.665	10.55	2.68	28.5
8	3.264	8.366	3.38	22.6
10	2.588	5.261	5.38	14.2
12	2.053	3.309	8.55	8.93
14	1.628	2.081	13.6	5.62
15	1.450	1.650	17.1	4.46
16	1.291	1.309	21.6	3.53
17	1.150	1.038	27.3	2.80
18	1.024	0.8231	34.4	2.22
19	0.9116	0.6527	43.3	1.76
20	0.8118	0.5176	54.6	1.40
21	0.7230	0.4105	68.9	1.11
22	0.6438	0.3255	86.9	0.879
23	0.5733	0.2582	110	0.697
24	0.5106	0.2047	138	0.553
25	0.4547	0.1624	174	0.438
26	0.4049	0.1288	220	0.348
27	0.3606	0.1021	277	0.276
28	0.3211	0.08098	349	0.219
29	0.2859	0.06422	440	0.173
30	0.2546	0.05093	555	0.138
31	0.2268	0.04039	700	0.109
32	0.2019	0.03203	883	0.0865
33	0.1798	0.02540	1110	0.0686
34	0.1601	0.02014	1400	0.0544
35	0.1426	0.01597	1770	0.0431
36	0.1270	0.01267	2230	0.0342
38	0.1007	0.007967	3550	0.0215
40	0.07987	0.005010	5640	0.0135

* Let C = per cent conductivity, R_{20} = resistance of 61 per cent conductivity wire at 20° C. (from table), R_t = resistance of wire of conductivity C at any temperature t° C, then

$$R_t = \frac{61\ R_{20}}{C}[1 + 0.004\ (t - 20)].$$

The temperature coefficient is approximate only.

SOLID STEEL WIRE

American Steel Wire Gage; English Units

12.5 Per Cent Conductivity; Density 7.78

Am. Steel Wire Gage No.	Diameter		Cross-section		Resistance at 20° C. or 68° F.*		Weight in pounds		Feet per pound
	In.	Mils	Circular mils	Square inches	Ohms per 1000 feet	Ohms per mile	per 1000 feet	per mile	
	½	500.0	250,000	0.1964	0.332	1.752	662.5	3499	1.51
7–0		490.0	240,100	0.1886	0.346	1.825	636.3	3360	1.57
	15/32	468.8	219,800	0.1726	0.378	1.993	582.4	3075	1.72
6–0		460.0	211,600	0.1662	0.392	2.07	560.8	2961	1.78
	7/16	437.5	191,400	0.1503	0.433	2.29	507.2	2678	1.97
5–0		430.0	184,900	0.1452	0.449	2.37	490.0	2587	2.04
	13/32	406.3	165,000	0.1296	0.503	2.65	436.8	2306	2.28
4–0		393.8	155,100	0.1218	0.535	2.82	411.9	2175	2.42
	⅜	375.0	140,600	0.1104	0.590	3.12	372.6	1967	2.68
3–0		362.5	131,400	0.1032	0.631	3.33	348.2	1839	2.87
	11/32	343.8	118,200	0.09280	0.702	3.71	313.1	1653	3.19
2–0		331.0	109,600	0.08605	0.757	4.00	290.3	1533	3.44
	5/16	312.5	97,660	0.07670	0.850	4.49	258.8	1366	3.86
0		306.5	93,940	0.07378	0.883	4.66	249.0	1315	4.02
1		283.0	80,090	0.06290	1.036	5.47	212.2	1121	4.71
	9/32	281.3	79,100	0.06213	1.049	5.54	209.6	1107	4.77
2		262.5	68,910	0.05412	1.204	6.36	182.6	964.1	5.48
	¼	250.0	62,500	0.04909	1.328	7.01	165.6	874.5	6.04
3		243.7	59,490	0.04665	1.397	7.38	157.4	831.0	6.35
4		225.3	50,760	0.03987	1.635	8.63	134.5	710.2	7.43
	7/32	218.8	47,850	0.03758	1.734	9.15	126.8	669.5	7.89
5		207.0	42,850	0.03365	1.936	10.22	113.6	599.5	8.81
6		192.0	36,860	0.02895	2.25	11.88	97.7	515.8	10.23
	3/16	187.5	35,160	0.02761	2.36	12.46	93.2	491.9	10.73
7		177.0	31,330	0.02461	2.65	13.98	83.0	438.4	12.04
8		162.0	26,240	0.02061	3.16	16.69	69.6	367.2	14.38
	5/32	156.3	24,410	0.01917	3.40	17.95	64.7	341.6	15.46
9		148.3	21,990	0.01727	3.77	19.92	58.3	307.8	17.16
10		135.0	18,200	0.01431	4.55	24.0	48.3	255.0	20.70
	⅛	125.0	15,630	0.01227	5.31	28.0	41.4	218.6	24.15

* Let C = per cent conductivity,
R_{20} = resistance of 12.5 per cent conductivity wire at 20° C. (from table),
R_t = resistance of wire of any conductivity C at any temperature $t°$ C,
then

$$R_t = \frac{12.5\, R_{20}}{C} [1 + 0.006\,(t - 20)].$$

The temperature coefficient is approximate only.

SOLID STEEL WIRE — *Continued*

American Steel Wire Gage; English Units

12.5 Per Cent Conductivity; Density 7.78

Am. Steel Wire Gage No.	Diameter		Cross-section		Resistance at 20° C. or 68° F.*		Weight in pounds		Feet per pound
	In.	Mils	Circular mils	Square inches	Ohms per 1000 feet	Ohms per mile	per 1000 feet	per mile	
11		120.5	14,520	0.01140	5.71	30.2	38.5	203.2	25.98
12		105.5	11,130	0.00874	7.45	39.4	29.5	155.7	33.90
	3/32	93.8	8,789	0.00690	9.44	49.8	23.3	123.0	42.94
13		91.5	8,372	0.00658	9.91	52.3	22.1	117.2	45.16
14		80.0	6,400	0.00503	12.96	68.5	17.0	89.55	58.97
15		72.0	5,184	0.00407	16.01	84.5	13.7	72.53	72.80
16		62.5	3906	0.00307	21.2	112.1	10.4	54.66	96.60
	1/16	62.5	3906	0.00307	21.2	112.1	10.4	54.66	96.60
17		54.0	2916	0.00229	28.5	150.2	7.73	40.80	129.5
18		47.5	2256	0.00177	36.8	194.2	5.98	31.57	167.2
19		41.0	1681	0.00132	49.4	261	4.45	23.52	224.4
20		34.8	1211	0.00095	68.5	362	3.21	16.95	311.5
21		31.8	1008	0.00079	82.3	435	2.67	14.11	374.4
	1/32	31.3	977	0.00076	85.0	449	2.59	13.66	386.5
22		28.6	818	0.00064	101.4	536	2.17	11.45	461.1
23		25.8	666	0.00052	124.6	658	1.76	9.31	567.0
24		23.0	529	0.00042	156.8	828	1.40	7.40	713.5
25		20.4	416	0.00033	199.4	1053	1.10	5.82	907.0
26		18.1	328	0.00026	253	1337	0.87	4.58	1152
27		17.3	299	0.00024	277	1464	0.79	4.19	1261
28		16.2	262	0.00021	316	1669	0.70	3.67	1438
29		15.0	225	0.00018	369	1947	0.60	3.15	1677
30		14.0	196	0.00015	424	2240	0.52	2.74	1925
31		13.2	174	0.00014	476	2510	0.46	2.44	2166
32		12.8	164	0.00013	506	2670	0.43	2.30	2303
33		11.8	139	0.00011	596	3150	0.37	1.95	2710
34		10.4	108	0.00008	767	4050	0.29	1.51	3489
35		9.5	90	0.00007	919	4850	0.24	1.26	4193
36		9.0	81	0.00006	1023	5410	0.21	1.13	4659

*Let C = per cent conductivity,
R_{20} = resistance of 12.5 per cent conductivity wire at 20° C. (from table),
R_t = resistance of wire of any conductivity C at any temperature t° C.,
then

$$R_t = \frac{12.5\,R_{20}}{C}[1 + 0.006\,(t - 20)].$$

The temperature coefficient is approximate only.

Copper-clad Steel Wire.—This wire consists of a steel core and a concentric coat of copper permanently welded thereto. It is used chiefly for long span transmission and telephone wire. It is made in several grades, which differ in the relative amounts of steel and copper. The grades are designated by the corresponding conductivity expressed as per cents of Matthiessen's Standard; e.g., 40 per cent grade has a conductivity of 40 per cent.

COPPER-CLAD STEEL WIRE

A. W. G. or B. & S. Gage; English Units

40 Per Cent Conductivity; Density 8.26

Gage No.	Diameter in mils	Cross-section		Resistance at 23.9° C. or 75° F.*		Weight in pounds		Feet per pound
		Circular mils	Square inches	Ohms per 1000 feet	Ohms per mile	per 1000 feet	per mile	
0000	460.0	211,600	0.1662	0.123	0.649	595	3140	1.68
000	409.6	167,800	0.1318	0.154	0.813	471	2490	2.12
00	364.8	133,100	0.1045	0.195	1.03	374	1970	2.67
0	324.9	105,500	0.08289	0.246	1.30	297	1570	3.37
1	289.3	83,690	0.06573	0.310	1.64	235	1240	4.26
2	257.6	66,370	0.05213	0.390	2.06	186	982	5.38
3	229.4	52,630	0.04134	0.492	2.60	148	781	6.76
4	204.3	41,740	0.03278	0.622	3.28	117	618	8.55
5	181.9	33,100	0.02600	0.782	4.13	92.9	491	10.76
6	162.0	26,250	0.02062	0.987	5.21	73.7	389	13.57
7	144.3	20,820	0.01635	1.25	6.60	58.5	309	17.09
8	128.5	16,510	0.01297	1.57	8.29	46.4	245	21.6
9	114.4	13,090	0.01028	1.98	10.5	36.8	194	27.2
10	101.9	10,380	0.008155	2.50	13.2	29.2	154	34.2
11	90.74	8,234	0.006467	3.15	16.6	23.1	122	43.3
12	80.81	6,530	0.005129	3.97	21.0	18.3	96.6	54.6
13	71.96	5,178	0.004067	5.00	26.4	14.6	77.1	68.5
14	64.08	4,107	0.003225	6.31	33.3	11.5	60.7	87.0

* Let C = per cent conductivity,
$R_{23.9}$ = resistance of 40 per cent conductivity wire at 23.9° C. (from table),
R_t = resistance of wire of conductivity C at temperature t° C.,

then

$$R_t = \frac{40\, R_{23.9}}{C}\,[1 + 0.00432\,(t - 23.9)].$$

The temperature coefficient is approximate only.

Alloy Wires of High Tensile Strength.—Copper alloys having a low conductivity, but having a tensile strength from 50 per cent to 100 per cent greater than that of copper are sometimes used where strength or hardness is a primary requisite, as in long spans of small wires or for trolley wires. The Bridgeport Brass Co. make a wire known as "phono-electric wire" which has a conductivity of 25 per cent and a tensile strength ranging from 68,000 pounds per square inch for No. 0000 B. & S. gauge to 85,000 pounds per square inch for No. 18 B. & S. gage.

Trolley Wire. — Trolley wires of two different sections are in use in the United States. The sections shown in Fig. 1 are known as the "American Standard" and the sections shown in Fig. 2 as the "Figure 8" sections. The

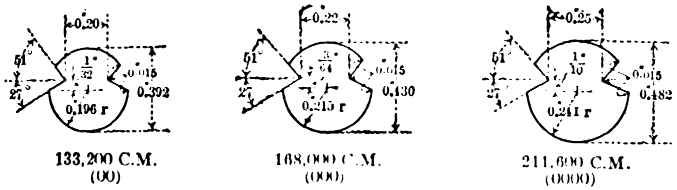


Fig. 1. American Standard Trolley-wire Sections

American Society for Testing Materials recommend that the sizes be specified in circular mils, and not as gage numbers; the sizes shown in the figure differ in the area of the cross-section from the gage numbers given in parentheses by less than 5 parts in 1000.

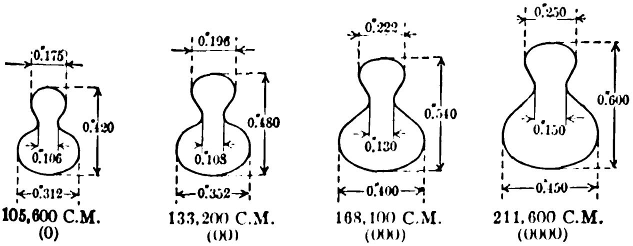


Fig. 2. "Figure 8" Trolley-wire Sections

Trolley wires are usually of hard-drawn copper; the electrical and mechanical properties are the same as for round hard-drawn wire of the same cross-section. Copper alloys, such as phono-electric wire (*see above*), are sometimes used for trolley wires.

FACTORS AFFECTING DIMENSIONS, WEIGHT AND RESISTANCE OF STRANDED WIRES. — Individual stranded wires or cables are of four different types, namely: (a) bunched wire; (b) wire braids; (c) concentric-lay cables and (d) rope-lay cables.

Bunched Wires. — Bunched wires are used especially for those extra flexible cables known as cords, wherein the individual wires are so small that concentric stranding is not necessary to keep them together. The wires are assembled parallel and then generally given a slight twist. Sometimes they are kept together by being wound with soft cotton thread which also serves to prevent adhesion between the insulation and wires.

Wire Braids. — In the flat form, wire braids are used for potential leads, etc., in lighting cables, where a flexible flat conductor is necessary. Tubular wire braids are also frequently formed over the insulation of cables in order to afford mechanical protection. Cables for naval or military purposes and for automobile work are frequently thus protected.

Concentric-lay Cables. — A concentric-lay cable is a stranded conductor composed of a central core surrounded by one or more layers of helically laid

wires. A rope-lay cable is a stranded wire made up in the same manner by using stranded wires instead of individual solid wires for the core and layers. The cores of concentric-lay cables may be composed of one, two, three or four wires of equal diameter. A five or six wire core would not be symmetrical and seven wires would themselves constitute a core and a layer.

Number of Wires in Concentric-lay Cables. — Hence the following table gives all the possible concentric-lay cables with eight or less layers of equal size wires and formulæ for calculating the number of wires with any number of layers.

NUMBER OF WIRES IN CONCENTRIC-LAY CABLES

(All wires of same diameter)

Number of layers over core	Number of wires in core			
	1	2	3	4
0	1	2	3	4
1	7	10	12	14
2	19	24	27	30
3	37	44	48	52
4	61	70	75	80
5	91	102	108	114
6	127	140	147	154
7	169	184	192	200
8	217	234	243	252
n	$3n^2+3n+1$	$3n^2+5n+2$	$3n^2+6n+3$	$3n^2+7n+4$

The number of wires per layer increases by six for each successive layer when the core has one wire, the first layer over the core having six. With cores having more than one wire, the increment per layer is not constant.

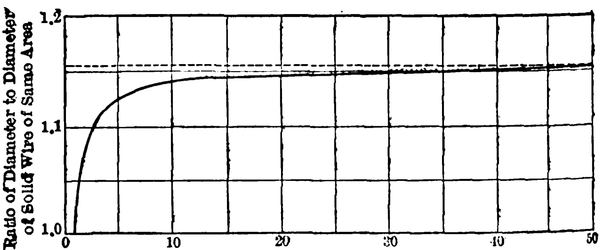


Fig. 3.

Diameter of Concentric-lay Cables. — The diameter of the circumscribing circle of any of the above cables is equal to $(2n + b)$ times the diameter of each wire, where n is the number of layers over the core and b has the following values: 1 wire in core, $b = 1$; 2 wires in core, $b = 2$; 3 wires in core, $b = 2.155$; 4 wires in core, $b = 2.414$.

The relation between the number of component wires and the diameter of the cable is shown in Fig. 3.

COMMERCIAL STRANDED CONDUCTORS

Area of conductor C. M.	Number of wires in the stranded conductor								
	7	19	37	7×7 =49	61	91	127	169	217
	Diameter, in inches, of each wire in the cable								
2,000,000	0.5345	0.3244	0.2325	0.202	0.181	0.1482	0.1255	0.1086	0.096
1,750,000	0.5000	0.3035	0.2175	0.189	0.169	0.1387	0.1174	0.1020	0.090
1,500,000	0.4629	0.2810	0.2013	0.175	0.157	0.1285	0.1087	0.0940	0.083
1,250,000	0.4226	0.2565	0.1838	0.1507	0.143	0.1174	0.0992	0.0860	0.076
1,000,000	0.3779	0.2294	0.1644	0.1429	0.1285	0.1048	0.0887	0.0769	0.0678
950,000	0.3684	0.2236	0.1602	0.1392	0.1247	0.1021	0.0864	0.0749	0.0661
900,000	0.3585	0.2176	0.1559	0.1355	0.1214	0.0995	0.0841	0.0729	0.0644
850,000	0.3484	0.2115	0.1515	0.1317	0.1180	0.0966	0.0818	0.0709	0.0625
800,000	0.3380	0.2050	0.1470	0.1278	0.1145	0.0937	0.0793	0.0687	0.0607
750,000	0.3273	0.1986	0.1423	0.1237	0.1108	0.0907	0.0769	0.0666	0.0588
700,000	0.3163	0.1919	0.1375	0.1195	0.1071	0.0887	0.0742	0.0643	0.0567
650,000	0.3047	0.1849	0.1325	0.1152	0.1032	0.0845	0.0715	0.0620	0.0547
600,000	0.2927	0.1776	0.1273	0.1107	0.0991	0.0812	0.0687	0.0595	0.0525
550,000	0.2803	0.1701	0.1219	0.1060	0.0949	0.0777	0.0658	0.0571	0.0503
500,000	0.2673	0.1622	0.1162	0.1010	0.0905	0.0741	0.0628	0.0543	0.0480
450,000	0.2535	0.1538	0.1103	0.0958	0.0858	0.0703	0.0595	0.0516	0.0455
400,000	0.2390	0.1457	0.1039	0.0904	0.0809	0.0663	0.0561	0.0486	0.0429
350,000	0.2236	0.1357	0.0972	0.0845	0.0757	0.0620	0.0526	0.0455	0.0401
300,000	0.2070	0.1256	0.0903	0.0783	0.0701	0.0574	0.0486	0.0421	0.0371
250,000	0.1889	0.1147	0.0824	0.0714	0.0640	0.0524	0.0443	0.0384	0.0339
Size A. W. G.									
0000	0.1739	0.1055	0.0756	0.0657	0.0589	......	......	......	......
000	0.1548	0.0940	0.0674	0.0586	0.0525	......	......	......	......
00	0.1379	0.0837	0.0600	0.0521	0.0467	......	......	......	......
0	0.1228	0.0745	0.0534	0.0464	0.0416	......	......	......	......
1	0.1094	0.0664	0.0475	0.0413	......	......	......	......	......
2	0.0974	0.0591	0.0424	0.0369	......	......	......	......	......
3	0.0867	0.0525	0.0377	0.0327	......	......	......	......	......
4	0.0772	0.0468	0.0335	0.0291	......	......	......	......	......
5	0.0688	0.0418	0.0299	0.0260	......	......	......	......	......
6	0.0612	0.0372	0.0266	0.0231	......	......	......	......	......
7	0.0545	0.0331	0.0237	0.0206	......	......	......	......	......
8	0.0484	0.0294	0.0211	0.0184					
9	0.0432	0.0263	0.0188	0.0164	......	......	......	......	......
10	0.0386	0.0233	0.0168	......	......	......	......	......	......
12	0.0306	0.0185	0.0133	......	......	......	......	......	......
14	0.0242	0.0148	0.0105	......	......	......	......	......	......

Weights in lb. per 1000 ft. of all bare copper cables are computed by multiplying the circular mils by 0.00309.

Rope-lay Cables. — As already noted, a rope-lay cable is made up in the same way as a concentric-lay cable except that stranded wires are used for the core and layers instead of individual solid wires. Rope strands are used for large conductors which would be too stiff if stranded concentrically. The formulas for regular concentric-lay cables may be readily modified to apply to rope-lay cables, as each stranded wire bears the same relation to the rope as each individual solid wire does to the concentric-lay cable. The following table gives the principal forms of rope-lay cables.

WIRES IN ROPE-LAY CABLES

Number of layers over core	Number of strands*	Total number of wires				
		Wires per strand*				
		7	19	37	61	91
0	1	7	19	37	61	91
1	7	49	133	259	427	637
2	19	133	361	703	1159	1,729
3	37	259	703	1369	2257	3,367
4	61	427	1159	2257	3721	5,551
5	91	637	1729	3367	5551	8,281
6	127	889	2413	4699	7747	11,557
n	$3(n^2+n)+1$	$21(n^2+n)+7$	$57(n^2+n)+19$	$111(n^2+n)+37$	$183(n^2+n)+61$	$273(n^2+n)+91$

* By "strand" is here meant the stranded wires of which the rope is built up.

The number of wires in a rope-lay cable is frequently designated by a product; thus, 7×19 indicates a conductor made up of 7 strands, each strand containing 19 wires.

Hemp-core Cables. — Cables with hemp centers have caused serious trouble and are not being recommended. (D. B. Rushmore, *G. E. Review, June, 1912.*)

Diameters of Component Wires in Commercial Stranded Conductors. — The table on p. 1875 gives the diameters of the component wires in the types of stranded conductors ordinarily used.

Effect of Lay on Resistance and Weight. — In the tables given on pp. 1860, 1862 and 1868, for stranded cables, the values given for "ohms per unit length" and "weight per unit length" are 2 per cent greater than for a solid rod of cross-section equal to the total cross-section of the wires of the cable. This increment of 2 per cent means that the values are correct for cables having a lay of 1 in 15.7. For any other lay, equal to 1 in n, resistance or mass may be calculated by increasing the above tabulated values by

$$\left(\frac{484}{n^2} - 2\right) \text{ per cent.}$$

General Formulas for Properties of Cables in Terms of the Properties of the Constituent Wires. — The following table gives the principal formulas for concentric-lay cable having a core of one wire.

A = total area in circular mils of the component wires measured at right angles to their axes, when laid out straight.

D = diameter of cable over-all, in inches.

d = diameter of each of the component wires, in inches.

d = diameter of core, in inches.

d_p = pitch diameter, in inches, of any layer (= mean diameter of the helix made by any layer).
e = elongation, per cent, at which the wires (other than the core) break.
l = number of wires in any layer having pitch diameter d_p.
N = total number of wires except where the core is of special size, in which case N is the number of wires exclusive of the core.
n = number of layers of wire over the core.
P = pitch of any layer of wires = distance in inches measured along the axis of the cable for one complete turn of the helix formed by any wire of this layer.
p = pitch-factor of any layer of wires = ratio of the actual length of a wire to the corresponding axial length of the cable.
R = ratio of wire area to the total area of the circle circumscribing the outside of the conductor.
s = stress in pounds per square inch in the core when the elongation is e.
t = tensile strength of each outer wire, in pounds per square inch.
T = tensile strength of conductor, in pounds.
W = weight of conductor, in pounds per foot.
w = weight of each wire of the cable, in pounds per foot.
w_c = weight of the core of the cable, in pounds per foot.

PROPERTIES OF CONCENTRIC-LAY CABLES

		Regular; $d_c = d$	Special; $d_c \neq d$
I.	Number of wires in terms of number of layers (and core diameter).	$N = 3(n^2+n)+1$ (including core)	$N = 3\left(n\frac{d_c}{d}+n^2\right)$ (excluding core)
II.	Diameter of cable in terms of diameter of wires and number of layers.	$D = d(1+2n)$	$D = d_c + 2nd$
III.	Diameter of cable in terms of total area and number of wires.	$D = 10^{-3}\sqrt{\frac{1}{3}\left(4-\frac{1}{N}\right)}\cdot\sqrt{A}$	
IV.	Ratio of wire area to area of circle circumscribing the outside of cable.	$R = \frac{3(n^2+n)+1}{(2n+1)^2}$	
V.	Weight of cable in terms of weight of wire, number of layers and pitch factors.	$W = w(1+6p_6+12p_{12}+\text{etc.})$	$W = w_c + w(1+6p_6+12p_{12}+\text{etc.})$
VI.	Strength of cable in terms of strength of the component wires and the pitch factors.	$T = \frac{\pi}{4}d^2\left[s+t\left(\frac{6}{p_6}+\frac{12}{p_{12}}+\text{etc.}\right)\right]$	$T = \frac{\pi}{4}\left[sd_c^2+d^2t\left(\frac{6}{p_6}+\frac{12}{p_{12}}+\text{etc.}\right)\right]$

PROPERTIES OF CONCENTRIC-LAY CABLES—*Continued*

		Regular; $d_o = d$	Special; $d_o \neq d$
VII.	Minimum pitch in terms of wire diameter and core diameter.	$\frac{3\pi d_p}{\sqrt{(\pi+3)(\pi-3)}} = 10.1$ times pitch diameter	$\frac{\pi d_p l d}{\sqrt{(\pi d_p)^2-(ld)^2}}$
VIII.	Diameter of wires in terms of total conductor area and number of wires.	$d = \frac{1}{1000}\sqrt{\frac{A}{N}}$	

STRENGTH, ELASTICITY AND EXPANSION COEFFICIENT OF WIRES.—The strength and elasticity of a wire of any material depends to a considerable extent upon the method of manufacture, heat treatment, etc. The tensile strength of soft copper is between 25,000 and 35,000 pounds per square inch, as against 60,000 pounds per square inch for hard-drawn copper. Again, due to the greater relative thickness of the hard "skin" and comparatively soft "core" of small hard-drawn copper wires as compared with large wires, the tensile strength, in pounds per square inch, of a small hard-drawn copper wire is greater than the tensile strength of a large hard-drawn wire. For example, a No. 0000 B. & S. hard-drawn copper wire has a tensile strength of about 50,000 pounds per square inch as against approximately 65,000 pounds per square inch for a No. 18. A similar but smaller variation holds for soft-drawn wires. The tensile strength of steel wire depends to a very great extent upon the composition of the steel.

The following table gives, for a No. 0 A. W. G. wire, representative values of the various quantities stated. These values do not hold, except to a rough approximation, for other sizes of wire. For further information see articles on *Copper*, *Aluminum*, etc., and the section on *Specifications* below.

STRENGTH, ELASTICITY AND COEFFICIENT OF EXPANSION
Of a No. 0 A. W. G. or B. & S. Wire

Kind of wire	Tensile strength, lb. per sq. in.	Elastic limit, lb. per sq. in.*	Modulus of elasticity, lb.-in. units	Coefficient of linear expansion per °F.	Coefficient of linear expansion per °C.
Copper, soft-drawn	36,000	......	....	9.6×10^{-6}	17×10^{-6}
Copper, hard-drawn	54,500	30,000	16×10^6	9.6×10^{-6}	17×10^{-6}
Aluminum, soft-drawn	16,000	......	....	12.8×10^{-6}	23×10^{-6}
Aluminum, hard-drawn	25,000	25,000	9×10^6	12.8×10^{-6}	23×10^{-6}
Copper-clad steel, 40% grade	60,000	51,000	22×10^6	6.7×10^{-6}	12×10^{-6}
Phono-electric	75,800	55,000	18×10^6	8.3×10^{-6}	14.9×10^{-6}
Steel, ordinary	68,000	40,000	24×10^6 to 30×10^6	7.0×10^{-6}	12.6×10^{-6}
Steel, Siemens-Martin	90,000	45,000		7.0×10^{-6}	12.6×10^{-6}
Steel, high strength	150,000	82,000		7.0×10^{-6}	12.6×10^{-6}
Steel, extra high strength	225,000	135,000		7.0×10^{-6}	12.6×10^{-6}

* There is no elastic limit for soft annealed copper and the elastic limits of hard-drawn copper and aluminum are doubtful.

The tensile strength in pounds for solid wires from 1/16 inch to 1/2 inch in diameter are given in the following table.

BREAKING LOAD FOR SOLID WIRES IN POUNDS PER WIRE

Gage No. A.W.G. or B. & S.	Diameter		Hard-drawn copper (Am. Soc. for Test. Mat.)*	Hard-drawn aluminum (23,000 to 33,300 lb. per sq. in.)	Copper-clad steel, 40 per cent grade (Duplex Metals Co.)	Steel (100,000 lb. per sq. in.) †
	Inches	Mils				
	1/2	500	9310	4520	11,400	19,640
0000		460	8140	3820	10,000	16,620
	7/16	437	7500	3460	9,250	15,030
000		410	6720	3030	8,300	13,180
	3/8	375	5800	2540	7,150	11,040
00		365	5540	2400	6,850	10,450
0		325	4520	1910	5,700	8,289
	5/16	312	4220	1770	5,400	7,670
1		289	3680	1530	4,800	6,573
2		258	3000	1240	4,000	5,213
	1/4	250	2830	1170	3,780	4,909
3		229	2420	1000	3,200	4,134
4		204	1950	810	2,600	3,278
	3/16	187	1680	693	2,300	2,761
5		182	1570	655	2,200	2,600
6		162	1270	532	1,800	2,062
7		144	1020	432	1,450	1,635
8		129	822	351	1,200	1,297
	1/8	125	780	335	1,150	1,227
9		114	660	287	975	1,028
10		102	528	234	800	816
11		91	423	191	650	647
12		81	337	155	510	513
13		72	268	126	410	407
14		64	213	103	330	323
	1/16	62	203	98	310	307

* Tensile strength in pounds per square inch ranging from 49,000 for No. 0000 to 66,200 for No. 14; see below.

† For wires having a tensile strength of S pounds per square inch, multiply by $S/100{,}000$. The tensile strength of steel varies from 60,000 to 225,000 pounds per square inch.

Strength, Elasticity and Expansion Coefficient of Cables.—The following is a summary of the results of tests made on stranded copper wires at the Massachusetts Institute of Technology in 1912. Each figure is the average of a number of individual tests. In determining the modulus of elasticity each cable was given a preliminary stretch before readings were taken.

MECHANICAL PROPERTIES OF BARE COPPER CABLES

Designation	Size of cable A. W. G. and cir. mils	Number of wires in cable	Pitch of layers, inches		Modulus, lb.-in. units	Elastic limit, lb. per sq. in.	Tensile strength, lb. per sq. in.
			First layer	Second layer			
A	0	7	4.5	...	16.6×10^6	22,000	53,500
B	300,000	19	7	5	16.4×10^6	21,300	59,800
C	300,000	19	9	4.5	16.3×10^6	25,000	56,000
D	300,000	19	9	9	16.5×10^6	24,000	52,600
E	300,000	19	4.5	3	13.6×10^6	25,300	53,800

PROPERTIES OF COMPONENT WIRES OF ABOVE CABLES

Wires from cable	Size of component wires, mils	Modulus, lb.-in. units	Elastic limit, lb. per sq. in.	Tensile strength, lb. per sq. in.
B	125	16.9×10^6	27,000	62,800
C	125	18.9×10^6	27,000	61,900
D	125	16.6×10^6	26,000	61,800
E	125	17.5×10^6	26,000	59,300

No data are available on the expansion coefficient of stranded wires, but it seems reasonable to assume that it would be approximately the same for stranded as for solid wires.

Steel Cable for Catenary Construction.— In the report of the Committee on Power Distribution of the American Street and Interurban Ry. Assoc., 1908, a modulus of 22×10^6 pound-inch units is given as representative of ordinary steel messenger cable.

The following tables are compiled from tables published by the General Electric Co. (*Bulletin 4538, 1907*). "High" and "Extra high" strength steel should only be used where absolutely necessary, as, on account of its stiffness, it requires special mechanical fastenings.

STEEL CABLE FOR CATENARY SUSPENSION

Extra-galvanized Siemens-Martin Steel Strand, 90,000 Pounds Per Square Inch.				
Diameter, inches	Tensile strength, pounds	Elastic limit, pounds	Elongation, per cent	Lay, inches
1/4	3,060	1,830	6 to 9	3
5/16	4,860	2,910	6 to 9	3 1/2
3/8	6,800	4,080	5 to 8	4
7/16	9,000	5,300	5 to 8	4 1/2
1/2	11,000	6,600	5 to 8	4 1/2
5/8	19,000	11,400	4 to 6	5

STEEL CABLE FOR CATENARY SUSPENSION — (*Continued*)

Extra-galvanized High-strength Crucible Steel Strand				
Diameter, inches	**Tensile strength, pounds**	**Elastic limit, pounds**	**Elongation, per cent**	**Lay, inches**
¼	5,100	3,315	3 to 5	3½
5/16	8,100	5,265	3 to 5	4
⅜	11,500	7,475	3 to 5	4½
7/16	15,000	9,500	3 to 5	5
½	18,000	11,700	3 to 5	5
⅝	25,000	16,250	2 to 4	5½
Extra-galvanized Extra-high-strength Plow Steel Strand.				
Diameter, inches	**Tensile strength, pounds**	**Elastic limit pounds**	**Elongation, per cent**	**Lay, inches**
¼	7,600	5,700	2½ to 4	4
5/16	12,100	9,075	2½ to 4	4½
⅜	17,250	12,930	2½ to 4	5
7/16	22,500	16,800	2½ to 4	5½
½	27,000	20,250	2½ to 4	5½
⅝	42,000	31,500	1½ to 3	6

COMPARISON OF COPPER AND ALUMINUM FOR EQUAL LENGTH AND EQUAL RESISTANCE. — (*See Aluminum.*)

CURRENT-CARRYING CAPACITY OF BARE WIRES. — (*See also articles on Rheostats; Wires, Resistance.*) Let d = diameter of conductor in inches, T = permissible temperature rise in ° C. above surrounding medium (air, earth, or water), r = resistance of conductor in ohms per mil foot at final temperature, I = current per conductor in amperes. Assuming that the rate of heat radiation per unit length of wire is proportional to the difference of temperature between the conductor and the surrounding medium and also proportional to the surface of the conductor, then for solid conductors

$$I = K\sqrt{\frac{Td^3}{r}},$$

and for stranded conductors

$$I = 0.85\,K\sqrt{\frac{Td^3}{r}},$$

where K is a constant, which depends upon the condition of the surface of the wire, and upon the amount of heat convection due to air currents. Values of the constant K for air given by different authorities vary from 800 to 1000, the former referring to still air and the latter to open air.

TESTS OF BARE WIRES AND CABLES. — The usual tests on bare wires are gaging diameter, measuring tensile strength, elongation, modulus, elastic limit and electrical conductivity. These tests are:

Gaging Diameter. — (*See also section on Specifications, below.*) The best type of gage for measuring wire diameters is that shown in Fig. 4. The wire is placed between the measuring surfaces and the screw adjusted until a click occurs. The number of large divisions exposed on the axis is multiplied by 100; the number of small divisions exposed on the axis is multiplied by 25; and the sum of these two items added to the number indicated on the revolving scale. The sum will be the diameter of the wire in mils.

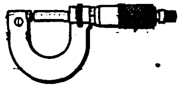


Fig. 4.

Tensile Strength, Elongation, Modulus and Elastic Limit. — The essential features of a wire-testing machine are a means of applying a measurable pulling force to the wire, and a means of taking up the elongation. Accordingly, the usual testing machine consists of two pairs of jaws for gripping the wire, one pair being connected to a balance lever and the other pair to a power-driven mechanism which draws it in the direction of the axis of the wire. A typical machine is shown diagrammatically in Fig. 5, where *A* and *B* are the two pairs of jaws between which the wire is stretched. The machine is operated by setting in motion the mechanism which makes the jaw *A* move steadily in the direction indicated. The operator then moves the counterpoise *C* by hand, in the direction indicated, so as to keep the beam balanced. This operation is continued until the wire breaks, when the elongation of the sample is measured by the travel of the jaw *A* and its breaking strength by the weight indicated on the balance beam at the counterpoise *C*.

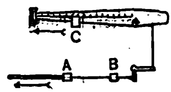


Fig. 5.

Measurement of Strain. — The amount by which the wire is stretched is measured by means of an extensometer which consists of a pair of clamps to grip the wire at points a definite distance apart, and a magnifying scale for measuring the increase of distance between these clamps as the wire stretches. The stress-strain curve obtained by plotting the elongations thus measured against the stresses measured by the machine described above is not a true one, as there is initially an abnormal elongation due to the straightening of the wire. It is, therefore, necessary to plot an initial curve as shown by *A* in Fig. 6, then to continue the line of true linear extension backward and replot the curve so that the exterpolated line passes through zero.

Modulus of Elasticity. — The modulus of elasticity is obtained from the slope of the straight part of the redrawn curve. In Fig. 6 the modulus of elasticity is *OD/CD* pound-inch units.

Elastic Limit. — The true elastic limit can be obtained only by applying a series of increasing loads, releasing the load (leaving, however, a sufficient load to keep the wire straight) and measuring the elongation between successive loads. The load at which a permanent elongation begins is the elastic limit.

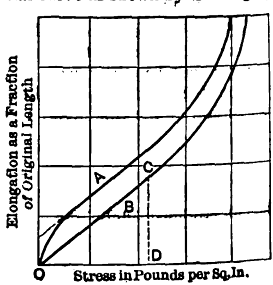


Fig. 6.

Conductivity. — In order to maintain a wire at a uniform and known temperature, it must be short. Unless the wire is very small the test sample will

therefore have a very low resistance, and an ordinary Wheatstone bridge will not be sufficiently accurate to measure it. This difficulty is avoided by using a bridge of either the Kelvin, Hoop, Willyoung or Reeves type (*see Bridges for Electrical Measurements; Resistance and Conductance*).

SPECIFICATION FOR SOFT OR ANNEALED COPPER WIRE. — The following specifications for bare copper wire are those prepared by the American Society for Testing Materials, August 21, 1911 (hard-drawn copper), and June 1, 1912 (soft-drawn copper). See also the article on *Specifications and Contracts*, and the section on *Specifications* in the article on *Wires and Cables, Insulated.*

1. **General.** — The copper shall be of such quality and purity that, when drawn and annealed, it shall have the properties and characteristics herein required.

2. This specification covers untinned, drawn and annealed round wire.

3. (*a*) The wire must be free from all surface imperfections not consistent with the best commercial practice.

(*b*) Necessary brazes in soft or annealed wire must be made in accordance with the best commercial practice.

4. **Shipment; Coils, Spools and Reels.** — (*a*) Wire may be shipped in coils or on reels as agreed upon by the purchaser and manufacturer. In Table I (*below*) there are stated the maximum and minimum weights of wire of the stated sizes which may be shipped in any one package, whether coil, reel or spool; in the case of wire larger than 0.010 inch in diameter, the maximum and minimum package weights are net, and in the case of wire 0.010 inch and less in diameter, the maximum package weights are gross, and the minimum package weights are net. The table also states the limiting of the dimensions of the coils, reels and spools on which wire may be shipped. The length and diameter stated for reels and spools are to be measured over-all and are maximum sizes; reels or spools smaller than these may be used provided the minimum weights called for are carried by the reel or spool. In the table, there are also stated the diameters of the draw-block on which the final drawing of the wire is to be made, when wire is shipped in coils; it being understood that the wire is not to be rewound after final drawing. This provision is made to insure that coils of wire of a given gauge, when supplied by different manufacturers, will be of the same general dimensions.

Wire 0.204 inch in diameter and larger may be shipped in larger packages when agreed upon.

(*b*) The wire shall be protected against damage in ordinary handling and shipping.

5. **Specific Gravity.** — For the purpose of calculating weights, cross-sections, etc., the specific gravity of copper shall be taken as 8.90.

6. **Size and Gaging.** — (*a*) Size shall be expressed as the diameter of the wire in decimal fractions of an inch.

(*b*) Wire shall be accurate in diameter; permissible variations from nominal diameter shall be:

For wire 0.010 inch in diameter and larger, 1 per cent over or under.

For wire less than 0.010 inch in diameter, 0.1 mil (0.0001 inch) over or under.

(*c*) Each coil shall be gaged at three places, one near each end and one approximately at the middle; from spools, approximately twelve feet shall be reeled off, the wire shall be gaged in six places between the second and twelfth foot from the end. The coils or spools will be rejected if the average of the measurements obtained is not within the limits in (*b*).

TABLE I

Diameters, inches	Package weights, pounds		Diam. of draw-block, inches	Dimensions of reels and spools, inches		
	Max.	Min.		Max. diam.	Max. length	Diameter of hole for rod
0.460 to 0.360	520	290	24	32	21	1½ to 2½
0.359 to 0.258	430	290	24	32	21	1½ to 2½
0.257 to 0.129	290	140	22	24	12	1½ to 2½
0.128 to 0.102	230	95	22	24	12	⅝ to 1½
0.101 to 0.083	230	75	22	24	12	⅝ to 1½
0.082 to 0.081	200	75	16	24	12	⅝ to 1½
0.080 to 0.064	200	50	16	24	12	⅝ to 1½
0.063 to 0.051	120	50	16	24	10	⅝ to 1½
0.050 to 0.041	100	50	16	24	10	⅝ to 1½
0.040 to 0.032	50	20	8	24	8	⅝ to 1⅛
0.031 to 0.020	25	15	8	10	6½	⅝ to ⅞
0.019 to 0.011	10	5	8	5½	4	⅜ to 1 1/16
0.010 to 0.008	5	2½	8	4	4	⅜ to 1 1/16
0.007 to 0.0056	2½	1	6	2½	4	⅜ to 1 1/16
0.005	1½	⅝	6	2½	4	⅜ to 1 1/16
0.004	1½	⅜	6	2½	4	⅜ to 1 1/16
0.003	1	¼	6	2½	4	⅜ to 1 1/16

7. **Tensile Strength and Elongation.**—Wire shall be so drawn and annealed that its tensile strength shall not be greater and its elongation not less than the values stated in Table II. Tensile tests shall be made upon fair samples, and the elongation shall be determined as the permanent increase in length, due to the breaking of the wire in tension, measured between bench marks placed upon the wire originally 10 inches apart. The fracture shall be between the bench marks and not closer than 1 inch to either bench mark. If, upon testing a sample from any coil, reel or spool of wire, the results are found to be below the stated value in elongation or above the stated value in tensile strength, tests upon two additional samples shall be made, and the average of the three tests shall determine acceptance or rejection of the coil. For wire whose nominal diameter is between listed sizes, the requirements shall be those of the next larger size included in the table.

TABLE II.—ANNEALED COPPER WIRE

Diameter, inches	Tensile strength, lb. per sq. in.	Elongation in 10 in., per cent
0.460–0.290	36,000	35
0.289–0.103	37,000	30
0.102–0.021	38,500	25
0.020–0.003	40,000	20

8. **Conductivity.** — Electric conductivity shall be determined upon fair samples by resistance measurements at a temperature of 20° C. (68° F.) and it shall not exceed 891.58 pounds per mile-ohm.

9. **Testing and Inspection.** — All testing and inspection shall be made at the place of manufacture. The manufacturer shall afford the inspector representing the purchaser all reasonable facilities to satisfy him that the material conforms to the requirements of these specifications.

SPECIFICATION FOR HARD-DRAWN COPPER WIRE.

1. **General.** — The material shall be copper of such quality and purity that, when drawn hard, it shall have the properties and characteristics herein required. (*See § 1 of preceding specification.*)

2. This specification covers hard-drawn round wire, grooved trolley wire, figure-eight trolley wire, and hard-drawn cable or strand, as hereinafter described.

3. (*a*) The wire, in all shapes, must be free from all surface imperfections not consistent with the best commercial practice.

(*b*) Necessary brazes in hard-drawn wire must be made in accordance with best commercial practice, and tests upon a section of wire containing a braze must show at least 95 per cent of the tensile strength of the unbrazed wire. Elongation tests are not to be made upon test sections including brazes.

4. **Shipment.** — (*a*) Package sizes for round wire and for cable shall be agreed upon in the placing of individual orders; standard packages of grooved trolley wire shall be shipped upon reels holding about 2500 pounds each.

(*b*) The wire shall be protected against damage in ordinary handling and shipping.

5. **Specific Gravity.** — For the purpose of calculating weights, cross-sections, etc., the specific gravity of copper shall be taken as 8.90.

6. **Testing and Inspection.** — All testing and inspection shall be made at the place of manufacture. The manufacturer shall afford the inspector representing the purchaser all reasonable facilities to enable him to satisfy himself that the material conforms to the requirements of these specifications.

7. **Size and Gaging of Round Wire.** — (*a*) Size shall be expressed as the diameter of the wire in decimal fractions of an inch, using not more than three places of decimals; i.e., in mils.

(*b*) Wire is expected to be accurate in diameter; permissible variations from nominal diameter shall be:

For wire 0.100 inch in diameter and larger, one per cent over or under;
For wire less than 0.100 inch in diameter, one mil over or under.

(*c*) Each coil is to be gaged at three places, one near each end, and one approximately at the middle; the coil may be rejected if, two points being within the accepted limits, the third point is off gage more than 2 per cent in the case of wire 0.064 inch in diameter and larger, or more than 3 per cent in the case of wire less than 0.064 inch in diameter.

8. **Tensile Strength and Elongation of Round Wire.** — Wire shall be so drawn that its tensile strength and elongation shall be at least equal to the value stated in Table III. Tensile tests shall be made upon fair samples, and the elongation of wire larger in diameter than 0.204 inch shall be determined as the permanent increase in length, due to the breaking of the wire in tension, measured between bench marks placed upon the wire originally 10 inches apart. The elongation of wire 0.204 inch in diameter and smaller shall be determined by measurements made between the jaws of the testing machine. The zero length shall be the distance between the jaws when a load equal to 10 per cent of the required ultimate breaking strength shall have been applied, and the

final length shall be the distance between the jaws at the time of rupture. The zero length shall be as near 60 inches as possible. The fracture shall be between the bench marks in the case of wire larger than 0.204 inch in diameter and between the jaws in the case of smaller wire, and not closer than 1 inch to either bench mark or jaw. If, upon testing a sample from any coil of wire, the results are found to be below the values stated in the table, tests upon two additional samples shall be made, and the average of the three tests shall determine acceptance or rejection of the coil. For wire whose nominal diameter is between listed sizes, the requirements shall be those of the next larger size included in the table.

TABLE III.—HARD-DRAWN COPPER WIRE

Approx. Gage No., B. & S.	Diameter, inches	Area, circular mils	Tensile strength, lb. per sq. in.	Elongation, per cent
				in 10 in.
0000	0.460	211,600	49,000	3.75
000	0.410	168,100	51,000	3.25
00	0.365	133,225	52,800	2.80
0	0.325	105,625	54,500	2.40
1	0.289	83,520	56,100	2.17
2	0.258	66,565	57,600	1.98
3	0.229	52,440	59,000	1.79
				in 60 in.
4	0.204	41,615	60,100	1.24
5	0.182	33,125	61,200	1.18
	0.165	27,225	62,000	1.14
6	0.162	26,245	62,100	1.14
7	0.144	20,735	63,000	1.09
	0.134	17,956	63,400	1.07
8	0.128	16,385	63,700	1.06
9	0.114	12,995	64,300	1.02
	0.104	10,815	64,800	1.00
10	0.102	10,404	64,900	1.00
	0.092	8,464	65,400	0.97
11	0.091	8,281	65,400	0.97
12	0.081	6,561	65,700	0.95
	0.080	6,400	65,700	0.94
13	0.072	5,184	65,900	0.92
	0.065	4,225	66,200	0.91
14	0.064	4,096	66,200	0.90
15	0.057	3,249	66,400	0.89
	0.051	2,601	66,600	0.87
16	0.045	2,025	66,800	0.86
18	0.040	1,600	67,000	0.85

9. **Conductivity of Round Wire.**—Electric conductivity shall be determined upon fair samples by resistance measurements at a temperature of 20° C. (68° F.).

The wire shall not exceed the following limits:

For diameters 0.460 inch to 0.325 inch, 900.77 pounds per mile-ohm at 20° C.

For diameters 0.324 inch to 0.040 inch, 910.15 pounds per mile-ohm at 20° C.

10. **Grooved Trolley Wire.** — Standard sections shall be those known as the "American Standard" grooved trolley-wire sections, the shape and dimensions of which are as shown in Fig. 1, above.

11. (*a*) Size shall be expressed as the area of cross-section in circular mils, the standard sizes being as follows:

211,600 circular mils, weighing 3386 pounds per mile.
168,100 circular mils, weighing 2690 pounds per mile.
133,200 circular mils, weighing 2132 pounds per mile.

(*b*) Grooved trolley wire may vary 4 per cent over or under in weight per unit length from standard, as determined from the nominal cross-section.

12. The physical tests shall be made in the same manner as those upon round wire. The tensile strength of grooved wire shall be at least 95 per cent of that required for round wire of the same sectional area; the elongation shall be the same as that required for round wire of the same sectional area.

13. The requirements for electric conductivity shall be the same as those for round wire of the same sectional area.

14. **Figure-eight Trolley Wire.** — Standard sections of figure-eight trolley wire shall be as shown in Fig. 2, above.

15. The requirements for weight, physical properties and electric conductivity of figure-eight trolley wire shall be the same as for the same sizes of grooved trolley wire.

16. **Hard-drawn Copper Wire Cable or Strand.** — For the purposes of these specifications, standard cable shall be that made up of hard-drawn wire laid concentrically about a hard-drawn wire center. Cable laid up about a hemp center or about a soft-wire core is to be subject to special specifications to be agreed upon in individual cases.

17. The wire entering into the construction of stranded cable shall, before stranding, meet all the requirements of round wire, hereinbefore stated.

18. The tensile strength of standard cable shall be at least 90 per cent of the total strength required of the wires forming the cable.

19. Brazes, made in accordance with the best commercial practice, will be permitted in wire entering into cable; but no two brazes in wire in the cable may be closer together than 50 feet.

20. The pitch of standard cable shall not be less than 12, nor more than 16, diameters of the cable. The cable shall be laid left-handed or right-handed, as shall be agreed upon in the placing of individual orders.

Extract from Notes to Above Specifications. — The permitting of brazes in wire entering into the construction of copper cable was discussed at considerable length, and it is finally the opinion of the Committee that, provided no two brazes are closer together than 50 feet, the cable has fully 90 per cent of the theoretical strength obtained by adding together the required strengths of the constituent wires. This is due, in such long lengths, to the frictional gripping of the wires in the cable. The construction of long lengths of cable without brazes is costly, and it has been thought best, therefore, to permit their use, provided they are sufficiently widely spaced as not to be detrimental to the strength of the cable.

INSTALLATION OF BARE WIRES AND CABLES. — For the installation of wires and cables in buildings see article on *Wiring of Buildings for Light and Power*. Below is given a brief description of modern practice in erecting bare wires and cables on pole or tower lines. For the construction of the latter see articles on *Distribution Lines; Transmission Lines*. Catenary construction is described in the article on *Trolley Systems, Overhead*.

Simple Span Construction with Pin Insulators. — Starting at an anchored pole, a rope about twice the length of the span is put over the cross arm and the wire pulled up by means of it. The rope is then put over the cross arm of the next pole and the wire drawn up in the same way. This is repeated from pole to pole until the reel, which remains at the starting point, is exhausted. The pulling may be done by men, horses, automobile, or locomotive. Care should be taken to prevent the wire unwinding too rapidly from the reel and the end of the wire must be prevented from slipping away.

The wire is placed on the insulators by means of a block and tackle attached either to the arm above or to a temporary boom. The next step is to draw the wires to the proper tension. Commencing at the first pole after the anchorage, the wire is gripped by a clamp attached to a rope which is pulled until the wire is drawn to the sag indicated by a table or curve showing the proper sag for different temperatures and spans (*Spans, Wire*). The sag is gauged by sighting from pole to pole, using a gauge or sight on each pole, and drawing the wire until the bottom of the span is tangential to the sight line.

Tying to Insulator.— (*See also Insulators for Overhead Lines.*) The wires are tied to their insulators by small wires, usually of the same metal, as shown in Fig. 7. The two ends of the tie wire (not visible in Fig. 7) are twisted together for three or four turns. The tie wire must be held quite taut while it is being installed.

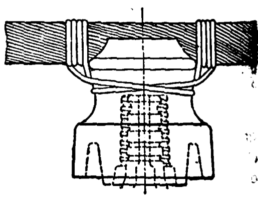


Fig. 7.

Simple Span Construction with Suspension Insulators. — Where suspension insulators are used, the wire, instead of being initially placed over the cross arms, is temporarily suspended therefrom on snatch blocks provided with wooden rollers. Prior to running the cables, linemen are sent ahead to attach the snatch blocks to the arms of the towers. The cables are then strung loosely through the snatch blocks, in much the same way as they are strung over the cross arms in the construction described above. The cables are anchored to the first pole and the necessary tension applied to the cable between the first and second towers, a dynamometer being used to indicate when the required stress point has been reached. The cable is then anchored to the second tower and the operation repeated for the different spans until the cable length is exhausted. The cable is then transferred to the insulator clamps and the anchorages removed except at the ends.

Clamping to Insulator. — The usual type of insulator clamp (Fig. 8) consists of a curved, malleable-iron wire seat about nine inches long, a saddle which fits over the wire and a pair of hook bolts to hold the wire tightly between the saddle and wire seat. The wire seat is supported from the insulator through a swivel joint. Copper wire or cable is laid in the wire seat, the saddle placed on it and the hook bolts placed in position. In the case of aluminum wire a 1/16-inch aluminum sleeve should be placed around the wire in the clamp. The entire clamp is then attached to the insulator by a bolt and socket or pin joint.

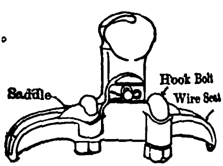


Fig. 8.

Jointing Conductors. — The following methods are used:

Western Union Joint. — The two ends of the wire are brought together so that they lap from 3 to 8 inches, depending upon the size. Then

beginning half-way between the two ends, each wire is wound around the other wire in a tight helix. With hard-drawn copper, excessive stress is avoided by giving the helix a long pitch for the first turn or two and then gradually reducing the pitch until a tight helix is obtained.

It would appear from tests on galvanized-iron wire by C. T. Rashman (*E. W., 1910, Vol. 56, p. 1187*), that in order to make the joint as strong as the wire, it should have the form shown in Fig. 9 which may be described as consisting of a neck of five turns and five end turns at each end.

Fig. 9.

Fig. 10.

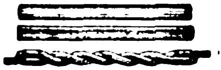


Fig. 11.

In order to obtain five turns in the neck, the following length of neck was found necessary:

Size wire, B.W.G.	Length of neck, inches
14	3-1/2
12	3-3/4
10	4
8	6
6	7

The form of splice shown in Fig. 10 should be avoided.

Dovetailing Strands. — The cables are unwound for say three or four feet, dovetailed or interlaced, and the strands individually wound round the unopened part of the cable. The joint may be soldered or not, as desired.

Aluminum Tube Joint. — Aluminum conductors of sizes up to about one-half inch diameter are usually jointed by inserting the two ends side by side into a flat aluminum tube, which is then given 3 or 4 twists as shown in Fig. 11 by means of a special kind of tongs called connectors.

Pressed Aluminum Joint. — Aluminum conductors larger than one-half inch in diameter are more often jointed by dovetailing the strands, or by mechanical clamps such as the Dossert connector (*see below*), or by inserting the wires into a cast aluminum sleeve which is squeezed hydraulically until the conductors and sleeve flow into a solid homogeneous mass.

Dossert Joint and Connector. — The Dossert joint, shown in Fig. 12, consists of a compression sleeve which is slipped over the conductor, a screw on

Fig. 12.

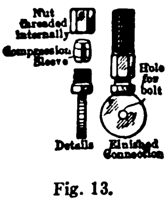


Fig. 13.

the lug proper and a nut threaded with a taper thread. The sleeve containing the conductor is thrust into the lug, with the nut over it. The nut is then tightened until the reaction of the tapered surface and the sleeve gives rise to a pressure of several thousand pounds per square inch, thereby making good electrical contact.

The Dossert connector or terminal is similar in construction, and is shown in Fig. 13.

COSTS. — The "base" price per pound of bare wire is usually about 1 cent more than the market price of ingot copper or "wire-bar." The following table (*from the American Steel & Wire Co.'s catalogue*) gives the extra charge in cents per pound for drawing, tinning and stranding.

CENTS PER POUND ABOVE BASE PRICE FOR DRAWING, TINNING AND STRANDING

Size A.W.G.	Drawing	Tinning	Stranding into cable or cord.
0000–8	—	¾	½
9	⅛	¾	¾
10	⅛	¾	¾
11	¼	¾	¾
12	¼	¾	¾
13	½	1	¾
14	½	1	1¼
15	¾	1	1¼
16	¾	1	1¼
17	1	1	2
18	1	1¼	2
19	1¼	1¼	2
20	1¼	1½	2
21	1⅝	1¾	5
22	1⅝	2	5
23	2½	2½	5
24	2½	3	5

Example. — If the market price of ingot copper were 16 cents per pound a cable composed of tinned No. 20 wires would cost 16 + 1 + 1¼ + 1½ + 2 = 21¾ cents per pound. For price of aluminum wire, see remarks in article on *Aluminum*.

BIBLIOGRAPHY. — (*See also bibliographies in the articles on Aluminum; Copper; Steel; Wires and Cables, Insulated.*) American Society for Testing Materials, *Standard Specifications for Hard Drawn Copper Wire*, Proc. 1909, Vol. 9, p. 311; Buck, H., *The Use of Aluminum as an Electrical Conductor*, Trans. Int. Elect. Congress, St. Louis, 1904, Vol. 2, p. 313; Feussner, K., and St. Lindeck, *Elektrischen Normal-Drahtwiderstände*, Wissenschaftliche Abhandlung der Phys. Tech. Reischsanstalt, 1895, Vol. 2, p. 501; Fowle, F. F., *Electrical Properties of Compound Wires*, El. W., 1910, Vol. 56, p. 1471, 1521; El. W., 1911, Vol. 57, p. 108; Huber-Stockar, E., *Aluminum for Electric Conductors*, Int. Elect. Congress, Turin, 1911 (contains a bibliography); Kennelly, A. E., Wright, C. A., and Van Bylevelt, J. S., *The Convection of Heat from Small Copper Wires*, Trans. A.I.E.E., 1909, Vol. 28, p. 363 (contains a bibliography); Rashman, C. T., *Some Tests on Splices in Galvanized Iron Wire*, El. W., 1910, Vol. 56, p. 1187; Wolff, F. A., and Dellinger, J. H., *Electrical Conductivity of Commercial Copper*, Bureau of Standards Circ. No. 148.

[W. A. Del Mar.]

WIRES AND CABLES, INSULATED. — (*See also Aluminum; Capacity and Charging Current; Copper; Distribution Lines; Electrolysis; Inductance and Inductive Reactance; Insulating Materials; Resistance; Skin Effect; Standardization Rules of the A.I.E.E.; Transmission Lines; Wires and Cables, Bare; Wires, Resistance.*)

The following is a brief table of contents of this article:

TERMINOLOGY. — The terminology employed in referring to wires and cables is at present in a very confused state, the same term being used with several different meanings. The following terminology is based upon a study of this subject by the Bureau of Standards, and upon a canvass of wire manufacturers and users made by the author of this article, and is quoted from the *Standardization Rules of the A.I.E.E.* (July 1914).

321.* **Wire.** — A slender rod or filament of drawn metal.

The definition restricts the term to what would ordinarily be understood by the term "solid wire." In the definition the word "slender" is used in the sense that the length is great in comparison with the diameter. If a wire is covered with insulation, it is properly called an insulated wire. While primarily the term "wire" refers to the metal, nevertheless when the context shows that the wire is insulated the term "wire" will be understood to include the insulation.

322. **Conductor.** — A wire, or combination of wires not insulated from one another, suitable for carrying a single electric current.

The term "conductor" is not to include a combination of conductors insulated from one another, which would be suitable for carrying several different electric currents.

Rolled conductors (such as bus-bars) are, of course, conductors, but are not considered under the terminology here given.

323. **Stranded Conductors.** — A conductor composed of a group of wires or any combination of groups of wires.

The wires in a stranded conductor are usually twisted or braided together.

324. **Cable.** — (1) A stranded conductor (single-conductor cable); or (2) a combination of conductors insulated from one another (multiple-conductor cable).

The component conductors of the second kind of cable may be either solid or stranded, and this kind of cable may or may not have a common insulating covering. The first kind of cable is a single conductor, while the second kind is a group of several conductors. The term "cable" is applied by some manufacturers to a solid wire heavily insulated and lead-covered; this usage arises from the manner of the insulation, but such a conductor is not included under this definition of "cable." The term "cable" is a general one and in practice it is usually applied only to the larger sizes. A small cable is called a "stranded wire" or a "cord," both of which are defined below. Cables may be bare or insulated, and the latter may be armored with lead or steel wires or bands.

* Numbers are the paragraph numbers of the Standardization Rules of the A.I.E.E. (q. v.).

325. **Strand.**—One of the wires or groups of wires of any stranded conductor.

The majority of American copper wire manufacturers have used the word "strand" to designate a group of bare wires twisted together.

326. **Stranded Wire.** — A group of small wires, used as a single wire.

A wire has been defined as a slender rod or filament of drawn metal. If such a filament is subdivided into several smaller filaments or strands and is used as a single wire, it is called a "stranded wire." There is no sharp dividing line of size between a "stranded wire" and a "cable." If used as a wire, for example, in winding inductance coils or magnets, it is called a stranded wire and not a cable. If it is substantially insulated, it is called a "cord," defined below.

327. **Cord.** — A small cable, very flexible and substantially insulated to withstand wear.

There is no sharp dividing line in respect to size between a "cord" and a "cable," and likewise no sharp dividing line in respect to the character of insulation between a "cord" and a "stranded wire."

328. **Concentric Strand.** — A strand composed of a central core surrounded by one or more layers of helically laid wires or groups of wires.

329. **Concentric-lay Cable.** — A single-conductor cable composed of a central core surrounded by one or more layers of helically laid wires.

330. **Rope-lay Cable.** — A single-conductor cable composed of a central core surrounded by one or more layers of helically laid groups of wires.

This kind of cable differs from the preceding in that the main strands are themselves stranded.

331. **N-Conductor Cable.** — A combination of N conductors insulated from one another.

It is not intended that the name as here given be actually used. One would instead speak of a "3-conductor cable" and a "12-conductor cable," etc. In referring to the general case, one may speak of a "multiple-conductor cable" (as in definition § 324 above).

332. **N-Conductor Concentric Cable.** — A cable composed of an insulated central conducting core with (N-1) tubular stranded conductors laid over it concentrically and separated by layers of insulation.

This kind of cable usually has only 2 or 3 conductors. Such cables are used in carrying alternating currents. The remarks on the expression "N-Conductor" given for the preceding definition apply here also.

333. **Duplex Cable.** — Two insulated single-conductor cables twisted together.

They may or may not have a common insulating covering.

334. **Twin Cable.** — Two insulated single-conductor cables laid parallel, having a common covering.

335. **Triplex Cable.** — Three insulated single-conductor cables twisted together.

They may or may not have a common insulating covering.

336. **Twisted Pair.** — Two small insulated conductors twisted together without a common covering.

The two conductors of a "twisted pair" are usually substantially insulated, so that the combination is a special case of a "cord."

337. **Twin Wire.** — Two small insulated conductors laid parallel, having a common covering.

CONSTRUCTION OF INSULATED WIRES AND CABLES.— Data on the manufacture and properties of insulating materials will be found in the articles on *Cambric, Varnished; Gutta-percha; Paper, Impregnated; Rubber; Insulating Materials, Miscellaneous.* Data on the conductor itself will be found in the articles on *Copper; Aluminum; Wires and Cables, Bare.*

Conductors. — Round copper wire, solid or stranded, is almost invariably used for insulated wires and cables. Aluminum, requiring a larger cross-section for the same conductance per unit length, requires more insulating material for the same thickness of insulation. Aluminum cables, however, are sometimes provided with a weatherproof covering.

Sector-shaped Conductors. — Multiple-conductor cables with sector-shaped conductors are in successful use in Europe, their advantage being that they may be assembled in less space than round conductors. P. Humann (*Elek. Zeit., 1910, vol. 31, p. 1265-1207*) says that tests and theory both demonstrate that the sharp corners give rise to excessive dielectric stresses at high voltages.

Preparation of Conductors for Insulation. — Particularly where rubber insulation is used it is necessary to cover the conductor with a thin film of tin or with a layer of soft cotton threads.

Tinning. — Low-grade and perhaps other rubber compounds react with copper to the detriment of both insulation and wire. It is, therefore, the invariable practice to tin copper under rubber, although it is not definitely known whether tinning is necessary with high-grade 30 per cent Para compounds. It is often stated that the copper is corroded by the *free* sulphur, but the author has known of cases where over ten times as much copper was consumed as could combine into Cu_2S, with the free sulphur actually present.

Separators. — Small stranded conductors are usually covered with a winding of soft cotton threads to prevent adhesion between the wire and insulation, and thereby facilitate the removal of insulation at joints. Larger conductors are sometimes provided with a dry muslin separator for the same purpose.

Insulation. — The materials used for insulation are vulcanized rubber, gutta-percha, varnished cloth, impregnated paper, asbestos, cotton and silk thread, enamel, etc.

Rubber Insulation. — There are three processes by which rubber compound is put on wire, the straight strip, the helical tape and the tube or seamless processes. By the first, the compound is first made into long narrow strips and then pressed around the wire; by the second, a rubber tape is wound spirally around the conductor; by the last, the wire is run through a die through which the compound is pressed on to the wire. Insulation made by the former process shows a seam or ridge where the sides of the strip have united, unless a tape is applied before vulcanization, while that made by the tube process is seamless.

It might be expected that the seam of strip-laid insulation would be electrically weak, but in practice this is found not to be the case with high-grade compound, the insulation practically never puncturing at the seam.

Gutta-Percha is applied to the wire in the same manner as rubber.

Varnished Cloth. — The prepared cloth is applied to the conductor in the form of tape wound on helically and reversed every two layers, with overlapping joints staggered in successive layers. A thin layer of a non-hardening viscous filler is applied between layers.

Impregnated Paper. — Strips of Manila paper are applied helically and then thoroughly impregnated with an insulating compound.

Asbestos is applied in the form of threads or tape, usually in several layers. One or more layers of varnished cloth are sometimes used to separate the groups of asbestos tapes, and the whole is covered with a protecting braid.

Cotton and Silk. — Cotton or silk insulation consists of one to three layers of threads spun on to the wire.

Enamel. — This type of insulation is suitable only for wires under 5 mils in diameter. Cellulose acetate is a common form of enamel. A solution of cellulose acetate is applied to revolving coating rolls, which, in turn, deposit it on the moving wire. The quantity of solution placed on the wire must be adjusted to a nicety. After receiving its coating of solution, the wire passes through an oven at the rate of several hundred feet per minute. The volatile solvent is here driven off and the coating hardened, after which the process is repeated until numerous coatings have been applied. This method insures an even distribution of the film over the whole surface of the wire (*R. Fleming*).

Fillers. — Dry jute is used as a filler between the insulated conductors of multiple-conductor cables, being placed directly in contact with the insulation. Tarred or asphalted jute is used as a filler between the sheath and armor of armored cables.

Protective Coverings. — Coverings of treated cotton, hemp, paper, reinforced rubber, and lead are used to protect cables from mechanical injury and arcing.

Cotton Braid. — The most common covering for rubber-insulated conductors is a cotton braid saturated with a weatherproof compound composed principally of mineral wax. For some purposes, flameproof paint is used instead of wax compound.

Hemp Braid. — Hemp is sometimes used instead of cotton for braids on large cables which are likely to be exposed to unusually rough treatment. Six-lea is the most usual size for this service. The hemp is saturated with mineral wax compound, in the same way as cotton braid.

Weatherproofing. — Treated cotton braid is also put on uninsulated hard-drawn copper wire for overhead service, in order to protect the wire from destructive arcing due to accidental contact with tree branches and other foreign bodies.

Fireproof Paper. — Johnson & Phillips of Charlton, England, make an impregnated paper covering, which, under the influence of a flame, gives off gases in which a flame cannot exist. Tests have shown that the cable may become red hot from an internal short-circuit without setting fire to the insulation. (*Elect. Age, July, 1907.*)

Reinforced Rubber. — A form of cable covering, combining both insulation and mechanical protection, is composed of alternate layers of rubber compound and rubber impregnated fabric, which are vulcanized together after assembling, the finished product being similar to air-brake hose. The electrical and mechanical properties may be varied by varying the thickness and quality of rubber compound and the number of layers of fabric. It is practically waterproof and very strong mechanically. (*Safety Insulated Wire & Cable Co.*)

Lead Sheath. — The only thoroughly waterproof cable covering yet devised is a lead sheath. It is put over the insulation by passing the cable through a die while hot lead is pressed hydraulically around it through an annular die, forming a continuous and close-fitting pipe. Were it not for electrolytic corrosion, it would be practically permanent. Unfortunately lead is very subject to electrolytic and even chemical corrosion. It is also rendered brittle and eventually breaks into pieces when exposed to vibration, this effect being due to crystallization. Pure lead is suitable for cables which are hard and compact. Other cables require a small quantity of tin to harden the lead. For a

given thickness pure lead is cheaper, but for a given tensile strength a 3 per cent tin alloy is cheaper.

Antimony has recently been used instead of lead, with very satisfactory results and a material saving in cost, especially with telephone cable.

Armor Wire and Tape. — Submarine cables and cables made to be buried direct in the ground are usually given a mechanical protection of galvanized-steel armor in the form of wires or tapes.

APPLICATIONS OF VARIOUS TYPES OF CABLES. — The type of insulation and protection to employ in any instance depends upon the purpose for which the conductor is to be used and the place in which it is to be installed.

Power Cables, that is, cables for the transmission of electric energy, may be installed in or on buildings, in cars, on pole lines under ground or under water.

Power Cables on Overhead Structures. — Insulated conductors supported on structures subject to vibration are usually made with rubber, varnished cloth or a combination of the two, with a steel armor or a heavy braiding. Insulated conductors in buildings and on cars are almost invariably made with rubber, covered with a saturated cotton braid.

Underground Cables are usually installed in ducts and are commonly made with paper or varnished cambric and a lead sheath. Where mechanical injury is to be guarded against, the sheath should be covered with asphalted jute and a steel band or wire armor.

Long-distance transmission by underground cables is seldom resorted to on account of the expense of conduit lines. There is no especial difficulty attending the manufacture of cables for three-phase voltages from 25,000 to 50,000 volts between conductors, the neutral being grounded. For such high potentials, it is generally necessary to grade the insulation so as to eliminate steep potential gradients.

According to L. Lichtenstein (*Elek. Zeit., 1910, Vol. 31, p. 773*) a reliable three-conductor cable for 50,000 volts is now available with a perforation e.m.f. above 330,000 volts. M. J. Grosselin (*La Revue Elec., 1910, Vol. 14, p. 92*) says that 2.6 kilometers and 4.5 kilometers of 3-phase 40,000-volt ungraded cable are in use by the Girod plant at Ugine and by the Grenoble Lighting & Power Co., respectively.

Submarine Cables are usually insulated with rubber, sheathed in lead and armored with wire or steel bands. With high-grade rubber compounds, the sheath may be omitted. Recently, however, paper-insulated lead-sheathed and armored submarines have come into use.

Insulated Conductors for Instrument and Machine Windings. — Enameled wire or wire insulated with cotton or silk is used for the former, while varnished cambric, mica and asbestos compounds are used for the latter.

Insulated Conductors for Telephony, Telegraphy and Signaling. — Underground telegraph and telephone cables are almost invariably insulated with dry paper and sheathed in lead, although rubber is used as insulation where a lead sheath is not permissible. Submarine telegraph cables are usually insulated with gutta-percha, protected with a lead sheath and armor. Rubber insulation of the highest grade is demanded for railroad signal purposes, the usual mechanical protection being cotton braid saturated with mineral wax compound.

THICKNESS OF INSULATION. — In the case of a single-conductor cable a simple formula can be deduced for the proper thickness of insulation to withstand a given electric stress. Such a formula, however, involves certain

assumptions which are only partially realized in practice. These assumptions are:

a. That the radial depth of the insulation or dielectric is the same at all points, i.e., that the cross-section of the cable is a perfect circle with the conductor section (also a perfect circle) exactly in the center of the insulation. Due, however, to the crinkling of tape, the pressure of the braid, or other accidents of manufacture, this is never the case. The eccentricity of the conductor with respect to the insulation may, however, be allowed for by adding to the theoretical thickness of insulation an additional thickness, known as the "error thickness" or excess thickness.

b. That the dielectric is perfectly homogeneous throughout. This is only partially realized in practice. It is probable that even in the most carefully made insulation there is a minute amount of moisture, but sufficient to modify considerably any conclusions based on absolute homogeneity of the dielectric.

c. That when the electric stress, or potential gradient (per inch, say) at any part of the dielectric exceeds a certain value F, known as the dielectric strength of the dielectric, that part of the dielectric becomes a conductor even though there is no actual rupture or puncturing. The nature of this partial breakdown is not understood, but there is considerable evidence to show that such an effect exists (*see discussion by Middleton, Trans. A.I.E.E., 1910, Vol. 29, p. 1587, and Del Mar, Trans. A.I.E.E., 1911, Vol. 30, p. 238*). As far as is known this partial breakdown is not permanent, but the insulation returns to its original state when the stress is removed.

Theoretical Thickness of Insulation. — Let

H = potential gradient, in volts per inch, at any point P in the dielectric at a distance x from the center of the wire, Fig 1,

V = volts between wire and sheath (or outside surface of insulation),

r = radius of wire, in inches,

R = outside radius of insulation, in inches,

F = dielectric strength of the insulation, in volts per inch.

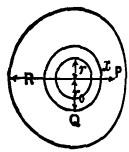


Fig. 1.

Then, on the assumption of a perfectly homogeneous dielectric and perfect symmetry between conductor and insulation,

$$H = \frac{V}{x \log_e \left(\frac{R}{r}\right)}.$$

That is, the potential gradient is the greatest at the surface of the conductor and decreases toward the outer surface of the insulation as shown in Fig. 2. The potential gradient at the surface of the conductor is

$$H_s = \frac{V}{r \log_e \left(\frac{R}{r}\right)}.$$

For the same outside diameter of the insulation this stress at the surface varies with the radius of the conductor as shown in Fig. 3, and has a ***minimum*** value when the radius of the conductor r equals the outside radius of the insulation R, divided by the base ϵ of the natural system of logarithms, that is, when $r = R/2.72$.

Consequently, if the breakdown of a cable is a progressive action, i.e., if the insulation breaks down close to the wire and then the breakdown spreads out through the insulation, it is evident, from Fig. 3, that if r is greater than $R/2.72$,

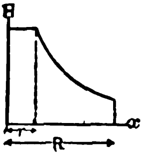


Fig. 2.

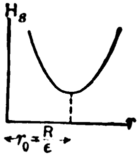


Fig. 3.

and the impressed voltage V is such as to produce a stress H_s at the surface of the conductor in excess of F, then as successive layers of the insulation break down, H_s at each new conducting surface thus formed will be greater than F, and the breakdown will rapidly extend to the sheath. If, however, r is less than $R/2.72$, then a value of H_s in excess of F will break down only a sufficient layer of insulation to render the value of H_s at the new conducting surface thus formed just equal to F.

Therefore, on the basis of the assumptions noted in the preceding section, the maximum voltage which a cable can withstand is equal to

$$V_m = Fr \log_\epsilon \left(\frac{R}{r}\right),$$

provided r is greater than $R/2.72$, but when r is less than $R/2.72$ the maximum voltage is

$$V_m = Fr_0 \log_\epsilon \left(\frac{R}{r_0}\right) = \frac{RF}{2.72},$$

and is *independent of the radius of the conductor.*

A more complete account of the theory of ungraded cables is in the *Trans. A.I.E.E., 1910, Vol. 29, p. 1614*, and a paper on graded cables by H. S. Osborne is in the *Trans. A.I.E.E., 1910, Vol. 29, p. 1553.*

Practical Determination of Insulation Thickness. — On account of the numerous assumptions which must be made in determining the necessary thickness of insulation from any simple theory, such as outlined above, a rule-of-thumb method, based on experience, is usually employed in practice.

The following tables represent current practice. With respect to rubber insulation the Underwriters' Rules give minimum thickness, and the Signal Engineers' specifications give maximum thickness. Similar tables are given by J. Langan (*Trans. A.I.E.E., 1906, Vol. 25, p. 200*) and W. A. Del Mar (*Electric Power Conductors, 1914*) the theoretical basis of the last-named tables being also given. See also report of Wire Committee of the Association of Railway Electrical Engineers, 1913.

RUBBER INSULATION

Thickness of Insulation in 64ths of an Inch

Working pressure, volts between wires	Nat. Elec. Code * Size of wire, A. W. G. or B. & S. and Cir. Mils	Nat. Elec. Code * Insulation thickness
600	18–16	2
600	14–8	3
600	7–2	4
600	1–0000	5
600	225,000–500,000	6
600	525,000–1,000,000	7
600	1,100,000–2,000,000	8
1500	14–8	4
1500	7–2	5
1500	1–0000	6
1500	225,000–500,000	7
1500	525,000–1,000,000	8
1500	1,100,000–2,000,000	9
2500	14–2	6
2500	1–0000	7
2500	225,000–500,000	8
2500	525,000–1,000,000	9
2500	1,100,000–2,000,000	10
2500	14–0000	8
2500	225,000–500,000	9
2500	525,000–1,000,000	10
2500	1,100,000–2,000,000	11
3500	14–0000	8
3500	225,000–500,000	9
3500	525,000–1,000,000	10
3500	1,100,000–2,000,000	11
5000	14–1,000,000	12
5000	1,100,000–2,000,000	14
7000	14–1,000,000	16
7000	1,100,000–2,000,000	18

Railway Signal Association †
666 volts or less

Single conductor		Multi-conductor	
Size of wire, A. W. G. or B. & S.	Insulation thickness, 64ths inch	Size of wire, A. W. G. or B. & S.	Insulation thickness, 64ths inch
18–16	4	16	3
14–9	5	14–10	4
8–4	6	9–6	5
2–0	8	4	6

* The rubber to comply with the rubber specification of the National Board of Fire Underwriters, which assumes the use of 20% of rubber gum.

† The rubber insulation to comply with the specifications of the R.S.A., which calls for a very high-grade 30% Para compound.

VARNISHED CLOTH AND IMPREGNATED PAPER

Insulation Thickness

(*G. E. Practice.*)

Working pressure, volts between wires	Varnished cloth		Impregnated paper	
	Size, A. W. G. or B. & S. and Cir. Mils	Insulation thickness* 64ths inch	Size, A. W. G. or B. & S. and Cir. Mils	Insulation thickness* 64ths inch
600	....	..	14–2	4
600	....	..	1–0000	5
600	....	..	225,000–500,000	6
600	....	..	550,000–1,000,000	7
1,000	6–2	4	12–2	5
1,000	1–0000	5	1–0000	6
1,000	250,000–500,000	6	....	..
1,000	550,000–1,000,000	7	225,000–500,000	7
1,000	1,100,000 and over	8	550,000–2,000,000	8
2,000	6–0000	6	10–0000	7
2,000	250,000–500,000	7	225,000–500,000	8
2,000	550,000–2,000,000	8	550,000–2,000,000	9
3,000	All sizes	9	8 and larger	10
4,000	All sizes	10	8 and larger	12
5,000	All sizes	12	6 and larger	14
6,000	All sizes	14	6 and larger	16
7,000	All sizes	16	5 and larger	18
8,000	All sizes	18	....	..
9,000	All sizes	18	5 and larger	20
10,000	All sizes	20	....	..
11,000	All sizes	22	4 and larger	22
13,000	All sizes	24	....	..
14,000	All sizes	24	4 and larger	24
15,000	All sizes	26	3 and larger	26
16,000	All sizes	26	....	..
17,000	All sizes	28	3 and larger	28
18,000	All sizes	28	....	..
19,000	All sizes	30	2 and larger	30
20,000	All sizes	32	....	..
21,000	All sizes	32	2 and larger	32
22,000	All sizes	34	....	..
23,000	All sizes	34	1 and larger	34
24,000	All sizes	36	....	..
25,000	All sizes	36	0 and larger	36

* Above working voltages are based on all conductors of the circuit being insulated. For d-c. 600-volt railway single-conductor, leaded cables, use 2,000-volt class. For three-phase "Y"-connected circuits with grounded neutral and three-conductor cables, thickness of insulation between conductors and ground need only be 7/10 of that between conductors. (*G. E. Bulletin No. 4787.*)

INSULATION THICKNESS USED BY SOME IMPORTANT COMPANIES FOR THREE-CONDUCTOR HIGH-TENSION CABLES

(*Conductors are from No. 0 A. W. G. to 250,000 Cir. Mils.*)

Company	Volts between wires	Kind of insulation	Thickness of insulation in mils: Over 1 conductor	Thickness of insulation in mils: Over 3 conductors
New York-Edison	6,600	Paper	156	156
New York-Metropolitan	6,600	Paper	218	125
New York-3rd Ave. R. R.	6,600	Paper	156	156
New York-Subway Co.	11,000	Paper	218	250
Buffalo-Niagara Lines	11,000	Paper	203	203
Buffalo-Niagara Lines	11,000	Rubber	281	...
Chicago-Edison	9,000	Paper	203	140
Chicago-Edison	20,000	Paper	281	187
Milwaukee	15,000	Paper	250	187
Saint Paul	25,000	Paper	281	203
Saint Paul	25,000	Rubber	218	156
Boston Edison	6,900	Paper	219	219
Providence	12,500	Rubber	281	...
Brooklyn Edison	6,600	Paper	172	172
Philadelphia Edison	6,600	Paper	156	156
Hudson & Manhattan Ry., N. Y.	11,000	Paper	219	156
N. Y. C. R. R.	11,000	Paper	219	187
N. Y. C. R. R.	11,000	V. C.	187	187
Philadelphia Rapid Transit	13,200	Paper	187	187
Public Service Cor. of N. J.	13,200	Paper	219	219
Twin City Rapid Transit	13,000	Paper	189	189

Cotton, Silk and Enamel.—The thicknesses rated by various manufacturers as "single," "double" and "triple" covered differ by several mils; see article on *Electromagnet Windings*. Enamel insulation will stand a working potential of about 500 volts alternating per mil of thickness. Silk or cotton will stand about one-quarter as much.

WEIGHT OF INSULATED WIRES AND CABLES.—So many variables enter into the weight of an insulated wire or cable, and there are so many forms in use, that it is impossible to give comprehensive tables here. The reader is referred to the catalogues and circulars of the manufacturing companies. When such sources of data are not available the weight may be calculated from the dimensions of the cable by finding the sum of the weights of the conductors and the weights of the insulation, braid, tape, sheath, etc. The weight of the conductor may be found in the wire tables under *Wires and Cables, Bare*. The weight of the insulation, braid, tape or sheath, may be found by calculating the cross-section of each of these materials and multiplying by the length of the cable and a factor proportional to the density of the material; see table accompanying.

The cross-section of a tube having an internal diameter d and thickness t is $\pi t(d + t)$. When the diameter and thickness are in inches and the specific

gravity of the material forming the tube is δ, then the weight in pounds per 1000 ft. of tube is

$$W = 1362\,\delta t\,(d + t).$$

The values of the specific gravity δ, the product $1362\,\delta$, and the weight per cu. in., for the various materials used in the construction of cables are given in the accompanying table. From this relation the weight per 1000 ft. of any tubular (circular cross-section) layer of insulation, braid, tape or sheath may be readily calculated. The formula is, therefore, directly applicable to single-conductor cables.

The weight of duplex or triplex cables is calculated as for a group of single-conductor cables, except with regard to the fillers. The cross-section of the filling material is most readily calculated by subtracting from the cross-section of the entire cable the cross-sections of the individual conductors and the tubular insulation, sheath, etc. The weight of twin cables can also be readily found by calculating the cross-section of the various parts in inches and multiplying by the length and the proper density factor.

The weight of separators in lb. per 1000 ft. is usually between 5 and 10 times the mean diameter, expressed in inches, of the conductor and separator.

WEIGHTS OF CABLE INSULATION

Material	Specific gravity	Pounds per cubic inch	$1362\,\delta$
	δ	$0.03613\,\delta$	
Rubber compound, 30 per cent para:			
Organic base	1.2	0.0434	1635
Mineral base	1.3 to 1.8	0.047 to 0.065	2040 to 2455
Varnished cloth	1.14 to 1.18	0.0412 to 0.0426	1550 to 1610
Paper, impregnated	1.10 to 1.32	0.0398 to 0.0476	1500 to 1800
Lead	11.4	0.411	15,530
Braid, untreated	.....	....	......
Braid, saturated	.....	....	......
Tarred jute, in cable	0.534	0.0193	728
Dry jute, in cable	0.267	0.00965	364
Dry hemp, in cable	0.267	0.00965	364
Gutta percha	1.0	0.0361	1362

INSULATION RESISTANCE. — The insulation resistance of a cable is usually expressed in megohm-miles, sometimes erroneously called megohms per mile. The total insulation resistance of a cable varies inversely as its length, e.g., a cable two miles long has half the resistance between conductor and sheath of a length of one mile of this cable. The formulas for insulation resistance of various types of cables are given below.

Single-conductor Cable. — In absolute units the insulation resistance of a length of l centimeters of such a cable is

$$R' = \frac{\rho}{2\pi l}\log_e\frac{D}{d},$$

where d = diameter of conductor, D = outside diameter of insulation, R = insulation resistance, l = axial length, ρ = specific resistance. From the above formula the megohm-miles are

$$R = k \log_{10} \frac{D}{d},$$

where $k = 5.8 \times$ (specific resistance in millions of megohms per inch cube) and the two diameters are expressed either in inches or in centimeters. The value of k for various types of insulation at 60° F., after an electrification of approximately one minute under a constant d-c. voltage, is given in the accompanying table. k varies both with the time of electrification and with the temperature and is, as a rule, several thousand times greater for alternating currents than for direct currents (*see Insulating Materials, Miscellaneous*).

Insulation	k at 60° F.	
	Limits	Usual values
Vulcanized rubber.....	780 to 23,000	3000 to 8000
Gutta-percha..........		2500
Varnished cloth.......	500 to 4,000	700 to 1200
Impregnated paper.....	1000 to 3,000	1500

The tables on the following pages give the value of $\log_{10} \frac{D}{d}$ for various sizes of wire and thicknesses of insulation.

Two- and Three-conductor Cable in Lead Sheath. — The insulation resistance between the two conductors of a two-conductor cable is practically double that between each conductor separately and the sheath.

The insulation resistance of a three-conductor cable between any one conductor and the other two conductors and the sheath is approximately one-half of that between each conductor separately and the sheath. Simple formulas for these resistances in terms of the dimensions of the cable are not available, for the various parts of the coverings (insulation, fillers and braids) have different resistivities.

LOG$_{10}$ $\frac{D}{d}$ FOR SOLID WIRES

Size wire, A. W. G. or B. & S.	d inches	Insulation thickness, 64ths of an inch							
		3	4	5	6	7	8	9	10
14	0.064	0.393	0.470	0.537	0.595	0.645	0.691	0.732	0.770
12	0.081	0.334	0.405	0.467	0.521	0.568	0.613	0.650	0.687
10	0.102	0.283	0.348	0.403	0.453	0.498	0.538	0.574	0.610
8	0.129	0.238	0.294	0.344	0.391	0.431	0.468	0.502	0.535
6	0.162	0.199	0.248	0.292	0.331	0.371	0.405	0.436	0.467
4	0.204	0.164	0.207	0.246	0.283	0.316	0.346	0.377	0.403
2	0.258	0.134	0.170	0.207	0.238	0.267	0.294	0.320	0.344
1	0.289	0.124	0.155	0.188	0.217	0.243	0.270	0.294	0.318
0	0.325	0.111	0.140	0.170	0.199	0.223	0.248	0.270	0.292
00	0.365	0.100	0.127	0.155	0.182	0.204	0.225	0.248	0.270
000	0.410	0.090	0.114	0.140	0.164	0.185	0.207	0.228	0.246
0000	0.460	0.079	0.104	0.127	0.149	0.170	0.188	0.207	0.225

Size wire, A. W. G. or B. & S.	d inches	Insulation thickness, 64ths of an inch							
		11	12	14	16	18	20	22	24
14	0.064	0.804	0.836	0.894	0.945	0.991	1.032	1.070	1.104
12	0.081	0.720	0.751	0.807	0.856	0.900	0.941	0.977	1.011
10	0.102	0.640	0.670	0.723	0.771	0.814	0.853	0.889	0.922
8	0.129	0.565	0.592	0.642	0.688	0.729	0.766	0.803	0.833
6	0.162	0.494	0.520	0.568	0.611	0.650	0.687	0.720	0.751
4	0.204	0.428	0.453	0.498	0.538	0.575	0.609	0.640	0.670
2	0.258	0.367	0.389	0.431	0.468	0.502	0.534	0.565	0.592
1	0.289	0.340	0.362	0.401	0.436	0.470	0.500	0.529	0.556
0	0.325	0.314	0.332	0.371	0.405	0.436	0.465	0.496	0.520
00	0.365	0.288	0.307	0.342	0.377	0.405	0.433	0.461	0.486
000	0.410	0.265	0.281	0.316	0.346	0.375	0.401	0.428	0.452
0000	0.460	0.243	0.260	0.290	0.320	0.346	0.373	0.398	0.420

d = diameter of wire; D = outside diameter of insulation.

LOG$_{10}$ $\frac{D}{d}$ FOR CABLES (STRANDED WIRES)

Size wire, A. W. G. or B. & S. and C.M.	d inches	Insulation thickness, 64ths of an inch							
		3	4	5	6	7	8	9	10
14	0.073	0.360	0.433	0.497	0.553	0.602	0.646	0.686	0.724
12	0.092	0.307	0.373	0.431	0.483	0.529	0.571	0.607	0.643
10	0.116	0.258	0.318	0.369	0.418	0.461	0.498	0.534	0.568
8	0.146	0.215	0.267	0.316	0.360	0.398	0.433	0.470	0.497
6	0.184	.....	0.225	0.267	0.305	0.340	0.373	0.403	0.431
4	0.232	.....	0.188	0.223	0.258	0.288	0.316	0.344	0.371
2	0.296	.....	0.152	0.185	0.215	0.241	0.267	0.290	0.314
1	0.332	.....	0.140	0.167	0.196	0.220	0.243	0.267	0.288
0	0.373	.....	0.127	0.152	0.179	0.201	0.225	0.246	0.267
00	0.419	.....	.....	0.137	0.161	0.182	0.201	0.223	0.243
000	0.471	.....	.....	0.124	0.146	0.167	0.185	0.204	0.220
0000	0.529	.....	.....	0.114	0.130	0.149	0.167	0.185	0.201
250,000	0.575	.....	.....	0.104	0.124	0.140	0.158	0.173	0.190
300,000	0.632	.....	.....	0.097	0.114	0.130	0.146	0.161	0.176
350,000	0.681	.....	.....	0.090	0.104	0.121	0.137	0.149	0.164
400,000	0.728	.....	.....	0.083	0.100	0.114	0.127	0.143	0.155
450,000	0.772	.....	.....	0.079	0.093	0.107	0.121	0.137	0.149
500,000	0.815	.....	.....	0.076	0.090	0.104	0.117	0.130	0.140
600,000	0.833	.....	.....	.....	0.086	0.097	0.111	0.121	0.130
700,000	0.964	.....	.....	.....	0.079	0.090	0.100	0.111	0.124
750,000	0.998	.....	.....	.....	0.074	0.085	0.097	0.107	0.118
800,000	1.031	.....	.....	.....	0.073	0.084	0.094	0.104	0.115
900,000	1.094	.....	.....	.....	0.069	0.079	0.089	0.099	0.109
1,000,000	1.153	.....	.....	.....	0.066	0.076	0.085	0.095	0.104
1,250,000	1.289	.....	.....	.....	0.059	0.068	0.077	0.086	0.094
1,500,000	1.413	.....	.....	.....	0.054	0.063	0.071	0.078	0.087
1,750,000	1.526	.....	.....	.....	0.050	0.058	0.066	0.073	0.081
2,000,000	1.632	.....	.....	.....	0.047	0.055	0.062	0.069	0.076

$\text{LOG}_{10} \frac{D}{d}$ FOR CABLES (STRANDED WIRES)

Size wire, A.W.G. or B. & S. and C. M.	d inches	Insulation thickness, 64ths of an inch							
		11	12	14	16	18	20	22	24
14	0.073	0.757	0.788	0.845	0.895	0.940	0.980	1.018	1.051
12	0.092	0.676	0.705	0.760	0.809	0.852	0.892	0.928	0.961
10	0.116	0.598	0.626	0.679	0.725	0.767	0.806	0.841	0.873
8	0.146	0.525	0.553	0.602	0.645	0.686	0.723	0.757	0.788
6	0.184	0.458	0.483	0.529	0.571	0.609	0.643	0.676	0.705
4	0.232	0.394	0.418	0.461	0.498	0.534	0.567	0.598	0.626
2	0.296	0.334	0.356	0.394	0.430	0.462	0.493	0.521	0.548
1	0.332	0.310	0.328	0.365	0.398	0.431	0.459	0.487	0.513
0	0.373	0.286	0.305	0.338	0.371	0.401	0.430	0.455	0.480
00	0.419	0.260	0.276	0.310	0.340	0.369	0.396	0.422	0.438
000	0.471	0.238	0.255	0.286	0.314	0.342	0.367	0.391	0.415
0000	0.529	0.217	0.233	0.260	0.290	0.314	0.338	0.362	0.384
250,000	0.575	0.204	0.217	0.246	0.272	0.297	0.320	0.342	0.364
300,000	0.632	0.190	0.204	0.230	0.255	0.276	0.299	0.320	0.340
350,000	0.681	0.176	0.190	0.215	0.238	0.262	0.283	0.303	0.322
400,000	0.728	0.167	0.182	0.204	0.228	0.250	0.270	0.290	0.307
450,000	0.772	0.161	0.173	0.196	0.217	0.238	0.258	0.276	0.294
500,000	0.815	0.152	0.164	0.188	0.210	0.228	0.248	0.267	0.283
600,000	0.833	0.143	0.152	0.173	0.193	0.212	0.230	0.248	0.265
700,000	0.964	0.134	0.143	0.164	0.182	0.201	0.217	0.236	0.250
750,000	0.998	0.129	0.138	0.158	0.176	0.196	0.211	0.228	0.243
800,000	1.031	0.125	0.135	0.154	0.172	0.189	0.206	0.222	0.237
900,000	1.094	0.118	0.128	0.146	0.163	0.180	0.196	0.212	0.227
1,000,000	1.153	0.113	0.122	0.140	0.157	0.173	0.188	0.203	0.217
1,250,000	1.289	0.103	0.111	0.127	0.142	0.157	0.172	0.186	0.199
1,500,000	1.413	0.094	0.102	0.117	0.132	0.146	0.159	0.172	0.185
1,750,000	1.526	0.088	0.096	0.110	0.123	0.136	0.149	0.162	0.173
2,000,000	1.632	0.083	0.090	0.103	0.116	0.129	0.141	0.153	0.164

CAPACITY AND CHARGING CURRENT. — See article on *Capacity and Charging Current.*

INDUCTANCE, REACTANCE AND IMPEDANCE. — The formulas for bare wires and cables (*see Inductance and Inductive Reactance*) are also applicable to lead-sheathed insulated cables, the insulation and sheath producing practically no effect on the self inductance.

Resistance and Impedance of Three-conductor Cables. — The following table, published by the G. E. Co., is based upon 100 per cent conductivity copper at 75° F., with an allowance of 3 per cent for spiral path of the component wires, 60 cycles per second, and standard thicknesses of varnished cambric insulation.

RESISTANCE AND IMPEDANCE OF THREE-CONDUCTOR COPPER CABLES

Size, A. W. G. or B. & S. and Cir. Mils	Resistance ohms of each conductor per mile R	Impedance of each conductor at 60 cycles, ohms per mile, z					
		Working voltage between wires					
		3000	5000	7000	10,000	15,000	20,000
2	0.850	0.858	0.859	0.863	0.867	0.872	0.884
1	0.674	0.692	0.696	0.700	0.706	0.712	0.724
0	0.535	0.545	0.547	0.552	0.558	0.565	0.580
00	0.424	0.436	0.439	0.444	0.452	0.460	0.478
000	0.336	0.352	0.352	0.357	0.365	0.374	0.396
0000	0.267	0.280	0.283	0.288	0.296	0.306	0.332
250,000	0.227	0.245	0.245	0.252	0.261	0.272	0.299
300,000	0.188	0.210	0.210	0.217	0.227	0.241	0.270
350,000	0.161	0.187	0.187	0.194	0.204	0.217	0.250
400,000	0.141	0.166	0.166	0.174	0.185	0.199	0.234
450,000	0.127	0.148	0.148	0.156	0.167	0.182	0.221
500,000	0.113	0.137	0.137	0.144	0.156	0.172	0.212

The reactance is $x = \sqrt{z^2 - r^2}$; the inductance is $L = x \div 2\pi f$ where f is the frequency in cycles per second.

Resistance and Reactance of a Pair of Armored Single-conductor Cables. — Dr. J. B. Whitehead (*Trans. A.I.E.E., 1909, Vol. 28, p. 737*), gives the following formula for the self inductance of each of two parallel armored single-conductor cables:

$$L = 0.161 \left\{ 2 \log_e \frac{D}{r} + \frac{1}{2} \right\} + 0.078\,(k - 1),$$

where L = millihenrys per mile of each conductor,

D = distance between centers of conductors, any unit,

r = radius of each conductor, in same unit as D,

k = 14 to 36, depending upon the permeability of the armor wires.

The above-cited paper concludes as follows: 1. Theoretically the reactance of the usual type of single-conductor, iron-armored power cable may be from 2 to 4 times that of unarmored cable, depending on the distance to return conductor; the factor increases with decreasing distance. 2. In cables as manufactured, the reactance is about 0.7 of the maximum theoretical value as calculated by the above formula. 3. The effective resistance at 25 and 60 cycles is about 1.6 and 2 times respectively the d-c. resistance for a current density of 1 ampere per 1000 circular mils, and varies little with the distance between conductors. 4. The impedance for distances under 12 inches is about 3 times the d-c. resistance at 60 cycles and 1.7 to 2 times the d-c. resistance at 25 cycles. With the sheathing and the armor grounded at both ends the impedance is somewhat lessened and is largely of the nature of resistance. 5. The impedance of a double-conductor iron-armored cable with conductors located 0.875 inch apart between centers and at the current density given above is about 5 per cent greater at 60 cycles and 3 per cent greater at 25 cycles than the calculated value. The increase is in effective resistance, due to eddy currents in the lead sheathing.

H. W. Fisher (*Trans. A.I.E.E., 1909, Vol. 28, p. 747*) says that the impedance of armored cables cannot be calculated with any degree of certainty owing to the effect of slight differences in the air gaps between armor wires. He shows that a single-conductor No. 0 B. & S. cable armored with two flat bands of steel (0.39 × 1.5) may have a maximum effective resistance 8 times the d-c. resistance when the lead and armor are disconnected, and that under practical conditions of operation the ratio is about 3.5. With larger cables the ratio will be greater.

Unless the armor and lead of single-conductor cables are bonded together, arcs may occur between them and under water they would be injured by alternating-current electrolysis.

CURRENT-CARRYING CAPACITY OF WIRES AND CABLES. — The permissible temperature rise of insulated conductors usually depends upon the effect of heat on the insulation. In the case of rubber there is no well-defined temperature at which deterioration becomes especially rapid. The usual order of events when rubber is heated is as follows: Oxidation is accelerated if air is present; "devulcanization" commences, i.e., the rubber molecule breaks up without, however, liberating any sulphur; vulcanization proceeds, until all the free sulphur has combined with the rubber or mineral matter. The maximum working temperature is given as 130° F. (54.4° C.) by Melsom & Booth and 122° F. (50° C.) by the G. E. Co.

In the case of varnished cambric a temporary weakening occurs at a much lower temperature than any permanent deterioration, the insulation resistance falling quite rapidly and the dielectric strength decreasing, owing probably to the ionization of the separating oils. The same applies to paper in a somewhat less degree. These cables may therefore be worked at higher temperatures at low voltages than at high voltages, especially in the case of yellow varnished cambric which does not contain gilsonite or petroleum products. Paper is desiccated and impregnated at between 220° and 230° F. and no deterioration of the materials is observable even if the processes are considerably prolonged but the material does not insulate effectively for low-tension work above about 167° F. (75° C.)

Approximate Formula for Permissible Current. —

Let A = cross-sectional area of conductor, circular mils,
r = resistance per mil-inch of conductor at final temperature,
I = current in conductor, amperes,
C = circumference of conductor, inches,
w = watts dissipated per square inch of conductor surface, per degree Centigrade temperature rise. This will depend upon the thermal resistivity of the insulation and upon the nature of the radiating surface,
T = temperature rise, degrees C.

When the watts dissipated become equal to the watts generated, the temperature of the conductor will no longer rise and the current will have the value

$$I = \sqrt{\frac{ACwT}{r}}.$$

The above formula does not hold very accurately in practice on account of the approximations in the equation for watts dissipated. It is, however, useful for estimating the influence of various factors upon the carrying capacity.

Table of Current-carrying Capacity. — The following table gives the safe current per conductor as recommended by the authorities cited.

CURRENT-CARRYING CAPACITY OF COPPER CABLES

Amperes per Conductor

(For Aluminum Cables multiply the currents given in Table by 0.84)

Size wire, A. W. G. or B. & S. and Cir. Mils	Single-conductor cables Low Tension					Three-conductor cables
	Nat. Elec. Code		General Electric Co.		Stand. Und. Cab. Co.	G. E. Co.
	Rubber insulation	Other insulations	Rubber insulation	Paper or varnished cambric	Paper insulated lead covered	Rubber, paper or cambric
	In-doors	In-doors	In-doors exposed	In-doors exposed	In tile ducts	In-doors exposed
18	3	5	....	....	....	...
16	6	8	....	....	....	...
14	12	16	....	....	18	...
12	17	23	....	....	24	...
10	24	32	20	24	33	16
8	33	46	30	36	45	24
6	46	65	50	60	64	40
5	54	77	65	75	76	50
4	65	92	80	90	91	65
3	76	110	100	110	108	80
2	90	131	120	140	125	...
1	107	156	145	170	146	110
0	127	185	170	200	168	130
00	150	220	200	235	195	155
000	177	262	240	280	225	185
0000	210	312	280	340	260	220
200,000	200	300	270	320	....	210
300,000	270	400	370	450	323	290
400,000	330	500	460	560	390	360
500,000	390	590	550	660	450	440
600,000	450	680	630	750	505	...
700,000	500	760	710	850	558	...
800,000	550	840	790	950	607	...
900,000	600	920	850	1050	650	...
1,000,000	650	1000	900	1150	695	...
1,200,000	730	1150	....	....	780	...
1,500,000	850	1360	1200	1500	895	...
1,700,000	930	1490	....	....	970	...
2,000,000	1050	1670	1400	1750	1085	...
Initial temp.	....	....	20° C.	20° C.	21.5° C.	20° C.
Temp. rise	....	....	30° C.	60° C.	44.5° C.	30° C.*

* 35° C. for paper.

Notes on above Table. — The columns headed National Electric Code give the requirements of the National Board of Fire Underwriters for interior wiring in buildings.

The columns headed General Electric Company are made up from a table published by this company in their bulletin No. 4787. The following notes are appended to this table in the bulletin.

"Concentric cables (one conductor inside the other, the latter forming a tube) will safely carry about 20 per cent less current in each conductor than the same size of single-conductor cable. Four-conductor cables, 10 per cent less than same size triple conductor. All temperatures refer to temperatures of copper core."

The values of the current given for three-conductor cables are approximately 78 per cent of the values given for single-conductor rubber-insulated cables for a rise of temperature of 30° C. above surrounding air at 20° C. This same factor may be applied to the values given for single-conductor cables in ducts in order to obtain the current-carrying capacity of three-conductor cables in ducts. The current-carrying capacity *per conductor* of twin-conductor cables is approximately the same as for single-conductor cables, while the current-carrying capacity *per conductor* of duplex cables (round) is about 15 per cent less.

The column headed Standard Underground Cable Co. is taken from the handbook issued by that company and applies to each of four equally loaded cables installed in adjacent ducts in the usual type of conduit system.

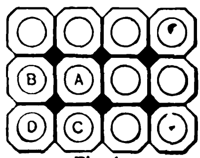


Fig. 4.

Cables in Ducts. — The following is also taken from the Standard Underground Cable Co's. handbook:

"Assuming that not more than twelve cables, arranged as shown in Fig. 4, can be used, the average carrying capacity may be taken as the criterion for the proper size of conductor; and for cables of a given type and size the carrying capacities of all cables, even though placed in adjacent ducts, will be represented by the following figures, taking unity as the average carrying capacity of four cables:

Number of Cables........	2	4	6	8	10	12
Multiplier...............	1.16	1.00	0.88	0.79	0.71	0.63

"**Rubber versus Paper.**— Rubber insulation is a somewhat better heat conductor than dry or saturated paper, and therefore, when applied to the same size conductor in equal thickness, will permit of a larger current flowing in the conductor for the same rise of temperature above the surrounding air. On the other hand, rubber deteriorates much more rapidly at high temperatures than saturated paper, and, while this disadvantage is apparently compensated for up to about 150° F. by its superior heat-dissipating qualities, at higher temperatures deterioration takes place and becomes so serious that its value as an insulating medium disappears in a comparatively short time.

"**Insulation Thickness.** — As the thickness of insulation is increased, the temperature of the conductor, with any given current flowing, gradually increases and therefore the current-carrying capacity becomes reduced. The reduction in capacity, however, is not very great, being in the ratio of about 93 for 14/32 insulation to 100 for 7/32 insulation, so that the values in the table given above should be slightly decreased when greater thicknesses than 7/32 are used.

"**Initial Temperature.** — As it is the final temperature reached which really affects the carrying capacity, the initial temperature of surrounding medium must be taken into account. If, for instance, the conduit system parallels steam or hot-water mains, the temperature of 150° F. (which is assumed in the above table to be the maximum for safe continuous work on cables) will be reached

with lower values of current than would otherwise be the case; and as 70° F. is the actual temperature we have assumed to exist in the surrounding medium prior to loading the cables, any increase over 70° F. must be compensated for by reducing the current carried.

"For rough calculations it will be safe to use the following multipliers to reduce the current-carrying capacity given in the table to the proper value for the corresponding initial temperatures:

Initial temperature, °F. . .	70	80	90	100	110	120	130	140	150
Multipliers.	1.00	0.93	0.86	0.78	0.70	0.60	0.48	0.34	0.00."

German Practice. — Tables giving the rules of the Verband Deutcher Elekiloteclniker for underground cables will be found in the *Elek. Zeit., 1907, Vol. 28, p. 500,* and the practice of the Siemens-Schuckert Werke for cables buried in the ground in the *Elek. Zeit., 1909, Vol. 30, p. 389.*

Cables Buried Underground, Under Water, etc. — A cable buried in earth will carry from 10 to 25 per cent more current with a given rise in temperature than it will when run in a dry duct, depending upon the character of the soil; immersed in water it will carry about 50 per cent more (*G. E. Bulletin, 4591*).

Current-carrying Capacity of Lead Sheath. — The carrying capacity of a lead sheath is given approximately by the following formula:

$$C = 128\sqrt{dp\,(d-p)T},$$

where

d = outside diameter of sheath in inches,
p = thickness of sheath in inches,
T = temperature rise, ° C.

Rise of Temperature with Time. — Fig. 5 shows the rise of temperature with time for a single conductor, 0.5 square inch in cross-section, carrying 750 amperes. Curves I to VI are from a paper by C. Beaver (*Jour. I.E.E., 1911*) and curve VII from Del Mar's *Electric Power Conductors.*

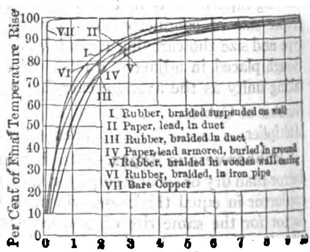


Fig. 5.

TESTS AND INSPECTION OF INSULATED WIRES AND CABLES.

— (*See also section on Specifications, below.*) Wires and cables are submitted to factory tests to ascertain whether they meet specified requirements. For the tests on the conductors themselves, see the article on *Wires and Cables, Bare.* The usual factory tests of insulation are the high-potential test and the measurement of insulation resistance and capacity. In the case of rubber insulation, the tensile strength and elasticity of the insulation are also measured. Insulated wires and cables are also tested after installation to discover incipient faults and to locate existing faults.

The tests described below in greatest detail are those employed by the majority of American cable manufacturers and operating engineers. Methods of measuring insulation resistance are described in the article on *Resistance and Conductance,* and methods of measuring capacity and inductance in the articles on *Capacity* and *Inductance.*

High-potential Tests. — The voltage which will break down the insulation of a given cable decreases as the length of the conductor is increased. For

this reason the potential tests of the National Board of Fire Underwriters, which are made upon one-foot lengths, are much too severe for commercial lengths. The chance of weak spots also increases with the conductor diameter making large cables dielectrically weaker than would be expected from theory based upon the assumption of homogeneous dielectrics. Large cables in long pieces are also likely to be overstressed momentarily upon the application of the voltage on account of the transient voltages caused by their electrostatic capacity. Furthermore, the manufacturers are unwilling to test large cables as near the limiting voltage as small cables on account of the greater commercial risk.

The breakdown voltage also decreases with the time of application. For example, a paper-insulated cable which breaks down at 2000 volts applied for 5 minutes will break down at about 1600 volts applied continuously for 30 minutes and at 1300 volts applied continuously for 60 minutes. On this account, the time of application of the test voltage as well as the value of this voltage should always be specified.

For the methods of measuring the voltage, see articles on *Insulating Material, Testing of; Spark Gap for Measuring High Voltage.*

Standard High-potential Tests. — In the following tables are given the test voltages recommended by the authorities noted for given thicknesses of various kinds of insulation.

POTENTIAL TESTS FOR 30 PER CENT PARA COMPOUNDS

Potentials in Kilovolts

(*Association of Railway Electrical Engineers, 1913*)

For 5-minute factory test.

Size wire, A. W. G. or C. M.	Thickness of insulation, 64ths inch											
	3	4	5	6	7	8	10	12	14	16	18	20
18	2.5	4.5	5.5	6.5	7.5	8.5	10.5	....	....	....	....	....
16	2.5	4.5	5.5	6.5	7.5	8.5	10.5	....	....	....	....	....
14	2.5	5.0	6.0	7.0	8.0	9.0	11.0	....	....	....	....	....
12	...	5.0	6.0	7.5	8.5	9.5	11.5	....	....	....	....	....
10	...	5.0	6.5	8.0	8.5	10.0	12.0	....	....	....	....	....
8	...	5.0	7.0	8.0	9.5	11.0	13.0	....	....	....	....	....
6	...	5.0	6.5	8.5	10.0	11.5	14.0	16	18	20	22	23
4	...	4.5	6.5	8.5	10.0	11.5	14.5	17	19	22	24	25
2	...	4.0	6.0	8.0	10.0	12.0	15.0	17	19	22	24	25
1	...	4.0	6.0	8.0	10.0	12.0	15.5	18	21	23	25	27
0	...	...	5.5	8.0	10.0	12.0	15.5	18	21	23	25	27
00	...	...	5.0	7.5	9.5	11.5	15.5	18	21	23	25	27
000	...	...	5.0	7.5	9.5	11.5	15.5	18	21	23	25	27
0000	...	...	4.5	7.0	9.0	11.5	15.5	18	21	23	25	27
250,000	...	...	4.0	6.5	9.0	11.0	15.5	19	22	24	26	28
500,000	...	...	2.5	5.0	7.5	10.0	14.5	19	22	24	26	28
750,000	...	...	...	...	6.5	9.0	14.0	19	22	24	26	28
1,000,000	...	...	...	...	5.5	8.0	13.0	19	22	24	26	28
1,250,000	...	...	...	...	...	7.5	12.5	19	22	24	26	28
1,500,000	...	...	...	...	...	7.0	12.0	19	22	24	26	28
1,750,000	...	...	...	...	...	6.5	11.5	19	22	24	26	28
2,000,000	...	...	...	...	...	5.0	10.5	19	22	24	26	28

For cables having conductors of a size not listed, use figures for next larger size.

POTENTIAL TESTS FOR VARNISHED-CLOTH INSULATION

Potentials in Kilovolts
(*G. E. Co. Bulletin 4787*)

Size wire, A. W. G. or B. & S. and Cir. Mils	Thickness of insulation, 64ths in.	At factory			After installation		
		5 min.	30 min.	60 min.	5 min.	30 min.	60 min.
6–2	4	2.5	2	1.6	2	1.6	1.3
1–0000	5	2.5	2	1.6	2	1.6	1.3
250,000–500,000	6	2.5	2	1.6	2	1.6	1.3
550,000–1,000,000	7	2.5	2	1.6	2	1.6	1.3
1,100,000 and over	8	2.5	2	1.6	2	1.6	1.3
6–0000	6	5.0	4	3.2	4	3.2	2.6
250,000–500,000	7	5.0	4	3.2	4	3.2	2.6
550,000–2,000,000	8	5.0	4	3.2	4	3.2	2.6
All sizes	9	7.5	6	4.2	6	4.8	3.8
All sizes	10	10.0	8	6.4	8	6.4	5.1
All sizes	12	12.5	10	8.0	10	8.0	6.4
All sizes	14	15.0	12	9.6	12	9.6	7.7
All sizes	16	17.5	14	11.2	14	11.2	9.0
All sizes	18	20.0	16	12.8	16	12.8	10.2
All sizes	18	22.5	18	14.4	18	14.4	11.5
All sizes	20	25.0	20	16.0	20	16.0	12.8
All sizes	22	27.5	22	17.6	22	17.6	14.1
All sizes	24	30.0	24	19.2	24	19.2	15.4
All sizes	24	32.5	26	20.8	26	20.8	16.6
All sizes	26	35.0	28	22.4	28	22.4	17.9
All sizes	26	37.5	30	24.0	30	24.0	19.2
All sizes	28	40.0	32	25.6	32	25.6	20.5
All sizes	28	42.5	34	27.2	34	27.2	21.7
All sizes	30	45.0	36	28.8	36	28.8	23.0
All sizes	30	47.5	38	30.4	38	30.4	24.3
All sizes	32	50.0	40	32.0	40	32.0	25.5
All sizes	32	52.5	42	33.6	42	33.6	26.8
All sizes	34	55.0	44	35.2	44	35.2	28.1
All sizes	34	57.0	46	36.8	46	36.8	29.4
All sizes	36	60.0	48	38.4	48	38.4	30.7
All sizes	36	62.5	50	40.0	50	40.0	31.9

Tests on three-conductor cables for circuits with grounded neutral, in proportion to thickness of insulation: Example, three-phase, 12,000-volt circuit "Y," neutral grounded, insulation on each conductor 3/16 inch (total between conductors 3/8 inch), outer belt 3/32 inch (total 9/32 inch); test pressure at factory for 5 minutes, between conductors 30,000 volts, each conductor to earth 22,500 volts. For mechanical reasons, thickness of insulation on individual conductors of three-conductor cables 3000 volts and less is made somewhat greater than required by working pressure on some sizes.

(The above tests are very conservative, some manufacturers advocating far severer tests.)

POTENTIAL TESTS FOR PAPER INSULATION

Potentials in Kilovolts

(G. E. Co. Bulletin 4787)

Size wire, A. W. G. or B. & S. and Cir. Mils	Thickness of insulation, 64ths in.	At factory			After installation		
		5 min.	30 min.	60 min.	5 min.	30 min.	60 min.
14–2	4	2.0	1.6	1.3	1.6	1.3	1
1–0000	5	2.0	1.6	1.3	1.6	1.3	1
225,000–500,000	6	2.0	1.6	1.3	1.6	1.3	1
550,000–1,000,000	7	2.0	1.6	1.3	1.6	1.3	1
12–2	5	2.5	2	1.6	2	1.6	1.3
1–0000	6	2.5	2	1.6	2	1.6	1.3
225,000–500,000	7	2.5	2	1.6	2	1.6	1.3
550,000–2,000,000	8	2.5	2	1.6	2	1.6	1.3
10–0000	7	5.0	4	3.2	4	3.2	2.5
225,000–500,000	8	5.0	4	3.2	4	3.2	2.5
550,000–2,000,000	9	5.0	4	3.2	4	3.2	2.5
8 and larger	10	7.5	6	4.8	6	4.8	3.8
8 and larger	12	10.0	8	6.4	8	6.4	5.1
6 and larger	14	12.5	10	8.0	10	8.0	6.4
6 and larger	16	15.0	12	9.6	12	9.6	7.7
5 and larger	18	17.5	14	11.2	14	11.2	9.0
5 and larger	20	22.5	18	14.4	18	14.4	11.5
4 and larger	22	27.5	22	17.6	22	17.6	14.1
4 and larger	24	32.5	26	20.8	26	20.8	16.6
3 and larger	26	37.5	30	24.0	30	24.0	19.2
3 and larger	28	42.5	34	27.2	34	27.2	21.7
2 and larger	30	47.5	38	30.4	38	30.4	24.3
2 and larger	32	52.5	42	33.6	42	33.6	26.8
1 and larger	34	57.0	46	36.8	46	36.8	29.4
0 and larger	36	62.5	50	40.0	50	40.0	31.9

Tests on three-phase cables for grounded neutral in proportion to thickness of insulation: Example, three-phase 13,000-volt circuit "Y," neutral grounded, insulation on each conductor 5/16 inch (total between conductors 5/8 inch), outer belt 5/32 inch (total 15/32 inch); test pressure at factory for 5 minutes, between conductors 32,500 volts, each conductor to earth 17,500 volts.

High-potential Test at Factory. — The entire reel of cable, unless lead covered, should be set, with its ends projecting, in a tank of water. The braid, tapes, etc., should be removed from the insulation for a length depending upon the potential to be applied and the surface of the insulation thoroughly dried or dipped in hot paraffine. One terminal of a source of alternating e.m.f. is then connected to the conductor and the other to the water. In the case of lead-covered cables the second terminal is connected directly to the lead. In the case of multiplex cables, the potential is sometimes applied between conductors.

The source of e.m.f. is invariably a transformer, but various means are adopted for obtaining variations of potential. Among these means are:

1. Supplying the transformer from a special alternator whose voltage is varied by means of a field rheostat.

2. Making the primary and secondary windings movable in relation to one another, so as to vary the linkage of magnetic lines.

3. Applying the current to suitable taps in the low-tension windings of the transformer by means of a multi-point switch.

Tests for Faults in Sheath. — There are sometimes faults in the lead of lead-encased cables. In order to detect them the usual practice has been to place the cable for twenty-four hours in water and then measure its insulation resistance. V. Planer (*Elek. Zeit., Jan. 4, 1912*) has investigated whether this method is a really reliable test with modern cables. In several lead-encased cables insulated with impregnated paper, a number of holes were artificially made in the lead armor and the cables placed in water. Every twelve hours the insulation resistance of the cable was measured and the cable was tested every hour at double normal voltage. After the cables had been in water for twenty-four hours or forty-eight hours the insulation resistance was just the same as before. The measurements were repeated daily and the results remained unchanged after five weeks. A second cable was placed in the water without a lead covering so that the impregnated paper was directly in contact with the water. After four weeks the cables could still withstand the test with double the normal voltage, while the insulation resistance had decreased by 10 per cent. These experiments prove that to place a modern cable for twenty-four hours in water before testing it is not a reliable method for detecting a fault in the lead armor. The best way to test the lead armor is with water under pressure. Some German cable manufacturers use pressures of several atmospheres.

Mechanical Tests of Rubber. — The test piece should be carefully prepared avoiding incisions and irregularities. The best way to obtain a test piece from a small conductor is to wet a sharp knife and run it tangentially along the wire so as to cut off a strip of rubber of segmental cross-section. In the case of large conductors, the best plan is probably to remove the insulation, spread it out flat and cut off a piece of uniform width by means of a special cutter consisting of a pair of parallel knife blades attached to a common frame. The diagonal projections of rubber which occur on the inside of insulation stripped from stranded conductors are neglected in computing the cross-sectional area.

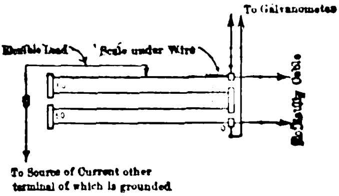
In order to measure its tensile strength the sample of rubber is held in some sort of grip and stretched in a machine similar to that used for wire (*see Bureau of Standards Bulletin No. 37*). It is usual to reduce the results thus obtained to pounds per square inch.

Seals. — Large purchasers of wire and cable assure themselves that they receive the material which they have inspected by sealing it at the time of inspection. Two sealing stamps are usually necessary, one for wire from No. 14 to No. 10 A.W.G. and one for wire from No. 8 to No. 4 A.W.G. These stamps press the purchaser's mark into the copper, Wires smaller than No. 14 A.W.G. are too small to show such a mark and are therefore sealed by looping a loose end into a lead seal and pressing this tight with a stamp which leaves the purchaser's mark upon the lead.

Tests after Installation. — A high-potential test with the voltages given above in the tables is usually applied to the full length of cable immediately after installation and at regular intervals to detect incipient faults. Double line voltage is frequently used for the periodic tests. Grounds or crosses of relatively low resistance may be detected by bridge methods or by exploration, as described below.

Some engineers make periodic insulation-resistance tests and plot the results as a curve. If the curve manifests any decided downward tendencies the cause **is immediately** sought for. The methods of making these tests are given below.

Testing for Faults by Murray Bridge Methods. — The bridge used in the Murray loop tests is shown diagrammatically in Fig. 6. It consists essentially of a resistance wire graduated into two equal scales each reading from 0 to 100, the zero points being at the ends of the wire. A galvanometer is connected across the zero points and a flexible lead connected to the positive terminal of a source of direct current is put in sliding contact with the wire. The negative terminal of the direct-current source is grounded.



Arrangement of Bridge
Fig. 6.

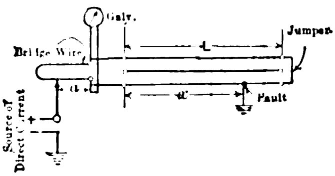


Fig. 7.

One Conductor Grounded. — When one conductor of a multiple-conductor cable is grounded, the bridge is connected as shown in Fig. 7, a jumper being placed between the faulty conductor and one of the others. The sliding contact is moved along the bridge wire until the galvanometer deflection becomes zero. Let a = distance of sliding contact, on bridge scale, to the terminal of the bridge connected to faulty wire, as a percentage of the length of the bridge wire, and L = length of cable; then if x is the distance from the bridge to the fault,

$$x = \frac{aL}{100}.$$

Two Conductors Crossed. — When one conductor of a three-conductor cable is crossed with another, the bridge is connected as shown in Fig. 8, a jumper

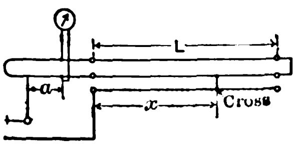


Fig. 8.

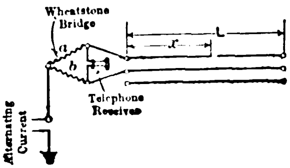


Fig. 9.

being again used, although connected differently. Zero reading is obtained on the galvanometer as before, and, using the same notation,

$$x = \frac{aL}{100}.$$

One Conductor Open. — When one conductor is open, the bridge is connected as shown in Fig. 9, a telephone receiver being substituted for the galvanometer, and alternating current for the direct current. The sliding contact is moved along the bridge until silence is obtained in the telephone.

Let a = distance from sliding contact to the terminal of bridge connected to open conductor,

b = distance from sliding contact to the terminal of bridge connected to whole conductor: then,

$$x = \frac{b}{a} L.$$

Ayrton's Method to Detect Crosses. — In Fig. 10, a, b and r represent three arms of a Wheatstone bridge, the fourth arm being the conductor N, which is in contact with a second conductor M of the same cable at the point P. Ground the conductor N at the far end and let x and y be the resistances

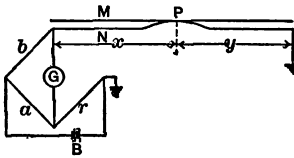


Fig. 10.

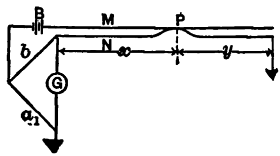


Fig. 11.

of the grounded conductor from the bridge to P and from P to the grounded end respectively. Let a, b and r be the resistances of the other three arms of the bridge. Make $a = b$, and adjust the variable resistance r until the galvanometer shows no deflection; then

$$x + y = r,$$

neglecting the resistance of the ground.

Then connect battery to M instead of to earth as shown in Fig. 11, and adjust a to a value a_1 such that the galvanometer shows no deflection. Then $a_1 x = by$ or

$$\frac{x}{x+y} = \frac{b}{b+a_1}.$$

But from first arrangement $x + y = r$, whence

$$x = \frac{br}{b+a_1}.$$

From the resistance of the conductor per foot, the distance to the point P is then immediately determined.

Varley Loop Test for Grounds. — Grounds may be located by the arrangement shown in Fig. 12. Loop the bad wire at the distant end with a good one of equal resistance. Call the resistance of the good wire c and the resistance to the fault on the bad wire x. Adjust the rheostat until the galvanometer balances. Then

$$x = \frac{2\,bc - ar}{a+b},$$

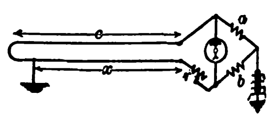


Fig. 12.

a, b and r being the values of the bridge arms and rheostat respectively when the galvanometer shows no deflection. (*S. G. McMeen.*)

Varley Loop Test for Crosses. — This is made by substituting one of the crossed wires for the earth path of Fig. 12, using one good wire as before.

Cable Tests by Exploring Coil. — Grounds on wires in cables can be located by the use of a telephone receiver connected to an exploring coil. To do this, connect a source of interrupted current to one end of the wire in trouble, the other terminal of the source being grounded. Remove from the wire in trouble all the grounds other than the fault. Then move along the cable a coil such as one spool of an 80-ohm telephone ringer (*see Telephony*), across whose terminals a receiver is connected. Hold the exploring coil with its iron core at right angles to the cable. A tone from the interrupter should be heard when the exploring coil is between the fault and the source of sound, but not when it is beyond the fault. (*S. G. McMeen.*)

Listening Tests on Telephone Lines. — A long telephone line will be noisy if one of its wires is grounded, open or crossed with a third wire, whether the latter be grounded or clear. To determine where the fault is, listen on the line while assistants short-circuit it at different places. The fault is beyond the short-circuit if the short-circuit makes the line quiet (*S. G. McMeen.*)

Use of Various Methods of Testing for Faults. — Two electric lighting companies (*N. E. L. A., 1911*) gave the following data on testing for faults: the relative number of tests for faults made by the different methods, and the cost per test of each type.

	Company A	Company B	Cost to Company B
	Per cent	Per cent	
Loop test	15.0	46	$12.00
Examination	36.5	8	12.00
Cut and try	17.5	28	60.00
Reported	24.3	11	2.50
Exploring coil	1.3	7	20.00
Miscellaneous	5.4	..	.....
	100.0	100	

SPECIFICATIONS. — (*See also article on Wires and Cables, Bare.*) The following forms are recommended for insulated wire and cable specifications. The general idea is for the purchaser to have printed or mimeographed forms and to fill them in according to specific requirements. These tabular specifications are supplemented by general specifications for workmanship and materials, which should be included when bids are sought or orders given. This arrangement minimizes the chance of accidental omission on the part of the purchaser and manufacturer and saves the bidder time in reading long specifications.

Items to be Covered in Detail Specification. —
Specification No.,
Title,
State briefly the service for which the conductor is intended,
Number of separately insulated conductors,
Material of conductors,
Type of stranding (parallel, concentric or rope),
Number of strands per conductor,
Number of wires per strand (if rope strand),
Total area of conductor or A.W.G. No.,
Conductor shall *not* be tinned (omit the word "not" as required),
Type of separator, if any,

Insulation material, each conductor,
belt,
Minimum thickness of insulation, each conductor,
belt,
Tape, material,
Number of tapes, each conductor,
over each layer,
over-all,
Pressure wire (if any), size,
insulation,
location,
Servings, material,
number,
thickness, inches,
Braid, material,
number on each conductor,
number over-all,
Tracers (if required), material,
number,
arrangement,
Laterals (if required), material,
Filling under sheath,
Sheath material,
Thickness of sheath,
Filling between sheath and armor, material,
thickness, measured after armoring,
Armor, material, wire or band,
Jute over armor,
Maximum over-all diameter (if limit is necessary),
Length, maximum,
minimum,
Whether to be supplied in coils or on reels,

All workmanship and materials shall conform with the General Wire and Cable Specifications given in next paragraph.

GENERAL WIRE AND CABLE SPECIFICATIONS. — The following general specifications are to be used in conjunction with the above. These are offered merely as suggestions, as every engineer should be guided by personal experience and local requirements. The specifications of the Association of Railway Electrical Engineers have been largely drawn upon in preparing them.

Tests and Inspection. — The wire or cable shall be open for inspection by an authorized representative of the Purchaser, who shall be afforded all the necessary facilities for making the specified electrical tests and to assure him that the materials used and the process of manufacture conform with the specifications.

The Contractor shall notify the designated representative of the Purchaser sufficiently in advance of the completion of the wire or cable to enable inspection to be arranged for.

Measurement of Cross-section. — The combined area of the wires when laid out straight and measured at right angles to their axes shall be not less than the specified gage or circular mils.

Thickness of Insulation. — Unless otherwise specified, wires and cables shall be insulated in accordance with the National Electric Code requirements as given above in the section on *Thickness of Insulation.*

Stranding. — Unless otherwise specified, cables shall be stranded in accordance with the accompanying table (which is that proposed by the Bureau of Standards).

Size wire, A.W.G. or B. & S. and Cir. Mils	Number of component wires per conductor
2,000,000–1,600,000	127
1,500,000–1,100,000	91
1,000,000–550,000	61
500,000–250,000	37
0000–1	19
2–8	7

Tinning. — The wire shall be provided with a heavy uniform coating of tin without projections.

Samples of wire, before stranding or covering, shall be thoroughly cleaned with alcohol and immersed in hydrochloric acid of specific gravity 1.088 and temperature 60° F. for one minute. They shall then be rinsed in clear water and immersed in a solution of sodium sulphide of specific gravity 1.142 for thirty seconds and again washed. This operation shall be repeated three times and upon the completion of the fourth cycle, the sample shall show no sign of blackening.

The sodium sulphide solution must contain an excess of sulphur and should have sufficient strength to thoroughly blacken a piece of clean untinned copper wire in five seconds.

Separator. — The separator may consist of soft cotton yarn (which may be braided), or of paper or muslin tape. With untinned conductors, the separator shall completely cover the conductors; with tinned conductors, it is desirable that the separator shall allow the insulation sufficient contact with the conductor to prevent the conductor sliding in the insulation.

Rubber Insulation. — *(See also article on Rubber.)* Various grades of rubber insulation may be used with advantage, according to the degree of permanence desired. Rubber compounds, being composed of a mixture of organic and inorganic substances and their compounds, are too complex to permit of their permanency being predicted from theoretical considerations based upon tests of chemical and physical properties. It is therefore necessary to rely entirely upon experience and to write specifications which will require all new material to be substantially identical with insulation which has been found satisfactory in service.

At the date of going to press, the subject of standard rubber specifications is receiving a great deal of attention from committees of technical societies and by committees appointed by the large consumers and manufacturers. The general tendency is to specify a combination of chemical, mechanical and electrical requirements. The idea of the chemical clauses is that by specifying close limits upon the following analytical results (obtained by a standard method of analysis), the use of non-permanent substances will be substantially prevented.

Analytical results upon which limits are placed	Deleterious substances whose use is prevented by limits upon analytical results
Total sulphur.	Reclaimed rubber.
Alcoholic potash extract.	Rubber substitutes.
Chloroform extract.	Tars.
Mineral matter.	Carbon, cellulose, etc.
Saponifiable acetone extract.	Unextracted high resin rubbers.
Unsaponifiable acetone extract.	Deresinated rubber.

The mechanical clauses ensure sufficient vulcanization and assist in limiting the grade of rubber. The electrical clauses are to ensure the proper insulating qualities.

Where a high-grade rubber compound is to be purchased on the basis of competitive bids, the chemical clauses recommended by the "Joint Rubber Insulation Committee" (*Proc. A.I.E.E.*, Jan. 1914 and *Jour. Ind. & Eng. Chem.*, Jan. 1914) may be used in combination with the following mechanical and electrical clauses.

Concentric Application. — The compound shall be applied concentrically about the conductor and shall fit closely thereto. If necessary, in order to achieve this result on conductors of greater diameter than three-quarters of an inch, a tape shall be applied over the insulation before vulcanization. Such tape will be additional to any which may be required in the accompanying wire specification.

Repairs and Joints. — If exigencies of manufacture require repairs of joints in the insulation, the compound employed shall meet the mechanical and electrical requirements of this specification, and the work shall be done in such a way as to leave the repaired part or joint and all parts affected by it as strong and durable electrically as the remainder of the insulation. The repairs or joints shall be properly vulcanized in a mold of approximately the same diameter as the rest of the insulation.

Elasticity. — A sample of the insulation shall be stripped from the conductor, and cut to such a width as to give a cross-sectional area as near as practicable to one thirty-second square inch. The covering having been removed, marks shall be placed two inches apart on the sample, which shall then be stretched at the rate of twelve inches per minute until the marks are six inches apart and one end immediately released; thirty seconds after release the marks shall be not further apart than is specified in the following table.

Tensile Strength. — A strip of segmental cross-section shall be cut from the cable by means of a sharp knife held tangential to the conductor. The sample shall be bent in every direction to magnify and reveal any surface incision or imperfection which may exist. A portion of the sample without such defects and having a free length of not less than two inches shall then be stretched at the rate specified above until it breaks. Five samples from every 25,000 feet or less, shall be tested in this way and their average tensile strength shall be not less than one thousand pounds per square inch, or their minimum tensile strength less than nine hundred pounds per square inch. At the moment of fracture the average distance between marks shall be not less than specified in the following table:

Insulation thickness	Length after stretching and release, in.	Length at moment of fracture, in.
Less than 8/64 in.	2 7/16	9
8/64 in. to 12/64 in.	2 7/16	8
12/64 in. or over	2½	7

Electrical Tests. — Electrical tests shall be made upon all wire or cable after at least twelve (or 24) hours immersion in water while still immersed and before the application of any covering other than the tape or braid used in vulcanization. The insulated conductor shall successfully meet the specified

high-potential tests. The insulation resistance shall then be measured. In the case of rubber-insulated multiplex cables, the insulation resistance shall be measured before assembling the conductors. An additional electrical test shall be made on lead-covered cable only and shall consist of a high-potential test to be made upon the cable after assembling and leading and without immersion in water. In the case of multiplex cables this test shall be made successively between each conductor and the other conductors and sheath.

The insulation resistance shall be not less than specified (*see section above on Insulation Resistance*).

The temperature coefficient for correcting the insulation resistance to the standard temperature of sixty (60) degrees Fahrenheit shall be in accordance with the value given in the article on *Insulating Materials, Miscellaneous.*

Varnished-cloth Insulation. — (*Assoc. Ry. Elect. Eng., 1912.*) The insulation shall consist of a closely woven cotton cloth and a viscous filler. Each surface of the cloth shall have smooth continuous films of varnish and shall be free from wrinkles, blisters and all other imperfections. It shall be thoroughly impregnated with insulating compound, shall be pliable and shall have no tendency to crack when doubled on itself.

Filler. — The filler shall be a viscous moisture repelling insulating compound whose dielectric constant is similar to that of the varnished-cloth insulation and of such a nature that it will prevent the tapes from unwrapping when cut, but will allow the layers to slide upon each other when the cable is bent.

Assembly. — The insulating cloth shall be applied in the form of tape wound on helically and reversed at least every two layers. The tapes shall be of such widths that for different diameters of cable they will lie smoothly and be free from wrinkles; the turns shall overlap and the joints in successive layers shall be staggered. The filler shall be so applied between layers as to exclude all air, the whole forming a hard homogeneous semi-flexible wall of insulation.

Tape. — A layer of cloth tape thoroughly filled with a rubber compound lapping one-fourth of its width shall be applied over the cloth insulation.

Electrical Tests. — Each and every length of cable shall conform to the specified electrical tests. Braided wires to be tested after twelve (12) hours immersion in water and before the braid is applied; lead-covered conductors to be tested against the sheath with sheath grounded. Multiplex cables shall be tested between each conductor, and the other conductors and sheath or ground in multiple.

Impregnated Paper Insulation. — (*Assoc. Ry. Elect. Eng., 1912.*) The insulation shall consist of a Manila paper applied helically and evenly to the conductor, and then thoroughly impregnated with an insulating compound. The cable shall be pliable and show no tendency to harden injuriously at 32° F. The paper or compound shall contain no acids, alkalis or metallic salts.

The compound shall be applied so as to exclude all air and moisture.

Tensile Strength. — The tensile strength of the paper measured lengthwise before impregnation shall be equivalent to the weight of a strip not less than 20,000 feet long, and its elongation shall be not less than 2 per cent. Acceptance will be based upon the average of six tests on samples selected at random from every order.

Electrical Tests. — Each and every length of cable shall conform to the specified electrical tests. No immersion is required before testing. The potential test to be made between conductor and sheath with the sheath grounded. Multiplex cables shall be tested between each conductor, and the other conductors and sheath or ground in multiple.

Braids. — (*Assoc. Ry. Elec. Eng., 1912.*) Braid shall be of closely woven cotton thread, at least two-ply, thoroughly impregnated with an insulating weatherproof compound and finished with a black insulating compound thoroughly slicked down. The compounds shall be neither injuriously affected by nor have injurious effect upon the braid at a temperature of 200° F. The thickness shall be not less than given in the following table.

Diameter over the insulation, inches	Thickness of braid, inches, not less than
0.160	0.018
0.290	0.028
0.530	0.038
1.000 and over	0.053

For intermediate diameters use next smaller diameter. (Note: The above does not apply to fancy or special braids for fixtures, etc.)

Tests. — A six-inch sample of wire with carefully paraffined ends shall be weighed and submerged in fresh water of a temperature of 70° F. for a period of twenty-four hours. The increase in weight after submersion and removal of surface water shall be not more than 8 per cent of the weight exclusive of copper and insulation before submersion. The compound shall not drip at a temperature of 125° F.

Rubber-filled Cloth Tape. — (*Assoc. Ry. Elec. Eng., 1912.*) The tape shall consist of a cotton cloth not lighter than one pound per four yards 36 inches wide with not less than 56 by 60 picks per inch, thoroughly filled with a rubber compound. The tape shall be applied spirally overlapping not less than one-quarter of its width. The following table gives the maximum width of tape allowed.

Diameter over insulation, inches	Maximum width of tape, inches	Diameter over insulation, inches	Maximum width of tape, inches
2.00	5	0.62	2
1.75	4½	0.50	1¾
1.50	4	0.44	1½
1.25	3½	0.38	1⅜
1.15	3¼	0.31	1⅛
1.00	3	0.25	⅞
0.88	2¾	0.19	¾
0.75	2⅝	0.16	⅝

For intermediate diameters use next smaller diameter.

Sheath. — (*Assoc. Ry. Elec. Eng. 1912.*) Unless otherwise specified the sheath shall have an average thickness of not less than that indicated in the tabulation next following, and the minimum thickness shall in no place be less than ninety per cent of the required average thickness.

Composition. — The sheath shall consist of commercially pure lead for all cables having a core diameter (i.e., internal diameter of the sheath) less than two inches; for cables having a core diameter equal to two inches or more the sheath shall consist of an alloy of lead and commercially pure tin containing not less than one per cent of tin.

Wire Armor. — (*Assoc. of Ry. Elec. Eng., 1912.*) The core of the cable shall be run through a hot asphalt compound, served with a layer of jute yarn, run through hot asphalt again, and then laid with galvanized wire armor.

Size of Armor Wire. — The proper size of armor wire will depend upon the conditions of service; the latitude allowed in the following table represents the difference arising from such difference in service conditions. Unless otherwise specified the armor wire shall be the minimum size.

Internal diameter of sheath, inches	Corresponding thickness of sheath in 64ths of an inch	
	Paper insulation	Rubber or var. cloth insulation
0.00 to 0.29	5	4
0.30 to 0.69	6	5
0.70 to 1.24	7	6
1.25 to 1.99	8	7
2.00 to 2.69	9	8
2.70 and over	10	9

For intermediate diameters use next smaller diameter.

Lay. — The armor shall be applied closely without appreciable space between adjacent wires. The lay shall be from eight to twelve times the pitch diameter.

Finish. — The armored cable shall be run through hot asphalt compound, served with a layer of the best three ply 14 pound hard-twisted jute yarn applied in a close short lay, run through hot asphalt compound, then served with a second layer of three-ply, 14-pound jute yarn, run through hot asphalt compound and finally run through some material to prevent sticking.

(Under certain conditions, the purchaser may find it advisable to specify the omission of the outer jute covering.)

Diameter of cable under jute bedding, inches	Armor wire A. (Steel) W. G.	Jute bedding under armor measured in finished cable, inches
		minimum
0.00 to 0.50	14-13	2/32
0.44 " 0.69	12	2/32
0.63 " 1.00	10	2/32
0.88 " 1.50	8	2/32
1.25 " 2.00	6	2/32
1.30 and over	4	2/32

Direction of Lay. — Successive layers of jute or jute and armor shall be laid in opposite directions. In the case of multiple-conductor cable armored without lead, the outside layer of conductors shall be put on with a right-hand lay. The armor shall be put on with a left-hand lay. The direction of lay is defined as the lateral direction in which the wires run over the top of the cable as they recede from an observer looking along the axis of the cable.

Tests of Armor Wire. — The armor wire shall consist of a galvanized mild steel wire of uniform diameter, free from all cracks, splits or other flaws. Samples shall be taken at random from 10 per cent of the coils to be used, for each of the following tests. At least 80 per cent of the samples shall fulfill the conditions of the tests in order that the whole lot may be considered satisfactory.

Tensile Test. — The wire must have a tensile strength of 50,000 pounds per square inch and an elongation of not less than ten per cent in eight inches.

Galvanizing Test. — Thoroughly clean a sample to remove all dirt and grease, rinse in clean water and wipe dry with a clean cloth or cotton waste. Immerse for one minute in a solution of copper sulphate, rinse in clean water and wipe dry. Repeat this operation until the samples have been immersed

four times. After these immersions no sample shall show any bright deposit of copper. The samples shall be approximately straight and the ends protected with paraffine. The solution shall be saturated with copper sulphate to which an excess of C. P. cupric oxide has been added, shall have a specific gravity of 1.186 at 65°F. and shall be maintained at 60° to 65° F. during the test.

Flexibility Test. — The armor wire shall admit of bending around a spindle of ten times the diameter of the wire and back again without developing cracks of the galvanizing which are visible to the naked eye.

Steel-tape Armor. — (*Assoc. Ry. Elec. Eng., 1912.*) The core of the cable shall be run through a bath of hot asphalt compound, served with a layer of jute yarn spun on with a close short lay, run through hot asphalt compound, armored with a steel tape, armored with a second steel tape, run through hot asphalt compound, served with a layer of one-hundred pound jute yarn with a close short lay, run through hot asphalt compound and finished by running through some material to prevent sticking. Each layer of jute shall be applied in the reverse direction to the adjacent layer. The space between adjacent turns of steel tape shall not exceed one-tenth the width of the steel tape.

Armor Tape. — The tape and jute under armor, after armoring, shall conform to the following table:

Cable diameter before armoring, inches	Maximum width steel tape, inches	Minimum thickness each tape, inches	Minimum jute thickness under armor, inches
0.00 to 0.45	½	0.02	0.06
0.46 to 0.75	¾	0.02	0.06
0.76 to 1.00	1	0.03	0.07
1.01 to 1.40	1¼	0.03	0.07
1.41 to 1.70	1½	0.04	0.08
1.71 to 2.00	1¾	0.04	0.08
2.01 and over	2	0.05	0.09

Cable Reels. — Each reel shall consist of a wooden drum with wooden disks or heads securely fastened thereto.

Bushing. — Each disk or head of the reel shall be provided with an iron plate or cast-iron bushing in the center of which shall be a hole 2½ inches in diameter. The plates or bushings shall be secured to the head by means of bolts or lags through the head.

Covering. — Insulated cable shall be thoroughly covered with burlap before lagging is applied.

Lagging. — When used for insulated cable the reels shall be suitably lagged; when used for bare cable, the lagging of the reel shall be replaced by a burlap covering securely bound to the cable.

Chocking. — Reels shall be properly chocked in the car so that there shall be no movement of reels during transit.

INSTALLATION OF INSULATED WIRES AND CABLES. — For the installation of wires and cables in buildings see articles on *Wiring of Buildings for Light and Power; Wiring of Buildings for Miscellaneous Devices.* Below is given a brief description of modern practice in installing insulated wires and cables on messenger wires and in underground conduit systems. For the construction of the pole lines carrying the messenger wire see articles on *Poles*

for Overhead Lines; Towers, Transmission. For the construction of conduit systems see article on *Conduits and Conduit Lines.* See also the articles on *Distribution Lines* and *Transmission Lines.*

Messenger Construction. — The messenger wire is erected in the same manner as an ordinary line wire; see article on *Transmission Lines.* A "leading-up" wire is stretched from the bottom of one pole to the messenger wire on the starting pole, forming an incline on which to pull up the cable. A pulling rope is then fastened to the end of the cable by means of a cable grip and carried alongside of the messenger to the point where the cable is to reach, thence through a snatch-block down to the terminal pole to a second block at the bottom and thence to a capstan, winch, locomotive or whatever is to be used for pulling. Either temporary rollers should be provided on the poles over which the rope runs, or the rope should be suspended from the messenger by wire hooks. The cable is then slowly drawn up the inclined wire and along the messenger, attaching temporary carriers to the cable as it is paid out and hooking them over the messenger to carry the weight of the cable. Linemen must be stationed on each pole to pass these carriers around the messenger clamp or insulator. The final suspension of the cable may be accomplished in either of the following ways: (1) When the end of the cable arrives at the beginning of the last span, the lineman on each pole replaces the temporary carriers by permanent hangers, spacing them regularly along the cable, so that when the last span is pulled, all the hangers will be in place. (2) When the cable has been pulled all the way, a lineman rides along the messenger wire in a carriage replacing each temporary carrier by a hanger. This plan is preferable as the hangers may be attached more tightly to the messenger wire, and are less likely to slip on the cable.

Installation of Cables in Ducts. — The conduit system having been constructed, it must be prepared for the reception of the cable by being cleaned out or rodded as described in the article on *Conduits and Conduit Lines.*

If there are several ducts available, the choice of the particular duct to be used should be governed by the following considerations: 1. Avoiding unnecessary crossings of cable in the splicing chambers, substations, etc; 2. Avoiding the obstructing of empty ducts; 3. Keeping the cables cool; and 4. Keeping d-c. cables away from others. The following statement in the catalogue of the A. S. & W. Co. is pertinent with regard to keeping the cables cool. "Usually the coolest and best heat-radiating ducts are those located at the lower corners of the system, next are those nearest to the outside of the system and lastly the middle and top ducts which not only take up heat from the lower cables, but must dissipate heat through adjoining ducts. Attention to these points when planning a new system may prove very profitable in the end."

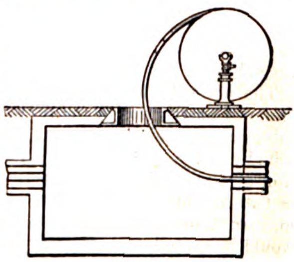


Fig. 13.

The next step is to set the cable reel on the shaft of a pair of wheels of slightly greater diameter than the reel itself or on jacks and raise it slightly above the ground, taking care to locate it as shown in Fig. 13, so that the cable will unreel into the manhole without making a reverse bend.

Fig. 14.

The pulling rope having been left in the duct after rodding, the cable is unreeled sufficiently to bring its end close to the mouth of the latter and a wire pulling grip (Fig. 14) is drawn over its end. The

end of the grip is hooked to the rope and the rope pulled from the other end. Fig. 15 shows an arrangement of pulleys for guiding the pulling rope. The pulling may be done by capstan, winch, motor truck, horse or by hand, depending upon the size and amount of cable to be pulled, and upon local conditions. The cable should be carefully guided into the duct so as to avoid sharp bends and abrasions, and a small quantity of grease may with advantage be spread over the cable as it enters the duct. R. J. Robb (*Journ. I.E.E., 1911, Vol. 47, p. 350*) says that 112 pounds of petroleum jelly is required per mile of cable. It is also necessary to cover the edges of the duct with pieces of lead to prevent abrasion of the cable as it passes into the mouth of the duct.

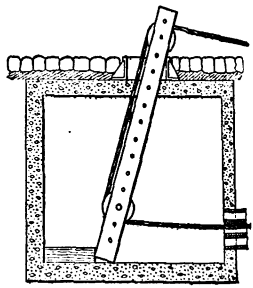


Fig. 15.

Protection of Cables in Splicing Chambers. — (*See also article on Conduits and Conduit Lines.*) Wherever there are several large cables in a splicing chamber, there is always danger that a burn-out of one cable will involve some or all of the remainder. Hence it is usual to protect such cables by means of one or more of the following methods:

1. Concrete shelves,
2. Open-face conduits,
3. Cement coating with ¼-inch rope bond,
4. Asbestos tape saturated with silicate of soda,
5. Asbestos tape covered with soft steel-tape armor,
6. Asbestos rope,
7. Split tile duct.

Methods Nos. 1 and 2 are applicable only to chambers containing few cables. No. 3 has given good results in many cases, but the cement is liable to crack and fall away from the cable unless carefully bonded. On the other hand, the location of a burn-out is plainly indicated by the cement covering being blown off. No. 4 combined with No. 5 is very generally used and gives very satisfactory results. No. 6 is but little used. No. 7 is very satisfactory, but is very clumsy, and cannot be used where there are many cables in the chambers.

The necessity for such protection (according to the Committee on Underground Construction of the N. E. L. A., 1911) is equally great with low-tension cables as with high-tension cables, for, while a breakdown of the insulation of a high-tension cable is often attended with a violent explosion and the generation of a very high temperature, the duration of the trouble is limited to one or two seconds; whereas in the case of a short circuit on a low-tension cable, it may continue to burn with a considerable flame for several minutes, which would be quite long enough to damage adjacent unprotected cables.

In the case of direct-current feeders for electric railways, where the return is grounded, the danger of burn-outs spreading is much greater than with the insulated lighting systems referred to in the N. E. L. A. report. This subject is treated at greater length in the author's *Electric Power Conductors*, from which the following is quoted: "If it is necessary to put direct- and alternating-current cables in the same duct line, it is well to isolate the direct-current cables as much as possible in the splicing chambers. The racks on which direct-current cables are supported should not be in metallic contact with other racks. If, however, this is unavoidable, the cables should not lay directly on the racks, but on insulating pads or blocks."

Jointing Insulated Conductors. — (*See also article on Wires and Cables, Bare.*) Having selected the corresponding cable ends, they should be inspected for mechanical defects along the entire exposed length and bushings placed over them at the ducts so that the sheath will not be cut by the edges of the conduits. The cables should then be bent until the ends overlap, the bends being of ample radius throughout, and the position of the cable such that the joint, when completed, will not have to bear any elastic stress or weight.

Drying Out Cable Ends. — In the case of paper or cambric insulated cables, the ends should be examined for moisture before they are finally cut short and if any exists or is suspected, heat should be applied to the sheath, beginning at the duct end and slowly working to the open end. This is usually done with a gasoline torch but sometimes by pouring on hot insulating compound.

Overlap. — This operation having been completed, the cable ends are cut so as to leave an overlap, depending upon the method of joining the conductors (*see below*) and upon the number of separately insulated conductors in the cable. If the cable has but a single conductor, and a butt joint is to be used, the overlap should be merely sufficient to allow for cutting off the ends, whereas if an interlaced joint is contemplated, the overlap should be greater by about 3 or 4 times the diameter of the conductor. In the case of multiple-conductor cables, the separate joints of which must be staggered, the overlap should be correspondingly greater.

Removal of Sheath. — The next operation is to cut off a sufficient length of lead sheath to permit the joint to be made. This length may be judged from the size of the sleeve to be used (*see below*). The operation is performed with a chipping knife and hammer, or with a special tool designed for the purpose, and the greatest care should be exercised to avoid cutting the insulation to the slightest degree. It is usual to make a cut around the sheath and gradually increase its depth until the lead is cut through. The lead must then be cut lengthwise from the circular cut to the end, injury to the insulation being avoided by holding the knife tangent thereto. The lead may then be pulled off with a pair of pliers and loose particles carefully removed. In the case of high-tension cables the ends of the lead should then be turned up slightly to a bell-mouth shape by means of a special tool.

Putting on Lead Sheath. — Before jointing the conductors, the lead sleeve should be slipped over one of the cable ends and pushed out of the way. The ends of the sleeve should have been previously scraped for a length of about a couple of inches along the outside and the cleaned surfaces smeared with tallow.

Removal of Insulation. — The next step is to cut back the insulation for a length between ¼ and ½ inch greater than half the length of the connector to be used. In the case of multiple-conductor cables having a belt, the belt must be cut back sufficiently to expose all the joints, the greatest care being exercised to avoid cutting the inner insulation in doing so.

Jointing the Conductors. — The next process is to prepare the conductor or conductors for jointing. The most usual type of joint is made by butting the ends of the conductors and enclosing them in a cylindrical copper connector. The conductors are first cleaned with gasoline and then tinned by pouring molten solder over them, using tallow for flux. All burrs should be removed with a file and the ends smoothed so that they will butt together perfectly. The connector is then put over one conductor end and the other end slipped in until the two ends butt. Solder is then poured over the joint until it is thoroughly saturated, when the surplus is wiped off so as to leave no sharp projections.

Connectors. — The connector should have a cross-section not less than that of the conductor and a length in accordance with the following table.

A length of 2¾ inches is recommended by a Committee of the N. E. L. A. (1911) for high-tension cable joints.

From points ½ inch back from each end, it should be pencilled down to thin edges at the ends, and it should be split the entire length on one side.

Size of conductors	Length of connector, inches
0 to 000 B. & S.	1-2
0000 B. & S. to 1 million Cir. Mils.	2½-3
1¼ million to 2 million Cir. Mils.. .	3½-4

Jointing Stranded Conductors. — While the connector joint is by far the most common, stranded conductors may be joined by cutting the wires alternately long and short, and fitting the two conductor ends into one another. The joint is then bound with small wire. Stranded conductors are sometimes joined by cutting the wires of successive layers alternately long and short, and telescoping the ends into one another. Yet another method is to lay strips of flexible copper braid longitudinally along the butted conductors and bind them securely with copper wire. In any case, the joint must be saturated with solder and wiped, as described above.

Insulation of Joint. — The next step is to insulate the joint. The usual process is to cover it with a tape of similar material to the remainder of the cable insulation, although this method is not universal. If of oiled paper or varnished cambric, the tape should be cut on the bias. Assuming this to be the process, the cable insulation is tapered gradually to the conductor, by means of a sharp knife, leaving about ½ inch between it and the connector. After this has been done, the exposed insulation, if paper, is to be dried by pouring over it melted paraffin heated to a temperature of 125° C. Narrow strips of tape should first be wrapped in the space between the insulation and the connector until this space is built up to the diameter of the connector. Wrapping should be continued back and forth in the space between the two ends of the original paper insulation until it is built up to the level of this insulation. The tape is then wound on until a thickness about 40 per cent greater than that of the cable insulation is reached, the tape running up the sloping part of the insulation until firmly attached to it.

In the case of three-conductor high-tension cables the wrapping should commence at a point on the covered conductor 3 inches from the outer belt and extend to a corresponding point at the other end. The wrapping is then continued back and forth, stopping each successive layer ½ inch short of the end of the preceding layer. In applying the tape, each turn should be drawn tight to exclude air and should overlap the preceding turn by two-thirds of its width. It is important that the tape should be applied tightly and evenly. In the case of rubber tape the tension should be such as to stretch it to about half its width. In this process care must be taken to have everything perfectly clean.

"Boiling Out." — Where cotton or linen tape is used, each layer must be "boiled out" by pouring hot compound over it until all the moisture is expelled and the tape wrapping is thoroughly saturated. Before applying the compound the jointer should assure himself that it is *not* hot enough to ignite a piece of paper dipped in it; otherwise the tape is likely to be charred. This should be done before entering the splicing chamber, as the accidental ignition of a pot of compound is dangerous in such a confined space.

Vulcanizing. — Where rubber tape containing sulphur is used, it should be partially vulcanized by the application of heat from a spirit lamp, care being

taken to apply the heat evenly and to avoid burning the insulation. This process is usually complete in about one minute with conductors up to one-half inch diameter.

Spreaders for Three-conductor Cables.— In the case of three-conductor cables, after all the conductors are joined and insulated as described, a roll of tape ½-inch diameter is inserted between them at the center of the joint to serve as a spreader. A band of tape is then wrapped around all three conductors to such a diameter that the whole will slide nicely into the lead sleeve, as shown in Fig. 16.

Use of Insulating Tubes. — Another way of insulating joints is to slip an insulating tube over the conductors before soldering and bring it over the joint when the latter is completed. In order to get the sleeve out of the way during soldering, it must be large enough to slip easily over the insulation. Such tubes are made of prepared paper, varnished cloth, or micanite. The jointed conductor should first be wound with cotton tape up to the level of the original insulation, "boiled out" as described above, and then the sleeve slipped on. A further "boiling out" with hot compound completes the job. In the case of belted multiple-conductor cables, in addition to the insulating tube over each conductor a large tube must be slipped over the belt before splicing the conductors.

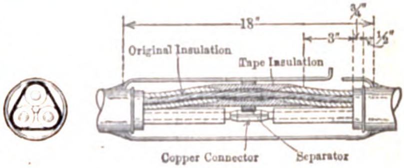


Fig. 16.

Owing to the absence of convolutions of tape and to the thorough impregnation of the tubes at the factory, this type of insulation is claimed to be more reliable than tape.

English Practice.—A type of joint used largely in England for multiple-conductor cables, has the conductors separated from one another by an insulating spreader or separator, which may be of ebonite, china or similar material. The shrinkage of solid compound gives rise to air holes near the conductors, with consequent low dielectric strength. By insulating the cores separately with insulating tapes before the compound is run in, this trouble can be avoided, but it is better to use a viscous compound which never sets (Vernier). This type of joint has two serious defects, 1st, breakdown is likely to occur on account of defects of the separators or on account of moisture on their surfaces; 2nd, a longitudinal pull of the cable, such as is likely to arise when the cable shrinks in cold weather, puts the separators in compression and may stress them unduly.

Lead Sleeves. — The joint having been insulated, the lead sleeve should be brought symmetrically over it, and the ends beaten down into contact with the sheath, taking care to make the sleeve concentric with the cable.

The sleeve and sheath are then joined by pouring solder over the ends from a ladle and wiping the joint with a cloth. This process should be continued until perfectly air-tight joints are obtained, the under side being examined for defects by means of a hand mirror.

When the sleeve is well wiped on, two small holes should be made in the top of it and hot compound poured in one hole until it appears at the other. If any frothing appears, the compound should be poured in one hole and allowed to escape from the other until this defect ceases. The joint should then be allowed to cool for about an hour, and if then the compound has settled, more should be added until the sleeve is full. The sleeve is then closed by soldering

The following data on lead sleeves are given by the Standard Underground Cable Co.

DATA ON LEAD SLEEVES

	Outside diam. of cable, mils	Inside diam. of sleeve, inches	Length of sleeve, inches	Gals. of compound per joint	Wiping solder per joint, lbs.
Single-conductor, light and power, up to 6600 volts.	Up to 550	1	8	0.05	0.9
	551–950	1½	10	0.1	1.7
	951–1350	2	12	0.2	2.8
	1351–1750	2½	12	0.3	4.2
	1751–2150	3	14	0.5	5.5
	2151–2550	3½	14	0.6	6.8
Single conductor, light and power, above 6600 volts.	Up to 550	1	10	0.05	0.9
	551–950	1½	12	0.1	1.7
	951–1350	2	14	0.2	2.8
	1351–1750	2½	16	0.4	4.2
	1751–2150	3	18	0.6	5.5
	2151–2550	3½	18	0.8	6.8
Multi-conductor, light and power, all voltages.	Up to 800	1½	14	0.2	1.5
	801–1200	2	16	0.25	2.5
	1201–1600	2½	16	0.35	3.7
	1601–2000	3	18	0.6	5.0
	2001–2400	3½	18	0.8	6.3
	2401–2800	4	18	1.0	7.6
	2801–3200	4½	20	1.4	8.3

small patches of lead over the holes. When the joint has thoroughly cooled and solidified, it may be pushed gently into its permanent place.

Compounds for Filling Lead Sleeves. — Various compounds have been used for filling the sleeves, such as paraffin wax, G. E. No. 227 compound, voltax, ozite, etc. Paraffin does not adhere to smooth surfaces and has an excessive contraction coefficient, causing voids which have a dielectric strength lower than ordinary air. The Commonwealth Edison Co. of Chicago uses a compound developed by the engineers of the company. It is poured in at a temperature of 150° C.

A good compound should have a high melting point, adhesiveness, high dielectric strength, low coefficient of contraction and should not be brittle at ordinary temperatures.

Stamping the Joint. — After the joint is completed the jointer should stamp his initials at each end of the sleeve.

Taped Joints. — Where the cable has no lead sheath, but is merely braided, it is usual to finish the joint with friction tape.

Terminals. — Conductors may be soldered to lugs or terminals or they may be clamped mechanically by means of a Dossert or similar connector.

In the former case the insulation is cut from the end of the conductor, which is then brightened by scraping or sandpapering and smeared with soldering flux. The conductor is then tinned by plunging into molten solder. The lug

must also be tinned internally and heated so that it will hold some solder in the molten state. The conductor, also heated above the melting point of solder, is then pushed into the lug and the latter cooled by the application of wet waste. When cool the shreds and globules of solder are filed off and the surfaces brightened with sandpaper. Some jointers prefer to hold the lug in the molten solder until it attains the same temperature as the latter. In this case, if the lug has been previously treated with flux, its outside as well as its inside surface will be tinned and it will make better contact with its accompanying lug or terminal. If, however, the copper or brass surface is desired for appearances, it may be preserved by coating the outside of the lug in a light oil of high flash point, before dipping it into the solder.

In either case it is advisable to wrap the end insulation in a rag previously wrung out in cold water to prevent it being melted or charred.

Grounding Sheaths. — The conductors of high tension cables induce electrostatic charges on the sheaths, often raising the latter to dangerously high potentials. It is, therefore, customary to ground the sheaths of cables at suitable points in order to carry off the "static," as these induced charges are called. The sheaths of low-tension cables do not have to be grounded to drain off static electricity. The sheaths of direct-current railway feeders which are used in conjunction with a grounded return system should, however, be grounded to the negative return system, except when the tracks have insulated sections for automatic block signals, in order to afford a low-resistance path for a short-circuit current. Unless this is done, the escaping current will return through devious paths, inflicting damage without attaining sufficient strength to trip the station circuit breakers.

BURN-OUTS OR PUNCTURING OF INSULATION. — Cable burn-outs may be caused by mechanical injury, exposure to excessive heat or cold, by chemical deterioration, or by transient high-voltage phenomena.

Burn-outs Due to Mechanical Injury. — Paper insulation differs from rubber and varnished cambric in depending upon the integrity of the lead sheath which incloses it. Hence in the case of paper-insulated conductors, a puncture of the lead sheath will sooner or later result in a burn-out even though it may take a week or longer for the moisture to penetrate the insulation sufficiently to accomplish this. (*Burch, Trans. A.I.E.E., 1903, vol. 22, p. 433.*) The moisture-resisting qualities of paper insulation are further treated above in the section on *Tests and Inspection.*

Mechanical injury often occurs in the process of installing underground cables. A slight projection in a duct will cut a groove in the lead sheath, thereby reducing its effective thickness and rendering it liable to crack open. The bending of cable in splicing chambers may crack the insulation, especially if tightly wound paper insulation is used. A loosely insulated cable is also liable to injury because bending will flatten the cable, compress the insulation in one direction and flare it out in the other, causing voids or air spaces between the layers, which under the influence of high voltages will cause electric discharges, heating, ozone and consequent chemical action. Mechanical injury after installation may be due to settlement of the conduit line, to careless stepping upon the cable, to vibration setting up crystallization of the lead sheath, etc.

Exposure to alternate heat and cold leads to expansion and contraction which may introduce mechanical stresses into the insulation and thereby injure it. Excessive heat (120 to 150° F.), such as would result from direct exposure to summer sunlight, is likely to dangerously reduce the insulating qualities of varnished cambric and to a less extent that of oiled paper. A higher degree of heat (200° F.), such as would occur in the proximity of steam pipes, has the effect of making rubber insulation become brittle.

Chemical deterioration may be due to electrolysis (q. v.) or merely to ordinary chemical reaction between the sheath and the material in contact with it, the former cause being the more common.

Burn-outs Due to Static Discharges. — Failure of high-tension rubber-insulated conductors is sometimes due to electrostatic discharges from the charged conductor to its supports, the irregular potential gradient giving rise to local static discharges with consequent formation of ozone and oxidation of the rubber.

Burn-outs Due to Imperfect Manufacture or Splicing. — Insulated conductors also burn out because of defects of manufacture or of splicing. The former class of defects is happily rare, but dirt, moisture and jagged edges of metal are frequently responsible for the failure of joints, especially on high-tension cables.

Transient High Voltages and Currents. — "Surges," i.e., transient high voltages and heavy currents, are sometimes responsible for very serious cable failures, such as described in *Trans. A.I.E.E., Vol. 24, p. 297.* To determine the origin and cause of high-voltage disturbances, so as to be able to guard against their recurrence, the most important thing seems to be to very carefully observe and record all the details of the phenomena, even those which appear unessential. The existence of static (i.e., high-voltage discharges of small currents) on switchboards, lines, etc., and the existence of voltages and currents different from those which may be expected require special attention. Either of these is sufficient to raise the suspicion of some dangerous fault in the system or some dangerous arrangement of apparatus, which requires consideration. The severity of the phenomena depends almost entirely upon the power momentarily available in the system and very rapidly increases with the size of the generating stations (*C. P. Steinmetz*).

After considering the likelihood and severity of such potential disturbances, P. Junkersfeld and E. Schweitzer reached the following conclusions with respect to their influence upon the use of high-tension cables.

1. Where local and commercial conditions justify, pressures as high as 25,000 volts can be satisfactorily used even for systems aggregating as much as a hundred miles of cable. No single line of such a system would be much longer than twenty miles. If higher voltages are needed to meet operating requirements and can be justified commercially, special construction will be necessary to overcome limitations in paper, rubber or varnished-cambric insulation, and also in the standard forms of underground conduit or subways used in this country.
2. On comparatively short lengths, underground or under water, as a part of a long overhead transmission line, cables operating at 40,000 volts can be used.
3. Potential rises of 50 per cent and 100 per cent are not uncommon in large underground cable systems, although this fact may not always be manifest, due to the high factor of safety in the insulation.

Occurrence of Cable Burn-outs. — The following table gives the number of cable burn-outs on some important cable systems. In each case the cables were three-conductor cables and were operated at 25 cycles.

REPAIRS. — The principal work of repairing cables consists in replacing lengths, for details of which see above under *Installation*. Sometimes a burn-out in a splicing chamber merely necessitates the insertion of a short length of cable, in which case two splices are necessary.

Lead-sheathed cable, which has been removed from ducts after having been in service, may often be releaded and made practically as good as new. The sheath is first stripped from the cable by passing the latter continuously over

CABLE BURN-OUTS

	New York Edison Co.	Commonwealth Edison Co.		Cataract Power and Conduit Co.	New York Inter-borough System
Period covered	1898–1907	1903–1908	1907–1908	1899–1906	2 years
Miles of cable	3,200	51.261	11	12	330
Line voltage	6600	9,000	20,000	11,000	11,000
Neutral, grounded or ungrounded	Ungrounded	Grounded	Grounded	....	Grounded
Kind of insulation	Paper	....	....	Rubber	Paper
Thickness of insulation:					
Between conductors	10/32 in.	....	....	....	....
Conductor and sheath	10/32 in.	....	....	....	....
Burn-outs due to:					
Mechanical injuries	38	25	1	2	....
Faults in or at splices	18	2	1	1	....
Faults in bends	4	2	....	....	....
Faults in run of cable	6	15	2	1	....
Absence of end bells	....	....	....	2	....
Total	66	44	4	6	16
Burn-outs manifested by:					
Opening circuit breakers	32	....	....	....	....
Insulation test	14	....	....	....	....
Reported by line inspectors	20	....	....	....	....

a pair of wheels between which a stationary knife is set, as shown in Fig. 17. The knife cuts nearly through the sheath, which is then pulled off by means of special tools. The core is re-saturated and a new sheath put on in the usual way.

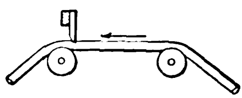


Fig. 17.

COST.—The cost of cables depends upon the price of their constituent metal, insulation, protection, sheathing, etc., and upon the degree of congestion in the factories. As rubber, copper and aluminum vary greatly in price from time to time, it is useless to give any specific cable costs, but the relative cost of different size cables of the same type and for the same voltage usually follows a curve such as shown in Fig. 18; the actual costs given in this curve refer to double-braided 30% Para rubber insulated cables and are approximate only. The cost per foot C, except for small conductors, practically follows the equation $C = A + Bx$ where A and B are constants depending upon the type of cable and the cost of materials and x is the cross-section of the conductor in circular mils. A curve of the same shape is applicable to multiple-conductor cables.

Life.—The life of cables depends so much upon the type, excellence of manufacture and conditions of service, that specific figures are of little use. An

average life of 20 years is suggested by A. W. Welch, provided the cables are not disturbed and are properly protected from electrolysis. Vernier says from 20 to 30 years. Fernie says that low-tension rubber-insulated cables in wet ducts last only 7 or 8 years. (*See article on Depreciation.*)

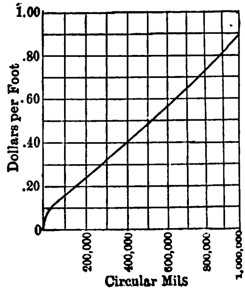


Fig. 18. Typical Cost Curve

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[W. A. Del Mar.]

WIRES, RESISTANCE. *(See also Electromagnet Windings; Resistance and Conductance, Electric; Rheostats and Resistors; Wires and Cables, Bare; Wires and Cables, Insulated.)* Metals and alloys having high specific resistance or low temperature coefficient of resistance are largely used for resistors. Some of the principal metals used for this purpose are listed below. They can usually be obtained in the form of wires, ribbon or sheets. The data given for Nichrome, Climax, Advance, Therlo, Yankee Silver, Ferro nickel, Monel Metal and Nickel were supplied by Mr. L. O. Hart of the Driver Harris Wire Co. The data on Krupp metal were supplied by T. Prosser and Son, those on Calido and Ideal materials were supplied by the Electrical Alloys Co., and those on Excello, Ia Ia, and Superior resistance metals were taken from the 1914 catalogue of Hermann, Boker & Co.

Advance. — This material is a copper-nickel alloy, containing no zinc. It is uniform in its composition and constant in its resistance under all conditions of service. It is especially recommended for measuring instruments and apparatus in which the wire is subjected to repeated heating and cooling.

Calido. — This is a high percentage nickel chromium alloy containing a small percentage of iron. The melting point is about 1550° C. It is recommended for electrically heated devices.

Climax. — This is a high resistance nickel steel alloy. It is especially well suited for use in rheostats. It is one of the cheapest resistance metals.

Excello is adapted for use in electric heating devices.

Ferro-nickel. — This alloy has a high current-carrying capacity, on account of its low specific resistance. As it will rust, it can only be used where it is not attacked by moisture.

German Silver.— This is an alloy of copper, nickel and zinc. The "grade" of the wire designates the percentage of nickel. The 18 per cent grade is the most common. The resistance of any particular grade depends upon the degree of annealing; hard wire is slightly higher in resistance than soft. German silver was for many years the only resistance alloy obtainable, but it is now being generally displaced by materials of the same specific resistance but of superior qualities.

Ia Ia is recommended for use in instruments and electrical devices where a low temperature coefficient is desired.

Ideal. —This is an alloy of nickel and copper, and contains no zinc. The manufacturers state that its temperature coefficient is "nil." It may be used at an incipient red heat of 520° C. It is adapted for resistors and measuring instruments.

Krupp Metal. — This is a special grade of nickel-steel adapted for resistors.

Manganin. — This is a material developed by the Reichsanstalt, for use in instruments and standards. The alloy which was shown to be the best for ordinary purposes is one containing 85 per cent of copper, 12 per cent of manganese and 3 per cent of nickel.

Monel Metal. — Monel metal contains approximately three parts nickel to one of copper. In smelting and refining the ore from which monel metal is made, the nickel and copper are not separated, and, therefore, appear in the finished alloy in the same relative proportions. The treatment of this ore consists merely in eliminating the impurities, excepting a small percentage of reduced iron. As a result the metal is tough, strong, as non-corrosive as pure nickel, and is the same in appearance; whereas nickel, as a pure metal, is relatively expensive, owing to the difficulty of isolating it. This alloy is produced at a cost which permits favorable competition with German silver, etc. The

TABLE I. — PROPERTIES OF RESISTANCE METALS

Material	Maximum working temp., °C.	Microhms per $cm.^3$ at 20° C. ρ	Temperature coefficient per ° C.: α_{20}	Specific gravity: δ	Tensile strength, lb. per sq. in.	Linear expansion coefficient per° C.
Advance (*a*)	480	48.8	0.000018	8.9	120,000	14.4×10^{-6}
Calido	1100	100	0.00034	8.2	....	....
Climax	540	87.2	0.00054	8.14	150,000	17.1×10^{-6}
Excello	(*c*)	91.6	0.00016	8.9	95,000	....
Ferro-nickel	340	28.2	0.00207	7.8	175,000	....
German silver (18%) (*a*) (*b*)	260	33.3	0.00031	8.5	....	17.3×10^{-6}
Ia Ia, soft	......	47.1	0.000005	8.92	....	....
Ia Ia, hard drawn	......	50.2	−0.000011	8.92	....	....
Ideal (*a*)	520	49.2	0.0000±	8.9	....	....
Krupp metal	600	85.1	0.00069	8.1	85,000	....
Manganin (*a*)	100	41.4 / 73.8	0.000011 / 0.000039	8.9	150,000	....
Monel metal	480	42.6	0.00198	8.9	160,000	13.8×10^{-6}
Nichrome	900	99.6	0.00044	8.15	150,000	16.4×10^{-6}
Nichrome II	1100	109.5	0.00016	8.02	150,000	....
Nickel	540	10.7	0.0037	8.9	120,000	....
Superior	550	87.2	0.00081	8.04	....	....
Therlo (*a*)	100	46.7	0.000006	8.15	....	19.4×10^{-6}
Yankee silver	480	33	0.000154	8.6	....	15.9×10^{-6}

(*a*) **Thermo-electric Power** of these metals with copper, in microvolts per ° C. is approximately: Advance, 40; German silver, 20 to 30; Ideal, 45 (M.I.T. tests); Manganin, 1 to 2; Nickel, 19.5; Therlo, 0.3 (*see also Pyrometers, p. 1153*).

(*b*) 30 per cent German silver has substantially the same properties as Advance metal.

(*c*) The melting point of Excello is 1500° C.

resistance varies somewhat in different lots, and according to temper. The variation is, however, no greater than that of 18 per cent German silver.

Nichrome. — This alloy is practically *non-corrosive*, has an extremely high melting point (about 1550° C.) and is far superior to nickel in its ability to withstand high temperatures. It is especially recommended for use in electrically heated appliances and resistance elements generally where extreme conditions are encountered.

Nichrome II. — This alloy is strongly resistant to oxidation. It has been especially developed for use in carbon combustion furnaces, and other laboratory furnaces where the more extreme temperatures are to be met.

Nickel. — Due to its high temperature coefficient nickel is very efficient for use in resistance thermometers and owing to its non-corrosive qualities it may be employed for rheostats where acid fumes are to be met with.

Superior is recommended for use in rheostats, arc lamp resistances, etc.

Therlo. — An alloy of copper, manganese and aluminum for work where low thermo-electric effect against copper is demanded. Compared with manganin, this alloy gives a higher specific resistance, does not oxidize so fast, and is more stable in its electrical and mechanical behavior. This material is especially suitable for shunts. Temperature coefficient is + 0.0000031 per 1° F.

Yankee Silver. — This is a new alloy with most of the qualities of "18 per cent German Silver." It will withstand repeated heating and cooling, and often gives satisfactory service where German silver fails.

TABLE II. — RESISTANCE, WEIGHT AND CURRENT-CARRYING CAPACITY OF WIRES

$$\text{Resistance, ohms per foot at } 20^\circ \text{ C.} = \frac{K\rho}{1000},$$

where ρ is the resistivity of the metal in microhms per centimeter cube, taken from Table I, and K is given in the table below.

$$\text{Weight, pounds per foot} = \frac{H\delta}{1000},$$

where δ is the specific gravity of the metal, taken from Table I, and H is given in the table below.

$$\text{Current for given temperature rise, amperes} = \frac{10\, I_0}{\sqrt{\rho}},$$

where ρ is the resistivity of the metal in ohms per centimeter cube, taken from Table I, and I_0 is given in the table below. This formula is approximate only. For insulated wire wound in coils of several layers see article on *Electromagnet Windings*.

A. W. G. or B. & S. Gage	Diameter, inches	Resistance factor (a) K	Weight factor (b) H	Current-carrying Cap. I_0 for $\rho = 100$ (c)			
				100° F. rise	200° F. rise	500° F. rise	1000° F. rise
6	0.162	0.229	8.91	14.0	19.6	31.5	45.5
8	0.128	0.364	5.62	11.5	15.9	25.4	37.5
10	0.102	0.579	3.51	8.95	12.2	19.4	29.0
12	0.0808	0.921	2.22	7.0	9.35	15.3	22.6
14	0.0641	1.46	1.40	5.35	7.25	11.8	17.5
16	0.0508	2.33	0.880	4.1	5.45	8.9	13.4
18	0.0403	3.70	0.553	3.1	4.2	6.75	10.1
20	0.0320	5.89	0.348	2.38	3.15	5.15	7.75
22	0.0254	9.36	0.219	1.75	2.35	3.85	5.85
24	0.0201	14.9	0.138	1.4	1.85	3.03	4.55
26	0.0159	23.7	0.0865	1.09	1.46	2.35	3.55
28	0.0126	37.6	0.0544	0.84	1.08	1.81	2.75
30	0.01003	59.9	0.0342	0.67	0.87	1.41	2.15
32	0.00795	95.2	0.0215	0.51	0.68	1.08	1.65
34	0.00631	151	0.0135	0.39	0.50	0.84	1.25
36	0.00500	241	0.00851	0.30	0.39	0.63	0.98
38	0.00396	383	0.00537	......	....	....	....
40	0.00314	608	0.00337	......	....	....	....

(a) The factor K is the resistance per 1000 feet for a resistivity of 1 microhm per cm.^{3}
(b) The factor H is the weight per 1000 feet for a specific gravity of 1.
(c) Amperes for a rise of temperature above 75° F. equal to the temperatures stated at head of column, for a single-layer spiral, without insulation and freely exposed to air, the diameter of the coil being 10 times the radius of the wire. The currents given in the table are those given by the Electric Alloys Co., for Calido wire, for which $\rho = 100$.

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[W. A. Del Mar.]

WIRING OF BUILDINGS FOR LIGHT AND POWER. — (*See also Distribution of Electric Energy; Distribution Lines; Transmission Lines; Wires and Cables; Wiring of Buildings for Miscellaneous Devices.*)

The following is a brief table of contents of this article:

General Requirements. — Wires and fittings designed to conduct electricity in a building should be selected as to size and insulation and installed in such a manner that: (1) the attending fire risk and the possibility of an electric shock to the inhabitants shall be a minimum; (2) the electric power efficiency of the system shall be reasonable; (3) the voltage at the receiver shall approximate the rated voltage of the receivers and shall remain sensibly constant; (4) the mechanical arrangement of the system shall be simple and convenient for inspection and use; (5) the conductors shall be mechanically protected from external injury; (6) the service shall not be interrupted under normal load; (7) the cost of the materials, labor of installation and replacement due to depreciation shall not be excessive, and (8) the entire wiring system shall conform to the rules and regulations of any authority having jurisdiction over the building in question.

SYSTEMS OF WIRING. — For direct-current and single-phase distribution the two-wire system (Fig. 1) and the three-wire system (Fig. 2) are

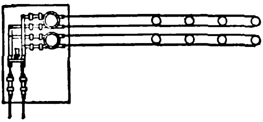


Fig. 1.

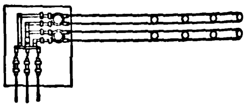


Fig. 2.

employed. For two-phase distribution either three of four wires are used. For three-phase distribution three wires are usually employed, although a fourth or neutral wire is sometimes installed. Fig. 3 shows typical methods of controlling a group of lamps from two or more switches.

Direct-current and Single-phase Distribution. — In direct-current or single-phase alternating-current circuits, the two-wire or the three-wire system is used. The two-wire system is used for the greater part of interior wiring, the three-wire system being used chiefly for feeders and mains. The three-wire system possesses the advantage over the two-wire system that for the

same power transmitted at the same efficiency to receivers of the same voltage, the three-wire system requires less weight of conductor. The neutral wire of a three-wire system is usually of the same size as either outside wire, the saving in copper over the two-wire system then being 5/8 or 62.5 per cent (*see Distribution of Electric Energy*). In some cases, buildings are wired with the three-wire system in which the neutral wire is made twice the size of either outside wire so that if necessary, the system may be operated either as a two-wire or a three-wire system; in the former case the two outside wires are connected in parallel. Power may then be supplied to the building from a local two-wire source of supply (isolated plant in the building) or from an emergency three-wire street service.

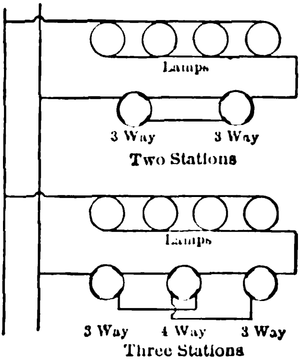


Fig. 3.

In this case wires of such size would be used as would give normal efficiency and regulation when used as a two-wire system; the loss when used as a three-wire system would then be only half as great.

The first cost of a three-wire system may not be less than that of a two-wire system because of the increased cost of the fittings, insulation and the labor of installation. The three-wire system is not as simple as the two-wire system, and is subject to more disturbances unless the load is kept balanced. Some electric power companies limit the power which they will supply to a two-wire system and in such cases in new installations buildings taking power above the two-wire limit must be wired with the three-wire system. The two-wire system is used for either lighting or power loads while the three-wire system is used for either lighting or mixed lighting and power loads, motors in the latter case being connected between the outside wires.

Two-phase and Three-phase Distribution. — For distributing two-phase alternating currents, either three or four wires are employed. A four-wire two-phase system may be treated as two separate two-wire systems, which cannot in any case be connected in parallel. A single wire 41 per cent larger than either of the wires it displaces may be substituted for any two of the wires of the four-wire two-phase system thus making a three-wire two-phase system. Either three or four wires may be used for two-phase lighting or power loads.

Three wires are usually employed on three-phase alternating-current circuits supplying power to lighting or power loads. If the neutral is accessible, a neutral wire may also be used, making a four-wire three-phase system for lighting loads, the lamps being connected between any outside wire and the neutral wire.

STANDARD VOLTAGES FOR LIGHTING AND POWER LOADS. — On all standard lighting loads the voltage across the lamps ranges from 100 to 125 volts in accordance with the rating of the lamps. In a few cases, lamps rated between 200 and 250 volts are used with the corresponding voltages. The voltage between the service wires of a constant-current series-arc or series-incandescent lamp system must not be greater than 3500 volts, the maximum voltage for inside work allowed by the National Electrical Code (see p. 1951.) Constant-current systems are used chiefly in lighting large areas as in mills, factories, armories, etc. On power loads standard voltages of approximately 110, 220 or 550 volts are used depending on the magnitude of the load supplied.

STANDARD FREQUENCIES ON ALTERNATING-CURRENT CIRCUITS. — Alternating currents are usually supplied to lighting loads at a frequency of 60 cycles and to power loads at 25 cycles. In some cases, a frequency of 40 cycles is used when the lighting and power loads are of about the same magnitude.

PRELIMINARY LAYOUT OF THE WIRING SYSTEM. — If power is to be supplied to the building from some outside source, the service entrance must first be located from plans or by an inspection of the building itself. In most cases, service entrances are made in the basement, although first-floor entrances may be substituted when basement entrances are impracticable. In overhead-service connections, the service wires are usually run in conduit on the outside of the building from the point where the wires are attached to the building to the service panel-board. In underground-service connections, the service wires are run in conduit from the street mains through the basement walls. In the latter case, the conduit should be tightly closed at the outlet to prevent gases from entering the building. Fig. 4 shows two types of service connections.

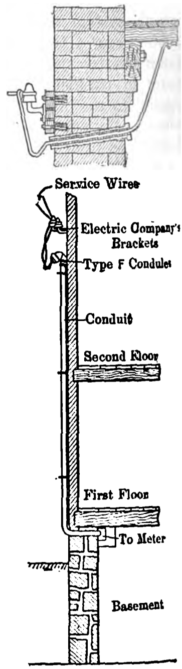


Fig. 4.

Location of Panel-Board. — The service panel-board should be placed securely in an accessible location and should contain space for the service cut-outs, service switch and meter. When power is supplied from within the building, the switchboard of the power plant must be located as a starting point in the design of the feeder system.

Location of Outlets, Switches, etc. — Outlets, control switches and cut-out cabinets should then be located throughout the building. Great care must be exercised in locating outlets and control switches in such positions as to accommodate the receiving devices planned for the building. Cut-out cabinets, which constitute the local distributing centers, should be located as near as possible to the dependent receiving devices and should at the same time be easily connected by feeders to the main switchboard. Fig. 5 shows a riser diagram and Fig. 6 a diagram of the wiring for one floor of an office building.

Number of Feeders. — Having located the outlets, control switches, distribution centers and the main switchboard, a feeder system must be planned to suit the conditions under which the system is to operate. The number of feeder sets required depends upon: (1) the power taken by the receiving devices; (2) the degree of control desired at the main switchboard; (3) the number of receiving devices which may rely upon one set of feeders; (4) the desired uniformity of voltage at the receiving devices, and (5) the character of the receiving devices. If the total current supplied to the building is roughly calculated and is found to exceed 650 amperes (the allowable carrying capacity of a 1,000,000-circular-mil rubber-insulated conductor), more than one set of feeders should be used, since it is impracticable to install feeders larger than one inch in diam-

(*Text continued on p. 1943.*)

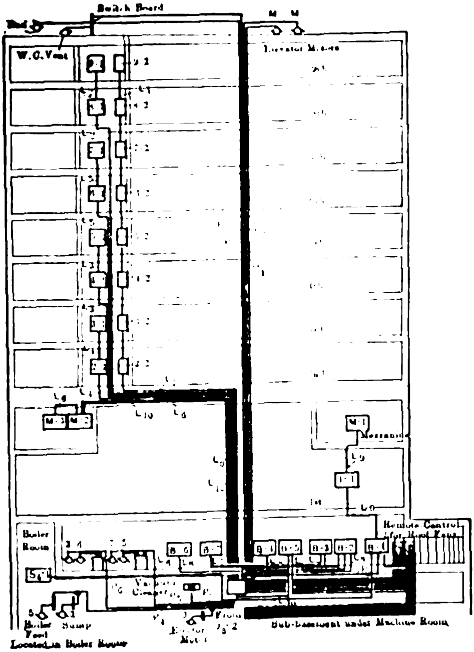

Switch Board
W.C. Vent
Boiler Room
Remote Control
Sub-basement under Machine Room
Located in Boiler House

Fig. 5.

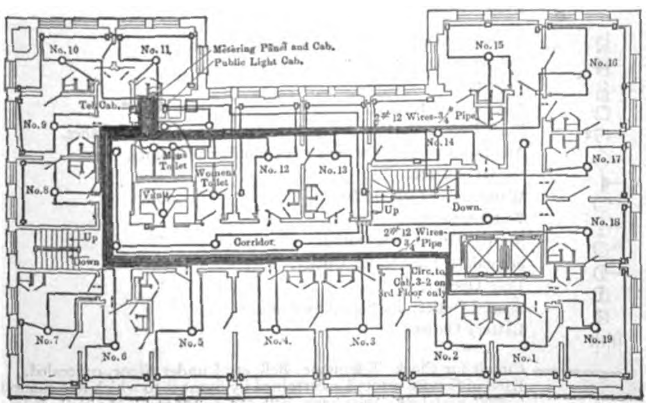
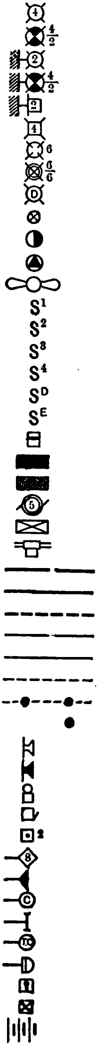

No. 10
No. 11
Metering Panel and Cab.
Public Light Cab.
Tel. Cab.
No. 9
No. 8
No. 15
No. 16
No. 17
No. 18
No. 14
No. 12
No. 13
Women's Toilet
Corridor
Up
Down
Circ. to Cab. 3-2 on 3rd Floor only
No. 7
No. 6
No. 5
No. 4.
No. 3
No. 2
No. 1
No. 19

Fig. 6.

STANDARD WIRING SYMBOLS

(Adopted by the National Electrical Contractors' Association and the American Institute of Architects.)



Ceiling Outlet; Electric only.*
Ceiling Outlet; Combination.† If gas only
Bracket Outlet; Electric only.*
Bracket Outlet; Combination.† If gas only
Wall or Baseboard Receptacle Outlet.*
Floor Outlet.*
Outlet for Outdoor Standard or Pedestal; Electric only.*
Outlet for Outdoor Standard or Pedestal; Combination.†
Drop Cord Outlet.
One Light Outlet, for Lamp Receptacle.
Arc Lamp Outlet.
Special Outlet, for Lighting, Heating and Power Current, as described.
Ceiling Fan Outlet.
S^1 S. P. Switch Outlet.
S^2 D. P. Switch Outlet.
S^3 3-Way Switch Outlet.
S^4 4-Way Switch Outlet.
S^D Automatic Door Switch Outlet.
S^E Electrolier Switch Outlet.

Show as many Symbols as there are Switches. Or in case of a very large group of Switches, indicate number of Switches by a Roman numeral, thus; S^1 XII; meaning 12 Single Pole Switches.
Describe Type of Switch in Specifications, that is, Flush or Surface, Push Button or Snap.

Meter Outlet.
Distribution Panel.
Junction or Pull Box.
Motor Outlet; Numeral in center indicates Horse-Power.
Motor Control Outlet.
Transformer.
Main or Feeder concealed under Floor.
Main or Feeder concealed under Floor above.
Main or Feeder run exposed.
Branch Circuit concealed under Floor.
Branch Circuit concealed under Floor above.
Branch Circuit run exposed.
Pole Line.
Riser.

Heights of Center-of-wall Outlets (unless otherwise specified):	
Living Rooms	5 ft. 6 in.
Chambers	5 ft. 0 in.
Offices	6 ft. 0 in.
Corridors	6 ft. 3 in.
Height of Switches (unless otherwise specified)	4 ft. 0 in.

Telephone Outlet; Private Service.
Telephone Outlet; Public Service.
Bell Outlet.
Buzzer Outlet.
Push Button Outlet; Numeral indicates number of Pushes.
Annunciator; Numeral indicates number of Points.
Speaking Tube.
Watchman Clock Outlet.
Watchman Station Outlet.
Master Time Clock Outlet.
Secondary Time Clock Outlet.
Door Opener.
Special Outlet; for Signal Systems.
Battery Outlet.
Circuit for Clock, Telephone, Bell, etc.§ under Floor, concealed.
Kind of Service wanted ascertained by Symbol to which line connects.
Circuit for Clock, Telephone, Bell, etc.,§ under Floor above, concealed.

* Numeral indicates number of standard 16 C. P. incandescent lamps.
† Upper numeral indicates number of standard 16 C. P. incandescent lamps, lower numeral number of gas burners, e.g., $\frac{4}{2}$ indicates 4 incandescent lamps and 2 gas burners.
§ Kind of service wanted ascertained by symbol to which line connects.

eter. In fact, in most cases, it is not considered good practice to install feeders larger than No. 0000 B. & S. gauge (0.46 inch in diameter). Unless controlled by remote control switches, separate feeders must be installed from the switchboard to receiving devices which are controlled at the main switchboard.

If a single set of feeders is used on a lighting load, the possible extinction of all the lights in a building by an open circuit in the main feeders may cause a panic; on a power load an interruption of all the machinery for any length of time may cause a considerable loss of money. To avoid such a discontinuance of the service, it is customary to connect separate feeders to sectionalized parts of the load, so that the entire load may not be interrupted at one time.

The voltage at receiving devices may be made more uniform by the use of separate feeders than by the use of one set of feeders, except when a single set of feeders may be run to a distributing panel to which branch feeders of nearly equal length are connected. When the power taken by certain receiving devices varies extensively and rapidly, the voltage at all other devices connected to the same feeder will vary accordingly, and in such cases lamps must be connected to separate feeders to avoid undesirable flickering.

Wiring Diagram. — After deciding upon the number of feeders to be used in any installation, a diagram should be made showing the location and length of all feeders and branch circuits. When the wiring system is sectionalized, each section should be treated by itself as a complete system. Figs. 5 and 6 are typical wiring diagrams for an office building.

WIRING CALCULATIONS. — The proper size of wire for any feeder is determined by three factors: (1) mechanical strength; (2) current-carrying capacity, and (3) the allowable potential drop in the feeder.

Minimum Size of Wire. — The National Electrical Code specifies No. 14 B. & S. (A.W.G.) as the minimum allowable size for all classes of wiring, except that in fixture work and for pendant cords wires as small as No. 18 B. & S. may be used.

Current-carrying Capacity. — The following table, from the National Electrical Code, gives the allowable carrying capacity of copper wires and cables of 98 per cent conductivity, according to the standard adopted by the American Institute of Electrical Engineers, and must be followed in placing interior conductors. See also *Wires and Cables, Insulated.*

CURRENT-CARRYING CAPACITY OF COPPER WIRES

National Electrical Code Standard

Size of wire, B. & S. gage	Max. allowable amperes		Size of wire, cir. mils	Max. allowable amperes*	
	Rubber insulation	Other insulations		Rubber insulation	Other insulations
18	3	5	200,000	200	300
16	6	10	300,000	275	400
14	15	20	400,000	325	500
12	20	25	500,000	400	600
10	25	30	600,000	450	680
8	35	50	700,000	500	760
6	50	70	800,000	550	840
5	55	80	900,000	600	920
4	70	90	1,000,000	650	1000
3	80	100	1,100,000	690	1080
2	90	125	1,200,000	730	1150
1	100	150	1,300,000	770	1220
0	125	200	1,400,000	810	1290
00	150	225	1,500,000	850	1360
000	175	275	1,600,000	890	1430
0000	225	325	1,700,000	930	1490
....	....	....	1,800,000	970	1550
....	....	....	1,900,000	1010	1610
....	....	....	2,000,000	1050	1670

* For insulated aluminum wire the maximum allowable current is 84 per cent of that given in the table for the corresponding type of insulation.

Currents Taken by Various Receiving Devices. — The current taken by each receiving device if not stated in the specifications may be determined roughly from the following tables.

CURRENT TAKEN BY INCANDESCENT LAMPS AT 114 VOLTS

Candle power	Amperes per lamp		
	Carbon	Gem	Tungsten
16	0.5	0.35	....
20	....	....	0.22
32	1.0	0.7	....
48	....	....	0.53
50	1.4	1.1	....
80	....	....	0.88
100	2.7	2.2	....

CURRENT TAKEN BY ORDINARY ARC LAMPS

½-inch carbons

Type	Multiple*		Series	
	Volts	Amperes	Volts per lamp	Amperes
Direct current	110	5 to 6.5	70	5 to 6.6
	220	3		
Alternating current	110	6 to 8.5	70	6.6 to 7.5
	220	6		

* On multiple-arc lamp circuits, the conductors must be designed to carry 150 per cent of the normal current taken by the lamps.

CURRENT TAKEN BY DIRECT-CURRENT MOTORS*

Horse-power	Amperes at		
	110 volts	220 volts	500 volts
½	4.5	2.2	1
¾	6.8	3.4	1.5
1	9.0	4.5	2
1½	13.6	6.8	3
2	16.9	8.5	3.8
3	25.4	12.7	5.6
4	33.8	16.9	7.5
5	42.3	21.1	9.3
7½	56.5	32.2	12.4
10	75.3	37.6	16.6
15	113	56.5	24.9
20	150	75.3	33.1
25	188	94.1	41.6
30	226	113	49.7
40	301	150	66.3
50	376	188	82.8
60	452	226	99.4
70	527	263	116
80	602	301	132
90	678	339	149

* For single-phase a-c. motors divide current given by power factor of motor.

CURRENT TAKEN BY THREE-PHASE INDUCTION MOTORS

Power factor taken as 80 per cent

Horse-power	Amperes per wire at line voltages of		
	110 volts*	220 volts*	550 volts*
1	6	3	1.2
2	12	6	2.4
3	18	9	4
4	24	12	5
5	28	14	6
10	56	28	11
15	85	42	17
20	112	56	22
25	140	70	28
30	167	83	33
40	222	110	44
50	278	140	58
60	330	165	66
70	385	192	77
80	440	220	88
90	490	245	98
100	550	275	110
150	790	395	158
200	1050	525	210
250	1320	660	264
300	1580	790	316

* These are volts between wires; the corresponding volts to neutral are 63.5, 127 and 318, respectively.

CURRENT TAKEN BY HEATING DEVICES

Device	Amperes at	
	110 volts	220 volts
Flatirons, 3-pound	2.3	1.1
Flatirons, 6-pound	5.5	2.7
Water-heaters, pint	3.6	...
Water-heaters, quart	4.5	...
Water-heaters, 2-quart	9.1	4.5
Chafing dishes	5.2	2.6
Coffee percolaters, 3-pint	4.5	...
Toasters	5.5	...
Heating pads	0.5	...
Cigar lighters	0.7	...
Glue pots, 2-quart	0.8	0.4
Luminous radiators	6.8 to 13.6	3.4 to 6.8

Potential Drop. — In most installations the potential drop *per conductor* between service-entrance or switchboard and the farthest receiver is taken as approximately 3 per cent of the *voltage to neutral* at the service-entrance or switchboard. This is equivalent to a *total drop* (both wires) of 3 per cent of the *voltage between* wires in the case of a two-wire direct-current or single-phase system. In three-wire systems the two outside wires are to be regarded as the feeders.

In direct-current or single-phase alternating-current systems the maximum potential drop *per conductor* will then be $0.015 \times 110 = 1.65$ volts for a 110-volt system, 3.3 volts for a 220-volt system, etc. In three-phase systems the potential drop *per conductor* will be $0.03 \times \frac{110}{\sqrt{3}} = 1.91$ volts for a 110-volt system, 3.82 volts for a 220-volt system, etc. In feeders and branch circuits the drop in the feeders is usually made two-thirds and the drop in the branch circuits one-third of the total drop.

Note that in the case of a lamp load the allowable drop of 3 per cent is based on the current corresponding to the *total connected load*, i.e., this is the maximum drop when *all* lamps are burning. In ordinary buildings the actual maximum load is seldom more than one-third the connected load; consequently when the wiring is designed on this basis the voltage at the lamps will seldom be more than 1 per cent lower than the voltage at the service connection.

Calculation of Size of Wire. — Let

I = current per conductor in amperes,
v = allowable potential drop *per conductor* in volts,
l = length of the conductor in feet,*

then the required cross-section of a copper wire in circular mils is

$$A = \frac{KlI}{v}, \tag{1}$$

where K is a factor depending upon the specific resistance of the wire, the size and spacing of the wires, the frequency and the power factor of the receiver. The factor K for alternating-current circuits is therefore not a constant but for preliminary calculations its value for the sizes of wires and spacings (1 to 6 inches) ordinarily used for interior wiring is approximately as given in the following table. The alternating-current values apply to single-phase, 2-phase 4-wire

System	Power factor	Values of K for copper
Direct current.............	....	11*
Alternating current.........	....	....
Lighting load only.......	1.00	11
Lighting and power loads	0.95	12
Lighting and power loads.	0.90	13.5
Power loads............	0.85	15
Power loads............	0.80	17

* This value is practically exact, the others are approximate only.

and 3-phase 3-wire systems at any frequency from 25 to 60 cycles per second. For a direct-current or single-phase 3-wire system proceed as for a 2-wire system, neglecting the presence of the neutral wire; the neutral wire should then be made the same size as each outside wire as thus calculated.

* Note that l is the length of *each conductor;* the total length of wire for a two-wire line is $2l$, for a three-wire line $3l$, etc.

Commercial sizes of wire differ successively by about 25 per cent in the larger sizes and 60 per cent in the smaller sizes; the odd-numbered sizes smaller than No. 1 are not generally manufactured, although No. 3, 5 and 9 are sometimes made. From the table below * select the size of wire corresponding to the calculated area. Unless the calculated area in circular mils agrees very closely with the area of one of the conductors given in the table, the next larger size of wire should be selected.

600–VOLT RUBBER–INSULATED COPPER WIRE
(Single Braid, N.E.C. Standard; Conductivity of Copper, 98 per cent)

Size, B. & S. gage	Cross-section of copper, cir. mils	Over-all diameter, inches	Ohms per 1000 ft. at 68°F.	Pounds per 1000 ft. of insulated wire	Reactance per 1000 ft. of each wire			
					Wires in contact†		6 inches between wires‡	
					25 cycles	60 cycles	25 cycles	60 cycles
		stranded	stranded	stra'd'd				
	1,000,000	1.531	0.0110	3721	0.0130	0.0313	0.0283	0.0679
	900,000	1.469	0.0122	3390	0.0132	0.0317	0.0288	0.0692
	800,000	1.406	0.0138	3051	0.0134	0.0320	0.0292	0.0702
	700,000	1.344	0.0157	2707	0.0136	0.0328	0.0298	0.0717
	600,000	1.250	0.0184	2354	0.0138	0.0332	0.0306	0.0732
	500,000	1.125	0.0220	1958	0.0135	0.0324	0.0312	0.0749
	400,000	1.063	0.0275	1617	0.0141	0.0340	0.0322	0.0773
	300,000	0.969	0.0367	1283	0.0144	0.0347	0.0333	0.0800
	250,000	0.906	0.0441	1095	0.0148	0.0354	0.0341	0.0820
0000	212,000	0.813	0.0520	869	0.0144	0.0347	0.0348	0.0835
000	168,000	0.752	0.0656	716	0.0149	0.0358	0.0358	0.0860
00	133,000	0.688	0.0827	595	0.0151	0.0362	0.0368	0.0883
0	106,000	0.657	0.104	493	0.0159	0.0381	0.0378	0.0908
1	83,700	0.626	0.133	410	0.0165	0.0396	0.0390	0.0936
		solid	solid	solid				
1	83,700	0.594	0.126	390	0.0159	0.0381	0.0388	0.0933
2	66,400	0.532	0.159	298	0.0160	0.0385	0.0398	0.0958
3	52,600	0.501	0.201	254	0.0166	0.0400	0.0410	0.0986
4	41,700	0.454	0.253	210	0.0168	0.0403	0.0421	0.1011
5	33,100	0.438	0.320	180	0.0176	0.0422	0.0432	0.1037
6	26,300	0.407	0.403	150	0.0179	0.0430	0.0442	0.1061
8	16,500	0.329	0.641	99	0.0181	0.0434	0.0464	0.1112
10	10,400	0.297	1.02	74	0.0193	0.0464	0.0495	0.1165
12	6,530	0.282	1.62	58	0.0210	0.0505	0.0507	0.1218
14	4,110	0.251	2.58	47	0.0221	0.0532	0.0529	0.1270
16	2,580	0.219	4.10	40	0.0231	0.0554	0.0550	0.1320
18	1,620	0.204	6.51	33	0.0245	0.0588	0.0572	0.1372

* More extended wire tables will be found in the articles on *Wires and Cables*.

† Two insulated wires side by side, the insulation of the two wires in contact.

‡ Measured from insulation to insulation, equals distance between centers minus twice the thickness of insulation on either wire.

Check on Calculation of Size of Wire. — Calculation of Actual Potential Drop. — (*See also Alternating Currents.*) From the table, corresponding to the size of wire selected, take

r = the resistance per 1000 feet of conductor,
x = the reactance per 1000 feet of conductor,

and put

l = length of each conductor in feet,
I = amperes per conductor,
V = volts to neutral at receiver,*
$\cos\phi$ = power factor of receiver,

$$R = \frac{rl}{1000} = \text{total resistance per conductor,}$$

$$X = \frac{xl}{1000} = \text{total reactance per conductor.}$$

Then the actual volts drop *per conductor* is

$$v = \sqrt{(V\cos\phi + RI)^2 + (V\sin\phi + XI)^2} - V. \tag{2}$$

To a very close approximation in all ordinary cases this is equal to

$$v = RI\cos\phi\left(1 + \frac{X}{R}\tan\phi\right). \tag{3}$$

If the drop as calculated does not check within a reasonable value with the drop assumed in calculating the size of the wire, select another size of wire and recalculate, etc. This refinement will be found necessary, only in the case of low power factors and when the size of wire as calculated is either exceptionally large or exceptionally small, in which case the values given for the factor K in the formula for A, equation (1), may be in error by a large amount (in limiting cases 50 per cent or more).

In addition to making sure that the drop will be within a reasonable amount, one should also note whether the conductor selected is large enough mechanically and has the proper current-carrying capacity (*see above*), remembering also that the National Electrical Code requires that conductors through which power is supplied to a motor shall have a current-carrying capacity equal to 125 per cent of the full-load current taken by the motor.

Example of Calculation for a D-C. or Single-phase System. — Incandescent lamps taking 0.5 ampere each are supplied with power from a 115-volt 60-cycle service through a set of feeders 200 feet in length which terminate at a cut-out cabinet as shown in Fig. 7. The lamps are grouped in sets of 10 lamps each (only one set is shown) and each set is connected to the cut-out cabinet by branch circuit conductors averaging 25 feet in length. Assuming a total potential drop of 3 volts, the drop per conductor will be 1.5 volts. The drop per conductor in the branch circuit should then be one-third of 1.5 or 0.5 volt and the drop per conductor in the feeder circuit should be two-thirds of 1.5 or

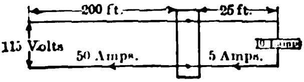


Fig. 7.

* For d-c. or a-c. single-phase or a-c. 2-phase, 2-wire systems $V = \frac{1}{2} \times$ (volts between wires); for a 3-phase system $V = \frac{1}{\sqrt{3}} \times$ (volts between wires); for a 2-phase 3-wire system $V = \frac{1}{2} \times$ (volts between either outside wire and middle wire). In calculating the normal drop in a single-phase 3-wire system pay no attention to the middle wire, i.e. assume a balanced load.

1 volt. The respective calculations of the areas of the conductors in the feeder and branch circuits may then be carried on independently. In the branch circuit, referring to equation (1), $I = 10 \times 0.5 = 5$, $l = 25$ and $v = 0.5$, whence for copper wire

$$A = \frac{11 \times 5 \times 25}{0.5} = 2750 \text{ circular mils.}$$

From the wire table it is found that the next larger commercial size is No. 14 B. & S., which has a resistance of 2.58 ohms per 1000 feet, and a reactance (assuming a 2.5-inch spacing) of 0.106 ohms per 1000 feet.

Referring to equation (3), $V = 57.5$, $\cos\phi = 1$, $\tan\phi = 0$, $R = 2.58 \times 25/1000 = 0.0645$, $X = 0.106 \times 25/1000 = 0.00265$, whence

$$v = 0.0645 \times 5 = 0.323.$$

Equation (2) gives the same value. This voltage is 35.4 per cent less than the assumed voltage of 0.5 volt, but since No. 13 wire is not a commercial size a closer realization of the assumed voltage is impracticable.

In the same manner, the required area of the conductors used in the feeder circuit is determined by substituting the following values in equation (1): $I = 100 \times 0.5 = 50$ amperes, $l = 200$ feet, and $v = 1$ volt. Hence

$$A = \frac{11 \times 50 \times 200}{1} = 110{,}000 \text{ circular mils.}$$

From the wire table it is found that a No. 00 B. & S. wire must be used.

The above calculations although made for a lighting load apply equally well to a motor load, except that in the final selection a wire must be chosen which will carry 125 per cent of the full-load current of the motor.

Example of Calculation for a Three-phase System. — Power is to be supplied by three feeders each 400 feet in length to a three-phase, 60-cycle, alternating-current motor (power factor 80 per cent), the voltage between any two line wires at the service entrance being 550 volts. At full load the current taken by the motor is 50 amperes. The voltage to neutral at the service entrance is $\frac{550}{\sqrt{3}}$ or 318 volts. The wires are to be spaced 6 inches apart.

Allowing a drop in each conductor of 3 per cent of the voltage to neutral, the drop will be 3 per cent of 318 or 10 volts approximately. The following values should then be substituted in equation (1): $K = 17$, $I = 50$ amperes, $l = 400$ feet and $v = 10$ volts, giving

$$A = \frac{17 \times 50 \times 400}{10} = 34{,}000 \text{ circular mils.}$$

From the wire table it is found that a No. 5 B. & S. wire might be used considering the potential drop alone but since the carrying capacity of the conductor must be 1.25×50 or 62.5 amperes, it would be necessary to use a No. 4 wire.

Adopting a No. 4 wire and referring to equation (3), $V = 318$, $\cos\phi = 0.8$, $\tan\phi = 0.75$, $R = 0.253 \times 400/1000 = 0.101$, $X = 0.101 \times 400/1000 = 0.040$, whence

$$v = 0.101 \times 50 \times 0.8 \left(1 + \frac{0.04 \times 0.75}{0.101}\right) = 5.3 \text{ volts,}$$

which is a little more than half the allowable drop of 10 volts. Equation (2) also gives 5.3 volts.

AUTHORITIES GOVERNING THE INSTALLATION OF WIRING IN BUILDINGS. — As noted at the beginning of this article all interior wiring must be installed in accordance with the regulations of the authorities having jurisdiction over the building in question. In general, these authorities are: (1) the fire underwriters; (2) the municipal authorities; and (3) the power company supplying the current.

National Electrical Code. — This code is a set of instructions published in odd years by the National Board of Fire Underwriters. A List of Electrical Fittings approved by the Underwriters' Laboratories, Inc. is published by the National Board in April and October of each year. In order that fire insurance on any building wired for electric light or power may be obtained from an insurance company, the wiring must be installed in accordance with the rules of the National Electrical Code or such modifications of it as are required by law in certain municipalities (*see Municipal Regulation, below*). Permission must also be obtained to use any fitting not included in the list of approved fittings.

Both the Code and the List of Approved Fittings may be obtained gratis by applying to the National Board of Fire Underwriters, 135 William St., New York City. In view of this fact, and the frequent revisions of the rules and list it is not deemed advisable to give them in detail here. The general requirements, however, are covered in the section below on *Methods of Installing Wiring.*

Municipal Inspection. — Many state legislatures have passed laws regulating the installation of electric wiring and empowering the appointment of municipal inspectors of wiring. In some states fines are imposed by legislative enactment upon those who violate the inspector's rules. Although free to decide upon the proper installation of electric wires, municipal inspectors have generally adopted the National Electrical Code as a standard with modifications suited to their desires. A copy of these modified rules can usually be obtained gratis from the municipal authorities.

Regulations of the Light and Power Companies. — The National Electrical Code is usually adopted by electric power companies with the addition of specific requirements pertaining to service connections, placing of meters, type of system, etc.

METHODS OF INSTALLING WIRING. — The National Electrical Code approves the use of the following methods of wiring: open work, moulding (wooden and metallic), concealed knob and tube work, rigid conduit, flexible conduit, armored cable and flexible tubing (in short runs). These methods are described in detail below, but first are given the more important general requirements regarding joints, protection, cut-outs, switches, etc., which apply to all classes of wiring.

General Requirements. — All joints must be mechanically and electrically secure without solder and unless made with some form of approved splicing device must be soldered and covered with an insulation equal to that on the conductor. Wires must be separated from contact with walls, floors, timbers or partitions through which they may pass by non-combustible, non-absorptive insulating tubes, such as glass or porcelain, except at outlets where flexible tubing is required. Unless inclosed in conduit or moulding, wires must be separated from any conducting material or from any other electric wire not more than two inches away by some continuous and firmly fixed non-conductor creating a permanent separation. In damp places wires must be separated from pipes by an air space and should be run over rather than under pipes upon which

moisture may gather. Wires must be protected from mechanical injury by running-boards or guard-strips on low ceilings and by wooden boxing or metal conduit on side walls. Protection on side walls must extend not less than five feet from the floor. No method other than open wiring may be used when the difference of potential between any two wires exceeds 550 volts.

Automatic Cut-outs (fuses or circuit breakers) must be placed in all service wires (i.e., the leading-in wires from the street circuit), and at every point where a change is made in the size of wire unless the cut-out in the larger wire will protect the smaller wire. The cut-out in the neutral of a three-wire system may be omitted if the neutral wire is grounded and is of the same size as the outside wires. No set of incandescent lamps requiring more than 660 watts, whether grouped on one fixture or on several fixtures must be dependent upon one cut-out. Cut-outs must in general be inclosed in an approved cabinet although circuit breakers and inclosed fuses may be placed in plain sight in dry places where there is no danger of igniting any combustible material.

Switches which disconnect *all* wires (i.e., switches having 2, 3 or more poles) must be placed in the service wires as near as possible to the point where they enter the building and in all circuits supplying current to motors, heating devices or lamps; except that in the case of motors of one-fourth horse-power or less in circuits where the voltage does not exceed 300 volts, and in circuits supplying not more than 660 watts of power to heating devices or lamps, single-pole switches may be used in two-wire systems or branches. Switches must always be placed in dry, accessible places and must be grouped as far as possible. When used in rooms where combustible flyings are liable to exist, switches must be placed in dust-tight cabinets and when flush switches are used, they must be inclosed in an approved steel box.

Open Wiring. — When the appearance of the wiring is not important, exposed surface wiring supported on cleats or insulators furnishes one of the safest and best methods of wiring. The wires must be insulated in dry places with rubber, slow-burning weatherproof or slow-burning insulation, in damp places with rubber insulation, and in locations where the wires are exposed to corrosive vapors with weatherproof or rubber insulation.

Voltage	Inches from surface to insulation on wire		Inches between wires* from insulation to insulation
	Dry	Damp	
0–300	0.5	1	2.5
301–550	1	1	4

* In the case of 3-wire d-c. or single-phase systems the neutral may be placed between the two outers, these latter being 2.5 inches apart.

Spacing of Wires and Supports. — Wires must be separated from each other and spaced from the surface wired over as given in the accompanying table.

Wires must be supported under ordinary conditions at least every 4.5 feet, except that in mill construction, wires of not less than No. 8 B. & S. gage, if separated about six inches, may be run from timber to timber and be supported at each timber only. Open wiring cannot be used in elevator shafts.

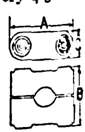


Fig. 8

Cleats. — Cleats are made in a variety of forms. The following are the dimensions and costs of cleats of the form shown in Fig. 8, all dimensions being in inches.

DIMENSIONS AND COSTS OF CLEATS

Size of wire, cir. mils or B. & S. gage	1-wire cleats				
	Dimensions, inches			Size of screw, inches	Cost* per 100
	A	*B*	*C*		
14-12	2	$1\frac{1}{4}$	$\frac{3}{4}$	3×8	$2.50
10- 8	2	$1\frac{5}{8}$	$\frac{3}{4}$	3×8	2.90
6- 3	$2\frac{1}{4}$	$1\frac{3}{4}$	1	3×10	3.15
2	$2\frac{1}{2}$	$1\frac{3}{4}$	$1\frac{1}{8}$	$3\frac{1}{2}$×12	3.40
1-0	$2\frac{1}{2}$	2	$1\frac{1}{8}$	$3\frac{1}{2}$×12	3.90
00	$2\frac{3}{4}$	$2\frac{1}{8}$	$1\frac{1}{4}$	$3\frac{1}{2}$×12	4.50
000	3	$2\frac{1}{4}$	$1\frac{1}{4}$	4×14	4.85
0000	3	$2\frac{3}{8}$	$1\frac{1}{4}$	4×14	6.50
225,000-500,000	$3\frac{1}{4}$	$2\frac{3}{4}$	$1\frac{1}{2}$	$4\frac{1}{2}$×16	7.70
600,000-800,000	4	$3\frac{1}{4}$	2	5×18 or	10.30
900,000-1,000,000	$4\frac{3}{4}$	$3\frac{5}{8}$	2	$\frac{3}{8}$×5 lag	18.30
	2- and 3-wire cleats				
14-12	$3\frac{3}{8}$	$1\frac{1}{8}$	$\frac{5}{8}$	3×8	2.48
10- 6	$3\frac{3}{8}$	$1\frac{3}{8}$	$\frac{5}{8}$	3×8	3.20

* Based on purchase price (1913) of large quantities, screws included.

Wooden Moulding. — When exposed wiring is not desired, and when it is impracticable in old buildings or too expensive in new buildings to install concealed wiring, wooden moulding may be used, provided the difference of potential between any two wires in the same moulding does not exceed 300 volts. Its use is forbidden in elevator shafts and in concealed or damp places. Wooden moulding is manufactured in the following sizes. See table on next page.

Moulding may be made inconspicuous by matching the wood of the moulding to the finish of the room and by using a capping, which will conceal the purpose of the moulding. In this manner, moulding may be made to simulate picture moulding, and on ceilings a panel effect can be obtained by the use of dead moulding. All wires used in wooden moulding must have a rubber insulation and must not be jointed or tapped in the moulding. Branch taps may be made with tap fittings designed for the purpose. Many other moulding fittings are manufactured, which, if used, reduce the labor of installation and improve the general appearance of the work. Wooden moulding is fastened to walls by means of thin screws or toggle-bolts and the capping is nailed on with brads. "Kicking boxes" are usually placed around the end of moulding on floors to protect the wires and porcelain tubes from possible injury.

Metallic Moulding. — Metallic moulding may also be used to conceal and protect wires when the difference of potential between any two wires in the moulding is not more than 300 volts and the power transmitted through the wires contained in the moulding does not exceed 1320 watts. Metallic moulding is manufactured in one width only, namely, 1 inch wide by ⅜ to ½ inch deep, the depth depending upon the type of moulding. The two types of metal

STANDARD WOODEN MOULDING

No. of wires	Size of groove, inches	Dimensions, inches		Size of wire		Cost* of moulding and screws per 1000 ft.
		Width over all	Depth, cap to base	B. & S. gage or cir. mils		
				Solid, 1 braid	Strd'd, 1 braid	
2	1/4	1 1/2	13/16	14–12		$12.75
2	5/16	1 11/16	15/16	10–8	8	16.00
2	7/16	2	1	6–4	6–5	19.50
2	9/16	2 7/16	1 1/8	3–2	4–2	40.15
2	3/4	3	1 3/8		1–000	56.00
2	7/8	3 7/8	1 3/4		0000–250,000	72.75
2	1	4 3/4	2 1/8		300,000–400,000	88.00
3	1/4	2 1/8	13/16	14–12		12.25
3	5/16	2 7/16	15/16	10–8	8	14.50
3	7/16	2 13/16	1 1/8	6–4	6–5	19.25
3	9/16	3 1/2	1 1/4	3–2	4–2	37.25
3	3/4	4 3/16	1 7/16		1–000	51.75
3	7/8	5 1/8	1 3/4		0000–250,000	66.50
3	1	6 11/16	2 3/16		300,000–400,000	81.25

* Based on purchase price (1913) of large quantities, screws included.

moulding in common use differ principally in the method of attaching the capping. In the "Lutz" metal moulding the capping consists of a flexible metal strip, which is slid into the two grooves that form the upper edge of the moulding. In the "National" metal moulding the capping is snapped over the base. "Lutz" moulding is made in 10-foot lengths while the National moulding is made in 8-foot 4-inch lengths. "Lutz" moulding is listed at $6.50 per 100 feet and "National" moulding at $8.00 per 100 feet, with discounts of about 40 per cent for large quantities.

Metallic moulding cannot be used in concealed or damp places except that, in passing through a floor, such moulding may be used if carried through an iron pipe extending from the ceiling below up through the floor to a point five feet above the floor or to a point at least three inches above the floor where appearance is an essential feature. All parts of a metallic moulding system including outlet boxes, junction boxes and cabinets must be electrically connected and grounded. Fittings employed with metallic moulding must be so constructed as to protect the insulation of the wires from abrasion. Wires placed in metallic moulding must be insulated and installed as in wooden moulding with the exception that in alternating-current systems wires of the same circuit must be installed in the same moulding. It is suggested that this also be done for direct-current systems when there is a possibility that an alternating-current system may be used at some future time.

Concealed Knob and Tube Work. — This method of wiring is forbidden by the inspectors in many large cities, but when approved, it is employed in buildings of frame construction when it is desired to install the wiring at a minimum first cost. The wires when running parallel to beams or studding are supported on knobs and when running through beams, studding or floors are insulated by porcelain tubes. When passing through floors at the bottom of plastered partitions or through braces, the porcelain tubes insulating the wires must project at least four inches above the floor or brace. Split knobs must be used for the support of conductors smaller than No. 8 B. & S. gage except at the end of runs where a solid knob or strain insulator must be used. For conductors larger than No. 8 B. & S. gage, solid knobs may be used. Wires must have a rubber insulation and tie wires when used must have an insulation equal to that of the conductors they confine.

Distance between Wires. — Wires must be installed in such a manner that the distance between *any* two wires is at least 5 inches and each wire must be separated from the surface wired over by a distance of at least 1 inch. If, in any place, the 5 inch separation cannot be maintained, each wire must be separately incased in a continuous length of approved flexible tubing. At outlets, wires must be protected by flexible tubing extending in continuous lengths from the last porcelain support to at least 1 inch beyond the outlet.

Distance between Supports. — Wires must be supported under ordinary conditions every 4.5 feet and if the wires are liable to be disturbed the distance between the supports must be shortened.

Knobs and Tubes. — Two types of knobs are used, namely, the solid knob and split knob. The following table gives the dimensions and costs of one type of split knobs and 4-inch tubes.

Size of wire, B. & S. gage	Split knobs *			Tubes		
	Height, inches	Diameter, inches	Cost † per 100	Inside diameter, inches	Outside diameter, inches	Cost † per 100
14-5	$1\frac{7}{8}$	$1\frac{1}{8}$-$1\frac{1}{2}$	$1.50	$\frac{5}{16}$	$\frac{9}{16}$	$0.70
4-00	2	2	2.00	$\frac{3}{8}$	$\frac{11}{16}$	0.75
				$\frac{5}{8}$	$\frac{15}{16}$	0.85
				$\frac{3}{4}$	$1\frac{3}{16}$	1.00

* Holding single wire 1 inch away from wall.
† Based on purchase price (1913) of large quantities; screws included in cost of knobs.

Rigid Conduit. — Although the most expensive method, rigid conduit is considered to be the the best method of wiring and its use is required by inspectors in certain districts of many large cities. Two kinds of conduit are manufactured; lined conduit, having a lining of insulating material and unlined conduit, having an inner coating of insulating enamel. Unlined conduit is used more often than lined conduit because unlined conduit is cheaper to buy and to install and because wires are drawn in unlined conduit more easily than in lined conduit. In lined conduit corrosive action on the conductors due to a possible leak in the conduit is prevented by the insulating lining. The conductors used in lined conduit cost slightly less than the conductors used in unlined conduit, because the conductors used in unlined conduit must have an

additional layer of braid to allow for abrasion in drawing in the wires. Rigid conduit is manufactured in the following sizes:

STANDARD RIGID CONDUIT

Made in 10-foot lengths

Standard size of pipe, inches	Internal diameter, inches	External diameter, inches	Nominal weight per 100 feet, lb.	No. of threads per in. of screw	Cost * per 100 feet	Max. size of double-braided wire used in a single conduit, B. & S. gage or cir. mils		
						1 wire	2 wires	3 wires
½	0.62	0.84	85	14	$4.55	8	14	......
¾	0.82	1.05	112	14	5.85	2	10	12
1	1.04	1.31	167	11½	7.65	00	6	8
1¼	1.38	1.66	224	11½	10.90	200,000	3	5
1½	1.61	1.90	268	11½	12.40	400,000	1	3
2	2.06	2.37	361	11½	17.15	800,000	200,000	00
2½	2.46	2.87	574	8	26.70	1,300,000	350,000	250,000
3	3.06	3.50	754	8	39.10	2,000,000	500,000	400,000
3½	3.54	4.00	900	8	51.50	....	800,000	500,000
4	4.02	4.50	1066	8	63.90	....	1,200,000	850,000

* Based on purchase price (1913) of large quantities.

Bushings, Bends, etc., in Rigid Conduit. — All parts of the conduit system including outlet or junction boxes must be mechanically secured in place and must be electrically connected and grounded. At outlets or junction boxes rigid conduits must be provided with bushings or nipples to protect the wires from abrasion. The radius of curvature of the inner edge of an elbow or bend must not be less than 3.5 inches and there must not be more than the equivalent of 4 quarter bends from outlet to outlet, the bends at the outlet not being counted. Bends should be used whenever possible in place of elbows as the wires will pass more easily around a bend than an elbow. The various bending tools that are on the market consist of some form of lever, the end of which may be slipped over the conduit while it is in position. When the number of bends between outlets becomes excessive or when a large number of conduits placed side by side must be deflected, pull boxes are installed so that the direction of the conduit may be changed and the wires may be drawn in more easily. In fireproof buildings conduit is frequently installed just after the steel work has been erected and is then covered up by concrete or tiling.

Insulation of Wires Used in Rigid Conduits. — Wires used in rigid conduit must have an approved rubber insulation and must not be spliced or tapped within the conduit. In alternating-current systems the two or more wires of a circuit must be drawn in the same conduit. It is suggested that this be done for direct-current systems also when there is a possibility that an alternating-current system may be used at some future time. The same conduit must not contain more than four two-wire or three three-wire circuits of the same system and circuits of different systems must be run in separate conduits.

Wires should not be drawn in until all mechanical work on the building has been completed. For short runs wires may be pushed in at one opening until they come out at the other opening, but on long runs or where there are many bends, a spring-steel "fish-wire" or "snake" is first pushed through the conduit and a piece of sash cord attached to the "fish-wire" is pulled through. The

conductor is then attached to the sash cord and pulled through. When the wires cannot be pulled through a conduit easily, powdered soapstone is blown into the conduit to reduce the friction of the conductors.

Supports for Wires. — In vertical runs conductors must be supported in the conduit in accordance with the following table:

No. 14 B. & S. to 0.	every	100	feet
No. 00 B. & S. to 0000,	"	80	"
0000 B. & S. to 350,000 cir. mils,	"	60	"
350,000 cir. mils to 500,000 cir. mils,	"	50	"
500,000 cir. mils to 750,000 cir. mils,	"	40	"
750,000 cir. mils,	"	35	"

Conductors may be supported by a turn of 90 degrees in the conduit system or by junction boxes designed for the purpose.

Flexible Conduit. — In distinction to flexible tubing, a non-metallic tubing, flexible conduit is a *metallic* tubing built up of spiral, convex and concave steel strips which interlock in such a manner as to form a fairly smooth surface, externally and internally. "Flexible conduit," which is metallic, is usually distinguished from "flexible tubing" (*see below*), which is non-metallic tubing. Flexible conduit possesses the advantage over rigid conduit in that it can be installed continuously and quickly from outlet to outlet and if necessary may be fished between partitions or floors. Its chief disadvantages are that it is neither nail-proof nor moisture-proof. Flexible conduit is manufactured in the following sizes:

STANDARD FLEXIBLE CONDUIT

Size, inches	Approx. outside diameter, inches	Weight per 100 feet, pounds	Approx. feet in coil	Cost * per 100 feet	Max. size of double-braided wire used in single conduit, B. & S. gage or cir. mils		
					1 wire	2 wire	3 wire
5/16	0.485	20	250	$2.65	14	......	......
3/8	0.61	34	250	4.00	12	......	......
1/2	0.92	68	100	5.25	8	12	......
3/4	1.18	95	50	6.85	2	10	12
1	1.49	141	50	11.00	00	6	8
1 1/4	1.75	182	50	16.60	200,000	3	5.
1 1/2	2.06	217	25-50	22.10	400,000	1	3
2	2.56	265	25-50	30.00	800,000	200,000	00

* Based upon purchase (1913) of large quantities.

All requirements as to installation and wires used in rigid conduit apply as well to flexible conduit.

Armored Cable. — In place of flexible conduit in which wires must be drawn in after the conduit is installed, armored cable consisting of rubber-insulated conductors protected by interlocking spiral steel strips may be used. When installed in damp places, leaded armored cable, which has a lead covering between the insulation and the steel armor, must be used. Armored cable is employed extensively in wiring old buildings where it would be impossible to install rigid conduit without cutting up the walls and floors. Armored cable for interior wiring is manufactured in the following sizes:

STANDARD ARMORED CONDUCTORS

Size of each conductor, B. & S.	Number of bunched conductors		Approx. outside diameter, inches	Weight per 100 feet, pounds	Approx. feet in coil	Cost,† per 100 feet
14	1	Steel armored	0.37	20	250	$3.85
12	1		0.40	21.5	250	4.50
10	1		0.435	23	250	5.70
8	1		0.500	28	250	6.70
6	1		0.640	54	250	9.20
10*	1		0.435	23	250	6.00
8*	1		0.500	28	250	7.25
6*	1		0.640	54	250	10.00
4*	1		0.690	59	200	13.00
2*	1		0.770	71	200	17.00
1*	1		0.860	98	100	20.00
10*	1	Lead covered and steel armored	0.555	48	250	7.50
8*	1		0.620	54	200	9.10
6*	1		0.760	81	200	12.40
4*	1		0.820	90	150	15.70
2*	1		0.900	120	150	20.00
1*	1		0.985	165	100	24.50

* Stranded. † Based on purchase price (in 1913) of large quantities.

STANDARD ARMORED CONDUCTORS — *Continued*

Size of each conductor, B. & S.	Number of bunched conductors		Approx. outside diameter, inches	Weight per 100 feet, pounds	Approx. feet in coil	Cost,† per 100 feet
14	2	Steel armored	0.630	44	150–250	$6.55
12	2		0.640	45	150–250	8.50
10	2		0.675	52	150–250	11.70
8*	2		0.818	81	100–150	13.20
6*	2		1.066	114	100	23.30
14	2	Lead covered and steel armored	0.636	75	100–200	10.30
12	2		0.688	78	100–200	13.30
10	2		0.861	108	100–150	16.00
8*	2		0.900	125	100–150	20.50
14	3	Steel armored	0.648	53	150–250	8.70
12	3		0.675	55	150–250	10.70
10	3		0.755	66	100–200	14.80
8*	3		0.900	99	100–150	20.50
14	3	Lead covered and steel armored	0.694	79	100–150	14.00
12	3		0.766	88	100–150	16.20
10	3		0.901	117	100–150	18.40

* Stranded. † Based on purchase price (in 1913) of large quantities.

All requirements as to mechanical and electrical connections to outlet boxes, grounding, splicing and tapping of wires, assembling of conductors in alternating-current systems, etc., applying to rigid conduit also pertain to armored cable. Bends in armored cable may be of smaller radius than in rigid conduit, however, the minimum allowable radius of a bend being 1.5 inches for armored cable.

Flexible Tubing. — In addition to its use in all places where it is necessary to add to the insulation of the conductors, if the difference of potential between the wires is not over 300 volts, continuous flexible tubing may be used to protect individual wires when fished for short distances in dry places. Although not allowed by some inspectors, flexible tubing is extensively used in this manner in conjunction with moulding or knob and tube work, especially in old buildings where it is necessary to fish the wires to avoid cutting into walls, floors or ceiling. Flexible tubing is manufactured in the following sizes:

STANDARD FLEXIBLE TUBING

Inside diameter, inches	Feet in coil	Weight per 1000 feet, pounds	Cost per 100 feet	Largest size of wire accommodated, B. & S.
1/4	250	75	$1.65	14
3/8	250	110	2.45	12
1/2	200	125	3.25	8
5/8	200	155	4.05	4
3/4	150	200	4.90	2
1	100	275	6.75	00
1 1/4	100	360	8.95	200,000
1 1/2	100	400	10.80	400,000
1 3/4	100	440	12.70	600,000
2	Odd lengths	600	14.90	800,000
2 1/4	Odd lengths	700	17.60	1,100,000

COST OF WIRING. — The total cost of wiring a building is made up of the three elements: materials, labor and engineering. The labor and engineering charge will depend upon local conditions, and these costs may be estimated accurately only by reference to similar costs in completed work of the same general character. In regard to labor, the local conditions must include the factor of wages paid in the vicinity and the amount of work completed in a given time by the class of labor available. In determining the engineering charge the contractor must estimate the amount for each contract which should be charged to office expense and solicitation of business.

Office Estimates. — The cost of the materials is usually determined from a supply catalogue; approximate costs of moulding, cleats, conduits, etc., are given above. This may be accomplished by finding (1) the costs of the wiring materials between outlets and (2) the costs of the special devices such as outlet boxes, switches, cut-out cabinets, etc., and adding the two together.

Cost of Rubber-covered Copper Wire. — The cost of insulated wire depends upon (1) the cost of copper-wire bars, (2) the cost of rubber gum, (3) the cost of the other ingredients in the insulation, and (4) upon the cost of manufacture. Items 1, 2 and 3 are subject to considerable variation from year to year, and consequently any costs given at the present time must be used with caution at any later time. In Table A is given the present (1913) cost of rubber-

insulated copper wire (*National Electrical Code Standard*) on the basis of copper at 15 cents per pound, together with the amount by which these costs must be increased or decreased for each cent increase or decrease in the cost of copper, assuming all other items unchanged in cost.

Cost of Wiring Material per 100 Feet of Single Wire. — Tables B and C give the cost of all wiring material, exclusive of insulated wire, switches, cut-outs, etc., per 100 feet of *single* wire for all methods of wiring. Table D gives approximate costs of switches, cut-outs, etc. Costs of armored conductors are given above.

The costs are based upon the purchase price (1913) of large quantities.

TABLE A. — COST OF ONE HUNDRED FEET OF RUBBER-COVERED COPPER WIRE

Single Conductor, 600 volts

Stranded, covered with double braid				Stranded, covered with single braid			
Size of wire, cir. mils or B. & S.	Cost per 100 ft., copper at 15 cts.	Add for each cent increase in price of copper	Add for each extra braid	Size of wire, B. & S.	Cost per 100 ft., copper at 15 cts.	Add for each cent increase in price of copper	Add for each extra braid
1,000,000	$85.10	$3.15	$2.60	1	$9.55	$0.25	$0.85
900,000	77.45	2.80	2.50	2	7.15	0.20	0.65
800,000	69.85	2.50	2.40	3	6.00	0.15	0.60
700,000	62.10	2.20	2.30	4	5.08	0.13	0.55
600,000	54.55	1.90	2.20	5	4.30	0.10	0.50
500,000	45.65	1.60	2.00	6	3.72	0.08	0.45
400,000	37.60	1.30	1.80	8	2.42	0.05	0.40
300,000	29.60	0.80	1.60	10	1.77	0.03	0.35
250,000	25.55	0.75	1.50	12	1.37	0.02	0.30
225,000	23.70	0.70	1.40	14	1.07	0.01	0.25
0000	21.25	0.65	.....	**Solid, covered with single braid**			
000	17.75	0.50	.....				
00	14.75	0.40	.....	1	8.40	0.25	0.85
0	12.35	0.30	.....	2	6.30	0.20	0.65
				3	5.20	0.15	0.60
				4	4.28	0.13	0.55
				5	3.67	0.10	0.50
				6	3.11	0.08	0.45
				8	1.98	0.05	0.40
				10	1.46	0.03	0.35
				12	1.10	0.02	0.30
				14	0.87	0.01	0.25

TABLE B. — COST OF WIRING MATERIAL PER ONE HUNDRED FEET OF SINGLE WIRE

Exclusive of Wire, Cut-Outs, Switches, Receptacles, Etc.

Size of wire, cir. mils or B. & S.	Open wiring on cleats			Wooden moulding		Knob and tube	
	1-wire cleats	2-wire cleats	3-wire cleats	2-wire	3-wire	25 knobs	80 tubes
1,000,000	$4.57	....	....	....	....	....	....
900,000	4.57	....	....	....	....	....	....
800,000	2.57	....	....	....	....	....	....
700,000	2.57	....	....	....	....	....	....
600,000	2.57	....	....	....	....	....	....
500,000	1.92	....	....	$5.00	....	....	....
400,000	1.92	....	....	4.40	$4.06	....	....
300,000	1.92	....	....	4.40	4.06	....	....
250,000	1.92	....	....	3.64	3.33	....	....
0000	1.62	....	....	3.64	3.33	....	....
000	1.21	....	....	2.80	2.59	....	....
00	1.12	....	....	2.80	2.59	$0.50	$0.79
0	0.97	....	....	2.80	2.59	0.50	0.79
1	0.97	....	....	2.80	2.59	0.50	0.69
2	0.85	....	....	2.08	1.86	0.50	0.69
3	0.79	....	....	2.08	1.86	0.50	0.69
4	0.79	....	....	2.08	0.96	0.50	0.69
5	0.79	....	....	0.98	0.96	0.40	0.69
6	0.79	$0.40	$0.27	0.98	0.96	0.40	0.69
8	0.72	0.40	0.27	0.80	0.72	0.40	0.60
10	0.72	0.40	0.27	0.80	0.72	0.40	0.55
12	0.62	0.31	0.21	0.64	0.61	0.40	0.55
14	0.62	0.31	0.21	0.64	0.61	0.40	0.55

Example of Use of Tables. — A three-phase, 550-volt, 25-horse-power, 80-per-cent power-factor induction motor is to be connected to a switchboard by conductors drawn in rigid conduit, the length of the conduit being 250 feet. The current taken by the motor is approximately 28 amperes. Assuming a potential drop in each conductor equal to 3 per cent of the voltage to neutral, the drop will be 9.5 volts per conductor. From equation (1), the required cross-section is then $A = 12,500$ circular mils. From the table of current-carrying capacity (see above), the corresponding size of wire which will carry 1.25×28 or 35 amperes is a No. 8. Assuming the base price of copper to be 14 cents per pound;

From table A, 100 feet of solid No. 8 wire costs 1.98–0.05 =	$1.93
Add for double braid	0.40
From table C, 100 feet of conduit costs per wire	2.56
Total cost per 100 feet of each wire	$4.89

Cost of 250 feet and for three wires is $\$4.89 \times 2.5 \times 3 = \$36.60$. To this **amount must be added the cost of switches, cut-outs, etc., labor and engineering.**

TABLE C.—COST OF WIRING MATERIAL PER 100 FEET OF SINGLE WIRE

Exclusive of Wire, Cut-Outs, Switches, Receptacles, Etc.

Size of wire, cir. mils or B. & S.	Flexible tubing containing 1 wire	Rigid conduit containing		Flexible conduit containing	
		2 wires	3 wires	2 wires	3 wires
1,000,000	$17.60	$31.95	....	....	....
900,000	17.60	31.95	....	....	....
800,000	14.90	25.75	$21.30	....	....
700,000	14.90	25.75	21.30	....	....
600,000	12.70	25.75	21.30	....	....
500,000	12.70	19.55	17.17	....	....
400,000	10.80	19.55	13.03	....	....
300,000	10.80	13.35	13.03	....	....
250,000	10.80	13.35	8.90	....	....
0000	8.95	8.57	8.90	....	....
000	8.95	8.57	8.90	$15.00	....
00	6.75	8.57	5.72	15.00	$10.00
0	6.75	8.57	5.72	15.00	10.00
1	6.75	6.20	5.72	11.00	10.00
2	4.90	6.20	5.72	11.00	10.00
3	4.90	5.47	4.13	8.30	7.35
4	4.05	5.47	4.13	8.30	7.35
5	4.05	5.47	3.65	8.30	5.55
6	4.05	3.83	3.65	5.50	5.55
8	3.25	3.83	2.56	5.50	3.65
10	3.25	2.92	2.56	3.43	3.65
12	2.45	2.92	1.95	2.62	2.28
14	1.65	2.28	1.95	2.62	2.28

TABLE D.—APPROXIMATE COST OF "HARDWARE"

	Each
Switches:	
Flush push button, single pole	$1.00
Standard snap, single pole	0.50
Automatic door switch	1.50
3-way flush switch	1.10
3-way snap switch	0.50
Cut-Outs:	
Standard double pole	0.12
3- to 2-wire double-branch	0.26
Receptacles:	
Waterproof floor receptacle	3.00
Flush baseboard receptacle	1.50
Standard wall socket	0.50
Drop cords:	
Drop cords, including spun brass canopy, cord and socket	1.00
Drop cord (without canopy)	0.75

Field Estimates. — In order to enable its representative to make a rapid estimate of the cost of wiring a building while he is interviewing the owner or agent, the contract department of a lighting company usually provides their solicitors with a schedule of wiring costs, based (1) upon the number of lamp sockets and *switch* outlets to be provided, or (2) upon the *total* number of outlets (both lamp and switch), or (3) upon the number of lamp sockets and the number of *lamp* outlets required. Such wiring schedules are usually applicable only to finished buildings; the cost of wiring a building while it is being erected is usually much less than the cost of wiring the same building after it has been completed.

First Method. — The buildings to be wired are divided into classes and the cost of wiring a building in any class is based upon the number of lamp sockets, the number of *switch* outlets, and the type of switches and receptacles installed. The disadvantage of this method is that it does not take into account the number of lamp outlets. In two buildings in which the same number of lamp sockets are installed, the number of lamp outlets may differ widely, and as a result the labor and material required for the work will differ in the two buildings although the cost of the wiring *may* be the same. In most instances, however, this defect is not serious, because of the close relationship between the number of lamp sockets and the number of lamp outlets which are usually installed in the buildings for which the schedule is intended.

Second Method. — The various openings for switches, receptacles, drop cords and fixtures are considered as outlets and on the number of these is based the cost of work in houses of different classes of construction. An extra charge is made for hardware and for work installed under double or hardwood floors. With this method the number of lamp sockets installed in two houses for which the cost of wiring is the same may differ considerably. For schedule of cost see *Elec. World, 1910, Vol. 56, p. 1131.*

Third Method. — The buildings to be wired are divided into classes and the cost of wiring a building in any class is based upon the number of lamp outlets and lamp sockets and the type of switches installed. This method is an improvement on Methods I and II as it does take into account the average number of lamp sockets per lamp outlet, and while the cost of the wiring in two buildings requiring the same labor and material may differ slightly, such differences will not be as frequently encountered as in the other two methods. As an example of this type of schedule see article in *Elec. World, 1912, Vol. 59, p. 548,* descriptive of the method used by the Consolidated Gas, Electric Light and Power Company of Baltimore, Md.

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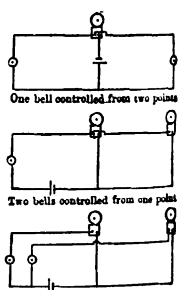
[R. G. HUDSON.]

WIRING OF BUILDINGS FOR MISCELLANEOUS DEVICES. — (*See also Wiring of Buildings for Light and Power.*) In addition to the wires designed to conduct electrical energy to lamps, motors and heating devices in a building, wires may be installed in connection with telephone (q.v.), telegraph (q.v.), district messenger and call-bell circuits, fire and burglar alarms, door-opening devices, gas lighters, watchman's clocks and electric clocks. Since all of these devices are operated at low voltage, it is unnecessary to use the same care in selecting and installing the wires as in the higher voltage systems, except that in all low-voltage systems care must be exercised that the conductors shall not become crossed with light and power circuits.

Protection of Low-voltage Wiring. — When the conductors of any low-voltage system are brought into a building from the outside, an approved protective device must be located as near as possible to the entrance of the wires to the building. With the exception of instrument circuits of telegraph systems, where cut-outs only are required, protective devices must contain a lightning arrester with a ground connection and a cut-out or heat-coil (*see Telephone Instruments and Circuits*). The conductors beyond the protective device in low-voltage systems need be insulated only sufficiently to prevent short-circuits and the consequent interruption of service. When bunched together in vertical runs the wires must be inclosed in a fire-resisting covering to prevent the wires from carrying fire from floor to floor. Low-voltage circuits may be run in the same shaft with light and power circuits, provided the two classes of wires are separated by at least two inches or one of the classes is run in a non-combustible tubing. Low-voltage wires may not be run in any case in the same tube with lighting or power wires.

Telephone Wiring. — (*See also Telephone Instruments and Circuits.*) Except in large buildings, rubber-insulated twisted pairs of wire are used extensively for telephone work. The twisted wires may be fished between partitions or concealed in moulding and if open wiring is not objectionable, the wires are carried along the finish of the room and fastened with insulating staples or tacks. In new buildings the wires are frequently installed in conduit and local connections are made at connection boxes designed for the purpose. In large buildings, when the number of telephones in use becomes very great, lead-covered insulated cable containing multiple pairs of conductors is installed in wire-ways provided for the purpose. Distributing connection boxes to which branch circuits may be connected are then installed on each floor.



One bell controlled from two points

Two bells controlled from one point

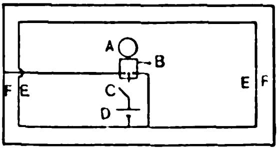
Selective ringing

Fig. 1.

Bell Wiring. — In its simplest form a bell-wiring system consists of a battery (usually a Leclanché type or dry battery), a bell-push, a bell or buzzer and the connecting wire. Paraffine-impregnated double-cotton covered wires ranging in size from No. 16 to No. 22 B. & S. gage are usually employed, either singly or in the form of twin wires. The wires may be fished, concealed in moulding or run exposed along the finish of the room. When a large number of bell-wires must be run together through a building as in the case of a hotel annunciator system, the wires are frequently inclosed in a cotton braid or in a tube made of zinc or impregnated paper. In many buildings supplied with an alternating current source of power a small step-down transformer is used in place of the batteries, thus effecting a saving in the cost of the installation and eliminating the possibility of run-down bat-

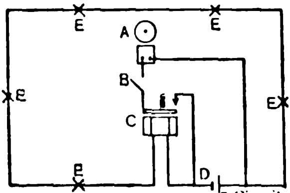
teries. In Fig. 1 are shown several systems of bell wiring; these figures are self-explanatory.

Burglar Alarms. — In place of the bell-push used in bell-wiring systems, the bell circuit may be closed by some circuit-closing device attached to windows, doors, etc., so that the opening of any window or door in the building will be made known by the ringing of a bell. As such a system has the objection that it will not operate if the battery is run down or if the wires are cut, a closed-circuit system is preferable, in which the opening of a window or door opens the circuit



A Bell B Constant Ringing Drop
C Day Switch D Battery
E and F Wires to which closing springs are connected

Fig. 2. Open-circuit Burglar Alarm



A Bell C Relay E Circuit-opening Springs
B Day Switch D Battery

Fig. 3. Closed-circuit Burglar Alarm

and a relay in turn closes the bell circuit. In Fig. 2 is shown a typical open circuit and in Fig. 3 a typical closed-circuit system. In Fig. 4 are shown a door spring *A* and a window spring *B*.

Fire Alarms. — In the manually-operated system glass disks are placed at convenient points throughout the building and with each disk is provided a hammer, with which the disk may be broken in case of fire. The breaking of the disk opens or closes an electric circuit by means of which electric gongs are rung throughout the building. In the automatic system the expansion or fusion by heat of elements in circuit-closing or opening devices usually placed on ceilings, causes gongs to ring throughout the building or in the watchman's office.

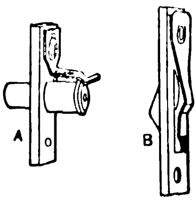


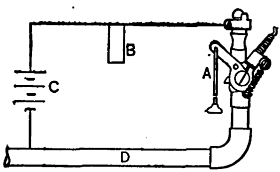
Fig. 4.

Door-opening Devices and Electric Clocks. — Doors may be opened by pressing a button at some distant point and thereby closing a circuit, through which current flows, energizing a magnet, which releases the nosing in the door frame. In the same manner, a moving disk on a master clock may periodically close a circuit, through which a magnet is energized which operates a pawl attached to the hands of another clock.

Watchman's Clock. — Circuit-closing devices are placed at different points throughout a building so that a watchman on his rounds may close the circuit by means of a handle or special key. The closing of the circuit at any station makes a record on a dial in the watchman's clock indicating the time at which the circuit was closed at each particular station, the stations being designated by numbers. To obviate the possibility of run-down batteries, magneto systems are often installed, a small magneto being placed at each station.

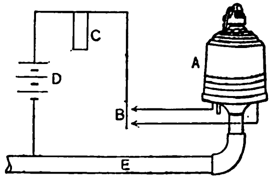
Gas Lighters. — In the "pull-burner" and "automatic-burner" systems, when the gas is turned on, an electric circuit is made and broken by means of contacts placed at the tip of the burner. To intensify the spark at the burner, a spark coil is always connected in series with the circuit. In the "pull-burner" system (Fig. 5) the pulling of a chain or the turning of a key on the gas jet turns on the gas and at the same instant makes and breaks the circuit by contacts

placed near the burner. In this system one wire connects the battery in series with a spark coil to the contact device, the other terminal of the contact device being grounded to the gas pipe through which connection is made to the other terminal of the battery.



A Pull Burner
B Spark Coil
C Battery
D Gas Pipe

Fig. 5.



A Automatic Burner
B Two-Button Push
C Spark Coil
D Battery
E Gas Pipe

Fig. 6.

In the "automatic-burner" system (Fig. 6) three wires are run from a two-button push to one pole of the battery and one terminal each of the two magnets on the automatic burner, the other terminals of the battery and the two magnets being grounded on the gas pipe. The two push buttons are usually made black and white respectively, the white button being pushed to light the gas and the black one to extinguish it. When the white button is pushed, a circuit is closed connecting the battery, the magnet which turns on the gas, the vibrating contactor and the spark coil in series. When the black button is pushed, a circuit is closed connecting the battery and the magnet, which turns off the gas, in series.

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[R. G. HUDSON.]

X-RAYS. — (*See also Generators, Static; Induction Coils.*) X-rays or Röntgen rays are generated when the cathode rays of a vacuum tube impinge upon any solid substance. The form of X ray tube commonly used, known as a "focus tube," is shown in Fig. 1. When a high unidirectional e.m.f. is impressed between the anode (*A*) and the cathode (*C*), the cathode rays emitted from (*C*) strike upon the anti-cathode (*B*) from which the X-rays originate. The anode and cathode are made of aluminum because of its low rate of disintegration under vacuous discharge. The anti-cathode is made of platinum to withstand the intense heat of cathodic bombardment. The anti-cathode must be electrically connected to the anode to prevent the possible emanation of cathode rays from its surface. The cathode is concaved spherically so that its rays are focussed upon the anti-cathode, the rays falling within a spot not more than 0.08 inch in diameter. Owing to the repulsion between the particles constituting the cathode rays, the optical focus point of the cathode should fall just in front of the surface of the anti-cathode, so that the actual focus point will fall upon the surface of the anti-cathode. The vacuum tube, made of thin glass to reduce the absorption of the X-rays, is blown in spherical form to withstand the external pressure most effectively.

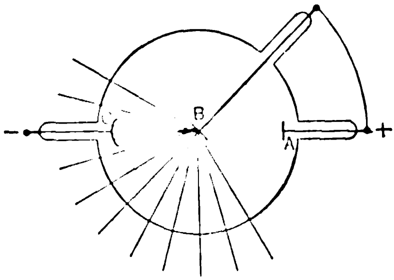

Fig. 1. X-ray Tube

Character of X-Rays. — Numerous attempts made to prove that X-rays are of undulatory form, similar to that of light, have been unsuccessful. It has been found that X-rays are neither reflected nor refracted and are not deflected like cathode rays by a magnetic force. Barkla (*Phil. Trans., 1905, p. 467*) found that such rays may be polarized and his conclusions have been confirmed by other investigators. According to the existing information, it may only be presumed that, when the negatively electrified particles constituting the cathode rays strike any solid substance, they lose their charge and give rise to the extremely high vibrations known as X-rays. It has been shown that such rays do not emanate from the impact spot on the anti-cathode but from a ring surrounding the spot, and travel in all directions from the surface of the anti-cathode, the densest emanation being in a direction normal to the surface.

Properties of X-Rays. — All substances, many of which are opaque to ordinary light, transmit X-rays to a greater or less degree. In most cases the transparency of any substance to such rays is (roughly) inversely proportional to its specific gravity. In the accompanying table are given the relative transparencies of various substances and their specific gravities, the transparency of water being taken as unity.

When X-rays are passed through gases, the gases become ionized and their electrical conductivity is accordingly increased. This property has been used as a means of measuring the relative strengths of X-rays.

Certain substances fluoresce under the action of X-rays making the existence of X-rays visible to the eye. The fluorescent screens ordinarily used for anatomical examinations are coated with barium platino-cyanide, potassium platino-cyanide or calcium tungstate.

X-rays are to a considerable degree chemically active and photographs may be made of objects generally opaque to the eye but built up of substances of

TRANSPARENCY OF VARIOUS SUBSTANCES TO X-RAYS

Material	Specific gravity, water = 1	Transparency, water = 1
Pine wood	0.56	2.21
Walnut	0.66	1.50
Paraffin	0.87	1.12
Rubber	0.93	1.10
Cardboard	....	0.80
Ebonite	1.14	0.80
Wool-cloth	....	0.76
Celluloid	....	0.76
Silk	....	0.74
Cotton	....	0.70
Charcoal	....	0.63
Bone	1.9	0.56
Aluminum	2.67	0.38
Glass	2.6	0.34
Tin	7.28	0.118
Zinc	7.20	0.116
Iron	7.87	0.101
Nickel	8.67	0.095
Brass	8.70	0.093
Copper	8.96	0.084
Silver	10.5	0.070
Lead	11.38	0.055
Mercury	13.59	0.044
Gold	19.36	0.030
Platinum	22.07	0.020

different transparencies to the X-rays, so that the internal structure of the object is revealed in the photograph. Such photographs made with highly sensitive plates are called radiographs or skiagraphs.

Considerable use is also made of X-rays in the electro-therapeutic treatment of certain diseases especially those of the skin.

Manipulation of the Tube. — It is important that the tube be connected in the proper polarity as a reverse current in the tube will cause it to blacken. The voltage used with the tube depends upon its size and the degree of vacuum attained. The ordinary tube requires a voltage of from 50,000 to 100,000 volts, or, as usually stated, a voltage which will puncture a needle gap of from 3 to 10 inches in air. The source of e.m.f. used with the tube is usually an induction coil (q.v.) and in some cases a static generator (q.v.). The voltage required to puncture a tube is materially reduced if a sheet of tinfoil electrically connected to the cathode terminal is wrapped around the outside of the tube at the cathode end and covering about one-third of its surface. The current taken by a tube is extremely small so that the connecting wires need only be large enough to withstand breakage but must be heavily insulated. Under continuous use the internal pressure of the gas in the tube decreases due to the occlusion of the gas content by the metals and glass of which the tube is constructed.

As the internal pressure in the tube decreases, it may become inoperative and should then be heated in an oven to release the occluded gases and reestablish the proper pressure. Certain "regenerative" tubes contain palladium which occludes gas at ordinary temperatures and gives off gas under heat, thereby maintaining a constant pressure within the tube.

"Hard" and "Soft" Tubes; Condition of Tubes.—The penetrating power of an X-ray tube is proportional, within certain limits, to its vacuity. A high-vacuum tube, known as a "hard" tube, will emit a few rays of great penetrating power, whereas a low vacuum tube, known as a "soft" tube will emit a greater number of rays of less penetrability. In most cases better contrast is obtained upon the photographic plate or fluorescent screen when a "soft" tube is used than when a "hard" tube is used, since the rays emitted by a "hard" tube may penetrate all the constituent parts of the object examined with nearly equal strength.

The condition of a tube under operation may be easily detected by observing the color of the vacuous discharge. When running under normal conditions, the tube will glow with a greenish-yellow light; the face of the anode will emit a canary-yellow light, but in the space behind the anode, a violet glow will appear. The question of correct polarity may then be checked by this appearance of the anode. A pinkish light within the tube indicates that the vacuum is too low, and the absence of any glow indicates too high a vacuum. A needle gap connected in parallel with the tube will show when the voltage is too high and protects the tube from excessive voltages.

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[R. G. Hudson.]

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Note. — Titles of articles are given in bold face type when principal references and in italics when cross references; other cross references are given in plain Roman type.

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